The Regime-Switching Policy for the RMB*

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Abstract

The RMB exchange rate policy is governed by a two-pillar rule. This paper presents novel evidence of regime-switching in the policy coefficient on the market pillar, but not on the basket pillar. The switching is closely tied to the implementation of the counter-cyclical factor (CCF) policy, which is designed to address market irrationality. The frequency of regime-switching is significantly higher than what is officially announced, suggesting that the *de facto* policy differs from the *de jure* policy. Using a simple model, this paper demonstrates that the regime-switching policy has an expectation formation effect and allows the CCF policy to effectively prevent self-fulfilling depreciation and achieve its objective. Yet, the *de jure* CCF policy is unable to stabilize the RMB exchange market due to its weak expectation effect.

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1 Introduction

With rapid economic growth over the last 40 years, China's exchange rate policy has become increasingly important in international trade and the financial market. The People's Bank of China (PBC) currently enforces a *managed floating* policy.[1] This policy relies on the formation of the central parity rate. Since 2016, this formation has depended on two pillars: market flexibility and RMB index stability.[2] In mid-2017, PBC incorporated a counter-cyclical factor (CCF, hereafter) into the central parity rate's formation mechanism to handle market irrationality. And this CCF factor is phased in and out over time.

In recent work, Jermann et al. [2022] (JWY, hereafter) document a two-pillar policy using the expression

$$\Delta c_t = \alpha d_{t-1} + \beta \Delta x_t^{USD} + \epsilon_t \tag{1}$$

where Δc_t is the percentage change of the RMB/USD central parity rate; d_{t-1} captures the previous day's market deviation of the RMB/USD rate from its parity rate, thus reflecting the market flexibility; and Δx_t^{USD} is the percentage change in the implied dollar index, which captures the RMB index stability or basket pillar.[3] JWY also discover that policy coefficient α differs greatly between subperiods, which are classified according to official announcements about the implementation of CCF.

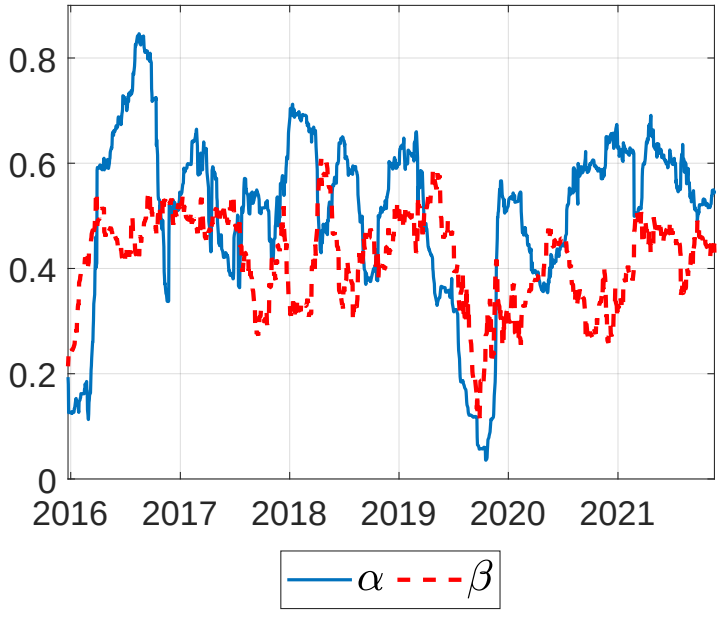
In this paper, we delve deeper into the two-pillar policy by studying its regime-switching patterns and potential effects through a simple model. Our empirical findings reveal that the PBC's implementation of CCF policy leads to α switching. Interestingly, the frequency of regime-switching in the *de facto* policy is higher than that in the official announcement (*de jure* policy), indicating that the PBC may be using CCF policy without public communication. To evaluate the effectiveness of CCF policy in managing market irrationality, which is its main objective, we propose a rational expectation model. In the absence of regime-switching policies, ensuring a unique equilibrium of the economy requires coordination between monetary and exchange rate policies that may be too difficult to achieve in practice. Notably, switching the CCF on and off in the RMB policy can prevent self-fulfilling

[1] The policy allows RMB spot rates to fluctuate within a predetermined band around the central parity rate in the intraday market.

[2] The first pillar is defined as " the closing rates of the previous business day," and the second pillar is "a theoretical rate that maintains the stability of the RMB index." See the PBC's Monetary Policy Reports of 2016Q1 and of 2017Q2 for details of the reform and details on the CCF, respectively.

[3] Here, we simplify the notation for the purpose of presentation. To map this expression to JWY's definition of the implied dollar index Δx_t, note that $\Delta x_t^{USD} = (1 - w_{usd})\Delta x_t$, where w_{usd} is the PBC-announced USD weight in the RMB index calculation. We return to the more precise notation in the main text from Section 2.

Figure 1: $t\pm$30D Rolling-window Estimates of the Two-Pillar Rule



Note: This figure plots the 60-day rolling window regression estimates of the policy coefficients α and β for each rolling sample.

RMB depreciation and make the coordination less demanding due to the expectation effect of switching policy coefficients. However, the *de jure* CCF policy has a low switching frequency, which results in a weak expectation effect and failure to achieve its objective.

Regime-switching coefficient is a readily observable feature of the RMB policy. We use a motivational 60-day rolling window estimation and observe the changing dynamics of policy coefficients from November 2015 to December 2021. The results, depicted in Figure 1, show that policy coefficients undergo significant shifts and follow a clear regime-switching pattern. Except during early 2016 and mid-to-late 2019, the coefficients α and β are negatively correlated and have a sum of approximately one.

To understand this regime-switching pattern, we study a natural candidate: CCF policy and its pattern of phasing in and out. Following JWY's specification, we focus on the CCF and the non-CCF regimes. As stated in the PBC's Monetary Policy Report (2017Q2), *"after the introduction of the counter-cyclical factor the central parity formation mechanism has increased the weight of the reference to the currency basket."* Presumably, one would expect the CCF regime to be characterized by a combination of low-flexibility and high-stability policies, while the non-CCF regime should involve a combination of high-flexibility and low-stability policies. To determine whether the data supports the *de jure* CCF policy, we

conduct the following empirical exercises.

We begin by presenting the results of a rolling-window regression analysis of the extended two-pillar rule, which includes a third factor.[4] According to JWY, this third factor is the daytime component of the basket pillar and captures the use of CCF. Our findings reveal a clear trade-off between market flexibility and the CCF factor and show that this relationship follows a regime-switching pattern even in late 2019. Additionally, we observe a significant weakening of the negative relationship between market flexibility and RMB index stability. Moreover, heteroskedasticity is empirically relevant because of the significant variations in the coefficients of the rule's explanatory power over time.

Based on the rolling-window regression results, we develop a benchmark coefficient-switching model in the style of Chang et al. [2023]. This model generalizes the extended policy rule to allow the switching of both the policy coefficients and the volatility of policy discretion. We define two states for the market and CCF coefficients, which correspond to the CCF and non-CCF regimes. We also specify two states for the stability coefficient and the volatility of policy discretion. Switching is driven by a three-dimensional vector autoregressive (VAR) latent factor. Each element of the vector drives the switching of its corresponding coefficient. Importantly, we ask the market and CCF coefficients to be influenced by the same policy factor but do not require the market and stability pillar coefficients to be driven by the same factor. Rather, we rely on the data to determine whether implementing the CCF is comparable to moving away from the market pillar and toward the basket pillar.

By conducting maximum likelihood (ML) estimation over the full sample, we gain valuable insights into the regime-switching patterns for the RMB policy rule. Highlights of our estimation include significantly different policy coefficients and volatility regimes, as well as the parameters of latent policy factors that drive the switching of each policy coefficient. Using these estimated latent policy factors, we discover that of the three coefficients, the market pillar coefficient switches most frequently, while the stability pillar coefficient and policy discretion volatility remain relatively stable. We observe that as the market pillar coefficient becomes high (low), the CCF coefficient becomes low (high). Furthermore, the estimated changes in the market pillar coefficient occur at substantially higher frequencies than those that have been publicly disclosed. This finding suggests that PBC may be using the CCF policy without informing the public. However, the evidence regarding the trade-off between market flexibility and RMB index stability is mixed. The two factors driving these variables are negatively correlated, with a correlation coefficient of -0.82, but

[4]Throughout the paper, the terms "extended" and "modified" are used interchangeably when referring to a policy rule.

the stability coefficient rarely switches. Therefore, we differentiate the *de facto* CCF policy, whose regime-switching pattern is revealed by data, from the *de jure* CCF policy, whose regime-switching is publicly announced.

The main empirical findings are fairly robust to different model specifications.[5] Among different alternative specifications, the simple two-pillar model is particularly informative. JWY shows that a simple two-pillar policy is overall accurate. Although the public is aware of the CCF policy, they do not know its operational detail. The actual implementation of the CCF policy may involve switching policy coefficients in a simple two-pillar framework. We thus estimate this coefficient-switching two-pillar model. We find that the resulting market pillar factor is strongly correlated with the benchmark market pillar factor. However, the benchmark specification appears to identify the announced CCF regime much better than the simple rule. This result implies that including the daytime component in the policy rule improves the accuracy of the threshold rather than the policy factors themselves. Our finding further suggests that our regime-switching framework can effectively capture the PBC's policy stance.

To analyze the economic effects of both the *de facto* and *de jure* CCF policies, we build a rational expectation model similar to that of Svensson [1994] and JWY. We evaluate the role of CCF policy in ruling out the self-fulfilling prophecy in the foreign exchange market. This aligns well with PBC's intention to propose the CCF. In its Monetary Policy Report (2017Q2), PBC stated that *"This factor is used to partially offset the deviation of the previous closing rate from the daytime component of the basket pillar, which may be driven by sentiment-induced "procyclicality" in the foreign exchange market."*

We first present a linear rational expectation (LRE) model—an economy with a constant exchange rate policy rule. The model includes the goods market, money market, and foreign exchange rate. For simplicity, the monetary policy only reacts to the foreign interest rate and central parity rate. We distinguish between noise traders and informed traders in the intraday exchange rate market. In this economy, if the monetary policy is muted, a sentiment shock would lead to self-fulfilling depreciation in the RMB exchange market unless noise traders lean against the wind. We also demonstrate the importance of policy coordination. For instance, a mild monetary policy (i.e., an *internal* policy that is insensitive to the central parity rate) and an aggressive exchange rate policy (i.e., an *external* policy that is sensitive to market conditions) can ensure a unique equilibrium and eliminate the possibility of multiple equilibria and self-fulfilling depreciation by jointly influencing the excess return of RMB asset. However, the internal-external coordination required for a unique equilibrium still depends on the noise trader's market force (i.e., the relative size of

[5] We applied a plethora of model specifications, and the results are available upon request.

the noise trader and their sensitivity to the spot rate when forming their expectations).

In reality, the coordination required of the monetary policy department may be too demanding, given that the policy is also responsible for output and employment. We next explore a model with a constant monetary policy and a Markov-switching exchange rate policy. Our empirical estimation of the structural parameters suggests that a CCF regime (a low α) itself leads to indeterminacy in our simple LRE model. This result is partially due to our minimalist specification of monetary policy. In future research, it will be important to entail a more realistic monetary policy rule and the role played by a low α regime in stabilizing the exchange market. For transparency, all analyses herein are limited to the minimalist model to illustrate the consequence of switching policies.

We model the switching process of policy coefficients as an exogenous Markov chain. Our main model only considers the switching of the market pillar coefficient.[6] This model is consistent with our empirical results. Using the Markovian transition probability matrix from our empirical exercise, the model solution shows that the region of determinacy is substantially expanded relative to the one implied by the constant policy rule. This expansion is due to the effects of regime change expectation formation, which are highlighted in the work by Davig and Leeper [2007]. In our environment, the exchange market participants realize that the future policy regime may differ from the prevailing regime. Irrational behavior in this market cannot be self-fulfilled even if the current policy combination is in the low α regime, as the rational market agent internalizes a nonzero probability of a high α regime in the future. The switching policy reduces the requirement for coordination between monetary policy and exchange rate policy.

The *de facto* CCF policy, but not the *de jure* policy, guarantees a determinate equilibrium in the Markov-switching rational expectations (MSRE) model. For a daily frequency model, the announced CCF policy is introduced and lifted several times at a very low frequency. To examine the consequence of the *de jure* policy, we solve the model at a very low switching probability between the CCF and the non-CCF regimes; all other elements are identical to those in the benchmark exercise. We find that extremely persistent regimes weaken the expectation formation effect. This result is not surprising, as the CCF regime itself cannot stabilize the market in the LRE model. Yet, our result emphasizes the importance of managing market expectations. Even in a model where the CCF regime helps rule out the self-fulfilling phenomenon, a highly persistent non-CCF regime would lead to indeterminacy when the expectation formation effect is weak.

Our emphasis on the *de facto* CCF policy and the induced market expectation formation

[6] In an extended model, we also show that the switching of the stability pillar policy coefficient does not change the main results of our analysis.

effect is practically relevant. Since 2022, no official announcement has been made regarding the use of the CCF regime. Anecdotally, however, some market participants believe that the CCF policy is used in the period when the RMB continues to depreciate.[7]

Literature review: Topic-wise, our study is positioned within a fast-growing body of literature that aims to understand China's recent reforms of macroeconomics policies and of the financial system. For examples, see Chen et al. [2018], Liu et al. [2021] and Brunnermeier et al. [2022]. Regarding the literature that focuses on the RMB exchange rate policy and reform, Frankel and Wei [2007], Frankel [2009] and Sun [2010] study policy after the RMB exchange rate regime reform in 2005. Cheung et al. [2018a,b] study the determinants of central parity after the reform in 2015 and Clark [2017] explore the role of dollar in driving the change of RMB. Liang et al. [2019] studies the spread between the onshore and offshore markets after the reform. Su and Qian [2021] test a structural break in the exchange rate mechanism. Lei et al. [2022] study the optimal trading band. Our paper is most closely related to JWY. Their work is the first paper to study empirically and theoretically the two-pillar policy and demystify the mechanism of the CCF. Inspired by these preceding works, we study a generalized two-pillar policy by allowing time-varying policy coefficients. Our work complements the findings of JWY and contributes to this important literature by documenting the difference between *de jure* and *de facto* CCF policies and showing the economic consequences of using the CCF.

In terms of methodology, our paper is related to the general literature on regime-switching models and a vast literature on monetary economics that employs such methods in studying time-varying monetary and fiscal policies. Regarding the empirical methodology, regime-switching models are widely used and proven useful in many contexts. For instance, Davig and Leeper [2006a,b], Bianchi [2012], Bianchi and Ilut [2017], and Chang et al. [2023] focus on the studies of US monetary and fiscal policies. Kaminsky [1993] applies the regime-switching monetary policy to the exchange market. Bianchi et al. [2018] uses the regime-switching model to discuss uncertainty shocks. Falck et al. [2021] discuss the switching regimes of professional forecasters. Bianchi et al. [2022] discuss the implications in asset pricing.

Our work is also related to the literature on MSRE model. In an otherwise canonical New-Keynesian model, Davig and Leeper [2007] use a quasi-linear solution of the MSRE model to highlight the possibility of using regime-switching monetary policy to rule out self-fulfilled inflation. We are inspired by their insights, and solve a simple MSRE model of the RMB exchange rate to demonstrate that regime-switching RMB policy rules may help to rule out self-fulfilling depreciation in the market. Notably, our solution is nonlinear, and

[7] See Bloomberg News: Some China Banks Change Fixing Models as Yuan Weakness Deepens.

we borrow the solution method from the following literature. Farmer et al. [2009] discusses the general solution of a forward-looking MSRE model without predetermined variables. Cho [2016] and Cho [2021] discuss the general solution in a model with predetermined variables. In our analysis, the central parity rate is partly determined by the closing rate of the previous day, and thus our system includes a predetermined variable. We apply Cho's solution technique in the RMB policy context.

The rest of the paper is organized as follows. Section 2 reviews the extended central parity rule and generalizes it to the regime-switching model, followed by discussions of the empirical findings and robustness checks. Section 3 presents tne baseline LRE model. Section 4 highlights the self-fulfilling phenomenon in this model. In section 5, we extend the baseline LRE model to a MSRE model and evaluate the consequence of the regime-switching policy. Section 6 concludes the paper.

2 Empirical Analysis

This section focuses on analyzing regime-switching policy rules for the RMB central parity rates to establish stylized facts of RMB policy regimes. The section is divided into four parts. The first part empirically examines an extended JWY rule that accounts for CCF policy in rolling samples. In the second part, we specify a benchmark regime-switching policy rule based on the rolling-window results. The third part offers strong empirical evidence of policy coefficient switching and reveals a significant difference between the announced (*de jure*) and *de facto* policy regimes. The last part revisits JWY's simple two-pillar rule without intra-day RMB movements but allows switching policy coefficients to absorb the effects of underlying RMB regimes. Our findings show that the estimated regime dynamics of the two-pillar policy with switching coefficients are quite robust, and the intra-day RMB data provide important insights into the timing of policy regimes.

