Disaggregated Economic Accounts*

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Abstract

We develop and analyze a new system of disaggregated economic accounts. The system breaks down national accounting positions into bilateral flows among consistently defined subgroups of consumers ("consumer cells"), subgroups of producers ("producer cells"), the government, and the rest of the world. We disaggregate the full circular flow of money, including consumption, labor compensation, firm surplus, foreign trade, taxes, and trade in intermediates. The measurement is comprehensive, so that the disaggregated flows add up to national aggregates and fulfill all national accounting identities. We implement the disaggregated system for small region-by-industry cells in Denmark. We present new facts on the structure of disaggregated flows across the economy, for example that spending flows into cities, city residents spend more abroad, and the government on net transfers resources into cities. Using a macroeconomic model, we highlight that disaggregated economic accounts change our understanding of shock propagation in general equilibrium. In particular, we find that the structure of disaggregated flows shapes the aggregate and distributional consequences of export demand shocks.

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I Introduction

National accounts, pioneered by Simon Kuznets and Richard Stone in the 1930s and 1940s, measure aggregate flows—most notably national consumption, income, and output—as well as input-output trade among producer industries. However, modern national accounts contain little data on flows connecting smaller subgroups in the economy, for example which consumers purchase goods from which producers, which producers pay income to which consumers, and how consumers and producers transact with the government and the rest of the world. The absence of comprehensively disaggregated economic accounts limits our understanding of how shocks propagate across the economy and how heterogeneity in the direct incidence of shocks affects aggregate and distributional outcomes.

Assembling disaggregated economic accounts raises conceptual and measurement challenges. For instance, there exist no nationally representative data linking consumers to individual retail establishments and no method describing how such data can be constructed. In this paper, we take a step toward developing a system of comprehensively disaggregated economic accounts. The aim of such a system is to break down all national accounting positions into bilateral flows among consistently defined subgroups of consumers ("consumer cells"), subgroups of producers ("producer cells"), the government, and the rest of the world. The system reveals the sources of all inflows into each cell (e.g., which producer cells pay labor and profit income to each consumer cell) and the destinations of all outflows leaving a cell (e.g., which producer cells receive spending from each consumer cell). The disaggregated system is comprehensive, in the sense that individual flows add up to a corresponding national aggregate and that all accounting identities are satisfied (e.g., each cell's inflows equal its outflows).

We implement the system of disaggregated economic accounts for Denmark using various transaction and government microdata.[1] Our unit of analysis are consistently defined region-by-industry cells of consumers and producers, allowing us to capture rich heterogeneity in flows and shock incidence across regions and industries. We use the new system to present facts on the circular flow of money across cells. On average, consumer spending travels from rural regions into urban services, city residents allocate a larger share of their spending abroad, and net government flows into cities are positive.

We complement the measured system with a neoclassical macroeconomic model at the same level of disaggregation. In two applications, we show that disaggregated economic accounts can change our understanding of the distributional and aggregate consequences of export demand shocks. First, we find that producer cell-specific export shocks have strongly heterogeneous aggregate welfare effects, independent of their size. The cell-specific aggregate multiplier depends

[1]Disaggregated data are available under disaggregatedaccounts.com.

on the full chain of disaggregated flows between the shocked cell and consumers and producers that import from abroad. Second, we find that a uniform export demand shock to all producers has stronger direct incidence on rural consumers, but improves the welfare of city consumer by more. We need to know the disaggregated system to arrive at both these findings and cannot replicate them with less comprehensive or coarser national accounting systems.

In the first part of the paper, we describe a general framework for a system of disaggregated economic accounts. Such a system assigns all adults to distinct "consumer cells" and all firm establishments to "producer cells." For example, in our implementation, we will assign consumers to cells based on their industry of employment and region of residence, while assigning establishments to producer cells based on their industry and location. However, the proposed framework is general and allows for any type of subgrouping, down to the individual level at the most extreme. We develop a cross-walk between merchant categories (MCC) and industry codes (NACE), which allows us to link consumer spending flows to production industries and to define a consistent set of producer and consumer industry groupings.

Disaggregated economic accounts comprehensively measure bilateral flows between different cells. Flows travel from consumer to producer cells (e.g., consumer spending), producer to consumer cells (e.g., labor and mixed income), between producer cells (e.g., intermediates trade), to and from the government (e.g., taxes and transfers), to and from the rest of the world (e.g., exports and imports), and to and from a capital accumulation cell (e.g., saving and borrowing). Each type of flow is captured in one disaggregated dataset (e.g., consumer spending matrix). We show how flows are linked through accounting identities within each cell (e.g., income equals expenditures plus saving), across cell types (e.g., consumer sales of a producer cell equal total consumer spending on that cell), and with respect to national accounts (e.g., total consumer spending of all cells equals national consumer spending). We lay out two general approaches to disaggregating a national accounting flow. A "bottom-up" approach uses microdata on individual flows (e.g., consumer payment transactions linking consumers to retail establishments) to calculate cell-to-cell flows. Alternatively, a "top-down" approach distributes an aggregate flow across cells using an assignment algorithm informed by relevant microdata.

In the second part of the paper, we implement a system of disaggregated accounts in Denmark for 5,400 region-industry cells. Cells are small, with the median consumer cell containing 658 adults and the median producer cell containing 47 establishments. We use the "bottom-up" method to disaggregate consumer spending, labor compensation, mixed income (unincorporated business profits), government transfers, and tax payments, among others. For instance, to measure the disaggregated consumer spending matrix, we rely on transaction-level data containing region and industry of both consumers and retail establishments, provided by the largest Danish retail bank. We also use government microdata to capture spending that is not covered by the bank data,

such as spending on owner-occupied housing and financial services. Thanks to the representative microdata, the distribution of spending across consumer regions and receiving industries in the disaggregated spending matrix is similar to data based on the national accounts and household surveys. To disaggregate a few flows not covered by rich microdata, we rely on "top-down" algorithms, such as a gravity algorithm based on Leontief and Strout (1963) for the disaggregated intermediates trade matrix.

In the third part of the paper, we analyze the system of disaggregated economic accounts to present facts on the circular flow of money across regions and industries. First, distance has a strong effect on consumer spending, labor compensation, and intermediates trade. Distance matters most for regular, in-person consumer spending (e.g., fuel, groceries) and less for travel-related spending (e.g., hotels) and remote services (e.g., insurance and telecommunication). Second, consumer spending flows toward cities—the population size of a consumer cell's home region is almost always lower than the average size of regions receiving its spending (Glaeser et al. 2001; Handbury and Weinstein 2015). Third, we directly measure spending abroad and on foreign retailers in the bank data, complementing recent work on the consumption of imported goods (Borusyak and Jaravel 2021). We find that city consumers allocate around 12 percent of their consumer spending abroad, while it is 8 percent for rural consumers. Fourth, net exports make up a larger share of rural producers' output (mostly manufacturers), while domestic sales are more important for city producers (mostly services, see Glaeser and Kohlhase 2004). Finally, net transfers by the government to consumers (transfers minus taxes) are larger in rural regions, but the government employs and purchases more in cities. On net, the government transfers resources into cities.

In the fourth part of the paper, we develop a static neoclassical model of an open macroeconomy, inspired by Acemoglu et al. (2012), Caliendo and Parro (2015), Caliendo et al. (2018), and Baqaee and Farhi (2019b, 2022a).[2] The model contains many region-by-industry domestic consumer and producer cells, foreign consumers and producers, and a government. We can use the disaggregated economic accounts to calibrate consumers' preferences across producer cell's output, producers' income and surplus payments to consumers, intermediates trade across producers, as well as consumers' and producers' transactions with the government and the rest of the world.

The calibrated model allows us to study how shocks propagate across region-industry cells. Empirical work has used quasi-experimental techniques to estimate spillover effects, but typically cannot disentangle all general equilibrium propagation channels (e.g., Greenstone et al. 2010;

[2]Related literatures have investigated the macroeconomic effects of cross-industry and cross-regional trade among producers (Long Jr. and Plosser 1983; Horvath 2000; Jones 2011; Caliendo and Parro 2015; Baqaee and Farhi 2019a, 2020; Caliendo et al. 2019; Bigio and La'O 2020; Rubbo 2022) as well as spillovers across regions and industries in the presence of Keynesian frictions (Miyazawa 1976; Farhi and Werning 2014; Flynn, Patterson and Sturm 2021; Guerrieri, Lorenzoni, Straub and Werning 2021, 2022; Baqaee and Farhi 2022b; Rubbo 2022; Woodford 2022).

Huber 2018; Giroud and Mueller 2019; Aghion et al. 2020; Carvalho et al. 2021; Gabaix and Koijen 2022; review in Huber 2022). A large literature has developed general equilibrium models to quantify shock propagation across regions and industries (e.g., Nakamura and Steinsson 2014; Farhi and Werning 2016; Beraja et al. 2019; Chodorow-Reich 2019; Adão et al. 2020; Galle et al. 2022). Disaggregated economic accounts reveal heterogeneity in a range of flows across cells, thereby facilitating fine model calibrations and improving our understanding of the channels of shock propagation.

In two applications, we show that disaggregated economic accounts change our understanding of how shocks propagate. We focus on the aggregate and distributional effects of export demand shocks. First, we find that producer cell-specific export demand shocks have vastly heterogeneous aggregate welfare effects. Ultimately, the aggregate effect depends on the shocked cell's position in the disaggregated circular flow of money relative to consumers and producers that import from abroad, going beyond the cell's size (see Hulten 1978).

Second, we show that a uniform export demand shock to all producer cells has stronger direct incidence on sales of rural producers (because they export more) and therefore on incomes of rural consumers (because labor is mostly local). However, spending by rural consumers disproportionately flows into cities, so urban consumers also benefit to a large extent. Ultimately, the welfare of city consumers rises even more than that of rural consumers because the foreign spending of city residents is greater, despite the direct incidence favoring rural consumers.

For both applications, we calculate the findings under alternative national accounting systems, such as those that do not contain disaggregated flows of consumer spending, the government, and the rest of the world. The results suggest that no alternative system delivers similar predictions to our calibrated model. Hence, disaggregated economic accounts substantially change our understanding of shock propagation and may aid in the design of policy interventions. Much of the raw data required to construct disaggregated economic accounts are already being collected in many advanced economies (e.g., Chetty et al. 2022a), but would require further data processing (e.g., linking consumers and retailers). In light of our findings, the social benefits of constructing disaggregated economic accounts may outweigh the costs.

