MARKET DESIGN FOR SOCIAL JUSTICE: A CASE STUDY ON A CONSTITUTIONAL CRISIS IN INDIA

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ABSTRACT. In a 3-2 split verdict, the Supreme Court approved the exclusion of India's socially and economically backward classes from its affirmative action measures to address economic deprivation. Dissenting justices, including the Chief Justice of India, protested the Majority opinion for sanctioning "an avowedly exclusionary and discriminatory principle." In order to justify their controversial decision, majority justices relied on technical arguments which are categorically false. The confusion of the justices is due to a combination of two related but subtle technical aspects of the affirmative action system in India. The first aspect is the significance of overlaps between members of protected groups, and the second one is the significance of the processing sequence of protected groups in the presence of such overlaps. Conventionally, protected classes were determined by the caste system, which meant they did not overlap. Addition of a new protected class defined by economic criteria alters this structure, unless it is artificially enforced. The majority justices failed to appreciate the significance of these changes, and inaccurately argued that the controversial exclusion is a technical necessity to provide benefits to previously-unprotected members of a new class. We show that this case could have been resolved with three competing policies that each avoids the controversial exclusion. One of these policies is in line with the core arguments in the Majority opinion, whereas a second one is in line with those in the Dissenting opinion.

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This paper replaces and subsumes Sönmez and Ünver (2022), an earlier version of the paper that was circulated before the judgment *Janhit Abhiyan vs Union of India (2022)* was delivered in a 3-2 split verdict of the Supreme Court of India. Both Sönmez and Ünver are affiliated with the Department of Economics, Boston College, 140 Commonwealth Ave, Chestnut Hill, MA, 02467. Emails: sonmezt@bc.edu, unver@bc.edu. We thank Ashwini Deshpande and Manshu Khanna for their detailed comments.

1. Introduction

Affirmative action for India's socially and educationally disadvantaged classes is embedded into its Constitution through a powerful positive discrimination policy called *vertical reservations (VR)*. Prior to the 103rd Constitutional Amendment,[1] the sole beneficiaries of the VR policy were classes who faced various degrees of social marginalization on the basis of their hereditary caste identity. Under the conventional VR policy, envisioned as a reparative and compensatory mechanism to level the playing field, 17.5% of the positions at government jobs and institutions of higher education are set aside for members of Scheduled Castes (SCs), 7.5% of the positions are set aside for members of Scheduled Tribes (STs), and 27% of the positions are set aside for members of Other Backward Classes (OBCs).[2] With a highly contentious Amendment to the Constitution in 2019, additional VR protections of up to 10% of the positions is granted for the members of a new category called Economically Weaker Sections (EWS). Eligibility for EWS is provided to individuals in financial incapacity, but controversially it was restricted to individuals who remain outside the scope of the earlier reparative and compensatory VR protections. As a result, EWS reservations serve as a positive discrimination policy for a subclass of India's forward classes.

The Amendment was immediately challenged by several groups, and ultimately advanced to a five-judge Constitution Bench of the Supreme Court in August 2020. In a first ever live-streamed Supreme Court hearings in India, on September 7th, 2022 the Bench announced its decision to address the following three main issues to determine whether the Amendment violates the basic structure of the Constitution:[3]

(1) Can reservations be granted solely on the basis of economic criteria?
(2) Can states provide reservations in private educational institutions which do not receive government aid?
(3) Are EWS reservations constitutionally invalid for excluding SCs, STs, and OBCs, from its scope?

In relation to the third (and the most contentious) issue, advocates for the petitioners repeatedly argued that the exclusionary clause in the Amendment violates the country's

[1]The Amendment is officially called the *Constitution (One Hundred and Third Amendment) Act, 2019*.

[2]Scheduled Castes is the official term for Dalits or "untouchables," who endured millennia-long oppression and discrimination due to their lowest status under the caste system. Scheduled Tribes is the official term for the indigenous ethnic groups of India, whose faced oppression due to their isolation and exclusion from mainstream society. Other Backward Classes is the official term that describes lower-level castes who were engaged in various marginal occupation assigned to them by the society to serve castes higher to them in the caste hierarchy.

[3]See the coverage of the case in the *Supreme Court Observer*, last retrieved on 11/14/2022.

Equality Code. On the last day of the hearings and as a "compromise" between the two sides, Prof. Mohan Gopal–representing the petitioners–suggested an alternative way forward that did not involve striking down the Amendment. Under this compromise, individuals who are covered by the existing VR protections are not excluded from the scope of the EWS reservations.[4] This compromise policy, henceforth referred to as the *scope-expanded EWS policy*, plays a central role in our analysis.

In a landmark judgment *Janhit Abhiyan vs. Union of India (2022)* that fundamentally changed the role of affirmative action in India, the Constitution Bench reached its verdict on November 7th, 2022. The Supreme Court upheld the 103rd Constitutional Amendment, although in a 3-2 split verdict.[5] While all five justices agreed that reservations can be granted solely on the basis of economic criteria, the two dissents–including the Chief Justice of India–strongly disagreed with the majority justices on the constitutionality of the exclusion of socially and educationally disadvantaged classes from the scope of EWS. The extent of the disagreement can be vividly seen in the following opening paragraph of the Dissenting opinion by Justice Ravindra Bhat:

> 1. I regret my inability to concur with the views expressed by the majority opinion on the validity of the 103rd Amendment on Question No. 3, since I feel - for reasons set out elaborately in the following opinion - that this court has for the first time, in the seven decades of the republic, sanctioned an avowedly exclusionary and discriminatory principle. Our Constitution does not speak the language of exclusion. In my considered opinion, the Amendment, by the language of exclusion, undermines the fabric of social justice, and thereby, the basic structure.

While the verdict was declared as a major victory for the central government led by Prime Minister Narendra Modi,[6] according to many media outlets, it also created an uproar in the country.[7] Many elements of the turmoil caused by the judgment is summarized as

[4]See, the document on EWS Reservation Day #8: Responses to Governments Arguments on Last Day of Hearing by the *Supreme Court Observer*, last retrieved on 10/01/2022.

[5]There are four separate judgments for *Janhit Abhiyan vs. Union of India (2022)*, three for each of the three majority justices, and one for the two dissents. The judgment that contains all four opinions is available in the following link: https://www.scobserver.in/wp-content/uploads/2021/10/EWS-Reservations-Judgment.pdf. See also the Majority opinion of Justice Dinesh Maheshwari, the Majority opinion of Justice Bela Trivedi, the Majority opinion of Justice J.B. Pardiwala, and the Dissenting opinion of Justice Ravindra Bhat for himself and the Chief Justice of India Uday Umesh Lalit. All links are last accessed on 11/11/2022.

[6]See, for example, the *The Hindu* story "EWS quota verdict historic, a victory for PM's mission: BJP."

[7]See, for example, the *Al Jazeera* story "Why 10% quota for 'economically weak' in India has caused uproar," and the *LiveLaw* interview"EWS Quota Is Reservation For The Over-Represented, Excludes The Real Unrepresented : Mohan Gopal On Supreme Court Judgment."

follows by Anup Surendranath, a Professor of Law at the National Law University at Delhi:[8]

> It is constitutionally perverse that the compelling need for measures to address social backwardness has become a justification for the exclusion of backward classes from measures to address economic deprivation.
>
> The historical confusions in the Supreme Court's reservation jurisprudence have come home to roost in the EWS judgment. Despite seven decades of constitutional adjudication on reservation, fundamental questions remain unexamined resulting in a jurisprudence that permits constitutionally perverse outcomes. More precisely, the constitutionally perverse outcomes resulting from the majority judgment are: India's most marginalised sections that comprise a significant proportion of India's poor stand excluded from reservation meant for the poor, and second, it is now far easier to provide reservation for this narrowly constructed EWS than it is to do the same for India's most marginalised sections. These outcomes are fuelled by a flawed constitutional logic that does tremendous disservice to the founding constitutional agreement, social history and lived reality of India's most vulnerable sections.

The main outrage against the judgment stems from the widespread perspective that it betrays the philosophy of affirmative action in India. We agree with this perspective. In order to contribute to this important debate, we present an analysis of the judgment from a completely different angle. Our paper can be seen as a complementary effort that presents the flaws of the Majority opinion from a market design perspective. Following our formal analysis in Section 2, in Section 3 we refute the main (and what appears to be pivotal) arguments in the Majority opinion that is used to justify the exclusion of socially and educationally backward classes from the scope of EWS reservation. Therefore, we conclude that the majority justices fundamentally altered the structure of affirmative action in India on completely false premises. In contrast, our formal analysis highlights the virtue of the Dissenting opinion, and shows how it could have been operationalized through three alternative refinements of the scope-expanded EWS policy, each with a distinct normative interpretation.

The confusion of the majority justices is largely due to a subtle technical aspect of the county's reservation system, which we discuss next.

1.1. Implementation of VR Policy. The landmark Supreme Court judgment *Indra Sawhney vs Union Of India (1992)*, widely known as the *Mandal Commission Case*, serves as the

[8]See *The Indian Express* opinion "EWS reservation: Supreme Court has not clarified tricky questions at the intersection of equality, non-discrimination, and affirmative action," last accessed 11/18/2022.

main reference for the reservation system in India.[9] The term *vertical reservations* was also coined in this ruling, which formulated the defining characteristics of this primary affirmative action provision in India as follows:

- No VR-protected position should be allocated to an individual who deserves an open (i.e. unreserved) position based on merit without invoking the provisions of the VR policy. VR-protected positions should instead be awarded to highest merit individuals who do not merit an open position.
- VR-protected positions are exclusive to members of the protected group.

When no individual is eligible for multiple VR-protected groups, as it has always been, these two defining characteristics together imply that positions should be allocated with the following *Over-and-Above* (O&A) choice rule: First, open positions are awarded to individuals with highest merit scores, and next, for each VR-protected category, the protected positions are awarded to remaining members of the category with highest merit scores.

Because (i) there is no overlap between the members of any two VR-protected categories and (ii) VR-protected positions are exclusively reserved for their beneficiaries, it does not matter in what sequence (or other form) the VR-protected positions are allocated for each category under the O&A choice rule. That is because, provided that both conditions hold, no individual competes for positions at multiple VR-protected categories, thus rendering the competitions at VR-protected categories completely independent from each other. Barring some rare exceptions, both conditions hold in India. Once either condition is relaxed, however, this conclusion no longer holds. In that case allocation of positions at VR-protected categories interfere with each other, and thus potentially affect the distribution of VR-protected positions. This is why the scope-expanded EWS policy fundamentally alters a key aspect of the reservation system. Without the specification of an additional (and admittedly subtle) aspect of the system (i.e. when EWS positions are allocated in relation to positions at other VR-protected categories), a mere expansion of the scope of the EWS reservation to cover all individuals with financial disability no longer results in a well-defined system under the current legislation. In our view, this technical oversight is one of the main culprits for the flawed arguments in the Majority opinion.

1.2. Consequences of Overlapping VR Protections. So, how important is this technical aspect of the reservation system? The short answer to this question is, it is very important! To explain why that is the case, we next consider the following two alternative refinements of the scope-expanded EWS policy:

[9]The judgment is available in `https://indiankanoon.org/doc/1363234/`, last retrieved on 03/10/2022.

(1) *EWS-last VR processing policy*: EWS positions are processed after all other VR-protected positions.
(2) *EWS-first VR processing policy*: EWS positions are processed prior to all other VR-protected positions.

First, recall that the two policies overlap under the current legislation with non-overlapping VR protections. What is the effect of an expansion of the scope of EWS eligibility under the *EWS-last VR processing policy*? The dissenting justices concluded that the Equality Code is violated by the exclusion of members of SC/ST/OBC from the scope of EWS. It is formally possible to identify the specific members of these classes who are directly affected by this violation by losing a position they would have received in the absence of the violation (Lemma 4). In Theorem 2 we show that, under the *EWS-last VR processing policy*,

- this very group of the "compromised" members of SC/ST/OBC replace those individuals from forward classes who each receive a position due to violation with lower scores,
- but otherwise, the rest of the positions are allocated exactly to the same individuals as the current policy.

Thus, the *EWS-last VR processing policy* is literally the smallest possible deviation from the current controversial policy that avoids a violation of the Equality Code. In particular, if no member of SC/ST/OBC loses a position due to their exclusion from the scope of EWS under the current policy, then the outcome of the *EWS-last VR policy* is identical to that of the current policy (Corollary 2). Essentially, the *EWS-last VR processing policy* continues to provide its primary benefits to those who are ineligible for earlier conventional VR-protected categories, but it does so in a way that avoids a violation of the Equality Code. As we later argue in Section 3.2, in our view, the arguments made in the Majority opinion–once the technical oversight on the impact of overlapping VR protections are ignored–are in line with a refinement of the expanded-scope EWS policy through the EWS-last VR processing policy.

What about the effect of a potential expansion of the scope of EWS-category eligibility under the *EWS-first VR processing policy*? This effect is especially easy to see under the current and very laxed income eligibility level for EWS. According to Deshpande and Ramachandran (2019), 98% of the Indian population earns below the annual income limit Rs 8 lakh to be eligible for the EWS reservation. Hence, it is fairly informative to consider a scenario where everyone is eligible for the EWS reservation. Under this assumption, the *EWS-first VR processing policy* is equivalent to completely striking down the EWS reservation! Thus, under the current income limit from EWS eligibility, the compromise policy

has a version that pretty much accounts to eliminating the EWS reservation. Therefore, as we emphasize in Section 3.2, the arguments in the Majority opinion are technically accurate under a refinement of the expanded-scope EWS policy through the EWS-first VR processing policy. On the other hand, these arguments are completely false under a refinement of the expanded-scope EWS policy through the EWS-last VR processing policy.

1.3. Normative Implications of the Three Focal VR Processing Policies. The contrast between the EWS-first and EWS-last VR-processing policies presented in Section 1.2 is not meant to be one that endorses one policy or another, but rather an illustration of the range of policies that can be supported under various refinements of the scope-expanded EWS policy. Indeed, EWS-first or EWS-last VR processing policies are not the only policies that are normatively plausible. By addressing the main disagreement between the Majority and Dissenting opinions, i.e., by the removal of the exclusion clause from the Amendment, this range of policies offer a more fertile ground for reaching a consensus. Our approach here is in the spirit of maintaining *informed neutrality* between reasonable but competing ethical principles (Li, 2017), and minimizing the *normative gap* between intended and implemented normative objectives (Hitzig, 2020).

If the normative objective of the court is to avoid a violation of the Equality Code, but otherwise, to minimally interfere with the Amendment, then EWS positions have to be allocated after all other VR-protected positions (Theorems 2 and 4). If the normative objective of the court is to maintain the elevated status of caste-based VR protections (as reparatory and compensatory provisions) by allowing mobility from caste-based VR-protected categories to EWS, then EWS positions have to be allocated prior to all other VR-protected positions (Theorems 3 and 5). If the normative objective of the court is to maintain neutrality between all VR-protected categories (including EWS) and to enforce a policy that awards the positions to highest merit individuals subject to the earlier base mandates of the Supreme Court, then positions at all VR-protected categories have to be allocated simultaneously through a maximal matching algorithm (Theorem 1). Given that the normative justifications of these seemingly similar policies are vastly different, we believe it is in the best interests of the society to understand their distinction. This is especially the case given the very different distributional implications the EWS-first VR processing policy has compared to the other two policies.

We circulated the first version our analysis in the working paper Sönmez and Ünver (2022), after the hearings for *Janhit Abhiyan (2022)* was completed in September 2022, but before a verdict was reached in November 2022. Since much of the questions were asked by the two dissenting justices during the hearings, at the time we came to the false conclusion that the Bench may likely give a decision that is parallel to the compromise policy

of Prof. Mohan Gopal, thus they may revoke the exclusion clause but allow for reservation based on economic criteria.[10] In Sönmez and Ünver (2022), we caution against a major loophole that would emerge under this scenario. Collaborating with Ashoka University economist Ashwini Deshpande, a renowned expert in the reservation system, we also brought this possibility to attention of the general population and judiciary in India in an opinion piece at *The Hindu* a week before the announcement of the verdict. While the focus of the opinion piece was on the potential loophole, it also clearly discussed the richness of the policies that can be accommodated under the scope-expanded EWS policy. Hence, our findings that directly contradict some of the main arguments in the Majority opinion were available in India prior to the verdict.

The rest of this paper is organized as follows: In Section 2, we present and analyze the basic version of our model with VR policy only. Related literature in market design is presented at the end of this section. In Section 3, we relate our formal analysis to the Supreme Court judgment *Janhit Abhiyon (2022)*, and present a critical analysis of Majority and Dissenting opinions. We conclude in Section 4. Presentation and analysis of a more general model that also includes *horizontal reservations*–a secondary affirmative action policy that provides minimum guarantees for various protected groups–is relegated to Appendix A. Finally, technical preliminaries are presented in Appendix B, and the proofs of formal results are presented in Appendix C.

2. Model and Analysis

There is a finite set $\mathcal{I}$ of individuals who are competing for $q^{\Sigma} \in \mathbb{N}$ identical positions. Each individual $i \in \mathcal{I}$ is in need of a single position, and has a distinct merit score $\sigma_i \in \mathbb{R}_+$. Let $\sigma = (\sigma_i)_{i \in \mathcal{I}}$ denote the vector of merit scores. In the absence of an affirmative action policy, individuals with higher merit scores have higher claims for a position. Throughout the paper, we fix the set of all individuals $\mathcal{I}$, the number of positions q^{Σ}, and the vector of merit scores σ.

There are two types of affirmative action provisions in India: the primary VR policy and the secondary HR policy. We start our analysis by focusing on VR policy only.[11] This version of our model is a refinement of Kominers and Sönmez (2016). An extended model that includes both VR and HR policies is later presented in Appendix A.

[10] Our conclusion was presented as one of the main possibilities in The Pre-Judgment Matrix of the *Supreme Court Observer*, last accessed 11/19/2022.

[11] Focusing on the primary VR policy enables us to relate our analysis to Supreme Court judgments and policy discussions in India, because, the discussions in the country on the 103rd Amendment completely abstract away from the secondary HR policy.

2.1. Vertical Reservations. Let $\mathcal{R}$ denote the set of **VR-protected categories**. Given an individual $i \in \mathcal{I}$, let $\rho_i \in 2^{\mathcal{R}}$ denote the (possibly empty) set of VR-protected categories she belongs as a member. Let $\rho = (\rho_i)_{i\in\mathcal{I}} \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$ denote the profile of **category memberships**. For each VR-protected category $c \in \mathcal{R}$, let $\mathcal{I}^c(\rho) = \{i \in \mathcal{I} : c \in \rho_i\}$ denote the set of members of category c. We sometimes refer to these individuals as the **beneficiaries** of VR protections at category c. Individuals who do not belong to any VR-protected category are members of a **general category** $g \notin \mathcal{R}$. Let $\mathcal{I}^g(\rho) = \{i \in \mathcal{I} : \rho_i = \emptyset\} = \mathcal{I} \setminus \cup_{c\in\mathcal{R}}\mathcal{I}^c(\rho)$ denote the set of individuals in the general category.

