Housing Privatization as Intergenerational Redistribution

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April 6, 2024

Abstract

This paper argues that in economies with temporary fast-growing wage incomes, housing purchase subsidy to initial cohorts serves as an effective tool redistributing from future towards current generations. Using China's housing reforms in the 1990s as a policy experiment, we show quantitatively that housing purchase subsidy to cohorts entering the labor market before 1994 increased substantially their homeowner rates in the initial years, which later on allowed them to enjoy the capital gain from selling their houses to future generations. Our welfare analysis suggests that, despite welfare loss for cohorts born in the 1980s, housing purchase subsidy is more desirable for future generations than alternative social security reforms as an intergenerational transfer scheme, as the latter involves high social security tax burdens for those generations experiencing a slowdown in wage growth.

Keywords: housing purchase subsidy, intergenerational transfer, social security, emerging economies

JEL Classification: G28, E02, E5, G11, G12

Acknowledgment: We thank Dirk Krueger, Xincheng Qiu, Fang Yang and participants of 2024 ASSA Meeting for helpful comments. The views expressed herein are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of Atlanta, the Federal Reserve System.

1 Introduction

Many emerging economies, such as Korea, Taiwan and other East Asian Miracle economies, had experienced fast-growing, but eventually slowing down wage income growth. The fast wage growth during a period of economic transition might benefit the young more than the old cohorts, since the latter may not be able to enjoy the full benefit of rising income, before they retire at mandatory age. It is therefore natural to call for intergenerational redistribution toward the old, so that they can also share the benefits of the fast growth of the economy. A conventional wisdom is for governments to use redistributive policies, such as social security, to transfer resources from the future young generations to the current old ones. However, given the eventually slowing down growth in wage income, such redistributive policy may necessarily imposes a heavy tax burden for future generations. Adding to such challenge is the issue of population aging, which implies higher old-age dependence ratio faced by future taxpayers.

This paper argues that during a transition stage featuring by fast wage growth, subsidized housing purchase during housing market privatization plays the role of inter-generational transfer from the young to the initial old generations, who are poor in income when the transition starts. The idea is as follows. Housing purchase subsidy allows those initial generations to get cheap housing when housing privatized starts. The access of initial cohorts to owner-occupied houses drove up growth in housing prices as they trade up for larger commercial houses later on. When the initial generations get retired, they can sell the houses to reap the capital gain to finance retirement consumption. A particular feature of using housing subsidy to the initial old generation as a way to transfer wealth from future generations is that housing prices will incorporate and adjust to the future income growth: when the future income growth is fast, the housing price will rise; and when the income growth slows down, the housing price will flatten.

China offers an ideal setting for such a policy experiment. Since the early 1990s, China has experienced fast growth in wage incomes until recent years. Between 1992

and 2007, the urban real wage grew at an average rate of 10.2%. Since then, wage growth has started to slow down, reaching 4.63% in 2022. Accompanied with China's fast wage growth was housing market privatization, featured by two key reforms. In 1994, the Chinese government allowed employees in the state-owned sector to purchase the full or partial property rights to their current apartment units at subsidized prices, the so-called "reformed housing." In 1998, the central government further requested housing provision as in-kind benefits to be terminated. Instead, employers had to include the provision of all implicit and in-kind housing benefits in the salaries of their workers.[1]

In this paper, we quantify the role of China's housing market privatization as intergenerational transfer. To fix ideas, we first construct a simple two-period-lived Overlapping-Generations (OLG) model that features temporarily fast wage growth. We show in such an economy even a utilitarian planner that has no incentive to redistribute across generations at the steady state would find it optimal to redistribute resources from future to current generations. Moreover, the optimal redistribution can be implemented by charging a housing price lower than the equilibrium prices as implicit transfer to the initial cohorts.

To quantify the welfare implications of the housing market privatization experienced by China, we then develop a dynamic housing model with household heterogeneity and incorporate the two major housing market reforms, so called "94 reform" and "98 reform", during the 1990s. A key model ingredient is the introduction of two types of housing: government housing and commercial housing, with government housing smaller in size than the latter. In the initial steady states, all households rent government houses at discounted rental rates. When the transition with housing reforms starts, households born in the initial steady state are eligible to purchase government housing at discounted prices, or to purchase commercial housing at market

[1]The two decrees issued by the Chinese State Council are "A Decision on Deepening the Urban Housing Reform (July, 1994)" and "Notice on Further Furthering the Urban Housing System Reform and Accelerating Housing Construction (1998)."

prices, while having the option to continue to rent government houses. By contrast, households who were born after the initial steady state but before the 98 reform lose the option to purchase government housing at discounted prices and those born after the 98 reform lose both options of purchasing or renting government housing at discounted prices. The model features Pay-As-You-Go social security system as in China, which implies no intergenerational transfer via social security.

We calibrate the model to the Chinese economy during 1994-2014 to target key aggregate moments such as the growth rates of wage and house prices and the aggregate ownership rates of different types of housing. Our quantitative model can replicate the ownership rate of government housing across different age groups and skill types along the transition path reasonably well. We then conduct counter-factual analysis to quantify the impacts of the 94 housing reform, i.e., purchase subsidy to government houses, on various cohorts alive in 1994. Our analysis suggests that the 94 reform significantly increased the ownership rate for government housing, especially among those skilled households, in the 1990s, and that for commercial housing during the first decade of the 2000s. The key mechanism is that housing purchase subsidy facilitates the initial cohorts to become homeowners, who later on trade up to purchase commercial housing.

What would be the distributive effects of housing purchase subsidy compared with those of social security reforms that serve a similar role of intergenerational transfer? To address this question, we construct a counterfactual economy in which the replacement rates of initial cohorts are chosen so that they are as well off living in the counterfactual economy as in our benchmark economy with housing reforms. We set the contribution rate for all cohorts starting to work after the 94 reform to balance the intertemporal government budget constraint. We find that while cohorts entering the labor force during 1994-2010 are better off in the counterfactual economy with the social security reform than in our benchmark economy, all future cohorts are better off in the benchmark economy than in the counterfactual economy. The intuition is as

follows. In our benchmark economy, cohorts entering the labor force during 1994-2010 need to purchase housing at a higher price, since the trade-up demand of the initial cohorts pushes up the total housing demand. Therefore, they are worse off in the benchmark economy than in the counterfactual economy, despite that the payroll tax rates in the former is lower. For cohorts starting to work after 2010, however, while they face similar housing price dynamics between the two economies, the payroll tax burden is much heavier in the economy with social security reforms than in the benchmark economy, since in the former economy, their payroll tax revenue are used to finance the intergenerational transfer to initial cohorts, despite slowdown in wage growth during their working life.

Our paper is close in spirit to Glover et al. (2020). Both papers study the role of asset prices in intergenerational redistribution, though in different contexts. Glover et al. (2020) show that during Great Recession, a decline in asset prices benefits younger generations who buy these assets at low prices, potentially compensating them for the fall in earnings they experience. By contrast, old households experienced large welfare loss, as they rely on sales of risky assets to finance consumption. Our paper shares a similar message on the intergenerational redistributive effects of asset price dynamics. However, it differs from Glover et al. (2020) in several dimensions. First, our paper explores the redistributive effects of asset prices in the context of an emerging economy, which features temporarily fast growth in wage incomes and increasing asset prices, while by contrast, Glover et al. (2020) study the redistributive effects of asset prices during Great Recession that features large declines in labor incomes and asset prices. Second, the source for asset prices to redistribute intergenerationally is very different. In our paper, the redistribution works through housing purchase subsidy during a period of housing market privatization.[2] By contrast, in Glover et al. (2020),

[2]Since households are borrowing constrained, we show that without hosing purchase subsidy, the incentives for initial cohorts to become homeowners are much weaker. With a similar logic, Hur (2018) finds that with young generations subject to binding borrowing constraints, the welfare benefit of declining asset prices during the Great Recession is much dampened.

aggregate risks, which temporarily dampened asset prices, are the main driver for intertemporal redistribution. Accordingly, the direction for intergenerational transfers via asset prices are different. In our paper, intergenerational transfer happens from the young generations to the old, while in Glover et al. (2020).

Our paper contributes to the emerging literature on China's housing reforms and its macroeconomic impacts. Wang (2011) is the first to study the effects of China's 1994 privatization of state-owned housing, and argues that the impacts of such reforms on housing prices depends on the degree of misallocation of housing prior to the reform. The focus of the paper, however, is not the role of housing privatization as intergenerational redistribution, which is the key of the paper. Logan et al. (2010) study the winners of privatization of public housing, with a focus on the distributional effects among initial generations. Existing studies in the literature also explore the impacts of housing prices on different households. For example, Fang et al. (2016) argue that anticipating of risking housing prices, potential home buyers may be forced to purchase housing early in life, with high down payment, thus sacrificing other consumption. Furthermore, for mortgage borrower, this would increase their monthly mortgage payment as a fraction of income if they purchase housing early in life.[3] All these findings are in line with our model's prediction on the welfare effects of rising housing prices on cohorts entering the labor force in the 2000.

Our paper also contributes to the discussion of alternative social security reforms as intergenerational transfer in China.[4] Song et al. (2015) is the first to explore social security reforms as intergenerational transfer in a fast-growing economy like China.[5] They find that in such an environment, a delayed reform of the current

[3]See also Chen and Wen (2017) and Jiang et al. (2022), which study the welfare effects of housing bubble bursts in general equilibrium model.

[4]See, for example, Song et al. (2015), Imrohoroglu and Zhao (2018b) and Deng et al. (2023), among others. For an overview of China's pension system, see Fang and Feng (2020).

[5]Similarly, for the U.S., Peterman and Sommer (2019) quantifies the role of the social security system in the U.S. as intergenerational transfer when they were initially established in the 1930s. The government establish the social security system to tax the young and transfer to the old by exempting the initial old from paying payroll taxes.

pension system, which involves paying generous pensions to the generations who are currently working or already retired, and negative pensions to subsequent generations, is a second-best policy. In this paper, we show that housing purchase subsidy for initial cohorts can provide an alternative approach to transfer to the initial old from future cohorts via the dynamics of house prices. Our paper takes seriously that political constraints faced by the government in reducing the social security replacement rate. We show that under such constraints, the high payroll tax rates paid by future generations as intergenerational transfers would make social security less desirable for these later generations as an intergenerational redistributive scheme than housing purchase subsidy for initial cohorts when housing privatization starts.

The remainder of the paper is structured as follows. Section 2 constructs a simple model to characterize the role of subsidized housing purchase as intergenerational transfer in an economy with temporary fast-growth wage income. Section 3 and Sections 4 develop a full-blown multi-period OLG framework and incorporate two housing reforms occurring in China in the 1990s and calibrate this model to match key aggregate and cross-sectional moments. Section 5 uses this model to quantify the distributive effects of housing privatization on households of different generations. Section 6 compares the welfare effects of housing purchase subsidy to initial cohorts on future generations with those of a counterfactual security reform that serves a similar role of intergenerational transfer. Section 7 concludes.

2 A Simple Model

In this section, we construct a simple dynamic model that illustrates the main point of the paper that in emerging economies with fast but declining wage growth, even a planner with a very low discount rate would find it optimal to redistribute resources from future to current generations. Moreover, the optimal redistribution can be implemented by subsidizing the housing purchase of the current generations (and tax housing expenditure

by future generations).

Consider an economy populated by two-period lived overlapping-generations of households who work when young and live off savings when old. For simplicity, we assume that the household value both nondurable consumption and housing service when young and only nondurable consumption when old.

In period 0, the planner is endowed with a fixed stock of public housing, $\overline{h}$, to be sold to the initial cohort of household.

We capture the notion of a temporarily high wage growth by assuming that wages grow at steady state rate g from period $t = 1$ and onward, while the growth rate is high in the first period: $w_1 > (1 + g) w_0$. At this stage, we assume a constant population growth rate n (which matters for housing price growth later. To endogenize housing prices, we assume that each period the government provides (inelastically) a fraction n of new housing such that the total housing supply, which is the sum of existing housing supplied by the old-generation and the new housing supplied by the government, grows at the population growth rate. As a result, per capita housing supply is constant, denoted as $\overline{h}$.

The preference of a household born at time $t \geq 0$, indexed as cohort t, is given by

$$U_t = \log c_t^y + \alpha \log h_t + \beta \log c_{t+1}^o \tag{1}$$

where c_t^y and c_{t+1}^o denote the nondurable consumption when young and old, respectively. h_t denotes housing services, produced by housing stock on a one-to-one basis; Note that households retire when old.

At this stage, we assume that the planner could not affect housing supply. Accordingly, both the planner and the households in the decentralized economy take housing price as exogenously given. After we implement the planner's solution with housing policies in the decentralized economy, we solve for the equilibrium housing price.

2.1 The Planner's Solution

We first solve the planner's problem. In this simple model, the planner takes housing prices and wage rates as given. The planner is endowed with an initial stock of wealth, A_0, and can borrow and lend in an international bond market at the gross interest rate R. In addition, the planned is endowed with a fixed stock of land/housing each period $\overline{h}$. We assume that all land sales revenues is used for government consumption, which corresponding to the idea that in reality the land sales revenues are used for infrastructure investment. In other words, the government could not finance transfer from the revenue from selling the public housing. The planner's resource constraint is given by

$$\sum_{t=0}^{\infty}\left(\frac{1+n}{R}\right)^{t}\left[c_t^y+\frac{c_{t+1}^o}{R}+\left(p_t-\frac{p_{t+1}}{R}\right)h_t-w_t\right]\leq A_0 \tag{2}$$

where $p_t-\frac{p_{t+1}}{R}$ is the user cost of one unit of housing consumption.

Moreover, given the inelastic supply of public housing in the initial period and the government's use of lower price than market price to subsidize housing, it is likely that the demand for housing at the subsidized price is higher than the supply. Hence we impose the constraint $h_0\leq\overline{h}$ to clear the market.

