Preparing for the Storm: Firm Policies and Time-varying Recession Risk*

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Abstract

How do firms respond to changes in recession risk? We study a rich dynamic model with *time varying* recession risk and *heterogeneous* firm size. In recessions, cash flows decline, cash-flow volatility increases, external financing becomes unavailable, and liquidation costs increase. Recession risk leads to preemptive equity issuances by low-cash firms, investment cuts by intermediate-cash firms, and payout cuts by high-cash firms. Interestingly, *large* firms' policies and values co-vary more with changes in recession risk because small firms prepare more when recession risk is low. We provide empirical support for these predictions.

JEL classification: E22, E44, G30

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"We continue to have strong demand from tenants under 10,000 square feet who dominate our markets, but because *larger tenants* have become more conservative in *response to recessionary concerns*, we leased less total square footage."

Jordan Kaplan, CEO of Douglas Emmett, Inc.
Market Capitalization: $2.6 billion
May 7, 2023
Source: Q1 2023 Earnings Call

1 Introduction

Firms operate in cyclical economies with periods of economic expansion interrupted by periods of contraction. Recent recessions include the 2008 financial crisis, the European sovereign debt crisis in 2011, and the COVID-related downturn in 2020. During recessions, firms' cash flows decrease, cash-flow volatility increases, liquidation becomes costlier, and external financing becomes largely unavailable. As the opening quote suggests, when recession risk increases, rational firms may choose to implement different real and financing policies.

To mitigate the *time-varying* risk of a *future* recession, firms can tinker with cash holdings, issuances, investments, and payout policies among others *today*. Do all firms respond similarly to changes in recession risk or are there subtle differences in how large/small and/or more/less financially constrained firms deal with recession risk? While the existing literature extensively examines recessionary periods, the body of theoretical and empirical work that contrasts how time-varying recession risk affects these closely-intertwined firm policies and how these responses differ based on a firm's size and degree of financial flexibility is nonexistent. For instance, the distinction in priorities between small/large firms with low/high cash holdings when reducing investment, issuing equity, and/or cutting payouts in response to increases in recession risk remains unexplored. For policymakers, comprehending the cross-sectional impact of recession risk on firms is crucial to evaluating the costs and benefits associated with policies that influence recession risk, such as banking regulations, economic stimulus programs, and monetary policy.

There exist several reasons why it is challenging to develop a rich model capable of shedding light on how firm policies change with recession risk differently in the cross-section of firm size and financial flexibility. First, it is difficult to jointly model a firm's closely intertwined policies for cash holdings, investments, payouts, and issuances in the presence of costly financing. Second, to examine how firm policies vary with recession risk, the model must allow for time-varying recession risk and stochastic transitioning between recessions and expansions. Third, the relations between firm policies, recession risk, and firm characteristics such as size and cash holdings must arise endogenously in the model to provide cross-sectional predictions.

Our main contribution is to develop and solve such a rich dynamic model of a firm. We uniquely characterize and solve for the firm's optimal value via dynamic programming, with productive capital stock k, cash holdings c, and the regime of the economy s as state variables. To model time-varying recession risk, we allow for two expansion regimes in addition to a recession regime. One expansion regime has a low probability of transitioning directly to a recession regime, while the other has a high probability of transitioning to a recession regime. Firms may transition between any of these three regimes. An important feature of the model is that it allows for heterogeneous firm sizes. Correspondingly, there exist differences between large and small firms in the model. Small firms face higher financing costs than large firms because we model a fixed component of issuance costs that does not scale with firm size (e.g., Altınkılıç and Hansen, 2000). Small firms also have better investment opportunities than large firms because companies operate capital equipment with diminishing returns to scale (e.g., Caballero, 1991).

A methodological contribution of our paper is to present an algorithm for solving the model based on the policy iteration method. It is important to note that policy iteration methods, which alternate between policy updates and payoff evaluations, are not straightforward in this context due to the impulsive and singular structure of our problem. We demonstrate that the firm's value function is the unique solution of this dynamic programming equation and obtain numerical convergence to the value function by proving a comparison theorem for viscosity solutions to the Hamilton–Jacobi–Bellman (HJB) equation.

We solve the model for a reasonable set of parameters based on the empirical literature. The availability of costly financing, cash flows, cash flow volatility, and liquidation costs are cyclical. As supported by the data, the parameters do not change significantly across

the low-risk and high-risk expansion regimes but do change between the expansion and recession regimes. To provide a quantitative link between our model and the data, we follow Livdan and Nezlobin (2021) and show that the calibrated model moments and sensitivities are of the same sign and order of magnitude as their empirical counterparts. The model reveals that a firm's real and financing policies vary nontrivially with recession risk and characterizes how these responses vary with both a firm's size and the degree of financial flexibility.

While firms facing increased recession risk may intuitively issue equity preemptively, slash investment, and cut payouts, the model further predicts that large firms' policies are more sensitive to *changes* in recession risk. Small firms preemptively respond more to recession risk in the low-risk regime and therefore need less adjustment of their policies when entering the high-risk regime. The early response in the low-risk regime is due to their attractive investment opportunities and therefore more aggressive investment, which reduces their cash holdings and increases their exposure to liquidation risks during economic downturns. In contrast, larger firms invest at lower rates, which increases their cash over time and lowers their exposure to liquidation risks. However, when the recession becomes imminent (high-risk regime), larger firms cannot wait to bump their cash holdings to the optimal level and thus lose the option to delay precautionary actions. Consequently, larger firms focus more on managing recession risk by adjusting their issuances, investments, and payouts with regime changes between low-risk and high-risk expansions. The model thus predicts a more significant covariance between the policies of larger firms and recession risks, as evidenced in empirical data.

We find support in the data for these cross-sectional predictions. To proxy for recession risk in the data, we use a measure of the probability of a recession *one year in the future* derived from the term spread of U.S. Treasury rates. Comparing a variety of possible predictors of a recession, Estrella and Mishkin (1998) shows that the term spread emerges as the clear individual choice and typically performs better by itself out of sample than in conjunction with other variables. Specifically, they show that the term spread's out-of-sample predictive power for U.S. recessions dominates a host of other predictors with pseudo R^2 values of approximately 30%.[1] To focus on how firms respond to changes in the risk of a future recession, we exclude the National Bureau of Economic Research

[1]Firms are aware of this measure. For example, Jason Serrano (CEO of New York Mortgage Trust, Inc.) said in the Q1 2023 earnings call, "We highlighted the obvious fact that the entire yield curve is inverted, but the not so obvious fact is that the months of inversion are now beyond or very close to when recessions were previously triggered."

(NBER) designated recessions from our analyses.

In the data, during expansionary regimes, we find large firms' issuances, investments, payouts, and values respond more to changes in recession risk. Specifically, as predicted by the model, a large firm's preemptive issuances are more sensitive to recession risk. That is, for large firms, cash holdings immediately prior to issuance are significantly more positively related to recession risk. Within a firm, a one-standard-deviation increase in firm size almost doubles the positive sensitivity of cash holdings prior to issuance to recession risk. Also, within a firm, a one-standard-deviation increase in firm size predicts 66% (73%) more negative sensitivity of investment (payouts) to recession risk. Lastly, large firms' monthly stock returns are 33% more negatively related to recession risk. Importantly, because firm policies vary with the state of the business cycle (Covas and Den Haan, 2011; Zetlin-Jones and Shourideh, 2017; Begenau and Salomao, 2018), in addition to dropping NBER recessions, we control for more subtle changes in the current state of the business cycle (e.g., Volatility Index, market return, the likelihood of being in a recession *today* Chauvet and Piger (2008)). Our measure of the risk of a future recession has effectively zero correlation with these proxies.

Literature Review. The goal of this paper is to study how policies of firms heterogeneous along size and cash holdings dimensions change with *time-varying* recession risk. A major twist in our model, and a point of departure from the existing literature, is the time-varying feature of recession risk. This innovation allows us to examine theoretically and empirically the *sensitivity* of firm policies to changes in recession risk and to characterize the sensitivity in the cross-section of firm size. Our paper contributes to three major themes in the literature.

First, our paper advances the theoretical literature on recession risk and firm policies. Chen, Xu and Yang (2021) solves a dynamic capital structure model with static recession risk to link firms' maturity choices to their systematic risk exposure and macroeconomic conditions. In their conclusion, they call for future work like ours that examines how firms' adjust cash holdings, real investments, and payouts to prepare for a future recession. Bolton, Chen and Wang (2013) models static stochastic financing risk and predicts low-cash firms preemptively issue when issuance costs are low.

Second, more broadly, our paper adds to the sizeable literature on risk management (e.g., Whited, 1992; Holmström and Tirole, 2000; Caballero and Krishnamurthy, 2008; Rampini and Viswanathan, 2010; Jermann and Quadrini, 2012). Froot, Scharfstein and Stein (1993) develops a general framework for analyzing corporate risk management

policies. Bachmann, Elstner and Sims (2013) empirically examines the predictions in Bloom (2009) that dramatic increases in uncertainty after major economic shocks generate short-run drops in industrial production. Several papers suggest that firms should adjust their policies for the current state of the business cycle (e.g., Bhamra, Kuehn and Strebulaev, 2010; Begenau and Salomao, 2018). For example, Hackbarth, Miao and Morellec (2006) examines dynamic capital structure choice and aggregate risk.

Third, our work examining how firms respond to changes in the risk of a future recession complements the sizeable literature examining how firm policies change during recessions. Garvey (1992) and Haushalter, Klasa and Maxwell (2007) find that in a crisis, cash allowed firms to maintain or make new investments and emerge stronger out of the recession. Duchin, Ozbas and Sensoy (2010) finds that investment declines more in a crisis for firms more dependent on external financing. Kahle and Stulz (2013) document a decline in cash holdings during the most recent financial crisis followed by an increase in cash holdings to pre-crisis levels. Bliss, Cheng and Denis (2015) finds that firms in recessions cut dividends.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 presents the model solution. Section 4 describes the data and presents the empirical results. Section 5 outlines the numerical algorithm for solving the model. And, Section 6 concludes. The Appendix includes omitted proofs.

2 Model

The firm's cash flows are stochastic and are a function of the size of its productive capital stock and a cash-flow shock. We assume that the firm's cash flow shock Z_{st} evolves according to

$$dZ_{st} = \mu_s dt + \sigma_s dW_t, \tag{1}$$

where W_t is a one-dimensional Brownian motion under the risk-neutral measure and μ_s and σ_s are positive constants that depend on the business cycle, denoted by s. Thus, shocks dZ_{st} are assumed to be i.i.d. with mean $\mu_s dt$ and variance $\sigma_s^2 dt$. The firm's cumulative cash flows Y_{st} follow the dynamics

$$dY_{st} = k_t^{\alpha} dZ_{st}, \tag{2}$$

where k is the size of the firm's capital stock and $\alpha \in (0,1)$ is a scale parameter following Bertola and Caballero (1994).[2] Therefore, production exhibits decreasing returns to scale.

The economic state s varies between expansions (l), recessions (h), and an intermediate state (m) in which the economy has expansive properties, but the risk of entering a recession is high. Specifically, we assume that the firm is in only one of these three (observable) states of the world. A firm's financing and investment opportunities may differ across states h, m, and l. This is modeled by a Markov chain s taking the value $s_t \in \{h, m, l\}$ at time t. The transition rate from state s to s' is denoted $q_{s,s'}$, i.e., $\sum_{s' \neq s} q_{s,s'}$ is the rate of transition away from s.

The productive capital stock depreciates over time at a rate $\delta \geq 0$, and the firm can invest in productive capital. As is standard in capital accumulation models, for an investment process i, the dynamics of the productive capital stock follows

$$dk_t = \Big(i_t - \delta k_t\Big)dt. \tag{3}$$

We assume that investment is irreversible, i.e., $i \geq 0$, and that the depreciation rate does not depend on the business cycle.

As is also standard in capital accumulation models, the investment is subject to convex adjustment costs:

$$g(k, i, s) = \frac{\theta_s}{2}\left(\frac{i}{k}\right)^2 k, \tag{4}$$

where θ_s is a positive state-dependent constant that measures the degree to which convexity in adjustment costs matters.

The firm determines its investment and cash management strategies, which include when to pay a dividend, the size of the dividend, when to raise equity, and the amount of equity to raise. The value of the cash reserve follows the dynamics

$$dc_t = (r - \lambda_c)c_t dt + dY_{st} - i_t dt - g(k_t, i_t, s_t)dt - dD_t + dI_t. \tag{5}$$

Here, r is the interest rate assumed to be state- and time-independent, λ_c is the cash holding cost (liquidity premium) also assumed to be state- and time-independent, D_t is the cumulative dividend payout, and I_t is the cumulative equity issuance. Both D_t and I_t are non-decreasing processes. Cash earns a return equal to the risk free rate (r) net of a

[2] Our model can accommodate $\alpha = 1$ or $\alpha > 1$. Evidence of diminishing returns to scale is quite common in the literature. See Caballero (1991), Basu and Fernald (1997), Gomes (2001), and Grullon and Ikenberry (2021).

carry cost of holding cash (λ_c).[3] Even though cash earns a lower rate of return, the firm holds cash for precautionary reasons to lower the expected issuance or liquidation costs if it runs out of liquid funds. The firm manages an optimal cash policy to trade off the risk management benefits of maintaining a cash reserve against the delay in dividend payouts.

Equity issuance is costly. The issuance spread is the compensation paid to the underwriter for selling a firm's security issue, calculated as a percent of capital raised. We characterize the nominal cost of issuing a lump sum of size I as:

$$\lambda(I, s) = \lambda_{f,s} + \lambda_p \times I. \tag{6}$$

The first term, λ_p, is the fixed spread or flat percentage fee. Thus, a \$1 billion issue will have higher costs than a \$1 million issue. The second term, $\lambda_{f,s}$, captures the economies of scale reasoning. When I is smaller, the $\lambda_{f,s}$ costs increase the spread more. However, as I increases, the fixed costs as a proportion of the amount issued decrease. The cost $\lambda_{f,s}$ may be state dependent. Together, $\lambda_{f,s}$ and λ_p can be thought of as summarizing the information, incentive, and transaction costs that a firm incurs whenever it chooses to issue external equity. These costs imply that the firm will optimally tap equity markets only intermittently, and, when doing so, it raises funds in lumps, consistent with observed firm behavior. To replicate the fact that issuances are procyclical and largely dry up in recessions (Covas and Den Haan, 2011), we make the fixed component of issuance costs state-dependent.

Even if a firm neither pays out cash nor invests, its cash reserve can run out due to negative productivity shocks. When this happens, the firm compares the benefit of equity issuance and continuing (continuation value) with the value for equityholders after liquidation (liquidation value). If the latter outweighs the former, the firm liquidates. Therefore, $\tau = \inf\{t \geq 0 : c_t < 0\}$ is the firm's liquidation time. When the firm liquidates, its capital stock k_τ is fire sold. The recovery rate ℓ_s depends on the regime of the business cycle and is constant across sizes of productive capital.

[3] If $\lambda_c = 0$, then the firm finds it optimal to hold as much cash as it can (indefinitely postponing the dividend) to prevent costly equity issuance. The equity is still valuable because equityholders could always choose to extract the cash via a dividend. The more realistic case is when $\lambda_c > 0$. Cash may earn low returns because interest earned on a firm's cash holdings is taxed at the corporate tax rate, which generally exceeds the personal tax rate (Graham, 2000; Faulkender and Wang, 2006). Agency problems may lower cash returns (Jensen, 1986; Harford, 1999; Dittmar and Shivdasani, 2003; Pinkowitz, Stulz and Williamson, 2006; Dittmar and Mahrt-Smith, 2007; Harford, Mansi and Maxwell, 2008; Caprio, Faccio and McConnell, 2011; Gao, Harford and Li, 2013).

2.1 The firm's problem

The firm chooses policies for investment i_t, dividend payout D_t, equity issuance I_t, and when to liquidate to maximize the present value of dividend payouts net of equity issuance costs:

$$\sup_{i\geq 0, D, \{\sigma_j, I_j\}} \mathbb{E}_0\left[\int_0^\tau e^{-rt} dD_t - \sum_j e^{-r\sigma_j}\Big(I_j + \lambda(I_j, s_{\sigma_j})\Big) + 1_{\{\tau<\infty\}} e^{-r\tau}\ell_{s_\tau} k_\tau\right], \tag{7}$$

where $\{\sigma_j\}$ is a sequence of stopping times when the lump sum of equity of size I_j is issued at each σ_j.[4]

The capital stock size k, the cash reserve value c, and the state of the economy s are the three state variables for the firm's problem. The firm's value function is

$$\begin{aligned} V(k_t, c_t, s_t) = \sup_{i\geq 0, D, \{\sigma_j, I_j\}} \mathbb{E}_t\Bigg[\int_t^\tau e^{-r(\rho-t)} dD_\rho - \sum_{\sigma_j \geq t} e^{-r(\sigma_j - t)}\Big(I_j + \lambda(I_j, s_{\sigma_j})\Big) \\ + 1_{\{\tau<\infty\}} e^{-r(\tau-t)}\ell_{s_\tau} k_\tau\Bigg]. \end{aligned} \tag{8}$$

The firms has a number of options—control variables—to choose from. Each control variable corresponds a component of the Hamilton–Jacobi–Bellman (HJB) equation that the value function satisfies (see (12) below). The option to pay a lump sum of $\Delta D \leq c$ as dividends implies that

$$V(k, c, s) \geq \underbrace{V(k, c - \Delta D, s) + \Delta D}_{\text{value if lump sum dividend is paid}},$$

satisfied with equality if a lump sum dividend of (at least) size ΔD is optimal, otherwise the value is greater than paying the lump sum. This inequality has two implications. First, subtract the right hand side, divide by ΔD, and let $\Delta D \to 0$ to obtain

$$\partial_c V - 1 \geq 0. \tag{9}$$

Again, equality holds whenever dividend payouts are optimal. Second, paying a lump sum $\Delta D = c$ allows the firm to liquidate:[5]

$$V(k, c, s) \geq V(k, 0, s) + c \geq \ell_s k + c.$$

[4]Note that because there is no information asymmetry between existing and new investors, one can simply think of the problem through the lens of a representative investor.