II Disaggregated Economic Accounts in Relation to Existing Measurement

Richard Cantillon (1755) and François Quesnay (1758) first envisioned the national economy as a complex circular flow of money, with different disaggregated consumer and producer groups (e.g., farmers, artisans, property owners) connected through bilateral consumption, income, and output flows. This early work suggests that a system of measurement should adhere to three principles. First, it should satisfy accounting identities linking the consumption, income, and

output of each disaggregated group as well as at the aggregate level. Second, it should capture transactions between groups comprehensively, so that measured disaggregated flows add up to national aggregates. And third, it should reveal how bilateral flows connect different disaggregated groups.

Economists in the early 20th century developed these ideas further. Lahn (1903), Foster (1922), and Knight (1933) visualized aggregate accounting identities using diagrams of the national circular flow of money. Kuznets et al. (1941, at the US National Bureau of Economic Research) and Meade and Stone (1941, at the British Statistical Office) comprehensively measured the components of the national circular flow (i.e., consumption, income, and output). Leontief (1928; 1966) instead focused on disaggregation and measured bilateral trade between producer industries.

Building on this existing work, Richard Stone chaired the first United Nations (UN) Committee on National Income Statistics in 1947 and defined a general system of national accounts (Stone 1961). Modern national accounts, standardized by the UN System of National Accounts (SNA), still follow his concept. That means, however, that existing systems do not fully implement the principles of Cantillon and Quesnay because they only disaggregate flows between producer groups. There is currently no system that comprehensively documents bilateral consumption and income flows between disaggregated consumer and producer groups. The absence of bilateral consumption data is particularly striking given that measuring consumption is a chief aim of national accounting (Barro 2021). The system of disaggregated economic accounts developed in this paper fills this gap.

Our work complements several recent innovations in the measurement of consumer and producer flows. First, Chetty et al. (2022a) develop high-frequency accounts for consumer and producer subgroups, which can support policy in real time. Second, distributional national accounts document income and wealth across consumer groups (Saez and Zucman 2016; Piketty et al. 2018; Blanchet et al. 2021). Recent work in this area focuses on top wealth shares (Saez and Zucman 2022; Smith et al. forthcoming) and saving rates of the rich (Mian et al. 2020). Third, financial, online, and smartphone transaction data can improve our understanding of national consumption dynamics (Aladangady et al. 2022; Ehrlich et al. 2022; Buda et al. 2022), heterogeneous consumption responses to shocks (e.g., Baker 2018; Vavra 2021; Cox et al. 2020; Andersen et al. 2022; Baker and Kueng 2022), business entry and exit (Glaeser et al. 2022), spending patterns across space (Davis et al. 2019; Agarwal et al. 2020; Dunn and Gholizadeh 2020; Allen et al. 2021; Miyauchi et al. 2022), and living standards across regions (Diamond and Moretti 2021). Fourth, Gabaix (2011) highlights the importance of studying granular patterns for macroeconomic outcomes. Fifth, government registers document income flows between producers and consumers (e.g., Card et al. 2013; Adão et al. 2022) and intermediates trade between producers (e.g., Huneeus 2018; Dhyne et al. 2021; Bernard et al. 2022). Sixth, the Social Connectedness Index (Bailey et al. 2018) and Social Capital

Atlas (Chetty et al. 2022b) measure friendships across regions and socioeconomic groups.

Four features distinguish disaggregated economic accounts from these existing approaches. First, we disaggregate the entire circular flow of money, rather than focusing on a subset of national account positions. Second, we focus on bilateral flows between producers, consumers, government, and the rest of the world, rather than just heterogeneity on the consumer or producer side. Third, the disaggregated system is comprehensive, so that the sum of disaggregated flows equals national aggregates and satisfies national accounting identities within and across groups. Finally, we provide measurement tools allowing researchers to implement disaggregated economic accounts in other countries.

III Concept and Methods for Disaggregated Economic Accounts

In this section, we outline the concept of disaggregated economic accounts and general methods to construct a disaggregated system.

III.A Defining Disaggregated Cells

The ultimate disaggregated system would record all transactions that occur in an economy between every individual and firm establishment. However, the necessary microdata on all individual-level flows are usually not available. In practice, a disaggregated system can be implemented by assigning individuals and firm establishments to non-overlapping groups, which we term consumer and producer cells, respectively, and by measuring flows between these cells. In principle, economists can choose cell classifications flexibly depending on the research question and setting at hand. For example, consumer cells could depend on region, work industry, age, or income as well as interactions of these characteristics, whereas producer cells could depend on region or industry.[3]

At least one cell needs to represent the rest of the world, but there could be multiple cells, for instance, different country-by-industry cells. The system also includes a capital accumulation cell. It intermediates saving, interest, and financial transfers. It also conducts investment transactions, by purchasing investment goods from producers.[4] Moreover, the system contains a government cell representing the entire public sector as well as the small group of non-profit institutions serving households (NPISH).

[3]The consumer and producer cells do not need to be defined symmetrically. While we choose a region-industry cut for both consumer and producer cells in our implementation (Section IV), one could, for instance, also choose a regional cut for consumers and an industry cut for producers.

[4]Note that national accounts differentiate between the finance industry, which sells financial services, versus the capital accounts, which record saving and investment. Our implementation disaggregates the financial industry into many regional cells and contains one capital accumulation cell. An alternative system could include multiple capital accumulation cells, one each for type of capital-accumulating institution (e.g., commercial versus investment banks etc.) and for different production industries trading investment goods with each other (vom Lehn and Winberry 2022). We leave such disaggregation to future work.

III.B Components of Disaggregated Economic Accounts

A full system of disaggregated accounts contains several individual datasets. Each breaks down one flow from the aggregate national accounts into a comprehensive set of bilateral flows between pairs of cells. In Table I, we list all disaggregated datasets needed to measure flows between consumer cells, producer cells, a government cell, the rest of the world, and a capital accumulation cell.

Each disaggregated dataset is comprehensive, in the sense that the sum of its individual observations equals the national aggregate (e.g., the total of the consumer spending matrix equals national consumer spending). The observations in different disaggregated datasets are linked by accounting identities that hold within each cell (i.e., a cell's inflows equal its outflows) and across cells, as we describe in detail in Section IV.

Table I: Components of disaggregated economic accounts

	Disaggregated dataset	Financial outflow from	Financial inflow to
1	Domestic consumer spending matrix	Consumers	Producers
2	Foreign consumer spending	Consumers	Rest of world
3	Consumer taxes	Consumers	Government
4	Consumer interest, transfers, and saving	Consumers / Capital acc.	Capital acc. / Consumers
5	Consumer labor compensation matrix	Producers	Consumers
6	Consumer dividend, mixed income, and surplus matrix	Producers	Consumers
7	Government benefits to consumers	Government	Consumers
8	Intermediates trade matrix	Producers	Producers
9	Government dividend and surplus income	Producers	Government
11	Producer net taxes	Producers	Government
12	Producer net interest, transfers, and saving	Producers	Capital acc.
13	Producer imports	Producers	Rest of world
14	Domestic government spending	Government	Producers
15	Domestic capital accumulation spending	Capital acc.	Producers
16	Producer exports	Rest of world	Producers
17	Government imports	Government	Rest of world
18	Government net interest, transfers, and saving	Government	Capital acc.
19	Capital accumulation imports and trade balance	Rest of world / Capital acc.	Capital acc. / Rest of world
20	Consumption of government output	Provided free of charge to consumers	

Notes: The table lists the datasets that make up a system of disaggregated economic accounts. A matrix is a dataset composed of multiple cells along both the outflow and inflow dimensions. The other datasets are vectors containing only one inflow or one outflow dimension. The consumption values in "Consumption of government output" are provided free of charge, so that no money flows from consumers to the government cell in exchange for the goods. The government cell includes all NPISH (non-profit institutions serving households). The outflows and inflows are defined in terms of financial flows, so that goods flow in the opposite direction.

III.C Relation to the UN System of National Accounts

Our disaggregated system, in particular the definitions of individual disaggregated flows, builds on global standards defined in the UN System of National Accounts (SNA). However, an important difference between the SNA and disaggregated accounts lies in the definition of the units of analysis. The SNA groups all individuals and organizations into five "institutional sectors": non-financial corporations, financial corporations, government, NPISH, and households (including unincorporated businesses). The SNA government, NPISH, and household sectors are both producers and consumers of output. In contrast, our disaggregated system assigns all establishments engaged in production to a producer cell. This implies that our disaggregated producer cells

include production units from different SNA institutional sectors and that our disaggregated consumer, government, and capital accumulation cells do not produce output.

A further difference between the SNA and the disaggregated system is that we assign NPISH to the government cell, since the NPISH sector is very small and, just like the government cell, purchases output from producers and provides it to consumers free of charge (or at low nominal fees). Finally, the SNA contains capital accounts within each of the five sectors, which record investment and asset accumulation. We include just one joint capital accumulation cell in the disaggregated system that carries out all these capital transactions.

III.D "Bottom-Up" and "Top-Down" Disaggregation Approaches

There are two approaches to disaggregating a national accounting flow. First, the "bottom-up" approach uses detailed microdata on individual transactions between consumers and producers, such as individual spending transactions. Adding up all transactions between two cells and, if appropriate, weighting to match the underlying population produces a cell-to-cell bilateral flow. The second disaggregation approach is "top-down." It decomposes an aggregate quantity into bilateral flows using an algorithm based on cell characteristics like group size, geographic distance, and other microdata on consumer and producer behavior.

III.E Measurement Challenges

Disaggregated accounts typically raise several measurement challenges. One challenge lies in the definition of producer cells. Different datasets do not report the activities of producer establishments in a consistent way. Consumption datasets typically classify consumer-facing establishments using the type of merchant (e.g., Merchant Category Code, MCC). In contrast, firm and employment datasets usually classify firm establishments into production industries (e.g., NACE or NAICS). We develop a new cross-walk allowing researchers to define producer cells consistently across consumption and firm/employment datasets (see Appendix E.C).

Another challenge is that national accounts data need to be transformed before they can inform a disaggregated system. For instance, standard national accounts measure transactions among producers in terms of output and consumption in terms of gross sales. We need a consistent definition of transaction values to construct a disaggregated system, so that inflows and outflows satisfy accounting identities and add up to national aggregates. We show how researchers can transform national accounts and raw microdata to ensure consistency across data sources.

Finally, top-down algorithms need to be fine-tuned to the requirements of each national accounting flow. We develop algorithms for individual flows and estimate flexible parameters that researchers can use to inform their top-down algorithms (e.g., the effect of distance on consumer

spending for consumers living in different types of regions).

IV Measurement of Disaggregated Economic Accounts in Denmark

We implement a comprehensive measurement exercise for the Danish economy in 2018. We outline the main steps in this section, with methodological details described in Appendices C to P. We share disaggregated data under disaggregatedaccounts.com.