Based on the conventional structure of VR-protected categories in India, papers in the literature assume that no individual belongs to multiple VR-protected categories. Motivated by the Dissenting opinion in *Janhit Abhiyan (2022)*, we drop this assumption. We refer to VR-protected categories as **overlapping** if some individuals are members of multiple VR-protected categories, and as **non-overlapping** if each individual is a member of at most one VR-protected category.

For any VR-protected category $c \in \mathcal{R}$, let $q^c \in \mathbb{N}$ be the number of positions that are exclusively set aside for the members of category c.[12] These provisions are referred to as **VR-protected positions at category *c***. For any VR-protected category $c \in \mathcal{R}$, let $\mathcal{E}^c(\rho) = \mathcal{I}^c(\rho)$ denote the set of individuals who are **eligible for VR-protected positions at category *c***. The total number of VR-protected positions is no more than the number of all positions. That is,

$$\sum_{c\in\mathcal{R}} q^c \leq q^{\Sigma}.$$

All individuals are eligible for the remaining

$$q^o = q^{\Sigma} - \sum_{c\in\mathcal{R}} q^c$$

positions, which are referred to as **open-category** (or **category-*o***) positions. Let $\mathcal{E}^o(\rho) = \mathcal{E}^o = \mathcal{I}$ denote the set of individuals who are **eligible for open-category positions**.

Let $\mathcal{V} = \mathcal{R} \cup \{o\}$ denote the set of **vertical categories for positions**.

2.2. Solution Concepts and Primary Axioms. We next present the solution concepts used in our paper, and the primary axioms imposed on them. Throughout this section, we fix a profile of category memberships $\rho \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$.

Definition 1. Given a category $v \in \mathcal{V}$, a **single-category choice rule** is a function $C^v\left(\rho;.\right) : 2^{\mathcal{I}} \to 2^{\mathcal{I}}$, such that, for any set of individuals $I \subseteq \mathcal{I}$,

$$C^v(\rho; I) \subseteq I \cap \mathcal{E}^v(\rho) \quad \text{and} \quad |C^v(\rho; I)| \leq q^v.$$

[12] This type of protective policy is sometimes referred to as *hard reserve* policy.

That is, for any set of individuals, a single-category choice rule selects a subset from those who are eligible, up to capacity.

Definition 2. A **choice rule** is a multidimensional function $C(\rho;.) = \left(C^v(\rho;.)\right)_{v\in\mathcal{V}} : 2^{\mathcal{I}} \to (2^{\mathcal{I}})^{|\mathcal{V}|}$ such that, for any set of individuals $I \subseteq \mathcal{I}$,

(1) for any category $v \in \mathcal{V}$,

$$C^v(\rho;I) \subseteq I \cap \mathcal{E}^v(\rho) \quad \text{and} \quad |C^v(\rho;I)| \leq q^v,$$

(2) for any two distinct categories $v, v' \in \mathcal{V}$,

$$C^v(\rho;I) \cap C^{v'}(\rho;I) = \emptyset.$$

That is, a choice rule is a list of interconnected single-category choice rules for each category of positions, where no individual is selected by more than a single category.

Definition 3. For any choice rule $C(\rho;.) = \left(C^v(\rho;.)\right)_{v\in\mathcal{V}}$, the resulting **aggregate choice rule** $\widehat{C}(\rho;.) : 2^{\mathcal{I}} \to 2^{\mathcal{I}}$ is given as, for any $I \subseteq \mathcal{I}$,

$$\widehat{C}(\rho;I) = \bigcup_{v\in\mathcal{V}} C^v(\rho;I).$$

For a given profile of category memberships and for any set of individuals, the aggregate choice rule yields the set of chosen individuals across all categories.

As it is discussed in depth in Sönmez and Yenmez (2022a), the following three axioms are mandated in India with the Supreme Court judgment *Indra Sawhney (1992)*.[13] Throughout our analysis, we focus on choice rules that satisfy all three axioms.

Definition 4. A choice rule $C(\rho;.) = \left(C^v(\rho;.)\right)_{v\in\mathcal{V}}$ satisfies **non-wastefulness** if, for any $I \subseteq \mathcal{I}$, $v \in \mathcal{V}$, and $j \in I$,

$$j \notin \widehat{C}(\rho;I) \text{ and } |C^v(\rho;I)| < q^v \quad \Longrightarrow \quad j \notin \mathcal{E}^v(\rho).$$

The first axiom requires no position to remain idle for as long as there is an eligible individual.

Definition 5. A choice rule $C(\rho;.) = \left(C^v(\rho;.)\right)_{v\in\mathcal{V}}$ satisfies **no justified envy** if, for any $I \subseteq \mathcal{I}$, $v \in \mathcal{V}$, $i \in C^v(\rho;I)$, and $j \in \left(I \cap \mathcal{E}^v(\rho)\right) \setminus \widehat{C}(\rho;I)$,

$$\sigma_i > \sigma_j.$$

[13]While *Indra Sawhney (1992)* formulated VR and HR protections separately, it did not specify how the two policies should be implemented jointly. As thoroughly discussed in Sönmez and Yenmez (2022a), this omission was later addressed in the Supreme Court judgment *Saurav Yadav (2020)*, but not before 25 years of confusion and thousands of litigations in India. See Section A.2 in Appendix A for generalizations of these axioms (along with a fourth axiom) for the extended model with VR and HR policies, each formulating the more refined mandates of the Supreme Court judgment *Saurav Yadav (2020)* in relation to joint implementation of the two protective policies.

The second axiom requires that no individual receives a position at any category $v \in \mathcal{V}$ at the expense of another individual who is both eligible for the position and at the same time has higher merit score.

Definition 6. A choice rule $C(\rho;.) = \left(C^v(\rho;.)\right)_{v\in\mathcal{V}}$ satisfies **compliance with VR protections** if, for any $I \subseteq \mathcal{I}$, $c \in \mathcal{R}$, and $i \in C^c(\rho; I)$,

(1) $|C^o(\rho; I)| = q^o$, and
(2) for every $j \in C^o(\rho; I)$,

$$\sigma_j > \sigma_i.$$

The third axiom requires that, an individual who is "deserving" of an open-category position due to her merit score, should be awarded an open-category position and not one that is VR-protected.

2.3. Sequential Choice Rules. In this section we present *sequential* choice rules, a class of choice rules that plays a prominent role in most real-life applications of reserve systems. The name of the class captures the idea that, under each member of the class, all positions in any category (including the open category) are processed in blocks following a given sequence of categories.

Let Δ denote the set of all linear orders on $\mathcal{V}$. Each element of Δ represents a linear processing sequence of vertical categories, and referred to as an **order of precedence**. Let

- $\Delta^o \subsetneq \Delta$ denote the set of processing sequences where the open category is processed first.

For any VR-protected category $c \in \mathcal{R}$, let

- $\Delta^o_c \subsetneq \Delta^o \subsetneq \Delta$ denote the set of processing sequences where the open category is processed first and category c is processed last, and
- $\Delta^{o,c} \subsetneq \Delta^o \subsetneq \Delta$ denote the set of processing sequences where the open category is processed first and category c is processed second.

Given a category $v \in \mathcal{V}$, define the following single-category choice rule: For any set of individuals $I \subseteq \mathcal{I}$, **category-v serial dictatorship** $C^v_{sd}(\rho;.)$ selects the set of highest merit-score individuals in I who are eligible for category v, up to capacity.

Fix an order of precedence $\rhd \in \Delta$. Given a set of individuals $I \subseteq \mathcal{I}$, the outcome of the sequential choice rule $C_S(\rhd, \rho;.)$ is obtained with the following procedure.

Sequential Choice Rule $C_S(\rhd, \rho;.) = \left(C^v_S(\rhd, \rho;.)\right)_{v\in\mathcal{V}}$

Step 0 (Initiation): Let $I_0 = \emptyset$.

Step k ($k \in \{1, \ldots, |\mathcal{V}|\}$): Let v_k be the category which has the k^{th} highest order of precedence under $\rhd$.

$$C_S^{v_k}(\rhd, \rho; I) = C_{sd}^{v_k}\Big(\rho; \big(I \setminus I_{k-1}\big) \cap \mathcal{E}^{v_k}\Big)$$

Let $I_k = I_{k-1} \bigcup C_S^{v_k}(\rhd, \rho; I)$.

Following their order of precedence under $\rhd$, category-v serial dictatorship $C_{sd}(\rho; .)$ is applied sequentially under this choice rule for each vertical category $v \in \mathcal{V}$.

2.3.1. *Relation to Over-and-Above Choice Rule and Preliminary Results on Sequential Choice Rules.* **Over-and-Above** (O&A) choice rule $C_{OA}(\rho; .) = \left(C_{OA}^{v}(\rho; .)\right)_{v \in \mathcal{V}}$ is a special case of a sequential choice rule in that,

(1) it is defined only for environments with non-overlapping VR protections, and
(2) it processes the open category before any other category.

Thanks to these two restrictions, the relative processing sequence of the VR-protected categories becomes completely immaterial under the O&A choice rule.[14]

The following characterization, first formally stated in Sönmez and Yenmez (2022a), is the starting point of our formal analysis.

Proposition 0. *Fix a profile of category memberships $\rho = (\rho_i)_{i \in \mathcal{I}} \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$ such that VR-protected categories are non-overlapping. Then a choice rule $C(\rho; .)$ satisfies non-wastefulness, no justified envy, and compliance with VR protections if and only if $C(\rho; .) = C_{OA}(\rho; .)$.*

Therefore, for as long as VR protections are non-overlapping, the O&A choice rule is the only choice rule that satisfies the mandates of *Indra Sawhney (1992)*. Due to the Dissenting opinion in *Janhit Abhiyan (2022)*, however, the more general version of the problem with overlapping VR protections is also important to explore. Therefore, we next turn our attention to other sequential choice rules.

The following lemmata show that each sequential choice rule satisfies the first two axioms mandated by *Indra Sawhney (1992)*, and for as long as it processes the open category before other categories, it also satisfies the third axiom.

Lemma 1. *For any $\rho \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$ and $\rhd \in \Delta$, the sequential choice rule $C_S(\rhd, \rho; .)$ satisfies non-wastefulness and no justified envy.*

Lemma 2. *For any $\rho \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$ and $\rhd \in \Delta^o$, the sequential choice rule $C_S(\rhd, \rho; .)$ satisfies compliance with VR protections.*

[14] Since the outcome of the O&A choice rule is independent of the choice of an order of precedence in Δ^o, the parameter $\rhd$ is suppressed in $C_{OA}(\rho; .)$.

The next lemma further shows that, any choice rule that satisfies the Supreme Court's mandates has to award the open-category positions to the same individuals who would receive them under O&A. Thus, any deviation from the outcome of the O&A choice rule is due to the allocation of VR-protected positions only.

Lemma 3. *Fix a profile $\rho \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$ of category memberships, and another a profile $\rho' \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$ of category memberships that is non-overlapping. Let $C(\rho;.) = \left(C^{v}(\rho;.)\right)_{v\in\mathcal{V}}$ be any choice rule that satisfies non-wastefulness, no justified envy, and compliance with VR protections. Then, for any set of individuals $I \subseteq \mathcal{I}$,*

$$C^{o}(\rho; I) = C^{o}_{OA}(\rho'; I).$$

2.4. Meritorious Over-and-Above Choice Rule and the Case for the Simultaneous VR Processing Policy. As presented in Section 2.3, provided that they pick an order of precedence in Δ^{o}, the sequential choice rules abide by the mandates of *Indra Sawhney (1992)*. Despite the prominence of these rules in real-life applications, however, they are not the only viable extensions of the O&A choice rule when VR protections are overlapping. On the contrary, absent of additional normative criteria, there is another choice rule which can be considered "superior" to any other for this more general version of the problem. We next introduce and analyze this alternative that we refer to as the *meritorious Over-and-Above* (mO&A) choice rule.

Fix a profile of category memberships $\rho \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$. The following pair of auxiliary definitions simplify the formulation of the mO&A choice rule.

Definition 7. Let $\beta : 2^{\mathcal{I}} \to \mathbb{N}$ denote the **VR-maximality function** that gives the maximum number of VR-protected positions that can be awarded to eligible individuals in I, for any set of individuals $I \subseteq \mathcal{I}$.

Assuming VR protections are non-overlapping, for any set of individuals $I \subseteq \mathcal{I}$ this number is given as

$$\beta(I) = \sum_{c\in\mathcal{R}} \min\left\{|I \cap \mathcal{E}^{c}(\rho)|, q^{c}\right\}.$$

For the general case with overlapping VR protections, we formally define this function in Definition 23 of Appendix B.

Definition 8. For any set of individuals $I \subseteq \mathcal{I}$ and an individual $i \in \mathcal{I} \setminus I$ (who is not a member of set I), **individual i increases the VR-utilization of I** (upon joining individuals in set I) if,

$$\beta(I \cup \{i\}) = \beta(I) + 1.$$

Given a set of individuals $I \subseteq \mathcal{I}$, the outcome of the mO&A choice rule $C_{\text{(OA)}}(\rho;.)$ is obtained in two steps with the following procedure.

Meritorious Over-and-Above Choice Rule $C_{\textcircled{OA}}(\rho;.) = \left(C^{\nu}_{\textcircled{OA}}(\rho;.)\right)_{\nu\in\mathcal{V}}$

Step 1. Open category is processed. Choose the highest merit score individuals in I one at a time until $\min\{|I|, q^o\}$ individuals are chosen. Let $C^o_{\textcircled{OA}}(\rho; I)$ be the set of chosen individuals.

Step 2. All VR-protected categories are processed in parallel.
Let $J = I \setminus C^o_{\textcircled{OA}}(\rho; I)$ be the set of remaining individuals in I.

Step 2.0 (Initiation): Let $J_0 = \emptyset$.

Step 2.k ($k \in \{1, \ldots, \sum_{c\in\mathcal{R}} q^c\}$): Assuming such an individual exists, choose the highest merit score individual in $J \setminus J_{k-1}$ who increases the VR-utilization of J_{k-1}. Denote this individual by j_k and let $J_k = J_{k-1} \cup \{j_k\}$. If no such individual exists, then end the process.

For any $c \in \mathcal{R}$, let $C^c_{\textcircled{OA}}(\rho; I)$ be the set of individuals who each received a category-c position in this step.

Remark 1. While the set of individuals $\widehat{C}_{\textcircled{OA}}(\rho;.)$ who are chosen under the above-given procedure is uniquely defined, there may be multiple ways to assign some individuals to their VR-protected categories. This potential multiplicity is benign for our analysis.

The mO&A choice rule abides by the mandates of *Indra Sawhney (1992)*.

Proposition 1. *The meritorious Over-and-Above choice rule $C_{\textcircled{OA}}(\rho;.)$ satisfies non-wastefulness, no justified envy, and compliance with VR protections.*

We need the following terminology to present our first main result.

Definition 9 (Gale (1968))**.** Let members of two sets of individuals $I = \{i_1, \ldots, i_{|I|}\}, J = \{j_1, \ldots, j_{|J|}\} \subseteq \mathcal{I}$ be each enumerated such that the higher the merit score of an individual is the lower index number she has. Then, the set of individuals I **Gale dominates** the set of individuals J if,

(1) $|I| \geq |J|$, and

(2) for each $\ell \in \{1, \ldots, |J|\}$,
$$\sigma_{i_\ell} \geq \sigma_{j_\ell}.$$

When a set of individuals I Gale dominates another set of individuals J,

(1) there are at least as many individuals who are admitted under I as under J, and
(2) the highest merit-score individual in I is at least as meritorious as the highest merit-score individual in J, the second highest merit-score individual in I is at least as meritorious as the second highest merit-score individual in J, and so on.

Thus, a group that Gale dominates another group is more meritorious in a strong way. Our first main result establishes that, while the uniqueness of a choice rule that satisfies the Supreme Court's mandates on *Indra Sawhney (1992)* is lost under overlapping VR protections, there is still a choice rule that fares better than any other under a system that promotes meritocracy.

Theorem 1. *Let $C(\rho;.)$ be any choice rule that satisfies non-wastefulness, no justified envy, and compliance with VR protections. Then for any set of individuals $I \subseteq \mathcal{I}$, the set of individuals $\widehat{C}_{\textcircled{O\&A}}(\rho;I)$ admitted by the meritorious Over-and-Above choice rule Gale dominates the set of individuals $\widehat{C}(\rho;I)$ admitted under choice rule $C(\rho;.)$.*

Theorem 1 justifies the naming of the meritorious Over-and-Above choice rule.

Given the mandates of *Indra Sawhney (1992)* and Theorem 1, it may be tempting to declare the meritorious Over-and-Above choice rule as an unambiguous "winner" for the more general version of the problem with overlapping VR protections. We, nevertheless, caution against a hasty dismissal of other alternatives for there may be application-specific normative considerations which may deem other choice rules to be strong contenders. Focusing on the specifics of the Supreme Court judgment *Janhit Abhiyan (2022)*, in Sections 2.5 and 2.6 we present two other generalizations of the O&A choice rule that also deserve serious consideration.

Our formal analysis in the rest of the paper is based on a perspective that differs from the Majority opinion in *Janhit Abhiyan (2022)*, and it is instead based on the Dissenting opinion which declared the exclusion of SC, ST, and OBS from the scope of EWS reservation as a violation of the Equality Code. Based on the Dissenting opinion, this exclusion has to be removed regardless of which generalization of the O&A choice rule is adopted. From this perspective, Theorem 1 can be interpreted as a normative justification for processing all VR categories simultaneously. In Sections 2.5 and 2.6, in contrast, we present alternative normative justifications for processing the EWS category after all caste-based VR-protected categories and before all caste-based VR-protected categories respectively. Since Sections 2.5 and 2.6 directly pertain to the crisis on EWS reservation, for much of the remaining analysis, we assume that there is only one VR-protected category $e \in \mathcal{R}$ (i.e. EWS) that shares common members with other VR-protected categories. Let $\mathcal{R}^0 = \mathcal{R} \setminus \{e\}$ denote the set of caste-based VR-protected categories. By assumption, categories in $\mathcal{R}^0$ are non-overlapping among themselves, but they each overlap with category e.

2.5. The Case for the EWS-last VR Processing Policy. In this section we consider a policy that removes the exclusion of caste-based VR categories from the scope of EWS, and

processes EWS category after all other categories. The first part of this reform directly addresses the violation of the Equality Code under the Dissenting opinion in the judgment *Janhit Abhiyan (2022)*. Therefore, through the second part of this policy, a refinement based on the Dissenting opinion is presented. This policy merits serious consideration for it avoids a violation of the Equality Code though the smallest possible interference with the existing Constitutional Amendment. In order to present why that is the case, we need the following additional analysis.

2.5.1. *Which Individuals are Adversely Affected by the Violation of the Equality Code?* Let $\mathring{\rho} = (\mathring{\rho}_i)_{i\in\mathcal{I}} \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$ denote the original (i.e. existing) profile of category memberships in the absence of overlapping VR protections. Therefore, $|\mathring{\rho}_i| \leq 1$ for each $i \in \mathcal{I}$. Fix a set of individuals $\mathcal{J} \subseteq \bigcup_{c\in\mathcal{R}^0} \mathcal{E}^c(\mathring{\rho}) \subseteq \mathcal{I}$ whose Right to Equality is violated due to exclusion from the scope of EWS reservation.

