# Demographics and Real Interest Rates Across Countries and Over Time*


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**Abstract**

We explore the implications of demographic trends for the evolution of real interest rates across countries and over time. To that end, we first develop a tractable three-country general equilibrium model with imperfect capital mobility and country-specific demographic trends. We calibrate the model to study how low-frequency movements in a country's real interest rate depend on its own demographics and on global factors, given a certain degree of financial integration. The more financially integrated a country is, the higher the sensitivity of its real interest rate to global developments is, and the less its own real rate determinants matter. We then estimate panel error-correction models relating real interest rates to many of its possible determinants—demographics included—imposing some restrictions motivated by lessons from our structure model. Results indicate a meaningful role for life expectancy in determining real interest rates.



**JEL codes:** E52, E58, J11

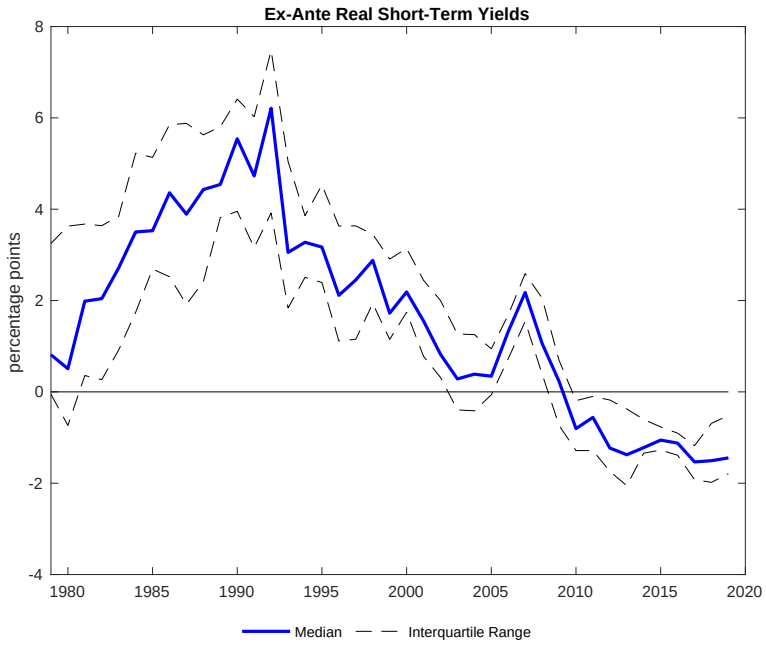
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**Figure 1:** Real interest rates.




*Note:* Median and interquartile range of ex-ante real short-term interest rates for 19 OECD countries between 1979 and 2019. See Section 4 for details about calculations.

# 1 Introduction

Between 1990 and the onset of the Covid-19 pandemic, real interest rates in advanced economies exhibited a pronounced and persistent decline (Figure 1). Since last year, however, short-term real rates have increased meaningfully, as central banks worldwide had to tighten monetary policy to fight high inflation. More intriguingly, long-term real rates have also increased markedly, raising the specter of persistently higher real interest rates in the future. Will advanced economies return to an environment of low real interest rates once central banks manage to tame inflation and normalize monetary policy? This is a key macroeconomic question, with important implications for both fiscal and monetary policy.

The pattern of real rate declines prior to the pandemic may indicate a return to a low-rate environment is a plausible conjecture (*e.g.* Blanchard, 2023). The reason is its low-frequency nature suggests long-term factors are likely important drivers of equilibrium real rates (Rachel and Smith, 2017).[1] In this paper, we revisit the role of one such factor—demographics—as a low-frequency driver of real interest rates.

Existing literature, including our earlier work (Carvalho et al., 2016), show past demographic developments and available projections can explain a significant portion of the real interest rate decline observed after 1990. We raise the bar for that conclusion, with the observation that if demographics are indeed an important driver of low-frequency movements in real interest rates, their pattern in cross-sectional and

[1]The debate in the literature, however, is far from being one-sided. At the other end of the spectrum, Hamilton et al. (2016) find little evidence of permanent factors in the decline in global real interest rates. Their empirical findings attribute the decline to temporary factors, including deleveraging, tighter financial regulation, and inflation trends.

time-series data should align well with demographic developments across countries and over time.

At least two issues, however, make that kind of inference challenging. First, in a world with international capital mobility, a country's real rate should depend not only on its own, bul also on global demographic developments. Second, other variables may affect real rates, so that uncovering the role of any given driver requires carefully controlling for other potential explanations. To handle these two issues, we resort to both a novel model and econometric analysis.

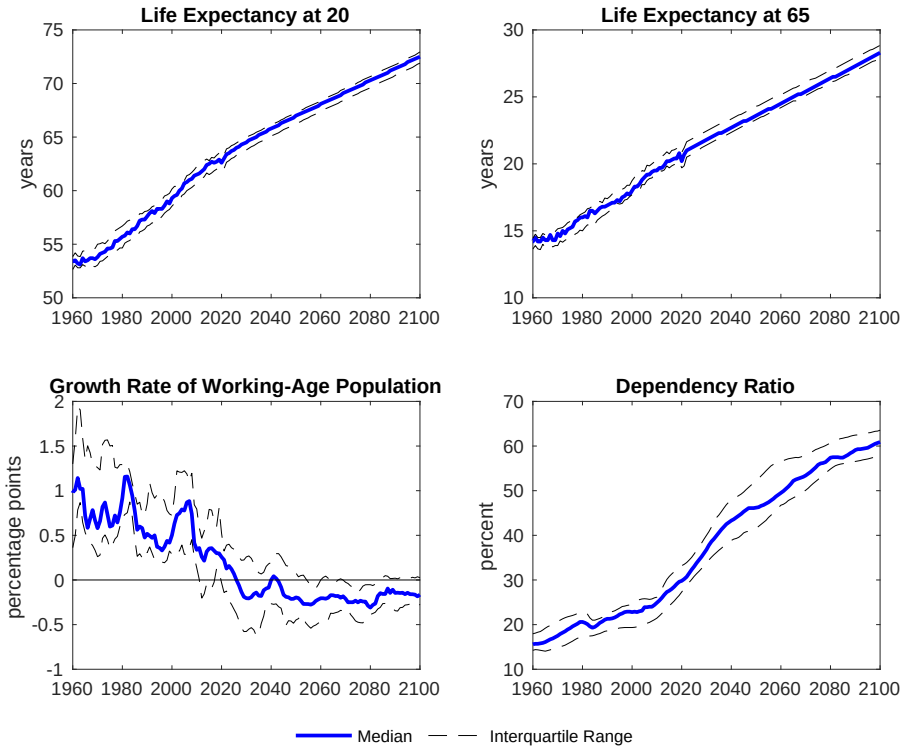
First, we develop a multicountry life-cycle model with imperfect capital mobility, and use it to study how a country's equilibrium real interest rate depends on its own demographics and on global factors. The main lesson from the model is that increased financial integration shifts the sensitivity of a country's real interest rate away from its own demographic developments and towards global determinants. Hence, integration tends to narrow the range of real rates observed across different countries, consistent with the pattern illustrated in Figure 1.

Drawing on the results of the model, we then turn to an empirical analysis of the long-run relationship between demographics and real interest rates. To address the challenge that other factors may drive real interest rates, we estimate panel error-correction models with a set of controls motivated by the literature, on top of the demographic variables our model suggests. Consistently with our theoretical analysis, we control for global factors through a measure of the relevant global real interest rate faced by each country, and take into account each country's time-varying degree of financial integration. The global rate is always statistically significant. Among the domestic factors, demographic variables are significant in most specifications. We also estimate analogous specifications without controlling for the time-varying degree of financial integration in order to showcase its importance.

The data clearly show advanced economies are aging at a fast pace (Figure 2). Between 1960 and 2020, median life expectancy at 20 has increased by about 9 years, from 53.4 to 62.6 (top-left panel). Over the same period, older generations have also experienced significant longevity gains, with median life expectancy at 65 increasing from 14.2 years to 20.2 years (top-right panel). Meanwhile, fertility rates have fallen sharply, implying a decline in the median growth rate of the working-age population from approximately 1% in 1960 to 0.26% in 2020 (bottom-left panel). The combination of lower fertility and higher longevity has roughly doubled the old-age dependency ratio (the ratio between people 65 years old and above to people 15 to 64 years old) from 15.6 in 1960 to 29.8 in 2020 (bottom-right panel). Going forward, the available demographic projections suggest that in advanced economies working-age population will contract at a rate of approximately 0.2% per year, while life expectancy will continue to increase, so that, by the end of this century, the dependency ratio will be over 60%.

Our goal is to study how such demographic developments shape real interest rates across countries in a world with imperfect capital mobility. We begin by developing a multicountry life-cycle model in which households can invest in both domestic and foreign assets. Investment in foreign assets is subject to portfolio-holding costs which proxy for various factors that can hinder capital flows in practice. As

**Figure 2:** Demographic variables.




*Note:* Top-left panel: Years of life expectancy at 20. Top-right panel: Years of life expectancy at 65. Bottom-left panel: Growth rate of working-age population. Bottom-right panel: Number of people 65 and older relative to the number of people 15 to 64 years old. Sample: 19 OECD countries between 1960 and 2100 (projections after 2020). Source: United Nations World Population Prospects (2021 Revision).

a result, international capital mobility is imperfect in the model.[2] In our framework, demographic trends affect the equilibrium real interest rate through changes in the growth rate of the labor force and life expectancy. Population composition effects arise endogenously from these two fundamental forces. Crucially, because households can trade assets internationally, demographic developments in one country can affect the others as well.

A calibrated three-country version of the model captures the salient features of the demographic transition in developed economies. We use this framework to study the low-frequency relationship between demographics and real interest rates, and how the degree of financial integration shapes this relationship across countries and over time. In particular, we focus on how a country's real interest rate depends on its own and on global demographic developments as financial integration changes over time as it does in the data. The main lesson from the model is that, as financial integration increases, a country's real interest rate becomes relatively more sensitive to global determinants, to the detriment of its own demographic developments.

[2]The limiting cases of the two-country version of our model with zero and infinite portfolio-holding costs correspond to Ferrero (2010) and Carvalho et al. (2016), respectively.

Our paper belongs to a recent wave of research that investigates, both theoretically and empirically, the determinants of real interest rates. A number of existing contributions focus on demographics, calibrating overlapping generation models to individual economies, such as the U.S. (Gagnon et al., 2021), the euro area (Kara and von Thadden, 2016), and Japan (Ikeda and Saito, 2014).[3] Our focus on the open economy dimension is closer in spirit to Lisack et al. (2021) and, especially, to Krueger and Ludwig (2007), who also discuss the interaction between demographics and financial integration. A key difference relative to our paper is that those contributions only consider the two extreme cases of closed economies or fully-integrated capital markets, while we allow for imperfect capital mobility.

Empirically, Lunsford and West (2019) conclude demographic variables can explain some of the variability in US real interest rates over more than one hundred years, while Fiorentini et al. (2018) highlight the importance of the share of young workers in accounting for the rise and fall of real rates between 1960 and 2016. Our empirical analysis expands on this second paper by pursuing an econometric specification that is informed by our structural model and by considering a number of additional candidate explanations, such as productivity growth (Holston et al., 2017), fiscal variables (Rachel and Summers, 2019), convenience yields (Del Negro et al., 2017; Del Negro et al., 2019), and inequality (Eggertsson et al., 2019; Mian et al., 2021). Despite this additional set of potential drivers, demographic variables remain an important determinant of real interest rates in most specifications of our panel analysis.

In our model, the real rate is the return on both government bonds, physical capital, and private claims. In practice, these returns differ. As Gomme et al. (2015) have documented for the US, while the return on safe assets (primarily government bonds in advanced economies) has declined, the return on risky assets (in particular equity) has remained roughly constant. Reis (2022) finds this result is robust across countries, and also to different measures of capital and income. By abstracting from aggregate uncertainty and imperfect competition, our model fails to capture the rise of macroeconomic risk and markups that Farhi and Gourio (2018) and Eggertsson et al. (2021) argue are key drivers of the wedge between the return on equity and the return on government bonds.[4] Therefore, we focus on the comparison between the real interest rate in the model with the real yield on government bonds in the data.

The rest of the paper proceeds as follows. Section 2 presents the model. Section 3 discusses the calibration and the quantitative experiments that illustrate how the relationship between demographics and real rates varies with the degrees of financial integration across countries. Section 4 reports our empirical analysis. Finally, Section 5 concludes.

[3]Demographic variables feature prominently also among the factors that can explain the secular stagnation hypothesis (Eggertsson et al., 2019). Goodhart and Pradhan (2017) express a contrarian view, arguing that demographic trends will contribute to revert recent observed macroeconomic trends, including for real interest rates. As noted in Carvalho et al. (2016) and Blanchard (2023), this view neglects the role of increased life expectancy on workers' savings decisions during their employment spell to finance a longer retirement period.

[4]In addition, Farhi and Gourio (2018) also find a role for the rising importance of intangibles in production.

## 2 The Model

This section presents an open-economy life-cycle model with imperfect capital mobility. Building on Gertler (1999), we allow for time-varying differential demographic trends across countries. Portfolio-holding costs, as in Chang et al. (2015), hamper the free flow of capital across countries. The resulting framework nests the closed-economy model of Carvalho et al. (2016) and the open-economy model of Ferrero (2010) as special cases.

### 2.1 Demographics

The economy consists of $\mathcal{M}$ regions. In each region $m = 1, \ldots, \mathcal{M}$, $(1-\omega_{mt}+n_{mt})N^w_{mt-1}$ new workers $(w)$ are born in every period $t$, where $N^w_{mt-1}$ is the number of workers at time $t-1$ and $\omega_{mt}$ is the probability a worker remains in the labor force between periods $t-1$ and $t$. Therefore, the number of workers evolves according to

$$N^w_{mt} = (1 - \omega_{mt} + n_{mt})N^w_{mt-1} + \omega_{mt}N^w_{mt-1} = (1 + n_{mt})N^w_{mt-1},$$

so that $n_{mt}$ is the net growth rate of the labor force.

