Inequality and Optimal Monetary Policy in the Open Economy*

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Abstract

We study optimal monetary and exchange-rate policy in a tractable Small Open Economy Heterogeneous-Agent New Keynesian (SOE-HANK) model in which households face with uninsurable idiosyncratic risk. We derive conditions under which a *SOE-HANK divine coincidence* prevails – a generalisation to HANK of the open-economy divine coincidence result in Galí and Monacelli (2005). Away from this benchmark, fluctuations in consumption inequality generate new tradeoffs for monetary policy, relative to those arising under complete markets. With countercyclical consumption risk, monetary policy puts additional weight on stabilizing output and the real exchange rate relative to the Representative-Agent benchmark.

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1 Introduction

How do aggregate shocks affect domestic inequality in a small open economy, and what should the central bank do about it? In this paper, we answer this question by constructing a Small-Open Economy Heterogenous-Agent New Keynesian (SOE-HANK) model in which aggregate shocks (to domestic productivity or capital flows) affect the distribution of earnings at home and, ultimately, the distributions of wealth and consumption – in addition to affecting the exchange rate, output and aggregate income as they would in the absence of distributional considerations. Following recent SOE-HANK literature – e.g., Auclert et al. (2021b); Druedahl et al. (2022); Guo et al. (2023) – our framework of analysis is a variant of Galí and Monacelli (2005)'s framework, augmented with incomplete markets with respect to both aggregate shocks and idiosyncratic shocks affecting the labour earnings of domestic households. This strand of the literature (reviewed below) has so far focused on how household heterogeneity may alter – and, in some cases, overturn – the predictions of the benchmark Representative-Agent model ("SOE-RANK") regarding the propagation of aggregate shocks in the open economy.[1] In contrast with this line of work, we take a normative, rather than positive, perspective: our goal here is to understand how incomplete markets and the implied time variations in household heterogeneity affect optimal monetary and exchange rate policies, relative to the complete-market benchmark. As far as we know, this paper is the first to undertake this investigation.

Our critical insight is that, under incomplete markets, aggregate shocks trigger potentially inefficient fluctuations in consumption dispersion across households that a benevolent central bank would want to stabilise. How much the home central bank effectively leans against such variations in inequality ultimately depends on how strongly this objective conflicts with the traditional central bank's objectives in the absence of household heterogeneity. As is now well understood (and synthesised in Galí (2015)), in the basic closed-economy New Keynesian model, the central bank should stabilise nominal prices as well as the "welfare-relevant output gap", i.e., the difference between actual output and level of output that annihilates the labour wedge. In the open economy, an additional motive arises, namely the efficient manipulation of the terms of trade – made possible by the fact that the central bank exerts some monopoly power over the international price of home-produced goods. The concern for inequality adds another term to this trilemma because optimally stabilising inequality may require departing from price stability, a zero labour wedge, and otherwise optimal terms of trade – at least temporarily. Our analysis uncovers the situations where such departures occur and explains why they do.

Analysing monetary policy tradeoffs from a normative perspective when several frictions interact is usually intricate, and the new tradeoffs that inequality creates make no exception. To dissect how inequality plays out as clearly and transparently as possible, we construct our baseline SOE-HANK model so that one of its specialisations exactly replicates optimal monetary policy under the corresponding SOE-RANK. This replication arises when consumption risk is present but acyclical

[1] For example, Auclert et al. (2021b) show that capital outflows may contract output in SOE-HANK, while they always stimulate output in SOE-RANK.

– hence, unresponsive to monetary policy. When such is the case, the best the central bank can do is focus on its other objectives and disregard consumption risk altogether, even though inequality is welfare-reducing. While consumption risk is an endogenous object in our model, it is (tautologically) constant whenever two conditions are met, namely (i) income risk is constant and (ii) the pass-through from idiosyncratic income shocks to consumption is constant – both of which are satisfied under a suitable parameterisation of the model.[2] This benchmark scenario allows us, *inter alia*, to generalise to SOE-HANK the well-known result of Galí and Monacelli (2005) that exact producer price stability is optimal when markets are complete, all elasticities of substitutions (between exported goods, between imported goods, and intertemporal) are equal to one – the so-called "Cole-Obstfeld" (1991) parameterisation –, and only domestic aggregate productivity shocks hit the small open economy. This "open-economy divine coincidence" arises under *complete* markets because, under the said parameters and shocks, (i) there would be no conflict between stabilising nominal prices and the output gap in the corresponding closed economy and (ii) the nominal exchange rate endogenously adjusts in the open economy in a way that makes movements in the terms of trade optimal. Under *incomplete* markets with respect to *aggregate* productivity shocks, the very same parameters imply that the net foreign asset position of the SOE does not respond to the shock so that the allocation under incomplete markets is identical to that under complete markets. Under incomplete markets with respect to *idiosyncratic* income shocks, the allocation obviously differs from that under complete markets. Still, the central bank can do nothing about it if consumption risk is constant. In that situation, the central bank operates "as if" under complete markets and a form of *SOE-HANK divine coincidence* prevails.[3]

Any departure from this benchmark scenario generates tradeoffs for the central banks. To isolate the specific role that consumption risk and inequality play in those tradeoffs, the first departure from the conditions of the SOE-HANK divine coincidence we consider is by assuming that idiosyncratic income risk is (realistically) *countercyclical* rather than acyclical. This feature makes consumption risk countercyclical, urging the central bank to lean against changes in consumption dispersion triggered by aggregate productivity shocks. For example, after a contractionary aggregate productivity shock (say), the central bank does not equate inflation and the output gap to zero because the contraction in output necessary for doing so would raise inequality and bring welfare losses, relative to SOE-RANK, it lets producer price inflation go in the short run to smooth the adverse impact of the shock on inequality over time. This is similar to what would occur in the closed economy (see Acharya et al. (2023)), with the additional feature that the net foreign asset position of the domestic economy and the adjustment of the real exchange rate now affect

[2] As discussed in more detail later, constant consumption risk arises when (i) income risk is time-invariant and (ii) income shocks are permanent - implying that households cannot use the bond market for self-insurance. Consumption risk becomes countercyclical when income risk is countercyclical, when income shocks are transitory, or both.

[3] This "as if" property is reminiscent of Werning (2015)'s equivalence result, according to which the aggregated Euler equation of an incomplete-market, zero-liquidity economy with acyclical income risk is the same as under complete markets. The reason is that, under those assumptions, the pass-through from income shocks to consumption is constant, making consumption risk and the implied precautionary motive time-invariant. In our model, too, acyclical consumption risk implies that the aggregate Euler equations in the baseline SOE-HANK model and the benchmark SOE-RANK model are identical.

both optimal policy and the implied aggregate dynamics. In particular, along the optimal path, the exchange rate responds significantly less to the aggregate shock than under Galí and Monacelli (2005)'s baseline (SOE-RANK) scenario. This property that the optimal monetary/exchange-rate policy in SOE-HANK implies less exchange-rate volatility than in SOE-RANK is robust across trade elasticities — i.e., within or away from the Cole-Obstfeld case –, provided that consumption risk is countercyclical.

Next, we turn to the optimal monetary response to capital-flow shocks.[4] In contrast to aggregate productivity shocks, the optimal response to such shocks implies departures from divine coincidence even in SOE-RANK. For example, in our SOE-RANK benchmark, a capital inflow (triggered by a fall in the world interest rate) generates a real appreciation and an output contraction (driven by expenditure switching) that the central bank partly fends off by cutting interest rates at home – but by less than the fall in the world interest rate. This is because cutting rates so much as to match the world interest rate and fully stabilise the exchange rate would be inflationary, and the SOE-RANK central bank trades off a lower inflation gap against and lower output gap. To the extent that the output contraction also raises inequality in SOE-HANK (because of countercyclical consumption risk), it brings additional welfare losses relative to SOE-RANK and provides an additional motive to stabilise the exchange rate. Quantitatively, the difference in optimal policy turns out to be small under the Cole-Obstfeld parameterisation – but it becomes larger when we realistically depart from it.

In summary, the broad conclusion from our analysis is that uninsurable, countercyclical income risk leading to countercyclical consumption risk leads the central bank of the small open economy to stabilise output and the exchange rate more than it would do under complete market – though the exact quantitative difference between SOE-HANK and SOE-RANK in general depends on the size of trade elasticities.

1.1 Literature review

Our paper bridges three strands of the literature: (i) one that analyses the propagation of aggregate shocks in open-economy HANK from a *positive* (rather than normative) perspective; (ii) one studying optimal monetary policy in *closed-economy* HANK models (rather than in the open economy); (iii) and the literature studying optimal monetary policy in SOE or two-country RANK models (rather than HANK).

De Ferra et al. (2020) and Auclert et al. (2021b) pioneered the *positive* literature on SOE-HANK models, focusing on how exchange-rate (i.e., capital-flow) shocks affect output and national income when a subset of the households have high marginal propensities to consume (MPCs). The main theme of these papers is the contractionary Keynesian multiplier effects that are set off by a currency depreciation when the latter hits household income – due currency mismatch in De Ferra et al. (2020) or the collapse in the relative price of home-produced goods in Auclert et al. (2021b).

[4] Capital flow shocks have motivated the development of SOE-HANK models in the first place, as epitomised in the work of De Ferra et al. (2020) and Auclert et al. (2021b).

Besides our focus on optimal policy, we abstract from high MPCs (which would also be at work in Two-Agent models) but stress the importance of cyclical consumption risk and precautionary savings in the propagation of aggregate shocks in SOE-HANK. Other contributions following their lead include Druedahl et al. (2022), who study the propagation of foreign demand shocks, as well as Guo et al. (2023) and Oskolkov (2023), who study how international integration and exchange-rate regimes shape the propagation of aggregate shocks to domestic inequalities. Relatedly, Müller et al. (2023) study international spillovers in a two-country HANK model (rather than a SOE-HANK).

The present paper naturally extends to the open economy the analysis of Ramsey-optimal monetary policy in *closed-economy* HANK models, as undertaken by Bhandari et al. (2021), Le Grand et al. (2022), McKay and Wolf (2022), Acharya et al. (2023) and Dávila and Schaab (2023). In contrast to the first three papers, which numerically solve for optimal policy, we here follow the approach of Acharya et al. (2023), who rely on the joint assumption of Constant Absolute Risk Aversion (CARA) utility for consumption and conditionally normally distributed income risk to achieve tractability. In that paper, tractability was key in deriving target criteria similar to those obtained in RANK. In the present paper, it is key in delivering the SOE-HANK divine coincidence result, which is pivotal in exploring monetary policy tradeoffs in the presence of household heterogneity.

Last but not least, our paper is in the continuation of the traditional literature examining optimal monetary and exchange-rate policy in various versions of the open-economy RANK model. As discussed above, the starting point of our analysis is the open-economy divine coincidence result of Galí and Monacelli (2005), which we generalise to incomplete markets – with respect to both aggregate and idiosyncratic shocks – and then branch out form. Related to Galí and Monacelli (2005), Clarida et al. (2001, 2002); Corsetti and Pesenti (2005); Benigno and Benigno (2003, 2006); Faia and Monacelli (2008) and De Paoli (2009a) study optimal monetary policy in SOE or two-country models under complete markets with respect to aggregate shocks (i.e., where the "Backus-Smith" (1993) condition relating the ratio of marginal consumption utilities to the real exchange rate holds), abstracting from uninsurable idiosyncratic shocks. Extending this line of work, Benigno (2009); De Paoli (2009b) and more recently Corsetti et al. (2023) have examined optimal monetary policy with imperfect risk sharing *internationally* – i.e., disposing of the Backus-Smith condition – but still in the absence of idiosyncratic risk. By and large, the main focus of this literature so far has been to clarify how terms of trade manipulation – granted by the central bank's ability to influence the relative price of home-produced goods – may affect optimal exchange-rate volatility and potentially conflict with domestic price stability.[5] Our focus instead is on the new monetary policy tradeoffs that uninsurable idiosyncratic risk may generate – potentially jointly with those generated by the terms-of-trade manipulation motive.

