# Artificial Intelligence in the Knowledge Economy*

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**Abstract**

How does Artificial Intelligence (AI) affect the organization of work? We incorporate AI into an economy where humans endogenously sort into hierarchical firms: Less knowledgeable agents become "workers" (execute routine tasks), while more knowledgeable agents become "managers" (specialize in problem-solving). We model AI as an algorithm that uses capital ("computing power") to mimic the behavior of humans with a given knowledge. AI leads to major reorganizations, affecting occupational choice, wages, and the size and productivity of firms. By explicitly incorporating the endogenous organization of knowledge, our results provide new insights into the effects of AI on the future of work.

## 1 Introduction

Artificial intelligence (AI) is a new, powerful form of automation based on machines that can perform cognitive tasks typically associated with the human mind. While the transformative impact of AI on the landscape of work is undeniable, its precise implications have become the center of a growing controversy. Indeed, predictions range from optimistic utopian visions to ominous apocalyptic scenarios (Meserole, 2018; Brynjolfsson, 2022; Johnson and Acemoglu, 2023; Autor, 2024).

Unlike previous waves of automation—that primarily led to the creation of tools proficient at handling repetitive tasks at scale—AI has demonstrated the capacity to address highly non-routine tasks previously reserved for highly skilled workers with sophisticated knowledge (Webb, 2020; Autor, 2024).[1] Consequently, it is not immediately apparent that experience with previous waves of automation will seamlessly extrapolate to the case of AI.[2]

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[1] According to Autor (2024, p. 7): "AI's capacity to depart from script, to improvise based on training and experience, enables it to engage in expert judgment — a capability that, until now, has fallen within the province of elite experts. Though only in its infancy, this is a superpower."

[2] For instance, Muro et al. (2019) state: ..." [When studying AI] most research has concentrated on an undifferenti-

With this issue in mind, in this paper we propose a new framework to study the effects of AI on the labor market. Our approach is motivated by two key observations. First, AI holds the potential to automate knowledge work. Second, as underscored by Hayek (1945), a fundamental economic challenge for society is the efficient utilization of the available knowledge. Therefore, to uncover the effects of AI on the future of work, it is imperative that we consider how economic agents organize themselves in terms of knowledge and time to achieve common production goals.

The overarching theme of our findings is that AI leads to major reorganizations in the economy. This leads to new insights. For example, while there is a growing interest in understanding the extent to which AI increases productivity at any given task (e.g., Brynjolfsson et al., 2023; Eloundou et al., 2023), we show how, once organizational effects are taken into account, AI may lead to a *reduction* in the productivity of a significant subset of the population. This occurs, for instance, when the best managers switch to work with AI, leaving a worse pool of managers to assist workers.

The starting point of our analysis is the seminal papers by Antràs et al. (2006), and Fuchs et al. (2015), who consider an economy where labor is the sole input for production. Humans are endowed with one unit of time and are heterogeneous in terms of knowledge. Production occurs when a human dedicates her full unit of time to production, and her knowledge exceeds the difficulty of the problems she confronts.

The competitive equilibrium in this setting—which we take as our pre-AI outcome—involves humans either trying to solve problems on their own (becoming "independent producers") or sorting into hierarchical firms to make more efficient use of their time and knowledge. These firms have two key features. First, they exhibit *management by exception*: Less knowledgeable humans become "workers" who try to solve problems first, while more knowledgeable humans become "managers" specializing in the exceptional problems the first layer of workers was unable to solve. Second, there is *positive assortative matching*: More knowledgeable workers are always matched to more knowledgeable managers.[3]

Our contribution is to incorporate AI into this otherwise canonical setting. We model AI as an algorithm that uses computing power (i.e., "capital") to mimic the behavior of a human with a given knowledge. As standard in the automation literature, we assume that all firms have access to this technology. Firms that use AI are identical to those that do not, except that they rent computing power instead of hiring labor to do either production or managerial work. Moreover, we assume that AI can be used at scale, in the sense that computing power is large relative to human time.

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ated array of 'automation' technologies including robotics, software, and AI all at once . . . [As a result,] past 'automation' analyses—including our own—have likely obscured AI's distinctive impact."

[3]There is both anecdotal and systematic empirical evidence showing the emergence of such "knowledge hierarchies" (see, e.g., Garicano and Hubbard, 2012; Caliendo and Rossi-Hansberg, 2012; Caliendo et al., 2015, 2020). For instance, Alfred Sloan (1924), a former head of General Motors (GM), once wrote: "We do not do much routine work with details. They never get up to us. I work fairly hard, but on exceptions."

We then characterize the competitive equilibrium when firms have access to AI. We show that if AI is used as a worker, it is necessarily the *most* knowledgeable worker in the economy, so it is supervised by the *most* knowledgeable humans. In addition, if AI is used as a manager, it is necessarily the *least* knowledgeable manager in the economy, so it assists the *least* knowledgeable humans.

A notable property of the equilibrium is that, as long as AI has unlimited potential applications, the rental rate of computing power is equal to AI's knowledge, and, as a result, AI does not lead to the complete destruction of human routine jobs. This result arises despite the fact that AI is more knowledgeable than a subset of the human population and can be used at scale. Intuitively, even though computing power is large relative to human time, it is still scarce relative to its potential uses. Hence, it continues to be worthwhile for every human to be employed in some capacity. The competitive equilibrium then allocates all those humans who are less knowledgeable than AI to be workers because they are the least likely to succeed on their own.[4]

We then turn to our main endeavor: Analyzing the effects of AI on labor outcomes by comparing the pre- and post-AI equilibrium. In particular, we study the implications of AI for (i) occupational choice, (ii) team formation and the size and productivity of firms, and (iii) labor income.

Regarding occupational choice, we show that when AI has the knowledge of a pre-AI worker, it displaces humans from routine to managerial work. In contrast, when AI has the knowledge of a pre-AI manager, the displacement goes in the opposite direction. Intuitively, when AI has the knowledge of a pre-AI worker, AI gives firms access to a relatively cheap technology to do routine work. This reduces workers' wages and increases the attractiveness of creating hierarchical firms. The result is a surge in the demand for managers, which induces the most knowledgeable routine workers of the pre-AI equilibrium to switch to managerial roles. A similar intuition explains why AI displaces the least knowledgeable managers into routine work when it has the knowledge of a pre-AI manager.

These displacement results have remarkable implications for the distribution of firm size (defined as firm output) and productivity (defined as firm output divided by the time/computing power it uses). For example, in the case of two-layer organizations, the maximum firm size is determined by the knowledge of the best worker, while the minimum firm size and the productivity of the least productive firm are determined by the knowledge of the worst manager. Hence, when AI has the knowledge of a pre-AI worker, the size of the biggest and the smallest firm and the productivity of the least productive firm decrease with AI (as the knowledge threshold to become a manager decreases with AI). Similarly, when AI has the knowledge of a pre-AI manager, the size of the biggest and the smallest firm and the productivity of the least productive firm increase with AI (as the knowledge

[4]In Section 5, we consider the case where computing power is so large that it is abundant not only relative to human time but also relative to its potential applications. In that case, the price of computing power is zero and AI leads to the complete destruction of human routine jobs. Firms, however, still have a hierarchical structure: All problems are initially attempted by AI, and all those humans who are more knowledgeable than AI specialize in solving problems that AI cannot solve.

threshold to become a manager increases with AI).[5]

After examining the repercussions of human displacement across occupations, we analyze the effects of AI on the workers and managers *who have not been displaced*. For workers, we focus on productivity (defined as their expected output). For managers, we focus on the size of the firm they supervise (again, defined as total output). We show that after the introduction of AI, all workers become less productive, except possibly the least knowledgeable ones, while all managers supervise larger firms, except possibly the very best ones. Interestingly, this result holds irrespective of whether AI has the knowledge of a pre-AI worker or a pre-AI manager.

To understand these results, note that a worker's productivity increases with her manager's knowledge, while the size of the firm that a manager supervises increases with her workers' knowledge. When AI has the knowledge of a pre-AI worker, the knowledge threshold to become a manager decreases, worsening the pool of managers available for non-displaced workers. At the same time, the pool of workers available for the worst non-displaced managers improves as the least knowledgeable agents match with the newly appointed managers. Similar forces arise when AI has the knowledge of a pre-AI manager: In such a case, the knowledge of the best worker increases, improving the pool of available workers for non-displaced managers and, therefore, leaving a worse pool of managers for non-displaced workers.

Finally, we study the effects of AI on labor income. We show that AI increases total labor income but that it creates winners and losers in the labor market. These distributional effects are shaped by two potentially countervailing forces: On the one hand, AI changes the composition of firms and, therefore, the quality of matches. On the other hand, AI changes the relative scarcities of different knowledge levels, affecting how each firm's output is divided between workers and managers. Which of these two forces dominates depends on the interaction between AI's knowledge and communication costs: Only the most knowledgeable humans benefit when AI's knowledge is low, while only the least knowledgeable humans benefit when AI's knowledge and communication costs are high. In any case, the winners from AI are always at the extremes of the knowledge distribution.

The rest of this paper is organized as follows. Following a brief discussion of the most closely related literature, Section 2 presents our model. After describing the pre-AI equilibrium in Section 2.3, we characterize the equilibrium with AI in Section 3. In Section 4, we turn to our main endeavor: Comparing the pre- and post-AI equilibrium; this section contains our main results. Section 5 discusses several extensions, including the cases in which computing power is (i) arbitrarily small and (ii) abundant not only relative to human time but also relative to its potential applications. Section 6 concludes.

[5] In Section 4.2, we also characterize the effects of AI on the size and productivity distribution of all firms. We show that the results are similar, except that the effects of AI on the size of the smallest firm are less clear-cut.

### Related literature

This paper makes two contributions. First, it introduces automation and artificial intelligence to the literature on knowledge hierarchies. Second, it incorporates organizations and a different way of thinking about AI to the automation literature. In this section, we discuss how these contributions relate to existing work.

The literature on knowledge hierarchies starts with Garicano (2000), which introduces the technology and describes the circumstances under which knowledge hierarchies are optimal when agents are homogenous. Garicano and Rossi-Hansberg (2004, 2006) embed this model in a setting with heterogenous agents to study how the endogenous organization of knowledge interacts with inequality. Fuchs et al. (2015), in turn, show that firm-like contractual arrangements uniquely deliver the first-best when there is double-sided asymmetric information about participants' ability to solve problems.[6] In contrast to our paper, none of the work in this literature considers automation or artificial intelligence.

In the context of knowledge hierarchies, the paper closest to us is Antràs et al. (2006), who study the effects of offshoring by comparing the equilibrium of a closed economy with one in which firms can form international teams. The main difference with our paper is that while offshoring gives firms access to a population of humans with different knowledge, artificial intelligence gives firms access to an algorithm that can solve problems *at scale* (up to a given difficulty). As we discuss in detail in Section 5.5, this implies that the effects of AI are qualitatively different than those of offshoring.

Our paper also contributes to the literature on automation (e.g., Zeira, 1998; Acemoglu and Autor, 2011; Aghion et al., 2017; Acemoglu and Restrepo, 2018, 2022; Acemoglu and Loebbing, 2024; Azar et al., 2023). This literature uses task-based models to study the effects of automation on labor outcomes, inequality, and economic growth.

The first important recent contribution to this literature is due to Zeira (1998), who shows how automation can lead to a decline in the labor share as the economy develops. Acemoglu and Restrepo (2018) counter that, by depressing the equilibrium wage, automation also encourages the creation of new tasks in which labor has a comparative advantage. Hence, even though the direct effect of automation is to decrease the labor share, the economy might still exhibit a balanced growth path in which the labor share stays constant in time. Expanding on this, Acemoglu and Restrepo (2022) delve into the effects of automation on wage inequality, and show that a substantial fraction of the changes in the US wage structure in recent decades can be attributed to relative wage declines among worker groups undergoing rapid automation.

We depart from this literature in two key respects. First, we explicitly integrate organizations into

[6]Other important contributions to this literature include Garicano and Hubbard (2007), Caliendo and Rossi-Hansberg (2012), Garicano and Rossi-Hansberg (2012), Caicedo et al. (2019), and Carmona and Laohakunakorn (Forthcoming). See Garicano and Rossi-Hansberg (2015) for an excellent survey on knowledge hierarchies.

the analysis by employing knowledge hierarchies. As explained by Garicano and Rossi-Hansberg (2015), this is a particular specification of the task-based framework, where tasks are hierarchical and the relationship between them arises from an explicit organizational problem. By incorporating organizations into our analysis, we present novel insights about the impact of AI on the size and productivity distributions of firms. Moreover, our approach offers a new perspective on how AI influences labor outcomes through its effects on organizational structure.

Second, we adopt a different approach to model AI. While existing literature often models automation as the capability of machines to replace workers in an exogenously determined subset of tasks, we assume that AI can perform the exact same tasks as humans but can only mimic the performance of a subset of the human population. This approach is motivated by the idea that—in contrast to previous waves of automation—artificial intelligence may be indistinguishable from human intelligence. Moreover, it offers a significant advantage: the tasks that are automated are determined endogenously as a function of (i) the knowledge of the population, (ii) the level of information and communication technologies, and (iii) the advancement of AI.[7]

# 2 The Model

We consider a perfectly competitive economy where producing output requires solving problems. The model builds on Garicano (2000), Antràs et al. (2006), and Fuchs et al. (2015). Our innovation is to introduce AI in this otherwise canonical setting. After describing the model in Section 2.1, we discuss its main assumptions in Section 2.2.

## 2.1 The Baseline Setting

*The Pre-AI Economy*.—There is a unit mass of humans, each endowed with one unit of time and exogenous knowledge $z \in [0, 1]$. The distribution of knowledge in the population is given by a continuous probability distribution with full support on $[0, 1]$, cumulative distribution function $G(z)$, and a bounded density $g(z)$. The knowledge of each individual is perfectly observable.

There is a large measure of potential and identical competitive firms that can enter the market. Entry is free, and firms are risk-neutral. Production occurs inside firms—the residual claimants of all output—and requires solving problems.

In particular, upon entering the market, firms hire humans to produce. Each of these "production workers" devotes her full unit of time and applies her knowledge to a single problem of difficulty $x$. The difficulty of each problem is ex-ante unknown and distributed uniformly on $[0, 1]$, independently

[7] Acemoglu and Loebbing (2024) provide an alternative assignment model where the tasks being automated are endogenously determined. Their model, however, is different than ours in that tasks are not hierarchical and there are no organizations.

across problems.[8] If the worker's knowledge exceeds the problem's difficulty, she solves the problem and produces one unit of output. Otherwise, the worker produces no output.

Additionally, the firm can hire another human to act as its workers' "manager." In that case, production workers can ask their manager for help when they are unable to solve a problem on their own. This exchange consumes $h \in (0, 1)$ units of the manager's time. If the manager's knowledge exceeds the problem's difficulty, the manager communicates the solution to the problem to the corresponding worker, who then produces a unit of output. Otherwise, no production takes place. We normalize the value of each unit of output to one.

*Artificial Intelligence*.— We model AI as an algorithm that can use computing power to mimic the behavior of a human with knowledge $z_{\text{AI}} \in (0, 1)$. Hence, AI is an automation technology. We refer to $z_{\text{AI}}$ as "AI's knowledge."

All firms have access to AI. Thus, in contrast to the pre-AI economy, firms decide not only their organizational structure but also whether to use this technology. Firms that use AI are identical to those that do not, except that they use AI instead of humans in either production work or problem-solving (i.e., managerial roles). To do this, they must rent one unit of computing power per production worker or manager they replace. The amount of computing power in the economy—which is exogenously given—is denoted by $\mu \geq 0$.

Our main endeavor is to analyze the effects of AI by comparing the pre-AI equilibrium with the post-AI equilibrium when computing power is abundant relative to human time:

$$\frac{1}{h(1 - z_{\text{AI}})} < \mu \tag{1}$$

This inequality guarantees that the binding constraint in human-AI interactions is human time, not computing power.[9] In Section 5.2, we explore the role of this assumption by comparing the pre- and post-AI equilibrium in the opposite extreme in which computing power is arbitrarily small.

*Wages, Prices, and Profits*.—Let $w(z)$ be the wage of a human with knowledge $z$ and denote by $r$ the rental rate of one unit of computing power. All agents in this economy are income maximizers.

The problem of an active firm is to decide (i) whether to use humans or AI in production (and the knowledge of the human workers, when appropriate), (ii) whether to operate as a one-layer or two-layer organization, and (iii) in the case of two-layer organizations, whether to use a human or AI as manager (and the knowledge of the human manager, when appropriate).

[8]The assumption that $x$ is uniformly distributed is without loss, given that the distribution of knowledge in the population is arbitrary (i.e., assuming a uniform distribution is simply a normalization).

[9]The most intensive (though not necessarily optimal) way to use computing power in organizations comprising humans and AI is to use it for production work. Since there is a unit mass of humans and the maximum number of production workers with knowledge $z_{\text{AI}}$ that a human can manage is $[h(1 - z_{\text{AI}})]^{-1}$, condition (1) implies that there are not enough humans to exhaust all computing power in the post-AI world.

The profit of a single-layer organization is simply:

$$\Pi_1 = \begin{cases} z - w(z) & \text{if the firm hires a human with knowledge } z \\ z_{\text{AI}} - r & \text{if the firm uses AI} \end{cases}$$

The profit of a two-layer organization, in turn, depends on whether it does not use AI (a "$n$A" firm), uses AI as a manager (i.e., automates the top layer, a "$t$A" firm), or uses AI as a worker (i.e., automates the bottom layer, a "$b$A" firm).[10] In either case, we restrict attention to matching arrangements in which all workers matched with a given manager have the same knowledge. This restriction is without loss since the equilibrium exhibits positive assortative matching.

Consequently, in order to exploit its manager's time fully, a two-layer organization that hires workers with knowledge $z$ optimally hires exactly $n(z) = [h(1-z)]^{-1}$ workers. The profits of a two-layer organization as a function of its type ($n$A, $t$A, or $b$A) are thus:

$$\begin{aligned} \Pi_2^{n\text{A}}(z,m) &= n(z)[m - w(z)] - w(m) \\ \Pi_2^{t\text{A}}(z) &= n(z)[z_{\text{AI}} - w(z)] - r \\ \Pi_2^{b\text{A}}(m) &= n(z_{\text{AI}})[m - r] - w(m) \end{aligned}$$

where $z$ and $m$ denote the knowledge of a human worker and manager, respectively.

For brevity, we will reserve the term "worker" for the agents engaging in production in two-layer organizations, while we use the term "independent producer" to refer to the agents working in production in single-layer organizations.

*Competitive Equilibrium*.— Denote by $I$ the set of humans hired as independent producers, and let $W_p$ and $W_a$ be the human workers managed by other humans and managed by AI, respectively.[11] Moreover, denote by $M_p$ the set of humans who manage other humans, and by $M_a$ the set of humans managing AI. Finally, let $\mu_i$, $\mu_w$, and $\mu_m$ be the mass of computing power rented for independent production, production in two-layer firms, and supervision of humans, respectively.

**Definition (Competitive Equilibrium).** An equilibrium consists of non-negative masses $(\mu_i, \mu_w, \mu_m)$, sets $(W_p, W_a, I, M_p, M_a)$, a feasible matching function $f : W_p \to M_p$, a wage schedule $w : [0,1] \to \mathbb{R}_+$ and a rental rate of computing power $r \in \mathbb{R}_+$, such that:

1. Firms optimally choose their structure (while earning zero profits).
2. $n$A firms that hire workers with knowledge $z$ hire a manager with knowledge $f(z)$.
3. $t$A firms hire workers with knowledge in $W_a$.
4. $b$A firms hire managers with knowledge in $M_a$.

[10] Note that a firm will never use AI at both layers of the organization. This is because an AI manager solves the exact same problems as its AI worker.

[11] We use the subscript "$p$" (for people) instead of "$h$" (for human), to avoid any confusion with the helping cost $h$.

5. Markets clear: (i) $\mu_i + \mu_w + \mu_m = \mu$, and (ii) the union of the sets $(W_p, W_a, I, M_p, M_a)$ is $[0,1]$ and the intersection of any two of these sets has measure zero.

