Neoclassical Growth in an Interdependent World

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Motivation

- Closed-economy neoclassical growth model remains a key benchmark for thinking about cross-country income dynamics
- In the closed-economy, each country converges to its own steady-state level of income per capita (conditional convergence)
- Open economy versions of this model often make strong assumptions about substitutability and/or frictions in goods and capital markets
  - Goods are assumed to be homogeneous across countries or trade is assumed to be costless
  - Capital is assumed to be homogeneous, implying large net capital flows to arbitrage away differences in rates of return
- We generalize the neoclassical growth model to allow for costly trade and capital flows with imperfect substitutability
- We simultaneously model
  1. Intra-temporal goods trade subject to trade frictions
  2. Intra-temporal capital allocations subject to capital market frictions
  3. Intertemporal consumption-savings choices (and current account)

This Paper

- We show that our framework is consistent with a number of features of observed data on trade flows and capital holdings
  - Gravity equation for trade in goods and capital holdings
  - Determinate predictions for gross and net capital holdings
  - Relatively low capital flows to capital-scarce countries
- Generalize existing dynamic exact-hat algebra techniques for counterfactuals to allow for bilateral trade and capital holdings
- Linearize the model to obtain closed-form solution for transition path
- Goods trade and capital holdings interact to shape the speed of convergence to steady-state in neoclassical growth model
  - Opening goods trade alone raises the speed of convergence
  - Opening capital flows alone raises the speed of convergence
  - Opening goods trade and capital flows reduces the speed of convergence
- Since our framework incorporates bilateral trade and capital holdings and intertemporal consumption-saving, it is well suited to counterfactuals for both goods and capital market integration
  - Decoupling of China and the United States

Related Literature

- Neoclassical models of growth
  - Ramsey (1928), Solow (1956), Swan (1956), Barro (1991), Mankiw et al. (1992), King & Rebelo (1993), Ventura (1997), Acemoglu & Ventura (2002)
- Quantitative international trade
  - Arkolakis et al. (2012), Adão et al. (2019), Baqaee & Farhi (2019), Huo et al. (2019), Barthelme et al. (2019), Kleinman et al. (2020, 2021)
  - Ju et al. (2014), Reyes-Heroles (2016), Eaton et al. (2016), Ravikumar et al. (2019)
- International finance and macroeconomics
  - Global imbalances and capital flows: Lucas (1990), Obstfeld & Rogoff (1996, 2000), Jin (2012), Gourinchas & Rey (2007), Gourinchas & Jeanne (2006, 2013), Maggiori et al. (2020), Auclert et al. (2020), Coppola et al. (2021), Atkeson et al. (2022)
  - Imperfect substitutability in capital markets: Koijen and Yogo (2019, 2020), Auclert et al. (2022) and Maggiori (2021)
  - International propagation of shocks: Backus et al. (1992), Kose et al. (2003), Cravino & Levchenko (2017), di Giovanni et a. (2022)
  - Home bias and international diversification: Obstfeld (1994), Cole & Obstfeld (1991), Martin and Rey (2004, 2006), Engel & Maysumoto (2009), Mendoza et al. (2009), Heathcote & Perri (2013), Coeurdacier & Gourichas (2016), Pellegrino et al. (2021), Jiang et al. (2022), Chau (2022), Hu (2022), Kucheryavyy (2022)
  - Gravity equation in finance: Portes & Rey (2005)

Outline

- Theoretical Framework
- Data
- Empirical Evidence
- Conclusions

Model Setup

- Economy consists of many countries n, $i \in \{1, \ldots, N\}$
- Time is discrete and indexed by $t \in \{0, \ldots, \infty\}$
- Each country supplies a differentiated good that is produced using labor and capital under constant returns to scale
- Markets are perfectly competitive
- Goods can be traded subject to bilateral trade costs
- Representative agent in each country endowed with labor ℓ_n
- At the beginning period t, representative agent in each country inherits a stock of wealth a_{nt}
- Wealth can be allocated to each country subject to capital market frictions and idiosyncratic productivity shocks
- Beginning period t: choose wealth allocation across countries and make consumption-saving decisions
- Beginning period $t + 1$: investment returns realized, depreciation occurs, and wealth again allocated across countries
- No aggregate uncertainty and perfect foresight

