The Natural Yield of Capital: Quasi-Experimental Evidence from UK Leaseholds*

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Abstract

Every month, a fraction of UK properties with long duration leases are extended for 90 years. We use new data on thousands of these lease extension experiments from 2003 onwards, to estimate the "natural yield of capital" y^*, the dividend-price ratio of capital in the long run. We have three main results. First, y^* falls from 4.5% at the onset of the Great Recession to 2.3% in 2023. Second, using real-time monthly data, we find that the recent rise of 30-year interest rates has not yet had an impact on y^*. Third, trend changes in y^* are stronger in areas with more inelastic construction, consistent with a simple equilibrium model of capital supply.

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1 Introduction

This paper studies the natural yield of capital, y^*. The *yield* is the dividend-price ratio for capital, and the *natural* yield is the market's long run expectation. Alternatively, the natural yield is defined as $y^* = r_K^* - g^*$, where r_K^* is the long run discount rate on capital including risk premia, and g^* is the long run dividend growth rate. The natural yield of capital is an important variable as it measures the market's expectations about the long run structure of the real economy. For example, while current bond market yields have generally risen, the more important question is whether the natural yield on real capital has moved away from its "secular stagnation" era level (Blanchard, 2023).

Estimating y^* is difficult for two reasons. First, y^* is supposed to "look through" medium-run factors that might influence current and more immediate valuations. Conceptually, y^* represents an economy's anchored long-run expectations once transitory and medium run effects of factors such as interest rate changes and adjustment costs in the real-economy have played out. Filtering out the effect of these factors on asset prices is challenging. Second, the dividend of capital is hard to observe for many assets—for instance, the service flow of owner occupied housing is unobservable.

This paper proposes a strategy to identify the natural yield of capital in real time for housing, which is an important class of capital, using a natural experiment from the UK housing market. Most apartments in U.K. are sold as "leaseholds"—long duration leases issued by the "freehold" owners of the property, which can be bought and sold. The Housing and Urban Development Act of 1993 granted leaseholder owners the right to extend the lease by 90 years, conditional on paying a negotiated payment to the freeholder. The act thus set in motion a natural experiment where every month a fraction of U.K. flat leaseholds get extended by 90 years. Leaseholds typically extend when they have long lease lengths remaining, in excess of 70 years.

We develop a difference-in-differences estimator to measure y^*. We study newly available administrative data on leasehold transactions between 2003 and the present, which provides tens of thousands of 90 year lease extensions with market transactions before and after the extension. We measure price gains from lease extensions, and how these gains vary with the duration of the lease before extension. Then we embed these statistics into a non-linear estimator based on a Gordon growth model of asset prices, in order to infer y^*.

Our data are available in real time, and provide roughly 900 new lease extensions per month. We also provide a method that allows the public to identify lease extensions using freely available public data. Hence our y^* estimate can be replicated and extended in real time by anyone interested going forward.

The estimator measures y^* over time with precision and has two additional advantages. First, by comparing the market value of the *same* property leasehold before and after lease extension, we are able control for unobserved characteristics that affect property values, such as the hard-to-measure service flow of housing. Second, the price increase after extension identifies the natural yield of housing, namely, a long-run valuation that "differences out" any shorter term volatility. Intuitively, the price of the leasehold before extension capitalizes discounted flows over the shorter term, and by differencing out this term, our estimate focuses only on the natural yield, which represents the very long run.

Our identification assumption is "parallel trends": the service flow of housing must grow similarly for extended properties and a suitably chosen control group. For each extending property, the control group is non-extending properties in the same neighborhood as the extender, of a similar duration before extension. We support the identification assumption in four ways. First, there are no "pre-trends", meaning prices of extending properties evolve similarly to the control before extension. Second, the treatment and control group are "balanced" on a rich set of hedonic characteristics. Third, market rents and hedonic characteristics evolve similarly for extenders and the control group. Fourth, the estimator is not sensitive to controlling for observed heterogeneity, suggesting bias from unobserved characteristics is small (Altonji et al., 2005; Oster, 2019).

Our estimator requires a measure of value of lease extensions that is extracted by freeholders, the owners of the property, in order to price the option value of extension. Our baseline assumption is that freeholders extract all of the value, consistent with institutional features of the British property market. However, we also able to directly estimate the value extracted by freeholders using a bunching estimator, and find that our estimates of y^* change little.

We find that extending a lease by 90 years increases property prices by roughly 20% compared with the control group of non-extended leaseholds in same location. Moreover lower duration properties enjoy greater price gains upon extension, consistent with our simple asset pricing model. Embedding the asset pricing model into a non-linear estimator converts these statistics into a single natural yield estimate.[1]

A key advantage of using quasi-experimental variation in lease extension within the same property is that we can estimate the dynamics of y^* with sufficient precision at quarterly or even monthly frequency in recent years. Our first main finding is that the natural yield on housing falls from about 4.5% on the eve of the Great Recession in 2006 to 2.5% in 2023. The magnitude of this trend decline is large, suggesting almost a doubling of the natural

[1]We also explore whether the illiquidity of short duration properties affects our estimates, because obtaining mortgages may be more difficult, and find small effects.

price-rent ratio.

The decline in natural yield thus broadly follows the decline in government bond yields post Great Recession. In principle, this did not have to be the case. For example, bond yields could have fallen due to factors such as quantitative easing that are specific to safe assets in segmented markets (e.g. Krishnamurthy and Vissing-Jorgensen, 2012; Caballero and Farhi, 2018; Jiang et al., 2023). Alternatively, as we show with a simple model, if the supply of capital is sufficiently elastic, then natural yields do not fall even when long-term discount rates falls. Instead, greater capital growth endogenously reduces the dividend growth rate. Our dynamic estimates tend to go against such hypotheses.

Our second main empirical result is about the real-time dynamics of y^* from 2022 through to the present day. We are able to estimate y^* at monthly frequency with precision in real-time. We find that y^* has remained low and stable thus far, despite safe asset yields having risen substantially over the last twelve months. Two scenarios are possible in the future: safe asset yields might fall back towards y^*, or, y^* starts to rise once market expects rising yields to be durable enough.

Understanding how such scenarios play out in real-time should be of interest to policy makers and economists alike. For example, possible upward movement in y^* represents duration risk embedded throughout the real economy that can adversely effect households, banks and hence the entire economy. The very low current level of y^*, combined with the large fall we document, suggests that the embedded duration risk may be huge and needs to be followed closely. Similarly, whether y^* ultimately rises will also inform the ongoing debate about whether economy is finally getting out of the long period of low yields and "secular stagnation" (e.g. Blanchard, 2023).

Our third main result relates the trend dynamics of y^* for UK housing to the supply elasticity of assets. As already mentioned, inelastic capital supply should result in a larger fall in y^* as long-run rates decline. We test this hypothesis by estimating the decline in y^* for about 100 local areas across the UK, and regressing it on known measures of local housing supply elasticity in the literature (Hilber and Vermeulen, 2016; Bahaj et al., 2020). We find that the trend decline in y^* is significantly greater in inelastic areas. In particular, areas with the supply elasticities in the top quartile experienced trend declines in y^* that were more than 1 percentage point greater than areas in the bottom quartile.

Our result suggests that the dynamics of y^* contains valuable information about the supply elasticity of capital. We formalize this argument using a simple and standard equilibrium model of capital formation with Tobin's Q adjustment costs. In the model, an exogenous decline in the natural rate of interest on safe assets, r^*, only leads to a fall in the natural yield of capital y^* if the supply of capital is not elastic. Otherwise a fall in r^* leads to an

expansion in the supply of capital, and this supply-side response leads to a slower growth in the service flow of capital. Formally, we define $y^* = r^* + \zeta^* - g^*$, where $\zeta^* \equiv r^*_K - r^*$ is the risk premium on capital. If supply of capital is elastic, then an exogenous fall in r^* endogenously leads to an equivalent fall in the growth rate of service flows g^*, due to higher investment, leaving y^* unchanged. Therefore the supply elasticity of capital affects the pass through of falling r^* into the real economy.

Related Literature. Our paper is most closely related to the seminal paper of Giglio, Maggiori and Stroebel (2015), who were the first to observe that because UK apartments vary in duration, they are particularly well suited to estimating the natural housing yield.[2] Giglio et. al. make a cross-sectional comparison of properties with different duration, and control for a rich set of hedonic characteristics. Our paper studies the dynamics of the natural housing yield, and in doing so builds on their work in two ways. First, we use a quasi-experimental design based on lease extensions, which allows us to use variation in leasehold duration and price *within the same property*. We show that within-property variation is critical to accurately estimating y^*, due to unobserved heterogeneity in the service flow of housing. This enables us to estimate the dynamics of y^* reliably in real time at quarterly frequency. Second, the possibility of lease extension creates option value that affects the price of UK leaseholds; we develop a bunching estimator to measure this option value.

Our work relates to papers measuring the natural rate of interest on safe assets, r^*, such as Laubach and Williams (2003) and Holston, Laubach and Williams (2017).[3] These papers estimate the natural rate of interest on long term government bonds, using time series methods combined with structural models. Our approach has two advantages. First, our quasi-experimental approach dispenses with the identification assumptions of the time series literature, leading to concrete advantages. For instance, the time series approach broke down with the start of pandemic due to model misspecification issues, and has standard errors that are an order of magnitude larger than our method. Second we measure the yield on capital $y^* = r^*_K - g^*$, instead of the yield on safe assets. Our measure encodes useful information not only about discount rates, but also information about the supply side of the economy via g^*.

Our results linking natural yields to supply-side elasticities relates to the large literature that estimates the pass through of r^* into the real economy. Several papers study the pass through into returns on physical capital (Farhi and Gourio, 2018; Eggertsson, Robbins and Wold, 2021; Caballero, Farhi and Gourinchas, 2016; Reis, 2022) whereas several other papers

[2] See also Giglio, Maggiori and Stroebel (2016) and Bracke, Pinchbeck and Wyatt (2018).

[3] See also Rachel and Smith (2017), Del Negro et al. (2017), Rachel and Summers (2019) and Del Negro et al. (2019) amongst others.

consider pass through into housing returns (Rognlie, 2015; Miles and Monro, 2019; Amaral, Dohmen, Kohl and Schularick, 2023). There is a debate about how much r^* has affected returns to capital, in part because common measures of the return on physical capital disagree (Reis, 2022; Vissing-Jorgensen, 2022). All these papers study the secular fall in r^* before 2020. Our contribution is threefold: (i) We offer a measure of the yield on capital with a tight conceptual mapping to r^*. (ii) Our model shows that the pass through of r^* to natural yields hinges on the supply elasticity of capital. (iii) We document a growing disconnect between the natural yield of capital and safe asset yields after 2020.

Outline. The rest of the paper is structured as follows. Section 2 defines the natural yield of capital. Section 3 describes the data. Section 4 presents our empirical methodology. Section 5 presents our estimates of lease extension price changes and estimates of y^*. Section 6 argues that y^* contains useful information about the supply side, by combining estimates with a simple equilibrium model. Section 7 measures the option value from lease extension and Section 8 concludes.

2 Introducing The Natural Yield of Capital

The yield of capital is its dividend-price ratio. The *natural* yield is markets' expectations of yields in the long run, after medium-run forces have subsided. Though this object is important, it is also difficult to measure.

Formally, consider the price of a unit of capital. The price at time t, P_t is given by the present value of its dividend, R_t, as

$$P_t = \int_0^\infty R_{t+S} \left(e^{-\int_0^S r_K(s)ds} \right) dS = R_t \int_0^\infty e^{-\int_0^S y(s)ds} dS. \tag{1}$$

In this equation, $r_K(s)$ is the discount rate, which discounts the present value of dividends at a rate including risk premia, all in real terms. The second equality defines $g(s)$ as dividend growth, and rewrites in terms of the current dividend R_t. We define $y(s) \equiv r_K(s) - g(s)$.[4]

We are interested in the yield of capital in the long run, that is, the long run dividend-price ratio. We assume that this "natural yield" exists, in which case it is given by

$$\frac{R_{t+\infty}}{P_{t+\infty}} \equiv \lim_{s \to \infty} y(s) \equiv y^* = r_K^* - g^*.$$

The final equality acknowledges that the natural yield is also the difference between the

[4]For simplicity this derivation omits a "rational bubble" term.

discount rate on capital and dividend growth in the long run. Crucially, y^* is independent of short run, transient shocks to yields. y^* also provides valuable information on supply side factors affecting the long-run equilibrium of the economy via g^*, as we will discuss in Section 6.

There are two difficulties in estimating y^*. First, current asset prices—even for very long duration assets—depend not only on y^* but also on shorter run yields $y(s)$, which include transitory factors such as monetary policy, credit booms, short run bubbles and adjustment costs. The reason is that, as equation (1) shows, the price of a long duration asset depends on the integral of short and longer term yields. Therefore the current dividend-price ratio, R_t/P_t, is affected by these transitory shocks and may greatly differ from the natural ratio of service flows to prices. As such the key challenge to estimating natural yields is to is to "difference out" shorter run yields.

Second, the dividend of capital R_t is often difficult to observe. For instance, the service flow of owner-occupied housing cannot be observed from market transactions, and must be imputed. The next section will describe a unique natural experiment from the UK housing market. This setting will let us overcome both challenges in order to estimate natural yields for an important class of capital.

3 Data & Lease Extension Details

This section introduces the setting—lease extensions for long duration leasehold properties in the United Kingdom—and explains the datasets we will use for our analysis, including analysis that can be done in real time with public data only.

3.1 Data

We use five data sets for analysis in this paper. Our main analysis will use newly available administrative data on lease extensions. The remaining datasets that are needed for the core analysis of the paper are publicly available and updated monthly by His Majesty's Land Registry. In the Appendix, we will provide additional details on how to access the data and replicate our real-time estimates, including providing a method to identify a subset of lease extensions using publicly available data only. This is an important feature of our method, since it allows for real-time updates and replication of our results. We now describe our five data sets.

(1) **Land Registry Transaction Data**: We obtain publicly available data on all property transactions registered in England and Wales between 1995 and April 2023 from the

Land Registry. The data set includes the exact date, price and address for each transaction. Properties are also subdivided into two categories: freeholds and leaseholds. Freeholds are a perpetual claim to the ownership of a property. Leaseholds are long duration leases to the property that can be bought and sold, and which typically last for many decades at origination. As we will discuss in detail below, leasehold durations are periodically extended, after negotiation with the freeholder. The distinction between freeholds and leaseholds dates to medieval England, during which permanent ownership of land and property, known as "freehold" ownership, was available only to feudal nobility.[5] During this time, leasehold estates were granted to serfs who would work the land for a set period of time and in exchange would pay a portion of the harvest to the freehold landowner. During the 20^{th} century, cash-poor landowners began to issue long leaseholds of 99 and 125 years, providing immediate liquidity without giving up ownership of the underlying land. To this day, leaseholds are very common in England and Wales, comprising approximately 5% of houses and 97% of flats.

(2) **Land Registry Lease Data**: Data on the length of contemporary lease terms, which vary significantly across leaseholds, are provided in a separate Land Registry dataset, also publicly available. The Land Registry lease term data set includes the property address associated to the lease, the term length and origination date of the lease, as well as the date in which the lease was registered with the Land Registry. A majority of leases were originated and registered after the 1950s. Starting in late 2003, lease registration became mandatory, so we capture all registered leases after this period. We will start our analysis in 2003, after mandatory registration.

Each entry in the lease term data set provides the length of the lease term, its registration date, and the start date of the lease. The Land Registry does not provide match keys to merge the transaction and lease term datasets, so we conduct a fuzzy merge based on provided addresses. In the UK, every property can be uniquely identified by three items: the first address number, the second address number and the 6-digit postcode. Therefore, we merge according to the following procedure: First, we conduct a perfect merge using address as our merge key. This methods accounts for 94% of our matches. Second, we conduct a fuzzy merge on all observations not matched by step 1. The fuzzy merge matches observations in which (1) all numeric elements of the address are the same, (2) all single letters (e.g. A, B, C, etc.) are the same, (3) a select set of identifying terms (e.g. first floor, second floor, basement, etc.) are the same and (4) the postcode is the same in both addresses. For example, this allows for the property "3 Swan Court 59-61 TW13 6PE" in the transaction dataset to be matched to "Flat 3 Swan Court 59-61 Swan Road Feltham TW13 6PE" in the

[5]The first known use of the term "freeholder" is in the Domesday Book published in 1086 under the reign of William the Conqueror.

lease term dataset. This method accounts for 6% of our matches.

We exclude 0.02% of our transactions, which have implied negative lease terms at the time of transaction. We also exclude 0.6% of properties which are sold both as a leasehold and a freehold within our sample. These include particular cases where both the leasehold and freehold title are sold simultaneously, as it is unclear how to interpret these properties.

The Land Registry classifies properties into five types: flats, detached houses, semi-detached houses, terraced houses, and others. The three categories of houses are for the most part freehold properties. Even when they are leaseholds, houses tend to have very long lease terms relative to flats, with median remaining lease term over 800. Since leaseholds are mostly flats, we will drop non-flats for the bulk of our analysis.

(3) **Land Registry Closed Lease Files**: We have obtained all closed leasehold titles from the Land Registry, which are not publicly available before September 2021 and have not previously been used in academic research to our knowledge. These files include all extended leases in England and Wales for which there is a recorded transaction before extension. In Appendix Section A.3 we will provide a method to identify a subset of lease extensions, using only publicly available data.

(4) **Rightmove Hedonics Data**: We obtain data on housing characteristics and rental prices from Rightmove, Inc. spanning 2006 to the present, which include the number of bedrooms, number of bathrooms, number of living rooms, floor area, property age, parking type, heating type and property condition (rated as Good, Average, or Poor) of listed properties. It also includes rents for rented properties.

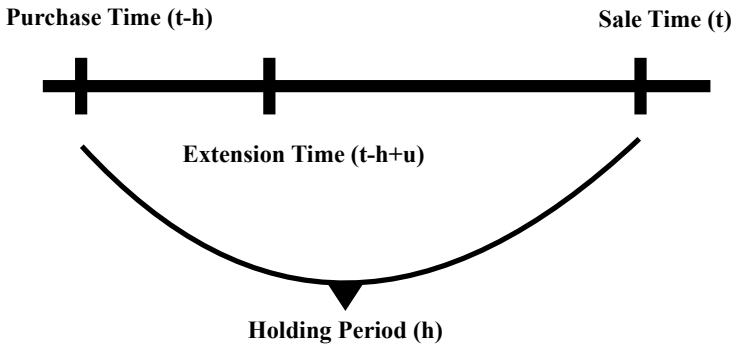
(5) **Zoopla Hedonics Data**: We supplement this with data from Zoopla, Inc. which is provided by the Urban Big Data Centre (UBDC). This data set also provides number of bedrooms and bathrooms and rental yields. Additionally, it includes the number of floors and receptions of the property. We are able to match approximately 80% of transactions to the Rightmove and Zoopla listing data. Rental data is available for about 40% of properties.

3.2 Lease Extension Details

This section provides additional details about leaseholds in England and Wales and the circumstances under which the duration of leases can be extended. As we described in the previous section, leaseholds are properties with very long but finite leases, moreover these leases can be bought and sold. The lease length at the time it was issued is denoted its "initial lease term" and the lease length it has at any future point in time is denoted its "remaining lease term." The distribution of leases can be divided into two groups; about 70% of leasehold flats in our sample are *short leaseholds* with remaining terms of 250 years or

less and the other 30% are *long leaseholds* with remaining terms of 700 years or more. There are practically no properties with remaining terms between 250 and 700. This distribution is illustrated in Figure A.1. The most common initial terms for short leaseholds are for 99 and 125 years, which account for 77% of short leaseholds. The most common initial term for long leaseholds is 999 years, which account for 96% of all long leaseholds.

Figure 1: Diagram of Extension Time



The figure is a diagrammatic representation of the notation we will use in the paper. We say a property is purchased at time $t-h$, sold at time t and held for an amount h of years. We say that a property extends at time $t-h+u$, where $u < h$.

Beginning in 1993, the Leasehold Reform, Housing and Urban Development Act (1993 Act) granted flat leasehold owners the right "to acquire a new lease" 90 years longer than the original lease, conditional on a one-off negotiated payment to the freeholder. This negotiation is potentially costly, since both the freeholder and the leaseholder may hire qualified surveyors in order to assess the value of the property, and the negotiation can be lengthy. We denote these cases as *lease extensions*. This option is particularly relevant for leasehold owners of short leaseholds for whom lease expiration may be more of a concern than for owners of long leaseholds.

Although the 1993 Act legally provides the option to extend, almost all extensions are negotiated out of court by leaseholders and their freehold landlords. Despite this, the court-recommended amount of 90 years is the most common extension length (accounting for about 30% of all extensions). A fraction of extensions, however, are for approximately 900 years, which effectively convert short leaseholds into long leaseholds.

We will now introduce some notation that we will use throughout the paper to refer to extended properties in our sample, which we show in a diagram in Figure 1. Consider a lease that transacts twice within the Land Registry Transaction Data Set. We say that a property was purchased at time $t-h$ and sold at time t, where h is the amount of time

between purchase and sale. We are interested in properties which were extended at some time $t - h + u$ where $u < h$.

We denote the lease term to maturity, henceforth referred to interchangeably as lease duration, at purchase time as $T + h$ and its duration at sale time as $T + 90$ (notice that its duration would have been T at sale had the lease failed to be extended). We denote the price of a property i of duration T at time t by P_{it}^{T}. The transacted prices before and after lease extension, and the lease duration before extension, will be the key inputs into our estimation.[6]

As we see in Figure A.2, properties with a very short holding period, h, are very likely to be "flippers" who purchase and sell properties with the explicit intention of making a quick profit and therefore behave differently from other owners. Unless otherwise specified, we exclude properties with $h \leq 1$ year from our analysis.

Table 1: Number of Extensions

Extension Amount	90	700+	Other	Total
2003-2005	601	1,867	1,973	4,441
2006-2010	3,094	6,751	4,987	14,832
2011-2015	10,713	15,296	10,110	36,119
2016-2020	15,676	18,708	11,461	45,845
2021-2023	9,496	10,457	4,722	24,675
Total	39,580	53,079	33,253	125,912

The table reports the number of extended leases that have transaction data for each time period. The first column includes 90 year extensions, the next column includes 700+ year extensions, and the third column includes others, which are almost all non-90 under 200 year extensions.

In Table 1 we present the distribution of extensions for different amounts over time. In total, there are 125,899 lease extensions in our main sample, which is 5% of flats.

How common are these lease extensions? In Figure A.3 we show the hazard rate of extension, defined as the conditional probability a property will extend given how long its remaining term to maturity is. We can see that almost no properties extended with more than 90 years remaining. After a property hits 80 years remaining, its extension probability jumps to a probability of extension of about 5%, and then slowly falls back to 2% or so. We elaborate more on the particulars of this hazard rate in Appendix A.7.

For most of our analysis, we will focus on leases that extend by the typical amount of 90 years. For these leases, the median duration before extension is large, at around 70 years,

[6] We do not observe the payment extension paid by leaseholders to freeholders.

as shown in Figure A.5. The median time between transactions is 10 years, with 7 years between purchase and lease extension (see Figure A.6 and Figure A.7). In Appendix A.1, we provide additional summary statistics about lease extensions, including a heatmap of the extension rate by region; and the distribution of transaction dates and extension amounts. We also show that leasehold owners are broadly representative of the overall UK housing market in terms of owner and mortgage characteristics and price-to-rent ratio business cycles in Table A.1 and Figure A.8.

4 Empirical Methodology

This section explains how to use the price gain from lease extension to estimate the natural yield of capital, for the major asset class of UK property. We also explain how we select the control group for our estimator, and present evidence in support of our identification assumption.

4.1 Pricing Leaseholds

We start by deriving the price of a leasehold. The price of a leasehold P_t^T, with T years until expiration, and an option to extend by 90 years on expiration, is

$$P_t^T = R_t \int_0^T e^{-\int_0^S y(s)ds} dS + \max\left[0, (1-\alpha)\, R_t \int_T^{T+90} e^{-\int_0^S y(s)dS} dS + ...\right] \tag{2}$$

This equation starts with the asset pricing identity of equation (1), since the first term is the present value of service flows from housing over the first T periods before the lease expires. The second term represents the option value of additional extensions. $(1-\alpha)$ is the share of the price gain from extension, after deducting the negotiated payment to the freeholder and various costs that this negotiation entails. These terms multiply the present value of service flows from the lease, over the 90 year period after the extension. The ellipsis refers to the value of future extensions after $T+90$, which have a similar structure. The max operator acknowledges that option value is non-negative—instead of extending the lease, the leaseholder can choose not to extend and receives zero payoff.

This equation clarifies that the option value of lease extension raises the value of a leasehold. Consider two cases. First, suppose $\alpha = 0$. Then, the leaseholder receives the entire value of a lease extension. Provided that the value of an extend lease is positive, the leaseholder will always choose to extend—the option of lease extension is always "in the money". Then $P_t^T = R_t \int_0^\infty e^{-\int_0^S y(s)ds} dS$, meaning the price of a finite duration leasehold is

the same as the price of an equivalent, but infinite duration asset. Since the property always costlessly exercises its option to extend, then the effective duration of the asset is infinite. A second instructive case is $\alpha = 1$. In this case, the leaseholder receives none of the value from extension, and the price of a leasehold is $P_t^T = R_t \int_0^T e^{-\int_0^S y(s)ds} dS$. This price is the same as an asset with a duration of exactly T periods. The service flows after T have no value to the leaseholder, since they go to the freeholder. With intermediate values $\alpha \in (0, 1)$, the price of a T duration leasehold is between the duration T price, and the infinite duration price.