[5]The precise behavior of V for $c = 0$ is described in (13).

In particular, this is a special case of the condition $\partial_c V - 1 \geq 0$, as $\partial_c V - 1 \geq 0$ implies $V(k,c,s) - V(k,0,s) - c = \int_0^c \partial_c (V(k,c',s) - c')dc' \geq 0$.

Next, the firm may issue equity. The issuance of I increases the cash by as much, at the shareholder expense of $I + \lambda(I,s)$. Hence, the shareholders are left with the value

$$V(k,c+I,s) - I - \lambda(I,s),$$

which, if issuing they aim to optimize. The value of the firm is at least as great as the largest value post-issuance:

$$V(k,c,s) \geq \sup_{I \geq 0} \Big[V(k,c+I,s) - I - \lambda(I,s) \Big]. \tag{10}$$

Finally, the firm may decide to neither pay a dividend nor issue equity, in which case it needs only pick its optimal instantaneous investment. By standard control arguments, V must satisfy

$$\begin{aligned} rV - \sup_{i \geq 0} \Big\{ \Big[i - \delta k \Big] \partial_k V + \Big[(r - \lambda_c)c + k^\alpha \mu_s - i - g(k,i,s) \Big] \partial_c V \\ + \frac{1}{2} k^{2\alpha} \sigma_s^2 \partial_{cc}^2 V + \sum_{s'} q_{s,s'} V(k,c,s') \Big\} \geq 0, \end{aligned} \tag{11}$$

with equality where optimal. Here, rV represents the required rate of return on equity, which equals the risk-free rate demanded by risk-neutral investors. The term $\partial_k V$ is a firm's marginal benefit of capital; hence, $[i - \delta k]\partial_k V$ captures the marginal effect of net investment on equity value. The term $\partial_c V$ is the firm's marginal cost of cash; hence the term

$$[(r - \lambda_c)c + k^\alpha \mu_s - i - g(k,i,s)]\partial_c V,$$

is the effect of a firm's expected savings on equity value. The term $\frac{1}{2} k^{2\alpha} \sigma_s^2 \partial_{cc}^2 V$ captures the effect of the volatility of cash holdings due to volatility in production on equity value. The last term in the first expression, $\sum_{s'} q_{s,s'} V(k,c,s')$, captures the value gain or loss due to a potential state transition from s to s'.

The three inequalities (9), (10), and (11), when stated together, gives us the HJB equation. As each is satisfied with equality when the corresponding action is optimal and at least one action is optimal, it is clear that at least one holds with equality. This observation is effectively summarized by the min in the following HJB equation for the

value function $V(k, c, s)$:

$$0 = \min \Bigg\{ rV - \sup_{i \geq 0} \Bigg\{ \Big[i - \delta k \Big] \partial_k V + \Big[(r - \lambda_c)c + k^{\alpha} \mu_s - i - g(k, i, s) \Big] \partial_c V + \frac{1}{2} k^{2\alpha} \sigma_s^2 \partial_{cc}^2 V + \sum_{s'} q_{s,s'} V(k, c, s') \Bigg\}, \partial_c V - 1, V(k, c, s) - \sup_{I \geq 0} \Big[V(k, c + I, s) - I - \lambda(I, s) \Big] \Bigg\}. \tag{12}$$

The equation highlights that the firm chooses among three alternatives: investment (the group of terms on the first line of the right-hand side of the equation), dividend payout (the first group of terms on the second line), and equity issuance (the second group on the second line).

The boundary condition at $c = 0$ is determined by comparing the liquidation and issuance values:

$$0 = \min \Big\{ V(k, 0, s) - \ell_s k, \; V(k, 0, s) - \sup_{I \geq 0} \Big[V(k, I, s) - I - \lambda(I) \Big] \Big\}. \tag{13}$$

In the equation above, the boundary value $V(k, 0, s)$ dominates the liquidation value $\ell_s k$ and the best issuance value $\sup_{I \geq 0}[V(k, I, s) - I - \lambda(I, s)]$, and for each value of k is equal to one of the terms, depending on which is largest. If $V(k, 0, s)$ equals the former value, it is optimal for the firm to liquidate; otherwise, issuance is optimal with optimal size.

3 The Model Solution

In this section, we present and discuss the model solution with a series of figures illustrating how recession risk affects a firm's investments, payouts, issuances, and value. We also examine how the effects of recession risk vary with a firm's size and financial flexibility captured by cash holdings.

To solve the model numerically, we need to determine several parameters, including the transition probabilities between regimes s. We estimate the transition probabilities empirically. Specifically, we use the monthly probability of a future recession in 12 months from 1959 to 2022, derived from the term spread and discussed in Section 4.1. Ignoring months containing NBER recessions, we identify the 75th percentile cut-off recession probability (20.3%), and use this threshold to separate months into "expansion,

Table 1: Transition Probabilities

From		To: Expansion, Low-Risk	To: Expansion, High-Risk	To: Recession
	Expansion, Low-Risk (l)	0.74	0.18	0.08
	Expansion, High-Risk (m)	0.37	0.42	0.21
	Recession (h)	0.53	0.11	0.36

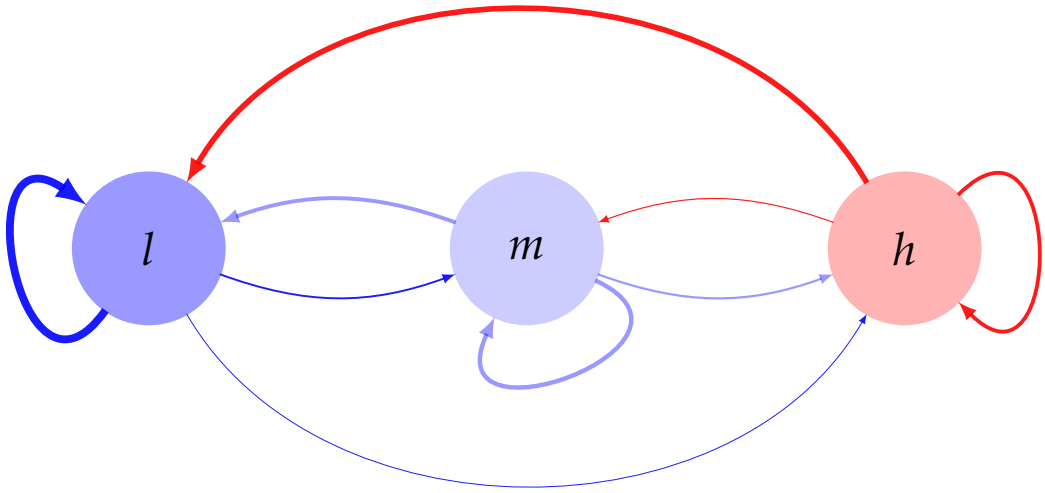


Figure 1: **Transition probabilities diagram**

This figure illustrates transition probabilities for the expansion-low risk (l), expansion-high risk (m), and recession (h) regimes. Edge thickness is adjusted by the corresponding probabilities (see Table 1).

low risk" and "expansion, high risk" regimes. Months with an NBER recession are labeled "recession." Then, we examine the transition intensity from one regime to each of the others. Table 1 shows the annual transition probabilities between regimes. The "expansion, low-risk" regime is the most stable. When a firm is in that regime, there is about a 74% chance of staying in that regime next period. By contrast, in the "expansion, high-risk" regime, there is only a 42% chance of being in that regime next period. In a "recession," there is a 36% probability of remaining in a recession and a 53% probability of transitioning to the low-risk expansion regime.

For the remaining parameters, we select plausible numbers based on existing empirical evidence to the extent that it is available. The last column of Table 2 contains a discussion

of the model parameter choice. In Section 4.6, we perform a quantitative analysis to show various moments of the model are reasonable relative to the data.

Table 2: Model Parameters

Parameter	Name	Values	Comments
r	Interest rate	6%	In line with a long-term average yield to maturity on 30-year U.S. Treasuries..
λ_c	Cash holding cost, liquidity premium	1%	Cash may earn low returns because interest earned on a firm's cash holdings is taxed at the corporate tax rate, which generally exceeds the personal tax rate (Graham, 2000; Faulkender and Wang, 2006). Also, agency problems may lower cash returns (Jensen, 1986; Harford, 1999; Dittmar and Shivdasani, 2003; Pinkowitz, Stulz and Williamson, 2006; Dittmar and Mahrt-Smith, 2007; Harford, Mansi and Maxwell, 2008; Caprio, Faccio and McConnell, 2011; Gao, Harford and Li, 2013).
μ_s	Expected productivity shock	$l = m = 0.18$, $h = 0.14$	In line with the estimates of Eberly, Rebelo and Vincent (2009) for large U.S. firms.
σ_s	Volatility of productivity shock	$l = m = 0.09$, $h = 0.14$	In line with the estimates of Eberly, Rebelo and Vincent (2009) for large U.S. firms.
θ	Degree of adjustment costs	1.5	See Whited (1992).
δ	Depreciation rate	10.07%	In line with the estimates of Eberly, Rebelo and Vincent (2009) for large U.S. firms.
α	Curvature of the production function. When $\alpha < 1$, then diminishing returns to scale	0.7	$\alpha = 0.75$ in Riddick and Whited (2009) and $\alpha = 0.627$ with std 0.219 for the full sample of firms in Hennessy and Whited (2007).
λ_p	Variable issuance cost	6.4%	In line with the estimates of Altınkılıç and Hansen (2000). We keep this parameter constant across the two regimes for simplicity and focus only on changes in the fixed cost of equity issuance to capture changes in the firm's financing opportunities.
$\lambda_{f,s}$	Fixed issuance cost	$l = m = 0.005$, $h = \infty$	For regimes l and m, the estimate is in line with the estimates of Altınkılıç and Hansen (2000). For the recession regime h, no empirical study exists on which we can rely for the estimates of issuance costs in a financial crisis for the obvious reason that there are virtually no initial public offerings or secondary equity offerings in a crisis. That is, issuances are procyclical and largely dry up in recessions (Covas and Den Haan, 2011; Bolton, Chen and Wang, 2013). Our choice for the parameter reflects the fact that raising external financing becomes extremely costly in a financial crisis.
ℓ_s	Recovery rate in liquidation of capital	$l = m = 1.0$, $h = 0.3$	The choice of ℓ is consistent with Hennessy and Whited (2007), where the average recovery rate is estimated to be 0.896 for the full sample of firms, so that the liquidation value in the expansion regimes should be somewhat higher. In the recession regime, the capital liquidation value is set to 30% to reflect the severe costs of asset fire sales during a crisis, when few investors have sufficiently deep pockets or the risk appetite to acquire assets.

Importantly, while the parameter values may be cyclical and change between expan-

sions and recessions, we assume that the parameters are the same across the low-risk and high-risk expansion regimes. To support this assumption, in the data, we show no significant difference in several key parameters across the low-risk and high-risk expansion regimes. By contrast, there are significant differences between the expansion regimes and the recession regime. Specifically, there are significant differences in the VIX, idiosyncratic volatility, sales growth, and return on equity.

Table 3: Parameter Values Across Regimes

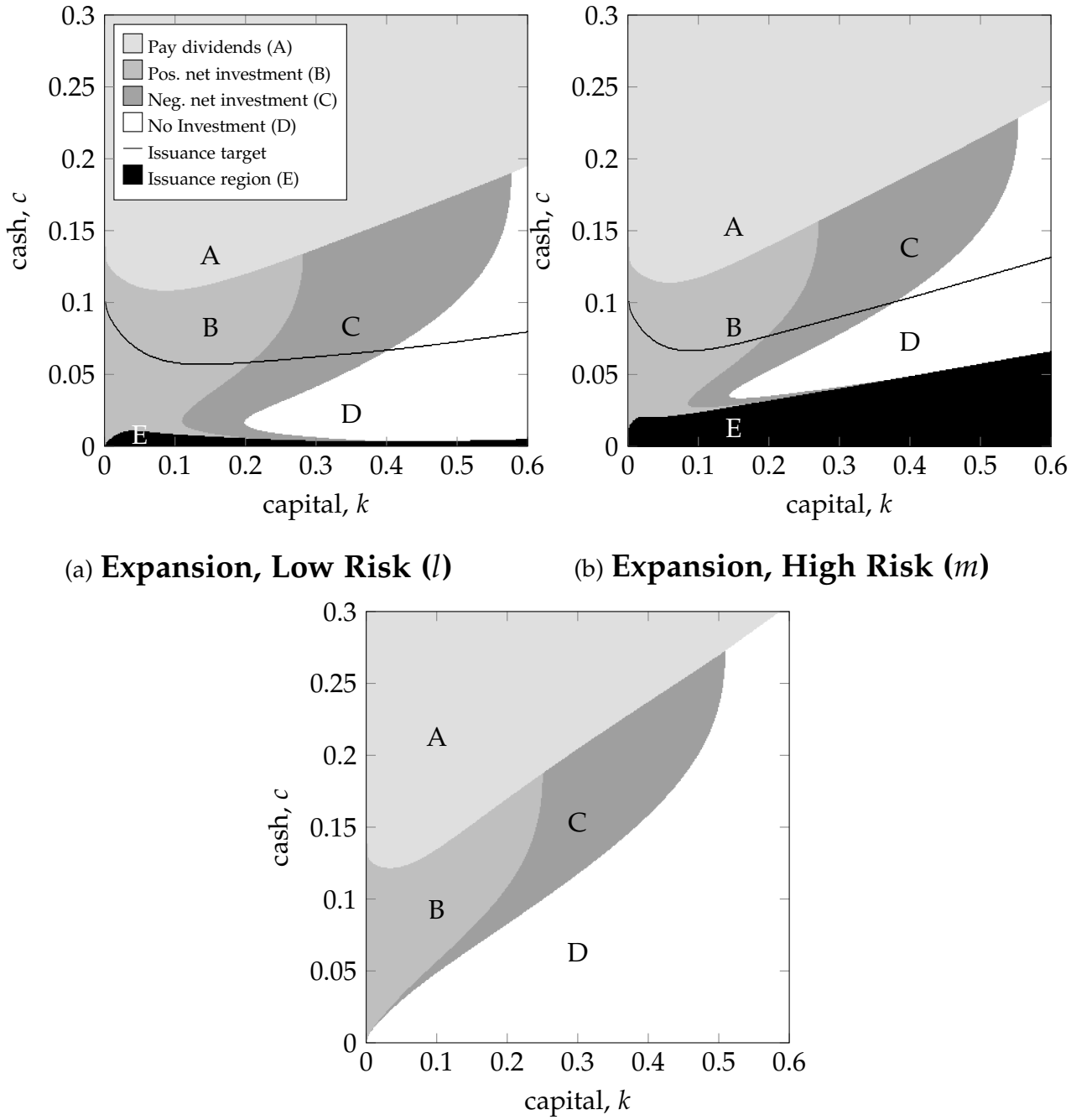
We test for significant differences in these variables across all possible pairings of economic regimes. *VIX* is the average volatility index in a quarter. *Idio Vol* is a firm's idiosyncratic volatility of stock returns after adjusting for the Fama-French three factor model Fama and French (1992). *Sales Growth* is quarter-over-quarter sales growth. *ROE* is the quarterly net income scaled by total book equity. *COGS Margin* is the ratio of cost of goods sold to total revenue in the quarter. *SGA Margin* is the ratio of selling, general, and administrative expenses to total revenue in the quarter. We regress these variables on an indicator for the regime, report the betas, and cluster the standard errors by quarter.

	VIX	Idio Vol	Sales Growth	ROE	COGS Margin	SGA Margin
Expansion, Low Risk (l)	18.1	12.3	2.6	1.2	72.5	27.3
Expansion, High Risk (m)	20.4	15.9	1.4	0.8	72.4	29.0
Recession (h)	29.9	20.8	-1.8	0.2	82.0	29.9
Pairwise T-Tests						
$m - l$	2.3	3.6	-1.3	-0.4	-0.1	1.7
$h - l$	11.8***	8.5***	-4.5***	-1.0**	9.6	2.6
$h - m$	9.5***	4.9	-3.2**	-0.6	9.7	0.9

3.1 Firm policies in the expansion regimes

Panels (a) and (b) of Figure 2 depict a firm's optimal policies in an expansion regime with low and high recession risk, respectively. The vertical axis captures the size of a firm's cash reserve, c, and the horizontal axis captures the size of a firm's capital stock, k. The legend notates which regions of k and c correspond to which firm behaviors.

The payout region, labeled A, characterizes when a firm pays a dividend in (k, c) space. The model firm only pays out a dividend when the marginal cost of reducing its cash reserve matches the marginal benefit of the dividend payout. For lower levels of cash c, the firm retains cash to economize over the issuance costs, but the value of this precautionary savings declines as the cash balance increases. Eventually, as cash c increases, holding additional cash is not economical because of the liquidity premium λ_c and the firm begins making payouts. If c is initially higher than the payout boundary, which is where the payout region (A) touches the two investment regions (B and C), a lump-sum dividend



(a) **Expansion, Low Risk (l)**

(b) **Expansion, High Risk (m)**

(c) **Recession (h)**

Figure 2: **Optimal policies with time-varying recession risks**

This figure shows the firm's optimal policies in the (k, c) state space for low-risk expansion (panel a), high-risk expansion (panel b), and recession (panel c) regimes. Parameters used are summarized in Table 2.

is paid out so that the state process (k, c) lands exactly on the boundary after dividend payout. When the state process (k, c) reaches the payout boundary from below, a minimal dividend is paid out to reflect the state process below so that the state process remains lower than the payout boundary.