The planner cares about all present and future generations, and discount future generations' utilities with a discount factor $\phi\in(0,1)$. For the resource constraint to be well-defined, we assume $R>(1+g)(1+n)$. Moreover, we assume $\phi(1+n)<1$, such that the planner's problem is well defined.

The planner's problem is

$$\max_{\left\{c_t^y,c_{t+1}^o,h_t,l_t\right\}_{t=0^\infty}}\sum_{t=0}^{\infty}\left(\phi(1+n)\right)^t\left[\log c_t^y+\alpha\log h_t+\beta\log c_{t+1}^o\right]$$

subject to (2) and

$$h_0\leq\overline{h}. \tag{3}$$

The first-order conditions for consumption at all $t \geq 0$,

$$c_t^y = \lambda^{-1} (\phi R)^t \tag{4}$$

$$c_{t+1}^o = \beta R \lambda^{-1} (\phi R)^t \tag{5}$$

and for housing

$$\frac{\alpha}{h_0} - \kappa = \lambda \left(p_0 - \frac{p_1}{R} \right) \tag{6}$$

$$\left(p_t - \frac{p_{t+1}}{R} \right) h_t = \alpha \lambda^{-1} (\phi R)^t \text{ for } t \geq 1 \tag{7}$$

where λ is the Lagrangian multiplier for the resource constraint, capturing the shadow value of wealth and κ is the Lagrangian multiplier for (3).

Both (4), (7) reflects the planner's incentive for inter-generational redistribution. Note that the optimal sequence of both nondurable consumption and housing expenditure is independent of the wage sequence and only depends on planner's discount factor and interest rate, ϕR. Over the life cycle, the planner chooses the same consumption growth as individual households, where across cohorts, consumption growth by the factor ϕR. Similarly, housing expenditure across cohorts grows by the factor ϕR.

2.2 Decentralization

The planner's solution can be decentralized in the competitive equilibrium. For simplicity, assume that there is no rental market for housing. A household purchases housing when young, and sells it when old. In addition to housings, household can save in bank with a fixed interest rate R.

At the initial period, the government holds public housing. Hence, we assume that the policy instrument for the government at period 0 is housing prices it sells to the households (public housing). This may capture, in reality, that in the initial stage

housing market privatization (1994-2000), the Chinese government allowed the initial generations to purchase housing at a discount rate (the so-called "standard price").[6] For simplicity, we rule out cash subsidy at period 0. For period $t \geq 1$, since the government does not have public housing anymore, the only instrument at the disposal of the government is a cash subsidy for housing purchased at market equilibrium prices. We call this economy the economy with initialized subsidized housing. We start with the problem for cohort born at or after $t \geq 1$. and then solve the problem for the initial cohort.

2.2.1 Cohort born at $t \geq 1$

For generality, we assume that the government can impose a lump-sum transfer (or tax) for each of the cohorts born or after $t \geq 1$.To avoid distorting the consumption allocation between nondurable goods and housing services, we further assume housing subsidy takes the form of lump-sum transfer (or tax) that is proportional to the housing expenditure:

$$T_t = \varsigma_t \left(p_t - \frac{p_{t+1}}{R} \right) h_t \tag{8}$$

where ς_t denotes the housing subsidy rate. Here, for tractability, the magnitude of transfers depend on the user cost of consuming housing h_t, rather than the purchasing

[6]In the initial stage of housing market privatization, there were three types of housing prices, the market price, the break-even price and the standard price. The market price is targeted toward the high-income households, whereas the break even price and the standard price is applied to low income households. The standard price comprises of two parts. The first part, called affordable (*fudan)* price, which is no less than three times the average household income of the city where housing is located. The second part, called redeeming price, is computed as the present value of the housing provident funds contributed by the household that is comprised of two full-time workers over the entire 65 years. However, Chinese government explicitly mandated that the standard price should be only temporary and should be replaced by the break-even price by 2000. Evidence shows that most households who purchased housing during this period (1994-2000) with the standard prices.

cost $p_t h_t$.[7] The lifetime budget constraint of a household born at time t is

$$c_t^y + \frac{c_{t+1}^o}{R} + \left(p_t - \frac{p_{t+1}}{R}\right) h_t = w_t + T_t \tag{9}$$

Maximizing the household problem (1) subject to (9) give the following first-order condition on housing

$$\left(p_t - \frac{p_{t+1}}{R}\right) h_t = \frac{\alpha w_t}{1 + \alpha + \beta - \alpha \varsigma_t} \tag{10}$$

Note that the higher is ς_t, the housing subsidy rate, the higher is the housing expenditure. In addition, we have consumption for each cohort as

$$\begin{aligned} c_t^y &= \frac{1}{\alpha}\left(p_t - \frac{p_{t+1}}{R}\right) h_t \\ c_{t+1}^o &= \frac{\beta R}{\alpha}\left(p_t - \frac{p_{t+1}}{R}\right) h_t \end{aligned}$$

The following proposition establishes that the first-best solution can be implemented by setting a suitable sequence of housing subsidy rates.

Proposition 1. *The first-best solution is implemented by setting the sequence of housing subsidy rate as*

$$\varsigma_t = \frac{1 + \alpha + \beta}{\alpha} - \frac{w_1 \lambda}{(1+g)\,\alpha}\left(\frac{\phi R}{1+g}\right)^{-t} \tag{11}$$

where λ is the Lagrangian multiplier associated with the planner's resource constraint.

Proof: See Appendix.

Note that for $t \geq 1$, the housing subsidy rate may increase or decrease over time depending on whether $\phi(1+g) - R$ is positive or negative. In particular, when $\phi = (1+g)/R$, $\varsigma_t = \frac{1+\alpha+\beta}{\alpha} - \frac{w_1\lambda}{(1+g)\alpha}$ is constant, implying that the planner has no incentive for inter-generational redistribution in steady state. Under this particular value of planner's discount factor, we have the following optimal housing subsidy policy. We

[7] In our numerical analysis, we will let the transfer to be proportional to the purchasing cost of housing and our main results would not change.

may solve for p_t by plugging (11) into (10).

$$\left(p_t - \frac{p_{t+1}}{R}\right) h_t = \frac{\alpha w_t (1+g)}{\lambda w_1}$$

And forward iterate the above equation, we get for $t \geq 1$

$$p_t = \frac{\alpha}{\lambda \overline{h}} \frac{1}{1-(1+g)/R} \tag{12}$$

2.2.2 Cohort born at period 0

By assumption, cohort 0 has no cash subsidy. Instead, the government chooses p_0 as implicit transfer, which might lead to binding housing demand constraint. Hence the constraints for cohort 0 are

$$c_0^y + \frac{c_1^o}{R} + \left(p_0 - \frac{p_1}{R}\right) h_0 = w_0 \tag{13}$$

and (3) . In the decentralized economy, (3) captures the fact that in reality, the Chinese government only allows a SOE employee to purchase the house she currently lives at a subsidized price, which essentially impose a demand constraint under the subsidized housing prices. The first order conditions are

$$\begin{aligned} \frac{1}{c_0^y} &= \beta R \frac{1}{c_1^o} \\ \frac{\alpha}{h_0} - \mu &= \frac{1}{c_0^y}\left(p_0 - \frac{p_1}{R}\right) \end{aligned} \tag{14}$$

where μ is Lagrangian multiplier associated with the housing demand constraint. Since p_0 is a policy variable and in equilibrium $h_0 = \overline{h}$, we have

$$c_0^y = \frac{w_0 - \left(p_0 - \frac{p_1}{R}\right)\overline{h}}{1+\beta} \tag{15}$$

To solve for the level of housing prices, we detrend all per capital variables $\widehat{x}_t = \frac{x_t}{(1+g)^{t-1}}$. After detrending, we have

$$\widehat{c}_0^y = \frac{\widehat{w}_0 - \left(\widehat{p}_0 - \widehat{p}_1 \left(1+g\right)/R\right)\overline{h}}{1+\beta}$$

Now, we establish the optimal housing price $\widehat{p}_0$.

Corollary 2. *The optimal housing price chosen by the government at period 0 is*

$$\widehat{p}_0 = \frac{\left[\widehat{w}_0 - \lambda^{-1}\left(1+\beta\right)\right]}{\overline{h}} + \frac{\alpha}{\lambda\overline{h}} \frac{\left(1+g\right)/R}{1-\left(1+g\right)/R} \tag{16}$$

In (16), $\widehat{w}_0 - \lambda^{-1}\left(1+\beta\right)$ is the user cost of housing for cohort 0. According to (16), the lower is $\widehat{w}_0$, the lower is the user cost of housing for initial cohort and the lower should be $\widehat{p}_0$ chosen by the government. Plugging (16) into (6), we can see for the demand constraint to be binding ($k > 0$), it is necessary that

$$\lambda \leq \frac{1+\alpha+\beta}{w_0}. \tag{17}$$

Finally, we can solve for λ using the planner's intertemporal budget constraint. Note for period 0, since there is no explicit transfer, (13) holds. Hence, plugging (4), (5) and (7) into the the intertemporal budget constraint becomes

$$\sum_{t=1}^{\infty} \left(\frac{1+n}{R}\right)^t \left[\left(1+\alpha+\beta\right)\lambda^{-1}\left(\phi R\right)^t - w_t\right] = A_0$$

which gives

$$\lambda = \frac{\left(1+n\right)\phi\left(1+\alpha+\beta\right)}{1-\left(1+n\right)\phi}\left[A_0 + \sum_{t=1}^{\infty}\left(\frac{1+n}{R}\right)^t\right] \tag{18}$$

2.2.3 Transfer Comparison

We can construct the implicit transfer for cohort 0 via subsidized housing price as the difference of user cost for housing between the case where $\widehat{p}_0$ is chosen by the

government and the case $\widehat{p}_0$ is decided by the market equilibrium. This difference is used to finance the consumption when young and old when young. Hence, the implicit transfer, denoted as $\widehat{T}_{r0}$, is

$$\begin{aligned}
\widehat{T}_{r0} &= \left(\widehat{p}_0^F - \frac{\widehat{p}_1^F(1+g)}{R}\right)\overline{h} - \left(\widehat{p}_0 - \frac{\widehat{p}_1(1+g)}{R}\right)\overline{h} - \\
&= \frac{\alpha\widehat{w}_0}{1+\alpha+\beta} - \left[\widehat{w}_0 - \lambda^{-1}(1+\beta)\right] \\
&= \lambda^{-1}(1+\beta) - \frac{1+\beta}{1+\alpha+\beta}\widehat{w}_0,
\end{aligned}$$

where $\widehat{p}_t^F$ denote the housing prices in an economy without government intervention. The first argument on the third row of RHS is the user cost of housing for initial cohort under the equilibrium price for $\widehat{p}_0$. To understand the source of implicit transfer via subsidized housing price, we compute the return for housing investment under decentralized economy.

$$\begin{aligned}
& \frac{\widehat{p}_1(1+g)}{\widehat{p}_0 - \frac{\alpha\widehat{c}_0^y}{h_0}} \\
= & \frac{\frac{\alpha\lambda^{-1}}{\overline{h}}\frac{R(1+g)}{R-(1+g)}}{\left[\widehat{w}_0 - \lambda^{-1}(1+\beta)\right]/\overline{h} + \frac{\alpha}{\lambda\overline{h}}\frac{(1+g)/R}{1-(1+g)/R} - \frac{\alpha}{\lambda\overline{h}}} \\
> & \frac{\frac{\alpha\lambda^{-1}}{\overline{h}}\frac{R(1+g)}{R-(1+g)}}{\frac{\alpha}{\lambda\overline{h}} + \frac{\alpha}{\lambda\overline{h}}\frac{(1+g)/R}{1-(1+g)/R} - \frac{\alpha}{\lambda\overline{h}}} \\
= & R
\end{aligned}$$

where the first equality is obtained by plugging in (16) and (42), the second inequality comes from the assumption (17) .

Given the only policy tool available to government for transferring to initial cohort is the subsidized housing prices, the government would choose a lower housing price than the market equilibrium price as implicit transfer to the initial cohort Accordingly, the initial cohort benefits from a higher return to housing investment than R by selling the public housing purchased at subsidized price later to future cohorts at the market

equilibrium prices. Such a capital gain would allow the initial cohort to enjoy higher consumption despite their low wage income.

Now we would like to compare the implicit transfer for cohort 0 with the transfer for cohorts born at $t \geq 1$. Note that for cohorts born at $t \geq 1$, part of transfer is used to purchase housing services, while for the initial cohort, the implicit transfers are used to finance consumption only. Hence, make the transfers between cohort 0 and the rest of cohorts comparable, we need to construct the transfer measure by excluding the part of transfers for future cohorts that are used to finance the user cost of housing from the total transfer they receive, denoted as

$$\begin{aligned} \widehat{T}_{rt} &= \widehat{T}_t - \left\{ \left(\widehat{p}_t - \frac{\widehat{p}_{t+1}(1+g)}{R} \right) \overline{h} - \left(\widehat{p}_t^F - \frac{\widehat{p}_{t+1}^F(1+g)}{R} \right) \right\} \\ &= \widehat{T}_t - \left[\alpha \lambda^{-1} \left(\frac{\phi R}{1+g} \right)^{-t} - \frac{\alpha \widehat{w}_1}{1+\alpha+\beta+\gamma} \right] \end{aligned}$$

where the argument in the solid bracket on is the difference of user cost for housing with and without government transfer. With (8) and (11) , we have

$$\widehat{T}_{rt} = (1+\alpha+\beta)\lambda^{-1} \left(\frac{\phi R}{1+g} \right)^{-t} - \widehat{w}_1 \left(\frac{\phi R}{1+g} \right)^{-t} - \left[\alpha \lambda^{-1} \left(\frac{\phi R}{1+g} \right)^{-t} - \frac{\alpha \widehat{w}_1}{1+\alpha+\beta} \right] \tag{19}$$

Proposition 3. *If the planner's discount factor is $\phi = (1+g)/R$, then the housing subsidy for $t \geq 1$ less than the implicit housing subsidy for the initial generation via subsidized housing price.*

Proof. Plugging $\phi = (1+g)/R$ into (19) , we have

$$\widehat{T}_{rt} = \lambda^{-1}(1+\beta) - \frac{1+\beta}{1+\alpha+\beta} \widehat{w}_1$$

for all $t \geq 1$. Since by assumption $\widehat{w}_1 > \widehat{w}_0$, $\widehat{T}_{r0} > \widehat{T}_{rt}$ for all $t \geq 1$. □

Intuitively, as the initial generation is poorer, even a utilitarian planner that has

no incentive to redistribute across generations in steady state will find it optimal to transfer resources from future to the current generations. This is achieved by charging a housing price lower than the equilibrium price as implicit transfer to the initial cohort.