In the main analysis, we will assume that $\alpha = 1$, so that the value of extension goes entirely to freeholders and not leaseholders. Therefore there is no option value from lease extension. This assumption is appropriate given the institutional features of the UK property market. As we discuss at length in Section 7, the law recommends that leaseholders pay freeholders the entire value of lease extensions. In Section 7 we will also consider a bunching estimator, which identifies α using discontinuities in legally mandated lease extension payments when $T = 80$. Our estimates of natural yields change by little when we estimate α directly.

4.2 A Difference-in-Differences Estimator of y^*

Next, we embed the formula for the lease extension price into a difference-in-differences estimator, in order to identify y^* for UK housing. We will study a difference-in-differences estimator Δ_{it} for the price change after lease extension:

$$\Delta_{it} \equiv \left[\log P_{it}^{T+90} - \log P_{i,t-h}^{T+h}\right] - \left[\log P_{jt}^{T} - \log P_{j,t-h}^{T+h}\right].$$

In this equation, $\log P_{it}^{T+90} - \log P_{i,t-h}^{T+h}$ is the price change for a property i bought h periods previously, which extends by 90 years from a duration of T years remaining, at time t. Property j is a suitably chosen control property, bought and sold in the same periods, with the same duration as property i prior to extension. Substituting the formula (2) for the price of a leasehold into the difference-in-differences estimator implies

$$\Delta_{it} = \log\left(\int_0^{T+90} e^{-\int_0^S y(s)ds} dS\right) - \log\left(\int_0^{T} e^{-\int_0^S y(s)ds} dS\right) + \Delta_{t,t-h}\left(\log R_{it} - \log R_{jt}\right).$$

In this equation, the first two terms represent discounting of the extended lease versus its control property. The final term is the difference between the growth rate of the service flow of housing, for the treatment versus the control group.

The identification assumption of our estimator is a form of "parallel trends". The growth in service flows for the treatment and control properties, before versus after extension, must

be the same. If so, then the final term from equation (3) vanishes. Helpfully, in this case the difference-in-differences estimator no longer depends on the holding period h, meaning the estimator automatically corrects for the size of the holding period.[7]

Finally, in order to implement the estimator we parameterize the shape of the yield curve $y(s)$. We make the simple assumption that $y(s) = y^*$, so that the yield curve is horizontal. Shortly, we will see that this assumption accurately prices leaseholds in our quasi-experiment, even when there is significant variation in the slope of the yield curve at short horizons. The reason is that when T is large, the difference-in-differences estimator is primarily identified from long duration flows between T and $T+90$. With this parameterization and the parallel trends assumption, we arrive at the final form of our estimator

$$\Delta_{it} = \log\left(1 - e^{-y^*(T+90)}\right) - \log\left(1 - e^{-y^*T}\right). \tag{3}$$

4.3 Advantages of the Estimator

Our estimator has two advantages which allow us to estimate the natural yield of capital. First, provided that that parallel trends assumption holds, the estimator differences out the service flow of housing. Terms related to service flow do not appear in the value of the difference-in-differences estimator (3). The service flow includes taxes, depreciation, as well as the utility from consuming housing. This property of the estimator is appealing because the service flow is difficult to measure directly, especially for owner occupied housing. Moreover there may well be significant unobserved heterogeneity in this service flow across properties.

The second advantage of our estimator is that it differences out the effect of short term yields on asset prices, in order to estimate the natural yield. Intuitively, suppose that the duration before extension T is large. The price before extension capitalizes flows over the first T periods. The price after extension capitalizes flows over $T + 90$ periods. Therefore the price change at extension identifies the price of cashflows *after* T, only in the far future. In practice, this argument is hard to demonstrate analytically.[8]

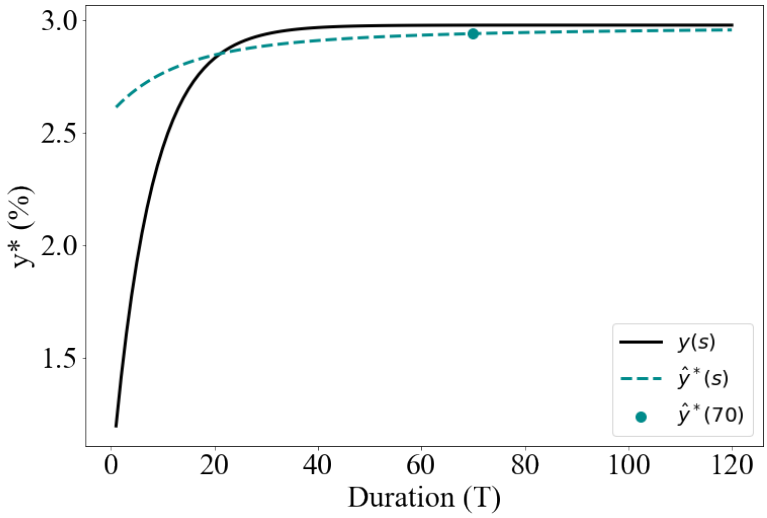
We illustrate the success of our estimator in differencing out short term yields numerically. In general, the success of our estimator depend on the shape of the yield curve before it converges to the natural rate y^*, and on the duration of the property before it extends. Numerically, our estimator successfully estimates y^* under an empirically reasonable yield curve, and when it is applied to long duration properties. The black solid line in Figure 2 presents one possible parameterization of the forward yield $y(s)$, where the forward yield

[7] Appendix Figure A.11 shows that estimates of Δ_{it} are uncorrelated with h.

[8] In effect, we are studying the price of a positive coupon bond, which is not analytically tractable.

curve $y(s)$ flattens out to y^* for $s \geq 40$ years, with y^* equal to 3.0 pp.[9] Given our pa-

Figure 2: A Parameterization of the Housing Yield Curve



The black line presents one parameterization of the forward yield curve, $y(T)$, which is chosen so that its shape matches the forward curve implied by the UK 3-month LIBOR, the 10 year gilts and the 30 year gilts. $\hat{y}^*(T)$ is our estimator of y^* for each T, described below.

rameterization of $y(s)$, we can solve for $\log P_{it}^{T+90} - \log P_{it}^{T}$ for all T. Then, for each T, we can solve for our estimator of y^* as a function of T numerically. The resulting values of our estimator, which we term $\hat{y}^*(T)$, are plotted in blue in Figure 2. Our estimator $\hat{y}^*(T)$ closely approximates the true natural yield y^* for durations T after which the forward curve has flattened. We also plot the point estimate of $\hat{y}^*$ at $T = 70$, which is approximately the median duration of leaseholds at extension.

Our estimator produces tight estimates for a wide range of yield curves. In Figure 3a we present a time-varying yield curve for which the short-end fluctuates tremendously over time but the long end (y^*) is constant. Then, the solid line in Figure 3b shows how our estimator $\hat{y}^*(70)$ reacts to changes in the short-end of the yield curve. The points corresponding to each instance of the yield curve in Figure 3a are labelled accordingly. Our estimator is relatively stable despite the fluctuations in the short end of the yield curve, and remains within 0.1% of y^*.[10] This logic suggests that our estimator can successfully estimate the dynamics of y^*,

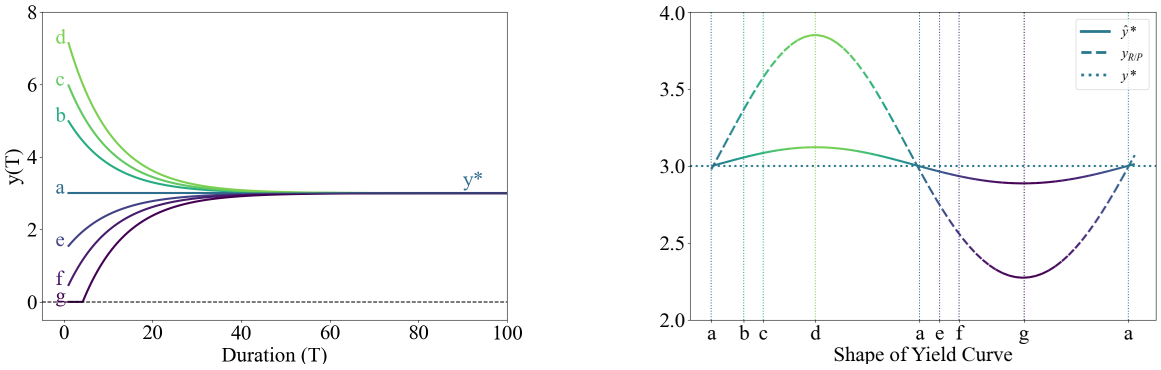
[9] We choose a flexible functional form $y(s) = \beta_1 - \beta_2 \cdot \beta_3^{-\beta_4(s-\beta_5)}$ and estimate the β parameters such that $y(0)$ is equal to the 3-month London Interbank Offered Rate (LIBOR), $\rho(10)$ is equal to the 10-year gilt yield, and the average of $y(s)$ for $10 \leq s \leq 30$ is equal to the 10 Year 20 Year gilt forward yield. For all the bond yields, we use the mean yield for our sample period. We present other possible parameterizations of $y(s)$ in Appendix A.4.

[10] In Appendix A.4 we show that we can use $\hat{y}^*(s)$ to bound the approximation error of our estimator, and therefore get a lower and upper bound for y^*.

even in the presence of volatile shocks to short term rates.

The reason why our estimator is able to provide a close approximation of y^* is because T is large, which effectively differences out most of the yield curve $y(s)$ for $s < T$. As T becomes smaller, the effect of the short-end on $\hat{y}^*(T)$ increases, meaning estimates of y^* become increasingly biased. To see this, we can consider an alternative estimator: the rent-to-price ratio of a freehold property, $y_{R/P} \equiv \frac{R_{it}}{P_{it}^{\infty}}$. Like our estimator, the price-to-rent estimator cancels out the flow value of housing; it does not, however, difference out the short-end of the yield curve and is therefore far more susceptible to changes in short-term forward rates. The dashed line in Figure 3b indicates the value of $y_{R/P}$, as the low-end of the yield curve shifts. Changes in the short-end of the yield curve affect $y_{R/P}$ by almost an order of magnitude more than they affect $\hat{y}^*(70)$. These results demonstrate that to effectively capture y^*, we must take the difference between *two* long duration assets; one does not suffice.

Figure 3: "Differencing Out" the Short End



(a) $y(s)$ **(b)** $\hat{y}^*$ and $y_{R/P}$

The figure illustrates the effect of fluctuations at the low-end of the yield curve on $\hat{y}^*$ and the rent-to-price rate, $y_{R/P}$. Panel (a) presents several instances of the yield curve, $y(s)$ over time. Panel (b) indicates the estimates $\hat{y}^*$ and $y_{R/P}$ at each of these instances. The $y_{R/P}$ is estimated such that $P_{it}^{\infty}/R_{it} = \int_0^{\infty} e^{-\int_0^S y(s)ds} dS \equiv \frac{1}{y_{R/P}}$.

Our approach is agnostic about the source of volatility in short run yields. Short run yields may fluctuate due to cyclical movements in housing risk premia, safe interest rates or liquidity conditions. Bubbles of the form studied by Harrison and Kreps (1978) also manifest in short run yields, provided that these bubbles disproportionately affect short duration valuations. Regardless, our approach differences out this short run volatility in order to estimate the natural housing yield. As an additional, third, advantage, our estimator also differences out any variation associated with "rational bubbles".[11]

[11] Our approach is related to how forward yields are calculated on financial assets such as zero coupon

4.4 Implementing the Estimator and Selecting a Control Group

We now describe how to implement our estimator via nonlinear least squares, as well as the procedure to identify "control" properties. We also present evidence supporting our assumption of parallel trends.[12]

According to equation (3), for each individual property i our difference-in-differences estimator is

$$\Delta_{it} = \log\left(1 - e^{-y_t^*(T_{it}+90)}\right) - \log\left(1 - e^{-y_t^* T_{it}}\right). \tag{4}$$

Here, we have generalized the expression of the estimator to allow a time varying natural yield, and to acknowledge that the duration of the property before extension, T_{it}, can vary. Equation 4 shows that we can estimate y_t^* by nonlinear least squares. The estimator is valid at any point in time, hence we can estimate the dynamics of y_t^*. Two statistics inform y_t^* in the estimator. First, the difference-in-difference Δ_{it} can be calculated for every property i, as the difference in price growth between the extending property and its control. Second, the covariance between Δ_{it} and the duration before extension T_{it} also helps to identify y^*.

We select a control group separately for each extending property, from neighboring properties of a similar duration that did not extend. Let property p be a flat which was purchased at time $t-h$, sold at time t, and extended for 90 years at some time $t-h < t-h+u \leq t$. Suppose p has duration $T+h$ at purchase and duration $T+90$ at sale. The control pool for property p is the set, $Q^{(p)} = \{q : \text{Haversine Distance}(p,q) \leq x\text{km}\}$ where $x \in \{0.1, 0.5, 1, 5, 10, 20\}$, using the traditional Haversine Distance formula based on latitude and longitude coordinates.[13] In general, we choose x to be the smallest possible value such that both the control sets described below are non-empty. The controls for the purchase transaction are $Q_{t-h}^{(p)} = \left\{q \in Q_p : t_q - h_q = t-h \text{ and } \left|\frac{(T+h)-(T_q+h_q)}{T+h}\right| \leq 10\%\right\}$ where $t_q - h_q$ is the purchase date and $T_q + h_q$ is the duration at purchase for property q. The controls for the sale transaction are $Q_t^{(p)} = \left\{q \in Q_p : t_q = t \text{ and } \left|\frac{T-T_q}{T}\right| \leq 10\%\right\}$ where t_q is the sale date and T_q is the sale duration for property q. Note that we choose controls that have a lease term close to what the extended property p would have had at sale had it had *not* been extended.

Then, the control purchase price for extended property p is given by $P_{p,t-h} = \text{mean}\{P_{q,t_q-h_q} : q \in Q_{t-h}^{(p)}\}$ and the control sale price is given by $P_{p,t} = \text{mean}\{P_{q,t_q} : q \in Q_t^{(p)}\}$. Under this

bonds, but is available at far longer horizons than are normally available for financial assets.

[12] We are careful to select control properties that are not treated, to avoid the "forbidden comparisons" problem of Borusyak et al. (2021).

[13] The Haversine formula is given by $d(p,q) = 2r \arcsin\left(\sqrt{\sin^2\left(\frac{\phi_q-\phi_p}{2}\right) + \cos\phi_p \cdot \cos\phi_q \cdot \sin^2\left(\frac{\lambda_q-\lambda_p}{2}\right)}\right)$ where (ϕ_p, λ_p) and (ϕ_q, λ_q) are latitude-longitude pairs for points p and q respectively and r is the radius of the Earth.

methodology, we are able to identify controls for 125,151 of our 125,899 lease extension experiments. This serves as our primary control measure.

Our identification assumption is parallel trends: growth in the service flow of housing should not differ for extending properties and controls. We provide two tests in support of this assumption. First, our treatment and control groups are close to being balanced on observable characteristics. We use the property characteristics data provided by Rightmove and Zoopla.[14] We regress our hedonic characteristics on Local Authority x 5-Year Duration Bin x Year fixed effects and plotting the density distribution of the residuals in Figure 4. These residual plots visually confirm how similar the amenities distribution is between extensions and controls, with only minor differences between the two. The results of regressing the same hedonic characteristics on an extension dummy are also presented in Panel A of Table 2. Flats that have been extended at some point tend to have slightly better amenities than neighbouring flats of similar lease length. The differences in magnitude are small, however. For instance, the mean number of bedrooms, after controlling for geographical region and lease length, in an extended property is 1.82, as opposed to 1.74 in a property that has not been extended. Average floor area is 70.2 vs 66.2 square meters for extended and non-extended properties, respectively. As a whole, rental prices, which ought to be representative of the aggregate quality of the property, are about 3% higher in extended properties.

As a second test of parallel trends, we show that extending properties are no more likely to renovate than controls; nor do rents grow by more for extensions. As such, the service flow of housing seems to behave similarly for control and treated groups, in support of the identification assumption. We study Rightmove and Zoopla data for properties that have listings corresponding to two separate transactions. For these properties we estimate the following specification,

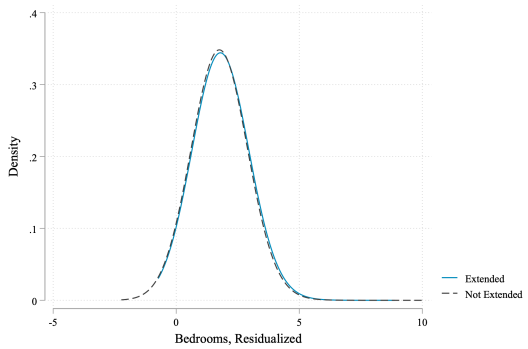
$$\Delta_h \text{Hedonic Control}_{it} = \alpha + \beta \times \mathbb{1}\text{Extension}_{it} + \Gamma_{it} + \epsilon_{it} \tag{5}$$

where Γ_{it} are Local Authority$_i$ $\times$ 5-Year Duration Bin$_{it}$ $\times$ Purchase Year $\times$ Sale Year fixed effects where the year fixed effects correspond to the year when the hedonic characteristics or rental prices were recorded. The regression results are presented in Panel B of Table 2. We find no significant evidence that extended properties renovate at a different rate than other properties. Importantly, Column (5) also indicates that the rental value of the property does not change before and after extension.

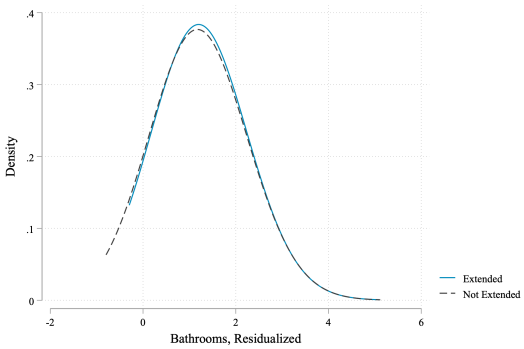
We also construct a second control measure, that explicitly accounts for hedonic differ-

[14] In Appendix Figure A.12 we show how our main hedonic variables vary with log price, controlling for Local Authority fixed effects. Notice that property size, bedroom number, and the log of rental prices all vary approximately linearly with log price.

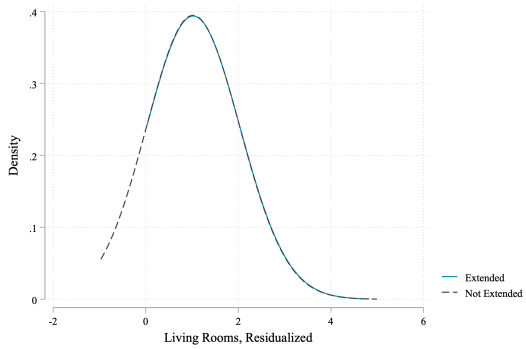
Figure 4: Hedonics Residuals Density Curve for Extended and Non-Extended Properties



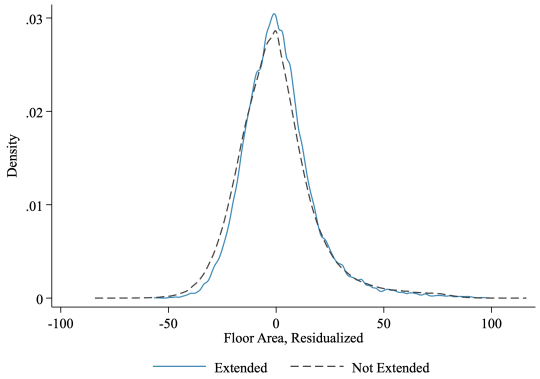
(a) Bedrooms



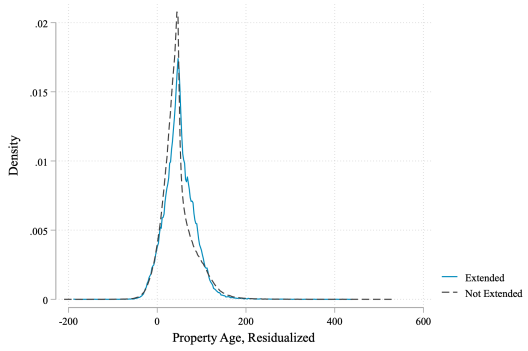
(b) Bathrooms



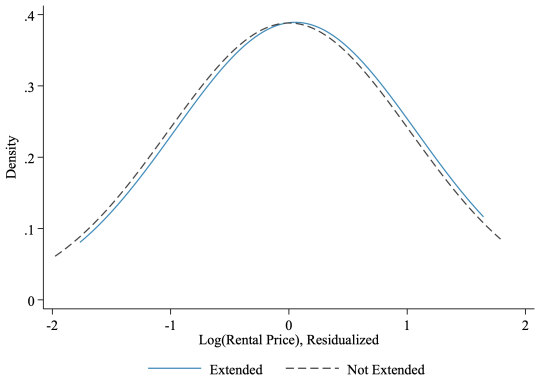
(c) Living Rooms



(d) Floor Area



(e) Age



(f) Log(Rent)

The figures show the distribution of residuals for extended and non-extended flats after regressing hedonic characteristics on 5-year duration bin × Local Authority × year fixed effects. The sample is leasehold flats which appear at least once in the Land Registry Transaction Data Set.

Table 2: Hedonic Characteristics in Extended vs Non-Extended Flats

Panel A: Levels

	(1) Num Bedrooms	(2) Num Bathrooms	(3) Num Living Rooms	(4) Floor Area	(5) Age	(6) Log Rental Price
Extension	0.08***	0.04***	0.01***	4.12***	6.52***	3.11***
	(0.002)	(0.002)	(0.001)	(0.129)	(0.184)	(0.137)
Fixed Effects	✓	✓	✓	✓	✓	✓
N	1,739,655	1,361,887	1,193,843	1,374,150	1,055,821	958,595

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Panel B: Differences

	(1) Δ Num Bedrooms	(2) Δ Num Bathrooms	(3) Δ Num Living Rooms	(4) Δ Floor Area	(5) Δ Log(Rent)
Extension	0.00	-0.00	-0.00	0.42	-0.01
	(0.00)	(0.00)	(0.00)	(0.35)	(0.01)
Fixed Effects	✓	✓	✓	✓	✓
N	222,285	162,604	141,993	158,930	79,065

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Panel A reports level of hedonic characteristics for extended properties relative to their non-extended counterparts. It presents estimates of β for the specification, $X_{it} = \alpha + \beta \mathbb{1}(\text{Extended}_{it}) + \Gamma_{it} + \epsilon_{it}$, for several hedonic characteristics X_{it} where Γ_{it} are 5-year duration bin × Local Authority × year of hedonic characteristic listing fixed effects. The sample is leasehold flats which appear at least once in the Land Registry Transaction Data Set. Panel B reports renovation rates in extended properties relative to their non-extended counterparts. It presents estimates from (5). For Columns 1-4, the sample includes properties with two separate Rightmove or Zoopla listings corresponding to two separate transactions. For Column 5, the sample includes properties that transact twice and have distinct rental price data around each transaction. Standard errors are clustered at the purchase year, sale year and postcode area level.

ences between the treatment and the control group. In particular, we run the following regression,

$$P_{it} = \alpha + \Gamma \times \text{Hedonic Controls}_{it} + \epsilon_{it}$$

where the hedonic controls are number of bedrooms, number of bathrooms and floor area. Then, we create a residualized price variable, $P_{it}^{(res)} = \overline{P} + \epsilon_{it}$ where $\overline{P}$ is the mean price level across all flats. $P_{it}^{(res)}$ represents the price of property i at time t, taking out the average effect of its hedonic characteristics. We can use the previously described procedure to condition on hedonic characteristics, by residualizing prices for both extensions and their controls.

The advantage of this second measure is that it controls for potentially confounding sources of heterogeneity in price. The disadvantage is that we only observe hedonic characteristics for a fraction of our experiments (e.g. we have data on bedroom count for 101 thousand and data on floor area for 80 thousand of our 125 thousand experiments), so it reduces our already limited sample. As we will see in Section 5, hedonic controls do not significantly affect our estimates, so the majority of our analysis is conducted using the first

control measure. We use the hedonic control measure for robustness on all our results.[15]

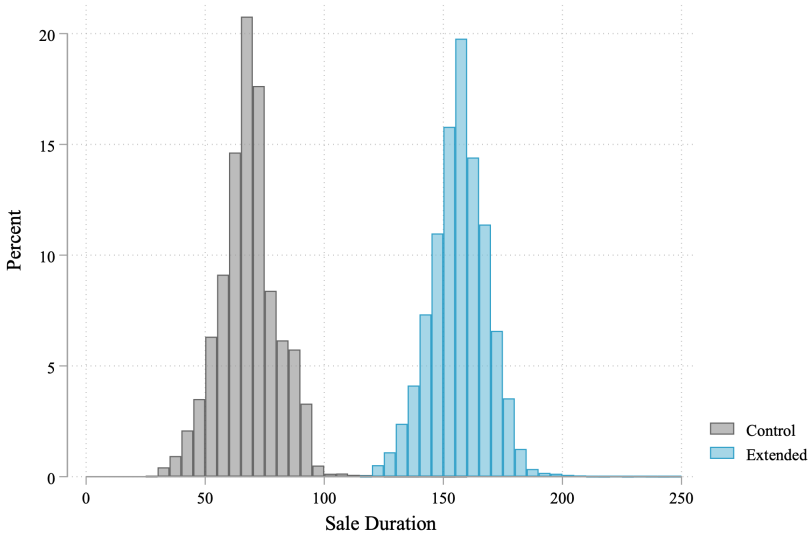
5 Empirical Results

This section presents our main empirical results. Using our difference-in-differences approach, we estimate the level of the natural yield for UK property to be 3%. We also investigate the dynamics of the natural yield between 2003 and 2023. We find a decline from over 4.5% in 2006 to 2.3% in 2023. Moreover the fall takes place before 2020 while the natural yield are stable afterwards—despite rising yields on safe safe assets over the same period.

