The Growth Dividend and Excess Interest

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Abstract

The deficit as a share of GDP increasingly exceeds each year's change in the debt-to-GDP ratio. The debt ratio change equals the deficit plus the *growth dividend*: the amount by which GDP growth shrinks the debt ratio, equal to last year's debt ratio times $\frac{-y}{1+y}$ where y is the nominal GDP growth rate. The growth dividend has more than doubled since the Great Recession because the debt ratio has more than doubled. Adding a growth dividend row to CBO's main budget table enables readers to exactly decompose America's debt ratio path. Current debt increases the debt ratio each year by the amount that interest payments exceed the growth dividend. This *excess interest* remains negative in CBO baseline's projection, implying that America's current debt is sustainable despite currently high interest payments. Instead, large ongoing primary deficits – the current and future imbalance between tax revenue and non-interest spending – entirely drive America's unsustainable debt ratio path. Focusing on the primary deficit provides a good guide to how the debt ratio is projected to change, while the deficit does not.

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1 Introduction

Mirroring numerous policymakers and commentators of all partisan and non-partisan backgrounds, the Wall Street Journal Editorial Board recently used the deficit to try to explain the February 2024 Congressional Budget Office's (CBO) ten-year projection of America's debt-to-GDP ratio:

> Debt as a share of GDP will rise to 116% in 2034 from 97.3%...Revenues are expected to average 17.8% of GDP through 2034...The problem is that spending over the next decade will average 23.5% of GDP. (Wall Street Journal Editorial Board, 2024)

Unfortunately, this common characterization of the debt ratio change does not add up.

Table 1 reprints CBO's June 2024 update to its February budget outlook (CBO, 2024c). Similar to the above quote, this latest projection shows the debt ratio rising from 99% of GDP in 2024 to 122% in 2034, which is an average annual change of 2.3% of GDP per year. Also similar to the above quote, the deficit – the difference between revenues and spending – averages 6.3% of GDP per year 2025-2034, which massively exceeds the 2.3% annual change in the debt ratio.

Figure 1 shows that the amount by which the deficit exceeds the debt ratio change has grown over time. Panel A chooses two example years: 2005 using historical data from CBO (2024b) and 2024 using the June 2024 CBO outlook. In 2005, the deficit was 2.5% of GDP while debt rose by only 0.1% of GDP, a difference of 2.4% of GDP. In 2024, the CBO projects that the deficit will be 6.7% of GDP but that debt will rise by only 1.7% of GDP, a difference of 5.0% of GDP. Panel B presents averages over the 2003-2007 period and over the 2025-2034 period. The deficit exceeds the debt ratio change in both periods, but the excess is projected to be twice as large over the next decade as it was before the Great Recession.

This article explains why. Aside from usually minor miscellaneous transactions, each year's change in the debt ratio equals the deficit plus what Bohn (2008) termed the *growth dividend*: the amount by which nominal GDP growth reduces the debt ratio. The growth dividend equals the prior year's debt ratio times $\frac{-y}{1+y}$ where y is the nominal GDP growth rate. The growth dividend has more than doubled as the debt ratio has more than doubled, which explains why the deficit increasingly exceeds the change in the debt ratio.

To see why the growth dividend is key to decomposing the debt ratio change, imagine a scenario with a $10 trillion debt at the start of the year, $10 trillion GDP in the prior year, a zero deficit, and 4% nominal GDP growth. The debt numerator would remain fixed over the year at $10 trillion. However, the GDP denominator would rise to $10.4

trillion. As a result, the debt-to-GDP ratio would decline from 100% to 96.2% ($= 10/10.4$), a decline of 3.8% of GDP which exactly equals $100\% \times \frac{-.04}{1+.04}$.

To see why the growth dividend has more than doubled now that the debt ratio has more than doubled, let us repeat the prior scenario except let us start the year with only $5 trillion of debt. The debt numerator would remain fixed over the year at $5 trillion. However, the GDP denominator would rise to $10.4 trillion. As a result, the debt-to-GDP ratio would decline from 50% to 48.1% ($= 5/10.4$), a decline of 1.9% of GDP which exactly equals $50\% \times \frac{-.04}{1+.04}$.

The preceding examples convey the fact that both interest payments and the growth dividend are proportional to the debt-to-GDP ratio. In fact, interest payments plus the growth dividend exactly equals the amount by which past debt grows on itself.[1] Economists and budget analysts distinguish between the deficit and the primary deficit: the deficit equals revenues minus spending, while the primary deficit equals revenues minus non-interest spending. If the primary deficit were zero, the debt-to-GDP ratio would change only because we have past debt. The amount by which the debt ratio would change exactly equals the amount by which interest payments exceed the growth dividend, a quantity I call *excess interest*.[2]

Adding a growth dividend row to CBO's ten-year outlook table enables readers to exactly decompose America's unsustainable fiscal path. Excess interest remains negative in CBO's outlook, implying that the debt ratio would fall on its own if primary deficits were zero. That fact is true despite net interest payments averaging 3.7% of GDP, because the growth dividend is even larger: an average of -4.0% of GDP. Instead, the debt ratio is projected to rise purely because of large ongoing primary deficits. Excess interest in CBO's projections is negative but close to zero, so the average debt ratio change of 2.3% of GDP approximately equals the average primary deficit of 2.6% of GDP. Hence, focusing on the primary deficit rather than the deficit provides a good guide to how the debt ratio is projected to change, while the deficit does not.

Based on empircal evidence, CBO assumes that the rising debt ratio will steadily increase the interest rate relative to the growth rate, such that excess interest turns positive in 2040. The only reason that the debt ratio rises in CBO's projections before 2040 is primary deficits. If primary deficits were zero, CBO would project a declining debt ratio forever. Hence, America's unsustainable fiscal path according to CBO projections is caused entirely by large ongoing primary deficits, despite currently high interest payments.

The above facts suggest caution around recent emphasis on interest payments, such as the following quote from

[1] Importantly, by phrases like "the amount by which past debt grows on itself" and "the effect of past debt on the debt ratio", I mean the mechanical accounting effect of debt on the debt ratio over and above any effect on the primary deficit. I therefore exclude any behavioral effects of debt on the primary deficit, such as investment crowd out or political feedback (Hicks, 1937; Auerbach, 2003).

[2] Haque and Montiel (1994) use "excess interest" in a different context, comparing debt yields to capital returns.

the New York Times Editorial Board:

> Borrowing is expensive. A mounting share of federal revenue, money that could be used for the benefit of the American people, goes right back out the door in the form of interest payments to investors who purchase government bonds. Rather than collecting taxes from the wealthy, the government is paying the wealthy to borrow their money. By 2029, the government is on pace to spend more each year on interest than on national defense. (New York Times Editorial Board, 2023)

By focusing on interest payments and not excess interest, policymakers and commentators risk missing the fact that America's unsustainable fiscal path in the CBO baseline is driven entirely by large ongoing primary deficits.