A worker who exits the labor force becomes a retiree $(r)$. The probability of a retiree surviving between periods $t-1$ and $t$ is $\gamma_{mt}$. Therefore, the number of retirees evolves according to

$$N^r_{mt} = (1 - \omega_{mt})N^w_{mt-1} + \gamma_{mt}N^r_{mt-1}.$$

The (old) dependency ratio, $\psi_{mt} \equiv N^r_{mt}/N^w_{mt}$, measures the number of retirees per worker, and evolves according to

$$(1 + n_{mt})\psi_{mt} = (1 - \omega_{mt}) + \gamma_{mt}\psi_{mt-1}. \tag{1}$$

The growth rate of the labor force and the probability of surviving as a retiree are the fundamental demographic variables in the model. In our quantitative exercises, we will measure the growth rate of the labor force directly from the data. Conditional on a given retirement age, we will back out the probability of surviving from the evolution of the (old) dependency ratio, which is an observable variable, using equation (1).

### 2.2 Preferences

Retirees and workers value consumption streams according to recursive nonexpected utility preferences (Epstein and Zin, 1989)

$$V^z_{mt} = \max\left[(C^z_{mt})^{\frac{\sigma-1}{\sigma}} + \beta^z_{mt+1}\mathbb{E}_t\left(V^{1-\theta}_{mt+1}|z\right)^{\frac{\sigma-1}{\sigma(1-\theta)}}\right]^{\frac{\sigma}{\sigma-1}},$$

where $z = \{w, r\}$ represents the agent's type, $V^z_{mt}$ represents the value of utility for an agent of type $z$ at time $t$, $C^z_{mt}$ denotes consumption of the single good, $\sigma > 0$ is the elasticity of intertemporal substitution,

and $\theta$ is the coefficient of risk aversion. The discount factor $\beta^z_{mt+1} > 0$ differs between retirees and workers:

$$\beta^z_{mt+1} = \begin{cases} \beta_m & \text{if } z = w \\ \\ \beta_m \gamma_{mt+1} & \text{if } z = r. \end{cases}$$

With standard preferences (e.g., constant relative risk aversion), the assumption that the transition probability into retirement is independent of age would give rise to excessive precautionary savings early in life. To mitigate this effect while retaining the sensitivity of consumption decisions to changes in the interest rate, we assume risk neutrality with respect to income fluctuations and set $\theta = 0$.[5]

### 2.3 Retirees

The problem of a retiree from country $m$ born in period $j$ and retired in period $k$ is

$$V^{rjk}_{mt} = \max_{C^{rjk}_{mt}, \left\{A^{rjk}_{m\ell t}\right\}_{\ell=1}^{\mathcal{M}}} \left[ \left(C^{rjk}_{mt}\right)^{\frac{\sigma-1}{\sigma}} + \beta_m \gamma_{mt+1} \left(V^{rjk}_{mt+1}\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{2}$$

subject to

$$C^{rjk}_{mt} + \left[1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} \left(\eta^{rjk}_{m\ell t} - \bar{\eta}_{m\ell}\right)^2\right] \sum_{\ell=1}^{\mathcal{M}} A^{rjk}_{m\ell t} = \frac{1}{\gamma_{mt}} \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A^{rjk}_{m\ell t-1} + E^{rjk}_{mt}, \tag{3}$$

where $\eta^{rjk}_{m\ell t} \equiv A^{rjk}_{m\ell t} / (\sum_{p=1}^{\mathcal{M}} A^{rjk}_{mpt})$ are portfolio shares, $A^{rjk}_{m\ell t}$ are assets a retiree of country $m$ holds against country $\ell = 1, \ldots, \mathcal{M}$ and pay a gross return $R_{\ell t}$. At the beginning of each period, retirees turn their wealth to a perfectly competitive mutual fund that pools the risk of death and pays an extra return equal to the inverse of the survival probability, as in Yaari (1965) and Blanchard (1985). In addition, a retiree receives a lump-sum pension benefit $E^{rjk}_{mt}$ from the government.

In forming their portfolios, retirees incur a cost that depends on the difference between the actual share invested in foreign assets $\eta^{rjk}_{m\ell t}$ and an exogenous target level $\bar{\eta}_{m\ell}$ that we assume to be independent of type and that pins down steady state gross foreign asset positions.[6] Following Chang et al. (2015), we assume portfolio-holding costs are quadratic and their level depends on a (possibly time-varying) parameter $\Lambda_{m\ell t} \geq 0$. By limiting capital flows across countries, portfolio-holding costs capture, in reduced form, all the factors that prevent equalization of real interest rates across countries, even after controlling for risk premia.[7] One interpretation of our adjustment cost formulation is that retirees delegate their investment

[5] The resulting preferences imply linear consumption decisions (Farmer, 1990), thus facilitating aggregation among workers and retirees of different ages.

[6] The portfolio-holding cost "rate" is positive as long as the share of foreign assets deviates from $\bar{\eta}_{mn}$. The assumption that this cost rate applies to total assets keeps the model tractable, allowing us to make substantial analytical progress based on a guess-and-verify approach.

[7] When $\Lambda_{m\ell t}$ tends to infinity, the countries in the model become closed, as in Carvalho et al. (2016). Conversely, when these parameters are equal to zero, the model corresponds to a multi-country version of Ferrero (2010).

to portfolio managers who charge a fee to invest in foreign assets.[8]

Appendix A.1 shows the share a retiree invests in country-$p$ assets (with $p \neq m$) is independent of age and time since retirement ($\eta^{rjk}_{mpt} = \eta^{r}_{mpt}$, $\forall j$ and $k$), and satisfies

$$\left[1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} \left(\eta^{r}_{m\ell t} - \bar{\eta}_{m\ell}\right)^2\right] \left(R_{pt} - R_{mt}\right) = \Lambda_{mpt} \left(\eta^{r}_{mpt} - \bar{\eta}_{mp}\right) R_{mt}. \tag{4}$$

In addition, Appendix A.2 shows the same condition holds for workers. Therefore, retirees and workers invest the same share of their total financial wealth in country-$p$ assets ($\eta^{r}_{mpt} = \eta^{w}_{mpt} = \eta_{mpt}$).

The solution of the optimization problem for a retiree yields a consumption function linear in the sum of total financial wealth and the present discounted value of pension benefits ($S^{rjk}_{mt}$):

$$C^{rjk}_{mt} = \xi^{r}_{mt} \left(\frac{1}{\gamma_{mt}} \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A^{rjk}_{m\ell t-1} + S^{rjk}_{mt}\right), \tag{5}$$

where

$$S^{rjk}_{mt} = E^{rjk}_{mt} + \frac{\gamma_{mt+1} S^{rjk}_{mt+1}}{\tilde{R}_{mt}},$$

and where we have defined the adjusted gross return

$$\tilde{R}_{mt} \equiv \frac{\sum_{\ell=1}^{\mathcal{M}} \eta_{m\ell t} R_{\ell t}}{1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} (\eta_{m\ell t} - \bar{\eta}_{m\ell})^2}. \tag{6}$$

The marginal propensity to consume is also independent of individual characteristics (birth and retirement age), and evolves according to the Euler equation

$$\frac{1}{\xi^{r}_{mt}} = 1 + \gamma_{mt+1} \beta^{\sigma}_{m} \tilde{R}^{\sigma-1}_{mt} \frac{1}{\xi^{r}_{mt+1}}. \tag{7}$$

### 2.4 Workers

In every period, workers need to take into account the possibility of retirement. Thus, the continuation value for an individual born in period $j$ and currently employed is $V^{wj}_{mt+1}$ with probability $\omega_{mt+1}$ and $V^{rjt+1}_{mt+1}$ with the complementary probability. The full optimization problem is

$$V^{wj}_{mt} = \max_{C^{wj}_{mt}, \{A^{wj}_{m\ell t}\}_{\ell=1}^{\mathcal{M}}} \left\{ \left(C^{wj}_{mt}\right)^{\frac{\sigma-1}{\sigma}} + \beta_m \left[\omega_{mt+1} V^{wj}_{mt+1} + (1 - \omega_{mt+1}) V^{rjt+1}_{mt+1}\right]^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}}, \tag{8}$$

[8] In our formulation, the government collects the costs so we can think of government regulation limiting foreign financial investment. Alternatively, we could write these costs in terms of real resources, possibly devoted to research about foreign financial markets.

subject to

$$C_{mt}^{wj} + \left[1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t}^{wj} - \bar{\eta}_{m\ell}\right)^2\right] \sum_{\ell=1}^{\mathcal{M}} A_{m\ell t}^{wj} = \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1}^{wj} + W_{mt}^{w} - T_{mt}^{w}, \tag{9}$$

where $W_t^w$ is the real wage and $T_{mt}^w$ are lump-sum taxes, which only workers pay.[9]

As already mentioned, all workers optimally choose the same portfolio shares, which also equal the choice of retirees. Workers' consumption is linear in the sum of total financial wealth, the present discounted value of wage income net of taxes ("human wealth"), and the present discounted value of pension benefits:

$$C_{mt}^{wj} = \xi_{mt}^{w} \left(\sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1}^{wj} + H_{mt}^{wj} + Z_{mt}^{wj}\right), \tag{10}$$

where human wealth is given by

$$H_{mt}^{wj} = W_{mt}^{w} - T_{mt}^{w} + \frac{\omega_{mt+1} H_{mt+1}^{wj}}{\Omega_{mt+1} \tilde{R}_{mt}},$$

and the present discounted value of pension benefits for workers is

$$Z_{mt}^{wj} = \frac{1}{\Omega_{mt+1} \tilde{R}_{mt}} \left[(1 - \omega_{mt+1}) \left(\frac{\xi_{mt+1}^{r}}{\xi_{mt+1}^{w}}\right)^{\frac{1}{1-\sigma}} S_{mt+1}^{rjt+1} + \omega_{mt+1} Z_{mt+1}^{wj}\right].$$

Workers discount both variables by the adjusted gross return, defined in equation (6), times an additional factor that takes into account the probability of retiring and the heterogeneity in the marginal propensity to consume between the two groups:

$$\Omega_{mt} \equiv \omega_{mt} + (1 - \omega_{mt}) \left(\frac{\xi_{mt}^{r}}{\xi_{mt}^{w}}\right)^{\frac{1}{1-\sigma}}. \tag{11}$$

Because retirees and workers discount the future at different rates, Ricardian equivalence does not hold in this model, even though taxes are lump-sum.

Finally, as for retirees, the marginal propensity to consume for workers is independent of individual characteristics and evolves according to

$$\frac{1}{\xi_{mt}^{w}} = 1 + \beta_m^{\sigma} \left(\Omega_{mt+1} \tilde{R}_{mt}\right)^{\sigma-1} \frac{1}{\xi_{mt+1}^{w}}. \tag{12}$$

[9] As in Ferrero (2010) and Carvalho et al. (2016), we assume workers inelastically supply one unit of labor and retirees do not work. Gertler (1999) shows how to relax both these assumptions. With endogenous labor, the optimal response to a declining growth rate of the labor force and an increase in life expectancy would be to supply more hours. Such a behavior of hours worked would be inconsistent with the data for most OECD economies (OECD, 2018).

### 2.5 Aggregation

Since marginal propensities to consume are independent of individual characteristics and consumption functions are linear, we can aggregate among workers and among retirees by simply adding over individuals in each group.[10]

Aggregate retirees' consumption is given by

$$C^r_{mt} = \xi^r_{mt}\left(\sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A^r_{m\ell t-1} + S^r_{mt}\right).$$

Note that, in the aggregate, the extra-return the mutual fund offers corresponds to the fraction of retirees who survive between two periods because of the law of large numbers. Similarly, aggregate workers' consumption is given by

$$C^w_{mt} = \xi^w_{mt}\left(\sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A^w_{m\ell t-1} + H^w_{mt} + Z^w_{mt}\right).$$

Aggregate consumption in country $m$ is simply the sum of retirees' and workers' consumption:

$$C_{mt} \equiv C^w_{mt} + C^r_{mt}.$$

Finally, because of the heterogeneity between workers and retirees over the life cycle, we need to keep track of the distribution of wealth between these two groups. The result that retirees and workers from a given country choose the same portfolio shares is useful in this respect. First, given the total amount of country-$n$ assets held by country-$m$ agents, we define the share held by retirees as

$$\lambda_{mpt} \equiv \frac{A^r_{mpt}}{A^r_{mpt} + A^w_{mpt}}. \tag{13}$$

Second, from the definition of portfolio shares, $A^z_{mpt} = \eta_{mpt} A^z_{mt}$ for $z = \{r, w\}$, where $A^z_{mt} = \sum_{\ell=1}^{\mathcal{M}} A^z_{m\ell t}$. Using this definition in equation (13), we obtain

$$\lambda_{mpt} = \frac{A^r_{mt}}{A^r_{mt} + A^w_{mt}} = \lambda_{mt},$$

that is, the retirees' share of country-$p$ assets held by country-$m$ agents corresponds to the share of wealth accruing to retirees in country $m$. In Appendix A.3, we show that, combining the budget constraints of

[10] In dropping reference to the birth and retirement period, we use the notation $\sum_r C^{rjk}_{mt} = N^r_{mt} C^{rjk}_{mt} \equiv C^r_{mt}$ and $\sum_w C^{wj}_{mt} = N^w_{mt} C^{wj}_{mt} \equiv C^w_{mt}$. The same notation applies to asset holdings, human wealth, and the present discounted value of pensions for retirees and workers.

retirees and workers, we can derive the evolution of the distribution of wealth, which obeys

$$\left[1+\sum_{\ell=1}^{\mathcal{M}}\frac{\Lambda_{m\ell}}{2}\left(\eta_{m\ell t}-\bar{\eta}_{m\ell}\right)^2\right]\left[\lambda_{mt}-(1-\omega_{mt+1})\right]A_{mt}$$
$$=\omega_{mt+1}\left[(1-\xi^r_{mt})\lambda_{mt-1}A_{mt-1}\sum_{\ell=1}^{\mathcal{M}}R_{\ell t-1}\eta_{m\ell t-1}+E_{mt}-\xi^r_{mt}S_{mt}\right], \quad (14)$$

where $A_{mt}\equiv A^r_{mt}+A^w_{mt}$ represents the total amount of assets held by country-$m$ residents.