5.1 Event Study Representation of Lease Extension

Figure 5 shows that the duration of properties when they extend is long. The figure presents a histogram of duration at sale time for the extensions and controls in each experiment, for the leaseholds that extend by 90 years. We can see that the majority of extensions have duration of 130-180 at sale (median is 157), so their controls tend to have duration of 40-90 at sale (median is 68).

Figure 5: Histogram of Remaining Lease Term At Sale



The figure shows a histogram of remaining duration at sale time t for the approximately 40 thousand experiments which were extended for 90 years. The blue-shaded histogram includes the sale duration for the controls in the experiment and the grey-shaded histogram includes the sale duration for the extensions.

[15] In much of our analysis, we focus on extensions of 90 years. Appendix Table A.2 repeats the analysis of Table 2 on that subsample only.

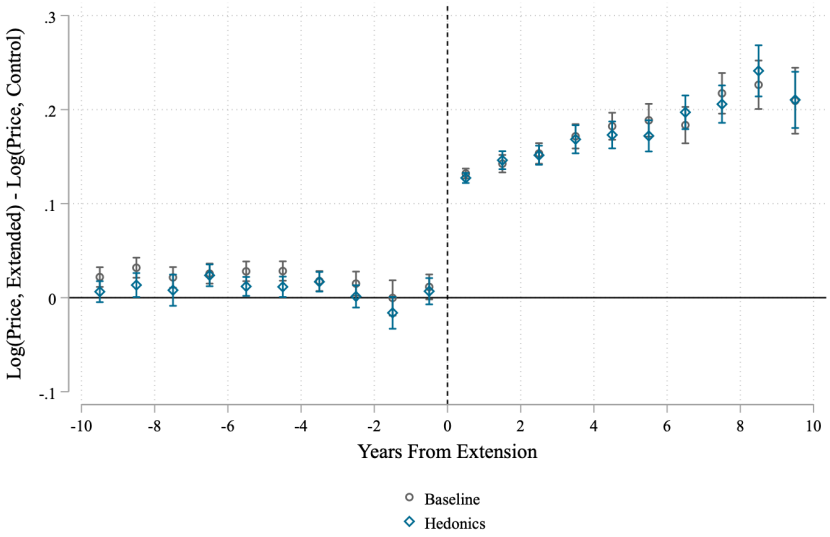


Figure 6: Event Study Representation of Lease Extension

The figure presents the difference in transaction price between extended properties and their controls before and after extension. The x-axis is the difference between transaction time and extension time, which we identify using the lease registration date for extended properties. The grey circles use mean log(price) as the y-axis variable and the blue diamonds use log(price) residualized on hedonic characteristics as the y-axis variable. We include 95% confidence intervals for each point estimate.

What effect does lease extension have on property valuations? To visualize this, we create a pseudo event study for the leases that extend by 90 years. We calculate the mean difference in log price between the extending property and its controls. We can calculate this price difference twice: once before and once after extension. We plot these price differences for all of the extension experiments, using time to extension, u, as our x-axis variable. The result is in gray in Figure 6. Though we only observe each extension experiment once on either side of extension, we have stacked multiple experiments together according to their distance from extension. Therefore we can study the trends in prices versus the control before and afterward. The result is similar to an event study. Upon extension, the difference in log price between extended properties and their controls jumps by 0.12 log points, or about 12.7%. The difference continues to grow after this point because as time from extension increases, the lease term falls more, which results in a greater predicted difference in price between extension and control according to our simple asset pricing model. Moreover, consistent with our identification assumption, there are small and statistically insignificant "pre-trends" before lease extension. The difference-in-difference price increase at extension, Δ_{it}, is a key input into our estimation of y^*.

As we saw in Section 4.4, extended properties tend to have slightly better amenities than their non-extended counterparts. We can see this in Figure 6, where extended properties con-

sistently sell at a time-invariant premium relative to controls before being extended. Given the parallel trends assumption and the absence of a pre-trend, this level shift is differenced out by our difference-in-difference estimate. However, the estimates plotted in blue in Figure 6 assuage any lingering worries by showing that the premium is largely accounted for by differences in property characteristics; i.e. once we control for hedonic variables, even small pre-extension differences in price between extensions and their controls disappear. Most importantly, our difference-in-difference estimates are essentially identical with and without hedonics, as we shall see formally in Section 5.2.

5.2 Estimating the Level of y^*

We present estimates of the level of y^*. We estimate equation (4),

$$\Delta_{it} = \log\left(1 - e^{-y_t^*(T_{it}+90)}\right) - \log\left(1 - e^{-y_t^* T_{it}}\right)$$

using all lease extension experiments i. There are two sources of variation identifying y^*. First, the estimating equation predicts that the price gain is larger when y_t^* is lower. Second, the gain from extension varies with duration, T_i. Leaseholds with shorter remaining terms will receive the benefit of extension sooner, which leads to greater price growth upon extension.

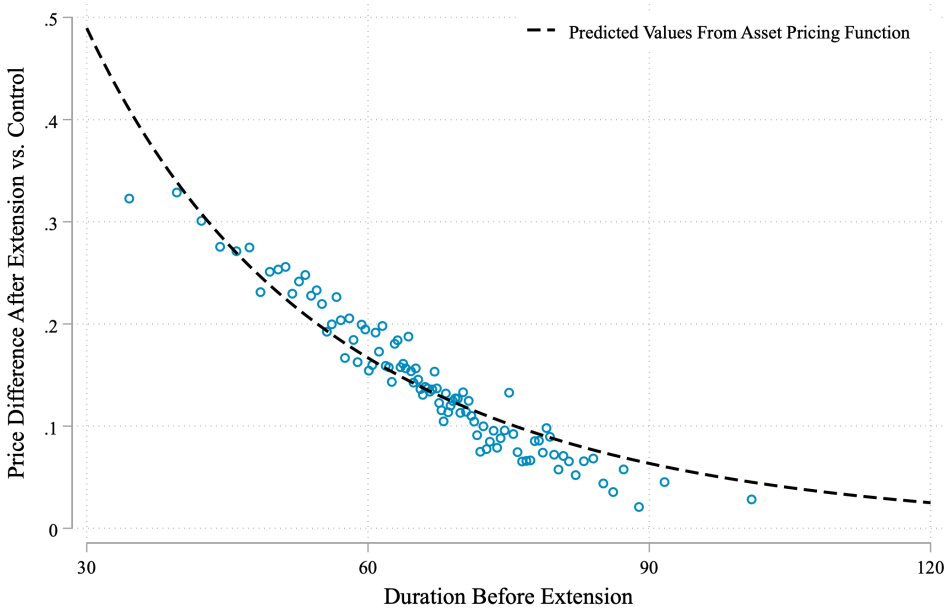
Now we plot the variation from the data that identifies y^* according to our estimator. Figure 7 binscatters the difference-in-difference estimates Δ_{it} for various durations T_i, for the sample of leaseholds that extend by 90 years. As predicted by the estimating equation, the percent increase in property value as a result of extension increases in the duration before extension—ranging from only 7% for 90-year duration properties to more than 30% for properties with duration of 40 years at extension. The black curve in Figure 7 presents fitted estimates from our nonlinear estimator. Inspection of Figure 7 indicates that our estimate of y^* is a good fit across both the average price gain after an extension, and variation in the price gain across T—the two statistics that identify y^*.

Next, we present estimates of the y^* implied by Figure 7. We estimate as $y^* = 3.0\%$, indicated in Column 1 of Table 3. The second and third rows of Table 3 show that these estimates are largely unchanged by inclusion of hedonic controls, as already demonstrated by the event study plot in Figure 6.[16]

As we discussed in Section 4.3, our estimator is not perfectly accurate when the short end of the yield curve for housing is upward sloping. In this case, our estimated y^* will vary with duration T (i.e. $y^*(T)$ will be upward sloping if $y(T)$ is upward sloping, and vice versa

[16] In Appendix Table A.3, we repeat this for all extension lengths, which produces very similar results.

Figure 7: Duration Before Extension vs. Price Gain After Extension



The figure is a binscatter of our difference-in-difference estimator against duration before extension, T, with 100 bins. The black line shows fitted estimates of (4).

if $y(T)$ is downward sloping). Therefore, we can estimate the degree to which the short-end of $y(s)$ affects our estimates by parameterizing our estimate of the natural yield of housing as $y^*(T) = \gamma + \beta \cdot T$. We exploit variation in duration at the time of extension to estimate both γ and β. In Columns 2-5 of Table 3, we present estimates of $y^*(T)$ under this more flexible form for our sample range of T. We can see that $y^*(T)$ varies little over the range we observe it, suggesting our estimates are close to the true y^*.[17]

In Appendix Section A.5, we investigate whether liquidity premia affect our estimates, because short duration leaseholds may have difficulty obtaining mortgages. These considerations do not seem to affect our results: from survey data, short duration leaseholds seem to have similar characteristics to other leaseholders and seem to be similarly liquid. Explicitly incorporating liquidity frictions into our estimator affects the value of y^* little.

5.3 Trend Dynamics of y^*

Now we present our estimates of the trend dynamics of y^*. Interest rates have been falling in the UK, as in much of the rest of the world, for at least four decades. Can the same be

[17] We can also choose more flexible parameterizations of y^* which capture its curvature. For example, the forward sloping parameterization of $y^*(T) = \gamma + \beta \frac{(1-e^{-T})}{T}$ estimates that $y^*(T)$ is even more flat that the linear parameterization.

Table 3: Estimated y^*

	Constant y^*	Flexible y^*				
	(1)	(2) $T = 50$	(3) $T = 60$	(4) $T = 70$	(5) $T = 80$	(6)
No Hedonics	3.03***	2.98***	3.02***	3.06***	3.09***	39,426
	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	
No Hedonics (Hedonics Sample)	2.99***	2.93***	2.98***	3.04***	3.09***	24,943
	(0.02)	(0.03)	(0.02)	(0.03)	(0.04)	
Hedonics	2.97***	2.87***	2.96***	3.05***	3.14***	24,943
	(0.02)	(0.02)	(0.02)	(0.03)	(0.04)	

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 01$

Column 1 presents estimates of y^* from (4) for 90 year extensions. Column 2-5 present estimates of $y^*(T)$ for $T \in \{50, 60, 70, 80\}$, where we parameterize y^* linearly as a function of T, $y^*(T) = \gamma + \beta \cdot T$. The first row presents estimates using the baseline procedure to find controls described in Section 4.4, which utilize the raw transaction price variable as the main outcome variable. The second row presents estimates using the raw transaction price variable, but restricted to the sample of observations which are not missing hedonic characteristics. The third row presents estimates of y^* and $y^*(T)$ produced using transaction price residualized on hedonic characteristics as the main outcome variable, as described in Section 4.4.

said for y^*? We find a sizable fall in y^* across the entire yield curve between the start and end of our sample period. This decline in y^* is depicted in Figure 8. In this figure we show that at all durations, the price growth associated with lease extension Δ_{it}, has increased by approximately 20pp between the beginning and end of our sample. The opacity of the bar graph is shaded by the number of observations in the bar, and contains only 90 year extensions. An increase in the price growth from lease extension implies that the value of long duration cashflows has increased—that is, y^* must have fallen.

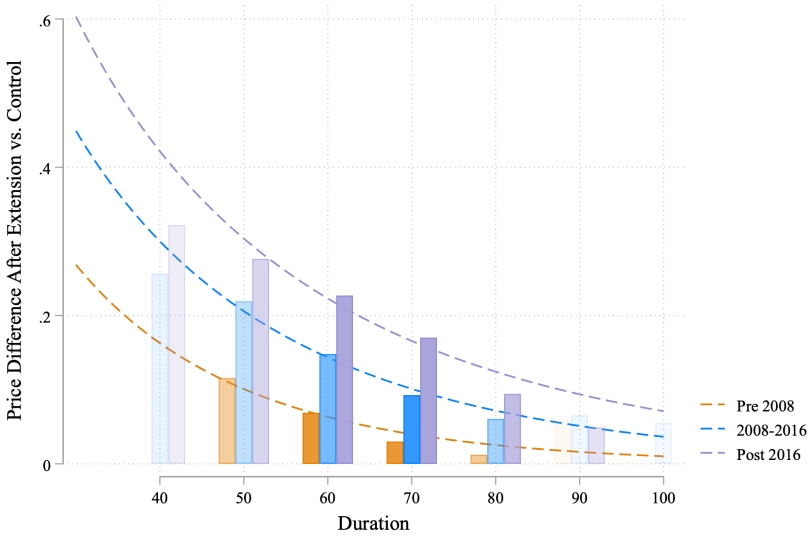
Given our large dataset, we are able to estimate the dynamics of y_t^* for precision. We start in 2003, at the beginning of our sample. We present the estimates in Figure 9, which presents estimates of y_t^* at annual frequency. The solid line includes only leases that extend for 90 years. The dashed line includes both extensions for 90 years, and leases of other extension sizes. We include the additional data in order to raise the precision of our estimates; the shaded region is 95% confidence intervals for the full sample of leases.

The estimates on both samples consistently show that the fall in y^* has been relatively consistent throughout this entire sample period.[18] Between 2003 and 2023, we estimate y^* has fallen from around 4.5% to 2.5% — almost a 50% decline. The magnitude of this decline is large, corresponding to a near doubling of the long run expected price-rent ratio. Notably, our estimates are stable during the 2020 pandemic, despite considerable volatility in shorter term asset prices during this period.

Our estimate of a declining *capital* yield before 2022 is related to the well known fact

[18] Appendix Figures A.13 and A.14 show the event studies for lease extensions of sizes other than 90 years.

Figure 8: y^* Yield Curve Before and After 2012



The figure shows the mean difference-in-difference estimate for various durations in the pre-2008, 2008-2016 and post-2016 periods. The bars show the level of Δ_{it} for each bin and the dashed lines shows fitted estimates. The sample includes only 90-year extensions. Bars are shaded proportionally to the number of observations that make up the bar.

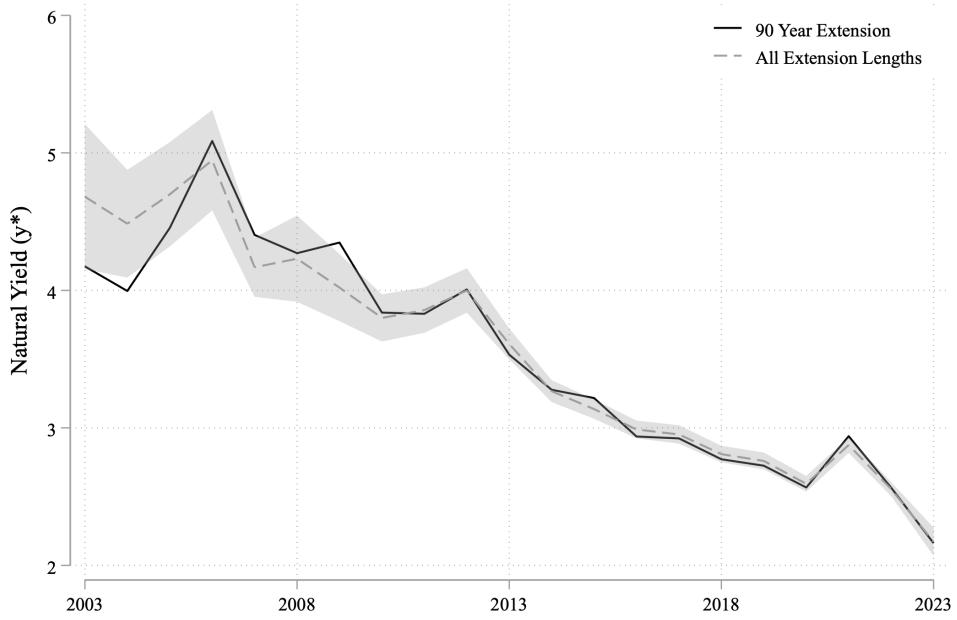
that *government bond* yields fell around the world before 2022 (e.g. Holston et al., 2017). However bond yields could have fallen due to factors that are specific to safe assets, such as quantitative easing or rising demand for the safety and liquidity of government bonds (e.g. Krishnamurthy and Vissing-Jorgensen, 2012; Caballero and Farhi, 2018; Jiang et al., 2023). These safe asset-specific factors may be unrelated to the yield on capital in the real economy. To the contrary, our estimates show that capital yields also experienced large trend declines in yields before 2022, similar to bonds.

5.4 Real Time Dynamics of y^*

We now study the implications of our real-time estimates for the current period of rising yields. As mentioned earlier, one useful feature of our methodology is that it can be implemented in real-time, and a subset of lease extensions can be identified in publicly available data. This is especially useful for understanding how significant changes in central bank policy and communication impact markets' expectations of long term yields and, hence, asset prices.

Figure 10 plots our estimate of the natural yield of capital, as well as long term real interest rates for safe assets, as proxied by the 10 year 20 year forward interest rate on UK government bonds. We plot y_t^* estimates at monthly frequency until April 2023, the

Figure 9: Time Series of the Natural Yield for Housing



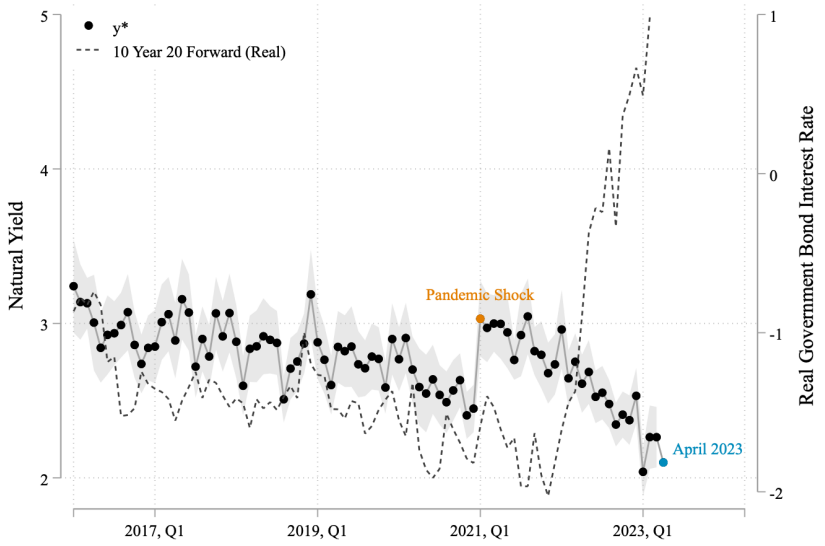
The figure shows estimates of y^* for each year of data. The solid line presents estimates for 90 year extensions. The dotted line presents estimates using all 125 thousand lease extensions, which include shorter and longer extension lengths. The shaded area shows 95% confidence intervals for the estimates using all the data. Standard errors are heteroskedasticity robust.

last month for which UK housing data are available. The shaded area is the confidence interval of our estimate of natural yields—given the large sample size of around 900 new lease extensions per month, the estimate is precise, even at monthly frequency.

Our estimate of the natural yield is stable since the start of 2022, even falling slightly. However real yields on safe assets, even at relatively long durations, have risen sharply since the start of 2022—by 300 basis points for the long term UK government bond. This increase in interest rates may in part be due to factors relating to monetary policy, since the Bank of England (as well as many other central banks) began to hike interest rates over this period.

The contrasting behavior of natural yields and safe asset yields suggests that post-pandemic monetary policy has created a wedge between long-term bond rates and y^*. Time will tell how this wedge will close. There are two possibilities. First, high interest rates may gradually lead to a rising natural housing yield, and a collapse in house prices. Or second, long-run interest rates will fall. According to this second possibility, the current rise in the long term forward rate is a temporary deviation from r^*, which has remained low. Our real-time estimates of y^* will allow us to track this convergence process on a monthly basis. We expect approximately three thousand extensions to be added to our sample every quarter, which will allow us to estimate y^* with robust standard errors of about 0.06pp at

Figure 10: Quarterly estimates of y^* and 30-year bond yield



The solid line presents y_t^* estimated at a monthly frequency. The dotted line presents the real 10 year 20 year forward rate. The lines are plotted from January 2016 to April 2023. 95% confidence interval is shaded.

a quarterly level in real time, with estimates updated at the end of every month (which is when the Land Registry publishes updates).

There is an active debate about whether the current increase in yields around the world will prove transitory. For instance, Blanchard (2023) argues that after the current period of monetary tightening, yields will revert to the low levels of the "secular stagnation" era. The real time dynamics of our measure of y^* will help to inform this debate.

Additionally, Figure 10 shows an moderate increase in the natural yield at the start of 2021, during the Pandemic Recession. This increases is entirely accounted for by lease extensions from London. We conjecture that this increase represents a temporary period during which the market expected long lived effects of the pandemic on the value of living in London; this period seems to have ended as the pandemic dissipated. In the future, our estimates should help policymakers to understand in real time whether other shocks have also affected long run expectations.

Our microdata and quasi-experimental approach stands in direct contrast to the prevailing structural approach to estimate long-run discount rates (or r^*) proposed by Laubach and Williams (2003) and Holston et al. (2017). Most notably, the Laubach & Williams estimate of r^* breaks down after the sharp decline in GDP at the start of the Pandemic Recession and fails to produce valid results *for any time period* if post-2020 data is included. Additionally, even before the Pandemic Recession, standard errors are much larger than ours, especially

for the UK where the mean standard error between 2000 and 2019 is 4.3pp — an order of magnitude greater than our real-time quarterly standard errors. Moreover the structural approach requires a model of inflation expectations, whereas our estimates are automatically in real terms. We elaborate on the differences between our estimates and those of Laubach & Williams in Appendix A.6.

5.5 The Advantages of a Quasi-Experimental Design

Our approach to estimate y^* builds on the key insight of Giglio et al. (2015), which was the first paper to observe that UK properties are uniquely well suited to estimating long term housing yields, because of their varying duration. Giglio et al use a cross-sectional comparison of freeholds and leaseholds with different duration to estimate the level of y^*. Building on their insight, our approach uses the quasi-experiment of lease extensions to estimate the dynamics of y^*. In this section, we elaborate on some advantages of the quasi-experimental approach.

An important difference between the cross-sectional approach and the quasi-experimental approach is that in the former, the primary source of variation for duration is across properties rather than within properties. Long duration properties might have differences in the service flow of housing. For example, freehold flats might have higher quality construction. As such, the cross-sectional approach relies on detailed hedonic characteristics to control sources of variation in property price associated with the service flow. However with either the cross sectional or the quasi-experimental approach, unobserved heterogeneity may bias the estimates.

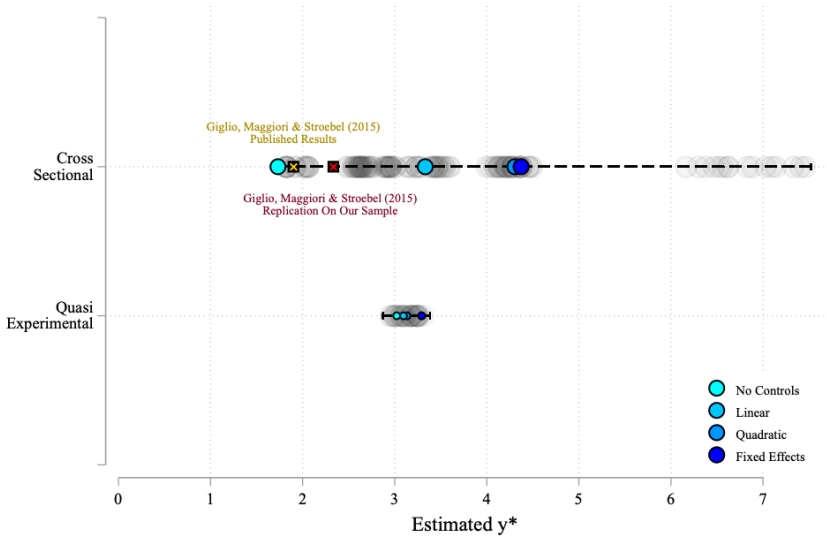
To gauge the effect of unobserved heterogeneity on the quasi-experimental and the cross sectional estimates, we study the sensitivity of the estimates to *observed* heterogeneity, in the spirit of Altonji et al. (2005) and Oster (2019). We therefore estimate y^* using both the quasi-experimental and the cross-sectional approach, controlling for over 100 different variations of hedonic characteristics. In one variation we do not include any controls. In another, we allow price to vary linearly with number of bedrooms and floor area, and in yet another we allow price to vary quadratically with these same controls. In the most extreme case, we control for fixed effects of the following seven characteristics: number of bedrooms, number of bathrooms, floor area, age, heating type, property condition rating, and availability of parking.[19] The other variations include all possible subsets of these seven

[19]The fixed effects controls are the same as the main specification Giglio et al. (2015). Giglio et. al. add an indicator variable for properties with missing hedonics and includes them in the main sample — whereas the current exercise restricts the sample with controls to properties that have hedonic characteristics. We remove properties without hedonics because these properties will not be affected by different ways of adding controls. Moreover, if controls are important, then including properties with missing control information

characteristics.

