Heterogeneous Downward Nominal Wage Rigidity: Foundations of a Nonlinear Phillips Curve*

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Motivation

• Two recent major economic events have rekindled interest in a nonlinear Phillips curve:

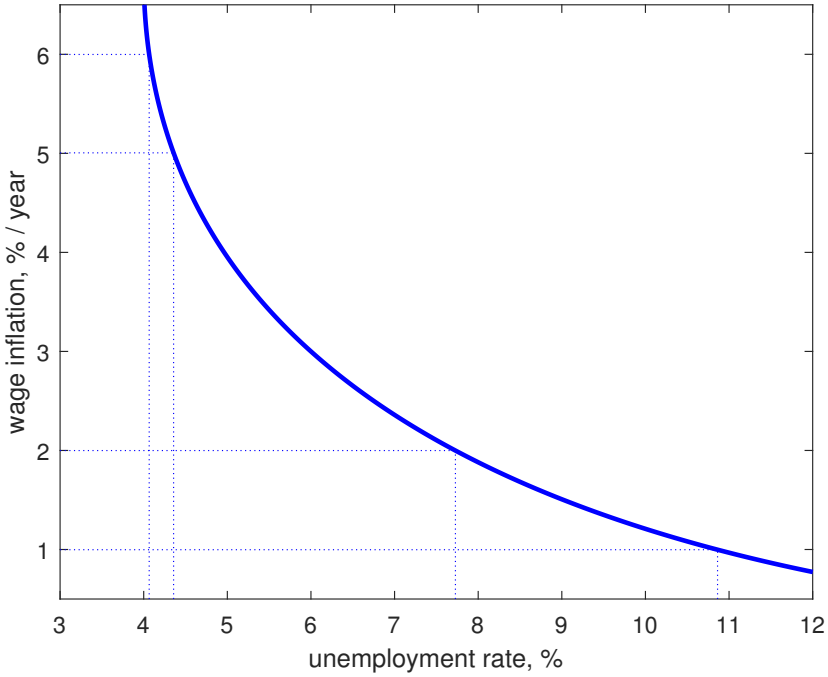
(1) Resilience of the labor market during the post-Covid-19 monetary tightening (the "missing unemployment" puzzle)
$\Rightarrow$ Is the Phillips curve steeper at high inflation? If so, then low cost of fighting high inflation in terms of unemployment.

(2) No significant increase in inflation in the recovery from the Global Financial Crisis of 2008 (the "missing inflation" puzzle)
$\Rightarrow$ Is the Phillips curve flatter at high unemployment rates? If so, then low cost of reducing high unemployment in terms of inflation.

This Paper proposes a model with heterogeneous downward nominal wage rigidity (HDNWR)

- The model implies a nonlinear wage Phillips curve.

- Calibrated to the US economy, the model predicts that lowering wage inflation from 6 to 5 percent raises unemployment by 0.3 percentage points, whereas lowering wage inflation from 2 to 1 percent raises unemployment by 3 percentage points.



• Yet, for regular fluctuations around the inflation target, impulse responses (approximated to first-order accuracy) to conventional monetary and technology shocks are fairly similar to those predicted by a canonical NK sticky-wage model.

$\Rightarrow$ The proposed model globally delivers a nonlinear Phillips curve, but locally preserves the dynamic properties of standard new-Keynesian models—at least for standard calibrations of monetary and productivity shocks.

Related Literature

• Phillips (1958): empirically documents a negative and "highly nonlinear" relation between wage inflation and unemployment; conjectures that the source of nonlinearity is downward nominal wage rigidity, but does not offer a model.

• Erceg, Henderson, and Levin (2000): new-Keynesian framework with sticky wages à la Calvo.

• Casares (2010) and Galí (2011) derive the log-linear wage-Phillips curve of that model.

• Schmitt-Grohé and Uribe (2016, 2017): homogeneous downward nominal wage rigidity in open and closed economies; L-shaped Phillips curve; not amenable to perturbation.

• Benigno and Eggertsson (2023): downward nominal wage rigidity in new-Keynesian labor search model; nonlinearity (piecewise linearity) arises from the assumption that wages are flexible when $v/u > 1$, but downwardly rigid when $v/u \leq 1$.

• Harding, Lindé, and Trabandt (2022, 2023) characterize numerically the Phillips curve of a (non-linearized) NK model with price and wage rigidity and a Kimball aggregator.

• Leduc and Wilson (2017) relate the missing inflation post Great Recession to a flattening of the Phillips curve; Crust, Lansing, and Petrosky-Nadeau (2023) interpret the missing unemployment post Covid-19 as a steepening of the Phillips curve.