The reason that "the government is on pace to spend more each year on interest than on national defense" is that the debt ratio has nearly tripled since the mid-2000s.[3] The growth dividend has mechnically also grown with the debt ratio. Indeed, just as accurately but incompletely for the debt ratio trajectory, one could write: "the government now enjoys a growth dividend greater than its spending on national defense". On net, excess interest – the amount by which past debt increases the debt ratio – has not increased since the mid-2000s. America's debt is still not building on itself, despite high interest payments.

However, the CBO baseline projection is not a risk scenario. Our tripled debt ratio means that we are three times as vulnerable to the risk that economic forces cause interest rates to exceed GDP growth rates, as they did 1980-2000. The tripled debt ratio means that any given persistent swing in interest rates relative to growth rates will cause past debt to add three times as much to our debt ratio each year as it would have before the Great Recession. Numerous papers analyze such scenarios (e.g., Ball et al. 1998; Mehrotra and Sergeyev 2021; Auerbach and Yagan 2024).

The substance of this note is already obvious to economists and budget experts (e.g., Hall and Sargent 2011; Auerbach and Gale 2023; Blanchard 2023). However, there has a been disconnect between economists and budget experts one the one hand and policymakers and commentators on the other, because the former often operate in real (inflation-adjusted) terms in continuous time while the latter operate in nominal terms in discrete time, with Bohn (2008) being a notable exception. Economists judge fiscal sustainability based on whether the debt ratio is stable, and the change in the debt ratio can be expressed primarily in real terms based on the difference between the real interest rate and the real GDP growth rate. Congress, on the other hand, operates in nominal terms: reconciliation bills have nominal dollar limits to cumulative ten-year deficits, and the debt limit is expressed as a nominal dollar amount. This

[3] In the 2003-2007 period, the debt ratio averaged 35% of GDP, interest payments averaged 1.5% of GDP, and the interest rate on government debt averaged 4.4% of GDP. Over the next decade, CBO projects that the debt ratio will average 111% of GDP and interest payments will average 3.6% of GDP, while the interest rate on government debt will average only 3.4%.

article bridges the presentational divide by adding one line – the growth dividend – to CBO's existing budget outlook and uses it to understand growing deficits and interest payments after the trippling of the U.S. debt ratio.

Section 2 decomposes debt dynamics in nominal terms. Section 3 adds a growth dividend row to CBO's budget outlook table in order to decompose debt ratio changes. Section 4 compares net interest to the growth dividend to determine whether current U.S. debt would be sustainable under zero future primary deficits. Section 5 uses hypothetical economies to explain why excess interest and not interest equals the effect of past debt on the debt ratio. Section 6 discusses sustainability risk. Section 7 relates the growth dividend and excess interest to "$r-g$" accounting and real net interest. Section 8 concludes.

2 Debt-to-GDP dynamics in nominal terms

Debt held by the public grows from year $t-1$ to year t according to the accounting identity:

$$\underbrace{DEBT_t - DEBT_{t-1}}_{\text{Debt change}} = \underbrace{\underbrace{NONINTEREST OUTLAYS_t - REVENUES_t}_{\text{Primary deficit}} + \underbrace{i_t \times DEBT_{t-1}}_{\text{Net interest}}}_{\text{Deficit}} \tag{1}$$

where i_t is the average nominal interest rate on government debt. See the appendix for the exactly true formula, which includes two additional minor terms: interest on current-year new borrowing due to new primary deficits, and "other means of financing" that affect borrowing from the public like changes in Treasury's operating cash balance.[4] All terms are nominal (i.e., not inflation adjusted) and are exactly as they appear in CBO's main budget table.

Dividing equation 1 through by GDP and rearranging terms, one obtains the following accounting identity for the change in the debt ratio as in Bohn (2008):

$$\underbrace{\frac{DEBT_t}{GDP_t} - \frac{DEBT_{t-1}}{GDP_{t-1}}}_{\text{Debt ratio change}} = \underbrace{\underbrace{\frac{NONINTEREST OUTLAYS_t}{GDP_t} - \frac{REVENUES_t}{GDP_t}}_{\text{Primary deficit}} + \underbrace{\frac{i_t \times DEBT_{t-1}}{GDP_t}}_{\text{Net interest}}}_{\text{Deficit}} + \underbrace{\frac{-y_t}{1+y_t} \times \frac{DEBT_{t-1}}{GDP_{t-1}}}_{\text{Growth dividend}} \tag{2}$$

where $y_t = GDP_t/GDP_{t-1} - 1$ is the nominal growth rate of GDP.[5] See the appendix for the derivation and for the exactly true version with two additional minor terms.

The unfamiliar term is the "growth dividend": $\frac{-y_t}{1+y_t}$ times the prior year's debt ratio. The reason that one needs the

[4] The President's Budget refers to other means of financing as "other transactions affecting borrowing from the public".

[5] I follow Bohn (2008) in using i and y to denote the nominal interest rate and GDP growth rate, while Mian et al. (2022)use R and G.

growth dividend term in the debt ratio change equation (2) and not the debt change equation (1) is that the debt ratio divides the debt by GDP, which grows by the rate of economic growth and thereby reduces the debt ratio. The growth dividend equals the percentage points of GDP by which the debt ratio would fall if the deficit were zero.

Equation (2) illuminates that past debt $DEBT_{t-1}$ appears in both the net interest term and the growth dividend term. Hence, the equation can be rearranged to separate the the $DEBT_{t-1}$ terms from the primary deficit, as in my preferred grouping of terms:

$$\underbrace{\frac{DEBT_t}{GDP_t} - \frac{DEBT_{t-1}}{GDP_{t-1}}}_{\text{Debt ratio change}} = \underbrace{\frac{NONINTERESTOUTLAYS_t}{GDP_t} - \frac{REVENUES_t}{GDP_t}}_{\text{Primary deficit}} + \underbrace{\underbrace{\frac{i_t \times DEBT_{t-1}}{GDP_t}}_{\text{Net interest}} + \underbrace{\frac{-y_t}{1+y_t} \times \frac{DEBT_{t-1}}{GDP_{t-1}}}_{\text{Growth dividend}}}_{\text{Excess interest}} \tag{3}$$

"Excess interest" – which equals interest plus the growth dividend – is the net effect of debt on the debt ratio. The equation shows that if $DEBT_{t-1}$ were zero, the debt ratio change would equal the primary deficit.

Holding fixed i_t and y_t, a rise in the debt ratio increases net interest and the growth dividend exactly in tandem. This fact can be seen especially easily when expressing net interest in terms of the prior year's debt ratio, just like the growth dividend:

$$\underbrace{\frac{DEBT_t}{GDP_t} - \frac{DEBT_{t-1}}{GDP_{t-1}}}_{\text{Debt ratio change}} = \underbrace{\frac{NONINTERESTOUTLAYS_t}{GDP_t} - \frac{REVENUES_t}{GDP_t}}_{\text{Primary deficit}} + \underbrace{\underbrace{\frac{i_t}{1+y_t} \times \frac{DEBT_{t-1}}{GDP_{t-1}}}_{\text{Net interest}} + \underbrace{\frac{-y_t}{1+y_t} \times \frac{DEBT_{t-1}}{GDP_{t-1}}}_{\text{Growth dividend}}}_{\text{Excess interest}} \tag{4}$$

For example, suppose that i_t and y_t remain fixed, and suppose that the debt ratio triples as it approximately has in the United States. Equation (4) makes clear that net interest will triple. However, equation (4) also makes clear that the growth dividend will triple. Hence, the sign of the net effect of past debt on the debt ratio will not change: the sign of excess interest is determined only by $i_t - y_t$.

