When Portfolio Choice Meets Hat Algebra: An Integrated Approach to International Finance and Trade

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Abstract

This paper develops an approach for solving a multi-country model where financial and trade flows are both endogenously determined under bilateral frictions. The approach combines the perturbation method for portfolio choice analysis in a DSGE framework and the hat algebra technique from the trade literature. It captures the interaction of financial and trade channels in the general equilibrium: Portfolio allocation reflects agents' risk-sharing incentives influenced by the global trade pattern. Meanwhile, countries' asset positions shift the world demand system in the commodity market. Comparative static analyses employing the approach elucidate the impacts of tariffs and financial frictions as barriers to globalization.

JEL Codes: F10, F41, F60

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1 Introduction

Cross-country commodity and capital flows serve as two paramount engines of globalization. Nonetheless, few models have been proposed to characterize both trade and financial linkages in a multi-country structural framework. This paper develops an approach for solving a general equilibrium model where trade and finance channels interact. This new approach can be employed to examine how international trade influences finance, such as how a trade war reshapes the pattern of financial flows across economies, and how input-output linkages draw the map of global capital allocation. Meanwhile, the approach can be used to study the effects of international finance on trade, including how regional financial integration influences the direction and volume of global trade flows.

The approach not only combines the recent breakthroughs from both international macro and trade literatures, but also mitigates the methodological challenges faced by each strand and yields different predictions from existing works. Compared to the trade literature that typically takes countries' asset positions as exogenous, this paper solves global financial allocation under agents' intertemporal utility maximization decisions and cross-country frictions. Compared to the international macro literature that usually studies a small number of countries, this paper examines the multilateral linkages across many economies of uneven sizes. Specifically, I embed portfolio choice analysis in a quantitative macro-trade model, and examine 43 calibrated economies linked through trade and financial exchanges. The endogenous portfolio captures agents' risk-sharing motives shaped by the global trade pattern. Furthermore, countries' asset positions shift demand in the world commodity market. Therefore, this approach permits a higher degree of interplay between trade and financial channels in the general equilibrium, which is difficult to be fully captured by standard international macro or trade models.

To illustrate the main idea of the approach, I develop a model that builds on the Eaton and Kortum (2002) trade framework with immobile human and physical capital endowments and a single tradable sector subject to iceberg trade costs. Besides intermediate goods, financial assets which are claims to countries' capital income are traded across countries. Financial frictions that vary across country pairs add costs to households' repatriation of foreign returns. Nonetheless, households hold different assets to reduce the impact of country-specific productivity shocks on their consumption through international risk sharing. To derive asset positions, I follow Devereux and Sutherland (2011)'s method which solves portfolios in open economy macro models. It combines a second-order approximation of agents' Euler equation with a first-order approximation of

other model equations to determine a steady-state portfolio decided by agents' forward-looking utility maximization decisions. The method is flexible enough to be applied to a wide range of DSGE frameworks,[1] but it is computationally challenging to implement when the number of variables rises substantially with the number of countries.[2]

I employ the 'exact hat algebra' technique developed by Dekle et al. (2007) from the trade literature to overcome this computational challenge. This tractable technique enables comparative static analyses in a rich environment with numerous economies of uneven sizes connected through multilateral economic linkages.[3] In response to changes in cross-country trade or financial frictions, the total changes of any variable may include 1) the change of its steady-state value under different policy regimes (inter-regime changes), and 2) the deviation of the variable from its steady state under shocks within a specific policy regime (intra-regime changes). I use the hat algebra method to characterize both types of changes: one globally to measure the distance between steady states across original and counterfactual regimes, and the other locally within each regime around its steady state. Countries' asset positions, determined by intra-regime changes of second-moment variables in response to shocks, will simultaneously influence inter-regime changes by shifting countries' expenditure. The counterfactual outcome is obtained as the solution to a joint fixed-point problem of changes to wage on the real side and to portfolio on the financial side of the economy. By characterizing both the size and composition of countries' financial allocation in the general equilibrium, this approach makes an important contribution to the trade literature, which typically treats financial flows either as exogenously determined with values taken from the data or endogenously derived under extreme circumstances such as financial autarky or complete markets.[4]

[1] Devereux and Sutherland (2011)'s method can be used to analyze multiple types of assets (such as equities or bonds) under different financial structures (complete or incomplete markets) in a DSGE model of any dimension solvable by the local approximation technique.

[2] This is because the local perturbation method typically requires the computation of steady-state values of all the variables and the loglinearization of all the equations that characterize the variables' relations. Moreover, its numerical results may not be accurate since the large matrix that links all the countries is badly scaled with sparse bilateral trade and financial ties. This is a common challenge for international macro models, which normally have to limit their inquiry scope to a small open economy or two symmetric economies (for example, Mendoza (1991) and Backus et al. (1992)). Such frameworks cannot capture all the cross-country bilateral linkages and multilateral resistances in the spirit of Anderson and Van Wincoop (2003).

[3] Hat algebra is tractable because it adopts an iterative procedure that solves numerical solutions efficiently and accurately. Plus, it only requires few sufficient statistics to predict counterfactuals.

[4] For example, Dekle et al. (2007) examines counterfactual trade patterns under financial autarky. Eaton et al. (2016) study the puzzles in international macroeconomics under complete markets. It is challenging to analyze general cases between these two extreme financial arrangements due to the difficulty of solving the portfolio choice problem in a multi-country general equilibrium model.

The intra-regime analysis for portfolio choice is able to explain observed cross-country financial flows when the model is calibrated to the data. Therefore, I use this rich structural framework to evaluate the drivers for countries' asset positions proposed in the home bias literature (see a comprehensive survey by Coeurdacier and Rey (2013)), namely risk sharing, risk hedging, and financial frictions. Through quantitative exercises that compare model predictions from complete markets with two economies and from incomplete markets with 43 economies, I highlight the departure of this paper from the home bias literature. Furthermore, the model predicts bilateral asset positions besides countries' holdings of domestic assets. I conduct counterfactual analysis based on the model to explain the strong comovement of bilateral trade and financial linkages in the data. By exploiting variations across country pairs, this analysis elucidates the mechanisms that build the deep connection between financial and trade channels.

Another contributor to the connection originates from the joint dynamics of the two channels when responding to global economic conditions. To capture their interactions, I perform both inter- and intra-regime analyses to examine how financial and real variables react to higher tariffs. The numerical results suggest that, most countries witness greater declines to real wage yet increases to asset holdings when financial positions are adjustable. This happens because higher tariffs impair international output synchronization, which reduces cross-country asset covariances in the financial market. The increased financial holdings allow countries to raise expenditure, therefore their welfare loss would be overestimated if endogenous financial adjustments in response to the real side of the economy were not considered. This comparison between fixed and adjustable asset positions highlights the importance of incorporating a financial channel in trade models. Meanwhile, the feedback effects of financial allocations on real variables are typically missing from the home bias literature. Therefore, the predictions made with this new approach differ from the standard international macro and trade literatures.

Last but not least, I conduct two event studies as applications of the new approach. The first examines a universal rise of cross-country financial frictions to replicate the tightening of international markets during a global financial crisis. The model predicts that most countries reduce asset positions under greater barriers to global investment. Nevertheless, they turn out to adjust portfolios less to financial frictions when assets' covariance structure offers risk-sharing benefits. The second event study calibrates the model to the tariff increase during the 2018 China-US trade war. The model predicts that most countries suffer a decline in real wage due to the rise in commodity prices, yet they raise asset holdings which partially offset the welfare shortfall. Moreover, I examine

a counterfactual scenario where China sells off its holding of US assets and reconstructs the portfolio under the asset covariance structure. The model predicts that, the welfare of China will decrease by another 1.3 percent if this financial retaliation occurs. Therefore, the decoupling of the two major economies in trade and financial channels is very likely to bring more costs than benefits to themselves and to the world economy.

This paper contributes to both international macro and trade literatures. The portfolio choice analysis employs the method developed by Devereux and Sutherland (2011) who use a second-order approximation to overcome the certainty equivalence in a first-order approximation. Similar insights are also formulated by Samuelson (1970), Judd and Guu (2001), and Tille and Van Wincoop (2010). The method offers a powerful tool in open economy macro to solve perplexing puzzles like asset home bias (for example, Coeurdacier (2009), Coeurdacier and Gourinchas (2016), and Hu (2022)). Compared to the portfolio techniques driven by investors' specific preference for assets, such as the asset demand system employed by Liu et al. (2022) and the rational inattention logit demand system adopted by Pellegrino et al. (2021), this method does not require separate utility assumptions for agents' intratemporal financial allocation, which is determined by the endogenous asset covariance structure instead. My approach extends the method by combining it with hat algebra, in order to perform quantitative analysis across many economies with both trade and financial linkages. Furthermore, inter-regime analyses using this approach predict the influences of financial allocation on the level of real variables and hence characterize two-way interactions of trade and financial channels, while other applications of the Devereux and Sutherland (2011) method focus more on portfolio choice taking the world trade structure as given (Heathcote and Perri (2013), Steinberg (2018), Chau (2020), and Hu (2020)).

This paper contributes to the trade literature by modeling cross-country financial flows endogenously. Most trade models are static and ignore intertemporal decisions, with recent exceptions Alvarez (2017) and Kleinman et al. (2021) who introduce forward-looking physical capital accumulation decisions. Nevertheless, they follow the standard trade approach by assuming balanced trade or taking the asset positions as exogenous from data. My approach instead generates predictions for how a country's asset position is formed by households' forward-looking investment decisions. Plus, I derive counterfactual financial linkages with the hat algebra technique (Dekle et al. (2007)), which has been commonly used in quantitative spatial models (see Redding and Rossi-Hansberg (2017) for a survey). I add financial flows, which exhibit similar geographic patterns to trade and migration flows according to Portes and Rey (2005), to complete the spatial analysis.

The remainder of the paper proceeds as follows: Section 2 develops a multi-country model with trade and financial linkages, and describes the method employed to solve the model. Section 3 starts with intra-regime analysis to explain the drivers for countries' asset positions in the data, and then adds inter-regime analysis under higher tariffs to study the joint dynamics of trade and financial channels. Section 4 conducts two event studies with either financial or trade frictions as applications of the model to elucidate the impacts of barriers to globalization. Section 5 concludes.

2 Theory

2.1 Model Setup

The world comprises I countries indexed $i = 1, ..., I$. Each country i produces a final composite good using a continuum of intermediate goods $u \in [0, 1]$ traded across countries

$$Q_{i,t} = \int_0^1 [q_{i,t}(u)^{\frac{\epsilon-1}{\epsilon}} du]^{\frac{\epsilon}{\epsilon-1}}, \tag{1}$$

where ϵ is the elasticity of substitution in the CES aggregator. The composite good can be used either for consumption $C_{i,t}$ or for the production of intermediate goods $Y_{i,t}$. Country i's productivity when producing u, denoted as $Z_{i,t}(u)$, consists of two components:

$$Z_{i,t}(u) = S_i(u)T_{i,t}, \tag{2}$$

where $S_i(u)$ is the firm-level productivity drawn from a time-invariant Frechet distribution governed by constants θ and Υ

$$\Pr(S_i \leq z) = \exp(-\Upsilon z^{-\theta}), \tag{3}$$

and $T_{i,t}$ is the country-level productivity that follows an AR(1) process[5]

$$T_{i,t} = \rho T_{i,t-1} + (1 - \rho)\bar{T}_i + \epsilon_{i,t}. \tag{4}$$

[5]This assumption about productivity process is commonly used in the international real business cycle literature, such as canonical works by Mendoza (1991) and Backus et al. (1992). Besides productivity shocks, this model can be adapted to accommodate other risks which drive countries' output fluctuations that induce agents to construct portfolios for international risk sharing.

To characterize the risks of the economy for portfolio analysis, I assume $T_{i,t}$ is subject to serially independent shocks $\epsilon_{i,t}$ drawn from a joint normal distribution with a cross-country variance-covariance matrix Σ_T that contains $cov(\epsilon_{i,t}, \epsilon_{j,t}) = \sigma_{ij},\ \forall i, j \in [1, I]$. ρ is the persistence of productivity and $\bar{T}_i$ is country i's mean productivity over time.

Production of intermediate goods combines country-specific labor and capital endowments denoted as L_i and K_i which are assumed to be fixed in supply.[6] Moreover, it uses country i's composite good as an input for production. Let $w_{i,t}$, $r_{i,t}$, and $P_{i,t}$ be the prices of these inputs, and let τ_{ij} be the iceberg trade cost for exports from country i to j, country i's cost of serving a specific good u to country j at time t is hence given by

$$p_{ij,t}(u) = \frac{\tau_{ij}[(r_{i,t}^{\mu} w_{i,t}^{1-\mu})^{\eta} P_{i,t}^{1-\eta}]}{Z_{i,t}(u)}, \tag{5}$$

where μ is the share of capital and $1-\eta$ is the share of the composite good in production. The share of i's goods in j's expenditure is

$$\pi_{ij,t} = \frac{T_{i,t}[\tau_{ij}(r_{i,t}^{\mu} w_{i,t}^{1-\mu})^{\eta} P_{i,t}^{1-\eta}]^{-\theta}}{\Phi_{j,t}}, \qquad where \qquad \Phi_{j,t} = \sum_{k=1}^{I} T_{k,t}[\tau_{kj}(r_{k,t}^{\mu} w_{k,t}^{1-\mu})^{\eta} P_{k,t}^{1-\eta}]^{-\theta}, \tag{6}$$

while $\Phi_{j,t}$ is linked to the price level in country j through

$$P_{j,t} = \Gamma \Phi_{j,t}^{-\frac{1}{\theta}}, \tag{7}$$

in which Γ represents a Gamma function: $\Gamma(\frac{1-\epsilon}{\theta} + 1)^{\frac{1}{1-\epsilon}}$.

The goods market clearing condition of country i hence follows

$$Y_{i,t} = \sum_{j=1}^{I} \pi_{ij,t} X_{j,t}. \tag{8}$$

$X_{j,t}$ is country j's expenditure and $Y_{i,t}$ is i's output. $X_{i,t}$ and $Y_{i,t}$ are also connected through $D_{i,t}$ which is country i's aggregate asset position that adds up its bilateral hold-

[6] To deliver the main idea of the method, I assume physical capital is a fixed endowment for tractability and focus on financial capital as means of consumption risk sharing. Deriving physical capital accumulation in such a DSGE framework under incomplete markets typically employs local perturbation or global value (or policy) function iteration, both of which face the curse of dimensionality in a large-scale multi-country environment. Future extensions of the model may consider deep-learning algorithm proposed by Fernandez-Villaverde et al. (2020) to overcome this challenge.

ings of all the countries' assets denoted as $\alpha_{ij,t}$:

$$X_{i,t} = Y_{i,t} D_{i,t}, \qquad where \qquad D_{i,t} = \sum_{j=1}^{I} \alpha_{ij,t}. \tag{9}$$

On the financial side, I follow the international macro literature including Coeurdacier and Rey (2013) to assume countries issue one-period equities whose dividends are claims to their capital income

$$d_{i,t} = \mu Y_{i,t}, \tag{10}$$

which together with equity prices $p^E_{i,t}$ define financial returns

$$R_{i,t+1} = \frac{d_{i,t+1} + p^E_{i,t+1}}{p^E_{i,t}}. \tag{11}$$

Households hold assets to maximize expected lifetime utility

$$\max \sum_{t=0}^{\infty} \beta^t \frac{C_{i,t}^{1-\gamma}}{1-\gamma}, \tag{12}$$

where β is the discount factor and γ is the coefficient of relative risk aversion in the CRRA utility function, both of which appear in the intertemporal Euler equation

$$\frac{C_{i,t}^{-\gamma}}{P_{i,t}} = \beta E_t [\frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}} e^{-f_{ij}} R_{j,t+1}], \qquad \forall i, j \in [1, I]. \tag{13}$$

Markets are incomplete due to the existence of barriers to global financial investment. In particular, financial frictions potentially vary across country pairs, which justifies the gravity model of capital flows documented by Portes and Rey (2005). I follow Heathcote and Perri (2004) and Aviat and Coeurdacier (2007) by introducing financial frictions as iceberg transaction costs $f_{ij} \geq 0$, so that households in country i expect to collect $e^{-f_{ij}} R_{j,t+1}$ when repatriating asset returns from country j.[7] Besides, these frictions

[7] This modeling assumption about financial friction, which is analogous to trade cost in the commodity market, makes the portfolio choice problem tractable. However, I do not take a strong stand on either the underlying structure or the theoretical foundation of these barriers to international financial investment. Specifically, the bilateral friction f_{ij} can reflect a mix of worldwide factors including global financial liquidity, country-specific factors including capital account openness, and pair-specific factors including geographic distance and bilateral financial agreements. It can take alternative forms such as informational frictions, as Okawa and Van Wincoop (2012) find that these types of frictions yield comparable implications for the gravity model of financial flows.

are second-order in magnitude (proportional to the variance of shocks in the model) so that I can employ the solution method for portfolio choice in an open economy DSGE framework developed by Devereux and Sutherland (2011). Acknowledging that assets are distinguishable by their risk characteristics, these authors develop a method that combines a second-order approximation of the portfolio choice equation derived from the Euler equation (13) with a first-order approximation of other equations of the model in order to determine a zero-order (i.e. steady-state) portfolio.

