Liquidity, Debt Denomination, and Currency Dominance

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January 2023

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Abstract

We provide a liquidity-based theory for the dominant use of the US dollar as the unit of denomination in global debt contracts. Firms need to trade their revenue streams for the assets required to extinguish their debt obligations. When asset markets are illiquid, as modeled via endogenous search frictions, firms optimally choose to denominate their debt in the unit of the asset that is easiest to obtain. This gives central importance to the denomination of government-backed assets with the largest safe, liquid, short-term float and to financial market institutions that reduce trading frictions. Equilibria with a single dominant currency emerge from a positive feedback cycle whereby issuing in the more liquid denomination endogenously raises its liquidity, incentivizing more issuance. We rationalize features of the current dollar-dominant international financial architecture and relate our theory to historical experiences, such as the prominence of the Dutch florin and pound sterling, the transition to the dollar, and the ongoing debate about the potential rise of the Chinese renminbi.

Keywords: International Currencies, Dollar Dominance, Liquidity, Corporate Debt.

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For helpful comments and conversations, we thank Chris Clayton, Peter DeMarzo, Darrell Duffie, Hanno Lustig, Matteo Maggiori, Matias Moretti, William Roberds, Jesse Schreger, seminar participants at the Atlanta Fed, Harvard Business School, and Stanford, as well as participants at the Becker-Friedman International Macro-Finance and MIT Sloan Juniors Finance Conferences. We also thank Olena Bogdan, Oliver Xie, Peilin Yang, and Wenyi Yin for excellent research assistance. Financial support from the Stanford Graduate School of Business and the Stanford Institute for Economic Policy Research is gratefully acknowledged.

1 Introduction

There is a great deal of dollar-denominated debt outstanding in the world, outsized relative to the wealth or GDP share of the United States in the world (see Cetorelli and Goldberg 2012; Bruno and Shin 2015b; McCauley, McGuire and Sushko 2015; Maggiori, Neiman and Schreger 2020). Indeed, the US dollar is the dominant currency in international finance and trade, much as the British pound sterling was prior to the dollar (Lindert 1969; Eichengreen 2005), and as the Dutch florin was prior to the sterling (Quinn and Roberds 2014b). This paper offers a theory of currency dominance in the denomination of global financial contracts. We seek to explain why among many alternatives for debt denomination, in equilibrium, one currency emerges endogenously as dominant.

The theory in a nutshell is as follows. An issuer's unit of denomination of a debt contract reflects a choice over which asset to hand over to extinguish the debt at the time of settlement. The issuer, for example an oil-producing firm, could issue debt that is denominated in any arbitrary unit, such as barrels of oil. In this case, upon maturity the firm would be obliged to deliver barrels of oil. But if barrels of oil are hard to come by—that is, if the oil market is illiquid—this may be a costly decision. Perhaps an oil producer will issue such debt, but an automaker will certainly not. Hence the liquidity that is necessary for settlement, which we model via endogenous search frictions (Duffie, Gârleanu and Pedersen 2005), constitutes the key economic force in our model.

The appeal of issuing debt denominated in dollars is the large and liquid nature of the dollar money market that facilitates dollar settlement. Both an oil producer and an automaker can see that there is an ample supply of safe, short-term dollar-denominated claims that can be used to settle their own liabilities. This is because there is a significant number of investors who own a substantial amount of dollar-denominated, short-term money market instruments such as Treasury bills, repos, or high-grade bank and firm debt. As a result, issuers can easily trade their revenue streams with these investors to acquire the assets required to settle their debt obligations.

The ease of settling obligations in dollars begets new issuance in dollars. As more firms issue dollar debt, some of this issuance adds to the stock of dollar-denominated money market instruments and expands the supply of available assets that can be used by other issuers to extinguish their debt. Dollar debt begets dollar debt, bootstrapping itself, leading to currency dominance.

Our theory identifies a complementarity between the issuance incentives of private firms

that strengthens currency dominance, and explains why both safe and risky foreign firms tap the dollar market. Some firms, say BBB-rated international firms, issue dollar debt because of the settlement liquidity available in the dollar money market. However, the debt of these firms may not be sufficiently money-like to be used as settlement liquidity by other parties. Nevertheless, their demand for a settlement medium generates a liquidity premium—a "convenience yield" (Krishnamurthy and Vissing-Jorgensen, 2012)—which makes it profitable to issue those claims. As a result, a safe international issuer, say the German government-backed supranational bank KfW, issues dollar debt to capture this convenience yield, which in turn raises overall dollar liquidity.

The phenomenon of currency dominance is not unique to the US dollar, and there have been historical precedents. The emergence of the Dutch florin as a dominant currency in the 17th century highlights the central economic idea in our theory. The florin, created by the Bank of Amsterdam, was a denomination that existed purely on the Bank's ledgers. While the florin represented a claim to its specie backing, credit contracts in this period (i.e., bills of exchange) were typically denominated in *florin* rather than one of the many metallic coins in circulation. In particular, both the Spanish *real de ocho* ("pieces of eight"), the most common specie coin in the world, and the current guilder, the physical representation of florin, might have been natural alternatives. Yet parties around the world, even those not transacting directly with Amsterdam, chose florin denomination over guilders or pieces of eight because of the superior liquidity of the florin's ledger-based settlement technology.[1]

Financial technologies that enhance the liquidity of debt claims go hand-in-hand with currency dominance, as the example of the Dutch florin makes clear. Our model shows that the sovereign or central bank of a country with a dominant currency will have a greater incentive to improve liquidity than that of a non-dominant country. The investment in the financial technology leads to more issuance in the dominant currency: this multiplier effect endogenously reinforces the dominant equilibrium. This complementarity between improving financial technologies and currency dominance is also evident in the history of the pound sterling and Bank of England. Starting in the 1820s, the Bank of England's role shifted from being a monopoly issuer of private money to one that used its balance sheet to aid the government and private sector in increasing the supply of money-like settlement instruments.

[1]The florin's international dominance was evident in the fact that all foreign exchange rates were quoted in relation to it, and in certain places such as Russia, only the exchange rate relative to Amsterdam was quoted until 1763 (Van Dillen, 1934, p. 105). Moreover, the trade between England and Russia was conducted exclusively through payments in Amsterdam, reflecting how the "[Bank of Amsterdam was] the clearinghouse of world trade" (De Vries and Van der Woude, 1997, p. 87, 131).

Our theory explains why these changes emerged in London at the same time as the sterling was becoming the dominant world currency. The same principles apply when considering the current global financial architecture where a collection of financial technologies have developed in the United States to sustain a large and liquid money market.

The other primary element that generates dominance is the supply of safe and liquid short-term government-backed liabilities. Throughout history, the government backing has taken various forms. The Bank of Amsterdam engineered its float of safe and liquid florin through reserves of specie backing. Just as importantly, confidence that the City of Amsterdam would not appropriate these reserves was central to the florin's success. In contrast, Spain was unable to provide such commitment given its history of serial defaults. In the post-Bretton Woods period, the United States government has backed a large quantity of dollar-denominated Treasury bills—with more safe float than the government bonds of alternative currencies—in this case supported by fiscal backing rather than physical reserves.

Currency dominance in international trade and finance have been related throughout history, but international trade is not a necessary element in the bootstrapping mechanism that we present. For example, during Spain's global empire in the 17th century, its trade volumes were two to six times larger than Amsterdam's. Great Britain became dominant in the early 19th century before the major periods of international trade and imperial expansion. In our model, firms which finance in the dominant currency (because of the convenience yield and settlement benefits) also find it cost-minimizing to invoice their trade in the dominant currency. The joint decision to finance and invoice in the same currency is also present in Gopinath and Stein (2021), but in their analysis causality runs in the opposite direction, from dominance in trade invoicing to dominance in finance. Further, liquidity demand in our model arises more generally from firms' needs to settle their payment obligations, which can be both financial (e.g., paying off a corporate bond) and real (e.g., payment for goods and services). In practice, gross financial positions are enormous and dwarf the volume of flows in goods.

In our theory, the decision to denominate debt in the dominant currency carries a positive externality, as it improves market thickness for all other borrowers and lenders. As a result, the competitive equilibrium is not efficient: a global planner will want to subsidize even more entry into the dominant currency. The social optimum can be implemented with formal international arrangements, like the Bretton Woods system, in which the commitment devices backing government debt issuance (such as gold) are concentrated in one country.

In return, the dominant country can provide contingent liquidity to the rest of the world, such as with a dollar swap line. In today's world, there is a *de facto* dollar standard without explicit coordination. Our analysis highlights that it would be beneficial for the dominant country to provide explicit commitments such as to dollar swap lines, as in the *de jure* dollar standard, because such commitments reinforce the dominant equilibrium.

In seeking to explain why a currency becomes dominant among many alternatives, we relate most closely to the Gopinath and Stein (2021) and Chahrour and Valchev (2022) approaches to the unit of account in which agents demand liquidity in order to import a good, and these agents would prefer to purchase an asset in the same unit as the denomination of the traded good. As noted, our notion of liquidity demand is broader than just trade, since we emphasize that even in the absence of trade, the liquidity settlement forces driving dominance will still be present. In Chahrour and Valchev (2022), dominance arises from the need for both parties in an international trade transaction to collateralize their trade contract using a common denomination that matches the currency of the trade. Parties then optimally choose to denominate trade in the currency of the most liquid and safe financial assets readily available. Our theory also emphasizes the denomination of the liquid asset as central in dominance, but the reinforcing mechanism runs through the denomination of private debt.

This paper additionally relates to five broad strands of the literature. First, we connect to the classic literature on the functioning of the international monetary system. Keynes (1923) and Nurske (1944) are examples of early incarnations of these discussions. More recent analyses of the current dollar architecture, both empirical and theoretical, include Matsuyama et al. (1993), Obstfeld et al. (1995), Tirole (2002), Gourinchas and Rey (2007a,b), Eichengreen et al. (2017), Maggiori (2017), Farhi and Maggiori (2018), and He et al. (2019). One prominent role for the dominant currency is as an exchange rate peg, *de jure* during Bretton Woods or *de facto* today (Ilzetzki et al., 2019). Central banks maintain these pegs by holding large stocks of foreign reserves that allow them to intervene. Our model links these two phenomena to the liquidity needs in the dominant currency of domestic firms.

Second, our paper belongs to the literature on the importance of the quantity of safe assets. Theoretical work in this area explores the macroeconomic and asset pricing implications of such shortages (Holmström and Tirole 1998; Caballero et al. 2008; Caballero and Krishnamurthy 2009). There is also an empirical literature documenting safe asset shortages and their consequences (Krishnamurthy and Vissing-Jorgensen 2012; Greenwood et al.

2015). Our model shows that the supply and denomination of safe assets affects the determination of the dominant currency and the convenience yields of safe assets denominated in that currency.

Third, we connect to theoretical and empirical work on currency dominance in global finance. In analyzing the US dollar's current dominance finance, we relate to papers such as Krugman (1984), Frankel (1992), Bruno and Shin (2015a,b), Ivashina et al. (2015), Bahaj and Reis (2020), Maggiori et al. (2020), Jiang et al. (2021), Correa et al. (2022), Eren and Malamud (2022), and Jiang et al. (2022). We focus on the currency of debt denomination, which we link to the liquidity externalities of coordinating on the unit of account.

Our paper sheds light not only the current dollar system but also the evolution of reserve currencies over time. Thus, we also relate to the area of this literature that examines historical international currencies in the pre-WWI era. Quinn and Roberds (2014b,a) analyze the experience of the Dutch florin in the 17th and 18th centuries. Similarly, King (1972), Dickson (1967), Lindert (1969), Eichengreen (2008), and Kynaston (2015a,b) examine British pound dominance in the early 19th and early 20th centuries.

Fourth, our modeling approach follows in the spirit of the literature studying search frictions in financial markets, including in money markets, such as Kiyotaki and Wright (1989), Trejos and Wright (1995), Freeman (1996), and Doepke and Schneider (2017). Duffie et al. (2005, 2007), Garleanu and Pedersen (2007), Vayanos and Weill (2008), Lagos and Rocheteau (2009), Chaumont (2020), Moretti (2020), and Passadore and Xu (2022) study applications to over-the-counter markets, including sovereign debt markets. In our model, we trace out the implications of search-based illiquidity frictions for the corporate financing decisions of firms around the world.

Lastly, we connect to the literature examining the centrality of the US dollar in trade invoicing, which includes Engel (2006), Goldberg and Tille (2008), Gopinath et al. (2010, 2020), Amiti et al. (2022), and Mukhin (2022). As noted, we highlight how currency dominance in global debt denomination generates incentives to invoice trade in that same currency.

2 A Model of Liquidity and Debt Denomination

We consider a three-period ($t = t_0, t_1, t_2$) environment with two countries, indexed by $j \in \{A, B\}$. In each country, there is a government backing risk-free debt G_j denominated in units of the local currency, which can be traded to meet the liquidity needs of agents. In terms

of the Bank of Amsterdam history, we can think of the quantity of risk-free government debt as the quantity of coin-backed florin. More generally, G_j includes all money-like instruments denominated in currency j that the government has committed to guarantee.

There are also entrepreneurs that issue debt (e.g., bills of exchange or corporate bonds) F_j and make a choice of denominating the debt in their home currency or in the foreign currency. These firms have liquidity needs in a manner similar to Holmström and Tirole (1998). Trading occurs in a secondary market with endogenous trading frictions, modeled as in Duffie et al. (2005). Finally, there is a continuum I_j of homogeneous risk-neutral investors that buy the debt of firms and governments.

2.1 Within-Country Environment

We start by developing the model within a given country where we consider the choice to either borrow or not. In Section 2.2, we turn to the full case where firms may choose to issue debt in the foreign currency and characterize the general equilibrium.

2.1.1 Debt Issuance at t_0

There is a mass of entrepreneurs F_j in each country j. Each entrepreneur owns a firm that will generate profits of one at either t_1 or t_2. At t_0, the entrepreneur can sell debt with face value of one maturing at t_2, which will be repaid using the future profits. For example, a merchant in Amsterdam expecting goods to arrive can issue a bill of exchange, backed by those goods, at t_0. The benefit of borrowing, if the interest rate is sufficiently low, is to increase the entrepreneur's consumption at date t_0. The preference of a given entrepreneur i is to maximize:

$$u^F_{i,j} = c_0 + \beta c_1 + \beta^2 c_2, \quad c_t \geq 0. \tag{1}$$

The liquidity need in the model arises if the entrepreneur's goods arrive early at t_1 while his debt is due at t_2. In this case, his debt and its revenue streams are mismatched in time. The probability of receiving profits early is ϕ. The mass of liquidity-demanding firms is the total number of early firms, which is

$$m_{F,j} = \phi F_j. \tag{2}$$

With the possibility of early revenues, it may be beneficial for firms to obtain a financial asset at t_1. For example, if the merchant's goods arrive at t_1, he could potentially sell those

goods immediately for a financial asset that he can hold until t_2, which will allow him to clear the bill of exchange that he issued at t_0.

At t_0, the government also issues a quantity G_j of government-backed securities. We assume that both government and private bonds are safe (i.e., there is no default risk, as we elaborate on later), and that they have the same liquidity properties. We consider breaking this symmetry later in the analysis. We also consider the case of risky bonds later in the analysis. For now, as the bonds are identical they have the same endogenous price $P_{0,j}$.

The financial asset that the firm seeks to extinguish its t_2 debt obligation are the bonds issued by the government and other firms at date t_0. We model obtaining financial assets from the money market as search and matching.

2.1.2 Money Market Settlement With Search Frictions

There is a mass I_j of investors who have a large t_0 endowment and purchase bonds issued by the government and firms at t_0. The investors are risk neutral with preferences

$$u^I_{i,j} = c_0 + \beta_I c_1 + \beta_I^2 c_2, \qquad c_t \geq 0, \qquad 1 > \beta_I > \beta. \tag{3}$$

Each investor potentially owns one bond, and bonds are indivisible. The total mass of bonds is $G_j + F_j$. Define the total mass of bondholders to be

$$m_{I,j} = G_j + F_j \leq I_j, \tag{4}$$

where the last inequality is a restriction on parameters. That is, we assume there are enough investors to purchase all of the bonds at t_0.

Firms with early profits may trade with investors at t_1 to obtain the financial asset (i.e., a bond) for settlement at t_2. There are gains from trade in a meeting. We assume that if the firm does not trade with the investor, then it keeps its goods until t_2 and uses the goods to settle its debt. The effective return on keeping its goods is therefore zero. On the other hand, investors discount the future at rate $\beta_I < 1$. Therefore an investor who owns a bond is willing to sell the bond as long as he receives at least β_I goods, which he then consumes at t_1. The gains from trade in a match between investor and firm is thereby $1 - \beta_I$. We assume the firm receives a fraction η of this surplus and the investor keeps the remaining $1 - \eta$ share.

We now describe the search market at t_1. We posit a matching function such that

the number of meetings between liquidity demanders (firms) and liquidity suppliers (date t_0 investors) is

$$n_j = \lambda_j m_{F,j}^{\theta} m_{I,j}^{\theta}, \qquad \theta > \frac{1}{2}. \tag{5}$$

Here $\lambda_j > 0$ captures the overall degree of *liquidity* of the money market. In the continuous-time asset trading model of Duffie et al. (2005, henceforth DGP), λ_j corresponds to the Poisson probability that a given agent (say, a firm) will meet another agent (say, an investor). Given Poisson meeting rates, $\theta = 1$ so that the total number of matches is proportional to the masses of both firms and investors (Duffie et al., 2018).

