The Growth Dividend and Excess Interest

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January 31, 2024

Abstract

What makes America's fiscal path unsustainable: old debt spiraling on its own, or new primary deficits? This note decomposes the Congressional Budget Office's projection for America's rising debt-to-GDP ratio. The decomposition illuminates that large future primary deficits entirely drive the rising debt ratio, despite high interest payments on old debt. While old debt is sustainable in the CBO baseline, the tripling of our debt ratio since the Great Recession has tripled our vulnerability to interest rate and GDP growth risk.

1 Introduction

Policymakers and commentators correctly understand that the increase in government debt each year equals spending minus tax revenue. That spending includes interest on debt. Interest payments have grown in size, leading many to suggest that old debt building on itself is a key source of the United States's unsustainable fiscal path, in addition to large future primary deficits adding new debt.[1]

However, what matters for fiscal sustainability is not a stable *level* of debt but rather a stable *debt-to-GDP ratio*. Past debt always increases the debt level because interest payments are always positive. However, this note clarifies that the effect of past debt on the debt ratio is a horse race. Interest payments add to the numerator in proportion to the level of past debt, but GDP growth adds to the denominator and reduces the debt ratio in proportion to the level of past debt (which I call the "growth dividend"). Past debt increases the debt ratio only when "excess interest" – interest minus the growth dividend – is positive. As a result, applying debt-level intuitions to the debt ratio is faulty: past debt can raise the debt level without raising the debt ratio.

*UC Berkeley and NBER. I thank Marc Goldwein for especially valuable conversations that led to this manuscript. I also thank Alan Auerbach, Bill Gale, David Kamin, Bobby Kogan, Richard Kogan, Michael Linden, Brian Riedl, and economists at the Congressional Budget Office and Office of Management and Budget for helpful conversations on this topic generally or this manuscript specifically.

[1] For example, "Borrowing is expensive. A mounting share of federal revenue, money that could be used for the benefit of the American people, goes right back out the door in the form of interest payments to investors who purchase government bonds. Rather than collecting taxes from the wealthy, the government is paying the wealthy to borrow their money" (New York Times Editorial Board 2023).

Empirically, I show that adding a row for the growth dividend to CBO's budget outlook table illuminates the effect of past debt on the debt ratio. CBO projects that the debt level will rise due to both interest payments and primary deficits. However, CBO projects that excess interest will remain negative for twenty years, so new primary deficits and not past debt are the sole reason that CBO projects a rising debt ratio. If Congress were to reduce primary deficits from approximately 3.3% of GDP to 0.8%, I estimate that CBO would project that excess interest would remain negative forever as the debt ratio stabilizes. Hence, large primary deficits entirely explain CBO's debt spiral projection.

Put simply: the effect of past debt on the debt ratio equals the debt ratio change that would occur if the primary deficit were zero. Under CBO assumptions, the debt ratio would fall forever toward zero if the primary deficit were zero because GDP growth rates exceed interest rates. Hence, the effect of past debt on the debt ratio is negative, and America's fiscal path problem according to CBO projections is caused entirely by large primary deficits.

Intuitions have suddenly failed for a simple reason: the debt ratio has tripled since the Great Recession, which has tripled both interest and growth dividends. Policymakers and commentators have focused on the higher interest payments without considering the higher growth dividends. When the interest rate is less than the growth rate, excess interest is negative no matter how large or small interest payments are and thus past debt does not build on itself.

However, the CBO baseline projection is not a risk scenario. Our tripled debt ratio means that we are three times as vulnerable to the risk that economic forces cause interest rates to exceed GDP growth rates, as they did 1980-2010. The tripled debt ratio means that any given persistent swing in interest rates relative to growth rates will cause past debt to add three times as much to our debt ratio each year as it would have before the Great Recession.

The substance of this note is already obvious to economists and budget experts (e.g., Ball, Elmendorf, and Mankiw 1998; Hall and Sargent 2011; Auerbach and Gale 2023; Blanchard 2023). Yet presentationally, I show that combining CBO's existing net interest line with only one new line – the growth dividend – illuminates the exact effect of past debt on the debt ratio (please let me know of forerunners so that I can acknowledge them). Importantly, by "the effect of past debt on the debt ratio", I mean the mechanical accounting effect of debt on the debt ratio over and above the effect of the primary deficit. I therefore exclude any behavioral effects of debt on the primary deficit, such as investment crowd out or political

feedback (Hicks, 1937; Auerbach, 2003).

2 The puzzle: High interest that does not increase the debt ratio

Table 1 reprints Table 1 of CBO's May 2023 Budget Outlook 2023-2033 (CBO, 2023b) for our fiscal path under current policy, with red boxes surrounding four notable cells. First, interest payments are high: CBO projects that net interest payments ("interest" for short) over the next decade will average 3.2% of GDP, which is over twice as large as its 2010s average. Second, primary deficits are also high: CBO projects that the primary deficit – equal to non-interest outlays (i.e., spending) minus revenues – will average 2.9% of GDP. One might therefore have thought that the debt ratio will rise by 6.1 percentage points per year from 2023 to 2033. Instead, the remaining two red boxes show that CBO projects the debt ratio will rise by only 2.1 percentage points per year, from 98% to 119% over the decade. That is: the debt ratio will rise by substantially less than primary deficits, despite high interest. Why?

3 The answer: Excess interest – the effect of past debt on the debt ratio – is still negative

The effect of debt on the debt ratio is a horse race between interest payments on the one hand and what I call the "growth dividend" on the other: the amount by which economic growth reduces the debt ratio by growing the denominator. The reason that the CBO projects the debt ratio will rise by less than primary deficits is that the net effect of debt on the debt ratio will remain negative: the growth dividend will exceed interest payments – and thus "excess interest" will be negative.

Debt held by the public grows from year $t-1$ to year t according to the accounting identity:

$$\underbrace{DEBT_t - DEBT_{t-1}}_{\text{Deficit}} = \underbrace{INTERESTRATE_t \times DEBT_{t-1}}_{\text{Net interest}} + \underbrace{NONINTERESTOUTLAYS_t - REVENUES_t}_{\text{Primary deficit}} \tag{1}$$

where $INTERESTRATE_t$ is the average nominal interest rate on government debt. See the appendix for the exactly true formula, which includes two additional minor terms: interest on current-year new borrowing due to new primary deficits, and other miscellaneous transactions that affect borrowing from the public like changes in Treasury's operating cash balance. All terms are nominal (i.e., not inflation adjusted) and are exactly as they appear in CBO's main budget table.