*Some Notation.*— For future reference, we define $W \equiv W_a \cup W_p$ and $M \equiv M_a \cup M_p$ as the overall set of human workers and managers of this economy, respectively. We also denote by $e(m)$ the inverse of the matching function $f(z)$. That is, $e(m)$ is the "employee matching function" denoting the knowledge of the human worker matched with a human manager with knowledge $m$. This function always exists given that, as shown below, the equilibrium matching function is strictly increasing.

Finally, we denote by $\text{int}S$ the interior of the set $S$. We also use $S \preceq S'$ to symbolize the idea that the set $S \subseteq [0,1]$ "lies below" the set $S' \subseteq [0,1]$. Formally, $S \preceq S'$ if $\sup S \leq \inf S'$ (or either $S$ or $S'$ is empty).[12] For example, $W_a \preceq W_p$ means that the best worker managed by AI is weakly less knowledgeable than the worst worker managed by humans.

## 2.2 Discussion of the Model

Before moving on to the main analysis, we briefly comment on some assumptions underlying our model. First, we focus on the case where computing power is large relative to human time because the hallmark of AI lies in its ability to operate *at scale*. This scalability is a defining feature of automation technologies, distinguishing them from other input supply shocks, such as immigration and offshoring. As mentioned above, we explore the role of this assumption in Section 5.2, where we characterize the post-AI equilibrium in the opposite extreme in which computing power is small.

Second, our model assumes that there are unlimited uses for AI (i.e., computing power is abundant relative to time but scarce relative to its potential applications) and that AI can perform the same roles in the knowledge economy as humans (i.e., AI can be a worker, a manager, or an independent producer). In Section 5.3, we discuss how our results change when computing power is also abundant relative to its uses, while in Section 5.6, we provide current real-world examples of AI being used in the three different roles of the knowledge economy.

Third, our main goal with this paper is to analyze how AI affects human labor outcomes (i.e., its effects on job displacement, productivity, and wages, among others). For this reason, we take AI technology and the economy's processing capacity as given. An important implication of this assumption is that computing power is exclusively used for the deployment of AI systems rather than for their training.[13] Studying firms' incentives to develop AI or increase the economy's computing

[12] Defining $S \preceq S'$ even if $S$ or $S'$ may be empty allows us to be more concise. For example, we will write $W_a \preceq W_p$ and $M_p \preceq M_a$ even if $W_a$ and/or $M_a$ are empty (in which case $\sup W_a = \infty$ and $\inf M_a = -\infty$).

[13] The industry separates AI's use of computing power between "training" (i.e., teaching AI systems how to respond) and "deployment" or "inferencing" (i.e., processing and reacting to new bits of information). As of 2023, more than 40% of Nvidia's data center business (the leading supplier of specialized microchips for AI) was for the deployment of AI systems, and that share is predicted to grow in the future. See Asa Fitch, "How a Shifting AI Chip Market Will Shape Nvidia's Fu-

power are intriguing avenues for future research.

Fourth, we assume that AI cannot mimic the most knowledgeable human, i.e., $z_{\text{AI}} < 1$. Moreover, we follow Antràs et al. (2006), and Fuchs et al. (2015) in assuming that the distribution of human knowledge is exogenous and that organizations have at most two layers. We opt for these assumptions primarily for the sake of simplicity, although we believe they offer a good first approximation to the problem at hand. Understanding the effects of artificial superintelligence and the impact of AI when human retraining or more complex organizations are feasible is left for future research.

Fifth and finally, we have introduced a large measure of anonymous firms whose role in the economy is to organize production. The existence of such firms is not strictly needed: The same outcome in terms of allocations and income arises if workers or managers are the owners of firms.[14] However, introducing this large measure of anonymous firms allows us to present the model in the most concise and cleanest possible way.

### 2.3 Benchmark: The Pre-AI Equilibrium

We begin by presenting a partial characterization of the equilibrium without AI (for the full characterization, see Appendix A). This is also the equilibrium when computing power $\mu$ is zero and was originally described by Fuchs et al. (2015).[15] Note that in this case $W_a = M_a = \emptyset$, so $W = W_p$ and $M = M_p$.

**Proposition 1.** *In the absence of AI, there is a unique equilibrium. The shape of this equilibrium depends on whether the "helping cost" $h$ is above or below a cutoff $h_0 \in (0,1)$:*

- *If $h \leq h_0$, there is $\hat{z} \in (0,1)$ such that $W = [0, \hat{z}]$, $I = \emptyset$, and $M = [\hat{z}, 1]$.*
- *If $h > h_0$, there are cutoffs $0 < \underline{z} < \bar{z} < 1$ such that $W = [0, \underline{z}]$, $I = (\underline{z}, \bar{z})$ and $M = [\bar{z}, 1]$.*

*In either case, there is strictly positive assortative matching (that is, the equilibrium matching function $f : W \to M$ is strictly increasing), and the wage function $w(z)$ is continuous, strictly increasing, and (weakly) convex. Moreover, it satisfies:*

- *$w(z) > z$ for $z \in W$ (except possibly at $z = \sup W$), and $w'(z) \in (0,1)$, and $w''(z) > 0$ for $z \in \text{int} W$.*
- *$w(z) = z$ for all $z \in I$.*
- *$w(z) > z$ for $z \in M$ (except possibly at $z = \inf M$), and $w'(z) > 1$, and $w''(z) > 0$ for $z \in \text{int} M$.*

*Proof.* See Appendix A. □

ture," *The Wall Street Journal*, February 25, 2024, https://www.wsj.com/tech/ai/how-a-shifting-ai-chip-market-will-shape-nvidias-future-f0c256b1 (accessed February 26, 2024).

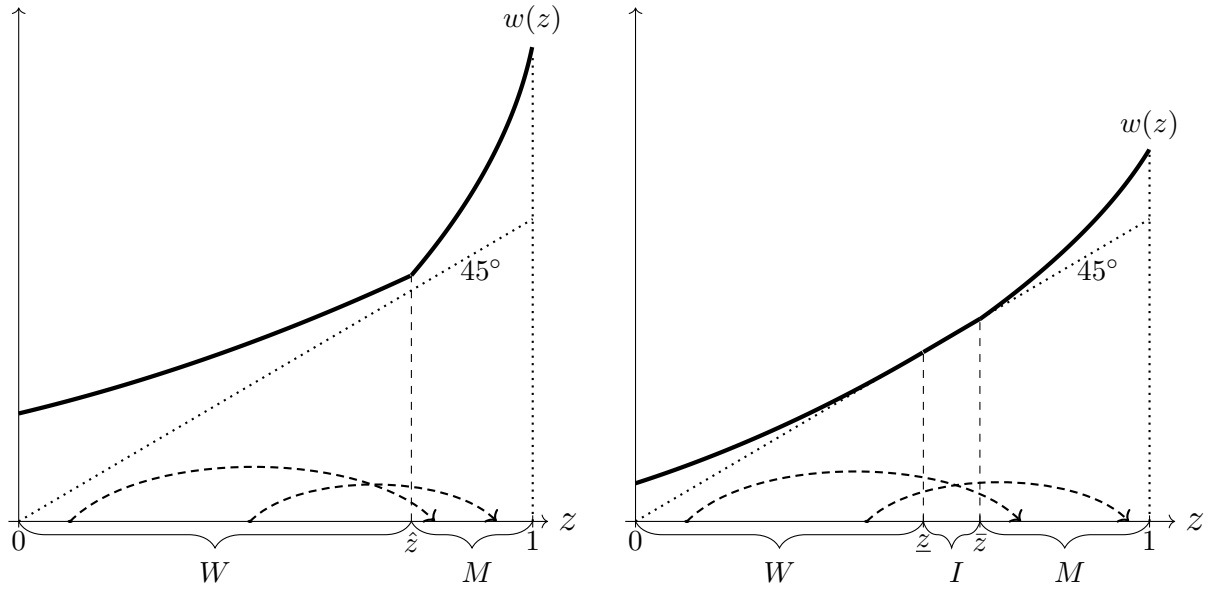
[14] Fuchs et al. (2015) refer to the case where workers are the owners of firms as a "consultant market arrangement," while they refer to the case where managers are the owners of firms as a "referral market arrangement."

[15] Note that an economy with $\mu = 0$ is different than an economy with $\mu > 0$ but $z_{\text{AI}} = 0$. This is because, even if AI cannot solve any problems, it can still draw them, enlarging the production possibility frontier of the economy.

The equilibrium without AI—which we illustrate in Figure 1—has several salient features. First, it exhibits occupational stratification: Managers are more knowledgeable than independent producers (i.e., $I \preceq M$), who, in turn, are more knowledgeable than workers (i.e., $W \preceq I$). This is because the marginal value of knowledge is strictly higher than 1 for managers (as they can leverage their knowledge by applying it to more than one problem), exactly equal to 1 for independent producers (as expected output equals knowledge for them), and strictly lower than 1 for workers (as their knowledge is used to free up managerial time).

Second, there is positive assortative matching: More knowledgeable workers are matched with more knowledgeable managers. The reason is that worker and managerial knowledge are complements, as a more knowledgeable manager helps produce more output (so she should manage a large team), and team size is increasing in the knowledge of workers (as more knowledgeable workers ask fewer questions).

Third, workers and managers earn strictly more than their output as independent producers (except possibly in the case of the most knowledgeable worker and the least knowledgeable manager). Indeed, since the marginal value of manager knowledge is larger than 1, if managers earned their expected output as independent producers, all two-layer firms would only want to hire the most




(a) Pre-AI Equilibrium when $h < h_0$

(b) Pre-AI Equilibrium when $h > h_0$

Figure 1: Illustration of the Pre-AI equilibrium.

*Notes.* Distribution of knowledge: $G(z) = z$. Parameter values: Panel (a) has $h = 1/2$ ($< h_0 = 3/4$), while panel (b) has $h = 0.8125$ ($> h_0 = 3/4$). The thick line depicts the wage function $w(z)$. The dashed arrows illustrate the matching function $f(z)$ at two arbitrary points. See Section 2 of the Online Appendix for more details on the construction of this figure.

knowledgeable agents as managers. Similarly, since the marginal value of worker knowledge is less than 1, if workers earned their expected output as independent producers, all two-layer firms would only want to hire the agents with the least knowledge.

Fourth and finally, the income function $w(z)$ is continuous, strictly increasing, and weakly convex. Monotonicity follows because the marginal value of knowledge is strictly positive, and its convexity (which is strict when $z \in W \cup M$) is due to the existence of complementarities between managers' and workers' knowledge. Continuity, on the other hand, is a necessary condition for market clearing.

For simplicity, in what follows, we restrict attention to $h < h_0$. This implies that there are no independent producers in the pre-AI equilibrium. As discussed in Section 5.1, virtually all of our results extend to $h \geq h_0$.

# 3 The AI Equilibrium

In this section, we present a partial characterization of the AI equilibrium with the essential information needed to understand our main results. We relegate the complete characterization to Appendix B since many of its details can be safely skipped on a first reading (but are needed for the formal arguments). For future reference, we index this equilibrium using the superscript "$*$" (note that the pre-AI equilibrium had no superscript).

**Proposition 2.** *In the presence of AI, there is a unique equilibrium. The equilibrium has the following features:*

- *The price of computing power is equal to AI's knowledge:* $r^* = z_{\text{AI}}$.
- *Occupational stratification:* $W^* \preceq I^* \preceq M^*$.
- *No worker better than AI; no manager worse than AI:* $W^* \preceq \{z_{\text{AI}}\} \preceq M^*$.
- *Positive assortative matching:* $f^* : W_p^* \to M_p^*$ *is strictly increasing and* $W_a^* \preceq W_p^*$ *and* $M_p^* \preceq M_a^*$.
- *AI does not lead to the complete destruction of human routine jobs:* $W^* \neq \emptyset$.

*Furthermore, AI is always used for independent production, and whether it is also used as a worker or as a manager depends on its knowledge* ***relative to the pre-AI equilibrium****.*

- *If* $z_{\text{AI}} \in W$*, then AI is necessarily used as a worker (and possibly also as a manager).*
- *If* $z_{\text{AI}} \in M$*, then AI is necessarily used as a manager (and possibly also as a worker).*

*Finally, the wage function* $w^*(z)$ *is continuous, strictly increasing, and (weakly) convex, and satisfies:*

- $w^*(z) = z_{\text{AI}}(1 - 1/n(z)) > z$ *for all* $z \in W_a^*$.
- $w^*(z) = f^*(z) - w^*(f^*(z))/n(z) > z$ *for all* $z \in W_p^*$.
- $w^*(z) = z$ *for all* $z \in I^*$.
- $w^*(z) = m_p + \int_{m_p}^{z} n(e^*(u))du > z$*, for all* $z \in M_p^*$*, where* $m_p \equiv \inf M_p^*$.
- $w^*(z) = n(z_{\text{AI}})(z - z_{\text{AI}}) > z$*, for all* $z \in M_a^*$.

*Proof.* Note that the statement implies that in the knife-edge case where AI has the knowledge of both a pre-AI worker and a pre-AI manager, i.e., $z \in W \cap M$, AI is necessarily used in all three roles. For the proof, see Appendix B. □

Proposition 2 has three parts. The first one states some fundamental properties of the equilibrium. The second one describes how firms use AI as a function of AI's knowledge. Finally, the third part characterizes the equilibrium wages. The remaining of this section provides intuition for each of these three parts.

### 3.1 Some Fundamental Properties of the AI Equilibrium

First, the fact that the equilibrium price of computing power is equal to AI's knowledge follows because computing power is abundant relative to human time. Indeed, by definition, this implies that there are not enough humans to interact with AI inside two-layer organizations. Hence, some computing power must be used for independent production. The zero-profit condition of the single-layer firms using AI then requires $r^* = z_{\text{AI}}$.

Second, the equilibrium continues to exhibit occupational stratification (as the marginal value of knowledge continues to be the smallest for workers, the second smallest for independent producers, and the highest for managers) and positive assortative matching (as there are still complementarities between worker and managerial knowledge).

Third, occupational stratification plus the fact that some computing power must be used for independent production implies that no worker can be better than AI and that no manager can be worse than AI, i.e., $W^* \preceq \{z_{\text{AI}}\} \preceq M^*$. Positive assortative matching then implies that if AI is used as a worker, then it is managed by the most knowledgeable humans, i.e., $M_p^* \preceq M_a^*$. Similarly, if AI is used as a manager, it manages the least knowledgeable humans, i.e., $W_a^* \preceq W_p^*$.

Finally, AI does not completely destroy worker positions for humans. This result arises even though computing power is abundant and AI is more knowledgeable than a fraction of the human population. Intuitively, even though computing power is large relative to human time, it is still scarce relative to its potential applications. Hence, it continues to be worthwhile for every human to be employed in some capacity. The competitive equilibrium then allocates all those humans who are less knowledgeable than AI to do routine work in two-layer organizations because they are the least likely to succeed on their own.

### 3.2 How Firms Use AI

As mentioned above, some computing power must necessarily be deployed in independent production. As stated in Proposition 2, whether AI is used in any other capacity depends on its knowledge

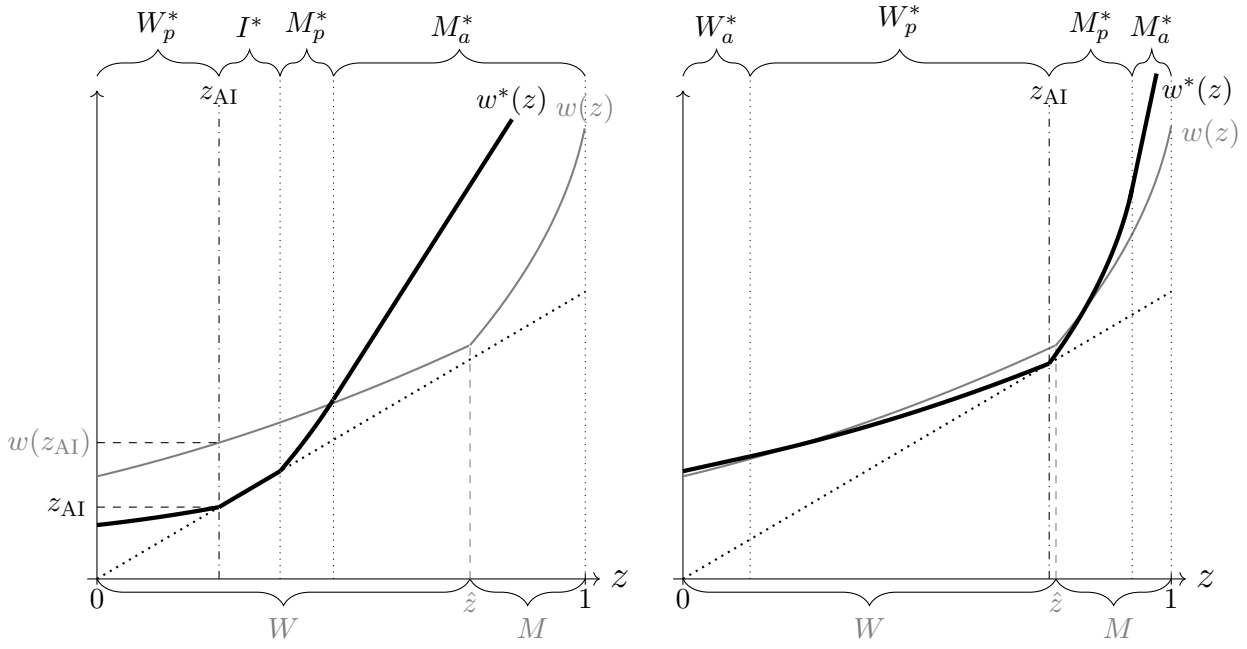
relative to the pre-AI equilibrium. For brevity, here we only explain the case where $z_{\text{AI}} \in W$ since the case where $z_{\text{AI}} \in M$ is analogous.

To start, note that AI cannot be exclusively used as an independent producer. To see this, suppose otherwise for contradiction. Then, AI does not affect labor outcomes, as it does not interact with humans in the workplace. This implies that the human with AI's knowledge is still being employed as a worker and is, therefore, earning strictly more than $z_{\text{AI}}$. Hence, the firm hiring this worker has incentives to replace her with one unit of computing power.

Next, we argue that AI must necessarily be used as a worker. To see this, suppose for contradiction that AI is exclusively used as a manager and as an independent producer. By occupational stratification, everyone with knowledge above $z_{\text{AI}}$ is also a manager, implying that the number of managers post-AI has increased while the number of workers has decreased. This violates market clearing.

Let us now show how AI may be used as both a worker and a manager. We do this via Figure 2, which illustrates the effects of introducing AI starting from the pre-AI scenario depicted in Figure 1 panel (a). The introduction of AI can be thought of as unleashing the following chain of events.

First, immediately upon its introduction, AI forces those humans with knowledge $z \in [z_{\text{AI}}, \hat{z}]$ to switch from workers to independent producers (recall that AI is always the most knowledgeable




(a) AI is used as a Worker and as an Independent Producer

(b) AI is Used in All Three Roles

Figure 2: From the Pre- to the Post-AI Equilibrium

*Notes*. Distribution of knowledge: $G(z) = z$. Parameter values: Both panels have $h = 1/2$ ($< h_0 = 3/4$), so $\hat{z} = 3 - \sqrt{5}$. Panel (a) has $z_{\text{AI}} = 0.25$, while panel (b) has $z_{\text{AI}} = 0.75$. See Section 2 of the Online Appendix for more details on the construction of this figure.

worker in equilibrium) and reduces the wages of all those workers who are (weakly) less knowledgeable than AI, i.e., those with $z \leq z_{\text{AI}}$. In both cases, there is a decline in the earnings of humans both to the left and to the right of AI's knowledge level, as AI provides firms with a relatively cheap technology to do routine work.

The reduction in workers' wages, coupled with the diminished income of the newly created class of independent producers, then increases the attractiveness of creating two-layer organizations. Consequently, there is a surge in demand for managerial roles: Firms begin hiring managers who are less knowledgeable than the least knowledgeable managers before the advent of AI.

If the surge in managerial positions is relatively modest compared to the mass of humans in $[z_{\text{AI}}, \hat{z}]$, then the last newly hired manager is still more knowledgeable than AI, so AI is used as a worker and as an independent producer. This is the case depicted in Figure 2(a). Otherwise, the class of human independent producers initially brought into existence by AI is completely absorbed by the newly created managerial positions, and firms begin using AI to manage the least knowledgeable workers. Hence, in this case, AI plays all three roles in equilibrium. This situation is illustrated in Figure 2(b).