• Cerrato and Gitti (2022) show that post Covid-19 the slope of regional Phillips curves was three times larger than pre Covid-19. Gitti (2024) documents nonlinearities in regional Phillips curves.

• Fehr and Goette (2005) estimate significant heterogeneity in downward nominal wage rigidity across individual workers. Bewley (1999) provides survey evidence that nominal fairness standards are a key determinant of downward nominal wage rigidity. Fehr and Gächter (2000) provide experimental evidence of significant heterogeneity in nominal fairness standards.

• Davis and Krolikowski (2024) document heterogeneity in downward nominal wage rigidity at the layoff margin using survey data.

• Heterogeneity in downward nominal wage rigidity in firm level administrative data has been documented by: Murray (2021) for the United States; Faia and Pezone (2023) and Fanfani (2023) for Italy; and Adamopoulou, Díez-Catalán, and Villanueva (2024) for Spain.

The HDNWR Model

Firms

- price and wage takers
- production

$$y_t = z_t F(h_t)$$

- labor input

$$h_t = \left[\int_0^1 h_{jt}^{1-\frac{1}{\eta}} dj \right]^{\frac{1}{1-\frac{1}{\eta}}}; \quad \eta > 0$$

- demand for labor of type j

$$h_{jt} = \left(\frac{W_{jt}}{W_t} \right)^{-\eta} h_t,$$

where

$$W_t^{1-\eta} = \int_0^1 W_{jt}^{1-\eta} dj.$$

- profits, $\Phi_t = P_t z_t F(h_t) - W_t h_t$. FOC:

$$P_t z_t F'(h_t) = W_t$$

Households

- price and wage takers
- preferences: $E_0 \sum_{t=0}^{\infty} \beta^t U(c_t)$
- inelastic labor supply:*

$$h_{jt} \leq \bar{h}(1 - u_t^n)$$

- budget constraint:

$$P_t c_t + \frac{B_t}{1 + i_t} + \tau_t = \int_0^1 W_{jt} h_{jt} dj + B_{t-1} + \Phi_t$$

*The case of endogenous labor supply will be presented below starting on slide 22.

Heterogeneous downward nominal wage rigidity

$$W_{jt} \geq \gamma(j)\, W_{t-1}$$

The Labor Market Slackness Condition

$$[\bar{h}(1-u_t^n)-h_{jt}]\,[W_{jt}-\gamma(j)W_{t-1}]=0$$

The Cutoff Variety j_t^* and the Cross-Sectional Determination of Labor and Wages

$$\bar{h}(1-u_t^n) = \left(\frac{\gamma(j_t^*)W_{t-1}}{W_t}\right)^{-\eta} h_t$$

$$\begin{cases} h_{jt} = \bar{h}(1-u_t^n) \quad \text{and} \quad W_{jt} = \gamma(j_t^*)W_{t-1} \quad \text{for } j \leq j_t^* \\ h_{jt} < \bar{h}(1-u_t^n) \quad \text{and} \quad W_{jt} = \gamma(j)W_{t-1} \quad \text{for } j > j_t^* \end{cases}$$

The Wage Phillips Curve: $\pi_t^W = f(u_t)$

Wage inflation and unemployment

$$W_t^{1-\eta} = \int_0^1 W_{jt}^{1-\eta} dj$$

$$u_t = \int_0^1 \frac{\bar{h} - h_{jt}}{\bar{h}} dj$$

Express as:

$$\begin{aligned}(1+\pi_t^W)^{1-\eta} &= j_t^* \gamma(j_t^*)^{1-\eta} + \int_{j_t^*}^1 \gamma(j)^{1-\eta} dj \\ u_t &= u_t^n + (1-u_t^n)\left[(1-j_t^*) - \int_{j_t^*}^1 \left(\frac{\gamma(j)}{\gamma(j_t^*)}\right)^{-\eta} dj\right]\end{aligned}$$

$\Rightarrow$ HDNWR model implies Phillips's Phillips Curve: a negative **nonlinear** relation between u_t and π_t^W (without a forward-looking component).