Dividing equation 1 through by GDP and rearranging terms, one obtains the following accounting identity for the change in the debt ratio:

$$\underbrace{\frac{DEBT_t}{GDP_t} - \frac{DEBT_{t-1}}{GDP_{t-1}}}_{\text{Debt ratio change}} = \underbrace{\underbrace{\frac{INTERESTRATE_t \times DEBT_{t-1}}{GDP_t}}_{\text{Net interest}} - \underbrace{\frac{GROWTHRATE_t \times DEBT_{t-1}}{GDP_t}}_{\text{Growth dividend}}}_{\text{Excess interest}} + \underbrace{\underbrace{\frac{NONINTERESTOUTLAYS_t}{GDP_t}}_{\text{Non-interest outlays}} - \underbrace{\frac{REVENUES_t}{GDP_t}}_{\text{Revenue}}}_{\text{Primary deficit}} \quad (2)$$

where $GROWTHRATE_t = GDP_t/GDP_{t-1} - 1$ is the nominal growth rate of GDP. See the appendix for the derivation and for the exactly true version with two additional minor terms. Excess interest – which equals interest minus a term I call the "growth dividend" – equals the effect of debt on the debt ratio. The primary deficit is the effect of current taxes and non-interest spending on the debt ratio. The sum of the two equals the change in the debt ratio. All terms are exactly as listed CBO's main budget table, except the growth dividend which CBO does not currently report.

The unfamiliar term is the growth dividend: the nominal growth rate multiplied by last year's debt, divided by this year's GDP. The reason that one needs the growth dividend term in the debt ratio equation and not the debt equation is that the debt ratio divides the debt by GDP, which grows by the rate of economic growth and therefore reduces the debt ratio. The growth dividend equals the percentage points of GDP by which the debt ratio would fall if government were to pay nothing and collect nothing this year. Excess interest – which equals interest minus the growth dividend – equals the net effect of debt on the debt ratio.

For example, suppose that the primary deficit is zero, the interest rate is 4%, the growth rate is 4%, and last year's debt and GDP were both \$100. Interest and the deficit this year will be \$4, so the debt will rise by \$4 from \$100 to \$104. However, the debt ratio exhibits zero change: GDP rises by \$4 as well from \$100 to \$104, so the debt ratio is stable at one. In terms of equation 2, both interest and the growth dividend equal 3.8% of GDP and exactly offset each other, leading to zero excess interest and zero debt ratio change.

4 Adding rows to CBO's budget outlook illuminates that excess interest is negative

Table 2 adds rows to the main CBO budget outlook table reprinted in Table 1 in order to decompose each year's change in the debt ratio into the terms in equation 2. These rows explain the puzzle of how the debt ratio rises each year this decade by less than the primary deficit despite high interest.

The key rows are the highlighted rows at the bottom. The row Change in Debt Held by the Public equals Excess Interest plus the Primary Deficit plus Other Means of Financing (CBO's term for other miscellaneous transactions). Most of the rows are self-explanatory and are either copied directly from other rows or are simple addition or subtraction of other values. Other means of financing is usually near zero, and CBO projects that it goes to zero by the end of the decade.[2]

The two unfamiliar rows are the Growth Dividend and Excess Interest. The growth dividend is computed using values from the original CBO table as defined above.[3] Excess Interest equals net interest minus the growth dividend. These new rows show that excess interest is negative in every year this decade. As a result, the debt ratio rises each year by less than the primary deficit. Although interest as a share of GDP is substantial, the growth dividend is even larger each year.

For example, the forecast for 2024 is that the debt ratio will rise by 2.2 percentage points. That 2.2 percentage point increase equals the sum of excess interest (-1.0% of GDP) and the primary deficit (3.0% of GDP), along with near-zero (0.2% of GDP) other means of financing. Excess interest is negative despite interest payments equaling 2.7% of GDP, because growth dividend is even larger: GDP growth from 2023 to 2024 shrinks the debt ratio by 3.7% of GDP. Negative excess interest means that without a primary deficit, the debt ratio will fall. Instead, the primary deficit is large, so the debt ratio rises.

See Table 3 for equivalent added rows to the key table from the President's Fiscal Year 2024 Budget (OMB, 2023a). Note that the Budget refers to other miscellaneous as "other transactions affecting borrowing from the public".

Figure 1 compares excess interest across decades. The figure plots annual excess interest, primary deficits,

[2]Other means of financing was large in 2022 because Treasury reversed extraordinary measures it undertook in 2021 in order to avoid a debt ceiling breach. Specifically, Treasury in 2021 spent down its cash reserves and suspended investments in the Thrift Savings Plan (TSP) Government Securities Investment Fund (G-Fund) for government workers' tax-preferred retirement savings accounts. In 2022, Treasury replenished cash reserves and fully reinvested the TSP G-Fund, which required a one-time increase in debt issuance. See OMB (2023b).

[3]Following equation 2, the growth dividend equals the formula above and equals the nominal growth rate of GDP (obtained from the Gross Domestic Product row) times the prior year's Debt Held by the Public, divided by the current year's Gross Domestic Product. Other means of financing appears in CBO's Table 3.

and other miscellaneous transactions as a share of GDP, from 1980 to the present combined with CBO projections for the next ten years (CBO, 2023a).[4] The figure shows that debt contributed to a growing debt ratio 1980-2010: excess interest averaged 0.3% of GDP in the 1980s, 0.6% of GDP in the 1990s, and 0.2% in the 2000s. However, debt contributed to a declining debt ratio during the 2010s: excess interest averaged −1.2% in the 2010s. That pattern continued 2020-2022 with average excess interest of −4.5%, in a time of high inflation and therefore high nominal GDP growth. Finally, the CBO projects average excess interest of −1.1% in the coming ten years 2024-2033.

5 An easy way to see that excess interest and not interest equals the effect of past debt on the debt ratio

Equation 2 showed mathematically why excess interest rather than interest equals the effect of past debt on the debt ratio. To solidify intuition, the following example considers two economies with no primary deficits, so the debt ratio can move only because of past debt. The two economies have very different levels of interest payments. However, they both have zero excess interest and therefore enjoy the same zero effect of past debt on the debt ratio: their debt ratios stay stable.

Suppose there is a Low Debt Economy with a 1% debt ratio last year: \$1 of debt and \$100 of GDP. Suppose there is a different High Debt Economy with a 1000% debt ratio: \$1,000 of debt and \$100 of GDP. Suppose the interest rate and the GDP growth rate are both 4% in both economies. Suppose both economies have zero primary deficit this year.