Consider the following question: Who among individuals in $\mathcal{J}$ can argue that she is adversely affected by the violation of the Equality Code, because she lost a position due to her exclusion from the scope of category e under $\mathring{\rho}$? An individual $i \in \mathcal{J}$ can make this argument, if she remains unmatched under the membership profile $\mathring{\rho}$, although she would have received a position under an alternative scenario where she is granted with a membership of the new category e rather than a membership of her existing category in $\mathcal{R}^0$. This observation motivates the following series of definitions.

Given an individual $j \in \mathcal{J}$, let $\tilde{\rho}_j = \{e\}$.

Definition 10. Given a profile of category memberships $\rho \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$, a choice rule $C(\rho;.)$, and a set of individuals $I \subseteq \mathcal{I}$, an individual $j \in \mathcal{J} \cap I$ **suffers from a violation of the Equality Code** under $C(\rho;.)$ for I, if

$$j \notin \widehat{C}\big(\rho; I\big) \quad \text{and} \quad j \in \widehat{C}\big((\rho_{-j}, \tilde{\rho}_j); I\big).$$

Given a profile of category memberships $\rho \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$, a choice rule $C(\rho;.)$, and a set of individuals $I \subseteq \mathcal{I}$, a set of individuals $J \subseteq \mathcal{J} \cap I$ **suffer from a violation of the Equality Code** under $C(\rho;.)$ for I, if, for each $j \in J$,

$$j \notin \widehat{C}\big(\rho; I\big) \quad \text{and} \quad j \in \widehat{C}\big((\rho_{-J}, \tilde{\rho}_J); I\big).$$

Definition 11. Given a profile of category memberships $\rho \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$ and a choice rule $C(\rho;.)$, a set of individuals $I \subseteq \mathcal{I}$ are **materially unaffected by the violation of the Equality Code** under the choice rule $C(\rho;.)$, if there exists no set of individuals $J \subseteq \left(\mathcal{J} \cap I\right)$ who suffer from a violation of the Equality Code under $C(\rho,.)$ for I.

By definition, for any set of individuals $J \subseteq \mathcal{J} \bigcap I$ who suffer from a violation of the Equality Code under the choice rule $C(\rho;.)$ for I, we have $J \bigcap \widehat{C}(\rho; I) = \emptyset$.

Definition 12. Given a profile of category memberships $\rho \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$, a choice rule $C(\rho;.)$ **abides by the Equality Code**, if, for any set of individuals $I \subseteq \mathcal{I}$, there exists no set of individuals $J \subseteq \left(\mathcal{J} \bigcap I\right)$ who suffer from a violation of the Equality Code under $C(\rho,.)$ for I.

By definition, for any set of individuals $J \subseteq \mathcal{J} \bigcap I$ who suffer from a violation of the Equality Code under the choice rule $C(\rho;.)$ for I, we have $J \bigcap \widehat{C}(\rho; I) = \emptyset$.

Observe that, a set of individuals may suffer from a violation of the Equality Code based on Definition 10, and yet some of its members may still not be deserving of a position, because there may be other individuals who also suffer from a violation of the Equality Code despite being even more meritorious. This observation motivates the next definition.

Definition 13. Given a profile of category memberships $\rho \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$, a choice rule $C(\rho;.)$, and a set of individuals $I \subseteq \mathcal{I}$, the set of individuals $J \subseteq \left(\mathcal{J} \bigcap I\right) \setminus \widehat{C}(\rho; I)$ is a **maximal set of individuals who suffer from a violation of the Equality Code** under $C(\rho;.)$ for I, if,

(1) the set of individuals J suffer from a violation of the Equality Code under $C(\rho;.)$ for I, and
(2) for any set of individuals $J' \subseteq \left(\mathcal{J} \bigcap I\right) \setminus \widehat{C}(\rho; I)$ with $J \subsetneq J'$,
   (a) J' does not suffer from a violation of the Equality Code under $C(\rho;.)$ for I, and
   (b) $J \subseteq \widehat{C}\Big(\left(\rho_{-J'}, \tilde{\rho}_{J'}\right); I\Big)$.

Our next result implies that, the maximal set of individuals who are adversely affected by the violation of the Equality Code is uniquely defined.

Lemma 4. *Fix the profile of category memberships as $\mathring{\rho}$. For any set of individuals $I \subseteq \mathcal{I}$, the maximal set of individuals who suffer from a violation of the Equality Code under the choice rule $C_{OA}(\mathring{\rho};.)$ for I is uniquely defined.*

2.5.2. *EWS-last Over-and-Above Choice Rule.* We are ready to formulate an sequential choice rule that not only abides by the Equality Code, but it is also a minimal deviation from the existing system in a well-defined sense.

Let $\rho^* = \left(\rho_i^*\right)_{i \in \mathcal{I}}$ be such that,

(1) $\rho_i^* = \mathring{\rho}_i \cup \{e\}$ for any $i \in \mathcal{J}$, and
(2) $\rho_i^* = \mathring{\rho}_i$ for any $i \in \mathcal{I} \setminus \mathcal{J}$.

Compared to the profile of category memberships $\mathring{\rho}$, the adjusted profile of category memberships ρ^* grants each member of set $\mathcal{J}$ an extra membership of category e. Since each of these individuals are already member of a caste-based VR-protected category, the structure of category memberships under this counterfactual involves overlapping VR protections. From a technical point of view, it also means that the O&A choice rule is no longer well defined. Therefore, as a second alternative to the O&A choice rule, we consider a sequential choice rule where the order of precedence is such that the open category is processed first and the "scope-extended" VR-protected category e is processed the last.

Note that, the first part of the resulting policy involves an increase in the scope of EWS category that is parallel to the Dissenting opinion in *Janhit Abhiyan (2022)*. The second part of the policy refines this aspect of the Dissenting opinion by processing the scope-extended EWS category after all other categories. We refer to this second (and more subtle) aspect of the resulting policy as **EWS-last VR processing policy**.

We next present, why this policy represents the smallest possible change from the contested Amendment for an authority whose objective is simply to avoid a violation of the Equality Code, but otherwise to remain true to the controversial Amendment.

Let $\trianglerighteq \in \Delta^o_e$, i.e., the order of precedence $\trianglerighteq$ orders the open category first and category e last. As an alternative choice rule that abides by the Equality Code through a "minimal interference" with the existing system, we formulate a sequential choice rule that is induced by the amended category membership profile ρ^* along with the order of precedence $\trianglerighteq$.[15] In the context of the crisis on EWS reservation, we refer to this rule as the **EWS-last Over-and-Above** (EWS-last O&A) choice rule.

Not only the EWS-last O&A choice rule satisfies the mandates of the Supreme Court in *Indra Sawhney (1992)*, but it also abides by the Equality Code.

Proposition 2. *Fix the profile of category memberships as ρ^* and the order of precedence as $\trianglerighteq \in \Delta^o_e$. Then, the sequential choice rule $C_S(\trianglerighteq, \rho^*; .)$ abides by the Equality Code and it satisfies non-wastefulness, no justified envy, and compliance with VR protections.*

We are ready to present our second main result.

Theorem 2. *Consider any set of individuals $I \subseteq \mathcal{I}$. Then, the set of individuals*

$$C_S(\trianglerighteq, \rho^*; I) \setminus C_{OA}\left(\mathring{\rho}; I\right)$$

is equal to the maximal set of individuals who suffer from a violation of the Equality Code under the choice rule $C_{OA}(\mathring{\rho}; .)$ for I.

[15] Here, the outcome of the resulting choice rule is independent of which order of precedence is picked from Δ^o_e, and therefore the formulation corresponds to a unique choice rule.

Corollary 1. *Consider any set of individuals $I \subseteq \mathcal{I}$. Then, we have*

$$C_S(\trianglerighteq, \rho^*; I) = C_{OA}\big(\mathring{\rho}; I\big)$$

if and only if the set of individuals I are materially unaffected by the violation of the Equality Code under the choice rule $C_{OA}(\mathring{\rho}; .)$.

Theorem 2 states that the outcome of the EWS-last O&A choice rule differs from the outcome of the existing O&A choice rule only if the latter involves a violation of the Equality Code, and when its outcome differs from O&A, it does so by merely replacing the material beneficiaries of the violation of the Equality Code with those who suffer from the violation. In that sense this particular choice rule is one that "minimally interferes" with the current system. Whereas the appeal of the meritorious Over-and-Above choice rule–presented in Section 2.4–is rooted in a more fundamental normative principle, the "proximity" of the EWS-last O&A choice rule to the existing system is its main appeal.

2.5.3. *Conditional Membership for the New Members Under the EWS-last VR Processing Policy.* It is worthwhile to highlight the important role the specific order of precedence $\trianglerighteq \in \Delta^o_e$ plays under the sequential choice rule $C_S(\trianglerighteq, \rho^*; .)$. Under this order of precedence,

(1) positions in the open category are allocated prior to positions in any other category (as it is mandated under under *Indra Sawhney (1992)*),
(2) but more critically, the positions in category e are processed after the positions in all other categories.

This selection has a key implication on the nature of benefits the extra category-e membership given to members of the set $\mathcal{J} \subseteq \bigcup_{c \in \mathcal{R}^0} \mathcal{E}^c(\mathring{\rho})$ under expanded category memberships in ρ^*. More specifically, the potential benefits of this new membership becomes largely diminished for individuals in $\mathcal{J}$ under the order of precedence $\trianglerighteq$, and the benefits kick in only if there would be a violation of the Equality Code in the absence of their category-e membership.[16] That is because, since positions at all other categories are already allocated prior to allocation of positions in category e under the order of precedence $\trianglerighteq$, relatively lower merit-score members of other VR-protected categories remain in competition for allocation of category-e positions. And if there wouldn't be any violation of the Equality Code in the absence of the extra memberships provided under ρ^*, then all these positions are awarded to original members of category e under $\tilde{\rho}$. Therefore, the extra membership provided to members of set $\mathcal{J}$ under the sequential choice rule $C_S(\trianglerighteq, \rho^*; .)$ can be interpreted as a "conditional membership" which only kicks in when it is absolutely necessary

[16] In contrast, the potential benefits would always fully kick in under an alternative order of precedence $\overline{\trianglerighteq} \in \Delta^{o,e}$, where category e positions are allocated immediately after the open-category positions. See Section 2.6 for a justification for this alternative policy.

to avoid a violation of the Equality Code. Importantly, in Section 3.2 we show that this aspect of the EWS-last VR processing policy directly contradicts with two of the main arguments in the Majority opinion in *Janhit Abhiyan (2022)*.

2.6. The Case for the EWS-first VR Processing Policy. So far we considered two choice rules under the scope-extended EWS category: The meritorious Over-and-Above choice rule under which all VR-protected categories are processed simultaneously, and the EWS-last Over-and-Above choice rule under which category EWS is processed after all other VR-protected categories. As a third policy that also deserves serious consideration under the scope-extended EWS category, we next consider one that processes EWS category immediately after the open category. We refer to this policy as the **EWS-first VR processing policy**.

Let $\overline{\rhd} \in \Delta^{o,e}$, i.e., the order of precedence $\overline{\rhd}$ orders the open category first and category e second. As a third choice rule that deserves serious consideration under the Dissenting opinion, we formulate a sequential choice rule that is induced by the expanded category membership profile ρ^* given in Section 2.5, along with the order of precedence $\overline{\rhd}$.[17] In the context of the crisis on EWS reservation, we refer to this rule as the **EWS-first Over-and-Above** (EWS-first O&A) choice rule.

2.6.1. *Should the Elevated Status of Caste-Based VR Protections be Maintained?* As we have emphasized before, the VR policy is intended as the strongest form of positive discrimination policy in India. The reason such a powerful policy is adopted is historical. The VR policy was originally intended for members of Scheduled Castes which is the official term for Dalits or "untouchables," whose members have suffered millennia-long systematic injustice due to their lowest status under the caste system, and Scheduled Tribes, which is the official term for the indigenous ethnic groups of India, whose members were both physically and socially isolated from the rest of the society. With *Indra Sawhney (1992)*, VR policy was also extended to members of Other Backward Classes who were historically engaged in various marginal occupation assigned to them by the society to serve castes higher to them in the caste hierarchy. The strength of the VR protection policy reflects the strong desire in Indian society to acknowledge and reverse the excessively disadvantaged status of these communities.

But other than the fraction of positions reserved under the VR policy, which other aspects of this policy makes it especially powerful? The answer to this question lays in the following two technical aspects of this policy. First of all, positions that are set aside for

[17] As in the case of the EWS-last Over-and-Above choice rule, the outcome of the resulting choice rule is independent of which order of precedence is picked from $\Delta^{o,e}$, and therefore the formulation corresponds to a unique choice rule.

a VR-protected category are exclusive to its beneficiaries, even when it means that some of these positions remain unassigned.[18] But more importantly, the landmark Supreme Court judgment *Indra Sawhney (1992)* explicitly mandated that these positions are not to be used for members of the VR-protected category who could already receive a unit from open category without invoking the benefits of VR policy (i.e., the axiom *compliance with VR policy*). Thus, the VR-protected positions are explicitly directed to those who could not receive an open position. It is this second aspect of the VR policy that makes it especially powerful.

A key question that needs to be addressed under the Dissenting opinion in *Janhit Abhiyan (2022)* is the following: What happens if a financially disadvantaged member of a caste-based VR category (say SC) does not merit a position from the open category, but she merits a VR-protected position both from EWS and SC? Should the principle that requires to allocate the open-category positions before the SC positions also be applied between EWS positions and SC positions? Equivalently, should an SC position be awarded to a financially-disadvantaged member of SC who already merits an EWS position or not? Since the Prime Minister Narendra Modi have repeatedly asserted that "the EWS reservation in the general category will not undermine the interests of dalits, tribals and the Other Backward Classes," it may be natural to answer this important question in the positive.[19] Indeed, the third point of the following discussion from paragraph 191 of the Dissenting opinion of Justice Shripathi Ravindra Bhat may also be interpreted as an argument in favor of allocating EWS positions before caste-based VR-protected positions, so that a caste-based VR-protected position is not allocated to an individual who merits an EWS position:

> The exclusionary clause operates in an utterly arbitrary manner. Firstly, it ‘‘others’’ those subjected to socially questionable, and outlawed practices -- though they are amongst the poorest sections of society. Secondly, for the purpose of the new reservations, the exclusion operates against the socially disadvantaged classes and castes, absolutely, by confining them within their allocated reservation quotas (15% for SCs, 7.5% for STs, etc.). Thirdly, it denies the chance of mobility from the reserved quota (based on past discrimination) to a reservation benefit based only on economic deprivation.

These observations motivate the following additional normative principle.

[18] These unassigned units are typically carried over to the next allocation period to be added to the VR-protected positions for the same category.

[19] See, for example, *The Print* story "EWS quota will not affect existing reservation, says PM Modi," last retrieved on 10/30/2022.

Definition 14. A choice rule $C(\rho;.) = \left(C^{\nu}(\rho;.)\right)_{\nu\in\mathcal{V}}$ **respects mobility from reparatory categories to EWS** if, for any $I \subseteq \mathcal{I}$, $c \in \mathcal{R}^0$, and $i \in C^c(\rho;I) \bigcap \mathcal{E}^e(\rho)$,

(1) $|C^e(\rho;I)| = q^e$, and
(2) for every $j \in C^e(\rho;I)$,

$$\sigma_j > \sigma_i.$$

Our third main result states that, EWS-first Over-and-Above choice rule is the only rule that satisfies this additional principle along with the mandates of *Indra Sawhney (1992)*.

Theorem 3. *Fix a profile of category memberships* $\rho = (\rho_i)_{i\in\mathcal{I}} \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$ *such that, for any* $i \in \mathcal{I}$,

$$|\rho_i| \leq 2 \quad \textit{and} \quad |\rho_i| = 2 \implies e \in \rho_i.$$

Fix an order of precedence as $\overline{\rhd} \in \Delta^{o,e}$. *Then, a choice rule* $C(\rho;.)$ *respects mobility from reparatory categories to EWS and it satisfies non-wastefulness, no justified envy, and compliance with VR protections if and only if* $C(\rho;.) = C_S(\overline{\rhd},\rho;.)$.

It is important to emphasize one important aspect of the income limit for EWS eligibility, and its impact on the EWS-first O&A choice rule. While EWS category is intended for individuals who are financially disabled, according to Deshpande and Ramachandran (2019), 98% of the Indian population earns below the annual income limit Rs 8 lakh to be eligible for the EWS reservation. Therefore, under the current scope of the EWS category, the Constitutional Amendment essentially provides a vertical category for members of forward castes. Under the Dissenting opinion of *Janhit Abhiyan (2022)* with scope-expanded EWS, however, the current income limit for EWS eligibility has a completely orthogonal implication under the EWS-first O&A choice rule. The outcome of the EWS-first O&A choice rule is virtually the same as a policy that completely removes the EWS category! In particular, if everyone were to be eligible for EWS (rather than the current 98%), then the outcome of the EWS-first O&A choice rule is exactly the same as transferring all EWS positions to open category!

2.7. Legislative Loophole under a Scenario which only Extends the Scope of EWS Category. So which extension of the O&A choice rule is plausible under the Dissenting opinion of *Janhit Abhiyan (2022)* with the scope-expanded EWS? Our purpose in this paper is not advocating for a specific choice rule, but rather formulating some of the most natural alternatives, and characterizing their policy implications. If the objective is to maintain neutrality between all VR-protected categories, then meritorious O&A choice rule would be a natural selection with the recognition that this sequencing tilts the EWS category in favor of forward castes. If the objective is to remove the violation of Equality Code

through the smallest possible modification of the Amendment, then the EWS-last O&A choice rule would be a natural alternative with the recognition that this selection tilts the EWS category in favor of forward castes. In Section 3 we argue that this objective is in line with the Majority opinion in *Janhit Abhiyan (2022)*. If the objective is to maintain the elevated status of caste-based VR protections, then the EWS-first O&A choice rule would be the only viable alternative. In Section 3 we argue that this objective is in line with the Dissenting opinion in *Janhit Abhiyan (2022)*.

If the scope of EWS is expanded but the laws stay silent on this subtle issue, then the outcome of the system would be subject to potential manipulation by politically-motivated authorities who can tilt the system to the benefit of any category they desire. While the difference between the EWS-first VR processing policy and EWS-last VR processing policy are already quite substantial, politically-motivated authorities could also adopt other and less natural policies that nonetheless better serve their own factions. For example, an authority who wants to aid members of OBC could choose an Over-and-Above policy which processes SC and ST prior to EWS, but OBC after EWS. Under this policy mobility to EWS is respected for OBC, but not for SC or ST. Similarly, an authority who wants to disadvantage members of OBC could choose an Over-and-Above policy which processes OBC prior to EWS, but SC, ST after EWS. Under this policy, in contrast, mobility to EWS is respected for SC and ST, but not for OBC.

2.8. Related Literature. On a methodological level, our paper benefits from insights of Rawls (1971), Sen (1987), Moulin (1991, 2004), Thomson (2001), Fleurbaey and Maniquet (2012), Li (2017), Schummer and Serizawa (2019), Jackson (2019), and Hitzig (2020).

Four papers that are especially related to our formal analysis are Kominers and Sönmez (2016), Sönmez and Yenmez (2022a), Dur et al. (2018), and Pathak et al. (2020a).