Country i's portfolio choice equation and wealth constraint are given by

$$E_t[\frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}}R_{i,t+1}] = E_t[\frac{C_{i,t+1}^{-\gamma}}{P_{i,t+1}}e^{-f_{ij}}R_{j,t+1}], \qquad \forall i,j \in [1,I], \tag{14}$$

$$\mathcal{W}_{i,t+1} = \mathcal{W}_{i,t}R_{I,t+1} + \sum_{j=1}^{I-1}\bar{\alpha}_{ij}(R_{j,t+1} - R_{I,t+1}) + \eta Y_{i,t+1} - P_{i,t+1}C_{i,t+1}. \tag{15}$$

In equation 15, $\bar{\alpha}_{ij}$ is i's holding of j's assets in the steady state of the economy and $\mathcal{W}_{i,t}$ is country i's net asset position at t. It is worth noting that portfolio choice derived from the Euler equations (13 and 14) captures both inter-temporal and intra-temporal investment decisions of households to maximize their expected lifetime utility (12). Inter-temporally, households decide between financial investment and current consumption, given their patience (β) and elasticity of intertemporal substitution ($\frac{1}{\gamma}$), upon expected asset returns $R_{j,t+1}$ and inflation ($P_{i,t+1}$). Intra-temporally, the covariance matrix of different countries' productivity shocks (Σ_T) and the matrix of bilateral financial frictions will be reflected in the second-order Taylor expansion of the Euler equation 14, evaluating which determines portfolio choice. Therefore, households will naturally prefer assets from countries whose shocks are less correlated with their home country's under risk-sharing motives, and whose assets are subject to lower transaction costs to maximize financial payoff.

To sum up the description of the model setup, the general equilibrium of the model consists of a set of prices and quantities such that 1) households choose consumption and construct portfolio to maximize expected lifetime utility, 2) firms set output and price to maximize profit, and 3) factor, commodity, and asset markets clear. The economy is deemed in a steady state when the endogenous variables that satisfy all the equilibrium conditions are constant over time.

2.2 Solution Techniques

Solving for the equilibrium portfolio choice in a DSGE framework with the perturbation method developed by Devereux and Sutherland (2011) requires log-linearizing the model around the steady state of the economy.[8] Let $\widetilde{A}_t$ represent the log-deviation of any variable A from its steady state $\bar{A}$ at t

$$\widetilde{A}_t = \ln(\frac{A_t - \bar{A}}{\bar{A}}), \tag{16}$$

then the cross-country ratio of any country-specific variable $B_{i,t}$ defined as

$$B_{i/j,t} = \frac{B_{i,t}}{B_{j,t}} \tag{17}$$

has its deviation from the steady state expressed as

$$\widetilde{B}_{i/j,t} = \widetilde{B}_{i,t} - \widetilde{B}_{j,t}. \tag{18}$$

I assume I is a numeraire country when solving the world matrix of portfolio weights[9]

$$\begin{bmatrix} \bar{\alpha}_{11} & \bar{\alpha}_{12} & \cdots & \bar{\alpha}_{1I\text{-}1} \\ \bar{\alpha}_{21} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \vdots & & & \bar{\alpha}_{I\text{-}2I\text{-}1} \\ \bar{\alpha}_{I\text{-}11} & \cdots & \bar{\alpha}_{I\text{-}1I\text{-}2} & \bar{\alpha}_{I\text{-}1I\text{-}1} \end{bmatrix} \tag{19}$$

whose elements in the i^{th} row are decided by country i's Euler equation (14) which satisfies

$$E_t[\frac{U'(C_{i,t+1})}{P_{i,t+1}} e^{-f_{iI}} R_{I,t+1}] = E_t[\frac{U'(C_{i,t+1})}{P_{i,t+1}} e^{-f_{i1}} R_{1,t+1}] = ... = E_t[\frac{U'(C_{i,t+1})}{P_{i,t+1}} e^{-f_{iI\text{-}1}} R_{I\text{-}1,t+1}]. \tag{20}$$

[8] Perturbation methods gain much popularity in macroeconomics because their solution is accurate and extremely close to the exact around an approximation point (e.g. steady state of the economy). Loglinearization performs a first-order perturbation whose results in percentage terms are easy to understand. In addition, it improves the accuracy of the solution in many circumstances (see a detailed discussion by Fernández-Villaverde et al. (2016) on the solution methods of DSGE models).

[9] The portfolio matrix's dimension is $(I\text{-}1) \times (I\text{-}1)$ instead of $I \times I$. For the remaining assets positions, country i's holding of the numeraire asset is decided by the difference between its aggregate wealth position $\bar{D}_i$ and its holding of other assets. Meanwhile, country I's holding of any asset j is decided by j's market clearing condition such that the supply of the asset equals its demand given by $\sum_{i=1}^{I} \alpha_{ij} \bar{Y}_i$.

Portfolio is derived from the second-order Taylor expansion of equation 20 while taking the difference between the numeraire asset I and all the other assets:

$$E_t[\widetilde{R}_{x,t+1} + \frac{1}{2}\widetilde{R}^2_{x,t+1} - \widetilde{R}_{x,t+1}(\gamma\widetilde{C}_{i,t+1} + \widetilde{P}_{i,t+1})] = -\frac{1}{2}F_i + \mathcal{O}(\epsilon^3), \tag{21}$$

where $R_{x,t+1}$ denotes a vector of excess returns relative to the numeraire asset

$$\widetilde{R}'_{x,t+1} = [\widetilde{R}_{1,t+1} - \widetilde{R}_{I,t+1}, \widetilde{R}_{2,t+1} - \widetilde{R}_{I,t+1}, ..., \widetilde{R}_{I\text{-}1,t+1} - \widetilde{R}_{I,t+1}], \tag{22}$$

$R^2_{x,t+1}$ denotes the vector of excess squared returns

$$\widetilde{R}^{2'}_{x,t+1} = [\widetilde{R}^2_{1,t+1} - \widetilde{R}^2_{I,t+1}, \widetilde{R}^2_{2,t+1} - \widetilde{R}^2_{I,t+1}, ..., \widetilde{R}^2_{I-1,t+1} - \widetilde{R}^2_{I,t+1}], \tag{23}$$

and F_i denotes i's vector of financial frictions defined as

$$F'_i = [f_{iI} - f_{i1}, f_{iI} - f_{i2}, ..., f_{iI} - f_{iI\text{-}1}], \tag{24}$$

whose k^{th} element represents the additional financial friction country i's households incur when holding I's relative to k's asset. $\mathcal{O}(\epsilon^3)$ captures all terms of order higher than two.

The difference between any country i's and the numeraire country I's expanded Euler equations (21) follows

$$E_t[(\gamma\widetilde{C}_{i/I,t+1} + \widetilde{P}_{i/I,t+1})\widetilde{R}'_{x,t+1}] = \frac{1}{2}F_{iI} + \mathcal{O}(\epsilon^3), \qquad \forall i \in [1, I\text{-}1], \tag{25}$$

where F_{iI} stands for the excess financial frictions faced by country i relative to by I

$$F_{iI} = F'_i - F'_I. \tag{26}$$

Equation 25 is country i's relative to I's portfolio determination equation: the variables on its left hand side covary with country i's asset positions which are also influenced by financial frictions F_{iI} on the right. When evaluating the equation to pin down the equilibrium portfolio, we consider the responses of $\gamma\widetilde{C}_{i/I,t+1} + \widetilde{P}_{i/I,t+1}$ and $\widetilde{R}_{x,t+1}$ to all the productivity shocks. A standard DSGE approach would take the first-order derivative of the two variables with respect to the vector of productivity shocks, denoted as G_{t+1} and

H_{t+1} respectively in

$$E_t(\underbrace{G_{t+1}}_{1\times I} \times \underbrace{\Sigma_T}_{I\times I} \times \underbrace{H'_{t+1}}_{I\times(I\text{-}1)}) = \frac{1}{2}\underbrace{F_{iI}}_{1\times(I\text{-}1)} + \mathcal{O}(\epsilon^3). \tag{27}$$

Nonetheless, solving the portfolio choice problem with a standard DSGE approach is challenging especially in a multi-country scenario, because it is a daunting task to compute all the steady-state values and loglinearize all the equations when their numbers grow exponentially with the number of countries. Moreover, the numerical results solved by the perturbation method can be inaccurate given the large matrix that covers the world economy is badly scaled with countries' uneven sizes and with their sparse bilateral linkages. I find that these challenges are significantly mitigated in a quantitative trade framework for two reasons. First, establishing the relationship among variables from a large number of countries in a closed form is easier.[10] Second, applying the 'hat algebra' method developed by trade economists makes the computation for counterfactual results more efficient and accurate.[11] Instead of taking a first-order perturbation to evaluate equation 27, I simulate productivity shocks and compute variables' responses directly around the steady state of the economy.

To solve for portfolios using this new approach, I rewrite equation 25 as[12]

$$E_t[(\gamma(1-\beta)\widetilde{Y}_{i/I,t+1} + (1-\gamma+\beta\gamma)\widetilde{P}_{i/I,t+1} + \gamma(1-\beta)(\check{\boldsymbol{\alpha}}_i - \check{\boldsymbol{\alpha}}_I)\widetilde{R}_{x,t+1})\widetilde{R}'_{x,t+1}] = \frac{1}{2}F_{iI}, \tag{28}$$

where portfolio weights are scaled by countries' value-added output and discount factor:

$$\check{\alpha}_{ik} = \frac{\bar{\alpha}_{ik}}{\beta\eta\bar{Y}_i}, \tag{29}$$

which constitute the country's vector of asset holdings

$$\check{\boldsymbol{\alpha}}_i = [\check{\alpha}_{i1}, \check{\alpha}_{i2}, ..., \check{\alpha}_{iI\text{-}1}]. \tag{30}$$

[10] This makes it sufficient to focus on the variables that appear in the portfolio determination equation (28) only. Other variables in the model which influence portfolio choice can be expressed as analytical functions of those variables in the equation.

[11] This is thanks to 1) the efficient iterative computation procedure, whose solution captures the dynamics of variables, replaces the need for loglinearing all the equations in the model to predict variables' responses to shocks, and 2) the fact that it is no longer necessary to solve for the steady-state values of all the variables; A few observable statistics are sufficient for comparative static analyses.

[12] See Appendix B for the detailed derivation. This step uses countries' wealth constraint (equation 15) to substitute out consumption as a function of other variables solvable by hat algebra. Productivity persistence is assumed to be zero in this baseline case ($\rho = 0$).

Equation 28, if stacked vertically with each row representing a country, constructs a system of equations for the world bilateral portfolio matrix (defined in 19) to be solved.

Policy experiments under counterfactual trade and financial frictions are conducted to examine the impacts of these barriers to globalization. Given the counterfactual frictions, equation 28 is evaluated in original and counterfactual cases, where variables are superscripted $s = \{org, ctf\}$ respectively:

$$E_t[(\gamma(1-\beta)\widetilde{Y}^{org}_{i/I,t+1}+(1-\gamma+\beta\gamma)\widetilde{P}^{org}_{i/I,t+1}+\gamma(1-\beta)(\check{\alpha}^{org}_i-\check{\alpha}_I)\widetilde{R}^{org}_{x,t+1})\widetilde{R}'^{org}_{x,t+1}] = \frac{1}{2}F^{org}_{iI}, \quad (31)$$

$$E_t[(\gamma(1-\beta)\widetilde{Y}^{ctf}_{i/I,t+1}+(1-\gamma+\beta\gamma)\widetilde{P}^{ctf}_{i/I,t+1}+\gamma(1-\beta)(\check{\alpha}^{ctf}_i-\check{\alpha}_I)\widetilde{R}^{ctf}_{x,t+1})\widetilde{R}'^{ctf}_{x,t+1}] = \frac{1}{2}F^{ctf}_{iI}. \quad (32)$$

If the numeraire country (I) does not adjust portfolio, the difference between equations 31 and 32 will decide changes in country i's bilateral asset holdings $\Delta\check{\alpha}_i = \check{\alpha}^{ctf}_i - \check{\alpha}^{org}_i$ in response to any change in the products of variables that appear in the equations including

$$\widetilde{Y}^{s}_{i/I,t+1}\widetilde{R}'^{s}_{x,t+1},\ \widetilde{P}^{s}_{i/I,t+1}\widetilde{R}'^{s}_{x,t+1},\ \widetilde{R}^{s}_{x,t+1}\widetilde{R}'^{s}_{x,t+1}, \qquad s \in \{org, ctf\} \quad (33)$$

as well as in financial frictions $\Delta F_{iI} = F^{ctf}_{iI} - F^{org}_{iI}$ across original and counterfactual scenarios. The resulting bilateral asset holdings will add up to countries' equilibrium aggregate position as shares of output $\bar{D}_i = \beta\eta\sum_{k=1}^{I}\check{\alpha}_{ik}$.

Counterfactual trade or financial frictions affect both the first moments (levels) and second moments (covariances) of variables. Using the relative output of country i to j at time $t+1$ as an example, I decompose its total dynamics under counterfactual versus original frictions into two parts

$$\overbrace{\underbrace{[\ln(Y^{ctf}_{i/j,t+1}) - \ln(Y^{org}_{i/j,t+1})}_{\Delta Y_{i/j}}}^{\text{Total changes}} = \overbrace{\underbrace{[\ln(\bar{Y}^{ctf}_{i/j}) - \ln(\bar{Y}^{org}_{i/j})]}_{\Delta \overline{Y}_{i/j}}}^{\text{Inter-regime changes}} + \overbrace{\underbrace{[\ln(Y^{ctf}_{i/j,t+1}) - \ln(\bar{Y}^{ctf}_{i/j})] - [\ln(Y^{org}_{i/j,t+1}) - \ln(\bar{Y}^{org}_{i/j})]}_{\Delta \widetilde{Y}_{i/j}}}^{\text{Intra-regime changes}}. \quad (34)$$

From this decomposition, the total changes of a variable reflect 1) the change of its steady-state value under different policy regimes of frictions (inter-regime changes), and 2) the deviation of the variable from its steady state under shocks within a specific policy regime (intra-regime changes). Inter-regime changes are derived in a similar way as in trade models, while examining intra-regime changes of how second-moment variables behave around a steady state is necessary to solve the equilibrium asset position. Now I describe how inter- and intra-regime changes can both be characterized by hat algebra.