Intertemporal Preferences

- In country n, the mass ℓ_n of representative consumers solve

$$\max_{\{c_{nt},k_{nit}\}} \sum_{s=0}^{\infty} \beta^{t+s} \frac{c_{nt+s}^{1-1/\psi}}{1-1/\psi}$$

$$\text{s.t.} \quad p_{nt} c_{nt} + p_{nt} \sum_{i=1}^{N} a_{nit+1} = \left(p_{nt}\left(1-\delta\right) + v_{nt}\right) \sum_{i=1}^{N} a_{nit} + w_{nt} \ell_n$$

$$\text{Or equivalently s.t.:} \quad c_{nt} + a_{nt+1} = \mathcal{R}_{nt} a_{nt} + \frac{w_{nt}\ell_n}{p_{nt}}$$

- δ is depreciation rate; v_{nt} is return to capital; p_{nt} is consumption price index; $\mathcal{R}_{nt} = 1 - \delta + v_{nt}/p_{nt}$ is real gross return to investment
- Consumption is linear function of current wealth (Angeletos 2007)

$$c_{nt} = \varsigma_{nt} \left(\mathcal{R}_{nt} a_{nt} + \frac{w_{nt}\ell_n}{p_{nt}} + h_{nt} \right)$$

- where ς_{nt} is defined recursively as

$$\varsigma_{nt}^{-1} = 1 + \beta^{\psi} \phi_{nt+1}^{\psi} \mathcal{R}_{nt+1}^{\psi-1} \varsigma_{nt+1}^{-1}$$

Capital Allocation Within Each Period

- Each unit of capital subject to idiosyncratic productivity shocks (φ_{nit})
- Iceberg capital market frictions: $\kappa_{nit} > 1$ for $i \neq n$; $\kappa_{nnt} = 1$
- Return to a unit of capital invested from source n in host i (v_{nit})

$$\frac{\varphi_{nit} r_{it}}{\kappa_{nit}}, \qquad \varphi \sim e^{-\eta_{it}\varphi^{-\epsilon}}, \qquad \epsilon > 1$$

- η_{it} controls average host capital productivity (e.g., property rights)
- Bilateral capital investments satisfy a gravity equation

$$b_{nit} = \frac{a_{nit}}{a_{nt}} = \frac{\left(\eta_{it} r_{it} / \kappa_{nit}\right)^{\epsilon}}{\sum_{h=1}^{N} \left(\eta_{ht} r_{ht} / \kappa_{nht}\right)^{\epsilon}}, \qquad \epsilon > 1$$

- Imperfect substitutability in capital across countries Alternate
- Expected = realized return to capital is equalized across hosts i

$$v_{nit} = v_{nt} = \gamma \left[\sum_{h=1}^{N} \left(\eta_{ht} r_{ht} / \kappa_{nht}\right)^{\epsilon} \right]^{\frac{1}{\epsilon}}, \qquad \gamma \equiv \Gamma\left(\frac{\epsilon - 1}{\epsilon}\right)$$

- No aggregate uncertainty (continuous measure of units of capital)

Production and Trade

- Consumption and investment bundles follow CES (Armington):

$$c_{nt} = \left[\sum_{i=1}^{N} \left(c_{nit} \right)^{\frac{\theta}{\theta+1}} \right]^{\frac{\theta+1}{\theta}}, \qquad \theta = \sigma - 1, \qquad \sigma > 1$$

- Country n's expenditure share on good i:

$$s_{nit} = \frac{\tau_{nit} p_{it}^{-\theta}}{\sum_{h=1}^{N} \tau_{nht} p_{ht}^{-\theta}}$$

- Prices

$$p_{nit} = \frac{\tau_{nit} w_{it}^{\mu_i} r_{it}^{1-\mu_i}}{z_{it}}, \qquad p_{nt} = \left[\sum_{i=1}^{N} p_{nit}^{-\theta} \right]^{-1/\theta}$$

- Total payments for capital used in country i are proportional to payments for labor:

$$\sum_{n=1}^{N} v_{nt} a_{nit} = r_{it} k_{it} = \frac{1-\mu_i}{\mu_i} w_{it} \ell_i, \qquad k_{it} = \sum_{n=1}^{N} \gamma \eta_{it} b_{nit}^{-\frac{1}{\epsilon}} a_{nit}$$

Steady-State Equilibrium

- Steady-state equilibrium of the model:
  - Time-invariant values of the state variables $\{a_n^*\}_{n=1}^N$ and the other endogenous variables of the model $\{w_n^*, r_n^*, s_{ni}^*, v_{nt}^*, b_{ni}^*\}_{n=1}^N$
  - Given time-invariant values of country fundamentals $\{\ell_n, z_n, \eta_n\}_{n=1}^N$ and $\{\tau_{ni}, \kappa_{ni}\}_{n,i=1}^N$ (set $\phi_{nt} = 1$ for all n, t)
  - Denote the steady-state values of variables by an asterisk