2.1 Evidence of Time-varying Policy in RMB from a Rolling-Window Regression Analysis

We start by presenting empirical results based on an extended formulation of the two-pillar policy rule proposed by JWY. According to the simple version of the two-pillar policy rule (Equation (1)), the RMB/USD central parity rate S_t^{CP} is determined by the previous closing RMB/USD exchange rate[8], S_{t-1}^{CL}, and a US dollar index X_t with a 24-hour reference period

[8] All exchange rate levels in this paper are US dollar prices in terms of RMB.

that is implied by an RMB index.[9] According to JWY, the first component of the policy rule is known as the market flexibility pillar, while the second component is the stability or basket pillar of the RMB index. Two reforms were implemented in early 2017. In February 2017, the PBC changed the reference period of the basket pillar from 24 hours to 15 hours. Since then, only the nighttime component of the implied US dollar index has been used in the basket pillar. That is, $\Delta x_t^{NT} = \log(\frac{X_{t,7:30AM}}{X_{t-1,4:30PM}})$, where $X_{t,7:30AM}$ and $X_{t,4:30PM}$ are the implied dollar index at 7:30 AM and 4:30 PM Beijing time, respectively. In May 2017, the PBC introduced an additional component in policymaking called the "counter-cyclical factor" (CCF) to help manage market sentiment and irrationality.

As a novel finding, JWY proposes to use the change of the daytime component of the implied US dollar index Δx_t^{DT} to proxy for the market sentiment:

$$\Delta x_t^{DT} = \log(\frac{X_{t,4:30PM}}{X_{t,7:30AM}}).$$

Based on the construction of X_t (See Footnote 9), Δx_t^{DT} can be regarded as an exogenous sentiment process outside of the RMB onshore market. Due to the liberalization of China's financial account, these market sentiment shocks affect the RMB market. As a result, a CCF policy is proposed and implemented by the PBC. Upon including the CCF, the extended RMB policy rule takes the following expression:

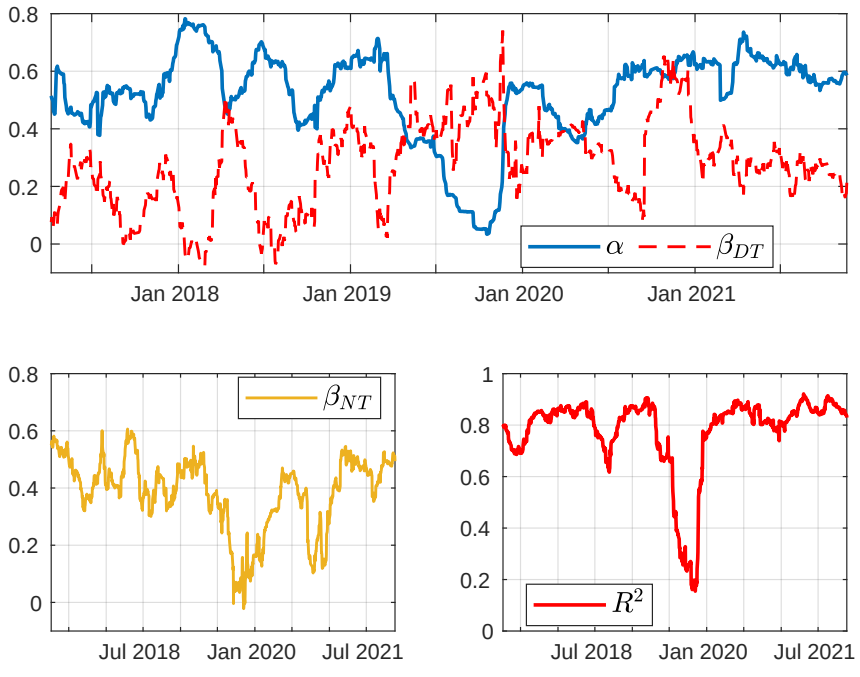
$$\Delta c_t = \alpha d_{t-1} + \beta_{DT}\Delta x_{t-1}^{DT} + \beta_{NT}(1 - w_{usd})\Delta x_t^{NT} + \sigma\varepsilon_t \tag{2}$$

where $c_t = \log(S_t^{CP})$ and $\Delta c_t = c_t - c_{t-1}$ is the percentage change in the RMB/USD central parity rate; $d_{t-1} = \log(S_{t-1}^{CL}) - \log(S_{t-1}^{CP})$ captures the market deviation of RMB/USD spot rate from its parity rate. Δx_t^{NT} and Δx_t^{DT} are the changes in the nighttime and daytime components of the basket pillar, respectively. The policy coefficients $\alpha, \beta_{DT}, \beta_{NT}$ can arise as functions of PBC's policy weights on various policy objectives. A larger α corresponds to a greater emphasis on the market-driven RMB rate; a larger β_{DT} reflects a greater effort to counter the market irrationality; and a larger β_{NT} corresponds to a heavier weight on RMB index stability. Throughout the paper, w_{usd} denotes the index weight of USD in the RMB index.[10] $\sigma\varepsilon_t$ represents the policy discretion, and ε_t is an innovation with unit variance.

[9]The RMB index is defined as $B_t = \chi\frac{X_t^{(1-w_{usd})}}{S_t^{CP}}$. χ is a constant parameter. $\Delta x_t = \Delta\log(X_t)$ is the change of X_t within 24 hours. Specifically, the following definition is obtained from JWY: $X_t = C_x(\frac{S^{CP,EUR/CNY}}{S^{CP,USD/CNY}})^{\frac{w_{eur}}{1-w_{usd}}}(\frac{S^{CP,JPY/CNY}}{S^{CP,USD/CNY}})^{\frac{w_{jpy}}{1-w_{usd}}}$.... C_x is some constant parameter.

[10]The RMB index, which is based on CFETS, changes its currency basket weight periodically. The weight of USD, w_{usd}, is 0.224 from 1/1/2017 to 12/31/2019, 0.2159 from 1/1/2020 to 12/31/2020, and 0.1879 from

Figure 2: $t\pm$30D Rolling-Window Estimates of the Extended "Two-Pillar" Rule



Note: This figure plots the 60-day rolling window regression estimates of the extended policy coefficients and the R^2 for each rolling sample.

We gathered daily data on the central parity rate covering the period 11/10/2015 to 12/31/2021 from the China Foreign Exchange Trade System (CFETS). The data on the market closing rate is collected from Bloomberg. The 24-hour implied dollar index is constructed based on the data from CFETS, with its daytime and nighttime components constructed using the BFI data from Bloomberg, following JWY's method. In line with the CCF policy reform, we limited our benchmark sample to data from 2/20/2017 to 12/31/2021 to maintain consistency. Figure 2 displays the policy coefficients that vary over time, obtained from rolling-window regressions for Equation (2). The rolling sample includes 30 days before and after a trading day t. Additionally, the associated coefficient of determination (R^2) is presented for each rolling sample.

According to the regression results, policy coefficients exhibit a pattern of regime-switching and vary significantly over time. Two major recurrent policy regimes emerged from our sample, namely the non-CCF regime and the CCF regime. The former is char-

1/1/2021 to 12/31/2021.

acterized by a combination of high α and low β_{DT}, while the latter is characterized by a combination of low α and high β_{DT}. The correlation coefficient between the two estimated policy coefficients is -0.48. Additionally, we find frequent and quantitatively significant time variations in policy coefficients α and β_{DT}, with α values ranging from 0.1 to approximately 0.8 and β_{DT} values ranging from 0 to approximately 0.4. Furthermore, our estimation reveals that $\beta_{DT}\Delta x_{t-1}^{DT}$ accounts for about 15% of the predicted variation in Δc_t.[11] This discovery suggests that PBC frequently and considerably changes its RMB policy to counter market irrationality. However, the correlation between α and β_{NT} - the stability pillar coefficient - appears to vary over time, with an overall correlation coefficient of 0.40. It is noteworthy that α and β_{NT} are negatively correlated, as per JWY, except during mid-2017, 2019, and mid-2021. Lastly, the rolling R^2 indicates the presence of heteroskedasticity over time. Specifically, the model performs poorly during mid-to-late 2019, which coincides with the peak of the US-China trade war.[12]

To summarize, despite potential sampling errors due to rolling windows in previous estimates, it is plausible to consider an econometric model where a shared factor drives the switching of the market pillar and sentiment coefficients while two other factors drive the stability pillar and heteroskedasticity coefficients.

2.2 Benchmark Specification for Regime-Switching Rules

To thoroughly analyze the dynamics observed in our rolling-window estimations, we investigate a benchmark regime-switching RMB policy rule that generalizes Equation (2). This policy rule allows changes in the coefficients and volatility:

$$\Delta c_t = \alpha\left(r_t^{\alpha}\right) d_{t-1} + \beta_{DT}(r_t^{\alpha})\Delta x_{t-1}^{DT} + \beta_{NT}(r_t^{\beta_{NT}})(1-w_{usd})\Delta x_t^{NT} + \sigma(r_t^{\sigma})\varepsilon_t \quad (3)$$

with $\varepsilon_t \sim_{i.i.d} \mathbb{N}(0,1)$ and state-dependent coefficients

$$\begin{aligned}
\alpha(r_t^{\alpha}) &= \alpha_0(1-r_t^{\alpha}) + \alpha_1 r_t^{\alpha}, \quad 0 \le \alpha_0 < \alpha_1 \le 1, \\
\beta_{DT}(r_t^{\alpha}) &= \beta_{DT,0}(1-r_t^{\alpha}) + \beta_{DT,1} r_t^{\alpha}, \quad 0 \le \beta_{DT,1} < \beta_{DT,0} \le 1, \\
\beta_{NT}(r_t^{\beta_{NT}}) &= \beta_{NT,0}(1-r_t^{\beta_{NT}}) + \beta_{NT,1} r_t^{\beta_{NT}}, \quad 0 \le \beta_{NT,0} < \beta_{NT,1} \le 1, \\
\sigma(r_t^{\sigma}) &= \sigma_0(1-r_t^{\sigma}) + \sigma_1 r_t^{\sigma}, \quad 0 < \sigma_0 < \sigma_1.
\end{aligned}$$

[11] For each rolling sample, we quantify the contribution of $\beta_{DT}\Delta x_{t-1}^{DT}$ to the predicted variation of Δc_t by $R^2_{DT} = \frac{\sum(\widehat{\beta_{DT}\Delta x_{t-1}^{DT}})^2}{\sum(\widehat{\Delta c_t})^2}$. Then, we compute the total contribution rate across all rolling samples.

[12] In August 2019, China was labeled as a "currency manipulator." During this period, the central parity rate of RMB/USD remained stable, reducing the effectiveness of a linear model in explaining the dependent variable.

For each policy coefficient, there is an associated binary regime indicator, represented by $r_t^i \in \{0,1\}$, where i belongs to the set $\{\alpha, \beta_{NT}, \sigma\}$. At any given time t, the exchange rate policy mix of PBC can be denoted by the vector $\mathbf{R}_t = (r_t^{\alpha}, r_t^{\beta_{NT}}, r_t^{\sigma})'$. The switching of α and β_{NT} reflects changes in policies regarding RMB market flexibility and RMB index stability. Meanwhile, the switching of σ captures the heteroskedasticity in policy discretion.

We use a Markovian regime-switching approach to model the dynamics observed in rolling regressions. To achieve this with a reasonable number of parameters, we introduce a latent vector autoregressive process represented by the vector $\mathbf{w}_t = (w_t^{\alpha}, w_t^{\beta_{NT}}, w_t^{\sigma})$ such that

$$\mathbf{w}_t = \mathbf{A}\mathbf{w}_{t-1} + \mathbf{v}_t \tag{4}$$

with a 3×3 stable autoregressive coefficient matrix

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

and innovation $\mathbf{v}_t \sim_{i.i.d} \mathbb{N}(0, \mathbf{V})$ that is independent of policy shock ε_t at all leads and lags, and with a covariance matrix

$$\mathbf{V} = \begin{pmatrix} 1 & \rho_{\alpha,\beta} & \rho_{\alpha,\sigma} \\ \rho_{\alpha,\beta} & 1 & \rho_{\beta,\sigma} \\ \rho_{\alpha,\sigma} & \rho_{\beta,\sigma} & 1 \end{pmatrix}.$$

We require $\mathbf{V}$ to be a proper correlation matrix for parameter identification.[13] For each t, the latent regime factors determine the regime by

$$r_t^i = \mathbf{1}\{w_t^i \geq \tau^i\} \quad i \in \{\alpha, \beta_{NT}, \sigma\} \tag{5}$$

where $\mathbf{1}(\cdot)$ is an indicator function and $\boldsymbol{\tau} = (\tau_{\alpha}, \tau_{\beta_{NT}}, \tau_{\sigma})$ is a vector of the threshold parameters to be estimated.[14]

We view the latent factors as indicators of PBC's policy positions that are unknown to the public. The inclination towards a more market-driven RMB is measured by w_t^{α},

[13] See Chang et al. [2023] for a detailed discussion.

[14] The latent factors and thresholds imply a time-invariant regime transition matrix. Our model admits $2^3 = 8$ regimes in total. The transition matrix can be alternatively modeled using an 8×8 transition probability matrix that involves 56 free parameters. Although we do not expect all parameters in the transition matrix to be significant with respect to zero, it is generally unclear what assumptions to impose. Our specification offers a set of interpretable assumptions on the transition matrix with 15 free parameters.

the policy stance towards stabilizing the RMB index is measured by $w_t^{\beta_{NT}}$, and the stance towards using discretionary policy is measured by w_t^{σ}. The coefficient matrix $\mathbf{A}$ describes the dynamic relationships among these policy factors, and the correlation matrix $\mathbf{V}$ characterizes the relationships among innovations in policy factors. The assumption that $\mathbf{w}_t$ is stationary leads to an unconditional distribution of w_t, which is centered around zero. This assumption appears to be reasonable, given the recurring policy regimes suggested in Figure 2.

Several specifications are worth emphasizing. First, the regime indicator of β_{DT} is also driven by w_t^{α}. This is consistent with our rolling-window regression results, where we find that α and β_{DT} are negatively correlated. This specification also represents one of our key interpretations regarding the CCF policy. When forming the central parity rate, the policymaker will learn from the market and evaluate the degree of market irrationality. w_t^{α} captures the policymaker's judgment. When w_t^{α} is below the threshold, the econometric model implies a low α, accompanied by a high β_{DT}, and vice versa. In the following empirical analysis, we label the regime with a combination of low α and high β_{DT} as the CCF regime. Alternatively, the non-CCF regime is associated with a policy combination of high α and low β_{DT}. This is also consistent with JWY. Specifically, they find that in the sub-periods with the announced CCF policy, the estimated value of α is much lower than in other sub-periods.

Second, the α and β_{NT} regimes are determined by separate policy factors. However, we allow the policy factors w_{α} and $w_{\beta_{NT}}$ to be related; this is captured by the parameters a_{21} and $\rho_{\alpha,\beta}$. This specification enables us to test one aspect of the *de jure* CCF policy, namely that the introduction of CCF implies shifting of the policy weight on market flexibility toward the currency basket pillar. In other words, we explore whether the CCF regime is accompanied by a high-β_{NT}.

Last, Equation (5) implies that regime-switching may happen at a daily frequency. With this specification, we test another aspect of the *de jure* CCF policy, namely that the introduction of CCF policy is extremely persistent.

2.3 Benchmark Estimation Results

The model is estimated using the maximum likelihood (ML) method with the regime-switching filter proposed by Chang et al. [2023]. The estimated parameters and their bootstrapped standard errors can be found in Table 1. To determine the PBC's policy regimes, we generate a filtered series of policy factors, $E(w_t^i|\mathcal{F}_t), i \in \{\alpha, \beta_{NT}, \sigma\}$, shown

Table 1: Maximum Likelihood Estimates of Benchmark Regime-Switching Parity Rule

Threshold	Est.	S.E.	Factor	Est.	S.E.	Policy	Est.	S.E.
τ_α	0.744	2.374	a_{11}	0.731	0.248	α_0	0.338	0.058
$\tau_{\beta_{NT}}$	-2.069	1.436	a_{21}	0.078	0.517	α_1	0.806	0.117
τ_σ	6.044	2.739	a_{31}	-0.464	0.352	$\beta_{DT,0}$	0.377	3.89E-16
			a_{12}	-0.208	0.413	$\beta_{DT,1}$	0.069	1.25E-16
			a_{22}	-0.065	0.517	$\beta_{NT,0}$	0.000	0.041
			a_{32}	0.169	0.495	$\beta_{NT,1}$	0.507	0.063
			a_{13}	0.009	0.269	σ_0	0.061	0.009
			a_{23}	0.151	0.408	σ_1	0.200	0.029
			a_{33}	0.902	0.302			
			$\rho_{\alpha,\beta}$	-0.487	0.662			
			$\rho_{\alpha,\sigma}$	-0.952	0.701			
			$\rho_{\beta,\sigma}$	0.731	0.689			
obs	1187							
llk/obs	1.005							
p-value	0.000							

Note: We use a non-derivative-based global optimizer in our ML estimation over a sample spanning from 2/20/2017 to 12/31/2021. Standard errors are computed from 1000 samples with a length of 1000 that are generated through stationary block-bootstrapping with an average block size of 60, which is selected by Politis and White [2004]. The p-value reports the statistical significance of a likelihood ratio test against the linear model with llk/obs = 0.768.