IV.A Region-Industry Cells

We assign all Danish adults to consumer cells (indexed by i) based on their region of residence and the industry paying the largest share of their income. The region-by-industry breakdown is useful because, as we will show below, many flows depend on geography and because many recent large-scale shocks (e.g., foreign trade shocks, Covid, technology) varied by industry. There is also a practical benefit to our cell definition, as we can observe region and industry across all underlying datasets.

There are 2,744 domestic consumer cells in total. The cells are formed from the interaction of 98 regions, one for each of the Danish municipalities, and 28 industries (listed in Table A.I). The industry classification includes industries selling directly to consumers and producers (e.g., food away from home, grocery stores, airlines), non-consumer facing industries selling to producers (e.g., wholesale, manufacturing), and four industries for the non-working parts of the population (retired, students, unemployed, out of workforce).

We assign firm establishments to 2,646 producer cells (indexed by j), based on their region and production industry. There are 24 producer industries paying labor compensation to consumers ("work industries") as well as three producer industries providing housing services without any employees (private landlords, owner-occupied housing, government-owned housing).

The individual cells are small, with 658 adults in the median consumer cell and 47 establishments in the median producer cell. In addition to the consumer and producer region-industry cells, we include cells for the government, rest of the world, and capital accumulation.

IV.B Disaggregated Flows from and to Consumers

The accounting identity for consumer cell i says that total outflows equal total inflows,

$$\begin{aligned}&\text{Domestic consumer spending}_i + \text{Foreign consumer spending}_i \\ &+ \text{Consumer taxes}_i + \text{Interest, transfers, and saving paid}_i \\ &= \text{Labor comp}_i + \text{Producer dividends, mixed inc, and surplus}_i \\ &+ \text{Government benefits}_i + \text{Interest and transfers rec}_i. \end{aligned} \tag{1}$$

We describe how we break down every component of equation 1 into bilateral flows.

IV.B.1 Disaggregated Consumer Spending Matrix

The consumer spending matrix measures how much consumer cell i spends on goods produced by every domestic producer cell and by the rest of the world. The closest corresponding position in the UN SNA is P.3, national final consumption expenditure. However, P.3 includes both spending flowing to producers and product taxes (e.g., value added taxes, import duties). In contrast, our spending matrix only records spending flowing to producers, whereas taxes appear in a separate disaggregated dataset.

We take four steps to construct the spending matrix, as detailed in Appendix E. The first step uses Danske Bank data. The sample consists of all adults who held their main bank account at Danske Bank and conducted at least one spending transaction per month in 2018 and 2019. The sample covers 20% of the population and is representative in terms of age, income, and asset holdings (Table A.II). We observe the full range of consumer and merchant microdata necessary to construct the spending matrix only for the years 2018 and 2019. We include both years in the sample to maximize observations.

Crucially, text strings in the bank's internal system allow us to identify the consumers and merchant establishments involved in a wide range of individual transactions, including cards, direct debits, bank transfers to firms, and mobile payments. We infer consumers' home region from customer records and their work industry from incoming salary payments. We extract the merchant's address and category code (MCC, indicating the type of merchant) for transactions involving credit and debit cards, bank transfers, direct debits, and mobile applications. We then develop a novel cross-walk between MCC and industry codes (ISIC/NACE), so that we can identify the industry of the merchant in terms of the industry codes used in the disaggregated production and income datasets. Thanks to the standardized MCC and industry classifications, the cross-walk can easily be adopted by other researchers who want to jointly study consumption, production, and income.

In a few cases, we do not observe merchants' region and/or industry. We assign these transactions to merchants in proportion to the observed spending of the same consumer cell using the same means of payment. Moreover, we assume that consumers spend cash withdrawals in proportion to in-store card payments (separately for withdrawals in Denmark and abroad). Cash withdrawals are 7% of aggregate transaction value, so the matrix is not sensitive to this assumption. To complete the first step, we aggregate all spending transactions going from a consumer cell to a producer cell and then scale up by each cell's ratio of consumers in the Danske Bank sample to the population.

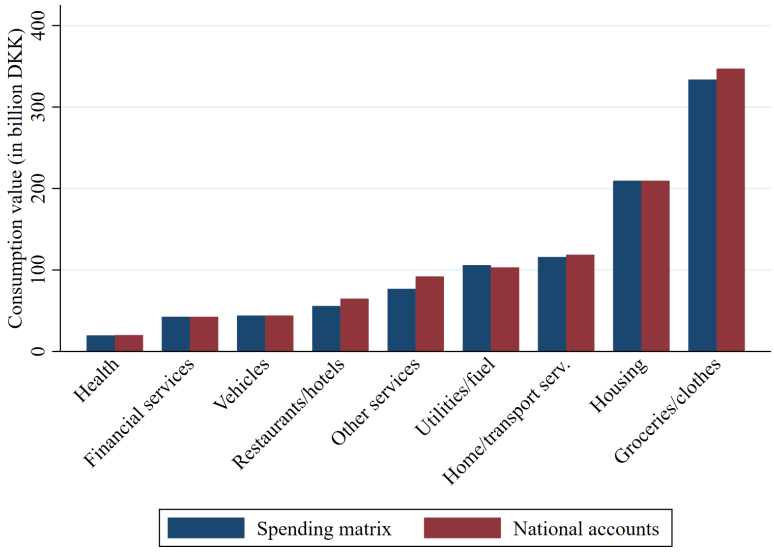
In the second step, we augment the matrix with government data on housing, financial services, vehicles, and water and waste services (details in Appendix E.E). Spending on these four goods is often not cleanly observed in bank transactions. However, we can measure such spending by combining government and bank data. Notably, the government income register directly records the rental value of owner-occupied housing, interest expenses, and vehicle registrations for every individual, and allows us to infer whether individuals are renters or owners.[5]

We compare the matrix produced after the second step with national accounts consumption data from 2018. Both include product taxes paid. Spending patterns by industry are very similar (Figure I). It is not clear to what extent remaining differences reflect errors in the matrix or national accounts, since both contain some statistical error due to sampling and assumptions.[6] Similarly, spending shares by consumer region in the spending matrix and the Danish household budget survey are close (Figure A.I). We have access to data on aggregate card spending in the Danske Bank for a longer period, which we can compare to card spending as recorded by Statistics Denmark. The evolution of card spending is similar in both series (Figure A.II). Finally, we show that the regional distribution of the share of spending received by different regions within an industry is close to the within-industry share of labor compensation paid (Figure A.III). This finding indicates that spending as recorded in the matrix flows proportionately to where producers pay labor.

[5]According to national accounting conventions, homeowners living in their own property rent to themselves, so that imputed rents are counted as consumer spending and rental income. Differences in home ownership rates therefore do not affect national consumption.

[6]For instance, national accounts calculate spending on some industries using retail indices, which requires making assumptions on spending patterns by foreigners. In contrast, we directly observe spending by Danes and can exclude spending by foreigners.

Figure I: Spending on receiving industry in the spending matrix and national accounts



Notes: The figure compares spending aggregated by receiving industry from the spending matrix with national accounts consumption data. Housing spending is constructed using a bottom-up approach for owner-occupied housing and a top-down approach for rented housing. Vehicles, financial services, and water and waste (part of utilities) are constructed using a top-down approach, so aggregates in the transaction data and national accounts match by construction. The remaining categories are constructed using a bottom-up approach, so there is no mechanical reason that these aggregates should match. For details, see Appendix E.

In the third step, we scale every cell-to-cell spending observation by a common scaling factor, so that aggregate gross spending in the matrix matches national gross spending in 2018 (SNA P.3). Aggregate spending in the unscaled matrix is 3% larger than national value, largely because we include both 2018 and 2019 in the bank data sample.

In the fourth and final step, we subtract product taxes from each gross flow to produce the final spending matrix. We use national accounts data to calculate the product tax rate paid on each industry's products as well as the import tax rate by foreign industry (as detailed in Appendix F).

IV.B.2 Disaggregated Consumer Taxes

Danish consumers pay product taxes (SNA D.21, details in Appendix F), non-product taxes on income and pension wealth returns (SNA D.5), and social contributions (SNA D.61). For non-product taxes, we observe individual income tax returns in the government registers and use a bottom-up summation to calculate income taxes paid per cell, ultimately scaling each cell-level observation by a common scaling factor to match the national aggregate exactly. We use a top-down approach to distribute pension return taxes across cells, under the assumption that taxes paid are proportional to accumulated pension contributions. Finally, we use a bottom-up approach

to measure various pension contributions, which make up social contributions (details in Appendix G).

IV.B.3 Disaggregated Consumer Interest, Transfers, and Saving Paid

Consumer interest, transfers, and saving flow to the capital accumulation cell (details in Appendix H). We measure consumer interest payments (SNA D.41) bottom-up by aggregating individual-level interest payments on financial liabilities as reported in the government tax registers. We apply an adjustment factor to all interest payments, following the aggregate national accounts, to ensure that we measure interest net of the consumption value of financial services, which is a component of the consumer spending matrix. Apart from interest, two small flows go to the capital accumulation cell—rental payments for natural resources (SNA D.45) and other current transfers (SNA D.7)—which we disaggregate using a top-down approach based on population size. Finally, we measure saving of a consumer cell as the difference between all other inflows and outflows.

IV.B.4 Disaggregated Labor Compensation Matrix

The labor compensation matrix records income paid to consumer cell i by each producer cell (SNA D.1). We construct the matrix bottom-up, drawing on tax registers containing individual labor income (including employer pension contributions) and address and industry of the workplace (details in Appendix I). The aggregate of the matrix is slightly lower than in the national accounts, mostly because our sample contains only Danish adults. We scale each matrix entry by a common factor to match the national aggregate.

IV.B.5 Disaggregated Consumer Dividend, Mixed Income, and Surplus Matrix

We record dividend, mixed income, and surplus payments received by consumer cell i from corporate and non-corporate firms in each producer cell (details in Appendix J). To assign dividends of Danish corporations (SNA D.42), we use government data on the stock ownership of every Danish consumer. We sum stock wealth in each consumer cell and assign dividends of Danish corporations in proportion to a cell's share in national stock wealth.

Non-corporate firms are privately-owned and mixed income accrues to their owners (SNA B.3G). We observe mixed income received and workplace of every business owner in the government registers. This allows us to construct a matrix that directly measures how much each consumer cell receives in mixed income from each producer cell. Operating surplus (SNA B.2G) corresponds to the (imputed) surplus of homeowners letting housing to themselves. We disaggregate it using each consumer cell's owner-occupied housing rental income, which is directly

observed in government income registers.