[5]Within this strand of the literature, one step in the direction of heterogeneity has recently been taken by Chen et al. (2023), who study (in their Section IV) the optimal monetary policy implications of their two-country, Two-Agent model. They abstract, however, from uninsurable risk.

2 Environment

The model is a version of Galí and Monacelli (2005) allowing for uninsurable idiosyncratic risk. Time is discrete and extends indefinitely: $t = 0, 1, ...$ The world economy consists of a continuum of small open economies $j \in [0, 1]$, one of which is the home economy and is accordingly indexed by H. We first describe the behaviour of domestic agents (households, firms and financial intermediaries) before we turn to foreign agents and their interactions.

2.1 Households

The home economy is populated by a continuum of households in mass 1 who face uninsurable idiosyncratic shocks to their individual labour productivity. The demographic structure is as in Blanchard (1985): in every period, a fraction $1 - \vartheta \in [0, 1]$ of the households exits the economy and is replaced by an equal fraction of entering households. The size at time t of the cohort having entered the economy at time $s \leq t$ is thus $(1 - \vartheta)\vartheta^{t-s}$. Upon entry, households are transferred an amount of asset wealth equal to the average beginning-of-period wealth of all surviving households (who are lump-sum taxed accordingly).[6]

Household i born at time s solves:

$$\max \mathbb{E}_s \sum_{t=s}^{\infty} (\beta\vartheta)^{t-s} \left[-\frac{1}{\gamma} e^{-\gamma c_t^s(i)} - v(n_t^s(i)) \right] \tag{1}$$

where $\beta \in [0, 1)$ is the household subjective discount factor, $c_t^s(i)$ their consumption of the composite consumption good (described below), $n_t^s(i)$ their labour supply and $v(\cdot)$ a increasing and convex labour-disutility function. The utility from consumption is of the Constant Absolute Risk Aversion (CARA) class with risk aversion coefficient $\gamma > 0$.

The flow budget constraint of this household is given by:

$$c_t^s(i) + \frac{\vartheta}{R_t} a_{t+1}^s(i) = (1 - \tau^a) a_t^s(i) + (1 - \tau^w) w_t n_t^s(i) e_t^s(i) + \mathcal{D}_t + \mathbb{T}_t \tag{2}$$

where $a_t^s(i)$ denotes the household's holdings of real actuarial bonds, ϑ/R_t the unit price of those bonds, w_t the pre-tax real wage per unit of effective labour, τ^a and τ^w are (constant) asset-income and labor-income tax rates, $e_t^s(i)$ is individual productivity. $\mathcal{D}_t$ denotes firms' dividends (uniformly distributed to the households), and $\mathbb{T}_t$ is a uniform net transfer from the government.[7]

[6] Population turnover ensures that the model always generates a stationary distribution of wealth (for suitable values of ϑ) despite the repeated occurrence of idiosyncratic productivity shocks at the household level. The transfer to newborn households kills the inter-cohort heterogeneity that this demographic structure entails whenever the economy's net foreign asset position differs from zero. This assumption implies that idiosyncratic productivity risk is the only source of household heterogeneity so that our complete-market benchmark is identical to Galí and Monacelli (2005).

[7] The assumption that dividends and the lump-sum transfer is uniform is made for simplicity. Asymmetric transfers due to, say, unequal distribution of dividends, would generate unequal exposure of households to aggregate shocks, against which the central bank may want to lean (see Bhandari et al. (2021), Acharya et al. (2023) in the context of a closed economy).

Individual labour productivity $e_t^s(i)$ evolves stochastically according to the following process:

$$e_t^s(i) = 1 + \sigma_t \xi_t^s(i), \quad \text{with} \quad \xi_t^s(i) = \lambda \xi_{t-1}^s(i) + v_t^s(i) \quad \text{and} \quad v_t^s(i) \rightsquigarrow \mathcal{N}(0,1) \tag{3}$$

The process in (3) implies that the conditional variance of next-period individual productivity is equal to σ_t^2, while the conditional mean is $1+\lambda\sigma_t\xi_t^s(i)$, where $\sigma_t \geq 0$ and $\lambda \in [0,1]$. Time-variations is σ_t generate changes in idiosyncratic income risk, while the fact that $\lambda \in [0,1]$ allows to span the whole range of persistence for individual income, from i.i.d. ($\lambda = 0$) to permanent ($\lambda = 1$).

Under incomplete consumption insurance and a frictionless labour market, households use labour supply for self-insurance against idiosyncratic shocks, working relatively more (less) when financial assets and consumption are low (high). As Auclert et al. (2021a) show, however, if preferences are separable, the wealth effects on this frictionless labour supply are implausibly large. Accordingly, we assume that labour is demand-determined by a continuum of unions acting on behalf of the households and responding monopolistically to the demand for labour by firms. In doing so, unions collect hours uniformly from the households, so that $n_t^s(i) = n_t$ far all (s,i). However, there are no nominal wage rigidities, implying that the optimal aggregate labour supply condition is static.[8] We show in Appendix A that, under this structure the optimal labour supply condition is

$$\mathcal{M}_w v'(n_t) = (1-\tau^w) w_t e^{-\gamma c_t} \Sigma_t \tag{4}$$

where $\mathcal{M}_w > 1$ is the wage markup, c_t is aggregate (or, equivalently, average) consumption – so that $e^{-\gamma c_t}$ is the marginal utility of average consumption – while Σ_t is a correction capturing the dispersion of marginal utilities around the mean:

$$\Sigma_t = (1-\vartheta) \sum_{s=-\infty}^{t} \vartheta^{t-s} \int e^{-\gamma[c_t^s(i) - c_t]} di \tag{5}$$

where, intuitively, the $\sum$ operator averages consumption dispertion over cohorts while the $\int$ operator averages over households of the same cohort. Under complete markets, there is no consumption dispersion ($c_t^s(i) = c_t$ for all s,t) and consequently $\Sigma_t = 1$ in Equations (4)-(5). Under incomplete markets, consumption dispersion (and Jensen inequality) implies that Σ_t is strictly greater than one.

Given the variance of idiosyncratic productivity σ_t, the conditional standard deviation of household income is given by $\sigma_{y,t} = (1-\tau^w) w_t n_t \sigma_t$. While σ_t is not directly observable, there is substantial evidence on the counter-cyclicality of income risk $\sigma_{y,t}$. Motivated by this, we choose to implicitly specify σ_t so that the standard deviation of income can be expressed as:

$$\sigma_{y,t} = (1-\tau^w) w_t n_t \sigma_t \quad = \quad \sigma_y \exp\left\{-\varphi\left(\frac{y_t}{y} - 1\right)\right\}, \tag{6}$$

[8] We impose nominal price rather than wage rigidities here. This allows us to nest the Galí and Monacelli (2005) benchmark as a special case of our baseline model.

where σ_y denotes the steady state income risk that households face, and $\varphi = -\frac{d \ln \sigma_{y,t}}{d \ln y_t}$ denotes the elasticity of income risk with respect to aggregate output, and controls the cyclicality of income risk. In particular, income risk is *countercyclical* (households face higher income risk during downturns) whenever $\varphi > 0$; $\varphi = 0$ denotes the case when income risk is *acyclical*.

Finally, because households do not face hard borrowing constraints (only a no-Ponzi game condition), their optimal consumption-saving choice is given by the following Euler equation:

$$e^{-\gamma c_t^s(i)} = \beta(1-\tau^a) R_t \mathbb{E}_t e^{-\gamma c_{t+1}^s(i)} \tag{7}$$

where ϑ has dropped out by virtue of the actuarial nature of real bonds.

2.2 Firms

2.2.1 Home retailers

Following Galí and Monacelli (2005), the composite good consumed by the households results from the bundling (by home "retailers") of infinitely many consumption goods produced at home and abroad (by "producers"). Home retailers' production function for the composite consumption good is:

$$c_t = \begin{cases} [(1-\alpha)^{1-\alpha}\alpha^{\alpha}]^{-1} c_{H,t}^{1-\alpha} c_{F,t}^{\alpha} & \text{if } \eta = 1 \\ \left[(1-\alpha)^{\frac{1}{\eta}} c_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} c_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} & \text{otherwise} \end{cases} \tag{8}$$

where $c_{H,t}$ is a bundle of Home-produced varieties and $c_{F,t}$ is a bundle of Foreign-produced (i.e., imported) varieties (described below). The nominal Home-currency price of the composite consumption good is denoted P_t, and the Home-currency prices of the two input bundles are denoted $P_{H,t}$ and $P_{F,t}$. To produce c_t optimally, retailers combine the two input baskets as follows:

$$c_{H,t} = (1-\alpha)\, p_{H,t}^{-\eta} c_t \quad c_{F,t} = (1-\alpha)\, p_{F,t}^{-\eta} c_t \tag{9}$$

where $p_{H,t} = P_{H,t}/P_t$ and $p_{F,t} = P_{F,t}/P_t$ are the real prices (i.e., in terms of the Home consumption good) of the Home-produced and Foreign-produced goods respectively.

The retail sector is perfectly competitive and hence makes to profit. This implies that $p_{H,t}$ and $p_{F,t}$ are related as follows:

$$(1-\alpha) p_{H,t}^{1-\eta} + \alpha p_{F,t}^{1-\eta} = 1 \tag{10}$$

with the understanding that $p_{H,t}^{1-\alpha} p_{F,t}^{\alpha} = 1$ in the case where $\eta = 1$.

Under producer currency pricing (our baseline assumption), we have $P_{F,t} = \mathcal{E}_t P_t^*$, where P_t^* denotes the price of imports expressed in foreign-currency units and $\mathcal{E}_t$ is the nominal exchange rate between Home and Foreign currencies, while $Q_t = \mathcal{E}_t P_t^*/P_t$ denotes the real exchange rate, and is also equal to the real price of Foreign goods in Home $p_{F,t} = Q_t$. Then, (10) implies that $p_{H,t}$

can be expressed in terms of the real exchange rate as:

$$p_{H,t} = \left(\frac{1 - \alpha Q_t^{1-\eta}}{1-\alpha} \right)^{\frac{1}{1-\eta}} \tag{11}$$

or $p_{H,t} = Q_t^{\frac{\alpha}{\alpha-1}}$ if $\eta = 1$.

The home-produced bundle $c_{H,t}$ is itself the product of a continuum $j \in [0,1]$ of home-produced varieties, which are combined according to the following production function:

$$c_{H,t} = \left(\int_0^1 c_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}j \right)^{\frac{\varepsilon}{\varepsilon-1}}, \ \varepsilon > 1 \tag{12}$$

From equation (12), retailers' demand for variety j is

$$c_{H,t}(j) = (P_{H,t}(j)/P_{H,t})^{-\varepsilon} c_{H,t}, \tag{13}$$

where $P_{H,t}(j)$ is the nominal price of this variety. By retailers' zero profit condition, the $P_{H,t}(j)$s satisfy $P_{H,t}^{1-\varepsilon} = \int_0^1 P_{H,t}^{1-\varepsilon}(j) dj$.

On the other hand, the foreign-produced bundle $c_{F,t}$ aggregates goods from a continuum of countries $m \in [0,1]$, with each country-specific basket itself aggregating a continuum of country-specific varieties $j \in [0,1]$. We have:

$$c_{Ft} = \left(\int_0^1 c_{m,t}^{\frac{\nu-1}{\nu}} \mathrm{d}m \right)^{\frac{\nu}{\nu-1}} \quad \text{and} \quad c_{m,t} = \left(\int_0^1 c_{m,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}j \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{14}$$

where $\nu, \varepsilon > 0$ and $\nu \neq 1$.[9] Because all foreign prices are symmetric (equal to P_t^*), home retailers' demands for foreign varieties are symmetric across and within countries, i.e. for all (m,j) we have $c_{m,t}(j) = c_{F,t}$ in Equation (9).