Interest payments in the two economies are wildly different. In Low Debt Economy, interest equals only \$0.04, which is 0.038% of this year's \$104 GDP. In High Debt Economy, interest equals a whopping \$40, or 38% of this year's \$104 GDP. As a result, debt rises in Low Debt Economy to \$1.04 and in High Debt Economy to \$1,040. This example shows that the higher the debt ratio, the larger the interest payments in percentage points of GDP.

However, the growth dividends in the two economies are also wildly different. In Low Debt Economy, the growth dividend equals 0.038% of GDP, equal to the 4% growth rate times \$1 of last year's debt divided by \$104 of this year's GDP. That means that if there were no interest payments, the debt ratio would fall by 0.038% of GDP: if debt were held constant at \$1 while GDP grows to \$104, the new debt ratio would

[4]For the figures, I use CBO's long-term budget outlook projections from June 2023 (CBO, 2023a) which slightly different from the CBO (2023b) projections shown in the tables, in order to avoid a discontinuity in Figure 2. See footnote 4 for details.

be 0.962% which is 0.038 percentage points lower than last year. In contrast, in High Debt Economy, the growth dividend equals 38% of GDP, equal to the 4% growth rate times $1000 of last year's debt divided by $104 of this year's GDP. That means that if there were no interest payments, the debt ratio would fall by 38%: if debt were held constant at $1,000 while GDP grows to $1,040, the new debt ratio would be 962% which is 38 percentage point lower than last year. This example shows that the higher the debt ratio, the larger the growth dividend in percentage points of GDP.

The net result in both economies is that the debt ratio is stable. In Low Debt Economy, the new debt ratio equals $1.04 divided by $104, which equals 1% just like last year. In High Debt Economy, the new debt ratio equals $1,040 divided by $104, which equals 1000% just like last year.

Excess interest and not interest correctly communicates that the debt ratio is stable. Interest payments are much higher in High Debt Economy than in Low Debt Economy, but excess interest equals zero in both economies which is why neither economy experiences a change in its debt ratio. Hight Debt Economy adds much more to its debt than Low Debt Economy, but GDP growth in High Debt Economy reduces the debt ratio by many more percentage points than in Low Debt Economy because of its high debt ratio. Hence, only excess interest and not interest correctly indicates how past debt affects the debt ratio.

6 The higher debt ratio explains why intuitions are failing

Interest as a share of GDP has grown, which one might intuitively think implies that debt is looming larger on our debt ratio path than it did before. The previous sections show that this is not true. For example, as written above, the CBO projects that excess interest will be as negative in the coming decade as it was in the 2010s.

The reason that intuitions have failed is that both interest and growth dividends are larger when the debt ratio is larger, and the debt ratio has tripled since the Great Recession. For any given pair of interest and growth rates, a tripled debt ratio means tripled interest – and as well as tripled growth dividends. The previous section showed an extreme example. What matters for the debt ratio is the difference between the two, i.e. excess interest. When the interest rate is less than the growth rate, excess interest is negative no matter how large or small is the debt ratio.

However, intuitions are appropriate for a different source of higher interest: an increase in interest rates relative to the growth rate. In that situation, interest and excess interest both rise by the same amount, as

shown in the next section.

7 Our higher debt ratio means greater risk going forward

Despite the forecast for negative excess interest for the next decade, our higher debt ratio is not costless, even if one were to assume that investment crowd-out impacts are zero. The reason is that the higher is the debt ratio, the more susceptible we are to an increase in the interest rate relative to the growth rate.

To see this, return to the comparison between Low Debt Economy and High Debt Economy. Suppose that the interest rate is 4% but the GDP growth rate is only 3%. In this scenario, Low Debt Economy suffers only slightly: excess interest equals only 0.01% of GDP. As a result, in order for Low Debt Economy to keep its debt ratio stable, it would need to run a primary surplus of only 0.01% of GDP. In contrast, High Debt Economy suffers greatly: excess interest equals 10% of GDP. As a result, in order for High Debt Economy to keep its debt ratio stable, it would need to run a primary surplus of 10% of GDP.

Figure 2 shows that according to CBO's latest long-term projections, the average interest rate on government debt will exceed the GDP growth rate beginning in 2044. To compute each year's effective nominal interest rate on government debt, I reproduce Auerbach and Gale's (2023) calculations using the latest CBO long-term budget outlook from June 2023 (CBO, 2023a).[5] The nominal growth rate uses CBO's GDP projections and the formula above: $GROWTHRATE_t = GDP_t/GDP_{t-1} - 1$. The average nominal interest rate on government debt equals CBO's forecast for current year net interest, divided by the previous year's debt held by the public plus half of the current year's primary deficit. Adding half of the current year's primary deficit to the denominator is a simple way to account for interest paid on new debt from current year primary imbalance.[6]

8 Relationship to fiscal gaps

In CBO's projections, the reason that the interest rate rises above the growth rate in the long run is that primary deficits add to the debt ratio over time which slowly feeds back into a higher interest rate relative

[5] Auerbach and Gale's Figure 3 uses the CBO ten-year budget outlook from May 2023 (CBO 2023b, as I do in Tables 1-2) and add the remaining twenty years from CBO's long-term budget outlook from June 2023 (CBO, 2023a). For my figures, I use the full thirty-year project projection from CBO (2023a) in order to avoid a discontinuity in 2023; CBO (2023a) slightly updated its the ten-year projection in CBO (2023b) due to the Fiscal Responsibility Act of 2023 which was enacted on June 3, 2023. CBO (2023a) projects a slightly smaller rise in the debt ratio over the next ten years than CBO (2023a). I use CBO (2023a) in Tables 1-2 in order to use the most familiar ten-year budget table.

[6] Interest paid on new debt from current year primary imbalance appears as $OTHERINT_t$ in Appendix's complete debt ratio formula 4.

to the growth rate (Gamber and Seliski, 2019). If primary deficits were zero, the interest rate would never exceed the growth rate, and the debt ratio would fall to zero due to negative excess interest forever. If primary deficits were reduced toward but not completely to zero, the debt ratio would stabilize and excess interest would remain negative forever and be in perfect balance with the remaining primary deficit. I now estimate the reduction in primary deficits that CBO assumes would be necessary in order to keep excess interest negative forever, by stabilizing the debt ratio.