### 3.3 Equilibrium Wages

Finally, we turn to the third part of Proposition 2: Equilibrium wages. For insight, we informally construct them in the case in which AI is used in all three possible roles.

By the first part of Proposition 2, when AI is used in all three roles the human population must necessarily partition as follows:

$$W_a^* \preceq W_p^* \preceq \{z_{\text{AI}}\} \preceq M_p^* \preceq M_a^* \text{ with } \sup W_p^* = z_{\text{AI}} = \inf M_p^*$$

The wage of a worker with knowledge $z \in W_a^*$ is determined by the zero-profit condition of $t$A firms plus the fact that the equilibrium rental rate of computing power is $r^* = z_{\text{AI}}$. The wage of a manager with knowledge $z \in M_a^*$ is determined similarly, though using the zero profit condition of $b$A firms.

Constructing the wages of those hired by $n$A firms requires more work. First, recall that $f^* : W_p^* \to M_p^*$ is the function matching human workers with human managers and that $e^*(z) = (f^*)^{-1}(z)$. Now consider the problem of a $n$A firm that recruited $n(z)$ workers with knowledge $z \in W_p^*$ and is deciding which manager $m \in M_p^*$ to hire:

$$\max_{m \in M_p^*} n(z)[m - w(z)] - w(m)$$

The corresponding first-order condition evaluated at $m = f^*(z)$ implies that $w^{*\prime}(f^*(z)) = n(z)$, or, equivalently, $w^{*\prime}(z) = n(e^*(z))$ for any $z \in M_p^*$. Thus:

$$w^*(z) = C + \textstyle\int_{z_{\text{AI}}}^{z} n(e^*(u))du, \text{ for any } z \in M_p^*$$

The constant $C$ is then determined by the fact that the least knowledgeable manager has the same knowledge as AI, $\inf M_p^* = z_{\text{AI}}$, so her wage must equal the price of one unit of computing power:

$C^* = r^* = z_{\text{AI}}$. Finally, the wages of the workers being managed by other humans come from the zero profit condition of $n$A firms:

$$w^*(z) = f^*(z) - \frac{w^*(f^*(z))}{n(z)}, \text{ for any } z \in W_p^*$$

# 4 How AI Reorganizes Work (and Its Labor Market Implications)

In this section, we study how AI reorganizes work by comparing the pre-AI and post-AI equilibrium. The results depend on the knowledge of AI relative to the pre-AI equilibrium. In particular, they depend on whether:

- $z_{\text{AI}} \in \text{int}W$: "AI has the knowledge of a pre-AI worker."
- $z_{\text{AI}} \in \text{int}M$: "AI has the knowledge of a pre-AI manager."
- $z_{\text{AI}} \in W \cap M$: "AI has the knife-edge knowledge of both a pre-AI worker and a pre-AI manager."

## 4.1 Occupational Displacement

We begin by analyzing the effects of AI on occupational choices.

**Proposition 3.**

- *If $z_{\text{AI}} \in \text{int}W$, then AI displaces humans from routine to managerial work, i.e., $W^* \subset W$ and $M^* \supset M$.*
- *If $z_{\text{AI}} \in \text{int}M$, then AI displaces humans from managerial to routine work, i.e., $W^* \supset W$ and $M^* \subset M$.*
- *If $z_{\text{AI}} \in W \cap M$, then there is no human displacement between routine and managerial work, i.e., $W^* = W$ and $M^* = M$. However, AI leads to the creation of bA and tA firms, i.e., $W_a^* \neq \emptyset$ and $M_a^* \neq \emptyset$.*

*Proof.* Note that $M^* \supset M$ if and only if $(W^* \cup I^*) \subset (W \cup I)$, since $(W^* \cup I^*)$ is the complement of $M^*$ and $(W \cup I)$ the complement of $M$. For the proof of the proposition, see Appendix C. □

According to the proposition, while it is true that AI might lead to the displacement of humans from routine to complex jobs—as popular wisdom usually states—it might also lead to the opposite outcome. Interestingly, the nature of the displacement is determined by the knowledge of AI *relative to the pre-AI equilibrium*: When AI has the knowledge of a pre-AI worker, it displaces humans from routine work into managerial positions. In contrast, when AI has the knowledge of a pre-AI manager, the displacement goes in the opposite direction.

To provide intuition, consider the case in which $z_{\text{AI}} \in \text{int}W$. The result that $W^* \subset W$ follows directly from the fact that in the AI equilibrium, no worker is better than AI, i.e., $W^* \preceq \{z_{\text{AI}}\}$. The more interesting result is that $M^* \supset M$. This is because AI has two opposing effects on the demand for managers.

On the one hand, by destroying human worker positions, AI reduces manager demand. On the other hand, by incentivizing firms to use computing power to do routine work, AI increases manager demand. However, the second effect more than compensates for the first effect. This is because the added "workers" (or, more precisely, computing power hired to perform worker roles) are less knowledgeable than the human workers displaced (i.e., those $z \in W$ and $z \geq z_{\text{AI}}$), and therefore, it is more valuable to match them with managers to provide them help.

The intuition for when $z_{\text{AI}} \in \text{int}M$ is analogous, so let us discuss the knife-edge situation where $z_{\text{AI}} \in W \cap M$. In this case, by lowering the wages of the best workers and the worst managers, AI increases the demand for managers (to match with the cheaper workers) and increases the demand for workers (to match with the cheaper managers). As a result, AI is used in both worker and managerial roles, so it is both the best worker and the worst manager. It follows that, in this case, $W^* = W$ and $M^* = M$.

The fact that the nature of the occupational displacement is determined by the knowledge of AI relative to the pre-AI equilibrium has an additional important implication. Indeed, as shown in Section 1 of the Online Appendix, in a pre-AI equilibrium without independent production, the knowledge cutoff to become a manager decreases in $h$. Moreover, as shown in the same appendix, this knowledge cutoff is pointwise higher for any admissible $h$ if the human population is more knowledgeable (in a first-order stochastic dominance sense).

Hence, if more developed countries have better communication technologies and/or a more knowledgeable human population, then the same AI technology might displace humans from routine work into managerial roles in more developed countries but displace humans in the opposite direction in less developed ones. The reverse, however, cannot happen: If AI displaces humans into managerial roles in less-developed countries, then the same must happen in more developed ones.[16]

### 4.2 Team Formation and the Size and Productivity Distribution of Firms

We take *firm size* to be its output, and *firm productivity* to be its output divided by the units of time and/or computing power it uses.[17] This implies that (i) the size and productivity of a single-layer firm are equal to the knowledge of its associated producer, and (ii) the size and productivity of a two-layer firm whose workers have knowledge $z$ and whose manager has knowledge $m$ are $mn(z)$ and $m$, respectively.

The following lemma—which follows because the equilibrium exhibits occupational stratification and positive assortative matching—describes the connection between the number of layers of a firm,

[16] An implicit assumption underlying this argument is that there are no cross-national teams. Understanding the impact of AI on offshoring is an interesting and relevant area for future research.

[17] An alternative would be to define firm size as the number of humans engaged in production. We decided against this definition because, in our setting, it would lead to some firms having size zero.

its size, and its productivity:

**Lemma 1.** *Take two arbitrary firms $f$ and $f'$. Irrespective of the presence of AI, in equilibrium:*

- *If $f$ has two layers and $f'$ has one layer, then $f$ is strictly larger and strictly more productive than $f'$.*
- *Firm $f$ is strictly larger than $f'$ if and only if $f$ is strictly more productive than $f'$.*

*Proof.* See Appendix C. □

This lemma has two important implications. First, the largest firm is always the most productive, while the smallest firm is the least productive. Second, the biggest firm is always a two-layer firm, which, combined with positive assortative matching, implies that its productivity is always $1$ irrespective of the presence of AI (as it is managed by the most knowledgeable human).

With Lemma 1 at hand, we turn to analyze the implication of AI on the size and productivity distribution of firms. We begin by describing its effect on two-layer firms:

**Proposition 4.** *Regarding two-layer firms:*

- *If $z_{\text{AI}} \in \text{int}W$, then the size of the biggest firm and the size and productivity of the smallest/least productive firm decrease with AI.*
- *If $z_{\text{AI}} \in \text{int}M$, then the size of the biggest firm and the size and productivity of the smallest/least productive firm increase with AI.*
- *If $z_{\text{AI}} \in W \cap M$, then the size of the biggest firm and the size and productivity of the smallest/least productive firm do not change with AI.*

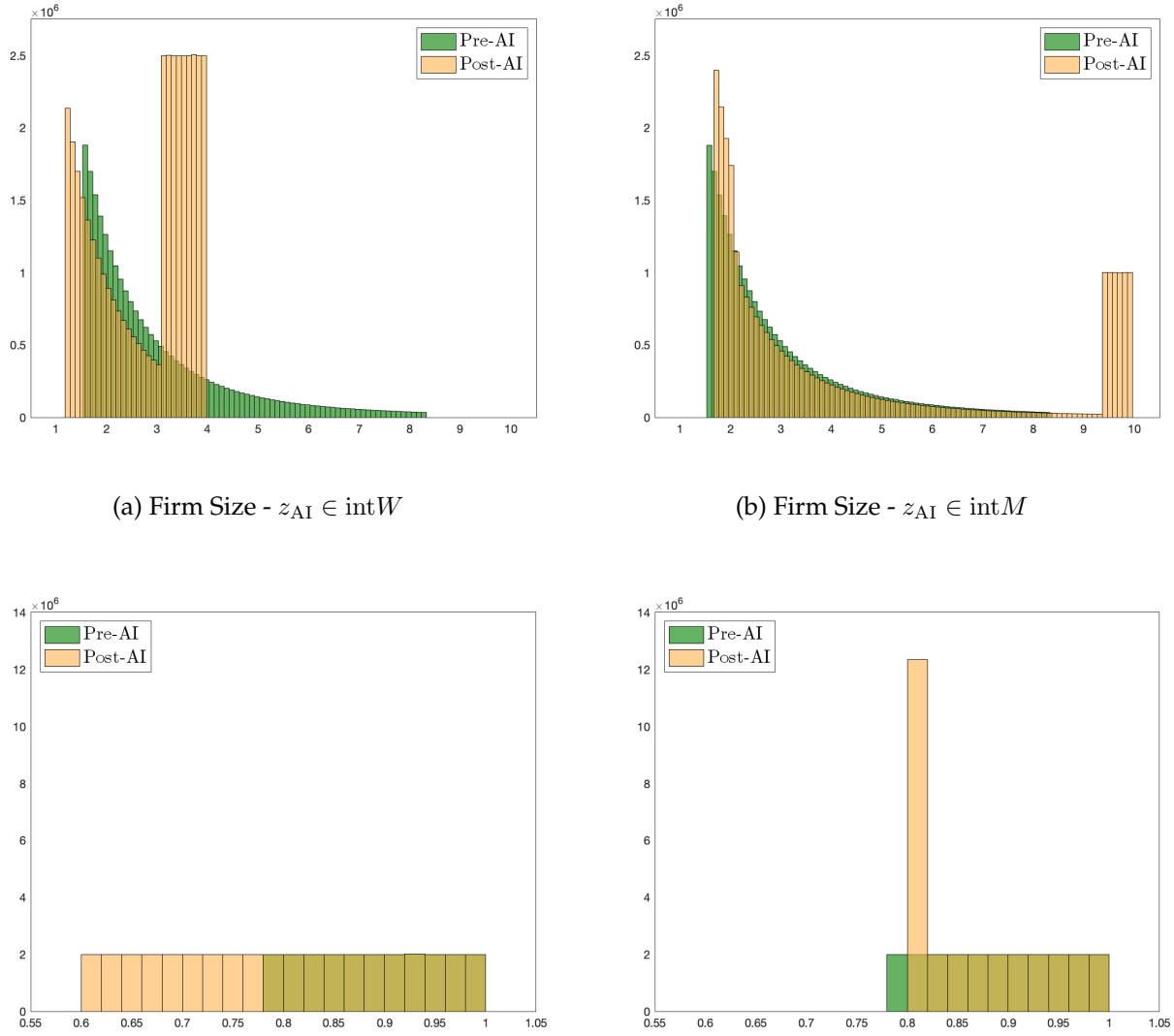
*Proof.* See Appendix C. □

Proposition 4 is illustrated in Figure 3, and its intuition is closely connected to the displacement result described in Proposition 3. In the case of two-layer organizations, the knowledge of the best worker determines the maximum firm size (as the best manager always has knowledge $1$), while the knowledge of the worst manager determines the minimum firm size and the productivity of the least productive firm (as the worst worker always has knowledge $0$). Hence, when AI has the knowledge of a pre-AI worker, the size of the biggest firm and the size and productivity of the smallest/least productive firm decreases with AI, as the best pre-AI workers become managers post-AI. A similar intuition explains the other two cases.

The effects of AI on the size and productivity distributions of *all firms* are similar:

**Proposition 5.** *The results regarding the productivity and size distribution of all firms mimic those of two-layer firms described in Proposition 4, except that:*

- *If $z_{\text{AI}} \in W \cap M$, the size of the smallest/least productive firm decreases with AI.*
- *If $z_{\text{AI}} \in \text{int}M$, the size of the smallest/least productive firm can increase or decrease with AI.*




(a) Firm Size - $z_{\text{AI}} \in \text{int}W$

(b) Firm Size - $z_{\text{AI}} \in \text{int}M$

(c) Firm Productivity - $z_{\text{AI}} \in \text{int}W$

(d) Firm Productivity - $z_{\text{AI}} \in \text{int}M$

Figure 3: The Effects of AI on the Size and Productivity Distribution of Two-Layer Firms

*Notes*. Histograms based on a human population of $N = 10^8$ individuals and $\mu = 10.01 \times N$ units of computing power. Distribution of knowledge: $G(z) = z$. Parameter values: Both panels have $h = 1/2$ ($< h_0 = 3/4$), so $\hat{z} = 3 - \sqrt{5}$. Moreover, panels (a) and (c) assume $z_{\text{AI}} = 0.5$, while panels (b) and (d) assume $z_{\text{AI}} = 0.8$. See Section 2 of the Online Appendix for more details on the construction of this figure.

*Proof.* See Appendix C. □

To understand these results, consider productivity first. In contrast to the case of two-layer firms, the productivity of the smallest/least productive firm is now determined by the knowledge of the worst independent producer (rather than by the knowledge of the worst manager).[18] Nevertheless,

[18]Note that this result holds irrespective of whether the smallest/least productive firm is a single- or a two-layer organization. Indeed, if the smallest/least productive firm is a two-layer organization, there cannot be any single-layer firms

even though these knowledge cutoffs need not coincide, they move in the same direction with the introduction of AI. This explains why the effects of AI on the productivity distribution of firms are the same whether we consider all firms or restrict the sample to two-layer organizations.

Second, consider size. Since the biggest firms are always two-layer firms, the results concerning the biggest firms remain unchanged. The difference arises in the results regarding the smallest/least productive firm: Intuitively, even in cases in which the size of the smallest two-layer firm increases with AI, the size of the smallest firm might decrease with AI because AI also leads to the creation of new, potentially smaller one-layer firms.

Propositions 4 and 5 are driven by the displacement of humans across occupations. We now analyze the effects of AI on the productivity of workers and the size of firms being managed by those *who have not been displaced*. In line with our definition of firm productivity, we define worker productivity as her expected output. For the proposition that follows, recall that $e(z)$ denotes the employee function of the pre-AI equilibrium:

**Proposition 6.**

- *If* $z_{\text{AI}} \in \text{int}W$*, then:*
  - *Each* $z \in W^* \subset W$ *is strictly less productive post-AI than pre-AI.*
  - *Consider* $z \in M \subset M^*$*. If* $e(z) < z_{\text{AI}}$*, then* $z$ *manages a strictly larger firm post-AI than pre-AI. If* $e(z) > z_{\text{AI}}$*, then* $z$ *manages a strictly smaller firm post-AI than pre-AI. Finally, if* $e(z) = z_{\text{AI}}$*, then* $z$ *manages a firm of equal size post-AI and pre-AI.*
- *If* $z_{\text{AI}} \in \text{int}M$*, then:*
  - *Consider* $z \in W \subset W^*$*. If* $z < e(z_{\text{AI}})$*, then* $z$ *is strictly more productive post-AI than pre-AI. If* $z > e(z_{\text{AI}})$*, then* $z$ *is strictly less productive post-AI than pre-AI. Finally, if* $z = e(z_{\text{AI}})$*, then* $z$ *is equally productive post-AI and pre-AI.*
  - *Each* $z \in M^* \subset M$ *manages strictly larger firm post-AI than pre-AI.*
- *If* $z_{\text{AI}} \in W \cap M$*, then:*
  - *Each* $z \in W^* = W$ *is less productive post-AI than pre-AI (strictly so for all* $z \neq 0$*).*
  - *Each* $z \in M^* = M$ *manages a larger firm post-AI than pre-AI (strictly so for all* $z \neq 1$*).*

*Proof.* See Appendix C. □

For intuition, note that a worker's productivity is the knowledge of her manager, so a worker's productivity increases if and only if her managerial match improves. Similarly, as more knowledgeable workers require less help, the size of the firm of a given manager increases if and only the knowledge of her workers increases.

in equilibrium. Thus, the productivity of the smallest/least productive, which is equal to the knowledge of the worst manager, is also equal to that of the best worker.

With this in mind, suppose first that AI has the knowledge of a pre-AI worker. In this case, the knowledge required to become a manager decreases, worsening the pool of managers available for non-displaced workers. At the same time, the match of the best managers worsens as they are now supervising production by AI (while pre-AI, they were managing humans more knowledgeable than AI), but the match of all the other managers improves because the worst workers are now managed by the newly appointed managers.

Similarly, when AI has the knowledge of a pre-AI manager, the match of the worst workers improves since they are now assisted by AI (while pre-AI, they were assisted by humans less knowledgeable than AI). However, the knowledge of the best workers increases, improving the pool of available workers for non-displaced managers and, therefore, leaving a worse pool of managers for the non-displaced workers that were originally managed by humans more knowledgeable than AI.

Finally, when AI has the knowledge of both a pre-AI worker and a pre-AI manager, there is no human displacement across occupations. However, the match of the least knowledgeable workers worsens (as they are now assisted by AI), while the match of the most knowledgeable managers improves (as they are now supervising AI). The first result then implies that the pool of workers available for the remaining non-displaced managers improves with AI (explaining why all managers are in charge of larger firms), while the second result implies that the pool of available managers for the remaining non-displaced workers worsens with AI (explaining why all workers become less productive).

### 4.3 Labor Income

In this section, we analyze the effects of AI on labor income. We begin with the following result:

**Lemma 2.** *Total output and total labor income increase with AI.*

Because the proof of this result is relatively straightforward, we provide it here as part of the main text.

*Proof.* The result that total output increases follows because (i) the First Welfare Theorem holds in this setting, and (ii) AI expands the production possibility frontier. The result that total labor income also increases with AI follows from two observations. First, if all computing power is assigned to independent production, total labor income does not change with AI (as AI does not interact with humans in the workplace). Second, capital income is equal to $\mu z_{\text{AI}}$ regardless of how computing power is used, so:

$$\text{Total output post-AI} = \text{Total labor income post-AI} + \mu z_{\text{AI}}$$

Consequently, given that the AI equilibrium is efficient, unique, and does not allocate all computing

power to independent production, it must be that:

$$\text{Total output post-AI} > \text{Total labor income pre-AI} + \mu z_{\text{AI}}$$

Hence, total labor income must be strictly larger post-AI than pre-AI. □

We now turn to analyzing the distributional effects of AI. The analysis is non-trivial due to the existence of two potentially countervailing forces: On the one hand, AI changes the composition of firms and, therefore, the quality of matches. On the other hand, by mimicking humans with knowledge $z_{\text{AI}}$, AI reduces the latter's wages and affects how each firm's output is divided between workers and managers.

We first show that *if* a subset of humans with knowledge below $z_{\text{AI}}$ win after the introduction of AI, then they must be the least knowledgeable humans in the population. Similarly, *if* a subset of humans above $z_{\text{AI}}$ are better off after the advent of AI, then they must be the most knowledgeable humans:

**Lemma 3.** *Define* $\Delta(z) \equiv w^*(z) - w(z)$. *Then* $\Delta(z_{\text{AI}}) < 0$ *and:*

- *If* $\Delta(z) > 0$ *for some* $z \in [0, z_{\text{AI}}]$, *then* $\Delta(z') > 0$ *for all* $z' \in [0, z]$.
- *If* $\Delta(z) > 0$ *for some* $z \in [z_{\text{AI}}, 1)$, *then* $\Delta(z') \geq 0$ *for all* $z' \in [z, 1]$, *with strict inequality for all* $z' \in [z, 1)$.