We then plot our estimates of y^* under each variation for both the quasi-experimental and cross-sectional methodologies in Figure 11. Under the cross-sectional approach, the estimates vary tremendously from 1.3% (in the case of no hedonic controls) to more than 7%. In contrast, our quasi-experimental estimates of y^* are highly stable around 3%. These results provide tentative evidence that our quasi-experimental methodology offers estimates of the natural housing yield that are relatively robust to unobserved heterogeneity.

Figure 11: Stability of y^*, Controlling for Observables



The figure presents estimates of y^* under various choices of hedonic controls. For the cross-sectional estimates, we estimate $\log P_{it}^T - \log P_{it}^\infty = \log(1 - e^{-y^* T})$ by NLLS, where P_{it}^∞ is the price of a freehold transacted in the same quarter and 3-digit postcode as a T duration leasehold. For the quasi-experimental estimates, we follow the methodology described in Section 4. For each methodology, we perform over 100 estimations, controlling for different combinations of hedonic characteristics. We indicate in various shades of blue four important sets of controls: no controls, linear controls, quadratic controls, and the full set of hedonic fixed effects. The gold cross presents the y^* estimate from Giglio et al. (2015), and the red cross presents our replication of their estimate, using the full data from 2003-2023.

6 Dynamics of y^* and the Supply Elasticity of Capital

This section relates the dynamics of y^* to the supply elasticity of capital. First, we show empirically that the trend decline in y^* was greater in areas with more inelastic construction. Second, we provide a model to rationalize this finding, and suggest some general implications of inelastic capital supply, beyond the specific setting of UK housing.

may lead to omitted variable bias.

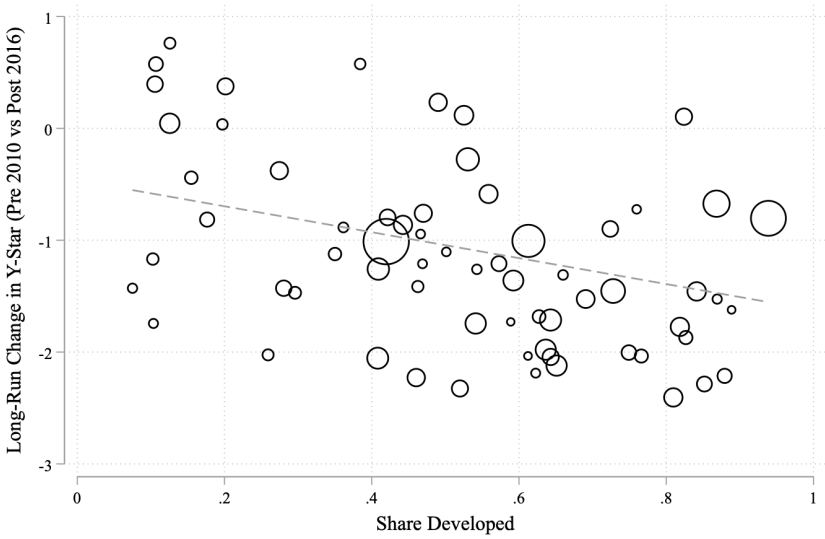
6.1 Estimates: Natural Yields and the Housing Supply Elasticity

We show that more inelastic regions experienced a greater decline in y^* over our sample period. We use two measures of supply elasticity in England, developed by Hilber and Vermeulen (2016): the share of developable land developed in 1990 and the share of applications for major housing construction projects rejected. The first is a measure of physical constraints, since more developed places have less room to build, and the second is a measure of legal impediments to construction. Both measures are captured at the Local Authority level.[20]

For each Local Authority for which we have sufficient data, we estimate y^* in the pre-2010 and post-2016 periods and take the long difference of the two, $\Delta y_t^* = y_{post-2016}^* - y_{pre-2010}^*$. We utilize 700+ year extensions in addition to 90 year extensions to increase our sample size, and we exclude leases with $T > 80$.[21]

In Figure 12, we present a scatter plot of this long difference against the share of developable land developed, which is our primary elasticity measure. We can see that more developed, and therefore more inelastic, Local Authorities experienced a significantly greater decline in y^*. The most elastic Local Authorities, in contrast, saw almost no decline in y^*.

Figure 12: Decline in y^* Depends on Elasticity



The first column of Table 4 presents the coefficient estimate of the slope in Figure 12. Column (2) shows that our second supply elasticity measure, the refusal rate of major con-

[20] There are 317 Local Authorities in England.

[21] We will see in Section 7 that leases with greater than 80 years remaining may be affected by the option value of lease extension.

struction applications, is also negatively correlated with Δy_t^*, although the effect is less statistically significant. This suggests that places with higher refusal rates also saw greater declines in y^*. In Column (3) we present estimates controlling for population density in the Local Authority. The share of land developed poses a constraint to housing supply elasticity to the degree that Local Authorities are unwilling to build upwards, which would thereby increase population density. Without controlling for population density, our estimates will therefore be biased downwards by areas that are very developed but are able to construct on the intensive margin. As expected, the estimated effect increases when we add this control. We utilize a historical measure of population density to avoid endogeneity concerns, but the effect is even stronger if we utilize contemporary population density. Column (4) shows that our results are robust to controlling for region fixed effects and Column (5) shows that the results hold if we exclude the pandemic years, which we know affected London differently. Appendix Figure A.15 shows that the results are robust to cutoff years other than 2010 and 2016 for the long difference.

Table 4: Decline in y^* Depends on Elasticity

	(1)	(2)	(3)	(4)	(5)
Share Developed	-1.00**	-1.54**	-2.19***	-1.62*	-1.97***
	(0.27)	(0.30)	(0.37)	(0.44)	(0.30)
Construction Refusal Rate		-2.60*	-2.29	0.70	-2.76*
		(0.78)	(1.04)	(0.96)	(0.93)
Population Density (1911)			1.01*	1.60***	0.71*
			(0.35)	(0.13)	(0.26)
Region FE				✓	
Excl. Pandemic Years					✓
Adjusted R2	0.09	0.14	0.18	0.22	0.16
N	109	109	109	109	116

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The table reports the results of regressing the long difference Δy_t^* on measures of supply elasticity. The long difference Δy_t^* is defined as the difference between y_t^* for the post-2016 period and y_t^* for the pre-2010 period. All three right-hand side variables–the share of developable land developed, construction refusal rate, and historical population density–are based on the original measures in Hilber and Vermeulen (2016). Regressions are weighted by the inverse variance of Δy_t^*, i.e. $w = 1/(var(y^*_{pre-2010}) + var(y^*_{post-216}))$. Standard errors are clustered at the region level.

6.2 A Simple Model of y^* and the Supply Elasticity of Capital

We now present a simple and standard model to relate the dynamics of y^* to the supply elasticity of capital. The model will explore how y^* responds to an exogenous decline in r^*, the rate of return on safe assets. Since $y^* = r^* + \zeta^* - g^*$, where $\zeta^* \equiv r_K^* - r^*$ is the

risk premium on capital, it may appear that changes in the natural rate of interest, r^*, pass through one-for-one into y^*. However, our model incorporates endogenous capital supply. Therefore any change in r^* will also endogenously effect the growth in the dividends of capital, g^*. If supply of capital is sufficiently elastic, then any decline in r^* would result in an equivalent decline in g^* such that y^* remains unchanged. Therefore the large decline in y^* observed in the data suggests that capital supply is inelastic, consistent with our reduced form estimates.

To remain consistent with our empirical setting, the model is cast in terms of the supply and demand for housing. In Appendix Section A.8, we consider a standard Tobin's Q model of firm investment, which we show is equivalent to the housing model in the main text.

Housing Demand There are $N(t)$ number of households at time t, which grow at a constant rate g_N. Individual households have the option of either buying a house at price $P(t)$ or renting at rental rate $R(t)$, which grows at a constant rate of g^*. Owning and renting are perfect substitutes, and as such the "no arbitrage" relationship between prices and rents gives us,

$$P(t) = \frac{R(t)}{r^* + \zeta^* - g^*}. \tag{6}$$

Aggregate household demand is given by,

$$H^d(t) = \Gamma N(t)^{\alpha} P(t)^{-\beta}, \tag{7}$$

so that aggregate demand increases with population, with elasticity α, and decreases in current house prices, with elasticity β.

Housing Supply There is a unit measure of construction firms $i \in [0, 1]$, and each firm maximizes net present values from construction,

$$V_i(t) = \int_0^{\infty} e^{-r^* s} \Pi_i(t+s) ds \tag{8}$$

where flow profits, $\Pi_i(t)$ are discounted at rate r^*. Construction firms make profits by building and selling houses, subject to standard Tobin's Q adjustment costs. As such, the flow profit of construction firm is,

$$\Pi_i(t) = P(t) B_i(t) - R(t) \Phi_i(t) \tag{9}$$

where $B_i(t)$ is the number of houses built at time t by firm i and $\Phi_i(t)$ are adjustment costs in the construction of new land, which are paid in units of rent. Adjustment costs are a constant returns to scale function of the aggregate housing stock $H(t)$ and the amount of building at time t by firm i, with

$$\Phi_i(t) = \Phi(B_i(t), H(t)) \tag{10}$$

where $H(t)$ is the aggregate housing stock and $\Phi(\cdot)$ is a constant returns to scale function.[22] The aggregate housing stock follows the law of motion,

$$\dot{H}(t) = B(t) - \delta H(t) \tag{11}$$

where housing stock grows due to building $B(t) = \int_0^1 B_i(t)di$ and depreciates at a rate δ.

Balanced Growth Equilibrium Since we are only interested in the relationship between long run variables, we can conveniently abstract away from shorter run transition dynamics and focus only on the steady state balanced growth path. The *balanced growth equilibrium* can be defined as the economy in which prices and rents satisfy the no arbitrage relationship (6), household demand for housing satisfies equation (7), construction firms $i \in [0, 1]$ choose a sequence $\{B_i(t+s)\}_{s\geq 0}$ in order to maximize profits in equation (8), and the market for land clears such that

$$H^d(t) + \Phi(t) = H(t) \tag{12}$$

Finally, given an exogenous and constant growth rate g_N for population $N(t)$, the endogenous variables $B(t)$, $H(t)$, $R(t)$, and $P(t)$ all grow at constant rates g_B, g_H, g^* and g_P, respectively along the balanced growth path.

Graphical Representation of Equilibrium The balanced growth path has a simple graphical representation, shown in Figure 13. First, the housing supply curve is

$$\frac{P(t)}{R(t)} = \phi'(g_H + \delta) \tag{13}$$

where $\phi\left(\frac{B(t)}{H(t)}\right) \equiv \Phi\left(1, \frac{B(t)}{H(t)}\right)$ is the re-scaled adjustment cost. The housing supply curve is an upward sloping relationship between the price-rent ratio and growth in the housing stock. As $P(t)$ rises relative to $R(t)$, building is more profitable and construction firms increase the

[22] For simplicity, we study external adjustment costs, which depend on the aggregate housing stock. Therefore we eliminate the dilution effect of capital on adjustment costs, which is present with internal adjustment costs.

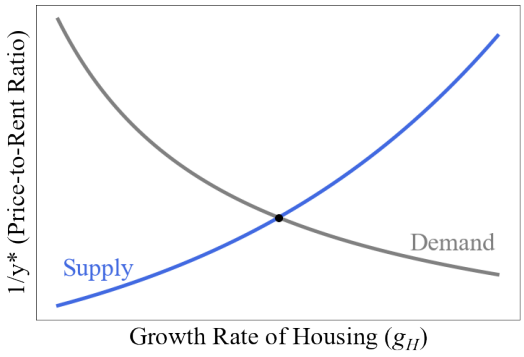
growth rate. The sizes of the increase depends on how the marginal adjustment cost $\phi'(\cdot)$ varies, so that the slope of the supply curve is $\phi''(\cdot)$.

Second, equation (6) yields the housing demand curve

$$\frac{P(t)}{R(t)} = \frac{1}{r^* + \zeta^* - g^*} = \frac{1}{r^* + \zeta^* - \frac{\alpha g_N - g_H}{\beta}}, \tag{14}$$

where the second equality replaces equilibrium rent growth g^* with equation (7) for housing demand, which evaluates to $g_H = \alpha g_N - \beta g^*$ on the balanced growth path. The demand curve is a downward sloping relationship between the price-rent ratio and growth in the housing stock. As the growth rate of building rises, more abundant housing lowers rent growth via housing demand. Therefore the present value of housing falls too, which lowers the price-rent ratio.

Figure 13: Supply and Demand Curves



The supply curve is given by $P(t)/R(t) = \phi'(g_H + \delta)$. The demand curve is given by $P(t)/R(t) = 1/(r^* + \zeta^* - \frac{\alpha g_N - g_H}{\beta})$.

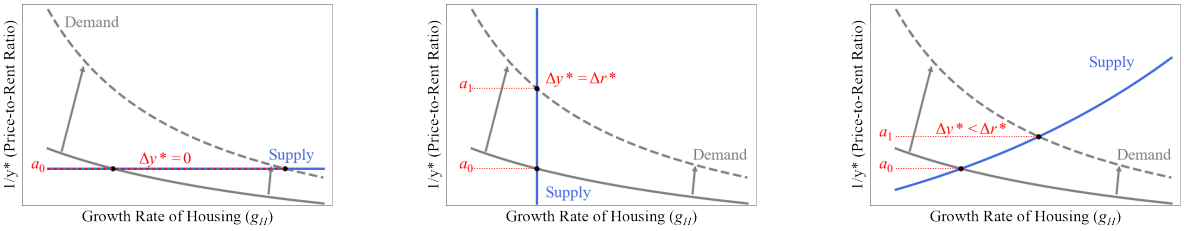
Pass Through of r^* into y^* The model shows that the pass through of the natural rate r^* into the natural housing yield y^* hinges on the supply elasticity of construction, which in turn affects the equilibrium response of rent growth g^* to r^*.

Proposition 6.1. *A fall in r^* weakly raises price to rent ratios, and the extent of the rise depends on the slope of the supply curve.*

See proof in Appendix A.11.1. The implications of Proposition 6.1 are illustrated Figure 14. Consider first the case of fully elastic supply in Figure 14a. In this case, a fall in r^* boosts demand for ownership, which is met with increased construction growth, resulting in a fall in g^*. Therefore, long-run price to rent ratios and y^* remain unchanged. On the other

extreme, if supply is fully inelastic as in Figure 14b, then the full effect of a fall in r^* will be channeled into increased valuations and r^* and y^* will fall by the same amount. Figure 14c presents an intermediate case in which y^* falls, but not by the same amount as r^*.

Figure 14: Effect of A Fall in r^* On y^*



(a) Fully Elastic Supply **(b)** Fully Inelastic Supply **(c)** Intermediate Case

The figures show the effect of a fall in r^* on y^* for the case of fully elastic, fully inelastic and partially elastic supply. The x-axis is the growth rate of the housing stock, g_H and the y-axis is the inverse of y^*, or equivalently the long-run price to rent ratio. The blue line is the supply curve. The grey line is the demand curve. The grey arrow indicates the direction in which the unexpected fall in r^* shifts the demand curve. a_0 is the initial long-term price to rent ratio and a_1 is the long-term price to rent ratio after the shock.

Therefore, the degree to which y^* co-moves with r^* depends on how elastically housing supply responds to changes in demand for housing. When the slope of the supply curve $\phi''(\cdot)$ is zero, supply is fully elastic and y^* does not vary with r^*; when $\phi''(\cdot) = \infty$, supply is fully inelastic and y^* and r^* move one-for-one. We have focused on a simple and standard microfoundation for upward sloping housing supply, via Tobin's Q adjustment costs. However we believe that in richer models of construction, the housing supply elasticity will remain a key determinant of the pass through of y^* into r^*.[23]

The main point of this section—consistently across the empirics and the model—is that the pass through of natural rates into natural yields may have a range of values because of the supply side. Therefore direct measures of y^*, as we presented in Section 5 are valuable, because they contain information about the supply side response of capital to falling r^*.

7 Measuring the Option Value of Lease Extension

So far, we have assumed that there is no option value from lease extension, because leaseholders pay the market value of extension to freeholders when they extend. This section studies option value and its consequences for y^*, in three steps. First, we present a simple

[23] Our emphasis on housing supply relates to work such as Glaeser (2008), which discusses how constrained supply raises house prices and rents in big cities. However we focus specifically on asset valuations, via the price-rent ratio, to connect to the natural rate of interest.

framework that encompasses a key feature of the UK law on extensions, namely discontinuities in the cost of lease extensions when leaseholds have 80 years remaining. Second, we use this framework to derive discontinuity based tests about whether there is option value. These tests indicate that the baseline estimate of the fall in y^*, which ignores option value, is a lower bound for the true fall in y^*. Third, we generalize our the difference-in-differences estimator to point identify the size of option value. We find that our baseline estimate of y^*, which ignores option value, is an excellent approximation.

7.1 A Framework for Lease Extension Costs

We now summarize the institutional framework of lease extensions and develop a simple model of this framework.

Tribunals. Leaseholders are legally entitled to a 90-year extension by the 1993 Leasehold Reform, Housing and Urban Development Act. According to the act, lease extensions ought to be priced at their market value, the present value of service flows from the lease. However the leaseholder and freeholder negotiate the size of the market value, by independently hiring surveyors. If there is no agreement, a Residential Property Tribunal determines the value of the extension after a costly legal process. Tribunals require that the extension is 90 years long, and price extensions using a two part formula, which requires an estimate of *reversion value* and *marriage value.*

Reversion value. The reversion value is the value of the lease extension according to a yield assumed by the tribunal. Therefore reversion value of property i at time t satisfies the formula

$$RV_t^T = \frac{R_{it}}{y_{RV}} \left(e^{-y_{RV}T} - e^{-y_{RV}(T+90)} \right), \tag{15}$$

which is the value of the lease extension according to a Gordon Growth model, with a yield y_{RV} and a current service flow R_{it}. In practice, the service flow R_{it} is imputed from the price of an observably similar freehold property. The yield is fixed by the tribunal at $y_{RV} = 5\%$.

Marriage value. The marriage value is the tribunal's estimate of the market value of the lease extension, given by

$$MV_{it}^T = P_{it}^{T+90} - P_{it}^T \tag{16}$$

which is the difference in price between property to be extended, with duration T, and the price of the same property with duration $T + 90$. The price of the property with $T + 90$ duration is imputed from the transacted prices of observably similar properties. Provided that the tribunal imputes correctly, the marriage value is the market value of the lease

extension, and satisfies

$$MV_{it}^{T} = \frac{R_{it}}{y_t^*}\left(e^{-y_t^* T} - e^{-y_t^*(T+90)}\right). \tag{17}$$

Here, we have written the marriage value as the market value of a lease extension according to a Gordon Growth model, where the natural yield expected by the market, y_t^*, enters the equation for market value.

Reversion value vs. marriage value. Our approach makes extensive use of the following observation. Compare equations (15) and (17) for reversion and market value. Reversion value calculated by the tribunal is greater than market value, if and only the natural yield is greater than the 5% yield assumed by the tribunal.

Discontinuity at 80 years duration. There is a discontinuity in the tribunal assessed cost of the lease extension when 80 years remain on the lease. For leases with more than 80 years remaining, the courts dictate that only the reversion value must be paid, whereas for leases with less than 80 years remaining, the cost is the average of the reversion value and the marriage value.

Tribunal costs to leaseholder. There are significant costs to the leaseholder of appealing to the tribunal. These costs include time and information costs, uncertainty from the outcome of the tribunal, costs of hiring a survey to value the lease, and legal costs associate with the tribunal. Moreover the law dictates that leaseholders must cover all of the freeholder's legal and surveyor fees associated with extension. We will denote these costs for property i by γR_{it}, which we assume for simplicity scales with the current service flow R_{it}.

Leaseholder's participation constraint. The leaseholder also has a participation constraint—if the tribunal assessed costs of extending are greater than the value of extending, the leaseholder can opt not to extend. Therefore the freeholder should reduce costs down to the market value of extension, so that a transaction can occur. The converse is not true: if the tribunal associated costs are less than the market value of extending, the leaseholder will choose the tribunal costs instead of paying the freeholder the market value of extension.

Total extension costs. We can summarize the cost of extension κ_{it}^T, of a property i with T years remaining at time t, as

$$\kappa_{it}^{T} = \begin{cases} \min\left\{RV_{it}^{T} + \gamma R_{it}, MV_{it}^{T}\right\} & T \geq 80 \\ \min\left\{\frac{RV_{it}^{T}+MV_{it}^{T}}{2} + \gamma R_{it}, MV_{it}^{T}\right\} & T < 80 \end{cases} \tag{18}$$

Equation (18) recognizes that the cost of extension κ_{it}^T is the minimum of the market value of extension from the present value of rents; and the tribunal recommended value of extension plus the costs of a tribunal settlement. For simplicity, we have equated the market value

of extension to MV_{it}^T, the marriage value.[24] The tribunal recommended settlement varies discontinuously at 80 years, provided that the market value and the reversion value are not equal.

Price of a T duration property. Property prices satisfy a simple no arbitrage assumption. Therefore the price of a property P_{it}^T, that has not extended, must equal the price of an otherwise similar property that has extended, after deducting lease extension costs. That is, prices satisfy

$$P_{it}^T = P_{it}^{T+90} - \kappa_{it}^T. \tag{19}$$

We denote $\alpha_t^T = \kappa_{it}^T / MV_{it}^T \leq 1$ as the share of the extension value that is lost by the leaseholder, noting that under our assumptions α does not depend on the service flow i.

Remark on option value. When $\alpha_t^T = 1$ we say there is *zero option value* and when $\alpha_t^T < 1$ there is *positive option value.* This terminology acknowledges that when $\alpha_t^T = 1$ for all T, then all of the gains of lease extension are lost to the leaseholder. As a result, the option to extend the lease has no value to the leaseholder ex ante.

7.2 A Discontinuity Based Test for Option Value

We now use our framework for lease extension costs to derive a discontinuity based test of whether there is positive option value. We summarize our predictions in the following proposition.

Proposition 7.1. *There exists some value $\bar{y} < y_{RV}$ such that:*

1. *If the natural yield satisfies $y_t^* \geq \bar{y}$ then*
   (a) *There is zero option value at all years of duration remaining, that is, $\alpha_t^T = 1$ for all T.*
   (b) *The price of a leasehold is continuous in duration as the property's duration falls below 80 years, so*
   $$\lim_{T \to 80^-} P_{it}^T = \lim_{T \to 80^+} P_{it}^T.$$
2. *If the natural yield satisfies $y_t^* < \bar{y}$ then*
   (a) *There is positive option value above 80 years in duration, that is, $\alpha_t^T < 1$ for all $T > 80$ and option value discontinuously falls at 80 years, so that α_t^T discontinuously increases at $T = 80$.*

[24] In effect, and for exposition, we assume that the tribunal, the leaseholder and the freeholder all agree on the market value of the extension. Our qualitative and quantitative results are not affected by this assumption.

(b) The price of a leasehold discontinuously falls as the property's duration falls below 80 years, so

$$\lim_{T \to 80^-} P_{it}^T < \lim_{T \to 80^+} P_{it}^T.$$

This proposition, which we prove in Appendix section A.11.2, has two implications. First, we should expect zero option value at all durations, including above 80 years remaining, only if the natural yield is relatively high. Second, we can test for the presence of zero option value by searching for discontinuities in the price of leaseholds at 80 years.

Part (1) of the proposition shows that when natural yields are high, there is zero option value at all durations. Intuitively, suppose that natural yields are greater than the yield assumed by the tribunal to calculate the reversion value of the extension. Then, the reversion value of the lease extension calculated by the tribunal is greater than the market value. The freeholder will only require the leaseholder to pay the market value, given their participation constraint. Beneath 80 years, the tribunal assessed value remains above the market value of the lease extension—again, by the participation constraint, the freeholder can only force the leaseholder to pay the market value. Part (1) of the proposition also shows how to detect whether the economy has zero option value everywhere—in that case prices are continuous around 80 years of duration. Note that we can extrapolate from the 80 year threshold to conclude that there is no option value at any durations, because we know the functional form of lease extension costs.

Part (2) of the proposition shows that when natural yields are low, there will be positive option value when lease durations are greater than 80 years. Suppose that the natural yield is significantly lower than the yield used by the tribunal to calculate the reversion value of the lease extension. Then the cost paid by leaseholders to extend via the tribunal, if more than 80 years remain, is less than the market value of extension plus time and legal costs—meaning positive option value. Beneath 80 years, the tribunal assessed cost of lease extension discontinuously increases, since the tribunal assessed extension cost is now a weighted average of the reversion value and market value plus additional costs, and market value is greater than reversion value. As a result, the price of leases must discontinuously fall. Importantly, Part (2) of the proposition does not rule out full holdup for leases with less than 80 years remaining.

We use proposition 7.1 to test for whether there is option value. The natural yield of housing seems to have been declining over time. As a result, our proposition suggests that there should be zero option value at all durations, only in the early part of the sample. Later on, there should be positive option value, at least for long duration leases. Our discontinuity based test confirms these predictions.