The two investment regions (B and C) exist because of diminishing returns to scale. Let i^* denote the firm's optimal investment policy. In the positive net investment region, labeled B, it is optimal for the firm to grow its capital stock and, therefore, investment i^* is larger than the depreciation δk. In the negative net investment region, labeled C, it is optimal for the firm to still invest, i.e., $i^* > 0$, but to keep the investment below the depreciation δk, thus resulting in the net disinvestment. Because of diminishing returns to scale, the firm is generally in the positive net investment region (B) when it is smaller, and it is in the negative net investment region (C) when it is larger. The interface between the positive (B) and negative (C) net-investment regions is where investment exactly offsets depreciation.

The no-investment region (D) exists because of costly financing and diminishing returns to scale. The no-investment region predominately emerges when cash is low and the capital stock is high. When cash is low, the probability of costly financing is high. When the capital stock is high, returns to investing are small and, therefore, halting investments to reduce the capital stock, hoard cash, and avoid the lumpy issuance cost is optimal for the firm.

The issuance target, shown in Figure 2 as a solid black line, denotes a firm's optimal cash holdings after raising external financing. The issuance boundary (∂E) is the upper boundary of the black region labeled E. When facing recession risk, the firm finds it optimal to issue at positive cash levels even when there is no immediate use of the funds for investment. The firm is timing the issuance for a period of lower issuance costs, anticipating that issuance costs may increase in the future when a recession occurs. Issuance is lumpy (difference between issuance boundary and issuance target) to economize over the fixed component of issuance costs.

Costs-benefit analysis helps rationalize why low-cash firms respond to recession risk by issuing equity preemptively. In terms of costs, firms anticipate the unavailability of external financing during a future recession and thus have incentives to preemptively issue during the expansion phase to hedge against the possibility of liquidation when the recovery rate on productive capital is low. Why do higher cash firms not also issue equity preemptively? Higher cash firms are further away from possible liquidation in a recession

and thus require less additional external financing to hedge the recession risk. However, issuing less equity is less cost-effective because of the fixed component of issuance costs.

3.2 Illustrating firm dynamics

To illustrate the firm's dynamics, Figure 3 shows the expected trajectories in (k, c) space for each expansion regime conditional on the regime not transitioning. Interestingly, when the productive capital stock k is small (e.g., $k = 0.05$) the firm sees a U-shape trajectory. That is because small firms invest aggressively, which uses cash to grow the firm. As the firm grows, the investment incentives decline and the firm starts to hoard cash to hedge against larger-scale cash-flow shocks. Thus, there is a negative covariance between size and cash when a firm is small and investing. By contrast, when a firm is large (e.g., $k = 0.5$), the trajectory is to the north west. That is, the firm invests less than depreciation because of the diminishing returns to scale assumption, which shrinks the firm, and also builds cash holdings.[6] The trajectories suggest that firms spend more time around the dividend boundary (lower boundary of region A) and around the point Z. In Section 4.6, we present a heat map of the firm's location in (k, c) space in simulations.

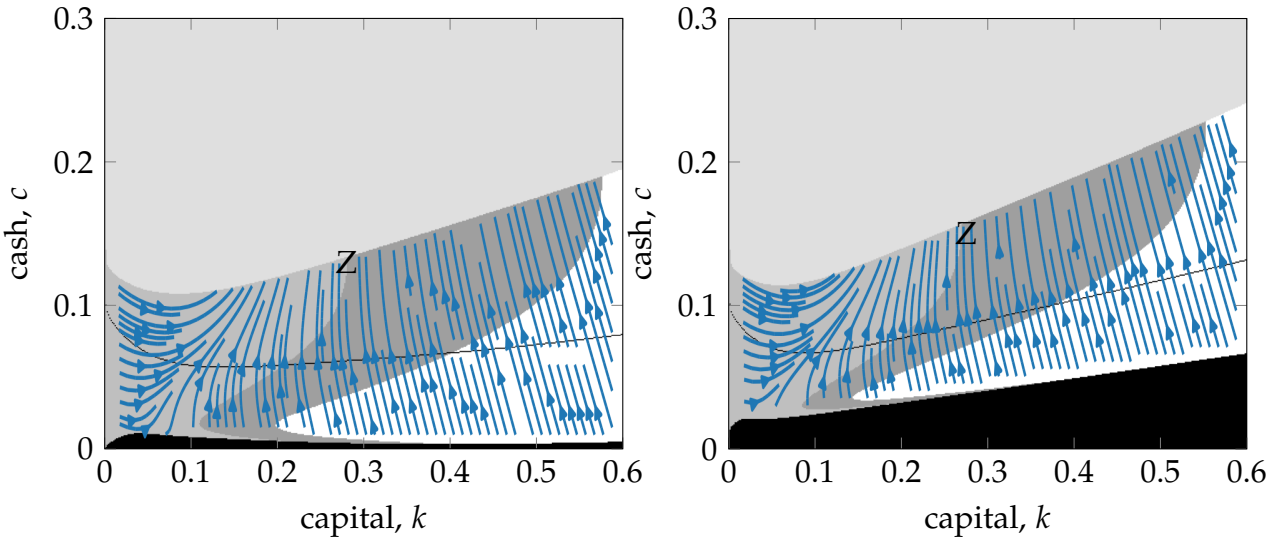
3.3 Firm policies in the recession regime

Firms may temporarily enter a recession regime. In a recession regime, the firm faces the chance of leaving the recession regime for an expansion regime with either a low-risk or high risk of a subsequent recession. Panel (c) of Figure 2 shows a firm's dynamics when the firm is in a recession regime.

One notable difference between the recession regime and expansion regimes is that in a recession regime, the issuance boundary region (E) and issuance target are absent because there is no opportunity for external financing. If a firm runs out of cash, it must liquidate. Indeed, there is no empirical study to measure issuance costs in a financial crisis for the obvious reason that there are virtually no initial public offerings or secondary equity offerings in a crisis (Bolton, Chen and Wang, 2013).

Another difference is that a firm is less willing to invest during the recession regime. The investment regions B and C shrink, while the no-investment reason D expands. The rationale is that the value of cash as a hedge against the higher issuance costs increases,

[6] We provide empirical support for the U-shaped relation between cash and capital in Kakhbod et al. (2022).



(a) **Expansion, Low Risk (l)** (b) **Expansion, High Risk (m)**

Figure 3: **Expected optimal state trajectories**

Illustration of *expected* expansion state trajectories of a firm while not paying dividends or issuing equity if the regime does not change. The actual trajectory depends on the Brownian productivity path, regime changes, possible excursions into the issuance or dividend region. Based on simulations, the firm's density is concentrated around the point Z.

which competes with investing. Additionally, the productivity of investment falls in a recession, while the riskiness of investments in terms of larger cash flow shocks increases.

3.4 Contrasting firm policies with low/high recession risk

The next subsections examine the effects of increases in recession risk on firm policies (comparing Panels (a) and (b) of Figure 2). Higher recession risk increases a firm's probability of liquidation. The liquidation likelihood increases in a recession because external financing is unavailable, cash flows become more volatile ($\sigma_{s=h} > \sigma_{s=l}$), and expected cash flows decline ($\mu_{s=h} < \mu_{s=l}$). Firms can manage recession risk by issuing equity preemptively and by curbing uses of cash such as investments and payouts. The extent to which firms use each of these levers to manage recession risk depends on a firm's cash holdings and size.

3.4.1 Issuances and recession risk

Figure 4 plots the percentage change in the issuance boundaries (upper boundary of region E) across the low-risk and high-risk expansion regimes for different firm sizes. The

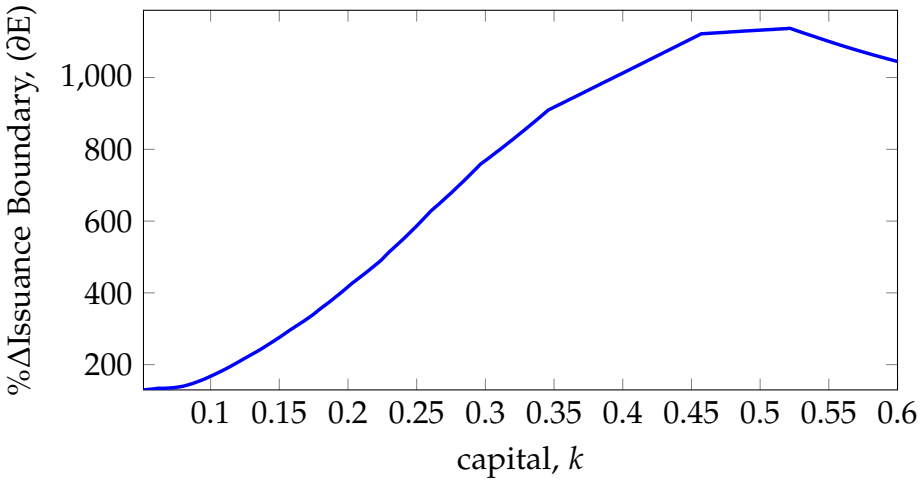


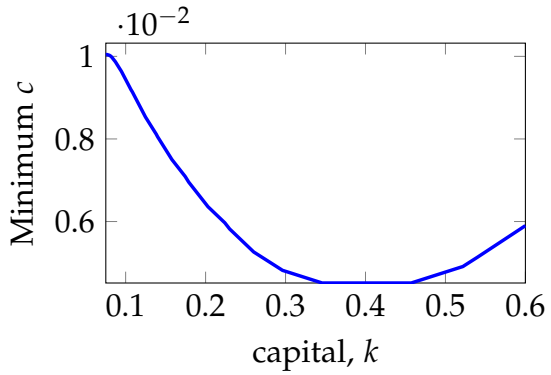
Figure 4: **How does recession risk change the level of cash at which different-size firms issue?**

This figure shows the percent change in the issuance boundary (∂E) when recession risk is high (mid regime) versus low (low regime) in Figure 2. In other words, firms facing recession risk optimally issue when cash holdings are positive and without an immediate need for the issuance proceeds to preempt the rise in issuance costs in a recession. This figure examines the change in cash holdings immediately prior to issuance when recession risk increases. Parameters used are summarized in Table 2.

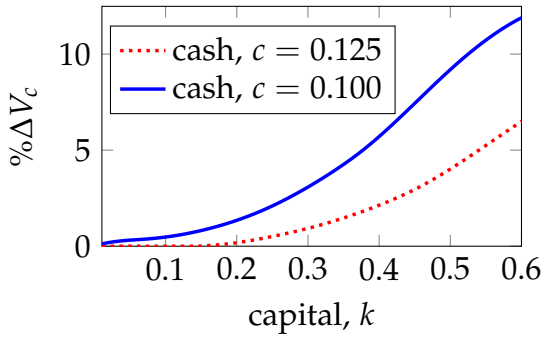
model evidently predicts a larger sensitivity of preemptive issuance behavior when a firm is larger. A firm of size $k = 0.2$ ($k = 0.4$) increases the minimum cash tolerated before preemptive issuance by about 400% (1000%) when recession risk increases. Thus, the model predicts a higher positive sensitivity of cash levels immediately prior to issuance to changes in recession risk for larger firms.

The rationale is that smaller firms more completely respond to recession risk already in the low-risk expansion regime than do larger firms. Note that even in the low-risk expansion regime, there is an 8% chance of transitioning directly into a recession regime. A small firm has incentives to respond more completely, even when recession risk is low because small firms have better investment opportunities and are investing more aggressively, which lowers cash holdings and raises the possibility of liquidation when a recession occurs.

To support this reasoning, note that the issuance region E exists only because of the potential for external financing to become unavailable in a recession. Thus, the boundary of the issuance region ∂E reflects the minimum cash holdings firms of various sizes tolerate because of recession risk before preemptively issuing new equity. Figure 5(a) plots the issuance boundary and shows that small firms require more cash in the low-risk



(a) Minimum cash holdings, c, because of recession risk in the low-risk, expansion regime



(b) Percent change in the marginal value of cash when recession risk increases

Figure 5: **Responses of cash to recession risk**

In Panel A, the vertical axis is the minimum cash holdings, c, that a firm tolerates before issuing equity preemptively because of recession risk. This minimum c is given by the issuance boundary (∂E) in the low-risk regime in Figure 2. In Panel B, the vertical axis is the change in a firm's marginal value of cash. We first compute the marginal value of cash ($\partial_c V = V_c$) as the difference in the value of the firm for a small increment in cash holdings scaled by the size of the small increment in cash holdings. Then, we compute the percent change in the marginal value cash ($\%\Delta V_c$) across the low-risk, expansion regime and the high-risk, expansion regime. Parameters used are summarized in Table 2.

regime, i.e., they respond more completely to the risk of a recession in the low-risk regime. As a result, the small firm's policies adjust less when recession risk does increase in the event of a transition to the high-risk regime.

Another way to support this reasoning is to examine how the marginal value of cash changes when recession risk increases. The marginal value of cash ($\partial_c V = V_c$) is the difference in the value of the firm for a small increment in cash holdings scaled by the size of the small increment in cash holdings. Then, we compute the percent change in the marginal value of cash ($\%\Delta V_c$) across the low-risk expansion regime and the high-risk expansion regime. Consistent with small firms responding more completely in the low-risk regime, Figure 5(b) shows small firms see a smaller percentage change in the marginal value of cash when recession risk increases.

Overall, because a small firm responds more completely when recession risk is low, the model predicts a larger sensitivity of firm policies to changes in recession risk when a firm is larger.

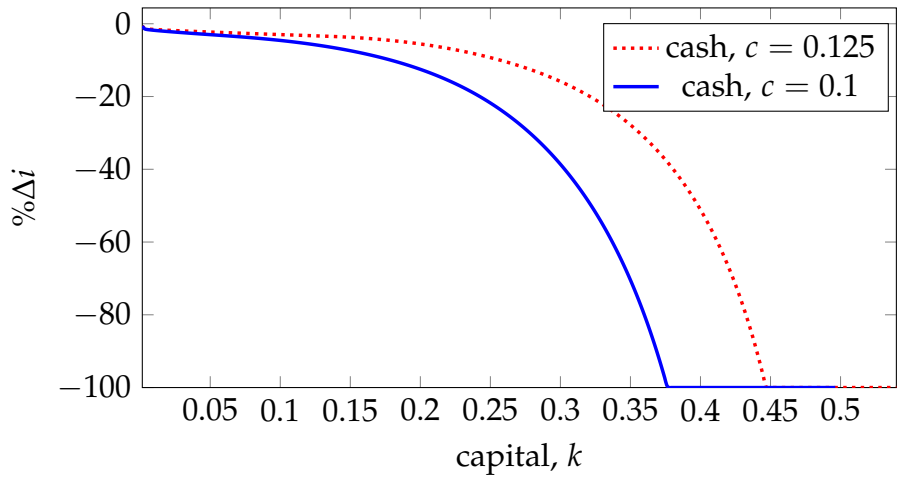


Figure 6: **Change in investment due to higher recession risk**

This figure plots the percent change in investment due to recession risk, $\%\Delta i$, when recession risk increases from the low-risk expansion regime (l) to the high-risk expansion regime (m) against the size of a firm's productive capital stock k. The parameters used are summarized in Table 2.

3.4.2 Investments and recession risk

Because of recession risk, firms reduce investments to preserve cash to avoid liquidation. Figure 6 plots the percent change in investment, $\%\Delta i$, when recession risk increases, against firm size for two levels of cash. Intuitively, higher cash firms cut investment less as they are more insulated from the effects of a recession. More interestingly, investment rates fall more because of recession risk when a firm is larger. This is because, as discussed in Section 3.4.1, smaller firms have stronger investment incentives because of the diminishing returns to scale assumption. Indeed, in Figure 2, the smaller firms are more likely to keep growing in the recession regime, while the larger firms are more likely to cut investment completely (the no-investment D region expands). Overall, the model predicts that the negative sensitivity of investment to recession risk is more negative for larger firms.

3.4.3 Payouts and recession risk

Because of recession risk, firms also reduce payouts to preserve cash to avoid liquidation. Firms facing recession risk anticipate that their profitability will be lower, that their cash flows will be more volatile, and that issuances will become unavailable at some point in the future when the firm enters a recession. Thus, during the expansion regimes, the demand for financial flexibility increases with recession risk, incentivizing firms to hold

more cash and cut payouts.

Figure 7 plots the percent change in the dividend boundary when recession risk increases against firm size. An increase in the dividend boundary corresponds with a drop in payouts. Again, interestingly, the dividend boundary increases by a larger percentage for larger firms. Thus, the model predicts that the negative sensitivity of payouts to recession risk is more negative for larger firms.

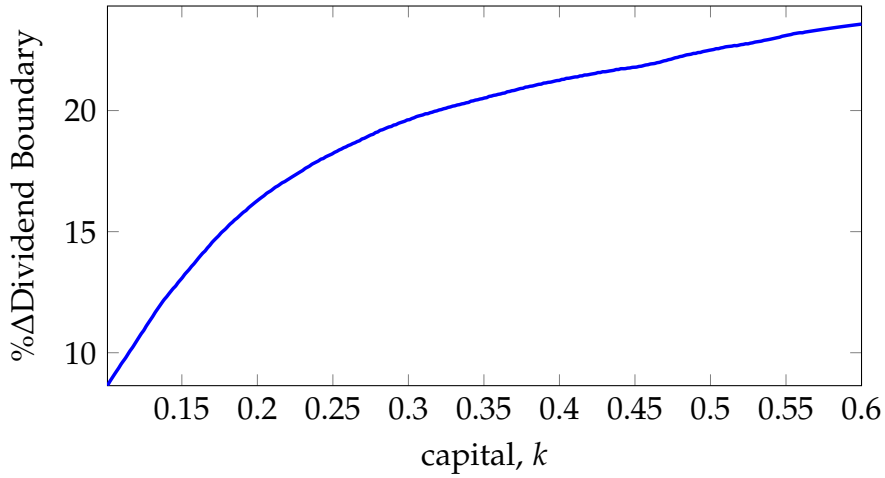


Figure 7: **Change in dividend boundary due to higher recession risk**
This figure plots the percent change in the dividend boundary when recession risk increases from the low-risk expansion regime (l) to the high-risk expansion regime (m) against against the size of a firm's productive capital stock k. Parameters used are summarized in Table 2.