Inter-regime changes are computed following Dekle et al. (2007)'s analysis where they introduced the hat algebra method to examine a scenario without global imbalances. Let the ratio of any variable A's counterfactual to original steady-state value under policy changes be denoted as

$$\widehat{\bar{A}} = \frac{\bar{A}^{ctf}}{\bar{A}^{org}}. \tag{35}$$

It follows that the vectors of all the countries' wage and price

$$\widehat{\bar{w}'} = [\widehat{\bar{w}}_1, \widehat{\bar{w}}_2, ..., \widehat{\bar{w}}_I], \qquad \widehat{\bar{P}'} = [\widehat{\bar{P}}_1, \widehat{\bar{P}}_2, ..., \widehat{\bar{P}}_I] \tag{36}$$

given countries' asset positions

$$\widehat{\bar{D}} = [\widehat{\bar{D}}_1, \widehat{\bar{D}}_2, ..., \widehat{\bar{D}}_I] \tag{37}$$

are obtained by an iterative computation procedure to solve a fixed-point problem for a pair of vectors $(\widehat{\bar{w}}, \widehat{\bar{P}})$ based on all the countries' price determination and goods market clearing conditions (see Appendix B for the derivation of equations):

$$\widehat{\bar{P}}_i^{-\theta} = \sum_{j=1}^{I} \bar{\pi}_{ji}^{org} \widehat{\tau}_{ji}^{-\theta} (\widehat{\bar{w}}_j^{\eta} \widehat{\bar{P}}_j^{1-\eta})^{-\theta}, \tag{38}$$

$$\widehat{\bar{w}}_i \bar{Y}_i^{org} = \sum_{j=1}^{I} \frac{\bar{\pi}_{ij}^{org} \widehat{\tau}_{ij}^{-\theta} (\widehat{\bar{w}}_i^{\eta} \widehat{\bar{P}}_i^{1-\eta})^{-\theta}}{\sum_{k=1}^{I} \bar{\pi}_{kj}^{org} \widehat{\tau}_{kj}^{-\theta} (\widehat{\bar{w}}_k^{\eta} \widehat{\bar{P}}_k^{1-\eta})^{-\theta}} \widehat{\bar{w}}_j \bar{Y}_j^{org} \bar{D}_j^{ctf}. \tag{39}$$

After solving for $\widehat{\bar{w}}, \widehat{\bar{P}}$ using the procedure, changes to other macroeconomic variables including output $\widehat{\bar{Y}}$ can be derived through their relations with $\widehat{\bar{w}}, \widehat{\bar{P}}$ based on the equilibrium equations in the model. The hat algebra method is easy to implement here and it only requires calibrating countries' initial output $(\bar{Y}_i^{org})$ and bilateral trade shares $(\bar{\pi}_{ij}^{org})$ in the original steady state, which can be regarded as their long-term average values directly observable in the data.

This paper departs from the trade literature in the characterization of $\bar{D}_j^{ctf}$ predicted by $\widehat{\bar{D}}$, which is determined by equations 31 and 32. To solve for portfolio choice, I apply the hat algebra method around original and counterfactual steady states respectively to determine 'intra-regime changes' in 34. For intra-regime analysis, the initial levels of output and trade shares using which hat algebra is performed are set as their steady-state values in each regime $\bar{Y}_i^s, \bar{\pi}_{ij}^s, s \in \{org, ctf\}$. As mentioned above, the original steady-

state values are calibrated to the data. The counterfactual steady-state values will be predicted under inter-regime changes $\widehat{\bar{w}}, \widehat{\bar{P}}$ imposed on the original steady state.

In terms of notations, if the ratio of any variable A in response to simulated productivity shocks to its steady-state value in the either original or counterfactual regime is denoted as

$$\widehat{A}_t^s = \frac{A_t^s}{\bar{A}^s}, \qquad where \qquad s \in \{org, ctf\}, \tag{40}$$

For any country-specific variable $B_{i,t}^s$, its cross-country ratio $B_{i/j,t}^s$ defined in equation 17 has its percentage deviation from the steady state approximated as

$$\widetilde{B}_{i/j,t}^s = \widetilde{B}_{i,t}^s - \widetilde{B}_{j,t}^s = \ln(\frac{\widehat{B}_{i,t}^s}{\widehat{B}_{j,t}^s}) = \ln(\widehat{B}_{i/j,t}^s). \tag{41}$$

Therefore, the 'tilde' variables that appear in the portfolio determination equations 31-32 can be converted from the corresponding 'hat' variables. Since the variables in the portfolio equations are all expected to be realized at t+1, I omit the time subscript for brevity in the following analysis.

The hat variables can be characterized as their responses to the vector of countries' productivity shocks around steady-state values

$$\widehat{T}' = [\widehat{T}_1, \widehat{T}_2, ..., \widehat{T}_I] = [\frac{T_1}{\bar{T}_1}, \frac{T_2}{\bar{T}_2}, ..., \frac{T_I}{\bar{T}_I}], \qquad \forall i \in [1, I]. \tag{42}$$

Elements in this vector of productivity shocks $\widehat{T}_i$ will drive the dynamics of variables around the steady state in each regime $s \in \{org, ctf\}$ including wage and price:

$$\widehat{P}_i^{-\theta,s} = \sum_{j=1}^{I} \bar{\pi}_{ji}^s \widehat{T}_j (\widehat{w}_j^{\eta,s} \widehat{P}_j^{1-\eta,s})^{-\theta}, \tag{43}$$

$$\widehat{w}_i^s \bar{Y}_i^s = \sum_{j=1}^{I} \frac{\bar{\pi}_{ij}^s \widehat{T}_i (\widehat{w}_i^{\eta,s} \widehat{P}_i^{1-\eta,s})^{-\theta}}{\sum_{k=1}^{I} \bar{\pi}_{kj}^s \widehat{T}_k (\widehat{w}_k^{\eta,s} \widehat{P}_k^{1-\eta,s})^{-\theta}} \widehat{w}_j^s \bar{Y}_j^s. \tag{44}$$

It is worth noting that equations 43 and 44 for intra-regime analysis are counterparts to equations 38 and 39 for inter-regime analysis. Within either regime, tariffs and equilibrium portfolios are constant and therefore do not influence how variables behave around the steady state. Instead, the main driver for the dynamics of variables are simulated productivity shocks $\widehat{T}$, under which $\widehat{w}^s, \widehat{P}^s$ are solved with the same iterative procedure as for inter-regime analysis. I then express the changes of the second-moment variables

in portfolio determination equations (listed in 33) as functions of $\widehat{w}^s, \widehat{P}^s$ to derive asset changes across original and counterfactual regimes ($\widehat{\widehat{D}}$). Asset positions in the financial channel, by affecting countries' expenditure in the trade channel (equation 9), will influence real variables in the economy. Hence, I update inter-regime dynamics $\widehat{\widehat{w}}, \widehat{\widehat{P}}$ with $\widehat{\widehat{D}}$ using equations 38 and 39. The general equilibrium of this framework, where trade and financial channels interact with each other, can be characterized by the solution to a joint fixed-point problem of $(\widehat{\widehat{w}}, \widehat{\widehat{P}}, \widehat{\widehat{D}})$.

Appendix B provides more technical details of the model and its solution: Section B.1 derives model equations used to perform hat algebra and portfolio analysis. Section B.2 discusses the existence and uniqueness of the model solution. Section B.3 outlines the algorithm which builds a recursive structure that combines inter- and intra-regime analyses to solve the fixed-point problem.

2.3 Calibration

I calibrate the model to a world economy that consists of 43 countries (listed in table A.1) plus the rest of the world (ROW). On the real side of the economy, I only need countries' GDP data from the Penn World Table (PWT) and bilateral trade shares from the Direction of Trade Statistics when using the hat algebra technique to predict counterfactuals. Analogously on the financial side, I obtain countries' net foreign asset positions (NFA) from the World Bank and bilateral portfolio weights from Factset/Lionshare (see Appendix C for details on data). The time-averaged values over the sample period 2001-2021 are used as the steady-state values of these variables in the original regime.

The risk characteristics of the economy are reflected as productivity fluctuations. Therefore, I estimate countries' dynamic productivity consistent with the Eaton and Kortum (2002) model following Levchenko and Zhang (2014)'s approach and compute its corresponding mean value and covariance matrix (see Appendix C). Productivity shocks necessary for intra-regime analyses can either be simulated with a joint normal distribution featuring the estimates or directly with the bootstrap method.[13] Other parametric assumptions include the annual discount factor $\beta = .9$, coefficient of relative risk aversion $\gamma = 2$, share of intermediate input in production $\eta = .312$ following Dekle et al. (2007), share of labor input $1 - \mu$ as country-specific labor income share from the PWT, and trade elasticity $\theta = 4$ based on Simonovska and Waugh (2014).

[13]Bootstrap is more feasible when the estimated covariance matrix is not positive semi-definite, due to the collinearity problem given the large-scale data, to simulate random samples from the specified multivariate normal distribution.

Several quantitative exercises compare model predictions in settings with 2 and with 43 countries. When calibrating the two-country setting, I collapse the multi-country framework to a model where each country is treated as the domestic economy and the sum of all the other countries from this country's perspective as the foreign economy. Following this rule, I calculate the two-by-two matrices of financial and trade shares, and re-estimate the productivity of domestic and foreign economies based on country sizes and trade flows.

3 Model Predictions

This section consists of two sets of quantitative analyse based on the model. The first performs intra-regime analysis of the calibrated original regime to evaluate the drivers of portfolio choice observed in the data. The second combines intra- and inter-regime analyses under counterfactual tariffs to elucidate the comovement of financial and trade channels in the general equilibrium.

3.1 Mechanisms for Portfolio Choice

To examine how trade influences finance in this multi-country framework, I compare the model predictions with those from the international macro literature on the topic of asset home bias including Coeurdacier (2009) and Heathcote and Perri (2013). I start with their assumption that there are two symmetric countries in a world with complete risk sharing.[14] Under the assumption, Coeurdacier and Rey (2013) derive the general expression for the share of domestic assets in a country's portfolio

$$\bar{\alpha}_{ii} = \underbrace{\frac{1}{2}}_{\substack{\text{Risk sharing} \\ \text{(Diversification)}}} \underbrace{- \frac{1}{2}\frac{1-\mu}{\mu}\frac{cov(\widetilde{w}_{i/j}, \widetilde{R}_{i/j})}{var(\widetilde{R}_{i/j})}}_{\text{Hedging labor income risk}} + \underbrace{\frac{1}{2}\frac{1-1/\gamma}{\mu}\frac{cov(\widetilde{P}_{i/j}, \widetilde{R}_{i/j})}{var(\widetilde{R}_{i/j})}}_{\text{Hedging RER risk}}. \tag{45}$$

The first term reflects households' incentive to diversify portfolios across different countries' assets in order to reduce the impact of country-specific shocks on consumption (as

[14] Risk sharing can be derived under various assumptions. For example, Coeurdacier and Rey (2013) follow the Backus and Smith (1993) condition implied by complete markets to solve for portfolio choice. Heathcote and Perri (2013) assume Cobb-Douglas preference for domestic and foreign goods in the expenditure bundle, which automatically changes terms-of-trade in response to shocks to make risk sharing achievable by time-invariant asset positions. Coeurdacier and Gourinchas (2016) show markets are locally complete to support risk sharing, as long as the set of asset returns spans the space of shocks.

in Lucas (1982)). The second term captures the hedging against labor income risk, which induces households to hold assets whose return decreases with domestic labor income to avoid the simultaneous shortfall in financial and wage income (Baxter and Jermann (1997)). The third term represents households' hedging of real exchange rate (RER) risk which motivates them to hold assets whose return increases with the domestic price level in order to sustain purchasing power (Kollmann (2006)).

Heathcote and Perri (2013) and Coeurdacier (2009) focus on the two hedging components in 45 respectively when studying the effects of international trade on asset home bias. Specifically, Heathcote and Perri (2013) assume log-utility ($\gamma = 1$) in their baseline case where the hedging of RER risk is turned off. For the hedging against the labor income risk, they argue that lower trade openness will alleviate the terms-of-trade (TOT) depreciation that decreases labor income in response to a positive productivity shock at home, which simultaneously reduces domestic dividends given higher capital investment.[15] Therefore, domestic labor and financial income become more negatively correlated to generate stronger asset home bias. In contrast, Coeurdacier (2009) considers capital as the only production factor ($\mu = 1$) to shut down the labor income risk. For the hedging against RER risk, he finds that higher trade costs weaken asset home bias because domestic financial income increases less closely with RER: A positive domestic productivity shock raises domestic dividends as claims to capital income but triggers RER depreciations when domestic and foreign goods are substitutes.[16] The contradiction of these two papers' predictions is resolved when the elasticity of substitution between domestic and foreign goods is above 4 (which is parametric assumption adopted by Eaton and Kortum (2002)) — Both labor income risk and RER risk will tilt portfolios towards foreign assets in the presence of trade costs.[17]

[15]Dividends are defined as the difference between capital income (μY) and physical capital investment expenditure, while labor income is proportional to nominal output ($(1-\mu)Y$) in their framework. Moreover, they assume domestic goods account for a greater share in the Cobb-Douglas bundle. Under these assumptions, domestics dividends will negatively correlate with labor income to provide a hedge.

[16]The reasoning is that, when the elasticity of substitution between domestic and foreign goods is above unity, in response to a positive domestic productivity shock, the output rise dominates the TOT drop to raise domestic financial income. When the consumption bundle is skewed more towards domestic goods due to higher trade costs, the TOT drop translates to a greater RER depreciation, which strengthens the negative comovement between RER and domestic return that induces households to hold foreign assets for the hedging against RER risk.

[17]See the sensitivity analysis conducted by Heathcote and Perri (2013) which shows their model with physical capital also predicts asset foreign bias when the elasticity is sufficiently large. TOT adjustment is too small in this case that a positive domestic productivity shock will increase both financial and labor income at home. The positive comovement between the two types of income will hence tilt portfolios towards foreign assets. Skewness of expenditure towards domestic goods in their model is generated by households' preference, which can also be replicated by the assumption of trade costs.

This paper departs from these home bias papers along three dimensions. First, I acknowledge the existence of financial frictions and solve for portfolio choice in incomplete markets. International risk sharing is hence not complete and the first term in equation 45 is no longer a constant that reflects a country's relative size in the world economy. Instead, households need to consider second-moment variables in the financial market including asset covariances and market frictions when constructing the optimal portfolio for the purpose of risk sharing. Second, in this framework with I economies, there are $I-1$ foreign countries instead of one. The correlations between individual foreign countries' asset returns and domestic macro fundamentals in this setting are lower while the diversification benefits offered by a large group of countries with non-perfectly correlated shocks are higher. For these two reasons, risk sharing becomes relatively more important than risk hedging in determining financial flows in this multi-country framework with incomplete markets than in a two-country framework with complete markets. Third, this paper examines inter-regime changes where the steady state of the economy shifts under the joint forces of real and financial variables in response to policy changes. In this process, portfolio allocation reflects agents' risk-sharing and risk-hedging motives shaped by the global trade pattern, while countries' asset positions shift the world demand system in the commodity market.[18,19] This general equilibrium analysis differs from the home bias literature which typically studies portfolios taking the real side of the economy as given around a fixed steady state.

I conduct two sets of numerical exercises to highlight these new mechanisms separately. The first focuses on the original regime to evaluate potential drivers for asset positions in the data. In particular, I will disentangle risk-sharing and risk-hedging mechanisms in a two-country and in a 43-country setting respectively to underscore the departure of this paper from the home bias literature. The second adds inter-regime changes under counterfactual tariffs to examine the two-way interactions of trade and financial channels.

To start the analysis of financial flows observed in the data, I write the portfolio

[18]Global trade linkages affect portfolio choice by influencing second-moment variables, such as cross-country output covariances, and comovments of macro variables (labor income and RER) with financial income. These second-moment variables will enter the portfolio determination equations to influence countries' financial allocations. Meanwhile, financial allocations influence international trade because a country's asset position determines the size of its expenditure in the commodity market (equation 9).

[19]It is noteworthy that portfolio choice does not influence the level of real variables within a regime as a condition to use the Devereux and Sutherland (2011) method. The level of variables only shifts across regimes driven by the joint changes of wage, price, and portfolio choice (derived in original and counterfactual regimes separately) until the fixed point problem of $(\widehat{\widehat{w}}, \widehat{\widehat{P}}, \widehat{\widehat{D}})$ is solved.

determination equation (31) in the original regime $s = \{org\}$ for simplicity as

$$E_t[(\gamma(1-\beta)\widetilde{Y}\widetilde{R}' + (1-\gamma+\beta\gamma)\widetilde{P}\widetilde{R}' + \gamma(1-\beta)\check{\alpha}\widetilde{R}\widetilde{R}'] = \frac{1}{2}F \tag{46}$$

where F and $\check{\alpha}$ stand for the vectors of all the countries' financial frictions and asset holdings (defined by 26 and 30) relative to the numeraire country I:

$$F' = [F_{1I}, F_{2I}, ..., F_{I-1I}], \qquad \check{\alpha}' = [\check{\alpha}_1, \check{\alpha}_2, ..., \check{\alpha}_{I-1}] - \alpha_I.$$

$\widetilde{Y}\widetilde{R}'$, $\widetilde{P}\widetilde{R}'$, and $\widetilde{R}\widetilde{R}'$ represent the second-moment variables listed in 33 for brevity.