*Proof.* See Appendix C. □

Lemma 3 is subtle because, even though $\Delta(z_{\text{AI}}) < 0$, $\Delta(z)$ need not be v-shaped around $z_{\text{AI}}$, as illustrated in Figure 4. This is because AI can sometimes worsen the matches of the best managers and improve the matches of the worst workers. Nevertheless, Lemma 3 states that in the interval $[0, z_{\text{AI}}]$, the $\Delta(z)$ function can only cross zero from above, while in the interval $[z_{\text{AI}}, 1]$, the $\Delta(z)$ function can only cross zero from below. Intuitively, this is because wages decrease the most at $z_{\text{AI}}$, and AI is always used as the best worker and/or the worst manager. Hence, the least and most knowledgeable humans (matched with the worst managers and the best workers, respectively) are the ones whose share of output increases the most after the reduction of wages at $z_{\text{AI}}$.

The next step is characterizing whether (and when) winners exist below or above $z_{\text{AI}}$. For the following proposition, recall that $\hat{z}$ is the knowledge level of the worst pre-AI manager (which is also the best pre-AI worker):

**Proposition 7.**

*(i) There always exists $z$ strictly greater than $z_{\text{AI}}$ such that $\Delta(z) > 0$.*

*(ii) There exists $z$ strictly smaller than $z_{\text{AI}}$ such that $\Delta(z) > 0$ if and only if $z_{\text{AI}} > \bar{z}_{\text{AI}}$, where $\bar{z}_{\text{AI}} \in (0, \hat{z})$.*

*Proof.* See Appendix C. □

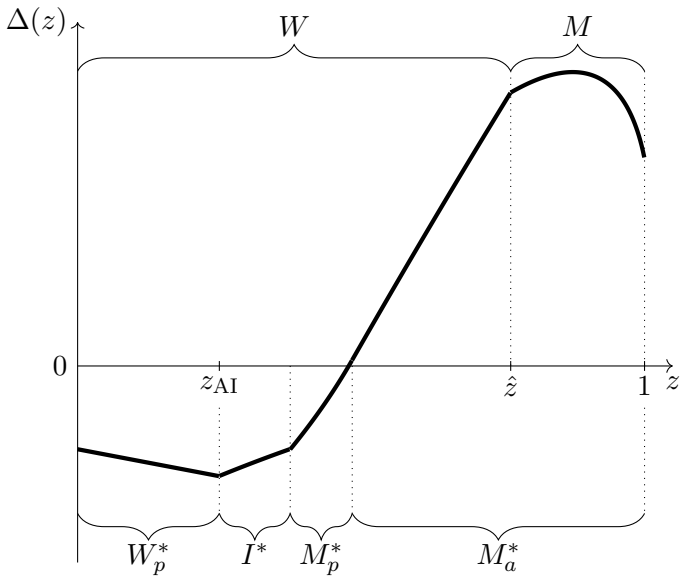



Figure 4: An Illustration of $\Delta(z)$ Function when $z_{\text{AI}} \in \text{int}W$

*Notes*. Distribution of knowledge: $G(z) = z$. Parameter values: $h = 1/2$ ($< h_0 = 3/4$), $z_{\text{AI}} = 1/4$. Thus, $\Delta(0) = -0.1695$, $\Delta(1) = 0.4222$, $\max_z \Delta(z) = 0.59823$ (with $\text{argmax}_z \Delta(z) = 0.87331$), and $\min_z \Delta(z) = -0.2243$ (with $\text{argmin}_z \Delta(z) = z_{\text{AI}} = 1/4$). See Section 2 of the Online Appendix for more details on the construction of this figure.

Proposition 7 reveals an interesting asymmetry between the effects of AI on the lowest and highest segment of the knowledge distribution. Indeed, while the most knowledgeable humans always benefit from the introduction of AI, the least knowledgeable humans benefit only when AI's knowledge is sufficiently high.

To understand this asymmetry, let us consider how AI's knowledge affects the wages of the least and most knowledgeable humans, i.e., $z = 0$ and $z = 1$. When $z_{\text{AI}}$ increases, (i) AI is a closer substitute to the most knowledgeable human, and (ii) the most knowledgeable human is better matched. While the first effect puts downward pressure on her wage, the second effect puts opposite pressure on her wage. Moreover, the latter effect is stronger the smaller is $h$, because a smaller helping cost allows each manager to supervise more workers. The assumption that $h < h_0$ then guarantees that the second effect more than compensates for the first.[19]

In contrast, when $z_{\text{AI}}$ decreases, (i) AI is a closer substitute to the least knowledgeable human, and (ii) the least knowledgeable human is worse matched. In this case, both effects put downward pressure on her wage, explaining why the least knowledgeable human benefits from AI only when $z_{\text{AI}}$ is sufficiently high.

[19]In section 5.1, we show that this is no longer the case when $h \geq h_0$, and hence in this case, the most knowledgeable humans in the population can lose from AI.

# 5 Discussion and Extensions

## 5.1 The Case $h \geq h_0$

For simplicity, we have focused on the case $h < h_0$. In Section 3 of the Online Appendix, we show that all our results remain unchanged when $h \geq h_0$, with the following two exceptions.

First, AI can have the knowledge of a pre-AI independent producer, in which case it does not affect labor outcomes. Intuitively, when $z_{\text{AI}} \in I$, the first unit of processing capacity is allocated in equilibrium to independent production (as prescribed by the pre-AI equilibrium), so it does not affect two-layer firms in any way. Hence, the same argument applies to the second unit, the third unit, and so on, so all units of computing power end up being allocated to independent production. Consequently, AI does not affect wages (as the $w(z_{\text{AI}}) = z_{\text{AI}}$ already in the pre-AI equilibrium) nor other labor outcomes since it does not interact with humans in the workplace.

Second, all humans that are more knowledgeable than AI can lose from AI's introduction when $h \geq h_0$. In particular, this occurs when $z_{\text{AI}} \in \text{int}M$. Intuitively, recall that a more knowledgeable AI has two opposing effects on the wages of the most knowledgeable humans: On the one hand, it reduces their wages, as AI is a closer substitute to them; on the other hand, it improves their matches. While the second effect always dominates when $h < h_0$ (as mentioned in Section 4.3), this is not necessarily the case when $h \geq h_0$, since each manager now supervises a small number of workers.

## 5.2 The Role of Abundance of Computing Power Relative to Human Time

Until now, we have considered the case in which computing power is large relative to human time. To understand the role of this assumption, in Section 4 of the Online Appendix we characterize the equilibrium in the opposite extreme in which computing power $\mu$ is arbitrarily small.

In this case, the nature of the displacement still depends on AI's knowledge relative to the pre-AI equilibrium: When AI has the knowledge of a pre-AI worker, AI displaces humans from routine work into problem-solving. In contrast, when AI has the knowledge of a pre-AI manager, humans are displaced in the opposite direction. This also implies that our results regarding the size and productivity distribution of firms continue to hold.

However, when $\mu$ is small, it is no longer the case that AI is the best worker and/or the worst manager. This difference has two important implications. The first one relates to the quality of matches of those who continue to be workers and/or managers. The second relates to who wins and who loses with the introduction of AI.

Regarding the quality of matches, it is no longer the case that the productivity of all workers decreases when $z_{\text{AI}} \in \text{int}W$, or that all managers supervise larger firms when $z_{\text{AI}} \in \text{int}M$. In particular, there is now a set of workers—those with knowledge above $z_{\text{AI}}$—who have better managers post-AI

than pre-AI when $z_{\text{AI}} \in \text{int}W$. Similarly, there is a set of managers—those with knowledge below $z_{\text{AI}}$—who manage smaller firms when $z_{\text{AI}} \in \text{int}M$. Intuitively, these differences arise because, when computing power is abundant, there are no workers with knowledge above $z_{\text{AI}}$ nor managers with knowledge below $z_{\text{AI}}$.

With regards to labor income, when computing power is small, the winners from AI are not necessarily at the extremes of the knowledge distribution. The reason is that AI can now be among the least knowledgeable workers, so its introduction might increase the wages of those in the middle of the knowledge distribution, thus harming the most knowledgeable humans (who manage them).[20] This intuition highlights the key role that abundant computing power plays in our analysis of the effects of AI on labor income.

### 5.3 What if Computing Power Exceeds its Potential Uses?

In the baseline setting, we assumed that computing power is abundant relative to human time but scarce relative to its potential uses. This has two noteworthy implications. First, the equilibrium rental rate of computing power is equal to AI's knowledge. Second, in the AI equilibrium, some humans are still doing routine work.

In Section 5 of the Online Appendix, we relax the assumption that computing power is scarce relative to its uses. In particular, we consider the case in which (i) there is a large but finite amount $Q$ of potential problems to be solved, and (ii) the computing power is abundant not only relative to human time but also relative to $Q$.[21]

In such a case, and in contrast to our baseline setting, the equilibrium price of computing power is zero, and all agents that are less knowledgeable than AI are unemployed. Organizations, however, still have a hierarchical structure: All problems are initially attempted by AI, and all humans who are more knowledgeable than AI specialize in solving problems that AI cannot solve. Intuitively, only the time of the humans that are more knowledgeable than AI is scarce, so only they get rewarded for their work.

Nevertheless, our results concerning occupational displacement still hold: If AI has the knowledge of a pre-AI worker, it still shifts humans from routine to managerial work, while if AI has the knowledge of a pre-AI manager, it still reduces the number of humans doing managerial work. In this extension, however, AI displaces everyone who is less knowledgeable than AI to unemployment.

Moreover, our results concerning the effects of AI on the size and productivity distribution of firms continue to hold with minor changes. In particular, the size of the smallest two-layer firm

[20] Similarly, AI can now be among the most knowledgeable managers, so their introduction can increase the wages of those in the middle of the knowledge distribution, harming the least knowledgeable humans (who are matched with them).

[21] Note that the equilibrium of the baseline setting is still the equilibrium of this alternative model in the region of the parameter space where $\mu$ is abundant relative to time but not to $Q$.

can increase or decrease with AI when $z_{\text{AI}} \in \text{int}W$ (while in the baseline, it always decreases), and necessarily increases when $z_{\text{AI}} \in W \cap M$ (in the baseline, its size remains unchanged). Intuitively, the difference arises because, in the baseline setting, the least knowledgeable worker has knowledge $z = 0$ both pre- and post-AI. In contrast, in this extension, AI is the least knowledgeable worker, so AI increases the knowledge of the workers of the smallest/least productive firm.[22]

Regarding the quality of matches, the results about non-displaced workers are vacuous because the set of non-displaced workers is empty in this case. In turn, the results concerning the size of the firms being supervised by the managers who are not displaced remain unchanged. Finally, regarding labor income, the wages of everyone who is less knowledgeable than AI drop to zero. Hence, the winners, if there are any, are always at the high end of the knowledge distribution.

### 5.4 The Effects of AI on the Labor Share

Pre-AI, the labor share is equal to one because labor is the sole input in production. Interestingly, whether AI lowers the labor share or not depends on whether the set of potential uses for AI is larger or smaller than the amount of computing power in the economy.

In the baseline setting—where the unlimited uses of computing power imply that $r^* = z_{\text{AI}}$—the labor share decreases when moving from the pre-AI to the post-AI equilibrium. Moreover, it converges to zero as $\mu \to \infty$ since, when computing power is abundant relative to human time, any additional unit of computing power is exclusively allocated to independent production. As a result, further increases in $\mu$ raise capital income but not labor income.

This result dramatically changes when there is a large but finite amount $Q$ of potential problems. In this case, the price of computing power is $0$ when $\mu$ is large enough (as discussed in the previous subsection). This has two implications. First, the post-AI labor share can be equal to one, just like pre-AI. Second, in a world with finitely many problems, the labor share is not monotone in $\mu$: it decreases with $\mu$ while computing power is abundant relative to time but not to problems and then increases with $\mu$ as computing power catches up with the number of problems. We leave a more detailed study of this race between computing power and its potential uses for future research.

### 5.5 AI versus Improvements in ICTs and Globalization

We now discuss how the effects of AI compare with the effects of two other important "shocks" previously studied in the literature: Improvements in communication and information technologies (ICTs) and the possibility of forming international teams ("globalization").

[22] As a result, in this extension, the smallest firm has a worse manager but better workers post-AI than pre-AI when $z_{\text{AI}} \in \text{int}W$ (while in the baseline, it only has a worse manager). Similarly, in this extension, the smallest firm has the same manager but better workers post-AI than pre-AI when $z_{\text{AI}} \in W \cap M$ (while in the baseline it has the same workers and manager).

Consider first a reduction in communication costs $h$, starting from $h < h_0$. This naturally allows managers to supervise larger teams, thus reducing the demand for managers. The latter has two immediate implications: There is occupational displacement from managerial to routine work (as the worst managers switch to worker roles), and all workers become better matched (and hence more productive). In contrast, the displacement generated by AI goes in the opposite direction when $z_{\text{AI}} \in \text{int}W$. Moreover, (i) AI reduces the productivity of the best non-displaced workers irrespective of AI's knowledge (and the productivity of all workers when $z_{\text{AI}} \in \text{int}W$), and (ii) decreases the size of the firms supervised by the best managers when $z_{\text{AI}} \in \text{int}W$.

Second, consider the effects of globalization. Antràs et al. (2006) consider a two-country model in which countries only differ in their knowledge distributions. In particular, one country, the North, has a distribution of knowledge with a relatively high mean, while the other country, the South, has a distribution of knowledge with a relatively low mean. They show that allowing the formation of international teams (or "offshoring") shifts humans from routine to managerial work in the North, while it shifts humans from managerial to routine work in the South.

Hence, one might conjecture that the effects of AI when $z_{\text{AI}} \in \text{int}W$ are qualitatively similar to the effects of offshoring from the North's perspective and that the effects of AI when $z_{\text{AI}} \in \text{int}M$ are qualitatively similar to the effects of offshoring from the South's perspective. This, however, is not the case. For instance, while offshoring increases the productivity of the best workers in the North (Antràs et al. (2006), Proposition 1), AI reduces the productivity of all workers when $z_{\text{AI}} \in \text{int}W$. Similarly, while offshoring decreases the size of the firms supervised by all southern managers (Antràs et al. (2006), Proposition 1), AI increases the size of the firms supervised by all managers when $z_{\text{AI}} \in \text{int}M$.

Intuitively, the key difference between AI and offshoring is AI's capacity to operate at scale (so AI is always the best worker and/or the worst manager in equilibrium). Indeed, when the North opens up to globalization, the best northern workers pre-globalization switch to managerial roles, thus freeing up the best northern managers to supervise the best non-displaced northern workers. In contrast, in the case of AI, while the best workers also switch to managerial roles when $z_{\text{AI}} \in \text{int}W$, the best managers do not switch to supervising the best non-displaced workers but to supervising AI instead.

Similarly, when $z_{\text{AI}} \in \text{int}M$, both globalization and AI induce the worst southern managers to switch to routine work, improving the overall pool of southern workers. However, in the case of globalization, the best southern workers are matched with the best northern managers, leaving a worse pool of workers for the non-displaced southern managers. In contrast, in the case of AI, it is the worst southern workers who end up being supervised by AI, leaving the best southern workers for the non-displaced southern managers.

We therefore conclude that irrespective of AI's knowledge, its effects on labor outcomes are qualitatively different from previous shocks, such as reductions in communication costs or opening up to

globalization.

### 5.6 Examples of AI Worker, Independent Producer, and Manager

We finish this section by providing real-world examples in which AI acts as a "worker," a "manager," or an "independent producer" in the sense of our framework.

To understand how AI can play the role of a "worker," consider legal services. Law firms are often hierarchical, partly reflecting the division of labor within the organization: Individuals with less knowledge and experience, such as associates and paralegals, execute tasks that are less knowledge-intensive, like reviewing old cases and searching records, among others, allowing senior lawyers and partners to use their expertise to solve the most important exceptions (Garicano and Hubbard, 2007).

These less knowledge-intensive tasks are the ones being increasingly automated by AI. For instance, according to a recent survey by Thomson Reuters (2021), 64% of law firms are deploying AI for legal research, 47% for document review, and 38% for document automation. In fact, as evidenced by the following quote from the American Bar Association, such automation has been occurring for some time:[23]

> AI is already having an impact on firms in the U.S. and around the world. Robots or machines are being utilized to do tedious, time-consuming tasks like collecting data, searching records, going through old cases, verifying facts, etc.— work currently done by junior lawyers and paralegals.

The legal services industry also provides vivid examples of how AI can be used as an "independent producer." Indeed, tax software like TurboTax, H&R Block, and SprintTax provide legal advice to millions of Americans every year—in most circumstances without any human assistance. More sophisticated AI independent producers can be found in U.S. cities like San Francisco, Phoenix, and Austin, where companies like Waymo and Cruise offer driverless taxi services ("robotaxis").[24]

Finally, to understand how AI can play the role of a "manager," consider its deployment in the customer service industry. Brynjolfsson et al. (2023) empirically study the effects of introducing a generative AI-based "conversational assistant" designed to help technical support agents of a Fortune 500 software company. As the authors emphasize, this AI system was designed to help these

[23] "Legal Tech Gurus Forecast how AI will Impact your Practice," American Bar Association, July 2017. https://www.americanbar.org/news/abanews/publications/youraba/2017/july-2017/artificial-intelligence-and-the-future-of-law-practice/ (accessed February 5, 2024)

[24] On October 26, 2023, Cruise halted its US robotaxi services nationwide (grounding its fleet of 400 robotaxis) after the California Public Utility Commission suspended its permit for service after an incident in San Francisco. Meghan Bobrowsky, "GM's Cruise Says U.S. Is Investigating Driverless Car's Collision With Pedestrian." *The Wall Street Journal*, January 25, 2024. https://www.wsj.com/business/autos/gms-cruise-confirms-doj-investigation-of-driverless-car-incident-b249c13b?st=v4m4h0mu9udh7ct (accessed February 12, 2024).

agents—for instance, by providing suggested answers to problems in real-time—rather than replace them. They show that AI increases the productivity of the least experienced and less knowledgeable agents by 35%, but that the effects on more experienced and knowledgeable workers were minimal.

In the context of our model, we can think of this experiment as providing an AI "manager" who can help, in real-time, the technical support agents.[25] In particular, in contrast to the legal application described above, AI's role, in this case, is not to free up time by handling the easiest problems—this should create productivity gains across the board—but rather to help agents resolve the most difficult problems they encounter in their work. Brynjolfsson et al. (2023) findings, moreover, are consistent with the results of Section 4.3: The least knowledgeable technical support agents are better off from the introduction of AI, while the most knowledgeable ones—whom the AI technology closely imitates—likely suffered a reduction in their current pay, as the firm's practice is to calculate their bonuses relative to other agents' performance.

## 6 Final Remarks

The transformative impact of Artificial Intelligence (AI) is undeniable, yet its precise implications for the nature of work are controversial. In this paper, we introduce a novel framework to examine the impact of AI on labor outcomes. Our approach is motivated by the observation that AI holds the potential to automate knowledge work and that a fundamental problem of society is the efficient utilization of knowledge.

Our framework provides novel insights about how AI influences labor outcomes through its effects on firm composition and organizational structure. Moreover, it has the attractive feature that the role that AI plays in the economy—and hence its effects on labor outcomes—is determined endogenously as a function of the knowledge of the population, the level of information and communication technologies, and the degree of advancement of AI.

This paper opens up several avenues for future research. For example, we believe that our approach may be fruitfully applied to investigate (i) the effects of AI on capital markets, (ii) firms' incentives to develop AI, (iii) the implication and effectiveness of different retraining schemes, and (iv) the organization of international trade, offshoring and, more generally, economic development.

[25] The technical support agents did have a human manager before the introduction of AI. However, they could not consult with her in real-time; the manager's role was to provide training and weekly feedback.

# APPENDIX

## A The Pre-AI Equilibrium: Complete Characterization

In this Appendix, we provide the full characterization of the equilibrium without AI, including the proof of Proposition 1. As noted in the main text, the equilibrium of an economy without AI was first described in general by Fuchs et al. (2015).