IV.B.6 Disaggregated Government Benefits to Consumers

Consumer income depends on the government through various benefit programs, including unemployment, social security, pensions, child support, and student benefits (SNA D.62). We aggregate these benefits bottom-up by summing individual-level benefits from the government registers and ultimately scaling to exactly match the national aggregate (details in Appendix K). We disaggregate other transfers (SNA D.7), which include disaster or accident relief, in proportion to each cell's population share. Last, national accounts include an adjustment to avoid double counting changes in pension fund entitlements (SNA D.8). We disaggregate this position by constructing individual pension contributions and payouts from the tax registers.[7]

IV.B.7 Disaggregated Consumer Interest and Transfers Received

Interest and transfers flow from the capital accumulation cell to consumers (details in Appendix L). We disaggregate interest (SNA D.41) and natural resource rents (SNA D.45) using a procedure analogous to the one for interest and rent paid (see Section IV.B.3). We distribute national pension investment income (SNA D.44) top-down in proportion to accumulated pension contributions.

IV.C Disaggregated Flows from and to Producers

The accounting identity for producer cell j is,

$$\begin{aligned}
&\text{Domestic intermediate spending}_j + \text{Labor comp}_j + \text{Producer dividends, mixed inc, and surplus}_j \\
&+ \text{Dividends and surplus of government producers} \\
&+ \text{Producer net taxes} + \text{Producer net interest, transfers, and saving paid}_j + \text{Imports}_j \\
&= \text{Domestic intermediate sales}_j + \text{Domestic consumer spending}_j \\
&+ \text{Domestic government spending}_j + \text{Domestic capital acc spending}_j + \text{Exports}_j.
\end{aligned}$$

Producer cells include all establishments producing market and non-market output, including financial and non-financial corporations, unincorporated businesses, government-owned firms (which are usually partly government-owned and sell output at market prices), and government-

[7] In our disaggregated system, all pension-related flows (SNA D.61, D.62, and D.8) originate and go to the government cell, which has the advantage that all double-counting of pension flows naturally nets out in the government cell. Aggregate national accounts in different countries deal with pensions in different ways. In the Danish national accounts, pensions flow between consumers and government as well as between consumers and the financial corporation sector, which creates the double-counting issue.

operated firms (which are run by government employees and provide mostly non-market output to consumers, e.g., in public administration).

IV.C.1 Disaggregated Intermediates Trade Matrix

The intermediates trade matrix measures flows of intermediate goods between producer cell j and every other domestic producer cell, net of product taxes. Its national counterpart is intermediate consumption (SNA P.2). Unlike for consumer spending, we do not have detailed bottom-up microdata for these positions but instead use a top-down approach based on a gravity model (details in Appendix M). This general method was pioneered by Leontief and Strout (1963) and is still frequently used (e.g., Rodríguez-Clare et al. 2022).

The aim of the approach is to convert the cross-industry table of intermediates trade to a region-by-industry matrix. Using a separate cargo trade dataset, we estimate an elasticity of trade with respect to distance of -0.6, which is roughly constant across different types of goods. We then apply an iterative algorithm that ensures the trade matrix is consistent with the estimated distance elasticity and with the salary shares of different regions within the same industry. For example, when disaggregating intermediates trade from the agricultural industry to the restaurant industry, we ensure that cells with a greater share of agricultural employment sell more intermediates to cells with a greater share of restaurant employment, all while the distance elasticity remains constant. Next, we aggregate the matrix to the level of our industry-by-region producer cells. Since we started with 173 fine industries, the resulting intermediates trade matrix contains substantial heterogeneity in both domestic and foreign trade flows within our industries.

We also face the challenge that national input-output tables do not report the sales value of goods sold by consumer-facing industries, but only the "trade margins" earned by these merchants (final value of goods sold minus their purchase value). We therefore convert the trade matrix so that it measures actual sales values for consumer-facing industries, thereby making it consistent with our consumer spending matrix. Finally, we ensure that exports are correctly disaggregated by allocating foreign tourists' spending in Denmark across Danish producers using information on tourists' purchases and overnight stays.

IV.C.2 Disaggregated Labor Compensation, Dividend, Mixed Income, and Surplus Payments

Labor compensation (SNA D.1), dividends (SNA D.42), mixed income (SNA B.3g), and surplus (SNA B.2G) payments by producer cell j to each consumer cell appear in the matrices described above. We identify the share of dividends paid to the government by each producer cell (SNA D.42) using hand-collected data on the location and industries of Danish government-owned corporations

(details in Appendix N). We measure the share of each producer cell's surplus accruing to the government using the share of government employees in each cell.

IV.C.3 Disaggregated Producer Sales, Imports, and Exports

Intermediates sales appear in the intermediates trade matrix, whereas sales to Danish consumers appear in the domestic consumer spending matrix. We additionally record sales of producer cell j to the government ("domestic government spending," SNA P.3) by assuming that a producer cell's share in the total sales of its industry to the government equals its salary share in the industry (at the level of 173 industries, details in Appendix M). We follow an analogous procedure to measure sales of investment goods, inventories, and valuables by producer cell j to the capital accumulation cell ("domestic capital accumulation spending", SNA P.1). We also employ this approach to disaggregate producer imports (SNA P.7) and exports (SNA P.6).

IV.C.4 Disaggregated Producer Net Taxes and Net Interest, Transfers, and Saving

We measure product taxes paid by each producer cell j by disaggregating product taxes paid by industry in the national input-output table (details in Appendix F). We calculate the net of producers' non-product taxes paid and subsidies received by summing aggregate positions in the national accounts of the financial, non-financial, and unincorporated (household) production sectors.[8] The distribution rules for the net of non-product taxes and subsidies depend on accounting profits in Denmark. Hence, we distribute non-product taxes and subsidies across cells in proportion to the accounting profits of each cell (total sales minus intermediates minus labor compensation). Finally, we calculate the net of interest, transfers, and saving by each producer cell as the difference between total inflows and outflows.[9]

[8]Producer taxes paid include the following SNA positions: D.29, other taxes on production; D.5, current taxes on income, wealth; D.62, social benefits other than social transfers in kind; D.8, adjustment for the change in pension entitlements, payable. Subsidies received include: D.39, other subsidies on production; D.61, net social contributions.

[9]The closest analog to producer net interest, transfers, and saving in the aggregate national accounts is the sum of the following SNA positions in the financial and non-financial corporate accounts: interest paid - received (net of D.41) + reinvested earnings on direct foreign investments other current transfers paid - received (net of D.43) + other investment income paid - received (net of D.44) + other current transfers paid - received (net of D.7) + natural resource rents paid (D.45) + gross saving (B.8g) + distributed income of corporations paid to rest of world (part of D.42) - distributed income of corporations received (D.42). This sum based on the aggregate national accounts differs slightly from "producer net interest, transfers, and saving" in our disaggregated accounts because we do not measure labor compensation from and to foreign countries, because we record inflows of consumer-facing merchants in terms of sales and not trade margins (see step 1 in Section IV.C.1), and because we treat pension-related flows differently (see Section IV.B.6).

IV.D Flows from and to Government

We include one cell for the Danish government, so the government flows are aggregate values for the entire economy and have no subscript,

$$\begin{aligned}&\text{Domestic government spending} + \text{Government imports}\\ &+ \text{Government benefits}\\ &+ \text{Government net interest, transfers, and saving paid}\\ &= \text{Consumer taxes} + \text{Producer net taxes}\\ &+ \text{Dividends and surplus of government producers.}\end{aligned}$$

The main function of the government cell is to purchase output from domestic government-operated producers (so-called "domestic government spending") and from abroad ("government imports") and to provide this output to consumers free of charge or at low nominal fees ("consumption of government output"). Government-operated producers are primarily in public administration, defense, education, and healthcare. We already disaggregated government spending on each producer cell (see Section IV.C.3). We observe government imports directly in the national accounts of the government sector.

We also measure the consumption of government output by each disaggregated consumer cell. Since there are no financial flows associated with this consumption, it does not appear in the consumer account (equation 1). We instead record consumption of government output in a separate dataset by combining individual-level government and Danske Bank data (details in Appendix O).

We sum the respective cell-measures to calculate total government benefits paid (Section IV.B.6), taxes collected from consumers (Sections IV.B.2 and IV.B.6), and producers (Section IV.C.4), and dividends and surplus from government-owned corporations (Section IV.C.2). Finally, we measure government net interest, transfers, and saving outflows as the difference between all other outflows and inflows.[10]

[10]The closest analog to government net interest, transfers, and saving in the aggregate national accounts is the sum of the following government SNA positions: interest paid - received (net of D.41) + other current transfers paid - received (net of D.7) + gross saving (B.8g) - other investment income (D.44) - natural resource rents received (D.45). However, this sum based on the aggregate national accounts does not equal "net interest, transfers, and saving" in our disaggregated accounts because we do not disaggregate taxes, benefits, and subsidies received by institutional sectors other than producer and consumers and because we treat pension-related flows slightly differently (see Section IV.B.6).

IV.E Flows from and to the Rest of the World and the Capital Accumulation Cell

We measure inflows and outflows for two remaining cells: the rest of the world and capital accumulation. This step ensures that all consumer, producer, and government flows disaggregated so far have a well-defined counter-party cell. The accounting identity for the rest of the world is,

$$\begin{aligned}&\text{Foreign consumer spending} + \text{Producer imports}\\&+ \text{Government imports} + \text{Capital acc imports} + \text{Trade balance}\\&= \text{Producer exports}.\end{aligned}$$

We have so far measured all these flows except capital accumulation imports, which we observe as imports of investment goods, valuables, and inventory in the national input-output table. The trade balance equals net exports (including foreign consumer spending) and captures the money entering Denmark due to its net exports.[11]

The accounting identity for the capital accumulation cell is,

$$\begin{aligned}&\text{Consumer interest, transfers, and saving rec} + \text{Domestic capital acc spending}\\&+ \text{Capital acc imports} + \text{Trade balance}\\&= \text{Consumer interest, transfers, and saving paid}\\&+ \text{Producer net interest, transfers, and saving paid}\\&+ \text{Government net interest, transfers, and saving paid}.\end{aligned}$$

We have already accounted for all these flows in the previous sections.

IV.F Overview of the Disaggregated Circular Flow in Denmark

We summarize and visualize the Danish system of disaggregated economic accounts. In Table II, we list all disaggregated flows along with the closest related SNA position.