2.2.2 Foreign retailers

Foreign retailers in every country $m \neq H$ behave symmetrically to those in the home economy, i.e., they aggregate varieties from the rest of the world using the aggregators in equations (8) and (14). It follows that the demand by any country $m \neq H$ for home-produced variety j is given by:

$$c_{Ht}^*(j) = \alpha \left(\frac{P_{H,t}^*(j)}{P_{H,t}^*} \right)^{-\varepsilon} \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\nu} c^* \tag{15}$$

where $P_{H,t}^*(j)$ is the foreign-currency price of variety j from country H, $P_{H,t}^{*1-\varepsilon} = \int_0^1 P_{H,t}^{*1-\varepsilon}(j) dj$ the foreign-currency price index for all goods from country H, and c^* is world consumption. Under PCP we have $P_{H,t}^*(j) = P_{H,t}(j)/\mathcal{E}_t$ for all j, while $Q_t = \mathcal{E}_t P_t^*/P_t$, so the previous equation can be

[9] In the case where $\nu = 1$, the expression for $c_{F,t}$ in Equation (14) should be replaced by $\ln c_{F,t} = \int_0^1 \ln c_{m,t} \mathrm{d}m$.

written as:

$$c^*_{Ht}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} c_{Ht}, \quad \text{where} \quad c^*_{Ht} = \alpha \left(\frac{p_{H,t}}{Q_t}\right)^{-\nu} c^* \tag{16}$$

is the home economy's volume of exports.

2.2.3 Home producers

Home varieties are produced by monopolists by means of a linear production function, turning one unit of labour into z_t units of the specialised good. Prices are nominally rigid: in every period, changing firm j's own nominal selling price from $P_{H,t-1}(j)$ to $P_{H,t}(j)$ costs $\frac{\Psi}{2}\left(\ln \frac{P_{Ht}(j)}{P_{Ht-1}(j)}\right)^2 y_t$ units of the good, where the scaling factors are Ψ (a parameter) and y_t (the total demand for home-produced goods). Eventually, net output by firm j is given by:

$$y_t(j) = z_t n_t(j) - \frac{\Psi}{2}\left(\ln \frac{P_{Ht}(j)}{P_{Ht-1}(j)}\right)^2 y_t \tag{17}$$

Firm j sets the price $P_{H,t}(j)$ facing the demand curve formed by home and foreign retailers. From equations (13) and (16), the total demand for good j is given by

$$y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} (c_{H,t} + c^*_{H,t}) \tag{18}$$

where $c_{H,t}$ is given by (9) and c^*_{Ht} by (16).

Firm j maximises the present value of real dividends $\mathcal{D}_t = p_{H,t}(j) y_t(j) - (1-\tau)\widetilde{w}_t n_t(j)$, where $p_{H,t}(j) = P_{H,t}(j)/P_t$ is the real price of good j, $n_t(j)$ is the labour input of firm j, and τ is a payroll subsidy (constant across time and firms), under the constraints (17)-(18). Solving for the first-order condition of firm j and imposing price symmetry across firms (which all face the same problem), we obtain the following New Keynesian Phillips curve:

$$\ln \Pi_{H,t} = \kappa \left[1 - \left(\frac{\chi - 1}{\chi - 1 + \alpha}\right) \frac{p_{H,t} z_t}{w_t}\right] + \beta \left(\frac{z_t w_{t+1} y_{t+1}}{z_{t+1} w_t y_t}\right) \ln \Pi_{H,t+1} \tag{19}$$

In equation (19), the payroll subsidy has been set to:

$$1 - \tau = \left(1 - \frac{1}{\varepsilon}\right)\left(\frac{\chi - 1 + \alpha}{\chi - 1}\right) \tag{20}$$

which achieves the optimal balance (in steady state) between reducing the monopolistic distortion induced by producers' market power and the domestic planner's desire to exploit its market power over the international price of home-produced goods (Farhi and Werning, 2012). Intuitively, in the closed economy ($\alpha = 0$), (20) collapses to $\tau = \varepsilon^{-1}$ and the subsidy eliminates the incidence of producers' market power, rendering the steady state optimal from the planner's perspective. In the open economy ($\alpha \in (0,1)$), the SOE planner finds it optimal to lower steady-state home output relative to the closed economy in order to raise the relative price of home-produced goods

and thereby the home country's national income. Therefore, the subsidy which renders the steady state optimal from the planner's perspective is lower than in the closed economy.

Finally, since producers are symmetric, equation (17) implies that home output is given by:

$$y_t = \frac{z_t n_t}{1 + \frac{\Psi}{2}(\ln \Pi_{H,t})^2} \tag{21}$$

2.2.4 Financial intermediaries

Because of imperfect international risk sharing, the Home economy's NFA generally responds to aggregate shock. Capital flows are operated by competitive financial intermediaries, which hold positions in international bonds (paying the composite consumption good of home and abroad) while trading actuarial bonds with home households. In the spirit of Schmitt-Grohé and Uribe (2003), we assume that the technology for turning domestic savings into international lending (or international borrowing into domestic consumption) involves transaction costs that create an (arbitrarily small) wedge between the home country's savings and its NFA. This will ensure that the home country's NFA eventually returns to steady state, despite the occurrence of aggregate shocks and the lack of international risk sharing.

To be more specific, at time t intermediaries issue a_{t+1} real actuarial bonds at a price $q_t = \vartheta/R_t$, which will pay one unit of the domestic composite consumption good to the ϑ surviving households at time $t+1$. Home savings $q_t a_{t+1}$ are turned into $\Phi(a_{t+1})\, q_t a_{t+1}$ good units available for international lending, where $\Phi(0) = 1, \Phi' < 0, \Phi'' > 0$. In other words, if the country is a creditor (i.e., $(a_{t+1} > 0)$, then $1 - \Phi(a_{t+1}) > 0$ per unit of savings is sunk into intermediation services, and hence only the amount $q_t a_{t+1}\left[1 - (1 - \Phi(a_{t+1}))\right] = \Phi(a_{t+1})\, q_t a_{t+1} < q_t a_{t+1}$ is effectively turned into international lending. Conversely, if the country is a debtor $(a_{t+1} < 0)$, then every unit of the domestic good made available for domestic consumption in excess of domestic sinks an additional quantity $\Phi(a_{t+1}) - 1 > 0$ of goods, i.e., it takes $q_t a_{t+1}\left[1 + (\Phi(a_{t+1}) - 1)\right] = \Phi(a_{t+1})\, q_t a_{t+1} > q_t a_{t+1}$ of international lending to make $q_t a_{t+1}$ available to domestic consumers. Denoting by $(q_{F,t}, q_{H,t})$ the time-t price of an international bond paying 1 unit of the foreign and home composite consumption good at time $t+1$, respectively, and $A_{F,t+1}$ the net quantity of foreign bonds purchased by home intermediaries, the relation between the home country's savings and its net foreign asset position (NFA) is given by:[10]

$$\underbrace{Q_t q_{F,t} A_{F,t+1}}_{\text{NFA}} = \underbrace{\Phi(a_{t+1})}_{\text{transaction costs}} \times \underbrace{q_t a_{t+1}}_{\text{Home savings}} \tag{22}$$

At time $t+1$, home intermediaries collect from their bond portfolio the amount $Q_{t+1} A_{F,t+1}$ and pay out ϑa_{t+1} to domestic survivors. Since intermediaries make no profits, we have:

$$Q_{t+1} A_{F,t+1} = \vartheta a_{t+1} \tag{23}$$

[10] Equation (22) implicitly assumes that, in every country, local financial intermediaries hold diversified portfolio of international bonds, in proportion to the relative size of the issuing country. Consequently, bonds denominated in the home consumption basket have negligible space in home intermediaries' portfolios.

Finally, by no arbitrage in the market for internationally traded bonds, the following Fisher relation must hold:

$$q_{H,t} = \frac{Q_t}{Q_{t+1}} q_{F,t} \tag{24}$$

Combining (22) and (23) gives the relation between the price of home actuarial bonds and that of international bonds denominated in the home consumption basket, i.e., $q_{H,t} = \Phi(a_{t+1}) q_t / \vartheta$. Combined with (24), we get the following Fisher equation:

$$R_t = \frac{Q_{t+1}}{Q_t} R_t^* \Phi\left(a_{t+1}\right) \tag{25}$$

where $R_t^* = 1/q_{F,t}$ is the world real interest rate, which is taken as given by the home economy. In what follows, we shall assume that $\Phi(x) = \mathrm{e}^{-\widetilde{\wp} x}$, $\widetilde{\wp} \geq 0$.

Finally, because of household heterogeneity, the dynamic of total home savings must be aggregated up from the budget constraints of individual residents. In Appendix B, we show that this yields the asset accumulation equation

$$\vartheta a_{t+1} = R_t \left\{ \vartheta a_t + p_{H,t} y_t - c_t \right\}, \tag{26}$$

where ϑa_{t+1} are the liabilities of financial intermediaries (i.e., the total quantity of goods to be delivered to time-$t+1$ survivors). Henceforth, with some abuse of language we will refer to equation (26) as the "NFA dynamics".

2.3 Government and central bank

The monetary authority sets the interest rate on home-currency nominal government debt, i_t. By no arbitrage between those bonds and the international bonds denominated in the home composite good (which return $R_t = 1/q_{H,t}$), we must have $R_t = (1 + i_t)/\Pi_t$, where Π_t denotes consumer price inflation. The government collects the income tax (at a rate τ^w), subsidies producers' wage bill (at a rate τ), transfers a_t to every newborn (to equate their beginning-of-period wealth to average wealth), and distributes a lump sum transfer ($\mathbb{T}_t$) to the households. We assume that home-currency nominal government debt is vanishingly small – so that the country's savings (minus the transaction cost) are held entirely in the form of real international bonds. Given tax collections, zero nominal government debt determines $\mathbb{T}_t$ as a residual in the government's flow budget constraint:

$$\mathbb{T}_t = \underbrace{-\tau w_t n_t}_{\text{payroll subsidies}} + \underbrace{\tau^w w_t n_t}_{\text{labour income taxes}} + \underbrace{\tau_t^a \int a_t^s(i) di}_{\text{asset income taxes}} \underbrace{- \left(1 - \vartheta\right) a_t}_{\text{transfers to newborn}} \tag{27}$$

3 Aggregate dynamics

3.1 From individual to aggregate consumption

As in Acharya and Dogra (2020) and Acharya et al. (2023), the CARA-Normal structure of our model allows us to linearly aggregate individual consumption rules into a single aggregate Euler equation summarising aggregate demand in the home economy. We discuss the main steps of this aggregation here and relegate the detailed derivations in Appendix C. Given our CARA-Normal structure, we conjecture and verify the existence of an individual consumption function of the form

$$c_t^s(i) = c_t + \mu_t \left[(1-\tau^a) \left(a_t^s(i) - a_t \right) + h_t^s(i) \right] \tag{28}$$

where

$$h_{k,t}^s(i) = \mathbb{E}_t \sum_{\tau=0}^{\infty} \frac{\vartheta^\tau}{\prod_{l=0}^{\tau-1}(1-\tau^a)R_{t+l}} \sigma_{y,t+\tau} \xi_{t+\tau}^s(i) \tag{29}$$

Intuitively, equations (28)-(29) express the deviation of individual consumption from the mean $(c_t^s(i) - c_t)$ as a linear function of the deviation of total wealth from the mean – namely, the sum of the deviation of asset wealth from the mean $(a_t^s(i) - a_t)$ and of human wealth from the mean $(h_t^s(i))$. The expression for human wealth in (29) comes from the fact that, from equations (2)-(3) as well as the fact that $\sigma_{y,t} = (1-\tau^w) w_t n_t \sigma_t$ and $n_t^s(i) = n_t$ (derived above), we can express the deviation of current labour income from the mean as $\sigma_{y,t} \xi_t^s(i)$. Moreover, equation (3) also implies that $\mathbb{E}_t \xi_{t+\tau}^s(i) = [\xi_t^s(i)]^\tau$. Using this substitution and factorising, we can write (deviations of) human wealth as $h_{k,t}^s(i) = \sigma_{h,t} \xi_t^s(i)$, where

$$\sigma_{h,t} = \sigma_{y,t} + \frac{\vartheta \lambda}{(1-\tau^a)R_t} \sigma_{h,t+1} \tag{30}$$

is the conditional standard deviation of shocks to human wealth. μ_t in equation (28) denotes the pass-through from lifetime income to consumption, or equivalently the MPC out of lifetime income (also the same as the MPC out of wealth). Appendix ?? shows that the evolution of μ_t can be described by:

$$\mu_t^{-1} = 1 + \frac{\vartheta}{(1-\tau^a)R_t} \mu_{t+1}^{-1} \tag{31}$$

The expression above shows that μ_t is declining in the future path of real interest rates. Lower real interest rates going forward make it easier for households who experience a temporary negative income shock to borrow. This reduces how much the temporarily lower income passes through to their consumption. Conversely, a lower path of real rate implies that households who experience temporarily higher income only observe a small increase in their *discounted* lifetime income. Since their consumption depends on lifetime income, rather than current income, the temporarily higher income does not increase their consumption a lot. Thus, μ_t reflects the ability of households to use the domestic (actuarial) bonds markets for self-insurance against uninsurable income shocks.