The key property of this matching function is increasing returns to scale with $\theta > \frac{1}{2}$. If the masses of both firms and bond-holding investors double, the number of matches more than doubles. Thus the search model embeds a thick-market liquidity externality as in Diamond (1982). This liquidity externality is at the heart of many of our results. We focus our analytical derivation on the case of $\theta = 1$. We also provide numerical results for the $\theta \in (1/2, 1)$ case.

Given the assumed matching function, the endogenous two-sided meeting probabilities are:

$$\underbrace{\alpha_{F,j} = \frac{n_j}{m_{F,j}} = \lambda_j m_{F,j}^{\theta-1} m_{I,j}^{\theta}}_{P(\text{Firm finds a bond seller})}, \qquad \underbrace{\alpha_{I,j} = \frac{n_j}{m_{I,j}} = \lambda_j m_{F,j}^{\theta} m_{I,j}^{\theta-1}}_{P(\text{Bond seller finds a firm})}. \tag{6}$$

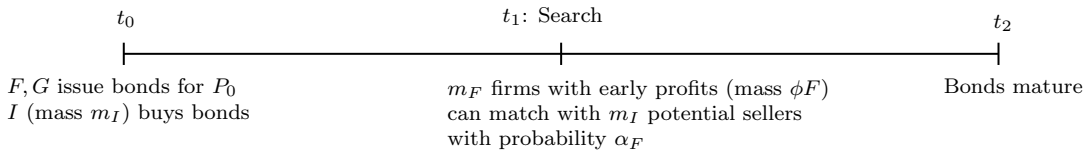
Here $\alpha_{F,j}$ is the probability of a firm meeting a bond seller (date t_0 investor in bonds) at time t_1, and $\alpha_{I,j}$ is the probability that the bond seller meets a firm. Figure 1 provides a timeline of the debt market.

The date t_1 liquidity market as described is an over-the-counter (OTC) bond market as in Duffie et al. (2005), where firms trade goods with investors for their one-period bonds. However, we do not take a stand on the market structure of this trade, and it is likely that this structure has varied across history.

For example, in Amsterdam, there were traders who owned specie who would deposit their specie at the Bank in a repo agreement and receive a florin deposit at the bank (Quinn and Roberds, 2014a). These traders would pay a fee to the Bank in this transaction and would therefore receive less florin than the value of the specie, as in an overcollateralized repo transaction. The traders would then lend florin to a merchant who needed florin for settlement, earning interest or a fee in this transaction. From the standpoint of the model, the repo technology and specie are "liquidity supply" and the merchant is the "liquidity

demander" at t_1.

Figure 1: **Timeline of debt issuance and demand for settlement**



Notes: This figure presents a schematic representation of timing in the model.

These labels can also apply in thinking about a modern money market and repo. For example, financial firms use Treasury securities as repo to obtain reserves that can then be lent to others as settlement instruments. We can also interpret this trade as occurring via a banking arrangement. The "liquidity suppliers" (m_I in the model) are banks that own the bonds as an asset and issue deposits to "liquidity demanding" firms (m_F) at t_1, which the firms then use to settle their t_2 debt.

Furthermore, in the world today there is an investor-side demand for liquidity, for example by foreign central banks and global banks. In the modern context, these entities hold safe dollar assets as foreign reserves. Our model can also be interpreted to include these entities. Suppose that firms which need liquidity at t_1 trade with a global bank or central bank to obtain the liquidity, paying a fee to these entities in the process. In this case, the global bank or central bank demands bonds at t_0 to resell these bonds at t_1 to the firms needing settlement. These intermediaries are "investors" at t_0, and the uncertain settlement needs of their firms is the motive for foreign central banks and other global banks to hold dollar safe assets as reserves. In this way, reserve holdings would tilt towards the dollar even without asymmetry in investor preferences for dollar assets—as in, for example Gourinchas et al. (2010) or Jiang et al. (2020).

The key economic force that the modeling captures is that the trade to obtain assets for settlement is frictional, and that a greater outstanding quantity of bonds makes obtaining this liquidity easier (higher $\alpha_{F,j}$).

2.1.3 Firm Issuance Decision and Asset Market Equilibrium

We formally present the entrepreneur's decision problem. The entrepreneur makes an issuance decision at date t_0. Denote D_i as an indicator function that takes the value one if the firm issues debt and zero if the firm does not. The firm decides at date t_1 to trade for

a bond or not. Denote T_i as an indicator function that reflects the decision to trade. Then the entrepreneur's problem is:

$$\max_{D_i, T_i} \; E[c_0 + \beta c_1 + \beta^2 c_2] \tag{7}$$

subject to

$$\begin{aligned} c_0 &= D_i P_{0,j}, && (8) \\ c_1 &= \begin{cases} 0, & \text{late}, \\ 0, & \text{early, but not matched}, \\ T_i\, \eta\, (1-\beta_I), & \text{early, and matched}, \end{cases} && (9) \\ c_2 &= 1 - D_i. && (10) \end{aligned}$$

As long as $P_{0,j} > \beta^2$ and $1 - \beta_I > 0$, the solution is to set $D_i = 1$ and $T_i = 1$. We have assumed that $1 - \beta_I > 0$, and as we show next, since $\beta_I > \beta$, we also have that $P_{0,j} > \beta^2$.

We solve for $P_{0,j}$ backwards. Consider the market at date t_1 first. If a match occurs, the total surplus is $1 - \beta_I$, of which a bond seller obtains $(1-\eta)(1-\beta_I)$. We assume that the date t_0 bond market is Walrasian. Each investor can bid for exactly one bond at date t_0. If an investor purchases a bond at t_0, the investor either resells the bond at date t_1 to earn $(1-\eta)(1-\beta_I)$, or the investor holds the bond to maturity. Thus the investor's valuation of the bond at t_0 is:

$$P_{0,j} = \underbrace{\alpha_{I,j}\beta_I\left[\beta_I + (1-\eta)(1-\beta_I)\right]}_{P(\text{Matched}) \times \text{PV of Sales Price}} \quad + \quad \underbrace{(1-\alpha_{I,j})\beta_I^2}_{P(\text{Not Matched}) \times \text{PV of 1}}, \tag{11}$$

or rewriting,

$$P_{0,j} = \beta_I^2 + \alpha_{I,j}\beta_I(1-\eta)(1-\beta_I). \tag{12}$$

We first note that since $\beta_I > \beta$, we have that $P_{0,j} > \beta^2$ so that $D_i = 1$ in the firm's issuance problem. Second, we note that the wedge $P_{0,j} - \beta_I^2$ is a *convenience yield* on bonds issued at t_0. That is, consider the pricing of a completely illiquid bond, which in our model is one for which $\lambda_j = 0$. This bond will be priced at β_I^2. The government and private firm bonds in our model trade at $P_{0,j} > \beta_I^2$ because they offer settlement liquidity to firms at date t_1. The convenience yield increases in the match probability and the surplus gained from the match.

Finally, consider the entrepreneur's expected utility from bond issuance at date t_0 at

an endogenous price $P_{0,j}$:

$$u_{i,j}^F = P_{0,j} + \underbrace{\beta\,\phi\,\alpha_{F,j}\ \times \eta(1-\beta_I)}_{\text{Benefit of liquidity at } t_1}. \tag{13}$$

The first term in this objective reflects the benefit from selling bonds at a high price at t_0. The second term reflects the benefit of settlement liquidity at t_1. The firm is early with probability ϕ and obtains the needed liquidity with probability $\alpha_{F,j}$. The share of the surplus in the trade that the firm receives is $\eta(1-\beta_I)$, and the firm discounts the future at β.

We simplify our expressions by taking the limit $\beta_I \to \beta$ and eliminate β_I. Given equilibrium bond prices, we can rewrite (13) as:

$$\begin{aligned} u_{i,j}^F - \beta^2 &= \beta(1-\beta)\left[(1-\eta)\alpha_{I,j} + \phi\eta\alpha_{F,j}\right] \\ &= \lambda_j\beta(1-\beta)\left(m_{F,j}m_{I,j}\right)^{\theta-1}\left[(1-\eta)m_{F,j} + \phi\eta\, m_{I,j}\right]. \end{aligned} \tag{14}$$

The two additive terms in this expression reflect the two ways in which firms benefit from money market liquidity: the first term reflects the benefit of capturing convenience yields on the firms' initial issuance which is increasing in $m_{F,j}$, the mass of firms demanding settlement liquidity, and $1-\eta$, the surplus share going to the owner of the bond if a trade happens. The second term reflects the benefit from a high probability of being able to find a match in the date t_1 money markets which is increasing in the mass of available bonds $m_{I,j}$, the surplus share η going to the firm needing settlement, and ϕ, the probability that the firm needs settlement.

In equation (14), both benefits accrue to the same firm because firms are homogeneous. In practice, there are some firms that are pure liquidity providers that harvest the convenience yield (e.g., firms with $\phi = 0$ but whose bonds are safe and liquid such as the German supranational KfW), and other firms whose debt does not offer liquidity benefits that do value settlement liquidity (e.g., firms with $\lambda = 0$ bonds but with $\phi > 0$ such as a BBB-rated firm). In Section 4.2, we consider an extension with heterogeneous firms.

From now on we simply set $\eta = \frac{1}{2}$, as we do not explicitly model the bargaining process and η plays no part in the analysis. We therefore write:

$$u_{i,j}^F - \beta^2 = \frac{1}{2}\lambda_j\beta(1-\beta)\left(m_{F,j}m_{I,j}\right)^{\theta-1}\left[m_{F,j} + \phi\, m_{I,j}\right]. \tag{15}$$

2.2 International Equilibrium Conditions

We next describe the international equilibrium. The two countries, $j = A, B$, have fundamentals $\{G_j, \lambda_j, F_j\}$. Firms earn revenues in domestic currency and choose the denomination of bonds, either domestic or foreign. For now we assume that the government only issues bonds in its domestic currency; we relax this assumption later in the analysis (see Section 3.5).

We assume that there is a fixed cost of issuing in a foreign currency proportional to K_i, which is heterogeneous across firms. This cost can be microfounded in several ways. In Appendix Section A.1, we provide a microfoundation in terms of costs associated with running a balance sheet currency mismatch, which is our leading example of this cost. We can also think of the cost in terms of fixed costs of finding a bank and underwriters of its bonds in the foreign currency, providing information so that investors can assess the firms' risks, and so on. We assume that K_i is distributed on $[\underline{K}, \infty)$ with cumulative distribution function $H(K_i)$, and corresponding density $h(K_i) = H'(K_i)$. This density is identical in the two countries.

We let the set of buyer and seller masses be $\mathcal{M} = (m_{F,A}, m_{I,A}, m_{F,B}, m_{I,B})$. We can compute expected utility, in excess of β^2, for the entrepreneurs in the two countries and for each of the possible denomination choices. These four expressions are as follows:

1. *Expected utility of entrepreneur in country B issuing in foreign currency (A)*:

$$U_{B\to A}(\mathcal{M}, K_i) \equiv \frac{\beta(1-\beta)}{2}\left[\lambda_A \left(m_{F,A} m_{I,A}\right)^{\theta-1}\left[m_{F,A} + \phi\, m_{I,A}\right] - K_i\right]. \tag{16}$$

2. *Expected utility of entrepreneur in country B issuing in home currency (B)*:

$$U_{B\to B}(\mathcal{M}) \equiv \frac{\beta(1-\beta)}{2}\lambda_B \left(m_{F,B} m_{I,B}\right)^{\theta-1}\left[m_{F,B} + \phi\, m_{I,B}\right]. \tag{17}$$

3. *Expected utility of entrepreneur in country A issuing in foreign currency (B)*:

$$U_{A\to B}(\mathcal{M}, K_i) \equiv \frac{\beta(1-\beta)}{2}\left[\lambda_B \left(m_{F,B} m_{I,B}\right)^{\theta-1}\left[m_{F,B} + \phi\, m_{I,B}\right] - K_i\right]. \tag{18}$$

4. *Expected utility of entrepreneur in country A issuing in home currency (A)*:

$$U_{A\to A}(\mathcal{M}) \equiv \frac{\beta(1-\beta)}{2}\lambda_A \left(m_{F,A} m_{I,A}\right)^{\theta-1}\left[m_{F,A} + \phi\, m_{I,A}\right]. \tag{19}$$

We index denomination choice by $\mathcal{D}_{i,j}$, where $\mathcal{D}_{i,j} = 1$ if firm i in country j issues in foreign currency, and $\mathcal{D}_{i,j} = 0$ otherwise. Each firm chooses its debt denomination optimally by comparing the expected utility functions given above:[2]

$$\mathcal{D}_{i,j} = \begin{cases} 1 & \text{if} \quad U_{j\to j'}(\mathcal{M}, K_i) > U_{j\to j}(\mathcal{M}), \\ 0 & \text{if} \quad U_{j\to j'}(\mathcal{M}, K_i) \leq U_{j\to j}(\mathcal{M}). \end{cases} \tag{20}$$

We then have three results. First, since $U_{j\to j'}$ is monotonically decreasing in K_i, we obtain:

Lemma 1. *Consider firms $\hat{i}$ and i in country j, where $K_i < K_{\hat{i}}$. If it is optimal for firm $\hat{i}$ to issue in foreign currency $j' \neq j$, then it is optimal for firm i to issue in foreign currency j'.*

Next, notice that the expressions for firm utility in country A have the same terms as the expressions for firms in country B. As a result:

Lemma 2. *Suppose that there is a positive mass of firms in j that find it optimal to issue in currency j'. Then, no firms in j' will issue in currency j.*

In other words, if some firms in B choose to pay a cost to issue in country A, then a firm in A for which there is no cost to issue in A will choose to only issue in A, and hence no firm from A issues in B. Together, these two lemmas imply optimal firm denomination choices must have a threshold structure, which we formalize as follows.

Lemma 3. *A necessary condition for a collection of firm denominations choices $\mathcal{D}_{i,j}$ to be consistent with firm optimality is that it must take the following threshold form:*

$$\mathcal{D}_{i,j'} = \begin{cases} 1 & \textit{if} \quad K_i < \bar{K}, \\ 0 & \textit{if} \quad K_i \geq \bar{K}, \end{cases} \qquad \mathcal{D}_{i,j} = 0. \tag{21}$$

In the lemma above, $\bar{K}$ corresponds to the threshold cost below which firms choose to issue in foreign currency: it is a scalar that provides a sufficient statistic summarizing the entirety of

[2] In writing a B firm's problem as one of comparing the expressions for utility to issuing in A to that of B, we require that conditional on a currency choice the optimal choices over issuance and settlement are as described in the previous within-country setting. With two currencies, there is another action that is feasible: a B firm could issue in currency B, then at date t_1 the early-firm converts its goods into currency A and searches for a bond in A, and finally at t_2, the firm converts its bond proceeds back into B and uses these proceeds for settlement. We assume that these two further foreign exchange transactions expose the firm to currency risk and thus the firm incurs additional costs. In this case, while this action is feasible, it is never optimal.

the set of all firms' denomination choices. A corollary is that the masses $\mathcal{M}$ can themselves be represented as functions of the threshold cost $\bar{K}$: $\mathcal{M} = \mathcal{M}(\bar{K})$. Further, we introduce the following notation for the expected utilities of the threshold firm (for which $K_i = \bar{K}$):

$$\bar{U}_{j'\to j}(\bar{K}) \equiv U_{j'\to j}(\mathcal{M}(\bar{K}), \bar{K}), \qquad \bar{U}_{j'\to j'}(\bar{K}) \equiv U_{j'\to j'}(\mathcal{M}(\bar{K})). \tag{22}$$

We denote the equilibrium value of $\bar{K}$ as $\hat{K}$. This is the cost which sets the marginal firm indifferent between the two currencies, equalizing the two expressions in equation (22).[3] Given the threshold structure for firms strategies, we can formally define equilibria.

Definition 1. *An equilibrium is a collection of an endogenous threshold $\hat{K}$, exogenous parameters $\Theta = (F_A, F_B, G_A, G_B, \lambda_A, \lambda_B, \phi, \theta)$, an exogenous firm size distribution $H(K)$, and endogenous masses $\mathcal{M} = (m_{F,A}, m_{I,A}, m_{F,B}, m_{I,B})$, satisfying:*

1. *(Market clearing) Given $\hat{K}$, the masses $\mathcal{M}$ satisfy:*

$$m_{F,j} = \phi\left[F_j + H(\hat{K})F_{j'}\right], \qquad m_{F,j'} = \phi\left(1 - H(\hat{K})\right)F_{j'}, \tag{23}$$

and,

$$m_{I,j} = G_j + F_j + H(\hat{K})F_{j'}, \qquad m_{I,j'} = G_{j'} + \left(1 - H(\hat{K})\right)F_{j'}. \tag{24}$$

2. *(Firm optimality) Given the masses $\mathcal{M}$, the threshold is optimal: firms in j' with $K_i < \hat{K}$ find it optimal to issue in currency j, while all other firms optimally issue in their own currency, so that*

$$\begin{cases} \bar{U}_{j'\to j}(\hat{K}) = \bar{U}_{j'\to j'}(\hat{K}) & \text{for} \quad \hat{K} > \underline{K}, \\ \bar{U}_{j'\to j}(\hat{K}) \leq \bar{U}_{j'\to j'}(\hat{K}) & \text{for} \quad \hat{K} = \underline{K}. \end{cases} \tag{25}$$

Further, throughout the analysis, we will examine the stability properties of the equilibria specified in the above definition, whenever the model features multiple equilibria. We formally specify our notion of stability using the following criterion.