The basic version of our model builds on Kominers and Sönmez (2016) which introduces a general model with slot-specific priorities. Sönmez and Yenmez (2022a) formulates the Indian reservation system with both the primary vertical and the secondary horizontal reservations, and shows that there is a unique mechanism which satisfies the mandates of the Supreme Court judgments *Indra Sawhney (1992)* and *Saurav Yadav vs The State Of Uttar Pradesh (2020)*.[20] The choice rule characterized in this paper, namely the *two-step minimum guarantee* choice rule, is endorsed by the Supreme Court in their judgment *Saurav Yadav (2020)*,[21] and it is further mandated in the state of Gujarat by its high court

[20]The judgment is available in `https://indiankanoon.org/doc/27820739/`, last retrieved on 10/06/2022.

[21]While this mechanism itself is merely endorsed by the Supreme Court, it is de facto mandated due to the uniqueness result by Sönmez and Yenmez (2022a).

in *Tamannaben Ashokbhai Desai vs Shital Amrutlal Nishar (2020)*.[22] Our general model in Appendix A extends the model of Sönmez and Yenmez (2022a) by allowing for overlaps between VR-protected groups. As we emphasized in Section 1.2, our generalization is relevant in the context of the Dissenting opinion of *Janhit Abhiyan (2022)*. Critically, in the absence of additional normative criteria, the uniqueness result by Sönmez and Yenmez (2022a) no longer holds under our generalization. And as we have thoroughly discussed in Sections 1.2 and 1.3, this observation has major policy implications in relation to the current debates in the country in relation to the controversial Amendment.

Practical and policy relevance of the processing sequence of categories was first documented in Dur et al. (2018) for allocation of seats at Boston Public Schools (BPS) between years 1999-2013. As a compromise between a faction which demanded neighborhood assignment and another which demanded more comprehensive school choice, in 1999 leadership at BPS announced that neighborhood students will receive preferential treatment in half of the seats at each public school. This policy was referred to as *walk-zone priority*. However, processing the walk-zone seats prior to remaining ones effectively negated this policy until 2013. After discovering that their policy was superfluous and misleading, the walk zone policy was abandoned at BPS altogether.

A related phenomenon for allocation of H1-B visas in the US is presented in Pathak et al. (2020a). With the *H-1B Visa Reform Act of 2004*, the US Congress reduced the number of annual H1-B visas from 195,000 to 65,000, but granted an exemption of 20,000 units for holders of advanced degrees. Reflecting a number of purely logistical constraints, the procedure that is used to implement this act was changed a few times over the years, although each time with significant (but likely unrealized) distributional implications. In response to the former President Trump's *Buy American and Hire American Executive Order in 2017*, however, the procedure was reformed by the U.S. Department of Homeland Security with an explicit objective of increasing the number of awards to recipients with advanced degrees. The latest reform, which led to an adoption of a new visa allocation rule for US Fiscal Year 2020, simply involved a reversal of processing sequence of advanced degree visas and general category visas.

Our main formal results are Theorems 1-5. From a technical perspective, the proof Theorem 1 builds on abstract research on matroid theory in Gale (1968), whereas the proof of Theorem 5 closely follows the proof strategy of the main characterization result in Sönmez and Yenmez (2022a). Both the conceptual formulation of Theorem 4 and its

[22]The judgment is available in `https://indiankanoon.org/doc/101656671/`, last retrieved on 10/06/2022.

proof are novel to our paper. Theorem 2 is a special case of Theorem 4 and Theorem 3 is a special case of Theorem 5.

Our paper contributes to a growing literature on reserve systems that starts with Hafalir et al. (2013). The role of processing order of different types of positions in this framework was first studied by Kominers and Sönmez (2016) in a theoretical framework, and subsequently by Dur et al. (2018) in the context of school choice. Other papers on reserve systems include Ehlers et al. (2014), Echenique and Yenmez (2015), Dur et al. (2020), Pathak et al. (2020a,c), Abdulkadiroğlu and Grigoryan (2021), Aygün and Bó (2021), Celebi and Flynn (2021, 2022), Celebi (2022), and Sönmez and Yenmez (2022a,b).

Finally, in addition to the literature on reserve systems, our paper also contributes to a large and growing literature on analysis and design of mechanisms that are deployed in settings in which issues of social, racial and distributive justice are particularly important. Some of the most related papers in this literature includes Abdulkadiroğlu and Sönmez (2003), Abdulkadiroğlu (2005), Erdil and Ergin (2008), Kesten (2010), Kojima (2012), Pycia (2012), Andersson and Svensson (2014), Kamada and Kojima (2015), Delacrétaz et al. (2016), Chen and Kesten (2017), Fragiadakis and Troyan (2017), Andersson (2019), Erdil and Kumano (2019), Andersson and Ehlers (2020), Ehlers and Morrill (2020), and Reny (2022).

3. Critical Analysis of Majority and Dissenting opinions in *Janhit Abhiyan (2022)*

Until recently, VR protections in India had been exclusive to members of socially and educationally disadvantaged classes who suffered from marginalization and discrimination due to their caste identities. As it is highlighted in the Dissenting opinion of Justice Bhat for the judgment *Janhit Abiyan (2022)*, it was embedded in the Constitution of India "as a reparative and compensatory mechanism meant to level the field." This norm has recently changed in India with the enactment of the 103rd Amendment of the Constitution in 2019. Under the Amendment, the *Economically Weaker Sections* of the society are awarded with VR protections for up to 10 percent of the positions in government jobs and seats in higher education on the basis of their financial incapacity. Controversially, the non-overlapping structure of the VR protections are maintained with the Amendment, and socially and educationally disadvantaged classes who are eligible for the existing VR protections, i.e. SCs, STs, and OBSs, are excluded from the new provisions.

In January 2019, along with many other petitioners, the two NGOs Janhit Abhiyan and Youth For Equality challenged the Amendment at a three-judge Bench of the Supreme Court.[23] Among their main objections was the exclusion of members of SCs, STs, and

[23]Details and progression of the case can be found in the following *Supreme Court Observer* link.

OBCs from the scope of the EWS category. Since the case involved substantial questions around the interpretation of the Constitution, it was referred to a five-judge Constitution Bench of the Supreme Court in August 2020. In a landmark judgment *Janhit Abhiyan (2022)* that fundamentally changed the meaning of affirmative action in India, the Supreme Court upheld the validity of the central law providing 10% reservation to EWS on November 7th, 2022. In a split verdict, the five-judge Constitution Bench ruled 3-2 in favor of the 103rd Constitutional Amendment of 2019.

The judgment *Janhit Abhiyan (2022)* is memorable in many ways. It became the first Supreme Court case which had its proceedings live-streamed.[24] The verdict was announced on the last working day of Justice Uday Lalit, who headed the Constitution Bench as the 49th Chief Justice of India (CJI).[25] According to Supreme Court Observer– a non-partisan platform that reports and analyses judgments of the Supreme Court–the role of the CJI in the judgment was also unconventional:[26]

> Chief Justice Lalit's role in this Judgment, on the eve of his retirement, is unusual for two reasons. First, he has not written an opinion of his own, choosing to express his agreement with Justice Bhat instead. Second, he forms part of the dissent in this case. Chief Justices more often form the Majority opinion in Constitution Bench cases.

But, perhaps, what made the judgment *Janhit Abhiyan (2022)* especially memorable is the tone of the the Dissenting opinion by Justices Bhat and Lalit who declared that " this court has for the first time, in the seven decades of the republic, sanctioned an avowedly exclusionary and discriminatory principle."[27]

In this section we relate both the Majority opinion and the Dissenting opinion in this historical case to our formal analysis in Section 2, and among others, we show that several of the main arguments that is used in the Majority opinion are outright false. Since the majority judgment on the third issue considered by the Bench–whether SC/ST/OBC can be excluded from the scope of EWS– is directly linked to these inaccurate arguments, we believe these mistakes played a key role in this judgment of historical significance. We

[24]See The Hindu story "In a first, Supreme Court live-streams Constitution Bench proceedings," last accessed 11/11/2022.

[25]See The Hindustan Times story "CJI Lalit: Leaving with a sense of accomplishment," last accessed 11/11/2022.

[26]See Supreme Court Observer article "EWS Reservation Judgment: SC Upholds 103rd Amendment in 3-2 Majority," last accessed 11/11/2022.

[27]See the opening paragraph of the Dissenting opinion.

also argue that, our analysis in Section 2 can be used in the future to formulate a less divisive resolution for the crisis.[28]

3.1. The Disagreement Between the Majority and Dissenting Opinions. All five justices declared that reservations can be granted solely on the basis of economic criteria. Their disagreement was mainly on the constitutionality of the exclusion of SC, ST and OBC from the scope of the EWS reservation. While the two dissents declared the exclusion of socially and educationally disadvantaged classes from the scope of EWS reservation to be unconstitutional, the three majority justices declared it to be constitutional, since (in their opinion) the exclusion was inevitable to be able to provide positive discrimination to groups who are not covered by earlier reservations.

The two minority justices characterized the exclusion as "Orwellian," and expressed their strong objection in the following concluding paragraphs of their Dissenting opinion:

> 191. A universally acknowledged truth is that reservations have been conceived and quotas created, through provision in the Constitution, only to offset fundamental, deep rooted generations of wrongs perpetrated on entire communities and castes. Reservation is designed as a powerful tool to enable equal access and equal opportunity. Introducing the economic basis for reservation - as a new criterion, is permissible. Yet, the ''othering'' of socially and educationally disadvantaged classes - including SCs/ STs/OBCs by excluding them from this new reservation on the ground that they enjoy pre-existing benefits, is to heap fresh injustice based on past disability. The exclusionary clause operates in an utterly arbitrary manner. Firstly, it ''others'' those subjected to socially questionable, and outlawed practices - though they are amongst the poorest sections of society. Secondly, for the purpose of the new reservations, the exclusion operates against the socially disadvantaged classes and castes, absolutely, by confining them within their allocated reservation quotas (15% for SCs, 7.5% for STs, etc.). Thirdly, it denies the chance of mobility from the reserved quota (based on past discrimination) to a reservation benefit based only on economic deprivation. The net effect of the entire exclusionary principle is Orwellian, (so to say) which is that all the poorest are entitled to be considered, regardless of their caste or class, yet only those who belong to forward classes or castes, would be considered, and those from socially disadvantaged classes for SC/STs would be ineligible. Within the narrative of the classification

[28]Leadership at DMK, the ruling party in the State of Tamil Nadu, have announced that they will file a review petition against the judgment. See The Hindu story "DMK to file review petition against EWS quota." Last accessed on 15/11/2022.

> jurisprudence, the differentia (or marker) distinguishing one person from another is deprivation alone. The exclusion, however, is not based on deprivation but social origin or identity. This strikes at the essence of the non-discriminatory rule. Therefore, the total and absolute exclusion of constitutionally recognised backward classes of citizens - and more acutely, SC and ST communities, is nothing but discrimination which reaches to the level of undermining, and destroying the equality code, and particularly the principle of nondiscrimination.
>
> 192. Therefore, on question 3, it is clear that the impugned Amendment and the classification it creates, is arbitrary, and results in hostile discrimination of the poorest sections of the society that are socially and educationally backward, and/or subjected to caste discrimination. For these reasons, the insertion of Article 15(6) and 16(6) is struck down, is held to be violative of the equality code, particularly the principle of nondiscrimination and non-exclusion which forms an inextricable part of the basic structure of the Constitution.

3.2. Alleged Necessity of the Exclusion to Avoid "Double Benefits". The majority justices are not unsympathetic to the above-given perspective of the two dissenting justices. In paragraph 77 of the Majority opinion, Justice Dinesh Maheshwari articulates their perspective as follows:

> 77.1. [...] Rather, according to the petitioners, the classes covered by Articles 15(4), 15(5) and 16(4) are comprising of the poorest of the poor and hence, keeping them out of the benefit of EWS reservation is an exercise conceptionally at conflict with the constitutional norms and principles.
>
> 77.2. At the first blush, the arguments made in this regard appear to be having some substance because it cannot be denied that the classes covered by Articles 15(4), 15(5) and 16(4) would also be comprising of poor persons within. However, a little pause and a closer look makes it clear that the grievance of the petitioners because of this exclusion remains entirely untenable and the challenge to the Amendment in question remains wholly unsustainable. As noticed infra, there is a definite logic in this exclusion; rather, this exclusion is inevitable for the true operation and effect of the scheme of EWS reservation.

Therefore, while agreeing that the exclusion may not correspond to an ideal scenario, the majority justices are of the (factually inaccurate) opinion that it is inevitable from an

operational perspective. In paragraphs 79-82 of their Majority opinion, Justice Dinesh Maheshwari articulate the alleged necessity of the exclusion as follows:

> 79. [...] The moment there is a vertical reservation, exclusion is the vital requisite to provide benefit to the target group. In fact, the affirmative action of reservation for a particular target group, to achieve its desired results, has to be carved out by exclusion of others. [...] But for this exclusion, the purported affirmative action for a particular class or group would be congenitally deformative and shall fail at its inception. Therefore, the claim of any particular class or section against its exclusion from the affirmative action of reservation in favour of EWS has to be rejected.
>
> 80. [...] It could easily be seen that but for this exclusion, the entire balance of the general principles of equality and compensatory discrimination would be disturbed, with extra or excessive advantage being given to the classes already availing the benefit under Articles 15(4), 15(5) and 16(4).
>
> 81. Putting it in other words, the classes who are already the recipient of, and beneficiary of, compensatory discrimination by virtue of Articles 15(4), 15(5) and 16(4), cannot justifiably raise the grievance that in another set of compensatory discrimination for another class, they have been excluded. It gets, perforce, reiterated that the compensatory discrimination, by its very nature, would be structured as exclusionary in order to achieve its objectives. Rather, if the classes for whom affirmative action is already in place are not excluded, the present exercise itself would be of unjustified discrimination.
>
> 82.1. [...] As said above, compensatory discrimination, wherever applied, is exclusionary in character and could acquire its worth and substance only by way of exclusion of others. Such differentiation cannot be said to be legally impermissible; rather it is inevitable. When that be so, clamour against exclusion in the present matters could only be rejected as baseless.

Observe that, the sole justification given by the majority justices for their support of the exclusion of the socially and educationally disadvantaged classes from the scope of EWS is a flawed technical argument that is made in paragraphs 79-82 of the Majority opinion.[29] They erroneously argue that, exclusion of the beneficiaries of earlier provisions from a

[29]Our observation–i.e., the justification offered by the majority Justice Dinesh Maheshwari in support of the exclusion–is also acknowledged by the following analysis in Supreme Court Observer analysis "EWS Reservations: Judgment Matrix," last accessed 11/11/2022.

new provision is absolutely necessary to be able to deliver benefits to individuals who are outside the scope of earlier provisions. In addition, they also argue that, inclusion of the socially and educationally disadvantaged classes to the scope of EWS as well would necessarily result in excessive advantage for members of these groups. This latter argument is also referred to as the "double-benefit argument" by the justices, government officials, and media.[30]

While the technical justification offered by the majority justices is accurate under non-overlapping VR protections, it is false under overlapping VR protections, i.e. the relevant version of the problem with scope-extended EWS category. As we have shown in Theorem 2, EWS positions are largely allocated to individuals who are not covered by the earlier reparative VR protections under the EWS-last VR processing policy. Moreover, as we have shown in Corollary 2, members of socially and educationally disadvantaged classes benefit from an additional EWS membership only when their membership to their caste-based VR-protected categories put them at a disadvantage compared to a membership to EWS. This is why we describe the EWS-last VR processing policy as a policy that effectively provides a conditional EWS membership to beneficiaries of caste-based VR-protections in Section 2.5.3. Hence, the technical justification offered by the majority justices for their controversial decision is entirely due to their oversight of the implications of the overlapping VR protections, a technical and subtle phenomenon the justices are not familiar with.

It is important to highlight two points at this point. First of all, as it is presented in Section 2.6, the scope-extended EWS policy indeed provides additional benefits for the socially and educationally disadvantaged classes under the alternative EWS-first VR processing policy. Thus, we are not suggesting that the technical point by the majority justices is never valid regardless of how EWS positions are processed. We do suggest, however, that their argument is completely false under the EWS-last VR processing policy. Therefore, in our view, the core of the arguments made by Justice Maheshwari in support of exclusion of SC/ST/OBC from the scope of EWS, are rather consistent with a removal of the exclusion, albeit under the EWS-last VR processing policy. Our view here is also consistent with our formal results in Theorem 2 and Corollary 2 which show that the EWS-last VR processing policy is the minimal possible deviation from the existing system that escapes a violation of the Equality Code. Indeed, Justice Maheshwari indicates the desirability of such a minimal interference in a case against a constitutional Amendment as follows:

[30]See, for example, Live Law story "SC/ST/OBC Exclusion From EWS Quota Logical, Necessary To Avoid Double Benefits: Supreme Court," last accessed 12/11/2022.

> 38.2. The reason for minimal interference by this Court in the constitutional Amendments is not far to seek. In our constitutional set-up of parliamentary democracy, even when the power of judicial review is an essential feature and thereby an immutable part of the basic structure of the Constitution, the power to amend the Constitution, vested in the Parliament in terms of Article 368, is equally an inherent part of the basic structure of the Constitution. Both these powers, of amending the Constitution (by Parliament) and of judicial review (by Constitutional Court) are subject to their own limitations. The interplay of amending powers of the Parliament and judicial review by the Constitutional Court over such exercise of amending powers may appear a little bit complex but ultimately leads towards strengthening the constitutional value of separation of powers. This synergy of separation is the strength of our Constitution.

Secondly, we are not suggesting that the dissident justices would necessarily be in favor of the EWS-last VR processing policy, had the majority justices supported this policy due to the advantage it provides to individuals who do not benefit from earlier provisions. On the contrary, we believe the Dissenting opinion is more consistent with the EWS-first VR processing policy. The basis of our belief is based on the following three arguments in the Dissenting opinion. In paragraph 100 of his Dissenting opinion, Justice Bhat refuses the "the double benefit argument" due to a completely different reason than our technical objection:

> 100. The characterisation of including the poor (i.e., those who qualify for the economic eligibility) among those covered under Articles 15(4) and 16(4), in the new reservations under Articles 15(6) and 16(6), as bestowing ''double benefit'' is incorrect. What is described as 'benefits' for those covered under Articles 15(4) and 16(4) by the Union, cannot be understood to be a free pass, but as a reparative and compensatory mechanism meant to level the field - where they are unequal due to their social stigmatisation. This exclusion violates the non-discrimination and the non-exclusionary facet of the equality code, which thereby violates the basic structure of the Constitution.

Justice Bhat argues that, the earlier provisions do not disqualify members of socially and educationally disadvantaged classes from deprivation-based provisions, because the earlier provisions are reparative and compensatory. He further elaborates on this point in paragraph 168 of his Dissenting opinion.

> 168. The characterisation of reservations for economically weaker sections of the population (EWS) as compensatory and on par with the existing reservations under Articles 15(4) and 16(4), in my respectful opinion, is without basis.

This latter argument by Justice Bhat suggests that he sees the earlier reparative and compensatory reservations as a "more important" provision, which is consistent with EWS-first VR processing policy. Moreover, in paragraph 191 of his Dissenting opinion, he further argues in favor of our axiom *respect for mobility from reparative categories to EWS* given in Definition 14:

> Thirdly, it denies the chance of mobility from the reserved quota (based on past discrimination) to a reservation benefit based only on economic deprivation.