Combining equations (30) and (31), we can derive an expression for how consumption risk

evolves over time:

$$\sigma_{c,t} = \mu_t \sigma_{y,t} + \lambda(1-\mu_t)\sigma_{c,t+1} \tag{32}$$

Consumption risk at date t depends not just on current income risk $\sigma_{y,t}$ but also on how much of it passes though to consumption risk, μ_t. This is captured by the first term on the RHS of (32). Furthermore, if individual income is persistent ($\lambda > 0$), then its not just today's income risk and passthrough that matter for consumption risk, rather its the entire path of future expected income risk and passthrough. This is captured by the last term on the RHS of (32).

Finally, since the consumption decision rule is linear in wealth, Appendix ?? shows that instead of having to keep track of the dynamics of the wealth distribution and the Euler equations of each individual household, the demand side of the economy can be summarized by a single aggregate Euler equation:

$$\Delta c_{t+1} = \frac{1}{\gamma} \ln \beta(1-\tau^a) R_t + \frac{\gamma}{2} \sigma^2_{c,t+1}, \tag{33}$$

where c_t denotes aggregate consumption. The first term on the RHS of (33) reflects the standard relationship between consumption growth and real interest rates - holding all else constant, a higher real interest rate incentivizes households to delay consumption, which causes the consumption growth rate to increase. The second term on the RHS reflects precautionary savings demand by households, since they face consumption risk. Higher expected consumption risk ($\sigma^2_{c,t+1} \uparrow$) causes households to increase their desired level of precautionary savings. This causes them to reduce consumption today, or equivalently a higher consumption growth.

3.2 Consumption dispersion

In SOE-RANK, aggregate domestic consumption is the only relevant consumption metric to the social planner. In SOE-HANK instead, cross-sectional consumption dispersion, due to the cumulation of uninsured idiosyncratic shocks, also matters. In particular, we show in Appendix D that individual wealth dynamics (conditional on survival) evolves as follows:

$$c^s_t(i) - c_t = c^s_{t-1}(i) - c_{t-1} + \sigma_{c,t} \upsilon^s_t(i) \tag{34}$$

This is an integrated process in $c^s_t(i) - c_t$, meaning that the repeated occurrence of idiosyncratic shocks takes individual consumption levels further and further apart over time. In particular, the time-t cross-sectional variance of consumption within cohort $s \leq t$ is a random walk:

$$\sigma^c(s,t)^2 = \sigma^c(s,t-1)^2 + (\sigma^c_t)^2 \tag{35}$$

Despite this, there may still exist a stationary distribution of consumption over the entire population thanks to population turnover, i.e., the fact that every cohort shrinks in size over time. In our economy, the overall dispersion of consumption can be summarised by a single scalar variable,

namely, Σ_t in equation (5). Intuitively, Σ_t (≥ 1) measures the *welfare cost of consumption dispersion* when all households are given equal welfare weight.[11] For Σ_t to be bounded above, it must be that a stationary distribution of consumption exists, and the steady state value of Σ_t corresponds to the welfare cost of inequality once the distribution of consumption has converged to its invariant counterpart. We show in Appendix D that Σ_t evolves as follows:

$$\ln \Sigma_t = \frac{\gamma^2}{2}\sigma_{c,t}^2 + \ln\left[1 - \vartheta + \vartheta \Sigma_{t-1}\right] \tag{36}$$

According to equation (36), Σ_t is a quasi auto-regressive process with innovations $(\gamma\sigma_{c,t})^2/2$. The latter term reflects the fact that consumption dispersion is ultimately driven by idiosyncratic productivity shocks, so that an increase in either their variance or their pass-through to consumption ultimately increases cross-sectional inequalities. The process is auto-regressive because inequality is inherently slow-moving: an increase in the variance of idiosyncratic income shocks translates into increased dispersion in individual savings (as the households partly smooth the impact of the shock over time), and hence in next period's consumption. Whenever $\vartheta < e^{-(\gamma\sigma_{c,t})^2/2}$, their exist a stationary distribution of consumption and Σ_t converges towards its steady state value of

$$\Sigma = \frac{1-\vartheta}{e^{-(\gamma\sigma_{c,t})^2/2} - \vartheta} \tag{37}$$

4 Optimal Policy Problem

Since we have an economy with heterogeneous agents, defining a welfare criterion is not trivial since there are multiple ways in which the planner can value the welfare of particular agents. We avoid this issue by studying the problem of a utilitarian planner who maximizes the average welfare of all agents. In particular, we assume that at date 0, the planner puts equal weights on all households that are alive and puts a weight of β^s on the lifetime welfare of households born at date $s > 0$. Because households can trade actuarial bonds which insure them against mortality risk, β is both the social and private rate of discount, and there is no time inconsistency arising because of a difference between the two rates of discount (Calvo and Obstfeld, 1988). As in Acharya et al. (2023), Appendix D then shows that the social welfare function can then be written as

$$\mathbb{W}_0 = \sum_{t=0}^{\infty} \beta^t (1 + \gamma\rho\Omega w_t) \left\{-\frac{1}{\gamma}e^{-\gamma c_t}\right\} \Sigma_t, \tag{38}$$

where, as explained above, Σ_t captures the fact that because of uninsurable income risk, households have different levels of consumption, and hence different marginal utilities. The remaining $(1 + \gamma\rho w_t)\left\{-\frac{1}{\gamma}e^{-\gamma c_t}\right\}$ is simply the utility of a notional agent who enjoys average consumption c_t and

[11] Σ_t appears in equation (4) because the assumed that unions were giving equal welfare weight to all members. In Section 4 below, we similarly assume that policy makers (fiscal and monetary) are benevolent and utilitarian.

supplies labor n_t.[12]

As discussed in Acharya et al. (2023), the planner, in general is concerned with not just cyclical fluctuations in inequality but also the long run level of inequality. To limit the burden on monetary policy, we assume that the average level of inequality is determined by fiscal policy, which we assume is set optimally absent shocks. This leaves monetary policy to address cyclical fluctuations of inequality around its long-run constrained optimal level, which is determined by fiscal policy. Appendix XXX provides details of how optimal fiscal policy sets τ^a and τ^w. Since the the rest of the world is assumed to be risk neutral, the steady state world interest rate is simply β^{-1}. Instead, since Home households face idiosyncratic risk, the precautionary savings demand pushes down Home real rates below β^{-1}, implying that holding all else constant, there would be capital outflows in steady state. The planner chooses a tax on savings $1-\tau^a = e^{\frac{\gamma^2\sigma_c^2}{2}}$, which simply offsets these capital flows in steady state. Second, as in Acharya et al. (2023) the planner uses the labor income tax τ^w to affect the steady state level of output and hence income risk faced by households. Without this tax instrument, would impart a positive inflation bias for monetary policy, since with countercyclical income risk, running the economy hot and having non-zero inflation would allow monetary policy to lower income risk faced by households. Instead, allowing fiscal policy to set this tax instead, allows a higher level of steady state output without requiring higher inflation.

Given the fiscal policy described above, the constraints facing the planner setting monetary policy has a single policy instrument: the sequence of real interest rates $\{R_t\}_{t=0}^{\infty}$ in order to maximize the objective function (38) subject to (i) the Aggregate Euler equation (33), (ii) the evolution of the net foreign asset position (??), (iii) the interest parity condition, (iv) market clearing for Home goods, (v) the Phillips curve (19), (vi) the evolution of consumption risk (32), (vii) the evolution of the social cost of inequality (??), and the evolution of passthrough μ_t (31). Appendix ?? presents the detailed problem. To avoid the standard time-inconsistency of Ramsey plans at date 0 (see Acharya et al. (2023) for details), we assume that there is no pre-existing inequality at date 0.[13]

In what follows, we study the optimal monetary policy response to a (i) temporary aggregate productivity shock and a (ii) temporary shock to the Foreign interest rate – triggering capital flows and eventually changes in the real exchange rate. We choose to study the productivity shock as it allows us to relate and compare our findings to the existing literature, allowing us to highlight how the introduction of uninsurable risk affects optimal policy. We choose the temporary shock to Foreign rates as it mimics a capital inflow or outflow scenario. As with the productivity shocks, the existing literature has studied the optimal response to this shock, allowing us to compare our results and identify exactly how the presence of uninsurable risk alters the optimal response relative to a perfect-insurance benchmark.

Before characterizing how uninsurable risk affects the optimal conduct of monetary policy, it is important to understand that simply the fact that households may face uninsurable consumption

[12]There is no n_t in the expression because we have used the labour supply (4) to substitute out for n_t in the disutility of labor, which yields then yields the expression in (38).

[13]Equivalently, we could also have endowed the planner with an additional wealth tax which is optimally set and levied at date 0. This is the approach taken in Acharya et al. (2023) and would yield the same outcome.