- Steady-state gross real return to capital ($\mathcal{R}_n^*$) and the steady-state saving rate (ς_n^*) are inversely related to discount factor (β):

$$\mathcal{R}_n^* = \frac{1}{\beta}, \qquad \varsigma_n^* = 1 - \beta$$

- Common steady-state realized real return to capital (v_n^* / p_n^*):

$$\frac{v_n^*}{p_n^*} = \beta^{-1} - 1 + \delta$$

- Solve for the economy's transition path using a generalization of existing dynamic exact hat algebra techniques

Linearized Transition Path

Proposition

Suppose that the economy at time $t = 0$ is on a convergence path toward an initial steady state with constant fundamentals $(\boldsymbol{z}, \boldsymbol{\eta}, \boldsymbol{\tau}, \boldsymbol{\kappa})$. At time $t = 0$, agents learn about one-time, permanent shocks to fundamentals $(\widetilde{\boldsymbol{f}} \equiv \begin{bmatrix} \widetilde{\boldsymbol{z}} & \widetilde{\boldsymbol{\eta}} & \widetilde{\boldsymbol{\kappa}}^{in} & \widetilde{\boldsymbol{\kappa}}^{out} & \widetilde{\boldsymbol{\tau}}^{in} & \widetilde{\boldsymbol{\tau}}^{out} \end{bmatrix}')$ from time $t = 1$ onwards. There exists a $N \times N$ transition matrix $(\boldsymbol{P})$ and a $N \times N$ impact matrix $(\boldsymbol{R})$ such that the second-order difference equation system above has a closed-form solution of the form:

$$\widetilde{\boldsymbol{a}}_t = \boldsymbol{P}\widetilde{\boldsymbol{a}}_{t-1} + \boldsymbol{R}\widetilde{\boldsymbol{f}}.$$

The transition matrix $\boldsymbol{P}$ satisfies:

$$\boldsymbol{P} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^{-\mathbf{1}},$$

where $\boldsymbol{\Lambda}$ is a diagonal matrix of N stable eigenvalues $\{\lambda_k\}_{k=1}^{N}$ and $\boldsymbol{U}$ is a matrix stacking the corresponding N eigenvectors $\{\boldsymbol{u}_k\}_{k=1}^{N}$. The impact matrix $(\boldsymbol{R})$ is given by:

$$\boldsymbol{R} = \left(\boldsymbol{\Psi}\boldsymbol{P} + \boldsymbol{\Psi} - \boldsymbol{\Gamma}\right)^{-1}\boldsymbol{\Pi},$$

where $(\boldsymbol{\Psi}, \boldsymbol{\Gamma}, \boldsymbol{\Theta}, \boldsymbol{\Pi})$ are the matrices from the system of second-order difference equations in the wealth state variables. more

Speed of Convergence

Proposition

Consider an economy that is initially in steady-state at time t=0 when agents learn about one-time, permanent shocks to fundamentals $(\widetilde{\boldsymbol{f}} \equiv \begin{bmatrix} \widetilde{\boldsymbol{z}} & \widetilde{\boldsymbol{\eta}} & \widetilde{\boldsymbol{\kappa}}^{in} & \widetilde{\boldsymbol{\kappa}}^{out} & \widetilde{\boldsymbol{\tau}}^{in} & \widetilde{\boldsymbol{\tau}}^{out} \end{bmatrix}')$ *from time* $t = 1$ *onwards. Suppose that these shocks are an eigen-shock* $(\widetilde{\boldsymbol{f}}_{(h)})$*, for which the initial impact on the state variables at time t=1 coincides with a real eigenvector* $(\boldsymbol{u}_h)$ *of the transition matrix* $(\boldsymbol{P})$*:* $\boldsymbol{R}\widetilde{\boldsymbol{f}}_{(h)} = \boldsymbol{u}_h$*. The transition path of the state variables* $(\boldsymbol{a_t})$ *in response to such an eigen-shock* $(\widetilde{\boldsymbol{f}}_{(h)})$ *is:*

$$\widetilde{\boldsymbol{a}}_t = \sum_{j=2}^{2N} \frac{1-\lambda_j^t}{1-\lambda_j} \boldsymbol{u}_j \boldsymbol{v}_j' \boldsymbol{u}_h = \frac{1-\lambda_h^t}{1-\lambda_h} \boldsymbol{u}_h \quad \Longrightarrow \quad \ln \boldsymbol{a}_{t+1} - \ln \boldsymbol{a}_t = \lambda_h^t \boldsymbol{u}_h,$$

and the half-life of convergence to steady-state is given by:

$$t_h^{(1/2)}\left(\widetilde{\boldsymbol{f}}\right) = -\left\lceil \frac{\ln 2}{\ln \lambda_h} \right\rceil,$$

for all state variables $h = 2, \cdots, 2N$*, where* $\widetilde{a}_{i\infty} = a^*_{i,new} - a^*_{i,initial}$*, and* $\lceil \cdot \rceil$ *is the ceiling function.*