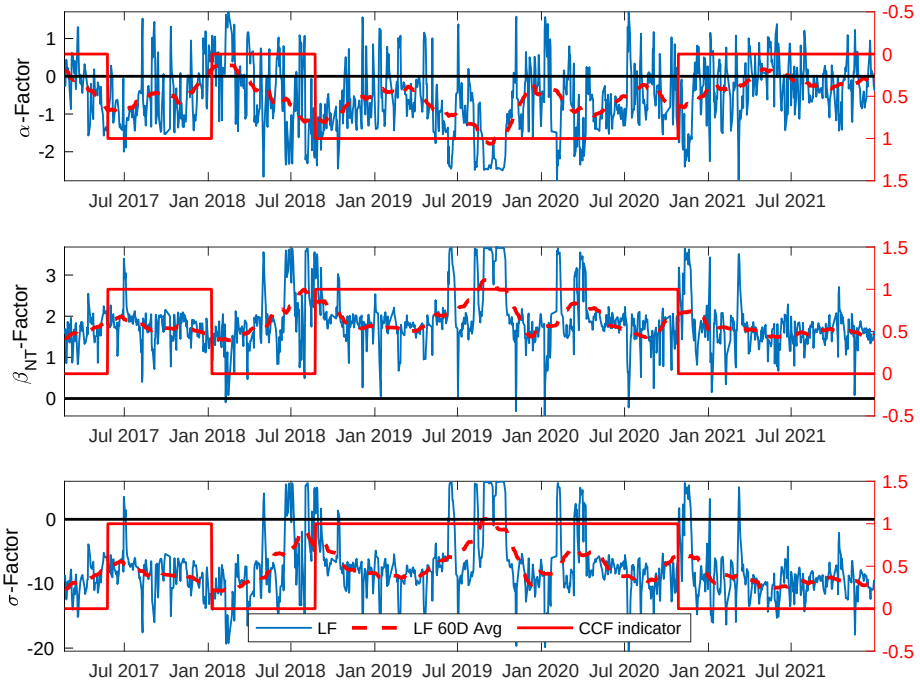
in Figure 3.[15] Additionally, we obtain smoothed regime probabilities for each coefficient, $\Pr(r_t^i = 0|\mathcal{F}_T), i \in \{\alpha, \beta_{NT}, \sigma\}$, which are presented in Figure 4.[16]

Our analysis shows that the results from our benchmark model are consistent with the findings from our rolling regressions and provide additional insights. We observe strong evidence of coefficient switching across α, β_{DT}, β_{NT}, and σ. Our regime-switching model significantly outperforms the linear model, as demonstrated by a likelihood ratio test with a p-value of 0.000. Interestingly, we find that the policy coefficients (α, β_{DT}) differ significantly depending on the market sentiment, which aligns with PBC's objective of countering such sentiment. In particular, these coefficients take values of (0.806, 0.069) in the non-CCF regime and (0.338,0.377) in the CCF regime. We identify two different β_{NT} regimes. One regime has a value of 0.507, the other regime has a value of 0.000. Additionally, our estimation reports a low-σ regime with a volatility of 0.061 and a high-σ regime with a volatility of 0.200. The small standard errors indicate that all of the coefficient pairs are

[15] For presentation, we plot $E(w_t^i|\mathcal{F}_t) - \tau_i, i \in \{\alpha, \beta_{NT}, \sigma\}$ in Figure 3, where τ_i's are ML estimates, so that all threshold parameters are normalized to zero.

[16] Throughout, we use $\mathcal{F}_t$ to denote the information set up to time t. In the empirical section, $\mathcal{F}_t = \sigma(\{\Delta c_s, d_{s-1}, \Delta x_{s-1}^{DT}, \Delta x_s^{NT}\}_{s=2}^t)$.

Figure 3: Benchmark Policy Regime Factors



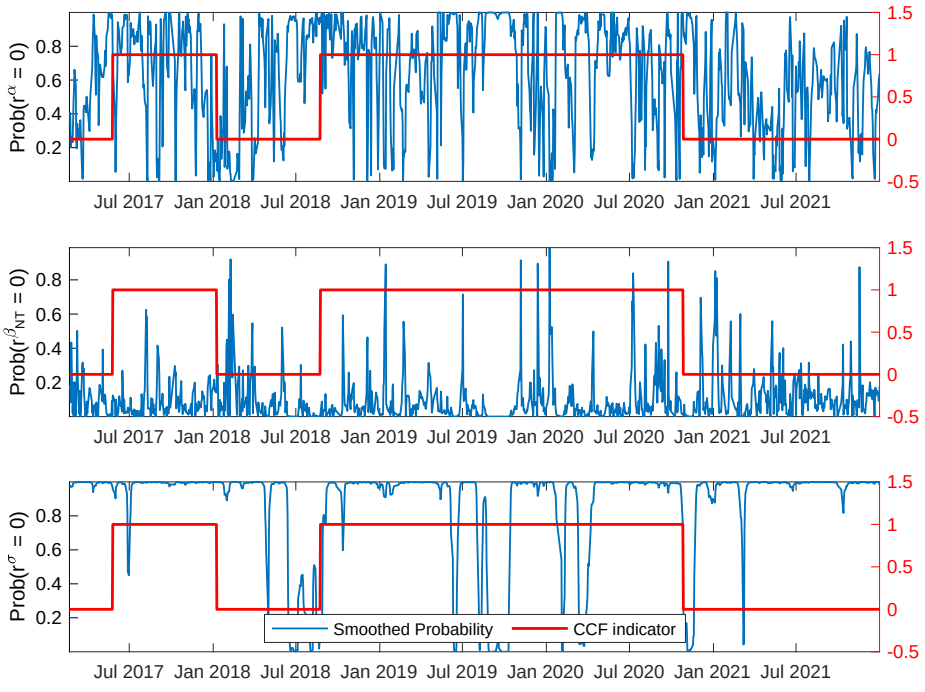
Note: This figure plots the filtered sequence of $w_t^{\alpha} - \tau_{\alpha}, w_t^{\beta_{NT}} - \tau_{\beta_{NT}}$ and $w_t^{\sigma} - \tau_{\sigma}$ (left scale), respectively, at the maximum likelihood estimates. The red dashed lines (left scale) plot the $t \pm 30$ moving average of the filtered w_t. The red solid lines (right scale) plot the announced periods of the introduction of the CCF.

well separated under different regimes.

Second, the *de facto* regimes in α are plausible. More importantly, it implies that policy switches occur at a frequency much higher than the *de jure* policy regimes. According to PBC's announcements, the CCF was introduced during two periods: 05/26/2017 to 01/08/2018 and 08/24/2018 to 10/26/2020. Figure 3 plots the estimated w_t^{α} at a daily frequency against the factor-specific threshold and PBC's announced dates of the CCF regime. Our estimated policy factor is generally consistent with the PBC's announcements. This is supported by the 60-day moving average of w_t^{α}, which helps eliminate the potential impact of daily policy innovations. The average policy factor identifies the official CCF regime and is typically low and below the threshold during the regime introduction.

However, the estimated policy regimes tend to change more frequently than those that are announced for CCF regimes. The high-frequency changes can potentially result from pure sampling error. However, even after smoothing the estimated factor by the moving average (Figure 3) and a backward filter (Figure 4), the movements at frequencies higher than the PBC announcements remain pronounced. In contrast, the heteroskedasticity regimes

Figure 4: Benchmark Smoothed Policy Regime Probabilities



Note: The red solid lines (right scale) plot the announced periods of the introduction of CCF.

after backward smoothing are rather persistent. These findings suggest that the *de facto* CCF policy is often implemented and lifted without any public announcement. Given these results, we are motivated to develop a daily frequency model in the theoretical section wherein market participants face policy uncertainty in the form of Markov switching α, through which we examine the policy implications of the *de facto* and *de jure* CCF policies.

Third, throughout the sample period, conflicting evidence arises regarding the relationship between the market pillar and RMB stability, as measured by α and β_{NT}. On one hand, a strong negative correlation between the α and β_{NT} policy factors is evident in Figure 3. The estimated factors, w_t^{α}, and w_t^{β}, are strongly negatively correlated, with a coefficient of -0.82. On the other hand, the switching of β_{NT} is much less frequent than that of w_t^{α}, as demonstrated in Figure 3. Moreover, the parameters that capture the correlations of policy factors, a_{21} and $\rho_{\alpha,\beta}$, are statistically insignificant. Therefore, the evidence on negative comovement of α and β_{NT} policy coefficients is inconclusive. As stated in the PBC's Monetary Report (2017Q2), the *de jure* CCF policy predicts a high β_{NT} regime, which is inconsistent with our finding. This observation further motivates us to differentiate the *de facto* CCF policy from the *de jure* CCF policy. In our quantitative analysis, we consider

the benchmark case with a constant β_{NT} and then explore the alternative scenario with switching β_{NT} regimes in Appendix E.

Fourth, it is clear that σ switching plays a crucial role in significant events. The threshold of the σ factor creates precisely identified regimes that correspond with major occurrences such as the U.S.-China trade war and the COVID-19 outbreak. Additionally, it is empirically necessary to include a switching σ to obtain reasonable estimates in the full sample estimation. Without this, a model would produce an unrealistic estimate of α, which would be close to zero in the low α regime.

Finally, and despite the correlated latent policy factors in Figure 3, the statistical proof for the connection between latent regime factors is inconclusive. The only notable estimates, in both economic and statistic senses, among all latent factor estimates in Table 1 are the diagonal elements a_{11} and a_{33}. These define the autoregressive dynamics of w_t^{α} and w_t^{σ}. Moreover, the off-diagonal elements in the correlation matrix of policy factor innovations are sizable but statistically insignificant. In view of this result, we can focus on modeling α in the theoretical section and regard other factors as independent of α.

2.4 CCF Regime as Switching Coefficients in A Simple Two-Pillar Rule

The two-pillar policy is a fairly accurate representation of the RMB policy, as clarified in JWY. However, the public is unaware of the exact implementation of the CCF policy. This policy can potentially be implemented by adjusting the policy coefficients. In this section, we analyze the RMB policy regimes using a simple two-pillar policy. We account for the CCF policy in the following model by including regime-switching in the policy coefficients[17]:

$$\Delta c_t = \alpha\,(r_t^{\alpha})\,d_{t-1} + \beta(r_t^{\beta})(1-\omega_{usd})\Delta x_t + \sigma(r_t^{\sigma})\varepsilon_t \tag{6}$$

where $\varepsilon_t \sim_{i.i.d} \mathbb{N}(0,1)$ and state-dependent coefficients

$$\begin{aligned}
\alpha(r_t^{\alpha}) &= \alpha_0(1-r_t^{\alpha}) + \alpha_1 r_t^{\alpha}, \quad 0 \le \alpha_0 < \alpha_1 \le 1,\\
\beta(r_t^{\beta}) &= \beta_0(1-r_t^{\beta}) + \beta_1 r_t^{\beta}, \quad 0 \le \beta_0 < \beta_1 \le 1,\\
\sigma(r_t^{\sigma}) &= \sigma_0(1-r_t^{\sigma}) + \sigma_1 r_t^{\sigma}, \quad 0 < \sigma_0 < \sigma_1.
\end{aligned}$$

Note that Δx_t in the above rule combines its nighttime component Δx_t^{NT} and daytime component Δx_{t-1}^{DT} in the extended rule (3). The dynamics of the regime are modeled in the same way as the benchmark model, with the required changes in notation.

[17] As discussed in the preceding section, we also include regime-switching volatility to obtain empirically plausible α estimates.

Table 2: Maximum Likelihood Estimates of the Two-Pillar Regime-Switching Parity Rule

Threshold	Est.	S.E.	Factor	Est.	S.E.	Policy	Est.	S.E.
τ_α	-1.012	1.456	a_{11}	0.878	0.386	α_0	0.351	0.089
τ_β	-1.565	0.967	a_{21}	0.290	0.543	α_1	0.747	0.062
τ_σ	9.575	2.787	a_{31}	-0.213	0.529	β_0	0.171	0.046
			a_{12}	-0.085	0.153	β_1	0.513	0.023
			a_{22}	0.734	0.385	σ_0	0.061	0.005
			a_{32}	0.067	0.292	σ_1	0.184	0.026
			a_{13}	0.002	0.230			
			a_{23}	0.204	0.422			
			a_{33}	0.906	0.339			
			$\rho_{\alpha,\beta}$	-0.969	0.029			
			$\rho_{\alpha,\sigma}$	0.092	0.158			
			$\rho_{\beta,\sigma}$	0.158	0.028			
obs	1498							
llk/obs	1.042							

Note: We use a non-derivative-based global optimizer in maximum likelihood estimation over a sample spanning from 11/10/2015 to 12/31 2021. Standard errors are computed from 1,000 samples with a length of 1000 that are generated through stationary block bootstrapping with an average block size 60. The block size is selected according to Politis and White [2004].

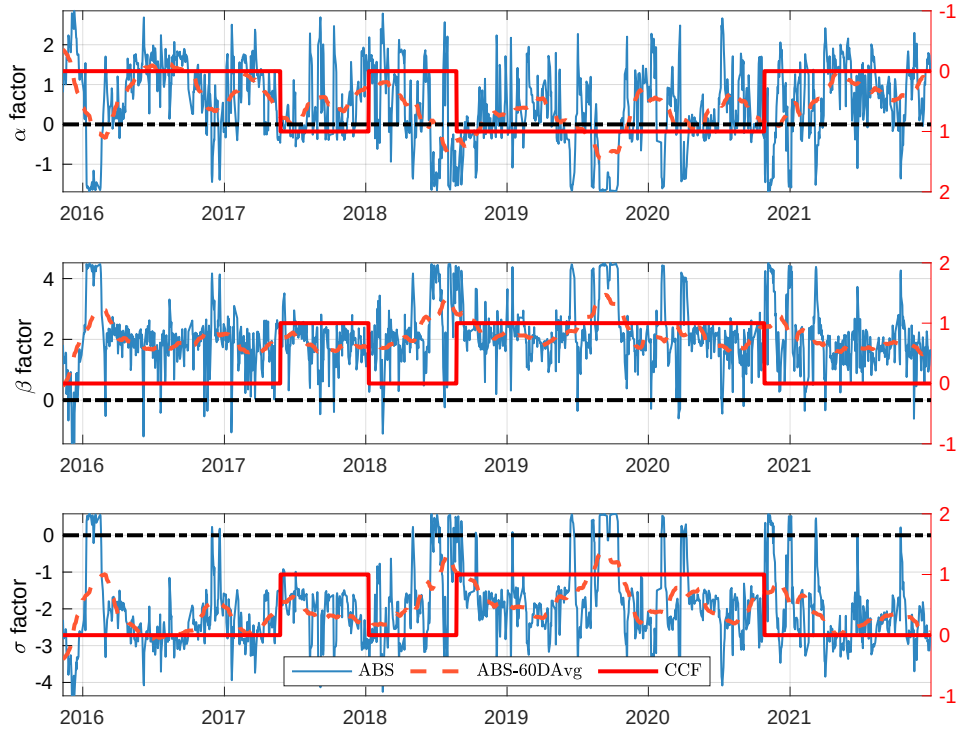
The ML estimates for the model (6) presented in Table 2 give a summary of the RMB policy that aligns well with our benchmark model.[18] In particular, the policy coefficients in different regimes resemble the benchmark result in Table 1. Moreover, the α regime remains central to the RMB policy switch. The estimated regime factors in Figure 5 reveal that all policy regime factors are similar to the benchmark estimates. Among all factors, the correlation between the w_t^α series in the two estimations is 0.71.[19] Our findings in the benchmark specification are robust: α regimes are well identified, with values of 0.35 and 0.75; the α regime switches at a high frequency; the evidence on shifting from the market flexibility toward the basket stability is mixed.

However, the α regimes induced from the two-pillar model are significantly less accurate in timing than the benchmark model. Evidently, the day factor is informative with respect to the RMB policy regime; however, the improved accuracy comes from better threshold

[18]This data sample covers the time period from 11/10/2015 to 12/31/2021 and includes various significant policy reforms that occurred after the PBC implemented the two-pillar RMB policy in 2015. To ensure consistency in our data analysis, the overnight implied dollar index (X_t) in this model is calculated using a 24-hour reference period following the PBC announcement before 2/20/2017.

[19]The correlation between $w_t^{\beta_{NT}}$ and w_t^β is 0.41, and the correlation between the two estimations of w_t^σ is 0.59.

Figure 5: Switching Two-Pillar Policy Regime Factors



Note: This figure plots the filtered sequence of $w_t^{\alpha} - \tau_{\alpha}, w_t^{\beta_{NT}} - \tau_{\beta_{NT}}$ and $w_t^{\sigma} - \tau_{\sigma}$ (left scale), respectively, at the maximum likelihood estimates. The red dashed lines (left scale) plot the $t \pm 30$ moving average of the filtered w_t. The red solid lines (right scale) plot the announced periods of the introduction of the CCF.

estimates rather than better estimates of w_{α} series itself. This finding implies that the latent policy factor *per se* is useful for capturing the CCF policy stance. In addition, according to Figure 5, the RMB policy may have begun to change in 2016, which is much earlier than the official communication in early 2017. This observation is not unexpected, as the PBC may have wanted to control market expectations, especially given the strong expectation of RMB depreciation throughout 2016.