Definition 2. *Consider an underlying dynamical system through which an out-of-equilibrium system converges to equilibrium (a tatonnement process) of the form $\partial_\tau \hat{K} = \delta\left[U_j(\hat{K}) - U_{j'}(\hat{K})\right]$*

[3] Alternatively, by complementary slackness, equilibria at the lower boundary ($\hat{K} = \underline{K}$) are also valid if $\bar{U}_{j'\to j}(\underline{K}) \leq \bar{U}_{j'\to j'}(\underline{K})$.

for $\delta > 0$ and where τ indexes a mass of continuous sub-periods within time t_0. An equilibrium featuring the endogenous threshold $\hat{K}$ is said to be stable if it fulfills the following condition:

1. *(Equilibrium stability) There exists an $\varepsilon > 0$ such that any trajectory beginning in the neighborhood $[\hat{K} - \varepsilon, \hat{K} + \varepsilon]$ converges to $\hat{K}$.*

2.3 Discussion of Modeling Choices

We further explain some of the modeling choices that we have made in this section.

1. *Rollover risk vs. saving:* The liquidity trade in our model at t_1 is firm "saving" but this is more for simplicity than it is substantive. The key component is that the firm trades its goods for a one-period bond. Consider a variant of the model in which there is a date t_3, and the firm's liquidity need stems from rollover risk. The timing mismatch in that problem is that with some probability, the firm will receive the goods at t_3 rather than t_2 while the debt is due at t_2. The firm will then want to trade its future revenues for bonds at t_1. If there are more bonds available, this trade will be less frictional and it will be less costly to rollover the debt. Again, the key economics our model captures is in linking liquidity with bond supply.

2. *Is search and liquidity a concern in money markets?* Our model links corporate financing decisions to money market illiquidity concerns. At a theoretical level, our model builds on a long tradition of using search to model money markets (see Kiyotaki and Wright, 1993; Lagos and Wright, 2005). Even in high-volume money markets such as the US dollar repo market, search models have been shown to capture price and quantity patterns well (Vayanos and Weill, 2008). To give one example of the empirical counterpart to such a consideration, in September 2019 the dollar money market turned illiquid and it was difficult for many actors to get their hands on dollar reserves (Copeland et al., 2021). If such events happen repeatedly, it is plausible that firms will pull back from issuing into the dollar money market. Of course, in this event, the Federal Reserve provided liquidity (expanded the supply of reserves, i.e. safe dollar-denominated assets) to allay these concerns. Our analysis turns on the relative liquidity of the money markets in different currencies rather than the absolute liquidity of any one market. At the macro level, during a period of global financial volatility,

the dollar money market remains more liquid than the markets of many emerging and even advanced economies. In our model, these considerations drive financing decisions.

3. *Theoretical understanding of denomination: payoff risk vs. settlement.* Currency denomination encompasses two distinct aspects: payoff risk and settlement. For example, a contract where payoffs vary with nominal exchange rates is one where denomination determines payoff risk. Our theory instead highlights the settlement aspect of denomination. To extinguish a dollar-denominated debt contract, the borrower needs to deliver dollars. As a concrete example, consider the payoffs of Swiss Franc or Canadian dollar debt. These currencies have exchange rates with similar payoff characteristics as the dollar: they appreciate during periods of economic turmoil. Yet, in terms of quantities, dollar debt is dominant, many orders of magnitude larger than that of Canadian or Swiss debt. Our settlement-liquidity theory can speak to this fact, whereas a payoff risk theory likely cannot. Historically, the classical gold standard era provides similar examples. Currencies like the German mark and French franc also provided a claim to the same underlying specie payoff as the British pound sterling, and yet there was much less foreign debt issuance in marks or francs relative to pounds.

4. *Investment incentives.* In the model as written, entrepreneurs maximize their discounted flow of consumption given an endowed revenue stream. An alternative formulation would be of firm profit maximization in which entrepreneurs can invest in a project with guaranteed revenues subject to raising enough financing initially (i.e., selling their bonds at a sufficiently high P_0). In this formulation, the benefit of issuing in a currency with high P_0 is to undertake more investment rather than to finance higher t_0 entrepreneurial consumption. Other than this alternative motivation for the benefit of issuing in a given currency, the analysis of equilibrium would remain the same.

5. *Interpretation of G.* While we define G_j as the supply of safe government bonds, it should be more broadly interpreted as the supply of money market instruments that depends on the government's commitment to maintain their value. Throughout history, government commitment has taken various forms: either purely fiscal, or through physical holdings of precious metals such as in a gold standard. For example, in the context of Amsterdam, the relevant government-backed debt supply is not the sovereign debt, but rather the specie-backed bank florin. The relevant government commitment

for firms was the City of Amsterdam's promise not to appropriate the underlying specie. When confidence in this commitment collapsed, as it did in 1795, there was a drop in G even though ultimately the default rate on florin was low. The relevance of the government commitment is also why Spain, despite having a large nominal debt in the 18th century, had low G. Its history of default reduced its ability to credibly commit to supporting a float of safe and liquid assets. We model government default risk in Section 4.1.

3 Currency Dominance and Denomination Incentives

Having specified the model environment and derived its equilibrium conditions in Section 2, we now turn to analyzing the properties of the resulting equilibria and examining the underlying economic forces. We outline how multiple equilibria naturally emerge as a consequence of increasing returns to scale, and how asymmetries in country fundamentals favor those equilibria that feature currency dominance. We also discuss the sovereign's incentive to invest in technologies that improve private market liquidity, relating it to historical experiences of transition between dominant currencies, and we analyze welfare aspects of the model from the perspective of a global planner. Throughout this section, we present analytical results for the $\theta = 1$ case.

3.1 Equilibria in the Symmetric $K_i = 0$ Case

Let us start by analyzing the simplest, fully symmetric case where fundamentals (λ_j, G_j, F_j) are the same for both $j = A, B$, and where we also set the foreign-currency issuance cost K_i to zero for all firms i.[4]

Proposition 1. *There are three equilibria in the symmetric, $K_i = 0$, $\theta = 1$ case:*

1. *No firm switches to issue in the foreign currency.*
2. *All firms from B switch to issuing in currency A.*
3. *All firms from A switch to issuing in currency B.*

[4]Therefore, in this case the distribution of costs K_i is degenerate with support only at zero: $H(K) = 1$ for $K \geq 0$, and $H(K) = 0$ otherwise.

We refer to equilibria in which firms in country B switch to currency A (equilibrium 2 above) as "class BA equilibria", and to equilibria in which firms in country A switch to currency B (equilibrium 3 above) as "class AB equilibria".

Consider each of these cases in turn:

1. No firm switches, so that the masses are symmetric across A and B. As a result the utility from issuing in the home currency equals that of issuing in the foreign currency. The convenience yields on bonds in both A and B are equal:

$$P_{0,j} - \beta^2 = \frac{\lambda_j \beta(1-\beta)}{2} m_{F,j}. \tag{26}$$

2. All firms switch to A. Starting from the no-firms-switch case, if a small mass of firms were to shift from B to A, then the masses $m_{F,A}$ and $m_{I,A}$ would rise while the corresponding masses in B would fall. Then because of increasing returns to scale in matching, the liquidity benefit from issuing in A rises relative to that of B. We can see this by examining the liquidity benefit of switching to A:

$$U_{B\to A} = \frac{\lambda_A \beta(1-\beta)}{2} \left[m_{F,A} + \phi\, m_{I,A} \right]. \tag{27}$$

   Equation (27) highlights the two forces that drive firm decisions. Consider the second term in brackets. A firm that needs liquidity at date t_1, which occurs with probability ϕ, benefits from having a larger pool of liquidity, which is linear in $m_{I,A}$. Next consider the first term in the bracket. At date t_0, firms sell their bonds at a convenience yield because these bonds are used for settlement purposes at date t_1. This convenience yield benefit is increasing in $m_{F,A}$, the mass of firms needing early settlement.

   As the liquidity benefit of moving to A rises, the liquidity benefit of remaining in B falls. The result is that the equilibrium resolves with all firms switching to A. In this equilibrium, the convenience yield on bonds in B is zero, while it is positive in A.

3. All firms switch to B. This is the symmetric case as that of (2).

The fully symmetric case illustrates the economics at work in the model. The dominant equilibria of all firms switching to either A or B are the only stable ones. The equilibrium in (1) is unstable: a movement in masses to either A or B shifts the benefits for all other firms in the same direction leading to the all-firms-switch equilibria.

3.2 Incentives and Equilibrium With Heterogeneous Costs

We next focus on a case where costs differ across firms. We parameterize $H(K)$ as a Pareto distribution. Therefore, the cumulative distribution function $H(K)$ takes the form

$$H(K) = 1 - \left(\frac{\underline{K}}{K}\right)^{\alpha}, \tag{28}$$

where $\alpha > 0$ is a shape parameter. This distribution captures several properties that are salient in the cross-section of firms. First, the size distribution of firms is fat-tailed and well-described by a Pareto form (Gabaix 2011, Chaney 2018). Second, we expect foreign currency issuance fixed costs (Maggiori et al. 2020) to give rise to a negative correlation between costs paid per unit of debt and firm size. The Pareto specification therefore captures the notion that most of debt issuance is done by a tail of very large firms with low per-unit costs K_i. These large firms will be the first to sort endogenously into foreign currency issuance, and therefore small increases in the threshold $\bar{K}$ in the neighborhood of the lower boundary $\underline{K}$ will give rise to disproportionately large movements in the masses $\mathcal{M}$, as compared to similar increases in $\bar{K}$ at higher levels of entry. Appendix Section A.3 formalizes the conditions required for equilibrium existence under a broader class of distributions, as well as the second-order conditions associated with the model.

We begin by studying the class of equilibria in which firms in country B switch to currency A (*class BA equilibria*). Firms in country B play the threshold strategy described in Lemma 3, and all firms in country A issue in home currency. The marginal firm in country B (one for which $K_i = \bar{K}$) is indifferent between issuing in the two currencies. Therefore an interior equilibrium threshold $\hat{K}$ satisies the following indifference condition, which is a specialization of equation (25):[5]

$$\underbrace{\lambda_A \left[m_{F,A} + \phi\, m_{I,A}\right] - \hat{K}}_{\bar{U}_{B\to A}\text{: Utility from issuing in foreign currency}} = \underbrace{\lambda_B \left[m_{F,B} + \phi\, m_{I,B}\right]}_{\bar{U}_{B\to B}\text{: Utility from issuing in home currency}}. \tag{29}$$

To obtain the masses of buyers and sellers, we also specialize the market clearing conditions (23) and (24) to this class of equilibria. The masses of liquidity demanders (buyers)

[5] For ease of exposition, here we don't separately write out the condition for equilibria at the left boundary $\hat{K} = \underline{K}$, although these are of course valid if the complementary slackness condition in the second row of equation (25) is satisfied.

in the two currencies are

$$m_{F,A} = \phi \left[F_A + H(\hat{K}) F_B \right], \qquad m_{F,B} = \phi \left(1 - H(\hat{K}) \right) F_B, \tag{30}$$

while the masses of liquidity suppliers (sellers) are

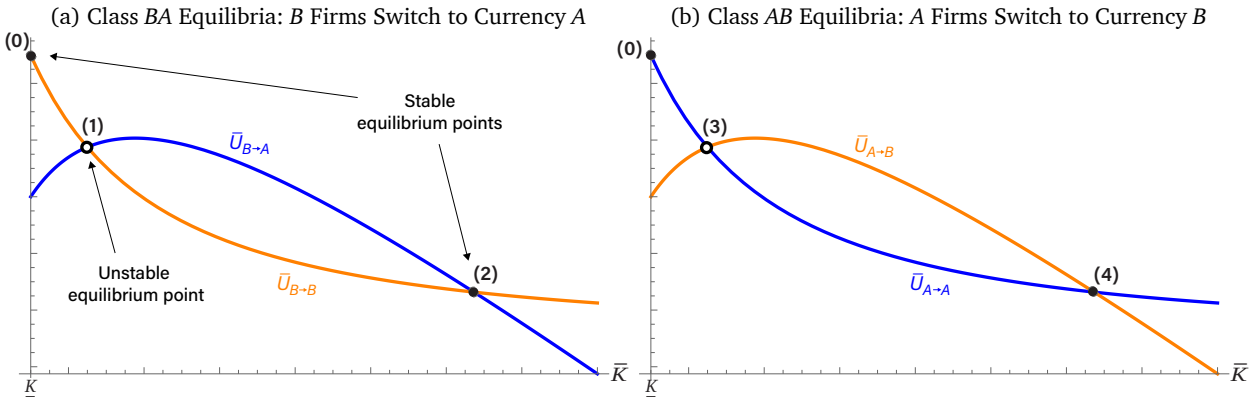
$$m_{I,A} = G_A + F_A + H(\hat{K}) F_B, \qquad m_{I,B} = G_B + \left(1 - H(\hat{K}) \right) F_B. \tag{31}$$

Figure 2a plots the curves $\bar{U}_{B\to A}$, and $\bar{U}_{B\to B}$ as functions of the threshold cost $\bar{K}$. These curves capture the expected utility of the marginal firm ($K_i = \bar{K}$) from issuing in foreign currency or home currency. The figure continues to keep the country fundamentals symmetric (λ_j, G_j, F_j). The shapes of these two curves reflect the economic forces at work. The curve $\bar{U}_{B\to B}$ is monotonically decreasing since higher values of $\bar{K}$ correspond to higher entry into currency A, which reduces the thickness of currency B markets, lowering the utility of home currency issuance. Conversely, higher entry raises the expected utility of foreign currency issuance, which is a force pushing $\bar{U}_{B\to A}$ higher. The curve $\bar{U}_{B\to A}$ is however also subject to a second force, since as $\bar{K}$ increases, the identity of the marginal firm changes: it is now a firm with higher foreign issuance cost K_i. This gives rise to a linearly decreasing component of $\bar{U}_{B\to A}$. Given the Pareto distribution of costs, there is a diminishing marginal impact of entry from increasing the threshold cost $\bar{K}$, which mediates the relative strength of the forces impacting $\bar{U}_{B\to A}$, giving rise to its non-monotonic (concave) shape.

The model features three equilibria of class BA. Two equilibrium points (labeled 1 and 2, and occurring at $\hat{K}_1$ and $\hat{K}_2$, respectively) lie at the intersections of the two curves $\bar{U}_{B\to A}$ and $\bar{U}_{B\to B}$. At these intersections, the expected utility from issuing in A and B is equalized for the marginal firm with threshold cost $\bar{K} = \hat{K}$. The interior equilibrium featuring low entry (point 1) is unstable. On the other hand, the high-entry equilibrium (point 2), featuring currency dominance, is stable. A further equilibrium point (labeled 0) is at $\bar{K} = \underline{K}$. In this equilibrium, no firms issue in foreign currency: this equilibrium is stable because of the presence of fixed costs $K_i > 0$, which make $\bar{U}_{B\to B}$ higher than $\bar{U}_{B\to A}$ in the neighborhood of $\bar{K} = \underline{K}$.

As also seen in Proposition 1, there is a second, symmetric class of equilibria as well. These are equilibria in which firms in country A issue in foreign currency, while all firms in country B remain in home currency (*class AB equilibria*). For this class of equilibria, $\bar{K}$ now characterizes the threshold strategy played by firms in country A. The analysis is analogous.

Figure 2: **Characterizing equilibria in the heterogenous cost case**



Notes: Panel A plots the expected utility of a firm in country B switching to issuing in foreign currency ($\bar{U}_{B\to A}$), and of a firm in B issuing in home currency ($\bar{U}_{B\to B}$), as a function of the threshold cost $\bar{K}$. Panel B plots the expected utility of a firm in country A switching to issuing in foreign currency ($\bar{U}_{A\to B}$) or issuing in home currency ($\bar{U}_{A\to A}$), also as a function of $\bar{K}$. The graph is for the case of symmetric fundamentals (λ_j, G_j, F_j) and positive currency-switching costs K_i distributed over $[\underline{K}, \infty)$ with a Pareto CDF $H(K)$. There are five equilibria.

Since it is now the marginal firm in country A that needs to be indifferent, the indifference condition determining an interior equilibrium threshold $\hat{K}$ in this case is:

$$\underbrace{\lambda_B \left[m_{F,B} + \phi\, m_{I,B}\right] - \hat{K}}_{\bar{U}_{A\to B}\text{: Utility from issuing in foreign currency}} = \underbrace{\lambda_A \left[m_{F,A} + \phi\, m_{I,A}\right]}_{\bar{U}_{A\to A}\text{: Utility from issuing in home currency}}, \tag{32}$$

and the masses $\mathcal{M}$ are obtained by performing the analogous specialization of (23) and (24).