Therefore the Dissenting opinion is more consistent with the expanded-scope EWS category under the EWS-first VR processing policy.

3.3. Interpretation of the Principle of Equality and the Cut-off Crisis. Another disagreement between the Majority opinion and the Dissenting opinion pertains to the interpretation of the *principle of equality.* Majority Justice Maheshwari explains this important principle and how it can be constitutionally applied–called *reasonable classification*–as follows:

> 44. In a nutshell, the principle of equality can be stated thus: equals must be treated equally while unequals need to be treated differently, inasmuch as for the application of this principle in real life, we have to differentiate between those who being equal, are grouped together, and those who being different, are left out from the group. This is expressed as reasonable classification. Now, a classification to be valid must necessarily satisfy two tests: first, the distinguishing rationale should be based on a just objective and secondly, the choice of differentiating one set of persons from another should have a reasonable nexus to the object sought to be achieved.

Majority Justice Bela Trivedi further elaborates on the principle of equality as follows:

> 20. [...] Treating economically weaker sections of the citizens as a separate class would be a reasonable classification, and could not be termed as an unreasonable or unjustifiable classification, much less a betrayal of basic feature or violative of Article 14. As laid down by this Court, just as equals cannot be treated unequally, unequals also cannot

> be treated equally. Treating unequals as equals would as well offend the doctrine of equality enshrined in Articles 14 and 16 of the Constitution.

According to majority justices, individuals facing economic deprivation is a reasonable classification as a class. This point is agreed upon by the dissenting justices as well. The majority justices, however, further refine this class by excluding the socially and educationally disadvantaged classes. This is where the fundamental difference lays between the majority justices and dissenting justices. In Section 3.2 we have presented the main and technical justification of this exclusion from the perspective of the majority justices. A second justification is offered by the majority justices based on the principle of justice, where they argue that "treating unequals as equals would as well offend the doctrine of equality." According to this argument, individuals from the socially and educationally disadvantaged classes are different than individuals from the upper classes, therefore economically deprived members of the two groups are also different, and consequently the two groups have to be treated differently. But what does the phrase "unequals cannot be treated equally" mean here? Does it allow for treating disadvantaged groups worse than advantaged groups? We are fairly confident that this is not what is intended in the constitution by the principle of equity. Clearly "unequals cannot be treated equally" must mean those that are disadvantaged must be treated better. But, if that is the correct interpretation, than a major violation of this principle is enabled by the exclusion of socially and educationally disadvantaged classes from EWS reservations.

The Amendment allows for an economically deprived member of a privileged forward class to receive a position with a low merit score, while it denies the same position for an even more economically deprived member of a disadvantaged class who has a higher merit score! This observation is also in line with one of the main objections of Justice Bhat to the Constitutional Amendment:

> 80. I am of the opinion that the application of the doctrine classification differentiating the poorest segments of the society, as one segment (i.e., the forward classes) not being beneficiaries of reservation, and the other, the poorest, who are subjected to additional disabilities due to caste stigmatization or social barrier based discrimination - the latter being justifiably kept out of the new reservation benefit, is an exercise in deluding ourselves that those getting social and educational backwardness based reservations are somehow more fortunate. This classification is plainly contrary to the essence of equal opportunity.

By Theorem 2, the scope-expanded EWS with EWS-last VR processing policy is the smallest possible deviation from the current Amendment that escapes this anomaly. Indeed, this clear violation of the principle of equality is not a mere theoretical possibility

but a regular anomaly under the Constitutional Amendment. This alarming observation is the main point of the story "EWS verdict shows merit matters only when it's 'their' children, not 'our' kids" from the digital platform *The Print*. Indeed, as it is also highlighted as follows in this story, this anomaly is a regular phenomenon under the contested Amendment.

> Finally, the judgment makes no mention of the most frequently cited principle when discussing reservation, 'merit'. This silence is stark in the face of the evidence of the first two years of application of EWS quota that the cut-off for EWS was even lower than the OBC. Clearly merit matters only when we discuss 'their' children, not when we discuss 'our' children benefiting from capitation fees, securing spurious degrees abroad or obtaining EWS quota.

That is, for the first two years of the implementation of EWS reservation, the minimum score needed to secure a position was lower for the economically deprived members of forward classes compared to the economically deprived members of the disadvantaged classes. Indeed, this anomaly is one of the three reasons for the Tamil Nadu government's opposition to EWS reservation.[31]

4. Conclusion

India has a constitutionally protected affirmative action system that involves a very complex set of normative goals and requirements. While justices at the Supreme Court and state high courts have historically done an exemplary job of rigorously formulating these normative principles and providing guidance on their implementation, due to the sheer complexity of the problem, in some cases they failed to identify the collective implications of these principles or how changes in various aspects of the applications may interfere with them (Sönmez and Yenmez, 2022a). These challenges, in turn, resulted in various unintended consequences, including major disputes between various groups, inconsistencies between judgments, loopholes in the system, and large scale disruption of recruitment and school admissions.

In this paper we report upon and analyze, perhaps, one of the most significant ones of these episodes. In a 3-2 split verdict, the Supreme Court has fundamentally altered the nature of affirmative action in India in their judgment *Janhit Abhiyan (2022)*. In the words of Justice Bhat from his Dissenting opinion, "the impugned amendment and the classification it creates, is arbitrary, and results in hostile discrimination of the poorest sections of the society that are socially and educationally backward, and/or subjected to

[31]See the *TNM* story "Three reasons why the DMK has opposed 10% EWS reservation."

caste discrimination." Prashant Padmanabhan describes the significance of this historical dissent as follows:[32]

> The dissenting view on fraternal obligations, non-exclusionary principles, social justice and equality must be taken forward in our future discourse in and out of Parliament. Such values alone can act as a barrier against further erosion of Constitutional values. [...]
>
> The 11th Chief Justice of the U.S. Supreme Court, Charles Evans Hughes wrote in 1936 that dissenting is "an appeal to the brooding spirit of the law, to the intelligence of a future day, when a later decision may possibly correct the error into which the dissenting judge believes the court to have been betrayed". [...]
>
> Any new policy of affirmative action ought to have been inclusive of the already marginalised sections. The existing reservation for a totally different cause, is no justification for their exclusion. The dissenting view on fraternal obligations, non-exclusionary principles, social justice and equality must be taken forward in our future discourse in and out of Parliament. Such values alone can act as a barrier against further erosion of Constitutional values.

What we find especially striking in the judgment is that, not only the majority justices use completely technical arguments to justify their controversial decision on the constitutionality of the exclusion of socially and educationally backward classes from the scope of EWS, but these arguments are also irrefutably false.[33] The irony of this situation is completely in line with the following important observation in Li (2017):

> A policymaker may be familiar with the details of their environment, and yet not know how to state their ethical requirements in precise terms.

More precisely, the majority justices failed to assess the implications of a changing feature of the fair allocation problem they were tasked to resolve, i.e., the structure of VR-protected categories changing from non-overlapping to overlapping. As it is further emphasized in Li (2017),

> In addition to studying cause and effect in markets, economists also have a comparative advantage in stating precisely the normatively-relevant properties of complex systems [...]

Utilizing this comparative advantage, in this paper we presented three alternative policies which could have been considered to reach a more amenable compromise for the crisis

[32]See *The Leaflet* story "Significance of dissent in Supreme Court's judgment in the EWS case." Last accessed on 11/16/2022.

[33]Confusion among policy makers seems to be especially common for real-life applications of reserve systems. See, for example, Dur et al. (2018), Pathak et al. (2020a,b), and Sönmez and Yenmez (2022a).

on EWS reservation. In case this important debate resurfaces in the future due to the Dissenting opinion, our analysis can be used to reach a less divisive resolution. The way we address the normative aspects of the crisis is parallel to the following three theses in Li (2017):

> Firstly, that the literature on market design does not, and should not, rely exclusively on preference utilitarianism to evaluate designs. Secondly, that market designers should study the connection between designs and consequences, and should not attempt to resolve fundamental ethical questions. Thirdly, that the theory and practice of market design should maintain an informed neutrality between reasonable ethical positions.

We believe our paper highlights the virtue of this approach in the broader context of market design, and shows that our expertise is not only valuable but also necessary for some important societal decisions that involve complex systems. This approach is very much influenced by Sen (1987), who advocates for the reunification of normative and positive origins of economics.[34] Moreover, in some cases, religiously following the three theses in Li (2017)–especially the *informed neutrality*–is absolutely necessary if market designers are aspiring to be part of these important decisions.

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[34] As Sen (1987) puts it, economics has two origins, *ethics* – the normative origin – and *engineering* – the positive origin –, which diverged over time at the cost of impoverishing economics as a science, and it is time to bring them back together.

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Appendices

Appendix A. Generalized Model with HR Protections and Extended Analysis

In addition to its primary vertical reservations, India also has horizontal reservations that serve as its secondary affirmative action policy. Since the discussions in the country on its contested Amendment and its resolution in *Jandit Abhiyan (2022)* are entirely focused on the primary VR policy, in Section 2 we also assumed away India's secondary HR policies. In this section we extend our analysis in Section 2 to the general version of the model with both VR and HR policies, and show that our analysis and conclusions persist in this more complex version of the model. The extended analysis in this section builds on Sönmez and Yenmez (2022a), and it is valuable not only because it establishes the robustness of our analysis in Section 2, but also because it provides the nuts and bolts mechanisms policymakers can use to implement their VR and HR policies if in the future VR protections becomes overlapping in the country.

A.1. Horizontal Reservation Policies. In addition to the VR-protected categories in $\mathcal{R}$ that are associated with the primary VR protections, there is a finite set $\mathcal{T}$ of (horizontal) **traits** associated with the secondary HR protections. Each individual $i \in \mathcal{I}$ has a (possibly empty) set of traits, denoted by $\tau_i \in 2^{\mathcal{T}}$. Let $\tau = (\tau_i)_{i\in\mathcal{I}} \in \left(2^{\mathcal{T}}\right)^{|\mathcal{I}|}$ denote the profile of individual traits. Individuals with these traits are provided with easier access to positions through a second (but less powerful) type of an affirmative action policy.

HR protections are provided within each vertical category of positions (including the open category).[35] For any trait $t \in \mathcal{T}$ and subject to availability of individuals with trait t, priority access is given to individuals with trait t for $q_t^o \in \mathbb{N}$ of the open-category positions. These are referred to as **open-category HR-protected positions for trait t**. Similarly, for any $\rho \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$, $c \in \mathcal{R}$ and $t \in \mathcal{T}$, subject to the availability of individuals in $\mathcal{E}^c(\rho)$ with trait t, priority access is given to individuals in $\mathcal{E}^c(\rho)$ with trait t for $q_t^c \in \mathbb{N}$ of the category-c positions. These are referred to as **category-c HR-protected positions for trait t**. Observe that, in contrast to VR protections which are provided on an Over-and-Above basis, HR protections are provided within each vertical category on a "minimum guarantee" basis. This means that positions obtained without invoking the benefits of the HR policy still accommodate the HR protections. In addition, unlike the VR-protected positions which are exclusively set aside for their beneficiaries, the HR-protected positions merely provide priority access for their beneficiaries. It is these two technical aspects which make the HR policy a secondary form of affirmative action policy.

[35] Because of this feature, HR protections are sometimes referred to as *interlocking reservations*.

Let $q = \left(\left(q^v\right)_{v \in \mathcal{V}}, \left(q^v_t\right)_{(v,t) \in \mathcal{V} \times \mathcal{T}} \right)$ denote the vector that specifies (ii) the number of all positions at each vertical category, and (iii) the number of HR-protected positions for each trait and category of positions. We refer to vector q as the **reservation vector**. Throughout this section, we fix the profile of individual traits τ and the reservation vector q.

A.1.1. *The HR Compliance Function.* Fix a profile of category memberships $\rho \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$ and a category $v \in \mathcal{V}$. The following technical construction, first formulated in Sönmez and Yenmez (2022a), is useful to formulate a measure of compliance with the HR policy.

Definition 15. Let $\eta^v : 2^{\mathcal{I}} \to \mathbb{N}$ denote a **category-v HR compliance function** that gives the maximum number of category-v HR-protected positions that can be awarded to eligible individuals in $I \cap \mathcal{E}^v(\rho)$, for any set of individuals $I \subseteq \mathcal{I}$.

If each individual has at most one trait, the case referred to as **non-overlapping HR protections** in Sönmez and Yenmez (2022a), this function is simply given as follows: For any $I \subseteq \mathcal{I}$,

$$\eta^v(I) = \sum_{t \in \mathcal{T}} \min \left\{ |\{i \in I \cap \mathcal{E}^v(\rho) \,:\, t \in \tau_i\}|, q^v_t \right\}.$$

Observe that, for each trait $t \in \mathcal{T}$, the function $\min \left\{ |\{i \in I \cap \mathcal{E}^v(\rho) \,:\, t \in \tau_i\}|, q^v_t \right\}$ gives the total number category-v and trait-t HR-protected positions that are honored by the set of individuals I, and therefore, when aggregated across all traits the formula gives the total number of HR-protected positions that are honored by I.

If an individual can have multiple traits, the case referred to as **overlapping HR protections** in Sönmez and Yenmez (2022a), then the formulation of HR compliance function is slightly more involved, and it involves a *maximal assignment of individuals to traits*. Fortunately, as further explained in Appendix B, it is straightforward to calculate the maximum number of HR-protected positions that can be accommodated by any set of individuals $I \subseteq \mathcal{I}$ for any category $v \in \mathcal{V}$ through various computationally efficient maximum cardinality matching algorithms in bipartite graphs.

A.2. Generalized Axioms. We next present extensions of the primary axioms in Section 2.2, and introduce an additional one that formulates the accommodation of HR protections. Throughout this section, fix a profile of category memberships $\rho \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$.

As it is discussed in depth in Sönmez and Yenmez (2022a), each of the following four axioms are mandated throughout India with the Supreme Court judgment *Saurav Yadav (2020)*.[36] Throughout this section, we focus on choice rules that satisfy all four axioms.

[36]Indeed, as it is thoroughly discussed in Sönmez and Yenmez (2022a), the failure of the *no justified envy* axiom under a choice rule that had been mandated in the country between years 1995-2020 resulted countless litigations in the country, and resulted enforcement of this axiom with *Saurav Yadav (2020)*. That

The first axiom is the *non-wastefulness*, and it has the same formulation with its original formulation in Definition 4.

The second axiom is new, and it requires that as many HR-protected positions to be honored as possible at each vertical category of positions. When HR-protected groups are non-overlapping, this simply means not to ignore HR protections. When HR protections are overlapping, it also implies a maximal assignment of individuals to HR-protected positions.

Definition 16. A choice rule $C(\rho;.) = \big(C^v(\rho;.)\big)_{v\in\mathcal{V}}$ satisfies **maximal accommodation of HR protections**, if for any $I \subseteq \mathcal{I}$, $v \in \mathcal{V}$, and $j \in \big(I \cap \mathcal{E}^v(\rho)\big) \setminus \widehat{C}(\rho;I)$,

$$\eta^v\big(C^v(\rho;I)\big) = \eta^v\big(C^v(\rho;I) \cup \{j\}\big).$$

The third axiom requires that no individual receives a position at any category $v \in \mathcal{V}$ at the expense of another eligible individual, unless she either has higher merit-score or awarding her the position increases the number of HR-protected positions that are honored at category v.

Definition 17. A choice rule $C(\rho;.) = \big(C^v(\rho;.)\big)_{v\in\mathcal{V}}$ satisfies **no justified envy** if, for any $I \subseteq \mathcal{I},\ v \in \mathcal{V},\ i \in C^v(\rho;I)$, and $j \in \big(I \cap \mathcal{E}^v(\rho)\big) \setminus \widehat{C}(\rho;I)$,

$$\sigma_i > \sigma_j \quad \text{or} \quad \eta^v\big(C^v(\rho;I)\big) > \eta^v\Big(\big(C^v(\rho;I) \setminus \{i\}\big) \cup \{j\}\Big).$$

The last axiom requires that, an individual who is "deserving" of an open-category position

- either because she has a sufficiently high merit score, or
- because she helps honor a higher number of open-category HR-protected positions,

should be awarded an open-category position and not a position that is VR-protected.

Definition 18. A choice rule $C(\rho;.) = \big(C^v(\rho;.)\big)_{v\in\mathcal{V}}$ satisfies **compliance with VR protections** if, for any $I \subseteq \mathcal{I}$, $c \in \mathcal{R}$, and $i \in C^c(\rho;I)$, we have

(1) $|C^o(\rho;I)| = q^o$,

(2) for each $j \in C^o(\rho;I)$,

$$\sigma_j > \sigma_i \quad \text{or} \quad \eta^o\big(C^o(\rho;I)\big) > \eta^o\Big(\big(C^o(\rho;I) \setminus \{j\}\big) \cup \{i\}\Big), \text{ and}$$

(3) $\eta^o\big(C^o(\rho;I) \cup \{i\}\big) \not\succ \eta^o\big(C^o(\rho;I)\big)$.

is, formulation and enforcement of the *no justified envy* axiom is the primary purpose of this important judgment. The judgment, however, also clarified what it means "to deserve an open-category position on the basis of merit" in the presence of HR protections, and also enforced the axiom *compliance with VR protections*.

A.3. Sequential Meritorious Horizontal Choice Rules. In this section we generalize the class of sequential choice rules to the more general version of the problem with HR policy. The core "engine" of this class is the *meritorious horizontal* choice rule, a single-category choice rule that is originally introduced in Sönmez and Yenmez (2022a), and which replaces the serial dictatorship to allocate positions at any vertical category.

Throughout this section, we fix a profile $\rho \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$ of category memberships.

The following auxiliary definition simplifies the formulation of the meritorious horizontal choice rule.

Definition 19. Given a category $v \in \mathcal{V}$ and a set of individuals $I \subseteq \mathcal{E}^v(\rho)$, an individual $i \in \mathcal{E}^v(\rho) \setminus I$ **increases the (category-v) HR utilization of I** if

$$\eta^v(I \cup \{i\}) = \eta^v\big(I\big) + 1.$$

Given a category $v \in \mathcal{V}$ and a set of individuals $I \subseteq \mathcal{I}$, the outcome of the meritorious horizontal choice rule $C^v_{\text{ⓜ}}\big(\rho;.\big)$ is obtained with the following procedure.

(Category-v) Meritorious Horizontal Choice Rule $C^v_{\text{ⓜ}}\big(\rho;.\big)$

Step 1.0 (Initiation): Let $I_0 = \emptyset$.

Step 1.k ($k \in \{1, \ldots, \sum_{t\in\mathcal{T}} q^v_t\}$): Assuming such an individual exists, choose the highest merit-score individual in $\big(I \cap \mathcal{E}^v(\rho)\big) \setminus I_{k-1}$ who increases the category-v HR utilization of I_{k-1}. Denote this individual by i_k and let $I_k = I_{k-1} \cup \{i_k\}$. If no such individual exists, proceed to Step 2.

Step 2: For unfilled positions, choose highest merit-score unassigned individuals in $\big(I \cap \mathcal{E}^v(\rho)\big)$ until either all positions are filled or all eligible individuals are selected.