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- Empirical Motivation
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- Quantitative Evidence
- Conclusions

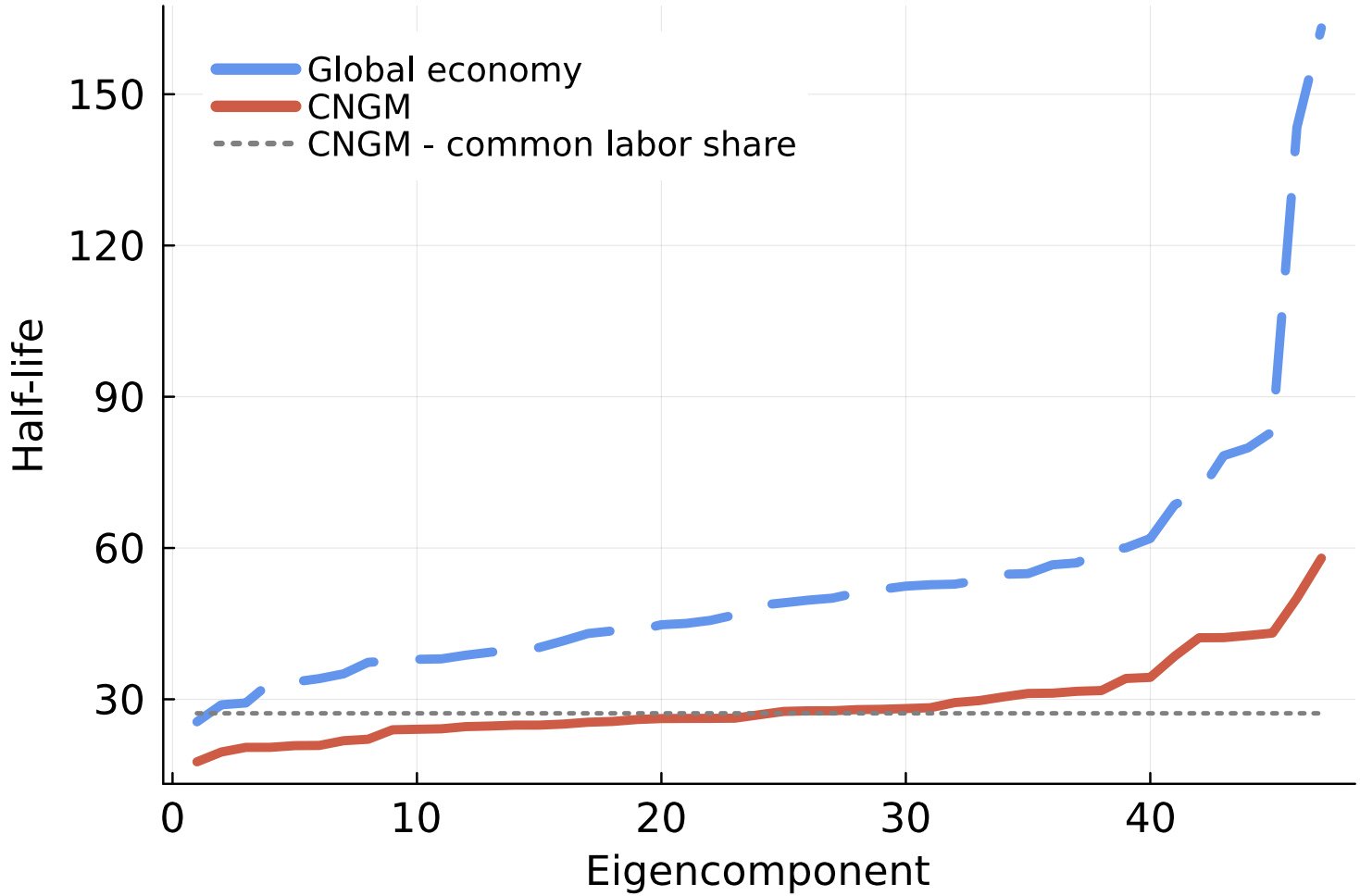
Data & Parameterization

- National Income Accounts (Penn World Tables)
- International Trade (UN COMTRADE)
- Capital Holdings (CPIS & Global Capital Allocation Project)
- Standard values for model parameters