3.4.4 Firm value and recession risk

Overall, recession risk leads to a drop in firm values. Figure 8 shows the percentage change in firm value V when the firm transitions from the low-risk to the high-risk regime. Intuitively, firm values decline with increases in recession risk, and also, as expected, higher cash firms see less of a drop in firm value. More interestingly, consistent with the prior discussion, the model predicts that the negative sensitivity of firm values to recession risk is more negative when a firm is larger.

4 Empirical Evidence

In this section, we empirically examine several model predictions discussed in Section 3.

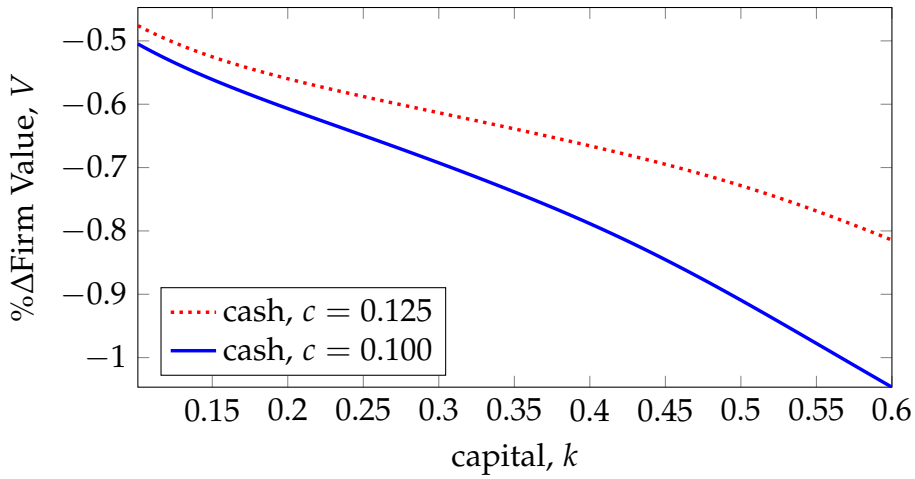


Figure 8: **Change in firm value due to higher recession risk**

Plots the percentage change in firm value when the firm transitions from the low-risk, expansion regime to the high-risk, expansion regime. Parameters used are summarized in Table 2.

4.1 Data and Variable Construction

To proxy for time-varying recession risk, we primarily rely on a monthly time series of recession probabilities from the *The Yield Curve as a Leading Indicator* at the Federal Reserve Bank of New York. The recession probabilities are derived from the term spread, defined as the difference between the 10-year and 3-month Treasury rates, and reflect the chance of the United States being in a recession in twelve months.[7]

Comparing a variety of possible predictors of a recession, Estrella and Mishkin (1998) shows that the slope of the yield curve emerges as the clear individual choice and typically performs better by itself out of sample than in conjunction with other variables.[8] The out-of-sample pseudo R^2 for the term-spread measure is approximately 30% at horizons of two-to-four quarters ahead. As is conventional, we use the one-year horizon. The one-year horizon also allows time for firms to adapt their slower-moving investment, payout, cash, and issuance policies. Additionally, relative to other possible predictors such as surveys, the term spread measure has more history, starting in 1959, allowing for

[7]The probability measure comes from a probit model, in which the outcome variable is an indicator variable that equals one if an NBER recession occurs 12 months later and the main explanatory variable is the current term spread.

[8]Other predictors examined in that paper include the commercial paper spread, the Stock and Watson (1989) and Stock and Watson (1993) indexes, market indexes like the *NYSE* and *S&P 500*, monetary base deflated by the consumer price index, a composite index of leading indicators from the U.S. Commerce Department, and lagged growth in real GDP.

richer analysis.[9] In addition, it is market-based and continuously observable rather than survey-based.[10] Table B.1 correlates our main recession probability measure based on the term spread with several other measures.[11] It is important to highlight that our recession probability measure contrasts with the frequent use of the phrase recession probability in the literature on business cycle dating, which is concerned with whether one is in a recession *today* (e.g., Chauvet and Piger, 2008).[12]

Figure 9 shows the probability that the U.S. economy is in a recession in 12 months. The grey bars indicate National Bureau of Economic Research (NBER)-designated recessions. The NBER identifies the dates of peaks and troughs that frame economic recessions and expansions. A recession is a period between a peak of economic activity and its subsequent trough, or lowest point. Between trough and peak, the economy is in an expansion. Expansion is the normal state of the economy; most recessions are brief. The figure shows that the probability of a recession generally spikes ahead of the NBER recessions. On December 31, 2022, the probability of a recession on December 31, 2023, is 47.3%.[13]

Our primary data source for firm fundamentals is the quarterly Compustat data file, which provides detailed financial statement information on public firms. Our sample

[9]Nevertheless, while the data start in 1959, Kessel (1965) presents graphical evidence that shows that the term spread tends to be negative at cyclical peaks, using data that go back as far as 1858. Bordo and Haubrich (2004) provide regression-based statistical evidence that the term spread predicts recessions using U.S. data from 1875 to 1997.

[10]There is one survey of recession probabilities that extends back to the 1960s. The Survey of Professional Economists has asked economists to estimate the probability of quarter-over-quarter chain-weighted real GDP growth less than zero for the current quarter (RECESS1) and the following four quarters (RECESS2 to RECESS5). RECESS2 is known as the "Anxious Index." See Andrade and Le Bihan (2013). However, consistent with Estrella and Mishkin (1998), the survey has a very low explanatory power for future recessions. Additionally, because we want to examine whether firms manage recession risk, predicting whether GDP will decline next quarter does not give firms much advance notice.

[11]We correlate it with the forecasts from the Survey of Professional Economists, CBOE Volatility Index, returns on the NYSE and S&P 500, the current state of the business cycle (Chauvet and Piger, 2008), the CBOE Volatility Index, leading indicators from Stock and Watson (1989) and Stock and Watson (1993), and the 3-month commercial paper spread over the federal funds rate. The VIX has a correlation of -0.0048 with our primary measure of recession risk. One reason may be that the VIX captures expected volatility over the next month, while the recession probability measure predicts recessions in 12 months. Another reason may be that the VIX measures short-term expected volatility (second moment), while the recession probabilities predict distant drops in GDP (first moment).

[12]For example, in December 2022, the Chauvet and Piger (2008) measure suggests a 5% probability of being in a recession in that month, while the term spread measure suggests a 47.3% probability of being in a recession one year later.

[13]When using the recession probabilities from the Federal Reserve Bank of New York it is important to shift the recession probabilities back 12 months to examine how firms manage current information about future recessions. Specifically, the data file associates each recession probability with the state of the business cycle one year later. For example, the recession probability based on the December 2022 term spread is associated with December 2023.

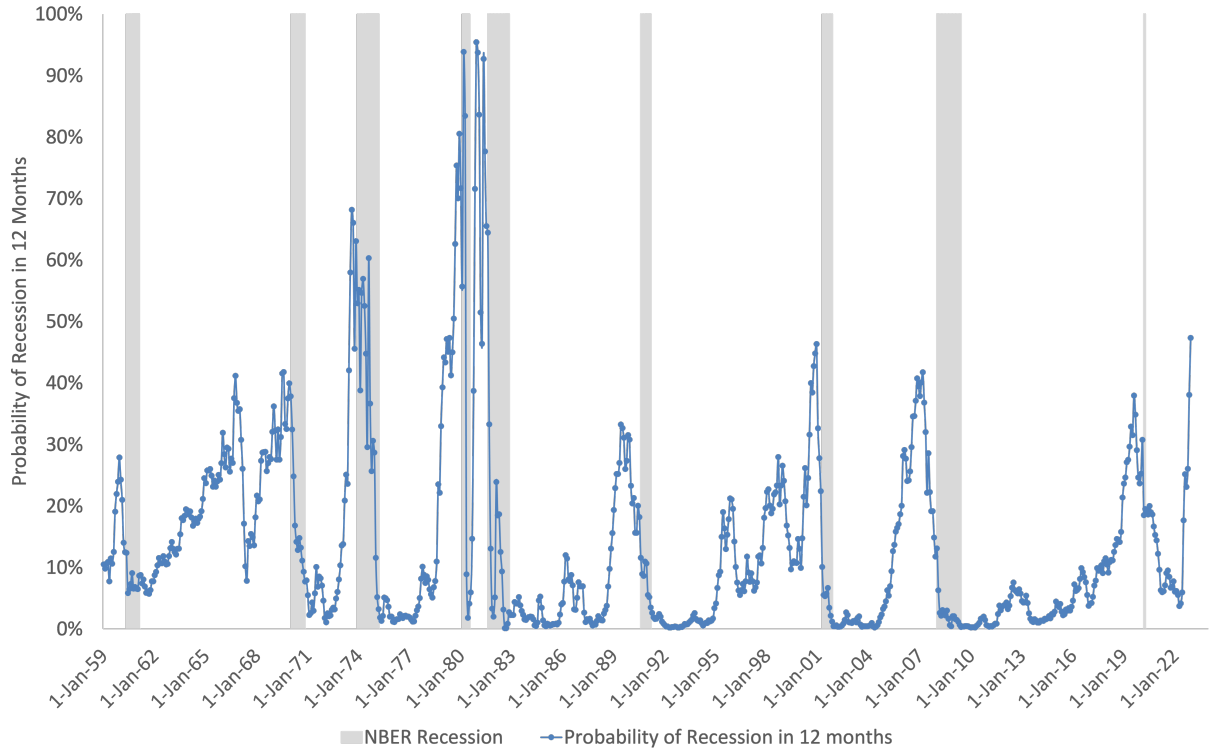


Figure 9: **Probability of Recession in One Year**

From the Federal Reserve Bank of New York. Source: https://www.newyorkfed.org/research/capital_markets/ycfaq#/.

period covers the fourth quarter of 1971 to the fourth quarter of 2021. While the data goes back to 1961, the availability of key outcome variables is largely unavailable until 1971. To examine how changes in recession risk affect firm dynamics, in our regressions, we remove from our sample the observations during NBER-designated recessions. See Appendix Table B.2 for details on the filtering. The final sample has 12,473 unique firms and a total of 443,336 firm quarters. Table 4 Panel A provides summary statistics.

The two continuous state variables in the model are the firm's cash holdings and capital stock. To proxy for the cash holdings, we use a firm's cash and cash equivalents (*cheq*) from the quarter-end balance sheet. To proxy for the size of a firm's productive capital stock, we use a firm's total assets less cash holdings (*atq-cheq*) from the quarter-end balance sheet.

The main firm outcome variables characterize a firm's issuance, payout, and investment activity. To proxy for a firm's issuance activity, we compute a firm's quarterly total sales of common stock from the cumulative total sales listed on the cash flow statement

Table 4: Summary Statistics
Variables winsorized at the 1% level.

Panel A: Firm-Quarter Panel						
	Obs.	Mean	Std. Dev.	P25	P50	P75
Recession Probability (%)	443,336	11.0	13.2	1.6	6.0	15.4
Total Assets-Cash (Million)	443,336	1,929	5,518	74.8	258.0	1,090.0
Cash and Short-Term Investments (Million)	443,336	173.8	530.3	3.7	19.8	92.5
Cash/(Total Assets-Cash) (%)	443,336	20.9	41.8	1.9	6.2	19.4
Net Property, Plant & Equipment (PP&E) (Million)	443,336	711.7	2249.1	19.4	69.2	331.9
Capital Expenditures (Million)	378,742	33.1	100.2	1.0	3.8	16.8
Capital Expenditures/(Total Assets-Cash) (%)	378,742	2.0	2.4	0.6	1.3	2.4
Capital Expenditures/PP&E (%)	378,742	6.1	5.6	2.5	4.5	7.9
%ΔCapital Expenditures from $(t-3,t)$ to $(t+1,t+4)$	236,057	26.5	87.0	-21.0	8.2	46.4
Dividend (Millions)	380,206	10.3	41.6	0.0	0.0	1.4
%ΔDividend from $(t-3,t)$ to $(t+1,t+4)$	113,033	13.6	83.8	-1.2	3.8	16.5
$\mathbb{1}$(Issuance Amount>30% of Total Assets-Cash) (%)	359,612	1.0	10.2	0.0	0.0	0.0

Panel B: Stock-Month Panel						
	Obs.	Mean	Std. Dev.	P25	P50	P75
Recession Probability (%)	1,401,062	12.2	14.6	1.6	6.6	19.3
Total Stock Return (%)	1,401,062	1.1	12.9	-6.0	0.2	7.4

(*sstky*). To proxy for a firm's payout activity, we compute a firm's quarterly amount of dividends from the cumulative dividend variable (*dvy*) and cumulative common stock repurchases (*prstkcy*).[14] To proxy for a firm's investment rate, we use a firm's capital expenditures on property, plants, and equipment (*capxq*).

To examine the effects of recession risk on firm values, we also rely on stock return data from the CRSP/Compustat merged database. Our stock analysis is at the monthly frequency because our recession probabilities are available monthly. Our stock sample covers 10,854 firms; spans June 1962 to December 2022; and contains about 1.4 million stock-month observations. Table 4 Panel B shows that the average monthly return is 1.1% with a standard deviation of 12.9%. In the stock panel, the recession probabilities have similar summary statistics as in the quarterly Compustat data. See Table B.3 for our sample selection criteria.

4.2 Issuance and recession risk

As discussed in Section 3.4.1, firms respond to higher recession risk by preemptively issuing equity. The model predicts that when a firm is larger, its preemptive issuance

[14]Because *sstky*, *dvy*, and *prstkcy* are cumulative over the fiscal year, we determine the quarterly values by differencing these variables across quarters in the same fiscal year.

behavior is more sensitive to changes in recession risk. That is, when a firm is larger, the cash holdings immediately prior to issuance increase more when recession risk increases.

To examine the prediction empirically, we examine cash holdings immediately prior to issuance activity. That is, we limit the sample to issuances at quarter $t+1$ that exceed various proportions of firm size (total assets less cash holdings) at the end of quarter t. Then, we estimate the following multivariate specification:

$$\begin{aligned}\log(\text{Cash})_{i,t} = \beta_0 &+ \beta_1 \log(\text{Recession Probability}_t) \\ &+ \beta_2 \log(\text{Recession Probability}_t) \times \log(\text{Size})_{i,t} + \beta_3 \log(\text{Size})_{i,t} \\ &+ X_t + \epsilon_{i,t}.\end{aligned} \tag{14}$$

Again, all of the variables are measured as of quarter t, which precedes a known issuance in quarter $t+1$. $Size_{i,t}$ is a firm's assets minus cash holdings at the end of quarter t. $Cash_{i,t}$ is a firm's cash holdings at the end of quarter t. $Recession\ Probability_t$ is the average monthly recession probability in quarter t. β_2 captures the extent to which cash holdings prior to issuance vary differently with recession risk because of differences in firm size. To account for the documented cyclicality in cash over the business cycle, X_t is a set of controls for the current state of the business cycle, including the average Volatility Index in quarter t, the CRSP market return in quarter t, and the probability from Chauvet and Piger (2008) of the economy being in a recession in quarter t (not $t+4$) from the business cycle dating literature. We interact these controls with $Size_{i,t}$ to allow small and large firms to have different sensitivities to the business cycle. Table B.1 shows that our measure of future recession risk has effectively zero correlation with these controls. We cluster standard errors by quarter because our variable of interest — recession risk – is constant across observations in a quarter.

In this and subsequent specifications, and because our variables of interest are a firm's levels of cash and capital, we scale variables by each firm's standard deviation of that variable, instead of scaling by some proxy for total firm size. In other words, we standardize these variables within a firm. Standardizing within a firm removes between-firm differences in the variances of variables of interest in addition to differences in their means to facilitate the interpretation of the magnitudes and is the recommended approach for within-firm analyses (Mummolo and Peterson, 2018; deHaan, 2021). Thus, any variation with firm size in the sensitivities of firm policies to changes in recession risk results from comparing a firm's policies when recession risk is high with that same firm's policies when recession risk is low.

Table 5 reports estimates from specification (14). The sample in columns (1)-(2), (3), and (4) require issuance amounts at $t+1$ of at least 30%, 50%, and 70% of total assets less cash holdings at the end of quarter t, respectively. Column (1) of Table 5 shows that, in agreement with the model predictions, recession risk in quarter t is positively related to cash holdings in quarter t immediately prior to issuance in quarter $t+1$ ($\beta_1 = 0.053$ and is significant at the 5% level). In other words, cash holdings prior to issuance are generally higher when recession risk is higher. This finding is consistent with firms issuing equity preemptively in response to changes in recession risk. More importantly, larger firms exhibit a higher positive sensitivity of cash holdings to recession risk prior to issuance ($\beta_2 = 0.059$ and is significant at the 1% level). A one standard deviation increase in recession risk corresponds with a 0.053 (0.112) standard deviation increase in cash prior to issuance for the average (one-standard-deviation larger) firm. This finding supports the prediction that larger firms' cash holdings at issuance are significantly and meaningfully more responsive to recession risk. Column (2) shows no change in these sensitivities when adding the controls for the current state of the business cycle. Columns (3) and (4) show that these findings hold for issuances exceeding 50% and 70% of firm size, respectively.