2.3 Second-best Ramsey Solution

In reality, the government may not be able to provide negative transfers to future cohorts (or tax future cohorts) based on their housing expenditure. That is, the government may face the constraint that $\varsigma_t \geq 0$, for $t \geq 1$. We now discuss whether this constraint changes our main results. According to (10), this is equivalent to put a constraint on the lower bound of h_t for the planner's problem.

$$h_t \geq \widetilde{h}_t \text{ for } t \geq 1 \tag{20}$$

Therefore, the planner solves the following problem

$$\max_{\left\{c_t^y, c_{t+1}^o, h_t\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \left(\phi\left(1+n\right)\right)^t \left[\log c_t^y + \alpha \log h_t + \beta \log c_{t+1}^o\right]$$

subject to (2), (3) and (20). The first-order conditions for consumption at all $t \geq 0$,

$$c_t^y = \lambda^{-1} \left(\phi R\right)^t \tag{21}$$

$$c_{t+1}^o = \beta R \lambda^{-1} \left(\phi R\right)^t \tag{22}$$

The first order conditions for h_t is

$$\frac{\alpha}{h_0} - \kappa = \lambda \left(p_0 - \frac{p_1}{R}\right) \tag{23}$$

$$\frac{\alpha}{h_t} = \left(p_t - \frac{p_{t+1}}{R}\right) \lambda \left(\phi R\right)^{-t} - \mu_t \text{ for } t \geq 1 \tag{24}$$

where κ and μ_t are the Lagrangian multiplier associated with the constraints (3) and (20), respectively.

Now, we assume that in the decentralized economy, the non-negative constraint for housing transfer is binding, $\varsigma_t = 0$, for $t \geq 1$. Hence, (10) becomes

$$\left(p_t - \frac{p_{t+1}}{R}\right) h_t = \frac{\alpha w_t}{1+\alpha+\beta} \geq \alpha \lambda^{-1} (\phi R)^t \tag{25}$$

This will give a lower bound for λ

$$\begin{aligned} \lambda &\geq \left(\frac{\phi R}{1+g}\right)^t \frac{(1+g)(1+\alpha+\beta)}{w_1} \\ &= \frac{(1+g)(1+\alpha+\beta)}{w_1} \end{aligned}$$

Similar to the logic above, for the initial cohort to receive a discounted housing price ($\kappa > 0$), it is necessary that

$$\lambda \leq \frac{1+\alpha+\beta}{w_0}. \tag{26}$$

Therefore, we have the following proposition

Proposition 4. *Suppose that $\phi = (1+g)/R$ and the planner cannot set negative housing subsidy. Assume further that $\lambda \in \left[\frac{(1+g)(1+\alpha+\beta)}{w_1}, \frac{1+\alpha+\beta}{w_0}\right]$, where λ is the Lagrangian multiplier for the planner's resource constraint:*

$$\lambda = \frac{(1+n)\phi(1+\alpha+\beta)}{1-(1+n)\phi}\left[A_0 + \sum_{t=1}^{\infty}\left(\frac{1+n}{R}\right)^t\right]. \tag{27}$$

Then, the Ramsey constrained (second-best) allocation is implemented by setting the following sequence of policy.

$$\widehat{p}_0 = \frac{[\widehat{w}_0 - \lambda^{-1}(1+\beta)]}{\overline{h}} + \frac{\alpha}{\lambda \overline{h}} \frac{(1+g)/R}{1-(1+g)/R} \tag{28}$$

$$\varsigma_t = 0, \text{ for all } t \geq 1. \tag{29}$$

In other words, in the second best solution, the planner simply applies the discounted housing price for the initial cohort and does not tax future generations. This is exactly

what the housing policy reform looks like in reality.

3 The Full-Blown Model

3.1 Household

Demographic Structure Each period a continuum of household is born. The new-born population grows at a exogenous rate n_t. Households live a maximum J periods, and each period represents one year. They enter the economy as adults, and are active as workers until age J_w. All households face a probability S_j of surviving up to age j, and they die at age J with certainty. $S_j = \prod_{k=1}^{j} \psi_k$, and ψ_k is the conditional survival probability from age $k-1$ to age k. Given this information, at the steady state, the fraction of households of age j, denoted as μ_j is

$$\mu_j = \frac{\psi_j}{1+n}\mu_{j-1}$$

where $\sum_{j=1}^{J} \mu_j = 1$. In the transitional dynamics, the equation becomes

$$\mu_{t,j} = \frac{\psi_j}{1+n_t}\mu_{t-1,j-1}$$

t is the calendar time, and j is the age.

Preference All agents have identical form of preference over aggregation of goods consumption and housing service,

$$U_t^i = \sum_{j=1}^{J} S_j \beta^{j-1} u\left(c_{t,j}^i, s_{t,j}^i\right),$$

where the utility function is separable in terms of goods and housing,

$$u\left(c_{t,j}^i, h_{t,j}^i\right) = \log c_{t,j}^i + \phi \log\left(s_{t,j}^i - \underline{h}\right),$$

where $\underline{h}$ is the subsistence level of housing, which we introduce to drive a difference of housing value ot income ratio across households of different permanent income levels.

$e^i_{t,j} = \epsilon^i \varepsilon_j \eta^i_{j,t}$, ϵ^i is the permanent efficiency shock to household i when he was born, ε_j is life-cycle income profile, and $\eta^i_{t,j}$ is the idiosyncratic efficiency shock to household i in age j; w_t is the wage rate per efficiency unit at t; τ_t is the labor income tax;

Finally, for individual household i, the social security benefit each period is determined as

$$b^i_{t,j} = \theta(0.6\widetilde{y}^i_{t-j+J_w,J_w} + 0.4\bar{y}_t),$$

where θ is the replacement rate at the time of retirement, and $\widetilde{y}^i_{t-j+J_w,J_w} = w_{t-j+J_w} e_{t-j+J_w,J_w}$ is the before-tax wage earnings of the household i before retirement. $\bar{y}_t$ is the current average yearly earning of workers.

$$\bar{y}_t = \frac{\sum_{j=1}^{J_w} \sum_i \mu_{t,j} w_t e^i_{t,j}}{\sum_{j=1}^{J_w} \mu_{t,j}}$$

Note that the above formula only allows the partial indexation of pension benefits to the current period earnings. We also assume that the social security program is self-financing where the social security payroll tax rate τ^{ss}_t each period is endogenously determined to balanced the social security budget each period. In other words, the social security tax rate is determined as

$$\tau^{ss}_t = \frac{\sum_{j=1}^{J_w} \sum_i \mu_{t,j} b^i_{t,j}}{\sum_{j=1}^{J_w} \sum_i \mu_{t,j} w_t e^i_{t,j}}$$

Finally, denote the after-tax income of a household as

$$y^i_{t,j} = \begin{cases} (1 - \tau_t - \tau^{ss}_t) w_t e^i_{t,j} \text{ for } j \leq J_w \\ b^i_{t,j} \text{ for } j > J_w \end{cases}$$

3.2 Housing Reform

Assume at the beginning of period 1 (1994), there is a housing reform, under which the government sells rental housing $h_{i,t}$ to existing households age j at a discounted price. All households born before year 1994 are eligible to purchase at the discounted price at any point of their life, unless they choose to move out of rental housing and move into a commercial housing. In particular, the discounted price computed as

$$p^g_{t,j} = p_t \left[1 - \frac{\min[\text{Age in 1994}, J_w]}{J_w} \times 65 \times 0.9\%\right]$$

Note that we could back out the age for a transitional cohort that is age j at time t as

$$\begin{aligned} t - j + 1 &= 1 - k + 1 \\ &\longrightarrow \\ k &= 1 - t + j. \end{aligned}$$

This formula is a simplified rule of the discounted price in practice, where p_t is the market price of the current period.[8] J_w is the maximum working age. According to this formula, the discounted price is age-specific: if a household was retired before 1994, then he can enjoy 58.5% (65*0.9%) of the price discount.[9] The longer an household worked before 1994, the more discount he or she can enjoy. This reflects the government idea to transfer resources for future cohorts to these cohorts who have fewer years to enjoy the higher wage growth. At the same time, all households revise their expectation about TFP growth and land supply in 2003. The first shock increase the resources of

[8] In reality, the price applied to the discount is called the cost price, which is lower than the market price that applied to the high income households. The cost price, however, adjusts every year upwards following the market prices. Another pricing rule is the standard price, under which the household only enjoyed partial ownership right. However, according to Wang (2011), the share of households, only 18 percent of households had partial ownership. So we do not consider it either.

[9] In reality, the total effective maximum working years of the household in the computation of the house price discount is 65 (35 years for the husband, plus 30 years for the wife, to be consistent with the fact that male retries at age 60 and female retires at age 55 at that time.)

future generations, and the second shocks help to increase the land prices, which further increase the housing prices. Note that the new born households has to purchase the housing from the market, which also creates the housing markets since 1994.[10] So we need to have market supply. The household who are eligible (alive in year 1994) for reformed housing can choose to purchase the housing at the discounted price, or keep renting the housing at the pre-reform prices, or purchase housing from the market.

Compare purchasing reformed housing versus purchasing commercial housing, the trade-off is as follows, reformed housing is cheaper than the market price, but fixed amount. On the other hand, commercial housing don't have size restriction. Therefore, the high income household expecting high income growth in the future and fast housing price growth, may choose to purchase commercial housing for more capital gain. Low income household, whose optimal housing size is small, may choose to purchase the reformed housing. However, the low income household may be subject to the credit constraint, and is not capable of purchasing discounted housing in year 1994.

Now, since higher TFP growth and lower land supply shocks implying higher future prices, households may prefer the reformed housing to the government rental housing for the potential capital gain in future. However, the down payment constraint and the transaction cost would make the reformed housing only affordable to the middle income households. Also, since they expect that the discounted housing purchase option would last throughout their life-cycle, then may postpone purchase the discounted housing in a later stage of life.

The 2nd stage is an expected reform in period 5 (year 1998), with the Stage 1 reform as the period 1, the government declared to eliminate the government rental housing for household born after period 5. All households born between period 1 and 5 are still eligible for government rental discount until they move out.

In summary, all households born no later than year 1998 can keep renting the

[10] According to our formula, for the new born, since their working years before 1994 is zero, they don't enjoy the price discount.

government assigned rental housing until they move out to a commercial housing. Households born before year 1994 have an additional option of purchasing rental housing with a discounted price, and the option is valid as long as they live in government assigned rental housing.

3.3 Households' Decision

Housing service can be obtained from the market by either renting a house at a rental rate ρ_h or buying houses at a price p. Housing size for both rental and purchased housing are discrete. For purchased houses, housing size belongs to the set H, while for rental housing, housing size belongs to the set $\tilde{\mathcal{H}}$.

To capture rental market friction on the demand side, we assume that renting generates service less than one-for-one with the size of house, i.e. $s = \omega h$. $0 < \omega < 1$. When a household sells its home, it incurs a proportional transaction cost $\kappa_h ph$, that is proportional to its housing value, and a fixed transaction cost, κ_0^j. Renter can adjust their housing size without cost. For renters who purchase commercial housing or government housing (if qualified), they incur a fixed cost of initiating the mortgage, m_j, which is time and age dependent. Households are subject to down payment constraint; $h^i_{t+1,j+1}$ is the stock of housing enjoyed by the household in t; households need to pay for the housing maintenance cost each period, $\delta_h p_t h^i_{t,j}$;

At the beginning of period 1, all transitional households hold no housing. At each period since period 2, the transitional household born before year 1994 started the period with three possible states: 1. renting government housing (Ng), 2. renting commercial housing (N), 3. owning housing (H). If the transitional household hold no housing, it can continue to rent public housing (r) at the government subsidized rental rate, or purchase the discounted (reformed) housing or commercial housing or rent from the market. If the household holds the discounted housing, it can keep the discounted housing or sell it and purchase commercial housing (or rent from the market). If the household holds the commercial housing, it can only choose to keep its commercial

housing or sell it and purchase another commercial housing (or rent from the market). Households can not hold discounted housing and market housing at the same time. Denote the index for the beginning of period housing status as $I \in \{N_g, N, H\}$.