We test for holdup by estimating whether there is a discontinuity in prices at 80 years, before and after 2010. To determine whether there is a discontinuity in property price at $T = 80$, we can estimate,

$$\frac{\Delta_h \log P_{it}^T}{h} = \alpha + \beta \cdot \mathbb{1}\text{Crossed } 80 + \Gamma_{i,t,t-h} \tag{20}$$

where the left hand side variable is the annualized log change in price of a property i between time $t-h$ and t and the right hand side variable is a dummy which checks whether i fell below 80 between time $t-h$ and t. More precisely, we say that a property crossed 80 if at time t, $T < 80$ and at time $t-h$, $T > 82$. We choose a cutoff of 82 because leaseholders cannot extend through the tribunals during the first two years of ownership, so any property purchased with less than 82 years remaining must pay the marriage value upon extension. $\Gamma_{i,t,t-h}$ are Purchase Year x Sale Year x Local Authority fixed effects. We restrict the sample to properties with duration between 70 and 90, to get the local effect around 80.[25]

Table 5 reports the estimated coefficient from equation (20). The first column presents estimates before 2010. There is no statistically significant discontinuity in price at 80. In fact, Figure 15 shows that in the pre-2010 period, the estimated coefficient is approximately in the middle of a distribution of "placebo" experiments using 30 placebo cutoffs between 70-100.

Table 5: Test for Discontinuity at 80

	(1)	(2)
Crossed Cutoff	0.02	-0.83**
	(0.12)	(0.22)
Sale Year x Purchase Year x LPA FE	✓	✓
Period	Pre 2010	Post 2010
N	63,259	11,808

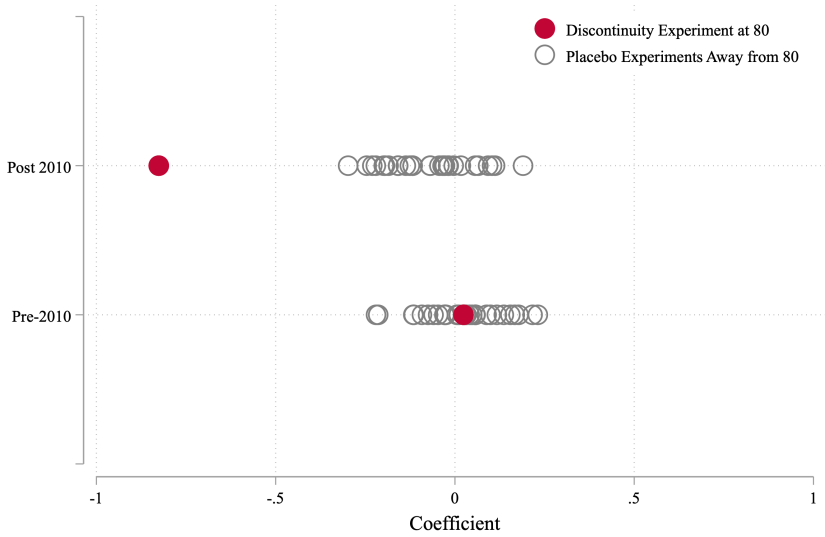
Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The table provides an estimate of the discontinuity in log price at $T = 80$. The sample includes properties with duration between 70 and 90. The first column is run on the pre-2010 data, and the second column is run on post-2010 data. Standard errors are clustered at the purchase year, sale year and Local Authority level.

[25] Table A.4 shows that rents for properties do not discontinuously fall as leases fall below 80 years, suggesting that service flows evolve smoothly around the discontinuity.

Figure 15: Test for Discontinuity at 80 Years (Placebos)



The figure presents the estimates from Table 5 in red. In grey, we run 30 placebo tests using alternative cutoffs between 70-100. In each case, the sample includes properties within 10 years above and below each cutoff.

Column 2 of Table 5 presents estimates after 2010. There is a significant discontinuity in the price of properties that fall below 80 years duration remaining. This fall is much greater in magnitude than any placebo test with cutoffs ranging from 70-100, shown in Figure 15. Taken together, the price discontinuity results show that there is zero option value at all durations before 2010, and positive option value for long duration leases after 2010. However the results do not pin down whether there is full holdup for leases with less than 80 years remaining.

Proposition 7.1 also suggests there should be time varying bunching of lease extensions. When the natural yield is relatively high, there is no gain to extending slightly before 80 years remaining; whereas when the yield is low there can be large gains to extending before 80 years. Therefore lease extensions should bunch in a time varying fashion—leases should be more likely to extend shortly before 80 years remain, only if natural yields are relatively low.

Figure 16 shows precisely this pattern of bunching. In the figure, we plot the likelihood that a lease extends when it has T years remaining, separately before and after 2010. Before 2010, the likelihood that a lease extends smoothly increases as leases cross the 80 year threshold. After 2010, the likelihood jumps just before 80 years, and there is a missing mass after 80 years. This time varying bunching strongly suggests that there is a difference in option value above versus below 80 years, only when natural yields are low—consistent with

Proposition 7.1.

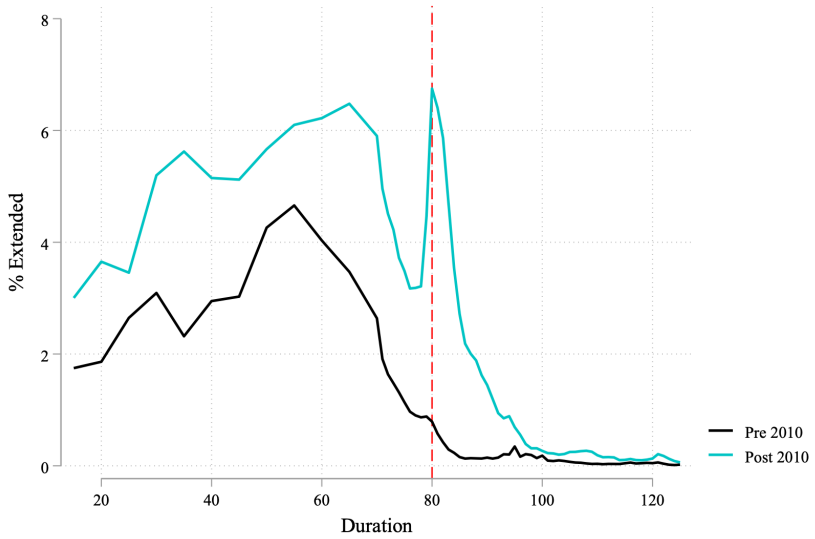


Figure 16: Hazard Rate in Pre and Post Period

The figure shows the conditional probability of extension for each T. The black line shows the probability before 2010 and the blue line shows the probability after 2010. We exclude the post-2020 pandemic period due to disruptions to the lease registration process, which resulted in lower extension rates than usual.

Our results on option value so far imply an informative bound—the estimates of the fall in y^* from the main analysis of Section 5 , which assumes zero option value at all durations and times, are a lower bound on the magnitude of the true fall. Intuitively, if there is positive option value then the *true* duration of non-extended leases is higher than their *notional* duration. As such the price gain from lease extension is associated with a smaller increase in duration after lease extension. Therefore incorrectly assuming zero option value biases estimates of the natural yield upward. Given that option value emerges later in the sample, this bias increases over time, meaning the estimates that ignore this source of bias will under-estimate the fall in y^*. We now introduce more structure to show that this bias is small.

7.3 A Difference-in-Differences Estimator of Option Value

This subsection directly estimates the degree of holdup before and after 2010, both above and below the 80 year threshold, and explores the implications for y^*. To do so, we introduce more structure by embedding the framework for lease extension costs into our difference-in-differences estimator of y^*. Doing so lets us point identify option value, using a different source of variation from the discontinuity based tests of the previous subsection.

In order to incorporate option value and the threshold in a tractable fashion, we assume the probability that a lease of duration T extends at time t is exogenous.[26] We also assume that the share of extension value lost by leaseholders is piecewise constant in duration and discontinuous at 80 years remaining, which captures the discontinuities imposed by the tribunal. Therefore we have $\alpha_t^T = \alpha_t^H$ if $T > 80$ and $\alpha_t^T = \alpha_t^L$ otherwise. In this case, Appendix Section A.9 shows that the difference-in-differences estimator of the price gain upon lease extension becomes

$$\begin{aligned}\Delta_{it}^T = \log\left(1 - e^{-y_t^*(T_{it}+90)}\right) - \log\Big(&\left(1 - e^{-y_t^* T}\right) \\ &+ \left[\Pi_{Tt}^H\left(1-\alpha_t^H\right) + \Pi_{Tt}^L\left(1-\alpha_t^L\right)\right] e^{-y_t^* T_{it}}\left(1 - e^{-y_t^* 90}\right)\Big)\end{aligned} \tag{21}$$

For simplicity, this derivation makes the additional assumption that leases extend only once.[27] In this equation, Π_{Tt}^H is the probability that a lease with $T > 80$ years remaining extends with more than 80 years remaining. Π_{Tt}^L is the probability that a lease extends with less than 80 years remaining. The cumulative probability of extension is derived from the observed extension hazard rate, as shown in Figure 16. Equation (21) is the same as the baseline estimator (3) either when $\alpha_H = \alpha_L = 1$, or $\Pi_H^T = \Pi_L = 0$. In either case, there is no option value from lease extension and the final term in square brackets vanishes.

We jointly estimate y_t^*, α_t^H and α_t^L using the difference-in-differences estimator with option value (21). Relative to the baseline estimation, we also add information on the probabilities of extension Π_H^T and Π_L, which helps to identify α_t^H and α_t^L.

The estimated y^*, α_t^H and α_t^L parameters are presented in Table 6. In the estimation, we constrain the α parameters to lie between zero and one. The results suggest, once again, that $\alpha = 1$ for all T in the pre-2010 period. In the post-2010 period, there is positive option value when leases have more than 80 years remaining, but there is zero option value below 80 years remaining.

[26] One possible microfoundation for this "Calvo" assumption is inattention to option value, as Berger et al. (2021) discuss in the context of mortgages. Section 5 showed that extension is orthogonal to observable characteristics of properties, consistent with inattention.

[27] The value of subsequent extensions, in the very far future, is quantitatively small but complicates the algebra.

Table 6: Estimating Alpha

	(1)	(2)
y^*	4.22***	2.85***
	(0.11)	(0.02)
α_t^H	1.00***	0.69***
	(0.31)	(0.15)
α_t^L	1.00***	1.00***
	(0.05)	(0.01)
Period	Pre 2010	Post 2010
N	19,199	105,952

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The table presents estimates of α_t^H and α_t^L, estimated jointly with y^*. Estimates of α_t^L and α_t^H are constrained to lie within [0,1]. If unconstrained, the estimates for α_t^L are slightly above but statistically equal to 1. Standard errors are bootstrapped.

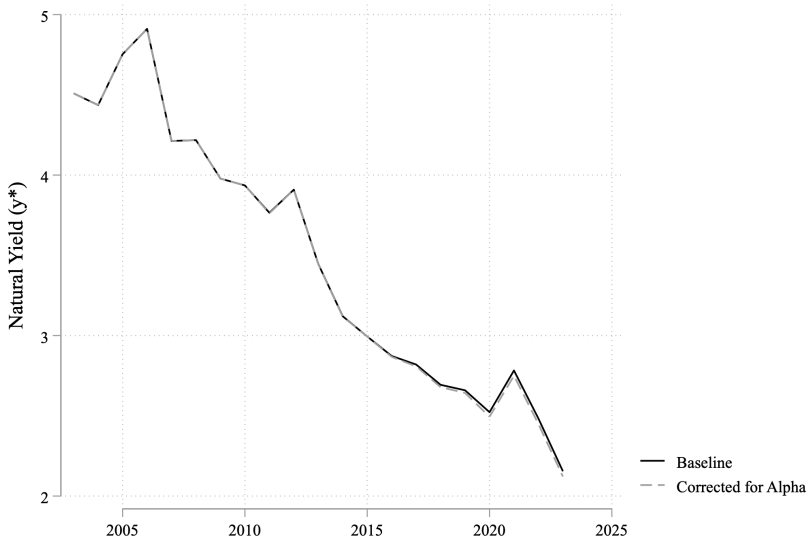
Our estimates of option value in this subsection are consistent with the results from the previous subsections—even though the two subsections use different sources of variation. Our estimates of option value in this subsection are also consistent with the change in option value implied by the discontinuity in price at 80. Based on the price discontinuity at 80, and assuming that $\alpha_t^L = 1$, the implied α_t^H for the post-2010 period is 0.56, which is within one standard error of α_t^H in Table 6 Column 2.[28]

7.4 Estimates of y^* Corrected for Option Value

Finally, we present estimates of y^* that correct for option value using the estimates of option value. Figure 17 presents a version of our y^* timeseries which corrects for $\alpha_t^H = 69\%$ in the post-2010 period. The solid line is the estimate of y^* from section 5, using our baseline assumption of zero option value at all times and durations. The dashed line is the estimate of y^* using our estimates of the degree of holdup.

[28]The details of this calculation are presented in Appendix A.10.

Figure 17: Estimate of y^* Correcting for Option Value



The figure presents a corrected y^* timeseries, adjusting for the fact that $\alpha_t^H = 0.69$ for the post-2010 period. The black line presents the unadjusted estimates from Section 5.3

The estimate of y^* that corrects for option value is very similar to the baseline estimates that assume no option value. The reason is that most leases do not extend with more than 80 years remaining, as Figure 16 shows. Therefore the possibility of extending with more than 80 years remaining has a small effect on equilibrium prices, meaning departures from zero option value are quantitatively small.

8 Concluding Remarks

This paper estimates the natural yield of capital and its dynamics from 2003 to the present day, for property. We exploit a natural experiment—extensions of long duration property leases in the British property market. Our findings imply an average natural yield of around 3%. We found a fall from 4.5% in 2003 to 2.3% in 2023, though the natural yield was stable after 2020.

The natural yield is a valuable statistic for at least two reasons. First, the natural yield encodes the pass through of safe asset yields into the real economy. As such, the dynamics of natural yields determine long term duration risk in the real economy. With a model, we showed that pass through depends on the supply elasticity of land; our estimate of a high pass through suggests inelasticity.

Second, natural yields are valuable because they "look through" the short term factors

affecting asset prices in real time. We documented a growing gap between natural yields and spot safe asset yields after 2020. This gap will narrow either by a fall in spot asset yields or a rise in natural yields. The consequences of these two scenarios for asset prices and the real economy will be different. For instance, if spot asset yields decline back towards the natural yield, then the factors currently affecting asset prices and the real economy will have proven to be temporary.

Therefore an important question for the future is how the gap between spot safe asset yields and y^* closes. Our real time estimates of y^* going forward should be helpful in this regard.

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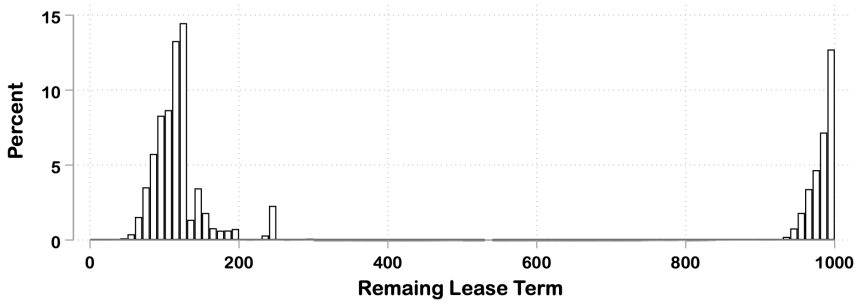
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A Appendix

A.1 Additional Figures

Figure A.1: Distribution of Lease Term for Leasehold Flats



The figure is a histogram of the remaining lease term at the time of transaction. The sample is all leasehold flats that transact at least once in the Land Registry Transaction Data Set.

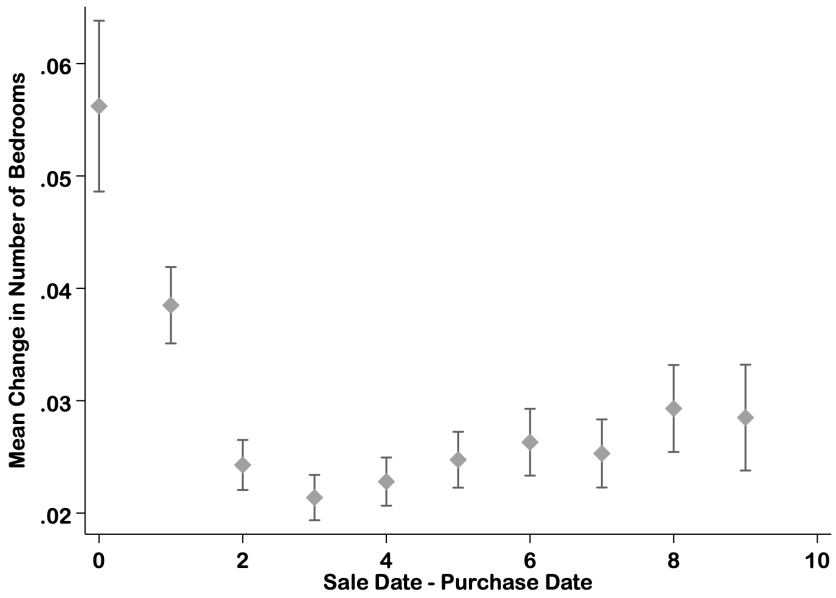
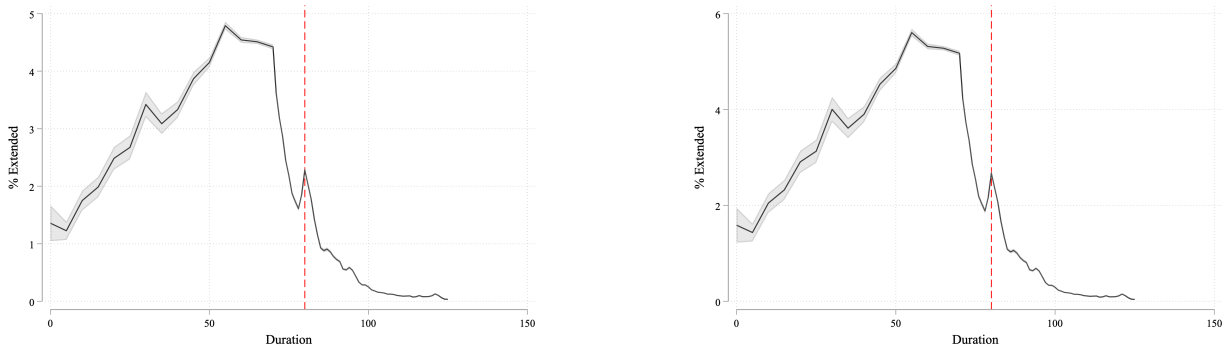


Figure A.2: Renovations By Holding Period

The figure shows the change in number of bedrooms reported relative to property holding period, where very short holds have a disproportionately high renovation rate. We take this as evidence that many of these are "flippers" who buy properties to re-sell them. The sample is all flats for which we observe two different Rightmove or Zoopla listings associated with two different property transactions.

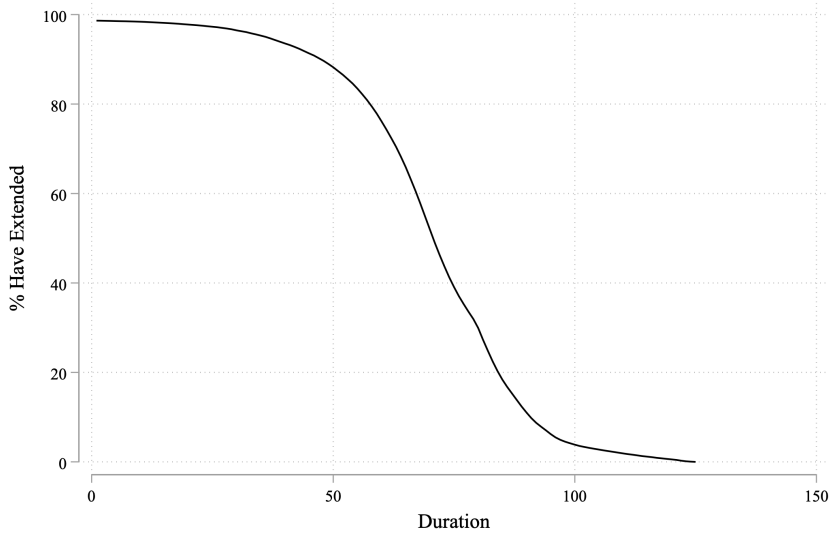
Figure A.3: Hazard Rate of Lease Extension



(a) Raw **(b)** Corrected

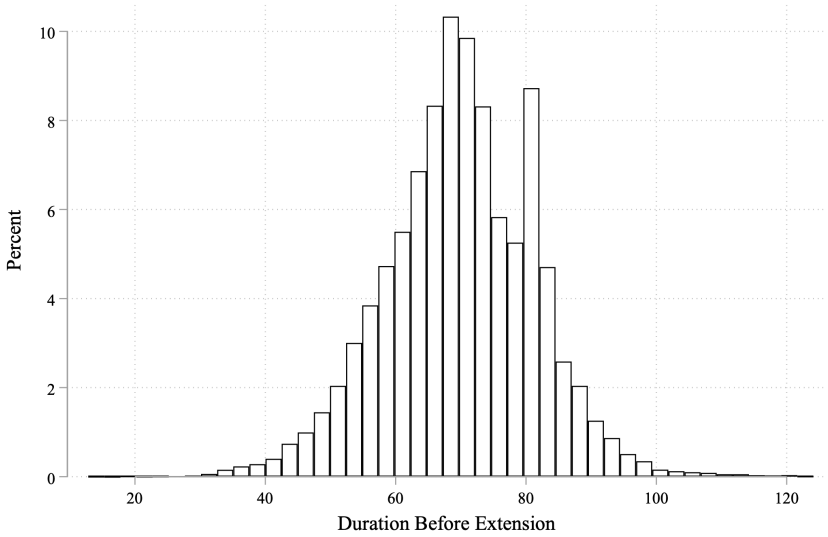
The figure shows the conditional probability of extension, $\theta(T)$ given that a property has duration T. In the first panel, the conditional probability of extension is given by $\theta_1(T) = \frac{N_T^{Ext}}{N_T}$ where N_T^{Ext} is the number of properties which extended with duration T and N_T is the number of properties that reached duration T. In the second panel, the conditional probability of extension is $\theta_2(T) = \gamma \frac{N_T^{Ext}}{N_T}$ where $\gamma = 1.17$ adjusts for the fact that our primary method does not identify properties which never transact before extension. The shaded area shows the 95% confidence interval.

Figure A.4: Cumulative Hazard Rate



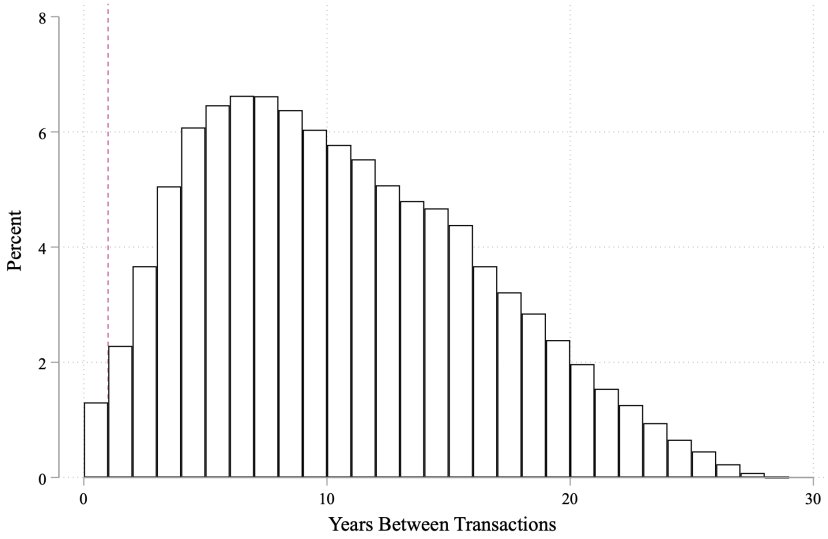
The figure shows the cumulative probability of extending over a property's lifetime. The sample includes all leases with at least one transaction and covers the 2003-2020 period. We exclude the pandemic period due to abnormally low extension rates.

Figure A.5: Histogram of Duration Before Extension



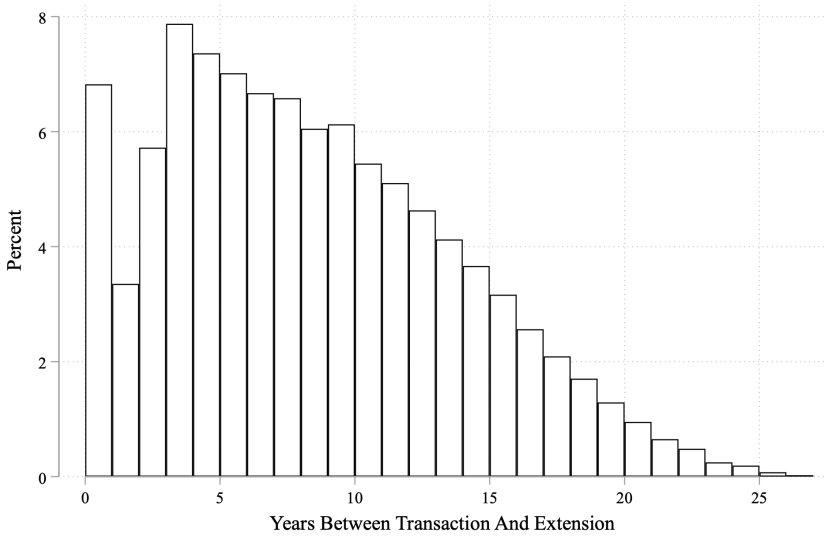
The figure presents a histogram of lease duration immediately before extension. The sample is all extended flats.