Note that Δx_{t-1}^{DT} is not included directly in this simple rule. By construction, this variable is the daytime component of Δx_t. For a more detailed discussion, please refer to Appendix A.1. As a sentiment shock, Δx_{t-1}^{DT} has a direct impact on the spot market, which in turn drives d_{t-1}. To provide evidence that the α regime is still driven by the CCF policy, we report additional results of ordinary least squares (OLS) regression in Appendix A.2. Here, the dependent variable is the policy factor $w_t^i, i \in \{\alpha, \beta, \sigma\}$, and the independent

variable of interest is Δx_{t-1}^{DT}. The results show that the CCF policy has a significant impact on w_t^{α}, which is estimated based on the simple rule. However, Δx_{t-1}^{DT} does not have a significant impact on w_t^{β}. This result provides further evidence that the regime-switching of α can be mapped to the CCF policy.

To summarize, the key regime dynamics remain robust when viewed through a policy framework consisting of two pillars with switching coefficients. The addition of intra-day data from the global market offers valuable insights into the RMB policy regimes.

3 Theoretical Model

In this section, we propose a model for exchange rate determinacy based on Svensson [1994] and Jermann et al. [2022]. We focus on three key markets: the goods market, the foreign exchange market, and the intraday market.[20] In particular, we include the noise trader in the benchmark model proposed by JWY. Noise traders do not possess complete information and rely on a regressive approach to form their expectations of exchange rates. The inclusion of noise traders is crucial as it enables us to deviate from the conventional uncovered interest parity condition and use the data to pin down structural parameters for quantitative analysis.

Using this model, we first discuss the indeterminacy issue of the RMB exchange rate market. Drawing inspiration from Davig and Leeper [2007], we then analyze the impact of regime-switching policy in terms of mitigating the self-fulfilling prophecy that occurs in the foreign exchange market.

3.1 The Goods Market

The goods market is characterized as in Flanders and Helpman [1979]. Overall, the RMB exchange rate affects the country's trade balance. The log of trade balance, $\log(TB_t)$, is a linear expression of the central parity rate:

$$\Delta \log(TB_t) = (1 - \omega_{usd})\Delta x_t - \Delta c_t. \tag{7}$$

[20] JWY model also includes the characterization of the money market and specifies the real exchange rate. However, for the purpose of our analysis, we are only focusing on a scenario where the real exchange rate remains constant. Readers are referred to Appendix C.1 for our demonstration that the process of the real exchange rate does not have an impact on our analysis.

The changes in the basket-implied dollar index Δx_t comprise an exogenous i.i.d process[21]:

$$\Delta x_t \sim_{i.i.d} N(0, \sigma^2_{\Delta x}).$$

3.2 The Foreign Exchange Market

The spot exchange rate (i.e., closing rate) is denoted as $e_t = \log(S_t^{CL})$, which measures the price of USD in RMB. Let the Foreign interest rate be i_t^*, which is an exogenous stationary autoregressive (AR) process:

$$i_t^* = \rho_{i^*} i_{t-1}^* + \varepsilon_{i^*,t}, \quad \varepsilon_{i^*,t} \sim_{i.i.d} N(0, \sigma^2_{i^*}) \tag{8}$$

where ρ_{i^*} is the AR coefficient, and $\varepsilon_{i^*,t}$ is the foreign interest rate shock.

Let i_t be the interest rate in the Home country (i.e., China). Home's domestic monetary policy is a linear function of the state variables i_t^* and c_t,

$$i_t = \phi_c c_t + \phi_{i*} i_t^* \tag{9}$$

where ϕ_c and ϕ_{i*} are the internal monetary policy coefficients.

The variable ρ_{t+1} represents the *ex-post* excess return of RMB-denominated bonds in a log-linearized form:

$$\rho_{t+1} \equiv (i_t - i_t^*) - (e_{t+1} - e_t). \tag{10}$$

As will be clear in the next subsection, the value of ρ_{t+1} plays an important role in determining the investors' position of RMB assets.

In the foreign exchange market, the authority announces the central parity rate c_t. We denote the market-closing exchange rate's deviation from the parity rate as $d_t = e_t - c_t$. The policy rule is a simple two pillar policy denoted as Equation (1). We will examine the behavior of an economy under this rule and highlight the problem of multiple equilibria. In Section 5, we present a regime-switching central parity rate policy to investigate whether this policy can alleviate the issue of indeterminacy.

3.3 The Intraday Market

In the intraday foreign exchange market, a continuum of investors on unit interval select portfolios consisting of RMB and dollar assets. For investor $j \in [0, 1]$, the first-order

[21] This specification is consistent with our dollar index data.

condition implies an optimal position

$$X_t^j = \frac{E_t^j(\rho_{t+1})}{\zeta var_t^j(\rho_{t+1})}. \tag{11}$$

The optimization problem of traders is standard and is explained in detail in Appendix B.1.

The information structure is the key to determining the RMB asset position. Following De Long et al. [1990], Jeanne and Rose [2002], and JWY, we consider two types of traders: informed traders and noise traders with incomplete information. The informed traders, denoted as $j \in [1-\overline{N}, 1]$, have rational expectations. That is, informed traders have accurate information regarding asset returns and risks. Their expected premium and conditional variance regarding the excess return of holding RMB assets are

$$\begin{aligned} E_t^j(\rho_{t+1}) &= E_t(\rho_{t+1}), \forall j \in [1-\overline{N}, 1], \\ Var_t^j(\rho_{t+1}) &= Var_t(\rho_{t+1}), \forall j \in [1-\overline{N}, 1]. \end{aligned}$$

Thus, the position of informed traders, denoted as X_t^I,[22] is

$$X_t^I = \frac{E_t(\rho_{t+1})}{\zeta Var_t(\rho_{t+1})}. \tag{12}$$

The noise traders, denoted as $j \in [0, 1-\overline{N})$, have precise information on the risks. Their conditional variance is identical to the informed traders,[23]

$$Var_t^N(\rho_{t+1}) = Var_t(\rho_{t+1}).$$

However, noise traders have imperfect information regarding asset returns. We follow Frankel and Froot [1987] and assume that noise traders form their expectation of exchange rate via *regressive learning*:

$$E^N(e_{t+1} - e_t) = \gamma(e_t - \overline{e}) \tag{13}$$

where $\overline{e}$ is the *log* long-run exchange rate (steady-state level), and γ is the learning parameter, which is the key to our analysis.[24] Thus, the position of noise traders, denoted as X_t^N,

[22] We suppress the individual index here because all informed traders choose the same position.

[23] This assumption is an innocuous simplification. Our results are not affected if we relax this assumption. See Appendix C.2 for details.

[24] In our setting, $exp(\overline{e}) = 1$.

is

$$X_t^N = \frac{E_t^N(\rho_{t+1})}{\zeta Var_t(\rho_{t+1})}. \tag{14}$$

In our specified information structure, informed traders and noise traders differ only in their expected exchange rate. Equations (12) and (14) highlight this distinction. This difference in exchange rate expectations results in distinct expectations of excess return between the noise and informed traders, which ultimately lead them to hold different asset positions.

In equilibrium, informed traders and noise traders are the only market forces in the intraday market.[25] Without government intervention, the market clearing condition of RMB asset is $\overline{N}X_t^I + (1-\overline{N})X_t^N = 0$. Substituting X_t^I and X_t^N using (12) and (14) respectively, we find that

$$\overline{N}E_t(\rho_{t+1}) + (1-\overline{N})E_t^N(\rho_{t+1}) = 0. \tag{15}$$

The equation presented above shows how the expectation of noise traders influences the equilibrium excess return. By substituting the value of ρ_{t+1} with its definition from (10), we can derive the modified uncovered interest parity condition

$$(i_t - i_t^*) - E_t(e_{t+1} - e_t) = -n(i_t - i_t^*) + n\gamma(e_t - \bar{e}), \tag{16}$$

where $n = \frac{(1-\overline{N})}{\overline{N}}$ denotes the size of noise traders relative to the size of informed traders. It is important to emphasize that E_t is the rational expectation operator conditional on the public information in period t.

4 The Indeterminacy Issue

To demonstrate the problem of multiple equilibria in the LRE model discussed in Section 3, we present the economy in the state-space form. Here, the vector of state variables is $X_t = [i_t^*, c_t]$. As in JWY, we introduce the forward looking variable e_t and define the vector

[25] The government intervention can be directly introduced as another policy instrument, similar to Brunnermeier et al. [2022] and Jermann et al. [2020]. With the government intervention G_t, the market clearing condition of RMB assets becomes $\overline{N}X_t^I + (1-\overline{N})X_t^N + G_t = 0$. This intervention can help to prevent a self-fulfilling cycle. For example, if a sentiment shock leads to RMB depreciation in the market, strong government policies can stabilize the economy at the cost of losing foreign reserves. However, our paper focuses on evaluating the consequences of regime-switching in the central parity rate rule. We aim to determine if it is possible to stabilize the market without government intervention and without losing foreign reserves.

$Z_t = [X_t', e_t]'$. The dynamics of Z_t follows

$$\begin{bmatrix} X_{t+1} \\ E_t e_{t+1} \end{bmatrix} = AZ_t + B\varepsilon_{z,t+1} \tag{17}$$

where $\varepsilon_{z,t+1} = [\varepsilon_{i*,t+1}, \Delta x_{t+1}, \sigma\varepsilon_{t+1}]'$ with matrices

$$A = \begin{bmatrix} \rho_{i^*} & 0 & 0 \\ 0 & 1-\alpha & \alpha \\ (1+n)(\phi_{i*}-1) & (1+n)\phi_c & 1-n\gamma \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \beta(1-\omega_0) & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

The discussion of the indeterminacy issue in Section 4 is based on Equation (17). Following Blanchard and Kahn [1980] (BK, hereafter), the determinacy is defined as the non-explosive expectations of X_t and e_t.[26] The BK condition requires the A matrix to have exactly one eigenvalue outside the circle because we have one forward-looking variable. Our general result suggests that determinacy depends on the coordination of the internal monetary policy with the central parity rate policy. However, the literature on the use of monetary policy to prevent multiple equilibria in the foreign exchange market is limited. To address this, we first estimate the key parameters using the forecast survey data. Then, we examine a special case wherein monetary policy is not influenced by the central parity rate to highlight the self-fulfilling prophecy. In the general case, we emphasize the significance of policy interactions in ensuring a unique equilibrium.

4.1 Estimation of the Key Parameters

The parameters α, $(1+n)\phi_c$, and $n\gamma$ determine the eigenvalue of A matrix and are the key parameters for our analysis. The full sample OLS estimation of the parity rule gives $\alpha = 0.42$. We further leverage Bloomberg's institutional-level forecasts of the CNY/USD

[26] We can rewrite A matrix in Jordan form $A = C^{-1}JC$, where the diagonal elements of J are the eigenvalues of A. Consider the transformation $[Y_t, Q_t]' = C[X_t, e_t]'$. The determinacy of the system is equivalent to the unique determination of Q_t. That is,

$$Q_t = -\sum_{i=0}^{\infty} (\mu_3)^{-i-1}(C_{11}B_1 + C_{12}B_2)E_t\varepsilon_{z,t+i} \tag{18}$$

The uniqueness of Q_t is guaranteed by the well-known BK condition. And $J = \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}$; $C^{-1} = \begin{pmatrix} C_{11} & C_{12} \\ (2\times2) & (2\times1) \\ C_{21} & C_{22} \\ (1\times2) & (1\times1) \end{pmatrix}$

spot rates over the same sample span to estimate the strength of noise trading and China's domestic monetary policy coefficients.

One important data feature is that all of the institutional forecasts are daily frequency forecasts of fixed targets, with quarterly moving targets instead of fixed horizons. To connect the data with our daily frequency model, we iterate the state space model (17) h periods forward to yield

$$E(Z_{t+h}|e_t, c_t, i_t^*) = A^h Z_t.$$

From this, we obtain

$$E^i(e_{t+h}|e_t, c_t, i_t^*) = A^h_{[3,1]} i_t^* + A^h_{[3,2]} c_t + A^h_{[3,3]} e_t. \tag{19}$$

where $A^h_{[p,q]}$ denotes the (p,q)-th element in the matrix A^h. $E^i(e_{t+h}|e_t, c_t, i_t^*)$ is the h-day forecast value of institution i.

We consider one-month ahead forecasts in the regression ($h = 23$). Table 7 in Appendix F reports the regression results. The estimates of γ_2 and γ_3 are quite robust and accurate over different specifications. With the value of α, we solve from the model with both individual- and time-fixed effects that $n\gamma = 1.6$, $(1+n)\phi_c = 1.58$. As will be clear soon, these two numbers are important for our quantitative analysis.

4.2 A Special Case When Monetary Policy Does Not Adjust

In this simple case, monetary policy has no effect, as ϕ_c equals zero. With the following proposition, we aim to explain the mechanism behind the self-fulfilling prophecy without the influence of monetary policy, and we emphasize the significance of noise traders.

Proposition 4.1. *If $\phi_c = 0$, the eigenvalues of matrix A are*

$$\mu_1 = \rho_{i^*}, \quad \mu_2 = 1 - \alpha, \quad \mu_3 = 1 - n\gamma.$$

The Blanchard-Kahn condition can only be satisfied if $\gamma < 0$ or $\gamma > \frac{2}{n}$.

Imagine a scenario where a non-fundamental shock causes rational market participants to anticipate a future depreciation of the RMB. As a result, the expected future value of RMB assets, $E_t\rho_{t+1}$, decreases. This outcome leads to a reduced optimal RMB position for rational traders, X_t^I. In the absence of noise traders and monetary policy adjustments, the decrease in demand for RMB assets leads to a decrease in the assets' value, causing the RMB to depreciate in the spot market. This is a typical self-fulfilling phenomenon in the foreign exchange market.

Noise traders may disrupt the self-fulfilling phenomenon. If they notice that e_t is rising, they may anticipate that the RMB will increase in value on the following day due to $\gamma < 0$. As a result, noise traders anticipate a greater premium for holding RMB assets, which is captured by $E_t^N(\rho_{t+1})$. This increases noise traders' willingness to hold RMB assets, which in turn increases the demand for RMB assets and prevents the self-fulfilling depreciation of the RMB in the foreign exchange market.

Alternatively, when $\gamma > \frac{2}{n}$, the noise traders anticipate a greater decrease in the value of the RMB when e_t increases. As a result, they are less likely to hold RMB assets. However, RMB cannot feasibly depreciate, as no traders are willing to hold RMB assets. Consequently, the equilibrium RMB exchange rate must appreciate the in the face of a non-fundamental depreciation shock.

4.3 A General Case When Monetary Policy Adjusts

When discussing determinacy, internal monetary policy is an important factor to consider. This is because the interest rate is a component of the modified uncovered interest parity condition and affects asset returns. When the monetary policy is specified by Equation (9), one of the eigenvalues of matrix A is $\mu_1 = \rho_{i^*}$. The other two eigenvalues μ_2 and μ_3, can be calculated using the following equation:

$$\mu = \frac{(2 - \alpha - n\gamma) \pm \sqrt{[\alpha - n\gamma]^2 + 4\alpha\phi_c(1+n)}}{2}. \tag{20}$$

As discussed in the previous subsection, when $\gamma < 0$, the government can always guarantee the determinacy by muting monetary policy's response to the central parity rate ($\phi_c = 0$). Therefore, in this part, we focus on the situation wherein $\gamma > 0$. According to the panel regression results in Subsection 4.1, the value of $n\gamma$ is approximately 1.6. We focus on this value in our quantitative exercises.

We use $|\mu|$ to denote the absolute value of μ. The proposition below outlines the requirements for policy coefficients to avoid indeterminacy in LRE. Specifically, only one value of $|\mu|$ can be greater than one.

Proposition 4.2. *If $\gamma > 0$, to guarantee the Blanchard-Kahn condition, the monetary policy coefficients ϕ_c and exchange rate policy coefficient α must satisfy one of the following*

conditions:

$$\phi_c(1+n) > n\gamma \quad \textit{and} \quad \phi_c(1+n)\alpha < (2-\alpha)(2-n\gamma),$$

Or,

$$\phi_c(1+n) < n\gamma, \quad \alpha < n\gamma, \quad \textit{and} \quad \phi_c(1+n)\alpha > (2-\alpha)(2-n\gamma).$$

Proof. See Appendix B.2 □

Proposition 4.2 includes Proposition 4.1 as a special case. To see this, suppose that ϕ_c approaches zero, and the above condition reduces to the required constraints listed in 4.1. In general, a constraint on $n\gamma$ is not sufficient to guarantee a unique equilibrium when the monetary policy responds to the central parity rate. Nonetheless, noise traders' learning behavior can affect the system's stability, potentially leading to a unique equilibrium even if $n\gamma < 2$.