Figure 2b again provides a graphical analysis by showing the curves $\bar{U}_{A\to B}$, and $\bar{U}_{A\to A}$ as functions of the threshold $\bar{K}$, continuing to keep country fundamentals symmetric. Like in the previous case, there are two interior equilibria: a low-entry unstable one (labeled 3) and a high-entry stable one (labeled 4). The equilibrium in which no firm switches (at $\bar{K} = \underline{K}$, labeled 0) also features. This is because the no-entry equilibrium has two isomorphic representations: one as a class BA equilibrium, and one as a class AB equilibrium.[6]

Summing up, the model in this case—with symmetric fundamentals but heterogeneous costs K_i—features a total of five equilibria, only three of which are stable. The three stable equilibria are the no-entry equilibrium (point 0), the equilibrium featuring high entry from

[6] If writing down the threshold strategy from the perspective of firms in country B (class BA representation), the no-entry equilibrium corresponds to the case in which $\bar{U}_{B\to B}(\underline{K}) \geq \bar{U}_{B\to A}(\underline{K})$. If expressing the strategies from the perspective of firms in country A (class AB representation), it corresponds to the case in which $\bar{U}_{A\to A}(\underline{K}) \geq \bar{U}_{A\to B}(\underline{K})$. In either case, each condition independently implies by Lemma 2 that *all* firms in *both* countries choose to denominate in home currency.

country A firms into currency B (point 2), and the equilibrium featuring high entry from country B intro currency A (point 4).

3.3 Asymmetries in Country Fundamentals and Transitions

We can now consider comparative statics with respect to country fundamentals, hence introducing asymmetry in fundamentals between the two countries A and B. The following proposition characterizes these comparative-statics results.

Proposition 2. *In the case with positive $K_i \in [\underline{K}, \infty)$, the following holds, respectively for class BA and class AB equilibria:*

1. *Class BA equilibria: An increase in country A's government bond supply G_A increases foreign-currency issuance by B-firms ($\frac{\partial \hat{K}}{\partial G_A} > 0$) in the stable high-entry equilibrium (point 2 in Figure 2). A sufficiently high increase in G_A dissolves the no-entry and low-entry equilibria (points 0 and 1), resolving equilibrium multiplicity in favor of the high-entry equilibrium. The same holds for increases in the overall matching intensity λ_A and increases in the mass F_A of firms in country A.*

2. *Class AB equilibria: An increase in country A's government bond supply G_A decreases foreign-currency issuance by A-firms ($\frac{\partial \hat{K}}{\partial G_A} < 0$) in the stable high-entry equilibrium (point 4 in Figure 2). A sufficiently high increase in G_A dissolves all class AB equilibria, resolving equilibrium multiplicity in favor of the high-entry class BA equilibrium point. The same holds for increases in the overall matching intensity λ_A. Increases in the mass F_A of firms in country A have ambiguous impact on entry, with the direction depending on parameter values.*

Figures 3a and 3b provide a graphical exposition of these comparative statics results. Consider first an increase in government bond supply G_A. As shown in Figure 3a, this acts as an outward shift of the blue curves $\bar{U}_{B\to A}$ and $\bar{U}_{A\to A}$: as government bond supply grows, the mass of investors $m_{I,A}$ that are sellers in the A money markets increases, which in turn benefits issuers in currency A's market as it expands the pool of liquidity available to them at date t_1. If this shift occurs starting from the high-entry class-BA equilibrium (point 2), the equilibrium threshold $\hat{K}$ shifts further to the right, which means entry is increasing, and currency A's dominance is more entrenched. If the shift occurs starting from the high-entry class-AB equilibrium (point 4), $\hat{K}$ conversely shifts to the left, as the resulting asymmetry in fundamentals weakens currency B's dominance.

Crucially, increasing G_A sufficiently will cause a qualitative shift in the configuration of equilibria in both equilibrium classes. First, a sufficient upward shift of the curve $\bar{U}_{A\to A}$ will raise $\bar{U}_{A\to A}$ above $\bar{U}_{A\to B}$ over the entire domain $[\underline{K}, \infty)$ for class AB equilibria, so that the two curves never intersect. Intuitively, this implies that sufficiently strong asymmetry between G_A and G_B dissolves all class AB equilibria, in favor of class BA equilibria. Simply put, equilibria in which B is dominant cannot survive once country A achieves a sufficiently large advantage in government debt supply, all other fundamentals equal.

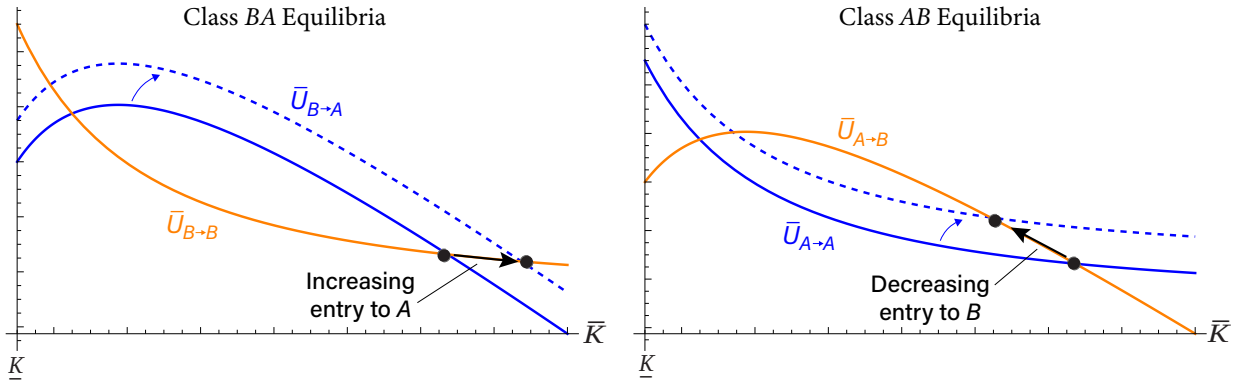
Second, large enough increases in G_A also impact the configuration of BA equilibria themselves. This happens once the $\bar{U}_{B\to A}$ curve crosses above $\bar{U}_{B\to B}$ at $\hat{K} = \underline{K}$. Once this threshold is crossed, the no-entry and low-entry equilibria (points 1 and 2) both disappear, leaving the high-entry equilibrium as the sole remaining one. These results illustrate another key economic point, which we discuss further below: multipolar equilibria (with denomination dispersed among multiple competitor currencies) can only survive in a world of roughly symmetrical fundamentals, while sharp asymmetries among countries bring about dominance.

Next, consider an increase in the mass of firms F_A in country A, the effects of which are shown in Figure 3b. Growing the size of the private sector is *not* equivalent to increasing safe government debt supply: while these have the same effect from the perspective of class BA equilibria, the same is not true for class AB equilibria. Similarly to an increase in G_A, increasing F_A shifts up the curve $\bar{U}_{A\to A}$, since a share of the additional mass of A firms will continue to issue in home currency, improving the liquidity of the currency A markets. On the other hand, some of the additional mass will go towards improving the liquidity of currency B markets, in proportion to the share of A firms that issue in foreign currency. For $\bar{K} > \underline{K}$, increasing F_A therefore tilts up the $\bar{U}_{A\to B}$ curve as well, which has the effect of *increasing*, rather than decreasing, entry of A firms into currency B.

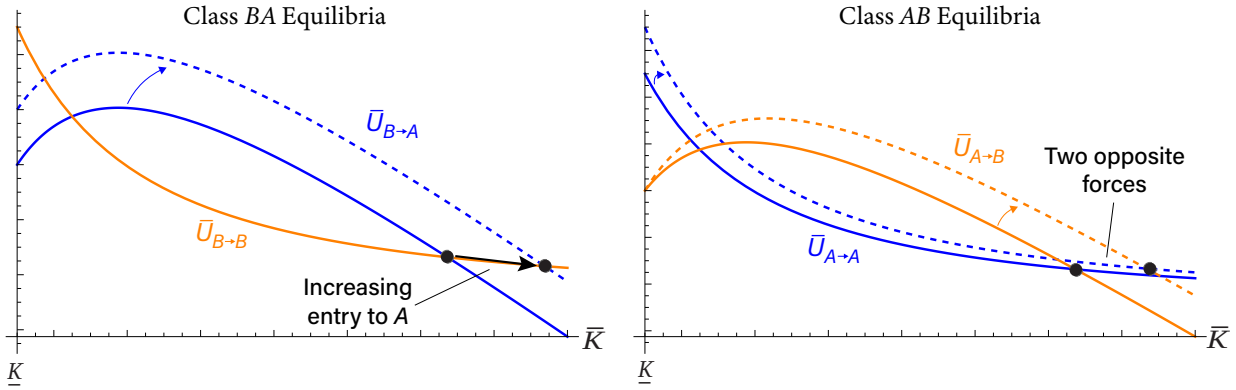
The net impact of increasing F_A on entry in class AB equilibria therefore depends on which of these two forces prevails. The relative strength of these two forces depends on the value of $\hat{K}$: for high value of $\hat{K}$, the second force will tend to be relatively stronger, and vice-versa. Figure 3b shows an example in which the starting value of $\hat{K}$ in the stable AB equilibrium is sufficiently high that the second force is stronger, and as a result the increase in F_A leads to overall higher entry, unlike in the case of increases in G_A. However, it is possible for the net direction of the comparative static to take the opposite sign. This illustrates a fundamental difference between sovereign and private issuance in our model:

Figure 3: **Comparative statics**

(a) Increasing government bond supply G_A



(b) Increasing firm mass F_A



Notes: We consider introducing asymmetry in country fundamentals by increasing government bond supply G_A (Panel A) or the mass of firms F_A (Panel B).

if a country does not start out as a dominant currency issuer, simply growing the size of the private sector is not guaranteed to facilitate the internationalization of its currency, and in fact can be counterproductive. Increasing the stock of safe government debt supply is instead a more reliable instrument, as it will facilitate international usage of the currency regardless of the starting equilibrium conditions.

3.3.1 Transitions Through History

Emergence of the First Global Currency. The asymmetries that can lead to dominance can be traced through history as well. Prior to the dominance of the Dutch florin in the 17th and 18th centuries, the European financial landscape was markedly different. Of particular salience is the experience of Italian city-states during the Renaissance. City-states

such as the Republics of Genoa, Venice, Florence, like Amsterdam, were highly prominent in both trade and finance. However, finance was conducted in a constellation of local currencies, such as the Genoese and Venetian lira and the Florentine florin, and none achieved the centrality that the Dutch florin would attain in subsequent centuries.

Our model rationalizes this difference between the Italian city-states and the later Dutch experience, and attributes it to the economic mechanisms centered on increasing returns to scale in liquidity provision that we discuss in Section 2. The multipolar equilibrium of the multiple Italian currencies corresponds to equilibrium point 0 of Figure 2, which features multiple currencies of roughly equal importance. This multipolar equilibrium is stable as long as these currencies have separate and approximately symmetric underlying liquidity pools (as determined by G_j).

We consider this to be an accurate representation of the historical context in which the constellation of Italian currencies circulated in the form of physical coins. They were in fluctuating and uncertain supply in any given place and time because they were often transported, debased, or re-minted. Indeed, Italy during the Renaissance faced the same settlement issues from a large variety of metallic coins that Amsterdam faced later on. While these city-states had large banks (including the Bank of San Giorgio dating from 1407 in Genoa and the Medici Bank from 1397 in Florence), none of them invested in creating a large and steady supply of safe debt in a common currency (G_j) as Amsterdam did.[7] The Genoese lira and the Florentine florin therefore remained unremarkable like the other coins of the era.

In contrast, the Dutch florin was a safe claim issued by the Bank of Amsterdam and backed by the City of Amsterdam with an initial supply sufficient to cover all large bills of exchange payments. In the context of our model, the financial innovations by the Bank of Amsterdam allowed it to increase florin-denominated G sufficiently to trigger a shift from the multipolar equilibrium to unipolar dominance. It did so by pooling disparate assets (such as the various kinds of specie and coins that were used prior to the florin in trade) to back a single, unified money market in the florin unit of account. Moreover, the fact that the Dutch cities did not all compete with each others' currencies, but rather followed Amsterdam's lead with its florin, provided liquidity agglomeration that the Italian city-states lacked.[8]

The usefulness of the florin could also be measured in part by the *agio*, the market

[7]Unlike Amsterdam, the Italian banks were slow to adopt ledgers and payments by account transfers. For instance, it was not until 1675 that the Bank of San Giorgio in Genoa issued depositors transferable vouchers reflecting deposit accounts (Willis, 1943, p. 12).

[8]See Appendix B.1 for more details on the arrangement between Rotterdam and Amsterdam.

exchange rate between locally circulating physical coin (current guilders) and bank florin. The *agio* was steadily around 4.5 to 5% for most of the 17th and 18th centuries (Van Dillen, 1934, p. 91, 102), reflecting the florin's liquidity premium.

Pound Sterling Dominance. The Bank of Amsterdam eventually collapsed following the French invasion of 1795, during which concerns about the florin's backing was reflected in a drop in the *agio* to -14%.[9] In our model, this is equivalent to a large and sudden contraction in G_A that shifts in the U_A curve and dissolves the dominant equilibrium of entry into A. The period that followed without a clear dominant currency was resolved by Great Britain's victory in the Napoleonic Wars. The UK at that time had the largest economy and a government debt in pound sterlings of approximately 200% of GDP, which was coupled with a tax base and credible governance that made this debt safe. As in the case of Amsterdam before it, having the largest pool of safe liquid assets for settlement initiated the process of dominance, which Great Britain held until the mid-20th century.

In contrast, France experienced several disruptions to its supply of safe government debt in the late 18th century, and its loss at Waterloo reduced its ability to float a large volume subsequently. Unlike Great Britain, it paid off its wartime debt with much higher taxes instead of new long-term issuances (Bordo and White, 1991). While the franc itself returned to the gold standard and maintained a stable value, the outstanding amounts of other forms of franc-denominated claims were much smaller than that of the pound's. One illustration of pound dominance is that French firms were the largest foreign contributors to the corporate bond issuance in London, accounting for 25% on average from the mid-19th century to WWI.

Following WWI, Eichengreen and Flandreau (2009) show that the pound sterling and US dollar were held in roughly equal proportion by central banks as foreign exchange reserves throughout the interwar period. They note that the dollar overtook the pound shortly after WWI (consistent with dollar liquidity growing following the centralization of US financial markets and a large war-time debt), but that its status also shrank after the 1933 dollar devaluation (consistent with a collapse in confidence in the US government's commitment to its currency).

[9]The Bank had secretly made large unsecured loans to the Dutch East India Company, which suffered large losses due to the naval blockade in the war. When these loans and losses came to light, the Bank suffered a run from which it never recovered.

F_j Throughout History. Throughout history, there have also been numerous examples of countries whose economic size alone was not sufficient to propel them to dominance. For example, Spain was the largest and wealthiest global economy during the 17th century, and yet the much smaller City of Amsterdam emerged as the center of the international financial system. During the 19th century, the US overtook Great Britain as the largest economy in the world in 1870. The projections now are that China will overtake US GDP in the next decade. Yet in all cases, these are examples of countries with large F_j but small G_j. Spain lacked the government commitment and financial technology to create a liquid money market; the US financial sector and monetary system was fragmented until 1913; and China lacks both an internationally tradable base of government-backed money market instruments and confidence in its government's commitment not to appropriate those assets.

3.4 Dominance in Financial Technologies

Following the initial emergence as a dominant currency, the government has further incentives to innovate in financial technologies that advantage its market liquidity. These innovations further entrench dominance. The history of the Bank of Amsterdam's innovations in the florin, and institutional changes in the London money market that impacted the pound sterling includes several dimensions of this additional source of complementarity.

We first specify the sovereigns' objective functions. The government of country j maximizes the following:[10]

$$W_j = \underbrace{F_j \int u_{i,j}^F(K_i)\,\mathrm{d}H(K_i)}_{\text{Domestic firms' welfare}} + \underbrace{G_j(P_{0,j} - \beta^2)}_{\text{Seignorage profits}}. \tag{33}$$

The first term in this objective function corresponds to the purely utilitarian welfare criterion that aggregates the preferences of domestic firms. In addition to this standard utilitarian objective, we allow the government to have a profit motive, which is reflected in the second term of equation (33): this term corresponds to the seignorage revenues earned from the convenience yield on sovereign debt issued at t_0, which scales linearly with the size of G_j government issuance.

For the rest of the analysis in the paper, we consider the case in which A is the higher liquidity market, by imposing the parameter restriction that $G_A > G_B$, $\lambda_A > \lambda_B$, and

[10] Note that investors always break even in equilibrium, so that we omit the utility of investors in writing the sovereign objective.

$F_A > F_B$. We therefore focus on class BA equilibria—the only ones that remain present once asymmetry in fundamentals is sufficiently large. The analysis is of course symmetric for the case in which B is the higher-liquidity country. Integrating firm preferences given the equilibrium masses $\mathcal{M}$ then yields the following expressions for welfare in the two countries.