Figure A.6: Histogram of Years Between Transaction



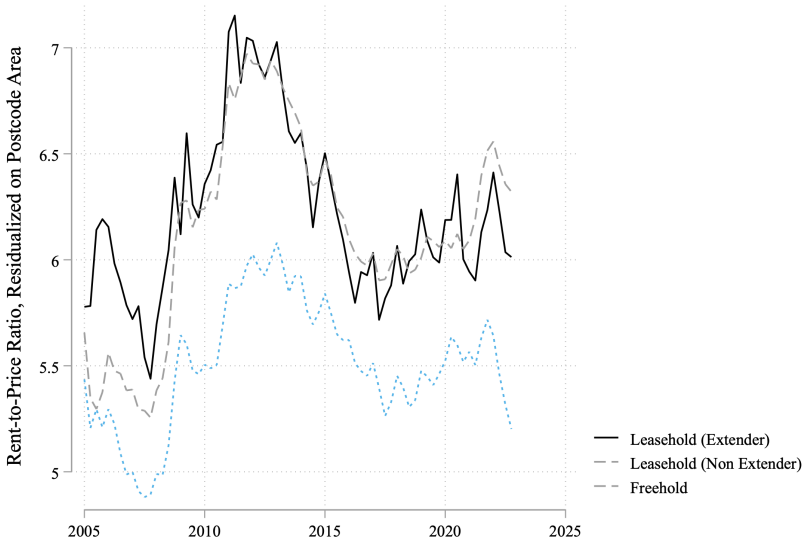
The figure shows a histogram of the holding period, h, for lease extensions which have a recorded transaction before and after extension. The dotted line shows the $h = 1$ cutoff; properties below the cutoff are not included in our primary sample. The sample is all extended flats.

Figure A.7: Histogram of Years Between Transaction and Extension



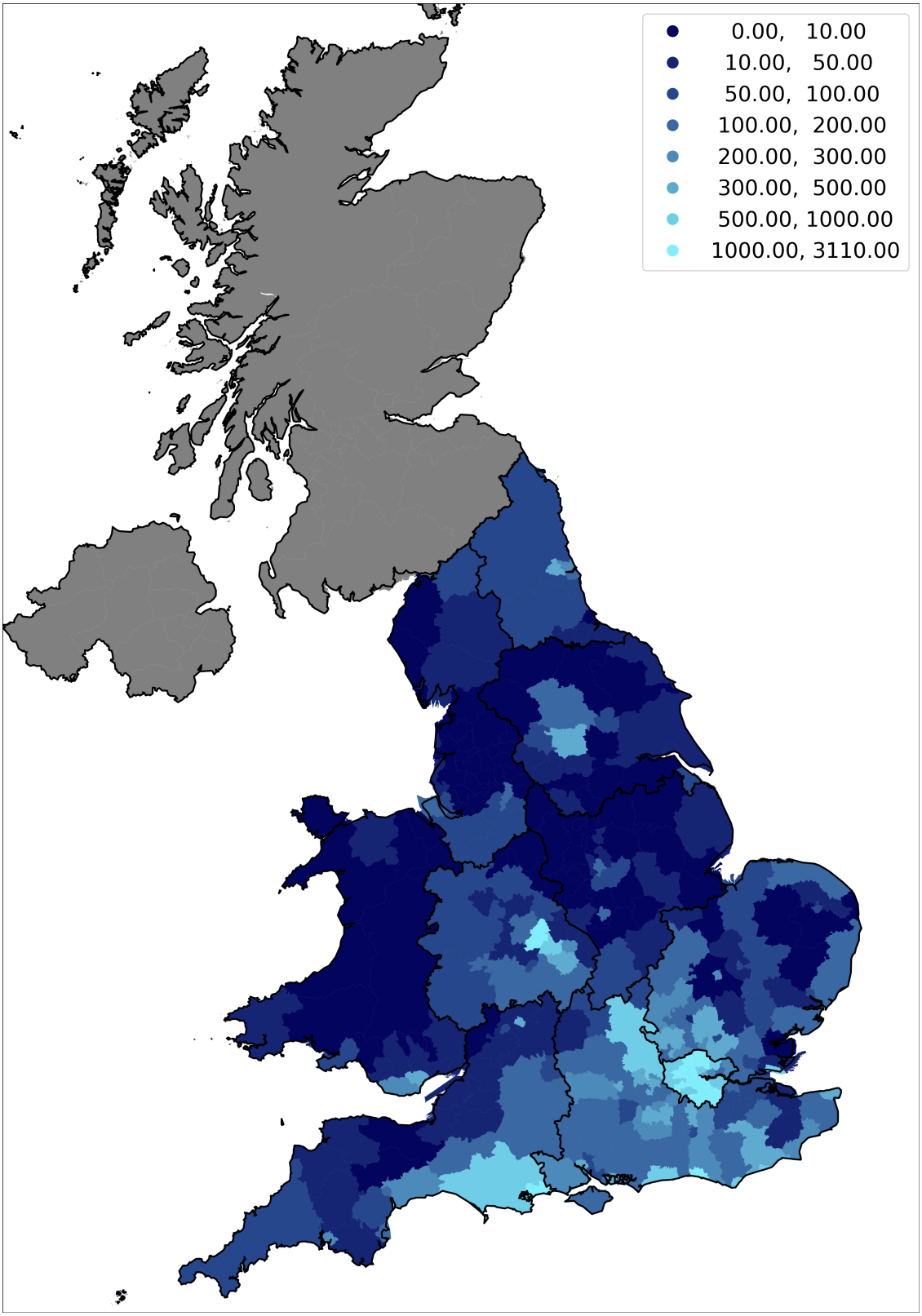
The figure presents a histogram of the number of years between purchase and extension time. The sample is all extended flats.

Figure A.8: Price to Rent (Freeholds vs Leaseholds)



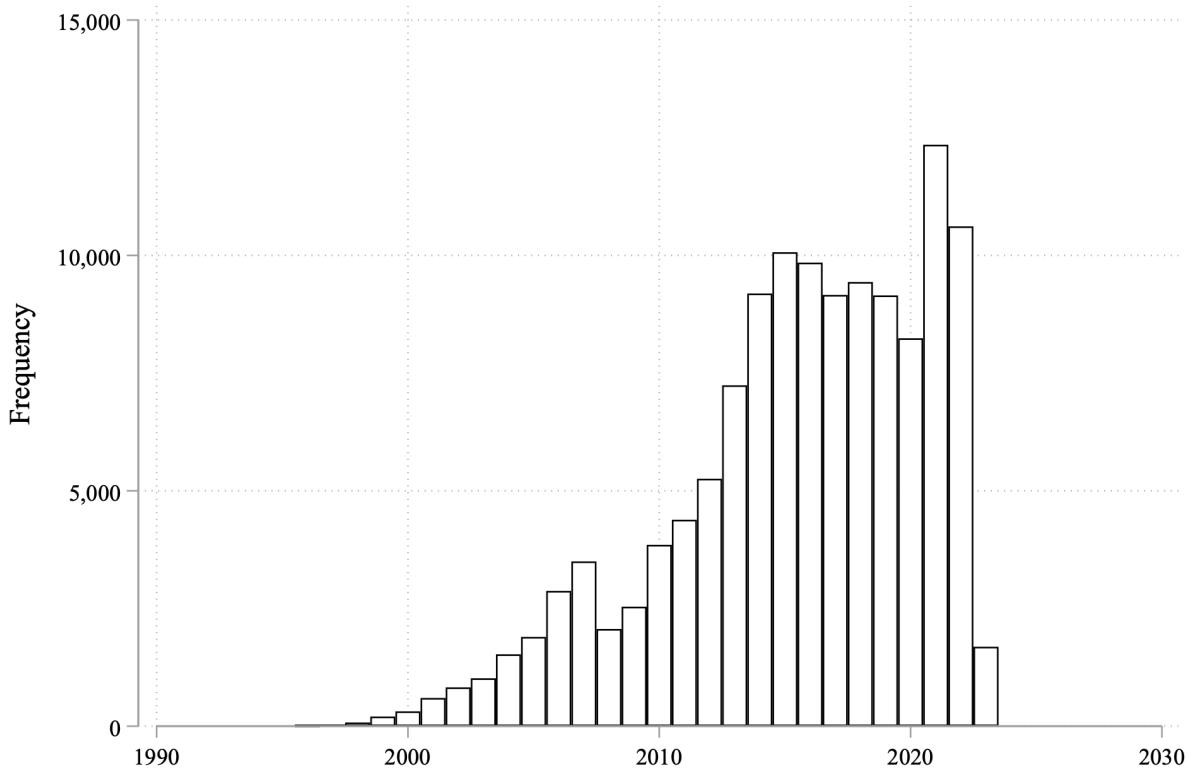
The figure shows the property-level price to rent ratio for leasehold and freehold properties. Leaseholds are subdivided into those which extend during our sample and those which do not. Property-level rental price data is collected from Rightmove and Zoopla.

Figure A.9: Heat Map of Extension Rate



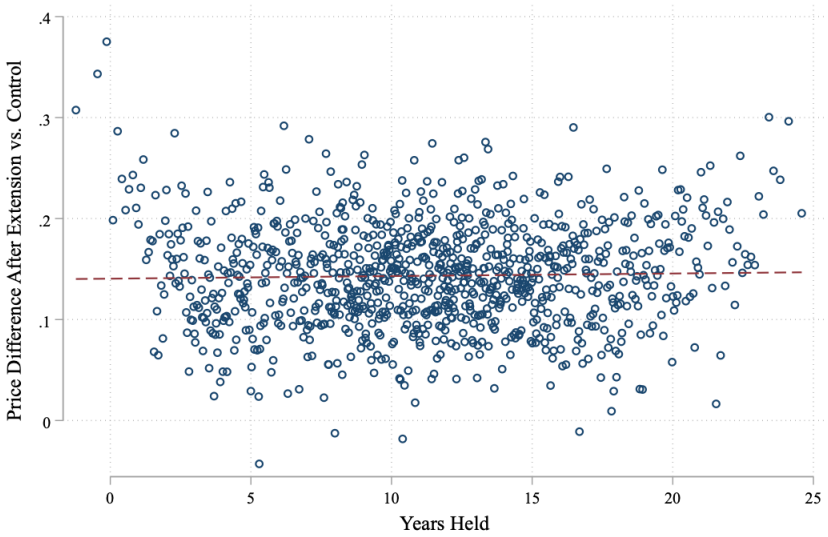
The figure shows a heatmap of the number of properties extended in each Local Authority in England and Wales.

Figure A.10: Transaction Date Histograms



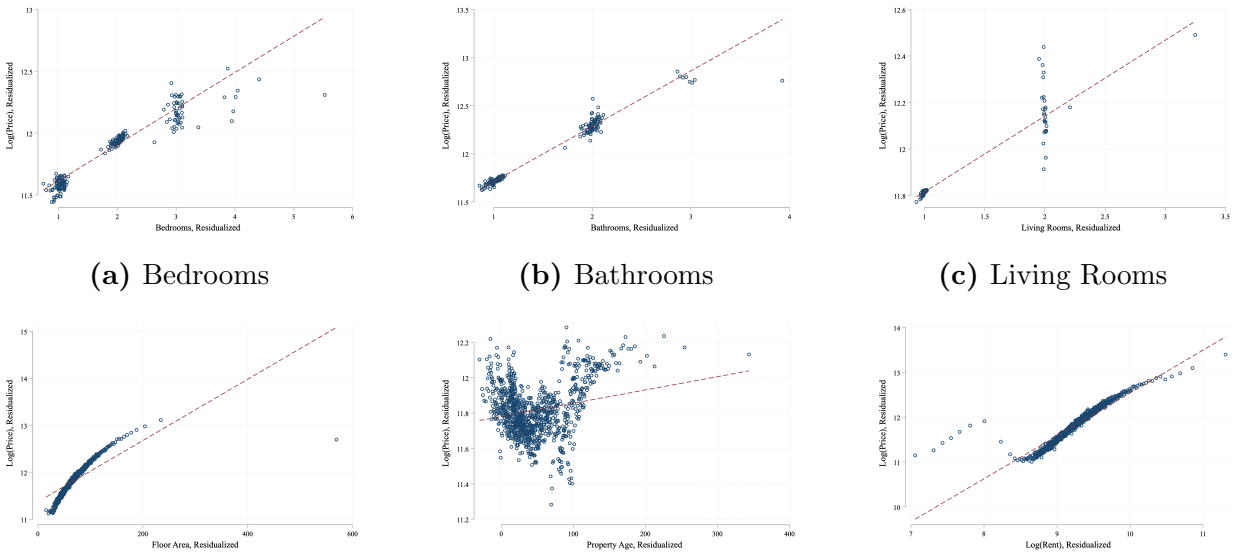
The figure presents a histogram of the post-extension sale date for leases that were extended. The sample is all extended leases.

Figure A.11: Relation Between Holding Period and Difference-in-Difference



The figure shows a binscatter of the difference-in-difference estimator, Δ_{it}, by holding period, controlling for sale year t fixed effects. The sample is all 90 year extensions.

Figure A.12: Binscatter Log(Price) on Hedonics

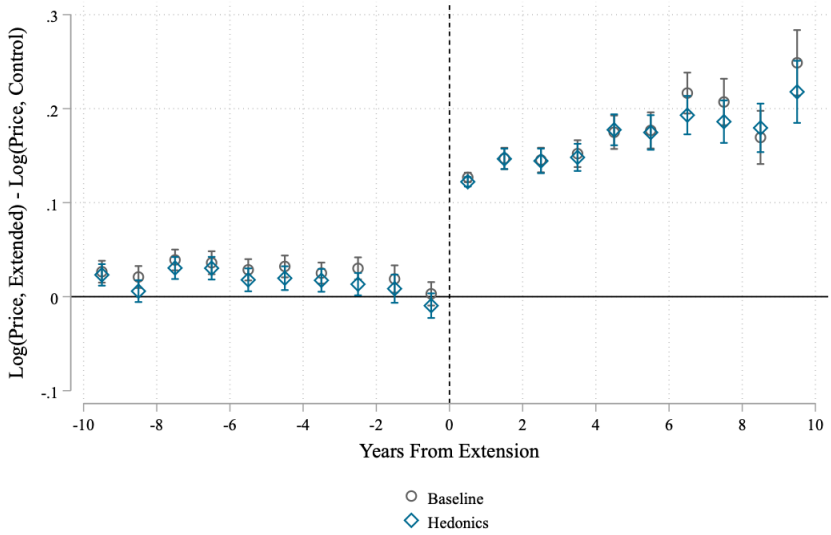


(a) Bedrooms **(b)** Bathrooms **(c)** Living Rooms

(d) Floor Area **(e)** Year Built **(f)** Log(Rent)

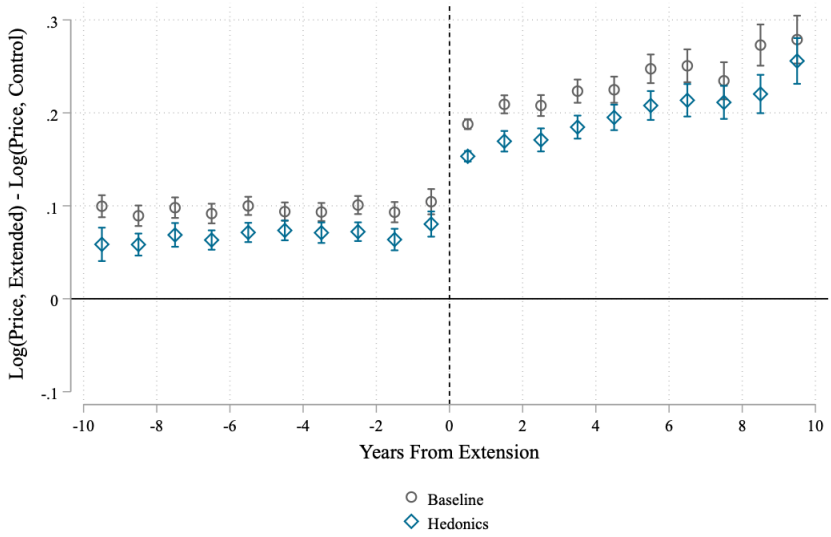
The figures are binscatters of log transaction price against the following hedonic characteristics: number of bedrooms, number of bathrooms, number of living rooms, floor area (sq. meters), year that the property was built, and log yearly rental price. Both the x and y-axis variables are residualized by Local Authority fixed effects, Γ_{it}. In particular, the y-axis variable is $\overline{\log(P_{it})} + \epsilon_{it}$ where $\overline{\log(P_{it})}$ is the mean log transaction price and ϵ_{it} is the residual from the following specification: $\log(P_{it}) = \Gamma_{it} + \epsilon_{it}$. The x-axis variable for each hedonic characteristic X_{it} is $\overline{X_{it}} + \eta_{it}$ where $\overline{X_{it}}$ is the mean level of X_{it} and η_{it} is the residual from the following specification: $X_{it} = \Gamma_{it} + \eta_{it}$. The sample is leasehold flats which appear at least once in the Land Registry Transaction Data Set.

Figure A.13: Event Study for Extension of Extension Length Less than 200 Years (Excluding 90 Year Extensions)



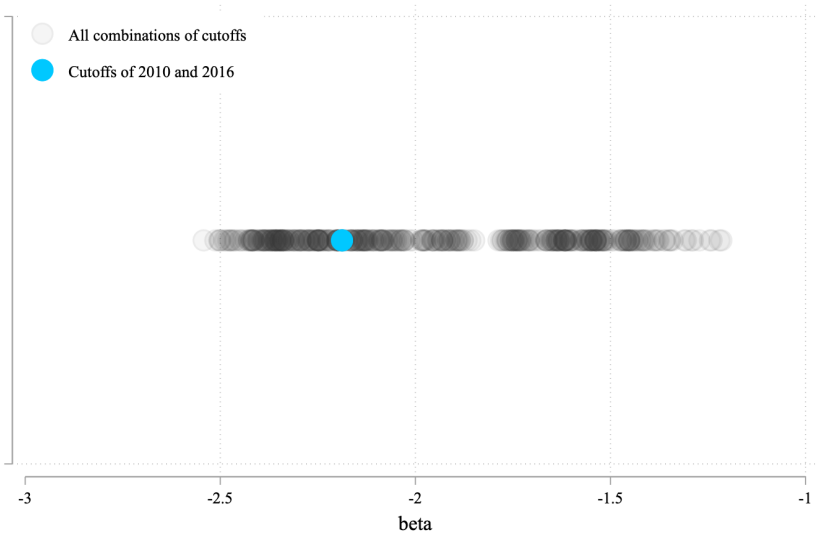
The figure replicates Figure 6 using extensions of lengths of less than 200 years, excluding 90 year extensions.

Figure A.14: Event Study for Extension of Extension Length More than 700 Years



The figure replicates Figure 6 using extensions for lengths of more than 700 years.

Figure A.15: Robustness of Elasticity to Various Cutoffs



The figure shows the coefficient estimate from Column (3) of Table 4 using every combination of pre-extension cutoffs (2008 Q1 - 2013 Q4) and post-extension cutoffs (2014 Q1 - 2018 Q4). We also label our baseline estimate obtained with cutoffs of 2010 and 2016. The variation in the estimate is driven both by changes in the cutoff years and compositional changes, since not all Local Authorities have data for every cutoff.

A.2 Additional Tables

Table A.1: Freehold vs Leasehold Statistics (English Housing Survey)

	Freehold	Leasehold
Income	29,628.73	25,653.20
	(52.95)	(138.48)
Age	53.95	51.49
	(0.03)	(0.10)
% Have Mortgage	54.82	59.07
	(0.10)	(0.28)
LTV	72.17	76.16
	(0.14)	(0.39)
N	305,135	

mean reported; standard error of mean in parentheses

Table A.2: Hedonic Characteristics in Extended vs Non-Extended Flats, Only 90 Year Extensions

Panel A: Levels

	(1) Num Bedrooms	(2) Num Bathrooms	(3) Num Living Rooms	(4) Floor Area	(5) Age	(6) Log Rental Price
Extension	0.03***	0.01***	-0.00	1.22***	1.57***	3.11***
	(0.004)	(0.003)	(0.001)	(0.210)	(0.295)	(0.137)
Fixed Effects	✓	✓	✓	✓	✓	✓
N	1,658,168	1,296,593	1,137,174	1,310,273	1,004,370	958,595

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Panel B: Differences

	(1) Δ Num Bedrooms	(2) Δ Num Bathrooms	(3) Δ Num Living Rooms	(4) Δ Floor Area	(5) Δ Log(Rent)
Extension	-0.01	0.00	0.00	0.74	-0.01
	(0.00)	(0.00)	(0.00)	(0.66)	(0.01)
Fixed Effects	✓	✓	✓	✓	✓
N	205,907	150,626	131,563	147,259	72,263

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The table repeats the results from Table 2 for the sub-sample of 90 year extensions.

Table A.3: Estimated y^* for All Extensions

	Constant y^*	Flexible y^*				
	(1)	(2) $T = 50$	(3) $T = 60$	(4) $T = 70$	(5) $T = 80$	(6)
No Hedonics	3.10***	3.10***	3.10***	3.10***	3.10***	125,159
	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	
No Hedonics (Hedonics Sample)	3.06***	3.06***	3.06***	3.07***	3.07***	74,517
	(0.01)	(0.02)	(0.01)	(0.02)	(0.02)	
Hedonics	3.10***	3.05***	3.10***	3.15***	3.19***	74,517
	(0.01)	(0.02)	(0.01)	(0.02)	(0.02)	

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 01$

The table repeats the estimates from Table 3 only for all extensions.

Table A.4: Test for Discontinuity at 80 (Placebo Test with Rents)

	(1)	(2)	(3)	(4)
	Δ Log(Rent)	Log(Rent)	Δ Log(Rent)	Log(Rent)
Crossed Cutoff	0.34	-0.01	0.44	0.05
	(1.06)	(0.01)	(3.07)	(0.03)
Sale Year x Purchase Year x LA FE	✓	✓	✓	✓
Period	Pre 2010	Pre 2010	Post 2010	Post 2010
N	1,263	22,419	180	3,003

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Columns (1) and (3) report the the average annualized change in rental price for properties that crossed duration $T = 80$ between time $t - h$ and t, relative to control properties which did not cross $T = 80$. Columns (2) and (4) report the average level of rent from properties that crossed duration $T = 80$ relative to their controls.

A.3 Identifying Extensions in Publicly Available Data

The analysis in the main text uses newly available data on lease extensions. However this lease extension data is not public: the publicly available Land Registry lease data does not record lease extensions before September 2021. This section develops and validates a method for identifying lease extensions that can be applied to public Land Registry data, meaning that our results can be replicated using public data only. We then present results using only these publicly available lease extensions and show similar findings to the main text.

In the publicly available Land Registry lease data, when leaseholders extend, the original lease is overwritten in the Land Registry with a new, longer lease term starting at the same date as the original lease. For instance, consider an original lease for "99 years from January 1, 1980". Suppose this lease was extended for 90 years in 2010. Then, the original lease would be removed from the land registry, and replaced with a new, longer lease that read "189 years from January 1, 1980", where 189 years is the sum of the initial lease term and the extension amount and the start date of the lease remains in 1980. As such, the public land registry only records a "snapshot", which does not single out leases that have been extended in the past.

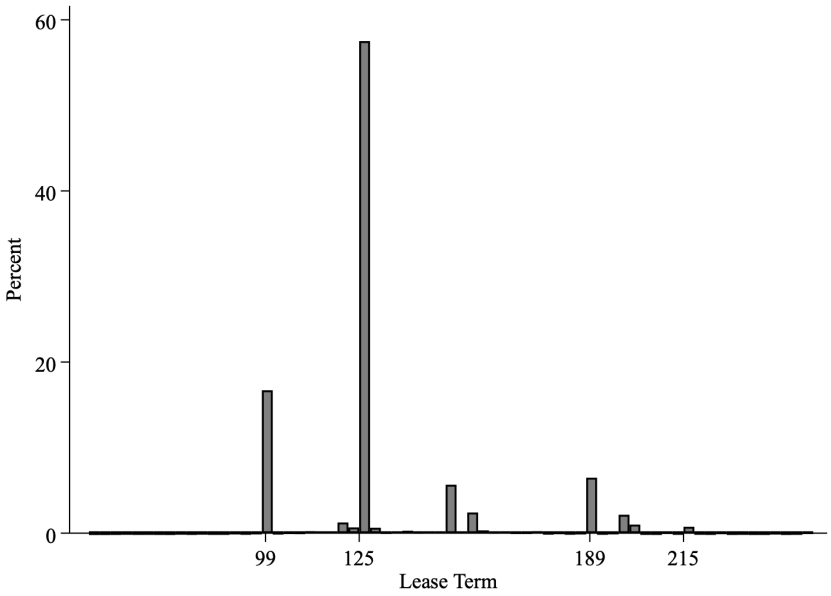
Starting in September 2021, the Land Registry began posting Change-Only Update files, which do capture extensions to new leases. We have also compiled this data into a publicly available resource, which will allow researchers to observe all extensions after this date. To infer extension before then, however, we develop an alternative method which we describe below.