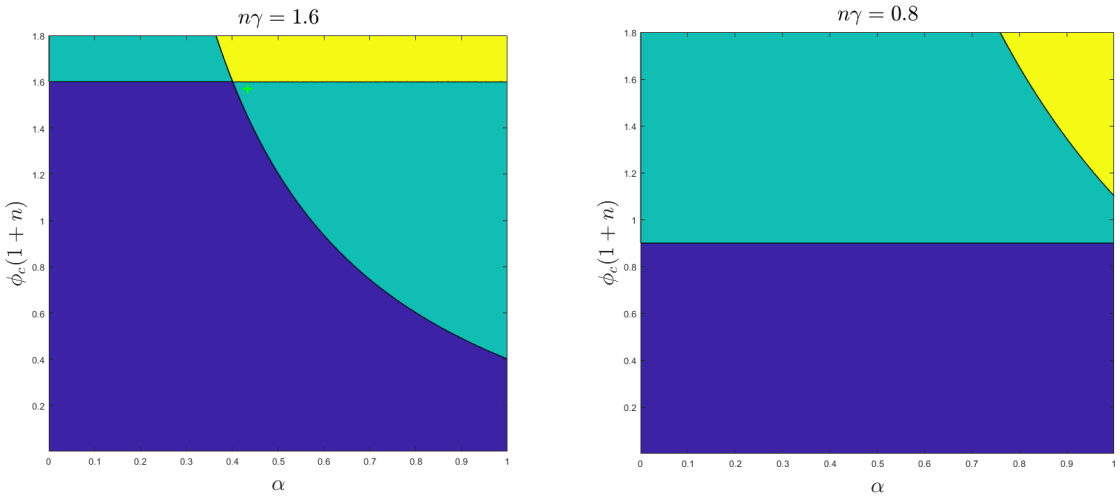
Let us start by examining two interesting special cases. The first case assumes that α approaches zero but remains greater than zero.[27] According to Equation (2), the central parity rate remains unaffected by the closing rate of the previous day. The equilibrium is unique when the monetary policy coefficient ϕ_c is greater than $(1-\overline{N})\gamma$. This occurs because a non-fundamental shock may trigger a self-fulfilling phenomenon in the market, where noise traders' learning behavior further enhances the self-fulfilling depreciation if γ is positive. As the interest rate is a significant factor that determines the excess return of holding RMB assets, the monetary policy rule can compensate investors who hold RMB assets by responding to the increasing parity rate, thereby increasing the asset demand. This self-fulfilling chain breaks down when the monetary policy stance is aggressive enough to dominate the amplification effect of noise traders.

In the second case, if α approaches one, the central parity rate responds fully to the closing rate of the previous day, and the monetary policy coefficient ϕ_c should fall within the range of $\frac{2-n\gamma}{1+n}$ and $\frac{n\gamma}{1+n}$. If both the monetary and exchange rate authorities are aggressive, any sentiment-driven depreciation will be fully reflected in the central parity rate and overcorrected by the interest rate, leading to no equilibrium solution. Conversely, if the monetary policy is too unresponsive to the central parity rate, it will fail to prevent self-fulfilling depreciation, resulting in an undetermined system.

Based on the estimated value of $n\gamma$, Figure 6a shows the combination of policy coefficients ϕ_c and α needed for a determinate equilibrium, using the eigenvalue expressions in (20). Assuming an overall linear exchange rate policy, the green cross represents the

[27] When $\alpha = 0$, the solutions of Equation (20) are $1-\mu\gamma$ and 1. There is no stable equilibrium in this system.

Figure 6: Determinacy Region of Benchmark LRE Model



(a) Benchmark Case (b) Alternative Case

Note: cyan: determinacy; blue: multiple solutions; yellow: no solution. Estimated values over the full sample at the green cross, where $\alpha = 0.42$ and $\phi_c(1+n) = 1.58$.

estimated values of ϕ_c and α from the panel regression in Subsection 4.1, indicating that the RMB policy is working and the economic system is marginally stable. The figure also highlights the coordination between the monetary policy and the central parity rate policy. When the monetary policy is less sensitive to the central parity rate, i.e., ϕ_c is smaller, the economy requires the central parity rate formation to better reflect the market conditions, i.e., α should be larger. This intuition is consistent with the previous special cases. The monetary authority works with the exchange rate authority to determine the extent to which the interest rate responds to sentiment-driven depreciation. The interest rate is a component of the excess return of holding RMB assets and may break down the self-fulfilling chain. In summary, a combination of aggressive (passive) monetary policy and passive (aggressive) exchange rate policy is necessary if the PBC wants to tackle self-fulfilling depreciation in the RMB exchange market and if the monetary policy can respond to the parity rate.

Importantly, the demand for policy coordination is influenced by the market forces of noise traders, which is measured by $n\gamma$. Recall that in this expression, n represents the number of noise traders in the market, and γ is the learning parameter that measures the trader's sensitivity to the spot rate. When $n\gamma < 2 - n\gamma$, as shown in Figure 6b, the amplification effect caused by the noise trader is relatively weak, and the exchange rate depreciation (driven by sentiment and amplified by noise traders) is not sufficiently significant. The monetary policy coefficient must be larger than $n\gamma$ to prevent depreciation

in the spot market and compensate for the holding of RMB assets. However, this policy parameter cannot be too large if α approaches one, or no solution can guarantee equilibrium.

To summarize, the analysis calls for coordination between the monetary policy and exchange rate policy to address indeterminacy. However, this coordination may be difficult in practice for two reasons. First, monetary policy may have its own goals, such as minimizing the output gap and stabilizing inflation. Although the regression results in Subsection 4.1 indicate that monetary policy responds to the central parity rate, we acknowledge that it is not explicitly designed to handle the exchange rate market. Second, the central parity rate and exchange rate rule are announced on a daily basis, while the monetary policy rule and corresponding adjustments occur at a lower frequency as they address the output gap and inflation. An important question concerns whether the exchange rate authority can handle sentiment shocks in the exchange rate market when monetary policy is binding (i.e., a constant rule). We explore this possibility in the next section.

5 Indeterminacy and the Regime-Switching Policy Rule

To understand the impact of regime-switching on the determination of exchange rates, we extend the linear model to include regime-switching RMB policy rules. Our focus is on a policy rule with switching coefficients specifically in the market pillar, [28]

$$c_t = (1-\alpha(r_t))c_{t-1} + \alpha(r_t)e_{t-1} + \beta(1-\omega_0)\Delta x_t + \sigma\varepsilon_t. \tag{21}$$

The exogenous regime r_t follows an ergodic Markov chain with a 2×2 transition matrix P where the transition probability from a regime i to regime j is $p_{ij} = \Pr(r_{t+1} = j | r_t = i)$ for $i, j \in \{0, 1\}$.

At time t, informed traders can access the observable information set denoted by $\mathcal{F}_t = \{Z_{t-l}, r_{t-l}, \varepsilon_{z,t-1}, l = 0, 1, 2, \cdots\}$, where Z_t and $\varepsilon_{z,t}$ are defined in the previous section. Our system of equations includes Equation (8) to describe the foreign interest rate process, Equation (21) to describe the RMB central parity policy, and Equation (16) to describe the modified uncovered interest rate parity condition with regime-switching coefficients. Then, the MSRE model can be expressed as

$$Z_t = \tilde{F}\mathbb{E}_t Z_{t+1} + \tilde{\Omega}(r_t)Z_{t-1} + \tilde{\Gamma}\varepsilon_{z,t} \tag{22}$$

[28] In Appendix E, we also consider the switching of β and show that our conclusions are not affected.

with coefficient matrices

$$\tilde{F} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \tilde{\gamma} \end{pmatrix}, \quad \tilde{\Gamma} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \beta(1-\omega_0) & 1 \\ -\tilde{\gamma}\frac{\phi_{i*}-1}{\tilde{N}} & -\tilde{\gamma}\frac{\phi_c}{\tilde{N}}\beta(1-\omega_0) & -\tilde{\gamma}\frac{\phi_c}{\tilde{N}} \end{pmatrix},$$

$$\tilde{\Omega}(r_t) = \begin{pmatrix} \rho_{i^*} & 0 & 0 \\ 0 & 1-\alpha(r_t) & \alpha(r_t) \\ -\tilde{\gamma}\frac{\phi_{i*}-1}{\tilde{N}}\rho_{i^*} & -\tilde{\gamma}\frac{\phi_c}{\tilde{N}}(1-\alpha(r_t)) & -\tilde{\gamma}\frac{\phi_c}{\tilde{N}}\alpha(r_t) \end{pmatrix},$$

where $\tilde{\gamma} = [1 - n\gamma]^{-1}$.

5.1 Equilibrium Selection

The MSRE model presents a non-trivial equilibrium selection problem. As demonstrated in Cho [2016, 2021], any solution to (22) can be written as a combination of a fundamental component and a sunspot component b_t, which is asymptotically covariance stationary and independent to Z_0 and r_t for all values of $t \geq 1$:

$$Z_t = [\Omega(r_t)Z_{t-1} + \Gamma(r_t)\varepsilon_{z,t}] + b_t \tag{23}$$

$$b_t = \mathbb{E}_t[F(r_t)b_{t+1}] \tag{24}$$

where

$$\Omega(r_t) = \Psi(r_t)\tilde{\Omega}(r_t) \tag{25}$$

$$\Gamma(r_t) = \Psi(r_t)\tilde{\Gamma} \tag{26}$$

$$F(r_t) = \Psi(r_t)\tilde{F} \tag{27}$$

for all values of $r_t, r_{t+1} \in \{0, 1\}$, if there exists matrix inverse

$$\Psi(r_t) = \left[I_3 - \mathbb{E}_t[\tilde{F}\Omega(r_{t+1})]\right]^{-1}$$

for all values of $r_t \in \{0, 1\}$. The minimum state variable (MSV) solution to (22) refers to the fundamental solution $Z_t = \Omega(r_t)Z_{t-1} + \Gamma(r_t)\varepsilon_{z,t}$ when the sunspot component $b_t = 0$. This MSV solution is discussed in Davig and Leeper [2007] and Farmer et al. [2009]. Briefly, the MSV solution excludes sunspots. In a more general setting, however, Equation (23) illustrates that rational expectations may also be affected by the non-fundamental components. The sunspot process b_t is an autoregressive process with regime-switching coefficients and stationary sunspot shocks. Note that, generally, $\Omega(r_t)$ is not unique because

finding solutions of (25) entails solving a quadratic vector equation.[29] Therefore, there generally exists a finite number of MSV solutions and possibly a continuum of mean-square stable (MSS) sunspot solutions. Additionally, in this system, $\Gamma(r_t)$ and $F(r_t)$ are uniquely defined when $\Omega(r_t)$ is specified. In the following text, we refer to $\Omega(r_t)$ as an MSV solution.

Consistent with the literature, we adopt mean-square stability as the stability concept.[30] Thus, the determinacy of the MSRE model refers to the uniqueness of the minimum of modulus mean-square stable (MOD-MSS) solution.[31] In the context of our study, Cho [2021] shows that the mean-square stability of the rational expectations model can be characterized by the following two types of matrix operators:

$$\bar{\Psi}_{G\otimes H} = [p_{ji}(G(j,i)')^T \otimes H(j,i)],$$
$$\Psi_{G\otimes H} = [p_{ij}(G(i,j)')^T \otimes H(i,j)],$$

for generic 3×3 matrices $G(i,j)$ and $H(i,j)$, with a given transition matrix P. Operators ^{T} and $'$ denote the non-conjugate transpose and conjugate transpose operators, respectively, and $\otimes$ denotes the Kronecker product. In our model, the expression in the squared bracket above represents the (i,j)-th $3^2 \times 3^2$ dimensional block for all $i,j \in \{0,1\}$. The two expressions at the center of the determinacy in our MSRE model are

$$\bar{\Psi}_{\Omega\otimes\Omega} = \begin{pmatrix} p_{00}\Omega(0)\otimes\Omega(0) & p_{10}\Omega(0)\otimes\Omega(0) \\ p_{01}\Omega(1)\otimes\Omega(1) & p_{11}\Omega(1)\otimes\Omega(1) \end{pmatrix},$$
$$\Psi_{F\otimes F} = \begin{pmatrix} p_{00}F(0)\otimes F(0) & p_{01}F(0)\otimes F(0) \\ p_{10}F(1)\otimes F(1) & p_{11}F(1)\otimes F(1) \end{pmatrix}.$$

Let $\mathbb{O}$ represent the set of $\{\Omega(r_t), r_t = 0,1\}$, satisfying (25); we then sort the solutions by the spectral radius $\rho(\cdot)$ so that

$$\rho(\bar{\Psi}_{\Omega_1\otimes\Omega_1}) \leq \rho(\bar{\Psi}_{\Omega_2\otimes\Omega_2}) \leq \cdots \leq \rho(\bar{\Psi}_{\Omega_M\otimes\Omega_M}). \tag{28}$$

Definition 5.1. *The MSV solution of model (22) is a MOD solution if* $\Omega(r_t) \in \mathbb{O}$ *and*

[29] Our three-equation model involves a maximum of four MSV solutions after applying the Gröbner basis technique proposed by Foerster et al. [2016].

[30] See Definition 1 in Farmer et al. [2009] for a rigorous definition of mean-square stability. Importantly, the mean-square stability in a regime-switching setting does not require each regime to be stable. As long as the unstable regime does not occur too frequently, the state variables will still converge to a well-defined ergodic distribution with finite first and second moments.

[31] For the LRE models, the MOD solution is identical to the standard one solved by Blanchard and Kahn [1980]. Also, compared with the forward solution proposed by Cho [2016], the MOD solution provides not only sufficient but also the necessary conditions for determinacy. Compared with Foerster et al. [2016], the MOD solution is computationally far more efficient.

$\Omega(r_t) = \Omega_1(r_t)$.

Proposition 3 of Cho [2021] classifies an MSRE model to be determinacy-admissible if $\rho(\bar{\Psi}_{\Omega_1 \otimes \Omega_1})\rho(\Psi_{F_1 \otimes F_1}) < 1$, under which the MSRE model is

1. determinate if and only if $\rho(\bar{\Psi}_{\Omega_1 \otimes \Omega_1}) < 1$ and $\rho(\Psi_{F_1 \otimes F_1}) \leq 1$,
2. indeterminate if and only if $\rho(\bar{\Psi}_{\Omega_1 \otimes \Omega_1}) < 1$ and $\rho(\Psi_{F_1 \otimes F_1}) > 1$,
3. of no stable solution if and only if $\rho(\bar{\Psi}_{\Omega_1 \otimes \Omega_1}) \geq 1$.

5.2 Determinacy Region under Various Policy Tools

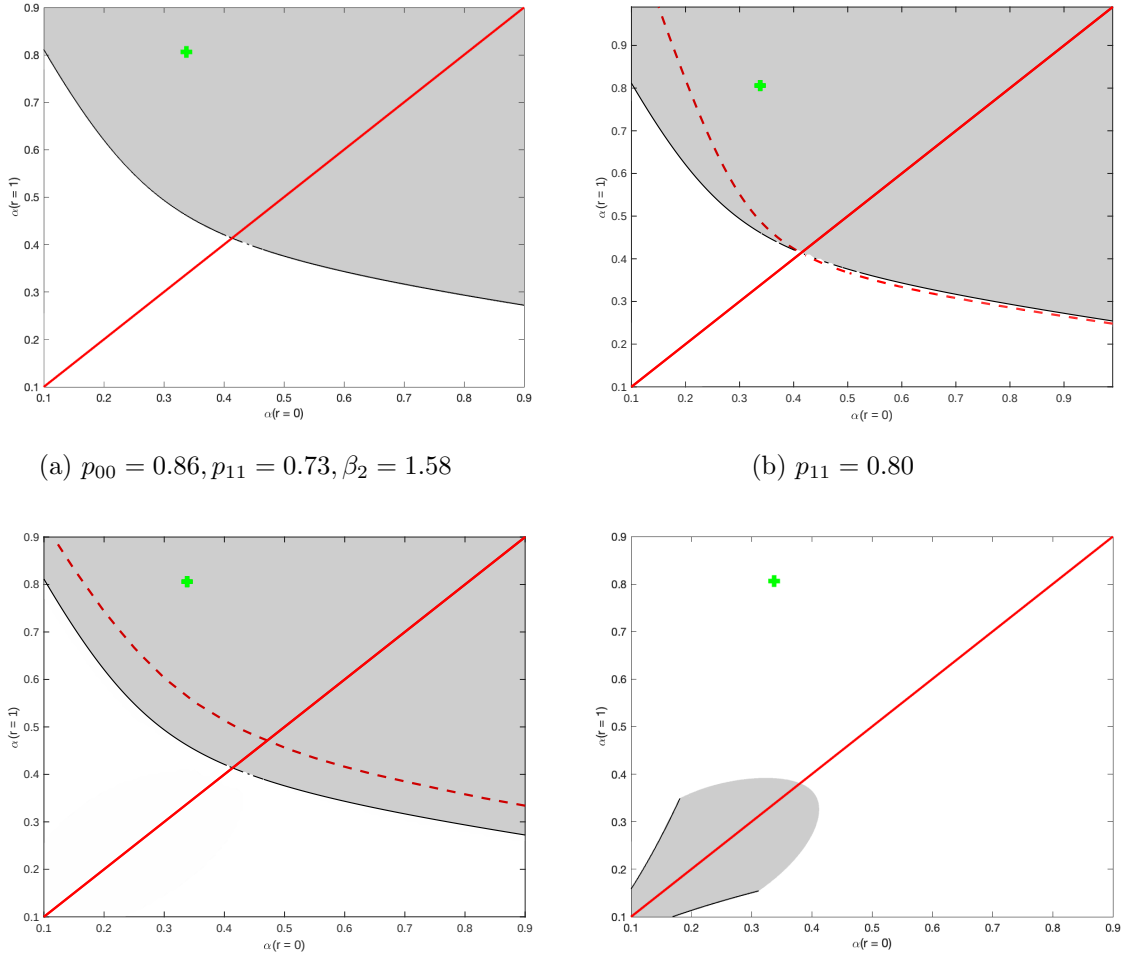
In this section, we illustrate how different monetary policy tools affect the RMB determinacy region using the MSRE framework, and present the results obtained using an LRE model for comparison.