3 Adding the growth dividend to CBO's budget outlook exactly decomposes each year's debt ratio change

Table 2 adds a growth dividend row to the main CBO budget outlook table reprinted in Table 1. It also reprints existing CBO rows alongside the growth dividend, in order to decompose each year's debt ratio change into the terms in equation (3). This decomposition explains the introduction's apparent puzzle of how the debt ratio is projected to rise each year this decade by less than half of the deficit.

The row Change in Debt Held by the Public equals the Primary Deficit plus Net Interest plus the Growth Dividend plus Other Means of Financing. Change in Debt Held by the Public equals year's value of Debt Held by the Public minus the prior year's value. Primary Deficit is the (negative of) the so-labeled row above. Net Interest is copied from above. Growth Dividend equals the prior year's Debt Held by the Public (i.e., the debt-to-GDP ratio) times $\frac{-y}{1+y}$, where y is CBO's assumed nominal GDP growth rate (3.8% on average 2025-2034). Finally, Other Means of Financing comes from CBO Table 1-3 and is usually near zero, and CBO projects that it goes to zero by the end of the decade.[6] Change in Debt Held by the Public exactly equals the sum of the other four rows.

For example, consider 2024, outlined in red. The deficit is projected to be 6.7% of GDP, equal to the sum of a 3.6% of GDP primary deficit and 3.1% of GDP in interest payments. However, the debt ratio is projected to increase in 2024 by only 1.7% of GDP, from 97.3% to 99.0%. The reason that the debt ratio change is so much lower than the deficit is the growth dividend. The projected growth dividend for 2024 is -5.1% of GDP, equal to the prior year's debt ratio 97.3% times $\frac{-.055}{1+.055}$.[7] Adding 6.7%, -5.1%, and 0.1% in Other Means of Financing exactly equals the 1.7% of GDP rise in the debt ratio.

The average values over the 2025-2034 period, also outlined in red, resolve the introduction's puzzle. The average deficit over the 2025-2034 period equals 6.3% of GDP, equal to the sum of an average 2.6% primary deficit and an average of 3.7% of GDP in interest payments. However, the debt ratio is projected to increase by only 2.3% of GDP per year, from 99% to 122% over the ten-year period. The reason that the average deficit exceeds the average change in the debt ratio by 4.0% of GDP is that the growth dividend averages -4.0% of GDP. The -4.0% figure approximately equals the average prior year's debt ratio of 109% times $\frac{-.038}{1+.038}$, where 3.8% is average nominal GDP growth.

[6] Other means of financing was large in 2023 because Treasury reversed extraordinary measures it undertook in 2021 in order to avoid a debt ceiling breach. Specifically, Treasury in 2021 spent down its cash reserves and suspended investments in the Thrift Savings Plan (TSP) Government Securities Investment Fund (G-Fund) for government workers' tax-preferred retirement savings accounts. In 2022 and 2023, Treasury replenished cash reserves and fully reinvested the TSP G-Fund, which required a one-time increase in debt issuance. See OMB (2023).

[7] The 2024 nominal growth rate y_{2024} of 5.5% equals the 2024 value of Gross Domestic Product divided by the 2023 value, minus one.

4 Projected excess interest implies that ongoing primary deficits entirely drive the rising debt ratio

Table 2 showed that CBO projects that net interest will be smaller than the (absolute value of the) growth dividend in every year in the ten-year outlook. On average 2025-2034, CBO projects that net interest will be 3.7% of GDP and that the growth dividend will be -4.0% of GDP. As a result, according to CBO projections, excess interest – net interest plus the growth dividend – will be negative throughout the decade.

Hence, the effect of past debt on the debt ratio is negative throughout the decade. If primary deficits were zero, the debt ratio would fall throughout the decade, despite spending 3.7% of GDP on interest payments. Instead, CBO projects primary deficits will be large throughout the decade, on average equal to 2.6% of GDP. Those large ongoing primary deficits entirely drive the rising debt ratio.

Figure 2 shows plots Table 2's values over time, illuminating that excess interest is slightly negative and thus primary deficits closely approximate CBO's projected debt ratio changes over the coming decade. Panel A combines primary deficit and net interest into a single deficit series. As is visually apparent, deficits are projected to be near 7% of GDP but are mostly offset by large negative growth dividends, yielding much smaller debt ratio changes of approximately 2.5% of GDP. Panel B combines net interest and the growth dividend into a single excess interest series. As is visually apparent, excess interest is near zero and slighly negative throughout the decade, implying that primary deficits closely approximate changes in the debt ratio.

Table 3 extends Table 2's decomposition backward in time to 2000. One sees that aside from recession years, net interest has remained consistently smaller than the (negative of the) growth dividend. Hence, excess interest has typically been negative this millenium. Notably, the growth dividend can be large both because of high real economic growth as well as because of high inflation. Those two forces combined to yield a growth dividend of -9.1% of GDP in 2022, its peak value since 2000. Surprise inflation can "inflate away the debt" via high growth dividends that are not matched by high nominal interest rates. However, CBO projects that inflation will return to normal, so stubbornly high inflation does not account for the large growth dividends and negative excess interest in CBO's outlook.

5 Why excess interest and not interest equals the effect of past debt on the debt ratio

Equation (3) showed mathematically why excess interest rather than interest equals the effect of past debt on the debt ratio. To solidify intuition, the following example considers two economies with no primary deficits, so the debt ratio can change only because of past debt. The two economies have very different levels of interest payments. However, they both have zero excess interest and therefore enjoy the same zero effect of past debt on the debt ratio: their debt ratios remain stable.

Suppose there is a Low Debt Economy with a 1% debt ratio at the end of last year: \$1 of debt and \$100 of GDP. Suppose there is a different High Debt Economy with a 1000% debt ratio: \$1,000 of debt and \$100 of GDP. Suppose the interest rate and the GDP growth rate are both 4% in both economies. Suppose both economies have zero primary deficit this year.

Interest payments in the two economies are wildly different. In Low Debt Economy, interest equals only \$0.04, which is 0.038% of this year's \$104 GDP. In High Debt Economy, interest equals a whopping \$40, or 38% of this year's \$104 GDP. As a result, debt rises in Low Debt Economy to \$1.04 and in High Debt Economy to \$1,040. The higher the debt ratio, the larger are interest payments as a percent of GDP.

Yet the net result in both economies is that the debt ratio is stable. In Low Debt Economy, the new debt ratio equals \$1.04 divided by \$104 which equals 1%, exactly like last year. In High Debt Economy, the new debt ratio equals \$1,040 divided by \$104 which equals 1000%, exactly like last year.