The following numerical exercises disentangle portfolios shaped by risk-sharing and by risk-hedging motives. First, to isolate the risk-sharing channel, I derive portfolios in complete markets where the portfolio equation satisfies the Backus and Smith (1993) condition

$$E_t[(\gamma\widetilde{C} + \widetilde{P})] = E_t[(\gamma(1-\beta)\widetilde{Y} + (1-\gamma+\beta\gamma)\widetilde{P} + \gamma(1-\beta)\check{\alpha}\widetilde{R})] = 0. \tag{47}$$

Second, to shut down the hedging of RER risk, I assume log-utility ($\gamma = 1$) under which assumption the portfolio equation becomes

$$E_t[((1-\beta)\widetilde{Y}\widetilde{R}' + \beta\widetilde{P}\widetilde{R}' + (1-\beta)\check{\alpha}\widetilde{R}\widetilde{R}'] = \frac{1}{2}F. \tag{48}$$

Lastly, labor income is excluded from households' wealth constraint (15) to turn off the hedging against labor income risk. The portfolio equation in this situation is

$$E_t[(\gamma(1-\beta)\mu\widetilde{Y}\widetilde{R}' + (1-\gamma+\beta\gamma)\widetilde{P}\widetilde{R}' + \gamma(1-\beta)\check{\alpha}\widetilde{R}\widetilde{R}'] = \frac{1}{2}F. \tag{49}$$

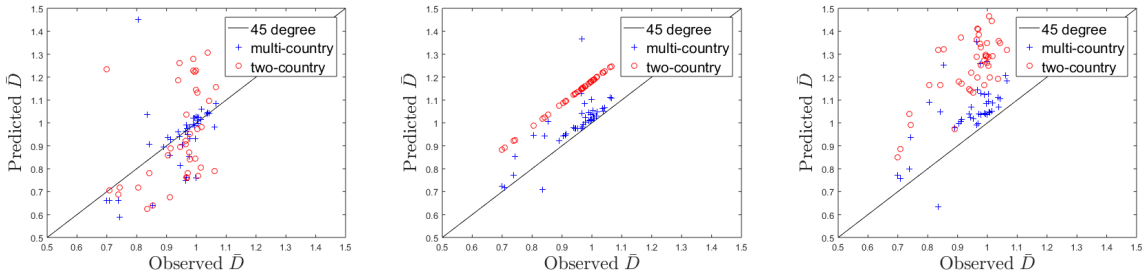
The shifts of asset positions characterized by equations 47-49 from those in the data reflect the potential impacts of these risk-sharing and -hedging factors on financial flows.

Table 1 and figure 1 report the numerical results for the size and composition of portfolios under different scenarios. When portfolios are derived under the assumption of complete markets (equation 47), the median holding of domestic assets drops from 0.427 to 0.373 in a multi-country setting and plummets to 0.088 in a two-country setting,[20] which contributes to the shrinkage of countries' aggregate asset positions to a

[20]The holding of domestic assets in complete markets with two countries is heavily influenced by the

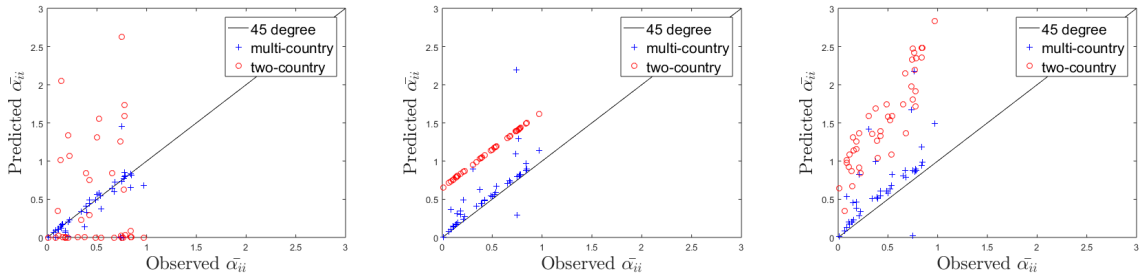
Figure 1: Disentangling Risk-sharing and Risk-hedging Channels

(I) Aggregate Asset Positions ($\bar{D}_i$)



(a) No risk sharing (b) No labor risk hedging (c) No RER risk hedging

(II) Domestic Asset Holdings ($\bar{\alpha}_{ii}$)



(d) No risk sharing (e) No labor risk hedging (f) No RER risk hedging

This figure examines countries' portfolio choice in three scenarios: 1) in complete markets with no additional need for risk sharing, 2) where there is no labor income risk hedging, and 3) where there is no RER risk hedging. Panel (I) reports the size of a country's portfolio ($\bar{D}_i$) and (II) reports its holding of domestic assets ($\bar{\alpha}_{ii}$) both as shares of GDP. Portfolios in the data are on the horizontal axis and model-predicted portfolios under counterfactual assumptions are on the vertical axis. Red circles and blue stars represent predictions from two- and forty-three-country settings respectively. See the complete results by country in table A.2.

Table 1: Median Asset Positions in Different Scenarios

	Data	Setting	No risk sharing	No labor risk hedging	No RER risk hedging
$\bar{\alpha}_{ii}$	0.427	multi-country	0.373	0.534	0.612
		two-country	0.088	1.077	1.395
$\bar{D}_i$	0.971	multi-country	0.965	1.010	1.048
		two-country	0.923	1.154	1.289

This table presents the median size of portfolios $\bar{D}_i$ and the median domestic asset holding $\bar{\alpha}_{ii}$ both as shares of GDP across countries in the sample in three scenarios: 1) in complete markets with no need for risk sharing, 2) where there is no labor income risk hedging, and 3) where there is no RER risk hedging. Results are reported for: 1) a multi-country case where there are 43 countries with bilateral trade and financial linkages, and 2) a two-country case with each of the countries in the sample treats itself as the domestic economy and all the other countries in the world as the aggregate foreign economy. See the complete results by country in table A.2.

median level of 0.965 and 0.923 as shares of GDP in the two cases respectively. The fact that counterfactual asset positions change little in a 43-country case suggests that the risk-sharing (diversification) benefits from holding assets of many economies with their non-perfectly correlated shocks almost replicate the complete market allocation. Such diversification benefits cannot be fully captured in a two-country setting, which explains the drastic portfolio reallocations from incomplete to complete markets.

In terms of risk hedging, when portfolio choice abstracts from labor income and RER risk considerations (equations 47 and 48 respectively), domestic asset holdings increase for almost all the countries. This result suggests that both labor income and RER risks tilt portfolios towards foreign assets, consistent with predictions from the home bias literature including Baxter and Jermann (1997), Coeurdacier (2009), and Heathcote and Perri (2013) under comparable parametric assumptions. Between the two risks, the magnitude of portfolio changes under the labor income risk is nearly identical across countries, which stems from the assumption of Cobb-Douglas production with fixed labor and capital endowments.[21] On the other hand, the RER risk generates more disparate patterns across countries, shaped by the sophisticated trade structure characterized in the Eaton and Kortum (2002) model. When comparing two- and multi-country predictions, we observe that the deviations of counterfactual from actual portfolios overall are substantially larger in a two-country than in a multi-country setting. This result implies that risk-hedging

distribution of country sizes. In a two-country symmetric case, the domestic share is $\frac{1}{2}$ in equation 45. Here country sizes are calibrated to the data where the home country is small as shares of world GDP, which contributes to the low shares of domestic asset holdings for most countries.

[21] Future extensions of the model can consider methods to reduce this strong comovement between labor and financial income, such as introducing endogenous labor supply as in Jermann (2002) and Matsumoto (2007) or endogenous capital accumulation as in Heathcote and Perri (2013).

mechanisms proposed in the home bias literature may play a less significant role in accounting for countries' portfolios given the multilateral trade and financial linkages in the world economy.

I proceed to establish the drivers for cross-country financial flows. Beyond countries' domestic holdings, the model also predicts bilateral asset positions across all the country pairs under the influences of second-moment variables shaped by the global trade pattern. To simplify the analysis, I rewrite the portfolio determination equation (46) as

$$E_t[(\gamma(1-\beta)\widetilde{R}\widetilde{R}'(1+\check{\alpha})+(1-\gamma+\beta\gamma)\widetilde{P}\widetilde{R}'] = \frac{1}{2}F \tag{50}$$

under the model assumption that financial income flows are proportional to output based on equation 10 such that $\widetilde{Y}\widetilde{R}' = \widetilde{Y}\widetilde{Y}' = \widetilde{R}\widetilde{R}'$.

In equation 50, $\widetilde{R}\widetilde{R}'$, $\widetilde{P}\widetilde{R}'$, F, and portfolio $\check{\alpha}$ are all $(I\text{-}1) \times (I\text{-}1)$ matrices. In particular, the matrix $\widetilde{R}\widetilde{R}'$ reflects cross-country asset covariances important for risk sharing in incomplete markets, while $\widetilde{P}\widetilde{R}'$ reflects the covariance of financial income with price fluctuations relevant for RER risk hedging. The elements in the i^{th} row j^{th} column of $\widetilde{R}\widetilde{R}'$ and $\widetilde{P}\widetilde{R}'$ capture the comovement of country i's financial income and price level respectively with country j's financial income under all the shocks in the economy. These elements will determine i's holding of j's assets affected also by the corresponding bilateral financial friction in the i^{th} row j^{th} column of the friction matrix F.

I use this structural model to explain the strong comovment between bilateral trade and financial ties observed in the data. When calculating bilateral trade and financial linkages measured as the mean value of bidirectional shares averaged over time for a pair of country:

$$\pi_{ij} = \frac{\bar{\pi}_{ij}+\bar{\pi}_{ji}}{2}, \qquad \alpha_{ij} = \frac{\bar{\alpha}_{ij}+\bar{\alpha}_{ji}}{2}, \qquad \forall i,j \in [1,I], \tag{51}$$

I find the correlation between trade (π_{ij}) and financial (α_{ij}) linkages to be as high as 0.835 in the sample of country pairs, which verifies the strong connection between the channels of globalization. There are two potential explanations for this empirical regularity based on the model.[22] First, trade linkages influence cross-country covariances of financial and macro variables, including $\widetilde{R}\widetilde{R}'$ and $\widetilde{P}\widetilde{R}'$, to the extent that bilateral trade flows predict financial flows. Second, cross-country financial and trade frictions are shaped by

[22]This analysis exploits the cross-sectional variation across country pairs to establish the interaction of trade and financial channels. In particular, I take the cross-country trade structure as given and investigate its implications of bilateral asset positions. In the following sections I will examine the joint dynamics of trade and financial flows under the impacts of global barriers including tariffs and financial frictions by adding the inter-regime analysis.

common factors such as geographic distance, cultural similarity, information accessibility, and regional economic integration, all of which are embedded in the friction matrix F.[23] I use the model to distinguish between these two explanations.

Table 2 reports the correlation coefficient between bilateral trade and financial linkages in different scenarios. By comparing the portfolio changes across counterfactual scenarios, risk sharing and bilateral frictions play a more significant role than risk hedging in contributing to the comvement of trade and financial channels. Under the assumption of complete markets, the correlation declines significantly from 0.835 to 0.587. This result suggests that cross-country financial flows are strongly directed by countries' need for risk sharing whose pattern is shaped by the global trade structure,[24] which explains the sharp drop in the comovement between trade and financial linkages under complete markets where portfolio choice is no longer necessary to ensure risk sharing. Moreover, in the case where all the elements in F are set as zero to turn off the variation in frictions, the correlation plummets to 0.524, which suggests the substantial impacts of common barriers facing both channels on their strong connection.

Table 2: Bilateral Financial and Trade Linkages

	Data	No risk sharing (1)	No labor risk hedging (2)	No RER hedging (3)	No bilateral friction (4)
Corr (α_{ij}, π_{ij})	0.835	0.587	0.798	0.696	0.524

This table presents the correlation coefficient between bilateral trade and financial linkages (defined in 51) across country pairs in the sample in three scenarios: 1) in complete markets with no need for risk sharing, 2) where there is no labor income risk hedging, 3) under log utility where there is no RER risk hedging, and 4) when the elements of the friction matrix F are set as zero.

To empirically validate the importance of risk sharing and bilateral frictions for portfolio choice, I test whether their measures in the data covary with the observed bilateral asset positions. To proxy risk sharing benefits, I consider cross-country productivity covariances (see Appendix C for estimation details). For bilateral frictions, I consider geographic distance, the mean value of a country-pair's Chinn-Ito index values to mea-

[23] As mentioned in footnote 7, cross-country barriers for financial flows in this model may take various forms including the factors mentioned here. These factors, besides through their potential inferences for $\widetilde{R}\widetilde{R}'$ and $\widetilde{P}\widetilde{R}'$, shape countries' portfolio choice with their impacts on the values of elements in F.

[24] Trade exerts influences on risk sharing through various channels. For example, trade matters for the degree of TOT adjustments in response to countries' idiosyncratic productivity shocks and hence affects the pattern of cross-country risk sharing, a mechanism widely examined in the international macro literature including Corsetti et al. (2008). I contribute to this literature by embedding the global trade structure in a multi-country DSGE framework, whose predictions for countries' TOT and RER variability differ from those in a standard two-country setup.

sure their capital account openness, and other gravity variables including dummies for contiguity, for regional trade agreements, for common official language, religion, and legal origin. All of these gravity variables are sourced from the CEPII dataset.

Table 3: Drivers for Bilateral Asset Positions

Dep. Var: log Bilateral Holdings	(1)	(2)	(3)	(4)
log(GDP_o)	1.245 ***	1.108 ***	1.085 ***	1.067 ***
	(0.034)	(0.061)	(0.062)	(0.061)
log(GDP_d)	1.442 ***	-0.012	0.042	0.048
	(0.032)	(0.093)	(0.094)	(0.090)
log(dist)	-0.709 ***	-1.167 ***	-1.202 ***	-1.099 ***
	(0.037)	(0.021)	(0.022)	(0.021)
Chinn-Ito			0.674 **	1.412 ***
			(0.298)	(0.298)
$cov(T)$				2.498 ***
				(0.224)
Chinn$\times cov(T)$				-2.093 ***
				(0.249)
Fixed Effects	N	Y	Y	Y
Gravity Var	N	Y	Y	Y
Observations	22,448	22,448	20,807	20,807
R^2	0.123	0.957	0.959	0.960

Robust standard errors in parentheses.***significant at 1% and ** significant at 5%. The dependent variable is bilateral asset holdings in logs from Factset/Lionshare. GDP_o and GDP_d are the GDPs of the investment origin (holders') country and of the destination (assets') country, and log(dist) is CEPII's log population-weighted distance between countries. Chinn-Ito is an index to measure capital account openness here averaged over the country pair. $cov(T)$ is the estimated covariance of productivity. Fixed Effects include those for the origin, destination, and time. Gravity variables include CEPII's dummy variables for contiguity, for regional trade agreements, for common official language, religion, and legal origin.

Table 3 reports the empirical results of the drivers for countries' asset positions. Column (1) shows that bilateral holdings increase in the GDPs of the holders' (origin) and assets' (destination) countries and decrease in geographic distance, consistent with the gravity model of international finance documented by Portes and Rey (2005). Column (2), by adding origin/destination/time fixed effects and the gravity variables, picks up most of the variability of asset positions in the data implied by the high R^2 value. Moreover, column (3) further improves the prediction and shows that capital account openness facilitates financial investment. Controlling for this institutional feature, stronger productivity comovement reduces bilateral asset positions, as suggested by the negative coefficient for the interaction term of productivity covariance with the Chinn-Ito index. These empirical findings support the theoretical predictions that bilateral barriers and risk sharing motives influence countries' portfolio choice.

So far I have performed intra-regime analysis to explain countries' asset positions in the data. The quantitative exercises from this section evaluate the implications of multilateral trade structure for portfolio choice. In the next step, I add inter-regime analysis under policy changes to examine the joint dynamics of trade and financial channels.