In Figure 7, we showcase the determinacy regions obtained using the MSRE and LRE models, conditional on various values of the exchange rate policies α and internal monetary policies $\beta_2 = \phi_c(1+n)$, as well as policy regime transition probabilities. The remaining parameters are consistent with the ML estimates of the benchmark regime-switching model, panel regression estimates from Bloomberg economic forecasts, and calibration by JWY. We set $\rho_i^* = 0.95$, as in JWY, and $\tilde{\gamma} = [1 - n\gamma]^{-1} = -1.67$ according to the panel estimation results from Bloomberg economic forecasts. For simplicity, we set $\frac{\phi_{i*}-1}{\bar{N}} = 0$ and $\beta = 0$ because they do not impact the determinacy region in our model. The benchmark policy regime transition probabilities $p_{00} = 0.86$ and $p_{11} = 0.73$ are computed based on the estimated transition matrix characterized by $(\boldsymbol{\tau}, \mathbf{A}, \mathbf{V})$.[32] The solid red line in each figure represents the 45-degree line, such that the determinacy region of an LRE model is given by the line segment that overlaps with the shaded area.

Figure 7a demonstrates that an aggressive exchange rate policy with a policy coefficient α greater than 0.4 is necessary for an LRE model if the internal monetary policy does not lean sufficiently against the wind by giving a coefficient $\beta_2 = 1.58$ when $\tilde{\gamma} = -1.67$. This message is already conveyed in Figure 6a. The CCF regime estimated in the empirical section (a low α) leads to indeterminacy in our LRE model. This result may be due to our monetary policy specification and the estimated value of β_2 and $n\gamma$. Again, a more realistic monetary policy rule should be employed to explore whether the low α regime itself can stabilize the exchange rate market. We reserve this question for future research.

[32] See Appendix D for the calculation in detail.

Figure 7: Determinacy Regions of the MSRE Model



(a) $p_{00} = 0.86, p_{11} = 0.73, \beta_2 = 1.58$

(b) $p_{11} = 0.80$

(c) $\beta_2 = 1.3$

(d) $\beta_2 = 1.7$

Note: This panel of figures plots the determinacy regions of the MSRE model at different parameter values on the $\alpha(r_t = 0) - \alpha(r_t = 1)$ plane. The grey-shaded areas marked by black and dotted red lines envelopes represent the determinacy region for different parameters. The solid red line in each figure represents the 45-degree line, on which the determinacy region of an LRE is given by the line segment that overlaps with the shaded area. The green cross reports the estimated policy weights in the benchmark empirical model. The transition probabilities in (a),(c), and (d) are estimated from the benchmark model, whereas the transition probabilities in (b) are counterfactual.

The following exercises are conducted to illustrate the consequence of switching policy, given our simple model.

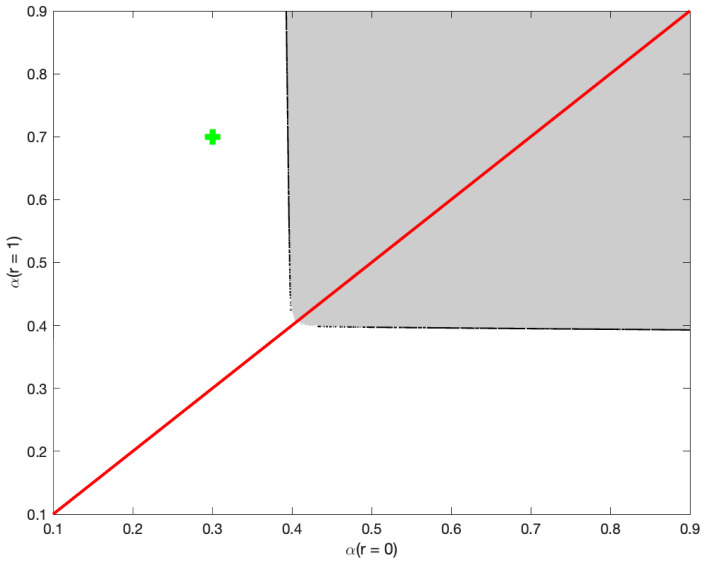
As shown in Figure 7a, there is less concern about coordinating monetary and exchange rate policies because the determinacy region of the economic system is substantially expanded. If the interest rate policy is constrained and insensitive to the exchange rate, the

monetary authority can implement time-varying RMB policy coefficients, occasionally allowing α to be below 0.4 while still achieving determinacy. For example, the green cross represents the coefficients of the *de facto* CCF policy. With estimated regime transition probabilities, a pair of RMB policies with both high and low α values (0.806 and 0.338) can satisfy the conditions for determinacy. The low-α regime may be a policy decision due to the expectation formation effect. In contrast to the LRE environment, the rational agent knows that the policy regime may switch to a high-α regime, thus ensuring that the state variables in this system will converge to a well-defined ergodic distribution and guarantee mean-square stability. Moreover, the edge of the determinacy region shows that, similar to the results in Davig and Leeper [2007], the central bank can lean more heavily against irrational trading of RMB in one regime by imposing an even lower market pillar coefficient if the other regime implements a larger market pillar coefficient.

Figure 7b illustrates how regime switching can be used as a tool for exchange rate policy. A central bank can manipulate the determinacy region by committing to certain probabilities of regime transitions and communicating them to the public. For the purpose of discussion, we focus on the triangular area above the 45-degree line in Figure 7b. Specifically, regime-0 (horizontal axis) and regime-1 (vertical axis) correspond to the low-α and high-α regimes, respectively. Transition probability has a subtle effect on determinacy. In a thought experiment, we increase p_{11}, i.e., the persistence of high-α regime, from 0.73 to 0.80; all other parameters remain identical to the benchmark model. Two effects are implied by the new determinacy region. First, a sufficiently high α in the high-α regime can not be coupled with a very low α in the other regime. To better understand this implication, recall that α can only take values in the unit interval. Second, at the boundary of the new determinacy region (red dashed curve), the central bank must impose a larger α in the high-α regime for any feasible low-α. According to Proposition 1 of Cho [2021], the problem with a more persistent high regime is that it may result in $\rho(\Psi_{F\otimes F}) > 1$, hence, a continuum of stable sunspot processes that is associated with the MOD solution.

Figure 7c and Figure 7d show how internal monetary policy affects the determinacy regions. These results are closely related to those in Figure 6a. Given the amount of noise trading in the intraday market characterized by γ and $\bar{N}$, the determinacy region of the MSRE model depends upon the strength of internal monetary policy ϕ_c, hence $\beta_2 = \frac{\phi_c}{\bar{N}}$, that leans against the central parity rate c_t. In this scenario, the determinacy region under a weaker monetary policy contracts toward higher market-pillar coefficients in both regimes. This scenario is shared by the LRE determinacy. The LRE intuition concerning determinacy under an interest rate policy stronger than noise trading continues to apply to the MSRE model. Additionally, when $\beta_2 > n\gamma$, there is a significant shift in the determinacy region,

Figure 8: Determinacy Region of MSRE under Persistent Regimes



Note: This figure plots the determinacy region of the MSRE model on the $\alpha(r_t = 0) - \alpha(r_t = 1)$ plane. The solid red line in the figure represents the 45-degree line, on which the determinacy region of an LRE is given by the line segment that overlaps with the shaded area. The green cross reports the estimated policy weights in the benchmark empirical model. The transition probabilities are set to be $p_{00} = 0.99$ and $p_{11} = 0.99$.

as shown in Figure 7d, which is consistent with Figure 6a.

5.3 The *de jure* Countercyclical Factor Policy

The RMB exchange rate policy underwent important reforms in February 2017. In particular, the PBC introduces the CCF intermittently as an intervention tool to discourage speculation. However, our estimates of the market pillar factor w_t^{α} in Figure 5 are not fully aligned with the announced CCF episodes. Rather, the *de facto* switching frequency is much higher than the frequency of announced episodes. In the next exercise, we illustrate one possible consequence of the officially announced CCF policy.

The *de jure* CCF regimes are very persistent regarding daily frequency data. The CCF was first introduced by the PBC on May 26, 2017, and subsequently removed on Jan 9, 2018. It was then reinstalled on Aug 24, 2018, and suspended on Oct 26, 2020. An actual RMB policy regime equivalent to the *de jure* CCF regime would suggest that the transition probabilities for policy regimes $p_{00}, p_{11} \approx 1$. In light of this argument, we let agents form the rational expectation using such a transition probability. In Figure 8, we display the determinacy region, where $p_{00} = 0.99, p_{11} = 0.99$. Our estimated policy coefficient does not

fall within the determinacy region, which implies a weak expectation formation effect due to the persistent regime. If the CCF regime (a low α) itself cannot guarantee determinacy, the rational agent realizes that the probability of switching to a high-α regime is very low, and therefore, the self-fulfilling prophecy cannot be ruled out.

This exercise demonstrates the importance of managing market expectations. The importance would not be undermined even if one develops an LRE model with estimated parameters to show that the CCF regime itself works to rule out self-fulfilling depreciation. In this scenario, the CCF is meaningful only if the non-CCF regime leads to indeterminacy. Our exercise shows that the *de jure* policy cannot be effective because it implies a very weak expectation formation effect in such a scenario.

6 Conclusion and Future Research

The formation of the RMB central parity rate follows a generalized two pillar policy that is affected by the CCF. In this paper, we exploit a regime-switching model to uncover the regime-switching policy weights. Our findings highlight the significance of the regime-switching pattern with respect to the policy coefficient of the market pillar. The empirical evidence suggests that the PBC may be implementing the CCF without issuing public announcements. Therefore, it is crucial to distinguish between the *de facto* and *de jure* CCF policies. In a simple MSRE model, we justify the introduction of a *de facto* CCF and illustrate the underlying economic logic. Conversely, we show that the *de jure* CCF policy is insufficient to stabilize the RMB exchange rate market in our quantitative exercises.

Some questions regarding the RMB policy switching remain unanswered. As a first step, in this study, we model the policy coefficients switching as an exogenous Markov chain. Future research could incorporate an endogenous regime-switching process. Policymakers may take the endogenous response of the market into consideration when designing the counter-cyclical policy. An optimal policy analysis could be conducted to examine the trade-offs associated with CCF policy implementation.

Moreover, as discussed in our MSRE analysis, our model specification and the implied panel regression estimates suggest that the CCF regime (low α) itself may be insufficient to guarantee determinacy. This result is driven by our simplified monetary policy. A more realistic monetary policy rule could be introduced to evaluate the consequence of a pure CCF regime.

Finally, our determinacy concept assumes that sentiment shocks are exogenous. A useful future study might consider an endogenous sentiment shock and explore the feedback effect between the market and the government.

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Appendices

Appendix A Daytime Factor and CCF Policy

A.1 Reference Period Reform and Daytime Factor

This section presents a detailed derivation of the RMB central parity rule with a 15-hour reference period and daytime factor and connects it to the simple two-pillar rule with a 24-hour preference period. The extended policy rule with a 15-hour reference period is given in Equation (10) of JWY. In the extended rule, with policy weights w_{NT}, w_{DT}, the central parity rate is a weighted average of the previous closing rate S_{t-1}^{CL}, and the daytime and nighttime components of the stability implied US dollar index, respectively, $\bar{S}_{t-1}^{DT}$ and $\bar{S}_t^{NT}$.

$$S_t^{CP} = \left(\bar{S}_t^{NT}\right)^{w_{NT}} \left(\bar{S}_{t-1}^{DT}\right)^{w_{DT}} \left(S_{t-1}^{CL}\right)^{1-w_{NT}-w_{DT}}. \tag{29}$$

Following the notation of JWY, the nighttime and daytime factors are

$$\begin{aligned} \bar{S}_t^{NT} &= S_{t-1}^{CP} \left(\frac{X_{t,7:30AM}}{X_{t-1,4:30PM}} \right)^{1-w_{usd}} \\ \bar{S}_{t-1}^{DT} &= S_{t-1}^{CP} \left(\frac{X_{t-1,4:30PM}}{X_{t-1,7:30AM}} \right)^{1-w_{usd}} \end{aligned}$$

where $X_{t,7:30AM}, X_{t,4:30PM}$ are the implied USD index with respect to a basket of currencies other than the USD computed at 7:30 AM and 4:30 PM Beijing Time. Importantly, X_t in

the simple two-pillar model equals $X_{t,7:30AM}$. With these,

$$
\begin{aligned}
S_t^{CP} &= \left(\bar{S}_t^{NT}\right)^{w_{NT}} \left(\bar{S}_{t-1}^{DT}\right)^{w_{DT}} \left(S_{t-1}^{CL}\right)^{1-w_{NT}-w_{DT}} \\
&= \left(S_{t-1}^{CP}\left(\frac{X_{t,7:30AM}}{X_{t-1,4:30PM}}\right)^{1-w_{usd}}\right)^{w_{NT}} \left(S_{t-1}^{CP}\left(\frac{X_{t-1,4:30PM}}{X_{t-1,7:30AM}}\right)^{1-w_{usd}}\right)^{w_{DT}} \left(S_{t-1}^{CL}\right)^{1-w_{NT}-w_{DT}} \\
&= \left(S_{t-1}^{CP}\left(\frac{X_{t,7:30AM}}{X_{t-1,7:30AM}}\right)^{1-w_{usd}}\right)^{w_{NT}} \left(\frac{S_{t-1}^{CP}}{S_{t-1}^{CP}}\left(\frac{X_{t-1,7:30AM}}{X_{t-1,4:30PM}}\right)^{1-w_{usd}}\right)^{w_{NT}} \\
&\quad \times \left(S_{t-1}^{CP}\left(\frac{X_{t-1,4:30PM}}{X_{t-1,7:30AM}}\right)^{1-w_{usd}}\right)^{w_{DT}} \left(S_{t-1}^{CL}\right)^{1-w_{NT}-w_{DT}} \\
&= \left(S_{t-1}^{CP}\left(\frac{X_{t,7:30AM}}{X_{t-1,7:30AM}}\right)^{1-w_{usd}}\right)^{w_{NT}} \left(S_{t-1}^{CP}\right)^{w_{NT}} \left(\frac{1}{S_{t-1}^{CP}}\left(\frac{X_{t-1,7:30AM}}{X_{t-1,4:30PM}}\right)^{1-w_{usd}}\right)^{w_{NT}} \\
&\quad \times \left(S_{t-1}^{CP}\left(\frac{X_{t-1,4:30PM}}{X_{t-1,7:30AM}}\right)^{1-w_{usd}}\right)^{w_{DT}} \left(S_{t-1}^{CL}\right)^{1-w_{NT}-w_{DT}} \\
&= \left(S_{t-1}^{CP}\left(\frac{X_{t,7:30AM}}{X_{t-1,7:30AM}}\right)^{1-w_{usd}}\right)^{w_{NT}} \left(S_{t-1}^{CP}\right)^{w_{NT}} \left(S_t^{DT}\right)^{w_{DT}-w_{NT}} \left(S_{t-1}^{CL}\right)^{1-w_{NT}-w_{DT}}
\end{aligned}
$$

Similar to the simple two-pillar model, let

$$\bar{S}_t = S_{t-1}^{CP}\left(\frac{X_{t,7:30AM}}{X_{t-1,7:30AM}}\right)^{1-w_{usd}}$$

to have

$$S_t^{CP} = \left(\bar{S}_t\right)^{w_{NT}} \left(S_{t-1}^{CP}\right)^{w_{NT}} \left(\bar{S}_{t-1}^{DT}\right)^{w_{DT}-w_{NT}} \left(S_{t-1}^{CL}\right)^{1-w_{NT}-w_{DT}}$$

Take first order log difference with respect to S_t^{CP} to have

$$
\begin{aligned}
\log\left(\frac{S_t^{CP}}{S_{t-1}^{CP}}\right) &= (1-w_{NT}-w_{DT})\log\left(\frac{S_{t-1}^{CL}}{S_{t-1}^{CP}}\right) \\
&\quad + w_{NT}\log\left(\frac{\bar{S}_t}{S_{t-1}^{CP}}\right) + (w_{DT}-w_{NT})\log\left(\frac{\bar{S}_{t-1}^{DT}}{S_{t-1}^{CP}}\right).
\end{aligned}
$$

This result in the notation of the simple two-pillar rule is

$$
\begin{aligned}
\Delta c_t &= (1-w_{NT}-w_{DT})d_{t-1} + (w_{DT}-w_{NT})\log\left(\frac{\bar{S}_{t-1}^{DT}}{S_{t-1}^{CP}}\right) + w_{NT}(1-w_{usd})\Delta x_t \\
&= (1-w_{NT}-w_{DT})d_{t-1} + (w_{DT}-w_{NT})\Delta x_{t-1}^{DT} + w_{NT}(1-w_{usd})\Delta x_t.
\end{aligned}
$$

It is important to note that the coefficients $(1 - w_{NT} - w_{DT})$ and w_{NT} have the same interpretation as α and β, respectively, in the simple two pillar rule. Compared to the simple rule, the policy rule above includes the additional daytime component Δx_{t-1}^{DT}.