A.3.1 Using Lease Terms to Identify Extensions in Public Data

Our primary method of identifying lease extensions in public Land Registry data relies on three aforementioned facts: (1) Two of the most common initial lease terms are for 99 and 125, (2) The official and most common extension length is for 90 years, and (3) When a lease is extended, the new lease has the same recorded length as the original lease, plus the amount of lease extension.

These three pieces of information suggest there should be a mass of lease terms for 99, 125, 189(= 99 + 90) years and 215(= 125 + 90) years, where the 189 and 215 year leases almost certainly refer to lease extensions. This is exactly what we observe in the data, as illustrated by Figure A.16. The most common initial lease terms are 99 and 125 (with a smaller peak at 150), but we also see masses at 189 and 215, which are very uncommon initial lease terms.

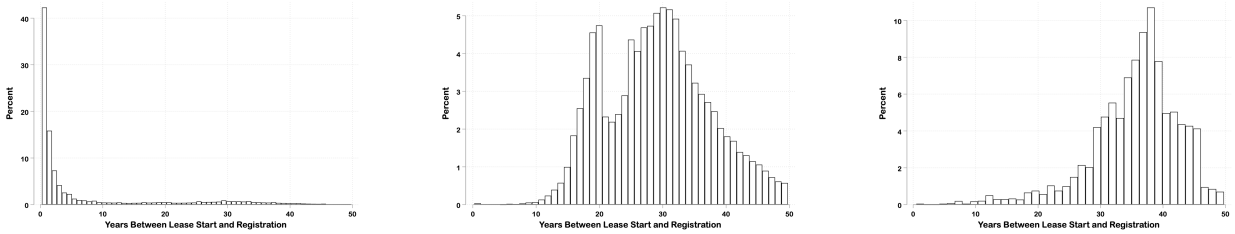
Figure A.16: Histogram of Registered Lease Terms



The figure is a histogram of the recorded initial lease term for flats in the UK that match with at least one transaction in the HMLR Transaction Data Set. The histogram is restricted to leases with initial term between 50 and 250.

The Land Registry also provides a registration date for each lease, which in the case of lease extensions will refer to the extension date. Figure A.17a shows that in the full sample, a vast majority of leases are registered shortly after their origination date. However, in Figure A.17b and A.17c we see that leases with initial terms of 189 and 215 are almost always registered several decades after the start of the lease, as expected.

Figure A.17: Time Elapsed Between Lease Origination and Registration



(a) Full Sample **(b)** Initial Term of 189 Years **(c)** Initial Term of 215 Years

The figures are histograms of the time in years between recorded lease initiation date and the recorded registration date. The first panel includes all leases belonging to flats which appear at least once in the Land Registry Transaction Data Set. The second panel includes the subset of leases from the first panel which have a recorded initial term of 189 years. The second panel includes the subset of leases from the first panel which have a recorded initial term of 215 years.

Therefore, it is safe to assume that leases with initial terms of 189 and 215 years have been extended for 90 years at the time of lease registration. Using our confidential data set on all lease extensions, we verify that over 97% of these cases are in fact extensions. Using this method, researchers can identify 105 thousand leases (2% of all registered leases) that have terms of 189 and 215 years. Twenty-five thousand of these leases have transactions before and after the extension time, and can be used to produce difference-in-difference estimators.

A.3.2 Results with Publicly Available Data

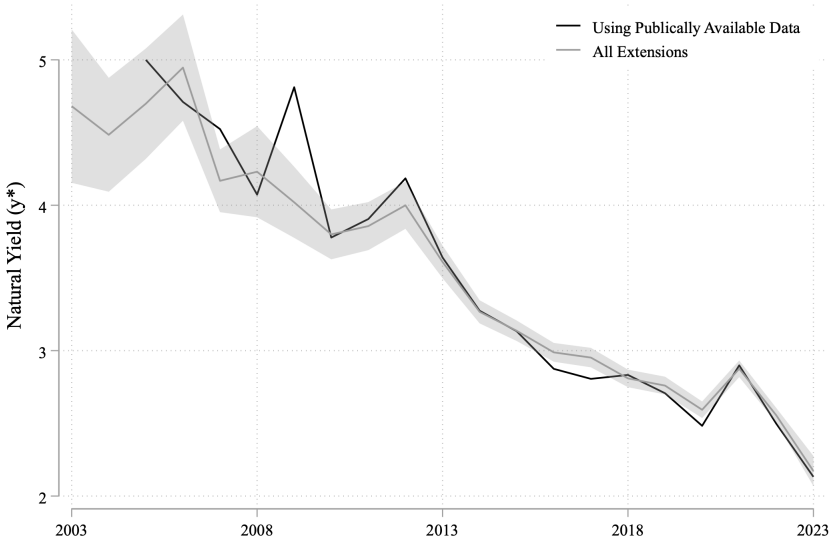
In Figure A.18, we present estimates of y_t^* using the extensions recovered by the algorithm described in Section A.3.1. We can see that the results are quantitatively very similar to those obtained using the full sample of extension experiments.

A.4 Simulation Results For Flexible Forward Curve

In Section 4.3 we presented one possible parameterization of $y(T)$. In this section, we explore other parameterizations and present several insights from our simulation. We assume that for $T \geq 40$, $y(T) = y^*$ is constant, since it is unlikely that individuals have strong expectations about economic conditions in these far away periods. We also presented evidence in Section 4.3 using bond yields that supports this assumption. Hence, for $T < 40$ we assume $y(\cdot)$ can have any shape as long as it asymptotes to y^* as $T \to 40$.

Figure A.19a shows several possible choices of $y(T)$, all of which asymptote to the same value. Figure A.19b presents the yield curves associated with each of these forward curves and Figure A.19c shows the corresponding $\hat{y}^*(T)$ curves that we estimate by (4) using simulation data. We also plot the point estimate of $\hat{y}^*$ we obtain at the median of our true distribution.

Figure A.18: Estimates Using Only Publicly Available Data



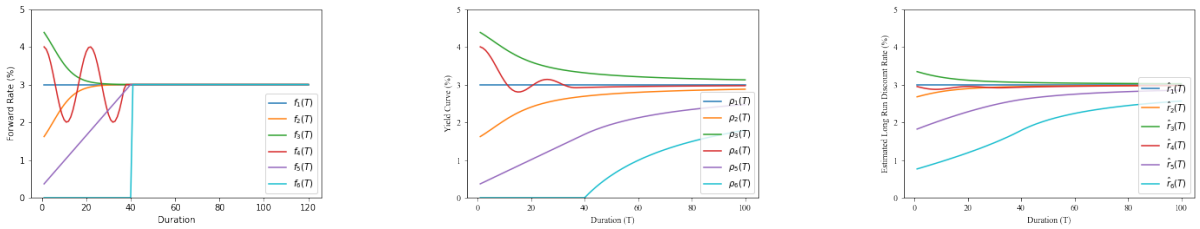
The figure presents estimates of y_t^* attained using only extensions identified with the method described in Section A.3. The dotted line shows the estimates using the full sample of data, with 95% confidence intervals shaded.

These simulation results yield several key insights. First, when the yield curve is flat, $y(T) = \rho(T) = \hat{y}^*(T) = y^*$, as exemplified with the by the dark blue line in Figure A.19. When the yield curve is not flat, however, our estimate will differ from the true asymptotic value of $y(T)$ by some amount $\eta \equiv y^* - \hat{y}^*$. When the yield curve is upwards sloping, $\eta > 0$ and when the yield curve is downwards sloping, $\eta < 0$.

Notice that for $T \geq 40$, $\hat{y}^*(T)$ converges to y^* much more quickly than $\rho(T)$. The reason for this is that our difference-in-difference estimate differences out a large portion of the short-end of the yield curve. To see this, consider a property with duration T that extends by k years to a total of 160 years ($T + k = 160$). The shorter T is, the less of the short-end that will be differenced out by our estimate. We present simulation evidence for this in Figure A.20.

We can bound the error term η by using the fact that discount rates must be non-negative. Let $F_{40} \equiv \int_0^{40} e^{-y(S)\cdot S} dS$ be the total discount rate for the first 40 periods. Given that $\hat{y}^* = 2.9\%$, our best-case scenario is a flat yield curve, as in $y_1(T)$ of Figure A.19a, in which case we would have $\underline{F}_{40} \equiv F_{40} = 23.6$ and $\underline{\eta} \equiv \eta = 0\%$. In the worst-case scenario, we have a step function like $y_6(T)$ of Figure A.19a, which is zero for $T < 40$ and y^* for $T \geq 40$. In this case, $\bar{F}_{40} \equiv F_{40} = 40$ and $\bar{\eta} \equiv \eta = 0.89\%$. Therefore, we must have $\underline{\eta} \leq \eta \leq \bar{\eta}$. We present the value of η for $\underline{F}_{40} \leq F_{40} \leq \bar{F}_{40}$ in Figure A.21. The results imply that y^* will be at most 90 basis points larger than our estimated $\hat{y}^*$.

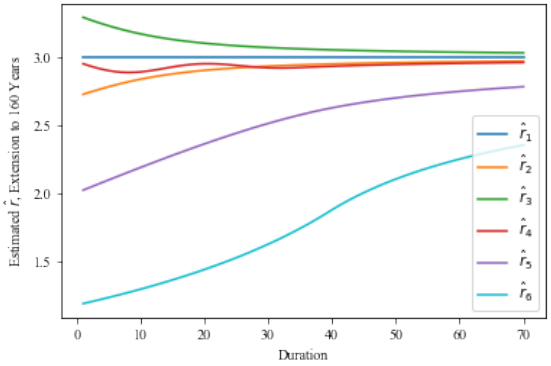
Figure A.19: Long Run Discount Rates Using Simulation Data



(a) $y(T)$ **(b)** $\rho(T)$ **(c)** $y^*(T)$

Panel (a) presents multiple possible parameterizations of $y(T)$, all of which asymptote to 3% by $T = 40$. Panel (b) presents the corresponding yield curve, $\rho(T)$, for each choice of $y(T)$. Panel (c) presents estimates of $y^*(T')$ obtained by NLLS corresponding to each choice of $y(T)$, where T' is the average between the control and extension sale duration. The point presents the estimate of y^* we would obtain at the median of our distribution.

Figure A.20: Estimated y^* When Extending From T to 160



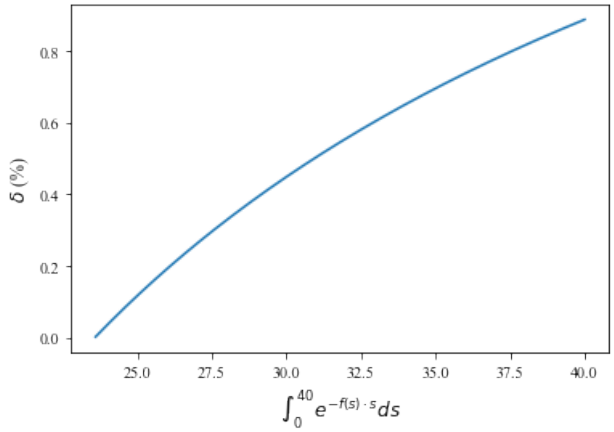
The figure indicates the point estimate of $\hat{y}^*$ we obtain by NLLS for an extension from duration T to duration 160. As the duration before extension increases, the estimate of $\hat{y}^*$ approaches the limit of the forward curve. We repeat this for each example forward curve, $y(T)$, from Figure A.19a.

A.5 Liquidity Premium

One institutional factor which could raise concerns about our estimator is the difficulty for owners of short leases to obtain a mortgage. If short leases have more limited access to financing than longer leases, we might worry that part or all of the observed price gain upon extension is a result of increased access to financing opportunities; in other words, we may wonder if the effect is driven by a "liquidity premium." Indeed, important lenders, such as Barclays, Halifax and The Co-Operative Bank, refuse to lend to leaseholds with less than 70 years remaining. Others have different thresholds, such as 55 years, and some have a preference for longer leases but allow for case-by-case exceptions.[29]

[29] A comprehensive list of lease length policies for banks in England can be found in the UK Finance Lenders' Handbook For Conveyancers.

Figure A.21: Estimated η, Given $y^* = 2.9\%$



The figure indicates the estimate of $\eta = y^* - \hat{y}^*$ for the range of possible upward sloping forward curves, given our estimate of $\hat{y}^* = 2.93\%$. In the best case, $y(T) = y^*$ for all T and so $\eta = 0$. In the worst case, $y(T)$ is a step function at 0 for $T < 40$ and at y^* for $T \geq 40$. In this case, $\int_0^4 0 e^{-y(s)s} ds = 40$ and $\eta \approx 0.9\%$.

Reassuringly, using detailed micro-data from the English Housing Survey 1993-2014, we find that mortgage access and conditions are not vastly different for shorter and longer leaseholds, especially for those with more than 30 years remaining, as indicated in Table A.5. Approximately 50-55% of short (under 80) duration leaseholds were purchased with a mortgage, relative to 60% of longer (over 80) duration leaseholds. The typical mortgage length and Loan-To-Value (LTV) ratios are similar across the duration spectrum, at around 23 years and 75-80%, respectively. Additionally, short leaseholds have similar interest rate types as long leaseholds, with 35-40% choosing adjustable rate mortgages (as opposed to mortgages with a fixed interest rate for a number of years, or tracker mortgages which are indexed to the Bank of England bank rate). These results suggest that financing constraints are unlikely to drive the very large extension price changes we observed in Section 5.

Another common way to test for the existence of a liquidity premium resulting from financing frictions is to use the amount of time a property was listed on the market (i.e. sale time minus the time of the first listing) as a proxy for its liquidity (Lippman and McCall, 1986; Lin and Vandell, 2007; Genesove and Han, 2012). The intuition is that properties for which the buyer cannot obtain a mortgage have a smaller pool of potential buyers, which ought to increase the amount of time that the property is on the market. We find limited evidence of this in the data, as illustrated in Figure A.22. After controlling for quarter $\times$ 3-digit postcode fixed effects and hedonics, the typical listing time for a 50-70 year lease is only 2-3 days longer than for a long lease of more than 100 years. This is negligible given that the average listing time is 5 months. For shorter leases, the listing period actually decreases further; all else equal, a leasehold with less than 50 years remaining will sell about

Table A.5: Mortgage Statistics For Short Leaseholds

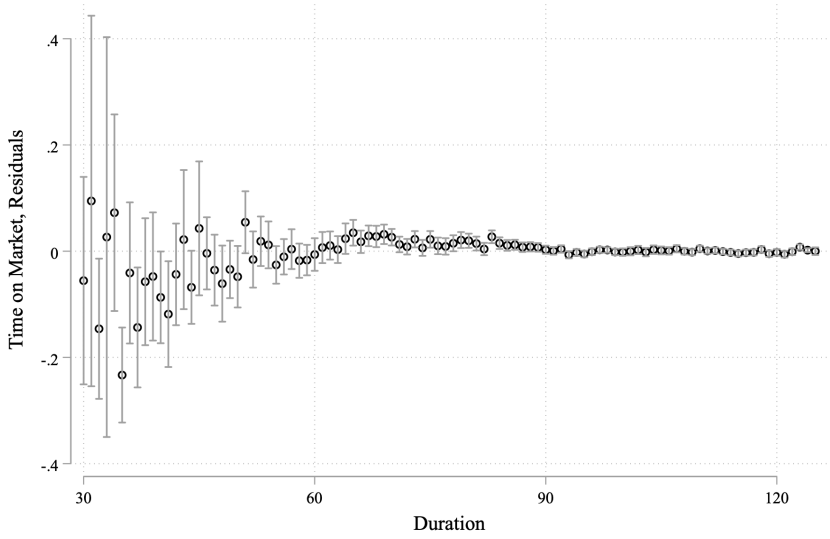
	Less Than 22 Years	22-30 Years	31-40 Years	41-60 Years	61-80 Years	80+ Years	Total
Mortgage Length	22.1	21.7	23.3	21.8	23.6	22.9	22.9
	(1.2)	(1.0)	(0.9)	(0.9)	(0.4)	(0.1)	(0.1)
LTV	73.1	85.4	79.0	76.8	79.7	75.4	75.9
	(5.6)	(4.3)	(4.8)	(3.4)	(1.4)	(0.5)	(0.5)
% Have Mortgage	58.8	54.8	49.4	46.9	51.0	60.7	60.0
	(8.6)	(3.1)	(4.0)	(3.5)	(1.7)	(0.4)	(0.3)
% Adjustable Rate	40.0	19.0	38.9	38.0	37.6	35.9	36.0
	(11.2)	(8.8)	(11.8)	(6.9)	(3.5)	(1.1)	(1.0)
N	20,028						

mean reported; standard error of mean in parentheses

The table reports several summary statistics for leases of various durations. The first row presents the average mortgage length; the second presents the average Loan-To-Value (LTV) ratio for the mortgage, calculated as the initial mortgage value divided by the market price of the property; the third row presents the percent of properties of that duration which have a mortgage; the fourth row presents the percent of properties with a mortgage that have a fully adjustable rate mortgage.

two weeks faster than a leasehold with more than 50 years.

Figure A.22: Time on Market by Lease Duration



The figure shows the mean time on market for every duration under 125, de-meaned by quarter × 3-digit postcode fixed effects and controls for bedroom count, floor area and property age.

To further test for the existence of a liquidity premium, we are able to model the case of discontinuous financing frictions and reject the existence of a liquidity premium using the data. Consider that under the threshold of 70 years–which is the most prominent bank mortgage cutoff duration—it becomes significantly more difficult to finance a leasehold property.

Then the price of a $T > 70$ duration leasehold is given by,

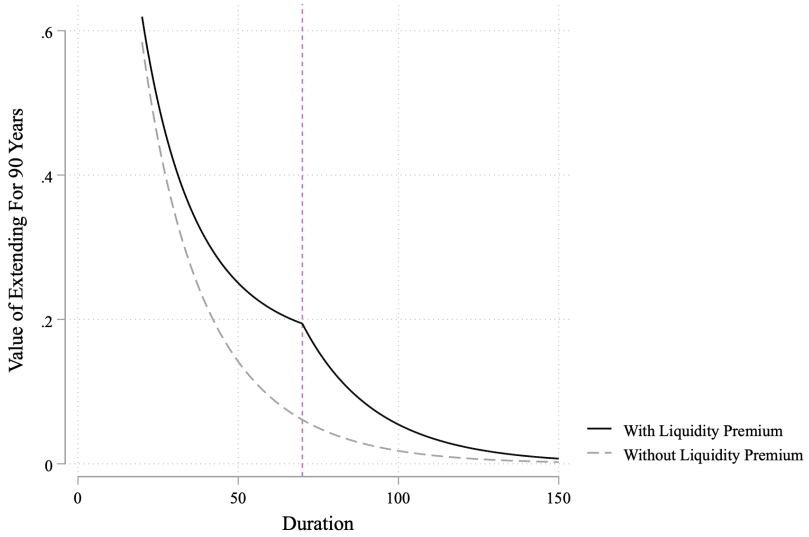
$$\begin{aligned} P_t^T &= R_t \left[\int_t^{t+(T-70)} e^{-(y^*)(s-t)} ds + e^{-(y^*)(T-70)} \int_{t+(T-70)}^{t+T} e^{-(y^*+\sigma)(s-(t+(T-70)))} ds \right] \\ &= R_t \left[\frac{1 - e^{-(y^*)(T-70)}}{y^*} + \frac{(1 - e^{-(y^*+\sigma)\cdot 70})}{y^* + \sigma} (e^{-(y^*)(T-70)}) \right] \end{aligned}$$

where rents below the cutoff of 70 are discounted at an additional rate σ. One way to think about σ is as the difference between the return on housing and outside investment options. Therefore, our difference in difference will yield the following equation:

$$\begin{aligned} \log P_t^{T+90} - \log P_t^T = &\log \left[\frac{1 - e^{-(y^*)(T+90-70)}}{y^*} + \frac{1 - e^{-(y^*+\sigma)70}}{y^* + \sigma} (e^{-(y^*)(T+90-70)}) \right] \\ &- \log \left[\frac{1 - e^{-(y^*)(\max\{0, T-70\})}}{y^*} + \frac{1 - e^{-(y^*+\sigma)(\min\{70, T\})}}{y^* + \sigma} (e^{-(y^*)(\max\{0, T-70\})}) \right] \end{aligned} \tag{22}$$

When $\sigma > 0$, the price change for extending a lease will have a kink at 70 as shown in Figure A.23. The reason for this is that as $T \to 70$, there are two incentives to extend: first is the value of 90 additional years of discounted rents and the second is the value of postponing the liquidity discount, σ. Once $T < 70$, the first incentive continues to grow, as we have discussed in earlier sections, but the second incentive becomes increasingly weaker, since there are less periods at which rents will be discounted at rate σ. Moreover, as $T \to 0$, $T + 90$ grows closer to 70, so the value of the extended lease also starts to decrease as it approaches the liquidity premium cutoff. This kink is not present in the data, which can be verified visually in Figure 7, and the existence of a discontinuous liquidity premium at $T = 70$ is rejected by estimating (22) by NLLS.

Figure A.23: Liquidity Premium Example



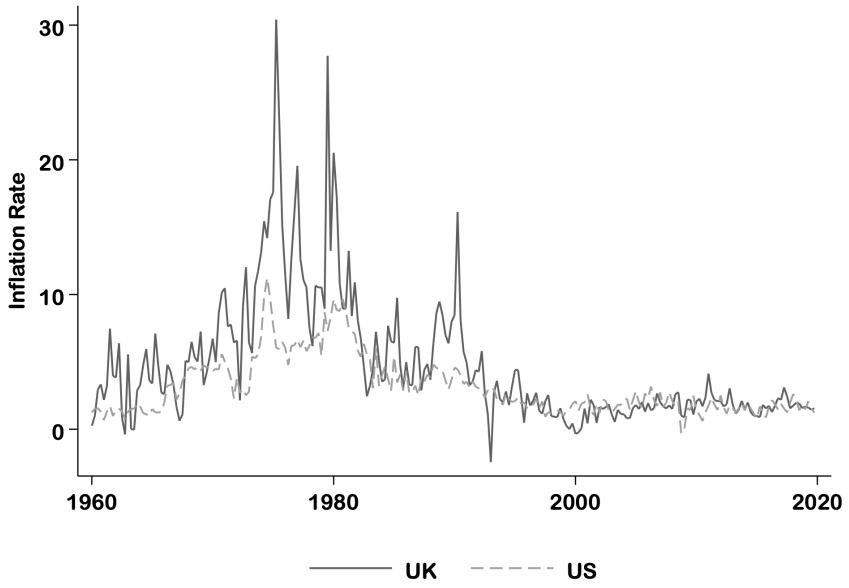
The figure shows the effect of a liquidity premium on the value of extending for 90 years ($P_t^{T+90} - P_t^T$). The dark line plots (22) when $\sigma = 0.7\%$ and the dashed light line plots the same equation absent a liquidity premium, i.e. $\sigma = 0\%$. The liquidity premium is assumed to start at $T = 70$. We can see that when there is a liquidity premium, the value of extension will exhibit a kinked shape, with a kink at $T = 70$.

A.6 Comparison with Laubach and Williams (2003) (LW)

Our estimates of y^* and its relationship to r^* are particularly important given the lack of robust methods to estimate the natural rate of interest in real-time. The most well-known approach to estimate r^* is the method proposed by Laubach & Williams (2003) and Holston, Laubach & Williams (2017). This method assumes that the output gap is an autoregressive process around it's natural rate value, which is itself a random walk with drift. This output gap is then linked to the real natural rate of interest via an Euler equation and a Phillips curve.

When estimated using US data, LW estimate that r^* has fallen by 2.9pp between 2000 and 2020, with a mean standard error on r^* of 1.35pp per quarter. When estimated with UK data, the decline of r^* over this time period is only 0.81pp, with average standard error of 4.3pp, as seen in Figure A.25a. The reason why the UK estimate has much larger standard errors has to do with higher inflation volatility in the UK, which is a small open economy and is therefore more responsive to exchange rate fluctuations (Figure A.24).

Figure A.24: Inflation Rate In US and UK

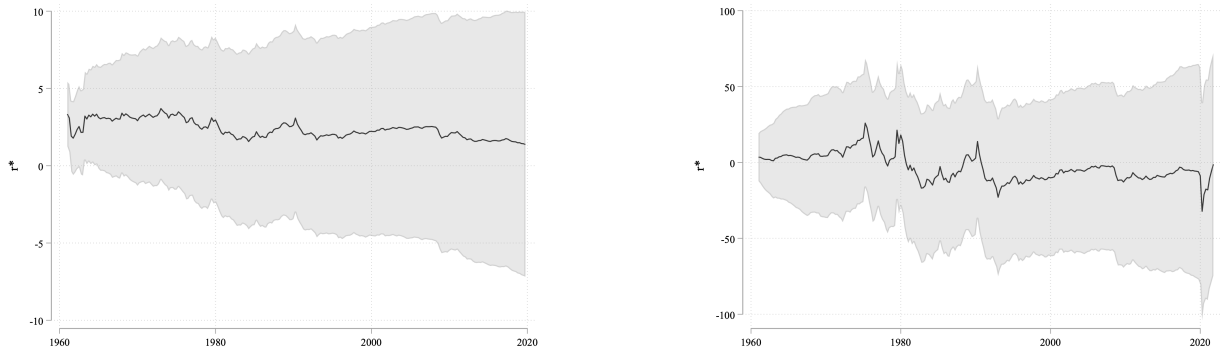


The solid dark grey line shows inflation in the UK between 1960-2020. The dashed light grey line shows inflation in the US between 1960-2020. The data is the same as that used by LW.