The reason that the debt ratio is stable in both economies despite the wildly different interest payments is that the two economies have wildly different growth dividends that exactly offset the interest payments. In Low Debt Economy, the growth dividend equals -0.038% of GDP: 1% times $\frac{-.04}{1+.04}$. Similarly, in High Debt Economy, the growth dividend equals -38% of GDP: 1000% times $\frac{-.04}{1+.04}$. The higher the debt ratio, the larger is the growth dividend as a percent of GDP, just like interest payments.

Excess interest and not interest correctly communicates that the debt ratio is stable. Interest payments are much higher in High Debt Economy than in Low Debt Economy, but excess interest equals zero in both economies which is why neither economy experiences a change in its debt ratio. Hight Debt Economy adds much more to its debt than Low Debt Economy, but GDP growth in High Debt Economy reduces the debt ratio by many more percentage points than in Low Debt Economy because of its high debt ratio. Hence, only excess interest and not interest correctly indicates how past debt affects the debt ratio.

6 Our higher debt ratio means greater risk going forward

Despite CBO's forecast for negative excess interest over the next decade, America's higher debt ratio is not costless, even if one were to assume that investment crowd-out impacts are zero. The reason is that the higher is the debt ratio, the more susceptible America is to an increase in the interest rate relative to the growth rate.

To see this, return to the previous section's comparison between Low Debt Economy and High Debt Economy. Suppose that the interest rate is still 4% but the GDP growth rate is only 3%, for example because immigration or technology growth slows. In this scenario, Low Debt Economy suffers only slightly: excess interest equals only 0.01% of GDP. In order for Low Debt Economy to keep its debt ratio stable, it would need to run a primary surplus of only 0.01% of GDP. In contrast, High Debt Economy suffers greatly: excess interest equals 10% of GDP. In order for High Debt Economy to keep its debt ratio stable, it would need to run a primary surplus of 10% of GDP.

Figure 3 shows that according to CBO's latest long-term projections (CBO, 2024a), the average effective interest rate on government debt will exceed the GDP growth rate beginning in 2040. Following Auerbach and Gale (2023), the average nominal interest rate on government debt i_t equals CBO's forecast for current year net interest, divided by the sum of the previous year's debt held by the public plus half of the current year's primary deficit. Adding half of the current year's primary deficit to the denominator is a simple way to account for interest paid on new debt from current year primary imbalance.[8] The nominal growth rate uses CBO's GDP projections and the formula above: $y_t = GDP_t/GDP_{t-1} - 1$.

In CBO's projections, the reason that the interest rate rises above the growth rate in the long run is that primary deficits add to the debt ratio over time which slowly feeds back into a higher interest rate relative to the growth rate (Gamber and Seliski, 2019). After 2040, the interest rate is projected to exceed the growth rate and excess interest is projected to turn positive, and thus both primary deficits and excess interest would increase the debt ratio. If economic forces happen to drive up interest rates relative to growth rates earlier than CBO expects, then past debt will build on itself sooner than 2040.

7 Relationship to "r-g" accounting and to real interest

Until now, this article has considered nominal interest and nominal GDP growth. It is well known that one can express the change in the debt ratio equivalently in terms of real rates of interest and growth. For example, many such as

[8] Interest paid on new debt from current year primary imbalance appears as $OTHERINT_t$ in Appendix's complete debt ratio formula 6.

Blanchard (2022) write equation (3) as follows:

$$\frac{DEBT_t}{GDP_t} - \frac{DEBT_{t-1}}{GDP_{t-1}} = \frac{PRIMARY\,DEFICIT_t}{GDP_t} + \frac{r_t - g_t}{1+y_t} \times \frac{DEBT_{t-1}}{GDP_{t-1}} \tag{5}$$

where r_t is the real average interest rate on government debt (i.e., nominal rate i_t minus GDP price index inflation) and g_t is the real GDP growth rate (i.e., nominal rate y_t minus GDP price index inflation).[9] The $r_t - g_t$ term equals excess interest. The advantage to my formulation in equation (3) is that it shoes how excess interest exactly equals net interest as already reported by CBO plus a new growth dividend term, so that one can see connect the interest costs listed in CBO tables to the change in the debt ratio.

The President's Budget for Fiscal Year 2022 added a line in its Table S-1 for real interest as a share of GDP, equal to part of the excess interest component of equation (5):

$$\frac{r_t}{1+y_t} \times \frac{DEBT_{t-1}}{GDP_{t-1}} = \frac{r_t \times DEBT_{t-1}}{GDP_t}$$

Based on uncertainty in $r-g$ and historical values for real interest as a share of GDP, Furman and Summers (2020) suggest that the United States should operate fiscal policy in order to keep expected real interest over the coming decade at or below two percent of GDP. The Furman-Summers fiscal rule effectively penalizes a country for the risk of having a high debt ratio: the higher the debt ratio, the lower must be the real interest rate in order for real interest to remain below two percent of GDP. As a result, events like the Great Recession or the COVID-19 pandemic that suddenly raise the debt ratio have the potential to force a country obeying the Furman-Summers rule to reduce its primary deficits in order to reduce its debt ratio and real interest rate.

Despite its potential use in a fiscal rule, real interest does not help to decompose debt ratio changes. Even if CBO tables reported real interest, they would still need to report a real growth dividend component equal to $g_t \times DEBT_{t-1}/GDP_t$ in order for readers to decompose each year's debt ratio change.

8 Conclusion

This article has shown how the addition of a growth dividend row to CBO's main budget table enables readers to exactly decompose each year's change in the debt-to-GDP ratio into the sum of the primary deficit, net interest, the growth dividend, and (usually minor) other means of financing as share of GDP. This decomposition helps to clarify whether and when debt adds to the debt ratio.

[9] Note that this equation is sometimes erroneously written with $1+g_t$ in the denominator instead of $1+y_t$.

While the deficit (the primary deficit plus net interest) far exceeds CBO's projected debt ratio changes, the primary deficit closely approximates those debt ratio changes. The reason is that the net effect of past debt on the debt ratio equals excess interest – net interest plus the growth dividend – which is projected to remain slightly negative and close to zero through 2040. Hence, according to the latest CBO projections, past debt does not add to the debt ratio in any year through 2040, as interest rates remain below GDP growth rates. Large primary deficits entirely drive the rising debt ratio, despite currently high interest payments. However, America's now-higher debt ratio has increased America's exposure to the risk that economic forces drive interest rates above growth rates.