A.2 Understanding RMB Policy Regimes in A Simple Two-Pillar Rule

In this section, we shed light on the driver of these policy factors. JWY suggests that the time variation of policy coefficients is due to the intermittent introduction of the daytime factor in the RMB policy rule. Their measure of countercyclical factor is defined as $\Delta x_t^{DT} = \log(\bar{S}_t^{DT}) - \log(S_t^{CP})$.[33] By replacing the definition of $\bar{S}_t^{DT}$, we find that $\Delta x_t^{DT} = (1-\omega_{usd}) * \log(\frac{X_{t,4:30PM}}{X_{t,7:30AM}})$. Intuitively, a higher value of Δx_t^{DT} implies a depreciation pressure on RMB in the intraday market. More importantly, the value of Δx_t^{DT} reflects the global market supply and demand condition outside China, as it captures the changes of the implied dollar index X_t during the trading hours of the RMB market.

Dividing the data into different subsamples according to the *de jure* introduction of CCF, JWY concludes that imposing the counter-cyclical factor essentially shifts the weight away from the market pillar toward the basket pillar. Besides, the regression coefficient of the countercyclical factor is insignificantly different from zero in the subperiod with no such factor. Yet, these exercises only focus on the *de jure* CCF policy. With our estimated policy factors, we are able to document the relationship between the *de facto* CCF policy and the daytime factor b_t. Specifically, we regress the estimated RMB policy factor w_t^i for $i \in \{\alpha, \beta, \sigma\}$ on the daytime factor. The empirical specification is the following:

$$w_t^i = \beta_i \Delta x_{t-1}^{DT} + \gamma_i w_{t-1}^i + \bar{\alpha}_i + \varepsilon_{i,t}, \quad i \in \{\alpha, \beta, \sigma\}. \tag{30}$$

The estimation results are reported in Table 3. In the same table, we also report the regression results based on the subsample. Overall, the regression results in Column One Panel A appear to lend support to the effect of the daytime factor on w^α. Yet, the full sample regression in Column One Panel B finds no significant relationship between the daytime factor and w^β. Intuitively, the policymaker may respond to the global market conditions, which may be driven by sentiment-induced "procyclicality." The response implies that a depreciation pressure of RMB is associated with a lower w^α, suggesting that a low-α regime

[33] where

$$\bar{S}_t^{DT} = S_t^{CP} \left(\frac{X_{t,4:30PM}}{X_{t,7:30AM}} \right)^{1-\omega_{usd}}$$

$X_{t,7:30AM}, X_{t,4:30PM}$ are the implied RMB index with respect to a basket of currencies modulo the USD computed at 7:30 AM and 4:30 PM Beijing Time.

is more likely to be implemented. Meanwhile, *de facto* CCF policy does not lead to a change of β policy factor. This is the key difference compared to the *de jure* CCF policy highlighted by PBC and studied by JWY.

We further explore the relationship using subsample regression. According to the authority, the *de jure* CCF policy is used in subperiod 3 and subperiod 5. The regression results in Panel A find that the daytime factor is negatively correlated with w^{α} in subperiod 3,4 and 6. The subperiod 3 and 4 results are consistent with JWY's (Table III of their work). The subperiod 6 result is interesting in the sense that it echoes the recent market view— PBC uses the CCF policy without public announcement. [34] Yet, we do not find a significant relationship in subperiod 5. Again, these subperiod results suggest that the PBC may watch the market conditions and determine the policy weights on the market pillar. The use of CCF policy appears more discretionary than commitment.

Panel C reveals a significant relationship between w^{σ} and the daytime factor Δx_{t-1}^{DT}. This result is also interesting. It suggests another way to impose the *de facto* CCF is to use the exchange rate policy shock and to surprise the market. In the following quantitative section, we show that the regimes of σ and β play the same role in the dynamics system and thus, we do not highlight the switching of w^{σ}.

[34] Also, our subperiod 6 data lasts from Oct 27, 2020, to Dec 31, 2021, making our sample much longer than JWY.

Table 3: Regression on Policy Factor

Panel A:						
	full sample	2	3	4	5	6
	w_t^α	w_t^α	w_t^α	w_t^α	w_t^α	w_t^α
Δx_{t-1}^{DT}	-0.112***	-0.0454	-0.186*	-0.172$^+$	-0.0704	-0.269*
	(-2.77)	(-0.40)	(-1.69)	(-1.64)	(-1.37)	(-1.83)
w_{t-1}^α	0.845***	0.891***	0.807***	0.820***	0.846***	0.854***
	(-56.44)	(-17.19)	(-16.41)	(-18.83)	(-33.06)	(-25.36)
$\bar{\alpha}_\alpha$	-0.0314	0.0307	0.00925	-0.0693	-0.0982*	0.00332
	(-0.98)	(-0.29)	(-0.12)	(-0.59)	(-1.81)	(-0.03)
N	1186	64	154	153	524	144
adj. R^2	0.718	0.755	0.667	0.678	0.721	0.719
Panel B:						
	full sample	2	3	4	5	6
	w_t^β	w_t^β	w_t^β	w_t^β	w_t^β	w_t^β
Δx_{t-1}^{DT}	0.00802	-0.154*	-0.00519	0.0451	-0.0216	0.152***
	(-0.43)	(-1.78)	(-0.10)	(-1.16)	(-0.87)	(-2.73)
w_{t-1}^β	0.756***	0.439***	0.581***	0.763***	0.791***	0.805***
	(-35.43)	(-4.21)	(-6.78)	(-12.45)	(-26.99)	(-16.63)
$\bar{\alpha}_\beta$	-0.0169	-0.200***	-0.111***	0.0317	0.00301	0.0125
	(-1.30)	(-3.13)	(-3.69)	(-0.67)	(-0.16)	(-0.29)
N	1186	64	154	153	524	144
adj. R^2	0.571	0.208	0.329	0.584	0.628	0.639
Panel C:						
	full sample	2	3	4	5	6
	w_t^σ	w_t^σ	w_t^σ	w_t^σ	w_t^σ	w_t^σ
Δx_{t-1}^{DT}	0.183*	-0.159	0.184	0.346	0.0507	0.728*
	(-1.94)	(-0.57)	(-0.83)	(-1.49)	(-0.47)	(-1.94)
w_{t-1}^σ	0.879***	0.856***	0.814***	0.853***	0.894***	0.892***
	(-63.01)	(-14.5)	(-16.27)	(-20.46)	(-41.27)	(-30.94)
$\bar{\alpha}_\sigma$	0.00599	-0.262	-0.2	0.2	0.0856	0.045
	(-0.09)	(-1.14)	(-1.45)	(-0.72)	(-0.85)	(-0.18)
N	1186	64	154	153	524	144
adj. R^2	0.774	0.697	0.666	0.728	0.805	0.783

Note: t statistics in parentheses, and $+p < 0.15, *p < 0.10, **p < 0.05, ***p < 0.01$. The first column is the regression with full sample data. Columns 2-6 correspond to regression for subperiod 2-6.

Appendix B Model and Derivation

B.1 The Optimization Problem of Traders

Now, we describe the intraday foreign exchange market. We assume that there is a continuum of investors in the foreign exchange market, indexed by $j \in [0,1]$, who trade choose to invest in RMB assets or in dollar assets. Following De Long et al. [1990], Jeanne and Rose [2002], and JWY, we consider two types of traders in the market: informed traders and noise traders with incomplete information. The informed traders have accurate information regarding the asset returns and risks. Each investor born at date t is endowed with wealth $\overline{W}$ and chooses optimal investment-consumption strategy to maximize the expected CARA utility over its next-period wealth W_{t+1}^j.

The optimal RMB position of the informed traders i is X_t^j:

$$\max_{X_t^j} E_t^j \left[-\exp(-\zeta W_{t+1}^j) \right]$$

subject to the following wealth dynamic:

$$W_{t+1}^j = (1+i_t^*)\overline{W} + X_t^j \rho_{t+1}. \tag{31}$$

ζ captures the risk aversion of the trader. Given that all shocks in our economy are all log-normal, the above maximization problem is equivalent to solving the following one:

$$\max_{X_t^j} E_t^j(W_{t+1}^j) - \frac{\zeta}{2} var_t^j(W_{t+1}^j)$$

where $E_t^j(W_{t+1}^j)$ is the expected wealth in the next period and $var_t^j(W_{t+1}^j)$ is the conditional variance for investor j:

$$\begin{aligned} E_t^j(W_{t+1}^j) &= (1+i_t^*)\overline{W} + E_t^j(X_t^j \rho_{t+1}), \\ var_t^j(W_{t+1}^j) &= (X_t^j)^2 var_t^j(\rho_{t+1}). \end{aligned}$$

The first-order condition implies the optimal position:

$$X_t^j = \frac{E_t^j(\rho_{t+1})}{\zeta var_t^j(\rho_{t+1})} \tag{32}$$

B.2 Proof of Proposition 4.2

$$\mu = \frac{(2-\alpha-n\gamma) \pm \sqrt{\Delta}}{2} \tag{33}$$

where $\Delta = [\alpha - n\gamma]^2 + 4\frac{\phi_c}{\overline{N}}\alpha$ and $n = \frac{(1-\overline{N})}{\overline{N}}$.

1. When $(1+n)\phi_c > n\gamma$, for any $\alpha > 0$, we have

$$\Delta = [\alpha - n\gamma]^2 + 4(1+n)\phi_c\alpha > [\alpha - n\gamma]^2 + 4\alpha n\gamma = [\alpha + n\gamma]^2 \tag{34}$$

Thus,

$$\mu_2 = \frac{(2-\alpha-n\gamma) + \sqrt{\Delta}}{2} \geq \frac{(2-\alpha-n\gamma) + \alpha + n\gamma}{2} > 1$$

To guarantee determinacy, we must have

$$-1 < \mu_3 = \frac{(2-\alpha-n\gamma) - \sqrt{\Delta}}{2} < 1$$

Note that (34) guarantee that

$$\mu_3 = \frac{(2-\alpha-n\gamma) - \sqrt{\Delta}}{2} < \frac{(2-\alpha-n\gamma) - \alpha - n\gamma}{2} = 1 - \alpha - n\gamma < 1$$

when $\alpha < 1$ and $n\gamma > 0$.

Then, we need to find the condition that guarantee $-1 < \mu_3$:

$$-2 < (2-\alpha-n\gamma) - \sqrt{\Delta} \quad \Leftrightarrow \quad \sqrt{\Delta} < (4-\alpha-n\gamma)$$

Rearranging the terms, it is easy to show that

$$(1+n)\phi_c\alpha \quad < \quad (2-\alpha)(2-n\gamma) \tag{35}$$

2. When $0 < (1+n)\phi_c < n\gamma$, for any $\alpha > 0$, we have

$$\Delta = [\alpha - n\gamma]^2 + 4(1+n)\phi_c\alpha < [\alpha - n\gamma]^2 + 4\alpha n\gamma = [\alpha + n\gamma]^2$$

Thus,

$$\mu_2 = \frac{(2-\alpha-n\gamma) + \sqrt{\Delta}}{2} < \frac{(2-\alpha-n\gamma) + \alpha + n\gamma}{2} < 1$$

There are two possibilities, depending on the relative value of α and $n\gamma$.

Case 2.1 When $\alpha < n\gamma$, we have

$$\mu_2 = \frac{(2-\alpha-n\gamma)+\sqrt{\Delta}}{2} > \frac{(2-\alpha-n\gamma)-\alpha+n\gamma}{2} > 1-\alpha > -1$$

To guarantee determinacy, we must have

$$\begin{aligned} -1 &> \mu_3 = \frac{(2-\alpha-n\gamma)-\sqrt{\Delta}}{2} \\ \text{or } \mu_3 &> 1 \end{aligned}$$

Note that we have

$$\mu_3 < \mu_2 < 1$$

So, we must find the condition that guarantees $-1 > \mu_3$. It is straightforward to get that

$$\begin{aligned} -2 &> (2-\alpha-n\gamma)-\sqrt{\Delta} \\ &\Leftrightarrow \\ (1+n)\phi_c\alpha &> (2-\alpha)(2-n\gamma) \end{aligned} \tag{36}$$

Case 2.2 When $\alpha > n\gamma$, we show that there is no condition that satisfies the Blanchard-Kahn condition. In this case,

$$\mu_2 = \frac{(2-\alpha-n\gamma)+\sqrt{\Delta}}{2} > \frac{(2-\alpha-n\gamma)+\alpha-n\gamma}{2} > 1-n\gamma$$

When $1-n\gamma > -1.$, we have $\mu_2 > 1-n\gamma > -1$.

The condition to guarantee determinacy is

$$(1+n)\phi_c\alpha > (2-\alpha)(2-n\gamma) \tag{37}$$

But the above condition cannot be satisfied since $(1+n)\phi_c < n\gamma$, which implies $0 > 4-2\alpha-2n\gamma)$. This cannot be true when $0 < \alpha < 1$, $\gamma > 0$ and $\alpha > n\gamma$.

Alternatively, when $1-n\gamma < -1.$. It implies $2 < n\gamma$, which contradicts to the condition that $\alpha > n\gamma$. So, the BK condition cannot be satisfied under the condition that $0 < (1+n)\phi_c < n\gamma$ and $\alpha > n\gamma$.

Appendix C Model Extensions

In this appendix, we extend the benchmark model. We specify the process of the real exchange rate, introduce the government intervention and release the assumption on the noise trader's information set.

C.1 The Role of Real Exchange Rate

A shock studied by JWY is the real exchange rate shock. By definition, the real exchange rate q_t is:

$$q_t = p_t^* + e_t - p_t \tag{38}$$

p_t^* is the foreign price level. The domestic price level is p_t. As in JWY, we assume that $p_t^* = 0$. Like i_t^*, q_t is an exogenous process.

Assuming the money stock to be m_t, the money demand function establishes a connection between the money stock, price level, and interest rate:

$$m_t - p_t = -\varpi i_t \tag{39}$$

In our benchmark model, we specify the monetary policy rule, see Equation (9). It is easy to see that the real exchange rate does not change the dynamics of the nominal exchange rate e_t. Thus, the real exchange rate is irrelevant to our analysis of the self-fulfilling prophecy in the exchange rate market. We summarize it as a lemma.

Lemma C.1. *In our model, the dynamics of the nominal exchange rate are not affected by the real exchange rate.*

Proof. The proof is straightforward. Whenever there is an exogenous shock to the real exchange rate, the domestic price level changes. The changing price level requires an adjustment of money supply so as to guarantee the equilibrium of the money market. Thus, the domestic interest rate still follows the rule Equation (9). The dynamics of the nominal exchange rate are still determined by the system, characterized by Equation (17). □

Our research aligns with the findings of JWY, particularly in their Proposition 3 which states that the value of d_t is determined by the foreign interest rate i_t^* and the central parity rate c_t in a model that includes the real exchange rate. Furthermore, their model indicates that the nominal exchange rate remains unaffected by changes in the real exchange rate.

C.2 An Alternative Conditional Variance of the Excess Return for the Noise Trader

In this appendix, we argue that our analysis will not be affected by changing the specification of the noise trader's conditional variance in the benchmark model.

We follow Jeanne and Rose [2002] and specify the conditional variance of noise trader to be the unconditional variance σ_ρ^2.

$$Var_t^N(\rho_{t+1}) = \sigma_\rho^2$$

With this specification, the position of noise trader becomes $X_t^N = \frac{E_t^N(\rho_{t+1})}{\zeta\sigma_\rho^2}$. The market clearing condition of RMB assets becomes

$$(i_t - i_t^*) - E_t(e_{t+1} - e_t) = -n/\sigma_\rho^2 Var_t(\rho_{t+1})(i_t - i_t^*) + n/\sigma_\rho^2 Var_t(\rho_{t+1})\gamma(e_t - \bar{e}) \quad (40)$$

Next, we show that $Var_t(\rho_{t+1})$ is time-invariant and conclude that our analysis is not affected if we change the specification of this conditional variance.

From the definition of risk premium, we have $\rho_{t+1} = (i_t - i_t^*) - (e_{t+1} - e_t)$. The conditional variance

$$Var_t(\rho_{t+1}) = (\rho_{t+1} - E_t\rho_{t+1})^2 = (e_{t+1} - E_t e_{t+1})^2. \quad (41)$$

From the system Equation (17), we know that $e_{t+1} - E_t e_{t+1} = A^{(3,)}\varepsilon_{z,t+1}$. Therefore, it is straightforward to show that the conditional variance of the excess return is time-invariant.