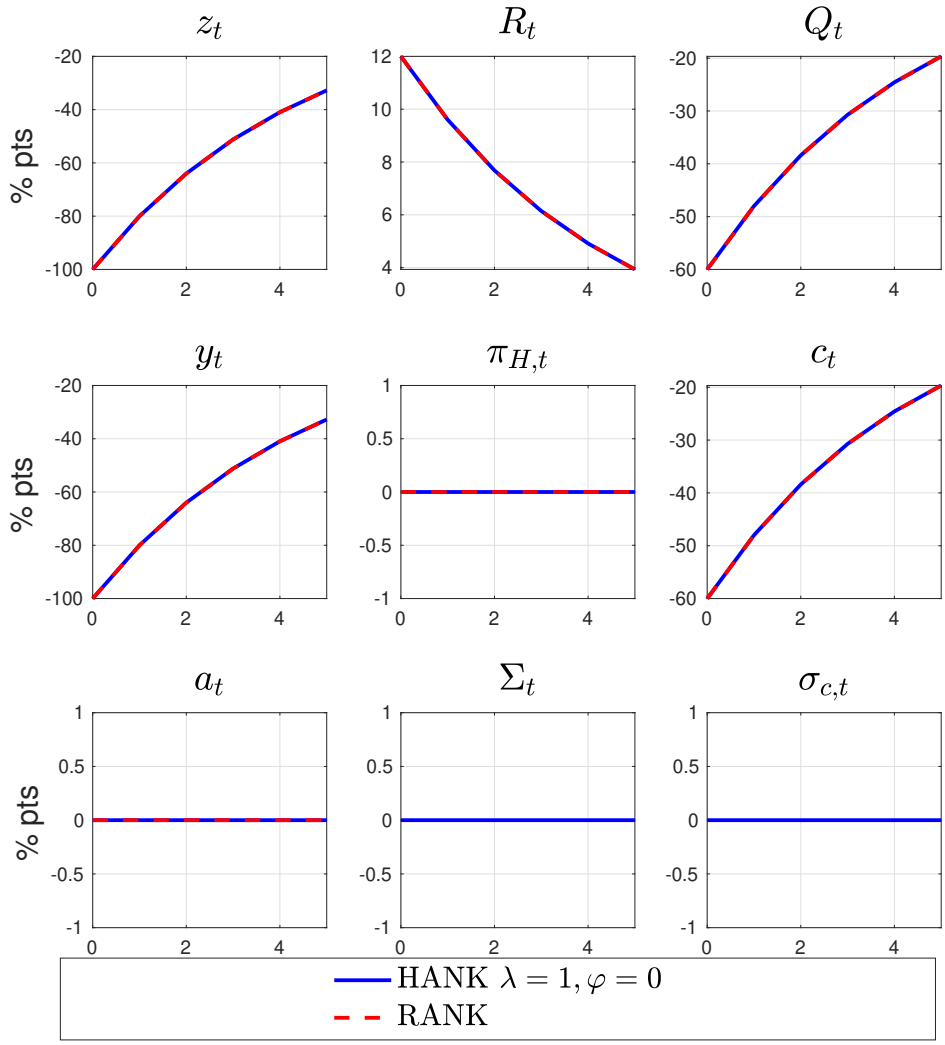


Figure 1: **Divine coincidence:** Under the $\gamma = \eta = \nu = 1$ configuration, with acyclical consumption risk ($\lambda = 1, \varphi = 0$), price stability is the optimal policy in both HANK and RANK in response to productivity shocks.

risk is not a reason that optimal policy should implement different prices and allocations than in the perfect-insurance benchmark. This is formalized by the following Proposition.

Proposition 1. *If individual income follows a random walk* ($\lambda = 1$) *and income risk is acyclical* ($\varphi = 0$)*, then consumption risk is acyclical and optimal monetary policy in response to a productivity shock* z_t *is the same as in HANK and RANK despite the fact that households face consumption risk in HANK.*

Proof. See Appendix XX. □

Intuitively, if consumption risk is acyclical, monetary policy has no ability to affect how consumption risk evolves. Consequently, even though the planner dislikes the fact that households face uninsurable consumption risk, given the policy instrument available to them, they cannot affect

the path of consumption risk and hence inequality. Consequently, monetary policy faces the same trade-offs as in RANK and hence optimal monetary policy in HANK with acyclical consumption risk implements the same path of allocations and prices as in RANK. Figure 1 depicts the optimal response of monetary policy to a persistent decline in aggregate productivity z_t under the Cole and Obstfeld (1991) parameterization $\gamma = \eta = \nu = 1$. As is well known from Galí and Monacelli (2005), the optimal response to a productivity shock in the RANK benchmark is to stabilize domestic PPI inflation perfectly. As can be seen in Figure 1, optimal policy in HANK also implements the same sequence of allocations and prices as in RANK.

Thus, while simply the presence of uninsurable consumption risk does not necessitate a change in the optimal conduct of monetary policy, away from the benchmark of acyclical consumption risk, optimal policy does differ in HANK compared to the corresponding RANK benchmark. Figure 2 shows that with countercylical income (as well as consumption) risk ($\varphi > 0$), optimal monetary policy stabilizes output and exchange rates more than in RANK in response to the RANK benchmark, even under the Cole and Obstfeld (1991) parameterization. Monetary policy in HANK prevents output from falling as much as in RANK. It does so by raising interest rates less than in RANK (even cutting interest rates if individual income is persistent enough; see the dotted line which represents the case in which individual income is a random walk). Furthermore, the smaller increase (or a cut) in Home real interest rates implies that the real exchange rate appreciates less than under RANK on impact. A smaller appreciation further supports higher demand for Home goods and prevents output from falling as much as in RANK. This higher output comes at the cost of higher inflation on impact, which the planner is willing to tolerate as it prevents an even larger increase in the welfare cost of inequality Σ_t (see the middle panel in the last row of Figure 2). Figure 2 plots the optimal dynamics for 3 different level of persistence of individual income: $\lambda = 0, 0.5, 1$. While the actual level of persistence does not qualitatively change the desire of monetary policy to stabilize output and exchange rates, a higher persistence does make the desire quantitatively stronger.

While we have restricted attention to the Cole and Obstfeld (1991) parameterization so far, the difference between optimal policy in HANK and RANK following a productivity shock are qualitatively similar if one departs from the unit elasticity parameterization. If consumption risk is countercyclical, optimal monetary policy policy in HANK following a productivity shock puts additional weight on stabilizing output and fluctuations in the real exchange order to prevent a larger increase in inequality. Figure 3 plots the optimal response to a negative productivity shock in HANK and RANK, this time under the parameterization of Itskhoki and Mukhin (2021), who set $\gamma = 1, \eta = 1.5$ and $\nu = 4$. As can be seen in the figure, price stability is no longer optimal even in RANK under this parameterization. However, the important thing to notice is that comparing HANK to RANK, optimal policy in HANK still continues to stabilize output and the real exchange rate relative to RANK.

Finally, Figure 4 shows the optimal response to a persistent decline in the Foreign interest rate $\widehat{R}^*_t < 0$. As with the productivity shock, with countercyclical income risk, at least qualitatively, optimal policy in HANK prevents the exchange rate from appreciating as much as in RANK by

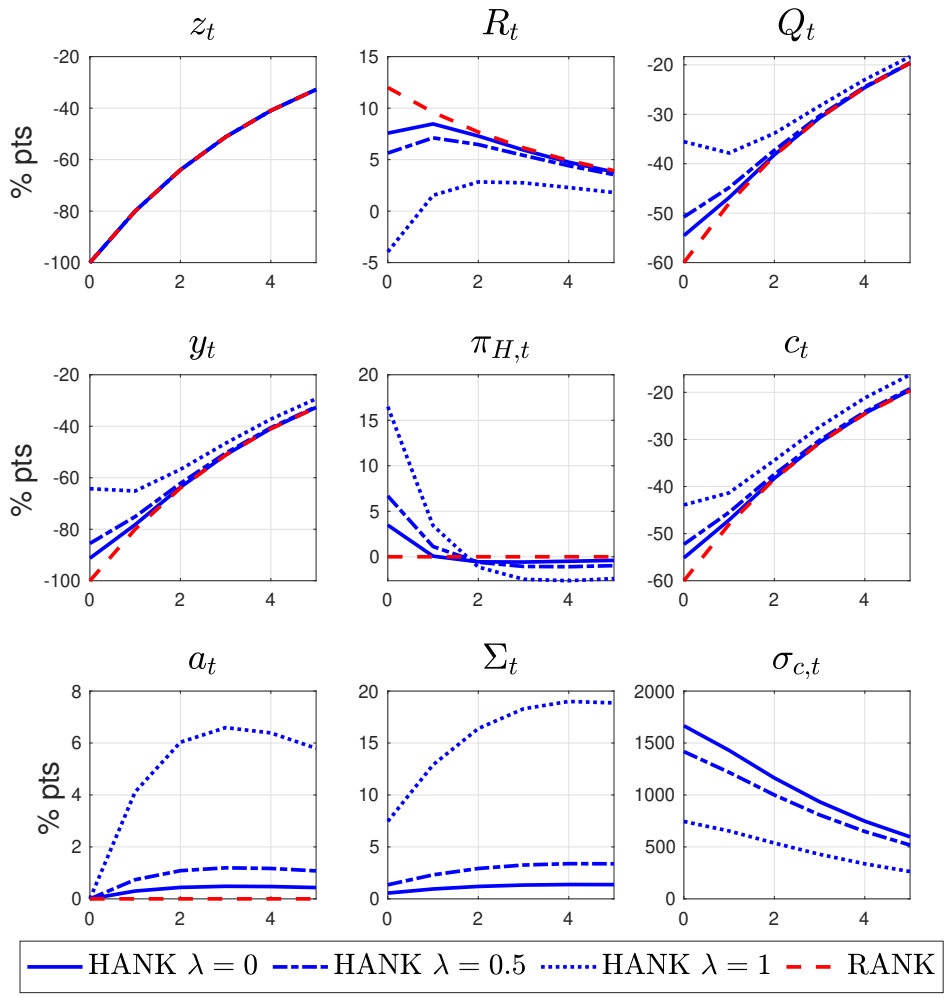


Figure 2: **Cole-Obstfeld case:** Even with $\gamma = \eta = \nu = 1$, price stability is not optimal in HANK if income risk is countercyclical, ($\varphi < 0$). The figure depicts three cases: the solid blue line depicts the case in which individual income is i.i.d across time ($\lambda = 0$), the dotted line is the case in which individual income follows a random-walk ($\lambda = 1$), and the dot-dashed line is the intermediate case in which the persistence of individual income is $\lambda = 0.5$. In response to a negative productivity shock, optimal policy stabilizes output and exchange rates more when individual income is more persistent.

cutting domestic interest rates more to reduce the interest rate differential between the Home and Foreign interest rates. This results in domestic output from falling by less in HANK relative to RANK, curtailing the increase in inequality. While these results do not seem quantitatively significant, it remains true that qualitatively, optimal policy in HANK puts more weight on stabilizing output and real exchange rates than in RANK.

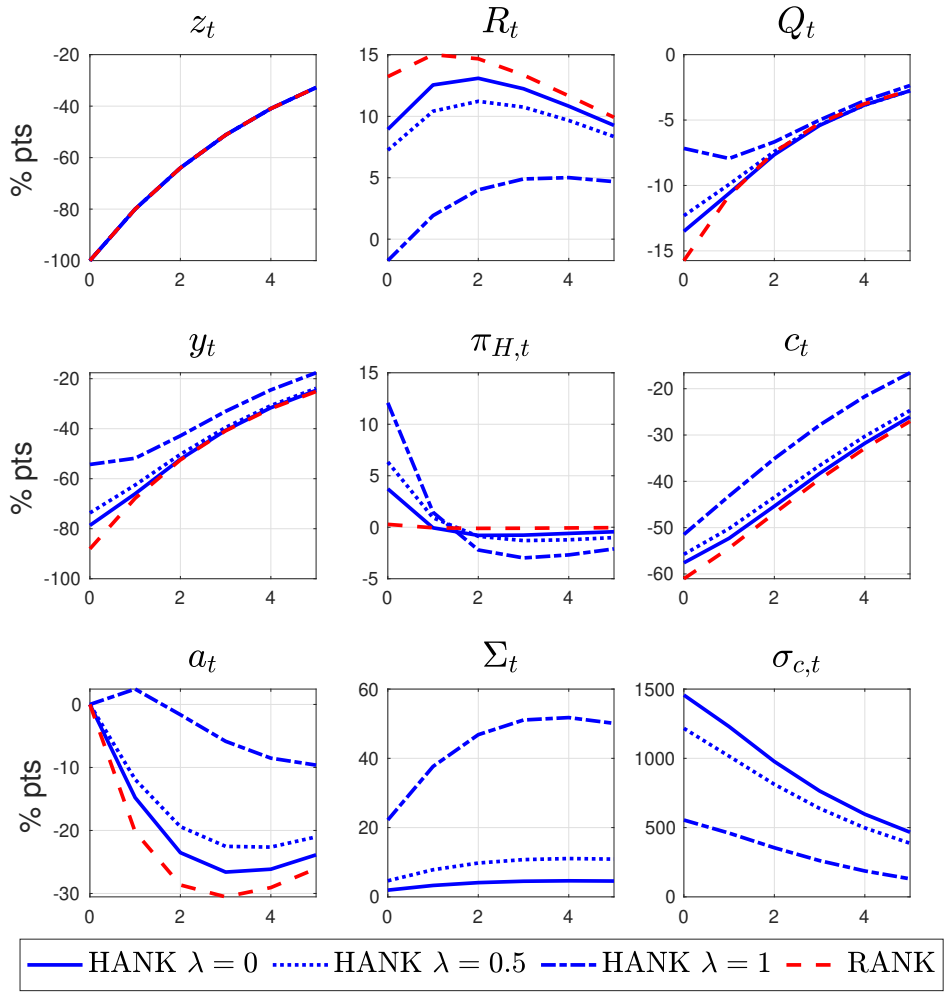


Figure 3: **Non Cole-Obstfeld case:** $(\gamma = 2, \eta = 1.5, \nu = 4)$. With countercyclical income risk $(\varphi > 0)$, optimal policy is qualitatively similar to the Cole-Obstfeld case. Optimal policy in HANK stabilizes output and exchange rates more relative to RANK. A higher λ raises the optimal degree of stabilization. The figure depicts three cases: the solid blue line depicts the case in which individual income is i.i.d across time $(\lambda = 0)$, the dotted line is the case in which individual income follows a random-walk $(\lambda = 1)$, and the dot-dashed line is the intermediate case in which the persistence of individual income is $\lambda = 0.5$.