### 2.6 Firms

A continuum of measure one of perfectly competitive firms operate in each country. Firms hire workers and accumulate capital $K_{mt}$ to produce the single good according to a labor-augmenting Cobb-Douglas technology, identical across countries:

$$Y_{mt}=\left(X_{mt}N^w_{mt}\right)^{\alpha}K^{1-\alpha}_{mt-1},$$

where $\alpha\in(0,1)$ and $Y_{mt}$ represents output. The productivity factor grows exogenously at a rate $x_{mt}$ between periods $t-1$ and $t$:

$$X_{mt}=(1+x_{mt})X_{mt-1}.$$

The law of motion of capital is standard:

$$K_{mt}=(1-\delta)K_{mt-1}+I_{mt},$$

where $I_{mt}$ stands for investment and $\delta\in(0,1)$ is the depreciation rate.

The first-order conditions for firms are standard. The wage is equal to the marginal product of labor

$$W^w_{mt}=\alpha\frac{Y_{mt}}{N^w_{mt}},$$

while the real interest rate is equal to the marginal product of capital

$$R_{mt}=(1-\alpha)\frac{Y_{mt+1}}{K_{mt}}+(1-\delta).$$

### 2.7 Government

In each period, the government issues one-period bonds, $B_{mt}$, and levies lump-sum taxes on workers $T_{mt}\equiv N^w_{mt}T^w_{mt}$ to fund pension benefits $E_{mt}\equiv N^r_{mt}E^r_{mt}$ and wasteful spending $G_{mt}$, and to repay maturing debt inclusive of interest to bondholders $R_{mt-1}B_{mt-1}$. The government also collects the amount of resources foreign investors forego to hold positions in country-$m$ assets (the portfolio-holding costs).

Its budget constraint is

$$B_{mt} = R_{mt-1}B_{mt-1} + G_{mt} + E_{mt} - \left[T_{mt} + \sum_{\ell \neq m} \frac{\Lambda_{\ell mt}}{2}(\eta_{\ell mt} - \bar{\eta}_{\ell m})^2 A_{\ell t}\right].$$

We assume debt, spending, and pensions are exogenous fractions of output:

$$G_{mt} = g_{mt}Y_{mt}, \qquad B_{mt} = b_{mt}Y_{mt}, \qquad E_{mt} = e_{mt}Y_{mt},$$

so that the government budget constraint determines taxes residually.

### 2.8 Balance of Payments and Equilibrium

Country-$m$ assets held by its residents correspond to the amount of capital and bonds not owned by foreigners:

$$A_{mmt} = K_{mt} + B_{mt} - \sum_{\ell \neq m} A_{\ell mt}.$$

Net foreign assets for country $m$ equal the gross amount of foreign assets owned by its residents net of country-$m$ assets held by foreigners:

$$F_{mt} \equiv \sum_{\ell \neq m}(A_{m\ell t} - A_{\ell mt}).$$

Net foreign assets evolve according to

$$F_{mt} = F_{mt-1} + \sum_{\ell \neq m}(R_{\ell t-1} - 1)A_{m\ell t-1} - (R_{mt-1} - 1)\sum_{\ell \neq m} A_{\ell mt-1} \\ - \sum_{\ell \neq m} \frac{\Lambda_{m\ell t}}{2}\left(\eta_{m\ell t} - \bar{\eta}_{m\ell}\right)^2 A_{mt} + \sum_{\ell \neq m} \frac{\Lambda_{\ell mt}}{2}(\eta_{\ell mt} - \bar{\eta}_{\ell m})^2 A_{\ell t} + NX_{mt},$$

where the trade balance is the difference between production and domestic absorption:

$$NX_{mt} \equiv Y_{mt} - C_{mt} - G_{mt} - I_{mt}.$$

Finally, the global asset market-clearing condition is

$$\sum_{\ell=1}^{\mathcal{M}} F_{\ell t} = 0.$$

Because the labor force and technology grow over time, we focus on a solution for detrended variables.[11] Given exogenous processes for the growth rate of the labor force, life expectancy, the growth rate

[11] For a generic variable $D_t$, the stationary counterpart is $d_t \equiv D_t/(X_t N_t^w)$. The model admits a well-defined steady state in terms of de-trended variables.

**Table 1:** Demographic variables in the initial steady state (in %).

| Parameter | Young Economy | Old Economy | Rest of the World |
|---|---|---|---|
| Relative size ($N^w_{m0}/N^w_{\mathcal{W}0}$) | 0.38 | 0.38 | 99.24 |
| Growth rate of the labor force ($n_{m0}$) | 1.13 | 0.59 | 0.75 |
| Dependency ratio ($\psi_{m0}$) | 20.98 | 24.46 | 22.29 |

of technology, and fiscal variables, a competitive equilibrium for the world economy requires that, in each country: retirees and workers maximize utility subject to their budget constraints; firms maximize profits given their technological constraints; the government satisfies its budget constraint; and labor markets clear. In addition, the asset market clears at the global level.

We solve the model with an "extended-path" approach, according to which agents form expectations in each period assuming that the exogenous processes will remain constant at their current values into the indefinite future. The extended-path solution concatenates the pointwise equilibrium values obtained in each period by solving for the perfect-foresight path from each period onward under those "constant beliefs." Like perfect foresight, the extended-path approach allows us to obtain a fully non-linear solution for the transition between steady states focusing on low-frequency dynamics (i.e., abstracting from fluctuations that are the focus of business cycle models with aggregate shocks). Due to the constant-beliefs assumption, however, the extended-path approach avoids the excessive "front-loading" of responses that characterizes the standard perfect-foresight solution.

# 3 Quantitative Analysis

Our main quantitative experiment characterizes the macroeconomic transition of the world economy between two steady states driven by differential demographic developments across countries and time-varying degrees of financial integration. For simplicity, we assume technology and fiscal variables remain constant and equal across countries.

## 3.1 Calibration

Each period corresponds to one year. We calibrate our multicountry model to a world with three regions: a small young economy ($\mathcal{Y}$), a small old economy ($\mathcal{O}$), and the rest of the world ($\mathcal{W}$). The "young" economy has a relatively high growth rate of the labor force and a relatively low dependency ratio, while the opposite is true for the "old" economy.

The source for the demographic data is the United Nation World Population Database (2019). We assume the initial size of the rest of the world is equal to the sum of the working-age population across the twenty OECD economies in our sample in 1990 (3.78 billion), and we set the initial relative size of both the young and the old economy to 0.4% to match the first quartile of the cross-sectional distribution

**Table 2:** Common parameters across countries.

| Parameter | value | Description |
|---|---|---|
| $\omega$ | = 0.978 | Average employment duration |
| $\sigma$ | = 0.500 | Elasticity of intertemporal substitution |
| $\alpha$ | = 0.667 | Labor share |
| $\delta$ | = 0.100 | Depreciation rate |
| $x$ | = 0.005 | Growth rate of productivity |
| $\bar{\eta}$ | = 0 | Target net foreign asset position |
| $b$ | = 0.600 | Debt/GDP |
| $g$ | = 0.250 | Government spending/GDP |
| $e$ | = 0.075 | Pensions/GDP |

of population sizes in the same year.

In the initial steady state, the young economy has a relatively high growth rate of the labor force and a relatively low dependency ratio. We match the initial growth rate of the working-age population and the dependency ratio in this region to the third and first quartiles, respectively, of their empirical counterparts, which gives $n_{\mathcal{Y}0} = 1.13\%$ and $\psi_{\mathcal{Y}0} = 20.98\%$. Conversely, for the old economy, we target the first quartile for the growth rate of the working-age population and the third quartile for the dependency ratio ($n_{\mathcal{O}0} = 0.59\%$ and $\psi_{\mathcal{O}0} = 24.46\%$, respectively). Finally, for the rest of the world, we simply target the weighted average of the countries in our sample for both growth rate of the working-age population and dependency ratio ($n_{\mathcal{W}0} = 0.75\%$ and $\psi_{\mathcal{W}0} = 22.29\%$).[12] Table 1 reports the values for the demographic variables in the initial steady state.

We follow Gertler (1999) and Carvalho et al. (2016) in setting most of the remaining parameters that are common to all countries (Table 2). Agents are born workers at the age of 20. We fix the probability of remaining employed $\omega$ at 0.978 to match an average employment duration of 45 years so that on average individuals retire at 65. We set the elasticity of intertemporal substitution $\sigma$ to 0.5, consistent with the evidence in Hall (1988) and Yogo (2004), who report values significantly lower than one. We further set the labor share $\alpha$ equal to 0.667 and the depreciation rate $\delta$ equal to 0.1, which are standard values in the literature. We assume the growth rate of technology is $x = 0.5\%$, roughly in line with the average growth rate for the countries in our dataset since 1990. We calibrate fiscal variables (debt, government spending and pensions) as a fraction of GDP to match the average values for OECD countries since 1990, which implies $b = 60\%$, $g = 25\%$ and $e = 7.5\%$, respectively.

The remaining parameters to calibrate are the target shares for foreign asset holdings, the initial value of the portfolio-holding costs and the individual discount factors. For simplicity, we assume the target shares for foreign asset holdings $\bar{\eta}$ to be zero in all countries.[13] Finally, for each country, we jointly choose

[12] Conditional on the growth rate of the working-age population and on the probability of retiring (which we discuss in the text), the steady state version of equation (1), $\psi_m = (1-\omega)/(1+n_m-\gamma_m)$, provides a unique mapping between the dependency ratio of country $m$ and the associated survival probability.

[13] Foreign asset positions in the initial steady state actually differ from zero because of cross-country demographic differ-

**Table 3:** Discount factors and portfolio-holding costs.

| Parameter | Young Economy | Old Economy | Rest of the World |
|---|---|---|---|
| Discount factor ($\beta_m$) | 0.987 | 1.013 | 1.003 |
| Portfolio-holding costs ($\Lambda_{\mathcal{Y}n0}$) | 0 | 300 | 32.8595 |
| Portfolio-holding costs ($\Lambda_{\mathcal{O}n0}$) | 300 | 0 | 68.5684 |
| Portfolio-holding costs ($\Lambda_{\mathcal{W}n0}$) | 32.8595 | 68.5684 | 0 |

the initial value of the portfolio-holding cost parameter $\Lambda_{mn0}(=\Lambda_{nm0})$ and of the individual discount factor $\beta_m$ to target the real interest rate and the external position in the initial steady state. The real interest rate measure that we use is a three-year moving average centered in 1990 of the ex-ante short yield (the same data plotted in Figure 1). For the external position, we target the cross-country distribution of gross foreign debt relative to GDP from Lane and Milesi-Ferretti (2017). The focus on debt aligns well our real interest rate measure with the appropriate asset class in the data. While the model only keeps track of net foreign assets, we use gross positions as the empirical counterpart, in order to limit the heterogeneity in discount factors necessary to match the initial dispersion of real interest rates. For the young economy, we match the third quartile of the empirical distribution of real interest rates (7.01%) and the first quartile of gross foreign debt liabilities relative to GDP (39.36%), which gives $\beta_{\mathcal{Y}} = 0.987$, $\Lambda_{\mathcal{Y}\mathcal{O}0} = 300$ and $\Lambda_{\mathcal{Y}\mathcal{W}0} = 32.8595$.[14] Similarly, for the old economy, we match the first quartile of the distribution of real interest rates (3.56%) and the first quartile of gross foreign debt assets relative to GDP (16.79%), which gives $\beta_{\mathcal{O}} = 1.013$ and $\Lambda_{\mathcal{O}\mathcal{W}0} = 68.5684$. Finally, for the global economy, we match the median real interest rate (5.28%) and a balanced external position, which gives $\beta_{\mathcal{W}} = 1.003$.[15] Table 3 summarizes this part of the calibration.

### 3.2 Experiment

In our baseline experiment, the size of the three economies and the degree of financial integration evolve over time in response to the evolution of the demographic variables and to changes in the portfolio-holding cost parameters. The targets that pin down the paths for these variables are the same as for the steady state (growth rate of working-age population, dependency ratio, and gross foreign debt positions relative to GDP). In order to obtain projections over an arbitrarily long horizon, we feed the model with the trend component of an HP filter (Hodrick and Prescott, 1997) applied to the data (see Figure 3).[16] We

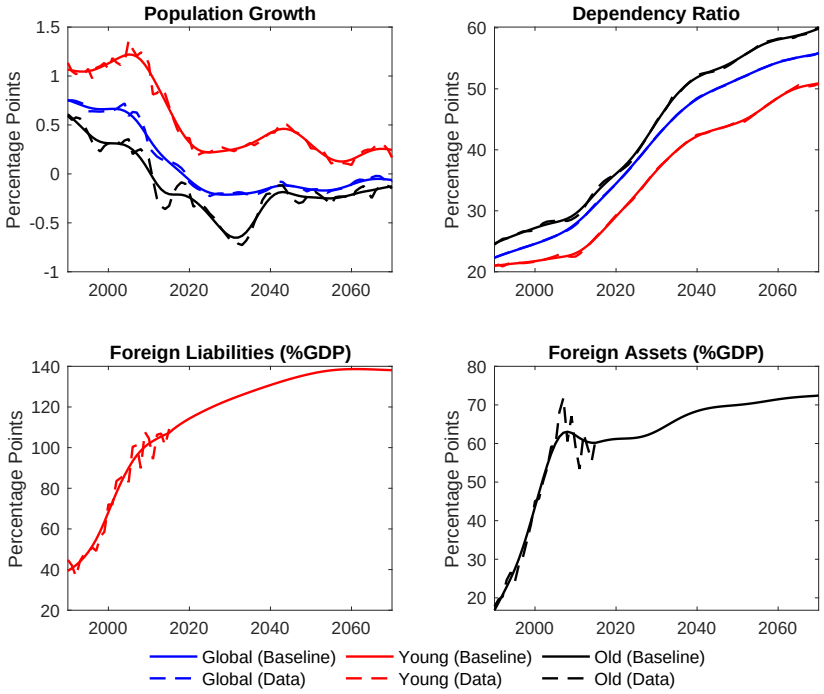
ences.