Parameter		Value
Discount rate	β	0.95
Intertemporal elasticity of substitution	ψ	0.2
Depreciation rate	δ	0.05
Trade elasticity	θ	5
Investment elasticity	ϵ	3.15

- Labor share (μ_i) equals observed value for each country in the Penn World Tables data

Half Lives



- Half lives of convergence to steady-state compared to closed-economy neoclassical growth model (NGM) (trade and capital autarky)

Speed of Convergence

- Consider special case of the model with a separation workers (hand to mouth) and capitalists (can save) with log utility
- Evolution of log deviations of wealth from steady-state

$$\widetilde{a}_{nt+1} - \widetilde{a}_{nt} = (1 - \beta + \beta\delta)\left(\widetilde{v}_{nt} - \widetilde{p}_{nt}\right)$$

- Speed of convergence to steady-state

$$\frac{\text{Cov}\left(\left(\widetilde{a}_{nt+1} - \widetilde{a}_{nt}\right), \widetilde{a}_{nt}\right)}{\text{Var}\left(\widetilde{a}_{nt}\right)} = (1 - \beta + \beta\delta)\frac{\text{Cov}\left(\left(\widetilde{v}_{nt} - \widetilde{p}_{nt}\right), \widetilde{a}_{nt}\right)}{\text{Var}\left(\widetilde{a}_{nt}\right)}$$

- First-order condition for cost minimization with common labor share

$$\widetilde{r}_{nt} = \widetilde{p}_{nnt} - \mu\widetilde{k}_{nt}$$

- where $\widetilde{p}_{nnt}$ is the log deviation for a country's own good and differs from the consumption price index $\widetilde{p}_{nt}$

Extreme Cases

- Closed-economy Neoclassical Growth (trade and capital autarky)
  - Capital autarky ($\kappa_{nit} \to \infty$ for $n \neq i$): $\widetilde{k}_{nt} = \widetilde{a}_{nt}$ and $\widetilde{v}_{nt} = \widetilde{r}_{nt}$
  - Trade autarky ($\tau_{nit} \to \infty$ for $n \neq i$): $\widetilde{p}_{nt} = \widetilde{p}_{nnt}$

$$\frac{\text{Cov}\left(\left(\widetilde{v}_{nt} - \widetilde{p}_{nt}\right), \widetilde{a}_{nt}\right)}{\text{Var}\left(\widetilde{a}_{nt}\right)} = -\mu$$

  - Intuition: Diminishing marginal physical productivity of capital

- Free Trade and Capital Autarky
  - Capital autarky ($\kappa_{nit} \to \infty$ for $n \neq i$): $\widetilde{k}_{nt} = \widetilde{a}_{nt}$ and $\widetilde{v}_{nt} = \widetilde{r}_{nt}$
  - Free trade ($\tau_{nit} = 1$ for all n, i): $\widetilde{p}_{nt} = \widetilde{p}_t$ but $\widetilde{p}_{nt} \neq \widetilde{p}_{nnt}$

$$\frac{\text{Cov}\left(\left(\widetilde{v}_{nt} - \widetilde{p}_{nt}\right), \widetilde{a}_{nt}\right)}{\text{Var}\left(\widetilde{a}_{nt}\right)} = -\frac{1}{\sigma} - \left(1 - \frac{1}{\sigma}\right)\mu$$

  - Intuition: Imperfect substitutability in goods markets ($1 < \sigma < \infty$)
  - Wealth accumulation expands domestic capital and output, which leads to a fall in the price of the domestic good, thereby reducing the marginal value product of capital and the real return to investment

Extreme Cases

- Trade Autarky and Free Capital
  - Trade autarky ($\tau_{nit} \to \infty$ for $n \neq i$): $\widetilde{p}_{nt} = \widetilde{p}_{nnt}$
  - Free capital ($\kappa_{nit} = 1$ for all n, i): $\widetilde{v}_{nt} = \widetilde{v}_t$

$$\frac{\text{Cov}\left(\left(\widetilde{v}_{nt} - \widetilde{p}_{nt}\right), \widetilde{a}_{nt}\right)}{\text{Var}\left(\widetilde{a}_{nt}\right)} = -\frac{1}{\epsilon} - \left(1 - \frac{1}{\epsilon}\right)\mu$$