### A.1 Proof of Proposition 1

Proposition 1 follows from Lemmas A.1, A.2, and Corollary A.1 (plus the fact that the First Welfare Theorem implies that the competitive equilibrium is efficient in our setting).

**Lemma A.1.** *In the absence of AI, there is a unique surplus maximizing allocation:*

- *When $h > h_0 \in (0,1)$, then $W = [0, \underline{z}]$, $I = (\underline{z}, \bar{z})$, and $M = [\bar{z}, 1]$. Moreover, $f(z; \bar{z})$ is strictly increasing and given by $\int_{\bar{z}}^{f(z;\bar{z})} dG(u) = \int_0^z h(1-u)dG(u)$, and the cutoffs $0 < \underline{z} < \bar{z} < 1$ satisfy:*

$$\frac{1}{h} - \bar{z} = \int_{\bar{z}}^1 n\big(e(u;\bar{z})\big)du \quad \text{and} \quad f(\underline{z};\bar{z}) = 1 \quad \text{where } e(z;\bar{z}) = f^{-1}(z;\bar{z}) \tag{2}$$

- *When $h \leq h_0$, then $W = [0, \hat{z}]$, $I = \emptyset$, and $M = [\hat{z}, 1]$. Moreover, $f(z; \hat{z})$ is strictly increasing and given by $\int_{\hat{z}}^{f(z;\hat{z})} dG(u) = \int_0^z h(1-u)dG(u)$ and the cutoff $\hat{z} \in (0,1)$ satisfies $f(0;\hat{z}) = \hat{z}$, $f(\hat{z};\hat{z}) = 1$, and:*

$$\begin{aligned} &\tfrac{1}{h} - \hat{z} > \textstyle\int_{\hat{z}}^1 n(e(z;\hat{z}))dz \text{ for all } h < h_0 \\ &\tfrac{1}{h} - \hat{z} = \textstyle\int_{\hat{z}}^1 n(e(z;\hat{z}))dz \text{ when } h = h_0 \end{aligned} \tag{3}$$

*Proof.* For the proof see Fuchs et al. (2015, Lemma 2). □

**Lemma A.2.** *In the absence of AI, the equilibrium wage function is given by:*

- *When $h \geq h_0$:*

$$w(z) = \begin{cases} f(z;\bar{z}) - \dfrac{1}{n(z)} w(f(z;\bar{z})) & \text{if } z \in W \\ z & \text{if } z \in I \\ \bar{z} + \int_{\bar{z}}^z n\big(e(u;\bar{z})\big)du & \text{if } z \in M \end{cases}$$

- *When $h < h_0$:*

$$w(z) = \begin{cases} f(z;\bar{z}) - \dfrac{1}{n(z)} w(f(z;\bar{z})) & \text{if } z \in W \\ \frac{1}{1+n(\hat{z})} \left\{ n(\hat{z}) - \int_{\hat{z}}^1 n\big(e(u;\hat{z})\big)du \right\} + \int_{\hat{z}}^z n\big(e(u;\hat{z})\big)du & \text{if } z \in M \end{cases}$$

*Proof.* See the Online Supplement of Fuchs et al. (2015, specifically pp. 1–4). □

**Corollary A.1.** *Irrespective of whether $h \gtrless h_0$, the equilibrium wage function $w(z)$ is continuous, strictly increasing, and (weakly) convex. Moreover, it satisfies:*

- $w(z) > z$ *for* $z \in W$ *(except possibly at* $z = \sup W$*), and* $w'(z) \in (0,1)$*, and* $w''(z) > 0$ *for* $z \in \text{int}W$*.*
- $w(z) = z$ *for all* $z \in I$*.*
- $w(z) > z$ *for* $z \in M$ *(except possibly at* $z = \inf M$*), and* $w'(z) > 1$*, and* $w''(z) > 0$ *for* $z \in \text{int}M$*.*

*Proof.* Consider first $h \geq h_0$. From Lemma A.2, it is immediate that $\lim_{z \downarrow \bar{z}} w(z) = w(\bar{z}) = \bar{z}$. Note than that for $z \in \text{int}M$, then $w'(z) = n(e(z;\bar{z})) > 1$, which also implies that $w''(z) > 0$ as both $n(z)$ and $e(z;\bar{z})$ are strictly increasing in their arguments. Given that $\lim_{z \downarrow \bar{z}} w(z) = \bar{z}$, the previous results then imply that $w(z) > z$ for all $z \in (\bar{z}, 1]$.

On the other hand, $\lim_{z \uparrow \underline{z}} w(z) = 1 - w(1)h(1 - \underline{z}) = \underline{z} = \lim_{z \downarrow \underline{z}} w(z)$, where we are using the fact that $f(\underline{z};\bar{z}) = 1$ and that $w(1) = \int_{\bar{z}}^{1} n\big(e(u;\bar{z})\big)du + \bar{z} = \frac{1}{h}$ (due to condition (2)). Note, moreover, that for $z \in \text{int}W$, then $w'(z) = hw(f(z;\bar{z})) > 0$, which immediately implies that $w''(z) > 0$, since both $w(z)$ and $f(z;\bar{z})$ are strictly increasing in their arguments. That $w(z) > z$ for all $z \in [0, \underline{z})$ then follows because $w(z)$ is strictly increasing and convex and $\lim_{z \uparrow \underline{z}} w(z) = \underline{z}$. Finally, that $w'(z) \in (0,1)$ comes from the fact that:

$$w'(z) = hw(f(z;\bar{z})) = h\left[\frac{f(z;\bar{z}) - w(z)}{h(1-z)}\right] < 1$$

where the second-to-last equality comes from the firms' zero-profit condition, and the last inequality because $f(z;\bar{z}) \leq 1$ and $w(z) > z$ when $z \in \text{int}W$.

Now consider $h \leq h_0$. Given that $f(\hat{z};\hat{z}) = 1$, from Lemma A.2, we have that:

$$\lim_{z \uparrow \hat{z}} w(z) = 1 - \tfrac{1}{n(\hat{z})} w(1) \quad \text{and} \quad \lim_{z \downarrow \hat{z}} w(z) = \tfrac{1}{1+n(\hat{z})}\left\{n(\hat{z}) - \int_{\hat{z}}^{1} n\big(e(u;\hat{z})\big)du\right\}$$

Note then that $w(1) = \lim_{z \downarrow \hat{z}} w(z) + \int_{\hat{z}}^{1} n\big(e(u;\hat{z})\big)du$. Combining the latter with the expression for $\lim_{z \uparrow \hat{z}} w(z)$ above and rearranging terms yields $\lim_{z \uparrow \hat{z}} w(z) = \lim_{z \downarrow \hat{z}} w(z)$. The proof that $\lim_{z \downarrow \hat{z}} w(z) > \hat{z}$, in turn, can be found in Fuchs et al. (2015, Online Supplement, p. 4). Finally, the proofs for the remaining properties of $w(z)$ (i.e., their monotonicity and convexity, among others) follow the exact same logic as in the case when $h > h_0$. □

# B The AI Equilibrium: Complete Characterization

In this Appendix, we provide a complete characterization of the AI equilibrium. As noted in the main text, we focus on $h < h_0$ (see Section 3 of the Online Appendix for $h \geq h_0$). The proof of Proposition 2 is a direct implication of the results that follow.

## B.1 Equilibrium Characterization

We begin the characterization with the following set of results:

**Lemma B.1.** *Any equilibrium with AI has the following features:*

- *Some computing power must be allocated to independent production:* $\mu_i^* > 0$.
- *The price of computing power is equal to AI's knowledge:* $r^* = z_{\text{AI}}$.
- *Occupational stratification:* $W^* \preceq I^* \preceq M^*$.
- *No worker better than AI; no manager worse than AI:* $W^* \preceq \{z_{\text{AI}}\} \preceq M^*$.
- *Positive assortative matching:* $f^* : W_p^* \to M_p^*$ *is strictly increasing and* $W_a^* \preceq W_p^*$ *and* $M_p^* \preceq M_a^*$.

*Proof.* • *Some computing power must be allocated to independent production.*— This result follows because computing power is abundant relative to human time. Hence, there are not enough humans to interact with AI inside two-layer organizations.

• *The price of computing power is equal to AI's knowledge.*— This follows because the single-layer firms using AI must obtain zero profits.

• *Occupational stratification.*— Notice that the First Welfare Theorem holds in our setting. Hence, a competitive equilibrium must be efficient. Occupational stratification then follows because any surplus maximizing allocation must satisfy it. The proof of this last result is analogous to the proof of Lemma 1 in Fuchs et al. (2015).

• *No worker better than AI; no manager worse than AI.*— This result follows from occupational stratification and the fact that some computing power must necessarily be used for independent production.

• *Positive assortative matching.*— The emergence of positive assortative matching—which follows from the supermodularity of the profits of two-layer organizations—is proven in Eeckhout and Kircher (2018, Proposition 1, p. 94) in a more general setting that encompasses ours. Positive assortative matching then implies that the matching function is strictly increasing and that $W_a^* \preceq W_p^*$ and $M_p^* \preceq M_a^*$ (since no worker is better than AI and no manager is worse than AI). □

The next corollary is a direct implication of Lemma B.1:

**Corollary B.1.** *An equilibrium allocation must take one of the following four potential configurations:*

- ***Type 1 configuration**:*

$$W_a^* = \emptyset,\ W_p^* = [0, z_{\text{AI}}],\ I^* = (z_{\text{AI}}, \underline{z}_1^*),\ M_p^* = [\underline{z}_1^*, \bar{z}_1^*],\ M_a^* = [\bar{z}_1^*, 1],\ \textit{where } z_{\text{AI}} < \underline{z}_1^* \leq \bar{z}_1^* \leq 1$$
$$\textit{So } \mu_w^* = \textstyle\int_{\bar{z}_1^*}^1 n(z_{\text{AI}})dG(z),\ \mu_m^* = 0,\ \mu_i^* = \mu - \mu_w^*$$

- ***Type 2 configuration**:*

$$W_a^* = [0, \underline{z}_2^*],\ W_p^* = [\underline{z}_2^*, z_{\text{AI}}],\ I^* \subseteq \{z_{\text{AI}}\},\ M_p^* = [z_{\text{AI}}, \bar{z}_2^*],\ M_a^* = [\bar{z}_2^*, 1],\ \textit{where } 0 \leq \underline{z}_2^* \leq z_{\text{AI}} \leq \bar{z}_2^* \leq 1$$
$$\textit{So } \mu_w^* = \textstyle\int_{\bar{z}_2^*}^1 n(z_{\text{AI}})dG(z),\ \mu_m^* = \int_0^{\underline{z}_2^*} n(z)^{-1}dG(z),\ \mu_i^* = \mu - \mu_w^* - \mu_m^*$$

- ***Type 3 configuration**:*

$$W_a^* = [0, \underline{z}_3^*],\ W_p^* = [\underline{z}_3^*, \bar{z}_3^*],\ I^* = (\bar{z}_3^*, z_{\text{AI}}),\ M_p^* = [z_{\text{AI}}, 1],\ M_a^* = \emptyset,\ \textit{where } 0 < \underline{z}_3^* \leq \bar{z}_3^* < z_{\text{AI}}$$
$$\textit{So } \mu_w^* = 0,\ \mu_m^* = \textstyle\int_0^{\underline{z}_3^*} n(z)^{-1}dG(z),\ \mu_i^* = \mu - \mu_m^*$$

- ***Type 4 configuration**:*

$$W_a^* = \emptyset,\ W_p^* = [0, \underline{z}_4^*],\ I^* = (\underline{z}_4^*, \bar{z}_4^*) \ni z_{\text{AI}},\ M_p^* = [\bar{z}_4^*, 1],\ M_a^* = \emptyset,\ \textit{where } 0 \leq \underline{z}_4^* < \bar{z}_4^* \leq 1$$

$$\textit{So } \mu_w^* = 0,\ \mu_m^* = 0,\ \mu_i^* = \mu$$

*Proof.* As mentioned above, the proof of this corollary is a direct implication of Lemma B.1. Note that in a Type 2 configuration, $I^*$ can either be $\{z_{\text{AI}}\}$ or $\emptyset$ because the human with knowledge $z_{\text{AI}}$ is indifferent between any of the three roles. However, this is irrelevant for all practical purposes because $I^*$ has measure zero. □

Intuitively, in a Type 1 configuration, AI is used as a worker and as an independent producer. In a Type 2 configuration, AI is used in all three possible roles (i.e., as a worker, an independent producer, and a manager). In a Type 3 configuration, AI is used as a manager and as an independent producer, while in a Type 4 configuration, AI is used exclusively as an independent producer.

Now, recall that $W$ and $M$ are the sets of human workers and managers in the pre-AI equilibrium. For $z_{\text{AI}} \in W$, define the function $f_w : [0, z_{\text{AI}}] \to [z_{\text{AI}}, 1]$ by $\int_{z_{\text{AI}}}^{f_w(z;z_{\text{AI}})} dG(u) = \int_0^z h(1-u)dG(u)$ and note that $z_{\text{AI}} \in W$ implies that $f_w(z_{\text{AI}}; z_{\text{AI}}) \leq 1$. Let then $e_w(z; z_{\text{AI}}) \equiv f_w^{-1}(z; z_{\text{AI}})$ and define:

$$\Gamma_w(x) \equiv n(x)(f_w(x;x) - x) - x - \int_x^{f_w(x;x)} n(e_w(u;x))du$$

Similarly, for $z_{\text{AI}} \in M$, define the function $e_m : [z_{\text{AI}}, 1] \to [0, z_{\text{AI}}]$ by $\int_z^1 dG(u) = \int_{e_m(z;z_{\text{AI}})}^{z_{\text{AI}}} h(1-u)dG(u)$, note that $z_{\text{AI}} \in M$ implies that $e_m(z_{\text{AI}}; z_{\text{AI}}) \geq 0$, and define:

$$\Gamma_m(x) \equiv \tfrac{1}{h} - x - \int_x^1 n(e_m(u;x))du$$

Consider the following partition of the knowledge space (note that $W \cup M = [0,1]$ when $h < h_0$): $\mathcal{R}_1 \equiv W \cap \{z \in W : \Gamma_w(z) < 0\}$, $\mathcal{R}_2 \equiv \{z \in W : \Gamma_w(z) \geq 0\} \cup \{z \in M : \Gamma_m(z) \geq 0\}$, and $\mathcal{R}_3 \equiv M \cap \{z \in M : \Gamma_m(z) < 0\}$. The next lemma provides some important properties of this partition that will be useful later on:

**Lemma B.2.** *$z \in \mathcal{R}_1$ if $z \in (0, \epsilon)$ with $\epsilon \downarrow 0$. Moreover, $\hat{z} \in \mathcal{R}_2$, where $\hat{z}$ is the knowledge of the best pre-AI worker/worst pre-AI manager.*

*Proof.* Note that $\lim_{x \downarrow 0} \Gamma_w(x) = 0$ (as $\lim_{x \downarrow 0} f_w(z;x) = 0$), and that $\lim_{x \downarrow 0} \Gamma_w'(x) = -1$, as:

$$\Gamma_w'(x) = \tfrac{1}{h} - 1 - \tfrac{1}{h(1-x)} + \tfrac{f_w(x;x)-x}{h(1-x)^2} + h\int_x^{f_w(x;x)} \tfrac{n(e_w(z;x))^3 g(x)}{g(e_w(z;x))} dz$$

Hence, if $z \in (0, \epsilon)$ with $\epsilon \downarrow 0$, then $z \in W$ and $\Gamma_w(z) < 0$, so $z \in \mathcal{R}_1$.

We now prove that $\hat{z} \in \mathcal{R}_2$ by showing that $\Gamma_m(\hat{z}) = \Gamma_w(\hat{z}) > 0$. Note that $f_w(z;\hat{z}) = f(z;\hat{z})$, where $f(z;\hat{z})$ is the matching function of the pre-AI equilibrium. This implies that $f_w(\hat{z};\hat{z}) = 1$, so $\Gamma_w(\hat{z}) = (1/h) - \hat{z} - \int_{\hat{z}}^1 n(e(z;\hat{z}))dz > 0$, where the last inequality follows from Lemma A.1. Similarly, note that $e_m(z;\hat{z}) = e(z;\hat{z})$, where $e(z;\hat{z}) = f^{-1}(z;\hat{z})$. Thus, $\Gamma_m(\hat{z}) = (1/h) - \hat{z} - \int_{\hat{z}}^1 n(e(z;\hat{z}))dz$, so $\Gamma_m(\hat{z}) = \Gamma_w(\hat{z}) > 0$. □

We then have the following result:

**Lemma B.3.** *In the presence of AI, there is a unique competitive equilibrium. It is given as follows:*

- *If $z_{\text{AI}} \in \mathcal{R}_1$, then the equilibrium allocation is Type 1. The equilibrium cutoffs $\underline{z}_1^*$ and $\bar{z}_1^*$ satisfy:*

$$\bar{z}_1^* = f_1^*(z_{\text{AI}}; \underline{z}_1^*) \quad \textit{and} \quad n(z_{\text{AI}})(f_1^*(z_{\text{AI}}; \underline{z}_1^*) - z_{\text{AI}}) = \underline{z}_1^* + \int_{\underline{z}_1^*}^{f_1^*(z_{\text{AI}}; \underline{z}_1^*)} n\big(e_1^*(z; \underline{z}_1^*)\big) dz$$

  *where $f_1^* : [0, z_{\text{AI}}] \to [\underline{z}_1^*, \bar{z}_1^*]$ is given by $\int_{\underline{z}_1^*}^{f_1^*(z;\underline{z}_1^*)} dG(u) = \int_0^z h(1-u) dG(u)$ and $e_1^*(z;\cdot) = (f_1^*)^{-1}(z;\cdot)$*

- *If $z_{\text{AI}} \in \mathcal{R}_2$, then the equilibrium allocation is Type 2. The equilibrium cutoffs $\underline{z}_2^*$ and $\bar{z}_2^*$ satisfy:*

$$\bar{z}_2^* = f_2^*(z_{\text{AI}}; \underline{z}_2^*) \quad \textit{and} \quad n(z_{\text{AI}})(f_2^*(z_{\text{AI}}; \underline{z}_2^*) - z_{\text{AI}}) = z_{\text{AI}} + \int_{z_{\text{AI}}}^{f_2^*(z_{\text{AI}}; \underline{z}_2^*)} n\big(e_2^*(z; \underline{z}_2^*)\big) dz$$

  *where $f_2^* : [\underline{z}_2^*, z_{\text{AI}}] \to [z_{\text{AI}}, \bar{z}_2^*]$ given by $\int_{z_{\text{AI}}}^{f_2^*(z;\underline{z}_2^*)} dG(u) = \int_{\underline{z}_2^*}^{z} h(1-u) dG(u)$ and $e_2^*(z;\cdot) = (f_2^*)^{-1}(z;\cdot)$*

- *If $z_{\text{AI}} \in \mathcal{R}_3$, then the equilibrium allocation is Type 1. The equilibrium cutoffs $\underline{z}_3^*$ and $\bar{z}_3^*$ satisfy:*

$$\bar{z}_3^* = e_3^*(1; \underline{z}_3^*) \quad \textit{and} \quad \tfrac{1}{h} = z_{\text{AI}} + \int_{z_{\text{AI}}}^{1} n\big(e_3^*(z; \underline{z}_3^*)\big) dz$$

  *where $f_3^* : [\underline{z}_3^*, \bar{z}_3^*] \to [z_{\text{AI}}, 1]$ given by $\int_{z_{\text{AI}}}^{f_3^*(z;\underline{z}_3^*)} dG(u) = \int_{\underline{z}_3^*}^{z} h(1-u) dG(u)$ and $e_3^*(z;\cdot) = (f_3^*)^{-1}(z;\cdot)$*

*The equilibrium matching function is given by $f^*(z) = f_j^*(z; \underline{z}_j^*)$ if $z_{\text{AI}} \in \mathcal{R}_j$, while the equilibrium wage $w^*(z)$ is continuous, strictly increasing, and (weakly) convex, and satisfies:*

*(i)* $w^*(z) = z_{\text{AI}}(1 - 1/n(z)) > z$ *for all* $z \in W_a^*$.
*(ii)* $w^*(z) = f^*(z) - w^*(f^*(z))/n(z) > z$ *for all* $z \in W_p^*$.
*(iii)* $w^*(z) = z$ *for all* $z \in I^*$.
*(iv)* $w^*(z) = m_p + \int_{m_p}^{z} n(e^*(u)) du > z$, *for all* $z \in M_p^*$, *where* $m_p \equiv \inf M_p^*$.
*(v)* $w^*(z) = n(z_{\text{AI}})(z - z_{\text{AI}}) > z$, *for all* $z \in M_a^*$.