3.2 Interaction of Trade and Finance under Tariffs

This section conducts policy experiments with varying trade costs to predict the general equilibrium changes of trade and finance both within and across regimes. In these experiments I assume that bilateral tariffs among all the country pairs uniformly increase by

$$\hat{\tau} = \frac{\tau_{ij}^{ctf}}{\tau_{ij}^{org}}, \qquad \forall i \neq j \in [1, I], \tag{52}$$

and examine inter-regime changes of variables denoted with a Δ. For any variable A,

$$\Delta A = A^{ctf} - A^{org}. \tag{53}$$

To derive countries' asset positions under the tariff changes, I examine the portfolio determination equation (50) in both original and counterfactual regimes

$$E_t[(\gamma(1-\beta)\widetilde{R}\widetilde{R}'^{s}(1+\check{\alpha}^{s}) + (1-\gamma+\beta\gamma)\widetilde{P}\widetilde{R}'^{s}] = \frac{1}{2}F^{s},\ s \in \{org, ctf\} \tag{54}$$

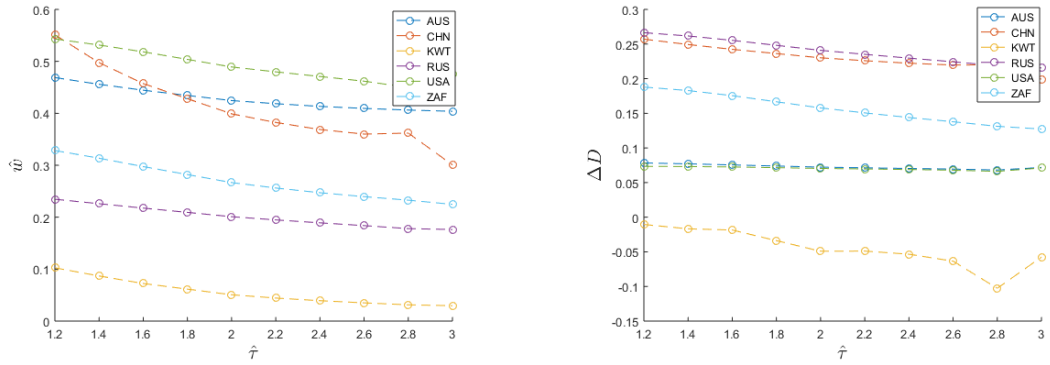
and take their difference to find the inter-regime portfolio changes after computing the second-moment variables from intra-regime analyses for both regimes [25]

$$E_t[(\gamma(1-\beta)\Delta(\widetilde{R}\widetilde{R}'(1+\check{\alpha})) + (1-\gamma+\beta\gamma)\Delta\widetilde{P}\widetilde{R}'] = \frac{1}{2}\widetilde{F}F^{org}. \tag{55}$$

Figure 2 presents the numerical results for a group of selected countries across continents. Under higher tariffs, wage across the selected countries declines in general but at different rates. Not surprisingly, China as a major exporter has wage losses most sensitive to tariff changes. In terms of finance, all the selected countries except Kuwait document an increase in asset positions $\Delta\bar{D} > 0$ by around 5% to 25% as shares of GDP under tariff changes, which suggests that these economies hold more assets to finance expendi-

[25] See Appendix B.3 for the detailed algorithm. $\widetilde{R}\widetilde{R}'^{s}$ and $\widetilde{P}\widetilde{R}'^{s}(s \in \{org, ctf\})$ are both computed with the hat algebra technique. Based on the finding from the previous section that $\widetilde{P}\widetilde{R}'^{s}$ for risk hedging is not as important in predicting financial flows in a multi-country incomplete-market setting, I will focus more on $\widetilde{R}\widetilde{R}'^{s}$ for risk sharing in the following discussion of results.

Figure 2: Wage and asset position changes under different tariffs



(a) $\hat{\tau}$ and $\widehat{\bar{w}}$ (b) $\hat{\tau}$ and $\Delta\bar{D}$

This figure plots the counterfactual wage $\widehat{\bar{w}}$ and as ratios of its original value and the change in asset position $\Delta\bar{D}$ as shares of output when the bilateral tariffs among all the country pairs formed by 43 countries and ROW are $\hat{\tau}$ times their initial values.

ture. Meanwhile, asset positions are not as responsive as wage to tariffs reflected in the smaller slope of the curves in (b), especially for Australia and the US. To diagnose the pattern, I compare countries' changes in asset positions ($\Delta\bar{D}$) and in their median asset covariance with others ($\Delta\widetilde{R}\widetilde{R}'$) in table 4. From the table, $\Delta\widetilde{R}\widetilde{R}'$ of Australia and the US is relatively stable which contributes to the flatness of their financial curves under tariff changes. Moreover, Kuwait is the only country that witnesses declines in asset holdings, which can be attributed to the sharp increase in the value of its $\Delta\widetilde{R}\widetilde{R}'$. Based on the earlier discussion, the increase implies fewer risk-sharing benefits offered by international assets, which induces the country to shrink holdings.

This analysis of how financial investment is affected by cross-country correlations echoes related works in international finance including Coeurdacier and Guibaud (2011) and Bergin and Pyun (2016). This paper contributes to this literature by proposing a multi-country structural framework useful to quantify the impacts of cross-country frictions on both portfolio choice and the underlying covariance structure.

Next I combine trade and financial channels to conduct welfare analysis. The trade literature including Dekle et al. (2007) measures welfare, with the cost and size of expenditure considered, as the wage-to-price ratio multiplied by the asset position:

$$\widehat{\mathbb{W}}_i = \frac{\widehat{\bar{w}}_i}{\widehat{\bar{P}}_i}\widehat{\bar{D}}. \tag{56}$$

Table 4: Asset covariance and position changes under different tariffs

Country	Variable	$\hat{\tau}=1.2$	$\hat{\tau}=2$	$\hat{\tau}=3$
Australia	$\Delta\widetilde{R}\widetilde{R}'$	-0.211	-0.199	-0.189
	$\Delta\bar{D}$	0.078	0.072	0.072
China	$\Delta\widetilde{R}\widetilde{R}'$	0.157	0.174	0.185
	$\Delta\bar{D}$	0.257	0.230	0.199
Kuwait	$\Delta\widetilde{R}\widetilde{R}'$	1.821	2.006	2.245
	$\Delta\bar{D}$	-0.011	-0.049	-0.058
Russia	$\Delta\widetilde{R}\widetilde{R}'$	-0.237	-0.206	-0.192
	$\Delta\bar{D}$	0.266	0.241	0.216
US	$\Delta\widetilde{R}\widetilde{R}'$	-0.508	-0.541	-0.559
	$\Delta\bar{D}$	0.073	0.070	0.071
South Africa	$\Delta\widetilde{R}\widetilde{R}'$	0.034	0.054	0.094
	$\Delta\bar{D}$	0.188	0.158	0.127

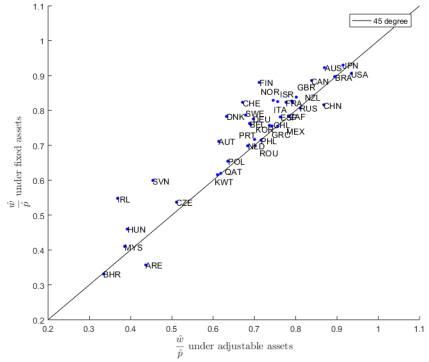
This table reports the changes in the median covariance of asset returns $\Delta\widetilde{R}\widetilde{R}'$ across pairs a country makes and in its asset position $\Delta\bar{D}$ under different tariff changes $\hat{\tau}$ relative to the original scenario.

The two components in this definition stand for the changes on the real and financial side of the economy respectively. Using this decomposition, I compare the welfare derived under fixed asset positions ($\widehat{\bar{D}}=1$) and under adjustable positions determined by households' optimal investment decisions under the tariff changes. The comparison quantifies the departure of this new approach, which endogenizes financial allocation, from the trade literature that treats countries' asset positions as exogenous and fixed.

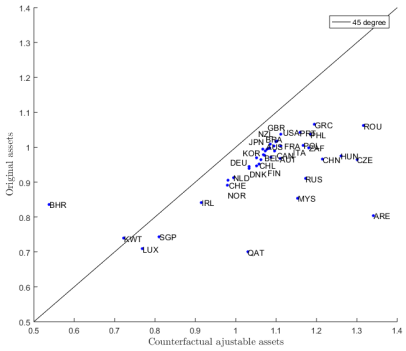
Figure 3a compares the counterfactual wage-to-price ratio under fixed and adjustable asset positions. Most countries' ratio is lower in either case ($\frac{\widehat{w}}{\widehat{\bar{P}}}<1$) given a tariff change $\hat{\tau}=1.2$ relative to in the original situation without tariffs. This is because higher tariffs prohibit cross-country commodity flows and raise the price paid by households more than their wage. Between the two cases, the wage-to-price ratio decreases by a greater magnitude under adjustable than under fixed asset positions as most countries lie above the 45 degree line in the diagram. This result suggests that households witness greater deteriorations to the purchasing power of their labor income when they can adjust their financial holdings in response to tariffs. Nonetheless, the financial holdings partially offset the impact of labor income loss on households' welfare, as figure 3b shows that most countries increase their holdings to raise expected lifetime utility.[26] This happens

[26]It is worth noting that figures 3a and 3b should be examined simultaneously because they are both derived from the model solution to a joint fixed-point problem of $(\widehat{w},\widehat{\bar{P}},\widehat{\bar{D}})$ under the normalization

Figure 3: Comparison of counterfactual welfare under fixed and adjustable asset positions

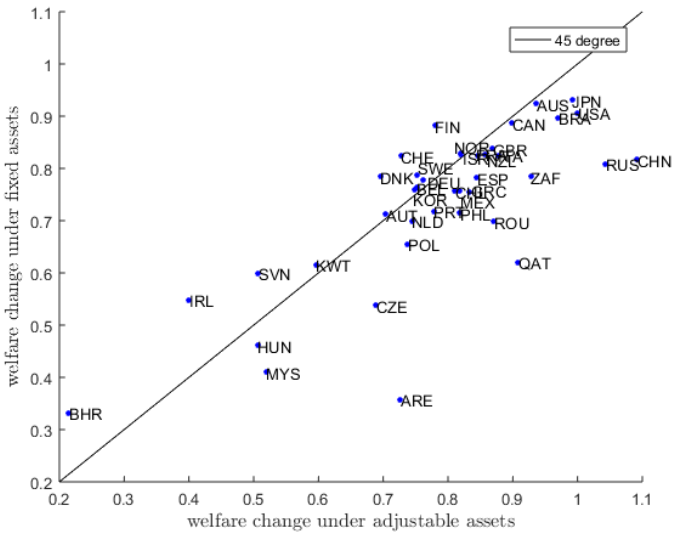


(a) $\frac{\hat{\widehat{w}}}{\hat{\widehat{P}}}$



(b) $\bar{D}$

=



(c) $\widehat{\mathbb{W}}$

This figure plots the counterfactual changes to wage-to-price ratio $\frac{\hat{\widehat{w}}}{\hat{\widehat{P}}}$ in (a), counterfactual equilibrium asset positions as shares of output $\bar{D}^{ctf}$ in (b), and counterfactual welfare changes $\widehat{\mathbb{W}}$ in (c) when bilateral tariffs in the world economy uniformly increase by 20%: $\hat{\tau} = 1.2$. Variables on the horizontal axis represent the case where asset positions are adjustable in response to tariffs and the variables on the vertical axis represent the case where asset positions are fixed.

since assets provide more risk-sharing benefits (reflected as $\Delta\widetilde{R}\widetilde{R}' < 0$) when the trade channel, which facilitates international output synchronization that generates a higher $\widetilde{R}\widetilde{R}'$, faces greater barriers. Consequently, most countries increase asset positions, in which process they resort to finance for risk sharing when trade relations deteriorate.

Figure 3c illustrates the welfare change which combines the wage-to-price ratio in figure 3a and the asset position in figure 3b. If the original welfare without tariff increase is normalized to 1, the median counterfactual welfare across countries is 0.782 under fixed and 0.818 under adjustable asset positions. Except for a few countries such as Finland and Bahrain, most countries have higher welfare when financial holdings can be adjusted. Hence, the welfare loss of these countries would be overestimated under the assumption that their asset positions were fixed given the tariff changes. For example, Qatar's welfare loss would be doubled if the country's financial adjustments were not considered.

The discrepancy between the two cases occurs because a standard trade model excludes finance, which is a major means for risk-averse agents to achieve consumption smoothing for utility maximization. When the world economy faces higher tariffs which hinder output synchronization, agents endogenously increase asset holdings under intertemporal decisions to take advantage of the risk-sharing benefits given lower cross-country covariances in the financial market. Ignoring this margin of adjustment may miscalculate welfare. Hence, this comparison between fixed and flexible asset positions underscores the importance of incorporating a financial channel in a trade model. Meanwhile, the feedback of countries' asset positions on real variables is missing in most home bias papers where portfolio is typically solved taking the the real side of the economy including trade as exogenous and fixed. Therefore, this paper also contributes to the international macro literature by capturing the two-way interactions of trade and finance.

4 Event Studies

This section conducts two event analyses as applications of the new approach. The first examines a scenario where cross-country financial frictions universally increase as in the 2008 global financial crisis. The second studies the implications of the 2018 China-US trade war under the calibrated tariff changes.

condition that treats the world output as a numeraire $\sum_{i=1}^{I} \widehat{w}\bar{Y}_i^{org} = 1$ and the world resource constraint $\sum_{i=1}^{I} \widehat{w}\bar{Y}_i^{org} = \sum_{i=1}^{I} \widehat{w}\bar{Y}_i^{org}\bar{D}_i^{ctf}$. Under these two conditions, countries' increased borrowing is sustainable since their output and wage income decline concurrently under tariffs. Countries' new asset positions reflect households' optimal utility maximization decisions given the tariff changes.

4.1 Tightening of Global Financial Markets

To examine how portfolio choice reacts to international financial frictions, I impose a uniform increase of bilateral frictions among all the country pairs by

$$\widetilde{F} = \frac{F^{ctf} - F^{org}}{F^{org}} \tag{57}$$

to replicate a scenario where countries face higher costs in foreign investment as in the global financial crisis. The changes in bilateral frictions are in relative terms to the frictions faced by the numeriare country I which is assumed to be ROW, as the element in the i^{th} row j^{th} column of F^s in regime $s \in \{org, ctf\}$

$$F^s(i,j) = (f_{iI} - f_{ij}) - (f_{II} - f_{Ij}). \tag{58}$$

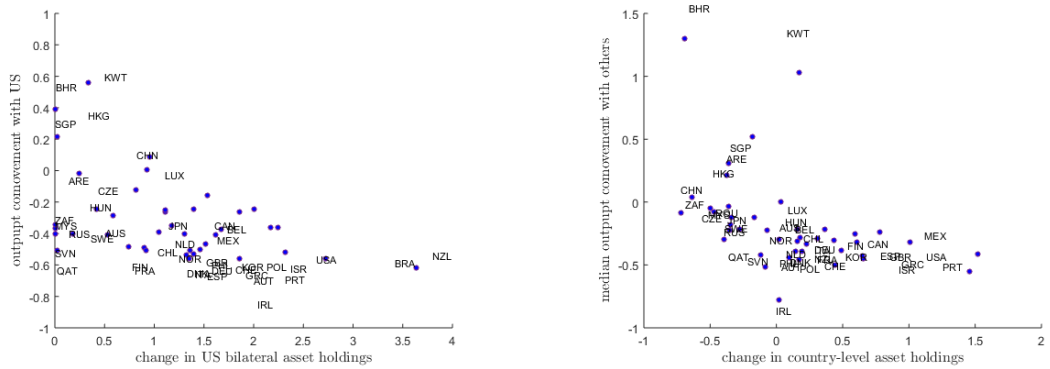
Everything else equal, an increase in bilateral friction f_{ij}, which stands for country i's friction when holding j's asset, will generate a negative $\widetilde{F}(i,j)$. The counterfactual exercise assumes $\widetilde{F}(i,j)$ = -0.2 for $\forall i \neq j \in [1, I]$, which represents a universal 20% increase of bilateral financial frictions relative to the numeraire country. Incorporating this increase in the cross-regime difference of the portfolio equations yields

$$E_t[(\gamma(1-\beta)\Delta(\widetilde{R}\widetilde{R}'(1+\check{\alpha})) + (1-\gamma+\beta\gamma)\Delta\widetilde{P}\widetilde{R}'] = \frac{1}{2}\widetilde{F}F^{org}. \tag{59}$$

From this equation, an increase in financial frictions will generate a decrease in the corresponding asset holdings $\check{\alpha}$. Nonetheless, the magnitude of the decrease also depends on $\Delta\widetilde{R}\widetilde{R}'$ whose values simultaneously shift across policy regimes under the influence of the frictions to restore equation 59. Specifically, asset holdings will drop less if $\Delta\widetilde{R}\widetilde{R}' < 0$. To understand the intuition, the term $\widetilde{R}\widetilde{R}'$ is the asset covariance matrix and therefore captures the comovement across countries' financial income. When its elements decrease, financial assets provide more risk-sharing benefits since their returns covary less across countries. Therefore, by constructing diversified portfolios of different countries' assets, households have more stable financial income less subject to country-specific risks for consumption smoothing. This creates greater incentives for households to hold financial assets despite higher financial frictions.