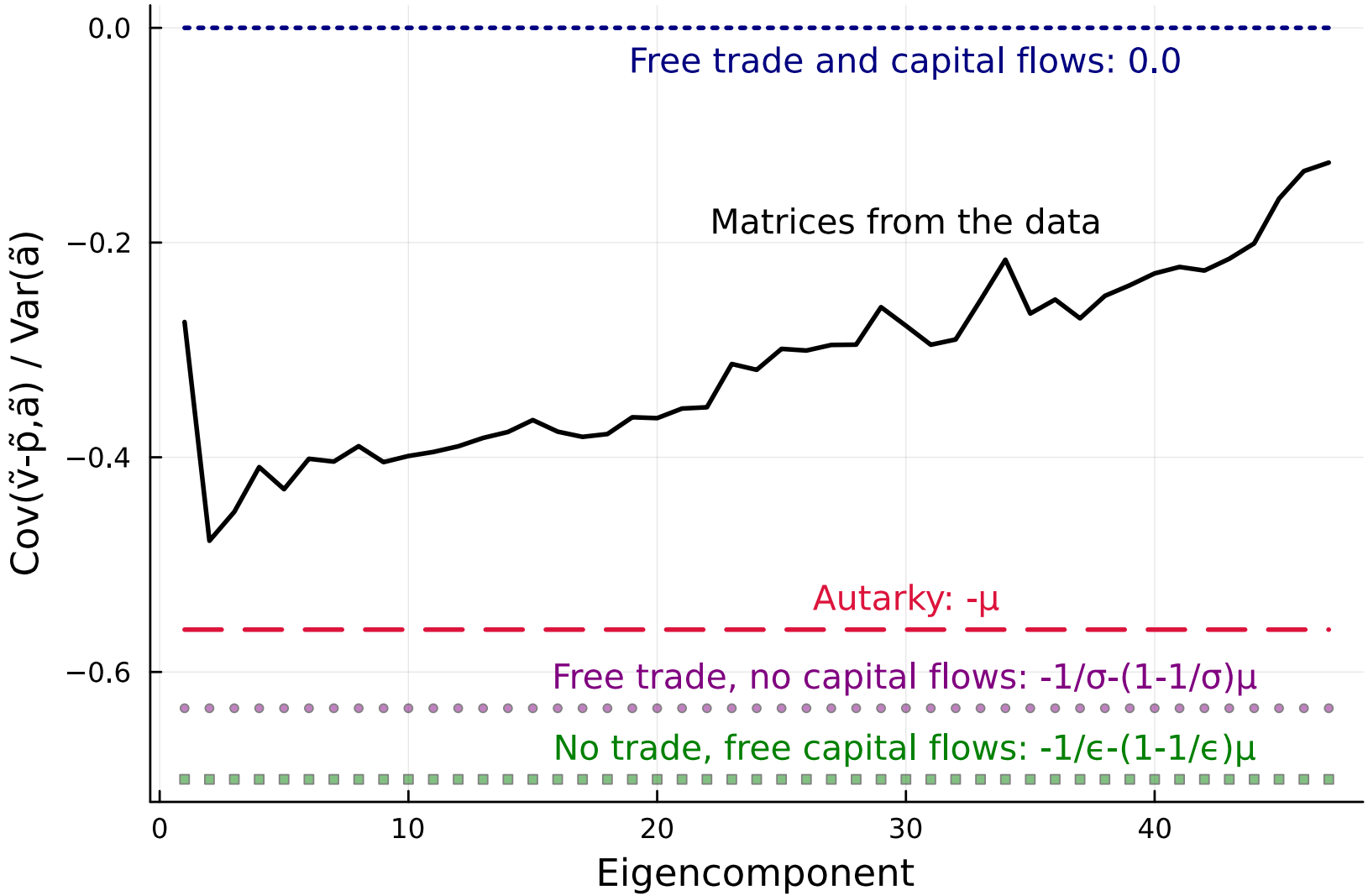
  - Intuition: Imperfect substitutability in capital markets ($1 < \epsilon < \infty$)
  - Wealth accumulation expands investments at home and abroad, which raises country income, and hence spending on domestic goods, thereby bidding up factor prices and the price of the domestic consumption index, and hence reducing the real return to investment

- Free Trade and Free Capital
  - Free trade ($\tau_{nit} = 1$ for all n, i): $\widetilde{p}_{nt} = \widetilde{p}_t$
  - Free capital ($\kappa_{nit} = 1$ for all n, i): $\widetilde{v}_{nt} = \widetilde{v}_t$

$$\frac{\text{Cov}\left(\left(\widetilde{v}_{nt} - \widetilde{p}_{nt}\right), \widetilde{a}_{nt}\right)}{\text{Var}\left(\widetilde{a}_{nt}\right)} = 0$$

  - Intuition: Equalization of real return to investment ($\widetilde{v}_{nt} - \widetilde{p}_{nt} = \widetilde{v}_t - \widetilde{p}_t$), which is therefore uncorrelated with initial country wealth