Calibration of the Predicted Wage Phillips Curve

Functional form for the wage lower bound

$$\gamma(j) = (1 + \pi^*)(\Gamma_0 + \Gamma_1 j)$$

Calibration of Γ_0 and Γ_1: Two targets

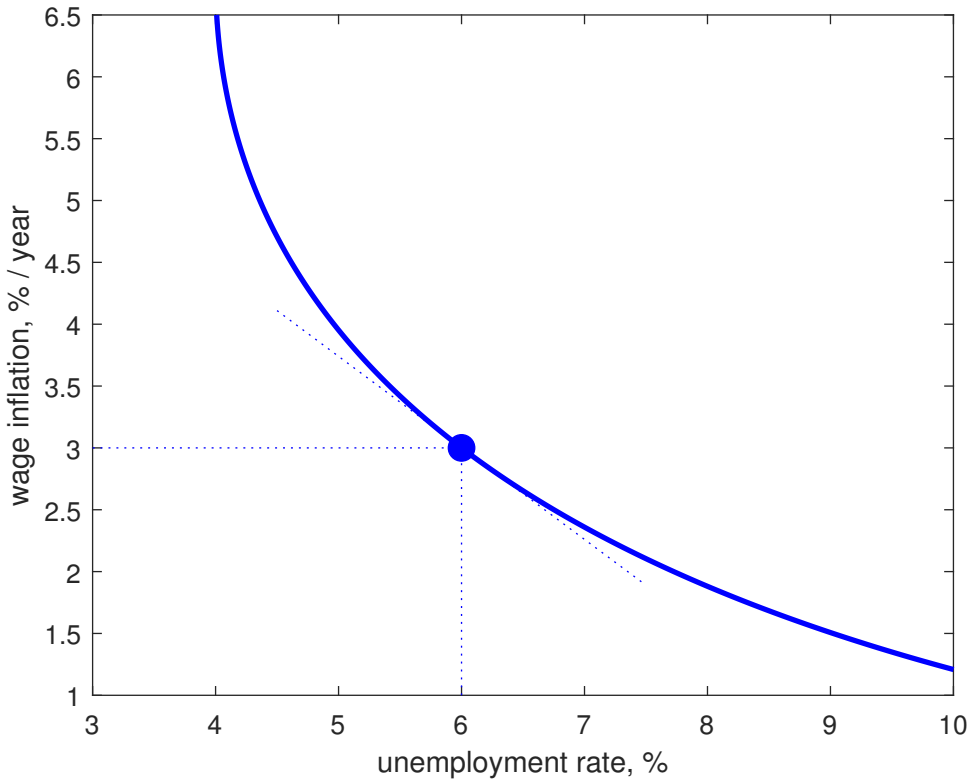
(1) the wage Phillips curve goes through $(u_t, \pi_t^W) = (0.06, 0.03)$, the median of US unemployment and wage inflation 1986–2007.

(2) at that point, the slope of the wage Phillips curve is -0.74 (Galí and Gambetti, 2019, estimate on 1986–2007 US data)

Set $u^n = 4\%$ (natural rate of unemployment) and $\eta = 11$ (elast. subs. across varieties), and $\pi^* = 0.03$ (annual inflation target).