A large literature studies fiscal gaps: the permanent change in primary deficits that would be needed to stabilize GDP. Those analyses require complicated calculations when primary deficits are rising in CBO's baseline. A convenient feature of the CBO's long-term budget outlook is that population aging is projected to plateau in the 2030s which stabilizes primary deficits at approximately 3.3% of GDP. That fact permits a simple and transparent approximation of the reduction in the primary deficit that would be needed to stabilize the debt ratio at its 2030 level.

CBO projects that the debt ratio in 2030 will be approximately 110%. CBO's nominal GDP growth rate stabilizes at approximately 3.7% in the 2030s. Finally, Figure 2 calculated that CBO estimates that in 2030 the average interest rate on government debt will be 0.8 percentage points below the growth rate, and I assume that that 0.8 percentage point difference equals CBO's assumed long-run difference if a primary deficit reduction were to stabilize the debt ratio.

Based on these assumptions, one can approximate the primary deficit $PRIMARYDEFICIT^*/GDP$ that would be needed to stabilize debt at 110% beginning in 2030 as:

$$\frac{PRIMARYDEFICIT^*}{GDP} = \frac{1.1}{1.037} \times 0.008 = 0.008$$

That is: a primary deficit of approximately 0.8% of GDP would stabilize the debt ratio at 108% beginning in 2030. Because the actual projected primary deficit in the 2030s is 3.3% of GDP, Congress would need to reduce primary the primary deficit by 2.5% in order to achieve the 0.8% primary deficit that would stabilize the debt ratio. This rough but transparent 2.5% estimate is close to other more precise fiscal gap estimates (e.g., Auerbach and Gale 2023).

9 Relationship to "r-g" accounting and to real interest

Until now, this note has considered nominal (i.e., not-inflation-adjusted) interest and nominal GDP growth. It is well known that one can express the change in the debt ratio equivalently in terms of real rates of interest and growth. For example, many such as Blanchard (2022) write equation 2 in real terms as follows:

$$\frac{DEBT_t}{GDP_t} - \frac{DEBT_{t-1}}{GDP_{t-1}} = \frac{r_t - g_t}{1+g} \times \frac{DEBT_{t-1}}{GDP_{t-1}} + \frac{PRIMARY DEFICIT_t}{GDP_t} \quad (3)$$

where r_t is the real average interest rate on government debt (i.e., nominal rate minus GDP price index inflation) and g_t is the real GDP growth rate (i.e., nominal rate minus GDP price index inflation). The term with $r - g$ exactly equals excess interest. The advantage to my equation 2 is that it shows how excess interest exactly equals interest as already reported by CBO minus a new growth dividend term, so that one can see connect the interest costs listed in CBO tables to the change in the debt ratio.

The President's Budget for Fiscal Year 2022 added a line in its Table S-1 for real interest as a share of GDP, equal to part of the excess interest component of equation 3: $r_t/(1+g_t) \times DEBT_{t-1}/GDP_{t-1} = r_t \times DEBT_{t-1}/GDP_t$. Based on uncertainty in $r - g$ and historical values for real interest as a share of GDP, Furman and Summers (2020) suggest that the United States should operate fiscal policy in order to keep expected real interest over the coming decade at or below two percent of GDP.

Real interest achieves half of the substantive advantage of excess interest: since inflation increases both nominal interest and nominal GDP growth, real interest leads to less mistaken intuition with respect to high interest costs and the change in the debt ratio. However, real interest omits the other half: when the real rate is less than the growth rate, real interest will be positive but the debt ratio will still fall in the absence of primary deficits. The Furman-Summers fiscal rule has the advantage of effectively penalizing a country for the risk of having a high debt ratio: the higher the debt ratio, the lower must be the real interest rate in order for real interest to remain below two percent of GDP. Auerbach and Yagan (in progress) are constructing a complementary fiscal rule that quantifies the risk exposure of a high debt ratio.

10 Conclusion

This note has shown how the addition of rows to CBO's main budget table can help readers decompose each year's change in the debt ratio into excess interest (i.e., the effect of past debt) plus the primary deficit

(i.e., the effect of current taxes and and non-interest spending) plus other miscellaneous transactions. This decomposition helps to clarify whether and when debt adds to the debt ratio. According to the latest CBO projections, past debt does not add to the debt ratio in any year over the coming two decades, as interest rates remain below GDP growth rates. Instead, large primary deficits drive the rising debt ratio. Our now-higher debt ratio has increased our exposure to the risk that interest rates rise above growth rates.

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Formula Appendix

Here are the detailed steps for going from equation 1 to equation 2:

$$DEBT_t - DEBT_{t-1} = INTERESTRATE_t \times DEBT_{t-1} + NONINTERESTOUTLAYS_t - REVENUES_t$$

$$\frac{DEBT_t}{GDP_t} - \frac{DEBT_{t-1}}{GDP_t} = \frac{INTERESTRATE_t \times DEBT_{t-1}}{GDP_t} + \frac{NONINTERESTOUTLAYS_t}{GDP_t} - \frac{REVENUES_t}{GDP_t}$$

$$\begin{aligned}\frac{DEBT_t}{GDP_t} - \frac{DEBT_{t-1}}{GDP_{t-1}} &= \frac{DEBT_{t-1}}{GDP_t} - \frac{DEBT_{t-1}}{GDP_{t-1}} + \frac{INTERESTRATE_t \times DEBT_{t-1}}{GDP_t} \\ &+ \frac{NONINTERESTOUTLAYS_t}{GDP_t} - \frac{REVENUES_t}{GDP_t} \\ &= \frac{DEBT_{t-1}}{GDP_t} - \frac{(1+GROWTHRATE_t) \times DEBT_{t-1}}{(1+GROWTHRATE_t) \times GDP_{t-1}} + \frac{INTERESTRATE_t \times DEBT_{t-1}}{GDP_t} \\ &+ \frac{NONINTERESTOUTLAYS_t}{GDP_t} - \frac{REVENUES_t}{GDP_t} \\ &= -\frac{GROWTHRATE_t \times DEBT_{t-1}}{GDP_t} + \frac{INTERESTRATE_t \times DEBT_{t-1}}{GDP_t} \\ &+ \frac{NONINTERESTOUTLAYS_t}{GDP_t} - \frac{REVENUES_t}{GDP_t} \\ &= \frac{INTERESTRATE_t \times DEBT_{t-1}}{GDP_t} - \frac{GROWTHRATE_t \times DEBT_{t-1}}{GDP_t} \\ &+ \frac{NONINTERESTOUTLAYS_t}{GDP_t} - \frac{REVENUES_t}{GDP_t}\end{aligned}$$