Covariances



more covvar

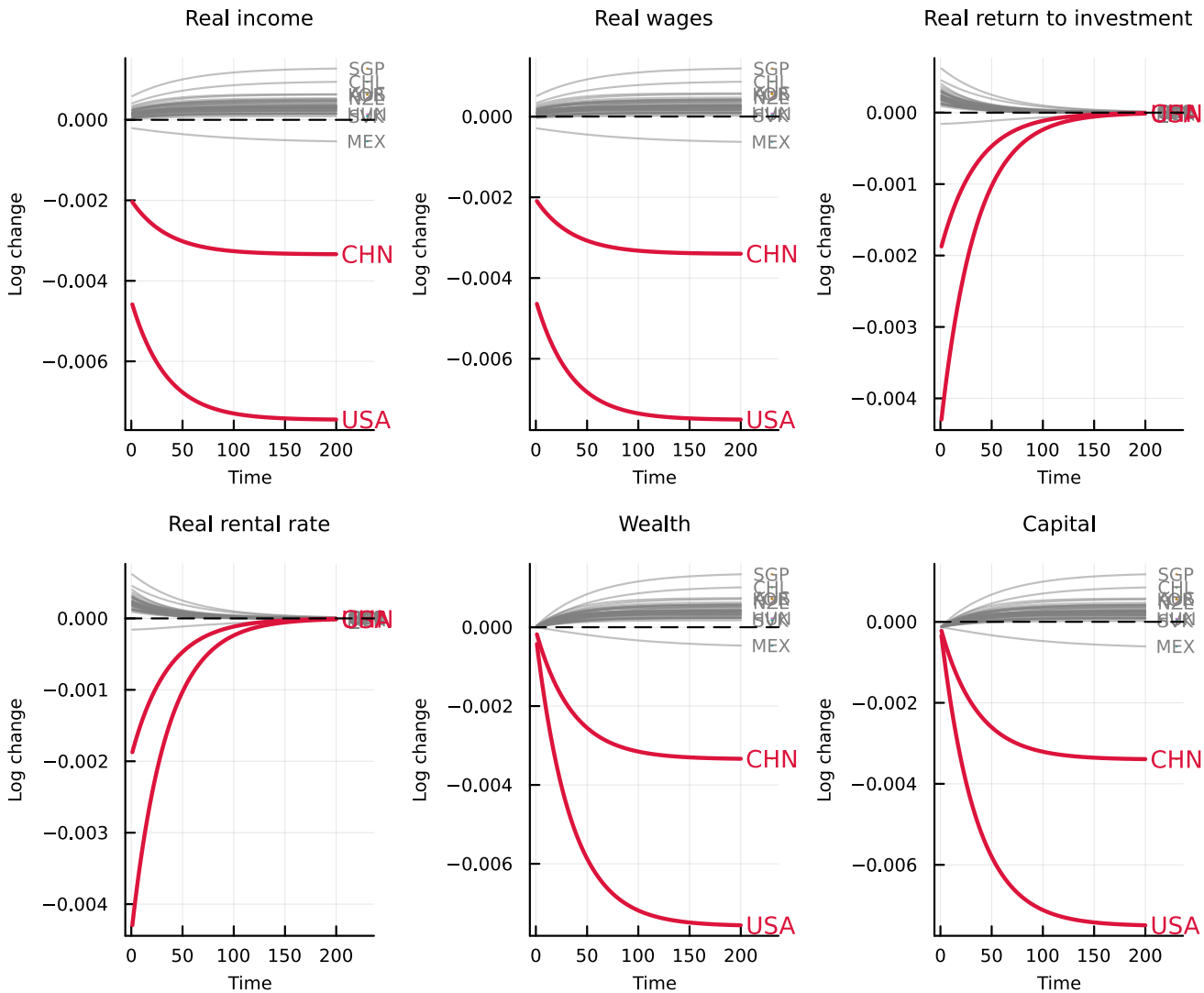
Counterfactuals

- Start at the observed equilibrium in the data and undertake counterfactuals for changes in goods and capital frictions
  - 50 percent increase in US-China trade frictions
  - 50 percent increase in US-China capital frictions
  - 50 percent increase in both frictions

- Undertake these counterfactuals in
  - Special case of model with goods openness (and capital autarky)
  - Baseline model with goods and capital openness