4.3 Investment and recession risk

As discussed in Section 3.4.2, firms respond to higher recession risk by cutting investment, especially larger firms. To examine this prediction empirically, we use the following multivariate specification:

$$\begin{aligned}\left(\frac{\sum_{t=1}^{4}\text{CAPX}_t}{\sum_{t=-3}^{0}\text{CAPX}_t}-1\right) &= \beta_0+\beta_1\log(\text{Recession Probability}_t)\\ &+\beta_2\log(\text{Recession Probability}_t)\times\text{Size}_{i,t}\\ &+\beta_3\text{Size}_{i,t}+X_{i,t}+\epsilon_{i,t}.\end{aligned} \tag{15}$$

The outcome variable is the change in capital expenditures on property, plant, and equipment. To account for seasonality in investment across quarters, we compare investment in the future four quarters ($t+1$, $t+2$, $t+3$, and $t+4$) to investment in the prior four quarters ($t-3$, $t-2$, $t-1$, and t). We standardize growth in capital expenditures within a firm to account for between-firm differences in average growth and its volatility. $Size_{i,t}$ is a firm's assets minus cash holdings at the end of quarter t, standardized within firm.

Table 5: Recession risk and cash holdings immediately prior to issuance

This table reports estimates from specification (14). The sample includes firm-quarter observations immediately preceding an equity issuance in quarter $t+1$ greater than 30% (columns 1-2), 50% (column 3), and 70% (column 4) of total assets less cash holdings at the end of quarter t. The outcome variable, $Cash_{i,t}$, is a firm's cash holdings at the end of quarter t, standardized within a firm. We drop the IPO year from our sample. *log(Recession Probability$_t$)*, is the quarter t log probability of a recession in twelve months. $Size_{i,t}$ is the log of firm i's total assets less cash holdings at the end of quarter t, standardized within a firm. Controls for the business cycle include the average Volatility Index in quarter t, the CRSP market return in quarter t, and the probability of the economy being in a recession in quarter t (not $t+4$) from Chauvet and Piger (2008). We interact these controls with $Size_{i,t}$ to allow small and large firms to have different sensitivities to the business cycle. We cluster standard errors by quarter. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	log(Cash)$_{i,t}$			
	(1)	(2)	(3)	
log(Recession Probability$_t$)	0.053**	0.059***	0.097***	0.091**
	(0.020)	(0.021)	(0.029)	(0.035)
log(Recession Probability$_t$) × log(Size)$_{i,t}$	0.059***	0.058***	0.098***	0.087***
	(0.016)	(0.015)	(0.020)	(0.026)
log(Size)$_{i,t}$	0.363***	0.295***	0.207***	0.183*
	(0.017)	(0.063)	(0.076)	(0.095)
Constant	0.714***	0.692***	0.582***	0.424***
	(0.025)	(0.092)	(0.104)	(0.115)
Issuance Sample $\left(\frac{\text{Issuance}_{i,t+1}}{\text{Assets-Cash}_{i,t}} > X\%\right)$	30%	30%	50%	70%
Business Cycle Controls	No	Yes	Yes	Yes
% Adjusted R^2	11.99	12.62	12.50	12.81
Observations	3768	3339	2093	1487

Recession Probability$_t$ is the average monthly recession probability in quarter t. β_1 captures the extent to which investment responds to the level of recession risk, and β_2 captures the extent to which the investment response to recession risk varies with a firm's size. We do not relate changes in investment to changes in recession risk because investment is unlikely to respond immediately to innovations in recession risk but rather over time to levels of recession risk. X_t are the aforementioned business cycle controls and their interactions with $Size_{i,t}$. In addition to clustering the standard errors by quarter because recession risk is quarterly, we also cluster standard errors by firm to account for the serial correlation induced by overlap in the outcome variable across quarters.

Table 6 reports estimates from specification (15). Column (1) of Table 6 shows that investment growth is negatively related to the level of recession risk as the regression coefficient ($\beta_1 = -0.030$ and is statistically significant at 1% level). Additionally, Column (1) shows that larger firms cut investment growth more in response to increases in

Table 6: Sensitivity of investment to recession risk

This table reports estimates from specification (15). Recession risk decreases investment growth, especially when a firm is larger. The outcome variable is the growth of capital expenditures on property, plant, and equipment. To account for seasonality in investment across quarters, we compare investment in the future four quarters ($t+1$, $t+2$, $t+3$, and $t+4$) to investment in the prior four quarters ($t-3$, $t-2$, $t-1$, and t). We standardize these changes in investment within a firm. *log(Recession Probability$_t$)*, is the quarter t average monthly probability of a recession in twelve months. $Size_{i,t}$ is the log of firm i's total assets less cash holdings at the end of quarter t, standardized within a firm. Controls for the business cycle include the average Volatility Index in quarter t, the CRSP market return in quarter t, and the probability of the economy being in a recession in quarter t (not $t+4$) from Chauvet and Piger (2008). We interact these controls with $Size_{i,t}$ to allow small and large firms to have different sensitivities to the business cycle. We double cluster standard errors by firm and quarter. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	$\left(\frac{\sum_{t=1}^{4} \text{CAPX}_t}{\sum_{t=-3}^{0} \text{CAPX}_t} - 1\right)$			
	(1)	(2)	(3)	
log(Recession Probability$_t$)	-0.030***	-0.030**	-0.028***	-0.034**
	(0.011)	(0.012)	(0.010)	(0.017)
log(Recession Probability$_t$) × log(Size)$_{i,t}$	-0.018***	-0.020***	-0.023***	-0.012
	(0.006)	(0.007)	(0.007)	(0.009)
log(Size)$_{i,t}$	-0.185***	-0.215***	-0.189***	-0.242***
	(0.006)	(0.021)	(0.021)	(0.029)
Constant	0.017*	0.091**	0.105***	0.091*
	(0.009)	(0.038)	(0.032)	(0.051)
Firm Size	All	All	Below-Median	Above-Median
Business Cycle Controls	No	Yes	Yes	Yes
% Adjusted R^2	2.71	3.02	2.80	2.73
Observations	235722	198524	99262	99262

recession risk ($\beta_2 = -0.018$ and is statistically significant at 1% level). Therefore, in agreement with model predictions, when a firm is larger, investment cuts because of recession risk are more sensitive to to the level of recession risk. A one standard deviation increase in recession risk corresponds with a 0.03 (0.048) standard deviation decrease in investment growth for the average (one-standard-deviation larger) firm. Column (2) shows that the results hold when a firm's size (total assets less cash holdings) is below the median in the cross-section of all firms. Column (3) shows that β_2 is negative but not significant in the sample of firms with a size above the median in the cross-section of all firms.

4.4 Payouts and recession risk

As discussed in Section 3.4.3, firms respond to increases in recession risk by cutting payout rates, especially large firms. To examine this prediction empirically, we use the

following multivariate specification:

$$\begin{aligned}\left(\frac{\sum_{t=1}^{4}\text{Payouts}_t}{\sum_{t=-3}^{0}\text{Payouts}_t}-1\right) &= \beta_0+\beta_1\log(\text{Recession Probability}_t)\\ &+\beta_2\log(\text{Recession Probability}_t)\times\text{Size}_{i,t}+\beta_3\text{Size}_{i,t}\\ &+X_{i,t}+\epsilon_{i,t}.\end{aligned} \tag{16}$$

The outcome variable is the change in a firm's dividend payments over the next four quarters relative to the past four quarters. We standardize dividend growth within a firm to account for between-firm differences in average growth and its volatility. $Size_{i,t}$ is a firm's assets minus cash holdings at the end of quarter t, standardized within the firm. *Recession Probability*$_t$ is the average monthly recession probability in quarter t. We do not relate changes in dividends to changes in recession risk because dividends are unlikely to respond immediately to innovations in recession risk but rather over time to levels of recession risk. $X_{i,t}$ are the aforementioned business cycle controls and their interactions with $Size_{i,t}$. β_1 captures the extent to which dividend growth responds to recession risk, and β_2 captures the extent to which the dividend response to recession risk varies with a firm's size. In addition to clustering the standard errors by quarter because recession risk is quarterly, we also cluster standard errors by the firm to account for the serial correlation induced by the overlap in the outcome variable across quarters.

Table 7 column (1) shows that changes in total payouts are negatively related to recession risk ($\beta_1 = -0.010$ but not statistically significant). Additionally, the drop in total payouts because of recession risk is larger when a firm is larger ($\beta_2 = -0.016$ and is significant at the 1% level). A one standard deviation increase in recession risk corresponds with a 0.01 (0.026) standard deviation decrease in payout growth for the average (one-standard-deviation larger) firm. Column (2) shows that the results are robust to controlling for the current state of the business cycle. Column (3) shows that the results are somewhat weaker when a firm's size is below the median in the cross-section of all firms. Column (4) shows that the relations are more significant for the sample of firms with a size above the median in the cross-section of all firms.

4.5 Firm value and recession risk

As discussed in Section 3.4.4, the model predicts that recession risk reduces firm values, especially when a firm is larger. To examine this prediction empirically, we use the

Table 7: Sensitivity of payouts to recession risk

This table reports estimates from specification (16). The outcome variable is dividend growth. Because dividend policies are sticky, we compare dividends over the future four quarters ($t+1$, $t+2$, $t+3$, and $t+4$) to dividends in the prior four quarters ($t-3$, $t-2$, $t-1$, and t). We standardize dividend growth within a firm. *log(Recession Probability$_t$)*, is the quarter t average monthly probability of a recession. *Size$_{i,t}$* is the log of firm i's total assets less cash holdings at the end of quarter t, standardized within a firm. Controls for the business cycle include the average Volatility Index in quarter t, the CRSP market return in quarter t, and the probability of the economy being in a recession in quarter t (not $t+4$) from Chauvet and Piger (2008). We interact these controls with *Size$_{i,t}$* to allow small and large firms to have different sensitivities to the business cycle. We double cluster standard errors by firm and quarter. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

			$\left(\frac{\sum_{t=1}^{4}\text{Payouts}_t}{\sum_{t=-3}^{0}\text{Payouts}_t}-1\right)$	
	(1)	(2)	(3)	(4)
log(Recession Probability$_t$)	-0.010	-0.015	-0.008	-0.024*
	(0.009)	(0.011)	(0.010)	(0.014)
log(Recession Probability$_t$) × log(Size)$_{i,t}$	-0.016***	-0.011**	-0.004	-0.013*
	(0.005)	(0.005)	(0.006)	(0.007)
log(Size)$_{i,t}$	-0.047***	-0.069***	-0.055***	-0.097***
	(0.006)	(0.018)	(0.020)	(0.027)
Constant	0.004	-0.006	-0.023	0.029
	(0.008)	(0.028)	(0.024)	(0.042)
Firm Size	All	All	Below-Median	Above-Median
Firm FE	Yes	Yes	Yes	Yes
Business Cycle Controls	No	Yes	Yes	Yes
% Adjusted R^2	0.19	0.21	0.15	0.30
Observations	157122	129031	64517	64514

following multivariate specification:

$$\begin{aligned}\text{Stock Return}_{i,t} =& \beta_0 + \beta_1 \Delta\log(\text{Recession Probability}_t) \\ &+ \beta_2 \Delta\log(\text{Recession Probability}_t) \times \text{Size}_{i,t} \\ &+ \beta_3 \text{Size}_{i,t} + X_{i,t} + \epsilon_{i,t}.\end{aligned} \tag{17}$$

The outcome variable is the stock return for firm i in month t. *Size$_{i,t}$* is a firm's assets minus cash holdings at the end of the prior fiscal year, standardized within a firm. *Recession Probability$_t$* is monthly recession probability in quarter t. In this specification, we use month-to-month changes in the recession probability because any changes in firm values (stock returns) because of changes in recession risk in an efficient market happen contemporaneously. By contrast, issuances, investments, and payouts are less likely to change immediately with recession risk. β_1 captures the extent to which stock returns

Table 8: Sensitivity of firm value to recession risk

This table reports estimates from specification (17). The outcome variable is firm i's total stock return in month t. Δ *log(Recession Probability$_t$)*, is the quarter $t-1$ to t change in the average monthly probability of a recession. *Size*$_{i,t}$ is the log of firm i's total assets net of cash holdings at the end of the prior year, standardized within a firm. Controls for the business cycle include the average Volatility Index in quarter t and the probability of the economy being in a recession in quarter t (not $t+4$) from Chauvet and Piger (2008). We interact these controls with *Size*$_{i,t}$ to allow small and large firms to have different sensitivities to the business cycle. Column (3) excludes the 2008-2009 financial crisis. We cluster standard errors by month. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Stock Return		
	(1)	(2)	
Δ log(Recession Probability$_t$)	-0.038*	-0.077***	-0.077***
	(0.020)	(0.020)	(0.020)
Δ log(Recession Probability$_t$) $\times$ log(Size)$_{i,t}$	-0.014**	-0.016***	-0.016***
	(0.006)	(0.005)	(0.005)
log(Size)$_{i,t}$	-0.029***	-0.024	-0.023
	(0.007)	(0.020)	(0.020)
Constant	-0.006	0.299***	0.303***
	(0.016)	(0.068)	(0.068)
Sample	All	All	Excl. 08/09
Controls	No	Yes	Yes
% Adjusted R^2	0.24	1.89	1.95
Observations	1368656	895656	883032

respond to recession risk, and β_2 captures the extent to which the stock return response to recession risk varies with a firm's size. To account for the documented cyclicality in firm outcomes over the business cycle, X_t is a set of controls for the current state of the business cycle, including the average Volatility Index in month t and the probability of the economy being in a recession in month t (not $t+4$) from Chauvet and Piger (2008). We interact these controls with *Size*$_{i,t}$ to allow small and large firms to have different sensitivities to the business cycle. We cluster the standard errors by month.

Table 8 column (1) shows that stock returns are negatively related to recession risk ($\beta_1 = -0.038$ and is significant at the 10% level). Additionally, stock returns are more negatively related to recession risk when a firm is larger ($\beta_2 = -0.014$ and is significant at the 5% level). Column (2) shows that the relations are robust to controls for the business cycle. Column (3) shows that the results hold outside of the 2008-2009 financial crisis.

4.6 Quantitative Exercise

To evaluate the model performance one would ideally estimate it in the data. Unfortunately, due to the complexity of our model, it is time-prohibitive to implement the Simulated Method of Moments (SMM) or even the Generalized Method of Moments (GMM). Therefore, to provide a quantitative link between our model and the empirical

evidence reported in Tables 5-8 we perform a calibration exercise in the spirit of Livdan and Nezlobin (2021). That is, we compare model-implied moments and sensitivities with their empirical counterparts. This exercise can shed light on the expected magnitude of the main variables and sensitivities in our model. In addition, this exercise helps cross-validate our parameter assumptions in Table 2.

When we compare the calibrated moments in the model to similar moments in the data, we need to compare the average model moment to the average empirical value. Because not every capital and cash position (k, c) is equally likely to occur, we determine the average model moment using simulations. We simulate firms for 10 years, starting in an expansion (50-50 likelihood of the two expansion states) and in each point of a uniform grid in (k, c)-space. Figure 10 is a density plot showing how often the firm is in a particular (k, c) point.

The first calibrated moment we consider is the cash-to-capital ratio. Using the model simulations, we determine the expected cash-to-capital ratio to be 50%. This quantity is similar to several empirical analogs using several proxies for the size of a firm's productive capital k. In the data, the average Cash/(Total Assets-Cash), Cash/Gross PP&E, and Cash/Net PP&E are 21%, 41%, and 103%.

Relatedly, the second calibrated moment we consider is the covariance between cash and capital in the model and in the data. For the model analysis, for each level of capital k, we determine the average cash holdings c in the simulations. We then standardize these cash and capital measures. Lastly, we regress cash on capital and get $\beta = 1$. For the data analysis, we use measures of cash and firm size standardized within a firm. Regressing cash on Total Assets-Cash, Gross PP&E, and Net PP&E yields uniformly close to 0.3.

Another calibrated moment to consider is the sensitivity of issuance amounts to firm size. In the model, we determine the issuance amount for each level of productive capital k as the distance between the issuance boundary (∂E) and the issuance target (black line) in Figure 2. We scale these issuance amounts by the firm's capital stock k. Then, we standardize the issuance amounts and capital k. Regressing issuance amounts on capital yields $\beta = -0.26$. In the data, we likewise look at quarters with the issuance and normalize the amount issued by the prior Total Assets-Cash, Gross PP&E, and Net PP&E. We standardize issuances proportional to firm size and firm size. Regressing issuances on firm size yields negative betas of -0.09, -0.06, and -0.02 respectively.