Figure 4 presents the changes in portfolios and in cross-country asset covariances under counterfactual financial frictions. (a) plots the US bilateral holdings of all the countries' assets ($\hat{\alpha}_{ij} = \frac{\alpha_{ij}^{ctf}}{\alpha_{ij}^{org}}$) and asset covariances with those countries. When faced with higher

Figure 4: Portfolio and cross-country covariance changes under financial frictions



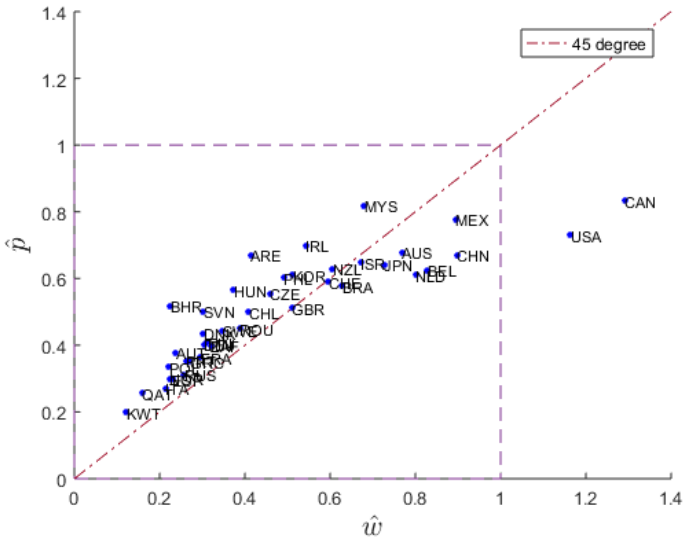
(a) US bilateral holding $\hat{\alpha}$ and $\Delta\widetilde{R}_{US}\widetilde{R}'$ (b) Country-level holding $\Delta\bar{D}$ and $\Delta\widetilde{R}\widetilde{R}'$

This figure plots the counterfactual changes in portfolio and in cross-country covariance matrix $\Delta\widetilde{R}\widetilde{R}'$ under changes in financial frictions $\tilde{F}$ = -0.2, $\forall i \neq j \in [1, I]$. (a) plots the US bilateral holdings of other countries' assets ($\hat{\alpha} = \frac{\alpha^{ctf}}{\alpha^{org}}$) and its asset return covariances with others. (b) plots all the countries' aggregate asset positions $\Delta\bar{D}$ and their median covariances with others.

financial frictions with foreign countries, the US holding of domestic assets more than doubles: $\hat{\alpha}_{US,US} > 2$, but the increase is not as large as its holding of Brazil's and New Zealand's assets with which the return covariances sharply decrease. In contrast, the US cuts holdings of assets from countries with which covariances increase significantly including Kuwait and Bahrain. The overall negative correlation between $\hat{\alpha}$ and $\Delta\widetilde{R}\widetilde{R}'$ verifies the risk sharing mechanism discussed earlier. Besides the composition of portfolio at the bilateral level, I also examine the size of portfolio which adds up bilateral positions to the country level. Figure (b) shows countries' aggregate asset holdings and their median asset covariance across partners. Overall there is a negative correlation between the changes in the two variables: Countries with a higher median asset covariance increase with others are expected to reduce financial holdings by a greater magnitude under counterfactual financial frictions. On the contrary, countries which yield more risk-sharing benefits, reflected by a lower $\Delta\widetilde{R}\widetilde{R}'$, are less likely to curtail asset positions in response to the increased financial frictions.

Besides financial variables, real variables shift across policy regimes under counterfactual financial frictions, because inter-regime changes are characterized as the simultaneous changes to the real and financial sides of the economy in the general equilibrium. Figure 5 plots cross-country wage ($\widehat{\bar{w}}$) and price ($\widehat{\bar{P}}$) changes under counterfactual financial frictions. Affected by the frictions, most countries in the diagram experience lower wage and price compared to in the original regime ($\widehat{\bar{w}}, \widehat{\bar{P}} < 1$). Between the two variables,

Figure 5: Wage and price changes under financial frictions



This figure plots the counterfactual changes in wage $\widehat{\tilde{w}}$ and price $\widehat{\tilde{P}}$ under changes in financial frictions $\tilde{F}$ = -0.2, $\forall i \neq j \in [1, I]$.

many countries' loss in wage is larger than that in price ($\widehat{\tilde{w}} < \widehat{\tilde{P}}$) as they lie above the 45 degree line. A few exceptions include Canada and US which witness wage increases. Several potential factors can contribute to this result, including the fact that these two countries' financial holdings which can be used to finance expenditure rise remarkably, which drives up domestic demand for goods that consequently raises labor compensation. This magnitude of the changes is jointly shaped by all their bilateral linkages with other countries through trade and financial channels. Except for these outliers, most countries are predicted to have lower wage under higher financial frictions.

4.2 China-US Trade War

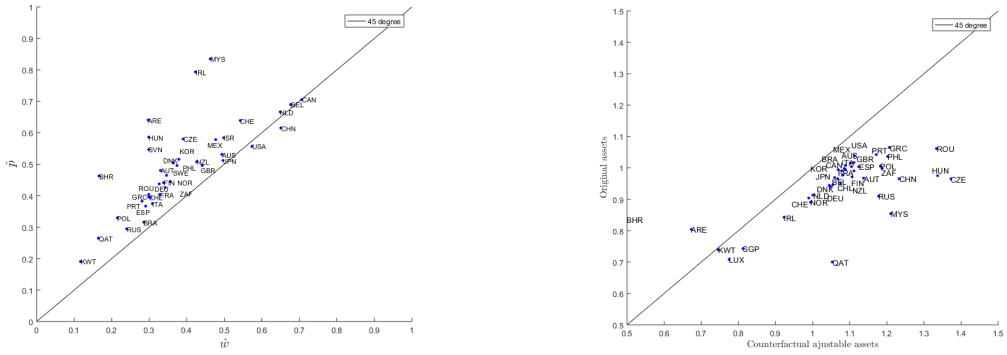
This section examines the recent China-US economic conflict as a real-world application of the model. In 2018, the Trump administration raised tariffs and other trade barriers on China, which took retaliatory actions in response. Besides reciprocal tariff increases, selling off US assets was considered a possible means for China to strike back.[27] I conduct analyses with both trade and financial measures based on the structural model

[27]Media coverage on this possibility can be found in New York Times, Forbes, Politico, among many others. Nearly all these media agree that although the possibility is not high, the consequences can be catastrophic for both countries once China weaponizes its portfolio ownership including over 1 trillion dollars US government debt.

to quantify the welfare implications of the trade war.

I obtain the bilateral tariff changes during the trade war from Li et al. (2020), who merge industry-level tariff changes from government announcements with sectoral trade data from the UN Comtrade, to calculate the trade-weighted cumulative tariff increases by March 2020. Based on their estimates, I set the country-level bilateral tariff changes as $\widehat{\tau}_{US,CN} = 1.189, \widehat{\tau}_{CN,US} = 1.186$ respectively. In the financial channel, besides the situation where all the countries including China adjust portfolios based on risk-sharing incentives shaped by the new global trade pattern under the tariffs, I also consider the extreme scenario where China sells all its US asset holdings and reconstructs the portfolio among the remaining investment destinations.

Figure 6: Changes in real and financial variables during the trade war



(a) $\widehat{\bar{w}}$ and $\widehat{\bar{P}}$ (b) $\bar{D}$

This figure plots the counterfactual changes in wage $\widehat{\bar{w}}$ and price $\widehat{\bar{P}}$ in (a), counterfactual equilibrium asset positions as shares of output $\bar{D}^{ctf}$ and original positions $\bar{D}^{org}$ in (b), under the calibrated tariffs during the trade war without restrictions on portfolio reallocation.

Figure 6 plots the model-predicted real and financial variables of the global economy during the trade war. Although the tariff changes in this analysis only occur between China and the US, all the countries besides the two major economies are affected. 6a shows that most countries witness lower wage and price due to the tariff increases as $\widehat{\bar{P}}, \widehat{\bar{w}} < 1$. Furthermore, the majority of the countries suffer loss in real wage as $\widehat{\bar{w}} < \widehat{\bar{P}}$. The median value of $\frac{\widehat{\bar{w}}}{\widehat{\bar{P}}}$ in the sample is 0.8, representing the mean value of Spain and Norway. The magnitude of wage changes is overall larger than what is predicted from the existing literature for two major reasons.[28] First, this model assumes a single tradable

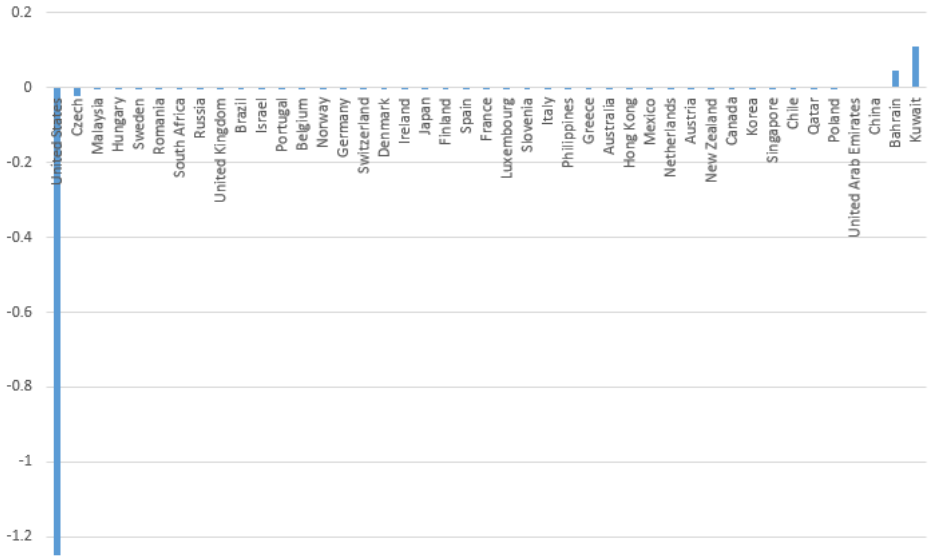
[28]For example, Caliendo and Parro (2021) estimate with a multi-sector multi-region quantitative spatial model that U.S. aggregate real wages decline by 0.16 percent due to the trade war. Li et al. (2020) build a computational general equilibrium (CGE) model to predict that China's welfare falls by

sector, while many trade models consider both nontradable and tradable sectors with global input-output linkages. Cross-country cross-sector substitution in a sophisticated quantitative trade framework can potentially reduce the impacts of industry-specific tariffs and leave wage and price less affected. Second and more importantly, this paper considers the endogenous changes to countries' asset positions while most existing literature ignores this margin of adjustment. Figure 6b shows that most countries increase their asset holdings in response to the tariff changes if cross-country financial frictions remain the same. The earlier explanation for figure 3b also applies here: cross-country asset covariances decrease when tariffs increase to impair output synchronization. Hence, countries have stronger incentives to hold each others' assets for international risk sharing. This financial allocation requires that wages be adjusted to a lower level to satisfy the world resource constraint, which magnifies the adjustment of real variables compared to in a standard trade model. When combining the changes in real wage and in asset positions, I find 32 out of the 43 countries in the sample suffer welfare losses, with a median welfare change $\widehat{\mathbb{W}} = 0.895$.

I proceed to discuss the possibility that China gives up its holding of US assets as means of financial retaliation. To conduct this counterfactual analysis, I set $\alpha_{CN,US} = 0$ and solve for China's portfolio choice among the remaining countries' assets under their covariance structure. Figure 7 plots the difference in the changes of China's asset holdings between the cases with and without financial retaliation. If China had to reconstruct its portfolio, the model predicts that it would replace US assets with either domestic assets or the assets of oil exporters including UAE, Bahrain, and Kuwait, while keeping most of the other asset positions slightly lower. Compared to the case with no financial retaliation, China's aggregate asset holding $\bar{D}^{ctf}$ would drop more by 0.16% as shares of GDP and its wage-to-price ratio would drop by 1.14%. Therefore, based on the welfare definition from equation 56, China's welfare loss during the trade war would be exacerbated by 1.3% if the country retaliated in the financial channel. Nevertheless, this estimate does not consider complications beyond the scope of this model, including the influence of the asset sale on Chinese exchange rate policy or on the US monetary policy (in particular interest rate). Understanding such policy-relevant questions requires incorporating additional modeling ingredients and techniques from the open economy macro literature, which will be interesting topics to explore for future research.

1.7%. Other countries have disparate but small changes to real wage.

Figure 7: Changes in China's Asset Holdings by Destination



This figure plots the difference in the changes to China's holdings of assets between the case with financial retaliation (denoted as ctf_1 below) where $\alpha_{CN,US} = 0$ and the case without relation (ctf_2), both relative to the situation where there is no trade war (org). The values presented in the figure are calculated as $\Delta\alpha_{CN,i} = \frac{\alpha^{ctf_1}_{CN,i}}{\alpha^{org}_{CN,i}} - \frac{\alpha^{ctf_2}_{CN,i}}{\alpha^{org}_{CN,i}}, \forall i \in [1, I]$

5 Conclusion

This paper combines the portfolio choice solution method and the hat algebra technique to solve a unified multi-country model where financial and trade channels influence each other. This new approach can readily be applied to a wide range of topics in both international macroeconomics and trade literatures, meanwhile it has many potentials for extensions. I hereby discuss two directions for future work.

First, this paper focuses on comparative statics across steady states under specific policy regimes, without tracing the dynamic path of the world economy between steady states. Although the portfolio choice problem considers agents' intertemporal investment decisions, the derived equilibrium portfolio is static in nature. If future research questions involve time-series patterns of economic activities, solving dynamic portfolios requires extending the current method to higher-order approximations of the model. Devereux and Sutherland (2010) show that the first-order dynamics of portfolios is obtained by combining a third-order approximation of the portfolio determination equation with a second-order approximation to the rest of the model. On the real side of the economy, either 'dynamic exact hat algebra' techniques from the quantitative trade literature

(Caliendo et al. (2019) and Kleinman et al. (2021) among others) or perturbation methods from the extensive DSGE literature (see Fernández-Villaverde et al. (2016) for a comprehensive survey) can be applied, depending on the specific context of interest. Such dynamic analyses characterize the world economy's pattern and speed of convergence towards a steady state, which is important in quantifying both persistent and transitory economic outcomes when equilibrium is being restored.

Second, this paper uses the local perturbation method to derive portfolio choice around a deterministic steady state. The method offers a powerful yet tractable toolkit widely applicable to DSGE models. Furthermore, the derived solution is extremely close to the exact around the point of approximation, but is less accurate when there are large deviations from the steady state or when the problem exhibits strong non-linearity (see a detailed discussion in Coeurdacier and Rey (2013)). Therefore, if global solution methods (such as policy or value function iterations) for the portfolio choice problem with comparable applicability and tractability become available in the future, financial investment can be endogenously determined in more general economic environments. Important questions including those related to sovereign defaults and disaster risks can be answered with the development of such solution techniques.

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Appendices

A Tables

Table A.1: List of Sample Countries

Name	Code	Name	Code	Name	Code	Name	Code
Australia	AUS	France	FRA	Luxembourg	LUX	Russia	RUS
Austria	AUT	Germany	DEU	Malaysia	MYS	Singapore	SGP
Bahrain	BHR	Greece	GRC	Mexico	MEX	Slovenia	SVN
Belgium	BEL	Hong Kong	HKG	Netherlands	NLD	Spain	ESP
Brazil	BRA	Hungary	HUN	New Zealand	NZL	Sweden	SWE
Canada	CAN	Ireland	IRL	Norway	NOR	Switzerland	CHE
Chile	CHL	Israel	ISR	Philippines	PHL	U.A.E.	ARE
China	CHN	Italy	ITA	Poland	POL	United Kingdom	GBR
Czech	CZE	Japan	JPN	Portugal	PRT	United States	USA
Denmark	DNK	Korea	KOR	Qatar	QAT	South Africa	ZAF
Finland	FIN	Kuwait	KWT	Romania	ROU		

B Details of the Model

B.1 Model Equations

This section derives the equations for hat algebra and portfolio analysis based on the model. Unless otherwise noted, a variable marked with a hat denotes the ratio of its value relative to the original steady state which is marked with a bar:

$$\widehat{A} = \frac{A}{\bar{A}}. \tag{B.1}$$

I do not distinguish between inter-regime ($\widehat{\bar{A}}$) or intra-regime ($\widehat{A}^s, s \in \{org, ctf\}$) changes because most equations in this section can be used to characterize either case with minor modifications. Moreover, I omit the time subscripts for brevity because these equations that contain hat variables characterize intra-temporal allocations.