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Formula Appendix

Here are the detailed steps for going from equation 1 to equation 3:

$$DEBT_t - DEBT_{t-1} = NONINTERESTOUTLAYS_t - REVENUES_t + i_t \times DEBT_{t-1}$$

$$\frac{DEBT_t}{GDP_t} - \frac{DEBT_{t-1}}{GDP_t} = \frac{NONINTERESTOUTLAYS_t}{GDP_t} - \frac{REVENUES_t}{GDP_t} + \frac{i_t \times DEBT_{t-1}}{GDP_t}$$

$$\begin{aligned}\frac{DEBT_t}{GDP_t} - \frac{DEBT_{t-1}}{GDP_{t-1}} &= \frac{NONINTERESTOUTLAYS_t}{GDP_t} - \frac{REVENUES_t}{GDP_t} \\ &\quad + \frac{i_t \times DEBT_{t-1}}{GDP_t} + \frac{DEBT_{t-1}}{GDP_t} - \frac{DEBT_{t-1}}{GDP_{t-1}} \\ &= \frac{NONINTERESTOUTLAYS_t}{GDP_t} - \frac{REVENUES_t}{GDP_t} \\ &\quad + \frac{i_t \times DEBT_{t-1}}{GDP_t} + \frac{DEBT_{t-1}}{(1+y_t) \times GDP_{t-1}} - \frac{1+y_t}{1+y_t} \times \frac{DEBT_{t-1}}{GDP_{t-1}} \\ &= \frac{NONINTERESTOUTLAYS_t}{GDP_t} - \frac{REVENUES_t}{GDP_t} \\ &\quad + \frac{i_t \times DEBT_{t-1}}{GDP_t} + \frac{-y_t}{1+y_t} \times \frac{DEBT_{t-1}}{GDP_{t-1}}\end{aligned}$$

The decomposition uses the fact that $GDP_t = (1+y_t) \times GDP_{t-1}$.

Finally, as noted above, there is actually an extra (usually-minor) "other means of financing" term for other transactions affecting borrowing from the public. In addition, the government must pay a relatively small amount of "other interest" on new current year borrowing – e.g., it pays some interest in February on new borrowing issued in January due to a primary deficit in January. As a result, the exactly true formulas – which I implement in the attached exhibits

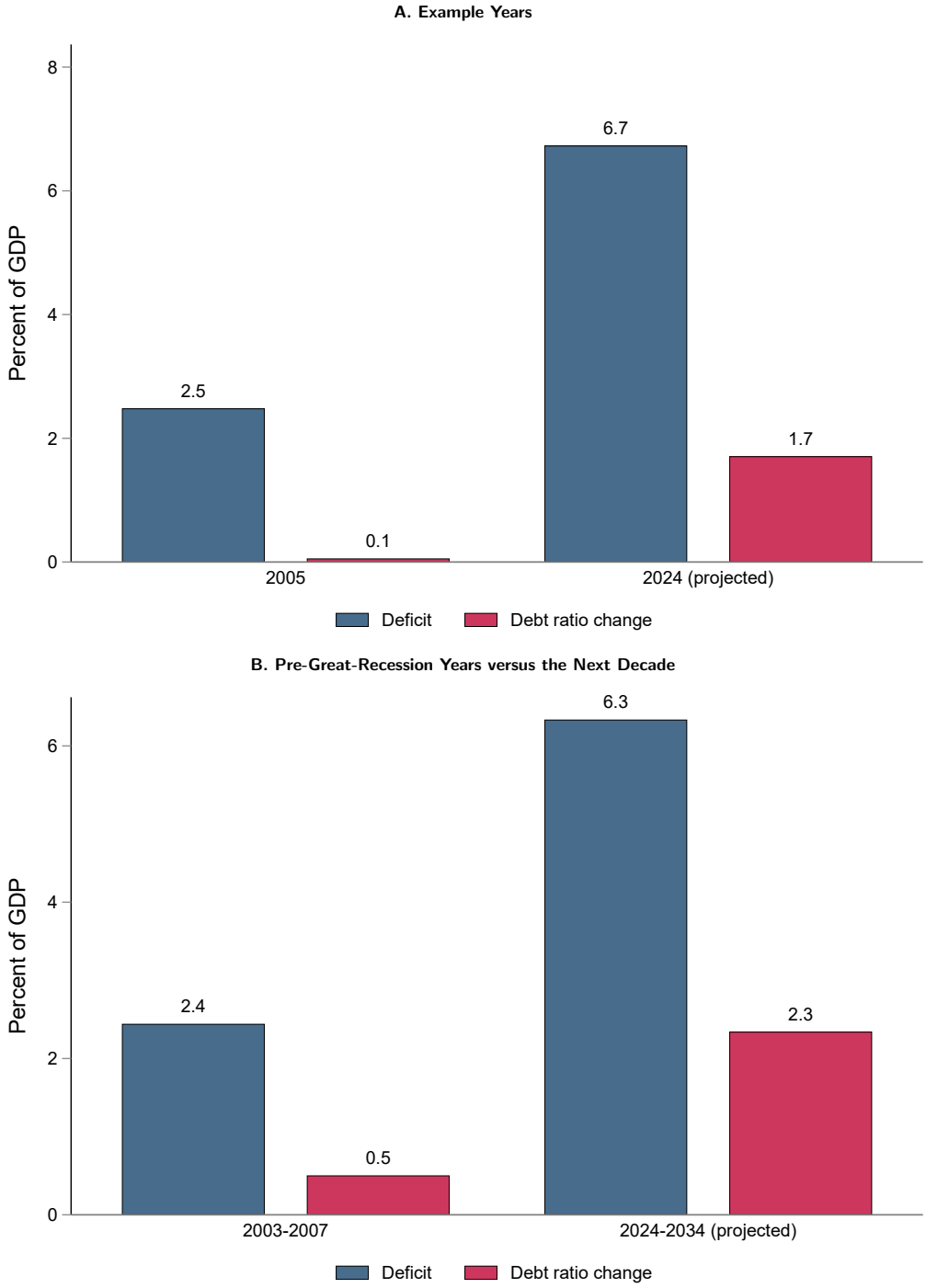
– include two additional terms:

$$\begin{aligned}\underbrace{DEBT_t - DEBT_{t-1}}_{\text{Deficit}} &= \underbrace{NONINTEREST OUTLAYS_t - REVENUES_t}_{\text{Primary deficit}} \\ &\quad + \underbrace{i_t \times DEBT_{t-1} + OTHERINT_t}_{\text{Net interest}} \\ &\quad + OTHERMEANS_t\end{aligned}$$

$$\begin{aligned}\underbrace{\frac{DEBT_t}{GDP_t} - \frac{DEBT_{t-1}}{GDP_{t-1}}}_{\text{Debt ratio change}} &= \underbrace{\frac{NONINTEREST OUTLAYS_t}{GDP_t} - \frac{REVENUES_t}{GDP_t}}_{\text{Primary deficit}} \\ &\quad + \underbrace{\underbrace{\frac{i_t \times DEBT_{t-1} + OTHERINT_t}{GDP_t}}_{\text{Net interest}} + \underbrace{\frac{-y_t}{1+y_t} \times \frac{DEBT_{t-1}}{GDP_{t-1}}}_{\text{Growth dividend}}}_{\text{Excess interest}} \\ &\quad + \frac{OTHERMEANS_t}{GDP_t}\end{aligned} \tag{6}$$