Before formally proving this lemma, we informally derive the equilibrium in one of the regions to provide insight into its construction:

*Informal Construction of the Equilibrium.*—- Suppose that $z_{\text{AI}} \in \mathcal{R}_2$. By Corollary B.1, we know that such equilibrium must lead to the following partition of the human population:

$$W_a^* = [0, \underline{z}_2^*],\ W_p^* = [\underline{z}_2^*, z_{\text{AI}}],\ I^* = \emptyset,\ M_p^* = [z_{\text{AI}}, \bar{z}_2^*],\ M_a^* = [\bar{z}_2^*, 1],\ \text{where } 0 \le \underline{z}_2^* \le z_{\text{AI}} \le \bar{z}_2^* \le 1$$

As mentioned in the main text, given that the equilibrium price of processing capacity is $r^* = z_{\text{AI}}$, the zero-profit condition of a $t$A firm pins down the wage $w^*(z) = z_{\text{AI}}(1 - 1/n(z))$ of a human worker with knowledge $z \in W_a^*$. Similarly, the zero profit condition of a $b$A firm determines the wage $w^*(z) = n(z_{\text{AI}})(z - z_{\text{AI}})$ of a human manager with knowledge $z \in M_a^*$.

Now consider the firms that do not use AI, i.e., the $n$A firms. First, let $f_2^*(z)$ be the equilibrium matching function in this case. This function must be strictly increasing and satisfy the following resource constraint $\int_{z_{\text{AI}}}^{f_2^*(z)} dG(u) = \int_{\underline{z}_2^*}^{z} h(1-u) dG(u)$ for all $z \in [\underline{z}_2^*, z_{\text{AI}}]$. This constraint states that

the total time required to consult on the problems left unsolved by workers in the interval $[\underline{z}_2^*, z]$ must equal the total time available of managers in the interval $[z_{\text{AI}}, f_2^*(z)]$. Moreover, given that $\sup W_p^* = z_{\text{AI}}$ and $\sup M_p^* = \bar{z}_2^*$, it must also be that $f_2^*(z_{\text{AI}}) = \bar{z}_2^*$.

Note then that for any given $z \in W_p$, there exists a unique increasing function $f_2^*(z)$ that satisfies both constraints. It is given by the solution to the differential equation $f_2^{*\prime}(z) = h(1-z)g(z)/g(f_2^*(z))$ with border condition $f_2^*(z_{\text{AI}}) = \bar{z}_2^*$ (which comes from differentiating both sides of the resource constraint). We denote such a unique function by $f_2^*(z; \underline{z}_2^*)$ (as it depends on $\underline{z}_2^*$ through its domain).

With this in mind, consider the problem of a $n$A firm that recruited $n(z)$ workers of type $z \in W_p^*$ and is deciding which manager $z \in M_p^*$ to hire: $\max_{m \in M_p^*} \Pi_2^{n\text{A}}(z, m) = n(z)[m - w(z)] - w(m)$. As mentioned in the main text, the corresponding first-order condition evaluated at $m = f_2^*(z; \underline{z}_2^*)$ implies that $w^{*\prime}(z) = n(e_2^*(z; \underline{z}_2^*))$ for any $z \in M_p^*$. Thus, $w^*(z) = C + \int_{z_{\text{AI}}}^{z} n(e_2^*(u; \underline{z}_2^*))du$ for any $z \in M_p^*$. The wages of the workers of such firms then come from the zero profit condition of $n$A firms: $w^*(z) = f_2^*(z; \underline{z}_2^*) - w^*(f_2^*(z; \underline{z}_2^*))/n(z)$ for any $z \in W_p^*$.

The final step is determining the constant $C$, the cutoff $\underline{z}_2^*$, and arguing that no firms have incentives to deviate. To do this, note that the least knowledgeable human manager has the same knowledge as AI, i.e., $\inf M_p^* = z_{\text{AI}}$. Hence, her wage must be equal to the price of one unit of computing power, so $C^* = z_{\text{AI}}$. Moreover, the most knowledgeable manager hired by a $n$A firm has the same knowledge as the least knowledgeable manager of a $b$A firm. As a result, they must also receive the same wage, i.e., $\lim_{z \uparrow \bar{z}_2^*} w^*(z) = \lim_{z \downarrow \bar{z}_2^*} w^*(z)$. Since $\bar{z}_2^* = f_2^*(z_{\text{AI}}; \underline{z}_2^*)$, we obtain:

$$n(z_{\text{AI}})(f_2^*(z_{\text{AI}}; \underline{z}_2^*) - z_{\text{AI}}) = z_{\text{AI}} + \int_{z_{\text{AI}}}^{f_2^*(z_{\text{AI}}; \underline{z}_2^*)} n(e_2^*(z; \underline{z}_2^*))dz \tag{4}$$

which is the equilibrium condition in the statement of Lemma B.3. It is then possible to prove that there is a unique cutoff $\underline{z}_2^*$ that satisfies this condition and that such a cutoff is contained in $(0, z_{\text{AI}}]$ if and only if $z_{\text{AI}} \in \mathcal{R}_2$. This explains why this equilibrium can only arise in such a region of the parameter space. The fact that $C^* = z_{\text{AI}}$ and that $\underline{z}_2^*$ satisfies (4) then implies that $w^*(z)$ is also continuous at the juncture between $W_p^*$ and $M_p^*$:

$$\lim_{z \downarrow \underline{z}_2^*} w^*(z) = f_2^*(\underline{z}_2^*; \underline{z}_2^*) - \tfrac{w^*(f_2^*(\underline{z}_2^*; \underline{z}_2^*))}{n(\underline{z}_2^*)} = z_{\text{AI}} \left(1 - \tfrac{1}{n(\underline{z}_2^*)}\right) = \lim_{z \uparrow \underline{z}_2^*} w^*(z)$$
$$\lim_{z \uparrow z_{\text{AI}}} w^*(z) = f_2^*(z_{\text{AI}}; \underline{z}_2^*) - \tfrac{w^*(f_2^*(z_{\text{AI}}; \underline{z}_2^*))}{n(z_{\text{AI}})} = z_{\text{AI}} = \lim_{z \downarrow z_{\text{AI}}} w^*(z)$$

This is sufficient sufficient to guarantee that $w^*(z)$ is continuous in all its domain, i.e., for all $z \in [0, 1]$.

From here, arguing that no firm has incentives to deviate is straightforward. Indeed, note that the wage function is continuous, strictly increasing, and weakly convex. This implies that if a firm does not have incentives to deviate "locally," then it does not have incentives to deviate globally either. That $b$A firms do not want to deviate locally, i.e., hire a different human manager in $M_p^*$, follows because such deviation also leads to no profits. Similar reasoning also explains why $t$A and $n$A firms do not have incentives to deviate locally either. □

We now formally prove the lemma. This is done in two steps. First, we show that the outcomes described in the lemma are indeed an equilibrium by verifying that (i) markets clear, and (ii) firms are maximizing their profits while obtaining zero profits. Then, we prove that the equilibrium is unique.

*Proof of Step 1.* We will only show this part for $z_{\text{AI}} \in \mathcal{R}_1$, as the other two cases are analogous. We begin by verifying market clearing in the market for computing power. By Corollary B.1, it is immediate that $\mu_i^* + \mu_w^* + \mu_m^* = \mu$. Moreover, the total time required to consult on the problems left unsolved by AI is equal to the total time available of managers in $M_a^*$, i.e., $h(1 - z_{\text{AI}})\mu_w^* = \int_{\bar{z}_1^*}^1 dG(z)$.

We now move to market clearing of the labor market. First, it must be that the total time required to consult on the problems left unsolved by the human workers in the interval $[0, z] \subseteq W_p^*$ is equal to the total time available of human managers in the interval $[\underline{z}_1^*, f_1^*(z; \underline{z}_1^*)] \subseteq M_p^*$. This resource constraint is satisfied as $f_1^*(z; \underline{z}_1^*)$ is given by $\int_{\underline{z}_1^*}^{f_1^*(z;\underline{z}_1^*)} dG(u) = \int_0^z h(1-u)dG(u)$ and $\bar{z}_1^* = f_1^*(z_{\text{AI}}; \underline{z}_1^*)$.

Second, it must be that the union of the sets $(W_p^*, W_a^*, I^*, M_p^*, M_a^*)$ is $[0, 1]$, and the intersection of any two of these sets has measure zero. By Corollary B.1, this occurs if and only if $z_{\text{AI}} < \underline{z}_1^* \leq \bar{z}_1^* \leq 1$. Verifying that that $\underline{z}_1^* \leq \bar{z}_1^*$ is straightforward: It follows because $f_1^*(z; \underline{z}_1^*)$ is strictly increasing in $z$ plus the fact that $\underline{z}_1^* = f_1^*(0; \underline{z}_1^*)$ and $\bar{z}_1^* = f_1^*(z_{\text{AI}}; \underline{z}_1^*)$.

Showing that $z_{\text{AI}} < \underline{z}_1^*$ requires more work. Note that $\underline{z}_1^*$ is given by the solution $\Gamma_1(\underline{z}_1^*; z_{\text{AI}}) = 0$, where $\Gamma_1(x; z_{\text{AI}}) \equiv n(z_{\text{AI}})(f_1^*(z_{\text{AI}}; x, z_{\text{AI}}) - z_{\text{AI}}) - x - \int_x^{f_1^*(z_{\text{AI}};x,z_{\text{AI}})} n\big(e_1^*(z; x, z_{\text{AI}})\big)dz$ (here we are making explicit that $f_1^*(\cdot)$ and $e_1^*(\cdot)$ also depend indirectly on $z_{\text{AI}}$ through the boundary of the set $W_p^* = [0, z_{\text{AI}}]$ to avoid any type of confusion[26]). It is then not difficult to prove that $\Gamma_1(x; z_{\text{AI}})$ is strictly increasing in $x$, so $z_{\text{AI}} < \underline{z}_1^*$ if and only if $\Gamma_1(0; z_{\text{AI}}) < 0$. Furthermore, note that $f_1^*(z; 0, z_{\text{AI}})$ satisfies $\int_0^{f_1^*(z;0,z_{\text{AI}})} dG(u) = \int_0^z h(1-u)dG(u)$ for $z \in [0, z_{\text{AI}}]$, which implies that $f_1^*(z; 0, z_{\text{AI}}) = f_w(z; z_{\text{AI}})$ (and, therefore, $e_1^*(z; 0, z_{\text{AI}}) = e_w(z; z_{\text{AI}})$). Hence, $\Gamma_1(0; z_{\text{AI}}) = \Gamma_w(z_{\text{AI}}) < 0$ as $z_{\text{AI}} \in \mathcal{R}_1$.

Finally, we show that $\bar{z}_1^* \leq 1$. This is more involved. To prove it, we show that $\underline{z}_1^* < \hat{z}$ and then use this result to conclude that $\bar{z}_1^* \leq 1$. As a first step, note that $f_1^*(z; \hat{z}, \hat{z})$ satisfies $\int_{\hat{z}}^{f_1^*(z;\hat{z},\hat{z})} dG(u) = \int_0^z h(1-u)dG(u)$ for $z \in [0, \hat{z}]$, so $f_1^*(z; \hat{z}, \hat{z}) = f(z; \hat{z})$ where $f(z; \hat{z})$ is the pre-AI matching function. Recall then that $\underline{z}_1^*$ is the unique solution $\Gamma_1(\underline{z}_1^*; z_{\text{AI}}) = 0$, where $\Gamma_1(x; z_{\text{AI}})$ is strictly increasing in $x$. It is not difficult to prove that $\Gamma_1(x; z_{\text{AI}})$ is also strictly decreasing in $z_{\text{AI}}$ for any given $x$. We then claim that $\Gamma_1(\hat{z}; z_{\text{AI}}) > 0$, which immediately implies that $\underline{z}_1^* < \hat{z}$. Indeed, note that $\Gamma_1(\hat{z}; z_{\text{AI}}) \geq \Gamma_1(\hat{z}; \hat{z}) = \frac{1}{h} - \hat{z} - \int_{\hat{z}}^1 n(e(z; \hat{z}))dz$, where the first inequality follows because $\Gamma_1(x; z_{\text{AI}})$ is strictly decreasing in $z_{\text{AI}}$ and $z_{\text{AI}} \leq \hat{z}$ (as $z_{\text{AI}} \in W$) and the last equality because $e_1^*(z; \hat{z}, \hat{z}) = e(z; \hat{z})$ for all $z \in [\hat{z}, 1]$. However, by Lemma A.1, we know that $\frac{1}{h} - \hat{z} - \int_{\hat{z}}^1 n(e(z; \hat{z}))dz > 0$ when $h < h_0$, so $\Gamma_1(\hat{z}; z_{\text{AI}}) > 0$.

Having proved that $\underline{z}_1^* < \hat{z}$, we now show that $\bar{z}_1^* \leq 1$. By construction $\bar{z}_1^*$ satisfies $\int_{\underline{z}_1^*}^{\bar{z}_1^*} dG(u) = \int_0^{z_{\text{AI}}} h(1-u)dG(u)$. Note, moreover, that $z \in W$ implies that $\int_0^{z_{\text{AI}}} h(1-u)dG(u) \leq \int_0^{\hat{z}} h(1-u)dG(u) = \int_{\hat{z}}^1 dG(u)$ (as $z_{\text{AI}} \leq \hat{z}$ and $\int_0^{\hat{z}} h(1-u)dG(u) = \int_{\hat{z}}^1 dG(u)$). Hence, $\int_{\underline{z}_1^*}^{\bar{z}_1^*} dG(u) \leq \int_{\hat{z}}^1 dG(u)$. Given that $\underline{z}_1^* < \hat{z}$, it must be that $\bar{z}_1^* < 1$.

[26] In the statement of Lemma B.3, we simply wrote $f_1^*(z; \underline{z}_1^*)$ instead of $f_1^*(z; \underline{z}_1^*, z_{\text{AI}})$ to avoid cluttering notation.

Having verified market clearing, we now show that in the candidate equilibrium firms maximize their profits while obtaining zero profits. Given how the wages are constructed (see Section 3.3 of the main text), it is clear that firms are optimizing their profits "locally" while obtaining zero profits. Thus, we only need to consider "global deviations." As discussed above, to discard such global deviations, it is sufficient to show that $w^*(z)$ is continuous, strictly increasing, and weakly convex. This is what we prove next.

To show continuity, it suffices to verify that $w^*(z)$ is continuous at the junctures of (i) $M_p^*$ and $M_a^*$, (ii) $I^*$ and $M_p^*$, and (iii) $W_p^*$ and $I^*$. For (i), note that $\lim_{z\uparrow\bar{z}_1^*} w^*(z) = \bar{z}_1^* + \int_{\underline{z}_1^*}^{\bar{z}_1^*} n(e_1^*(z;\underline{z}_1^*))dz$ and $\lim_{z\downarrow\bar{z}_1^*} w^*(z) = n(z_{\text{AI}})(\bar{z}_1^* - z_{\text{AI}})$. Hence, from the conditions determining $\underline{z}_1^*$ and $\bar{z}_1^*$, we obtain that $\lim_{z\uparrow\bar{z}_1^*} w^*(z) = \lim_{z\downarrow\bar{z}_1^*} w^*(z)$. For (ii) note that by construction, $\lim_{z\uparrow\underline{z}_1^*} w^*(z) = \underline{z}_1^* = \lim_{z\downarrow\underline{z}_1^*} w^*(z)$. Finally, for (iii) note that $\lim_{z\uparrow z_{\text{AI}}} w^*(z) = \bar{z}_1^* - w^*(\bar{z}_1^*)/n(z_{\text{AI}}) = z_{\text{AI}} = \lim_{z\downarrow z_{\text{AI}}} w^*(z)$ given that $w^*(\bar{z}_1^*) = n(z_{\text{AI}})(\bar{z}_1^* - z_{\text{AI}})$.

With continuity at hand, proving that $w^*(z)$ is strictly increasing and weakly convex is straightforward: The logic is analogous to the proof of Corollary A.1 (which shows that the pre-AI wage function satisfies these two properties). Consequently, in the case $z_{\text{AI}} \in \mathcal{R}_1$, the outcome described in the statement is indeed a competitive equilibrium. □

*Proof of Step 2.* We will only show this part for $z_{\text{AI}} \in \mathcal{R}_1$, as the other two cases follow the same logic. In particular, we show that if $z_{\text{AI}} \in \mathcal{R}_1$, then there cannot be any other type of equilibrium.

Suppose first by contradiction that there is a Type 3 equilibrium. By Corollary B.1, we know that such equilibrium must lead to the following partition of the human population:

$$W_a^* = [0, \underline{z}_3^*],\ W_p^* = [\underline{z}_3^*, \bar{z}_3^*],\ I^* = (\bar{z}_3^*, z_{\text{AI}}),\ M_p^* = [z_{\text{AI}}, 1],\ M_a^* = \emptyset,\ \text{where } 0 < \underline{z}_3^* \leq \bar{z}_3^* < z_{\text{AI}}$$

By Lemma B.2, we have that if $z_{\text{AI}} \in \mathcal{R}_1$, then $z_{\text{AI}} < \hat{z}$. However, if so, then:

$$\int_{\underline{z}_3^*}^{\bar{z}_3^*} h(1-u)dG(u) < \int_0^{z_{\text{AI}}} h(1-u)dG(u) < \int_0^{\hat{z}} h(1-u)dG(u) = \int_{\hat{z}}^1 dG(u) < \int_{z_{\text{AI}}}^1 dG(u)$$

which violates the resource constraint that the total time required to consult on the problems left unsolved by the human workers in the interval $W_p^*$ must equal to the total time available of human managers in the interval $M_p^*$, i.e., $\int_{\underline{z}_3^*}^{\bar{z}_3^*} h(1-u)dG(u) = \int_{z_{\text{AI}}}^1 dG(u)$. Hence, a Type 3 configuration cannot arise when $z_{\text{AI}} \in \mathcal{R}_1 \subset W$.

Now suppose for contradiction that there is a Type 2 equilibrium. As explained above (see "Informal Construction of the Equilibrium"), for this to be an equilibrium, there must exist a cutoff $\underline{z}_2^* \in [0, z_{\text{AI}}]$ such that $\Gamma_2(\underline{z}_2^*; z_{\text{AI}}) = 0$, where $\Gamma_2(x; z_{\text{AI}}) \equiv n(z_{\text{AI}})(f_2^*(z_{\text{AI}}; x) - z_{\text{AI}}) - z_{\text{AI}} - \int_{z_{\text{AI}}}^{f_2^*(z_{\text{AI}};x)} n(e_2^*(z;x))dz$. It is then not difficult to prove that $\Gamma_2(x; z_{\text{AI}})$ is strictly decreasing in $x$. Hence, there exists at most one $\underline{z}_2^*$ that satisfies $\Gamma_2(\underline{z}_2^*; z_{\text{AI}}) = 0$, and for $\underline{z}_2^* \geq 0$, it must be that $\Gamma_2(0; z_{\text{AI}}) \geq 0$, where $f_2^*(z;0)$ is given by $\int_{z_{\text{AI}}}^{f_2^*(z;0)} dG(u) = \int_0^z h(1-u)dG(u)$ for $z \in [0, z_{\text{AI}}]$. However, from this last condition we have that $f_2^*(z;0) = f_w(z; z_{\text{AI}})$ (as $f_2^*(z;0)$ and $f_w(z; z_{\text{AI}})$ satisfy the same condition), so $\Gamma_2(0; z_{\text{AI}}) = \Gamma_w(z_{\text{AI}})$. Consequently, for $\underline{z}_2^* \geq 0$, we need that $\Gamma_w(z_{\text{AI}}) \geq 0$, which contradicts the fact that $z_{\text{AI}} \in \mathcal{R}_1$.