Our estimates have a number of advantages to those of LW. First, our estimate relies on minimal structure and is largely model-free. Moreover, our standard errors are on average 0.2 percentage points over our sample period — an order of magnitude smaller than the LW r^* estimate for the UK.

Most importantly, our estimates persist throughout the pandemic and post-pandemic era and provide valuable insight on the growing wedge between discount rates of housing and of long-term government bonds. In contrast, when post-2020 data is included, the LW methodology is de-stabilized by the steep decline in GDP in the second quarter of 2020. In the case of the US, the model is incapable of converging when post 2020 data is included. In the UK, inclusion of post 2020 data results in implausible estimates; ranging from -32.2% in the third quarter of 2020 to 25.9% in the second quarter of 1975. This error not only affects the pandemic era, but also propagates to previous to previous decades (Figure A.25b).

Figure A.25: LW estimate of r^*



(a) Pre COVID-19 Pandemic **(b)** Post COVID-19 Pandemic

Panel (a) shows LW's estimates of r^* in the UK using pre-2020 data. Panel (b) shows LW's estiamtes of r^* using post-2020 data. The shaded area represents the 95% confidence interval.

A.7 Calculating the Extension Hazard Rate

In this section, we explain how we calculate the extension hazard rate, shown in Figure A.3 and Figure A.4. We define the conditional probability that a property i extends given that it has duration T as $\theta_i(T) = P(\text{Extended At } S|\text{Duration} = S)$. To get the total cumulative probability that a T duration property i will extend over the course of its lifetime, we must convert our conditional probabilities to unconditional probabilities as follows,

$$\begin{aligned}\pi_i(S) &= P_i(\text{Extended At } S) \\ &= P_i(\text{Extended At } i|\text{Duration} = S)P_i(\text{Duration} = i) \\ &= \theta_i(S)\prod_{U=S+1}^{T}(1-\theta_i(U))\end{aligned}$$

The cumulative probability that property i extends over its lifetime is then given by $\Pi_i(T) = \sum_{S=1}^{T}\pi_i(S)$. In Figure A.4, we scale the hazard rate up by a factor of 1.17. This is because our method to identify lease extensions does not capture extensions that have no transactions before extension. We estimate that there are about 17% more extensions that have been extended but do not have pre-extension transaction data.

Then, the price of a T duration property at time t is given by the following recursive formula,

$$P_{i,t}^{T} = \frac{R_{i,t+1} + \theta_i(T)(P_{i,t+1}^{T+90-1} - \kappa_{t+1}^{T-1}) + (1-\theta_i(T))P_{t+1}^{T-1}}{1 + r^* + \zeta^*} \tag{23}$$

Intuitively, the price of a T duration asset is the discounted dividends it yields next period,

plus with probability $1-\theta_i(T)$, the price of a $P_{i,t+1}^{T-1}$ asset, and with probability $\theta_i(T)$, the price of a $P_{i,t+1}^{T+90-1}$ duration asset minus the cost of extending, all appropriately discounted.

A.8 Tobin's Q Model of Firm Investment and Capital Supply

This section considers a standard Tobin's Q model of firm investment. We show that this model is equivalent to the model of the main text. In particular, the supply elasticity of capital affects the pass through of r^* to y^* in the same way as in the main text.

A.8.1 Setup

Consider a continuum of firms $i \in [0,1]$ who each have profit $\Pi_{it} = A_t K_{it}^{\alpha}$ where A_t is an exogenous productivity term that grows at a constant rate g_A, K_{it} is firm i's capital, and $\alpha \in (0,1)$. In general, variables without i subscripts are the aggregate of corresponding variables with i subscripts, for instance $K_t \equiv \int_0^1 K_{it} di$. The firm can either install capital or frictionlessly rent it at rental rate R_t. Capital depreciates at rate δ .

If the firm chooses to install capital, it faces the problem

$$V(K_{it}) = \max_{\{I_{is}, K_{is}\}_{s \geq 0}} \int_t^{\infty} e^{-r(s-t)} \left(A_s K_{is}^{\alpha} - R_s \Phi(I_{st}, K_s)\right) ds \tag{24}$$

subject to the law of motion for firm capital

$$\dot{K}_{it} = I_{it} - \delta K_{it}$$

where r is the exogenous discount rate, and $\Phi(I_{it}, K_t)$ is a function capturing capital adjustment costs. Capital adjustment costs are a constant returns to scale function in I_{it} and K_t, i.e. in firm level investment and aggregate capital. Therefore there are external adjustment costs. Capital adjustment costs are also paid in units of the rental rate of capital R_t. That is, firms must rent some capital, and pay an adjustment cost, in order to install more capital.

A.8.2 Solving For Equilibrium

The Hamiltonian of the firm problem (24) is

$$\begin{aligned}
\mathcal{H}_{it} &= A_t K_{it}^{\alpha} - R_t \Phi(I_{it}, K_t) + \lambda_t (I_{it} - \delta K_{it}) \\
&= A_t K_{it}^{\alpha} - R_t K_t \phi\left(\frac{I_{it}}{K_t}\right) + \lambda_t (I_{it} - \delta K_{it})
\end{aligned}$$

where$\phi\left(\frac{I_{it}}{K_t}\right) \equiv \Phi\left(\frac{I_{it}}{K_t}, 1\right)$.

The optimality conditions associated with the Hamiltonian are

$$A_t \alpha K_{it}^{\alpha-1} - \delta\lambda_t = r\lambda_t - \dot{\lambda}_t \tag{25}$$

and

$$-R_t K_t \phi'\left(\frac{I_{it}}{K_t}\right)\frac{1}{K_t} + \lambda_t = 0 \tag{26}$$

as well as a transversality condition and the law of motion of capital.

Solving equation (25) and imposing balanced growth implies

$$A_t \alpha K_{it}^{\alpha-1} - \delta\lambda_t = r\lambda_t - \dot{\lambda}_t$$

$$\implies A_t \alpha K_{it}^{\alpha-1} = (r+\delta)\lambda_t - \dot{\lambda}_t$$

$$\begin{aligned}
\implies \lambda_t &= \int_t^\infty e^{-(r+\delta)(s-t)} A_s \alpha K_{is}^{\alpha-1} ds \\
&= \int_t^\infty e^{-(r+\delta)(s-t)} e^{(g_A+(\alpha-1)g_K)(s-t)} A_t \alpha K_{it}^{\alpha-1} ds \\
&= \frac{A_t \alpha K_{it}^{\alpha-1}}{r+\delta-(g_A+(\alpha-1)g_K)}
\end{aligned} \tag{27}$$

where we use

$$\begin{aligned}
d\log\left(A_s \alpha K_{is}^{\alpha-1}\right) &= d\log(A_s) + d\log\left(K_{is}^{\alpha-1}\right) \\
&= d\log(A_s) + (\alpha-1)\, d\log(K_{is}) \\
&= g_A + (\alpha-1) g_K,
\end{aligned}$$

meaning

$$\frac{\lambda_t}{A_t \alpha K_{it}^{\alpha-1}} = \frac{1}{r+\delta-g_A+(1-\alpha)g_K}. \tag{28}$$

We also have from equation (26)

$$-R_t K_t \phi'\left(\frac{I_{it}}{K_t}\right)\frac{1}{K_t} + \lambda_t = 0$$

$$\implies \lambda_t = R_t \phi'\left(\frac{I_{it}}{K_t}\right)$$

$$\implies \lambda_t = R_t \phi'(g_K + \delta) \tag{29}$$

where we have used that the law of motion of capital is

$$\dot{K}_{it} = I_{it} - \delta K_{it}$$

$$\implies \frac{\dot{K}_{it}}{K_{it}} = \frac{I_{it}}{K_{it}} - \delta$$

$$\implies g_K = \frac{I_{it}}{K_{it}} - \delta$$

$$\implies g_K + \delta = \frac{I_{it}}{K_{it}}.$$

A.8.3 Characterizing Equilibrium

Therefore we can characterize the balanced growth path by

$$\frac{\lambda_t}{A_t \alpha K_{it}^{\alpha-1}} = \frac{1}{r + \delta - g_A + (1-\alpha) g_K}$$

$$\frac{\lambda_t}{R_t} = \phi'(g_K + \delta)$$

$$R_t = A_t \alpha K_{it}^{\alpha-1}$$

where we have stated equations (28) and (29), and the final equation is an equilibrium condition that requires the rental rate on capital to equal its marginal product. R_t/λ_t is the yield on capital, since λ_t is the present value of the marginal product of capital (see equation 27) and R_t is the current rental rate of capital. λ_t/R_t is also the same as Tobin's Q and/or the price-dividend ratio of capital. We can write this system more compactly as

$$\frac{\lambda_t}{R_t} = \frac{1}{r + \delta - g_A + (1-\alpha) g_K} \tag{30}$$

$$\frac{\lambda_t}{R_t} = \phi'(g_K + \delta) \tag{31}$$

which is a "demand curve" and a "supply curve" for capital.

This economy is also equivalent to our model with housing from the main text. All that is required to see the exact parallel is to replace λ_t, the marginal value of installed capital, with P_t, the price of capital in the main text.

A.9 Difference-in-Differences Estimator with Option Value

Let Π_{Tt}^H be the likelihood that a lease, with $T > 80$ years of duration remaining, extends before its duration reaches 80 years. Let Π_{Tt}^L be the likelihood that a lease with $T \leq 80$ years of duration remaining is extended at some point before expiration. Assume a constant discount rate r_K^*. Then the price of a leasehold is

$$P_t^T = \int_0^T e^{-r_K^* s} R_{t+s} ds + \Pi_{Tt}^H \left(1 - \alpha_t^H\right) \int_T^{T+90} e^{-r_K^* s} R_{t+s} ds + \Pi_{Tt}^L \left(1 - \alpha_t^L\right) \int_T^{T+90} e^{-r_K^* s} R_{t+s} ds.$$

In this equation, the first term is the present value of the first T years of service flow. The second term is the next 90 years of service flow, scaled by the share going to the leasholder, $\left(1 - \alpha^H\right)$; and the likelihood that the lease extends at any time before it falls below 80 years remaining, Π_H^T. The third term is the analogous option value if the lease extends with less than 80 years remaining. Rearranging this expression implies

$$\begin{aligned}
P_t^T &= \int_0^T e^{-r_{Kt}^* s} R_{t+s} ds + \Pi_{Tt}^H \left(1 - \alpha_t^H\right) \int_T^{T+90} e^{-r_{Kt}^* s} R_{t+s} ds + \Pi_{Tt}^L \left(1 - \alpha_t^L\right) \int_T^{T+90} e^{-r_{Kt}^* s} R_{t+s} ds \\
&= \int_0^T e^{-y_t^* s} R_t ds + \Pi_{Tt}^H \left(1 - \alpha_t^H\right) e^{-y_t^* T} R_t \int_0^{90} e^{y_t^* s} ds + \Pi_{Tt}^L \left(1 - \alpha_t^L\right) e^{-y_t^* T} R_t \int_0^{90} e^{-y_t^* s} ds \\
&= \int_0^T e^{-y_t^* s} R_t ds + e^{-y_t^* T} R_t \int_0^{90} e^{y_t^* s} ds \left[\Pi_{Tt}^H \left(1 - \alpha_t^H\right) + \Pi_{Tt}^L \left(1 - \alpha_t^L\right)\right] \\
&= \int_0^T e^{-y_t^* s} R_t ds + e^{-y_t^* T} R_t \int_T^{T+90} e^{y_t^* s} ds \left[\Pi_{Tt}^H \left(1 - \alpha_t^H\right) + \Pi_{Tt}^L \left(1 - \alpha_t^L\right)\right] \\
&= \frac{1 - e^{-y_t^* T}}{y_t^*} R_t + e^{-y_t^* T} R_t \frac{1 - e^{-y_t^* 90}}{y_t^*} \left[\Pi_{Tt}^H \left(1 - \alpha_t^H\right) + \Pi_{Tt}^L \left(1 - \alpha_t^L\right)\right].
\end{aligned}$$

Then for two properties i and j with identical service flow growth, where property i extends and j does not, we have

$$\begin{aligned}
\Delta_{it}^T &= \log\left(\frac{1 - e^{-y_t^*(T+90)}}{y_t^*}\right) - \log\left(\frac{1 - e^{-y_t^* T}}{y_t^*} + e^{-y_t^* T} \frac{1 - e^{-y_t^* 90}}{y_t^*} \left[\Pi_{Tt}^H \left(1 - \alpha_t^H\right) + \Pi_{Tt}^L \left(1 - \alpha_t^L\right)\right]\right) \\
&= \log\left(1 - e^{-y_t^*(T+90)}\right) - \log\left(\left(1 - e^{-y_t^* T}\right) + e^{-y_t^* T} \left(1 - e^{-y_t^* 90}\right) \left[\Pi_{Tt}^H \left(1 - \alpha_t^H\right) + \Pi_{Tt}^L \left(1 - \alpha_t^L\right)\right]\right),
\end{aligned}$$

which is the expression in the main text.

A.10 Estimating Change in Option Value Using Discontinuities

As before, assume that $\alpha_t^H = \alpha_t^T$ for $T \geq 80$ and $\alpha_t^L = \alpha_t^T$ for $T < 80$, such that the share of holdup in a given time period t is fixed above and below 80, separately. In this section,

we aim to estimate $\alpha_t^L - \alpha_t^H$ for the post-2010 period, using the discontinuity in prices at $T = 80$ observed in Table 5. From the preceding subsection, the price of a property i with duration T is

$$P_{it}^T = \frac{R_{it}}{y_t^*}\left(1 - e^{-y^*T} + \left[\Pi_{Tt}^H\left(1-\alpha_t^H\right) + \Pi^L\left(1-\alpha_t^L\right)\right]e^{-y_t^*T_{it}}\left(1-e^{-y_t^*90}\right)\right) \tag{32}$$

To condense notation, denote the option value term

$$\Omega(T) \equiv \left[\Pi_{Tt}^H\left(1-\alpha_t^H\right) + \Pi^L\left(1-\alpha_t^L\right)\right]e^{-y_t^*T_{it}}\left(1-e^{-y_t^*90}\right).$$

Then, the difference in change in price between time $t-h$ and t of properties i and j is,

$$\begin{aligned}\Delta \log P_{it}^T - \Delta \log P_{jt}^T = &\log(1 - e^{-y^*T_i} + \Omega(T_i)) - \log(1 - e^{-y^*(T_i+h)} + \Omega(T_i+h)) \\ &- (\log(1 - e^{-y^*T_j} + \Omega(T_j)) - \log(1 - e^{-y^*(T_j+h)} + \Omega(T_j+h_j))).\end{aligned}$$

This equation acknowledges that option value will discontinuously change around the 80 year threshold, via changes in the Ω terms. We can then estimate α_t^H by nonlinear least squares on the same sample as for regression equation 20, by setting $\alpha_t^L = 1$, which is what we estimate in Table 6, and y^* to its mean for that period. We obtain an estimate of $\alpha_t^H = 0.56$ in the post-2010 period.

A.11 Proofs

A.11.1 Model Proofs

Proposition A.1. *The supply curve for the housing market is given by,*

$$\frac{P(t)}{R(t)} = \phi'(g_H + \delta) \tag{33}$$

Proof. First, we substitute the re-scaled adjustment cost $H(t)\phi\left(\frac{B(t)}{H(t)}\right) = \Phi(B(t), H(t))$ into (9), which yields,

$$\Pi(t) = P(t)B(t) - R(t)H(t)\phi\left(\frac{B(t)}{H(t)}\right)$$

Taking the first order condition with respect to building, $B(t)$, yields,

$$\begin{aligned}\frac{d\Pi(t)}{dB(t)} &= P(t) - R(t)\phi'\left(\frac{B(t)}{H(t)}\right) = 0 \\ \frac{P(t)}{R(t)} &= \phi'\left(\frac{B(t)}{H(t)}\right)\end{aligned} \tag{34}$$

Moreover, from the housing stock law of motion, (11), we can see that,

$$\begin{aligned}\dot{H} &= g_H \cdot H(t) = B(t) - \delta H(t) \\ \frac{B}{H} &= g_H + \delta\end{aligned}$$

Substituting this into (34), we get the desired result,

$$\frac{P(t)}{R(t)} = \phi'(g_H + \delta)$$

□

Proposition A.2. *Under a balanced growth path,*

$$g_H = \alpha g_N - \beta g^* \tag{35}$$

Proof. We begin by taking the derivative of (7), with respect to time,

$$\dot{H} = \alpha\Gamma N(t)^{\alpha-1}\dot{N}P(t)^{-\beta} - \beta\Gamma N(t)^{\alpha}P(t)^{-\beta-1}\dot{P}$$

substituting $\dot{P} = g^* \cdot P(t)$, $\dot{H} = g_H \cdot H(t)$, and $\dot{N} = g_N \cdot N(t)$, we get,

$$\begin{aligned}g_H \cdot H(t) &= \left[\Gamma N(t)^{\alpha}P(t)^{-\beta}\right](\alpha g_N - \beta g^*) \\ g_H \cdot H(t) &= H(t) \cdot (\alpha g_N - \beta g^*) \\ g_H &= \alpha g_N - \beta g^*\end{aligned}$$

where the second step relies on substituting (7) for $H(t)$. □

Proposition A.3. *A fall in r^* weakly raises price to rent ratios, and the extent of the rise depends on the slope of the supply curve. More specifically,*

$$0 \leq \frac{\partial y^*}{\partial r^*} \leq 1$$

Proof. From the supply schedule,

$$y^* = \frac{1}{\phi'(g_H - \delta)}$$

and from the demand schedule,

$$y^* = \frac{R(t)}{P(t)} = r^* + \zeta^* - \frac{\alpha g_N - g_H}{\beta}$$

At equilibrium, the supply and demand curves are equal. Therefore,

$$r^* + \zeta^* - \frac{\alpha g_N - g_H}{\beta} = \frac{1}{\phi'(g_H - \delta)}$$

By the Implicit Function Theorem, we can substitute $g_H = g_H(r^*)$. Differentiating both sides with respect to r^*,

$$\begin{aligned} 1 + \frac{1}{\beta}\frac{\partial g_H}{r^*} &= -\left[\phi'(g_H + \delta)\right]^{-2} \phi''(g_H + \delta)\frac{\partial g_H}{\partial r^*} \\ \frac{\partial g_H}{\partial r^*} &= \frac{-1}{1/\beta + \left[\phi'(g_H + \delta)\right]^{-2} \phi''(g_H + \delta)} \end{aligned}$$

Then,

$$\begin{aligned} \frac{\partial y^*}{\partial r^*} &= \frac{1}{\partial r^*}\left(r^* + \zeta^* - \frac{\alpha g_N - g_H}{\beta}\right) \\ &= 1 - \frac{1}{\beta} \cdot \frac{\partial g_H}{\partial r^*} \\ &= 1 - \frac{1}{1 + \beta\left[\phi'(g_H + \delta)\right]^{-2} \phi''(g_H + \delta)} \end{aligned}$$

Observe that when $\phi''(g_H + \delta) = 0$, $\frac{\partial y^*}{\partial r^*} = 1$ and as $\phi''(g_H + \delta) \to \infty$, $\frac{\partial y^*}{\partial r^*} \to 0$. For all $0 \leq \phi''(g_H + \delta) \leq \infty$, $0 < \frac{y^*}{r^*} < 1$. □

A.11.2 Option Value Proofs

Proposition A.4. *There exists some value $\bar{y} < y_{RV}$ such that:*

1. *If the natural yield satisfies $y_t^* \geq \bar{y}$ then*
   (a) *There is zero option value at all years of duration remaining, that is, $\alpha_t^T = 1$ for all T.*

(b) *The price of a leasehold is continuous in duration as the property's duration falls below 80 years, so*

$$\lim_{T\to 80^-} P_{it}^T = \lim_{T\to 80^+} P_{it}^T.$$

2. *If the natural yield satisfies* $y_t^* < \bar{y}$ *then*

(a) *There is positive option value above 80 years in duration, that is,* $\alpha_t^T < 1$ *for all* $T > 80$ *and option value discontinuously falls at 80 years, so that* α_t^T *discontinuously increases at* $T = 80$.

(b) *The price of a leasehold discontinuously falls as the property's duration falls below 80 years, so*

$$\lim_{T\to 80^-} P_{it}^T < \lim_{T\to 80^+} P_{it}^T.$$

Proof. From equations (18) and (19), the price of a property is

$$P_{it}^T = \begin{cases} P_{it}^{T+90} - \min\left[RV_{it}^T + \gamma R_{it}, MV_{it}^T\right] & T \geq 80 \\ P_{it}^{T+90} - \min\left[\frac{RV_{it}^T + MV_{it}^T}{2} + \gamma R_{it}, MV_{it}^T\right] & T < 80. \end{cases} \tag{36}$$

Recall the definitions of reversion value and marriage value

$$MV_{it}^T = \frac{R_{it}}{y_t^*}\left(e^{-y_t^* T} - e^{-y_t^*(T+90)}\right) \tag{37}$$

$$RV_{it}^T = \frac{R_{it}}{y_{RV}}\left(e^{-y_{RV} T} - e^{-y_{RV}(T+90)}\right) \tag{38}$$

We will define $\bar{y}$ as the value of y_t^* such that $RV_{it}^T + \gamma R_{it} = MV_{it}^T$, that is, the tribunal costs for a lease above 80 years are exactly the market value. The value of $\bar{y}$ satisfies

$$\frac{R_{it}}{y_{RV}}\left(e^{-y_{RV} T} - e^{-y_{RV}(T+90)}\right) + \gamma R_{it} = \frac{R_{it}}{\bar{y}}\left(e^{-\bar{y} T} - e^{-\bar{y}(T+90)}\right)$$

$$\implies \frac{e^{-y_{RV} T} - e^{-y_{RV}(T+90)}}{y_{RV}} + \gamma = \frac{e^{-\bar{y} T} - e^{-\bar{y}(T+90)}}{\bar{y}} \tag{39}$$

where in the first line we have substituted in the definitions of marriage value (37) and reversion value (38). The right hand side of equation (39) is strictly decreasing in $\bar{y}$. Therefore there is a unique value of $\bar{y}$ satisfying the equation.

Now we will prove part (1) of the proposition, in which $y_t^* \geq \bar{y}$. Equation (39) implies that for all $y_t^* \geq \bar{y}$ we must have

$$RV_{it}^T + \gamma R_{it} \geq MV_{it}^T. \tag{40}$$

Equation (40) implies

$$\begin{aligned} RV_{it}^T + \gamma R_{it} &\geq MV_{it}^T \\ \implies RV_{it}^T &\geq MV_{it}^T \\ \implies \frac{RV_{it}^T + MV_{it}^T}{2} &\geq MV_{it}^T \\ \implies \frac{RV_{it}^T + MV_{it}^T}{2} + \gamma R_{it} &\geq MV_{it}^T \end{aligned} \tag{41}$$

Therefore for $y_t^* \geq \bar{y}$, prices satisfy

$$P_{it}^T = \begin{cases} P_{it}^{T+90} - MV_{it}^T & T \geq 80 \\ P_{it}^{T+90} - MV_{it}^T & T < 80, \end{cases} \tag{42}$$

where we have substituted equations (40) and (41) into equation (36) for $y_t^* \geq \bar{y}$.Recall the definition of α_t^T as the ratio of the lease extension cost to MV_{it}^T. Equation (42) shows that $\alpha_t^T = 1$ for all t, which proves part (1a) of the proposition. Since the top and bottom of equation (42) are equal at $T = 80$, prices are continuous at $T = 80$, which proves part (1b) of the proposition.

Now we will prove part (2) of the proposition, in which $y_t^* < \bar{y}$. Equation (39) implies that for all $y_t^* < \bar{y}$ we must have

$$RV_{it}^T + \gamma R_{it} < MV_{it}^T. \tag{43}$$

Then by equation (36), the price of a property with more than 80 years duration remaining is

$$P_{it}^T = P_{it}^{T+90} - \left(RV_{it}^T + \gamma R_{it}\right).$$

The price of a property with less than 80 years remaining is

$$P_{it}^T = P_{it}^{T+90} - \min\left[\frac{RV_{it}^T + MV_{it}^T}{2} + \gamma R_{it}, MV_{it}^T\right]. \tag{44}$$

Also, note that

$$RV_{it}^T + \gamma R_{it} < \min\left[\frac{RV_{it}^T + MV_{it}^T}{2} + \gamma R_{it}, MV_{it}^T\right], \tag{45}$$

since $RV_{it}^T + \gamma R_{it} < MV_{it}^T$ by inequality (43) and also by inequality (43) we have

$$RV_{it}^T + \gamma R_{it} < MV_{it}^T$$

$$\implies RV_{it}^T < MV_{it}^T$$

$$\implies 2RV_{it}^T < RV_{it}^T + MV_{it}^T$$

$$\implies RV_{it}^T < \frac{RV_{it}^T + MV_{it}^T}{2}$$

$$\implies RV_{it}^T + \gamma R_{it} < \frac{RV_{it}^T + MV_{it}^T}{2} + \gamma R_{it}.$$

Equations (44) and (45) imply that for $T < 80$

$$P_{it}^T < P_{it}^{T+90} - \left(RV_{it}^T + \gamma R_{it}\right)$$

Therefore prices discontinuously fall when T falls below 80 which proves part (2b) of the proposition. Since lease extension costs rise when T falls below 80, α_t^T also discontinuously rises when T falls below 80, which is part (2a) of the proposition.

□