We are ready to formulate the class of *sequential meritorious horizontal (SMH)* choice rules.

Fix an order of precedence $\rhd \in \Delta$. Given a set of individuals $I \subseteq \mathcal{I}$, the outcome of the sequential meritorious horizontal choice rule $C_{\text{Ⓢ}}(\rhd, \rho; .)$ is obtained with the following procedure.

SMH Choice Rule $C_{\text{Ⓢ}}(\rhd, \rho; .) = \big(C^v_{\text{Ⓢ}}(\rhd, \rho; .)\big)_{v\in\mathcal{V}}$

Step 0 (Initiation): Let $I_0 = \emptyset$.

Step k ($k \in \{1, \ldots, |\mathcal{V}|\}$): Let v_k be the category which has the k^{th} highest order of precedence under $\rhd$.

$$C^{v_k}_{\text{Ⓢ}}(\rhd, \rho; I) = C^{v_k}_{\text{ⓜ}}\Big(\rho; \big(I \setminus I_{k-1}\big) \cap \mathcal{E}^{v_k}(\rho)\Big)$$

Let $I_k = I_{k-1} \cup C^{v_k}_{\text{Ⓢ}}(\rhd, \rho; I)$.

Under this procedure the meritorious horizontal choice rule is applied sequentially for each vertical category $v \in \mathcal{V}$, following their order of precedence under $\rhd$.

A.3.1. *Preliminary Results on SMH Choice Rules.* The class of SMH choice rules generalizes the *two-step meritorious horizontal (2SMH)* choice rule $C^{2s}_{\textcircled{M}}(\rho;.) = \left(C^{2s,v}_{\textcircled{M}}(\rho;.)\right)_{v\in\mathcal{V}}$, originally introduced by Sönmez and Yenmez (2022a). 2SMH choice rule $C^{2s}_{\textcircled{M}}(\rho;.)$ is defined for environments with non-overlapping VR protections only, and in these environments it is equivalent to any SMH that where the open category has the highest order of precedence.[37] That is,

$$C^{2s}_{\textcircled{M}}(\rho;.) = C_{\textcircled{S}}(\rhd^o,\rho;.) \qquad \text{for any } \rhd^o \in \Delta^o.$$

As such, the open category is processed prior to any VR-protected category under the choice rule 2SMH. Just as the relative processing sequence of VR-protected categories are immaterial under the O&A choice rule when they are non-overlapping, the same is also true for the 2SMH choice rule.

The following characterization by Sönmez and Yenmez (2022a) generalizes Proposition 0, thus establishing that the 2SMH choice rule assumes the central role of the O&A choice rule in the presence of HR policy.

Theorem 0. *Fix a profile of category memberships $\rho = (\rho_i)_{i\in\mathcal{I}} \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$ such that VR-protected categories are non-overlapping. Then, a choice rule $C(\rho;.)$ satisfies non-wastefulness, maximal accommodation of HR protections, no justified envy, and compliance with VR protections if and only if $C(\rho;.) = C^{2s}_{\textcircled{M}}(\rho;.)$.*

Since VR-protected categories are currently non-overlapping in India, the legislation in India has airtight implications by Theorem 0. Indeed, focusing on a simpler version of problem with non-overlapping HR protections, the 2SMH choice rule was recently endorsed in the country by the Supreme Court judgment *Saurav Yadav (2020)*, and enforced in the state of Gujarat by the high court judgment *Tamannaben Ashokbhai Desai (2020)*.[38] Since the scope of the EWS category may soon be expanded in the country, some other elements of the class of SMH choice rules, especially those under EWS-first and EWS-last VR processing policies, are of particular interest.

Extending Lemmas 1 and 2, the next two lemmas show that each SMS choice rule satisfies the first three axioms mandated by *Saurav Yadav (2020)*, and for as long as open category has higher order of precedence than any other VR-protected category, it also satisfies the fourth axiom.

[37] Since the outcome of the 2SMH choice rule is independent of the choice of an order of precedence in Δ^o, the parameters is suppressed in $C^{2s}_{\textcircled{M}}(\rho;.)$.

[38] When HR protections are non-overlapping, Sönmez and Yenmez (2022a) refers to the resulting simpler version of the 2SMH choice rule as the *two-step minimum guarantee (2SMG) choice rule*.

Lemma 5. *For any $\rho \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$ and $\rhd \in \Delta$, the SMH choice rule $C_{\text{Ⓢ}}(\rhd, \rho; .)$ satisfies non-wastefulness, maximal accommodation of HR protections, and no justified envy.*

Lemma 6. *For any $\rho \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$ and $\rhd \in \Delta^o$, the SMH choice rule $C_{\text{Ⓢ}}(\rhd, \rho; .)$ satisfies compliance with VR protections.*

Extending Lemma 3, the next lemma further shows that, even if non-regular VR policies are adopted at some (or all) of potentially overlapping VR-protected categories, any choice rule that satisfies the Supreme Court's axioms still has to allocate the open-category positions to the same individuals who would have received them under the 2SMH choice rule. Thus, any deviation from the outcome of the 2SMH is due to the assignment of VR-protected positions.[39]

Lemma 7. *Fix a profile of category memberships $\rho \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$, and another profile of category memberships $\rho' \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$ that is non-overlapping. Let $C(\rho; .) = \left(C^v(\rho; .)\right)_{v \in \mathcal{V}}$ be any choice rule that satisfies non-wastefulness, maximal accommodation of HR protections, no justified envy, and compliance with VR protections. Then, for any set of individuals $I \subseteq \mathcal{I}$,*

$$C^o(\rho; I) = C^{2s,o}_{\text{Ⓜ}}(\rho'; I).$$

A.3.2. *Extended Results on EWS-last VR Processing Policy.* In this section we follow the notation and terminology used in Section 2.5, and directly extend the results in this section for the most general version of the problem with horizontal reservations. Below, Lemma 8 extends Lemma 4, Proposition 3 extends Proposition 2, Theorem 4 extends Theorem 2, and Corollary 2 extends Corollary 1 respectively. In all these results, the EWS-last SMH choice rule plays is the same role the EWS-last sequential choice rule plays in Section 2.5, and the role 2SMH choice rule plays is the same role the role O&A choice rule in Section 2.5.

Lemma 8. *Fix the profile of category memberships as $\mathring{\rho}$. For any set of individuals $I \subseteq \mathcal{I}$, the maximal set of individuals who suffer from a violation of the Equality Code under the choice rule $C^{2s}_{\text{Ⓜ}}(\mathring{\rho}; .)$ for I is uniquely defined.*

Proposition 3. *Fix the profile of category memberships as ρ^* and the order of precedence as $\trianglerighteq \in \Delta^o_e$. Then, the EWS-last SMH choice rule $C_{\text{Ⓢ}}(\trianglerighteq, \rho^*; .)$ abides by the Equality Code and it satisfies non-wastefulness, maximal accommodation of HR protections, no justified envy, and compliance with VR protections.*

[39] While this result is a slightly stronger version of Lemma 10 in Sönmez and Yenmez (2022a) (due to overlapping VR-protected categories and different profiles of category memberships used under the two choice rules), the changes are superfluous and the two proofs are analogous. Hence, we omit the proof of Lemma 7.

Theorem 4. *Consider any set of individuals $I \subseteq \mathcal{I}$. Then, the set of individuals*

$$C_{\textcircled{S}}(\trianglerighteq, \rho^*; I) \setminus C^{2s}_{\textcircled{M}}\big(\mathring{\rho}; I\big)$$

is equal to the maximal set of individuals who suffer from a violation of the Equality Code under the choice rule $C^{2s}_{\textcircled{M}}(\mathring{\rho}; .)$ for I.

Corollary 2. *Consider any set of individuals $I \subseteq \mathcal{I}$. Then, we have*

$$C_{\textcircled{S}}(\trianglerighteq, \rho^*; I) = C^{2s}_{\textcircled{M}}\big(\mathring{\rho}; I\big)$$

if and only if the set of individuals I are materially unaffected by the violation of the Equality Code under the choice rule $C^{2s}_{\textcircled{M}}(\mathring{\rho}; .)$.

A.3.3. *Extended Results on EWS-first VR Processing Policy.* In this section we follow the notation and terminology used in Section 2.6, and directly extend the main characterization in this section for the EWS-first VR processing policy for the most general version of the problem with horizontal reservations.

In order to do this, we first have to extend the axiom which expands the elevated status of caste-based VR protections to the general version of the problem with HR policy. The generalized version below requires that, a member of a caste-based VR category who is also eligible for EWS should not use up a position for her caste-based VR category if she merits an EWS position either due to her merit score or because she increases the HR utilization at EWS category.

Definition 20. A choice rule $C(\rho; .) = \big(C^{\nu}(\rho; .)\big)_{\nu \in \mathcal{V}}$ **respects mobility from reparatory categories to EWS** if, for any $I \subseteq \mathcal{I}$, $c \in \mathcal{R}^0$, and $i \in C^c(\rho; I) \bigcap \mathcal{E}^e(\rho)$,

(1) $|C^e(\rho; I)| = q^e$,

(2) for each $j \in C^e(\rho; I)$,

$$\sigma_j > \sigma_i \quad \text{or} \quad \eta^e\big(C^e(\rho; I)\big) > \eta^e\Big(\big(C^e(\rho; I) \setminus \{j\}\big) \cup \{i\}\Big), \text{ and}$$

(3) $\eta^e\big(C^e(\rho; I) \cup \{i\}\big) \not> \eta^e\big(C^e(\rho; I)\big)$.

Our final result states that, EWS-first SMH choice rule is the only rule that satisfies this additional axiom along with the mandates of *Saurav Yadav (2020)*.

Theorem 5. *Fix a profile of category memberships $\rho = (\rho_i)_{i \in \mathcal{I}} \in \big(2^{\mathcal{R}}\big)^{|\mathcal{I}|}$ such that, for any $i \in \mathcal{I}$,*

$$|\rho_i| \leq 2 \quad \textit{and} \quad |\rho_i| = 2 \implies e \in \rho_i.$$

Fix an order of precedence as $\overline{\trianglerighteq} \in \Delta^{o,e}$. Then, a choice rule $C(\rho; .)$ respects mobility from reparatory categories to EWS and it satisfies non-wastefulness, maximal accommodation of HR protections, no justified envy, and compliance with VR protections if and only if $C(\rho; .) = C_{\textcircled{S}}(\overline{\trianglerighteq}, \rho; .)$.

Appendix B. Technical Preliminaries for the Proofs

B.1. Matchings, Matroids, Greedy Algorithm, and Choice Rules. In this subsection, we introduce auxiliary mathematical structures that will be useful in formalizing some of our definitions and proving our results.

Fix a profile of category memberships $\rho \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$. Let $\mathcal{E}^c = \mathcal{E}^c(\rho)$ for each $c \in \mathcal{R}$.

Consider a reservation vector $q = \left(q^v, (q^v_t)_{t\in\mathcal{T}}\right)_{v\in\mathcal{V}}$ so that there are potentially HR-protected positions in each category. Let $\mathcal{Q}$ be the set of reservation vectors.

Define for each $v \in \mathcal{V}$,

$$q^v_{t_\emptyset} = q^v - \sum_{t\in\mathcal{T}} q^v_t$$

where $t_\emptyset$ refers to **nil trait**, which we use to denote a position of any category v that is not protected by horizontal reservations.

Definition 21. Let $I \subseteq \mathcal{I}$. A **matching** is a mapping $\mu : I \to \left(\mathcal{V} \times (\mathcal{T} \cup \{t_\emptyset\})\right) \cup \{\emptyset\}$ such that

- for each $i \in I$, $\mu(i) = \emptyset$ or $\mu(i) = (v, t)$ with $i \in \mathcal{E}^v$ and $t \in \tau_i \cup \{t_\emptyset\}$, and
- for each $v \in \mathcal{V}$ and $t \in \mathcal{T} \cup \{t_\emptyset\}$, $|\mu^{-1}(v, t)| \leq q^v_t$.

For an individual i, $\mu(i) = \emptyset$ refers to her remaining **unmatched**. Let $\mathcal{M}(q, I)$ be the set of matchings for I under q.[40]

The set of individuals matched by a matching $\mu \in \mathcal{M}(q, I)$ with a category $v \in \mathcal{V}$ is denoted as

$$I_{\mu,v} = \left\{i \in I : \mu(i) \in \{v\} \times \left(\mathcal{T} \cup \{t_\emptyset\}\right)\right\},$$

and the overall set of individuals matched by μ is denoted as

$$I_\mu = \bigcup_{v\in\mathcal{V}} I_{\mu,v}.$$

Using the auxiliary concept of matchings, we define the following two functions, which were formally defined for special cases before:

Definition 22. For each $v \in \mathcal{V}$, **the category-v HR-compliance function** $\eta^v : 2^{\mathcal{I}} \to \mathbb{N}$ is defined for any $I \subseteq \mathcal{I}$ as

$$\eta^v(I) = \max_{\mu\in\mathcal{M}(q,I)} \left|\{i \in I : \mu(i) \in \{v\} \times \mathcal{T}\}\right|.$$

[40] Although we do not change the real reservation vector q after it is set, we keep a reservation vector in the argument of the set of matchings. This is because we introduce auxiliary reservation vectors by modifying q, which play an important role in formal definitions and proofs.

Definition 23. The **VR-maximality function** $\beta : 2^{\mathcal{I}} \to \mathbb{N}$ is defined for any $I \subseteq \mathcal{I}$ as

$$\beta(I) = \max_{\mu \in \mathcal{M}(\tilde{q},I)} \Big| \{ i \in I : \mu(i) \in \mathcal{R} \times \{t_\emptyset\} \} \Big|.$$

where auxiliary reservation vector $\tilde{q} \in \mathcal{Q}$ is equal to reservation vector q in every component except for trait positions in VR-protected categories such that $\tilde{q}^c_t = 0$ for each $c \in \mathcal{R}$.

The auxiliary reservation vector $\tilde{q}$ in Definition 23 effectively converts all positions of a VR-protected category to nil-trait positions. Therefore, $\beta(I)$ is the maximum number of individuals in I that can be accommodated by VR-protected positions regardless of traits of the individuals.

The following mathematical structure is useful to understand properties of our choice rules.

Definition 24. A pair of finite sets $(E, \mathcal{X})$ is a **matroid** if $\mathcal{X} \subseteq 2^E$ and

(1) if $F \in \mathcal{X}$, then for any $G \subsetneq F$ we have $G \in \mathcal{X}$ (thus, $\emptyset \in \mathcal{X}$).
(2) if $F, G \in \mathcal{X}$ such that $|G| < |F|$, then there is an element $x \in F \setminus G$ such that $G \cup \{x\} \in \mathcal{X}$.

Each element of $\mathcal{X}$ is called an **independent set** of matroid $(E, \mathcal{X})$.

Next we define a specific matroid for our purposes.

For any set of individuals $I \subseteq \mathcal{I}$, define a collection of subsets of I

$$\boldsymbol{\mathcal{A}(q, I)} = \{ J \subseteq I : \exists\, \mu \in \mathcal{M}(q, I) \text{ s.t. } J \subseteq I_\mu \}. \tag{1}$$

We refer to each set $J \in \mathcal{A}(q, I)$ as an **assignable subset of I under q**.

Lemma 9 (Edmonds and Fulkerson (1965))**.** *For any set of individuals $I \subseteq \mathcal{I}$, the pair $\big(I, \mathcal{A}(q, I)\big)$ is a matroid.*

Matroid $\big(I, \mathcal{A}(q, I)\big)$ is called a **transversal matroid** (e.g., see Lawler, 2001). Observe that assignable subsets of I under q are exactly the independent sets of transversal matroid $\big(I, \mathcal{A}(q, I)\big)$.

Let $\Sigma \subsetneq \mathbb{R}_+^{|\mathcal{I}|}$ be the set of merit score vectors, which are non-negative real valued and no two individuals have the same score. Given a merit score vector $\sigma \in \Sigma$, we find a particular assignable subset of I under q that we denote as $G(q, I; \sigma)$ through an iterative procedure.[41]

Greedy algorithm for transversal matroid $\big(I, \mathcal{A}(q, I)\big)$ under merit score vector σ:

[41]The greedy algorithm is defined for general matroids, although we define it using special wording for a transversal matroid. Lemma 10 also holds for the greedy algorithm for general matroids.

Step 0 (Initiation): Let $I_0 = \emptyset$.

Step k $\left(k \in \{1, \ldots, |I|+1\}\right)$: Assuming such an individual exists, choose the highest σ-score individual whom we denote as $i_k \in I \setminus I_{k-1}$ such that $I_{k-1} \cup \{i_k\} \in \mathcal{A}(q, I)$ and let $I_k = I_{k-1} \cup \{i_k\}$. If no such individual exists, then end the process by setting $\boldsymbol{G(q, I; \sigma)} = I_{k-1}$.

We refer to the outcome set $G(q, I; \sigma)$ of this procedure as **the greedily assignable subset in $\mathcal{A}(q, I)$ under σ**.

While the greedily assignable subset is uniquely defined, there can be multiple matchings that assign the same set of individuals to different category – trait/nil-trait pairs. We refer to any matching $\mu \in \mathcal{M}(q, I)$ such that

$$I_\mu = G(q, I; \sigma)$$

as **a greedy matching in $\mathcal{M}(q, I)$ under σ**. The following important property of the greedy assignable subset is key in proving Theorem 1 .

Lemma 10 (Gale (1968))**.** *For any reservation vector $q \in \mathcal{Q}$, merit score vector $\sigma \in \Sigma$, set of individuals $I \subseteq \mathcal{I}$, and assignable subset $J \in \mathcal{A}(q, I)$, the greedily assignable subset $G(q, I; \sigma)$ Gale dominates assignable subset J.*

An immediate corollary to this result is as follows using the definition of Gale domination.

Corollary 3. *For any reservation vector $q \in \mathcal{Q}$, merit score vector $\sigma \in \Sigma$, set of individuals $I \subseteq \mathcal{I}$, and assignable subset $J \in \mathcal{A}(q, I)$, the greedily assignable subset $G(q, I; \sigma)$ satisfies $|G(q, I; \sigma)| \geq |J|$.*

Greedy algorithm has a special place in our choice rule constructions.

Remark 2 (Sönmez and Yenmez (2022a))**.** Let $v \in \mathcal{V}$. The category-v meritorious horizontal choice rule utilizes the greedy algorithm for any $I \subseteq \mathcal{I}$ for transversal matroid $\left(I, \mathcal{A}(q^*, I)\right)$ under σ in Step 1 of its procedure where q^* is obtained from q by setting (i) for each $\nu \in \mathcal{V} \setminus \{v\}$, $q^{*\nu} = 0$ and (ii) $q^{*v} = q^v$ and $q^{*v}_t = q^v_t$ for each $t \in \mathcal{T}$.

Remark 3. Assuming there are no HR-protected positions in q, the mO&A choice rule $C_{\text{(OA)}}(\rho; .)$ utilizes the greedy algorithm for any $I \subseteq \mathcal{I}$ for transversal matroid $\left(J, \mathcal{A}(q', J)\right)$ under σ in Step 2 of its procedure where

- $J = I \setminus C^o_{\text{(OA)}}(\rho; I)$, and
- q' is obtained from q by setting (i) $q'^o = 0$ and (ii) $q'^c = q^c$ for each $c \in \mathcal{R}$.