Finally, suppose by contradiction that there is a Type 4 equilibrium. By Corollary B.1, we know that such equilibrium must lead to the following partition of the human population:

$$W_p^* = \emptyset,\ W_p^* = [0, \underline{z}_4^*],\ I^* = (\underline{z}_4^*, \bar{z}_4^*) \ni z_{\text{AI}},\ M_p^* = [\bar{z}_4^*, 1],\ M_a^* = \emptyset$$

Moreover, following similar reasoning as the one developed above for a Type 2 equilibrium, for this to be an equilibrium, (i) it must be that $\bar{z}_4^* = f_4^*(0; \underline{z}_4^*)$, where $f_4^*(0; \underline{z}_4^*)$ satisfies $f_4^*(\underline{z}_4^*; \underline{z}_4^*) = 1$ and $\int_{f_4^*(0;\underline{z}_4^*)}^{f_4^*(z;\underline{z}_4^*)} dG(u) = \int_{\underline{z}_4^*}^{z} h(1-u)dG(u)$ for $z \in [0, \underline{z}_4^*]$, and (ii) there must exist a cutoff $\underline{z}_4^* < f_4^*(0; \underline{z}_4^*)$ such that $1/h - f_4^*(0; \underline{z}_4^*) - \int_{f_4^*(0;\underline{z}_4^*)}^{1} n(e_4^*(z; \underline{z}_4^*))dz = 0$. But by Lemma A.1, we know that the unique solution to these equilibrium conditions is $\underline{z}_4^* = \underline{z}$ and $\bar{z}_4^* = f_4^*(0; \underline{z}_4^*) = \bar{z}$, where $\underline{z}$ and $\bar{z}$ are the equilibrium cutoffs of the pre-AI equilibrium when $h > h_0$. This, however, implies that this configuration is an equilibrium only if $z_{\text{AI}} \in I^* = (\underline{z}, \bar{z})$, which contradicts the assumption that $z_{\text{AI}} \in \mathcal{R}_1$. □

# C Proofs Omitted from Section 4

## C.1 Proof of Proposition 3

• $z_{\text{AI}} \in \text{int}W$.— By Lemma B.3 the AI equilibrium is either Type 1 or Type 2. If it is Type 2, then $\sup W^* = \inf M^* = z_{\text{AI}}$, so $W^* \subset W$ and $M^* \supset M$, since $z_{\text{AI}} < \hat{z}$ when $z_{\text{AI}} \in \text{int}W$. If it is Type 1, then $\sup W^* = z_{\text{AI}}$ and $\inf M^* = \underline{z}_1^*$. That $\sup W^* = z_{\text{AI}}$ immediately implies that $W^* \subset W$ as $z_{\text{AI}} < \hat{z}$. That $\inf M^* = \underline{z}_1^*$ immediately implies that $M^* \supset M$ given that $\underline{z}_1^* < \hat{z}$ (as shown in the proof of Lemma B.3). □

• $z_{\text{AI}} \in \text{int}M$.— By Lemma B.3, the AI equilibrium is either Type 2 or Type 3. Suppose first that it is Type 2. Then $\sup W^* = \inf M^* = z_{\text{AI}}$, so $W^* \supset W$ and $M^* \subset M$ given that $z_{\text{AI}} > \hat{z}$ when $z_{\text{AI}} \in \text{int}M$.

Now suppose the equilibrium is Type 3. Then, $\inf M^* = z_{\text{AI}} > \hat{z}$, immediately implying that $M^* \subset M$. To prove that $W^* \supset W$, we show that $\sup W^* = \bar{z}_3^* > \hat{z}$. Indeed, recall that $\bar{z}_3^*$ is given by:[27]

$$\bar{z}_3^* = e_3^*(1; \underline{z}_3^*, z_{\text{AI}}) \quad \text{and} \quad \tfrac{1}{h} = z_{\text{AI}} + \int_{z_{\text{AI}}}^{1} n\big(e_3^*(z; \underline{z}_3^*, z_{\text{AI}})\big)dz$$
$$\text{where } \int_{z_{\text{AI}}}^{f_3^*(z;\underline{z}_3^*,z_{\text{AI}})} dG(u) = \int_{\underline{z}_3^*}^{z} h(1-u)dG(u) \text{ for } z \in [\underline{z}_3^*, \bar{z}_3^*]$$

Using the fact that $f_3^*(\bar{z}_3^*; \underline{z}_3^*, z_{\text{AI}}) = 1$, the equilibrium conditions that determine $\underline{z}_3^*$ and $\bar{z}_3^*$ can be written as follows:[28]

$$\underline{z}_3^* = \tilde{e}_3^*(z_{\text{AI}}; \bar{z}_3^*, z_{\text{AI}}) \quad \text{and} \quad \tfrac{1}{h} = z_{\text{AI}} + \int_{z_{\text{AI}}}^{1} n\big(\tilde{e}_3^*(z; \bar{z}_3^*, z_{\text{AI}})\big)dz$$
$$\text{where } G(z) = 1 - \int_{\tilde{e}_3^*(z;\bar{z}_3^*,z_{\text{AI}})}^{\bar{z}_3^*} h(1-u)dG(u) \text{ for } z \in [z_{\text{AI}}, 1]$$

Define $\Gamma_3(x; z_{\text{AI}}) \equiv \frac{1}{h} - z_{\text{AI}} - \int_{z_{\text{AI}}}^{1} n\big(\tilde{e}_3^*(z; x, z_{\text{AI}})\big)dz$. It is not difficult to prove that $\Gamma_3(x; z_{\text{AI}})$ is strictly decreasing in $x$ and strictly increasing in $z_{\text{AI}}$. Moreover, $\bar{z}_3^*$ is given by the unique solution

[27] To avoid any type of confusion, we are making explicit that $f_3^*(z; \underline{z}_3^*, z_{\text{AI}})$ and $e_3^*(1; \underline{z}_3^*, z_{\text{AI}})$ depend on both $\underline{z}_3^*$ and $z_{\text{AI}}$ (in the statement of Lemma B.3, we simply wrote $f_3^*(z; \underline{z}_3^*)$ instead of $f_3^*(z; \underline{z}_3^*, z_{\text{AI}})$ to avoid cluttering notation).

[28] Note that $\tilde{e}_3^*(z; \bar{z}_3^*, z_{\text{AI}})$ is the equilibrium employee function indexed by $\bar{z}_3^*$ instead of $\underline{z}_3^*$.

to $\Gamma_3(\bar{z}_3^*; z_{\text{AI}}) = 0$. To prove that $\bar{z}_3^* > \hat{z}$, it suffices to show that $\Gamma_3(\hat{z}; z_{\text{AI}}) > 0$. Since $z_{\text{AI}} > \hat{z}$, we have that $\Gamma_3(\hat{z}; z_{\text{AI}}) > \Gamma_3(\hat{z}; \hat{z}) = \frac{1}{h} - \hat{z} - \int_{z_{\text{AI}}}^1 n\big(e(z; \hat{z})\big)dz > 0$, where the second-to-last inequality follows because $\tilde{e}_3^*(z; \hat{z}, \hat{z}) = e(z; \hat{z})$ for all $z \in [\hat{z}, 1]$, and the last inequality comes from the pre-AI equilibrium characterized in Lemma A.1. □

• $z_{\text{AI}} \in W \cap M$.— By Lemma B.2, we know that $z_{\text{AI}} = \hat{z} \in \mathcal{R}_2$, so Lemma B.3 implies that the equilibrium is necessarily Type 2. This implies that $\sup W^* = \inf M^* = z_{\text{AI}} = \hat{z}$, so there is no occupational displacement.

Showing that $W_a^* \neq \emptyset$ and $M_a^* \neq \emptyset$ requires more work. First, it is not difficult to prove that $\underline{z}_2^* > 0$ if and only if $\bar{z}_2^* < 1$. Hence, to prove that $W_a^* \neq \emptyset$ and $M_a^* \neq \emptyset$ it suffices to show that $\underline{z}_2^* > 0$. To do the latter, suppose for contradiction that $\underline{z}_2^* = 0$ ($\underline{z}_2^* < 0$ immediately contradicts that we are in a Type 2 equilibrium). Then the equilibrium matching function is given by $\int_{\hat{z}}^{f_2^*(z;0)} dG(u) = \int_0^z h(1-u)dG(u)$ for $z \in [\hat{z}, 1]$, implying that $f_2^*(z; 0) = f(z; \hat{z})$ for all $z \in [\hat{z}, 1]$. However, if so, then:

$$n(z_{\text{AI}})(f_2^*(z_{\text{AI}}; 0) - z_{\text{AI}}) = \tfrac{1}{h} \neq \hat{z} + \int_{\hat{z}}^1 n\big(e(z; \hat{z})\big)dz = z_{\text{AI}} + \int_{z_{\text{AI}}}^{f_2^*(z_{\text{AI}};0)} n\big(e_2^*(z; 0)\big)dz$$

where the inequation follows because $1/h - \hat{z} > \int_{\hat{z}}^1 n(e(z; \hat{z}))dz$ (by Lemma A.1). Thus, the equilibrium condition for $\underline{z}_2^* = 0$ is not satisfied (see the statement of Lemma B.3). Contradiction. □

### C.2 Proof of Lemma 1

The size of a two-layer firm with a manager with knowledge $m$ and workers with knowledge $z$ is $mn(z)$. The size of a single-layer firm is the knowledge of its independent producer. The first part of the lemma then follows because $n(z) > 1$ for all $z \in [0, 1]$, and the fact that independent producers are always less knowledgeable than managers (due to occupational stratification).

Now the second part. Take two arbitrary firms $f$ and $f'$. Without loss of generality, there are three cases to consider: (i) $f$ and $f'$ are two-layer firms, (ii) $f$ and $f'$ are single-layer firms, and (iii) $f$ is a two-layer firm and $f'$ is a single layer firm. In case (i), the statement follows directly from positive assortative matching. In case (ii), the statement follows because size equals productivity in one-layer firms. Finally, in case (iii), $f$ is necessarily strictly more productive (due to occupational stratification) and bigger (because $f$ has more layers than $f'$). □

### C.3 Proof of Proposition 4

As noted in the main text, in the case of two-layer organizations, the best worker determines the maximum firm size (as the best manager always has knowledge 1), while the knowledge of the worst manager determines the minimum firm size and the productivity of the least productive firm (as the worst worker always has knowledge 0).

With this in mind, consider $z_{\text{AI}} \in \text{int}W$. Then $\sup W^* < \sup W$ and $\inf M^* < \inf M$ (as $W^* \subset W$

and $M^* \supset M$), so the size of the biggest firm and the size and productivity of the smallest/least productive firm decreases with AI. Analogously, when $z_{\text{AI}} \in \text{int}M$, then $\sup W^* > \sup W$ and $\inf M^* > \inf M$ (as $W^* \supset W$ and $M^* \subset M$). Hence, in this case, the size of the biggest firm and the size and productivity of the smallest/least productive firm increases with AI. Finally, if $z_{\text{AI}} \in W \cap M$, then $\sup W^* = \sup W$ and $\inf M^* = \inf M$, so the size of the biggest firm and the size and productivity of the smallest/least productive firm do not change with AI. □

### C.4 Proof of Proposition 5

First, since two-layer firms are always larger than one-layer firms, the results regarding the size of the largest firm among all firms mimic those of two-layer firms described in Proposition 4.

Consider now the smallest/least productive firm. For ease of exposition, we refer to this firm as the "smallest firm." Regarding productivity, note that the smallest firm pre-AI is a two-layer firm with productivity $\hat{z} = \sup W$, while the smallest firm post-AI is a single-layer firm with productivity $\sup W^*$. Consequently, the results in this case mimic those of two-layer firms, as $\sup W^* < \sup W$ when $z_{\text{AI}} \in \text{int}W$, $\sup W^* > \sup W$ when $z_{\text{AI}} \in \text{int}M$, and $\sup W^* = \sup W$ when $z_{\text{AI}} \in W \cap M$ (as formally shown in the proof of Proposition 4).

Now consider size. Pre-AI, the size of the smallest firm is $\hat{z}n(0)$, while post-AI, the size of the smallest firm is $\sup W^*$. First, take $z_{\text{AI}} \in \text{int}W$. By Proposition 4, $\sup W^* < \sup W = \hat{z} < \hat{z}n(0)$, so, in this case, the result mimics that of two-layer firms described in Proposition 4. Second, take $z_{\text{AI}} \in W \cap M$. Then $\sup W^* = z_{\text{AI}} = \hat{z} < \hat{z}n(0)$. Hence, in contrast to the results regarding two-layer firms, now the size of the smallest firm decreases with AI. Finally, take $z_{\text{AI}} \in \text{int}M$. Then $\sup W^*$ is either $z_{\text{AI}}$ (in a Type 2 equilibrium) or a $\bar{z}_3^*$ (in a Type 3). Hence, in contrast to the results regarding two-layer firms, now the size of the smallest firm can increase or decrease with AI, as $\hat{z}n(0) \gtrless z_{\text{AI}}$ and $\hat{z}n(0) \gtrless \bar{z}_3^*$. □

### C.5 Proof of Proposition 6

As noted in the main text, a worker's productivity increases if and only if her managerial match improves. Similarly, the size of the firm of a given manager increases if and only the knowledge of her workers increases.

• $z_{\text{AI}} \in \text{int}W$.— First, we show that every $z \in W^*$ has a worse manager post-AI. Note that if $z \in W_a^*$, then such a worker is matched with AI in the AI equilibrium. However, if so, then $f(z; \hat{z}) \geq \hat{z} > z_{\text{AI}}$, as $z_{\text{AI}} \in \text{int}W$.

Proving that every $z \in W_p^*$ also has a worse manager is more involved. Since $z_{\text{AI}} \in \text{int}W$, then the AI equilibrium is either Type 1 or Type 2. Suppose first it is Type 1. Then, the matching functions

pre- and post-AI are given by:

$$\int_{\underline{z}_1^*}^{f_1^*(z;\underline{z}_1^*)} dG(u) = \int_0^z h(1-u)dG(u) \text{ for } z \in W_p^* = [0, z_{\text{AI}}]$$
$$\int_{\hat{z}}^{f(z;\hat{z})} dG(u) = \int_0^z h(1-u)dG(u) \text{ for } z \in W = [0, \hat{z}]$$

Thus, if $z \in W_p^* \cap W = W_p^*$, then $\int_{\underline{z}_1^*}^{f_1^*(z;\underline{z}_1^*)} dG(u) = \int_{\hat{z}}^{f(z;\hat{z})} dG(u)$, so $f_1^*(z;\underline{z}_1^*) < f(z;\hat{z})$ as $\underline{z}_1^* < \hat{z}$.

Suppose instead that the AI equilibrium is Type 2. Then, the matching functions pre- and post-AI are given by:

$$\begin{aligned} &\int_{z_{\text{AI}}}^{f_2^*(z;\underline{z}_2^*)} dG(u) = \int_{\underline{z}_2^*}^z h(1-u)dG(u) \text{ for } z \in W_p^* = [\underline{z}_2^*, z_{\text{AI}}] \\ &\int_{\hat{z}}^{f(z;\hat{z})} dG(u) = \int_0^z h(1-u)dG(u) \text{ for } z \in W = [0, \hat{z}] \end{aligned} \tag{5}$$

Consequently, if $z \in W_p^* \cap W = W_p^*$, then $\int_{\hat{z}}^{f(z;\hat{z})} dG(u) - \int_{z_{\text{AI}}}^{f_2^*(z;\underline{z}_2^*)} dG(u) = \int_0^{\underline{z}_2^*} h(1-u)dG(u) > 0$, which implies that $f_2^*(z;\underline{z}_2^*) < f(z;\hat{z})$ as $z_{\text{AI}} < \hat{z}$.

We now turn to managers, i.e., those $z \in M \subset M^*$. We first claim that if $e(z;\hat{z}) = z_{\text{AI}}$, then $z \in M_a^* \cap M$. This immediately implies that if $e(z;\hat{z}) = z_{\text{AI}}$, then $z$ manages a firm of equal size pre- and post-AI. The proof is via the contrapositive. Suppose that $z \notin M_a^* \cap M$ (but that $z$ is a manager). Then $z \in M_p^* \cap M$. However, if so, then $z_{\text{AI}} \geq e_j^*(z;\underline{z}_j^*) > e(z;\hat{z})$, where the first inequality is because AI is the best worker, and the second inequality follows because $e_j^*(z';\underline{z}_j^*) > e(z';\hat{z})$ for all $z' \in M_p^* \cap M$ if $f_j^*(z'';\underline{z}_j^*) < f(z'';\hat{z})$ for all $z'' \in W_p^* \cap W = W_p^*$ (which we already showed is true for $j = 1, 2$). Hence, $e(z;\hat{z}) \neq z_{\text{AI}}$.

The previous claim then implies that if $e(z;\hat{z}) < z_{\text{AI}}$, then $z$ manages a strictly larger firm post-AI, while if $e(z;\hat{z}) > z_{\text{AI}}$, then $z$ manages a strictly smaller firm post-AI. Indeed, if $e(z;\hat{z}) < z_{\text{AI}}$, then either $z \in M_p^* \cap M$ or $z \in M_a^* \cap M$. If $z \in M_p^*$, we already know that $e_j^*(z;\underline{z}_j^*) > e(z;\hat{z})$, so $z$ manages a strictly larger firm post-AI, while if $z \in M_a^*$, then the knowledge of $z$'s workers also increases since she is now managing AI, while before, she was managing humans with knowledge $e(z;\hat{z})$. Similarly, if $e(z;\hat{z}) > z_{\text{AI}}$, then $z \in M_a^* \cap M$. Hence, the knowledge of $z$'s workers decreases since she is now supervising the work of AI, while before, she was managing humans with knowledge $e(z;\hat{z})$. □

• $z_{\text{AI}} \in \text{int}M$.— First, we will show that every $z \in M^* \subset M$ manages a strictly larger firm post-AI than pre-AI. Note that if $z \in M_a^*$, then such a manager is matched with AI in the post-AI equilibrium. However, if so, then $e(z;\hat{z}) \leq \hat{z} < z_{\text{AI}}$, as $z_{\text{AI}} \in \text{int}M$.

We now show that the same holds for every $z \in M_p^*$. Since $z_{\text{AI}} \in \text{int}M$, then the AI equilibrium is either Type 2 or Type 3. Suppose first it is Type 2. Then, the employee functions pre- and post-AI are given by:

$$\begin{aligned} &\int_z^{\bar{z}_2^*} dG(u) = \int_{e_2^*(z;\underline{z}_2^*)}^{z_{\text{AI}}} h(1-u)dG(u) \text{ for } z \in M_p^* = [z_{\text{AI}}, \bar{z}_2^*] \\ &\int_z^1 dG(u) = \int_{e(z;\hat{z})}^{\hat{z}} h(1-u)dG(u) \text{ for } z \in M = [\hat{z}, 1] \end{aligned} \tag{6}$$

Consequently, if $z \in M_p^* \cap M = M_p^*$, then $0 < \int_{\bar{z}_2^*}^1 dG(u) = \int_{e(z;\hat{z})}^{\hat{z}} h(1-u)dG(u) - \int_{e_2^*(z;\underline{z}_2^*)}^{z_{\text{AI}}} h(1-u)dG(u)$, which implies that $e_2^*(z;\underline{z}_2^*) > e(z;\hat{z})$ (since $z_{\text{AI}} > \hat{z}$).

Suppose instead that the AI equilibrium is Type 3. Then, the employee functions pre- and post-AI are given by:

$$\int_z^1 dG(u) = \int_{e_3^*(z;\underline{z}_3^*)}^{\bar{z}_3^*} h(1-u)dG(u) \text{ for } z \in M_p^* = [z_{\text{AI}}, 1]$$
$$\int_z^1 dG(u) = \int_{e(z;\hat{z})}^{\hat{z}} h(1-u)dG(u) \text{ for } z \in M = [\hat{z}, 1]$$

Consequently, for $z \in M_p^* \cap M = M_p^*$, then $\int_{e_3^*(z;\underline{z}_3^*)}^{\bar{z}_3^*} h(1-u)dG(u) = \int_{e(z;\hat{z})}^{\hat{z}} h(1-u)dG(u)$. However, if so, then $e(z;\hat{z}) < e_3^*(z;\underline{z}_3^*)$ since $\hat{z} < \bar{z}_3^*$.