On the production side, factor prices are determined by firms' profit maximization

$$\frac{w_i L_i}{r_i K_i} = \frac{1-\mu}{\mu}. \tag{B.2}$$

Therefore, in this economy with labor and capital as endowments, the change in capital rental fee equals that in wage:

$$\widehat{r}_i = \widehat{w}_i. \tag{B.3}$$

Table A.2: Disentangling Risk-sharing and Risk-hedging Channels

Country	Data		No Risk Sharing				No Labor Risk Hedging				No RER Risk Hedging			
			multi-co		two-co		multi-co		two-co		multi-co		two-co	
	α_{ii}	D_i	α_{ii}	D_i	α_{ii}	D_i	α_{ii}	D_i	α_{ii}	D_i	α_{ii}	D_i	α_{ii}	D_i
AUS	0.68	1.01	0.73	1.02	4.41	2.05	0.72	1.02	1.33	1.19	0.78	1.04	1.36	1.20
AUT	0.11	0.97	0.11	0.99	0.35	1.04	0.15	1.04	0.76	1.15	0.20	1.15	0.93	1.20
BHR	0.75	0.84	1.45	1.04	0.00	0.62	0.29	0.71	1.40	1.02	0.02	0.63	2.47	1.32
BEL	0.12	0.98	0.10	0.93	0.02	0.95	0.14	1.02	0.77	1.16	0.17	1.13	1.09	1.25
BRA	0.78	1.00	0.85	1.03	1.59	1.23	0.83	1.01	1.43	1.18	0.89	1.03	1.72	1.26
CAN	0.50	0.99	0.56	1.02	1.31	1.22	0.53	1.01	1.15	1.18	0.58	1.04	1.58	1.30
CHL	0.77	0.96	0.77	0.97	0.62	0.92	0.81	0.98	1.42	1.15	0.86	0.99	2.19	1.36
CHN	0.74	0.97	0.00	0.76	0.02	0.76	2.20	1.37	1.39	1.15	4.39	1.98	2.33	1.41
CZE	0.21	0.96	0.01	0.75	0.00	0.91	0.49	1.13	0.86	1.15	0.83	1.35	1.36	1.29
DNK	0.14	0.94	0.15	0.96	1.02	1.19	0.16	0.98	0.79	1.12	0.20	1.04	0.87	1.15
FIN	0.49	0.98	0.49	0.98	0.00	0.84	0.54	1.00	1.14	1.16	0.61	1.04	1.74	1.33
FRA	0.39	1.00	0.41	1.02	0.84	1.13	0.45	1.04	1.04	1.19	0.53	1.09	1.26	1.25
DEU	0.21	0.94	0.20	0.94	1.34	1.26	0.24	0.98	0.86	1.13	0.28	1.02	0.91	1.14
GRC	0.43	1.07	0.44	1.08	0.75	1.16	0.49	1.11	1.08	1.25	0.61	1.18	1.33	1.32
HKG	0.18	0.95	0.07	0.82	0.00	0.90	0.30	0.97	0.83	1.13	0.38	1.07	1.15	1.22
HUN	0.38	0.98	0.14	0.85	0.00	0.87	0.63	1.09	1.03	1.16	0.99	1.26	1.69	1.34
IRL	0.15	0.84	0.17	0.91	-0.06	0.78	0.31	0.94	0.80	1.02	0.46	1.05	1.31	1.17
ISR	0.75	0.99	0.73	0.99	2.62	1.52	0.80	1.01	1.40	1.18	0.88	1.04	1.81	1.29
ITA	0.22	0.99	0.23	0.99	1.07	1.23	0.27	1.04	0.87	1.17	0.34	1.12	0.85	1.17
JPN	0.54	1.00	0.37	0.93	0.00	0.84	0.67	1.04	1.19	1.18	0.82	1.08	1.59	1.29
KOR	0.79	0.97	0.81	0.98	0.03	0.76	0.82	0.98	1.44	1.15	0.88	1.00	2.35	1.41
KWT	0.18	0.74	0.09	0.66	0.00	0.69	0.34	0.77	0.83	0.92	0.47	0.80	1.25	1.04
LUX	0.01	0.71	0.01	0.66	0.00	0.71	0.01	0.72	0.66	0.89	0.01	0.76	0.64	0.89
MYS	0.76	0.85	0.00	0.64	0.00	0.64	1.30	1.00	1.41	1.04	2.17	1.25	2.42	1.32
MEX	0.84	1.02	0.83	1.01	0.09	0.81	0.88	1.03	1.49	1.20	0.94	1.05	2.36	1.44
NLD	0.09	0.91	0.09	0.93	0.00	0.89	0.11	0.95	0.74	1.10	0.13	1.01	1.01	1.17
NZL	0.52	0.99	0.58	1.02	1.55	1.28	0.57	1.01	1.17	1.17	0.63	1.04	1.53	1.27
NOR	0.06	0.89	0.07	0.89	3.08	1.74	0.07	0.92	0.71	1.07	0.09	0.98	0.35	0.97
PHL	0.54	1.04	0.54	1.04	6.03	2.58	0.59	1.06	1.19	1.22	0.68	1.11	1.09	1.19
POL	0.85	1.01	0.82	1.00	0.00	0.77	0.89	1.02	1.50	1.19	0.99	1.05	2.48	1.47
PRT	0.65	1.04	0.64	1.04	0.84	1.10	0.71	1.07	1.30	1.22	0.81	1.11	1.74	1.35
QAT	0.14	0.70	0.12	0.66	2.05	1.24	0.18	0.72	0.79	0.88	0.20	0.77	0.67	0.85
ROU	0.97	1.06	0.68	0.98	0.00	0.79	1.14	1.11	1.62	1.24	1.49	1.21	2.83	1.58
RUS	0.84	0.91	0.65	0.86	0.00	0.68	0.97	0.95	1.49	1.09	1.18	1.01	2.48	1.37
SGP	0.09	0.74	0.00	0.59	0.00	0.72	0.37	0.85	0.74	0.93	0.54	0.94	0.97	0.99
SVN	0.67	0.97	0.60	0.95	0.00	0.78	0.75	1.00	1.32	1.15	0.88	1.04	2.15	1.38
ESP	0.34	1.00	0.34	1.01	0.23	0.97	0.41	1.05	0.99	1.19	0.51	1.12	1.35	1.29
SWE	0.40	0.95	0.32	0.90	3.62	1.86	0.45	0.98	1.05	1.13	0.51	1.02	1.04	1.13
CHE	0.16	0.90	0.18	0.94	0.00	0.86	0.19	0.94	0.81	1.09	0.23	1.00	1.14	1.18
ARE	0.30	0.80	3.08	1.45	0.00	0.72	0.90	0.95	0.95	0.99	1.42	1.09	1.59	1.16
GBR	0.42	1.02	0.49	1.06	0.30	0.98	0.48	1.05	1.07	1.20	0.55	1.09	1.39	1.29
USA	0.78	1.04	0.80	1.05	1.74	1.31	0.82	1.05	1.43	1.22	0.88	1.07	1.91	1.36
ZAF	0.74	1.00	0.00	0.76	1.26	1.14	1.10	1.10	1.39	1.18	1.67	1.27	1.98	1.35

This table presents the model predictions for the size and domestic holding of portfolio choice in three scenarios: 1) in complete markets with no need for risk sharing, 2) where there is no labor income risk, and 3) where there is no RER risk. Results are reported for: 1) a multi-country case (labeled "multi-co") where there are 43 countries with bilateral trade and financial linkages, and 2) a two-country case (labeled "two-co") with each of the countries in the sample treats itself as the domestic economy and all the other countries in the world as the aggregate foreign economy. The median values and plots across countries are presented in table 1 and figure 1.

These factor prices are reflected in a country's total income

$$Y_i = w_i L_i + r_i K_i, \tag{B.4}$$

which, with the definition of hat variables (equation B.1) and B.3, is re-written as

$$Y_i = \widehat{w}_i \bar{w}_i L_i + \widehat{r}_i \bar{r}_i K_i = \widehat{w}_i (1-\mu)\bar{Y}_i + \widehat{w}_i \mu \bar{Y}_i. \tag{B.5}$$

Hence the change in income also equals that in wage

$$\widehat{Y}_i = \frac{Y_i}{\bar{Y}_i} = \widehat{w}_i. \tag{B.6}$$

Now I characterize the changes to wage ($\widehat{w}_i$) and price ($\widehat{P}_i$) which, with minor modifications, can be applied to the analysis both within (equations 38 and 39) and across (equations 43 and 44) regimes. With equations 6 and 7, price in country i is given by

$$P_i^{-\theta} = \Gamma^{-\theta} \sum_{j=1}^{I} T_j \tau_{ji}^{-\theta} (w_j^{\eta\mu} r_j^{\eta(1-\mu)} P_j^{1-\eta})^{-\theta}, \tag{B.7}$$

Dividing this by the steady-state price while imposing B.3 yields the change of price

$$\widehat{P}_i^{-\theta} = \sum_{j=1}^{I} \bar{\pi}_{ji} \widehat{T}_j \widehat{\tau}_{ji}^{-\theta} (\widehat{w}_j^{\eta} \widehat{P}_j^{1-\eta})^{-\theta}. \tag{B.8}$$

Meanwhile, wage $\widehat{w}_j$ in the equation is derived from the goods market clearing condition. Plugging equation 6 in 8 while using $X_i = Y_i \bar{D}_i$ gives

$$Y_i = \sum_{j=1}^{I} \frac{T_i [\tau_{ij} (r_i^{\mu} w_i^{1-\mu})^{\eta} P_i^{1-\eta}]^{-\theta}}{P_j^{-\theta} / \Gamma^{-\theta}} Y_j \bar{D}_j. \tag{B.9}$$

Combining equations B.3, B.6, and B.8 yields its change

$$\widehat{w}_i \bar{Y}_i = \sum_{j=1}^{I} \frac{\bar{\pi}_{ij} \widehat{T}_i \widehat{\tau}_{ij}^{-\theta} (\widehat{w}_i^{\eta} \widehat{P}_i^{1-\eta})^{-\theta}}{\sum_{k=1}^{I} \bar{\pi}_{kj} \widehat{T}_k \widehat{\tau}_{kj}^{-\theta} (\widehat{w}_k^{\eta} \widehat{P}_k^{1-\eta})^{-\theta}} \widehat{w}_j \bar{Y}_j \bar{D}_j. \tag{B.10}$$

In the financial channel, countries' income determines their financial return defined by equations 10 and 11. Around the steady state of the economy, it is sufficient to solve for the equilibrium portfolio by focusing on financial income flows (dividends).[29] Therefore, asset returns are proportional to income. It follows immediately that the changes of these

[29] Coeurdacier (2009) shows that the equilibrium portfolio derived from a static wealth constraint, where equity prices p^E drop out such that return $R_{i,t}$ is determined by dividends $d_{i,t}$, is identical to that in a dynamic model for two reasons. First, the first-order dynamics of non-portfolio equations in the dynamic model remain the same as in the static model. Second, the present value of the dynamic budget

variables are the same:

$$\widehat{R}_i = \widehat{d}_i = \widehat{Y}_i. \tag{B.11}$$

Lastly, change to welfare combines the wage-to-price ratio and asset position changes as in Dekle et al. (2007)

$$\widehat{\mathbb{W}}_i = \frac{\widehat{w}_i}{\widehat{P}_i}\widehat{\bar{D}}_i. \tag{B.12}$$

The derivation of equations for portfolio choice is modified from the two-country analysis by Devereux and Sutherland (2011). The first-order dynamics of the wealth constraints (equation 15) around a steady state in this model is

$$\widetilde{\mathcal{W}}_{i,t+1} = \frac{1}{\beta}\widetilde{\mathcal{W}}_{i,t} + \sum_{k=1}^{I-1}\frac{\alpha_{ik}}{\bar{Y}_i}(\widetilde{R}_{k,t+1} - \widetilde{R}_{I,t+1}) + \eta\widetilde{Y}_{i,t+1} - \eta\widetilde{P}_{i,t+1} - \eta\widetilde{C}_{i,t+1} + \mathcal{O}(\epsilon^2), \tag{B.13}$$

$$\widetilde{\mathcal{W}}_{I,t+1} = \frac{1}{\beta}\widetilde{\mathcal{W}}_{I,t} + \sum_{k=1}^{I-1}\frac{\alpha_{Ik}}{\bar{Y}_I}(\widetilde{R}_{k,t+1} - \widetilde{R}_{I,t+1}) + \eta\widetilde{Y}_{I,t+1} - \eta\widetilde{P}_{I,t+1} - \eta\widetilde{C}_{I,t+1} + \mathcal{O}(\epsilon^2), \tag{B.14}$$

where wealth is normalized by a country's output $\widetilde{\mathcal{W}}_{i,t} = (\mathcal{W}_{i,t} - \bar{\mathcal{W}}_i)/\bar{Y}_i$

Taking the cross-country difference between equations B.13 and B.14, iterating forward over the infinite time horizon to sum up the discounted consumption flows, and imposing a transversality condition to drop $\widetilde{\mathcal{W}}_{i/I,t+\infty}$ yields

$$\begin{aligned}\sum_{s=0}^{\infty}\beta^s(\eta\widetilde{C}_{i/I,t+1+s}) = \frac{1}{\beta}\widetilde{\mathcal{W}}_{i/I,t} + \sum_{s=0}^{\infty}\beta^s[\eta\widetilde{Y}_{i/I,t+1+s} - \eta\widetilde{P}_{i/I,t+1+s} \\ + \sum_{k=1}^{I-1}(\frac{\alpha_{ik}}{\bar{Y}_i} - \frac{\alpha_{Ik}}{\bar{Y}_I})(\widetilde{R}_{k,t+1+s} - \widetilde{R}_{I,t+1+s})] + \mathcal{O}(\epsilon^2).\end{aligned} \tag{B.15}$$

Under households' consumption smoothing characterized by the Euler equation (13), the expected consumption differential between the two countries at $t+1$ follows

$$\widetilde{C}_{i/I,t+1} = \frac{1-\beta}{\beta\eta}\widetilde{\mathcal{W}}_{i/I,t} + \frac{1-\beta}{1-\rho\beta}\widetilde{Y}_{i/I,t+1} - \frac{1-\beta}{1-\rho\beta}\widetilde{P}_{i/I,t+1} + \frac{1-\beta}{1-\rho\beta}(\check{\boldsymbol{\alpha}}_i - \check{\boldsymbol{\alpha}}_I)\widetilde{R}_{x,t+1} + \mathcal{O}(\epsilon^2), \tag{B.16}$$

where $\check{\alpha}$ denotes a country's vector of asset holdings scaled by its value-added output and discount factor

$$\check{\boldsymbol{\alpha}}_i = [\check{\alpha}_{i1}, \check{\alpha}_{i2}, ..., \check{\alpha}_{iI\text{-}1}], \qquad \text{where} \qquad \check{\alpha}_{ik} = \frac{\bar{\alpha}_{ik}}{\beta\eta\bar{Y}_i}. \tag{B.17}$$

If the baseline case assumes no productivity persistence ($\rho = 0$), then plugging the consumption differential in the second-order approximation of equation 25 yields the

constraint is satisfied up to a first-order if the static constraint holds. For this reason the financial income appears in their original expression of equity home bias (45) as dividends, whose covariances with labor income and real exchange rate decide portfolio choice.

portfolio determination equation:

$$E_t[(\gamma(1-\beta)\widetilde{Y}_{i/I,t+1}+(1-\gamma+\beta\gamma)\widetilde{P}_{i/I,t+1}+\gamma(1-\beta)(\check{\alpha}_i-\check{\alpha}_I)\widetilde{R}_{x,t+1})\widetilde{R}'_{x,t+1}] = \frac{1}{2}F_{iI}. \quad \text{(B.18)}$$