[14]The high value for $\Lambda_{\mathcal{Y}\mathcal{O}0} = \Lambda_{\mathcal{O}\mathcal{Y}0}$ is a by-product of the calibration strategy. A high enough value of the portfolio-holding cost parameter corresponds to a country being in autarky. Starting from autarky, we then progressively lower the friction to match the target for the gross foreign debt position. Given the relative size of the three countries, the bulk of financial flows occurs between each small country and the global economy so that we can simply adjust the portfolio-holding cost parameter $\Lambda_{m\mathcal{W}0}$.

[15]The household problem is well defined even when the individual discount factor is bigger than one as long as $\beta_m\gamma_{mt} < 1$, which is always the case in our experiments.

[16]We set the HP filter smoothing parameter for both the demographic variables and the financial integration to 40 so that the smoothed series approximate well the variables of interest.

**Figure 3:** Calibration of demographic processes and financial integration.




*Note:* Growth rate of working-age population (top-left panel), dependency ratio (top-right panel), foreign debt liabilities (bottom-left), and foreign debt assets (bottom-right). Red lines correspond to the young economy, black lines to the old economy, blue lines to the rest of the world. The dashed lines are data, the solid lines are the fitted processes (trend from a HP filter).

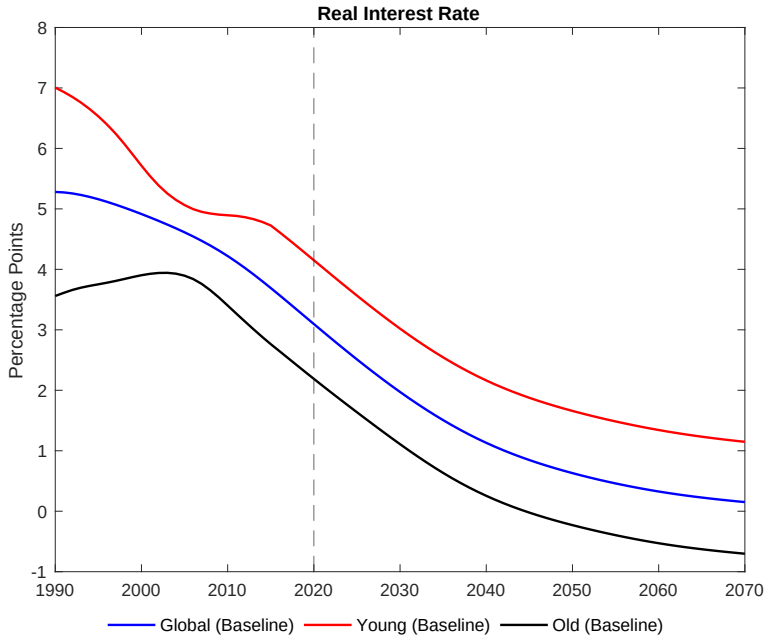
consider the period 1990-2020 as our "sample." The simulation, however, uses the projected trends for demographic variables and foreign debt until 2070. We refer to the results for the period 2021-2070 as "projections."

### 3.3 Results

Figure 4 shows the results of the basic experiment for the period 1990-2070. In sample (1990-2020), the real interest rate of the global economy (solid blue line) falls by more than two percentage points, from its initial value of 5.28% to 3.10%. In the young economy, the decline is almost three percentage points (from 7% to 4.15%). Over the entire sample, the interest rate falls also in the old economy, by about one and a half percentage point (from 3.56% to 2.19%). Differently from the other two countries, however, the dynamics for the old economy are not monotonic. Before starting to decline, the real interest rate actually increases for the first decade and a half of the sample to reach almost 4%.

These rich dynamics reflect the interaction between demographic variables and financial integration. The intuition for the initial increase of the real interest rate in the old economy corresponds to what happens in a Metzler diagram when two countries with different discount factors move from autarky to financial integration (Obstfeld and Rogoff, 1996). Initially, financial integration is low, the interest rate in

**Figure 4:** Real interest rates in the baseline simulation.



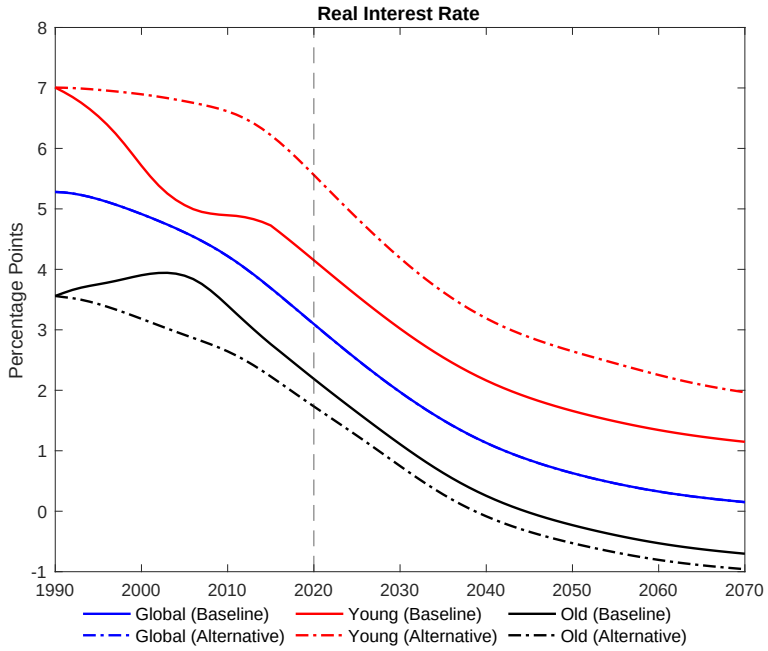

*Note:* The figure plots the simulated real interest rate from the model (baseline experiment) for the global economy (blue line), the young economy (red line), and the old economy (black line). The vertical line denotes the last period for which data on foreign debt are available.

the young economy is higher than in the rest of the world, and lower in the old economy. As the two small open economies become more integrated with the rest of the world, their real interest rates converge, so that the real interest rate falls for the young economy and increases for the old economy. Over time, when financial integration is sufficiently high, the evolution of domestic and foreign demographic variables exerts downward pressure on the real interest rate everywhere in the world.

The dynamics of foreign debt liabilities as a percentage of GDP (bottom-left panel of Figure 3) suggest the process of financial integration has slowed down following the financial crisis of 2008. In fact, foreign debt assets as a percentage of GDP fell slightly (bottom-right panel of Figure 3). This leads to a widening of interest rate differentials between both small regions and the global economy in the last few years of the sample.

Going forward, the projections for demographic variables by the United Nations suggest that the aging process will continue everywhere in the world, with some further decline in the growth rate of the labor force and a progressive increase in the dependency ratio. The implication is that real interest rates would continue to decline, reaching a level near zero globally in 2070. As for financial integration, we can only assess the evolution of foreign debt assets and liabilities based on the projected trends from our filtering procedure, which implies a further increase in liabilities and a stabilization of assets (the two bottom panels of Figure 3 after 2020). The model predicts that the differentials of about one percentage point

**Figure 5:** Real interest rates with constant financial integration.

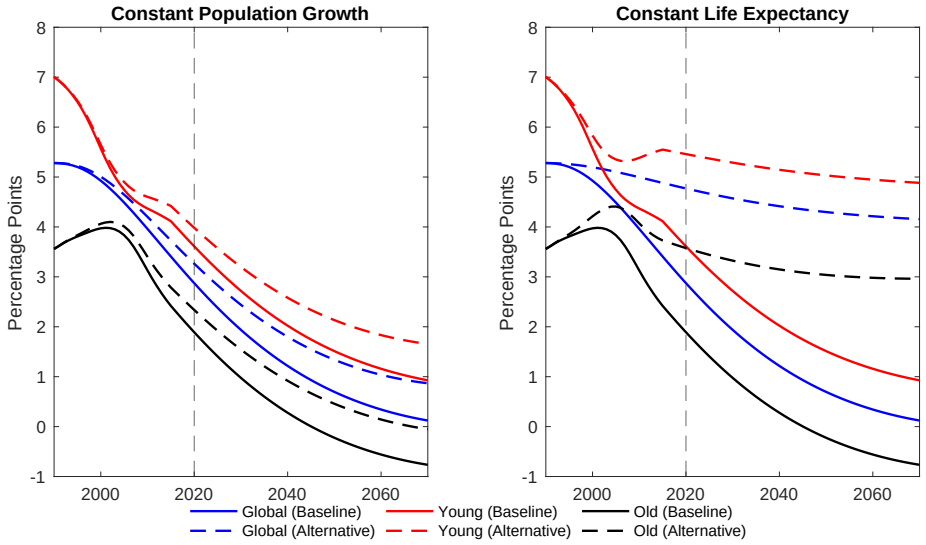



*Note:* The figure plots the simulated real interest rate from the model for the global economy (blue lines), the young economy (red lines), and the old economy (black lines). The solid lines correspond to the baseline simulation in Figure 4. The dashed-dotted lines are the counterfactual simulation in which the degree of financial integration remains at its initial value.

around the global real interest rate prevailing in 2020 will persist throughout the simulation horizon. Should financial integration experience further increases in the future, real interest rates will tend to converge once again, as observed especially between 1990 and 2005.

Figure 5 isolates the role of financial integration in determining the dynamics of real interest rates across regions. The solid lines are again the real interest rates in the three regions from the baseline experiment. The dashed-dotted lines correspond to the simulation that keeps the degree of financial integration at its initial value. Because the sizes of the three regions change over time, in the counterfactual simulation we adjust the portfolio-holding cost so that in each period we match the same target for foreign debt assets and liabilities as in the initial steady state. As expected, financial integration does not matter for the world economy (country $\mathcal{W}$), as its real interest rate is essentially unchanged in the two cases for. In contrast, financial integration makes a substantial difference for the two small economies. As discussed, the evolution of demographic variables still exerts downward pressure on the real interest rate in both regions. In line with the closed economy results in Carvalho et al. (2016), the decline is less pronounced in country $\mathcal{Y}$ and more so in country $\mathcal{O}$. The main difference relative to the baseline simulation is that, with constant financial integration, the real interest rate of the old economy now falls monotonically, and also by more. As a result, the real interest rates in the two small economies

**Figure 6:** Real interest rates in two demographics counterfactuals.




*Note:* The figure plots the simulated real interest rate from the model for the global economy (blue lines), the young economy (red lines), and the old economy (black lines). The solid lines correspond to the baseline simulation. The dashed-dotted lines are the counterfactual simulations in which the growth rate of the working-age population (left-hand side) and the probability of surviving (right-hand side) remain at their initial values in each country.

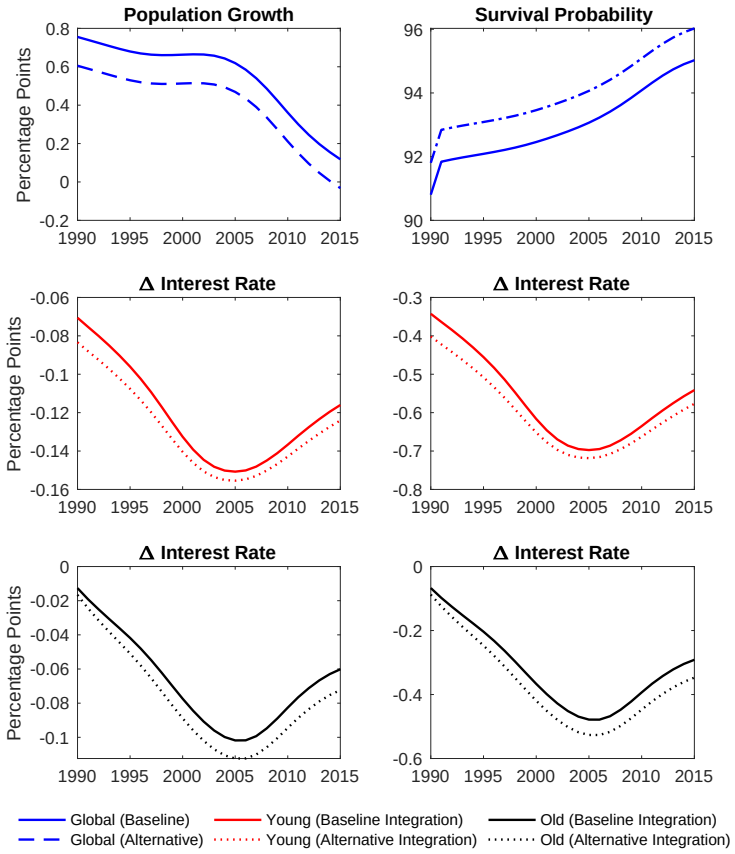
do not converge. With constant financial integration, domestic demographic developments dominate. Conversely, with increasing financial integration, global demographic trends become progressively more important over time.

The dashed-dotted lines in Figure 6 show the path of the real interest rate in the three regions in two counterfactual simulations. The left-hand side panel presents the result of the simulation in which we fix the growth rate of the working-age population at its initial value. The right-hand side panel displays the experiment in which we hold the probability of surviving constant at its initial value. Focusing on the left-hand side panel, the increase in life expectancy associated with the higher probability of surviving explains about 80% of the overall effect, depending on the region. Conversely, in the right-hand side panel, we can see that the lower growth rate of the labor force explains less than 20% of the overall effect.