We likewise compare several investment moments. To examine model investment, we keep track of the investment rate (i^*/k) in our simulations in Figure 10. First, the

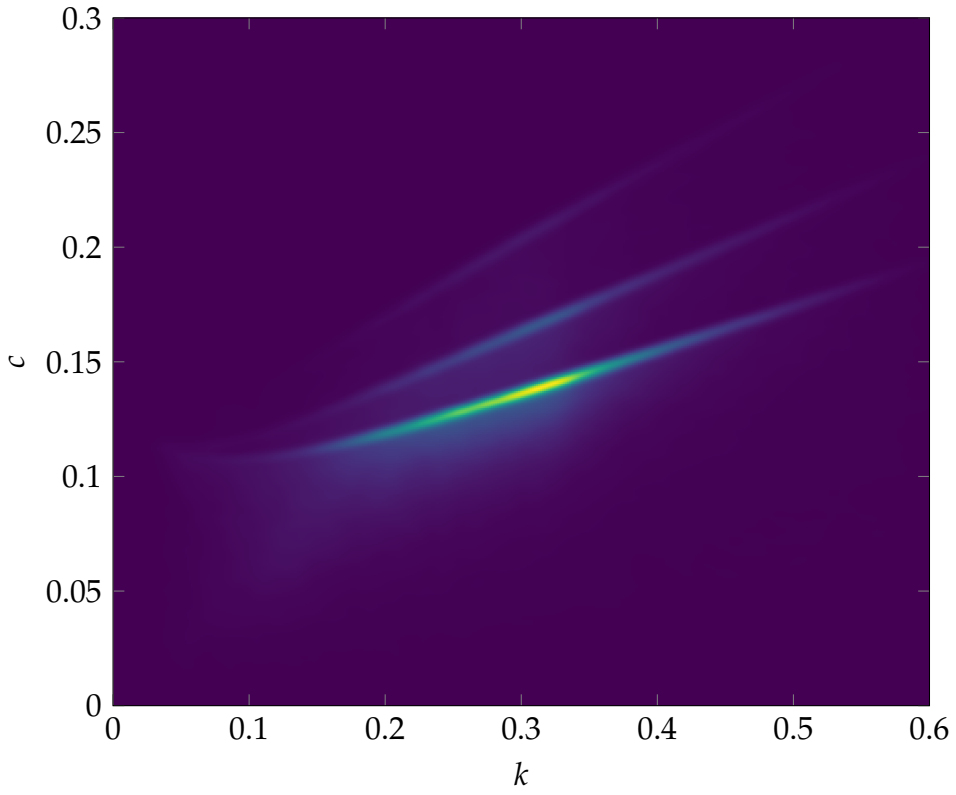


Figure 10: **Heat map of firm location in (k, c) space from simulations**

The (marginal) (k, c)-density of firms: Firms are simulated for 10 years, starting in an expansion (50-50 likelihood of the two expansion states) and in each point of a uniform grid in (k, c)-space. Yellow/light regions denote greater occupancy, whereas blue/dark denotes lower. The density is estimated using the full path of states, aggregated over the business cycle state. In other words, the figure is a heatmap of an estimate of the function $(k, c) \mapsto \int_{k_0,s_0} \sum_{c_0 \in \{m,l\}} \int_0^{10} \sum_s p_{t,k_0,c_0,s_0}(k, c, s) dt dc_0 dk_0$, where $p_{t,k_0,c_0,s_0}(k, c, s)$ is the density of the state at time t, staring in (k_0, c_0, s_0), and the (k_0, c_0) integral is over the domain of the figure. Parameters used are summarized in Table 2.

Table 9: Quantitative Exercise

The model values come from the simulation in Figure 10. Specifically, we find the average cash holdings, cash-to-capital ratio, and investment rate during the simulations. To estimate the model implied issuance amounts, we compute the distance between the exercise boundary ∂E and the issuance target in Figure 2. We also keep track of firm investment and cash in the simulations so that we can examine the sensitivities of these variables to capital. To compute the sensitivities, we regress the standardized variable of interest on standardized capital and report the β. In the data, we compute the empirical analogs. When estimating the sensitivities, we likewise standardize the y and x variables within a firm so that the coefficients are more comparable to those from the model.

Model		Empirical	
Variable	Value	Variable	Value
Cash-to-Capital Ratio (c/k)	50%	Cash/(Total Assets-Cash)	21%
		Cash/(Gross PP&E)	41%
		Cash/(Net PP&E)	103%
β(Cash, Capital)	1.0	β(Cash, Total Assets-Cash)	0.3
		β(Cash, Gross PP&E)	0.3
		β(Cash, Net PP&E)	0.3
β(Issuance Amt., Capital)	-0.26	β(Issuance Amt., Total Assets-Cash)	-0.09
		β(Issuance Amt., Gross PP&E)	-0.06
		β(Issuance Amt., Net PP&E)	-0.02
Investment/Capital (i^*/k)	10%	Investment/(Total Assets-Cash)	2%
		Investment/Gross PP&E	3%
		Investment/Net PP&E	6%
β(Investment Rate, Capital)	-0.86	β(Investment Rate, Total Assets-Cash)	-0.19
		β(Investment Rate, Gross PP&E)	-0.23
		β(Investment Rate, Net PP&E)	-0.13

average simulated investment rate is 10%. By contrast, in the data, we find investment rates of 2%-6% depending on how the productive capital stock is measured. Second, the sensitivity of investment rates to capital is -0.86 in the model and -0.19, -0.23, and -0.13 in the data using Total Assets-Cash, Gross PP&E, and Net PP&E to proxy for the productive capital stock.

Overall, the calibrated model predicts the correct signs for the sensitivities that we consider. And, the moments and sensitivities match the order of magnitude of their data counterparts.

5 Numerical Algorithm

In this section, we present an outline of the policy iteration method (Kushner and Dupuis (2001), Chapters 5 and 6) employed to solve the model. Importantly, the proposed algorithm converges to the unique solution of the HJB equation. The viscosity comparison

proof underlying the convergence of the algorithm, along with the uniqueness of the value function satisfying the HJB equation, can be found in Appendix A. It is important to point out that the application of policy iteration methods, which alternate between updating policies and evaluating payoffs, is not trivial in this context due to the impulsive and singular control of dividend payouts and equity issuance. We refer to this phenomenon collectively as 'singular'. In essence, we address the singularities in the problem by applying policy iteration to an approximation of the problem. This approximation involves two steps: the discretization of space and the imposition of a penalty on singularity.[15]

For some policy $\pi = (\pi_{\text{inv}}, \pi_{\text{div}}, \pi_{\text{iss}})$, let π_{inv} denote the investment intensity, π_{div} denote whether dividends are paid, and π_{iss} the issuance amount. Let $\mathcal{C}_{\pi_{\text{inv}}} V$ be the expression in (11) with $i = \pi_{\text{inv}}$, and let $\mathcal{I}_{\pi_{\text{iss}}} V(k, c, s) = V(k, c + \pi_{\text{iss}}, s) - \pi_{\text{iss}} - \lambda(\pi_{\text{iss}}, s)$. For an optimal policy, we can then write the HJB equation (12) as

$$0 = \min\{\mathcal{C}_{\pi_{\text{inv}}} V, \mathcal{D}V, \mathcal{I}_{\pi_{\text{iss}}} V\}.$$

Next, define

$$\mathcal{M}_\pi V = \mathcal{C}_{\pi_{\text{inv}}} V + m 1_{\pi_{\text{div}}} \mathcal{D}V + m 1_{\{\pi_{\text{iss}} > 0\}} \mathcal{I}_{\pi_{\text{iss}}} V,$$

for some large $m \gg 1$. The equation

$$0 = \inf_\pi \mathcal{M}_\pi V$$

is referred to as a penalized version of the problem and has a natural stochastic representation as randomized activation of the control actions.

Finally, denote by $\mathcal{B}$ the discretized domain of computation[16] and, with some abuse of notation, $\mathcal{M}_\pi(k, c, s, k', c', s')$ the coefficient in the discretization of $\mathcal{M}_\pi$ for point (k', c', s')

[15] For discussion about convergence and comparison to other methods, see, e.g., Azimzadeh and Forsyth (2016). See also Reppen, Jean-Charles and Soner (2020); Kakhbod et al. (2022).

[16] We select this domain to be substantially large and impose corresponding boundary conditions. At $k = 0$, the adjustment cost g becomes infinitely large for $i > 0$. Conversely, at $k = k_{max}$ (where k_{max} is a sufficiently large value), the advantage of investment becomes negligible in comparison to the adjustment cost. This is due to the effects of diminishing returns to scale and depreciation. Consequently, we set the value of no investment as the boundary condition at both extremes. Upon setting k_{max}, when the cash reserve is significantly high, we anticipate that the firm will optimally disburse excess cash. Thus, we impose the boundary condition $\partial_c V = 1$ at $c = c_{max}$ and $k \in [0, k_{max}]$, provided c_{max} is sufficiently large. We then verify that the first group of terms on the right-hand side of equation (12) is positive, ensuring that c_{max} is sufficiently magnitude. Please refer to the appendix for a more detailed discussion of the boundary conditions and the convergence of the numeric scheme.

in the equation for (k, c, s). With an initial policy π^0, we iterate the following.

Policy iteration algorithm (step i)

1. Compute V^i such that

$$\sum_{(k',c',s')\in\mathcal{B}} \mathcal{M}_{\pi^i}(k,c,s,k',c',s')V^i(k',c',s') = 0, \quad \forall (k,c,s) \in \mathcal{B}.$$

   Halt if $V^i = V^{i-1}$.

2. For each $(k, c, s) \in \mathcal{B}$, compute $\pi^{i+1}(k, c, s)$ according to

$$\pi^{i+1}(k,c,s) \in \arg\min_{\hat{\pi}} \sum_{(k',c',s')\in\mathcal{B}} \mathcal{M}_{\hat{\pi}}(k,c,s,k',c',s')V^i(k',c',s').$$

   Set $\pi^{i+1} = \pi^i$ if possible.

3. Return to step (i).

Finally, it is important to note that $\mathcal{M}$ is appropriately discretized to be weakly diagonally dominant, which is achieved if the discretized operator can be interpreted as the transition matrix of a (continuous time) Markov chain. Using these, Theorem 1 and Corollary 3 prove convergence (see Appendix A).

6 Conclusion

This paper examines theoretically and empirically how firm policies respond to recession risk. To characterize these sensitivities, one of our main contributions is to solve a dynamic model of a firm that uniquely features time-varying recession risk. Interestingly, large firm policies have a higher sensitivity to recession risk because small firms respond more when recession risk is low. The rationale is that small firms aggressively invest, which uses cash and makes liquidation more likely when a recession occurs. Thus, small firms have stronger incentives to require a higher minimum cash holding relative to their size even when recession risk is low. We then proceed by documenting empirical support for these predictions. Lastly, given the real effects of *anticipating* recessions, future research may examine the effectiveness of government policies aimed at mitigating expectations about the severity of future recessions.

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Appendix

This Appendix contains supplementary theoretical and empirical work. These include the following:

1. Appendix A provides proofs.
2. Appendix B provides additional empirical work.
   (a) Table B.1 shows the correlation of our recession probability measure with other leading indicators.
   (b) Table B.2 shows the Compustat sample selection criteria.
   (c) Table B.3 shows the CRSP sample selection criteria.
   (d) Table B.4 shows the annual Compustat sample selection criteria.
   (e) Robustness for Table 5, examining preemptive issuance to recession risk.
      i. Table B.5 repeats Table 5 using net plant, property, and equipment to proxy for firm size.
      ii. Table B.6 uses an indicator that equals one if recession risk exceeds the 75th percentile outside of NBER recessions.
   (f) Robustness for Table 6, examining investment and recession risk.
      i. Table B.7 uses net plant, property, and equipment to proxy for firm size.
      ii. Table B.8 adds firm fixed effects.
      iii. Table B.9 uses an indicator that equals one if recession risk exceeds the 75th percentile outside of NBER recessions.
   (g) Robustness for Table 7, examining payouts and recession risk.
      i. Table B.10 uses net plant, property, and equipment to proxy for firm size.
      ii. Table B.11 adds firm fixed effects.
   (h) Robustness for Table 8, examining stock returns and recession risk.
      i. Table B.12 uses net plant, property, and equipment to proxy for firm size.
      ii. Table B.13 adds firm fixed effects.

A Proofs

We simplify the setting of the comparison proof by making the following assumption: For each k and s, there exists a cash level such that dividend payouts are optimal whenever c is above that level, and this level is continuous in k. Furthermore, since the proof compactifies in k, we, for expositional simplification, assume that this level is a constant $\bar{C}$. We may thus write the HJB equation on the domain

$$\mathcal{O} = [0, \infty) \times [0, \bar{C}] \times \{l, m, h\}, \tag{18}$$

with the additional boundary condition

$$\partial_c V = 1 \quad \text{where } c = \bar{C}. \tag{19}$$

This is indeed satisfied by the value function, and in the numerical experiments, we verify that this is correct by solving on larger domains and observing that the dividend boundary does not move.

Theorem 1. *Let u and v be, respectively, continuous viscosity sub- and supersolutions to (12) in $\mathcal{O}$ with the boundary conditions (13) and (19). Assume further that u and v are both of linear growth in c and polynomial growth in k, i.e., they take values in $[c, c + M + p(k)]$ for some constant $M > 0$ and polynomial p. Then, $u \leq v$ everywhere in $\mathcal{O}$.*

In the proof, we will use the result from Grullon et al. (2022) or Altarovici, Reppen and Soner (2017) establishing that the functions obtained from applying the issuance operator to u and v are continuous.

Proof. In this proof, we drop the dependence of μ_s and σ_s on the cycle s from the notation, as this dependence does not affect the arguments.

Suppose there exists a point at which $u > v$. Fix some $\eta_k > 0$ and consider $e^{-\eta_k k}(u - v)$, which, by the growth condition is bounded and attains a maximizer. We may therefore restrict ourselves to maximizers in a compact domain on which k is bounded by k^*, the latter depending only on η_k. Let $\hat{v}_\omega = (1 - \omega)v + \omega(1 + \lambda_p(s))c$, for some $\omega > 0$ small enough that $u > \hat{v}_\omega$ somewhere. From here on, we omit the ω in the notation: $\hat{v} = \hat{v}_\omega$.

Denote by $(\bar{k}, \bar{c}, \bar{s})$ a maximizer of $e^{-\eta_k k}(u - \hat{v})$. We may choose it so that $\{(\bar{k}, c, s) \in \mathcal{O} : c > \bar{c}\}$ does not contain any other maximizer. As a consequence, by the compactness of $\mathcal{O}$ in c, all points above $\bar{c}$ are take strictly lower values.

Let δ_η be the corresponding maximum. Define $\bar{f}(k,c,s) = \|(k,c,s) - (\bar{k},\bar{c},\bar{s})\|^4$ and

$$\Phi^\epsilon(k,c,s,\ell,d,t) = e^{-\eta_k k}u(k,c,s) - e^{-\eta_k \ell}\hat{v}(\ell,d,t) \\ - \beta\bar{f}(k,c,s) - \frac{1}{2\epsilon}\Big((c-d)^2 + (k-\ell)^2 + (s-t)^2\Big) \quad \text{in } \mathcal{O}\times\mathcal{O}.$$

Clearly,

$$\sup_{\mathcal{O}\times\mathcal{O}} \Phi^\epsilon \geq \Phi^\epsilon(\bar{k},\bar{c},\bar{s},\bar{k},\bar{c},\bar{s}) = e^{-\eta_k \bar{k}}\Big(u(\bar{k},\bar{c},\bar{s}) - \hat{v}(\bar{k},\bar{c},\bar{s})\Big) = \delta_\eta.$$

In particular, Φ^ϵ has a maximizer $(k_\epsilon, c_\epsilon, s_\epsilon, \ell_\epsilon, d_\epsilon, t_\epsilon)$ because of the growth conditions on u and v. Moreover, the growth conditions give an upper bound for this maximizer, depending only on η_k. Therefore, $(k_\epsilon, c_\epsilon, s_\epsilon, \ell_\epsilon, d_\epsilon, t_\epsilon)$ converges along a subsequence as $\epsilon \to 0$. From here on, let us only consider ϵ along this subsequence. Because the lower bound at the maximum above is independent of ϵ,

$$0 < \delta_\eta \leq \liminf_{\epsilon\to 0} \Phi^\epsilon(k_\epsilon, c_\epsilon, s_\epsilon, \ell_\epsilon, d_\epsilon, t_\epsilon),$$

which implies

$$\limsup_{\epsilon\to 0} \frac{1}{2\epsilon}\Big((c_\epsilon - d_\epsilon)^2 + (k_\epsilon - \ell_\epsilon)^2 + (s_\epsilon - t_\epsilon)^2\Big) < \infty,$$

so $(k_\epsilon, c_\epsilon, s_\epsilon), (\ell_\epsilon, d_\epsilon, t_\epsilon) \to (\bar{k},\bar{c},\bar{s})$. Note that $\bar{k} \leq k^*$, again because of the growth condition.

Rearranging terms and letting $\epsilon \to 0$,

$$\begin{aligned}
&\lim_{\epsilon\to 0} \beta\bar{f}(k_\epsilon, c_\epsilon, s_\epsilon) + \lim_{\epsilon\to 0} \frac{1}{2\epsilon}\Big((c_\epsilon - d_\epsilon)^2 + (k_\epsilon - \ell_\epsilon)^2 + (s_\epsilon - t_\epsilon)^2\Big) \\
&\qquad \leq \limsup_{\epsilon\to 0} e^{-\eta_k k_\epsilon}u(k_\epsilon, c_\epsilon, s_\epsilon) - e^{-\eta_k \ell_\epsilon}\hat{v}(\ell_\epsilon, d_\epsilon, t_\epsilon) - \delta_\eta \\
&\qquad \leq e^{-\eta_k \bar{k}}\Big(u(\bar{k},\bar{c},\bar{s}) - \hat{v}(\bar{k},\bar{c},\bar{s})\Big) - \delta_\eta \\
&\qquad \leq 0.
\end{aligned}$$

That is,

$$\lim_{\epsilon\to 0} \beta\bar{f}(k_\epsilon, c_\epsilon, s_\epsilon) + \lim_{\epsilon\to 0} \frac{1}{2\epsilon}\Big((c_\epsilon - d_\epsilon)^2 + (k_\epsilon - \ell_\epsilon)^2 + (s_\epsilon - t_\epsilon)^2\Big) \leq 0. \tag{20}$$

If $\bar{k} = 0$, we directly obtain $u(\bar{c},0) \leq \bar{c} \leq v(\bar{c},0)$, which is a contradiction. If $\bar{c} = 0$, the situation is either similar or the issuance condition is active, in which case it can be

treated similarly to the issuance condition on the interior. Hence, we resume with the case that $(\bar{k}, \bar{c}, \bar{s})$ lies in the interior, and therefore also $(k_\epsilon, c_\epsilon, s_\epsilon)$ and $(\ell_\epsilon, d_\epsilon, t_\epsilon)$ for sufficiently small ϵ.