[11]The Danish trade balance in our data is slightly below the aggregate trade balance measured in the national accounts (SNA P.6 - P.7) because the spending matrix implies a slightly higher value for foreign consumer spending, and thus slightly higher total imports. An advantage of our approach is that we can directly observe foreign spending by Danish consumers in the Danske Bank data. In contrast, national accounts rely on balance of payments statistics, retail turnover, and consumer surveys to determine foreign spending.

Table II: Overview of all flows in the Danish disaggregated economic accounts

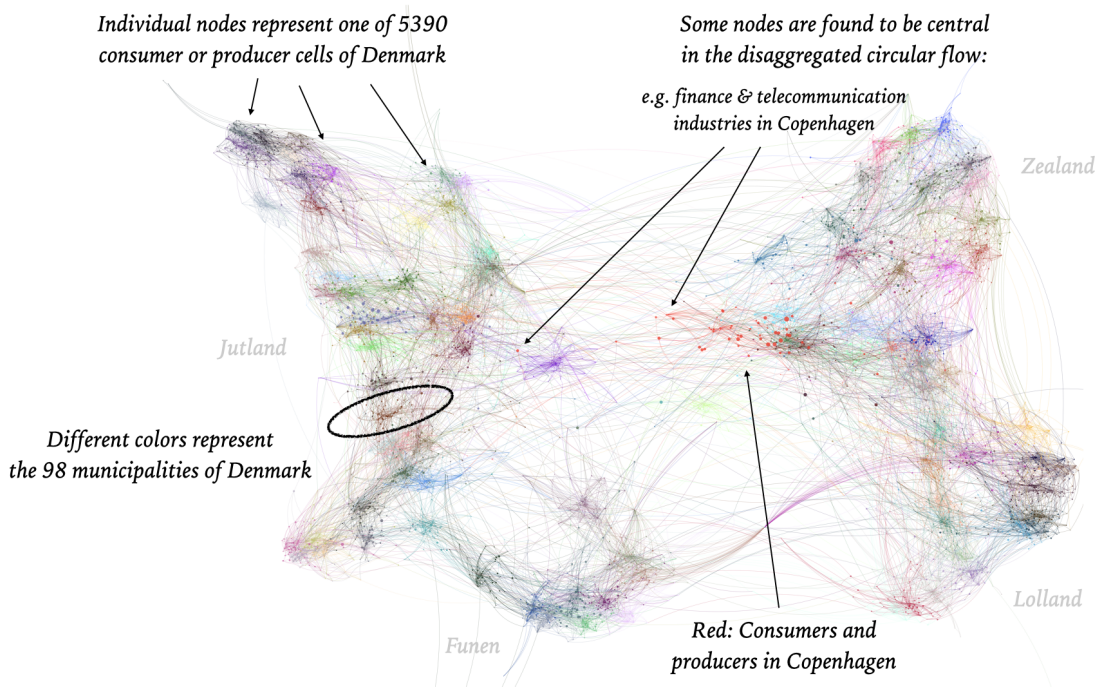
	Disaggregated flow name	Related SNA code	Outflow from	Inflow to	Disaggregated dataset	Total value (bn DKK)
1	Domestic consumer spending	P.3[a]	Consumers	Producers	Domestic consumer spending matrix	771.9
2	Foreign consumer spending	P.3[a]	Consumers	Rest of world	Foreign consumer spending	81.9
3	Consumer product taxes paid	D.21	Consumers	Government	Consumer taxes	173.2
4	Consumer non-product taxes paid	D.5	Consumers	Government	Consumer taxes	566.4
5	Consumer social contributions paid	D.61	Consumers	Government	Consumer taxes	181.1
6	Consumer interest paid	D.41	Consumers	Capital acc.	Consumer interest, transfers, and saving	29.7
7	Consumer natural resource rents paid	D.45	Consumers	Capital acc.	Consumer interest, transfers, and saving	3.4
8	Consumer other transfers paid	D.7	Consumers	Capital acc.	Consumer interest, transfers, and saving	44.8
9	Consumer gross saving	B.8g	Consumers	Capital acc.	Consumer interest, transfers, and saving	130.0
10	Labor compensation	D.1	Producers	Consumers	Consumer labor compensation matrix	1,141.8
11	Mixed income from non-corporate producers	B.3g	Producers	Consumers	Consumer dividend, mixed income, and surplus matrix	80.7
12	Surplus of corporate producers to consumers (dividends)	D.42	Producers	Consumers	Consumer dividend, mixed income, and surplus matrix	38.4
13	Surplus of owner-occupied housing to consumers	B.2g	Producers	Consumers	Consumer dividend, mixed income, and surplus matrix	83.3
14	Consumer social benefits received	D.62	Government	Consumers	Government benefits to consumers	422.2
15	Consumer adjustment for pension entitlements received	D.8	Government	Consumers	Government benefits to consumers	92.5
16	Consumer interest received	D.41	Capital acc.	Consumers	Consumer interest, transfers, and saving	5.3
17	Consumer pension investment income	D.44	Capital acc.	Consumers	Consumer interest, transfers, and saving	75.5
18	Consumer natural resource rents received	D.45	Capital acc.	Consumers	Consumer interest, transfers, and saving	3.4
19	Consumer other transfers received	D.7	Capital acc.	Consumers	Consumer interest, transfers, and saving	39.2
20	Domestic intermediates	P.2[b]	Producers	Producers	Intermediates trade matrix	1,414.5
21	Dividends and surplus of government-owned/operated producers to government	D.42	Producers	Government	Government dividend and surplus income	67.9
22	Producer product taxes paid	D.21	Producers	Government	Producer net taxes	71.9
23	Producer net non-product taxes paid and subsidies received	D.29 + D.5 + D.62 + D.8 - D.39 - D.61	Producers	Government	Producer net taxes	82.8
24	Producer net interest, transfers, and saving	(D.41 + D.43 + D.44+ D.7) [net outflow] + D.45 + B.8g - D.42 + D.42 to rest of world[c]	Producers	Capital acc.	Producer net interest, transfers, and saving	422.4
25	Producer imports	P.7[b]	Producers	Rest of world	Producer imports	792.3
26	Domestic government spending	P.3[b]	Government	Producers	Domestic government spending	572.3
27	Domestic capital accumulation spending	P.1[b]	Capital acc.	Producers	Domestic capital acc. spending	359.5
28	Producer exports	P.6[b]	Rest of world	Producers	Producer exports	1,077.9
29	Government imports	P.7[b]	Government	Rest of world	Government imports	4.3
30	Government net interest, transfers, and saving	(D.41 + D.7) [net outflow] + B.8g - D.44 - D.45[d]	Government	Capital acc.	Government net interest, transfers, and saving	52.0
31	Capital accumulation cell imports	P.7[b]	Capital acc.	Rest of world	Capital acc. imports and trade balance	98.9
32	Aggregate trade balance	P.6 - P.7[e]	Rest of world	Capital acc.	Capital acc. imports and trade balance	100.6
33	Consumption of government output	P.3	Provided free of charge to consumers		Consumption of government output	578.6

Notes: The table lists all the individual flows that make up the measured disaggregated economic accounts in Denmark. "Related SNA code" indicates the closest analog to the disaggregated flow in the standard national accounts. a: disaggregated flow excludes product taxes, unlike SNA (Section IV.B.1). b: disaggregated flow measures output of consumer-facing producers in terms of sales, whereas SNA measures trade margins (Section IV.C.1). c: disaggregated flow and SNA position defined slightly differently (Footnote 9). d: disaggregated flow and SNA position defined slightly differently (Footnote 10). e: foreign spending higher in disaggregated flow than in SNA (Footnote 11).

To visualize the system, we construct a disaggregated analog to the circular flow of money. The classical circular flow contains a handful of nodes for groups at the national level. In contrast, our disaggregated circular flow in Figure II contains 5,390 nodes, one for each region-by-industry consumer and producer cell in Denmark.[12] Nodes lying in the same region share the same color. Node size on the plot is proportional to the cell's economic size, measured as the square root of all inflows into the cell.

The disaggregated system allows us to calculate the total amount of money flowing from each cell to every other cell through any type of transaction (consumer spending, income, intermediates trade). We visualize cell-to-cell flows by drawing a link from a source cell to a receiving cell if the flow between the two cells is among the five largest outflows for the source cell or the single largest inflow for the receiving cell.[13] We let an algorithm ("ForceAtlas 2", Jacomy et al. 2014) arrange the nodes, so that cell pairs with larger pairwise flows are located next to each other.

Figure II: The disaggregated circular flow of money



Notes: Nodes are all consumer and producer cells in Denmark. We draw a link between two cells if the cell-to-cell flow is among the top five outflows per cell or the top inflow per cell. Nodes are arranged according to the "ForceAtlas 2" algorithm of Jacomy et al. (2014).

A few patterns are noteworthy. First, nodes in the same region (and color) are close together.

[12] We deliberately do not plot nodes for the government, capital accumulation, and rest of the world cells, since these cells are orders of magnitude larger than individual consumer and producer cells, making it difficult to see flows between consumer and producer cells.

[13] The plot looks similar but more crowded if these restrictions are relaxed.

The shape of the graph is strikingly similar to the geography of Denmark: the large cluster of nodes on the left is the continental Western part of Denmark, the small cluster in the bottom is the central island Funen (with major city Odense), and the large Eastern island Zealand with the capital Copenhagen (red) lies on the right. However, there are also notable deviations from the geography of Denmark. For example, the producer cells for airline and shipping industries in Copenhagen (red central nodes) sit much more centrally in the plot than the location of Copenhagen would suggest, mirroring their central position in the disaggregated circular flow.

V Facts Based on Disaggregated Economic Accounts in Denmark

We analyze the newly measured system of disaggregated economic accounts and present facts about how different parts of the the Danish economy are connected.