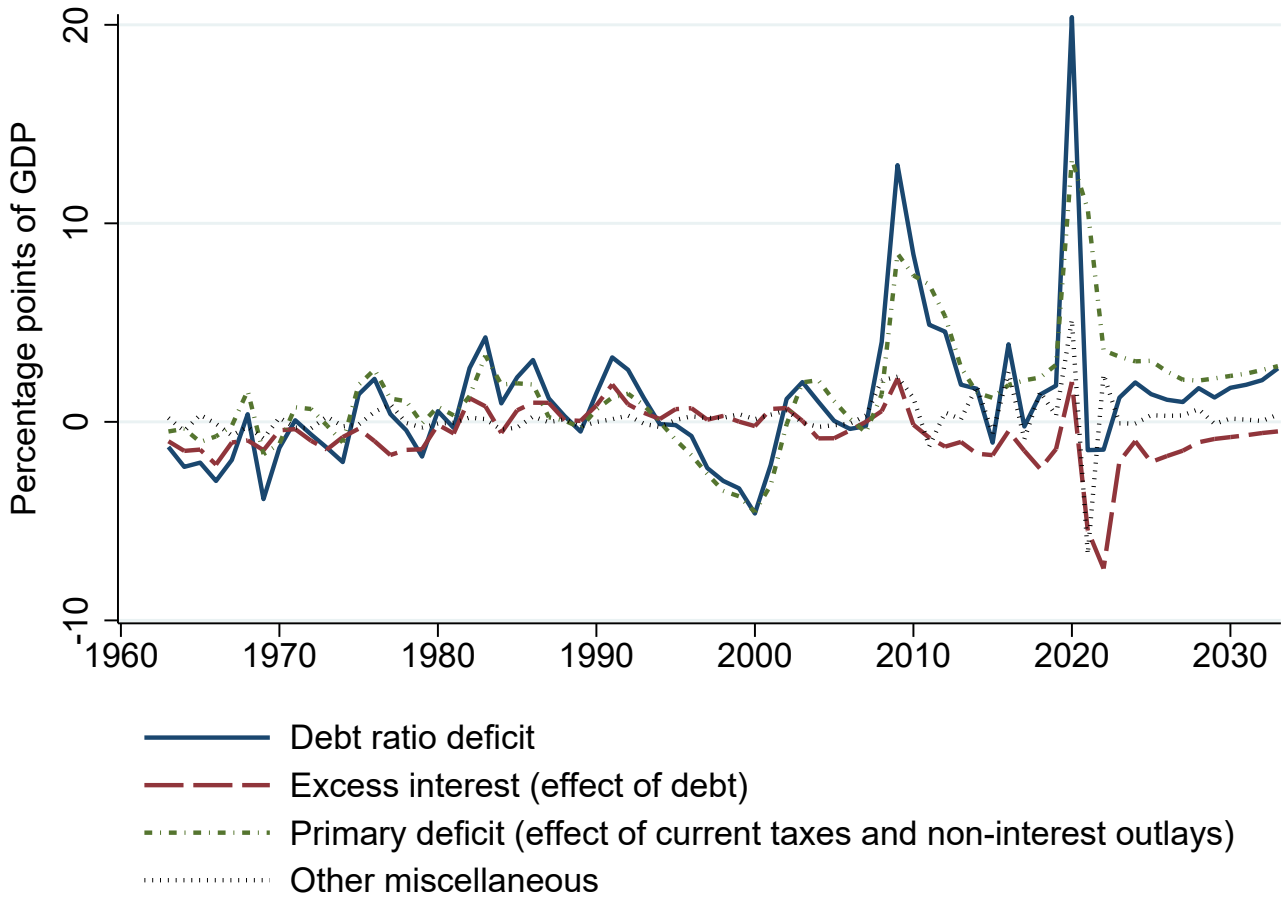
Finally, as noted above, there is actually an extra (usually-minor) other miscellaneous transactions term for other transaction affecting borrowing from the public. In addition, the government must pay a relatively small amount of other interest on new current year borrowing – e.g., it pays some interest in February on new borrowing issued in January due to a primary deficit in January. As a result, the exactly true formulas

– which I implement in the attached tables – include two additional terms:

$$\begin{aligned}\underbrace{DEBT_t - DEBT_{t-1}}_{\text{Deficit}} &= \underbrace{INTERESTRATE_t \times DEBT_{t-1} + OTHERINT_t}_{\text{Net interest}} \\ &+ \underbrace{NONINTERESTOUTLAYS_t - REVENUES_t}_{\text{Primary deficit}} \\ &+ OTHERMISCELLANEOUS_t\end{aligned}$$