Proposition 3. *For $\theta = 1$, up to an affine scaling the objective for the leading country (A) reduces to*

$$W_A = \lambda_A \left[G_A m_{F,A} + F_A (m_{F,A} + \phi m_{I,A}) \right], \tag{34}$$

while the objective for the follower country (B), again up to an affine scaling, becomes

$$W_B = G_B \lambda_B m_{F,B} + F_B \lambda_B (1 - H(\hat{K}))(m_{F,B} + \phi m_{I,B}) + U_{B \to A}, \tag{35}$$

where $U_{B \to A}$ is the utility of the B-firms that issue in foreign currency, given by

$$U_{B \to A} = F_B H(\hat{K}) \lambda_A (m_{F,A} + \phi m_{I,A}) - F_B \int_{\underline{K}}^{\hat{K}} K h(K) dK. \tag{36}$$

The expressions for the two countries' government objectives derived in Proposition 3 reveal several aspects of the structure of sovereign incentives in this environment. Consider, for example, the government's incentives to invest in technologies that improve private sector liquidity provision: in the context of the model, these can be captured as an increase in overall market liquidity λ_j, so that government incentives correspond to $\frac{\partial W_j}{\partial \lambda_j}$. These incentives to invest in financial innovation are increasing in measures of dominance ($\hat{K}$), size (G_j, F_j), and in the degree of the private sector's demand for liquidity (ϕ). Importantly, given this asymmetry, the incentive to increase λ_j is higher for the leader country (A) than the follower country.

Furthermore, our model identifies a complementarity between financial innovation and dominance: the incentive $\frac{\partial W_j}{\partial \lambda_j}$ is increasing with entry into A's markets ($\hat{K}$), and investments in financial innovation further spur entry, leading to an endogenous rise in $\hat{K}$, as formalized in the following proposition—which follows straightforwardly from differentiating W_A.

Proposition 4. *Issuance in the dominant currency and the leader's incentives to invest in financial innovation are complementary:*

$$\frac{\partial^2 W_A}{\partial \lambda_A \, \partial \hat{K}} > 0, \qquad \frac{\partial \hat{K}}{\partial \lambda_A} > 0. \tag{37}$$

Equipped with the theoretical result above, we now discuss several ways in which this interaction between currency dominance and incentives to invest in private sector financial innovation played out throughout history.

3.4.1 Financial Innovation Throughout History

Amsterdam's Repo Facility. The Bank's introduction of a specie-florin repo facility in 1683 is credited with propelling the florin to global prominence (Quinn and Roberds, 2014a). Before this facility, obtaining florin was a cumbersome process that required depositing specific, high-quality coins. This raised the cost of obtaining florins, and accounts were primarily held by those with liquidity needs in bills of exchange. The specie repo facility, however, greatly expanded access to florins by allowing individuals and businesses to monetize safe but illiquid assets.[11] For specie investors, issuing florin against their assets was profitable because of the convenience yield from the liquidity benefits of transacting in florin, reflecting the incentives to provide liquidity even by those without payment needs. In the context of the model, the repo facility was an investment in λ_A that generated entry into using the florin.

Bank of England's Changing Role. In Great Britain, the evolving role of the Bank of England from its creation in 1693 to the 19th century encapsulates the institution's changing incentives to facilitate financial market liquidity. At its founding, it was a private corporation that was granted several privileges in return for raising and administering the Crown's debt, and during the early part of its history, the Bank competed with other private banks such that it sometimes limited market liquidity in order to protect its own balance sheet.[12]

Beginning in the 1830s, a series of legislative reforms changed the Bank of England's role into one of a liquidity supplier. First, the Bank's notes became legal tender for pound sterling debts in 1833, which legally expanded the supply of pounds sterling.[13] Second,

[11] The facility was so popular that the quantity of florins at the Bank of Amsterdam doubled from approximately eight million to sixteen million from the mid 17th to the beginning of the 18th century, and is credited for drawing Europe's specie trade to Amsterdam, where it could be more profitably be conducted.

[12] The privileges restricted banking competition and gave the Bank of England a monopoly over note issuance. From 1697 until 1844, only the Bank of England could raise equity; all other banks were restricted to partnerships of six or fewer (after 1844, this was altered to a radius of 25 miles around London). In 1708, the Bank was granted an exemption to laws restricting bank note issuances to private partnerships (Broz and Grossman, 2004). The Bank was seen to have abused its monopoly in the 1825 crisis, after which The Banking Act of 1826 required the Treasury to monitor the amount of small-denomination notes issued by the Bank.

[13] Like most currencies during this era, the pound sterling referred to a specific metallic coin, and obligations

the entire note issuance was consolidated onto the Bank's balance sheet and given a large fiduciary issue in 1844, which again expanded the supply of pound-denominated settlement instruments. Third, by the mid-19th century, the Bank of England established its role as a reliable lender of last resort to the financial sector (an innovation that Bagehot (1873) credits to the Bank), where its balance sheet explicitly became the back stop to the private bills market.

Developments in the London Money Market. Institutionally, the private bills market also underwent changes. The legal codification of the contractual terms for bills of exchange coordinated the market on the terms of borrowing and the procedures for default, which reduced their information sensitivity in the sense of Dang et al. (2017) and collectively raised the safety and liquidity of the London money market. The growing clarity on the Bank of England's discount window rules also helped to homogeneize money market securities and raised the incentives to produce high quality "discountable" bills. The government lowered the costs for the banking sector to create bills of exchange by deregulating private banks' equity issuance in 1830. This fueled British bank overseas expansion, allowing them to pursue a business model of issuing bills of exchange collateralized on the large base of international trade around the world. As in Amsterdam, this technology monetized a pool of previously illiquid assets, further increasing market depth in the London money market.

3.5 Welfare Aspects: Natural Monopoly, Constrained Efficiency

We now discuss welfare aspects of the model from the perspective of a global planner. We continue to consider the case in which A is the higher liquidity market, with the restriction that $G_A > G_B$, $\lambda_A > \lambda_B$, and $F_A > F_B$, and focusing on class BA equilibria. We consider a planner whose objective is utilitarian over the preferences of the two governments:

$$\mathcal{W} = W_A + W_B. \tag{38}$$

Our first result is that the planner's choice of optimal entry features more entry into currency A than the competitive equilibrium.

denominated in sterling were contracted to be repaid in those coins. However, coins were inconvenient for the reasons already discussed, and private banks like the Bank of England found it profitable to issue paper notes denominated in sterling (i.e., claims on sterling coin).

Proposition 5. *Let K^* be the value of $\bar{K} \in [\underline{K}, \infty)$ that maximizes global welfare $\mathcal{W}$, and let $\hat{K}_{\max}$ be the stable class BA equilibrium point featuring highest entry.*[14] *It holds that*

$$K^* > \hat{K}_{\max}. \tag{39}$$

Proof. See Appendix Section A.2. □

At the heart of this result is the liquidity externality discussed in Section 2: entry into currency A by firms in country B carries social benefits in terms of improved market thickness, which are in excess of the private benefits of entry. Since these excess liquidity benefits are not internalized by the firms, there is too little private entry in equilibrium.

Conceptually, our setting presents an analogy with the theory of *natural monopoly* (Posner 1978). Here the higher-liquidity country (A) is effectively a natural monopolist, since consolidating issuance in its currency is welfare-improving from a global perspective. The planner's chosen equilibrium K^* achieves the *first best*, which can always be made into a Pareto improvement relative to the private equilibrium $\hat{K}_{\max}$ by introducing appropriate transfers.[15] Analogously to the normative results that feature in theories of natural monopoly, optimal policy in this setting will therefore feature a subsidy to entry into currency A.

In the results above we have considered the entry problem from the perspective of the global planner. A second, related question is how the shadow value of increasing government debt supply G_A in the leader country differs when viewed from the perspective of the global planner versus the sovereign in country A. That is, we are interested in comparing the two quantities $\frac{\partial \mathcal{W}}{\partial G_A}$ and $\frac{\partial W_A}{\partial G_A}$. If the shadow value from the global planner's perspective, $\frac{\partial \mathcal{W}}{\partial G_A}$, is higher, then the planner will prefer to increase G_A even beyond what is privately optimal for country A's government, leaving open the possibility of welfare-improving international coordination in sovereign liquidity provision.

Proposition 6. *The shadow value of increasing G_A is higher from the global planner's perspective, as compared to the perspective of the sovereign in the leading country (A), if and only if the following is satisfied:*

$$H(\hat{K})\frac{\lambda_A}{\lambda_B} > \frac{1}{2}\frac{G_B}{F_B} + \left[1 - H(\hat{K})\right] \quad \Longleftrightarrow \quad \frac{\partial \mathcal{W}}{\partial G_A} - \frac{\partial W_A}{\partial G_A} = \frac{\partial W_B}{\partial G_A} > 0. \tag{40}$$

[14] This corresponds to point 2 in Figure 2.

[15] We note that utility is transferable in our setting owing to the quasi-linear structure of preferences.

Proof. Differentiate (35) and (38) with respect to G_A, and rewrite to yield the inequality. □

In the expression above, note that if A is dominant, then $H(\hat{K})$ is high, tending to one, while $1 - H(\hat{K})$ tends to zero, and likewise the ratio $\frac{\lambda_A}{\lambda_B}$ is high, increasing the likelihood that this condition is satisfied. The direction of this result therefore hinges on the relative magnitudes of G_B and F_B. Improving liquidity in country A has two effects on welfare in country B: on the one hand, it improves the utility of the infra-marginal B firms that have already switched to foreign currency, but on the other hand it reduces the convenience yields earned on sovereign issuance G_B by inducing stronger entry. If country B has large private borrowing needs relative to the stock of safe government debt outstanding, the first effect outweighs the second, so that increasing G_A is also welfare-improving from the perspective of country B.

If the condition in equation (40) is satisfied, the global planner will want to engineer incentives for country A to further increase its liquidity supply, financing these with transfers from country B. These results provide a lens to interpret historical international liquidity provision arrangements, such as the Bretton Woods agreements of 1944. In the Bretton Woods system, major world economies effectively coordinated on having liquidity provided by the United States with the bulk of the gold reserves underpinning the Bretton Woods gold standard held at the Federal Reserve.

Throughout the Bretton Woods period, the United States held more than 90 percent of the world's gold reserves (Monnet and Puy 2020), with large transfers of gold from central banks outside of the United States to New York during and after WWII. These transfers and the resulting coordination on a US-backed gold convertibility sytem provide a historical counterpart to the possibility of welfare-improving coordination in international liquidity provision that features in our model. In response to the classic Triffin Dilemma that the US gold reserves would be insufficient to back its internationally held liabilities, our model would have prescribed more transfers of commitment (i.e., gold) to the United States.

Lastly, we note that our discussion so far has not drawn a distinction between state-contingent and non-contingent expansions of liquidity supply G_A, although state contingency also played an important role within the Bretton Woods architecture—for instance, through the role of central bank swap line arrangements, which remain a core feature of the international monetary system to this day. A formal discussion of this topic requires extending the model to incorporate a role for aggregate risk: we present this extension in Section 4.1, where we show explicitly how state-contingent arrangements such as swap lines can further

enforce currency dominance.

A key component of dominance is the ability of the central country to commit to the safety of its debt obligations. While the evolution of a government's commitment is outside the scope of our theory, our model has two features that pertain to a modern Triffin (1978) dilemma when the dollar is the *de facto* center of the international monetary system. First, as in Despres et al. (1966), financial institutions that speed up settlement, such as well-functioning repo markets, reduce the burden on government debt as the primary source of liquidity. Second, private firms, both domestic and foreign, contribute to the stock of dollar-denominated money-like liabilities. However, the latter does so by accumulating currency mismatch risk on balance sheets around the world, which can have costly financial spillovers, as discussed in Jiang et al. (2020).[16] Therefore while US GDP may not be the limiting factor, the world continues to face barriers in costlessly creating dollar liabilities.

3.6 Finance and Trade in the Model

It has been clear in history and today that trade and finance are intricately connected. Historically, bills of exchange were issued first and foremost to finance international trade transactions and today, trade finance continues to be an important part of the credit market. This section clarifies how trade invoicing impacts the forces we identify in our model. The next section discusses the history of trade and finance through the lens of our model.

In our model, firms only receive revenues in their domestic currency. However, we can consider the case where firms receive a portion of their revenue stream to be denominated in foreign currency, as in the case of traded goods that are invoiced in dollars. For a given firm, this will reduce K_i because the receipt of some profits in the foreign currency reduces the amount of currency mismatch that a firm faces if it also issues dollar-denominated debt, as in Gopinath et al. (2020) and in our Appendix Section A.1.

A reduction in K_i for all i is a leftward shift of $H(K)$, raising the utility of issuing abroad. Rewriting the equilibrium condition in equation (29) in terms of firm masses, the equilibrium condition is:

$$\underbrace{\lambda_A \phi \left[2F_A + G_A + F_B H(\hat{K})\right] - \hat{K}}_{\bar{U}_{B\to A}} = \underbrace{\lambda_B \phi \left[2F_B + G_B + F_B(1 - H(\hat{K}))\right]}_{\bar{U}_{B\to B}} \tag{41}$$

[16] On the topic of over-borrowing externalities in an open economy context, see also Caballero and Krishnamurthy (2001), Lorenzoni (2008), Bianchi (2011), and Korinek (2018).

Consider the case where there is an increase in the number of B firms receiving revenues in foreign currency. The cost distribution shifts left from $H_0(K)$ to $H_1(K)$ where H_0 f.o.s.d. H_1. This shift increases $H(\hat{K})$ and raises $U_{B\to A}$ while reducing $U_{B\to B}$ for the marginal firm at $\bar{K} = \hat{K}_0$. This benefit of entry will lead B firms to issue abroad until a new equilibrium is reached that equates the two sides, which will occur at $\hat{K}_1 > \hat{K}_0$. Therefore invoicing in foreign currency leads to more financial dominance.

Separately, we can also consider how trade invoicing might endogenously emerge in our model as a consequence of the liquidity forces discussed. Although this steps outside our formal analysis, if firms were also allowed to choose their currency of invoicing, the dominant currency invoicing would emerge *because* of the foreign exchange exposure firms have in their financial obligations. In that case, firms would choose to invoice in the foreign currency in order to reduce K_i.

We note that our theory implies particular prescriptions for the paths that lead a currency to become globally dominant. While in Gopinath and Stein (2021), as in the earlier work of Eichengreen (2011), a shift happens via a sequence beginning with increased invoicing of world trade, in our theory and Chahrour and Valchev (2022), a shift will occur when a country offers a large stock of safe and liquid assets denominated in its own currency that can be traded by the rest of the world. Our theory therefore suggests that in order to internationalize the renminbi, the Chinese government will need to open the capital account and invest in liquidity and financial innovation, rather than simply promoting trade invoicing in renminbi.

3.7 Finance and Trade Through History

The preceding argument is that trade invoicing is a natural consequence of the liquidity forces of our model. This contrasts with the argument in Gopinath and Stein (2021) that dominant currency trade invoicing is the source of financial dominance. History provides examples where trade was not the seed of currency dominance.

In the 17th and 18th centuries, the value of Spanish trade was a factor of two to six times larger than that of Dutch trade, and in fact the Dutch remained smaller throughout its centuries of florin dominance.[17] The pound sterling gained dominance in the 1820s during the first sovereign debt boom, before the mid-century surge in world trade and expansion of

[17] In 1700, the ratio of Spanish to Dutch trade was just over two; by 1750 it had grown to almost six, and in 1800 it was still approximately five. See Appendix B.1.1 for the full sources and calculation.

the British Empire.[18]

In both the Amsterdam and British cases, there were investments in financial technologies by the state and private sector that enhanced liquidity. As we have explained, the Bank of Amsterdam's ledger technology created liquidity in florin, which was the driving factor behind florin dominance in this period. For Britain, we described the legal and institutional developments around bills of exchange and the Bank of England.

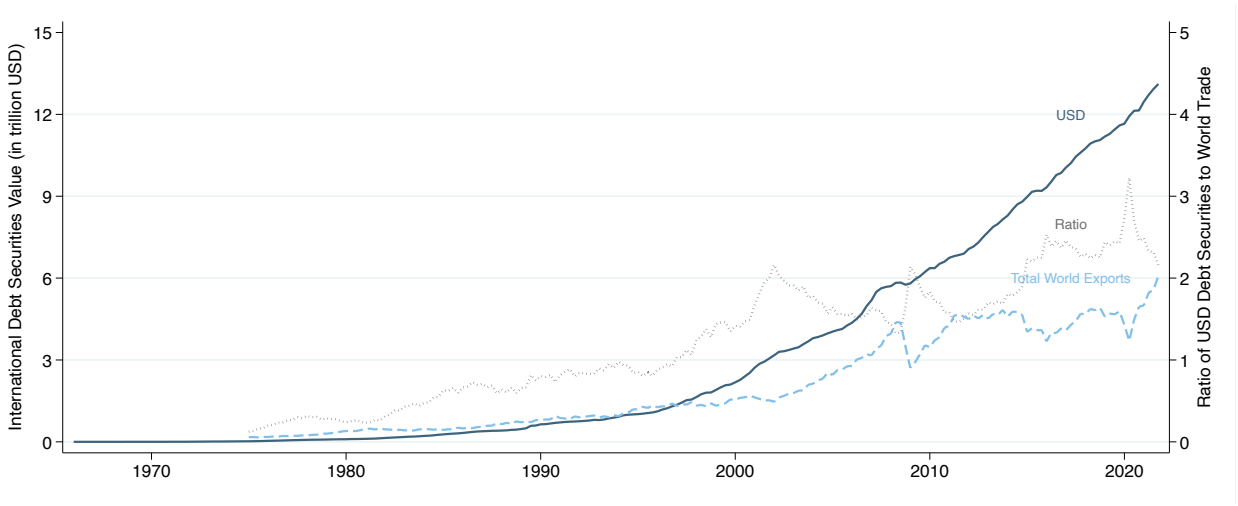
In addition, as world trade grew during this period, London banks provided a technology for generating secure money-like assets collateralized by these goods, analogous to Amsterdam's specie-florin repo technology. Specifically, in the 1830s, Parliament allowed British banks to issue equity for the first time, which fueled their expansion abroad. These banks provided firms around the world with a means of issuing short-term debt (bills of exchange) denominated in pounds sterling, which remitted to London and generated a large fraction of the circulating medium in the money market there. The series of institutional developments in British financial markets allowed these banks to securitize trade flows around the world into safe, liquid money market securities, just as the repo facility in Amsterdam securitized specie. During this period, British liquidity denominated in pounds underwrote 90% of world trade, while Britain in the goods market only comprised 50% of world trade (Xu, 2022).