The intuition for this decomposition is the same as in Carvalho et al. (2016). The increase in life expectancy induces households to save more in anticipation of a longer retirement period. This saving-for-retirement motive is stronger for workers, who face a longer expected lifespan, but also affects retirees since their life expectancy continues to increase even after leaving work.[17] The fall in the growth rate of the working-age population has only a modest effect on the real interest rate because two effects tend to counteract each other. On the one hand, a lower growth rate of the working-age population increases

[17] An increase in the retirement age would mitigate this effect. In many OECD countries, pension reforms are moving in this direction. In addition, people work for more years, even though the official retirement age has not changed (Scott, 2021). Yet, Carvalho et al. (2016) show that the increase in retirement age necessary to fully offset the consequences of higher life expectancy on the real interest rate is substantial—well above the changes currently being discussed and implemented in most countries.

**Figure 7:** Real interest rate sensitivity to alternative demographic profiles.




*Note:* Change in the real interest rate of the small economy (right) for baseline (solid red line) and high (dotted red line) level of financial integration in response to (i) a more pronounced decline of the growth rate of the labor force in the large economy (dashed blue line, top-left panel); and (ii) a more pronounced increase of the probability of surviving in the large economy (dashed-dotted blue line, bottom-left panel). The solid blue lines in the left column correspond to the baseline processes for demographic variables.

capital per-worker and thus tends to depress asset returns. On the other hand, the reduction in the number of workers implies a change in the composition of the population. Since retirees have a higher marginal propensity to consume, aggregate savings fall and the real interest rate rises. On balance, the first effect associated with the lower growth rate of the working-age population dominates in the simulation, but its quantitative implications are nonetheless small. As a result, the dynamics of real interest rates are dominated by the evolution of life expectancy.

### 3.3.1 Demographic Comparative Statics

In this section, we perform two comparative-static exercises to understand how global demographic developments affect the real interest rate in the two small economies, and how this effect depends on the degree of financial integration with the rest of the world. We perturb the path of one demographic variable at a time (growth rate of the labor force or probability of surviving) as to generate a parallel upward shift in the dependency ratio relative to the baseline by one percentage point, and assess the

impact of the change on the real interest rate of the three regions given different degrees of financial integration.

In the left column of Figure 7, we consider a downward shift of the growth rate of the working age population in the global economy by 15 basis points each year (dashed line) relative to the baseline scenario (solid line), displayed in the top-left panel. The middle-left panel reports the change of the real interest rate relative to the baseline scenario in the young economy. The solid line keeps the degree of financial integration at the same level as in the main simulation. The dotted line corresponds to a higher degree of financial integration (a 25% reduction of the portfolio-holding cost). The downward shift in the trajectory of global population growth leads to a lower real interest rate in the young economy, and this effect is more pronounced the more financially integrated the country is with the global economy. The bottom-left panel plots the same experiment for the old economy. In this case, the consequences of a downward shift of the growth rate of working-age population in the global economy for the domestic real interest rate are smaller. The reason is the old economy is less financially integrated with the global economy than the young one. As a consequence, its sensitivity to demographic developments in the rest of the world is lower.

The right column of Figure 7 shows the effects of an upward shift of the probability of surviving in the global economy by 1 percentage point each year (dashed-dotted line) relative to the baseline scenario (solid line), displayed in the top-right panel. As in the case of the growth rate of the working age population, the effect of a higher probability of surviving is stronger in the young economy than in the old economy, and is larger with higher financial integration.

Together with the main counterfactual analysis of Figure 5, both comparative static exercises in this section reinforce the message that global demographic developments influence the real rate of a small economy progressively more as its financial integration with the rest of the world increases. This point will serve as a guide to the empirical analysis that we present in Section 4.

### 3.3.2 Other Factors

The existing literature has identified a number of factors that contribute to explain the observed decline of global real interest rates over time. While our analysis focuses on demographic variables, our model is suitable to analyze the effects of at least a subset of these other potential drivers. We keep this analysis deliberately simple and only perform a few comparative statics exercises. Specifically, we modify the calibrated value of one factor at a time, and compare the resulting new steady state real interest rate with the one in the baseline simulation under the same demographic transition. The last two columns of Table 4 report the difference (in basis points) between the real interest rate in the alternative and in the baseline calibrations in 2020 for the young and old economy, respectively, in each of these comparative statics exercises.

The first row (TFP) shows the consequence of increasing TFP growth ($x$ in the model) from 0.5% as in the baseline calibration to 0.6%. The real interest rate increases by about 8 basis points in country $\mathcal{Y}$

**Table 4:** Real interest rate in 2020 relative to the baseline with different calibrations.

| **Factor** | **Baseline** | **Alternative** | $\mathbf{R}^{\mathbf{Alt}}_{\mathcal{Y},\mathbf{2020}} - \mathbf{R}^{\mathbf{Base}}_{\mathcal{Y},\mathbf{2020}}$ | $\mathbf{R}^{\mathbf{Alt}}_{\mathcal{O},\mathbf{2020}} - \mathbf{R}^{\mathbf{Base}}_{\mathcal{O},\mathbf{2020}}$ |
|---|---|---|---|---|
| TFP ($x$) | 0.5% | 0.6% | 8 | 10 |
| Debt/GDP ($b$) | 60% | 70% | 21 | 11 |
| Gov't Spending/GDP ($g$) | 25% | 26% | 32 | 17 |
| Pensions/GDP ($e$) | 7.5% | 8.5% | 66 | 39 |
| Age of Retirement ($\omega$) | 65 | 66 | 66 | 47 |

*Note:* For each experiment, the first column (Factor) reports the parameter that changes, the second (Baseline) the value in the baseline calibration, the third (Alternative) the value in the alternative calibration, the fourth ($R^{Alt}_{\mathcal{Y},2020} - R^{Base}_{\mathcal{Y},2020}$) the difference (in basis points) between the real interest rate in 2020 under the alternative calibration relative to the baseline in the young economy, and the fifth ($R^{Alt}_{\mathcal{O},2020} - R^{Base}_{\mathcal{O},2020}$) the difference (in basis points) between the real interest rate in 2020 under the alternative calibration relative to the baseline in the old economy.

and 10 basis points in country $\mathcal{O}$. The model-implied elasticity of the real interest rate to TFP growth is broadly consistent with some of the recent literature, for example, Holston et al. (2017) and Rachel and Smith (2017), that discusses the importance of the slowdown in trend GDP for the secular decline of the equilibrium real interest rate.

The next three rows (Debt/GDP, Pensions/GDP and Gov't Spending/GDP) correspond to changes in $b$, $e$, and $g$, respectively. In the case of debt, we consider a ten percentage point increase relative to the baseline, while for government spending and pensions the change is one percentage point. In all cases, a more expansionary fiscal policy causes a higher interest rate. A 10 percentage point increase in debt/GDP raises the real interest rate by 21 basis points in the small young economy and 11 in the small old economy. The effect of a one percentage point increase in government spending is slightly larger, 32 and 17 basis points, respectively. Finally, a one percentage point increase in pensions as a fraction of GDP causes a 66 basis points increase in the real interest rate of the young economy and 39 basis points for the old economy. Because of the life-cycle features of the model, government bonds are net wealth for the private sector. Therefore, a higher level of debt relative to GDP supports private sector's consumption, and thus contributes to increase the real interest rate. For government spending, the mechanism works through a crowding out of private consumption and investment. The increase in pensions has a large effect on the real interest rate because, effectively, the government transfers resources from agents with lower marginal propensity to consume (workers) to agents with a higher marginal propensity to consume (retirees). Overall, our results about fiscal policy are in line with the findings in Rachel and Summers (2019).

The last factor that we consider is the retirement age. In this experiment, we increase the parameter $\omega$ so that the average duration of employment is 46 years compared to 45 in the baseline. This exercise approximates the reforms that governments in many countries are implementing to make their public finances—and in particular their pension systems—more sustainable (OECD, 2019). The rise in interest rate in this case is 66 basis points in the young economy and 47 in the old. The intuition is that a

longer employment span reduces the incentives for workers to save during their working years, and thus diminishes the downward pressure on the real interest rate.

Overall, the message from these additional comparative statics exercises is that a rebound of TFP growth, a fiscal expansion, or an increase in the retirement age are all factors that could contribute to lift the real interest rate. While the exact magnitudes may depend on the details of the model, we take the direction of the effects as the main lesson to inform our empirical analysis below.

# 4 Empirical Analysis

The analysis presented in the previous section illustrates how demographic variables affect real interest rates in a world of imperfect capital mobility. In particular, the model shows that both domestic and global demographics affect the real interest rate of a given country. Furthermore, the relative importance of country-specific and global determinants varies with the degree of financial integration. The more financially integrated a country is, the higher the sensitivity of its real interest rate to global demographic developments. The process of financial integration, coupled with different initial conditions, may generate non-monotonic dynamics in the process of convergence towards the world real interest rate. Finally, size matters too. For given demographics, the larger the relative size of an economy is, the more domestic variables affect the equilibrium of the global economy.

These implications of the calibrated model serve as guidelines for our empirical analysis of the relationship between demographics and real interest rates. To this end, we exploit a panel of countries, thus leveraging variation both across countries and over time.

The model suggests a particular specification of the panel regressions. First, demographic trends should imply low-frequency movements in real interest rates. We take into account the long-run nature of this relationship by employing a panel error-correction model (panel ECM), which allows for cointegrating relationships between real interest rates and their determinants—in particular demographic variables. Second, the relative importance of domestic and global factors for a country's real interest rate should vary over time with the degree of financial integration. In light of this consideration, we interact global factors with a measure of the degree of financial openness that takes values between zero and one, and domestic factors with "1 - openness".

For the sake of parsimony, we summarize global factors with a measure of the foreign real interest rate faced by each country.[18] We then separately add country-specific demographic variables and other determinants of real rates, interacted with one minus the country's degree of openness. The resulting

[18] An alternative would be to include all determinants of global interest rates as regressors, interacted with openness. Due to the relatively small sample, we favor using the foreign real rate as a summary measure.

panel error-correction model (ECM) for country $m$ is

$$\Delta r_{m,t} = \alpha_m + \gamma r_{m,t-1} + \theta \Theta_{m,t-1} r^*_{m,t-1} + \sum_j \psi_j (1 - \Theta_{m,t-1}) D_{m,j,t-1} + \sum_k \Psi_k (1 - \Theta_{m,t-1}) X_{m,k,t-1}$$
$$+ \lambda \Delta(\Theta_{m,t} r^*_{m,t}) + \sum_j \phi_j \Delta[(1 - \Theta_{m,t}) D_{m,j,t}] + \sum_k \chi_k \Delta[(1 - \Theta_{m,t}) X_{m,k,t}] + \epsilon_{m,t}, \quad (15)$$

where $r_{m,t}$ is the ex-ante real interest rate, $r^*_{m,t}$ is the foreign real interest rate faced by the country, $\alpha_m$ is a country fixed effect, $\Theta_{m,t}$ is an index of financial openness, $D_{m,j,t}$ includes $j$ demographic variables (described below), $X_{m,k,t}$ collects $k$ other potential determinants of real interest rates, and $\Delta$ is the first-difference operator. Based on this specification, the estimated long-run effects of each variable on interest rate corresponds to its estimated coefficient ($\hat{\theta}$, $\hat{\psi}_j$ or $\hat{\Psi}_k$) divided by $-\hat{\gamma}$. The next section explains how we construct the variables in the regression.

### 4.1 Data and Results

We estimate the panel ECM in equation (15) using annual data for a set of 19 OECD countries. Our sample covers the period 1979-2019 and includes Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Spain, Sweden, Switzerland, the United Kingdom, and the United States.[19] As Figure 1 shows, starting in 1979 allows us to include in the sample a decade during which real interest rates increased, so that spurious correlations of purely trending variables should not contaminate the empirical analysis.

We rely on different data sources to construct the variables that enter our regression. The ex-ante short-term real interest rate $r_{m,t}$ equals the difference between the time-$t$ short-term nominal rate $i_{m,t}$ and one-period-ahead expected inflation $\mathbb{E}_t \pi_{m,t+1}$. The data for short-term nominal interest rate are either overnight or three-month official rates (see Table A1 for details). To construct expected inflation, we follow the approach in Hamilton et al. (2016) and calculate the one-year-ahead forecast from AR(1) regressions with rolling windows of 20 years, that is, we estimate the regression $\pi_{m,t} = a_m + b_m \pi_{m,t-1} + \varepsilon_{m,t}$ with OLS so that $\mathbb{E}_t \pi_{m,t+1} = \hat{a}_m + \hat{b}_m \pi_{m,t}$. For all countries, we use headline CPI inflation, from the OECD.

Using data from Lane and Milesi-Ferretti (2017), we build a measure of financial integration of country $m$ ($LMF_{m,t}$) as the sum of financial assets and liabilities expressed as a fraction of GDP. The index of financial openness that we use in the regression ($\Theta_{m,t}$) to weight the global interest rate is a transformation of $LMF_{m,t}$

$$\Theta_{m,t} = \frac{LMF_{m,t}}{100 + LMF_{m,t}},$$

which makes $\Theta_{m,t}$ an index between zero and one, consistent with the specification of equation (15).

The global real interest rate that an individual country faces is a weighted average of all other countries'

[19] The sample excludes countries that experienced episodes of high inflation (above 25% in a given year) between 1970 and 2019.

real interest rates, where the weight associated with each country is the working-age population ($POP_{m,t}$) share adjusted by the index of financial openness

$$r^*_{m,t} = \sum_{\ell \neq m} \left( \frac{\Theta_{\ell,t} POP_{\ell,t}}{\sum_{\ell \neq m} \Theta_{\ell,t} POP_{\ell,t}} \right) r_{\ell,t}.$$

The vector $D_{m,j,t}$ contains two demographic variables: life expectancy ($LE_{m,t}$) and the growth rate of working-age population ($DPOP_{m,t}$). The data source for both demographic variables is the United Nations World Population Prospects 2019. Life expectancy (measured at 20) comes straight from the database. We use the data on the number of individuals between 20 and 65 years old ($POP_{m,t}$) to construct the growth rate of working-age population, given by

$$100 \times \left( \frac{POP_{m,t}}{POP_{m,t-1}} - 1 \right).$$

The model introduced in Section 2 isolates the effects of demographic trends on real interest rates given financial openness. In practice, as discussed in Section 3.3.2, a growing literature has identified a number of other forces at play that may contribute to explain the dynamics of real interest rates. At the end of Section 3, we have discussed a subset of these forces that constitute parameters in our model through simple comparative static exercises. The vector $X_{m,k,t}$ in the panel ECM (15) includes these and other factors.