We now turn to workers, i.e., those with $z \in W \subset W^*$. We first claim that if $z = e(z_{\text{AI}};\hat{z})$, then $z \in W_a^* \cap W$. This immediately implies that if $z = e(z_{\text{AI}};\hat{z})$, then $z$ is equally productive pre- and post-AI. The proof is via the contrapositive. Suppose that $z \notin W_a^* \cap W$ (but that $z$ is a worker). Then $z \in W_p^* \cap W$. However, if so, then $z_{\text{AI}} \leq f_j^*(z;\underline{z}_j^*) < f(z;\hat{z})$, where the first inequality is because AI is the worst manager, and the second inequality follows because $f_j^*(z';\underline{z}_j^*) < f(z';\hat{z})$ for all $z' \in W_p^* \cap W$ if $e_j^*(z'';\underline{z}_j^*) > e(z'';\hat{z})$ for all $z'' \in M_p^* \cap M = M_p^*$ (which we already showed is true for $j = 2, 3$). Hence, $z \neq e(z_{\text{AI}};\hat{z})$.

The previous claim then implies that if $z < e(z_{\text{AI}};\hat{z})$, then $z$ is strictly more productive post-AI than pre-AI, while if $z > e(z_{\text{AI}};\hat{z})$, then $z$ is strictly less productive post-AI than pre-AI. Indeed, if $z < e(z_{\text{AI}};\hat{z})$, then $z \in W_a^* \cap W$. Hence, the knowledge of $z$'s manager increases since she is now being managed by AI, while before, she was being managed by a human with knowledge $f(z;\hat{z}) < z_{\text{AI}}$. Similarly, if $z > e(z_{\text{AI}};\hat{z})$, then $z \in W_a^* \cap W$ or $z \in W_p^* \cap W$. If $z \in W_a^* \cap W$, the knowledge of $z$'s manager decreases since she is now being managed by AI (while before, she was being managed by a human with knowledge $f(z;\hat{z}) > z_{\text{AI}}$), while if $z \in W_p^* \cap W$, the knowledge of her manager again decreases since we already established that $f_j^*(z;\underline{z}_j^*) < f(z;\hat{z})$ for $j = 2, 3$. □

• $z_{\text{AI}} \in W \cap M$.— We first show that each $z \in W^* = W$ is managed by a worse manager post-AI than pre-AI (strictly so for all $z \neq 0$). By Lemma B.2, we know that $z_{\text{AI}} = \hat{z} \in \mathcal{R}_2$, so Lemma B.3 implies that the equilibrium is necessarily Type 2. Moreover, as shown in the proof of Proposition 3, in this case we have that $\underline{z}_2^* > 0$ and $\bar{z}_2^* < 1$. Consequently, if $z \in W_a^*$, then $f(z;\hat{z}) \geq \hat{z} = z_{\text{AI}}$, where the first inequality is strict when $z > 0$. If $z \in W_p^*$ instead, then the matching functions pre- and post-AI are given by (5) evaluated at $z_{\text{AI}} = \hat{z}$. Consequently, for $z \in W^* = W$, $\int_{f_2^*(z;\underline{z}_2^*)}^{f(z;\hat{z})} dG(u) = \int_0^{\underline{z}_2^*} h(1-u)dG(u) > 0$, which implies that $f_2^*(z;\underline{z}_2^*) < f(z;\hat{z})$ given that $\underline{z}_2^* > 0$.

We now show that each $z \in M^* = M$ improves her match post-AI compared to pre-AI (strictly so for all $z \neq 1$). Indeed, if $z \in M_a^*$, then $e(z;\hat{z}) \leq \hat{z} = z_{\text{AI}}$, where the first inequality is strict inequality when $z < 1$. If $z \in M_p^*$ instead, then the employee functions pre- and post-AI are given by (6) evaluated at $z_{\text{AI}} = \hat{z}$. Consequently, for $z \in M^* = M$, $\int_{\bar{z}_2^*}^1 dG(u) = \int_{e(z;\hat{z})}^{e_2^*(z;\underline{z}_2^*)} h(1-u)dG(u) > 0$, which implies that $e_2^*(z;\underline{z}_2^*) > e(z;\hat{z})$ since $\bar{z}_2^* < 1$. □

## C.6 Proof of Lemma 3

For ease of exposition, we have divided the proof of the lemma into three smaller claims:

**Claim C.1.** *(i)* $\Delta(z_{\text{AI}}) < 0$*, (ii)* $\Delta(1) > 0$*, and (iii)* $\Delta(0) > 0$ *if* $z_{\text{AI}} \in M$*.*

*Proof.* That $\Delta(z_{\text{AI}}) = z_{\text{AI}} - w(z_{\text{AI}}) < 0$ follows directly from the fact that $w(z) > z$ for all $z \in [0,1]$ when $h < h_0$ (see Lemma A.1 and Corollary A.1). Consider next $\Delta(1) = w^*(1) - w(1)$. By Lemma B.3, we have that $w^*(1) = 1/h$, while by Lemma A.1 and Corollary A.1, we have that $w(1) < 1/h$. Hence, $\Delta(1) > 0$. Finally, consider $\Delta(0) = w^*(0) - w(0)$ and suppose that $z_{\text{AI}} \in M$. From Lemma B.3, we have that $w^*(0) = z_{\text{AI}}(1-h)$ as the human with zero knowledge is managed by AI irrespective of whether $z_{\text{AI}} \in \mathcal{R}_2$ or $z_{\text{AI}} \in \mathcal{R}_3$. Moreover, from Lemma A.1 and Corollary A.1 we have that $w(0) = \hat{z} - hw(\hat{z})$. Thus, $\Delta(0) = z_{\text{AI}}(1-h) - (\hat{z} - hw(\hat{z})) > (1-h)(z_{\text{AI}} - \hat{z}) \geq 0$, where the first inequality follows because $w(\hat{z}) > \hat{z}$, and the second inequality follows because $z_{\text{AI}} \geq \hat{z}$. □

**Claim C.2.** *If* $\Delta(z) > 0$ *for some* $z \in [z_{\text{AI}}, 1)$*, then* $\Delta(z') > 0$ *for all* $z' \in [z, 1)$*.*

*Proof.* Given that $\Delta(z_{\text{AI}}) < 0$ (by Claim C.1), it suffices to show that if $\Delta(z)$ crosses zero at some $z > z_{\text{AI}}$, then it always crosses zero from below. We first consider $z_{\text{AI}} \in \text{int}W$ and then $z_{\text{AI}} \in M$.

• $z_{\text{AI}} \in \text{int}W$.— Lemma B.3 implies that $\sup W^* = z_{\text{AI}}$, while Proposition 3 that $W^* \subset W$ and $M^* \supset M$. Hence, if $z > z_{\text{AI}}$, then $z$ can only belong to either $I^* \cap W$, $M^* \cap W$, or $M^* \cap M$.

Now, irrespective of the presence of AI, the marginal return to knowledge is higher for managers than for independent producers, and it is higher for independent producers than for workers. Hence, $\Delta'(z) = w^{*\prime}(z) - w'(z) \geq 0$ whenever $z$ is in either $I^* \cap W$ or $M^* \cap W$. This implies that if $\Delta(z)$ crosses zero in either of these sets, then it crosses it necessarily from below.

Consider then $z \in M^* \cap M$. Since $M^* = M_p^* \cup M_a^*$, here we have two cases to consider: $z \in M_p^* \cap M$ and $z \in M_a^* \cap M$. If $z \in M_p^* \cap M$, then $\Delta'(z) = n(e_j^*(z; \underline{z}_j^*)) - n(e(z; \hat{z}))$, where $e_j^*(z; \underline{z}_j^*)$ is the employee function in a Type $j = 1, 2$ equilibrium and $e(z; \hat{z})$ employee function in the pre-AI equilibrium. However, by Proposition 6, we know that $e_j^*(z; \underline{z}_j^*) > e(z; \hat{z})$ as every $z \in M_p^* \cap M$ manages better workers post-AI. Hence, in this case, $\Delta'(z) > 0$ also. Consequently, if $\Delta(z)$ crosses zero when $z \in M_p^* \cap M$, then it can only cross it from below.

Finally, consider the possibility that $\Delta(z)$ crosses zero at a $z \in M_a^* \cap M$. In this case, $\Delta'(z) = n(z_{\text{AI}}) - n(e(z; \hat{z})) \gtrless 0$, so $\Delta(z)$ is no longer monotone in $z$ in this set. Note, however, that $\Delta''(z) = -n'(e(z; \hat{z}))e'(z; \hat{z}) < 0$, so $\Delta(z)$ is concave. Moreover, if $M_a^* \neq \emptyset$, then $1 \in M_a^*$. The fact that $\Delta(z)$ is concave and that $\Delta(1) > 0$ then immediately implies that if $\Delta(z)$ crosses zero in this set, then it can only cross once and from below (otherwise, $\Delta(1) \leq 0$ contradicting the fact $\Delta(1) > 0$).

• $z_{\text{AI}} \in M$.— From Lemma B.3, we know that $\inf M^* = z_{\text{AI}}$. Moreover, from Proposition 3, we have that $W^* \supseteq W$ and $M^* \subseteq M$. Consequently, if $z > z_{\text{AI}}$, then $z \in M^* \cap M$ necessarily. Since

$M^* = M_p^* \cup M_a^*$, here we have two cases to consider: $z \in M_p^* \cap M$ and $z \in M_a^* \cap M$

If $z \in M_p^* \cap M$, then $\Delta'(z) = n(e_j^*(z; \underline{z}_j^*)) - n(e(z; \hat{z}))$, while if $z \in M_a^* \cap M$, then $\Delta'(z) = n(z_{\text{AI}}) - n(e(z; \hat{z}))$, where $e_j^*(z; \underline{z}_j^*)$ is the employee function in a Type $j = 2, 3$ equilibrium and $e(z; \hat{z})$ the employee function in the pre-AI equilibrium. In either case, $\Delta'(z) > 0$ since Proposition 6 states that $e_j^*(z; \underline{z}_j^*) > e(z; \hat{z})$ and $z_{\text{AI}} > e(z; \hat{z})$ when AI has the knowledge of a pre-AI manager, or if AI has the knife-edge knowledge of a pre-AI worker and a pre-AI manager. Consequently, $\Delta'(z) > 0$ for all $z \geq z_{\text{AI}}$, so if $\Delta(z)$ crosses zero at some $z > z_{\text{AI}}$, then it always crosses it from below. □

**Claim C.3.** *If $\Delta(z) > 0$ for some $z \in [0, z_{\text{AI}}]$, then $\Delta(z') > 0$ for all $z' \in [0, z]$.*

*Proof.* Given that $\Delta(z_{\text{AI}}) < 0$, it suffices to show that if $\Delta(z)$ crosses zero at some $z < z_{\text{AI}}$, then it always crosses zero from above. We first consider $z_{\text{AI}} \in W$, and then $z_{\text{AI}} \in \text{int}M$.

• $z_{\text{AI}} \in W$.— Lemma B.3 implies that $\sup W^* = z_{\text{AI}}$, while Proposition 3 that $W^* \subseteq W$ and $M^* \supseteq M$. Consequently, if $z < z_{\text{AI}}$, then $z \in W^* \cap W$, where $W^* = W_a^* \cup W_p^*$.

Now, if $z \in W_p^* \cap W$, then $\Delta'(z) = hw^*(f_j^*(z; \underline{z}_j^*)) - hw(f(z; \hat{z}))$, where $f_j^*(z; \underline{z}_j^*)$ is the matching function in a Type $j = 1, 2$ equilibrium, and $f(z; \hat{z})$ the matching function of the pre-AI equilibrium. However, using the firms' zero-profit condition:

$$\Delta'(z) = hw^*(f_j^*(z; \underline{z}_j^*)) - hw(f(z; \hat{z})) = \frac{f_j^*(z; \underline{z}_j^*) - f(z; \hat{z}) - \Delta(z)}{1 - z}$$

Consequently, if $\Delta(z) = 0$ at some $z$ in this interval, say at $z = \zeta$, then $\Delta'(\zeta) = f_j^*(\zeta; \underline{z}_j^*) - f(\zeta; \hat{z}) \leq 0$, where the last inequality follows because every worker is managed by a worse manager post-AI in this case, as shown in Proposition 6.

On the other hand, if $z \in W_a^* \cap W$, then following the same reasoning as before, we have that:

$$\Delta'(z) = hw(z_{\text{AI}}) - hw(f(z; \hat{z})) = \frac{z_{\text{AI}} - f(z; \hat{z}) - \Delta(z)}{1 - z}$$

Consequently, if $\Delta(z) = 0$ at some $z$ in this interval, say at $z = \zeta$, then $\Delta'(\zeta) = z_{\text{AI}} - f(\zeta; \hat{z}) \leq 0$, where the inequality follows, again, from Proposition 6.

• $z_{\text{AI}} \in \text{int}M$.— In this case, Lemma B.3 implies that $\inf M^* = z_{\text{AI}}$, while Proposition 3 that $W^* \supset W$ and $M^* \subset M$. Consequently, if $z < z_{\text{AI}}$, then $z$ can only belong to either $I^* \cap M$, $W^* \cap M$, or $W^* \cap W$.

Now, $\Delta'(z) \leq 0$ whenever $z$ is in either $I^* \cap M$ or $W^* \cap M$, given that the marginal return to knowledge is higher for managers than for independent producers, and it is higher for independent producers than for workers. Hence, if $\Delta(z)$ crosses zero in either of these sets, then it crosses it necessarily from above.

Consider then $z \in W^* \cap W$. Since $W^* = W_p^* \cup W_a^*$, we have two cases to consider: $z \in W_p^* \cap W$ and $z \in W_a^* \cap W$. If $z \in W_p^* \cap W$, then:

$$\Delta'(z) = hw^*(f_j^*(z; \underline{z}_j^*)) - hw(f(z; \hat{z})) = \frac{f_j^*(z; \underline{z}_j^*) - f(z; \hat{z}) - \Delta(z)}{1 - z}$$

where $f_j^*(z;\underline{z}_j^*)$ is the matching function in a Type $j = 2,3$ equilibrium, and $f(z;\hat{z})$ the matching function of the pre-AI equilibrium. Consequently, if $\Delta(z) = 0$ at some $z$ in this interval, say at $z = \zeta$, then $\Delta'(\zeta) = f_j^*(\zeta;\underline{z}_j^*) - f(\zeta;\hat{z}) \leq 0$, where the last inequality follows because every $z \in W^* \cap W$ is managed by a worse manager post-AI when AI has the knowledge of a pre-AI manager (as shown in Proposition 6).

Finally, consider the possibility that $\Delta(z)$ crosses zero at a $z \in W_a^* \cap W$. In this case, $\Delta'(z) = hz_{\text{AI}} - hw(f(z;\hat{z}))$, so $\Delta''(z) = -hw'(f(z;\hat{z}))f'(z;\hat{z}) < 0$, implying that $\Delta(z)$ is concave. Moreover, if $W_a^* \neq \emptyset$, then $0 \in W_a^*$, and we know that $\Delta(0) > 0$ in this case. Consequently, if $\Delta(z)$ crosses zero in this set, then it can only cross once and from above (otherwise, $\Delta(0) \leq 0$ contradicting the fact $\Delta(0) > 0$). □

### C.7 Proof of Proposition 7

Part (i) ("there always exists $z$ strictly greater than $z_{\text{AI}}$ such that $\Delta(z) > 0$") follows directly from Lemma 3 and the fact that $\Delta(1) > 0$ (see Claim C.1). Hence, it only remains to prove part (ii) ("there exists $z$ strictly smaller than $z_{\text{AI}}$ such that $\Delta(z) > 0$ if and only if $z_{\text{AI}} > \bar{z}_{\text{AI}}$, where $\bar{z}_{\text{AI}} \in (0,\hat{z})$"). To prove this part, we begin by constructing $\bar{z}_{\text{AI}}$ and then show that the statement is true.

Let $\Delta(0;z_{\text{AI}}) \equiv w^*(0;z_{\text{AI}}) - w(0)$, and define $\bar{z}_{\text{AI}}$ as the solution to $\Delta(0;\bar{z}_{\text{AI}}) = 0$. We first show that $\bar{z}_{\text{AI}}$ exists and is unique and that $\bar{z}_{\text{AI}} \in (0,\hat{z})$. We do this by showing that $\Delta(0;z_{\text{AI}})$ crosses zero once as we go from $z_{\text{AI}} = 0$ to $z_{\text{AI}} = 1$, and that this crossing point is at a $z_{\text{AI}} < \hat{z}$. Indeed, if $z_{\text{AI}} \geq \hat{z}$, then Claim C.1 states that $\Delta(0;z_{\text{AI}}) > 0$ (as $z_{\text{AI}} \in M$ in this case). Moreover, as shown in Lemma B.2, $z \in \mathcal{R}_1$ if $z \in (0,\epsilon)$ with $\epsilon \downarrow 0$, so when $z_{\text{AI}} > 0$ but $z_{\text{AI}} \to 0$, the equilibrium is always Type 1. The latter implies that:[29]

$$\lim_{z_{\text{AI}}\to 0} \Delta(0;z_{\text{AI}}) = \lim_{z_{\text{AI}}\to 0}[\underline{z}_1^*(z_{\text{AI}})(1-h) - w(0)] = -w(0) < 0$$

where the second-to-last equality follows because $\underline{z}_1^*(z_{\text{AI}}) \to 0$ when $z_{\text{AI}} \to 0$, as can be easily be proven from the condition that determines $\underline{z}_1^*(z_{\text{AI}})$ (see the statement of Lemma B.3).

Now, when $z \in (0,\hat{z}) = W$, the equilibrium is either Type 1, in which case $w^*(0;z_{\text{AI}}) = \underline{z}_1^*(z_{\text{AI}})(1-h)$, or Type 2, in which case $w^*(0;z_{\text{AI}}) = z_{\text{AI}}(1-h)$. Using the equilibrium condition that determines $\underline{z}_1^*(z_{\text{AI}})$, it is not difficult to prove that (i) $\underline{z}_1^*(z_{\text{AI}})$ is strictly increasing in $z_{\text{AI}}$, and that (ii) $\underline{z}_1^*(z_{\text{AI}}) = z_{\text{AI}}$ whenever we switch from a Type 1 into a Type 2 equilibrium (and vice versa). Consequently, irrespective of the equilibrium type in this region, $w^*(0;z_{\text{AI}})$ is continuous and strictly increasing in $z_{\text{AI}}$, which implies that so is $\Delta(0;z_{\text{AI}})$. This result, combined with the fact that $\Delta(0;0) < 0$ and $\Delta(0;\hat{z}) > 0$, immediately yields the desired result.

Having constructed $\bar{z}_{\text{AI}}$, we prove that there exists $z < z_{\text{AI}}$ such that $\Delta(z;z_{\text{AI}}) > 0$ if and only if

[29] To avoid any confusion, here we are making explicit that $\underline{z}_1^*(z_{\text{AI}})$ depends on $z_{\text{AI}}$ as can be seen from the condition that determines $\underline{z}_1^*(z_{\text{AI}})$ in the statement of Lemma B.3.

$z_{\text{AI}} > \bar{z}_{\text{AI}}$. First we show that if there exists a $z < z_{\text{AI}}$ such that $\Delta(z; z_{\text{AI}}) > 0$, then $z_{\text{AI}} > \bar{z}_{\text{AI}}$. To do this, we prove the contrapositive statement: If $z_{\text{AI}} \leq \bar{z}_{\text{AI}}$, there is no such $z$. Indeed, $\Delta(0; z_{\text{AI}}) \leq 0$ for all $z_{\text{AI}} \leq \bar{z}_{\text{AI}}$ as shown above. Hence, Lemma 3 immediately implies that $\Delta(z; z_{\text{AI}}) \leq 0$ for all $z < z_{\text{AI}}$.

Now we prove that if $z_{\text{AI}} > \bar{z}_{\text{AI}}$, then there exists a $z < z_{\text{AI}}$ such that $\Delta(z; z_{\text{AI}}) > 0$. Indeed, as shown above, $z_{\text{AI}} > \bar{z}_{\text{AI}}$ then $\Delta(0; z_{\text{AI}}) > 0$. Consequently, Lemma 3 immediately implies that there exists $\zeta \in [0, z_{\text{AI}})$ such that $\Delta(z; z_{\text{AI}}) > 0$ for $z \in [0, \zeta)$ and $\Delta(z; z_{\text{AI}}) < 0$ for $z \in (\zeta, z_{\text{AI}}]$. □

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