In today's dollar world, the volume of cross-border financial assets trade dwarfs the volume of real goods trade. Figure 4 plots the evolution of cross-border bank liabilities denominated in US dollars from 1975 until today relative to total world exports at a quarterly level. The average ratio in the last twenty years has been two. Accounting for the fraction of world trade that is denominated in US dollars (approximately 40% in recent years), the relative sizes are five-fold. Considering all debt securities in the world outstanding, the ratio of dollar-denominated financial claims to world trade annually has on average been eight-fold larger since 2015.

The relative volumes of financial assets and international trade in all currencies are similarly skewed. Annual global trade flows are approximately 24 trillion USD compared to global debt contracts outstanding of approximately 300 trillion USD. Assuming an aver-

[18] As of 1815, the British Empire was still relatively small with little presence in Africa, the middle east, and Asia outside of India. Its land mass and population as a share of the world did not reach their peaks until the early 20th century with most gains in the second half of the 19th century. More generally, world trade was approximately 5% of world GDP until the 1880s, at which point it grew to 10%. Following the interwar years, world trade did not regain this share of GDP until 1970, and it is now at approximately all-time highs of 25%.

Figure 4: **Values of cross-border financial claims in USD and total world exports**



Notes: This figure plots the total values of cross-border debt securities denominated in US dollars with the total values of world exports at a quarterly level, and the ratio of the two series.

age maturity of seven years and an average interest rate of 7%, annual debt payments are approximately 65 trillion USD, or three times global trade flows.

4 Additional Theoretical Results

This section discusses additional theoretical results which deepen and extend the analysis in the preceding sections.

4.1 Aggregate Risk, State-Contingent Liquidity, and Default Risk

We now turn back to the discussion of state-contingent liquidity provision and central bank swap line agreements, first introduced in Section 3.5. As noted, this discussion requires that we formally introduce aggregate risk in the model. Hence, we now suppose that ϕ is subject to an aggregate shock, realized at time t_1. The state is $\omega \in \Omega$, and in state ω, which has probability q_ω, the early profit realization probability ϕ takes on the value ϕ_ω. The state realization is a shock to aggregate liquidity demand: if the realized value of ϕ is higher, more firms experience timing mismatch and therefore there is more overall demand for liquidity.

We analyze this extended version of the model in the $\theta = 1$ case. We allow for state-contingency in the supply of government assets G^A in the leading country, but we assume

that G_B, F_A, and F_B are all not state-contingent. The value of G_A in each state ω is G^A_ω. The following result presents the equilibrium indifference condition in this extended version of the model.

Proposition 7. *In the model with aggregate risk, focusing on the case in which A is the dominant currency (and hence considering class BA equilibria), the equilibrium condition that determines the marginal firm-i with $K_i = \hat{K}$ is*

$$\lambda_A\Big(\mathbb{E}[\phi_\omega]\big(2(F_A+H(\hat{K})F_B)+\mathbb{E}[G^A_\omega]\big)+\mathrm{Cov}[\phi_\omega, G^A_\omega]\Big)-\hat{K} = \lambda_B\mathbb{E}[\phi_\omega]\Big(2(1-H(\hat{K}))F_B+G_B\Big). \tag{42}$$

Proof. See Appendix Section A.4. □

Hence with the addition of aggregate risk, the equilibrium condition is nearly the same as in the baseline case, except for the additional terms $\mathbb{E}[G^A_\omega]$ and $\mathrm{Cov}[\phi_\omega, G^A_\omega]$, which are respectively the expected value of G^A and its covariance with the timing mismatch probability ϕ_ω across states. The intuition for the expectations term is straightforward: higher average government debt supply G^A_ω shifts the left-hand side of this equality upwards and thereby increases the equilibrium entry threshold $\hat{K}$.

The covariance term is positive when liquidity supply G^A_ω increases in states of the world with high liquidity demand (and hence high ϕ_ω). When the covariance is zero, even with stochastic ϕ_ω, the model collapses to the baseline case with constant ϕ. When G^A_ω is positively correlated with ϕ_ω, the extra covariance benefit makes currency A more attractive.

Central bank institutions such as the discount window and swap lines arrangements achieve positive covariance: both technologies allow the dominant currency issuer to expand liquidity supply when liquidity demand is particularly high. Hence we obtain the result that dollar swap lines reinforce the dollar dominance equilibrium and can be an important part of the architecture of the international monetary system.

The analysis also allows us to discuss default risk in government debt and the importance of the safety of the settlement asset. From the standpoint of t_0, default risk is that G^A_ω can fall in some states of the world. One effect of this risk is to reduce $\mathbb{E}[G^A_\omega]$, thereby reducing government supply and entry $\hat{K}$. A second effect is that if risk is higher in high-liquidity states of the world, then the covariance term is negative, further reducing effective government supply and $\hat{K}$.

These effects concern risk realized between t_0 and t_1. There is a third effect that concerns risk realized between t_1 and t_2 that does not arise in the modeling of this section, but is likely

an important concern. Bonds purchased at t_1 as settlement instrument for private debt at t_2 are poor settlement instruments if these bonds carry default risk. For example, if between t_1 and t_2, the bond defaults completely, then the firm will not own an asset to extinguish its debt. At t_0, default risk in government debt will mean that the currency is a less attractive currency in which to denominate private debts.

4.2 Complementarities Between Liquidity Provision and Demand

We now consider the interactions among the debt denomination choices of different types of issuers. To do this, we focus on *class BA* equilibria and we separate the liquidity demand and liquidity provision roles in the cross-section of firms in country B. Specifically, we now let the overall mass of firms F_B be composed of two different groups of firms:

- A first mass F_B^+ consists of *pure liquidity suppliers*: these are issuers for whom $\phi = 0$, which therefore never experience an early realization profits and hence have no motive for demanding liquidity. Given that $\phi = 0$ for these firms, they will not contribute to the liquidity-demander masses $m_{F,j}$ in either country.
- A second mass F_B^- consists of *pure liquidity demanders*. These are firms whose bonds have no possibility of re-sale in the money market of date t_1, so that effectively λ_j (which is now heterogenous for different assets) is zero for these firms' issues. These firms therefore will not contribute to the liquidity-supplier masses $m_{I,j}$.

The cost K_i follows the same distribution $H(K)$ in these two subgroups of firms, and $F_B = F_B^+ + F_B^-$. These two groups of firms will now have two different endogenous equilibrium thresholds $(\hat{K}^+, \hat{K}^-)$: liquidity suppliers issue in foreign currency if and only if $K_i < \hat{K}^+$, while liquidity demanders issue in foreign currency if and only if $K_i < \hat{K}^-$.

We consider the equilibrium determination of $(\hat{K}^+, \hat{K}^-)$ in the DGP case, in which $\theta = 1$. The equilibrium conditions pinning down the two thresholds are

$$\lambda_A m_{F,A} - \hat{K}^+ = \lambda_B m_{F,B}, \tag{43}$$

$$\lambda_A \phi m_{I,A} - \hat{K}^- = \lambda_B \phi m_{I,B}. \tag{44}$$

To complete the characterization of the equilibrium, the liquidity-demand masses in the two countries are

$$m_{F,A} = \phi \left[F_A + H(\hat{K}^-) F_B^- \right], \qquad m_{F,B} = \phi \left(1 - H(\hat{K}^-) \right) F_B^-, \tag{45}$$

while the liquidity-supply masses are

$$m_{I,A} = F_A + G_A + H(\hat{K}^+)F_B^+, \qquad m_{I,B} = G_B + \left(1 - H(\hat{K}^+)\right) F_B^+. \tag{46}$$

The solution is characterized by a two-equation system for the equilibrium values of $(\hat{K}^+, \hat{K}^-)$:

$$\hat{K}^+ = \phi \left[\lambda_A F_A - \lambda_B F_B^- + (\lambda_A + \lambda_B) H(\hat{K}^-) F_B^-\right], \tag{47}$$

$$\hat{K}^- = \phi \left[\lambda_A (F_A + G_A) - \lambda_B (G_B + F_B^+) + (\lambda_A + \lambda_B) H(\hat{K}^+) F_B^+\right]. \tag{48}$$

Equations (47) and (48) express the best responses of liquidity suppliers and liquidity demanders, respectively, to the entry decisions of the other type of firm. Crucially, these coupled entry decisions are *complementary*, in that entry by one class of firms increases the entry incentives of the other class, and vice-versa, as formalized in the following proposition.

Proposition 8. *The equilibrium entry decisions of liquidity suppliers (F_B^+) and liquidity demanders (F_B^-) are complementary, as their respective best responses satisfy:*

$$\frac{\partial \hat{K}^+(\hat{K}^-)}{\partial \hat{K}^-} > 0, \quad \frac{\partial \hat{K}^-(\hat{K}^+)}{\partial \hat{K}^-} > 0. \tag{49}$$

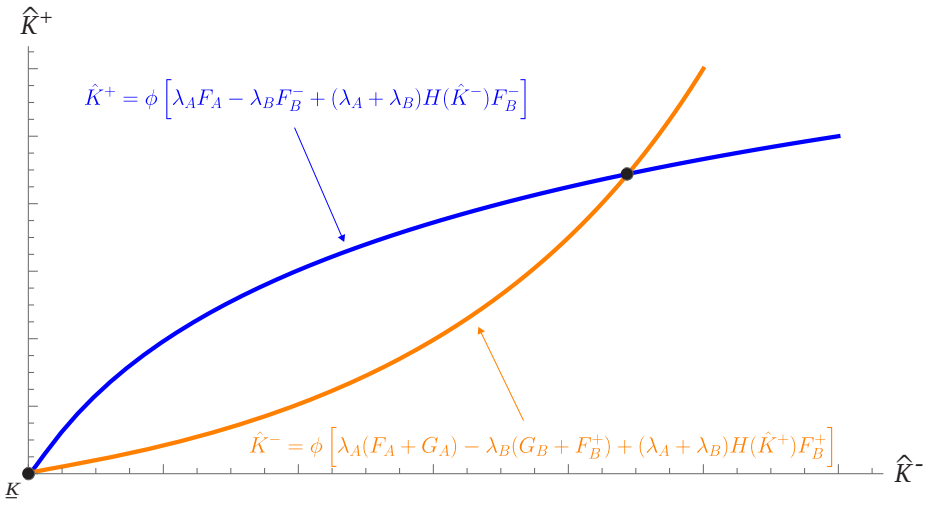
Proof. See Appendix Section A.2. □

Figure 5 provides a parametric plot of the best response functions (47) and (48) in the two-dimensional cost space of this extended model, continuing to use a Pareto distribution $H(K)$ with symmetric country fundamentals. In the symmetric-fundamentals case, the system features two stable equilibria, one in which neither kind of firm enters, and one in which both kinds have strong entry.

When we introduce asymmetry by increasing G_A (or λ_A or F_A), a sufficient rightward shift of the $\hat{K}^-$ curve will resolve equilibrium multiplicity in favor again of the high-entry equilibrium, in which currency dominance is now further reinforced by these strategic complementarities.

The growth of the London bond market in pounds sterling in the 19th century reflects these complementarities. Figure 6 plots the amounts of bonds issued by foreign governments and all corporates. We think of the foreign governments as suppliers of safe bonds in the pound market, so that their entry reflects the incentives of liquidity suppliers to harvest the pound convenience yield. We think of the corporates as liquidity demanders who choose to issue in pounds to access the easy settlement benefits of the pound money market. The

Figure 5: **Illustrating complementarity in the choices of liquidity demanders and liquidity suppliers**

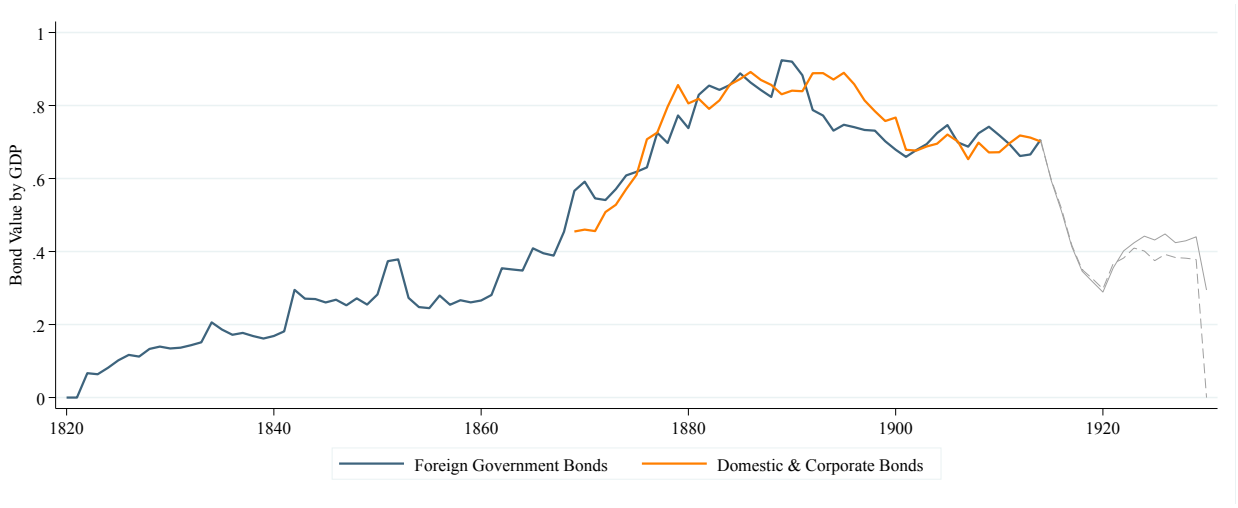


Notes: This figure plots the best-response functions for the coupled entry decisions of liquidity suppliers ($\hat{K}^+$) and liquidity demanders ($\hat{K}^-$) in the extended model of Section 4.2. These are shown in the cost space $(\hat{K}^+, \hat{K}^-)$ for the case of symmetric country fundamentals.

complementarity in the entry decisions is reflected in the positive correlation in issuances of these two types of bonds. Finally, note that the growth in corporate bond issuance prior to WWI reflects both pound issuance by domestic firms in Great Britain as well as foreign firms. This is because the liquidity benefits of agglomeration raise issuance utility for all firms in the dominant equilibrium, which encourages all forms of issuance.

The case of foreign governments issuing debt in the dominant currency illustrates another aspect of sovereign incentives. In our analysis, we have assumed that countries have no choice over the denomination of their debts. We can consider an extension of our model where country B's government (which has no liquidity demand) chooses the currency denomination of its debt. The benefit of denominating in currency A is that debt finance will be cheaper given the higher convenience yield in A than B. On the other hand, given the sovereign's objective in (35), country B's government will optimally be at an interior in its denomination choice. It will issue some debt in B since such debt provides liquidity to the high K_i firms who only issue in B. Our analysis indicate that sovereigns of non-dominant currencies will choose to issue some, but not all, of their debt in the dominant currency. This prediction of the model is consistent with issuance patterns historically and in the world today.

Figure 6: **Volumes of government and private debt denominated in pound sterling**



Notes: This figure plots the time series of the total outstanding amounts of pound sterling denominated debt traded in London, scaled by UK GDP. The series for foreign government bonds consists of sovereign debt collected by Meyer et al. (2022). The series for corporate debt comes from the publications of the Investor Monthly Manual (1869–1929) digitized by the International Center for Finance at Yale University.

4.3 The $\theta < 1$ Case and Asset Pricing Channels

The analysis in Section 3 has focuses on presenting analytical results for the $\theta = 1$ case, as in Duffie et al. (2005). In this section we discuss the case in which $\theta < 1$. When $\theta < 1$, the equilibrium condition that determines $\hat{K}$, again focusing for ease of exposition on BA equilibria, is

$$\underbrace{\lambda_A \left(m_{F,A} m_{I,A}\right)^{\theta-1} \left[m_{F,A} + \phi\, m_{I,A}\right] - \hat{K}}_{U_A\text{: Utility from issuing in foreign currency}} = \underbrace{\lambda_B \left(m_{F,B} m_{I,B}\right)^{\theta-1} \left[m_{F,B} + \phi\, m_{I,B}\right]}_{U_B\text{: Utility from issuing in home currency}}. \tag{50}$$

The market clearing expressions which characterize the masses $\mathcal{M}$ remain unchanged. This more general set of equilibrium conditions yields economic results that are equivalent to those studied in the baseline case, but it allows for a richer behavior of asset prices in response to variations in bond supply.[19]

A key difference between the positive implications of the $\theta = 1$ case and those of the $\theta < 1$ case concerns the mechanism of transmission of asset supply to convenience yields.