FIGURE 1

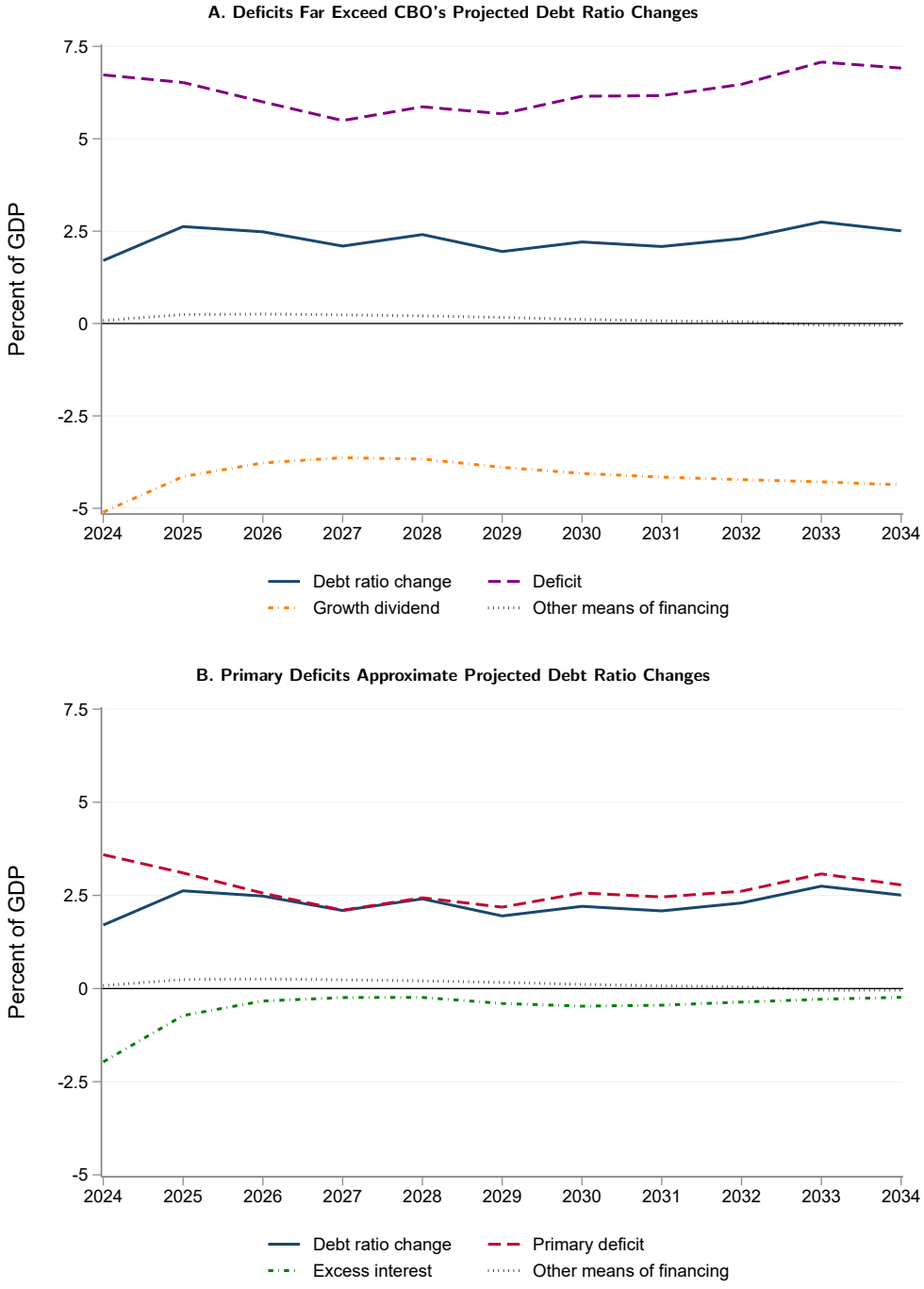
The Deficit Increasingly Exceeds the Change in the Debt-to-GDP Ratio



Notes: These graphs plot the deficit as a percent of GDP and the change in the debt-to-GDP ratio for select time periods. The deficit equals the primary deficit plus net interest. The change in the debt-to-GDP ratio equals the debt-to-GDP ratio at the end of the year, minus the debt-to-GDP ratio at the end of the prior year. Panel B plots evenly weighted averages of annual values. Historical data are from CBO (2024b) which derive from OMB historical tables. Next-decade projections are from CBO (2024c).

FIGURE 2

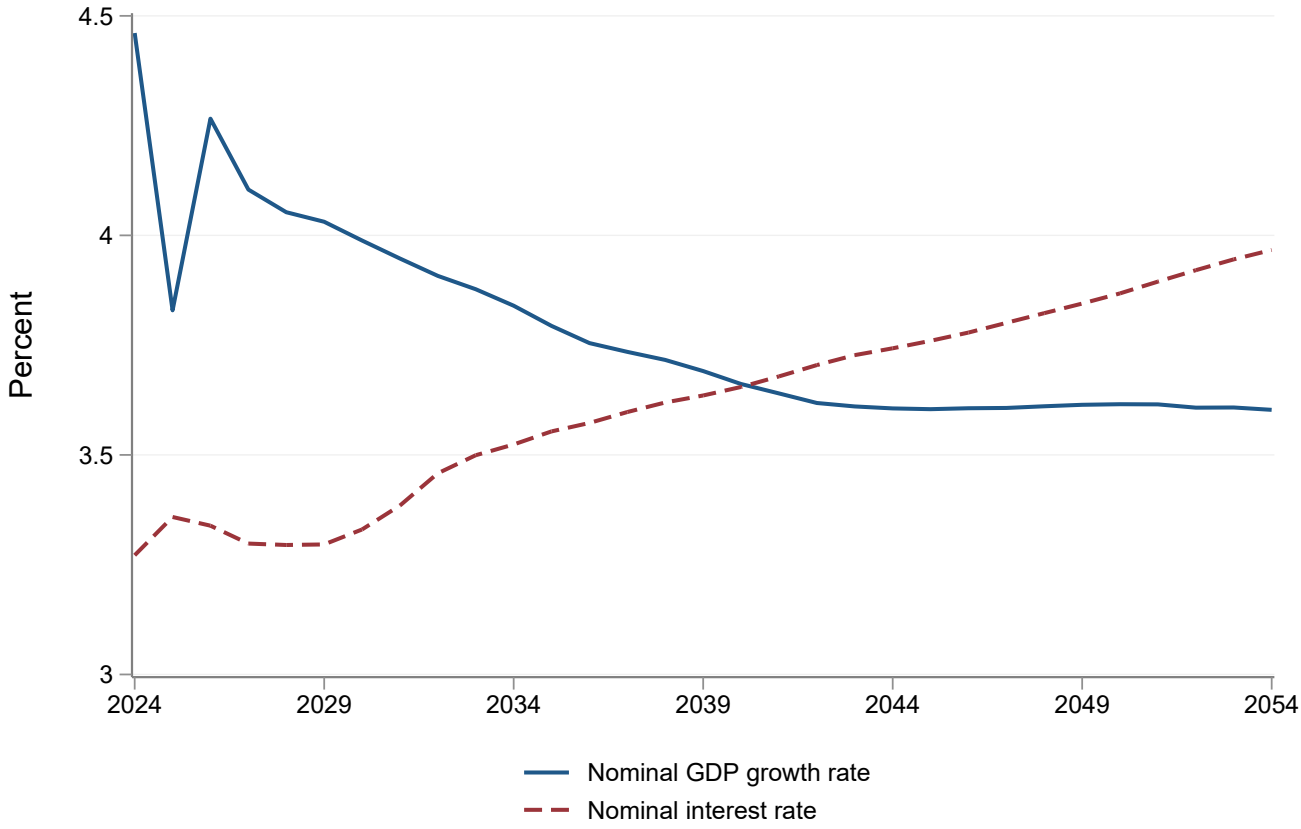
The Primary Deficit, Not the Deficit, Approximates CBO's Projected Debt Ratio Changes



Notes: These graphs use Table 2's values from CBO's latest ten-year budget outlook (CBO, 2024c) to decompose each year's debt-to-GDP ratio change into constituent compoents that exactly sum to the debt-to-GDP ratio change. In each panel, the debt ratio change series equals the sum of the other series. Panel A combines the primary deficit series and the net interest series into a single deficit series. Panel B combines net interest and the growth dividend into a single excess interest series.