Because the maxima are attained in interior points, we proceed to use Ishii's lemma. Since the equation only has a second derivative in c, we abuse notation and consider the corresponding elements of the jets as only the ∂_{cc}-component. We obtain $(p^u, X) \in \overline{J}^{2,+}(e^{-\eta_k k_\epsilon} u(k_\epsilon, c_\epsilon, s_\epsilon))$ and $(p^v, Y) \in \overline{J}^{2,-}(e^{-\eta_k \ell_\epsilon}(1-\omega)v(\ell_\epsilon, d_\epsilon, t_\epsilon))$ (Crandall, Ishii and Lions, 1992, Theorem 3.2), satisfying

$$
\begin{aligned}
p_c &= \frac{c_\epsilon - d_\epsilon}{\epsilon} \\
p^u &= (p_c^u, p_k^u) = \left(p_c + 4\beta(c_\epsilon - \bar{c})^3, p_k^v + 4\beta(k_\epsilon - \bar{k})^3 \right), \\
p^v &= (p_c^v, p_k^v) = \left(p_c - e^{-\eta_k \ell_\epsilon} \omega(1 + \lambda_p(t_\epsilon)), \frac{k_\epsilon - \ell_\epsilon}{\epsilon} \right)
\end{aligned}
$$

and

$$
k_\epsilon^{2\alpha} X - \ell_\epsilon^{2\alpha} Y \leq k_\epsilon^{2\alpha} 12\beta(c_\epsilon - \bar{c})^2 + \frac{(k_\epsilon^\alpha - \ell_\epsilon^\alpha)^2}{\epsilon} + o(1),
$$

where $o(1)$ denotes a term that converges to 0 as $\epsilon \to 0$.

Because u is a subsolution, $\tilde{u} = e^{-\eta_k k} u$ satisfies

$$
\begin{aligned}
0 \geq \min \Bigg\{ & r\tilde{u} - \sup_{i \in [0, i_{\max}]} \Bigg(\Big[i - \delta k_\epsilon \Big] (\eta_k \tilde{u} + \partial_k \tilde{u}) \\
& \qquad + \Big[(r - \lambda_c) c_\epsilon + k_\epsilon^\alpha \mu - i - g(k_\epsilon, i) \Big] \partial_c \tilde{u} \\
& \qquad\qquad + \frac{1}{2} k_\epsilon^{2\alpha} \sigma^2 \partial_{cc}^2 \tilde{u} \\
& \qquad\qquad + \sum_{s'} q_{s_\epsilon, s'} \tilde{u}(k_\epsilon, c_\epsilon, s') \Bigg), \\
& \tilde{u}(k_\epsilon, c_\epsilon, s_\epsilon) - \sup_{I \geq 0} \Big[\tilde{u}(k_\epsilon, c_\epsilon + I, s_\epsilon) - e^{-\eta k_\epsilon} (I + \lambda(I, s_\epsilon)) \Big], \\
& \partial_c \tilde{u} - e^{-\eta_k k_\epsilon} \Bigg\}.
\end{aligned}
\tag{21}
$$

Similarly, $\tilde{v} = e^{-\eta k}(1-\omega)v$ satisfies[17]

$$\begin{aligned} 0 \leq \min \Bigg\{ & r\tilde{v} - \sup_{i \in [0, i_{\max}]} \Bigg(\Big[i - \delta \ell_\epsilon \Big] (\eta_k \tilde{v} + \partial_k \tilde{v}) \\ & \qquad + \Big[(r - \lambda_c) d_\epsilon + \ell_\epsilon^\alpha \mu - i - g(\ell_\epsilon, i) \Big] \partial_c \tilde{v} \\ & \qquad\qquad + \frac{1}{2} \ell_\epsilon^{2\alpha} \sigma^2 \partial_{cc}^2 \tilde{v} \\ & \qquad\qquad + \sum_{s'} q_{t_\epsilon, s'} \tilde{v}(\ell_\epsilon, d_\epsilon, s') \Bigg), \\ & \hat{v}(\ell_\epsilon, d_\epsilon, t_\epsilon) - \sup_{I \geq 0} \Big[\hat{v}(\ell_\epsilon, d_\epsilon + I, t_\epsilon) - e^{-\eta \ell_\epsilon} (I + \lambda(I, t_\epsilon)) \Big], \\ & \partial_c \tilde{v} - (1-\omega) e^{-\eta_k \ell_\epsilon} \Bigg\}. \end{aligned} \tag{22}$$

We split into two cases, depending on which expression is smallest in Equation (21). We begin with the simple case of

$$p_c^u \leq e^{-\eta_c k_\epsilon}.$$

Subtracting the two equations (21) and (22) thus gives

$$e^{\eta_k \ell_\epsilon} \omega (1 + \lambda_p(t_\epsilon)) + 4\beta (c_\epsilon - \bar{c})^3 = p_c^u - p_c^v \leq e^{-\eta_k k_\epsilon} - (1-\omega) e^{-\eta_k \ell_\epsilon}.$$

Letting $\epsilon \to 0$, and dividing out equal factors, $\lambda_p(t_\epsilon) \leq 0$, which is a contradiction.

In the issuance case, because $\tilde{u}$ is continuous and $\lambda_f > 0$, there exist a (uniform as $\epsilon \to 0$) choice of $\underline{I}$ so that the optimization can be restricted to $I \geq \underline{I}$. We subtract the equations and pass to limits. Using the continuity of the issuance operator, $(\bar{k}, \bar{c}, \bar{s}) = (\bar{k}, \bar{c}, \bar{s})$, and the fact that $(\tilde{u} - \hat{v})(\bar{k}, \bar{c} + I, \bar{s})$ is strictly smaller than the maximum,

$$(\tilde{u} - \hat{v})(\bar{k}, \bar{c}, \bar{s}) \leq \sup_{I \geq \underline{I}} \Big[(\tilde{u} - \hat{v})(\bar{k}, \bar{c} + I, \bar{s}) \Big] < (\tilde{u} - \hat{v})(\bar{k}, \bar{c}, \bar{s}),$$

which is a contradiction.

[17] For convenience, we write the issuance expression in terms of $\hat{v}$, which we can do thanks to the growth rate of $1 + \lambda_p(s)$.

This leaves the final case, so we subtract the equations and get

$$
\begin{aligned}
r(\tilde{u}-\tilde{v}) \leq \sup_{i\in[0,i_{\max}]} \Bigg\{ & \Big[i-\delta_\zeta k_\epsilon\Big](\eta_k\tilde{u}(k_\epsilon,c_\epsilon,s_\epsilon)+p_k^v+4\beta(k_\epsilon-\bar{k})^3) \\
& + \Big[(r-\lambda_c)c_\epsilon+k_\epsilon^\alpha\mu-i-g(k_\epsilon,i)\Big](p_c+4\beta(c_\epsilon-\bar{c})^3)+\frac{1}{2}k_\epsilon^{2\alpha}\sigma^2 X \\
& + \sum_{s'} q_{s_\epsilon,s'}\tilde{u}(k_\epsilon,c_\epsilon,s') \\
& - \Big[i-\delta_\zeta\ell_\epsilon\Big](\eta_k\tilde{v}(\ell_\epsilon,d_\epsilon,t_\epsilon)+p_k^v) \\
& - \Big[(r-\lambda_c)d_\epsilon+\ell_\epsilon^\alpha\mu-i-g(\ell_\epsilon,i)\Big](p_c^u-e^{-\eta_k\ell_\epsilon}\omega(1+\lambda_p(t_\epsilon)) \\
& - \sum_{s'} q_{t_\epsilon,s'}\tilde{v}(\ell_\epsilon,d_\epsilon,s') \\
& - \frac{1}{2}\ell_\epsilon^{2\alpha}\sigma^2\Upsilon\Bigg\} \\
\leq \sup_{i\in[0,i_{\max}]} \Bigg\{ & i\eta_k(\tilde{u}(k_\epsilon,c_\epsilon,s_\epsilon)-\tilde{v}(\ell_\epsilon,d_\epsilon,t_\epsilon)) \\
& + \Big[i-\delta_\zeta k_\epsilon\Big]4\beta(k_\epsilon-\bar{k})^3 \\
& + \Big[(r-\lambda_c)c_\epsilon+k_\epsilon^\alpha\mu-i-g(k_\epsilon,i)\Big]4\beta(c_\epsilon-\bar{c})^3 \\
& + \Big[(r-\lambda_c)d_\epsilon+\ell_\epsilon^\alpha\mu-i-g(\ell_\epsilon,i)\Big]e^{-\eta_k\ell_\epsilon}\omega(1+\lambda_p(t_\epsilon)) \\
& - \delta_\zeta(\ell_\epsilon-k_\epsilon)p_k^u+\Big[(k_\epsilon^\alpha-\ell_\epsilon^\alpha)\mu-(g(k_\epsilon,i)-g(\ell_\epsilon,i))\Big]p_c^v \\
& + 6k_\epsilon^{2\alpha}\sigma^2\beta(c_\epsilon-\bar{c})^2+\frac{(k_\epsilon^\alpha-\ell_\epsilon^\alpha)^2}{\epsilon}\Bigg\}+o(1),
\end{aligned}
$$

where we use that $q_{s,s}=-\sum_{s'\neq s}q_{s,s'}$ and

$$
\begin{aligned}
\sum_{s'} q_{s_\epsilon,s'}\tilde{u}(k_\epsilon,c_\epsilon,s')-\sum_{s'} q_{t_\epsilon,s'}\tilde{v}(\ell_\epsilon,d_\epsilon,s') & \\
= \sum_{s'} q_{s_\epsilon,s'}e^{-\eta_k k_\epsilon}u(k_\epsilon,c_\epsilon,s')-\sum_{s'} q_{t_\epsilon,s'}e^{-\eta_k\ell_\epsilon}\hat{v}(\ell_\epsilon,d_\epsilon,s') & \leq o(1),
\end{aligned}
$$

the latter because $e^{-\eta_k k}(u-\hat{v})$ is maximized at the limit $(\bar{k},\bar{c},\bar{s})$.

Let $\eta_k<(r-\Delta)/i_{\max}$ for some $\Delta\in(0,r)$. Then, taking lim sup as $\epsilon\to 0$, and using that $g(\cdot,i)$ and $k\mapsto k^\alpha$ are Lipschitz in the neighborhood of $(\bar{k},\bar{c},\bar{s})$, i.e.,

$$
|g(k_\epsilon,i)-g(\ell_\epsilon,i)|+\mu|k_\epsilon^\alpha-\ell_\epsilon^\alpha|\leq R|k_\epsilon-\ell_\epsilon|,
$$

we get

$$\begin{aligned}\limsup_{\epsilon\to 0}\ &\Delta(\tilde{u}(k_\epsilon,c_\epsilon,s_\epsilon)-\tilde{v}(\ell_\epsilon,d_\epsilon,t_\epsilon))\\ &\leq \lim_{\epsilon\to 0}\Bigg[(\delta_{\bar{\zeta}}+R^2)\frac{(k_\epsilon-\ell_\epsilon)^2}{\epsilon}+R\frac{(c_\epsilon-d_\epsilon)}{\sqrt{\epsilon}}\frac{(k_\epsilon-\ell_\epsilon)}{\sqrt{\epsilon}}\\ &\qquad +R'(|c_\epsilon-\bar{c}|^2+|c_\epsilon-\bar{c}|^3+|k_\epsilon-\bar{k}|^3+d_\epsilon\omega)+o(1)\Bigg]=R'\bar{c}\omega,\end{aligned}$$

for some constant R', depending on k^* (i.e., η_k), $i_{\max}$, β, and the model parameters. Finally, because $\Delta>0$, for small enough ω,

$$\delta_\eta/2\leq e^{-\eta\bar{k}}(u-(1-\omega)v)(\bar{k},\bar{c},\bar{s})\leq\frac{R'}{\Delta}\bar{c}\omega,$$

which is a contradiction, because $\bar{c}\leq\bar{C}$ and ω can be chosen arbitrarily small. Hence, there cannot exist a point (c,k) such that $(u-v)(c,k)>0$.

□

The value function is bounded by the value of a firm that is permanently in an expansion, which is bounded by $M+c+k$ (cf. Kakhbod et al. (2022)). As a consequence, V satisfies the assumptions of Theorem 1. The following results are standard consequences of the comparison of viscosity solutions.

Corollary 2. *The value function V is the unique solution to Equation (12) on (18) with its boundary conditions.*

For computations, in addition to (13), the boundary conditions where $c=c_{\max}$ and $k_{\max}$ are given by

$$\begin{aligned}0&=\partial_c V-1 && \text{at } c=c_{\max}\\ 0&=\min\left\{rV+\delta k\partial_k V-\left[rc+k^\alpha\mu\right]\partial_c V-\frac{1}{2}k^{2\alpha}\sigma^2\partial^2_{cc}V,\partial_c V-1,\mathcal{I}V\right\} && \text{at } k=k_{\max}\end{aligned}$$

At the corners, the c-conditions are used.

Another consequence of the comparison result in Theorem 1 is the convergence of the numerical scheme (see Section 5).

Corollary 3. *Numerical solutions converge to the value function as the discretization gets finer.*

B Empirical Appendix

Table B.1: Correlation of the recession probability measure with other leading indicators

Recession Prob. is the month t probability of the U.S. being in a recession in one year according to the term spread, calculated as the difference between the 10-year and 3-month Treasury rates. It gives the probability of the U.S. being in a recession in one year. *VIX* is the month t level of the CBOE Volatility Index. *BC* is the current state of the business cycle, which is the month t probability that the U.S. is *currently* in a recession (Chauvet and Piger, 2008). *CPSB* is the 3-month commercial paper rate minus the federal funds rate. *XRI* is the month t value of the Experimental Recession Index from Stock and Watson (1989). It gives the probability of the U.S. being in a recession in six months. The index includes industrial production, real personal income, real manufacturing, total employee hours, housing permits, real manufacturers' unfilled orders, exchange rates, number of people working part-time, the 10-year Treasury bond yields, the spread between the 3-month commercial paper rate and the interest rate on 3-month Treasury bills, and the spread between the 10-year Treasury bonds and the 1-year Treasury bonds. *XRI-2* is the month t value of the Alternative Experimental Recession Index from Stock and Watson (1993). It gives the probability of being in a recession in six months. The index includes building permits, manufacturers' unfilled orders, exchange rates, help wanted advertising, average weekly hours of production workers, vendor performance, and manufacturing capacity utilization rates. *S&P 500* is the month t return on the S&P 500 index. *NYSE* is the month t return on the NYSE index. *AI* is the Anxious Index based on the Survey of Professional Economists, which has asked economists to estimate the probability of quarter-over-quarter chain-weighted real GDP growth less than zero for the current quarter (RECESS1) and the following four quarters (RECESS2 to RECESS5). RECESS2 is known as the "Anxious Index." See Andrade and Le Bihan (2013).

Variables	Recession Prob.	VIX	BC	CPSB	XRI	XRI-2	S&P 500	NYSE	AI
Recession Prob.	1.00								
VIX	-0.00	1.00							
BC	0.05	0.50	1.00						
CPSB	-0.18	0.23	0.44	1.00					
XRI	0.60	0.27	0.60	-0.16	1.00				
XRI-2	0.33	-0.29	0.73	-0.41	0.68	1.00			
S&P 500	0.03	-0.39	-0.15	-0.18	-0.07	0.05	1.00		
NYSE	0.05	-0.41	-0.16	-0.23	-0.08	0.06	0.97	1.00	
AI	0.16	0.39	0.60	0.24	0.58	0.55	-0.02	-0.01	1.00

Table B.2: Quarterly Compustat Sample Selection

This table presents the criteria used to prepare the firm-quarter dataset.

Criteria	Obs. Lost	Obs. Remaining
COMPUSTAT, 1961Q1 – 2021Q2		1,863,593
Less:		
Pre-IPO Data	(114,054)	1,749,539
Firms headquartered outside of USA	(321,773)	1,427,766
Firms incorporated outside of USA	(20,026)	1,407,740
Financials (SIC-1=6)	(396,540)	1,011,200
Utilities (SIC-2=49)	(72,154)	939,046
Public Administration (SIC-1=9)	(18,930)	920,116
Missing or zero assets	(114,561)	805,555
Missing cash and cash equivalents	(2,607)	802,948
Drop gvkey-quarter duplicates	(712)	802,236
PP&E less than $5M or missing PP&E	(274,469)	527,767
Negative cash and cash equivalents	(371)	527,396
Less than $1M in sales	(12,433)	514,963
Drop if data before 1971	(9)	514,954
Singleton Firms	(373)	514,581
SIC-4 industries-quarters with one firm	(9,226)	505,355
Drop quarters in NBER recessions	(62,019)	443,336
Final sample (12,473 firms, 1971Q4-2021Q4)		443,336

Table B.3: CRSP Sample Selection

This table presents the criteria used to prepare the monthly stock return dataset. We start with the full CRSP/Compustat Merged Database. However, these data do not include cash holdings and property, plant, and equipment (PP&E). We use cash and PP&E to sort firms. Also, we need to apply similar filtering across the CRSP stock return data and the Compustat data. To do so, we merge the CRSP/Compustat data file with the filtered annual Compustat data. See Table B.4 for the sample selection criteria for the annual Compustat data. We merge with the annual Compustat file as opposed to the quarterly Compustat file because the annual file extends farther back in time. We prefer the quarterly Compustat sample for our other main analyses because when examining how firm outcomes change with recession probabilities, it is better to use the quarterly frequency data since companies may adjust for recession risk within the year. Also, while data on cash holdings and PP&E are available since 1960, the firm outcomes of interest generally are not available until 1971.

Criteria	Obs. Lost	Obs. Remaining
CRSP/Compustat Merged Database, 1962-Jun — 2022-Dec		6,756,523
Drop identical duplicate returns within gvkey-cusip-month	(721,236)	6,035,287
Merge with annual Compustat sample lagged one year	(4,362,287)	1,673,000
Missing stock returns	(40,696)	1,632,304
Drop months in NBER recessions	(231,242)	1,401,062
Final sample, 1962-Jun — 2022-Dec (10,854 firms)		1,401,062

Table B.4: Annual Compustat Sample Selection

The table presents the criteria used to prepare the firm-annual dataset that is merged with the CRSP/Compustat dataset in Table B.3. The criteria are similar to those used in Table B.2 to construct the quarterly Compustat sample.