Diagram I: Beginning-of-period Housing Status and Housing Choices for transition cohorts born before 1994

Beginning of Period State			Intermediate/Choice					
w/o house	w/o house	w house	rent gov't H	buy gov't H	buy com. H	keep H	sell H	rent com. H
Ng	N	H	rg	bg	bm	p	s	r

$$N_g \longrightarrow \begin{cases} \text{rent gov't H}\rightarrow N_g \\ \text{buy gov't H}\rightarrow\text{H} \\ \text{buy com. H}\rightarrow\text{H} \\ \text{rent com. H}\rightarrow\text{N} \end{cases}$$

$$N \longrightarrow \begin{cases} \text{buy com. H}\rightarrow\text{H} \\ \text{rent com. H }\rightarrow\text{N} \end{cases}$$

$$H \longrightarrow \begin{cases} \text{keep H}\rightarrow\text{H} \\ \text{sell H}\longrightarrow \begin{cases} \text{buy com. H}\rightarrow\text{H} \\ \text{rent com. H}\rightarrow\text{N} \end{cases} \end{cases}$$

We now write the households' problem in recursive form. Each period, a household's idiosyncratic state vector is $\chi = (a, y, h)$. Denote the measure of households across individual states as $\Phi_t(\chi)$. The above household problem can be rewritten recursively as

- The choice of a transitional household without house and renting government housing (Ng) at the beginning of the period is

$$V_{t,j}^{Ng}(a,y) = \max\left\{V_{t,j}^{r}(a,e), V_{t,j}^{bg}(a,e), V_{t,j}^{bm}(a,e), V_{t,j}^{rg}(a,y)\right\}$$

- The choice of a transitional household without house and renting commercial housing (N) at the beginning of the period is

$$V_{t,j}^{N}(a,y) = \max\left\{V_{t,j}^{r}(a,e), V_{t,j}^{bm}(a,e)\right\}$$

- The intermediate optimization problems for transitional household renting government or commercial housing are as following

  – For a renter of government house, its problem is therefore

$$V_{t,j}^{rg}(a,y) = \max u\left(c_{t,j}^{i}, s_{t,j}^{i}\right) + \beta\psi_{j+1}E\left[V_{t+1,j+1}^{N}(a,y)\right]$$

  subject to

$$\begin{aligned} c_{t,j}^{i} + qa_{t+1,j+1}^{i} + R_{c,t}s_{t,j}^{i} &\leq y_{j,t}^{i} + a_{t,j}^{i}, \\ s_{t,j}^{i} &= \omega\widetilde{h}_{1,1+j-t}^{i} \end{aligned} \tag{30}$$

  where $a_{t+1,j+1}^{i}$ is the amount of asset purchase by a household of age j at t, which has an exogenous price $q = \frac{1}{1+r}$. $\widetilde{h}_{1,1+j-t}^{i}$ is the rental housing by agent i in the initial steady state

  – For a renter of commercial housing, its problem is

$$V_{t,j}^{r}(a,y) = \max u\left(c_{t,j}^{i}, s_{t,j}^{i}\right) + \beta\psi_{j+1}E\left[V_{t+1,j+1}^{N}(a',y')\right]$$

  subject to

$$\begin{aligned} c_{t,j}^{i} + qa_{t+1,j+1}^{i} + \rho_{h,t}\widetilde{h}_{t+1,j+1}^{i\prime} &\leq y_{j,t}^{i} + a_{t,j}^{i}, \\ s_{t,j}^{i} &= \omega\widetilde{h}_{t+1,t+1}^{i\prime},\ \widetilde{h}_{t+1,j+1}^{i\prime} \in \widetilde{H} \end{aligned} \tag{31}$$

– For a household who decided to buy the discounted government housing, the problem is

$$V_{t,j}^{bg}(a,y) = \max u\left(c_{t,j}^{i}, s_{t,j}^{i}\right) + \beta\psi_{j+1}E\left[V_{t+1,j+1}^{H}(a',y',h')\right]$$

subject to

$$\begin{gathered} c_{t,j}^{i} + qa_{t+1,j+1}^{i} + p_{t,j}^{g}h_{t+1,j+1}^{i} + m_j\mathbb{1}(a_{t+1,j+1}^{i} < 0) \leq y_{j,t}^{i} + a_{t,j}^{i}, \qquad (32) \\ s_{t,j}^{i} = h_{t+1,j+1}^{i}, h_{t+1,j+1}^{i} = \widetilde{h}_{1,1+j-t}^{i}. \\ a_{t+1,j+1}^{i} \geq -\Upsilon p_{t,j}^{g}h_{t+1,j+1}^{i}, \end{gathered}$$

– For a household who choose to buy the commercial housing, the problem is

$$V_{t,j}^{bm}(a,y) = \max u\left(c_{t,j}^{i}, s_{t,j}^{i}\right) + \beta\psi_{j+1}E\left[V_{t+1,j+1}^{H}(a',y',h')\right]$$

subject to

$$\begin{gathered} c_{t,j}^{i} + qa_{t+1,j+1}^{i} + p_t h_{t+1,j+1}^{i} m_j\mathbb{1}(a_{t+1,j+1}^{i} < 0) \leq y_{j,t}^{i} + a_{t,j}^{i}, \qquad (33) \\ s_{t,j}^{i} = h_{t+1,j+1}^{i}, h_{t+1,j+1}^{i} \in H \\ a_{t+1,j+1}^{i} \geq -\Upsilon p_t h_{t+1,j+1}, \end{gathered}$$

- A household who held housing at the beginning of the period (no matter government housing or commercial housing) can make the following choice, keep the housing (government housing or commercial housing) or sell it and purchase new commercial housing.

$$V_{t,j}^{H}(a,y,h) = \max\left\{V_{t,j}^{p}(a,e,h), V_{t,j}^{s}(a,y,h)\right\}$$

– If keep the government housing, in the next period, it still own the government

housing. As a result, the problem is

$$V_{t,j}^{p}\left(a,y,h\right)=u\left(c_{t,j}^{i},s_{t,j}^{i}\right)+\beta\psi_{j+1}E\left[V_{t+1,j+1}^{H}\left(a',y',h'\right)\right]$$

$$\begin{aligned} c_{t,j}^{i}+qa_{t+1,j+1}^{i}+\delta p_{t}h_{t,j}^{i} &\leq y_{j,t}^{i}+a_{t,j}^{i}, \\ s_{t,j}^{i} &= h_{t+1,j+1}^{i}, h_{t+1,j+1}^{i}=h_{1994}^{i}, \\ a_{t+1,j+1}^{i} &\geq -\Upsilon p_{t}h_{t+1,j+1}^{i}, \end{aligned} \tag{34}$$

Note that keeping the government housing means that selling the government housing at the market price and repurchase the same house at the market price.

– if the household sells the housing, then it can only purchase the commercial housing or rent from the commercial market.

$$V_{t,j}^{s}\left(a,y,h\right)=\max\left\{V_{t,j}^{sr}\left(a_{n},e\right),V_{t,j}^{sb}\left(a_{n},y\right)\right\} \tag{35}$$

subject to

$$a_{t,j}^{i,n}=a_{t,j}^{i}+(1-\delta_{h}-\kappa)p_{t}h_{t,j}^{i}-\kappa_{0}$$

where the fixed moving cost, κ_0, can depend on household specific types.

– For a household who choose to sell the housing and purchase commercial housing, the problem is

$$V_{t,j}^{sb}\left(a^{n},y\right)=u\left(c_{t,j}^{i},s_{t,j}^{i}\right)+\beta\psi_{j+1}E\left[V_{t+1,j+1}^{H}\left(a',y',h'\right)\right]$$

subject to

$$c_{t,j}^i + qa_{t+1,j+1}^i + p_t h_{t+1,j+1}^i \leq y_{j,t}^i + a_{t,j}^{i,n}, \tag{36}$$
$$s_{t,j}^i = h_{t+1,j+1}^i,\ h_{t+1,j+1}^i \in H$$
$$a_{t+1,j+1}^i \geq -\Upsilon p_t h_{t+1,j+1}^i.$$

– For a household who choose to sell the housing and rent commercial housing, the problem is

$$V_{t,j}^{sr}(a,y) = u\left(c_{t,j}^i, s_{t,j}^i\right) + \beta\psi_{j+1} E\left[V_{t+1,j+1}^N(a',y')\right]$$

subject to

$$c_{t,j}^i + qa_{t+1,j+1}^i + \rho_{ht}\widetilde{h}_{t+1,j+1}^{i\prime} \leq y_{j,t}^i + a_{t,j}^{i,n}, \tag{37}$$
$$s_{t,j}^i = \omega\widetilde{h}_{t+1,j+1}^{i\prime},\ \widetilde{h}_{t+1,j+1}^{i\prime} \in \widetilde{H}$$

For a household born at or after period 1 (1994) and no later than period 5 (1998), their problem is similar to households born before year 1994. The difference is that they do not have the option of purchasing government rental housing although they can choose to rent until they move out. The intermediate optimization problems are the same as above.

Diagram II: Beginning-of-period Housing Status and Housing Choices for cohorts born between 1994 and 1998.

Beginning of Period State			Intermediate/Choice				
w/o house	w/o house	w house	rent gov't H	buy com. H	keep H	sell H	rent com. H
Ng	N	H	rg	bm	p	s	r

$$N_g \longrightarrow \begin{cases} \text{rent gov't H}\rightarrow N_g \\ \text{buy com. H}\rightarrow\text{H} \\ \text{rent com. H}\rightarrow\text{N} \end{cases}$$

$$N \longrightarrow \begin{cases} \text{buy com. H}\rightarrow\text{H} \\ \text{rent com. H}\rightarrow\text{N} \end{cases}$$

$$H \longrightarrow \begin{cases} \text{keep H}\rightarrow\text{H} \\ \text{sell H}\longrightarrow \begin{cases} \text{buy com. H}\rightarrow H \\ \text{rent com. H}\rightarrow N \end{cases} \end{cases}$$

For a household born after period 5 (1998), their problem is

Diagram III: Beginning-of-period Housing Status and Housing Choices for cohorts born at or after 1998.

Beginning of Period State		Intermediate/Choice			
w/o house	w H	rent com. H	buy com. H	keep H	sell H
N	H	r	bm	p	s

$$N \longrightarrow \begin{cases} \text{buy com. H}\rightarrow\text{H} \\ \text{rent com. H}\rightarrow\text{N} \end{cases}$$

$$H \longrightarrow \begin{cases} \text{keep H}\rightarrow\text{H} \\ \text{sell H}\longrightarrow \begin{cases} \text{buy com. H} \rightarrow H \\ \text{rent com. H} \rightarrow N \end{cases} \end{cases}$$

3.4 Pre-reform Steady State

Before year 1994, we assume the economy is in the pre-reform steady state, and the government budget is in balance. We assume household' idiosyncratic shocks perfectly insured in the pre-reform stage, i.e. households with the same age and permanent efficiency shock are identical including the level of housing. This is equivalent to a deterministic case. For notation concision, we drop the time subscript.

For a household born before period 1 (1994), their problem is

$$V_{t,j}^{rg}(a,y) = \max u\left(c_{t,j}^{i}, s_{t,j}^{i}\right) + \beta\psi_{j+1}E\left[V_{t+1,j+1}^{rg}(a,y)\right]$$

subject to

$$\begin{aligned} c_{t,j}^{i} + qa_{t+1,j+1}^{i} + R_{c,t}s_{t,j}^{i} &\leq y_{j,t}^{i} + a_{t,j}^{i}, \\ s_{t,j}^{i} &= \widetilde{h}_{1,1+j-t}^{i} \end{aligned}$$

We use the households' choice of government rental housing in the pre-reform steady state as the government plan of assigning rental housing to households born between period 1 and 5.

For housing production, we assume that the government use land and labor to produce housing and then rent it out to households at a given rental rate.

3.5 Production Sectors

There are two production sectors in the economy: a consumption goods sector and a construction sector.

Final goods sector: The competitive final goods sector operates a constant return to scale technology

$$Y_t = z_t N_{ct}$$

where N_{ct} is the total units of labor services used to produce consumption goods, and $w_t = z_t$. z_t is an exogenous process.

Construction sector operates the production technology

$$I_t = (z_t N_{ht})^{\alpha} (L_t)^{1-\alpha}$$

A developer solves the static problem

$$\begin{aligned} &\max_{N_h} p_t I_t - w_t N_{ht} \\ &s.t. \\ &I_t = (z_t N_{ht})^{\alpha} (L_t)^{1-\alpha} \end{aligned} \tag{38}$$

We have the housing investment function

$$I_t = (\alpha p_t)^{\frac{\alpha}{1-\alpha}} L_t$$

The labor demand from the construction sector is

$$N_{ht} = (\alpha p_t)^{\frac{1}{1-\alpha}} \frac{L_t}{z_t} \tag{39}$$

The profit of the government leasing the land is

$$\pi_t = \frac{1-\alpha}{\alpha} (\alpha p_t)^{\frac{1}{1-\alpha}} L_t$$

Combine the labor demand of two sectors, $N_{dt} = N_{ct} + N_{ht}$, and the total labor supply is given by $N_{st} = \int e \Phi_t(s) ds$.

3.6 Rental Sector

To simplify the rental market, we assume that there is a representative real estate developer. Each period, a representative rental company purchases housing in the housing market and rents them to renters. The rental company can frictionlessly buy and sell housing units subject to an operating cost ψ for each unit of housing rented out. The problem of the representative rental company is

$$J(\tilde{H};\Phi) = \max_{\tilde{H}'} \left[\rho_h(\Phi) - \psi\right]\tilde{H}' - p(\Phi)[\tilde{H}' - (1-\delta_h)\tilde{H}] + \frac{1}{1+r}E_{\Phi'|\Phi}J(\tilde{H}';\Phi')$$

The zero profit condition gives the equilibrium rental rate as

$$\rho_h(\Phi) = \psi + p(\Phi) - \frac{1-\delta_h}{1+r}E_{\Phi'}[p(\Phi')|\Phi].$$

In other words, the rent is simply the user cost of housing plus the operating cost.

3.7 Government

We assume all the revenues by the governments by either renting or selling the public houses or land consumed by the government. In the transition path (in and after year 1994), the government's budget is more complicated. The expenditure includes discounted housing subsidy and rental housing subsidy. The revenue includes the income tax.

$$\sum_{t=1}^{\infty} R^{-t} \left[\begin{array}{c} \sum_{j=t+1}^{J} \mu_{j,t} \int_{I\in bg} \left(p_t - p^g_{t,j}\right) h^i_{1,1+j-t}\Phi_t(s)ds \\ +\sum_{j=t+1}^{J} \mu_{j,t} \int_{I\in rg} \left(\rho_t - R_{c,t}\right) h^i_{1,1+j-t}\Phi_t(s)ds \\ +\sum_{j=1}^{t} \mu_{j,t} \int_{I\in rg} \left(\rho_t - R_{c,t}\right) h^i_{1,j}\Phi_t(s)ds \\ -\sum_{j=1}^{J_w} \mu_{j,t}\tau_t w_t \int_s e\Phi_t(s)ds \end{array}\right] N_t = A_0.$$

where $h^i_{1,1+j-t}$ denotes the reformed housing purchase by a household age j at period t, which is the size of house resided by this household at period 1, when he was aged

$1+j-t$. This intertemporal government budget constraint implies that all transfer to the transitional cohorts need to be financed by future generations.