Appendix D Regime Transition Probabilities

In this section, we demonstrate the process of obtaining regime transition probabilities of r_t^α, which is at the core of our empirical and theoretical analysis.

Table 4: Regimes

	$R_t=1$	$R_t=2$	$R_t=3$	$R_t=4$	$R_t=5$	$R_t=6$	$R_t=7$	$R_t=8$
r_t^α	0	0	0	0	1	1	1	1
r_t^β	0	0	1	1	0	0	1	1
r_t^σ	0	1	0	1	0	1	0	1

The byproducts of the regime-switching filter include the regime transition probability $p(r_{t+1}^\alpha, r_{t+1}^\beta, r_{t+1}^\sigma | r_t^\alpha, r_t^\beta, r_t^\sigma)$, and the stationary regime probabilities $\pi(r_t^\alpha, r_t^\beta, r_t^\sigma)$. With a slight abuse of notation, we code the regime $\mathbf{R}_t$ as in Table 4.

Table 5: Benchmark Regime Transition and Stationary Probabilities

	$R_t=1$	$R_t=2$	$R_t=3$	$R_t=4$	$R_t=5$	$R_t=6$	$R_t=7$	$R_t=8$
$R_{t+1}=1$	0.09	0.00	0.03	0.00	0.02	0.00	0.01	0.00
$R_{t+1}=2$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$R_{t+1}=3$	0.52	0.16	0.78	0.04	0.10	0.06	0.29	0.00
$R_{t+1}=4$	0.00	0.58	0.04	0.87	0.00	0.16	0.00	0.42
$R_{t+1}=5$	0.13	0.00	0.03	0.00	0.37	0.00	0.12	0.00
$R_{t+1}=6$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$R_{t+1}=7$	0.26	0.19	0.15	0.02	0.51	0.60	0.58	0.27
$R_{t+1}=8$	0.00	0.06	0.00	0.03	0.00	0.15	0.00	0.25
$\pi(R_t)$	0.02	0.00	0.41	0.24	0.07	0.00	0.26	0.01

The ML estimates of the benchmark model in Table 1 imply a transition matrix in Table 5. From these components, we can easily obtain the joint regime probabilities

$$p(r_{t+1}^\alpha, r_{t+1}^\beta, r_{t+1}^\sigma, r_t^\alpha, r_t^\beta, r_t^\sigma) = p(r_{t+1}^\alpha, r_{t+1}^\beta, r_{t+1}^\sigma | r_t^\alpha, r_t^\beta, r_t^\sigma)\pi(r_t^\alpha, r_t^\beta, r_t^\sigma)),$$

and marginalize to get the joint probabilities

$$p(r_{t+1}^\alpha, r_t^\alpha) = \sum_{r_{t+1}^\beta, r_t^\beta, r_{t+1}^\sigma, r_t^\sigma} p(r_{t+1}^\alpha, r_{t+1}^\beta, r_{t+1}^\sigma, r_t^\alpha, r_t^\beta, r_t^\sigma).$$

In addition, we may marginalize the stationary distribution to have the stationary r_t^α prob-

abilities

$$\pi(r_t^\alpha) = \sum_{r_t^\beta, r_t^\sigma} \pi(r_t^\alpha, r_t^\beta, r_t^\sigma).$$

Table 6 collects the joint and stationary regime probabilities for r_t^α that is computed from

Table 6: Joint and Stationary Regime Probabilities for r_t^α

	$r_t^\alpha = 0$	$r_t^\alpha = 1$
$r_{t+1}^\alpha = 0$	0.5728	0.0908
$r_{t+1}^\alpha = 1$	0.0902	0.2462
$\pi(r_t^\alpha)$	0.6630	0.3370

the benchmark transition matrix.

Therefore, the corresponding r_t^α transition probabilities can be easily obtained as

$$p(r_{t+1}^\alpha | r_t^\alpha) = \frac{p(r_{t+1}^\alpha, r_t^\alpha)}{\pi(r_t^\alpha)}.$$

Specifically, we find $p(r_{t+1}^\alpha = 0 | r_t^\alpha = 0) = 0.86$ and $p(r_{t+1}^\alpha = 1 | r_t^\alpha = 1) = 0.73$

Appendix E Model with Switching α and β in RMB Rule

To explore the implication of Regime-Switching α and β coefficients on the exchange rate's determinacy, we extend the model to allow both coefficients to switch, possibly together. We will use most of the previous notation in this section when there is little risk of confusion. To begin with, we consider a regime-switching policy rule with switching coefficients,

$$c_t = (1-\alpha(r_t^\alpha))c_{t-1} + \alpha(r_t^\alpha)e_{t-1} + \beta(r_t^\beta)(1-\omega_0)\Delta x_t + \sigma\varepsilon_t \tag{42}$$

with exogenous binary state variables $r_t^\alpha, r_t^\beta \in \{0,1\}$. Let the exogenous regime $r_t = 1 + r_t^\alpha + 2r_t^\beta$ and assume it follows an ergodic Markov chain with a 4×4 transition matrix Q where the transition probability from a regime i to regime j is $q_{ij} = \Pr(r_{t+1} = j | r_t = i)$ for $i, j \in \{1,2,3,4\}$. Then, we can rewrite the policy rule as

$$c_t = (1-\alpha(r_t))c_{t-1} + \alpha(r_t)e_{t-1} + \beta(r_t)(1-\omega_0)\Delta x_t + \sigma\varepsilon_t \tag{43}$$

by defining

$$\begin{aligned}\alpha(r_t = 1 + r_t^\alpha + 2r_t^\beta) &= \alpha(r_t^\alpha) \quad \text{for } r_t^\beta = 0,1\\ \beta(r_t = 1 + r_t^\alpha + 2r_t^\beta) &= \beta(r_t^\beta) \quad \text{for } r_t^\alpha = 0,1\end{aligned}$$

Denote the information set observable to traders at time t as $\mathcal{F}_t = \{Z_{t-l}, r_{t-l}, \varepsilon_{z,t-1}, l = 0,1,2,\cdots\}$ with Z_t and $\varepsilon_{z,t}$ defined in the previous section. Our system of equations with regime-switching coefficients contains foreign interest rate process, Equation (8), RMB central parity policy, Equation (43), and modified uncovered interest rate parity condition, Equation (16). Then the Markov-switching rational expectation (MSRE) model has the expression

$$Z_t = \tilde{F}\mathbb{E}_t Z_{t+1} + \tilde{\Omega}(r_t)Z_{t-1} + \tilde{\Gamma}(r_t)\varepsilon_{z,t} \tag{44}$$

with coefficient matrices

$$\tilde{F} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \tilde{\gamma} \end{pmatrix}; \quad \tilde{\Gamma} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \beta(r_t)(1-\omega_0) & 1 \\ -\tilde{\gamma}\frac{\phi_{i*}-1}{\bar{N}} & -\tilde{\gamma}\frac{\phi_c}{\bar{N}}\beta(r_t)(1-\omega_0) & -\tilde{\gamma}\frac{\phi_c}{\bar{N}} \end{pmatrix}$$

$$\tilde{\Omega}(r_t) = \begin{pmatrix} \rho_{i^*} & 0 & 0 \\ 0 & 1-\alpha(r_t) & \alpha(r_t) \\ -\tilde{\gamma}\frac{\phi_{i*}-1}{\bar{N}}\rho_{i^*} & -\tilde{\gamma}\frac{\phi_c}{\bar{N}}(1-\alpha(r_t)) & -\tilde{\gamma}\frac{\phi_c}{\bar{N}}\alpha(r_t) \end{pmatrix}$$

where $\tilde{\gamma} = [1 - n\gamma]^{-1}$.

Similar to the benchmark model, an equilibrium selection issue arises. We apply the result of Cho [2016, 2021] again to find that any solution to (44) can be written as a combination of a fundamental component and a sunspot component b_t that is asymptotically covariance stationary and independent to Z_0 and r_t for all $t \geq 1$

$$Z_t = [\bar{\Omega}(r_t)Z_{t-1} + \bar{\Gamma}(r_t)\varepsilon_{z,t}] + b_t \tag{45}$$

$$b_t = \mathbb{E}_t[\bar{F}(r_t)b_{t+1}] \tag{46}$$

where

$$\bar{\Omega}(r_t) = \Psi(r_t)\tilde{\Omega}(r_t) \tag{47}$$

$$\bar{\Gamma}(r_t) = \Psi(r_t)\tilde{\Gamma}(r_t) \tag{48}$$

$$\bar{F}(r_t) = \Psi(r_t)\tilde{F} \tag{49}$$

for all $r_t, r_{t+1} \in \{1, 2, 3, 4\}$, if there exists

$$\Psi(r_t) = \left[I_3 - \mathbb{E}_t[\tilde{F}\bar{\Omega}(r_{t+1})]\right]^{-1}$$

for all $r_t \in \{1, 2, 3, 4\}$. Note that $(\bar{\Omega}(r_t), \bar{\Gamma}(r_t))$ is referred to as an MSV solution to the extended model, which depends on the values of $\alpha(r_t)$ and $\beta(r_t)$.

The model determinacy in the MOD-MSS sense only depends on $\bar{\Omega}(r_t)$ and the regime transition probability matrix because $\bar{F}(r_t)$ is determined once $\bar{\Omega}(r_t)$ is given. By the same argument in the main text, the model has a unique real MSS solution if and only if there

are minimum spectral radius $\rho(\bar{\Psi}_{\bar{\Omega}_1 \otimes \bar{\Omega}_1}) < 1$ and $\rho(\Psi_{\bar{F}_1 \otimes \bar{F}_1}) \leq 1$, where

$$\bar{\Psi}_{\bar{\Omega}_1 \otimes \bar{\Omega}_1} = \begin{pmatrix} q_{11}\bar{\Omega}_1(1) \otimes \bar{\Omega}_1(1) & \cdots & q_{41}\bar{\Omega}_1(1) \otimes \bar{\Omega}_1(1) \\ \vdots & \ddots & \vdots \\ q_{14}\bar{\Omega}_1(4) \otimes \bar{\Omega}_1(4) & \cdots & q_{44}\bar{\Omega}_1(4) \otimes \bar{\Omega}_1(4) \end{pmatrix}$$

$$\Psi_{\bar{F}_1 \otimes \bar{F}_1} = \begin{pmatrix} q_{11}\bar{F}_1(1) \otimes \bar{F}_1(1) & \cdots & q_{14}\bar{F}_1(1) \otimes \bar{F}_1(1) \\ \vdots & \ddots & \vdots \\ q_{41}\bar{F}_1(4) \otimes \bar{F}_1(4) & \cdots & q_{44}\bar{F}_1(4) \otimes \bar{F}_1(4) \end{pmatrix}$$

Clearly, model determinacy depends on $\alpha(r_t)$ and Q, but not on the values of $\beta(r_t)$. We may expect the determinacy condition to be identical to the model with only α switching in the vein of footnote (17) of Cho [2021]. Upon this, we assume without loss of generality that $\beta(r_t) = \beta$, and assume r_t^β does not switch between $0, 1$. This violates the ergodicity assumption of r_t but simplifies the discussion. Similar to the argument in Appendix E of Cho [2021], where he shows the equivalence of MOD determinacy conditions to the standard eigenvalue conditions in LRE, non-switching β reduces the system to one that only admits α switching with ergodic r_t^α process. That is, the extended model is identical to the benchmark model, hence, its determinacy region.

If we continue and examine the transition probabilities for the extended model assuming non-switching β, then there are

$$q_{11} = q_{33} = p_{00}, \tag{50}$$

$$q_{12} = q_{34} = p_{01}, \tag{51}$$

$$q_{21} = q_{43} = p_{10}, \tag{52}$$

$$q_{22} = q_{44} = p_{11}, \tag{53}$$

$$q_{13} = q_{14} = q_{23} = q_{24} = q_{31} = q_{41} = q_{32} = q_{42} = 0, \tag{54}$$

with p_{ij}'s the same transition probabilities in the benchmark model with α switching only. Additionally,

$$\Psi(k) = \left[I_3 - \sum_{j=1}^{4} q_{kj} \tilde{F} \bar{\Omega}(j) \right]^{-1} = \begin{cases} \left[I_3 - \sum_{j=1}^{2} q_{kj} \tilde{F} \bar{\Omega}(j) \right]^{-1} & \text{if} \quad k = 1, 2 \\ \left[I_3 - \sum_{j=3}^{4} q_{kj} \tilde{F} \bar{\Omega}(j) \right]^{-1} & \text{if} \quad k = 3, 4 \end{cases}$$

which implies

$$\bar{\Omega}_1(3) = \bar{\Omega}_1(1) \quad \bar{\Omega}_1(4) = \bar{\Omega}_1(2)$$
$$\bar{F}_1(3) = \bar{F}_1(1) \quad \bar{F}_1(4) = \bar{F}_1(2)$$

by equations (47), (50)-(53), and the fact that $\tilde{\Omega}(1) = \tilde{\Omega}(3)$ and $\tilde{\Omega}(2) = \tilde{\Omega}(4)$ in our construction. It is important to realize $\bar{\Omega}_1(1), \bar{\Omega}_1(2), \bar{F}_1(1), \bar{F}_1(2)$ here are the same solutions to the benchmark model, namely, $\Omega_1(0), \Omega_1(1), F_1(0), F_1(1)$. From these calculations, there are

$$\bar{\Psi}_{\bar{\Omega}_1 \otimes \bar{\Omega}_1} = \begin{pmatrix} p_{00}\Omega_1(0) \otimes \Omega_1(0) & p_{10}\Omega_1(0) \otimes \Omega_1(0) & & \\ p_{01}\Omega_1(1) \otimes \Omega_1(1) & p_{11}\Omega_1(1) \otimes \Omega_1(1) & & \\ & & p_{00}\Omega_1(0) \otimes \Omega_1(0) & p_{10}\Omega_1(0) \otimes \Omega_1(0) \\ & & p_{01}\Omega_1(1) \otimes \Omega_1(1) & p_{11}\Omega_1(1) \otimes \Omega_1(1) \end{pmatrix}$$

$$\Psi_{\bar{F}_1 \otimes \bar{F}_1} = \begin{pmatrix} p_{00}F_1(0) \otimes F_1(0) & p_{01}F_1(0) \otimes F_1(0) & & \\ p_{10}F_1(1) \otimes F_1(1) & p_{11}F_1(1) \otimes F_1(1) & & \\ & & p_{00}F_1(0) \otimes F_1(0) & p_{01}F_1(0) \otimes F_1(0) \\ & & p_{10}F_1(1) \otimes F_1(1) & p_{11}F_1(1) \otimes F_1(1) \end{pmatrix}$$

The expressions above show that the 4×4 matrices share the same eigenvalues, hence, the same spectral radius, of their 2×2 diagonal blocks, which is identical to the benchmark model with only α switching.

Appendix F Estimating Noise Trading Behavior and Domestic Monetary Policy

We use h-period ahead forecast data of the CNY/USD spot rate and consider panel regression

$$E^i(e_{t+h}|e_t, c_t, i_t^*) = \mu_i + \nu_t + \gamma_1 i_t^* + \gamma_2 c_t + \gamma_3 e_t + \epsilon_{it}$$

Where $E^i(e_{t+h}|e_t, c_t, i_t^*)$ denotes institution i's (out of 95) h-day ahead forecast of RMB/USD exchange rate at time t (out of 24, four times a year) in natural log; i_t^*: Daily LIBOR rate (Annualized rate/240); c_t: RMB/USD parity rate in natural log; e_t: Spot RMB/USD rate in natural log. With α estimated from the parity rule over the full sample spanning from 2015 to 2021, we may identify up to $\frac{1}{N}\phi_c$ and $n\gamma$ by solving for $A^h_{[3,2]} = \gamma_2$ and $A^h_{[3,3]} = \gamma_3$.

We consider one-month ahead forecasts in the regression. To that end, we note that an average month has 23 trading days, and let $h = 23$.[35] Table 7 reports the regression results. The estimates of γ_2 and γ_3 are quite robust and accurate over different specifications. With $\alpha = 0.42$ We use it to solve from the model with both individual- and time-fixed effects that $n\gamma = 1.6$, $\frac{1}{N}\phi_c = 1.58$.

Table 7: Panel Regression for One-month-ahead CNY/USD Forecasts

Forecast	1	2	3
LIBOR	-1.465	-1.060	-0.991
	(0.259)	(0.243)	(0.299)
Parity	3.089	2.356	2.184
	(0.234)	(0.247)	(0.169)
Spot	-2.084	-1.558	-1.396
	(0.234)	(0.242)	(0.166)
Constant		*	
Individual FE			*
Time FE			*
R^2		0.441	0.441
Obs	1511	1511	1511

Note: The robust standard errors for fixed effect models are reported in parenthesis.

[35] The selection of h is important because A^h with an even h imposes unnecessary sign restrictions on γ_2, γ_3 that prevent a solution for $\frac{1}{N}\phi_c$ and $n\gamma$.