Criteria	Obs. Lost	Obs. Remaining
COMPUSTAT, 1950 – 2020		578,958
Less:		
Pre-IPO data	(43,952)	535,006
Firms headquartered outside of USA	(92,031)	442,975
Firms incorporated outside of USA	(5,240)	437,735
Financials (SIC-1=6)	(139,978)	297,757
Utilities (SIC-2=49)	(20,971)	276,786
Public administration (SIC-1=9)	(4,962)	271,824
Missing or zero assets	(12,036)	259,788
Missing cash and cash equivalents	(450)	259,338
Drop duplicates gvkey-year	(623)	258,715
PP&E less than $5M or missing PP&E	(94,973)	163,742
Negative cash and cash equivalents	(13)	163,729
Less than $1M in sales	(1,348)	162,381
Singleton firms	(1,277)	161,104
SIC-4-by-year groups one firm	(3,739)	157,365
Missing observations after merge with NBER recession dates	(3,525)	153,840
Final sample (12,032 firms)		153,840

Table B.5: Robustness of Table 5 to using plant, property, and equipment to proxy for firm size

The sample includes firm-quarter observations immediately preceding an equity issuance in quarter $t+1$ greater than 30% (columns 1-2), 50% (column 3), and 70% (column 4) of total assets less cash holdings at the end of quarter t. The outcome variable, $Cash_{i,t}$, is a firm's cash holdings at the end of quarter t, standardized within a firm. We drop the IPO year from our sample. *log(Recession Probability$_t$)*, is the quarter t log probability of a recession in twelve months. $Size_{i,t}$ is the log of firm i's net property, plant, and equipment at the end of quarter t, standardized within a firm. Controls for the business cycle include the average Volatility Index in quarter t, the CRSP market return in quarter t, and the probability of the economy being in a recession in quarter t (not $t+4$) from Chauvet and Piger (2008). We interact these controls with $Size_{i,t}$ to allow small and large firms to have different sensitivities to the business cycle. We cluster standard errors by quarter. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	log(Cash)$_{i,t}$			
	(1)	(2)	(3)	(4)
log(Recession Probability$_t$)	0.042*	0.050**	0.078***	0.066*
	(0.021)	(0.020)	(0.028)	(0.035)
log(Recession Probability$_t$) × log(PP&E)$_{i,t}$	0.043**	0.042**	0.077***	0.062**
	(0.017)	(0.017)	(0.020)	(0.027)
log(PP&E)$_{i,t}$	0.318***	0.076**	0.019	0.042
	(0.020)	(0.033)	(0.036)	(0.042)
Constant	0.663***	0.570***	0.470***	0.336***
	(0.026)	(0.092)	(0.103)	(0.113)
Issuance Sample $\left(\frac{\text{Issuance}_{i,t+1}}{\text{Assets-Cash}_{i,t}} > X\%\right)$	30%	30%	50%	70%
Business Cycle Controls	No	Yes	Yes	Yes
% Adjusted R^2	9.40	12.23	12.06	12.51
Observations	3768	3339	2093	1487

Table B.6: Robustness of Table 5 to using an indicator for above 75th-percentile recession risk

The sample includes firm-quarter observations immediately preceding an equity issuance in quarter $t+1$ greater than 30% (columns 1-2), 50% (column 3), and 70% (column 4) of total assets less cash holdings at the end of quarter t. The outcome variable, $Cash_{i,t}$, is a firm's cash holdings at the end of quarter t, standardized within a firm. We drop the IPO year from our sample. $\mathbb{1}(High\ Risk)_t$ is an indicator that equals one if the quarter t average probability of a recession in twelve months exceeds the 75th percentile of recession risk outside of NBER recessions. $Size_{i,t}$ is the log of firm i's total assets less cash holdings at the end of quarter t, standardized within a firm. Controls for the business cycle include the average Volatility Index in quarter t, the CRSP market return in quarter t, and the probability of the economy being in a recession in quarter t (not $t+4$) from Chauvet and Piger (2008). We interact these controls with $Size_{i,t}$ to allow small and large firms to have different sensitivities to the business cycle. We cluster standard errors by quarter. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	$\log(\text{Cash})_{i,t}$			
	(1)	(2)	(3)	(4)
$\mathbb{1}(\text{High Risk})_t$	-0.017	0.003	0.020	-0.013
	(0.044)	(0.048)	(0.065)	(0.065)
$\mathbb{1}(\text{High Risk})_t \times \log(\text{Size})_{i,t}$	0.086**	0.094***	0.160***	0.159***
	(0.036)	(0.031)	(0.043)	(0.050)
$\log(\text{Size})_{i,t}$	0.354***	0.282***	0.177**	0.164*
	(0.021)	(0.063)	(0.077)	(0.096)
Constant	0.728***	0.688***	0.578***	0.433***
	(0.031)	(0.093)	(0.107)	(0.116)
% Adjusted R^2	11.88	12.48	12.19	12.87
Observations	3768	3339	2093	1487

Table B.7: Robustness of Table 6 to using plant, property, and equipment to proxy for firm size

Recession risk decreases investment growth, especially when a firm is larger. The outcome variable is the growth of capital expenditures on property, plant, and equipment. To account for seasonality in investment across quarters, we compare investment in the future four quarters ($t+1$, $t+2$, $t+3$, and $t+4$) to investment in the prior four quarters ($t-3$, $t-2$, $t-1$, and t). We standardize these changes in investment within a firm. *log(Recession Probability$_t$)*, is the quarter t average monthly probability of a recession in twelve months. *Size*$_{i,t}$ is the log of firm i's net plant, property, and equipment assets at the end of quarter t, standardized within a firm. Controls for the business cycle include the average Volatility Index in quarter t, the CRSP market return in quarter t, and the probability of the economy being in a recession in quarter t (not $t+4$) from Chauvet and Piger (2008). We interact these controls with *Size*$_{i,t}$ to allow small and large firms to have different sensitivities to the business cycle. We double cluster standard errors by firm and quarter. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

			$\left(\frac{\sum_{t=1}^{4} \text{CAPX}_t}{\sum_{t=-3}^{0} \text{CAPX}_t} - 1\right)$	
	(1)	(2)	(3)	(4)
log(Recession Probability$_t$)	-0.029***	-0.031***	-0.030***	-0.039**
	(0.011)	(0.012)	(0.010)	(0.016)
log(Recession Probability$_t$) × log(PP&E)$_{i,t}$	-0.018***	-0.018**	-0.026***	-0.006
	(0.006)	(0.007)	(0.008)	(0.009)
log(PP&E)$_{i,t}$	-0.276***	-0.382***	-0.403***	-0.358***
	(0.006)	(0.009)	(0.010)	(0.015)
Constant	0.025***	0.071**	0.039	0.092*
	(0.008)	(0.035)	(0.029)	(0.047)
Firm Size	All	All	Below-Median	Above-Median
Business Cycle Controls	No	Yes	Yes	Yes
% Adjusted R^2	5.74	6.67	7.35	5.36
Observations	235629	198453	99227	99226

Table B.8: Robustness of Table 6 to using firm fixed effects

This table reports estimates from specification (15). Recession risk decreases investment growth, especially when a firm is larger. The outcome variable is the growth of capital expenditures on property, plant, and equipment. To account for seasonality in investment across quarters, we compare investment in the future four quarters ($t+1$, $t+2$, $t+3$, and $t+4$) to investment in the prior four quarters ($t-3$, $t-2$, $t-1$, and t). We standardize these changes in investment within a firm. *log(Recession Probability$_t$)*, is the quarter t average monthly probability of a recession in twelve months. $Size_{i,t}$ is the log of firm i's total assets less cash holdings at the end of quarter t, standardized within a firm. Controls for the business cycle include the average Volatility Index in quarter t, the CRSP market return in quarter t, and the probability of the economy being in a recession in quarter t (not $t+4$) from Chauvet and Piger (2008). We interact these controls with $Size_{i,t}$ to allow small and large firms to have different sensitivities to the business cycle. We include firm fixed effects and double cluster standard errors by firm and quarter. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	$\left(\frac{\sum_{t=1}^{4} \text{CAPX}_t}{\sum_{t=-3}^{0} \text{CAPX}_t} - 1\right)$			
	(1)	(2)	(3)	(4)
log(Recession Probability$_t$)	-0.028**	-0.028**	-0.024*	-0.031*
	(0.012)	(0.014)	(0.013)	(0.018)
log(Recession Probability$_t$) × log(Size)$_{i,t}$	-0.019**	-0.027***	-0.036***	-0.020*
	(0.008)	(0.008)	(0.010)	(0.011)
log(Size)$_{i,t}$	-0.203***	-0.229***	-0.216***	-0.277***
	(0.007)	(0.027)	(0.031)	(0.036)
Constant	0.019**	0.105**	0.111***	0.110**
	(0.009)	(0.043)	(0.039)	(0.055)
Firm Size	All	All	Below-Median	Above-Median
Firm FE	Yes	Yes	Yes	Yes
% Adjusted R^2	-0.49	0.68	1.49	2.66
Observations	235722	198500	99186	99158

Table B.9: Robustness of Table 6 to using an indicator for above 75th-percentile recession risk

This table reports estimates from specification (15). Recession risk decreases investment growth, especially when a firm is larger. The outcome variable is the growth of capital expenditures on property, plant, and equipment. To account for seasonality in investment across quarters, we compare investment in the future four quarters ($t+1$, $t+2$, $t+3$, and $t+4$) to investment in the prior four quarters ($t-3$, $t-2$, $t-1$, and t). We standardize these changes in investment within a firm. $\mathbb{1}(\textit{High Risk})_t$ is an indicator that equals one if the quarter t average probability of a recession in twelve months exceeds the 75th percentile of recession risk outside of NBER recessions. $Size_{i,t}$ is the log of firm i's total assets less cash holdings at the end of quarter t, standardized within a firm. Controls for the business cycle include the average Volatility Index in quarter t, the CRSP market return in quarter t, and the probability of the economy being in a recession in quarter t (not $t+4$) from Chauvet and Piger (2008). We interact these controls with $Size_{i,t}$ to allow small and large firms to have different sensitivities to the business cycle. We double cluster standard errors by firm and quarter. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	$\left(\frac{\sum_{t=1}^{4} \text{CAPX}_t}{\sum_{t=-3}^{0} \text{CAPX}_t} - 1\right)$			
	(1)	(2)	(3)	(4)
$\mathbb{1}(\text{High Risk})_t$	-0.049**	-0.050**	-0.055***	-0.050
	(0.021)	(0.023)	(0.020)	(0.033)
$\mathbb{1}(\text{High Risk})_t \times \log(\text{Size})_{i,t}$	-0.004	-0.005	-0.014	0.009
	(0.013)	(0.015)	(0.016)	(0.019)
$\log(\text{Size})_{i,t}$	-0.185***	-0.215***	-0.181***	-0.251***
	(0.007)	(0.021)	(0.022)	(0.028)
Constant	0.024**	0.095***	0.110***	0.097**
	(0.011)	(0.036)	(0.031)	(0.048)
Firm Size	All	All	Below-Median	Above-Median
% Adjusted R^2	2.63	2.93	2.74	2.61
Observations	235722	198524	99262	99262

Table B.10: Robustness of Table 7 to using plant, property, and equipment

The outcome variable is dividend growth. Because dividend policies are sticky, we compare dividends over the future four quarters ($t+1$, $t+2$, $t+3$, and $t+4$) to dividends in the prior four quarters ($t-3$, $t-2$, $t-1$, and t). We standardize dividend growth within a firm. *log(Recession Probability$_t$)*, is the quarter t average monthly probability of a recession. *Size*$_{i,t}$ is the log of firm i's net plant, property, and equipment at the end of quarter t, standardized within a firm. Controls for the business cycle include the average Volatility Index in quarter t, the CRSP market return in quarter t, and the probability of the economy being in a recession in quarter t (not $t+4$) from Chauvet and Piger (2008). We interact these controls with *Size*$_{i,t}$ to allow small and large firms to have different sensitivities to the business cycle. We double cluster standard errors by firm and quarter. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	$\left(\frac{\sum_{t=1}^{4}\text{Payouts}_t}{\sum_{t=-3}^{0}\text{Payouts}_t}-1\right)$			
	(1)	(2)	(3)	(4)
log(Recession Probability$_t$)	-0.011	-0.018*	-0.009	-0.029**
	(0.009)	(0.011)	(0.010)	(0.014)
log(Recession Probability$_t$) × log(Size)$_{i,t}$	-0.013***	-0.005	-0.000	-0.005
	(0.005)	(0.005)	(0.006)	(0.007)
log(Size)$_{i,t}$	-0.051***	-0.048***	-0.054***	-0.041***
	(0.005)	(0.005)	(0.008)	(0.009)
Constant	0.003	-0.019	-0.032	-0.004
	(0.008)	(0.029)	(0.024)	(0.041)
Firm Size	All	All	Below-Median	Above-Median
% Adjusted R^2	0.21	0.25	0.22	0.29
Observations	157029	128960	64480	64480

Table B.11: Robustness of Table 7 to using firm fixed effects

This table reports estimates from specification (16). The outcome variable is dividend growth. Because dividend policies are sticky, we compare dividends over the future four quarters ($t+1$, $t+2$, $t+3$, and $t+4$) to dividends in the prior four quarters ($t-3$, $t-2$, $t-1$, and t). We standardize dividend growth within a firm. *log(Recession Probability$_t$)*, is the quarter t average monthly probability of a recession. *Size*$_{i,t}$ is the log of firm i's total assets less cash holdings at the end of quarter t, standardized within a firm. Controls for the business cycle include the average Volatility Index in quarter t, the CRSP market return in quarter t, and the probability of the economy being in a recession in quarter t (not $t+4$) from Chauvet and Piger (2008). We interact these controls with *Size*$_{i,t}$ to allow small and large firms to have different sensitivities to the business cycle. We include firm fixed effects and double cluster standard errors by firm and quarter. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	$\left(\frac{\sum_{t=1}^{4}\text{Payouts}_t}{\sum_{t=-3}^{0}\text{Payouts}_t}-1\right)$			
	(1)	(2)	(3)	(4)
log(Recession Probability$_t$)	-0.009	-0.020*	-0.016	-0.026*
	(0.010)	(0.011)	(0.010)	(0.014)
log(Recession Probability$_t$) × log(Size)$_{i,t}$	-0.019***	-0.018***	-0.011	-0.021**
	(0.006)	(0.006)	(0.008)	(0.008)
log(Size)$_{i,t}$	-0.056***	-0.065***	-0.059**	-0.108***
	(0.006)	(0.020)	(0.024)	(0.032)
Constant	0.006	0.005	-0.011	0.048
	(0.008)	(0.030)	(0.025)	(0.044)
Firm Size	All	All	Below-Median	Above-Median
Firm FE	Yes	Yes	Yes	Yes
% Adjusted R^2	-3.48	-2.29	-1.47	-0.36
Observations	157122	129008	64448	64449

Table B.12: Robustness of Table 8 to using property, plant, and equipment to proxy for firm size

The outcome variable is firm i's total stock return in month t. Δ *log(Recession Probability$_t$)*, is the quarter $t-1$ to t change in the average monthly probability of a recession. $Size_{i,t}$ is the log of firm i's net property, plant, and equipment at the end of the prior year, standardized within a firm. Controls for the business cycle include the average Volatility Index in quarter t and the probability of the economy being in a recession in quarter t (not $t+4$) from Chauvet and Piger (2008). We interact these controls with $Size_{i,t}$ to allow small and large firms to have different sensitivities to the business cycle. Column (3) excludes the 2008-2009 financial crisis. We cluster standard errors by month. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Stock Return		
	(1)	(2)	(3)
Δ log(Recession Probability$_t$)	-0.037*	-0.077***	-0.076***
	(0.020)	(0.020)	(0.020)
Δ log(Recession Probability$_t$) $\times$ log(PP&E)$_{i,t}$	-0.012**	-0.018***	-0.018***
	(0.006)	(0.005)	(0.005)
log(PP&E)$_{i,t}$	-0.027***	-0.001	0.000
	(0.006)	(0.005)	(0.005)
Constant	-0.005	0.295***	0.299***
	(0.016)	(0.066)	(0.067)
Sample	All	All	Excl. 08/09
Controls	No	Yes	Yes
% Adjusted R^2	0.21	1.89	1.95
Observations	1369460	894723	882109

Table B.13: Robustness of Table 8 to using firm fixed effects

The outcome variable is firm i's total stock return in month t. Δ *log(Recession Probability$_t$)*, is the quarter $t-1$ to t change in the average monthly probability of a recession. *Size*$_{i,t}$ is the log of firm i's total assets net of cash holdings at the end of the prior year, standardized within a firm. Controls for the business cycle include the average Volatility Index in quarter t and the probability of the economy being in a recession in quarter t (not $t+4$) from Chauvet and Piger (2008). We interact these controls with *Size*$_{i,t}$ to allow small and large firms to have different sensitivities to the business cycle. Column (3) excludes the 2008-2009 financial crisis. We include firm fixed effects and cluster standard errors by month. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Stock Return		
	(1)	(2)	(3)
Δ log(Recession Probability$_t$)	-0.038*	-0.078***	-0.077***
	(0.020)	(0.020)	(0.020)
Δ log(Recession Probability$_t$) $\times$ log(Size)$_{i,t}$	-0.014**	-0.017***	-0.017***
	(0.006)	(0.005)	(0.005)
log(Size)$_{i,t}$	-0.030***	-0.025	-0.023
	(0.007)	(0.023)	(0.023)
Constant	-0.006	0.327***	0.332***
	(0.016)	(0.071)	(0.071)
Sample	All	All	Excl. 08/09
Firm FE	Yes	Yes	Yes
Controls	No	Yes	Yes
% Adjusted R^2	-0.54	1.21	1.28
Observations	1368656	895646	883020