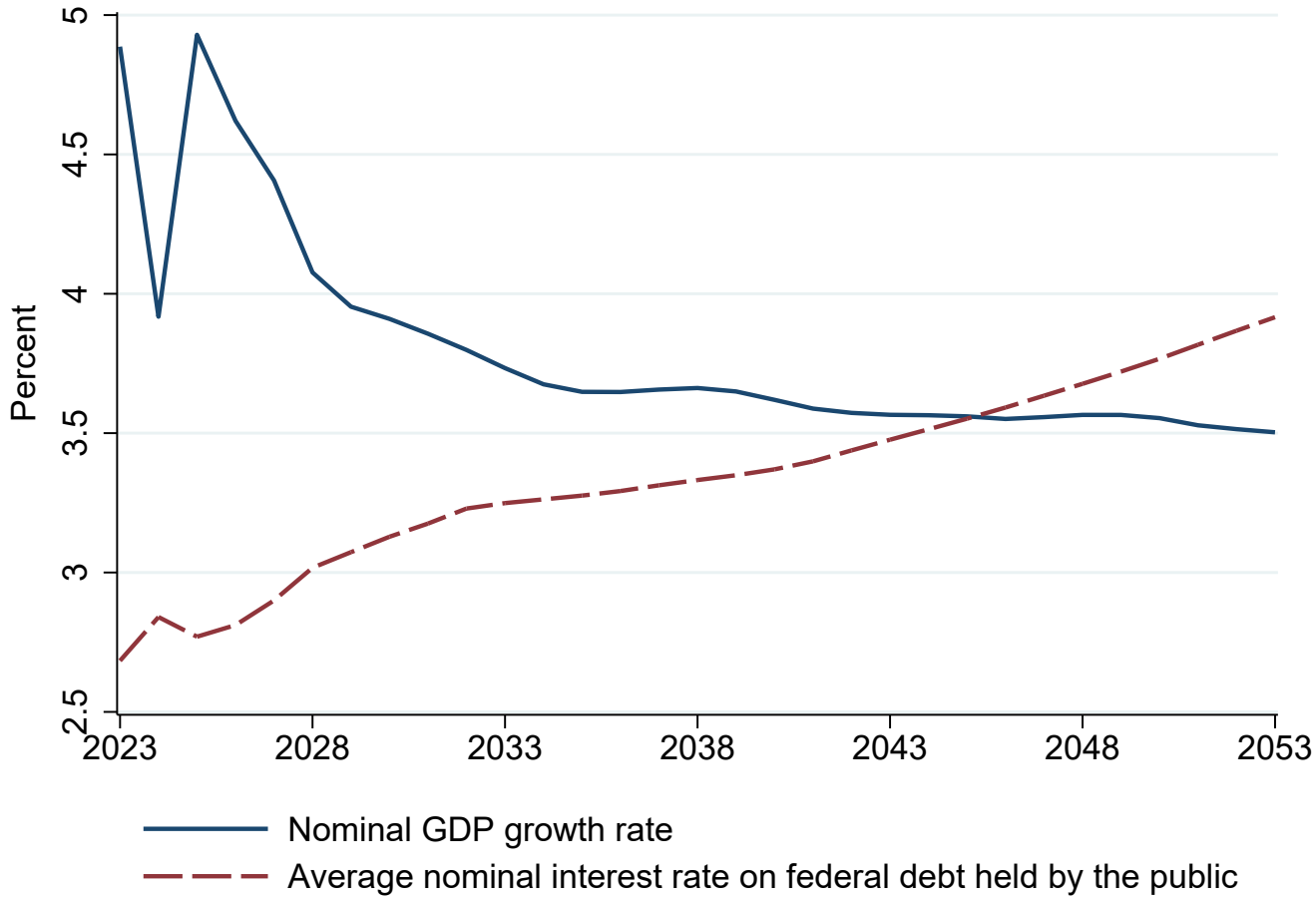
$$\begin{aligned}\underbrace{\frac{DEBT_t}{GDP_t} - \frac{DEBT_{t-1}}{GDP_{t-1}}}_{\text{Debt ratio change}} &= \underbrace{\underbrace{\frac{INTERESTRATE_t \times DEBT_{t-1} + OTHERINT_t}{GDP_t}}_{\text{Net interest}} - \underbrace{\frac{GROWTHRATE_t \times DEBT_{t-1}}{GDP_t}}_{\text{Growth dividend}}}_{\text{Excess interest}} \\ &+ \underbrace{\underbrace{\frac{NONINTERESTOUTLAYS_t}{GDP_t}}_{\text{Non-interest outlays}} - \underbrace{\frac{REVENUES_t}{GDP_t}}_{\text{Revenue}}}_{\text{Primary deficit}} \\ &+ \frac{OTHERMISCELLANEOUS_t}{GDP_t}\end{aligned} \tag{4}$$

FIGURE 1: Debt Ratio Change over Time



Notes: This figure decomposes each year's change in U.S. debt held by the public as a share of GDP, using equation 2. The debt ratio change equals each year's change in the debt ratio: the current year's debt-to-GDP ratio minus the previous year's debt-to-GDP ratio. Excess interest equals net interest minus the growth dividend. The growth dividend equals nominal GDP growth times the previous year's debt divided by the current year's GDP. The primary deficit equals non-interest outlays minus revenues. Other miscellaneous equals other miscellaneous transactions affecting borrowing from the public such as Treasury's cash balance (which CBO calls "other means of financing" and OMB calls "other transactions affecting borrowing from the public". Historical data are from CBO's February 2023 historical tables (https://www.cbo.gov/system/files/2023-02/51134-2023-02-Historical-Budget-Data.xlsx) which are based on OMB's historical tables. Years 2023-2033 are from CBO (2023b).

FIGURE 2: CBO Projects that Excess Interest Will Remain Negative through 2043



Notes: This graph reproduces the third figure in Auerbach and Gale's (2023) using the latest CBO long-term budget outlook (CBO, 2023a) to plot the projected nominal GDP growth rate and the average interest rate on government debt over the next thirty years. The nominal growth rate equals year-on-year growth in nominal GDP minus one: $GROWTHRATE_t = GDP_t/GDP_{t-1} - 1$. The average nominal interest rate on government debt equals CBO's forecast for current year net interest, divided by the previous year's debt held by the public plus half of the current year's primary deficit which is a simple way to account for interest paid on new debt from current year primary imbalance. The graph essentially reproduces Auerbach and Gale's Figure 3. The nominal GDP growth rate is projected to exceed the average nominal interest rate through 2043, which means that excess interest is projected to remain negative through 2043 and then be positive beginning in 2044.