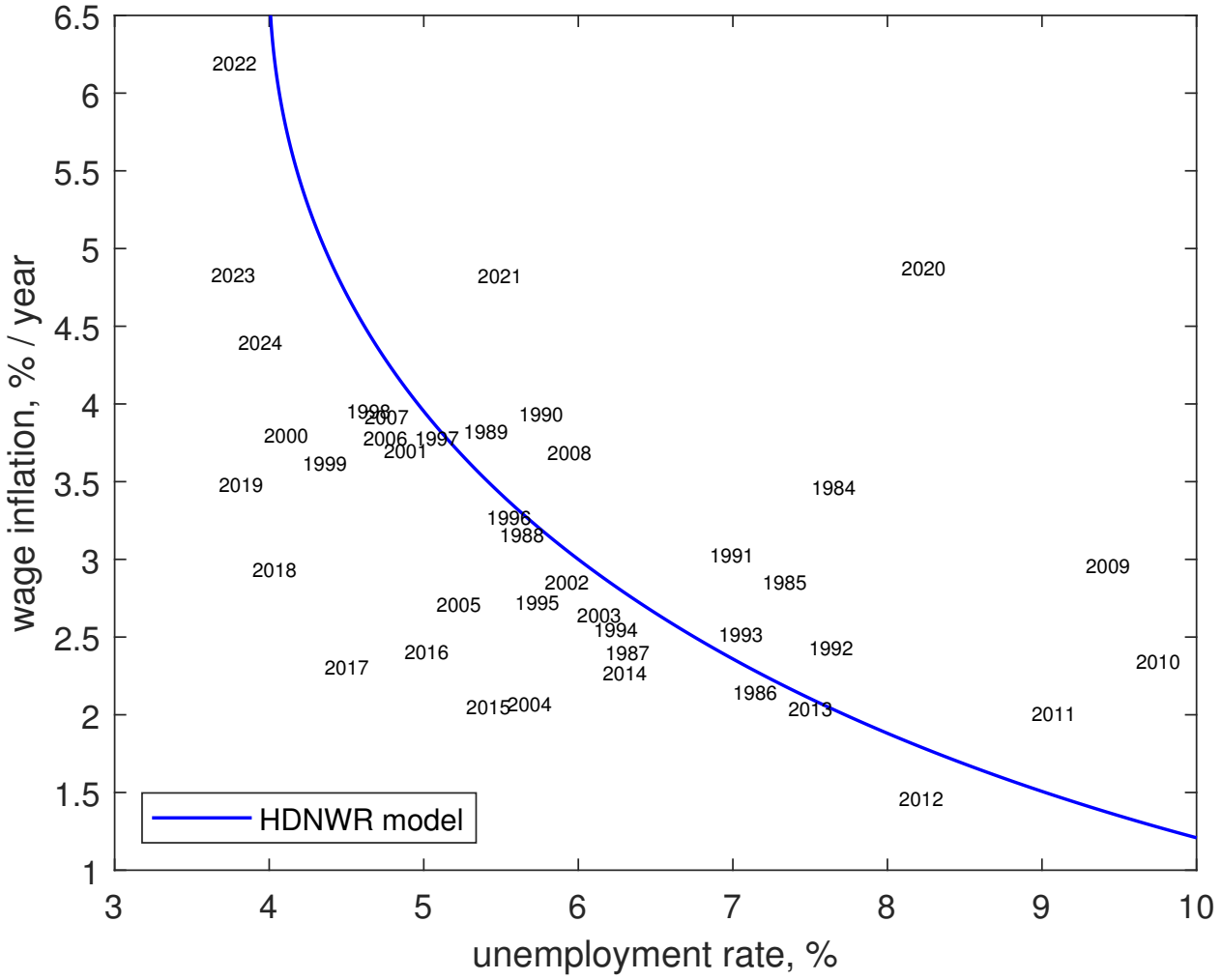
Result: $\Gamma_0 = 0.978$ and $\Gamma_1 = 0.031$ at quarterly frequency.

The Short-Run Wage Phillips Curve of the HDNWR Model



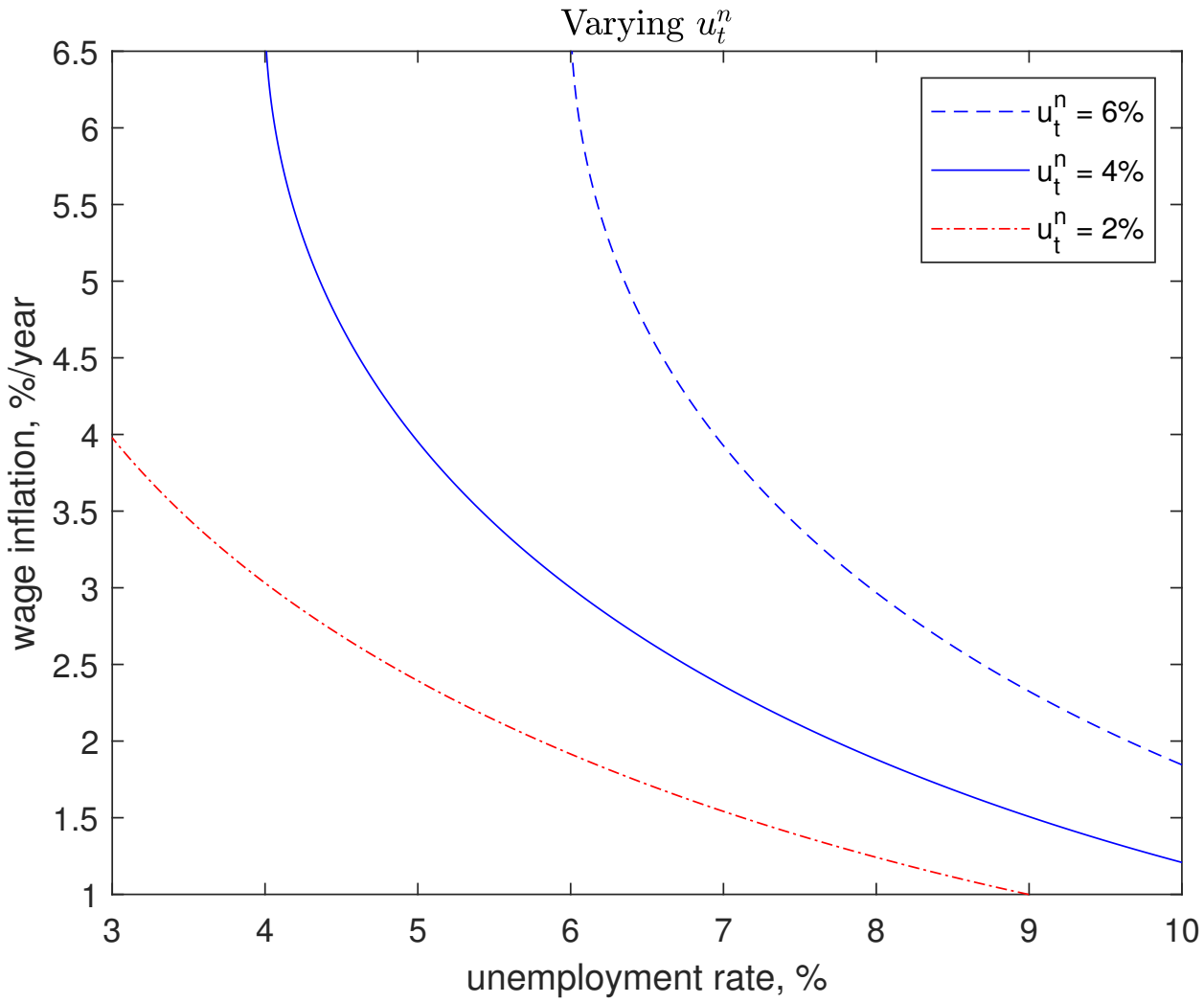
Nonlinearity: lowering inflation from 6 to 5 percent raises the unemployment rate by 0.3 percentage points, whereas lowering inflation from 2 to 1 percent raises the unemployment rate by 3 percentage points.