Increase US-China Trade Frictions

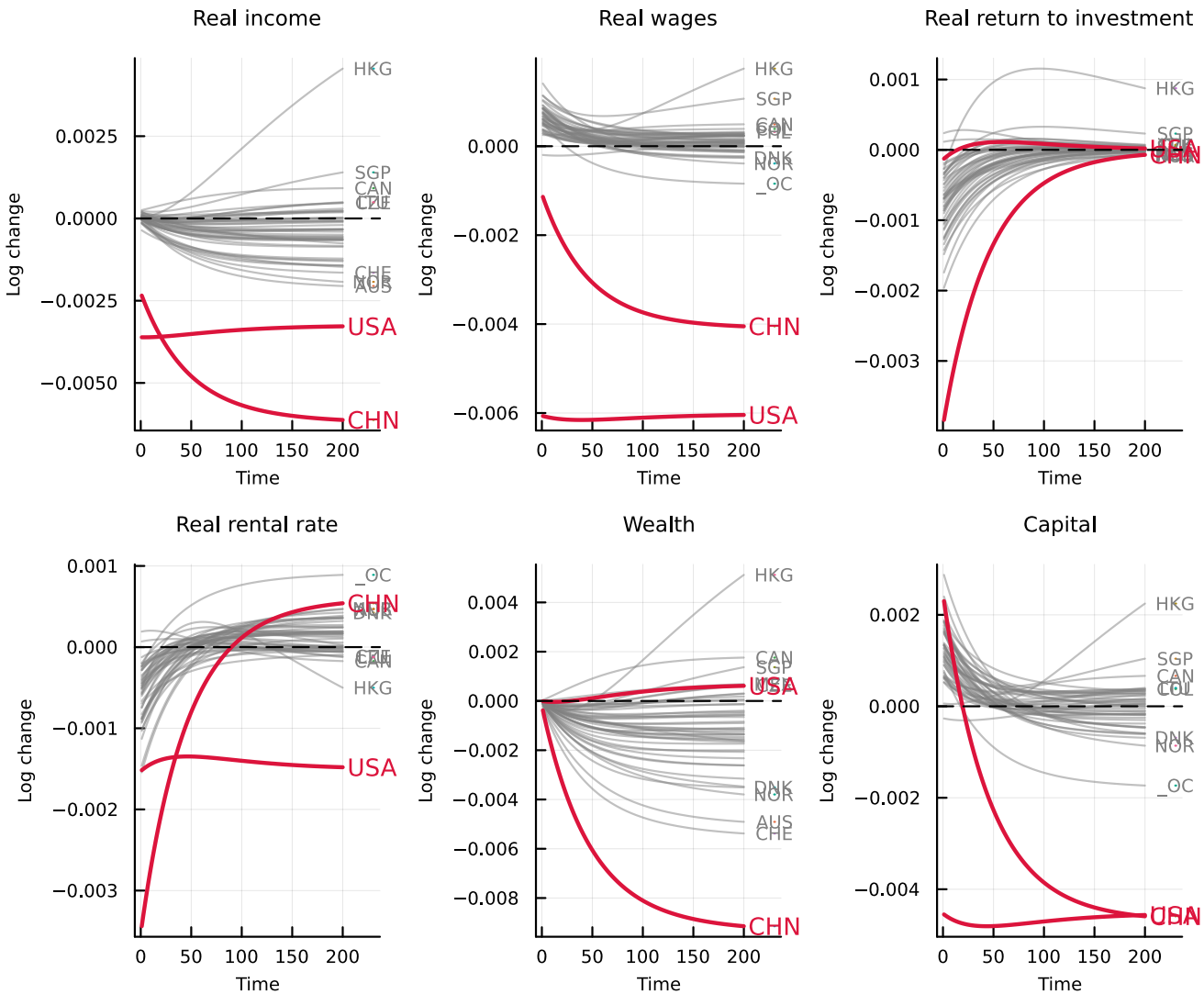
- Special case of model with goods openness (and capital autarky)



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Increase US-China Trade Frictions

- Baseline model with goods and capital openness



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Conclusions

- We generalize the open economy neoclassical model to allow for costly trade in goods and capital flows and imperfect substitutability
- We simultaneously model
  1. Intra-temporal goods trade subject to trade frictions
  2. Intra-temporal capital allocations subject to capital market frictions
  3. Intertemporal consumption-savings choice (hence current account)
- We show that our framework is consistent with a number of features of observed data on trade flows and capital holdings
  - Gravity equation for trade in goods and capital holdings
  - Determinate predictions for gross and net capital holdings
  - Relatively low capital flows to capital-scarce countries
- Goods trade and capital holdings interact to shape speed of convergence to steady-state
  - Goods openness & capital autarky: faster convergence than closed NGM
  - Capital openness & goods autarky: faster convergence than closed NGM
  - Goods & capital openness: slower convergence than closed NGM
- Incorporating bilateral capital holdings is consequential for the counterfactual impact of US - China decoupling

Thank You