V.A Most Flows are Regionally Concentrated and Sparse

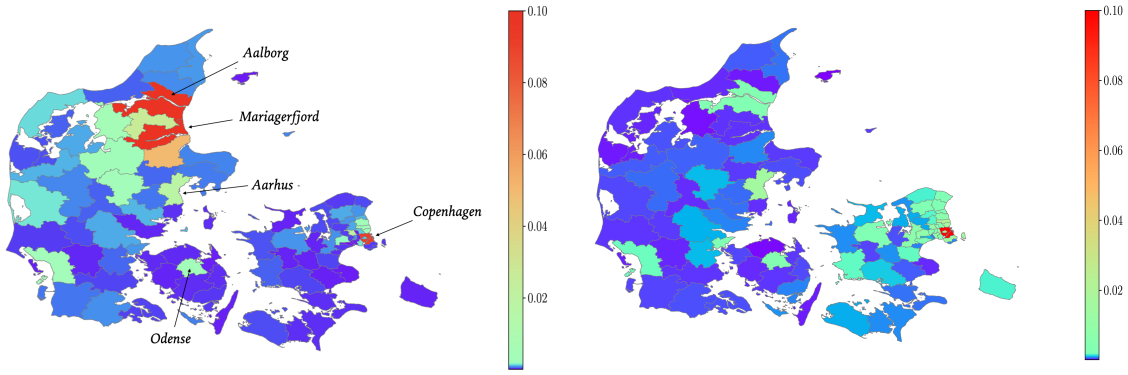
We begin by exploring the role of geographic distance for domestic consumer spending flows. Figure III plots the geographical distribution of domestic consumer spending for restaurant workers living in rural Mariagerfjord (Panel IIIa) and in the capital city Copenhagen (Panel IIIb). In both cases, we see that spending is regionally concentrated and decreases with distance. However, deviating from the pattern of regional concentration, a large share of spending from rural Mariagerfjord goes to the big cities Aalborg, Aarhus, Odense, and Copenhagen. In contrast, little spending from Copenhagen flows to rural regions and essentially no spending flows to far-away rural regions.[14]

[14] See Figure A.IV for similar maps of disaggregated labor compensation flows.

Figure III: Spending shares across regions: Examples

(a) Restaurant workers in Mariagerfjord

(b) Restaurant workers in Copenhagen



Note: We plot the fraction of spending received by producers in each region, for spending by restaurant workers in Mariagerfjord (Panel a) and Copenhagen (Panel b). The scale is truncated at 0.10. We include only spending to Danish producers in the calculations.

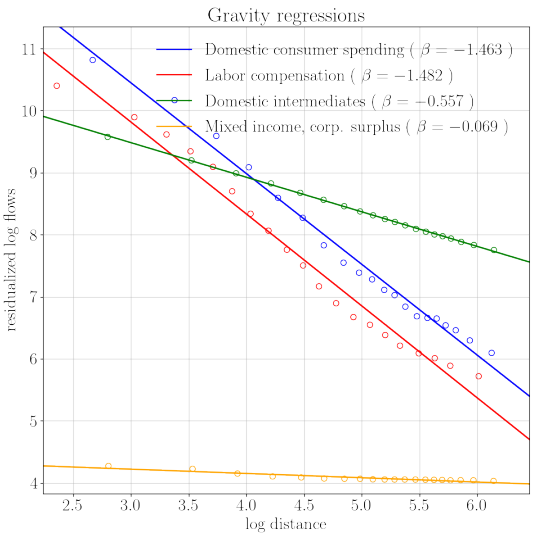
We formalize the role of distance by estimating gravity specifications for consumer spending. We regress spending from a consumer cell i on a producer cell j on driving distance (measured using the Google Maps API) and a full set of consumer and producer cell fixed effects,

$$\log \text{flow}_{i \to j} = \alpha_i + \delta_j + \beta \times \log \text{distance}_{ij} + \epsilon_{ij}. \tag{2}$$

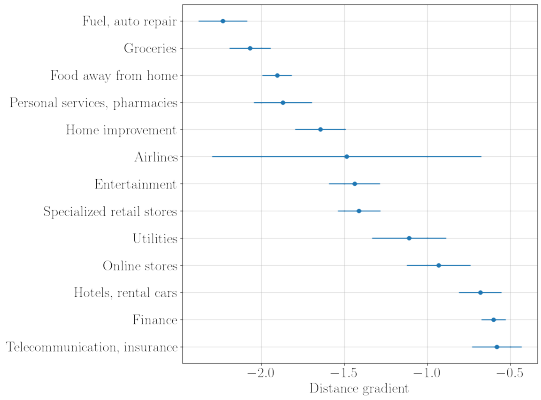
The fixed effects control for all cell-specific factors that determine total spending out of or into a cell, such as population, average income, or industry. Panel a of Figure IV shows a binned scatter plot and Table A.III the associated regression table. The relation between spending and distance is negative and close to linear, with a gradient of -1.5. Panel b shows that there is significant heterogeneity depending on the types of goods purchased (consistent with data on card spending in Agarwal et al. 2020). An advantage of the new spending matrix is that it contains the universe of consumption goods, not just those purchased with cards. Spending on fuel, auto repair, and groceries, which often involves in-person shopping trips, decreases steeply with distance. Spending on telecommunications, insurance, and financial services, which requires no in-person interaction and is often done via bank transfer, and spending on hotels and rental cars, which is often part of travel, still decline with distance but less steeply.

Figure IV: Gravity

(a) Across types of flows



(b) Across receiving industries for consumer spending



Notes: Panel (a) shows binned scatter plots estimated separately for different flows using equation (2). We plot averages of $\overline{\alpha} + \overline{\delta} + \beta \times \log \text{ distance}_{ij} + \epsilon_{ij}$ for 20 evenly-sized bins of distance. Flows are in log DKK and distance is in log driving km. Flows are residualized using fixed effects for source cells and receiving cells. We add the mean of log flows to the residualized variables. We exclude observations with zero distance or zero flow. The solid line is the line of best fit, estimated using the cell-level data. Panel (b) repeats the estimation for consumer spending by receiving industry. Figure A.V for labor compensation flows. The lines show 95% confidence intervals. All standard errors are clustered by region and industry.

We also display gravity results for labor compensation, intermediate inputs, and mixed plus surplus income (sum of mixed income, surplus of corporate producers, and surplus of owner-occupied housing payments to consumers). Labor compensation displays an average distance gradient close to that of consumer spending, although the labor compensation gradient is flatter at low and high distances and steeper for middle ranges.[15] Trade in intermediate inputs declines less with distance, as the distance coefficient estimated in the cargo data is -0.6. The low distance coefficient for mixed plus surplus income is mainly driven by corporate surplus payments (i.e., the relatively high distance between stock owners and corporate producers).

In general, the disaggregated matrices are sparse. For most cells, outgoing flows tend to go to a few destination cells while incoming flows tend to come from a few source cells. For instance, the largest 10% of the 7.3 million pairwise consumer spending flows account for roughly 98% of aggregate consumer spending (as spending mostly goes to nearby regions as well as near and far cities).

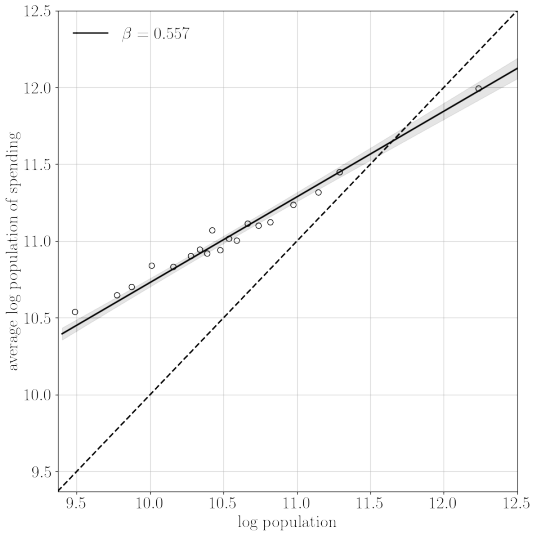
[15]The similarity in the distance coefficients between labor compensation flows and consumer spending flows is partly due to the correlation between labor compensation and consumer spending shares, see Figure A.VI.

V.B Consumer Spending Flows Toward Cities

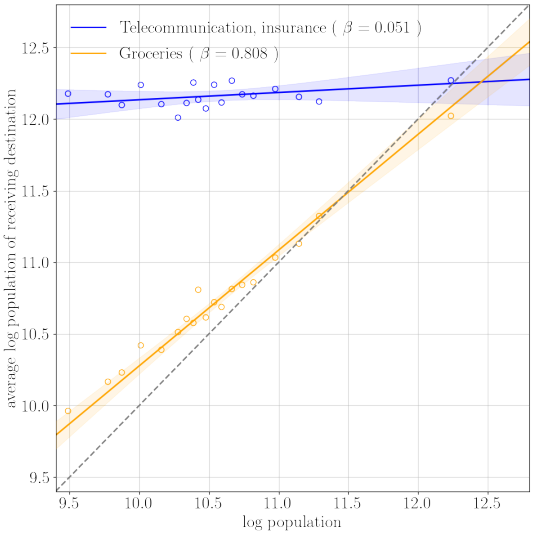
Figure III already illustrated that a larger share of spending goes to cities than to rural regions. We next show that spending within Denmark systematically travels to more urban regions. We use population size to identify more urban regions, but results are similar when we use density. Panel a of Figure V plots the average size of producer cells (which receive spending) for consumer cells binned by size. Almost all points lie above the 45 degree line, indicating that consumers tend to spend in more urban regions than in their home region.

Figure V: Population of home and receiving region

(a) All categories



(b) Only telecommunications and groceries



Notes: The panels show average log population of regions receiving consumer spending (vertical axis) for 20 evenly-sized bins of consumers' home region log population (horizontal axis). We include only spending to Danish producers in the calculations. The solid line is the line of best fit, estimated using the cell-level data. The shaded areas represented 95% robust confidence intervals.

There is heterogeneity in the degree of spending flowing to cities depending on the receiving industry, shown in Panel Vb. Consumers purchase a large share of groceries in their home region, implying that the line for groceries is close to the 45 degree line, though still above it. In contrast, spending on telecommunication services and insurance takes place in similar urban locations for almost all consumers, implying that the line for that category is relatively flat and above the 45 degree line.[16]

The flow of spending toward more urban regions represents a statistically and economically

[16]Figure A.VII shows the estimated population gradients for all consumer spending categories. Figure A.VIII shows that in-person spending also flows toward cities.

significant deviation from the standard gravity equation,

$$\log \text{flow}_{i\to j} = \alpha_i + \delta_j + \beta \times \log \text{distance}_{ij} + \gamma_{\text{destination}} \times \log \text{distance}_{ij} \times \log \text{density}_i + \gamma_{\text{origin}} \times \log \text{distance}_{ij} \times \log \text{density}_j + \epsilon_{ij}. \quad (3)$$

Table III shows that spending flowing into large regions is less sensitive to distance, indicating the consumers from near and far tend to spend in cities, whereas spending in rural areas is more likely to come from closer consumers (column 2). Since the gravity specification includes cell fixed effects, spending flows toward cities cannot be explained by differences in size, industrial composition, or other attributes of cities. We also find that spending by consumers from small regions is more sensitive to distance, indicating that rural consumers are less likely to spend at distant retailers, relative to urban consumers (column 3).