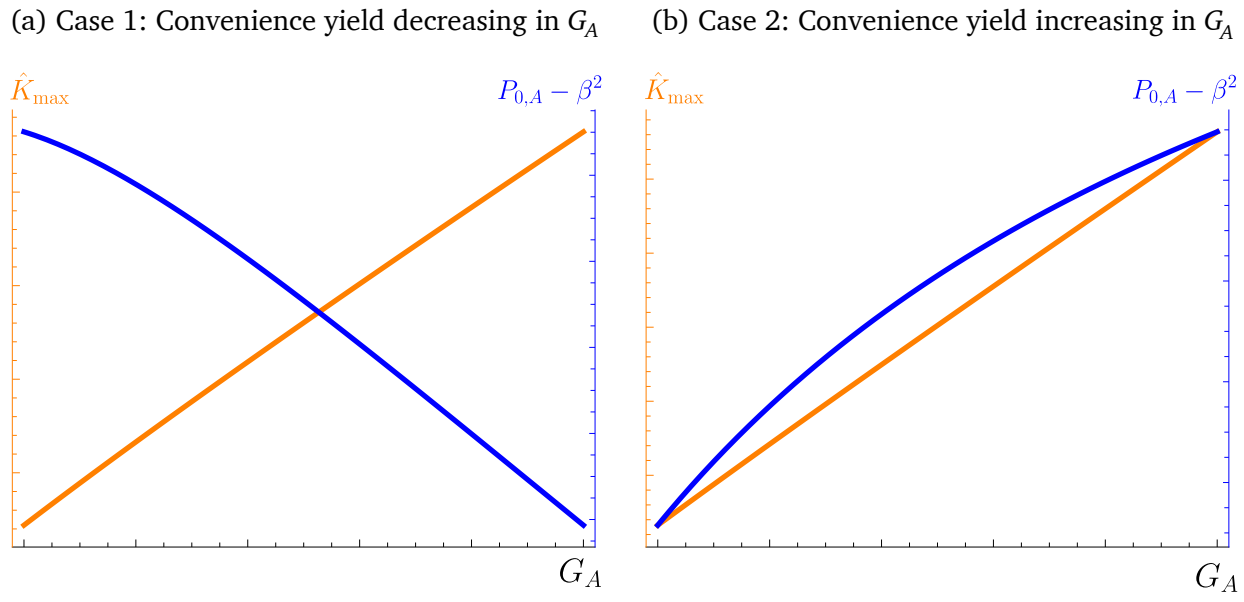
[19] In the $\theta < 1$ case, we need to make assumptions on the vector of parameters in order for the problem to be well-behaved and the equilibrium configuration to be equivalent to the one we present in Section 3.2. For example, we need θ to exceed a critical value $\theta^*(\Theta) \in (\frac{1}{2}, 1]$ which depends on parameters.

Recall the expression for the convenience yield in country A, which is

$$P_{0,A} - \beta^2 = \frac{\lambda_A \beta(1-\beta)}{2} m_{F,A}^{\theta} m_{I,A}^{\theta-1}. \tag{51}$$

In the general case ($\theta < 1$), both liquidity demand ($m_{F,A}$) and liquidity supply ($m_{I,A}$) impact the convenience yield: as intuitive, rising liquidity demand increases the convenience yield, while rising liquidity supply reduces it. The $\theta = 1$ case is a knife-edge parameter configuration in which the latter channel is shut off: in this special case, shifts in the liquidity supply schedule (such as changes in government debt supply G_A) do not have a direct effect on bond prices, but rather only impact them through their indirect effect on entry. In the $\theta = 1$ case, the convenience yield is therefore unambiguously increasing in government debt supply G_A, and this effect is entirely mediated through stronger equilibrium entry $\hat{K}$. Figure 6b illustrates this case for a parametric example with $\theta = 1$. Both the convenience yield (in blue) and entry, $\hat{K}_{\max}$ (in orange), are increasing functions of G_A.

Figure 7: **Convenience yields and government bond supply**



Notes: We focus on equilibria of the class BA, in which B firms switch to currency A. We show the behavior of the convenience yield $P_{0,A} - \beta^2$ for currency A as a function of government debt supply G_A. The analyses show these simulated comparative statics selecting the stable equilibrium with maximum entry threshold, $\hat{K}_{\max}$, which is also plotted. Panel A uses $\theta = .9$, while Panel B uses $\theta = 1$.

There is some tension between these results and the empirical findings in Krishnamurthy and Vissing-Jorgensen (2012) that convenience yields are decreasing in government debt supply. Note that comparing across countries, it is the case that country A has the higher

convenience yields: the tension regards the model's result within an equilibrium of entry into country A.

When $\theta < 1$, the liquidity supply effect re-emerges. Figure 6a illustrates this point for a parametric case where $\theta = 0.9$, but the other parameters are the same as in panel (b). Now we see that while entry is increasing in sovereign debt supply G_A, the convenience yield is decreasing in G_A.

5 Conclusion: Today's Dollar-Denominated World

The conceptual mechanisms that we have highlighted throughout this paper explain many of the features of US dollar dominance that we currently observe in world financial markets. To begin with, we view the large, liquid, and safe stock of US Treasury bills—largely first established to finance the war effort in World War I and subsequently expanded throughout the 20th century—as the kernel that seeded the process of dollar dominance.

Building on this initial stock of government-issued money market instruments, innovation in the US financial system allowed privately-issued short-term debt instruments to add to the pool of dollar-denominated money market liquidity. The growth of the US banking system and of the commercial paper market (Greenwood and Scharfstein 2013) are early examples of these liquidity-producing financial technologies. In more recent decades, securitization (Mian and Sufi 2009, Keys et al. 2010) allowed for further private-sector production of safe liquid assets, while the expansion of repo markets (Gorton and Metrick 2012, Krishnamurthy et al. 2014) broadened liquidity in the overnight segment of dollar money markets.

Correspondingly, the currency denomination choices of today's worldwide debt issuers feature the complementarities that we have illustrated through our theory. On the one hand, very safe global issuers—for instance, the German state-backed development bank KfW—are attracted to the US dollar because of the convenience yields attached to dollar-denominated assets, and they act as net liquidity suppliers to dollar money markets. On the other hand, lower-rated entities all around the world also issue debt in US dollars, drawn in by the ensuing liquidity benefits.

Other elements of the global financial architecture reinforce the current dollar-dominant equilibrium. Central bank swap line arrangements (Bahaj and Reis 2021) are an important example. Swap lines provide emergency liquidity supply by one central bank to others: overwhelmingly, the central bank on the supplying end of these arrangements has been the

US Federal Reserve, which provided US dollar liquidity in overseas markets during stress periods such as the global financial crisis of 2008-09 and the COVID crisis of 2020. Federal Reserve swap lines effectively promise a state-contingent expansion of dollar-denominated money market liquidity in times of need, hence increasing the expected liquidity benefits from issuing dollar debt.

Our study demonstrates that the financial and central banking advancements in the United States are a natural consequence of the incentives that come with being a dominant country, and that these developments further reinforce the country's dominance. An often-asked question in academic and policy discussions is whether the Chinese Renminbi (RMB) might be posed to displace the US dollar as the world's dominant international currency in the near future (Horn et al. 2021, Clayton et al. 2022). Our theory suggests that China will need to invest in all these factors which support currency dominance, and which are currently lacking in its financial system.

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Appendix for
"Liquidity, Debt Denomination, and Currency Dominance"

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January 2023

A Additional Microfoundations and Proofs

A.1 Microfoundation for Switching Cost K_i

Assume that at date t_1, the foreign exchange rate E either appreciates or depreciates with equal probability. Thus, a firm in country B can convert its revenue to currency A at a stochastic FX rate $E \in \{1+\gamma, 1-\gamma\}$. Suppose that a firm in B chooses to issue debt in A. At t_1, the exchange rate realizes, with a depreciation of currency B to $1-\gamma$ being a bad state for the firm: the firm has revenues in units of currency A of $1-\gamma$ and debt obligation of one. We assume that in this bad state, the firm-i can pay a disutility cost of $\kappa_i\gamma > 0$ to make up for the lost revenue. The disutility cost is a modeling device that ensures that firms reckon some cost due to currency mismatch, and to ensure that the firm does not default so that the bond is riskless (and hence private bonds are perfect substitutes for government-issued bonds). We assume that there is heterogeneity in this cost across firms. In this case,

$$K_i = \frac{1}{2}\gamma\kappa_i \tag{A.1}$$

which readily maps back to the model in the main text.

A.2 Proofs Not Included in Main Text

This section contains proofs that are omitted from the main text.

Proof of Proposition 5. Global welfare is $\mathcal{W} = W_A + W_B$. Call K^* the global welfare optimizing K. We wish to show that $K^* > \hat{K}_{\max}$. It is sufficient to show that,

$$\frac{\partial \mathcal{W}}{\partial K}\Big|_{K=\hat{K}} > 0. \tag{A.2}$$

We have,

$$\frac{\partial W_A}{\partial K} = G_A\lambda_A(\phi F_B h) + F_A\lambda_A 2(\phi F_B h), \tag{A.3}$$

and,

$$\begin{aligned}\frac{\partial W_B}{\partial K} \;=\; & -G_B\lambda_B(\phi F_B h) - (1-H)F_B\lambda_B 2(\phi F_B h) \\ & -\lambda_B(m_{F,B} + \phi m_{I,B})hF_B + \lambda_A(m_{F,A} + \phi m_{I,A})hF_B - \hat{K}hF_B \\ & +F_B H\lambda_A 2(\phi F_B h).\end{aligned}$$

Notice that the middle line here is equal to zero: it is the equilibrium condition that determines $\hat{K}$. Then,

$$\frac{\partial \mathcal{W}}{\partial K}\Big|_{K=\hat{K}} = \{G_A\lambda_A - G_B\lambda_B + 2F_A\lambda_A - 2F_B\lambda_B + 2F_BH\lambda_A + 2F_BH\lambda_B\}(\phi F_B h), \tag{A.4}$$

which is positive in the case of A dominance ($G_A > G_B, \lambda_A \geq \lambda_B$).[1] □

Proof of Proposition 7. The relevant derivations are given in Appendix Section A.4.

□

Proof of Proposition 8. Differentiating equation (47), we obtain:

$$\frac{\partial \hat{K}^+(\hat{K}^-)}{\partial \hat{K}^-} = \phi(\lambda_A + \lambda_B)h(\hat{K}^-)F_B^-, \tag{A.5}$$

where $h(\cdot) = H'(\cdot)$ is the probability density function (PDF) for the distribution of costs K_i. We assume that the density has non-zero mass over the entire the support $[\underline{K}, \infty)$, which is the case for the Pareto distribution that we use to derive our analytical results. Since h is a PDF, it is then positive over the full support: $h(\hat{K}^-) > 0$. All other terms are also positive ($\phi, \lambda_A, \lambda_B, F_B^- > 0$), so

$$\frac{\partial \hat{K}^+(\hat{K}^-)}{\partial \hat{K}^-} > 0. \tag{A.6}$$

By the same argument, differentiating (48) we have

$$\frac{\partial \hat{K}^-(\hat{K}^+)}{\partial \hat{K}^+} = \phi(\lambda_A + \lambda_B)h(\hat{K}^+)F_B^+ > 0. \tag{A.7}$$

□

A.3 Conditions for Equilibrium Existence and Convexity

This section discusses the formal conditions required for equilibrium existence as well as the second-order condition associated with the optimization problems that feature in the model. We derive conditions on the cost distribution $H(K_i)$ under which the model's objective function is well-behaved, and we show that the Pareto distribution featured in our baseline analysis satisfies these conditions.

Consider equilibria of class BA (as the analysis is symmetric for AB equilibria). The marginal firm with $K_i = \hat{K}$ satisfies

$$\lambda_A \left(m_{F,A}m_{I,A}\right)^{\theta-1}\left[m_{F,A} + \phi\, m_{I,A}\right] - \hat{K} = \lambda_B \left(m_{F,B}m_{I,B}\right)^{\theta-1}\left[m_{F,B} + \phi\, m_{I,B}\right], \tag{A.8}$$

[1] We also need $\mathcal{W}$ to be globally concave for this approach to be valid. It is straightforward to differentiate $\frac{\partial \mathcal{W}}{\partial K}$ and show that the second derivative is negative as long as $h' < 0$, which the Pareto distribution satisfies.

while the market clearing conditions are

$$m_{F,A} = \phi \left[F_A + H(\hat{K}) F_B \right], \qquad m_{F,B} = \phi \left(1 - H(\hat{K}) \right) F_B, \tag{A.9}$$

and

$$m_{I,A} = G_A + F_A + H(\hat{K}) F_B, \qquad m_{I,B} = G_B + \left(1 - H(\hat{K}) \right) F_B. \tag{A.10}$$

Define

$$\Delta(\hat{K}) = \lambda_A \left(m_{F,A} m_{I,A} \right)^{\theta - 1} \left[m_{F,A} + \phi \, m_{I,A} \right] - \lambda_B \left(m_{F,B} m_{I,B} \right)^{\theta - 1} \left[m_{F,B} + \phi \, m_{I,B} \right]. \tag{A.11}$$

The equilibrium condition is that

$$\Delta(\hat{K}) = \hat{K}. \tag{A.12}$$

Consider the $\theta = 1$ case. Then

$$\Delta(\hat{K}) = \lambda_A \left[m_{F,A} + \phi \, m_{I,A} \right] - \lambda_B \left[m_{F,B} + \phi \, m_{I,B} \right]. \tag{A.13}$$

Substituting in the masses:

$$\Delta(\hat{K}) = \phi \lambda_A \left[F_A + H(\hat{K}) F_B + G_A + F_A + H(\hat{K}) F_B \right] - \phi \lambda_B \left[2 \left(1 - H(\hat{K}) \right) F_B + G_B \right], \tag{A.14}$$

or

$$\Delta(\hat{K}) = \underbrace{\phi \lambda_A \left[2F_A + G_A \right] - \phi \lambda_B \left[2F_B + G_B \right]}_{(1)} + \underbrace{2 \phi F_B (\lambda_A + \lambda_B) H(\hat{K})}_{(2)}. \tag{A.15}$$

Under A dominance ($F_A > F_B, G_A > G_B, \lambda_A > \lambda_B$), the term marked (1) is positive and constant. The term marked (2) is zero at $\hat{K} = \underline{K}$, and it asymptotes to $2\phi F_B(\lambda_A + \lambda_B) > 0$ as $\hat{K}$ goes to infinity. The second term is also strictly increasing.

To get two crossings then we only need that $H(\cdot)$ is sufficiently curved for low values of $\hat{K}$. A necessary condition is that

$$\frac{d}{d\hat{K}} \left(2 \phi F_B (\lambda_A + \lambda_B) H(\hat{K}) \right) > 1, \tag{A.16}$$

or

$$2 \phi F_B (\lambda_A + \lambda_B) h(\hat{K}) > 1 \tag{A.17}$$

for low values of $\hat{K}$.

Candidate distributions are ones for which

$$h(\underline{K}) > \frac{1}{2 \phi F_B (\lambda_A + \lambda_B)}, \tag{A.18}$$

and where $h(\cdot)$ is uniformly decreasing. The Pareto distribution satisfies these criteria, as do many other distributions.

A.4 Derivations for the Model With Aggregate Risk

This section provides additional derivations for the extended version of the model with aggregate risk that we introduce in Section 4.1. We analyze this extended version of the model in the $\theta = 1$ case. To start, we consider the equilibrium at time t_1 in state ω. The masses of liquidity providers are

$$m^{\omega}_{I,j} = G^{\omega}_{j} + F_j \tag{A.19}$$

where we recall that we allow for state-contingency in the supply of government assets G^{A}_{ω}, but we assume that F_j is not state-contingent. Correspondingly, the masses of liquidity demanders are

$$m^{\omega}_{F,j} = \phi^{\omega} F_j. \tag{A.20}$$

The two-sided match probabilities are then

$$\underbrace{\alpha^{\omega}_{F,j} = \lambda_j m^{\omega}_{I,j}}_{P(\text{Buyer finds a seller})} \quad , \qquad \underbrace{\alpha^{\omega}_{I,j} = \lambda_j m^{\omega}_{F,j}}_{P(\text{Seller finds a buyer}).} \tag{A.21}$$

The surplus from are match remains $1 - \beta$, independent of the aggregate state. The date t_0 price of the private bond is therefore

$$P_{0,j} = E\left[\underbrace{\alpha^{\omega}_{I,j}\beta\left[\beta + (1-\eta)(1-\beta)\right]}_{P(\text{Matched}) \times \text{PV of Profit}} + \underbrace{(1-\alpha^{\omega}_{I,j})\beta^2}_{P(\text{Not Matched}) \times \text{PV of 1}}\right], \tag{A.22}$$

which we rewrite as

$$P_{0,j} = \beta^2 + (1-\eta)\lambda_j E[m^{\omega}_{F,j}]\beta(1-\beta). \tag{A.23}$$

Firm utility at date t_0 is

$$u^{F}_{i,j} = P_{0,j} + \beta E[\phi^{\omega}\alpha^{\omega}_{F,j}]\eta(1-\beta), \tag{A.24}$$

which, substituting in for $P_{0,j}$, becomes

$$u^{F}_{i,j} = \beta^2 + (1-\eta)\lambda_j E[m^{\omega}_{F,j}]\beta(1-\beta) + \beta\lambda_j E[\phi^{\omega} m^{\omega}_{I,j}]\eta(1-\beta). \tag{A.25}$$

As before, we take the case $\eta = \frac{1}{2}$ and rewrite:

$$u^{F}_{i,j} - \beta^2 = \frac{1}{2}\lambda_j\beta(1-\beta)\Big(E\big[m^{\omega}_{F,j} + \phi^{\omega} m^{\omega}_{I,j}\big]\Big). \tag{A.26}$$

This expression is similar to that of the non-stochastic case, except that the masses are now stochastic.