FIGURE 3

CBO Projects that Excess Interest Will Remain Negative through 2040



Notes: This graph reproduces the third figure in Auerbach and Gale (2023) using the latest CBO long-term budget outlook (CBO, 2024a) to plot the projected nominal GDP growth rate y_t and the average interest rate on government debt i_t over the next thirty years. The nominal growth rate equals year-on-year growth in nominal GDP minus one: $y_t = GDP_t/GDP_{t-1} - 1$. The average nominal interest rate on government debt equals CBO's forecast for current year net interest, divided by the previous year's debt held by the public plus half of the current year's primary deficit which is a simple way to account for interest paid on new debt from current year primary imbalance.

TABLE 1
CBO's Baseline Budget Projections Table

	Actual, 2023	2024	2025	2026	2027	2028	2029	2030	2031	2032	2033	2034	Total 2025–2029	Total 2025–2034
In Billions of Dollars														
Memorandum:														
Gross Domestic Product	26,974	28,467	29,711	30,856	31,972	33,115	34,346	35,654	37,018	38,432	39,890	41,398	160,000	352,392
As a Percentage of Gross Domestic Product														
Revenues														
Individual income taxes	8.1	8.6	8.6	9.2	9.8	9.7	9.7	9.7	9.7	9.7	9.7	9.7	9.4	9.6
Payroll taxes	6.0	5.9	5.8	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9
Corporate income taxes	1.6	1.8	1.6	1.5	1.4	1.4	1.3	1.3	1.3	1.2	1.2	1.2	1.5	1.3
Other	0.9	0.8	0.9	0.9	0.9	0.9	0.9	0.9	1.1	1.1	1.1	1.2	0.9	1.0
Total	**16.5**	**17.2**	**17.0**	**17.5**	**18.0**	**18.0**	**17.9**	**17.8**	**18.0**	**18.0**	**18.0**	**18.0**	**17.7**	**17.8**
On-budget	12.0	12.8	12.6	13.1	13.7	13.6	13.5	13.5	13.6	13.6	13.6	13.7	13.3	13.5
Off-budget	4.4	4.4	4.3	4.3	4.4	4.4	4.4	4.4	4.4	4.4	4.3	4.3	4.4	4.4
Outlays														
Mandatory	13.9	14.5	13.9	13.9	14.0	14.4	14.1	14.6	14.7	14.9	15.5	15.3	14.1	14.6
Discretionary	6.4	6.3	6.2	6.2	6.1	6.0	5.9	5.8	5.7	5.6	5.6	5.5	6.1	5.8
Net Interest	2.4	3.1	3.4	3.4	3.4	3.4	3.5	3.6	3.7	3.9	4.0	4.1	3.4	3.7
Total	**22.7**	**23.9**	**23.5**	**23.5**	**23.5**	**23.8**	**23.5**	**24.0**	**24.2**	**24.4**	**25.1**	**24.9**	**23.6**	**24.1**
On-budget	18.2	19.3	18.7	18.6	18.5	18.7	18.3	18.7	18.8	18.9	19.5	19.3	18.6	18.8
Off-budget	4.5	4.6	4.8	4.9	5.0	5.1	5.2	5.3	5.4	5.5	5.6	5.6	5.0	5.3
Deficit	**-6.3**	**-6.7**	**-6.5**	**-6.0**	**-5.5**	**-5.9**	**-5.7**	**-6.2**	**-6.2**	**-6.5**	**-7.1**	**-6.9**	**-5.9**	**-6.3**
On-budget	-6.2	-6.5	-6.1	-5.5	-4.9	-5.1	-4.8	-5.2	-5.1	-5.3	-5.9	-5.6	-5.3	-5.4
Off-budget	-0.1	-0.3	-0.4	-0.5	-0.6	-0.7	-0.8	-0.9	-1.0	-1.1	-1.2	-1.3	-0.6	-0.9
Primary Deficit	-3.8	-3.6	-3.1	-2.6	-2.1	-2.4	-2.2	-2.6	-2.5	-2.6	-3.1	-2.8	-2.5	-2.6
Debt Held by the Public	97.3	99.0	101.6	104.1	106.2	108.6	110.5	112.7	114.8	117.1	119.9	122.4	n.a.	n.a.

Notes: This table reprints the bottom half of "Table 1. CBO's Baseline Budget Projections, by Category", found in CBO's June 2024 report An Update to the Budget Outlook: 2024 to 2034. The difference between Deficit and Primary Deficit equals Net Interest: Deficit equals Total Revenues minus Total Outlays, while Primary Deficit equals Total Revenues minus the sum of Mandatory Outlays and Discretionary Outlays (i.e., revenues minus non-net-interest outlays). I add four red boxes to highlight an apparent puzzle: the deficit is projected to average 6.3% percent of GDP each year, but the debt-to-GDP ratio is projected to rise by only 2.3% of GDP each year (for a total of 23 percentage points over the decade). Table 2 resolves the puzzle.

TABLE 2
CBO's Baseline Budget Projections Table, with Exact Decomposition of the Debt-to-GDP Ratio Change