Table III: Gravity regressions with interactions

	log spending			
	(1)	*(2)*	*(3)*	*(4)*
log distance	-1.463^{***}	-1.781^{***}	-1.765^{***}	-2.098^{***}
	(0.028)	(0.119)	(0.046)	(0.126)
log distance × log dest. pop.		0.573^{***}		0.585^{***}
		(0.209)		(0.208)
log distance × log origin pop.			0.551^{***}	0.564^{***}
			(0.068)	(0.068)
Origin FE	Yes	Yes	Yes	Yes
Destination FE	Yes	Yes	Yes	Yes
Observations	2561029	2561029	2561029	2561029
R^2	0.332	0.333	0.333	0.334
F stat.	1268417.4	637166.1	637012.6	426738.8

$^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

Notes: Distances are measured as driving distances between region centers. For the interaction terms, we normalize log population to range from 0 to 1 across regions. The interaction coefficient therefore shows the change in the distance gradient when moving from lowest to highest population size. We include only spending to Danish producers in the calculations.

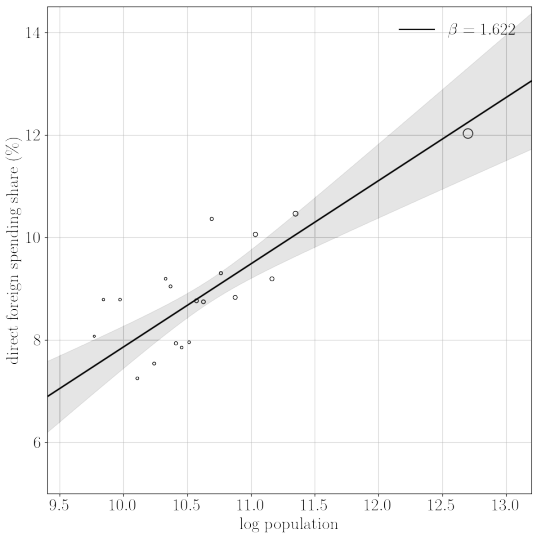
V.C Urban, Rich Consumers Spend More Abroad

We next document which Danish consumers spend abroad. Figure VI plots direct foreign spending shares by consumer cells' size and average spending. Our measure of foreign spending includes both in-person spending abroad as well as spending on foreign online shops. Across the two panels, we see that size and average spending predict direct foreign spending positively: urban,

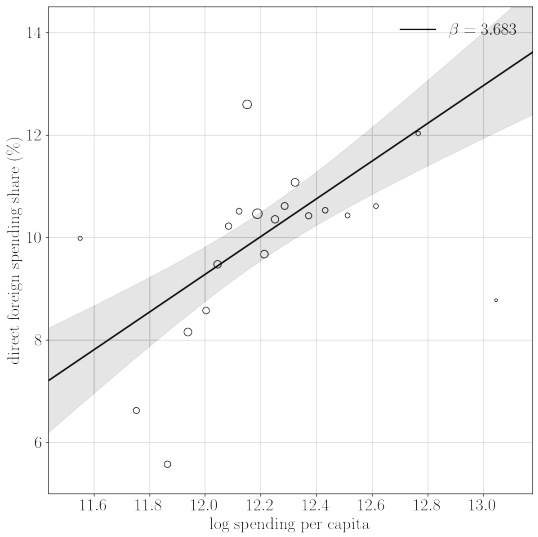
rich consumers spend a larger share of their total spending abroad.[17]

Figure VI: Foreign spending by population size and total spending

(a) By population size



(b) By total spending per capita



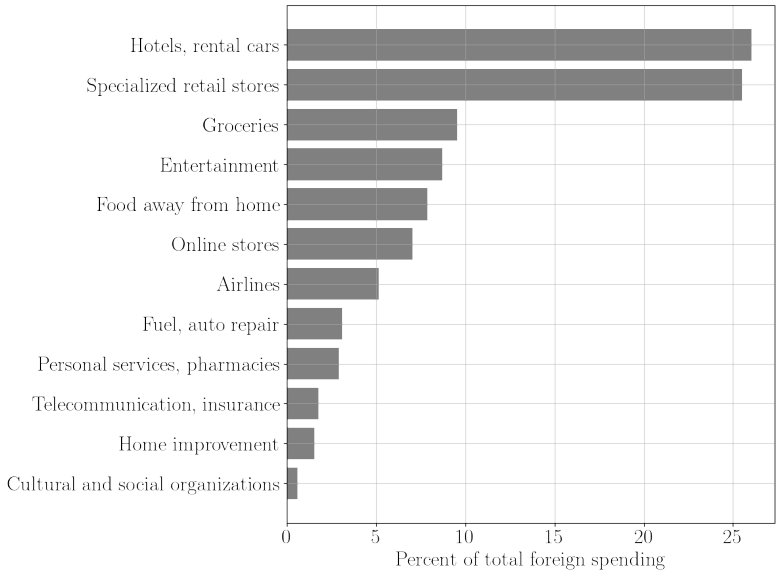
Notes: Panel (a) shows the direct foreign spending share of consumer cells by their home region's log population. Panel (b) shows the direct foreign spending share of consumer cells by their log spending per capita. Both plots are binned scatter plots with 20 bins. Weights are proportional to the number of bank customers in a municipality. The shaded area are 95% robust confidence intervals. See Figure A.IX for plots with population density and income per capita.

The spending matrix also contains information on which types of goods are purchased abroad. Figure VII shows that spending on hotels, rental cars, and airlines accounts for around 26 percent of total foreign spending, entertainment and food away from home for around 15 percent, and spending on groceries and specialized retail stores (e.g., books, computers, shoes, clothing) for about 35 percent.[18]

[17] In Table A.V, we show that population and average spending predict foreign spending also when controlling for driving or geographic distance to the Danish border.

[18] Figure A.X decomposes the slope in Panel (a) of Figure VI by categories, confirming the importance of travel-related expenses.

Figure VII: Foreign spending shares by spending category



Notes: This bar graph shows the distribution of direct foreign spending by consumers across spending categories.

Taken together, the facts established so far imply that rural consumers allocate a larger share of their spending to their home region and to cities. In contrast, urban consumers allocate a larger share to their home region and abroad. These patterns suggest that shocks to the spending of rural consumers affect rural and urban producers more strongly, while shocks to urban consumers impact urban and foreign producers. We will explore the asymmetric consequences of such shocks below using a formal model. Before doing so, however, we study how other flows, apart from consumer spending, differ between rural and urban regions.

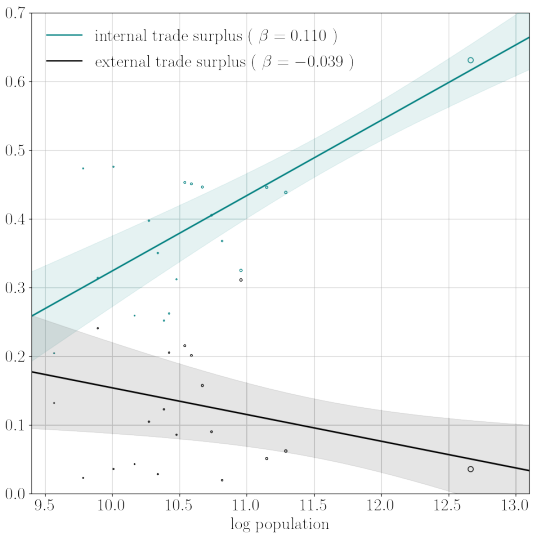
V.D Rural Regions Export Abroad, Urban Regions Domestically

We now investigate all other flows that enter and leave different regions. We use our disaggregated economic accounts to construct regional balance of payments (BOP) and GDP statistics. We compute five cross-regional flows, which jointly add up to zero: (i) *external trade surplus*, defined as a region's trade surplus with the rest of the world (producer exports - producer imports - foreign consumer spending); (ii) *internal trade surplus*, the trade surplus with the rest of the Danish economy (domestic intermediates + domestic consumer spending + domestic government spending + domestic capital accumulation spending - domestic intermediate outflows/purchases); (iii) *net factor payments* (net of: labor compensation + producer dividends, mixed income, and surplus + natural resources rents); (iv) *net transfers* (net of: social benefits - taxes - social contributions); and (v) *net borrowing* (negative of: consumer interest, transfers, and saving + net producer interest, transfers, and saving). We compute (nominal) regional GDP as total production value net of

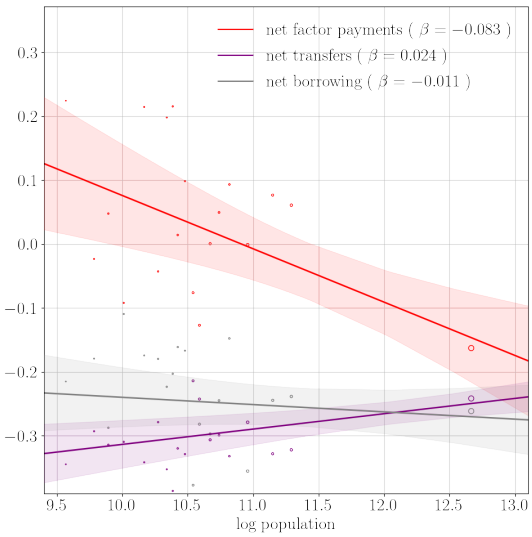
domestic and foreign intermediate purchases.

Figure VIII: Regional balance of payments (normalized by GDP)

(a) External and internal trade surplus



(b) Other items of regional balance of payments



Notes: The panels show the five components of the regional balance of payments (BOP) statistics, normalized by regional GDP. The five components add up to zero for each region. The "internal trade surplus" includes investment and government spending, see Footnote 19. 20 bins. Weights are region GDP. Shaded areas are 95% robust confidence intervals.

Figure VIII shows how each of the five components, normalized by regional GDP, varies with a region's size. The external trade surplus is largest in rural regions, indicating that rural regions export a significant fraction of their output to foreign countries (Panel VIIIa). The internal trade surplus is largest in urban regions, indicating that those regions export a large share of their output to the rest of Denmark, in particular in the form of intermediates supplied to rural regions.[19] Net transfers and net borrowing do not vary significantly, whereas net factor payments flow into rural regions and out of urban regions, on average (Panel VIIIb). Taken together, these facts reveal that, on net, money flows from abroad into domestic rural cells, while it flows from domestic rural cells to domestic urban cells (e.g., through consumer spending flowing toward cities).