We pick a greedy matching $\mu' \in \mathcal{M}(q', J)$ under σ to determine the VR-protected category components of $C_{\textcircled{OA}}(\rho; .)$. For each $c \in \mathcal{R}$, we set

$$C^c_{\textcircled{OA}}(\rho; I) = I_{\mu', c}.$$

Thus,

$$\cup_{c \in \mathcal{R}} C^c_{\textcircled{OA}}(\rho; I) = G(q', J; \sigma).$$

B.2. Some Properties of Choice Rules. We use the following property in our proofs.

Definition 25 (Aygün and Sönmez (2013))**.** For any $\rho \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$, a choice rule $C(\rho; .)$ satisfies **irrelevance of rejected individuals** if, for any $I \subseteq \mathcal{I}$ and $i \in I \setminus \widehat{C}(\rho; I)$,

$$\widehat{C}\big(\rho; I \setminus \{i\}\big) = \widehat{C}(\rho; I).$$

Lemma 11 (Sönmez and Yenmez (2022b))**.** *For any $\rho \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$ and $v \in \mathcal{V}$, the category-v meritorious horizontal choice rule $C^v_{\textcircled{m}}(\rho; .)$ satisfies irrelevance of rejected individuals.*

We also explicitly state an implication of Theorem 0 for single-category choice rules that we use in our proofs.[42]

Lemma 0 (Sönmez and Yenmez (2022a))**.** *For any $\rho \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$ and $v \in \mathcal{V}$, a single-category choice rule $C^v(\rho; .)$ maximally accommodates HR protections, satisfies no justified envy, and is non-wasteful if and only if $C^v(\rho; .) = C^v_{\textcircled{m}}(\rho; .)$.*

Appendix C. Proofs

Several results in Section 2 follow from their generalizations in Section A. In Section 2, Lemma 1 follows from Lemma 5, Lemma 2 follows from Lemma 6 and Lemma 3 follows from Lemma 7 (see Footnote 39). In Section 3, Lemma 4 follows from Lemma 8, Proposition 2 follows from Proposition 3, Theorem 2 follows from Theorem 4, and Theorem 3 follows from Theorem 5.

C.1. Proofs of Results in Section 2.4.

Proof of Proposition 1. Consider the reservation vector q such that $q^v_t = 0$ for each $v \in \mathcal{V}$ and $t \in \mathcal{T}$. Fix a profile of category memberships $\rho \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$ and a set of individuals $I \subseteq \mathcal{I}$. Let $\mathcal{E}^c = \mathcal{E}^c(\rho)$ for each $c \in \mathcal{R}$.

Non-wastefulness: Suppose $v \in \mathcal{V}$ be such that $|C^v_{\textcircled{OA}}(\rho; I)| < q^v$, and there exists some $j \in I \setminus \widehat{C}_{\textcircled{OA}}(\rho; I)$. If $v = o$, then this contradicts j not being selected in Step 1 of the mO&A choice rule, which is the category-o serial dictatorship. Suppose $v \in \mathcal{R}$. Thus, even

[42] Also see Theorem 2 in Sönmez and Yenmez (2022a).

though Step 2 of the procedure leaves vacant positions in category v, j does not receive a position, meaning that she cannot increase the VR-utilization of $\bigcup_{c\in\mathcal{C}} C^c_{\textcircled{OA}}(\rho; I)$. Then $j \notin \mathcal{E}^v$. We showed $C_{\textcircled{OA}}(\rho; .)$ is non-wasteful.

No justified envy: Let $v \in \mathcal{V}$ and $j \in I \setminus \widehat{C}_{\textcircled{OA}}(\rho; I)$. We have two cases:

(1) Consider the case $v = o$. By construction of Step 1 of the mO&A choice rule, if $i \in C^o_{\textcircled{OA}}(\rho; I) = C^o_{sd}(\rho; I)$, then i is chosen instead of j, in the category-o serial dictatorship. Hence, $\sigma_i > \sigma_j$.

(2) Consider the case $v \in \mathcal{R}$. Suppose $i \in C^v_{\textcircled{OA}}(\rho; I)$. Then $i \in \mathcal{E}^v$. Let $J = I \setminus C^o_{\textcircled{OA}}(\rho; I)$. Observe that Step 2 of the mO&A choice rule is the greedy algorithm for matroid $\big(J, \mathcal{A}(q', J)\big)$ where the auxiliary reservation vector q' is obtained from q by setting the open category positions to zero and keeping every other category's positions the same as in q (see Remark 3). Hence, in Step 2, the greedily assignable subset $G(q', J; \sigma)$ is the set of individuals chosen. In particular, $i \in G(q', J; \sigma)$.

Suppose, for a contradiction, $\sigma_j > \sigma_i$ and $j \in \mathcal{E}^v$.

We first show that set K defined as $K = \big(G(q', J; \sigma) \setminus \{i\}\big) \cup \{j\}$ is an assignable subset of J under q': To see this, pick a greedy matching $\mu \in \mathcal{M}(q', J)$ assigning i to v. Such a greedy matching exists as $i \in C^v_{\textcircled{OA}}(\rho; I)$ (see Remark 3). By definition of a greedy matching, $J_\mu = G(q', J; \sigma)$. Since both $i, j \in \mathcal{E}^v$ and there are no HR-protected positions in q', we can modify μ by assigning j to v instead of i, leaving i unmatched, and keeping all other assignments the same. Thus, we obtain a matching in $\mathcal{M}(q', J)$ that matches all individuals in K, showing that $K \in \mathcal{A}(q', J)$.

Since $\sigma_j > \sigma_i$, j is processed before i in the process of the greedy algorithm. Then j should be chosen instead of i, as K is an assignable subset of J under q', which includes j and all previously committed individuals in the greedy algorithm prior to j. This is a contradiction to $j \notin G(q', J; \sigma) \subseteq \widehat{C}_{\textcircled{OA}}(\rho; I)$. Hence, $\sigma_i > \sigma_j$ or $j \notin \mathcal{E}^v$.

Thus, $C_{\textcircled{OA}}(\rho; .)$ satisfies no justified envy.

Compliance with VR protections: Let $c \in \mathcal{R}$ and $i \in C^c_{\textcircled{OA}}(\rho; I)$. In executing the procedure of the mO&A choice rule, the open category is processed first and individual i is still available when the VR-protected categories are about to be processed in Step 2. Thus, it should be the case that $|C^o_{\textcircled{OA}}(\rho; I)| = q^o$, as otherwise i would have received a category-o position instead of a category-c position. Moreover, $C^o_{\textcircled{OA}}(\rho; I)$, which is formed by a serial dictatorship, includes the highest σ-score q^o individuals from I, implying that for every $j \in C^o_{\textcircled{OA}}(\rho; I)$, $\sigma_j > \sigma_i$. Thus, $C_{\textcircled{OA}}(\rho; .)$ satisfies compliance with VR protections. ■

Proof of Theorem 1. Consider the reservation vector q such that $q^v_t = 0$ for each $v \in \mathcal{V}$ and $t \in \mathcal{T}$. Fix $I \subseteq \mathcal{I}$. By Lemma 7, for any non-overlapping profile of category memberships

$\rho' \in \left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$ we have $C^o_{\text{(OA)}}(\rho; I) = C^{2s,o}_{\text{(M)}}(\rho'; I)$ and $C^o(\rho; I) = C^{2s,o}_{\text{(M)}}(\rho'; I)$, implying that

$$C^o_{\text{(OA)}}(\rho; I) = C^o(\rho; I). \tag{2}$$

Let $J = I \setminus C^o_{\text{(OA)}}(\rho; I)$. Define the auxiliary reservation vector $q' \in \mathcal{Q}$ such that the open category has no position and each VR-protected category has the same number of positions as it has under q: $q'^o = 0$ and $q'^c = q^c$ for each $c \in \mathcal{R}$. Construct the matching $\mu \in \mathcal{M}(q', J)$ by assigning nil-trait positions of each category $c \in \mathcal{R}$ under μ to the individuals in $C^c(\rho; I)$. Thus, the set $\bigcup_{c\in\mathcal{R}} C^c(\rho; I)$ is an assignable subset of J under q', i.e.,

$$\bigcup_{c\in\mathcal{R}} C^c(\rho; I) \in \mathcal{A}(q', J). \tag{3}$$

Step 2 of the mO&A rule is the greedy algorithm for matroid $\left(J, \mathcal{A}(q', J)\right)$ under σ (see Remark 3), implying that

$$\bigcup_{c\in\mathcal{C}} C^c_{\text{(OA)}}(\rho; I) = G(q', J; \sigma).$$

Thus, by Lemma 10 and Eq. (3), $\bigcup_{c\in\mathcal{C}} C^c_{\text{(OA)}}(\rho; I)$ Gale dominates $\bigcup_{c\in\mathcal{R}} C^c(\rho; I)$. This statement and Eq. (2) show that set $\widehat{C}_{\text{(OA)}}(q; I)$ Gale dominates set $\widehat{C}(q; I)$. ■

C.2. Proofs of Preliminary Results in Section A.3.1.

Proof of Lemma 5. Fix $\rho \in (2^{\mathcal{R}})^{|\mathcal{I}|}$ and $\rhd \in \Delta$. Let $\mathcal{E}^c = \mathcal{E}^c(\rho)$ for each $c \in \mathcal{R}$. Fix also $I \subseteq \mathcal{I}$.

Non-wastefulness: Suppose $v \in \mathcal{V}$ be such that $|C^v_{\text{(S)}}(\rhd, \rho; I)| < q^v$, and there exists some $j \in I \setminus \widehat{C}_{\text{(S)}}(\rhd, \rho; I)$. Then, just before v is processed in the sequence $\rhd$, j is still available. Moreover, she does not receive a category-v position. Thus, even though Step 2 of the procedure of $C^v_{\text{(m)}}(\rho; .)$ leaves some vacant positions in category v, j does not receive a position. Thus, $j \notin \mathcal{E}^v$. Since by definition

$$C^v_{\text{(S)}}(\rhd, \rho; I) = C^v_{\text{(m)}}\left(\rho; I \setminus \bigcup_{\nu\in\mathcal{V}:\nu\lhd v} C^\nu_{\text{(S)}}(\rhd, \rho; I)\right) \tag{4}$$

and the argument in previous sentence is true for each category v, the SMH choice rule is non-wasteful.

Maximal accommodation of HR protections: Suppose $v \in \mathcal{V}$ and $j \in \left(I \cap \mathcal{E}^v\right) \setminus \widehat{C}_{\text{(S)}}(\rhd, \rho; I)$. In processing the sequence $\rhd$ while executing the SMH choice rule for I, j was available before category v was processed and remains available after it was processed although $j \in \mathcal{E}^v$. Thus, as $C^v_{\text{(m)}}(\rho; .)$ is maximal for accommodation of HR protections for category v by Lemma 0 and Eq. (4) holds by definition, we have $\eta^v\left(C^v_{\text{(S)}}(\rhd, \rho; I)\right) = \eta^v\left(C^v_{\text{(S)}}(\rhd, \rho; I) \cup \{j\}\right)$. Thus, the SMH choice rule maximally accommodates HR protections.

No justified envy: Suppose $v \in \mathcal{V}$ and $i \in C^v_{\textcircled{s}}(\rhd, \rho; I)$ and $j \in \big(I \cap \mathcal{E}^v\big) \setminus \widehat{C}_{\textcircled{s}}(\rhd, \rho; I)$ such that $\sigma_j > \sigma_i$. By Lemma 0, as $C^v_{\textcircled{m}}(\rho; .)$ satisfies no justified envy we have

$$\eta^v\big(C^v_{\textcircled{m}}(\rho; J)\big) > \eta^v\Big(\big(C^v_{\textcircled{m}}(\rho; J) \setminus \{i\}\big) \cup \{j\}\Big)$$

where $J = I \setminus \bigcup_{\nu \in \mathcal{V}: \nu \lhd v} C^\nu_{\textcircled{s}}(\rhd, \rho; I)$. Since $C^v_{\textcircled{s}}(\rhd, \rho; I) = C^v_{\textcircled{m}}\Big(\rho; J\Big)$ (see Eq. (4)), the SMH choice rule satisfies no justified envy. ■

Proof of Lemma 6. Fix $\rho \in \big(2^{\mathcal{R}}\big)^{|\mathcal{I}|}$ and $\rhd \in \Delta^o$. Suppose $I \subseteq \mathcal{I}$, $c \in \mathcal{R}$, and $i \in C^c_{\textcircled{s}}(\rhd, \rho; I)$. In executing the procedure of the SMH choice rule according to $\rhd$, the open category is processed first and individual i is still available when the VR-protected category c is about to be processed. Thus, it should be the case that $|C^o_{\textcircled{s}}(\rhd, \rho; I)| = q^o$, as otherwise i would have received a category-o position instead of a category-c position by non-wastefulness of the $C^o_{\textcircled{s}}(\rhd, \rho, .) = C^o_{\textcircled{m}}(\rho; .)$ by Lemma 0. Moreover, $C^o_{\textcircled{m}}(\rho; .)$ satisfies no justified envy by Lemma 0. Since $C^o_{\textcircled{s}}(\rhd, \rho; I) = C^o_{\textcircled{m}}(\rho; I)$, for each $j \in C^o_{\textcircled{s}}(\rhd, \rho; I)$, we have $\sigma_j > \sigma_i$ or $\eta^o\big(C^o_{\textcircled{s}}(\rhd, \rho; I)\big) > \eta^o\Big(\big(C^o_{\textcircled{s}}(\rhd, \rho; I) \setminus \{j\}\big) \cup \{i\}\Big)$. Moreover, $C^o_{\textcircled{m}}(\rho; .)$ maximally accommodates HR protections for the open category also by Lemma 0, and $i \notin C^o_{\textcircled{m}}(\rho; I) = C^o_{\textcircled{s}}(\rhd, \rho; I)$. Thus,

$$\eta^o\big(C^o_{\textcircled{s}}(\rhd, \rho; I) \cup \{i\}\big) \not> \eta^o\big(C^o_{\textcircled{s}}(\rhd, \rho; I)\big).$$

These show that the SMH choice rule satisfies compliance with VR protections. ■

C.3. Proofs of Results in Section A.3.2.

Consider the category of memberships $\mathring{\rho} = (\mathring{\rho}_i)_{i \in \mathcal{I}} \in \big(2^{\mathcal{R}}\big)^{|\mathcal{I}|}$ with $|\mathring{\rho}_i| \leq 1$ for each $i \in \mathcal{I}$ so that the VR-protected categories are non-overlapping. Recall that $\mathcal{J} \subseteq \bigcup_{c \in \mathcal{R}^0} \mathcal{E}^c(\mathring{\rho})$ and $\tilde{\rho}_i = \{e\}$ for each $i \in \mathcal{J}$ where $\mathcal{R}^0 = \mathcal{R} \setminus \{e\}$.

We state and prove a more detailed version of Lemma 8 as we use this new lemma also in the proof of Theorem 4.

Lemma 12. *Fix $I \subseteq \mathcal{I}$. Define $\overline{J}$ as*

$$\overline{J} = (\mathcal{J} \cap I) \setminus \widehat{C}^{2s}_{\textcircled{M}}(\mathring{\rho}; I). \tag{5}$$

and J as

$$J = \widehat{C}^{2s}_{\textcircled{M}}\big((\mathring{\rho}_{-\overline{J}}, \tilde{\rho}_{\overline{J}}); I\big) \setminus \widehat{C}^{2s}_{\textcircled{M}}(\mathring{\rho}; I). \tag{6}$$

Then, J is the unique maximal set of individuals who suffer from a violation of the Equality Code under the choice rule $C^{2s}_{\textcircled{M}}(\mathring{\rho}; .)$ for I.

Proof of Lemma 12. First observe that Lemma 7 implies for any $K \subseteq \mathcal{J}$,

$$C^{2s,o}_{\text{Ⓜ}}\big((\mathring{\rho}_{-K}, \tilde{\rho}_K); I\big) = C^{2s,o}_{\text{Ⓜ}}(\mathring{\rho}; I). \tag{7}$$

Then, by the definition of the procedure of the 2SMH choice rule and non-overlapping nature of VR-protected categories at $\mathring{\rho}$ and $(\mathring{\rho}_{-K}, \tilde{\rho}_K)$ that only differ in the memberships of individuals in K for any fixed $K \subseteq (\mathcal{J} \cap I) \setminus \widehat{C}^{2s}_{\text{Ⓜ}}(\mathring{\rho}; I)$, we have

$$C^{2s,c}_{\text{Ⓜ}}\big((\mathring{\rho}_{-K}; \tilde{\rho}_K), I\big) = C^{2s,c}_{\text{Ⓜ}}(\mathring{\rho}; I) \quad \text{for each } c \in \mathcal{R}^0. \tag{8}$$

As (i) single-category meritorious horizontal rule used in the definition of the 2SMH choice rule satisfies irrelevance of rejected individuals by Lemma 11 and (ii) category e is processed after the open category

$$\begin{aligned} C^{2s,e}_{\text{Ⓜ}}\big((\mathring{\rho}_{-K}, \tilde{\rho}_K); I\big) &= C^{e}_{\text{ⓜ}}\big((\mathring{\rho}_{-K}, \tilde{\rho}_K); I \setminus C^{2s,o}_{\text{Ⓜ}}(\mathring{\rho}; I)\big) \\ &= C^{e}_{\text{ⓜ}}\big((\mathring{\rho}_{-K}, \tilde{\rho}_K); K \cup \widetilde{J}\big) \end{aligned} \tag{9}$$

where we define

$$\widetilde{J} = \mathcal{E}^e(\mathring{\rho}) \cap \big(I \setminus C^{2s,o}_{\text{Ⓜ}}(\mathring{\rho}; I)\big).$$

Here, $\widetilde{J}$ is the set of original members of the VR-protected category e in I who do not receive an open category position (i.e., not matched under 2SMH choice rule in Step 1 of its procedure). Thus, Eq. (9) holds as members of sets K and $\widetilde{J}$ are the only individuals who are eligible to be chosen under category e in Step 2 of the procedure of the 2SMH choice rule just before e is processed. Irrelevance of rejected individuals for the single-category meritorious choice rule (by Lemma 11) and Eqs. (7), (8), and (9) imply

$$\widehat{C}^{2s}_{\text{Ⓜ}}\big((\mathring{\rho}_{-K}, \tilde{\rho}_K); I\big) \setminus \widehat{C}^{2s}_{\text{Ⓜ}}(\mathring{\rho}; I) = C^{e}_{\text{ⓜ}}\big((\mathring{\rho}_{-K}, \tilde{\rho}_K); K \cup \widetilde{J}\big) \setminus C^{e}_{\text{ⓜ}}\big((\mathring{\rho}_{-K}, \tilde{\rho}_K); \widetilde{J}\big). \tag{10}$$

Therefore, by definition of J in Eq. (6) and by Eq. (10) after setting $K = \overline{J}$, which was defined in (5), we obtain J is also equal to

$$J = C^{e}_{\text{ⓜ}}\big((\mathring{\rho}_{-\overline{J}}, \tilde{\rho}_{\overline{J}}); \overline{J} \cup \widetilde{J}\big) \setminus C^{e}_{\text{ⓜ}}(\mathring{\rho}; \widetilde{J}). \tag{11}$$

Since the single-category meritorious horizontal choice rule satisfies irrelevance of rejected individuals by Lemma 11 and $\overline{J} \setminus J$ is a subset of individuals who do not receive any position at membership profile $(\mathring{\rho}_{-\overline{J}}, \tilde{\rho}_{\overline{J}})$ (by definitions in Eqs. (5) and (6)), making these individuals ineligible for category e will not change the choice for this category, i.e., $C^{e}_{\text{ⓜ}}\big((\mathring{\rho}_{-J}, \tilde{\rho}_J); J \cup \widetilde{J}\big) = C^{e}_{\text{ⓜ}}\big((\mathring{\rho}_{-\overline{J}}, \tilde{\rho}_{\overline{J}}); \overline{J} \cup \widetilde{J}\big)$. Hence, Eq. (11) implies that

$$\begin{aligned} J &= C^{e}_{\text{ⓜ}}\big((\mathring{\rho}_{-J}, \tilde{\rho}_J); J \cup \widetilde{J}\big) \setminus C^{e}_{\text{ⓜ}}(\mathring{\rho}; \widetilde{J}) \\ &= \widehat{C}^{2s}_{\text{Ⓜ}}\big((\mathring{\rho}_{-J}, \tilde{\rho}_J); I\big) \setminus \widehat{C}^{2s}_{\text{Ⓜ}}(\mathring{\rho}; I), \end{aligned}$$

where the last equality follows from Eq. (10). Thus, J is a set of individuals that suffer from a violation of the Equality Code.