Following Holston et al. (2017), the first variable that we consider is TFP growth, which we obtain from the Penn World Tables. Secondly, in line with the analysis in Rachel and Smith (2017) on the role of fiscal policy, we add government debt as a fraction of GDP (from the AMECO database). Still in relation to fiscal policy, albeit on a different dimension, we also include data on pension spending as a fraction of GDP (from the OECD database) as a separate potential explanatory variable. Papetti (2021) explores the possibility that the generosity of pensions may affect the real interest rate by reducing the incentive of workers to save while employed. Third, we consider versions of our regressions augmented with convenience yields obtained from Del Negro et al. (2019).[20] Since convenience yields are only available for seven major advanced economies, these regressions have a smaller sample size. Finally, Mian et al. (2021) have recently highlighted the potential importance of widening inequality in explaining the secular decline of real interest rates. To account for this potential explanation, we also control for the Gini coefficient of each country, which we obtain from the World Bank.

Table 5 presents the main results from the estimation of equation (15). For each variable, we report the estimated long-run coefficients from the panel ECM, that is, $\hat{\theta}$, $\hat{\psi}_j$, and $\hat{\Psi}_k$ divided by $-\hat{\gamma}$. Standard errors are clustered at the country level.

Column (1) considers a basic specification that only includes the demographic variables. The global real interest rate is statistically significant at the 1% level, with a point estimate just below 0.7. Life

[20] We thank the authors for kindly sharing their series for Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States.

**Table 5:** Panel Error Correction Model (ECM)

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| Global Rate | 0.68*** | 0.66*** | 0.70*** | 0.74*** | 1.00*** | 1.48*** |
| | (0.17) | (0.17) | (0.13) | (0.14) | (0.20) | (0.20) |
| Life Expectancy | 0.14*** | 0.14*** | -0.24*** | -0.17 | -0.36*** | -0.54* |
| | (0.04) | (0.04) | (0.06) | (0.19) | (0.09) | (0.28) |
| Growth Rate of Labor Force | 0.24 | 0.30 | 6.03*** | 6.12*** | 8.95*** | 11.59*** |
| | (1.02) | (1.01) | (0.98) | (1.10) | (1.51) | (1.49) |
| TFP Growth | | 0.49 | 0.02 | -0.14 | -0.02 | -0.01 |
| | | (0.34) | (0.30) | (0.37) | (0.39) | (0.41) |
| Government Debt | | | 0.03 | 0.01 | 0.07** | 0.10** |
| | | | (0.02) | (0.03) | (0.03) | (0.04) |
| Pension Spending | | | 2.31*** | 2.11*** | 2.12*** | 2.65*** |
| | | | (0.41) | (0.59) | (0.53) | (0.80) |
| Gini Coefficient | | | | -0.05 | | -0.03 |
| | | | | (0.23) | | (0.33) |
| Convenience Yield | | | | | 0.67 | 1.99 |
| | | | | | (1.35) | (1.68) |
| R-Squared | 0.24 | 0.24 | 0.39 | 0.36 | 0.53 | 0.55 |
| Adjusted R-Squared | 0.21 | 0.22 | 0.35 | 0.31 | 0.48 | 0.48 |
| Observations | 743 | 743 | 505 | 445 | 206 | 169 |
| Clusters | 19 | 19 | 19 | 18 | 7 | 7 |

*Note:* Results from the estimation of equation (15). Robust standard errors, reported in parenthesis, are clustered at the country level.

expectancy is statistically significant but enters with the wrong sign compared to the predictions of the model. The growth rate of working-age population has the right sign but is not statistically significant. Column (2) adds TFP growth, which is not statistically significant and does not meaningfully alter the results, although the sign of the coefficient is consistent with the prediction of our model. The picture changes once we control for fiscal variables (government debt and pension spending) in column (3). Pension spending, in particular, is significant at the 1% level. The sign is in line with the prediction of the model and the point estimate is economically large. More importantly, both life expectancy and the growth rate of the labor force now become significant at the 1% level and have the sign predicted by the model. Columns (4) and (5) introduce the Gini coefficient and the convenience yield, respectively. While the point estimates of the coefficients on these two variables are in line with the literature, neither is statistically significant. With the inclusion of the Gini coefficient, life expectancy loses statistical significance, although the sign and magnitude of the coefficient remains comparable to column (3). The inclusion of convenience yields significantly reduces the number of observations, due to a smaller number of countries. The results, however, remain comparable to those obtained in column (3). The coefficient

on life expectancy is negative and again statistically significant at the 1% level. In addition, government debt also becomes significant (at the 5% level), with a coefficient of the expected sign. Finally, column (6) reports the regression with both the Gini coefficient and the convenience yield, which are again not statistically significant. In this case, the results are largely comparable to column (5). The main difference is that the coefficient on life expectancy is larger in magnitude, although statistically significant only at the 10% level.

Because variables are interacted with financial openness, one can think of the estimated coefficients as pertaining to the interacted series, or, alternatively, as country-specific and time-varying coefficients that apply to the original (*i.e.* non-interacted) variables. Here we illustrate the latter interpretation with some simple calculations. To that end, we multiply every coefficient by the cross-country time average of $1 - \Theta_t$, except for the coefficient on the global rate, which should be multiplied by the average value of $\Theta_t$. Across the six regression specifications considered in Table 5, $\Theta_t$ ranges from 0.66 to 0.76. As a result, a one percentage point change in the global real interest rate is associated with a change of the domestic real rate between 45 and 113 basis points. In column (3) to (6), the effect of life expectancy ranges between 6 and 13 basis points and the effect of the growth rate of working-age population falls between 205 and 278 basis points. This effect may also account for why the point estimates become larger as we move to richer specifications in the last columns, given that the countries for which we have data on convenience yields are more financially integrated.

On balance, the panel error correction model suggests a stable link between the global rate and domestic real interest rates. Once we control for fiscal variables, the coefficients on life expectancy and the growth rate of working-age population become significant, in line with the predictions of the model, and broadly consistent across specifications.[21] TFP growth is never statistically significant, while government debt becomes significant only once we also add the convenience yield. Pension spending is highly significant in all specifications. Lastly, neither the Gini coefficient nor the convenience yield are statistically significant, whether included separately or together.[22]

Between 1990 and 2019, the median real interest rate in our sample went from 5.5% to -1.5%. In the data, the growth rate of working-age population for the median country in our sample fell from 0.52% in 1990 to 0.26% in 2020. Therefore, the regressions suggest that the contribution of this factor to the decline of the real interest rate for the median country in the data is between 53 and 72 basis points over the sample period. Over the same time span, median life expectancy at 20 increased by five years (from 57.6 to 62.6), which implies a total effect on the real interest rate between 30 and 65 basis points. Therefore, the mid-range of the estimated effects for demographic variables (63 basis points for the growth rate of working age population and 48 basis points for life expectancy) accounts for about 110 basis points of

[21] We have also experimented with a number of additional specifications that include only a subset of the control variables at a time. While the results are broadly robust, we have found cases in which statistical significance, or even the sign, of the coefficients changes. In general, controlling for both government debt and pension spending is important for the statistical significance and the sign of the coefficient on life expectancy, while introducing even just one of those two variables is sufficient to make the coefficient on the growth rate of working age population positive and statistically significant.

[22] The convenience yield is statistically significant if introduced without fiscal variables. In this case, all other variables are not statistically significant, except for life expectancy, which however enters with the wrong sign.

**Table 6:** Panel ECM without interaction.

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| Global Rate | 0.32** | 0.30** | -0.23 | -0.36*** | -0.24 | -0.21 |
| | (0.13) | (0.13) | (0.15) | (0.13) | (0.21) | (0.17) |
| Life Expectancy | -0.76*** | -0.72*** | -1.49*** | -1.43*** | -0.95*** | -1.07*** |
| | (0.14) | (0.13) | (0.16) | (0.16) | (0.30) | (0.31) |
| Growth Rate of Labor Force | -0.58 | -0.41 | 0.58 | 0.04 | 1.64** | 1.16* |
| | (0.35) | (0.36) | (0.41) | (0.36) | (0.74) | (0.63) |
| TFP Growth | | 0.31** | 0.24* | 0.05 | 0.34* | 0.16 |
| | | (0.14) | (0.14) | (0.14) | (0.20) | (0.19) |
| Government Debt | | | 0.01 | -0.02** | 0.01 | -0.02* |
| | | | (0.01) | (0.01) | (0.01) | (0.01) |
| Pension Spending | | | 0.47** | -0.03 | 0.71** | 0.05 |
| | | | (0.21) | (0.20) | (0.28) | (0.26) |
| Gini Coefficient | | | | 0.08 | | -0.00 |
| | | | | (0.09) | | (0.14) |
| Convenience Yield | | | | | -1.90** | -0.53 |
| | | | | | (0.85) | (0.78) |
| R-Squared | 0.24 | 0.24 | 0.30 | 0.31 | 0.50 | 0.54 |
| Adjusted R-Squared | 0.21 | 0.21 | 0.25 | 0.26 | 0.44 | 0.46 |
| Observations | 743 | 743 | 505 | 445 | 206 | 169 |
| Clusters | 19 | 19 | 19 | 18 | 7 | 7 |

*Note:* Results from the estimation of equation (16). Robust standard errors, reported in parenthesis, are clustered at the country level.

the decline of the real interest rate between 1990 and 2020. Empirically, the total effect of the decline in the growth rate of the working-age population is slightly larger than the effect of the increase in life expectancy.

Among the other controls, pension spending is the variable that is consistently significant across the various specifications. According to the panel ECM, a one percent increase in pension spending is associated with an increase of 64 and 72 basis points to the real interest rate. Interestingly, government debt becomes significant only with the smaller sample due to the limited availability of data for convenience yields. One possible explanation is that real interest rates are more sensitive to government debt for countries with liquid bond markets. In those cases, a 10 percentage point increase in government debt is associated with an increase of 17 to 24 basis points in the real interest rate.[23]

Table 6 reports the results for the same specifications as in Table 5, but without interacting the

[23]The implied sensitivity of the real interest rate to pension spending and government debt is in line with the comparative static exercises in section 3.3.2 for the small young economy in the model.

explanatory variables with the degree of financial openness:

$$\Delta r_{m,t} = \alpha_m + \gamma r_{m,t-1} + \theta r^*_{m,t-1} + \sum_j \psi_j D_{m,j,t-1} + \sum_k \Psi_k X_{m,k,t-1} + \lambda \Delta r^*_{m,t} + \sum_j \phi_j \Delta D_{m,j,t} + \sum_k \chi_k \Delta X_{m,k,t} + \epsilon_{m,t}. \quad (16)$$

In this case, for most variables the values and statistical significance of the associated coefficients are less stable across specifications. Life expectancy is the only variable that is consistently significant across specifications, with the correct sign. The global rate ceases to be statistically significant once we add fiscal variables and the convenience yield. When we include the Gini coefficient, the global rate is significant but with a negative sign. The growth rate of the labor force is significant only in two out of the six specifications. Among the control variables, TFP growth is significant in three out of five specifications, at the 5% level. Pension spending is significant in two out of four specifications and the convenience yield in one out of two. The good news is that the sign of the coefficient of these two variables is in line with the theoretical prediction of our model and the literature. Conversely, government debt is significant in two out of four specifications, but with the wrong sign. The Gini coefficient remains insignificant as in Table 5.

Overall, the comparison between the results reported in Tables 5 and 6 further highlights the importance of taking the degree of financial integration into account, as suggested by our model.

## 5 Conclusions

The demographic trends that most advanced economies are undergoing are a natural explanation for the prolonged decline of global real interest rates observed between 1990 and 2020. In this paper, we have explored the interaction of these trends with the process of increasing financial integration that took place globally over the same period. First, we have developed a multicountry, general-equilibrium model with imperfect capital mobility and differential demographic trends. A calibrated three-country version of the model highlights how low-frequency movements in a country's real interest rate depend on its own as well as on global demographic developments. The weight on global demographic variables is increasing in the degree of global financial integration. Conversely, domestic demographic developments matter less for the real interest rate of a highly financially integrated country. Drawing on the lessons from the model, we have then estimated several specifications of a panel error-correction model that relates real interest rates to demographic variables and other possible drivers, interacted with a measure of financial integration. A "world" real interest rate, which summarizes global factors, is consistently significant in all specifications. Nevertheless, domestic demographic variables remain important determinants of real interest rates, along with pension spending.

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# Appendix

## A Derivations

This section presents the derivations of retirees' and workers' problems.