3.8 Equilibrium

Denote $\chi^H = (a, y, h)$ and $\chi^N = (a, y)$ as the idiosyncratic state vectors for homeowners and non-homeowners.

A recursive competitive equilibrium is a sequence of age-dependent value functions $\left\{V_j^N\left(\chi^N;\Phi\right), V_j^H\left(\chi^H;\Phi\right), V_j^r\left(\chi^N;\Phi\right), V_j^{rg}\left(\chi^N;\Phi\right), V_j^{bm}\left(\chi^N;\Phi\right), V_j^{bg}\left(\chi^N;\Phi\right), V_j^p\left(\chi^H;\Phi\right), V_j^s\left(\chi^H\right.\right.$ households' decision rules, aggregate function for construction labor $N_h\left(\Phi\right)$, rental unit stocks $\widetilde{H}'\left(\widetilde{H};\Phi\right)$, home buyers' housing stock $H'\left(H;\Phi\right)$, housing investment $I_h(\Phi)$, rental prices, $\rho_h(\Phi)$, housing price $p(\Phi)$, and a law of motion for the aggregate state Φ such that:

1. Given the price functions and aggregate law of motion, the value function solve the recursive problem of the households, and are the associated policy functions.

2. Construction sector firms maximize profit with associated labor demand and housing investment function (N_h, I_h)

3. The labor market clears at wage rate $w = z$.

4. The rental market clears at price ρ_h.

5. Housing market clears at price p

$$\widetilde{H}' + H' = (1-\delta_h)\left(\widetilde{H} + H\right) + I$$

6. The aggregate law of motion is induced by the exogenous stochastic processes and all the decision rules, and it is consistent with individual behavior.

7. Government intertemporal budget constraint is satisfied.

4 Calibration

The model period is one year. We calibrate the benchmark economy to match the Chinese economy during 2003-2018.

4.1 Household sector

For the household sector, we need to calibrate the following:

(1) Demography:

- In the initial steady state, the population growth rate is set to be 2.4%, which is the average working-age population growth rate in 1995. Between 1995 and 2020, the population growth rate is UN working-age population growth rate. After 2020, the projected population growth rate is set to be zero.
- The conditional survival probability $\{\psi_j\}_{j=1}^{J}$ are taken from Imrohoroglu and Zhao (2018a).
- In our model, $j = 1$ corresponds to age 20 in real life, and $J_w = 40$ so that agents retire at age 60 in real life. We set $J = 70$, corresponding to a maximum lifetime of 89 years.

(2) Labor:

- The life-cycle income profile, ϵ_j are taken from He et al. (2017) who use the CHNS data to estimate them;
- Distribution for the permanent efficiency shock, ε^i is calibrated following Imrohoroglu and Zhao (2018a), We normalize $\varepsilon^L = 1$. The average wage rate of high school graduate is approximately 1.79 times that of high school dropout. So $\varepsilon^H = 1.79$. Finally, we calibrate the high-skill working aging population to match the the proportion of working-age population that are high school graduate, 46%.

- The stochastic component (until the retirement) is given by

$$\log(\eta_t) = \rho \log(\eta_{t-1}) + \nu_t$$

Agents face the same income risks in the steady state and in the transition. Based on Imrohoroglu and Zhao (2018a) to calibrate a three state Markov Chain, $\{0.36, 1.0.2.7\}$.

The transition probability matrix is

$$\begin{bmatrix} 0.9259 & 0.0741 & 0 \\ 0.0235 & 0.953 & 0.0235 \\ 0 & 0.0741 & 0.9259 \end{bmatrix}.$$

Alternatively, we can follow He et al. (2017), we take $\rho = 0.84$ and the variance σ_v^2 as 0.055. We then discretize this process into a five-state Markov chain using the Tauchen method. The results for ϵ are $\{0.2735, 0.5229, 1.0, 1.9123, 3.6567\}$. The transition probability matrix is

$$\begin{bmatrix} 0.6906 & 0.3088 & 0.0006 & 0.0000 & 0 \\ 0.0340 & 0.7923 & 0.1735 & 0.0001 & 0 \\ 0.0000 & 0.0834 & 0.8331 & 0.0834 & 0.0000 \\ 0 & 0.0001 & 0.1735 & 0.7923 & 0.0340 \\ 0.0000 & 0.0000 & 0.0006 & 0.3088 & 0.6906 \end{bmatrix}.$$

(3) Preference:

- The utility discount factor, β is calibrated to a to-be-determined target. One candidate is the wealth to income ratio.

- Utility parameter for housing service ϕ is to target the average share of housing consumption in total consumption expenditure (0.2). (note that this value is used for both steady state, where there is only rental markets, and the transitional

dynamics). We can check the share of housing wealth in total wealth along the transitional dynamics.

We calibrate the utility discount for commercial rental housing ω to match a home ownership rate in 2002 as 82.4 percent (UHS data).

(5) Housing:

- housing depreciation rate, δ_h is set to be 2%, the estimated depreciation rate on China's urban owner-occupied housing by OECD.
- housing transaction cost, $\kappa p_t h_t$, which is a fraction of the housing value sold. $\kappa = 3\%$; Υ is set to be 80 percent of the housing values, consistent with the mortgage policy in 2002, that is a 20% of the down payment ratio.
- The subsidized rent in the pre-reform steady state, we set government subsidy to rental housing to be 10% of the market rental rate. According to Wang (2012), in 1993 the rental subsidy per month is 66.31 RMB and monthly income for households who lived in state-owned housing is 456 RMB. Alternatively, We may calibrate R_c such that $\frac{(R_c - R^*)\bar{h}}{y} = \frac{66.31}{456}$.
- The mortgage initiation fixed cost, m_j, is set as a stage function of household age to match home ownership rate by age at year 2003. We set totally three stages over household life span, which are from 21 to 40, from 41 to 60, and from 60 to the maximum of life span.
- The operating cost for the rental company ψ is calibrated to match rent to price ratio of 2%.

(6) The annual deposit interest rate, $r = 0.02$,, which is the real interest rate in 1994.

4.2 Production sector

- The construction technology parameter α is set to be 0.8, so that price elasticity of new housing supply equals 4, about the average value by Wang et al. (2012)

for the 35 major cities in China).

- We calibrate TFP process for non-durable goods, $\{z_t\}_{t=0}^{T}$ to match the urban hourly wage growth rate. In particular, at the steady state, the TFP grow rate is set to be 4.2%, which is the average growth rate of urban wage rate growth rate between 1980-1993. Between 1994-2010, the wage growth rate is 10.8%. Between 2010-2020, the wage growth rate is 8.1%. Between 2021-2050, the wage growth is 5.0 percent. After than the wage growth at 2 percent per year.

4.3 Government sector

In addition, we need to calibrate the following government policy parameters:

- Social security policy: According to Song et al. (2015), the Chinese pension system provided a replacement of 60 percent to those retiring between 1997 and 2011 covered by the system. The replacement rate θ is calibrated to be 0.6.

- General budget account associated policy: At the initial steady state, since there is no intergenerational transfer via housing prices, and pension is self-financed, there is no need to impose the labor income tax. $\tau = 0$. We assume that during the transition, the income tax rate remains zero as the initial steady state value until period 70, where the transition cohorts alive before period 1 all die. After that, the income tax rates has a one-time change to balance the intertemporal government budget constraint.

- Along the transition path, we feed in the urban land supply (for residential use) from the data (up to 2017). Since 2018, the land supply is set to be 1. The land supply also takes into consideration the deterioration of the land quality. Following Jiang et al. (2022), we assume land quality diminishes by 8%.

Table 1 summarizes the calibrated parameter values and their targeted moments. Table 7 compares the targeted moments generated by the model and the data. Overall,

our calibrated model can match the data moments fairly well.

We now compare the cross-sectional moments generated by the model with those by the data, which are untargeted in calibration. Table 3 compares the distribution of the overall home ownership rate by both age and education group between the model and the data. Table 4 focuses on the ownership rate of the government house. Overall, the model has matched those untargeted moments reasonably well.

5 Benchmark Results

5.1 Trading-up vs. first-time home purchase

We are particularly interested the magnitudes of trade-up among those initial cohorts. In panel (a) of Figure 1 we compare the demand for commercial houses led by trade-up with those demand among first-time buyers during 1998-2005.[11] Evidently, trade-up takes up a significant proportion of demand for commercial houses — at least 50 percent of demand of commercial houses is led by trade-up.

Trade-up tends to be prevalent only within a relatively short periods. In panel (b) of Figure 1, we measure both the volume share and population share of trade-up demand in the total demand for commercial houses in selective years. It is shown that the magnitudes are more than 50-percent in 2005, then decline to about 30-percent in 2010. In 2020, we hardly observe any trade-up demand.

We also explore who trades up among those initial cohorts in Figure 2. Panel (a) suggests that middle-age cohorts trade up earlier than those young cohorts during the periods when trade-up is prevalent. A few of households who were very young in 1994 still catch the wave of trading up when they turn middle-age in 2020. Panel (b) implies that skilled households dominate trade-up in early years. Unskilled households trade

[11] We can only keep track of those who sell their reformed houses and purchase commercial houses within the same period. We cannot record those who may purchase commercial houses a few periods later than selling the reformed houses.

up later due to either late purchase of reformed houses or accumulating saving over a longer periods.

5.2 The Role of Housing Reforms

The 94 reforms affect the ownership rates of initial cohorts via two potential channels. First, it allows these cohorts to purchase government houses, which are smaller in size than the commercial houses. Second, it provides purchase subsidy for government houses. To evaluate these two channels, we have performed two counterfactual exercises. In the first one, we remove the housing purchase subsidy to those born before 1994, but they are allowed to purchase the reformed house that they were initially assigned to. We call such a counterfactual economy the economy without purchase subsidy. In the second exercise, we further eliminate this option so that those born before 1994 can no longer purchase reformed houses. If they plan to own a house, they have to purchase a commercial house.[12] We call this counterfactual economy the economy without 94 reform. The difference of home ownership rate between our benchmark economy and the economy without purchase subsidy captures the effects of purchase subsidy, while the difference of home ownership rate between the economy without purchase subsidy and the one without 94 reform captures the effects of the availability of government houses.

In Figure 3 we plot the dynamics of housing prices in the three scenarios, respectively. Housing price is initially higher than the other two scenarios due to the housing purchase subsidy to initial cohorts in the benchmark economy. Housing prices then appear to be highest in the scenario without housing purchase subsidy during 2010-2020. This is likely led by the fact that initial cohorts still purchase the reformed house despite without purchase subsidy. They just purchased the houses slightly later than

[12]We assume free conversion between consumption goods and rental apartment. Therefore, if households choose to move out of the reformed house that they were initially assigned to rent, those houses are then converted to consumption goods at no cost.

those in the benchmark economy. Therefore the trade-up demand during 2010-2020 are pervasive. In addition, when there is no purchase subsidy, the government is able to levy a lower payroll tax to balance the budget. This positive income effect tends to trigger a stronger demand for commercial houses among younger cohorts than those in the benchmark economy. Housing prices tend to be the lowest in the scenario when purchasing reformed house is not allowed. In this case, not only the demand for reformed house among initial cohorts is absent, but also the trade-up motive is missing. The younger cohorts are also unlikely to afford a commercial house. After year 2020, housing price evolves in a similar pattern among all scenarios. This suggests that the purchase of reformed house and trade-up for commercial houses are mostly completed before 2020. The demand for new houses are mainly from young cohorts after 2020.

In Figure 4, we compare the evolution of ownership rate among different groups of households. The ownership rate of reformed houses peaks earliest in the benchmark economy, but its value is less than that without purchase subsidy. This suggests less trade-up take place than the benchmark economy when initial cohorts are not given purchase subsidy. Ownership rate among skilled is strictly higher than those among unskilled but both follow very similar pattern over time. The ownership rate of commercial house tends to be increasing over time in all scenarios. It is the lowest in the scenario without purchase subsidy to reformed houses.

6 Welfare Effects of Housing Reforms v.s. Pension Reforms

In this section, we compare the roles of housing privatization with social security for intergenerational transfer in the economy with temporarily fast growing wage income growth. To this end, we construct two counterfactual economies without housing purchase subsidy for initial cohorts, but with alternative social security systems that serve as intergenerational transfer from the later generations to earlier generations.

The first counterfactual economy serves to highlight the different distributive effects of these two types of redistributive policies across generations that were alive in 1994, while the second one emphasizes the different distributive effects of these two types of redistributive policies across cohorts than enter into the labor market since 1994.