5 Conclusion

In this paper we have presented a tractable framework to study the additional trade-offs faced by monetary policy in an open economy context when households within the country face uninsurable consumption risk. We showed that simply the presence of uninsurable consumption risk is not sufficient to generate differences in the optimal policy response in HANK vs RANK. However, if consumption risk is countercyclical, optimal monetary policy in HANK puts a larger weight on stabilizing output and exchange rates compared to optimal policy in RANK. This higher weight reflects the desire of the planner to prevent fluctuations in the level of consumption risk and the resulting consumption inequality.

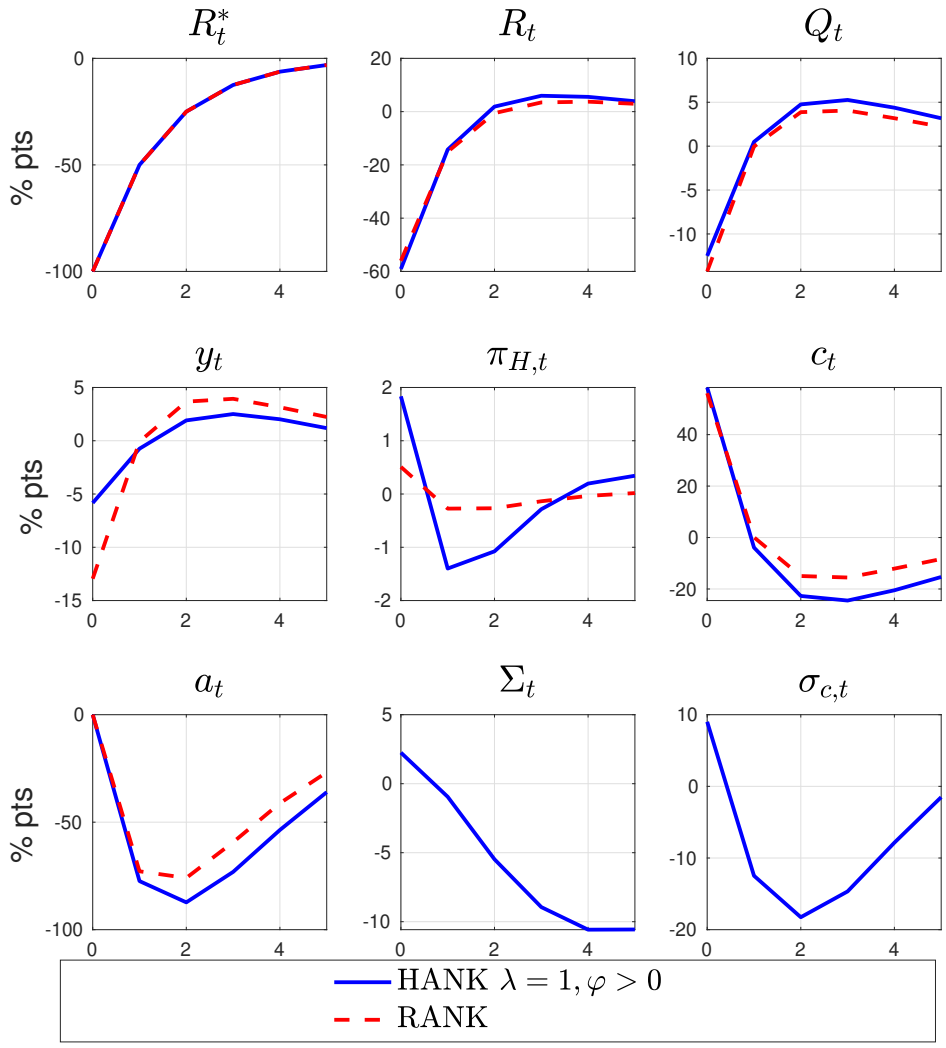


Figure 4: **Non Cole-Obstfeld case:** negative R^* shock

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Appendix

A Labour supply

The setting is as in Auclert et al. (2023): a continuum of monopolistically competitive unions $j \in [0,1]$ demands labour from the households and turn them into specialised labour types j sold to competitive labour agencies. Labour agencies then repackage those types into the final labour sent to producers.

Each worker i supplies $\ell_t^s(i,j)$ labour hours to each union $j \in [0,1]$, so that worker i's total labour supply is $n_t^s(i) = \int_j \ell_t^s(i,j)dj$. Each union j aggregates the effective labour supply of each household to produce a differentiated labour type $n_t(j)$. It demands the same quantity of raw hours $\ell_t(j)$ from all its members so that $n_t(j) = \ell_t(j)$. A competitive labour agency sector re-aggregate the $n_t(j)$ into a single composite labour type n_t sold to firms:

$$n_t = \left(\int_j n_t(j)^{\frac{\varkappa-1}{\varkappa}} \mathrm{d}j \right)^{\frac{\varkappa}{\varkappa-1}}, \quad \varkappa > 1$$

Let $W_t(j)$ be the nominal cost of labour type j sold to labour agencies and W_t the average wage level. The demand for labour input j by labour agencies is given by

$$n_t(j) = n_t \left(\frac{W_t(j)}{W_t} \right)^{-\varkappa} = n_t \left(\frac{w_t(j)}{w_t} \right)^{-\varkappa} \tag{39}$$

Union j sets wages (or equivalently labour supply $n_t(j)$) so as to maximise the expected utility of all its members, giving equal welfare weight to each member. Moreover, in the absence of nominal wage rigidities, the problem of the union is static: in every period, it maximises

$$\mathcal{O}_t(j) = (1-\vartheta) \sum_{s=-\infty}^{t} \vartheta^{t-s} \int \left[-\frac{1}{\gamma} e^{-\gamma c_t^s(i,j)} - v\left(n_t(j)\right) \right] \mathrm{d}i$$

We can rearrange the latter expression as follows:

$$\mathcal{O}_t(j) = \left(-\frac{1}{\gamma}e^{-\gamma c_t(j)}\right) \underbrace{(1-\vartheta)\sum_{s=-\infty}^{t}\vartheta^{t-s}\int\left[e^{-\gamma c_t^s(i,j)-c_t(j)}\right]\mathrm{d}i}_{\equiv\Sigma_t(j)} - v\left(n_t(j)\right) \tag{40}$$

$$= -\frac{1}{\gamma}e^{-\gamma c_t(j)}\Sigma_t(j) - v\left(n_t(j)\right) \tag{41}$$

where $c_t(j)$ is the average consumption of union j's members, given by

$$c_t(j) = (1-\tau^w)w_t(j)n_t(j) + (1-\tau^a)a_t(j) + \mathbb{T}_t - \frac{\vartheta}{R_t}a_{t+1}(j), \tag{42}$$

while $\Sigma_t(j)$ captures the dispersion in marginal utility among union j's member.

Maximising $\mathcal{O}_t(j)$ with respect to the relative real wage $w_t(j)/w_t$ and subject the union-level budget constraint (42) and the inverse demande curve (39) gives the first-order condition:

$$\Sigma_t(j)u'\left(c_t(j)\right)(1-\tau^w)w_t n_t(1-\varkappa)\left[\frac{w_t(j)}{w_t}\right]^{-\varkappa} + \varkappa v'\left(n_t(j)\right)n_t\left[\frac{w_t(j)}{w_t}\right]^{-\varkappa-1} = 0$$

Next, imposing symmetry ($c_t(j) = c_t, \Sigma_t(j) = \Sigma_t, n_t(j) = n_t, w_t(j) = w_t$), we eventually get equation (4), where $\mathcal{M}_w = \varkappa/(\varkappa-1)$.

B NFA dynamics

From household i's budget constraint, conditional on survival, individual wealth evolves as follows:

$$a_t^s(i) = \frac{(1-\tau^a)R_{t-1}}{\vartheta}\left\{a_{t-1}^s(i) + w_{t-1}n_{t-1}e_{t-1}^s(i) + \mathcal{D}_{t-1} + \mathbb{T}_{t-1} - c_{t-1}^s(i)\right\}$$

Aggregating the latter over individuals of the same cohort, we get:

$$a_t^s = \frac{(1-\tau^a)R_{t-1}}{\vartheta}\left\{a_{t-1}^s + w_{t-1}n_{t-1} + \mathcal{D}_{t-1} + \mathbb{T}_{t-1} - c_{t-1}^s\right\}$$

The amount a_t is transferred to every newborn agent at the beginning of time t. Aggregating

cohort-level budget constraints over cohorts, we get:

$$
\begin{aligned}
a_t &= (1-\vartheta)\sum_{s=-\infty}^{t}\vartheta^{t-s}a_t^s \\
&= (1-\vartheta)\underbrace{a_t^t}_{=a_t} + (1-\vartheta)\,\vartheta a_t^{t-1} + (1-\vartheta)\,\vartheta^2 a_t^{t-2} + ... \\
\vartheta a_t &= (1-\vartheta)\,\vartheta\left\{\frac{(1-\tau^a)R_{t-1}}{\vartheta}\left\{a_{t-1}^{t-1} + w_{t-1}n_{t-1} + \mathcal{D}_{t-1} + \mathbb{T}_{t-1} - c_{t-1}^{t-1}\right\}\right\} \\
&\quad + (1-\vartheta)\,\vartheta^2\left\{\frac{(1-\tau^a)R_{t-1}}{\vartheta}\left\{a_{t-1}^{t-2} + w_{t-1}n_{t-1} + \mathcal{D}_{t-1} + \mathbb{T}_{t-1} - c_{t-1}^{t-2}\right\}\right\} + ... \\
&= (1-\tau^a)R_{t-1}\left\{(1-\vartheta)\sum_{s=-\infty}^{t-1}\vartheta^{t-s-1}\left(a_{t-1}^s - c_{t-1}^s\right) + w_{t-1}n_{t-1} + \mathbb{T}_{t-1}\right\} \\
&= (1-\tau^a)R_{t-1}\left\{a_{t-1} - c_{t-1} + w_{t-1}n_{t-1} + \mathcal{D}_{t-1} + \mathbb{T}_{t-1}\right\}
\end{aligned}
$$

Aggregate dividends are given by $\mathcal{D}_t = p_{H,t}y_t - (1-\tau)w_t n_t$, while the transfer $\mathbb{T}_t$ is given by equation (27). Substituting those into the previous equation, rearranging and leading one period, we get the expression (26) for the dynamics of aggregate Home savings.