### A.1 Retirees

Retirees maximize (2) subject to (3). After substituting the constraint into the objective function, we can rewrite the unconstrained maximization problem as

$$V_{mt}^r = \max_{\{A_{m\ell t}^r\}_{\ell=1}^{\mathcal{M}}} \left\{ \left[ \frac{1}{\gamma_{mt}} \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1}^r + E_{mt}^r - \left( 1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t} \left( \eta_{m\ell t}^r - \bar{\eta}_{nm} \right)^2}{2} \right) \sum_{\ell=1}^{\mathcal{M}} A_{m\ell t}^r \right]^{\frac{\sigma-1}{\sigma}} + \gamma_{mt+1} \beta_m (V_{mt+1}^r)^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}}.$$

The first-order condition with respect to foreign assets ($A_{mpt}^r,\ p \neq m$) is

$$\left[ \left( 1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} \left( \eta_{m\ell t}^r - \bar{\eta}_{m\ell} \right)^2 \right) - \sum_{\ell=1}^{\mathcal{M}} \Lambda_{m\ell t} \left( \eta_{m\ell t}^r - \bar{\eta}_{m\ell} \right) \eta_{m\ell t}^r + \Lambda_{mpt} \left( \eta_{mpt}^r - \bar{\eta}_{mp} \right) \left( 1 - \eta_{mpt}^r \right) \right] \left( C_{mt}^r \right)^{-\frac{1}{\sigma}} = \beta_m \gamma_{mt+1} \left( V_{mt+1}^r \right)^{-\frac{1}{\sigma}} \frac{\partial V_{mt+1}^r}{\partial A_{mpt}^r},$$

while the first-order condition with respect to domestic assets ($A_{mmt}^r$) is

$$\left[ \left( 1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} \left( \eta_{m\ell t}^r - \bar{\eta}_{m\ell} \right)^2 \right) - \sum_{\ell=1}^{\mathcal{M}} \Lambda_{m\ell t} \left( \eta_{m\ell t}^r - \bar{\eta}_{m\ell} \right) \eta_{m\ell t}^r \right] \left( C_{mt}^r \right)^{-\frac{1}{\sigma}} = \beta_m \gamma_{mt+1} \left( V_{mt+1}^r \right)^{-\frac{1}{\sigma}} \frac{\partial V_{mt+1}^r}{\partial A_{mmt}^r}.$$

By the Envelope Theorem, the partial derivatives above are

$$\frac{\partial V_{mt+1}^r}{\partial A_{mpt}^r} = \left( V_{mt+1}^r \right)^{\frac{1}{\sigma}} \left( C_{mt+1}^r \right)^{-\frac{1}{\sigma}} \frac{R_{pt}}{\gamma_{mt+1}}, \quad \forall\ p = 1, ..., \mathcal{M}. \tag{A.1}$$

Substituting (A.1) into the first-order conditions above gives

$$\left[ \left( 1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} \left( \eta_{m\ell t}^r - \bar{\eta}_{m\ell} \right)^2 \right) - \sum_{\ell=1}^{\mathcal{M}} \Lambda_{m\ell t} \left( \eta_{m\ell t}^r - \bar{\eta}_{m\ell} \right) \eta_{m\ell t}^r + \Lambda_{mpt} \left( \eta_{mpt}^r - \bar{\eta}_{mp} \right) \left( 1 - \eta_{mpt}^r \right) \right] \left( C_{mt}^r \right)^{-\frac{1}{\sigma}} = \beta_m R_{pt} (C_{mt+1}^r)^{-\frac{1}{\sigma}}, \tag{A.2}$$

and

$$\left[\left(1+\sum_{\ell=1}^{\mathcal{M}}\frac{\Lambda_{m\ell t}}{2}\left(\eta^r_{m\ell t}-\bar{\eta}_{m\ell}\right)^2\right)-\sum_{\ell=1}^{\mathcal{M}}\Lambda_{m\ell t}\left(\eta^r_{m\ell t}-\bar{\eta}_{m\ell}\right)\eta^r_{m\ell t}\right]\left(C^r_{mt}\right)^{-\frac{1}{\sigma}}=\beta_m R_{mt}(C^r_{mt+1})^{-\frac{1}{\sigma}}. \quad \text{(A.3)}$$

Dividing (A.2) by (A.3) and rearranging yields:

$$\left[1+\sum_{\ell=1}^{\mathcal{M}}\frac{\Lambda_{m\ell t}}{2}\left(\eta^r_{m\ell t}-\bar{\eta}_{m\ell}\right)^2\right]\left(R_{pt}-R_{mt}\right)=\Lambda_{mp}\left(\eta^r_{mpt}-\bar{\eta}_{mp}\right)R_{mt},$$

which correspond to equation (4) in the main text.

Next, if we multiply equation (A.2) by $\eta^r_{mmt}$ and equation (A.3) by $\sum_{\ell=1}^{\mathcal{M}}\eta_{m\ell t}\eta^r_{m\ell t}$, and we add them up, we obtain the Euler equation for the optimal path of consumption of retirees

$$C^r_{mt+1}=\left[\frac{\beta_m\sum_{\ell=1}^{n}\eta^r_{m\ell t}R_{\ell t}}{1+\sum_{\ell=1}^{\mathcal{M}}\frac{\Lambda_{m\ell t}}{2}\left(\eta^r_{m\ell t}-\bar{\eta}_{m\ell}\right)^2}\right]^{\sigma}C^r_{mt}. \quad \text{(A.4)}$$

In order to find the difference equation for the marginal propensity to consume out of wealth for retirees, we substitute the retirees budget constraint (3) into the policy function (5). After rearranging, we obtain

$$\frac{1-\xi^r_{mt}}{\xi^r_{mt}}C^r_{mt}=\left[1+\sum_{\ell=1}^{\mathcal{M}}\frac{\Lambda_{m\ell t}}{2}\left(\eta^r_{m\ell t}-\bar{\eta}_{m\ell}\right)^2\right]\sum_{\ell=1}^{\mathcal{M}}A^r_{m\ell t}+\frac{\gamma_{mt+1}S^r_{mt+1}}{\tilde{R}_{mt}},$$

where the present discounted value of pension benefits to retirees $S^r_{mt}$ and the adjusted return $\tilde{R}_{mt}$ are defined in the text. Replacing for current consumption from the Euler equation (A.4) , we obtain

$$\frac{1-\xi^r_{mt}}{\xi^r_{mt}}C^r_{mt+1}(\beta_m\tilde{R}_{mt})^{-\sigma}=\left[1+\sum_{\ell=1}^{\mathcal{M}}\frac{\Lambda_{m\ell t}}{2}\left(\eta^r_{m\ell t}-\bar{\eta}_{m\ell}\right)^2\right]\sum_{\ell=1}^{\mathcal{M}}A^r_{m\ell t}+\frac{\gamma_{mt+1}S^r_{mt+1}}{\tilde{R}_{mt}}$$

Finally, we can substitute the guess of the consumption function at $t+1$ for $C^r_{mt+1}$ to obtain

$$\frac{1-\xi^r_{mt}}{\xi^r_{mt}}\xi^r_{mt+1}\left(\frac{1}{\gamma_{mt+1}}\sum_{\ell=1}^{\mathcal{M}}R_{\ell t}A^r_{m\ell t}+S^r_{mt+1}\right)(\beta_m\tilde{R}_{mt})^{-\sigma}$$
$$=\left[1+\sum_{\ell=1}^{\mathcal{M}}\frac{\Lambda_{m\ell t}}{2}\left(\eta^r_{m\ell t}-\bar{\eta}_{m\ell}\right)^2\right]\sum_{\ell=1}^{\mathcal{M}}A^r_{m\ell t}+\frac{\gamma_{mt+1}S^r_{mt+1}}{\tilde{R}_{mt}}$$

Dividing by $\sum_{\ell=1}^{\mathcal{M}}A^r_{m\ell t}$ and using the definition of $\eta_{mpt}$ allows us to obtain (7) in the text.

Finally, we guess and verify that the value function is linear in the level of consumption:

$$V^r_{mt}=\Delta^r_{mt}C^r_{mt}.$$

Substituting the guess into the functional equation (2) together with the Euler equation (A.4) to eliminate

$C^r_{mt+1}$, we obtain

$$\Delta^r_{mt}C^r_{mt} = \left[(C^r_{mt})^{\frac{\sigma-1}{\sigma}} + \beta_m\gamma_{mt+1}\left(\Delta^r_{mt+1}\right)^{\frac{\sigma-1}{\sigma}}(\beta_m\tilde{R}_{mt})^{\sigma-1}(C^r_{mt})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}.$$

We can simplify the last expression by eliminating the terms in $C^r_{mt}$. After rearranging, we obtain

$$\frac{1}{(\Delta^r_{mt})^{\frac{\sigma-1}{\sigma}}} = 1 - \gamma_{mt+1}\beta^\sigma_m\tilde{R}^{\sigma-1}_{mt}\left(\frac{\Delta^r_{mt+1}}{\Delta^r_{mt}}\right)^{\frac{\sigma-1}{\sigma}}. \tag{A.5}$$

Comparing (A.5) with the difference equation for the marginal propensity to consume (7), we can see that

$$\Delta^r_{mt} = (\xi^r_{mt})^{-\frac{\sigma}{\sigma-1}}.$$

## A.2 Workers

The workers' problem is to maximize (8) subject to (9). After substituting the constraint into the objective, the unconstrained maximization problem becomes

$$V^w_{mt} = \max_{\{A^w_{m\ell t}\}_{\ell=1}^{\mathcal{M}}}\left\{\left[\sum_{\ell=1}^{\mathcal{M}}R_{\ell t-1}A^w_{m\ell t-1} + W^w_{mt} - T^w_{mt} - \left(1+\sum_{\ell=1}^{\mathcal{M}}\frac{\Lambda_{m\ell t}}{2}\left(\eta^w_{m\ell t}-\bar{\eta}_{nm}\right)^2\right)\sum_{\ell=1}^{\mathcal{M}}A^w_{m\ell t}\right]^{\frac{\sigma-1}{\sigma}}\right.$$
$$\left. + \beta_m\left[\omega_{mt+1}V^w_{mt+1} + (1-\omega_{mt+1})V^{rt+1}_{mt+1}\right]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}}.$$

The first-order condition with respect to country-$p$ assets (with $p \neq m$) is

$$\left[\left(1+\sum_{\ell=1}^{\mathcal{M}}\frac{\Lambda_{m\ell t}}{2}\left(\eta^w_{m\ell t}-\bar{\eta}_{m\ell}\right)^2\right) - \sum_{\ell=1}^{\mathcal{M}}\Lambda_{m\ell t}\left(\eta^w_{m\ell t}-\bar{\eta}_{m\ell}\right)\eta^w_{m\ell t} + \Lambda_{mpt}\left(\eta^w_{mpt}-\bar{\eta}_{mp}\right)\left(1-\eta^w_{mpt}\right)\right](C^w_{mt})^{-\frac{1}{\sigma}}$$
$$= \beta_m\left[\omega_{mt+1}V^w_{mt+1} + (1-\omega_{mt+1})V^r_{mt+1}\right]^{-\frac{1}{\sigma}}\left[\omega_{mt+1}\frac{\partial V^w_{mt+1}}{\partial A^w_{mpt}} + (1-\omega_{mt+1})\frac{\partial V^r_{mt+1}}{\partial A^r_{mpt}}\right], \tag{A.6}$$

while the first-order condition with respect to domestic assets is

$$\left[\left(1+\sum_{\ell=1}^{\mathcal{M}}\frac{\Lambda_{m\ell t}}{2}\left(\eta^w_{m\ell t}-\bar{\eta}_{m\ell}\right)^2\right) - \sum_{\ell=1}^{\mathcal{M}}\Lambda_{m\ell t}\left(\eta^w_{m\ell t}-\bar{\eta}_{m\ell}\right)\eta^w_{m\ell t}\right](C^w_{mt})^{-\frac{1}{\sigma}}$$
$$= \beta_m\left[\omega_{mt+1}V^w_{mt+1} + (1-\omega_{mt+1})V^r_{mt+1}\right]^{-\frac{1}{\sigma}}\left[\omega_{mt+1}\frac{\partial V^w_{mt+1}}{\partial A^w_{mmt}} + (1-\omega_{mt+1})\frac{\partial V^r_{mt+1}}{\partial A^r_{mmt}}\right]. \tag{A.7}$$

As for retirees, we use the Envelope Theorem to calculate the partial derivatives above

$$\frac{\partial V^w_{mt+1}}{\partial A^w_{mpt}} = \left(V^w_{mt+1}\right)^{\frac{1}{\sigma}}\left(C^w_{mt+1}\right)^{-\frac{1}{\sigma}}R_{pt}. \tag{A.8}$$

To solve the workers' problem, we need to guess the functional form of the value function at this stage. Like for retirees, we conjecture that the value function is linear in consumption and the slope is the same function of the marginal propensity to consume

$$V_{mt}^{w} = \Delta_{mt}^{w} C_{mt}^{w}, \quad \text{with} \quad \Delta_{mt}^{w} = \left(\xi_{mt}^{w}\right)^{-\frac{\sigma}{\sigma-1}}. \tag{A.9}$$

By substituting equation (A.8) and the guess (A.9) into equation (A.7) we get

$$\begin{aligned}
&\left[\left(1+\sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2}\left(\eta_{m\ell t}^{w}-\bar{\eta}_{m\ell}\right)^{2}\right)-\sum_{\ell=1}^{\mathcal{M}} \Lambda_{m\ell t}\left(\eta_{m\ell t}^{w}-\bar{\eta}_{m\ell}\right)\eta_{m\ell t}^{w}\right]\left(C_{mt}^{w}\right)^{-\frac{1}{\sigma}} \\
&= \beta_m\left[\omega_{mt+1}\Delta_{mt+1}^{w}C_{mt+1}^{w}+(1-\omega_{mt+1})\Delta_{mt+1}^{r}C_{mt+1}^{r}\right]^{-\frac{1}{\sigma}}\left[\omega_{mt+1}\left(\Delta_{mt+1}^{w}\right)^{\frac{1}{\sigma}}+(1-\omega_{mt+1})\left(\Delta_{mt+1}^{r}\right)^{\frac{1}{\sigma}}\right].
\end{aligned} \tag{A.10}$$

Multiplying both sides of (A.10) by $(\Delta_{mt+1}^{w})^{\frac{1}{\sigma}}$ and rearranging yields

$$\begin{aligned}
&\left[\omega_{mt+1}C_{mt+1}^{w}+(1-\omega_{mt+1})\frac{\Delta_{mt+1}^{r}}{\Delta_{mt+1}^{w}}C_{mt+1}^{r}\right]^{\frac{1}{\sigma}} \\
&\qquad = \frac{\beta_m\left[\omega_{mt+1}+(1-\omega_{mt+1})\left(\frac{\Delta_{mt+1}^{r}}{\Delta_{mt+1}^{w}}\right)^{\frac{1}{\sigma}}\right]R_{mt}\left(C_{mt}^{r}\right)^{\frac{1}{\sigma}}}{\left[1+\sum_{\ell=1}^{\mathcal{M}}\frac{\Lambda_{m\ell t}}{2}\left(\eta_{m\ell t}^{w}-\bar{\eta}_{m\ell}\right)^{2}\right]-\sum_{\ell=1}^{\mathcal{M}}\Lambda_{m\ell t}\left(\eta_{m\ell t}^{w}-\bar{\eta}_{m\ell}\right)\eta_{m\ell t}^{w}}.
\end{aligned}$$