We use consumption equivalent variation (CEV) to measure the welfare effects of a particular redistributive policy. Specifically, CEV is defined as the uniform percentage change in expected consumption in each period over the remainder of an individual's lifetime that makes the individual from cohort s indifferent between the benchmark and the counterfactual scenario.

$$E\Big[u(c_s^B, h_s^B) + \sum_{j=1}^{J-s}\big[u(c_{s+j}^B, h_{s+j}^B)\big]\Big] = E\Big[u\big(c_s^C(1+\frac{CEV}{100}), h_s^C\big) + \sum_{j=1}^{J-s}\big[u\big(c_{s+j}^C(1+\frac{CEV}{100}), h_{s+j}^C\big)\big]\Big]$$

6.1 Exemption of Payroll Tax for Initial Cohorts

Our first counterfactual economy resembles the U.S. experience using social security as intergenerational transfer. The government lets the social security tax be zero among those working-age cohorts born before 1994, while the pension replacement is kept at 0.6 among all generations.

$$\tau_{st,j} = \begin{cases} 0, \text{ if } & t-j < 1994 \\ \tau_s, \text{ if } & t-j >= 1994 \end{cases}$$

τ_s is set to balance the government's intertemporal budget constraint:

$$\sum_{t=1}\frac{1}{(1+r)^t}\left[\sum_{j=1}^{J}\sum_i\int d\rho_t h_g^i(1-\mathbb{I}_{\text{buy}_{t,j}^g(y,z)})d\mu_{t-1,j-1}^{i,rg}(y,z) + \sum_{j=J_w+1}^{J}\sum_i b_{t,j}^i\mu_{t,j}^i\right]$$
$$= B_0 + \sum_{t=1}\frac{1}{(1+r)^t}\sum_{j=1}^{J_W}\sum_i \tau_s w_t^i\mu_{t,j}^i,$$

In Figure 5 we compare payroll tax obtained from pay-as-you-go pension scheme

in the benchmark economy with the single payroll tax obtained above. It is shown that after 2020s, working-population are subjected to lower payroll tax in the pension reform.

Figure 6 compares the welfare among those initial cohorts between pension and housing reform relative to an economy without housing purchase subsidy. A negative number implies the benchmark economy without subsidy is better off. The results suggest that younger cohorts benefit from pension reform but suffer from housing reform, visa versa for older cohorts.

In Table 5 we focus on newborn cohorts. Till 2020s, the new born suffer in both reforms, and suffer less from housing reforms during 2004-2013, but more in other years. After 2020s, pension reform benefits newborn. Eventually, pension reform benefits newborn more than housing reforms.

6.2 Increasing Replacement Rates for Initial Cohorts

To further compare the effects of housing privatization with those of social security on cohorts born after 1994, in the following exercise, we construct a counterfactual economy, in which the age-specific replacement rate to match the average utility level of each given cohorts in the benchmark economy. Specifically, for those born before 1994, we choose three possible values for pension replacement rate so that the average welfare among young-age, middle-age and old-age individuals in 1994 match their benchmark counterparts. The replacement rate is kept at 0.6 for all those cohorts born after 1994. The purpose is to have a pension reform that makes those cohorts born before 1994 be as good as those in the benchmark economy with housing reform. We then evaluate how such pension reform affects cohorts born after 1994, and compare the outcome with the benchmark economy.

Figure 7 shows that in such counterfactual economy, the payroll tax paid by the cohorts born after 1994 is uniformly higher than their counterparts in the benchmark economy. In particular, cohorts born after 2020 have to bear a heavy tax burden,

despite that their wage income growth slow down. Figure 8 shows that house prices in our benchmark economy are significantly higher than in the economy with social security reforms until 2015, after which these two economies share similar house price growth rates.

In Figure 9 we compare the ownership with the benchmark counterpart. Without housing reform, households own houses later, but the peak level is higher than the benchmark counterpart. This is likely due to less trade-up as discussed in the paragraph below. The results suggest that fewer households trade-up their reformed houses in the economy with pension reform, and thus the ownership rate of commercial house is also lower than the benchmark counterpart.

In Figure 10 we plot the consumption equivalence so that the newborn cohort after 1994 in the economy with pension reform obtains the same life-time utility as the benchmark counterpart. A positive number indicates newborn is better off in the economy with pension reform. Till 2020s, the new born is better off in the housing reform than pension reform except during early 2000s when trade-up is prevalent in the benchmark economy. The contrary is true in other years. After 2020s, the newborn is strictly better off from housing reform. Our results suggest that pension reform benefit newborn over time and unskilled are benefited more than the skilled counterparts. The largest benefits are found in early years when trade-up are prominent. The reason that pension reform makes newborn better off than housing reforms is likely due to a lower income tax rate and housing prices.

7 Conclusion

Many emerging economies experienced fast growing but eventually declining wage income growth, which makes intergenerational transfer from future generations to the initial generations challenging via conventional redistributive policies such as social security. This paper uses China's housing reforms in the 1990s as a policy experiment to

show that in such economies housing purchase subsidy to initial cohorts when housing privatization starts can serve as an alternative intergenerational redistributive scheme. By developing a quantitative general equilibrium model and calibrating it to Chinese economy, we find that housing purchase subsidy to these initial cohorts facilitates them, especially the skilled workers, to become homeowners, who later on reap the capital gain from selling their houses to future generations. Our welfare analysis suggests that for a majority of future generations, housing purchase subsidy is more desirable than an alternative social security reform that redistribute from future generations to the initial ones, as the latter involves high social security tax burdens for those generations experiencing slowing down in wage growth. This is despite the fact that cohorts born in the 1980s would prefer social security as intergenerational transfer to avoid higher housing prices caused by housing purchase subsidy to initial cohorts. Our findings provides a concrete step towards future research on how a combination of housing purchase subsidy to earlier generations with social security reforms can jointly redistribute from future towards current generations for economies that experience fast growing wage incomes during their economic transition.

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Table 1: Summary of Model Parameters

Description	Parameter	Value	Sources
Depreciation rate	δ_h	0.02	OECD estimates
Reformed H rental discount	d	0.2	Wang (2012)
Replacement rate	θ	0.6	Song et. al (2015)
Discount Factor	β	0.90	standard
Minimum Down Payment Ratio	γ	0.3	government policy
Interest Rate for Savings	r	0.02	Government policy
Mortgage Interest Rate	r_m	0.06	Government policy
Land share	α	0.7	Wang, Chan and Xu (2012)
TFP growth rate	g_w	0.0713	urban wage growth 1994-2050
Conditional survival prob	$\{\psi\}_j$		Imrohoroglu and Zhao (2018)
Life-cycle income profile	$\{\varepsilon\}_j$		He et.al. (2018)
Skill premium	λ^i	{1,1.79}	Population census
Initial gov assets	B_0	5.3	10-percent income tax rate
Minimum reformed house Size	h_g^1	0.8	ownership of gov. house in 2002[a]
Minimum commercial house Size	h^1	2.4	ownership of com. house in 2009[a]
Minimum rental apartment size	h_a^1	1.6	average house size of owners to renters[b]
Initial TFP level	A_1	1.3	price-to-income ratio in 2003[c]
Initial land supply	ℓ_0	98.2	normalized initial housing price
Weight on c	ϕ	0.35	expenditure share on rents[b]
Bequest motive	ι	10.0	median NW at age 75 to median NW at age 50[b]
Floor of housing	$\underline{h}$	0.08	average earnings of owners to renters[b]
Owning premium	$\overline{\zeta}$	2.3	ownership rate in 2004[a]
Commercial house Size	$\{h^i\}$		ratio of net housing wealth to net worth[b]
Land supply	ℓ_t		annual housing price growth rate[c]

[a] Urban Household Survey; [b] China Household Finance Survey; [c] Fang et.al. (2016).

Table 2: Targeted Moments in the Calibration

Description	Data	Model
Price-to-rent ratio	15	15
Price-to-income ratio in 2006	8.4	8.4
Expenditure share on rents	0.2	0.2
Median NW at age 75 to median NW at age 50*	0.81	0.80
Ownership rate in 2004	0.85	0.85
Average earnings of owners to renters*	2.1	2.1
Average house size of owners to renters*	1.5	1.5
Ratio of net housing wealth to net worth: 10 th percentile*	0.61	0.63
Ratio of net housing wealth to net worth: 50 th percentile*	0.93	0.93
Ratio of net housing wealth to net worth: 90 th percentile*	1	1

*Final equilibrium.

Table 3: Model Fit: Distribution of the Overall Ownership rate

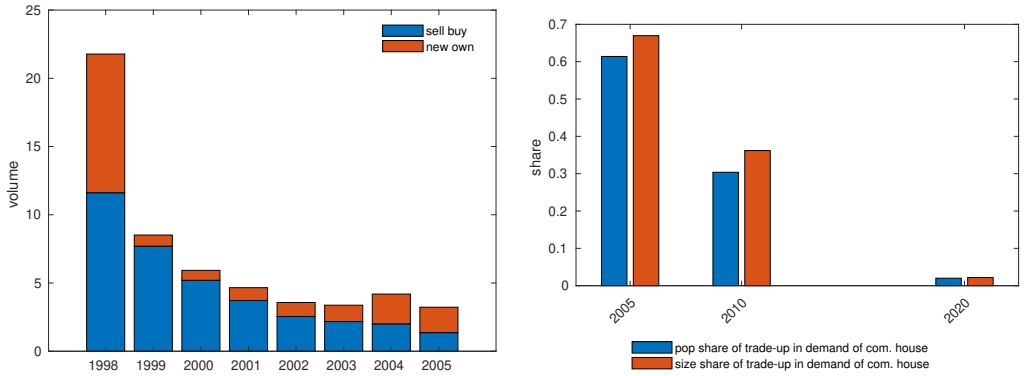
	2002		2009	
	Data	Model	Data	Model
21-30	0.736	0.107	0.819	0.246
31-40	0.827	0.759	0.906	0.737
41-50	0.813	0.903	0.897	0.980
51-60	0.824	0.969	0.907	0.995
61-70	0.862	0.994	0.926	0.995
	By edu group			
unskilled	0.750	0.769	0.861	0.782
skilled	0.839	0.799	0.910	0.855
Overall	0.820	0.758	0.903	0.792

Table 4: Model Fit: Distribution of Government House Ownership rate

	2002		2009	
	Data	Model	Data	Model
21-30	0.531	0.437	0.192	0.203
31-40	0.684	0.733	0.325	0.345
41-50	0.730	0.799	0.488	0.488
51-60	0.745	0.801	0.586	0.566
61-70	0.783	0.890	0.641	0.710
	By edu group			
unskilled	0.672	0.744	0.523	0.554
skilled	0.729	0.705	0.496	0.397
Overall	0.7820	0.717	0.491	0.441

Table 5: Welfare Effects of Housing Reforms v.s. Pension Reform For Cohorts born after 1994

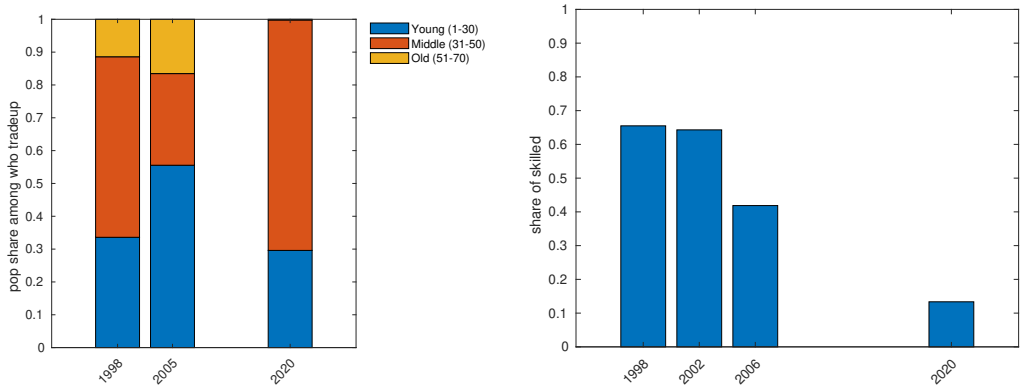
	Unskilled		Skilled	
	Housing reform	Pension reform	Housing reform	Pension reform
1994-2003	-0.14	-0.05	-0.12	-0.06
2004-2013	-0.05	-0.14	-0.06	-0.13
2014-2023	-0.07	-0.04	-0.07	-0.04
2024-2033	-0.00	0.03	-0.01	0.03
2034-2050	0.04	0.10	0.03	0.09



(a) 1998-2005 (b) selective years

Figure 1: Trade-up v.s. Newly Owned

Notes:



(a) age group (b) education group

Figure 2: Demographics of Trade-up

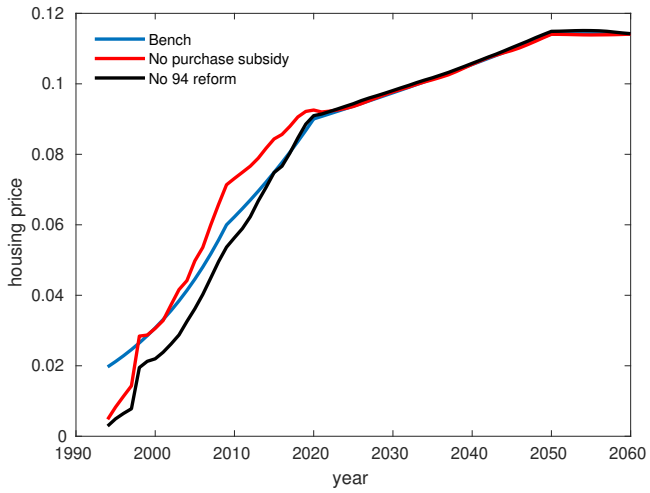
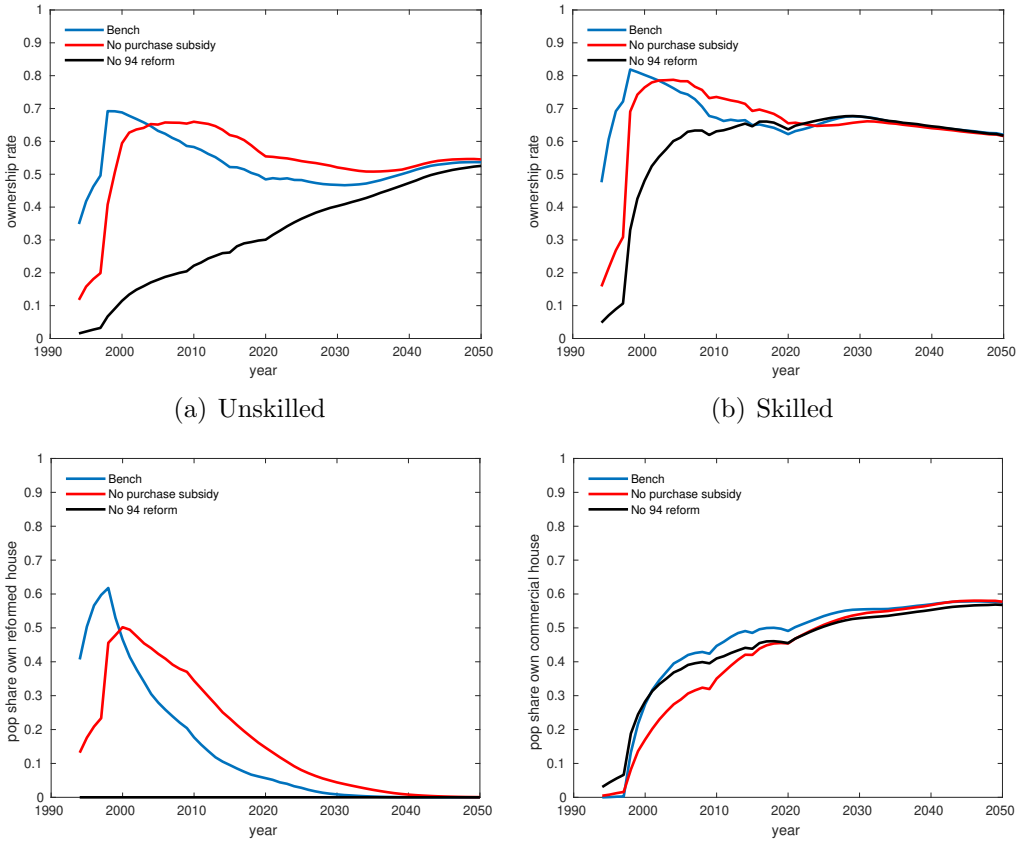