The HDNWR Wage Phillips Curve and U.S. Data



Notes. Annual wage inflation is computed as the average of year-over-year monthly wage inflation. The measure of monthly nominal wages is Average Hourly Earnings of Production and Nonsupervisory Employees, FRED series AHETPI. The annualized unemployment rate is the arithmetic mean of monthly unemployment rates, FRED series UNRATE. The observation labeled 2024 in the figure refers to unemployment and wage inflation in the first three months of 2024. Sample: 1984 to 2024.

Shifters of the Wage Phillips Curve
The Aggregate Supply Shock, u_t^n



Note. The solid line corresponds to the baseline calibration.

The Wage Phillips Curve:

$$(1+\pi_t^W)^{1-\eta} = j_t^* \gamma(j_t^*)^{1-\eta} + \int_{j_t^*}^1 \gamma(j)^{1-\eta} dj$$

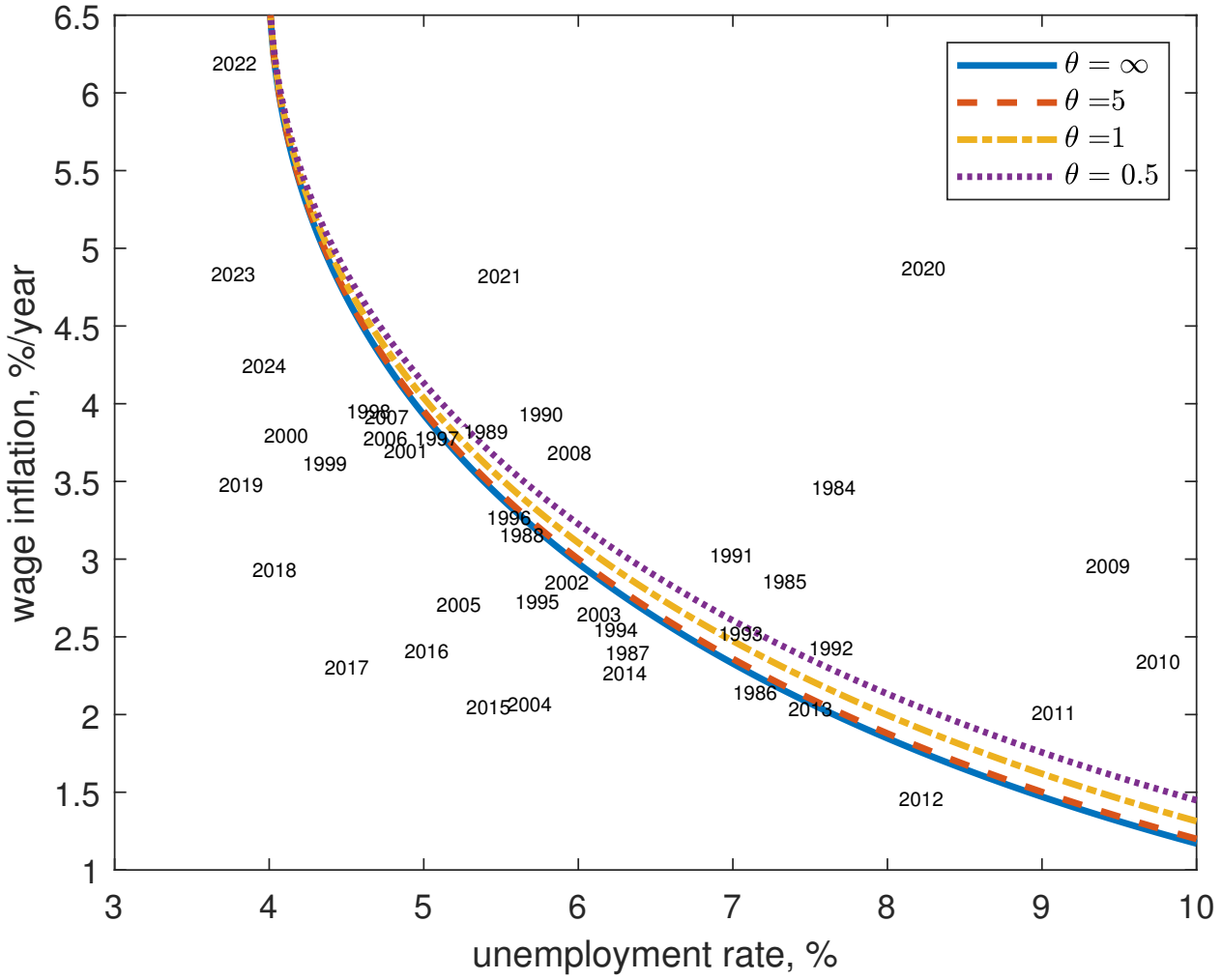
$$u_t = u_t^n + (1-u_t^n)\left[(1-j_t^*) - \int_{j_t^*}^1 \left(\frac{\gamma(j)}{\gamma(j_t^*)}\right)^{-\eta} dj\right]$$

Aggregate Supply Shocks in the Pandemic Era

Year	Actual Wage Inflation π_t^W	Actual Unemployment u_t	Predicted Supply Shock $u_t^n - u^n$
2020	4.88	8.09	3.70
2021	4.83	5.35	0.92
2022	6.20	3.63	-0.40
2023	4.84	3.63	-0.81

This analysis suggests that the predicted curvature of the Phillips curve is not at odds with the prediction that the economy was buffeted by significant supply shocks during the worst of the pandemic.

The HDNWR Wage Phillips Curve in the Model with Endogenous Labor Supply: $E_0 \sum_{t=0}^{\infty} \beta^t \left[U(c_t) - \int_0^1 \frac{h_{jt}^{1+\theta}}{1+\theta} dj \right]$