	Actual, 2023	2024	2025	2026	2027	2028	2029	2030	2031	2032	2033	2034	Total 2025–2029	Total 2025–2034
							In Billions of Dollars							
Memorandum:														
Gross Domestic Product	26,974	28,467	29,711	30,856	31,972	33,115	34,346	35,654	37,018	38,432	39,890	41,398	160,000	352,392
							As a Percentage of Gross Domestic Product							
Revenues														
Individual income taxes	8.1	8.6	8.6	9.2	9.8	9.7	9.7	9.7	9.7	9.7	9.7	9.7	9.4	9.6
Payroll taxes	6.0	5.9	5.8	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9
Corporate income taxes	1.6	1.8	1.6	1.5	1.4	1.4	1.3	1.3	1.3	1.2	1.2	1.2	1.5	1.3
Other	0.9	0.8	0.9	0.9	0.9	0.9	0.9	0.9	1.1	1.1	1.1	1.2	0.9	1.0
Total	**16.5**	**17.2**	**17.0**	**17.5**	**18.0**	**18.0**	**17.9**	**17.8**	**18.0**	**18.0**	**18.0**	**18.0**	**17.7**	**17.8**
On-budget	12.0	12.8	12.6	13.1	13.7	13.6	13.5	13.5	13.6	13.6	13.6	13.7	13.3	13.5
Off-budget	4.4	4.4	4.3	4.3	4.4	4.4	4.4	4.4	4.4	4.4	4.3	4.3	4.4	4.4
Outlays														
Mandatory	13.9	14.5	13.9	13.9	14.0	14.4	14.1	14.6	14.7	14.9	15.5	15.3	14.1	14.6
Discretionary	6.4	6.3	6.2	6.2	6.1	6.0	5.9	5.8	5.7	5.6	5.6	5.5	6.1	5.8
Net Interest	2.4	3.1	3.4	3.4	3.4	3.4	3.5	3.6	3.7	3.9	4.0	4.1	3.4	3.7
Total	**22.7**	**23.9**	**23.5**	**23.5**	**23.5**	**23.8**	**23.5**	**24.0**	**24.2**	**24.4**	**25.1**	**24.9**	**23.6**	**24.1**
On-budget	18.2	19.3	18.7	18.6	18.5	18.7	18.3	18.7	18.8	18.9	19.5	19.3	18.6	18.8
Off-budget	4.5	4.6	4.8	4.9	5.0	5.1	5.2	5.3	5.4	5.5	5.6	5.6	5.0	5.3
Deficit	**-6.3**	**-6.7**	**-6.5**	**-6.0**	**-5.5**	**-5.9**	**-5.7**	**-6.2**	**-6.2**	**-6.5**	**-7.1**	**-6.9**	**-5.9**	**-6.3**
On-budget	-6.2	-6.5	-6.1	-5.5	-4.9	-5.1	-4.8	-5.2	-5.1	-5.3	-5.9	-5.6	-5.3	-5.4
Off-budget	-0.1	-0.3	-0.4	-0.5	-0.6	-0.7	-0.8	-0.9	-1.0	-1.1	-1.2	-1.3	-0.6	-0.9
Primary Deficit	-3.8	-3.6	-3.1	-2.6	-2.1	-2.4	-2.2	-2.6	-2.5	-2.6	-3.1	-2.8	-2.5	-2.6
Debt Held by the Public	97.3	99.0	101.6	104.1	106.2	108.6	110.5	112.7	114.8	117.1	119.9	122.4	n.a.	n.a.
Change in Debt Held by the Public	**1.4**	**1.7**	**2.6**	**2.5**	**2.1**	**2.4**	**1.9**	**2.2**	**2.1**	**2.3**	**2.7**	**2.5**	**2.3**	**2.3**
Primary Deficit	3.8	3.6	3.1	2.6	2.1	2.4	2.2	2.6	2.5	2.6	3.1	2.8	2.5	2.6
Net Interest	2.4	3.1	3.4	3.4	3.4	3.4	3.5	3.6	3.7	3.9	4.0	4.1	3.4	3.7
Growth Dividend	-5.9	-5.1	-4.1	-3.8	-3.6	-3.7	-3.9	-4.1	-4.2	-4.2	-4.3	-4.4	-3.8	-4.0
Other Means of Financing	1.1	0.1	0.2	0.3	0.2	0.2	0.2	0.1	0.1	0.0	0.0	0.0	0.2	0.1

Notes: This table adds one line -- the Growth Dividend -- to CBO's Table 1-1 in order to exactly decompose CBO's projected changes in the debt-to-GDP ratio. Change in Debt Held by the Public equals each year's value of Debt Held by the Public, minus the prior year's. (The negative of) Primary Deficit and Net Interest equal their labeled rows above. Other Means of Financing comes from CBO Table 1-3. The Growth Dividend equals $y/(1+y)$ times the prior year's Debt Held by the Public, where y equals the growth rate of nominal GDP.

TABLE 3
Exact Decomposition of Debt Ratio Changes 2000-2034

			Deficit			
				Excess Interest		
Year	Debt-to-GDP Ratio	Change in Debt-to-GDP Ratio	Primary Deficit	Net Interest	Growth Dividend	Other Means of Financing
	(% of GDP)	(pp of GDP)	(% of GDP)	(% of GDP)	(% of GDP)	(% of GDP)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
2000	33.7	-4.6	-4.5	2.2	-2.4	0.1
2001	31.5	-2.2	-3.2	2.0	-1.3	0.4
2002	32.7	1.2	-0.1	1.6	-0.9	0.6
2003	34.7	2.0	2.0	1.4	-1.3	0.0
2004	35.7	1.0	2.1	1.3	-2.2	-0.3
2005	35.8	0.1	1.0	1.4	-2.3	-0.2
2006	35.4	-0.4	0.2	1.7	-2.1	-0.1
2007	35.2	-0.2	-0.5	1.7	-1.7	0.3
2008	39.2	4.0	1.4	1.7	-1.2	2.1
2009	52.2	12.9	8.5	1.3	0.9	2.3
2010	60.6	8.4	7.4	1.3	-1.5	1.2
2011	65.5	4.9	6.9	1.5	-2.3	-1.2
2012	70.0	4.5	5.3	1.4	-2.6	0.5
2013	71.8	1.8	2.7	1.3	-2.4	0.1
2014	73.3	1.5	1.5	1.3	-3.1	1.8
2015	72.2	-1.1	1.2	1.2	-3.0	-0.6
2016	76.0	3.8	1.8	1.3	-1.8	2.5
2017	75.7	-0.3	2.1	1.4	-2.9	-0.9
2018	77.1	1.4	2.2	1.6	-3.9	1.5
2019	79.0	1.9	2.9	1.8	-3.0	0.3
2020	98.7	19.7	13.1	1.6	-0.1	5.1
2021	97.1	-1.6	10.6	1.5	-7.1	-6.6
2022	95.8	-1.3	3.6	1.9	-9.1	2.3
2023	97.3	1.4	3.8	2.4	-5.9	1.1
2024	99.0	1.7	3.6	3.1	-5.1	0.1
2025	101.6	2.6	3.1	3.4	-4.1	0.2
2026	104.1	2.5	2.6	3.4	-3.8	0.3
2027	106.2	2.1	2.1	3.4	-3.6	0.2
2028	108.6	2.4	2.4	3.4	-3.7	0.2
2029	110.5	1.9	2.2	3.5	-3.9	0.2
2030	112.7	2.2	2.6	3.6	-4.1	0.1
2031	114.8	2.1	2.5	3.7	-4.2	0.1
2032	117.1	2.3	2.6	3.9	-4.2	0.0
2033	119.9	2.7	3.1	4.0	-4.3	0.0
2034	122.4	2.5	2.8	4.1	-4.4	0.0

Notes - This table exactly decomposes annual changes in the debt-to-GDP ratio from 2000-2023, as well as CBO's projected changes 2024-2034. Change in Debt-to-GDP ratio equals the current year's debt-to-GDP ratio minus the prior year's. Change in Debt-to-GDP ratio also equals Primary Deficit plus Net Interest plus Growth Dividend plus Other Means of Financing.