C Aggregate Euler equation

We here detail the steps going from (28) to (33). To ease notations, define $\widetilde{R}_t \equiv (1-\tau^a)R_t$ and take log of (7) to write:

$$
c_t^s(i) = -\frac{\ln\beta\widetilde{R}_t}{\gamma} + \mathbb{E}_t\left[c_{t+1}^s(i)\right] - \frac{\gamma}{2}\mathbb{V}_t\left[c_{t+1}^s(i)\right] \tag{43}
$$

From the individual budget constraint (2), conditional on survival, individual wealth evolves as follows:

$$
a_{t+1}^s(i) = \frac{R_t}{\vartheta}[(1-\tau^a)\,a_t^s(i) + (1-\tau^w)w_t n_t e_t^s(i) + \mathcal{D}_t + \mathbb{T}_t - c_t^s(i)]
$$

Using the shock process (3), we find

$$\begin{aligned}
a_{t+1}^s(i) &= \frac{R_t}{\vartheta}\left\{(1-\tau^a)a_t^s(i) + w_t n_t e_t^s(i) + \mathcal{D}_t + \mathbb{T}_t - c_t^s(i)\right\} \\
&= \frac{R_t}{\vartheta}\left\{(1-\tau^a)a_t^s(i) + w_t n_{k,t}\left[1+\sigma_t\xi_t^s(i)\right] + \mathcal{D}_t + \mathbb{T}_t - c_{k,t}^s(i)\right\} \\
&= \frac{R_t}{\vartheta}\left\{(1-\tau^a)a_t^s(i) + \underbrace{w_t n_t \sigma_t}_{=\sigma_t^y}\xi_t^s(i) + \underbrace{w_t n_t + \mathcal{D}_t + \mathbb{T}_t}_{=\frac{\vartheta a_{t+1}}{R_t}+c_t-a_t} - c_{k,t}^s(i)\right\} \\
&= a_{t+1} + \frac{R_t}{\vartheta}\left\{(1-\tau^a)\left(a_t^s(i)-a_t\right) + \sigma_t^y\xi_t^s(i) - \underbrace{\left(c_t^s(i)-c_t\right)}_{=\mu_t[a_t^s(i)-a_t+h_t^s(i)]}\right\} \\
&= a_{t+1} + \frac{R_t}{\vartheta}\left\{(1-\tau^a)(1-\mu_t)\left(a_t^s(i)-a_t\right) + \sigma_t^y\xi_t^s(i) - \mu_t\underbrace{h_t^s(i)}_{=\sigma_t^h\xi_t^s(i)}\right\} \\
&= a_{t+1} + \frac{R_t}{\vartheta}\left\{(1-\tau^a)(1-\mu_t)\left(a_t^s(i)-a_t\right) + \left(\sigma_t^y - \mu_t\sigma_t^h\right)\xi_t^s(i)\right\}
\end{aligned}$$

Next, using the conjectured consumption function (28), we can write individual consumption as:

$$\begin{aligned}
c_{t+1}^s(i) &= c_{t+1} + \mu_{t+1}(a_{t+1}^s(i) - a_{t+1} + h_{t+1}^s(i)) \\
&= c_{t+1} + \mu_{t+1}\left[\frac{R_t}{\vartheta}[(1-\tau^a)(1-\mu_t)(a_t^s(i)-a_t) + (\sigma_t^y - \mu_t\sigma_t^h)\xi_t^s(i)] + \sigma_{t+1}^h\xi_{t+1}^s(i)\right] \\
&= c_{t+1} + \mu_{t+1}\left[\frac{R_t}{\vartheta}[(1-\tau^a)(1-\mu_t)(a_t^s(i)-a_t) + (\sigma_t^y - \mu_t\sigma_t^h)\xi_t^s(i)] + \sigma_{t+1}^h\lambda\xi_t^s(i) + \sigma_{t+1}^h\upsilon_{t+1}^s(i))\right]
\end{aligned}$$

This implies that the conditional moments of individual consumption are:

$$\begin{aligned}
\mathbb{E}_t\left[c_{t+1}^s(i) - c_{t+1}\right] = \mu_{t+1}\frac{R_t}{\vartheta}(1-\tau^a)(1-\mu_t)\left(a_t^s(i)-a_t\right) \\
+ \mu_{t+1}(1-\tau)\frac{R_t}{\vartheta}\left(\sigma_{y,t}-\sigma_t^c\right)\xi_t^s(i) + \sigma_{t+1}^c\lambda\xi_t^s(i)
\end{aligned}$$

and

$$V_t\left[c_{t+1}^s(i) - c_{t+1}\right] = \left(\sigma_{t+1}^c\right)^2 \tag{44}$$

Plugging those into the individual Euler equation above gives:

$$\begin{aligned}
c_t + \mu_t\left(a_t^s(i)-a_t\right) + \sigma_{t+1}^c\xi_t^s(i) = -\frac{\ln\beta\widetilde{R}_t}{\gamma} + c_{t+1} \\
+ \mu_{t+1}\frac{\widetilde{R}_t}{\vartheta}(1-\mu_t)\left(a_t^s(i)-a_t\right) + \left[\mu_{t+1}\frac{\widetilde{R}_t}{\vartheta}\left(\sigma_{y,t}-\sigma_t^c\right) + \lambda\sigma_{t+1}^c\right]\xi_t^s(i) - \frac{\gamma}{2}\sigma_{t+1}^{c2}
\end{aligned}$$

Matching coefficients in $a_t^s(i) - a_t$, we get:

$$\mu_t = \mu_{t+1}\frac{\widetilde{R}_t}{\vartheta}[(1-\mu_t) \Rightarrow \mu_t^{-1} = 1 + \frac{\vartheta}{\widetilde{R}_t}\mu_{t+1}^{-1}$$

Matching coefficients in $\xi_t^s(i)$, we get:

$$\mu_t\sigma_t^h = \mu_{t+1}\sigma_{t+1}^h\lambda + \mu_{t+1}\frac{\widetilde{R}_t}{\vartheta}(\sigma_t^y - \mu_t\sigma_t^h) \Rightarrow \sigma_t^h = \sigma_t^y + \left(\frac{\vartheta\lambda}{\widetilde{R}_t}\right)\sigma_{t+1}^h$$

Eliminating those terms, we are eventually left with (33).

D Consumption dispersion

D.1 Within cohorts

We start by characterising the evolution of individual wealth, before we turn to consumption. From individual Euler equation, we infer that

$$c_t^s(i) - c_t = \mathbb{E}_t[c_{t+1}^s(i) - c_{t+1}]$$

Using the consumption rule and rearranging, we obtain:

$$\begin{aligned}
\mu_t(a_t^s(i) - a_t + \sigma_t^h\xi_t^s(i)) &= \mathbb{E}_t[\mu_{t+1}(a_{t+1}^s(i) - a_{t+1} + \sigma_{t+1}^h\xi_{t+1}^s(i))] \\
\frac{\mu_t}{\mu_{t+1}}(a_t^s(i) - a_t) + \frac{\mu_t}{\mu_{t+1}}\sigma_t^h\xi_t^s(i) &= a_{t+1}^s(i) - a_{t+1}(i) + \sigma_{t+1}^h\lambda\xi_{k,t}^s(i) \\
a_{t+1}^s(i) - a_{t+1} &= \frac{\mu_t}{\mu_{t+1}}(a_t^s(i) - a_t) + \left(\frac{\mu_t\sigma_t^h}{\mu_{t+1}} - \lambda\sigma_{t+1}^h\right)\xi_t^s(i)
\end{aligned}$$

Next, adding $\sigma_{t+1}^h\xi_{t+1}^s(i)$ on both sides and rearranging, we get:

$$\begin{aligned}
\underbrace{a_{t+1}^s(i) - a_{t+1} + \sigma_{t+1}^h\xi_{t+1}^s(i)}_{=c_{t+1}^s(i)-c_{t+1}} &= \frac{\mu_t}{\mu_{t+1}}(a_t^s(i) - a_t) + \left(\frac{\mu_t\sigma_t^h}{\mu_{t+1}} - \lambda\sigma_{t+1}^h\right)\xi_t^s(i) + \sigma_{t+1}^h\xi_{t+1}^s(i) \\
c_{t+1}^s(i) - c_{t+1} &= \frac{\mu_t}{\mu_{t+1}}\underbrace{(a_t^s(i) - a_t + \sigma_t^h\xi_t^s(i))}_{=c_t^s(i)-c_t} + \sigma_{t+1}^h\upsilon_{t+1}^s(i) \\
c_t^s(i) - c_t &= c_{t-1}^s(i) - c_{t-1} + \underbrace{\mu_t\sigma_t^h\upsilon_t^s(i)}_{\text{consumption innovation}}
\end{aligned}$$

This gives us the within-cohort s moments:

$$\begin{aligned}
\text{mean} &: c_t^s - c_t = c_{t-1}^s - c_{t-1} = 0 \\
\text{variance} &: \sigma^c(s,t)^2 = \sigma^c(s,t-1)^2 + (\sigma_t^c)^2
\end{aligned}$$

D.2 Across cohorts

Consider the calculation of aggregate welfare from the point of view of a domestic social planner given equal welfare weight to all residents. First, start with the utility brought at time t to the

planner by household i from cohort s:

$$\mathcal{V}_t(s,t,i) = u(c_t^s(i)) - v(n_t) = u(c_t)e^{-\gamma[c_t^s(i)-c_t]} - v(n_t)$$

where $u(c) = -e^{-\gamma c}/\gamma$. Aggregating over all individuals of the same cohort, we get:

$$\mathcal{V}_t(s,t) = u(c_t)\int e^{-\gamma[c_t^s(i)-c_t]}di - v(n_t)$$

which, given the within-cohort, cross-sectional variance of consumption (equation (35)), can be written as:

$$\mathcal{V}_t(s,t) = u(c_t)e^{\frac{1}{2}\gamma^2\sigma_c^2(s,t)} - v(n_t)$$

Now, aggregating over cohorts, we get:

$$\begin{aligned}
\mathbb{U}_t &= \sum_{s=-\infty}^{t}(1-\vartheta)\vartheta^{t-s}\mathcal{V}_t(s,t) \\
&= u(c_t)\underbrace{(1-\vartheta)\sum_{k=0}^{\infty}\vartheta^k e^{\frac{1}{2}\gamma^2\sigma_c^2(t-k,t)}}_{\equiv\Sigma_t} - v(n_t) \\
&= u(c_t)\Sigma_t - v(n_t)
\end{aligned}$$

Now, using the moments of the consumption distribution within cohorts (computed above), we can compute:

$$\begin{aligned}
\Sigma_t &= (1-\vartheta)\sum_{k=0}^{\infty}\vartheta^k e^{-\gamma[c_t^{t-k}-c_t]}e^{\frac{1}{2}[\gamma\sigma_c(t-k,t)]^2} \\
&= (1-\vartheta)\sum_{k=0}^{\infty}\vartheta^k e^{\frac{1}{2}\gamma^2\sigma_c^2(t-k,t)} \\
&= (1-\vartheta)e^{\frac{1}{2}[\gamma\sigma_t^c]^2} + \vartheta(1-\vartheta)\left\{e^{\frac{1}{2}[\gamma\sigma_c(t-1,t)]^2} + \vartheta e^{\frac{1}{2}[\gamma\sigma_c(t-2,t)]^2}...\right\} \\
&= (1-\vartheta)e^{\frac{1}{2}[\gamma\sigma_t^c]^2} + \vartheta e^{\frac{1}{2}[\gamma\sigma^c]^2}(1-\vartheta)\left\{e^{\frac{1}{2}[\gamma\sigma_c(t-1,t-1)]^2} + \vartheta e^{\frac{1}{2}[\gamma\sigma_c(t-2,t-1)]^2}...\right\} \\
&= (1-\vartheta)e^{\frac{1}{2}[\gamma\sigma_t^c]^2} + \vartheta e^{\frac{1}{2}[\gamma\sigma^c]^2}\Sigma_{t-1} \\
&= e^{\frac{1}{2}[\gamma\sigma_t^c]^2}\left[1-\vartheta+\vartheta\Sigma_{t-1}\right]
\end{aligned}$$

thereby arriving at equation (36).