An HDNWR Model with

Heterogeneity in Labor Productivity

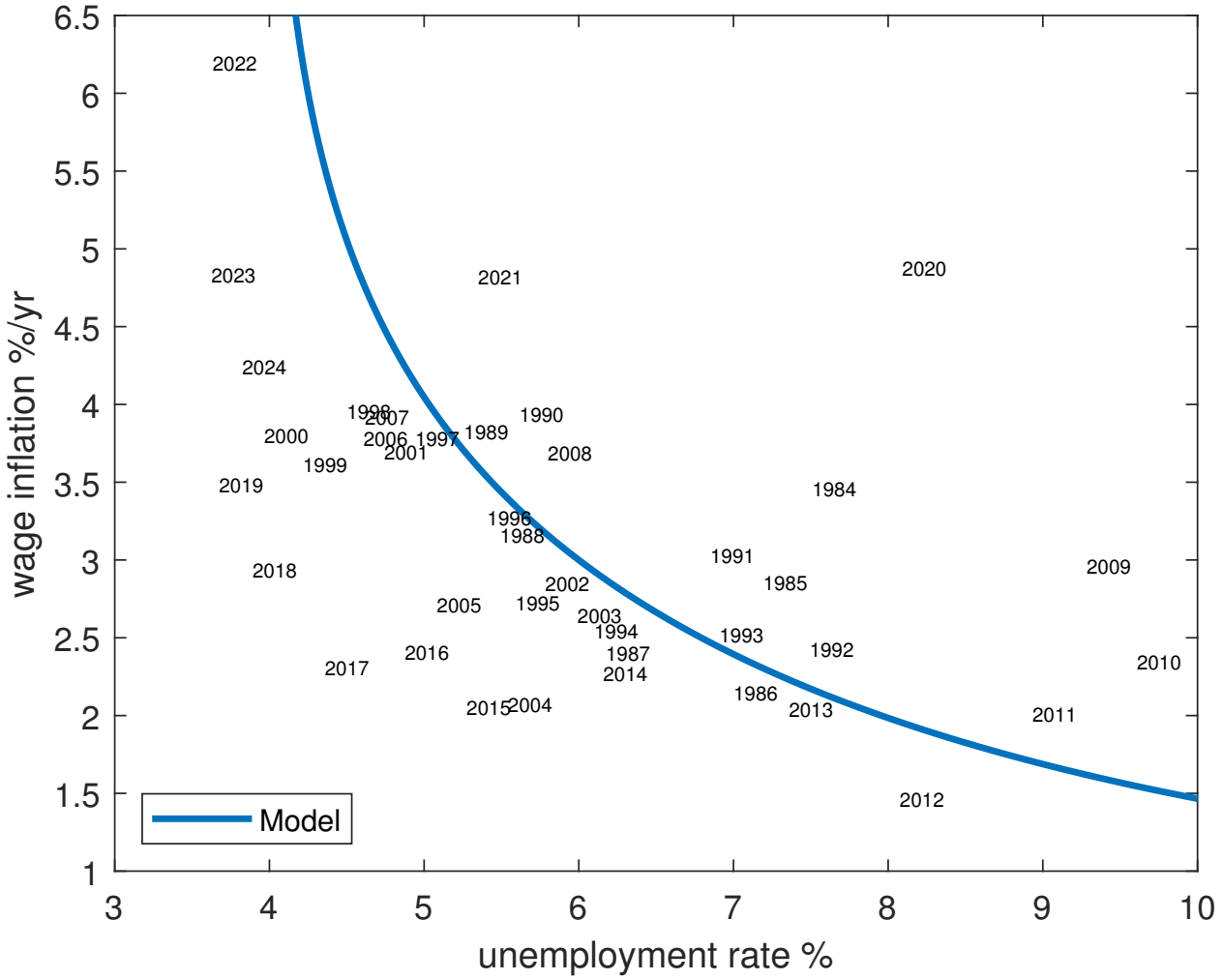
Firms

- production: $y_t = A_t F(h_t)$
- heterogeneous labor productivity, z, with log-normal density $f(z)$
- labor input: $h_t = \left[\int_0^\infty (z h_t(z))^{1-1/\eta} f(z) dz\right]^{\frac{1}{1-1/\eta}}$; $\quad \eta > 1$

Heterogeneous downward nominal wage rigidity

$$W_t(z) \geq z^{\xi} \gamma W_{t-1}$$

The Heterogeneous Productivity Wage Phillips Curve and U.S. Data



Notes. The figure shows with a solid line the short-run wage Phillips curve implied by the calibrated heterogeneous labor productivity model. The figure also shows the (u_t, π_t^W) pairs observed in annual U.S. data over the period 1984 to 2024. The observation labeled 2024 in the figure refers to unemployment and wage inflation in the first quarter of 2024.

Regular Dynamics

Determination of j_t^* in General Equilibrium

Equilibrium: 8 processes, j_t^*, y_t, h_t, w_t, i_t, π_t, π_t^W, and u_t, satisfying

$$y_t = z_t F(h_t) \tag{1}$$

$$z_t F'(h_t) = w_t \tag{2}$$

$$U'(y_t) = \beta(1+i_t) E_t \frac{U'(y_{t+1})}{1+\pi_{t+1}} \tag{3}$$

$$1 + i_t = \frac{1+\pi^*}{\beta} \left(\frac{1+\pi_t}{1+\pi^*}\right)^{\alpha_\pi} \left(\frac{y_t}{y}\right)^{\alpha_y} \mu_t \tag{4}$$