Figure 3: The Dynamics of House Prices under Different Scenarios



(a) Unskilled

(b) Skilled

(c) Reformed House

(d) Commercial House

Figure 4: Home Ownership Rate under Different Scenarios

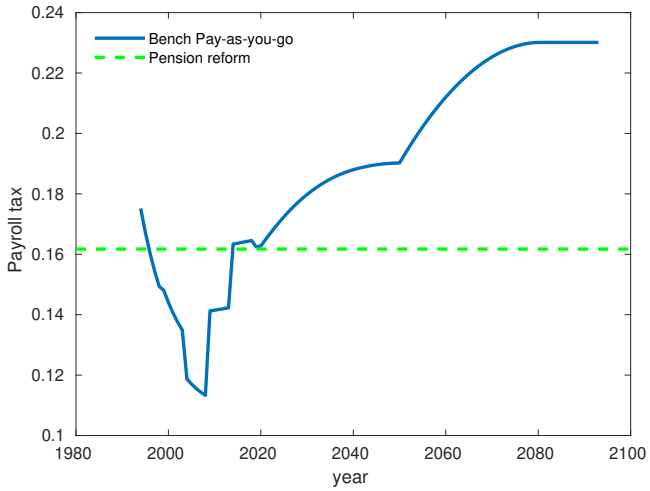
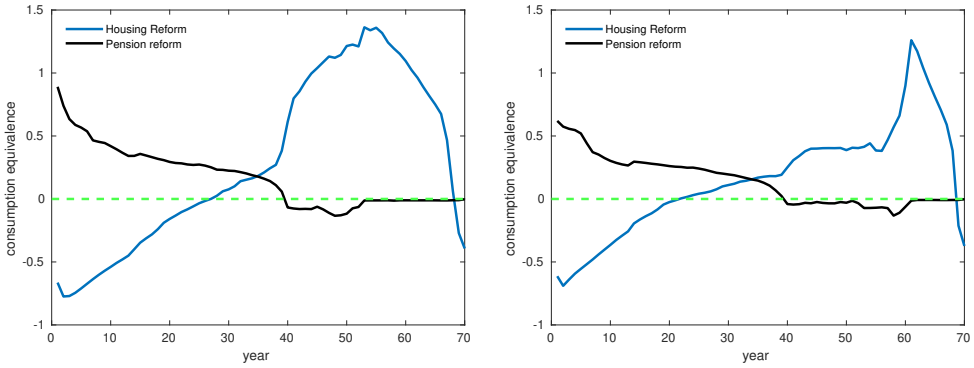


Figure 5: Payroll Tax Rates in the Benchmark Economy and the Economy with Counterfactual Payroll Tax for Initial Cohorts



(a) Unskilled (b) Skilled

Figure 6: Welfare Effects of Housing Reform vs. Social Security Reform For Initial Cohorts

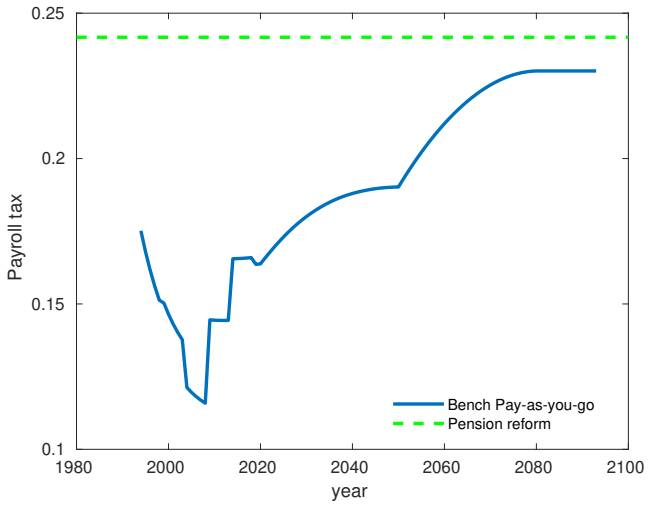


Figure 7: Payroll Tax Rates in the Benchmark Economy vs. the Economy with Counterfactual Replacement Rates for Initial Cohorts

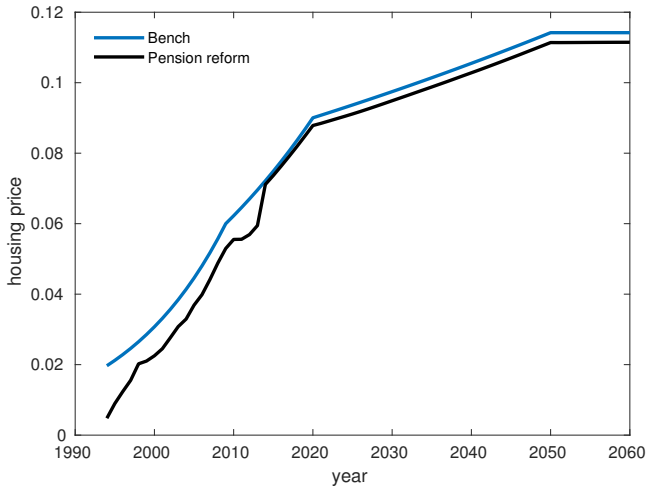
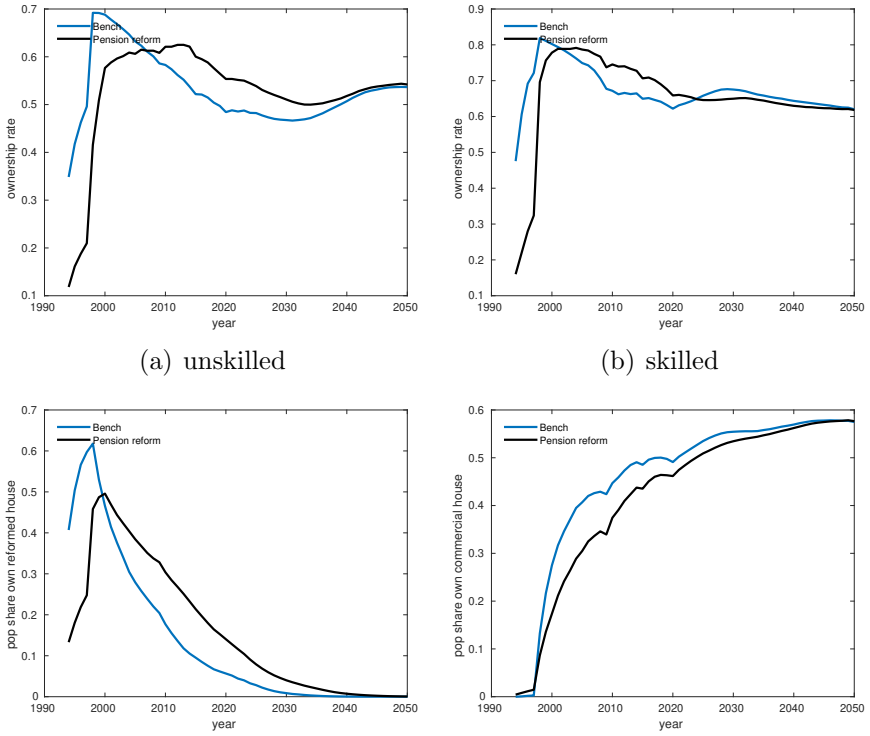


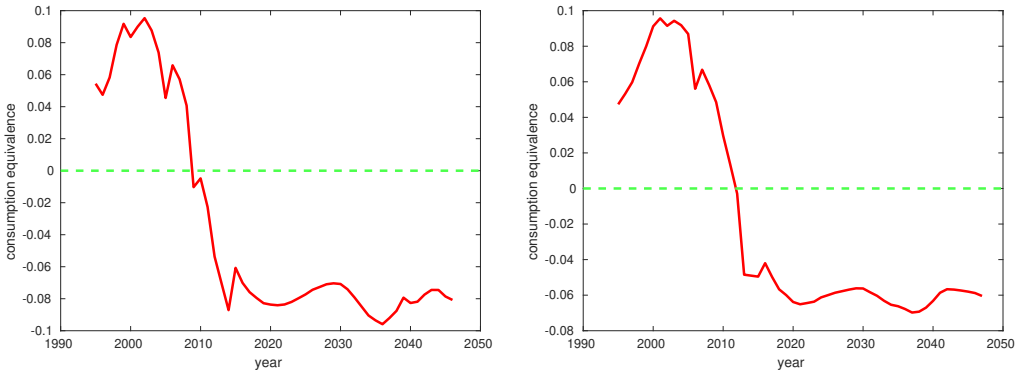
Figure 8: House Prices in the Benchmark Economy vs. the Economy with Counterfactual Replacement Rates for Initial Cohorts



(a) unskilled (b) skilled

(c) reformed house (d) commercial house

Figure 9: Ownership: Benchmark v.s. Pension Reform



(a) unskilled (b) skilled

Figure 10: Welfare relative to benchmark among those born after 1994

A Appendix

A.1 Proofs of Propositions and Lemmas

In this section, we prove the propositions in the main text.

Proof of Proposition 1: The strategy of the proof is to show that the housing subsidy policies (11) is consistent with the planner's allocation (5) , (4) and (7) . Plugging (11) into (10) , yields

$$\begin{aligned} c_t^y &= \frac{w_t}{1+\alpha+\beta-\alpha\varsigma_t} \\ &= \lambda^{-1}(\phi R)^t \\ c_{t+1}^o &= \frac{\beta R w_t}{1+\alpha+\beta-\alpha\varsigma_t} \\ &= \beta R \lambda^{-1}(\phi R)^t \\ \left(p_t - \frac{p_{t+1}}{R}\right) h_t &= \alpha\lambda^{-1}(\phi R)^t \end{aligned}$$

which is the same as the first-best allocation (5) , (4), (7) .

Proof of Corollary 2: Plugging $h_t = \overline{h}$ into (10) and reordering, we obtain the law of motion for equilibrium housing prices

$$p_t = \frac{w_t}{\overline{h}\,(1+\alpha+\beta-\alpha\varsigma_t)} + \frac{p_{t+1}}{R} \tag{40}$$

Plugging (11) into (40) , we have

$$p_t = \frac{\alpha(1+g)w_t}{\overline{h}\lambda w_1} + \frac{p_{t+1}}{R} \text{ for } t \geq 1 \tag{41}$$

Forward iterating (41) , we get

$$\widehat{p_t} = \frac{\alpha}{\overline{h}\lambda}\frac{R}{R-(1+g)}, \text{ for } t \geq 1. \tag{42}$$

Hence, p_t grows at a constant rate $(1+g)$ for $t \geq 1$.

Proof of Corollary 3: Equating (4) for the planner's solution and (15) for the decentralized economy, we have

$$\lambda^{-1} = \frac{\widehat{w}_0 - \left(\widehat{p}_0 - \widehat{p}_1 \left(1+g\right)/R\right)\overline{h}}{1+\beta}$$

Hence,

$$\widehat{p}_0 = \left[\widehat{w}_0 - \lambda^{-1}\left(1+\beta\right)\right]/\overline{h} + \widehat{p}_1 \left(1+g\right)/R$$

Plugging (42) into the above equation, we obtain (16).

A.2 Algorithm

We have two algorithms here. One algorithm is to solve the pre-reform steady state. The other is to solve jointly the steady state after 2013 and the transition path from some initial state of the economy to the steady state after 2013. For example, in the benchmark, we need to use transition path algorithm twice. Firstly, we solve the transition path from the pre-reform to the steady state after 2013. Secondly, we use the economy in year 1998 as the initial state and solve for the transition path again.

Pre-reform steady state:

To solve for the pre-reform steady state, we assume households taking the assigned housing service as given, and government plans for the life cycle consumption profile across permanent efficiency shock groups.

Given government policy, τ, b, and interest rate r and wage rate w in pre-reform steady state.

1. Parameterize the model, and calculate the density of retired workers in the population, μ_t, $t = J_{w+1}, ..., J$.

2. Given the government expenditure, $\bar{g}$, guess the pre-reform rental rate R_c;

3. Given the policy function (analytical) of households,

(a) guess the initial bequest.