TABLE 1
CBO's Baseline Budget Projections Table

	Actual, 2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032	2033	Total 2024–2028	Total 2024–2033
As a Percentage of Gross Domestic Product														
Revenues														
Individual income taxes	10.5	9.6	9.1	8.8	9.2	9.7	9.6	9.6	9.6	9.6	9.6	9.7	9.3	9.5
Payroll taxes	5.9	6.0	6.0	6.0	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9
Corporate income taxes	1.7	1.8	1.8	1.7	1.7	1.6	1.6	1.5	1.5	1.4	1.4	1.4	1.6	1.5
Other	1.4	1.0	1.0	0.9	0.9	0.9	1.1	1.1	1.1	1.1	1.2	1.1	1.0	1.1
Total	**19.6**	**18.4**	**17.8**	**17.4**	**17.8**	**18.1**	**18.2**	**18.2**	**18.1**	**18.1**	**18.1**	**18.1**	**17.9**	**18.0**
On-budget	15.3	14.0	13.4	13.0	13.4	13.7	13.8	13.8	13.8	13.8	13.7	13.8	13.5	13.6
Off-budget	4.3	4.3	4.4	4.4	4.4	4.4	4.4	4.3	4.3	4.3	4.3	4.3	4.4	4.3
Outlays														
Mandatory	16.5	15.2	14.0	14.1	14.0	14.1	14.6	14.1	14.6	14.8	15.0	15.6	14.2	14.5
Discretionary	6.7	6.5	6.8	6.8	6.7	6.6	6.5	6.4	6.3	6.2	6.1	6.0	6.6	6.4
Net Interest	1.9	2.5	2.7	2.7	2.8	2.9	3.1	3.2	3.3	3.4	3.6	3.7	2.9	3.2
Total	**25.1**	**24.2**	**23.5**	**23.5**	**23.5**	**23.6**	**24.1**	**23.7**	**24.2**	**24.4**	**24.7**	**25.3**	**23.7**	**24.1**
On-budget	20.8	19.6	18.7	18.6	18.5	18.5	19.0	18.4	18.8	18.9	19.1	19.7	18.6	18.8
Off-budget	4.3	4.6	4.9	5.0	5.0	5.1	5.2	5.2	5.4	5.5	5.6	5.7	5.0	5.3
Deficit	**-5.5**	**-5.9**	**-5.8**	**-6.2**	**-5.7**	**-5.5**	**-5.9**	**-5.5**	**-6.1**	**-6.3**	**-6.6**	**-7.3**	**-5.8**	**-6.1**
On-budget	-5.4	-5.6	-5.3	-5.6	-5.1	-4.7	-5.1	-4.6	-5.0	-5.2	-5.3	-5.9	-5.2	-5.2
Off-budget	-0.1	-0.3	-0.5	-0.6	-0.7	-0.7	-0.8	-0.9	-1.0	-1.1	-1.2	-1.3	-0.7	-0.9
Primary Deficit	-3.6	-3.3	-3.0	-3.5	-3.0	-2.6	-2.9	-2.3	-2.7	-2.8	-3.0	-3.6	-3.0	-2.9
Debt Held by the Public	96.9	98.2	100.4	102.2	103.7	105.2	107.3	108.9	111.0	113.3	115.8	118.9	n.a.	n.a.

Notes: This table reprints the bottom half of "Table 1. CBO's Baseline Budget Projections, by Category", found in CBO's May 2023 report An Update to the Budget Outlook: 2023 to 2033. I add four red boxes to highlight an apparent puzzle. First, interest payments are high: CBO projects that net interest payments ("interest" for short) over the next decade will average 3.2% of GDP, which is over twice as large as its 2010s average. Second, primary deficits are also high: CBO projects that the primary deficit – equal to non-interest outlayes (i.e., spending) minus revenues – will average 2.9% of GDP. One might therefore have thought that the debt ratio will rise by 6.1 percentage points per year from 2023 to 2033. Instead, the remaining two red boxes show that CBO projects the debt ratio will rise by only 2.1 percentage points per year, from 98% to 119% over the decade. That is: the debt ratio will rise by substantially less than primary deficits, despite high interest. Table 2 illuminates why.

TABLE 2
CBO's Baseline Budget Projections Table, Amended to Show how the Debt Ratio Changes Each Year

	Actual, 2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032	2033	Total 2024–2028	Total 2024–2033
As a Percentage of Gross Domestic Product														
Revenues														
Individual income taxes	10.5	9.6	9.1	8.8	9.2	9.7	9.6	9.6	9.6	9.6	9.6	9.7	9.3	9.5
Payroll taxes	5.9	6.0	6.0	6.0	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9	5.9
Corporate income taxes	1.7	1.8	1.8	1.7	1.7	1.6	1.6	1.5	1.5	1.4	1.4	1.4	1.6	1.5
Other	1.4	1.0	1.0	0.9	0.9	0.9	1.1	1.1	1.1	1.1	1.2	1.1	1.0	1.1
Total	**19.6**	**18.4**	**17.8**	**17.4**	**17.8**	**18.1**	**18.2**	**18.2**	**18.1**	**18.1**	**18.1**	**18.1**	**17.9**	**18.0**
On-budget	15.3	14.0	13.4	13.0	13.4	13.7	13.8	13.8	13.8	13.8	13.7	13.8	13.5	13.6
Off-budget	4.3	4.3	4.4	4.4	4.4	4.4	4.4	4.3	4.3	4.3	4.3	4.3	4.4	4.3
Outlays														
Mandatory	16.5	15.2	14.0	14.1	14.0	14.1	14.6	14.1	14.6	14.8	15.0	15.6	14.2	14.5
Discretionary	6.7	6.5	6.8	6.8	6.7	6.6	6.5	6.4	6.3	6.2	6.1	6.0	6.6	6.4
Net Interest	1.9	2.5	2.7	2.7	2.8	2.9	3.1	3.2	3.3	3.4	3.6	3.7	2.9	3.2
Total	**25.1**	**24.2**	**23.5**	**23.5**	**23.5**	**23.6**	**24.1**	**23.7**	**24.2**	**24.4**	**24.7**	**25.3**	**23.7**	**24.1**
On-budget	20.8	19.6	18.7	18.6	18.5	18.5	19.0	18.4	18.8	18.9	19.1	19.7	18.6	18.8
Off-budget	4.3	4.6	4.9	5.0	5.0	5.1	5.2	5.2	5.4	5.5	5.6	5.7	5.0	5.3
Deficit	**-5.5**	**-5.9**	**-5.8**	**-6.2**	**-5.7**	**-5.5**	**-5.9**	**-5.5**	**-6.1**	**-6.3**	**-6.6**	**-7.3**	**-5.8**	**-6.1**
On-budget	-5.4	-5.6	-5.3	-5.6	-5.1	-4.7	-5.1	-4.6	-5.0	-5.2	-5.3	-5.9	-5.2	-5.2
Off-budget	-0.1	-0.3	-0.5	-0.6	-0.7	-0.7	-0.8	-0.9	-1.0	-1.1	-1.2	-1.3	-0.7	-0.9
Primary Deficit	-3.6	-3.3	-3.0	-3.5	-3.0	-2.6	-2.9	-2.3	-2.7	-2.8	-3.0	-3.6	-3.0	-2.9
Debt Held by the Public	96.9	98.2	100.4	102.2	103.7	105.2	107.3	108.9	111.0	113.3	115.8	118.9	n.a.	n.a.
Change in Debt Held by the Public	**-1.4**	**1.3**	**2.2**	**1.8**	**1.5**	**1.4**	**2.1**	**1.6**	**2.1**	**2.3**	**2.5**	**3.1**	**1.8**	**2.1**
Excess Interest (Effect of past debt)	-7.4	-2.0	-1.0	-2.0	-1.7	-1.5	-1.0	-0.9	-0.8	-0.7	-0.6	-0.5	-1.4	-1.1
Net Interest	1.9	2.5	2.7	2.7	2.8	2.9	3.1	3.2	3.3	3.4	3.6	3.7	2.8	3.1
Growth Dividend	9.3	4.5	3.7	4.7	4.5	4.4	4.1	4.1	4.1	4.1	4.1	4.2	4.3	4.2
Primary Deficit (Effect of current imbalance)	3.6	3.3	3.0	3.5	3.0	2.6	2.9	2.3	2.7	2.8	3.0	3.6	3.0	2.9
Non-Interest Outlays	23.2	21.7	20.8	20.8	20.7	20.7	21.1	20.4	20.9	21.0	21.1	21.7	20.8	20.9
Revenues	19.6	18.4	17.8	17.4	17.8	18.1	18.2	18.2	18.1	18.1	18.1	18.1	17.8	18.0
Other Means of Financing	2.4	-0.1	0.2	0.3	0.3	0.3	0.3	0.2	0.2	0.1	0.0	0.0	0.3	0.2