B.2 Existence and Uniqueness of Solution

Dekle et al. (2007) follow the theorems of Alvarez and Lucas (2007) to establish the existence and uniqueness of the model solution to a fixed-point problem of $(\widehat{w}, \widehat{\bar{P}})$ given counterfactual asset positions.[30] Many properties of their numerical solutions are maintained under the assumptions specified in this model, for example there exists a unique price vector $\widehat{P}^s$ within a regime given the corresponding wage vector $\widehat{w}^s$. After solving the portfolio choice problem by evaluating the second-moment variables as functions of $\widehat{w}^s$ in each regime $s \in \{org, ctf\}$, the resulting portfolio $\bar{D}^{ctf}$ is then used to characterize an excess demand system across regimes

$$\mathbb{Z}_i(\widehat{\bar{w}}_i) = \frac{1}{\widehat{\bar{w}}}[\widehat{\bar{w}}_i\bar{Y}_i^{org} - \sum_{j=1}^{I}\frac{\bar{\pi}_{ij}^{org}\widehat{\tau}_{ij}^{-\theta}(\widehat{\bar{w}}_i^{\eta}\widehat{\bar{P}}_i^{1-\eta})^{-\theta}}{\sum_{k=1}^{I}\bar{\pi}_{kj}^{org}\widehat{\tau}_{kj}^{-\theta}(\widehat{\bar{w}}_k^{\eta}\widehat{\bar{P}}_k^{1-\eta})^{-\theta}}\widehat{\bar{w}}_j\bar{Y}_j^{org}\bar{D}^{ctf}]. \quad \text{(B.19)}$$

$\bar{D}^{ctf}$ appears as a finite multiplier and hence does not change most properties described in footnote 30 of $\widehat{\bar{w}}$ computed as the solution to equation B.19. For example, under the world resource constraint

$$\sum_{i=1}^{I}\widehat{\bar{w}}\bar{Y}_i^{org} = \sum_{i=1}^{I}\widehat{\bar{w}}\bar{Y}_i^{org}\bar{D}^{ctf}, \quad \text{(B.20)}$$

Walras's Law is satisfied:

$$\sum_{i=1}^{I}\widehat{\bar{w}}_i\mathbb{Z}_i(\widehat{\bar{w}}) = \sum_{i=1}^{I}(\widehat{\bar{w}}_i\bar{Y}_i^{org} - \sum_{j=1}^{I}\bar{\pi}_{ij}^{ctf}\widehat{\bar{w}}_j\bar{Y}_j^{org}\bar{D}^{ctf}) = 0, \quad \text{(B.21)}$$

which is necessary to establish the existence of the solution to inter-regime changes. The solution will characterize the counterfactual outcome $(\widehat{\bar{w}}, \widehat{\bar{P}}, \widehat{\bar{D}})$ which encompasses both real and financial sides of the economy.

B.3 Algorithm

Step 1. Collect data to calibrate the original steady state of the economy

[30] Alvarez and Lucas (2007) show that under the assumptions that $\eta < 1$, $1+\theta(1-\epsilon) > 0$, $\tau_{ij} \geq 1$, a unique solution to counterfactual w exists to ensure zero excess demand (denoted as $\mathbb{Z}_i(w)$ in equation B.19 for the level of wage) in the commodity market. They prove these theorems by showing that $\mathbb{Z}_i(w)$ is continuous, homogenous of degree zero, has the gross substitute property $\frac{\partial \mathbb{Z}_i(w)}{\partial w_j} > 0$, satisfies Walras's Law ($\sum_i w_i\mathbb{Z}_i(w) = 0$), faces a lower but not upper bound $\mathbb{Z}_i(w) > -\max_j L_j, \max_i \mathbb{Z}_i(w \to w^{org}) \to \infty$.

Obtain the data of country-level GDP and NFA, and of bilateral trade shares and bilateral portfolio weights (see Appendix C for data sources). The mean values over the sample period are used as the steady-state values of these variables in the original regime.

Step 2. Form initial guesses about inter-regime changes[31]

Start with the guess that original and counterfactual regimes have the same steady-state values for wage, price, and asset positions

$$\widehat{\widehat{w}}^0 = \widehat{\widehat{P}}^0 = \widehat{\widehat{D}}^0 = ones(I, 1), \tag{B.22}$$

Step 3. Characterize intra-regime changes[32]

Simulate productivity shocks (see Appendix C for its calibration), solve for the responses of $\widehat{w}^s, \widehat{P}^s$ with a recursive procedure based on 43 and 44 around original and counterfactual steady states respectively ($s \in \{org, ctf\}$) to the shocks. The original steady state is from step 1, and the counterfactual steady state including $\bar{Y}^{ctf}, \bar{\pi}_{ij}^{ctf}$ is predicted from inter-regime changes ($\widehat{\widehat{w}}$ and the corresponding $\widehat{\widehat{P}}$ that satisfies 38) imposed on the original steady state. Calculate other variables including $\widehat{Y}^s, \widehat{R}^s$ as functions of the solved $\widehat{w}^s, \widehat{P}^s$ (see appendix B for their relations) and these variables' products (listed in 33) in response to the simulated productivity shocks and take the mean values across simulations to get the second-moment variables.

Step 4. Solve the portfolio choice problem

Use the second-moment variables from step 3 in the original and counterfactual portfolio determination equations (31 and 32) to yield

$$E_t[(\gamma(1-\beta)\widetilde{Y}\widetilde{R}'^s + (1-\gamma+\beta\gamma)\widetilde{P}\widetilde{R}'^s + \gamma(1-\beta)\check{\alpha}^s\widetilde{R}\widetilde{R}'^s] = \frac{1}{2}F^s,\ s \in \{org, ctf\}. \tag{B.23}$$

where F^s and $\check{\alpha}^s$ stand for the vectors of all the countries' relative financial frictions and asset holdings (defined by 26 and 30):

$$F^{s'} = [F^s_{1I}, F^s_{2I}, ..., F^s_{I-1I}], \qquad \check{\alpha}^{s'} = [\check{\alpha}^s_1, \check{\alpha}^s_2, ..., \check{\alpha}^s_{I-1}] - \alpha^s_I. \tag{B.24}$$

$\widetilde{Y}\widetilde{R}'^s$, $\widetilde{P}\widetilde{R}'^s$, and $\widetilde{R}\widetilde{R}'^s$ represent the second-moment variables listed in 33 for brevity. Take the difference of equation B.23 between the two regimes to determine the shift of

[31] The values used for the initial guesses do not matter significantly due to the existence and uniqueness of the model solution discussed in Appendix B. Therefore, the iterative computation procedure described in the algorithm will update ($\widehat{\widehat{w}}, \widehat{\widehat{P}}, \widehat{\widehat{D}}$) to reach the unique solution from alternative initial guesses.

[32] The purpose of this step is to calculate second-moment variables that appear in portfolio determination equations (31 and 32). To do the calculation, we need the percentage deviation of variables from a steady state under the shocks of the economy.

bilateral asset holdings driven by second-moment variables including potential changes to financial frictions. Numeraire country I's holdings will be derived after other countries' relative holdings to satisfy the assets' market clearing condition. Add up bilateral holdings to yield country-level aggregate asset positions, which will update $\widehat{\widehat{D}}^{0}$ to $\widehat{\widehat{D}}^{1}$.

Step 5. Update Inter-regime changes given the solved portfolio

Update inter-regime changes of $\widehat{\bar{w}}$ using the portfolio $\widehat{\widehat{D}}$ from step 4. This involves employing an iterative computation procedure similar to that described by Alvarez and Lucas (2007) and Dekle et al. (2007) with function M, which denotes the mapping of $\widehat{\bar{w}}^{0}$ to $\widehat{\bar{w}}^{1}$ with a constant $\nu \in (0,1)$

$$\widehat{\bar{w}}^{1} = M(\widehat{\bar{w}}^{0}) = \widehat{\bar{w}}^{0}(1 + \nu \frac{\mathbb{Z}_i(\widehat{\bar{w}}^{0})}{\bar{Y}_i^{org}}), \tag{B.25}$$

and excess demand $\mathbb{Z}_i(\widehat{\bar{w}})$

$$\mathbb{Z}_i(\widehat{\bar{w}}_i) = \frac{1}{\widehat{\bar{w}}}[\widehat{\bar{w}}_i \bar{Y}_i^{org} - \sum_{j=1}^{I} \frac{\bar{\pi}_{ij}^{org} \widehat{\tau}_{ij}^{-\theta} (\widehat{\bar{w}}_i^{\eta} \widehat{\bar{P}}_i^{1-\eta})^{-\theta}}{\sum_{k=1}^{I} \bar{\pi}_{kj}^{org} \widehat{\tau}_{kj}^{-\theta} (\widehat{\bar{w}}_k^{\eta} \widehat{\bar{P}}_k^{1-\eta})^{-\theta}} \widehat{\bar{w}}_j \bar{Y}_j^{org} \bar{D}^{ctf}]. \tag{B.26}$$

Note that the mapping is bounded by one under Walras's Law (equation B.21)

$$\sum_{i=1}^{I} M(\widehat{\bar{w}}_i^{\,0}) \bar{Y}_i^{org} = \sum_{i=1}^{I} \widehat{\bar{w}}_i^{\,0} (1 + \nu \frac{\mathbb{Z}_i(\widehat{\bar{w}}_i^{\,0})}{\bar{Y}_i^{org}}) \bar{Y}_i^{org} = \sum_{i=1}^{I} \widehat{\bar{w}}^{0} \bar{Y}_i^{org} + \nu \sum_{i=1}^{I} \widehat{\bar{w}}_i^{\,0} \mathbb{Z}_i(\widehat{\bar{w}}^{0}) = 1, \tag{B.27}$$

and the normalization condition that treats the world output as a numeraire

$$\sum_{i=1}^{I} \widehat{\bar{w}} \bar{Y}_i^{org} = 1. \tag{B.28}$$

Step 6. Repeat steps 3-5 until convergence

Use the updated $\widehat{\bar{w}}^{1}$ from step 5 and $\widehat{\widehat{D}}^{1}$ from step 4 as new guesses, and repeat the procedures from step 3 to 5 for both inter-regime and intra-regime analyses to reach new updated $\widehat{\bar{w}}^{2}$, $\widehat{\bar{P}}^{2}$ and $\widehat{\bar{D}}^{2}$. This continues until the difference between the k^{th} and the $k{+}1^{th}$ iteration $\mid \widehat{\bar{w}}^{k+1} - \widehat{\bar{w}}^{k} \mid, \mid \widehat{\bar{P}}^{k+1} - \widehat{\bar{P}}^{k} \mid, \mid \widehat{\bar{D}}^{k+1} - \widehat{\bar{D}}^{k} \mid$ is sufficiently small, which solves the joint fixed-point problem of $(\widehat{\bar{w}}, \widehat{\bar{P}}, \widehat{\bar{D}})$ necessary to characterize counterfactual outcomes under alternative tariffs or financial frictions.

C Data and Calibration

This section describes the data source and calibration strategy for both the real and financial sides of the economy. The sample of economies includes 43 countries (listed in table A.1) and the rest of the world (ROW). The time-averaged values of variables over the sample period from 2001-2021 will be used as their original steady-state values,[33] including countries' output obtained from the Penn World Table (PWT) and net foreign asset positions from the World Bank. The values of the ROW's variables are the difference between the world aggregate values and those of the countries in the sample.

C.1 Bilateral Trade and Financial Shares

Cross-country trade data are obtained from the Direction of Trade Statistics (DOTS) compiled by the IMF. I use the bilateral import (CIF) data to calculate a country's spending on goods sourced from other countries. A country's spending on its own goods is computed as the difference between its gross expenditure and total imports, both available from the World Development Indicators (WDI) compiled by the World Bank.

Financial data are sourced from Factset/Lionshare, a dataset that provides information on institutional investors' asset holdings. It has comprehensive coverage of institutional holdings across countries. I describe its details in Hu (2022) and its consistency in terms of portfolio composition with macro-level datasets such as IMF's International Financial Statistics. Factset/Lionshare compiles financial investment by investors' origin and their investment destination, using which I calculate bilateral portfolio shares.

As the analysis in this paper covers two channels, the sample of countries includes those with trade and finance data both available (see table A.1).

C.2 Productivity

The estimation of productivity consistent with the Eaton and Kortum (2002) model is modified from the approach developed by Levchenko and Zhang (2014), who infer Ricardian productivity from bilateral trade data.

Let any country's production cost be denoted as

$$c_{i,t} = (r_{i,t}^{\mu} w_{i,t}^{1-\mu})^{\eta} P_{i,t}^{1-\eta}. \tag{C.1}$$

It follows from equation 6 that trade shares for any destination country j should satisfy

$$\frac{\pi_{ij,t}}{\pi_{jj,t}} = \frac{T_{i,t}}{T_{j,t}} \left(\frac{\tau_{ij,t} c_{i,t}}{c_{j,t}}\right)^{-\theta}. \tag{C.2}$$

As the left hand side is directly observable from the trade data, we can recover relative productivity $\frac{T_{i,t}}{T_{j,t}}$ after estimating bilateral trade friction $\tau_{ij,t}$ and relative input cost $\frac{c_{i,t}}{c_{j,t}}$.

[33] The selection of the sample period is mainly driven by the availability of financial data from Factset.

I follow the trade literature by estimating bilateral trade costs $\hat{\tau}_{ij,t}$ from a combination of gravity variables including geographic distance divided into intervals,[34] dummies for contiguity, common language, common colonizer, common religion, common legal system, and regional trade agreements. The values of these gravity variables are obtained from the CEPII.

I estimate a country's production cost (denoted as $\hat{c}_{i,t}$) based on the information from the PWT. Specifically, I compute a country's wage (w) as the ratio of its total labor compensation (output-side GDP ($rgdpo$) × share of labor compensation in GDP ($labsh$)) to total labor hours (number of employees (emp) × average hours per employee (avc)). Price of domestic absorption (pl_{da}) and price of capital services (pl_k) are used as the proxies for the price of intermediate inputs and capital rental fee respectively. Besides, I calibrate the share of intermediate input in production $\eta = .312$ based on Dekle et al. (2007) and the share of labor input $1-\mu$ as country-specific $labsh$ from the PWT. The production cost of ROW is calculated as the median cost across countries not included in table A.1.

The full estimating specification for all the country pairs in the sample follows

$$\ln(\frac{\pi_{ij,t}}{\pi_{jj,t}}) = \ln(T_{i,t}\hat{c}_{i,t}^{-\theta}) - \ln(T_{j,t}\hat{c}_{j,t}^{-\theta}) - \theta\hat{\tau}_{ij,t} + \gamma_{ij,t}, \tag{C.3}$$

The first two terms on the right $\ln(T_{i,t}\hat{c}_{i,t}^{-\theta})$ and $\ln(T_{j,t}\hat{c}_{j,t}^{-\theta})$ can be captured by the exporter and importer fixed effects respectively when running the estimation. $\hat{\tau}_{ij,t}$ represents the estimated bilateral trade costs as a linear combination of the gravity variables described above and $\gamma_{ij,t}$ stands for error terms. Exponentiating the importer fixed effects yields a term that combines country j's productivity and cost denoted as

$$\widehat{Tc}_{j,t} = T_{j,t}\hat{c}_{j,t}^{-\theta}. \tag{C.4}$$

If the US is the benchmark country, and its productivity ($T_{US,t}$) is obtained as the country's TFP from the PWT ($rtfpna$). Then other countries' Ricardian productivity can be calculated as

$$T_{j,t} = T_{US,t}\frac{\widehat{Tc}_{j,t}}{\widehat{Tc}_{US,t}}(\frac{\hat{c}_{j,t}}{\hat{c}_{US,t}})^{\theta}, \tag{C.5}$$

where trade elasticity is set as $\theta = 4$ as in Simonovska and Waugh (2014). After calculating countries' dynamic productivity $T_{j,t}$, I compute its mean value over time $\bar{T}$ and the cross-country covariance matrix Σ_T. Dynamic productivity with these moments can be used to simulate shocks for intra-regime analysis.

[34] I follow Eaton and Kortum (2002) by setting the distance intervals in miles as [0, 350], [350, 750], [750, 1500], [1500, 3000], [3000, 6000], [6000, ∞) and code dummies for these intervals as independent variables in the regression analysis.