(b) simulate the optimal path for consumption and saving for the new born generation by forward induction given the initial bequest.

(c) aggregate household's decision, and calculate the bequest leftover on the path.

(d) update the guess of the initial bequest until it converges. (Guass-Seidel method)

4. Aggregate government's tax revenue, renting revenue, and pension expenditure. Check whether government's intertemporal budget is balanced, and update the guess of the rental rate R_c.

5. Check whether R_c match the calibration target, if not, update $\bar{g}$. Derive the allocation of $\overline{H}$.

Transition path:

We need to find the equilibrium path of housing price and the tax rate in the final steady state. Assume we know the state of the economy at $t = 1$, and the economy reaches the final steady state after some periods T. (T is larger than the three times of the maximum lifespan.)

To solve the transition path, we have the following steps:

Given government policy, $\{\tau_t, b_t\}_{t=1}^{\infty}$, discount housing policy, land supply, $\{H_t\}_{t=1}^{\infty}$, and interest rate $\{r_t\}_{t=1}^{\infty}$ and wage rate $\{w_t\}_{t=1}^{\infty}$ on the path, and the initial distribution of household on the state space (initial state).

1. Choose the number of transition periods T.

2. De-trend the economy by the time T variables.

3. Provide an initial guess for tax rate in the steady state, τ.

4. Given all policy variables, solve for the final steady state housing price that clear the housing market by bisection method.

5. Provide an initial guess for housing price on the path, $\{p_t\}_{t=0}^{T}$, and solve household's problem backwards:
   At period t, compute the value functions and policy functions for the new born at t, which has a perfect foresight.

6. Compute the transition path: Compute the optimal path for consumption, housing, and saving by forward induction given the initial state in period $t = 1$. In initial state, households receive assignment of public housing from the government, $\bar{H}$. The bequests for period t newborn are collected from the household passing away at period t.

7. Aggregate household's net housing demand each period. Check if housing market in each period is clear. If not, update the guess of $\{p_t\}_{t=1}^{T}$, and go to step 5.

8. Aggregate government tax revenue, housing sale revenue, pension expenditure on the path. Combined with government's deficit/surplus in the steady state, check whether government's intertemporal budget is balanced. If not, update the guess of τ, and go to step 4.

9. Check whether p_T is close enough with the final steady state housing market price. If not, increase T, and go to step 2.

Long run equilibrium:

The price adjustment step. Because it is the long run equilibrium

$$I_t = \delta H$$

$$p_0 \Longrightarrow H(p_0) + \widetilde{H}(p_0) \Longrightarrow I_t(p_0) \Longrightarrow p_1 = 0.2 * \frac{1}{\alpha}\left(\frac{I_t(p_0)}{L}\right)^{\frac{1-\alpha}{\alpha}} + 0.8 * p_0$$

A.3 Detrending

This section describe how we detrend the individual and aggregate variables. Since the initial steady state and final steady state have different trend growth rates for productivity and population, we detrend the initial steady state and transitional paths separately. After we solve for the initial steady state, we back out the original values for the initial steady state and detrend it with the growth rate of the final steady state as the initial values in the transition path.

A.4 Transitional path and final steady state

Denote g as the balance-growth rate of the wage rate, g_p and g_I as the growth rate of housing prices and housing investment at the balance growth rate in the final steady state. Note that according to (38) , since there is no secular growth of land at the steady state, the growth rate of housing investment at balance growth path is

$$1 + g_I = [(1 + g)(1 + n)]^{\alpha}$$

or

$$g_I = \alpha(g + n)$$

Then, according to (39) , the growth rate of housing price at balance growth path is

$$1 + g_p = [(1 + g)(1 + n)]^{1-\alpha}$$

or

$$g_p = (1 - \alpha)(g + n)$$

For individual housing growth, notice that at balance growth path

$$\sum_{j=1}^{J} N_j \sum_{i} h_j^i = H$$

Therefore, the balance growth rate of h_j^i, denoted as g_h is simply as

$$g_h = g_I - n$$

Note that

$$1 + g = (1 + g_h)(1 + g_p)$$

For individual variables, we detrend as follows

$$\begin{aligned}
\widehat{c}_{t,j} &= c_{t,j}(1+g)^{T+j-t-1} \\
\widehat{a}_{t+1,j+1} &= a_{t+1,j+1}(1+g)^{T+j-t-1} \\
\widehat{h}_{t+1,j+1} &= h_{t+1,j+1}(1+g_h)^{T+j-t-1} \\
\widehat{s}_{t,j} &= s_{t,j}(1+g_h)^{T+j-t-1} \\
\widehat{\widetilde{h}}_{t+1,j+1} &= \widetilde{h}_{t+1,j+1}(1+g_h)^{T+j-t-1} \\
\widehat{b}_{t,j} &= b_{t,j}(1+g)^{T+j-t-1} \\
\widehat{y}_{t,j} &= y_{t,j}(1+g)^{T+j-t-1}
\end{aligned}$$

For aggregate variables, we detrend as follows

$$\begin{aligned}
\widehat{w}_t &= w_t(1+g)^{T-t} \\
\widehat{N}_{et} &= N_{et}(1+n)^{T-t} \\
\widehat{N}_{ht} &= N_{ht}(1+n)^{T-t} \\
\widehat{p}_t &= p_t(1+g_p)^{T-t} \\
\widehat{p}^g_{t,j} &= p^g_{t,j}(1+g_p)^{T-t} \\
\widehat{I}_t &= I_t(1+g_I)^{T-t} \\
\widehat{L}_t &= L_t \\
\widehat{H}_t &= H_t\,(1+g_I)^{T-t} \\
\widehat{\rho}_h &= \rho_h(1+g_p)^{T-t} \\
\widehat{R}_{c,t} &= R_{c,t}(1+g_p)^{T-t} \\
\widehat{\tilde{H}} &= \tilde{H}'\,(1+g_I)^{T-t} \\
\widehat{Y}_t &= Y_t\,[(1+g)\,(1+n)]^{T-t} \\
\widehat{z}_t &= z_t(1+g)^{T-t}
\end{aligned}$$

Finally, we assume that the operating cost ψ grows at a constant rate g_I along transition and at the final steady state. Therefore we have the detrended version of the above

equations as

$$
\begin{aligned}
\widehat{c}^{i}_{t,j} &= \widehat{y}^{i}_{j,t} + \widehat{a}^{i}_{t,j} - (q\widehat{a}^{i}_{t+1,j+1} + \widehat{R}_c\widehat{s}^{i}_{t,j}(1+g_p)^{j-1}) \\
\widehat{s}^{i}_{t,j} &= \widehat{\widetilde{h}}^{i}_{1,1+j-t}; \widehat{s}^{i}_{t,j} = \widehat{h}^{i}_{t+1,j+1} \\
\widehat{y}^{i}_{j,t} &= \begin{cases} (1-\tau_t-\tau^{ss}_t)\widehat{w}_t(1+g)^{j-1}e^{i}_{t,j} \text{ for } j \leq J_w \\ \widehat{b}^{i}_{t,j} \text{ for } j > J_w \end{cases} \\
\widehat{c}^{i}_{t,j} &= \widehat{y}^{i}_{j,t} + \widehat{a}^{i}_{t,j} - (q\widehat{a}^{i}_{t+1,j+1} + \widehat{\rho}_{h,t}\widehat{\widetilde{h}}^{i\prime}_{t+1,j+1}(1+g_p)^{j-1}), \\
\widehat{c}^{i}_{t,j} &= \widehat{y}^{i}_{j,t} + \widehat{a}^{i}_{t,j} - (q\widehat{a}^{i}_{t+1,j+1} + \widehat{p}^{g}_{t,j}\widehat{h}^{i}_{t+1,j+1}(1+g_p)^{j-1}) \\
\widehat{c}^{i}_{t,j} &= \widehat{y}^{i}_{j,t} + \widehat{a}^{i}_{t,j} - (q\widehat{a}^{i}_{t+1,j+1} + \widehat{p}_t\widehat{h}^{i}_{t+1,j+1}(1+g_p)^{j-1}), \\
\widehat{a}^{i}_{t+1,j+1} &\geq -\Upsilon\widehat{p}^{g}_{t,j}\widehat{h}^{i}_{t+1,j+1}(1+g_p)^{j-1} \\
\widehat{a}^{i,n}_{t,j} &= \widehat{a}^{i}_{t,j} + (1-\delta_h-\kappa)\widehat{p}_t\widehat{h}^{i}_{t,j}(1+g_p)^{j-1} \\
\widehat{\rho}_{h,t}(\Phi) &= \psi + \widehat{p}_t(\Phi) - \frac{(1-\delta_h)(1+g_p)}{1+r}E_{\Phi'}[\widehat{p}_{t+1}(\Phi)|\Phi] \\
\widehat{Y}_t &= \widehat{z}_t\widehat{N}_{ct} \\
\widehat{I}_t &= \left(\widehat{z}_t\widehat{N}_{ht}\right)^{\alpha}\left(\widehat{L}_t\right)^{1-\alpha} \\
\widehat{I}_t &= (\alpha\widehat{p}_t)^{\frac{\alpha}{1-\alpha}}\widehat{L}_t \\
\widehat{N}_{ht} &= (\alpha\widehat{p}_t)^{\frac{1}{1-\alpha}}\frac{L_t}{\widehat{\widetilde{z}}_t} \\
(1+g_I)\left(\widehat{\widetilde{H}}_{t+1}+\widehat{H}_{t+1}\right) &= (1-\delta_h)\left(\widehat{\widetilde{H}}_t+\widehat{H}_t\right)+\widehat{I}_t \\
(1+g_{Ipre})(h_{pre}) &= (1-\delta_h)(h_{pre})+\widehat{I}_t \\
\widehat{A}_0 &= \sum_{t=1}^{\infty}R^{-t}\begin{bmatrix} \sum_{j=t+1}^{J}\mu_{j,t}\int_{I\in bg}\left(\widehat{p}_t-\widehat{p}^{g}_{t,j}\right)\widehat{h}^{i}_{1,1+j-t}/\left((1+g)^{T-t}(1+g_h)^{j-1}\right)\Phi_t(\\ \sum_{j=t+1}^{J}\mu_{j,t}\int_{I\in rg}\left(\widehat{\rho}_t-\widehat{R}_{c,t}\right)\widehat{h}^{i}_{1,1+j-t}/\left((1+g)^{T-t}(1+g_h)^{j-1}\right)\Phi_t \\ \sum_{j=1}^{t}\mu_{j,t}\int_{I\in rg}\left(\widehat{\rho}_t-\widehat{R}_{c,t}\right)\widehat{h}^{i}_{1,j}/\left((1+g)^{T-t}(1+g_h)^{j-1}\right)\Phi_t(s) \\ -\sum_{j=1}^{J_w}\mu_{j,t}\tau_t\widehat{w}_t/(1+g)^{T-t}\int_s e\Phi_t(s)ds \end{bmatrix} \\
\widehat{b}^{i}_{t,j} &= \theta(0.6\widehat{\widetilde{y}}^{i}_{t-j+J_w,J_w}+0.4\widehat{\widetilde{y}}_t(1+g)^{j-1}) \\
\widehat{\widetilde{y}}^{i}_{t-j+J_w,J_w} &= \widehat{w}_{t-j+J_w}(1+g)^{J_w-1}e_{t-j+J_w,J_w} \\
\widehat{\widetilde{y}}_t &= \frac{\sum_{j=1}^{J_w}\sum_i\mu_{j,t}\widehat{w}_te^{i}_{t,j}}{\sum_{j=1}^{J_w}\mu_{j,t}} \\
\tau^{ss}_t &= \frac{\sum_{j=1}^{J_w}\sum_i\mu_{j,t}\widehat{b}^{i}_{t,j}(1+g)^{1-j}}{\sum_{j=1}^{J_w}\sum_i\mu_{j,t}\widehat{w}_te^{i}_{t,j}}
\end{aligned}
$$

A.5 Pre-reform Steady State

Denote g_0 as the balance-growth rate of the wage rate, g_{p0} and g_{I_0} as the growth rate of housing prices and housing investment at the balance growth rate in the initial steady state. Similar to the final steady state, we have

$$\begin{aligned} 1+g_{I0} &= [(1+g_0)(1+n_0)]^{\alpha} \\ 1+g_{p0} &= [(1+g_0)(1+n_0)]^{1-\alpha} \\ g_{h0} &= g_{I0}-n_0 \\ 1+g &= (1+g_{h0})(1+g_{p0}) \end{aligned}$$

For individual variables, we detrend as follows

$$\begin{aligned} \widehat{c}_{t,j} &= c_{t,j}(1+g_0)^{-t} \\ \widehat{a}_{t+1,j+1} &= a_{t+1,j+1}(1+g_0)^{-t} \\ \widehat{s}_{t,j} &= s_{t,j}(1+g_{h0})^{-t} \\ \widehat{y}_{t,j} &= y_{t,j}(1+g_0)^{-t} \end{aligned}$$

For aggregate variables, we detrend as follows

$$
\begin{aligned}
\widehat{w}_t &= w_t(1+g_0)^{-t} \\
\widehat{N}_{ht} &= N_{ht}(1+n_0)^{-t} \\
\widehat{I}_t &= I_t(1+g_{I0})^{-t} \\
\widehat{L}_t &= L_t \\
\widehat{H}_t &= H_t\left(1+g_{I0}\right)^{-t} \\
\widehat{H}_t^D &= H_t^D\left(1+g_{I0}\right)^{-t} \\
\widehat{R}_{c,t} &= R_{c,t}(1+g_{p0})^{-t} \\
\widehat{C}_t &= C_t\left[\left(1+g_0\right)\left(1+n_0\right)\right]^{-t} \\
\widehat{Y}_t &= Y_t\left[\left(1+g_0\right)\left(1+n_0\right)\right]^{-t} \\
\widehat{z}_t &= z_t(1+g_0)^{-t}
\end{aligned}
$$