Notes: This table adds lines to Table 1 in order to show why CBO projects that the debt ratio will rise by substantially less than primary deficits, despite high interest: excess interest, which is what actually matters for the effect of past debt on the debt ratio, is projected to be negative throughout the decade. They key rows are the four in darker blue at the bottom. They use equation 2 to show that each year's percentage-point change in the debt-to-GDP ratio equals excess interest plus the primary deficit plus other means of financing (a miscellaneous category that goes to zero in the long run). Excess interest equals net interest minus a new term: the growth dividend. The growth dividend equals the percentage points by which nominal GDP growth shrinks the debt ratio: the nominal growth rate times last year's debt, divided by this year's GDP. The debt ratio deficit is smaller than the primary deficit in every year because excess interest is negative in every year. Excess interest is negative in every year because the average nominal interest rate on government debt is projected to be smaller than the nominal GDP growth rate in every year.

TABLE 3
OMB Budget Totals, Amended to Show how the Debt Ratio Changes Each Year
(In billions of dollars and as a percent of GDP)

	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032	2033	Totals 2024-2028	Totals 2024-2033
Budget totals in billions of dollars:														
Receipts	4,897	4,802	5,036	5,419	5,773	6,080	6,400	6,669	6,953	7,264	7,601	7,991	28,708	65,187
Outlays	6,273	6,372	6,883	7,091	7,294	7,589	8,003	8,205	8,639	9,040	9,472	10,026	36,860	82,242
Deficit	1,376	1,569	1,846	1,671	1,521	1,509	1,604	1,536	1,686	1,776	1,871	2,035	8,151	17,054
Debt held by the public	24,252	25,910	27,783	29,592	31,233	32,851	34,517	36,106	37,838	39,650	41,553	43,619		
Debt held by the public net of financial assets	22,049	23,619	25,465	27,137	28,658	30,167	31,771	33,306	34,997	36,776	38,647	40,681		
Gross domestic product (GDP)	25,000	26,336	27,238	28,432	29,679	30,909	32,188	33,534	34,968	36,489	38,076	39,732		
Budget totals as a percent of GDP:														
Receipts	19.6%	18.2%	18.5%	19.1%	19.5%	19.7%	19.9%	19.9%	19.9%	19.9%	20.0%	20.1%	19.3%	19.6%
Outlays	25.1%	24.2%	25.3%	24.9%	24.6%	24.6%	24.9%	24.5%	24.7%	24.8%	24.9%	25.2%	24.8%	24.8%
Deficit	5.5%	6.0%	6.8%	5.9%	5.1%	4.9%	5.0%	4.6%	4.8%	4.9%	4.9%	5.1%	5.5%	5.2%
Debt held by the public	97.0%	98.4%	102.0%	104.1%	105.2%	106.3%	107.2%	107.7%	108.2%	108.7%	109.1%	109.8%		
Debt held by the public net of financial assets	88.2%	89.7%	93.5%	95.4%	96.6%	97.6%	98.7%	99.3%	100.1%	100.8%	101.5%	102.4%		
Memorandum, real net interest:														
Real net interest in billions of dollars	-1,064	-513	177	242	249	254	271	298	337	375	417	447	1,192	3,066
Real net interest as a percent of GDP	-4.3%	-1.9%	0.6%	0.9%	0.8%	0.8%	0.8%	0.9%	1.0%	1.0%	1.1%	1.1%	0.8%	0.9%
Memorandum, change in debt held by the public as a percent of GDP:														
Excess interest (Effect of past debt):														
Net interest	1.9%	2.5%	2.9%	2.9%	2.9%	2.9%	3.0%	3.0%	3.1%	3.2%	3.3%	3.3%	2.9%	3.1%
Growth dividend	10.4%	4.9%	3.3%	4.3%	4.4%	4.2%	4.2%	4.3%	4.4%	4.5%	4.5%	4.5%	4.1%	4.3%
Subtotal, excess interest	-8.5%	-2.4%	-0.4%	-1.4%	-1.5%	-1.2%	-1.2%	-1.3%	-1.3%	-1.3%	-1.2%	-1.2%	-1.1%	-1.2%
Primary deficit (Effect of current imbalance):														
Receipts	19.6%	18.2%	18.5%	19.1%	19.5%	19.7%	19.9%	19.9%	19.9%	19.9%	20.0%	20.1%	19.3%	19.7%
Non-interest outlays	23.2%	21.7%	22.4%	22.0%	21.7%	21.6%	21.9%	21.4%	21.6%	21.6%	21.6%	21.9%	21.9%	21.8%
Subtotal, primary deficit	3.6%	3.5%	3.9%	2.9%	2.2%	1.9%	2.0%	1.5%	1.7%	1.7%	1.6%	1.8%	2.6%	2.1%
Other transactions affecting borrowing from the public	2.4%	0.3%	0.1%	0.5%	0.4%	0.4%	0.2%	0.2%	0.1%	0.1%	0.1%	0.1%	0.3%	0.2%
Total, change in debt held by the public	-2.5%	1.4%	3.6%	2.0%	1.1%	1.0%	0.9%	0.4%	0.5%	0.5%	0.4%	0.7%	1.7%	1.1%

Notes: This table reproduces Table S-1 from the President's Fiscal Year 2024 Budget, with added rows to show how the debt ratio evolves from year to year. Like Table 2, the four key rows are in darker blue. See the notes to that table for details. OMB's "Other transactions affecting borrowing from the public" corresponds to CBO's "Other means of financing." (I have removed footnotes and also a memorandum due to October 1 falling on a weekend.)