E Optimal Policy Problem

The planner's per-period felicity function can be written as:

$$\mathbb{U}_t = -\frac{1}{\gamma} e^{-\gamma c_t} \Sigma_t - \rho e^{\frac{1}{\rho}(n_t - \overline{n})}$$

Using the labor supply equation (4) to substitute out the second term, we get:

$$\mathbb{U}_t = -\frac{1}{\gamma} \left(1 + \gamma \rho \Omega w_t\right) e^{-\gamma c_t} \Sigma_t,$$

where $\Omega = \frac{1-\tau^w}{\mathcal{M}_w}$. Then, the Ramsey planner's problem can be written as choosing the sequences $\{c_t, Q_t, R_t, \sigma_{c,t}, \mu_t, a_{t+1}, \Sigma_t, \Pi_{H,t}, w_t\}_{t=0}^{\infty}$ to maximize $\sum_{t=0}^{\infty} \beta^t \mathbb{U}_t$ subject to (i) the aggregate Euler equation

$$-c_t - \frac{1}{\gamma} \ln \beta (1 - \tau^a) R_t + c_{t+1} - \frac{\gamma}{2} \sigma_{c,t+1}^2,$$

(ii) the evolution of NFA

$$\frac{\vartheta}{R_t} a_{t+1} = \vartheta a_t + p_{H,t} y_t - c_t,$$

(iii) the interest parity condition

$$R_t e^{\wp \beta \vartheta a_{t+1}} = R_t^* \frac{Q_{t+1}}{Q_t},$$

(iv) the relationship between wages and aggregate output

$$y_t = z_t \frac{1 + \gamma\rho + \rho \ln\left(\frac{w_t}{w_t}\right)\left(\frac{\Sigma_t}{\Sigma}\right) - \gamma\rho c_t}{1 + \frac{\Psi}{2}\left(\ln \Pi_{H,t}\right)^2},$$

(v) the Home goods market clearing condition

$$y_t = (1 - \alpha) p_{H,t}^{-\eta} c_t + \alpha \left(\frac{p_{H,t}}{Q_t}\right)^{-\nu} c_t^*,$$

(vi) the Phillips curve

$$\ln \Pi_{h,t} = \kappa \left[1 - \frac{\chi - 1}{\chi - 1 + \alpha} \frac{p_{H,t} z_t}{w_t}\right] + \beta \frac{z_t w_{t+1} y_{t+1}}{z_{t+1} w_t y_t} \ln \Pi_{H,t+1},$$

(vii) the evolution of average consumption risk

$$\sigma_{c,t} = \mu_t \sigma_{y,t} + \lambda \left(1 - \mu_t\right) \sigma_{c,t+1},$$

(viii) the evolution of the welfare cost of inequality:

$$\ln \Sigma_t = \frac{\gamma^2 \sigma_{c,t}^2}{2} + \ln\left[1 - \vartheta + \vartheta \Sigma_{t-1}\right],$$

and (ix) the evolution of μ_t

$$\mu_t^{-1} = 1 + \frac{\vartheta}{(1-\tau^a)R_t}\mu_{t+1}^{-1},$$

This problem can be expressed as the following Lagrangian:

$$\begin{aligned}
\mathcal{L} &= \sum_{t=0}^{\infty}\beta^t\left\{-\frac{1}{\gamma}\left(1+\gamma\rho\Omega w_t\right)e^{-\gamma c_t}\Sigma_t\right\} \\
&+\sum_{t=0}^{\infty}\beta^t\gamma\mathbb{U}m_{1,t}\left\{-c_t-\frac{1}{\gamma}\ln\beta(1-\tau^a)R_t+c_{t+1}-\frac{\gamma}{2}\sigma_{c,t+1}^2\right\} \\
&+\sum_{t=0}^{\infty}\beta^t\gamma\mathbb{U}m_{2,t}\left\{-\frac{\vartheta}{R_t}a_{t+1}+\vartheta a_t+p_{H,t}y_t-c_t\right\} \\
&+\sum_{t=0}^{\infty}\beta^t\gamma\mathbb{U}m_{3,t}\left\{-\ln R_t+\ln R_t^*+\ln Q_{t+1}-\ln Q_t-\beta\wp\vartheta a_{t+1}\right\} \\
&+\sum_{t=0}^{\infty}\beta^t\gamma\mathbb{U}m_{4,t}\left\{1+\gamma\rho+\rho\ln w_t+\rho\ln\left(\frac{\Sigma_t}{\Sigma}\right)-\gamma\rho c_t-\frac{y_t}{z_t}\left[1+\frac{\varepsilon}{2\kappa}\left(\ln\Pi_{H,t}\right)^2\right]\right\} \\
&+\sum_{t=0}^{\infty}\beta^t\gamma\mathbb{U}m_{5,t}\left\{\left(1-\alpha\right)p_{H,t}^{-\eta}c_t+\alpha\left(\frac{p_{H,t}}{Q_t}\right)^{-\nu}c_t^*-y_t\right\} \\
&+\sum_{t=0}^{\infty}\beta^t\gamma\mathbb{U}m_{6,t}\left\{\kappa\left[1-\left(\frac{\chi-1}{\chi-1+\alpha}\right)\frac{p_{H,t}z_t}{w_t}\right]+\beta\left(\frac{z_t}{z_{t+1}}\right)\left(\frac{w_{t+1}}{w_t}\right)\left(\frac{y_{t+1}}{y_t}\right)\ln\Pi_{H,t+1}-\ln\Pi_{H,t}\right\} \\
&+\sum_{t=0}^{\infty}\beta^t\gamma\mathbb{U}m_{7,t}\left\{\sigma_{c,t}-\mu_t\sigma_{y,t}-\lambda\left(1-\mu_t\right)\sigma_{c,t+1}\right\} \\
&+\sum_{t=1}^{\infty}\beta^t\mathbb{U}m_{8,t}\left\{\frac{\gamma^2}{2}\sigma_{c,t}^2+\ln\left[1-\vartheta+\vartheta\Sigma_{t-1}\right]-\ln\Sigma_t\right\} \\
&+\sum_{t=0}^{\infty}\beta^t\gamma\mathbb{U}m_{9,t}\left\{1+\frac{\vartheta}{\left(1-\tau^a\right)R_t}\mu_{t+1}^{-1}-\mu_t^{-1}\right\},
\end{aligned}$$

where $\mathbb{U}=-\frac{1}{\gamma}e^{-\gamma\Sigma}<0$ is a scaling constant. The corresponding first order conditions are given by:

- FOC c_t

$$0 = \left(1+\gamma\rho\Omega w_t\right)e^{-\gamma(c_t-1)}\left(\frac{\Sigma_t}{\Sigma}\right)+m_{1,t}-\beta^{-1}m_{1,t-1}+m_{2,t}+\gamma\rho m_{4,t}-m_{5,t}\left(1-\alpha\right)p_{H,t}^{-\eta}$$

- FOC w

$$\begin{aligned} 0 &= \gamma\rho\Omega w_t e^{-\gamma(c_t-1)}\left(\frac{\Sigma_t}{\Sigma}\right)+\gamma\rho m_{4,t}+\gamma\kappa\left(\frac{\chi-1}{\chi-1+\alpha}\right)m_{6,t}\frac{p_{H,t}z_t}{w_t} \\ &\quad -\gamma m_{6,t}\beta\left(\frac{z_t}{z_{t+1}}\right)\left(\frac{w_{t+1}}{w_t}\right)\left(\frac{y_{t+1}}{y_t}\right)\ln\Pi_{H,t+1} \\ &\quad +\gamma m_{6,t-1}\left(\frac{z_{t-1}}{z_t}\right)\left(\frac{w_t}{w_{t-1}}\right)\left(\frac{y_t}{y_{t-1}}\right)\ln\Pi_{H,t} \end{aligned}$$

- FOC y_t

$$\begin{aligned} 0 &= \gamma m_{2,t}p_{H,t}y_t-\gamma\frac{y_t}{z_t}\left[1+\frac{\varepsilon}{2\kappa}\left(\ln\Pi_{H,t}\right)^2\right]m_{4,t}-\gamma y_t m_{5,t}+\varphi\gamma m_{7,t}\mu_t\sigma_{y,t}y_t \\ &\quad -\beta\gamma m_{6,t}\left(\frac{z_t}{z_{t+1}}\right)\left(\frac{w_{t+1}}{w_t}\right)\left(\frac{y_{t+1}}{y_t}\right)\ln\Pi_{H,t+1}+\gamma m_{6,t-1}\left(\frac{z_{t-1}}{z_t}\right)\left(\frac{w_t}{w_{t-1}}\right)\left(\frac{y_t}{y_{t-1}}\right)\ln\Pi_{H,t} \end{aligned}$$

- FOC R_t

$$0 = -m_{1,t}+\gamma m_{2,t}\frac{\vartheta}{R_t}a_{t+1}-\gamma m_{3,t}-\gamma m_{9,t}\left(\mu_t^{-1}-1\right)$$

- FOC Q_t

$$\begin{aligned} 0 &= \gamma m_{2,t}p'_{H,t}Q_t y_t-\gamma m_{3,t}+\gamma\beta^{-1}m_{3,t-1} \\ &\quad -\gamma m_{5,t}\left\{\eta\left(1-\alpha\right)p_{H,t}^{-\eta-1}p'_{H,t}Q_t c_t+\alpha\nu\left(\frac{p_{H,t}}{Q_t}\right)^{-\nu}\left(\frac{Q_t p'_{H,t}}{p_{H,t}}-1\right)c_t^*\right\} \\ &\quad -\gamma m_{6,t}\kappa\left(\frac{\chi-1}{\chi-1+\alpha}\right)\frac{p'_{H,t}Q_t z_t}{w_t} \end{aligned}$$

- FOC a_{t+1}

$$0 = \beta R_t m_{2,t+1}-m_{2,t}+\beta R_t\wp m_{3,t}$$

- FOC $\Pi_{H,t}$

$$0 = -m_{4,t}\frac{y_t}{z_t}\frac{\varepsilon}{\kappa}\ln\Pi_{H,t}-m_{6,t}+\left(\frac{z_{t-1}}{z_t}\right)\left(\frac{w_t}{w_{t-1}}\right)\left(\frac{y_t}{y_{t-1}}\right)m_{6,t-1}$$

- FOC μ_t

$$0 = m_{7,t}\left(\lambda\sigma_{c,t+1}-\sigma_{c,t}\right)+m_{9,t}\mu_t^{-1}-\beta^{-1}m_{9,t-1}\left(\mu_{t-1}^{-1}-1\right)$$

- FOC $\sigma_{c,t}$

$$0 = \left(-\beta^{-1}m_{1,t-1}+m_{8,t}\right)\gamma\sigma_{c,t}+m_{7,t}-\beta^{-1}\lambda\left(1-\mu_{t-1}\right)m_{7,t-1}$$

- FOC Σ_t

$$0 = (1+\gamma\rho\Omega w_t)\, e^{-\gamma(c_t-1)} \left(\frac{\Sigma_t}{\Sigma}\right) + \gamma\rho m_{4,t} - m_{8,t} + m_{8,t+1}\frac{\beta\vartheta\Sigma_t}{1-\vartheta+\vartheta\Sigma_t}$$