Fix $J' \subseteq (\mathcal{J} \cap I) \setminus \widehat{C}^{2s}_{\text{Ⓜ}}(\mathring{\rho}; I)$ such that $J \subsetneq J'$. Consider the profile of category memberships $(\mathring{\rho}_{\overline{J}}, \tilde{\rho}_{-\overline{J}})$. If we revoke the category-e membership of individuals in $\overline{J} \setminus J'$, who are matched under neither $C^{2s}_{\text{Ⓜ}}\big((\mathring{\rho}_{\overline{J}}, \tilde{\rho}_{-\overline{J}}); I\big)$ nor $C^{2s}_{\text{Ⓜ}}(\mathring{\rho}; I)$, and give them their original membership back, then by Eqs. (10) and (11), we have

$$J = \widehat{C}^{2s}_{\text{Ⓜ}}\big((\mathring{\rho}_{-J'}, \tilde{\rho}_{J'}); I\big) \setminus \widehat{C}^{2s}_{\text{Ⓜ}}(\mathring{\rho}; I) \subsetneq J';$$

thus, there exists some $i \in J'$ such that $i \notin \widehat{C}^{2s}_{\text{Ⓜ}}\big((\mathring{\rho}_{-J'}, \tilde{\rho}_{J'}); I\big)$, so J' is not a set of individuals that suffer from a violation of the Equality Code, and moreover, $J \subseteq \widehat{C}^{2s}_{\text{Ⓜ}}\big((\mathring{\rho}_{-J'}, \tilde{\rho}_{J'}); I\big)$. These two establish that J is a maximal set of individuals who suffer from a violation of the Equality Code.

Finally, we prove its uniqueness. Since $\overline{J}$ is a superset of any $J'' \neq J$ such that $J'' \subseteq \mathcal{J} \cap I$ is a set of individuals who suffer from a violation of the Equality Code, Eq. (6) implies that J'' cannot be maximal, as there exists some $i \in J''$ such that $i \notin \widehat{C}^{2s}_{\text{Ⓜ}}\big((\mathring{\rho}_{-\overline{J}}, \tilde{\rho}_{\overline{J}}); I\big)$. Therefore, this establishes that J is the unique maximal set of individuals who suffer from a violation of the Equality Code under $C^{2s}_{\text{Ⓜ}}(\mathring{\rho}; .)$ for I. ■

Proof of Lemma 8. It directly follows from Lemma 12. ■

Recall that $\rho^*_i = \mathring{\rho}_i \cup \{e\}$ for each $i \in \mathcal{J}$ and $\rho^*_i = \mathring{\rho}_i$ for each $i \in \mathcal{I} \setminus \mathcal{J}$. Consider the order of precedence $\trianglerighteq$ that orders category o first and category e last.

Proof of Proposition 3. Fix a set of individuals $I \subseteq \mathcal{I}$. Lemma 5 implies that $C_{\text{Ⓢ}}(\trianglerighteq, \rho^*; .)$ satisfies non-wastefulness, no justified envy, and maximal accommodation of HR protections. Lemma 6 implies that $C_{\text{Ⓢ}}(\trianglerighteq, \rho^*; .)$ satisfies compliance with VR protections.

Fix $J \subseteq \mathcal{J} \cap I$. We show that J is not a set of individuals that suffer from a violation of the Equality Code under $C_{\text{Ⓢ}}(\trianglerighteq, \rho^*; .)$. If there exists some $i \in J \cap \widehat{C}_{\text{Ⓢ}}(\trianglerighteq, \rho^*; I)$ then we are done. So assume that $J \cap \widehat{C}_{\text{Ⓢ}}(\trianglerighteq, \rho^*; I) = \emptyset$. We use induction in our proof to show that $\widehat{C}_{\text{Ⓢ}}\big(\trianglerighteq, (\rho^*_{-J}, \tilde{\rho}_J); I\big) = \widehat{C}_{\text{Ⓢ}}(\trianglerighteq, \rho^*; I)$.

Suppose that as the inductive assumption, for any $J' \subsetneq J$ with $|J'| \leq k$ for a fixed k with $|J| > k \geq 0$ we have $\widehat{C}_{\text{Ⓢ}}\big(\trianglerighteq, (\rho^*_{-J'}, \tilde{\rho}_{J'}); I\big) = \widehat{C}_{\text{Ⓢ}}(\trianglerighteq, \rho^*; I)$. (For $k = 0$, we have $J' = \emptyset$ in the initial step, and the inductive assumption is vacuously proven for this step.) We prove this statement for $k + 1$.

Let $J' \subsetneq J$ be such that $|J'| = k$ and let $i \in J \setminus J'$. We prove the statement holds for $J' \cup \{i\}$. When it is turn of category e to be processed in Step 2 of the procedure of the SMH choice rule at both $(\rho^*_{-J' \cup \{i\}}, \tilde{\rho}_{J' \cup \{i\}})$ and $(\rho^*_{-J'}, \tilde{\rho}_{J'})$, as i is not selected yet, the same set of

individuals are selected until that point at both cases, as only i's category membership is different at both profiles. Moreover, as i is not selected at $(\rho^*_{-J'}, \tilde{\rho}_{J'})$ by the inductive assumption, she will not receive a position at $(\rho^*_{-J'\cup\{i\}}, \tilde{\rho}_{J'\cup\{i\}})$, either, as $\tilde{\rho}_i \subseteq \rho^*_i$. Thus, $i \notin \widehat{C}_{\text{Ⓢ}}(\trianglerighteq, (\rho^*_{-J'\cup\{i\}}, \tilde{\rho}_{J'\cup\{i\}}); I)$, and moreover, $\widehat{C}_{\text{Ⓢ}}\big(\trianglerighteq, (\rho^*_{-J'}, \tilde{\rho}_{J'}); I\big) = \widehat{C}_{\text{Ⓢ}}\big(\trianglerighteq, (\rho^*_{-J'\cup\{i\}}, \tilde{\rho}_{J'\cup\{i\}}); I\big)$ completing the inductive step's proof.

We showed that $\widehat{C}_{\text{Ⓢ}}\big(\trianglerighteq, (\rho^*_J, \tilde{\rho}_J); I\big) = \widehat{C}_{\text{Ⓢ}}(\trianglerighteq, \rho^*; I)$ proving that $J \not\subseteq \widehat{C}_{\text{Ⓢ}}\big(\trianglerighteq, (\rho^*_J, \tilde{\rho}_J); I\big)$, and hence, J is not a set of individuals who suffer from a violation of the Equality Code. This proves that $C_{\text{Ⓢ}}(\trianglerighteq, \rho^*; .)$ abides by the Equality Code for I. ■

Proof of Theorem 4. Fix a set of individuals $I \subseteq \mathcal{I}$. Define

$$J = \widehat{C}_{\text{Ⓢ}}(\trianglerighteq, \rho^*; I) \setminus \widehat{C}^{2s}_{\text{Ⓜ}}(\mathring{\rho}; I).$$

We prove below in Claim 1

$$\widehat{C}_{\text{Ⓢ}}(\trianglerighteq, \rho^*; I) = \widehat{C}^{2s}_{\text{Ⓜ}}\big((\mathring{\rho}_{-\overline{J}}, \tilde{\rho}_{\overline{J}}); I\big),$$

where

$$\overline{J} = \big(\mathcal{J} \cap I\big) \setminus \widehat{C}^{2s}_{\text{Ⓜ}}(\mathring{\rho}; I),$$

so that the rest of the proof follows from Lemma 12.

Claim 1. $\widehat{C}_{\text{Ⓢ}}(\trianglerighteq, \rho^*; I) = \widehat{C}^{2s}_{\text{Ⓜ}}\big((\mathring{\rho}_{-\overline{J}}, \tilde{\rho}_{\overline{J}}); I\big)$.

Proof of Claim 1. Let

$$\rho' = (\mathring{\rho}_{-\overline{J}}, \tilde{\rho}_{\overline{J}}).$$

Observe that for individuals in I, we have

(1) for each $i \in \mathcal{J} \cap \widehat{C}^{2s}_{\text{Ⓜ}}(\mathring{\rho}; I)$,	$\rho^*_i = \mathring{\rho}_i \cup \{e\}$	&	$\rho'_i = \mathring{\rho}_i$,
(2) for each $i \in \big(\mathcal{J} \cap I\big) \setminus \widehat{C}^{2s}_{\text{Ⓜ}}(\mathring{\rho}; I)$,	$\rho^*_i = \mathring{\rho}_i \cup \{e\}$	&	$\rho'_i = \{e\}$,
(3) for each $i \in I \setminus \mathcal{J}$,	$\rho^*_i = \mathring{\rho}_i$	&	$\rho'_i = \mathring{\rho}_i$.

Lemma 7 implies that

$$C^o_{\text{Ⓢ}}(\trianglerighteq, \rho^*; I) = C^{2s,o}_{\text{Ⓜ}}(\rho'; I) = C^{2s,c}_{\text{Ⓜ}}(\mathring{\rho}; I). \tag{12}$$

Since VR-protected categories other than e do not overlap with each other at ρ^* and VR-protected categories do not overlap at all at ρ' and $\mathring{\rho}$, their order of precedence does not matter for $C^{2s}_{\text{Ⓜ}}$ under ρ' and $\mathring{\rho}$ and as long as e is processed last as it is done for choice rule $C_{\text{Ⓢ}}(\trianglerighteq, \rho^*; .)$ under $\trianglerighteq$. Therefore, for each $c \in \mathcal{R}^0$,

$$C^c_{\text{Ⓢ}}(\trianglerighteq, \rho^*; I) = C^{2s,c}_{\text{Ⓜ}}(\rho'; I) = C^{2s,c}_{\text{Ⓜ}}(\mathring{\rho}; I). \tag{13}$$

Thus, only the set of individuals who receive category-e positions could possibly differ under both choice rules $C^{2s,e}_{\textcircled{M}}(\rho';.)$ and $C^{e}_{\textcircled{S}}(\trianglerighteq,\rho^*;.)$. By Eqs. (12) and (13), in the procedures of $C^{2s}_{\textcircled{M}}(\rho';.)$ and $C_{\textcircled{S}}(\trianglerighteq,\rho^*;.)$ just before category e is processed, we have exactly the same set of eligible individuals available for category e by definitions of $\overline{J}$ and ρ'. Since each choice rule uses the single-category meritorious choice rule for category e, after using Eqs. (12) and (13),

$$\begin{aligned} C^{e}_{\textcircled{S}}(\trianglerighteq,\rho^*;I) &= C^{e}_{\textcircled{m}}\big(\rho^*;I\setminus\cup_{e\trianglerighteq\nu}C^{\nu}_{\textcircled{S}}(\trianglerighteq,\rho^*;I)\big)\\ &= C^{e}_{\textcircled{m}}\big(\rho';I\setminus\cup_{e\trianglerighteq\nu}C^{2s,\nu}_{\textcircled{M}}(\rho';I)\big)=C^{2s,e}_{\textcircled{M}}(\rho';I). \end{aligned} \tag{14}$$

Eqs. (12), (13), and (14) imply

$$\widehat{C}_{\textcircled{S}}(\trianglerighteq,\rho^*;I)=\widehat{C}^{2s}_{\textcircled{M}}(\rho';I).$$

□

■

C.4. Proof of Result in Section A.3.3.

Proof of Theorem 5. Fix $\rho=(\rho_i)_{i\in\mathcal{I}}\in\left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$ such that for any $i\in\mathcal{I}$, $|\rho_i|\leq 2$ and $|\rho_i|=2$ implies $e\in\rho_i$. Fix also $\overline{\trianglerighteq}\in\Delta^{o,e}$. Let choice rule $C(\rho;.)$ respect mobility from reparatory categories to EWS and satisfy non-wastefulness, maximal accommodation of HR protections, no justified envy, and compliance with VR protections.

Let $I\subseteq\mathcal{I}$. By Lemma 7, for any profile of non-overlapping category memberships $\rho'\in\left(2^{\mathcal{R}}\right)^{|\mathcal{I}|}$, $C^{o}(\rho,I)=C^{2s,o}_{\textcircled{M}}(\rho',I)$ and $C^{o}_{\textcircled{S}}(\overline{\trianglerighteq},\rho;I)=C^{2s,o}_{\textcircled{M}}(\rho',I)$. Thus,

$$C^{o}(\rho,I)=C^{o}_{\textcircled{S}}(\overline{\trianglerighteq},\rho;I) \tag{15}$$

Let

$$J=I\setminus C^{o}(\rho,I).$$

We prove the following claim:

Claim 2. $C^{e}(\rho,I)=C^{e}_{\textcircled{S}}(\overline{\trianglerighteq},\rho;I)$.

Proof of Claim 2. Recall that $C^{e}_{\textcircled{S}}(\overline{\trianglerighteq},\rho;I)=C^{e}_{\textcircled{m}}(\rho;J)$. To show that $C^{e}(\rho;I)=C^{e}_{\textcircled{m}}(\rho;J)$, we consider component $C^{e}(\rho;I)$ as a single-category choice rule executed on set J to invoke Lemma 0. To this end, $C^{e}(\rho;I)$ satisfies the following three properties:

$C^{e}(\rho;I)$ satisfies non-wastefulness on J: Let $j\in J\setminus C^{e}(\rho;I)$. Suppose $|C^{e}(\rho;I)|<q^{e}$. We show that $j\notin\mathcal{E}^{e}(\rho)$ to complete the proof.

Since $C(\rho;.)$ is non-wasteful, either (i) $j\notin\mathcal{E}^{e}(\rho)$ or (ii) $j\in\mathcal{E}^{e}(\rho)$ and $j\in\widehat{C}(\rho;I)$.

We show that (ii) does not hold. Contrary to the claim, suppose it does. As $j \notin C^o(\rho; I)$ by $j \in J$ and Eq. (15), then $j \in C^c(\rho, I)$ for some $c \in \mathcal{R}^0$. Since $C(\rho;.)$ respects mobility from reparatory categories to EWS then $|C^e(\rho; I)| = q^e$, a contradiction. Therefore, we showed that (ii) cannot hold. Thus, $j \notin \mathcal{E}^e(\rho)$.

$C^e(\rho; I)$ *satisfies no-justified envy on* J: Let $i \in C^e(\rho; I)$ and $j \in (\mathcal{E}^e(\rho) \cap J) \setminus C^e(\rho; I)$. Suppose

$$\eta^e\big(C^e(\rho; I)\big) \leq \eta^e\Big(\big(C^e(\rho; I) \setminus \{i\}\big) \cup \{j\}\Big). \tag{16}$$

We show that $\sigma_i > \sigma_j$ to complete the proof.

If $j \notin \widehat{C}(\rho; I)$ then no-justified envy property of $C(\rho;.)$ implies $\sigma_i > \sigma_j$. Consider the other case, $j \in \widehat{C}(\rho; I)$. Eq. (15) implies that $j \in C^c(\rho; I)$ for some $c \in \mathcal{R}^0$. Since $C(\rho;.)$ maintains the elevated status of caste-based VR protections and $i \in C^e(\rho; I)$, we have either (i) $\sigma_i > \sigma_j$ or (ii) $\eta^e\big(C^e(\rho; I)\big) > \eta^e\Big(\big(C^e(\rho; I) \setminus \{i\}\big) \cup \{j\}\Big)$. By Eq. (16), Case (ii) cannot hold. Therefore, $\sigma_i > \sigma_j$.

$C^e(\rho; I)$ *satisfies maximal accommodation of HR protections on* J: Let $i \in (\mathcal{E}^e(\rho) \cap J) \setminus C^e(\rho; I)$. We show that

$$\eta^e\big(C^e(\rho; I) \cup \{i\}\big) \not\succ \eta^e\big(C^e(\rho; I)\big) \tag{17}$$

to complete the proof.

If $i \notin \widehat{C}(\rho; I)$, then by the fact that $C(\rho;.)$ satisfies maximal accommodation of HR protections, Eq. (17) holds. If $i \in \widehat{C}(\rho; I)$, then Eq. (15) implies that $i \in C^c(\rho; I)$ for some $c \in \mathcal{R}^0$. Since $C(\rho;.)$ maintains the elevated status of caste-based VR protections, Eq. (17) holds.

Lemma 0 implies $C^e(\rho; I) = C^e_{\textcircled{m}}(\rho; J) = C^e_{\textcircled{s}}(\overline{\rhd}, \rho; I)$. □

Let

$$K = J \setminus C^e(\rho; I).$$

Let $c \in \mathcal{R}^0$. Since $|\rho_i| \leq 2$ and $|\rho_i| = 2$ implies $e \in \rho_i$ for each $i \in \mathcal{I}$, and both categories o and e precede all categories in $\mathcal{R}^0$, precedence order of categories in $\mathcal{R}^0$ is immaterial to the outcome of $C_{\textcircled{s}}$. In particular, $C^c_{\textcircled{s}}(\overline{\rhd}, \rho; I) = C^c_{\textcircled{m}}(\rho; K)$. To show that $C^c(\rho; I) = C^c_{\textcircled{m}}(\rho; K)$, we consider execution of the component $C^c(\rho; I)$ as a single-category choice rule on K. Given Eq. (15) and Claim 2, non-wastefulness, no justified envy, and maximal accommodation of HR reservations properties of $C(\rho;.)$ directly imply the corresponding properties are satisfied by $C^c(\rho; I)$ on K. Lemma 0 implies that $C^c(\rho; I) = C^c_{\textcircled{m}}(\rho; K) = C^c_{\textcircled{s}}(\overline{\rhd}, \rho; I)$. This together with Eq. (15) and Claim 2 imply that $C(\rho; I) = C_{\textcircled{s}}(\overline{\rhd}, \rho; I)$. ■