We next substitute in for the masses and rewrite:

$$u^F_{i,j} - \beta^2 = \frac{1}{2}\lambda_j\beta(1-\beta)\Big(E[\phi^\omega(2F_j + G^\omega_j)]\Big) \tag{A.27}$$

$$= \frac{1}{2}\lambda_j\beta(1-\beta)\Big(E[\phi^\omega]\big(2F_j + E[G^\omega_j]\big) + cov[\phi^\omega, G^\omega_j]\Big) \tag{A.28}$$

This yields the indifference condition presented in the main text:

$$\lambda_A\Big(\mathbb{E}[\phi_\omega]\big(2(F_A + H(\hat{K})F_B) + \mathbb{E}[G^A_\omega]\big) + \mathrm{Cov}[\phi_\omega, G^A_\omega]\Big) - \hat{K} = \lambda_B\mathbb{E}[\phi_\omega]\Big(2(1-H(\hat{K}))F_B + G_B\Big). \tag{A.29}$$

B Further Historical Details

This section provides additional details which supplement our discussion of the various historical episodes.

B.1 Bank of Amsterdam

The Dutch florin created by the Bank of Amsterdam in 1609 was the first global currency. For much of history, transactions and debts around the world were primarily settled in metallic coins. However, hundreds of domestic and foreign varieties existed, and using them entailed large transaction costs such as transportation, insurance, and assayance.[2] The difficulty of enforcing quality created incentives to debase the currency and reduce the circulating supply of high-quality coins. These costs compounded the difficulty of coordinating on the coins that were valid for settling a debt. While negotiable credit instruments such as the bill of exchange reduced the need to transfer coins, they still required a unit of denomination and so settlement ultimately relied on an uncertain supply of physical assets.

The Bank of Amsterdam was chartered by the City of Amsterdam to provide a high quality standardized currency that would reduce settlement frictions. The Bank primarily did so by creating a currency that existed on its ledgers ("bank florin") that was backed by coin and could be transferred across accounts freely. The City of Amsterdam initialized the pool of florin available for settlement by requiring that all large bills of exchange drawn and/or payable in Amsterdam had to be settled at the Bank, i.e., denominated in florin.[3] Relative to the uncertain supply of specific metal coins in circulation anywhere else in the world, the Bank provided florin the advantage that there would be a ready supply for payments in Amsterdam.

[2] An ordinance in the Dutch Republic from 1606 officially recognized 25 gold and 14 silver trade coins from 35 domestic mints, but many more varieties circulated, and the Republic officially published exchange rates for almost 1000 coins (Roberds and Velde, 2016, p. 344).

[3] The first ordinance in February 1609 applied to bills over 600 guilders; in 1642 year this was revised to include bills over 300 guilders. As a result, all merchants kept an account at the Bank, and the Bank maintained two to three thousand accounts at any given time (Van Dillen, 1934, p. 107).

Bank accounts were freely provided to anyone, and florins were credited to accounts for deposits of recognized coins. These coins backed the florins that could be withdrawn as current guilders in a narrow bank model. Since the Bank charged a fee for withdrawals, it was usually less costly to trade florin for current guilder in a secondary "open market."[4] In that market, the *agio* was the market exchange rate between the bank florin and the current guilder.

Rotterdam, a neighboring mercantile city, also created its own exchange bank modeled after the Bank of Amsterdam. While the two institutions maintained separate balance sheets, Rotterdam adopted Amsterdam's *agio* because merchants preferred florin (Van der Borght, 1896, p. 209). Rotterdam provided a system where all deposits and withdrawals of guilders were made allowing for the Amsterdam *agio* and thus used Amsterdam's florin as the unit of account (Carey, 1818, p. 369). Rotterdam also conceded to provide current accounts, which were the primary means for merchants to access florin by way of guilders and much more heavily used than its own bank money (Van der Borght, 1896, p. 210). In addition, the Bank of Rotterdam, despite requiring large bills of exchange to be settled in its own bank florin, also settled bills payable in florin in Amsterdam. In these ways, Rotterdam provided access to florins to the extent possible given its separate balance sheet.

In 1683, the Bank also introduced a *receipts* technology that operated like a modern day repurchase agreement. The Bank of Amsterdam advanced florin for short-term deposits of specie and metal bars. Depositors were issued a receipt, negotiable and renewable with an initial maturity of six months, for the right to withdraw the specific metal they deposited.[5] This technology broadened the set of assets that could be converted into florin beyond the original set of trade coins, and it was beneficial for both the Bank of Amsterdam and for private parties. The former gained from the metal deposits, which became part of the Bank's assets if the receipt expired, and the latter was able to obtain florin for settlement without needing to convert them into eligible trade coins for deposit at market value.[6]

Following the introduction of the repo facility, the quantity of florins at the Bank of Amsterdam doubled from approximately eight million to sixteen million from the mid 17th to the beginning of the 18th century (Quinn and Roberds, 2014b). These complementary innovation forces are also at play in the rise of Amsterdam. For instance, the receipts technology (i.e., repurchase facility) that

[4]The withdrawal fee of 1.5% covered the costs that the Bank of Amsterdam incurred to mint current guilders for deposits of inferior coins (judged by their metallic content).

[5]There was a large secondary market in receipts. Receipts could be redenominated in smaller face values, and they were renewable by paying the withdrawal fees. Withdrawal fees with receipts (0.125% for silver and 0.25% for gold) were much lower than that for current guilders (1.5%) because the Bank did not need to mint guilders to meet withdrawal demands. Around this time it appears the Bank of Amsterdam eliminated the right to withdraw from its accounts, which has led some authors to argue that the florin was an early fiat currency (Quinn and Roberds, 2014b).

[6]Given the wide variety of specie circulating, the demand and supply for specific coins varied significantly, and market prices were usually in flux. The receipts technology made it possible to transact on the Bank's mandated value for the specie while retaining the ability to withdraw and sell at a future date when prices rose. It also supported a large trade in precious metals since the freely-traded receipts were equivalent to advances on pledges of the underlying metals.

created florin claims out of raw specie was only introduced in 1683, almost seven decades after the Bank's establishment. The fact that florin balances doubled after its introduction is indicative of the force identifed in our model that increases in λ_j generate entry (Quinn and Roberds, 2014b).

After a century of dominance, the Bank of Amsterdam eventually became a victim of its own success. Intermediation in bank florins was profitable for the Bank, and it routinely turned over its wealth to the City of Amsterdam, leaving it with little capital buffer. It also made advances to the Dutch East India Company (VOC), which eventually led to runs on bank florin after the VOC came close to failure following the fourth Anglo-Dutch War in 1784. The French invasion in 1795 led to a drop in the *agio* to -14%, after which it never fully recovered, and eventually the Bank was formally dissolved in 1819.

B.1.1 Dutch versus Spanish Trade

The figures on Spanish trade per capita and the population are taken from Ortiz-Ospina et al. (2018) and Allen (2003) on p. 438 (Table A1) respectively. Data are available for four years (1600, 1700, 1750 and 1800). Dutch trade comes from Zanden and Leeuwen (2018) survey. We extract the four data points to compute Spanish trade relative to Dutch trade in the $17^{th} - 19^{th}$ centuries. Pound-guilder exchange rates are taken from Denzel (2017) (Figure 28.1).

Table B.I: Estimates to compute Spanish trade

Year	Trade Holland (in million guilder)	Trade Spain (in £ per capita)	Population Spain	Exchange rate £-guilder
1600	13.462	.18907407	8,700,000	0.095
1700	12.152	.31009123	8,600,000	0.1
1750	10.221	.57871836	9,600,000	0.095
1800	16.241	.48912659	13,000,000	0.09

In order to compare Spanish trade to Dutch trade volumes, we need the estimates for Spanish trade in guilder. Thus, we multiply the Spanish trade in £per capita by Spanish population and divide it by the £–guilder exchange rate to arrive at the final figures below.

Table B.II: Trade volumes (in million guilder)

Year	Holland	Spain
1600	13.462	17.315
1700	12.152	26.667
1750	10.221	58.481
1800	16.241	70.651

B.2 Great Britain

This initial point established the pound as a dominant currency with substantial entry from foreign sovereigns.[7]

The institutional developments in the legal structure of bills of exchange and the role of the Bank of England also reflect investments in overall market liquidity λ_A. These investments continued throughout the 19th century as dominance engendered entry, liquidity, and increased incentives to innovate.

The Bank of England was a key institution in the London money market, founded in 1694 as a note-issuing private corporation that was granted several privileges in return for raising and administering the Crown's debt.[8] During the early part of its history, the Bank competed with other private banks to increase its note circulation and raise its profits.[9] In this respect, the Bank was like any other private firm that was incentivized to issue safe debt in order to benefit from the yield premium in equation (A.23). It was very successful in establishing a sound reputation for its notes, and by the late 18th century, Bank notes became synonymous with the pound sterling (Thornton, 2017).[10]

In 1833 and subsequently 1844, the *de facto* equivalence between the pound and Bank notes became *de jure* with passage of the Bank Notes Act and the Bank Charter Act respectively. The former made Bank notes legal tender while the latter consolidated the entire note issuance onto the Bank of England's balance sheet where it was fully backed in gold reserves above the allowed fiduciary issue.[11] The full note circulation of the Bank of England therefore officially contributed to the supply of G_A following the 1840s.

A second important innovation was the legal codification of the contractual terms for bills of exchange, which coordinated the market on the terms of borrowing and the procedures for default. Bills of exchange were the primary London money market instrument, and each time

[7] The vast majority of the sovereign debt issued in London after the Napoleonic Wars were by Latin American and other European nations rather than the British colonies (Meyer et al., 2022).

[8] Like most currencies during this era, the pound sterling referred to specific metallic coins, and obligations denominated in sterling were contracted to be repaid in those coins. However, coins were inconvenient for the reasons already discussed, and private banks found it profitable to issue paper notes denominated in sterling (i.e., claims on sterling coin). The privileges restricted banking competition and gave the Bank of England a monopoly over note issuance. From 1697 until 1844, only the Bank of England could raise equity; all other banks were restricted to partnerships of six or fewer (after 1844, this was altered to a radius of 25 miles around London). In 1708, the Bank was granted an exemption to laws restricting bank note issuances to private partnerships (Broz and Grossman, 2004).

[9] The Banking Act of 1826 required the Treasury to monitor the amount of small-denomination notes issued by the Bank following the 1825 crisis in which the Bank was seen to have abused its monopoly (Scammell, 1968, p. 132).

[10] Estimates of historical convenience yields in the era of British pound dominance include Chen et al. (2022) and Payne et al. (2022).

[11] The limits of the Bank's note supply was therefore primarily governed by the gold reserves at the bank and secondarily by the government-determined fiduciary issue. Private bank notes already in circulation were allowed to remain, but no new notes could be issued, and banks lost their right when they merged. The Bank Charter Act could be suspended during financial crises when there was large demand for Bank notes.

one was traded ("discounted"), the seller guaranteed ("endorsed") the bill. These endorsements were legally equivalent to being the original borrower, and so each endorser was equally liable. The generality of these conditions were constantly tried at court and established a strong legal precedent.[12] These laws reduced the information sensitivity of bills and collectively raised the safety and liquidity of all bills of exchange with multiple endorsers, regardless of the idiosyncratic characteristics of the ultimate borrower. Thus, these innovations made private debt money-like in the sense of Dang et al. (2017).[13]

A third notable institution was the Bank of England's role as a credible and reliable lender of last resort to the financial sector. During the banking crises of 1847, 1857, and 1866, the Bank obtained permission from the Treasury to suspend the Bank Charter Act in order to meet all demand for Bank notes.[14] In fact, the Bank's behavior during the crisis of 1866 was the basis for Bagehot's rules for central banking (Bagehot, 1873). As a lender of last resort, the Bank provided liquidity at its discount window by converting private bills of exchange into pounds sterling, thereby *de facto* became a backstop to the private bills market. This backstop officially only applied to high-quality bills—those first guaranteed ("accepted") by large merchant banks that held accounts at the Bank of England—but like all liquidity backstops, its existence reduced the occurrence of market freezes and increased the willingness of private firms to lend in all states.[15] These forces together led issuers to prioritize denominating issues in sterling, thereby increasing the quantity of safe pound-denominated debt in the London money market.

The Bank of England acting as a lender of last resort was a major transition from its earlier history in the 18th century when discounting and note issuance was a profit-maximizing endeavor. At that point, the Bank's discount window followed the market and became similarly unavailable during downturns and crises.[16] As the London money market deepened and the pound sterling

[12] A Parliametary report from 1837 describes the legal protections against default: "a holder of a bill of exchange can bring actions at one and the same time, against every party whose name is attached to it, and in the event of the failure of them all, can prove upon the estate of each for the full value of the bill" (Joplin, 1837, p. 17).

[13] An additional factor is that the Act of 1833 exempted short-term bills of exchange from the Usury Laws, which also expanded the market's general willingness to hold them (Scammell, 1968).

[14] Since the Bank Charter Act limited the supply of Bank notes to the Bank's gold reserve, suspending it (and therefore the gold standard) was the only way to ensure they could meet demand. Even when the gold reserve was high, the presence of a limit reduced liquidity in the market. It is worth noting that obtaining permission to suspend the Bank Charter Act was sufficient, and Great Britain did not actually suspend the gold standard during this period.

[15] The high quality bills eligible at the Bank of England became a class of their own, and the financial press throughout this period reported the rates on "Bank" bills separately from "trade" bills (Xu, 2022).

[16] Scammell writes, "All in all the discounting of bills by the Bank in the early 19th century must be seen primarily as a prosperous business of the Bank and only very secondarily as a manifestation of credit policy," (Scammell, 1968, p. 144). For example, during the 1797 crisis, Parliament assumed the role of being a liquidity provider by issuing exchequer (treasury) bills to the market (Thornton, 2017, p. 98). The subsequent crisis in 1825 provides a microcosm into the transition that took place. Early in the year, the Bank of England closed its discount window because it anticipated a financial market downturn. This action in itself "created an atmosphere of misgiving and potential crisis," (Scammell, 1968, p. 131). However, when the crisis peaked in November with numerous failures in London, the Bank reversed its earlier decision and made discounts and advances on government securities and private bills. Thereafter starting in 1830, it

gained dominance in the 19th century, the Bank increasingly took on a more formal role within the government. This was despite the fact that it remained privately owned by stockholders until 1944 and run by Governors and Courts of Directors that primarily stemmed from the merchant and banking classes (Cassis, 1994, p. 85). The Bank's transitioning role in the money market given its dual identities reflects how the benefits of agglomeration accrued to both the government and to the private sector, all embodied in a single institution.

One final development during this period that contributed to maintaining the dominant equilibrium is the growth of international banking, which facilitated access to the pound sterling in locations around the world. British overseas banking institutions generally followed the business model of issuing deposits and shares domestically while lending via bills of exchange payable in London in their branches abroad. As in Amsterdam, the short term commercial bill became the dominant credit instrument internationally, with payments settled in London even for transactions that did not involve Great Britain.[17] The network of British banks increased the likelihood that foreign firms could hold pound obligations (or equivalently receive part of their profits in pounds), which in the context of the model we view as equivalent to reducing the cost of foreign currency issuance K_i, whether through a reduction of underlying FX exposures or via a reduction in the fixed cost of debt underwriting. Reducing this cost increases the mass of firms for which issuing in the foreign currency is profitable, as shown in equation (A.8).[18]

allowed bill brokers to access the discount window for the first time, after recognizing that these institutions were important conduits of liquidity.

[17]For example, "the bill on London enabled the banks [...] to finance a large share of international trade regardless of whether that trade touched Britain's shores," (Orbell, 2017, p. 8), and "wines from France, coffee from Brazil, sugar from the West Indies, and silk from Hong Kong were paid alike with bills on London," (Jenks, 1927, p. 69).

[18]Incidentally, both the French and the Germans followed the British model, often with explicit reference to expanding their currencies abroad. For instance, Edward Hurley, in his arguments for the creation for the US Federal Reserve System wrote, "The logical ambition of the German commercial policy is naturally to enthrone the Mark in the estimation of the world until it need pay no deference to the pound sterling."