Using the solution for the value function of retirees and the guess for the value function of workers, we can rewirte the last expression as

$$\begin{aligned}
&\left[\omega_{mt+1}C_{mt+1}^{w}+(1-\omega_{mt+1})\left(\frac{\xi_{mt+1}^{r}}{\xi_{mt+1}^{w}}\right)^{\frac{\sigma}{1-\sigma}}C_{mt+1}^{r}\right]^{\frac{1}{\sigma}} \\
&\qquad = \frac{\beta_m\left[\omega_{mt+1}+(1-\omega_{mt+1})\left(\frac{\xi_{mt+1}^{r}}{\xi_{mt+1}^{w}}\right)^{\frac{1}{1-\sigma}}\right]R_{mt}\left(C_{mt}^{r}\right)^{\frac{1}{\sigma}}}{\left[1+\sum_{\ell=1}^{\mathcal{M}}\frac{\Lambda_{m\ell t}}{2}\left(\eta_{m\ell t}^{w}-\bar{\eta}_{m\ell}\right)^{2}\right]-\sum_{\ell=1}^{\mathcal{M}}\Lambda_{m\ell t}\left(\eta_{m\ell t}^{w}-\bar{\eta}_{m\ell}\right)\eta_{m\ell t}^{w}}.
\end{aligned} \tag{A.11}$$

Following the same steps for equation (A.6), we obtain

$$\begin{aligned}
&\left[\omega_{mt+1}C_{mt+1}^{w}+(1-\omega_{mt+1})\left(\frac{\xi_{mt+1}^{r}}{\xi_{mt+1}^{w}}\right)^{\frac{\sigma}{1-\sigma}}C_{mt+1}^{r}\right]^{\frac{1}{\sigma}} = \\
&\qquad \frac{\beta_m\left[\omega_{mt+1}+(1-\omega_{mt+1})\left(\frac{\xi_{mt+1}^{r}}{\xi_{mt+1}^{w}}\right)^{\frac{1}{1-\sigma}}\right]R_{pt}\left(C_{mt}^{r}\right)^{\frac{1}{\sigma}}}{\left[1+\sum_{\ell=1}^{\mathcal{M}}\frac{\Lambda_{m\ell t}}{2}\left(\eta_{m\ell t}^{w}-\bar{\eta}_{m\ell}\right)^{2}\right]-\sum_{\ell=1}^{\mathcal{M}}\Lambda_{m\ell t}\left(\eta_{m\ell t}^{w}-\bar{\eta}_{m\ell}\right)\eta_{m\ell t}^{w}+\Lambda_{mp}(\eta_{mpt}^{w}-\bar{\eta}_{mp})}.
\end{aligned} \tag{A.12}$$

Dividing equation (A.11) by equation (A.12) shows that workers choose asset shares according to the

same condition as retirees

$$\left[1+\sum_{\ell=1}^{\mathcal{M}}\frac{\Lambda_{m\ell t}}{2}\left(\eta^w_{m\ell t}-\bar{\eta}_{m\ell}\right)^2\right]\left(R_{pt}-R_{mt}\right)=\Lambda_{mpt}\left(\eta^w_{mpt}-\bar{\eta}_{mp}\right)R_{mt},$$

which implies $\eta^w_{mpt}=\eta^r_{mpt}=\eta_{mpt}\ \forall p$.

We can find the Euler equation for workers' consumption following the same steps we did for retirees and get

$$\omega_{mt+1}C^w_{mt+1}+(1-\omega_{mt+1})\left(\frac{\xi^r_{mt+1}}{\xi^w_{mt+1}}\right)^{\frac{\sigma}{1-\sigma}}C^r_{mt+1}=\left(\beta_m\Omega_{mt+1}\tilde{R}_{mt}\right)^{\sigma}C^r_{mt}, \tag{A.13}$$

with $\Omega_{mt}$ defined in (11) in the text.

Next, we substitute the guesses for the policy functions, (5) and (10), for $C^r_{mt+1}$ and $C^w_{mt+1}$, respectively, in (A.13) to obtain

$$\begin{aligned}\omega_{mt+1}\xi^w_{mt+1}\left(\sum_{\ell=1}^{\mathcal{M}}R_{\ell t}A^w_{m\ell t}+H^w_{mt+1}+Z^w_{mt+1}\right)+(1-\omega_{mt+1})\left(\frac{\xi^r_{mt+1}}{\xi^w_{mt+1}}\right)^{\frac{1}{1-\sigma}}\xi^w_{mt+1}\left(\sum_{\ell=1}^{\mathcal{M}}R_{\ell t}A^r_{m\ell t}+S^r_{mt+1}\right)\\ =\left(\beta_m\Omega_{mt+1}\tilde{R}_{mt}\right)^{\sigma}\xi^w_{mt}\left(\sum_{\ell=1}^{\mathcal{M}}R_{\ell t-1}A^w_{m\ell t-1}+H^w_{mt}+Z^w_{mt}\right).\end{aligned}$$

Dividing this expression by $\xi^w_{mt+1}$ and $\sum_{\ell=1}^{\mathcal{M}}\eta_{m\ell t}R_{\ell t}$ gives us

$$\begin{aligned}&\omega_{mt+1}\left(\sum_{\ell=1}^{\mathcal{M}}A^w_{m\ell t}+\frac{H^w_{mt+1}+Z^w_{mt+1}}{\sum_{\ell=1}^{\mathcal{M}}\eta_{m\ell t}R_{\ell t}}\right)+(1-\omega_{mt+1})\left(\frac{\xi^r_{mt+1}}{\xi^w_{mt+1}}\right)^{\frac{1}{1-\sigma}}\left(\sum_{\ell=1}^{\mathcal{M}}A^r_{m\ell t}+\frac{S^r_{mt+1}}{\sum_{\ell=1}^{\mathcal{M}}\eta_{m\ell t}R_{\ell t}}\right)\\ &=\left[\frac{\beta_m\Omega_{mt+1}}{1+\sum_{\ell=1}^{\mathcal{M}}\frac{\Lambda_{m\ell t}}{2}\left(\eta^w_{m\ell t}-\bar{\eta}_{m\ell}\right)^2}\right]^{\sigma}\left(\sum_{\ell=1}^{\mathcal{M}}\eta_{m\ell t}R_{\ell t}\right)^{\sigma-1}\frac{\xi^w_{mt}}{\xi^w_{mt+1}}\left(\sum_{\ell=1}^{\mathcal{M}}R_{\ell t-1}A^w_{m\ell t-1}+H^w_{mt}+Z^w_{mt}\right).\end{aligned}$$

Note that, for a worker who retires, $\sum_{\ell=1}^{\mathcal{M}}A^w_{m\ell t}=\sum_{\ell=1}^{\mathcal{M}}A^r_{m\ell t}$. Therefore, we can simplify the previous equation using the workers' budget constraint as to obtain

$$\begin{aligned}&\sum_{\ell=1}^{\mathcal{M}}R_{\ell t-1}A^w_{m\ell t-1}+W^w_{mt}+T^w_{mt}-C^w_{mt}+\frac{\omega_{mt+1}(H^w_{mt+1}+Z^w_{mt+1})}{\Omega_{mt+1}\tilde{R}_{mt}}+\frac{(1-\omega_{mt+1})\left(\frac{\xi^r_{mt+1}}{\xi^w_{mt+1}}\right)^{\frac{1}{1-\sigma}}S^r_{mt+1}}{\Omega_{mt+1}\tilde{R}_{mt}}\\ &=\beta_m^{\sigma}\left(\Omega_{mt+1}\tilde{R}_{mt}\right)^{\sigma-1}\frac{\xi^w_{mt}}{\xi^w_{mt+1}}\left(\sum_{\ell=1}^{\mathcal{M}}R_{\ell t-1}A^w_{m\ell t-1}+H^w_{mt}+Z^w_{mt}\right).\end{aligned}$$

Substituting the guess for $C^w_{mt}$ and using the recursive definitions of $H^w_{mt+1}$ and $Z^w_{mt+1}$, we get the difference equation for the marginal propensity to consume of workers (12)

The last step to characterize the workers' problem is to verify the guess for the value function. After

substituting the guess into equation (8) and rearranging, we get

$$\Delta^w_{mt}C^w_{mt} = \left\{ (C^w_{mt})^{\frac{\sigma-1}{\sigma}} + \beta_m \left[ \omega_{mt+1} C^w_{mt+1} + (1-\omega_m t+1) \frac{\Delta^r_{mt+1}}{\Delta^w_{mt+1}} C^r_{mt+1} \right]^{\frac{\sigma-1}{\sigma}} \left(\Delta^w_{mt+1}\right)^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}}. \quad \text{(A.14)}$$

We can then substitute the Euler equation (A.13) into (A.14) and write

$$\Delta^w_{mt}C^w_{mt} = \left\{ (C^w_{mt})^{\frac{\sigma-1}{\sigma}} + \beta_m \left[ (\beta_m \Omega_{mt+1} \tilde{R}_{mt})^{\sigma} C^w_{mt} \right]^{\frac{\sigma-1}{\sigma}} \left(\Delta^w_{mt+1}\right)^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}}.$$

Simplifying $C^w_{mt}$ from the equation and rearranging leads to

$$(\Delta^w_{mt})^{\frac{\sigma-1}{\sigma}} = 1 + \beta^{\sigma}_m \left( \Omega_{mt+1} \tilde{R}_{mt} \right)^{\sigma-1} (\Delta^w_{mt+1})^{\frac{\sigma-1}{\sigma}}. \quad \text{(A.15)}$$

Comparing equation (A.15) to equation (12) shows that the guess for the policy function is correct provided that

$$\Delta^w_{mt} = (\xi^w_{mt})^{-\frac{\sigma}{\sigma-1}}.$$

### A.3 Assets

The heterogeneity between workers and retirees makes it necessary to keep track of the distribution of wealth between the two groups. We start by writing the law of motion of the amount of assets held by retirees

$$\sum_{\ell=1}^{\mathcal{M}} A^r_{m\ell t} = \frac{\sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A^r_{m\ell t-1} + E_{mt} - C^r_{mt}}{1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t} - \bar{\eta}_{m\ell}\right)^2} + (1-\omega_{mt+1}) \frac{\sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A^w_{m\ell t-1} + W_{mt} - T_{mt} - C^w_{mt}}{1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t} - \bar{\eta}_{m\ell}\right)^2}. \quad \text{(A.16)}$$

From the workers' aggregate budget constraint, we substitute the total amount of workers' assets into the second term of the right-hand side of equation (A.16). Next, we substitute out retirees' consumption, and rewrite retirees and workers' total value of non-human assets as shares of total assets using the definition in the text

$$\lambda_{mt} \sum_{\ell=1}^{\mathcal{M}} A_{m\ell t} - \frac{1-\omega_{mt+1}}{\omega_{mt+1}} (1-\lambda_{mt}) \sum_{\ell=1}^{\mathcal{M}} A_{m\ell t}$$
$$= \frac{\lambda_{mt-1} \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1} + E_{mt} - \xi^r_{mt} \left( \lambda_{mt-1} \sum_{\ell=1}^{\mathcal{M}} R_{\ell t-1} A_{m\ell t-1} + S_{mt} \right)}{1 + \sum_{\ell=1}^{\mathcal{M}} \frac{\Lambda_{m\ell t}}{2} \left(\eta_{m\ell t} - \bar{\eta}_{m\ell}\right)^2}.$$

After rearranging and using the definition of aggregate assets $A_{mt} \equiv \sum_{\ell=1}^{\mathcal{M}} A_{m\ell t}$, we obtain equation (14) in the text.

**Table A1:** Sources for nominal short-term interest rates used to construct ex-ante real interest rates.

| Country | Source | Description |
|---|---|---|
| Australia | World Bank | Lending interest rate |
| Austria | OECD | 1-day central bank yield |
| Belgium | OECD | 3-month interbank yield |
| Canada | OECD | 1-day central bank yield |
| Denmark | OECD | 1-day central bank yield |
| Finland | OECD | 1-day central bank yield |
| France | OECD | 3-month interbank rate |
| Germany | AMECO | Short term interest rate |
| Ireland | OECD | 3-month interbank rate |
| Italy | AMECO | Short term interest rate |
| Japan | OECD | 1-day central bank yield |
| Netherlands | AMECO | Short term interest rate |
| New Zealand | OECD | 3-month bankbill yield |
| Norway | OECD | 3-month interbank yield |
| Spain | OECD | 3-month interbank rate |
| Sweden | OECD | 3-month interbank rate |
| Switzerland | OECD | 3-month interbank loan rate |
| United Kingdom | OECD | 3-month interbank loan rate |
| United States | IFS | Money market rate |

# B Interest Rate Data

This section describes the sources of the data for short-term nominal interest rates.

For each country in our sample, Table A1 reports the source and the maturity of the nominal interest rate $i_t$ used to construct the ex-ante real interest rate $r_t$, according to

$$r_t = i_t - \mathbb{E}_t \pi_{t+1}.$$

As discussed in the text, we construct expected inflation following Hamilton et al. (2016). Specifically, for each country $m$, we first estimate a regression of inflation on its own lag with rolling windows of 20 years

$$\pi_{m,t} = a_m + b_m \pi_{m,t-1} + \varepsilon_{m,t}. \tag{B.17}$$

We then calculate the one-year-ahead forecast

$$\mathbb{E}_t \pi_{m,t+1} = \hat{a}_m + \hat{b}_m \pi_{m,t},$$

where $\hat{a}_m$ and $\hat{b}_m$ are the OLS estimates of the coefficients in (B.17). For all countries, we use the headline CPI inflation rate obtained from the OECD.