$$1 + \pi_t^W = \frac{w_t}{w_{t-1}}(1+\pi_t) \tag{5}$$

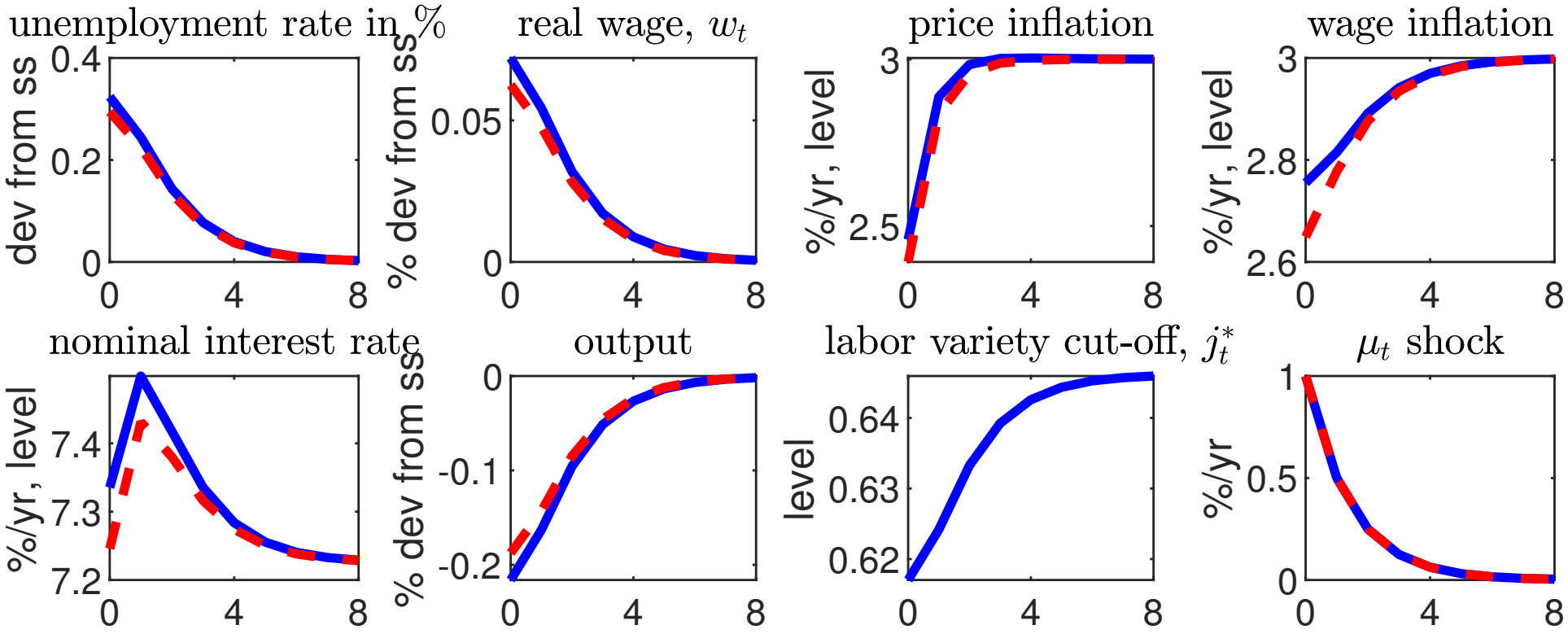
$$\boxed{\bar{h}(1-u_t^n) = \left(\frac{\gamma(j_t^*)}{1+\pi_t^W}\right)^{-\eta} h_t} \tag{6}$$

$$\boxed{(1+\pi_t^W)^{1-\eta} = j_t^* \gamma(j_t^*)^{1-\eta} + \int_{j_t^*}^1 \gamma(j)^{1-\eta} dj} \tag{7}$$

$$\boxed{u_t = u_t^n + (1-u_t^n)\left[(1-j_t^*) - \int_{j_t^*}^1 \left(\frac{\gamma(j)}{\gamma(j_t^*)}\right)^{-\eta} dj\right]} \tag{8}$$

• Note that although the equilibrium features occasionally binding constraints for individual labor types, there are no such constraints in the aggregate, making the model **amenable to perturbation analysis.**

Impulse Responses to a Monetary Tightening in the HDNWR and NK Models



Notes. Solid lines correspond to the HDNWR model and dashed lines to the NK model with Calvo wage stickiness. The size of the monetary shock is 1 percent per annum and its serial correlation is 0.5. The horizontal axes measure quarters after the shock.

Conclusions

• This paper proposes a model with heterogeneous downward nominal wage rigidity (HDNWR)

• The model implies a nonlinear convex wage Phillips curve.

• The model can account for both the missing inflation post GFC and the missing unemployment during the post Covid inflation stabilization.