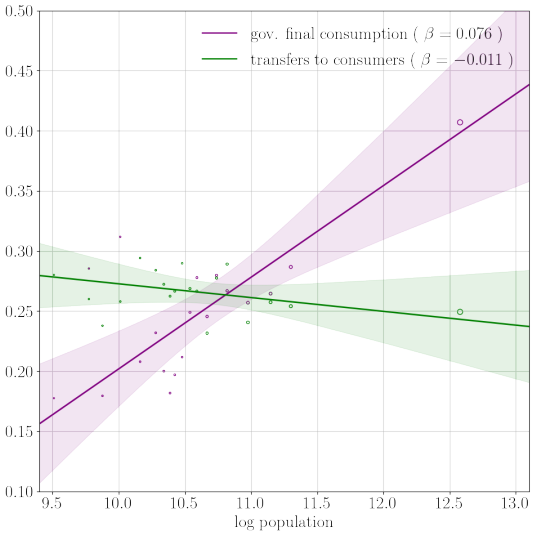
[19]The internal trade surplus does not average out to zero in Panel VIIIa because it includes capital accumulation spending and government spending, which are paid for by the capital accumulation and government cells, respectively. Since those two cells do not have a region assigned to them, they do not show up in Panel VIIIa. See Figure A.XI for decompositions of the internal and external trade surpluses by industry. They highlight that both are mostly driven by services, such as telecommunications, finance, shipping, and personal services.

V.E Government Redistributes Toward Cities

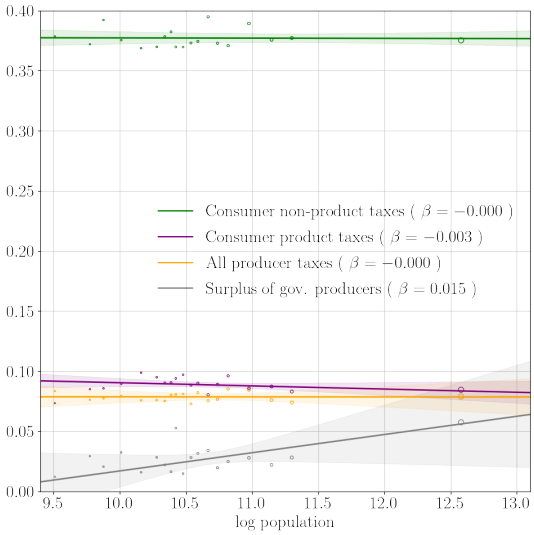
The regional BOP statistics also allow us document which regions rely more on transactions with the government. The government purchases more output from urban regions (including output produced in public administration, education, and healthcare), relative to regional GDP. However, it transfers more benefits to rural regions, relative to regional GDP. Overall, the size of purchases is larger than that of transfers, implying that, on net, the government transfers resources into urban regions.

Figure IX: Government consumption and benefits across regions (by region GDP)

(a) Outlays



(b) Taxes

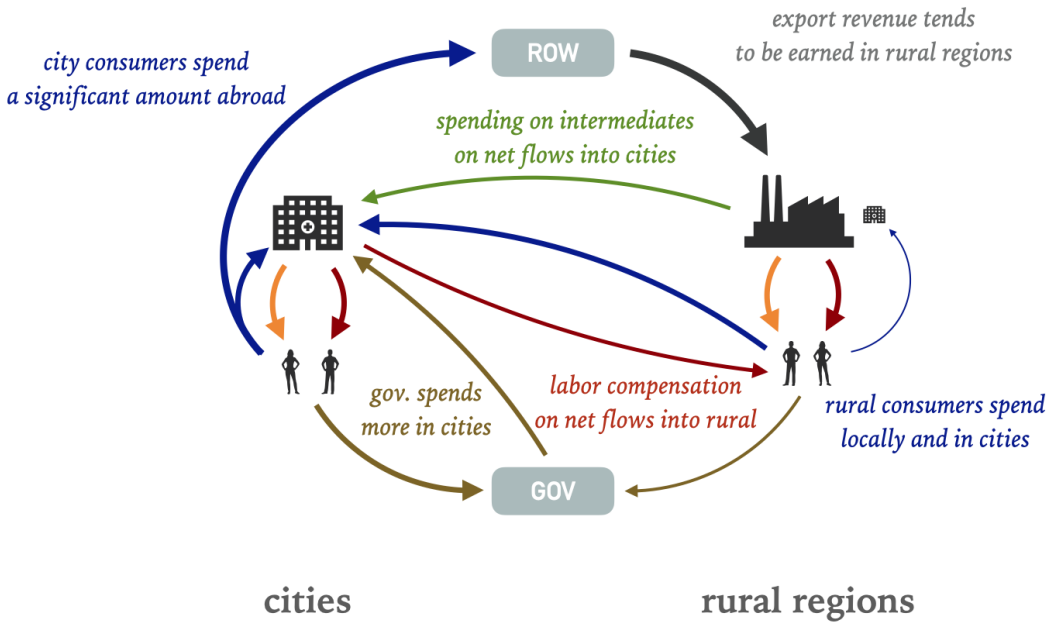


Notes: The panels show all flows between individual regions and the government, normalized by region GDP, as function of a region's population size. Binned scatters with 20 bins. Weights are region GDP. Shaded areas are 95% robust confidence intervals.

V.F Stylized Overview of the Disaggregated Circular Flow

Figure X provides a stylized overview of the facts we have documented. Export revenue is largest in rural regions where it contributes to rural consumers' income. On net, consumer spending leaves rural regions toward cities where it contributes to the income of urban consumers working in service industries. Similarly, spending on intermediate goods on net flows toward cities. In turn, urban consumers spend a relatively large share of income abroad. The government transfers income to rural consumers, but spends on urban industries, so that, on net, resources flow into cities.

Figure X: Stylized overview of the circular flow



VI A Model of Disaggregated Economics Accounts

We have so far discussed how to measure disaggregated economic accounts and that they reveal new facts about connections between different parts of the economy. Next, we study whether disaggregated economic accounts improve our understanding of the aggregate and distributional effects of shocks. We therefore develop a model that allows us to analyze how a range of shocks propagate across consumer and producer cells. We then calibrate the model using the measured disaggregated economic accounts.

VI.A Overview

We model a small open economy composed of region-industry consumer and producer cells, which are linked via consumer spending, factor payments to labor and capital owners, and trade in intermediate goods. The region-industry cells also transact with a government cell and the rest of the world. The model builds on seminal previous work, among others by Caliendo and Parro (2015) and Baqaee and Farhi (2019b). It is supposed to capture the medium-run behavior of an economy. Because we focus on medium-run effects, the model is static, abstracts from nominal rigidities, and does not include a capital accumulation cell (i.e., no saving and investment). We leave a dynamic extension of the model to future work.

There exists a set $\mathcal{I}$ of consumer cells (corresponding to our region-by-industry of work cells in

the data, indexed by $i \in \mathcal{I}$) and a set $\mathcal{J}$ of producer cells (corresponding to our region-by-industry cells in the data, indexed by $j \in \mathcal{J}$). The (large) representative consumer that populates the rest of the world is labeled $i = \mathcal{R} \notin \mathcal{I}$ and the foreign representative producer cell is $j = \mathcal{R} \notin \mathcal{J}$. We write $\mathcal{I} \cup \{\mathcal{R}\}$ and $\mathcal{J} \cup \{\mathcal{R}\}$ whenever the rest of the world is included in the set of indices at consideration. Each cell lies in an industry s, which is an element of the set $\mathcal{S}_\mathcal{I}$ for consumers and the set $\mathcal{S}_\mathcal{J}$ for producers. We denote by $s(i)$ and $s(j)$ the industries associated with cells i and j, respectively. We use the price level of the goods produced by the rest of the world as our numeraire and denote all prices and wages in those units. This is not only convenient but also realistic for the Danish economy, which effectively operates a fixed exchange rate against the Euro.

VI.B Setup

We describe consumer and producer cells in turn.

Consumer cells: utility. A mass L_i of consumers lives within each consumer cell i, all sharing the same preferences and maximizing utility over consolidated private consumption C_i and consumption of government-provided services G_i,

$$U_i = \log C_i + \chi_i \log G_i = \alpha_{i\mathcal{R}} \log c_{i\mathcal{R}} + (1 - \alpha_{i\mathcal{R}}) \sum_{s \in \mathcal{S}_\mathcal{J}} \alpha_{is} \log \left(\sum_{j:s(j)=s} \alpha_{isj}^{\frac{1}{\sigma}} c_{ij}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} + \chi_i \log G_i, \tag{4}$$

where $\sum_{s \in \mathcal{S}_\mathcal{J}} \alpha_{is} = \sum_{j:s(j)=s} \alpha_{isj} = 1$, and $\chi_i > 0$. $c_{i\mathcal{R}}$ is consumer cell i's consumption of foreign goods and c_{ij} is consumer cell i's consumption of goods produced in producer cell j. Equation 4 describes a nested CES utility function, with a Cobb-Douglas specification across industries and a CES specification with elasticity $\sigma > 0$ within industries.

Consumer cells: labor supply. Consumer industries $\mathcal{S}_\mathcal{I}$ are of two types: work industries or non-work industries (listed in Table A.I). If consumer cell i is in a work industry, it has a total labor endowment of N_i, for which it earns a wage W_i. Total labor income is then $N_i W_i$. Consumer cells in non-work industries do not supply labor.

Consumer cells: budget constraint. In addition to labor income, consumer cells earn profit income $\sum_{j \in \mathcal{J}} \kappa_{ij} \Pi_j$. The total profit of producer cell j is Π_j. The share of producer cell j's profits earned by consumer cell i is $\kappa_{ij} \in [0, 1]$. Further, consumer cell i pays a proportional income tax rate τ_i, a domestic product tax τ_i^{vat}, and receives a government transfer T_i. Total nominal pre-tax

income of consumer cell i is therefore

$$Y_i = N_i W_i + \sum_{j \in \mathcal{J} \cup \{\mathcal{R}\}} \kappa_{ij} \Pi_j + T_i. \tag{5}$$

The consolidated budget constraint of cell i is then

$$c_{i\mathcal{R}} + \sum_{j \in \mathcal{J}} \left(1 + \tau_i^{vat}\right) P_j c_{ij} \leq \left(1 - \tau_i\right) Y_i. \tag{6}$$

Producer cells. There is a representative firm in each producer cell $j \in \mathcal{J}$. It produces quantity Q_j of its good j with technology

$$Q_j = Z_j \left(\sum_{i \in \mathcal{I}} \lambda_{ij}^{\frac{1}{\zeta}} N_{ij}^{\frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}\gamma_j^N} \prod_{s \in \mathcal{S}_\mathcal{J}} \left(\sum_{j' \in \mathcal{J} \cup \{\mathcal{R}\}: s(j')=s} \omega_{jsj'}^{\frac{1}{\eta}} X_{j'j}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}\omega_{js}\gamma_j^X}, \tag{7}$$

where $\sum_s \omega_{js} = \sum_{j':s(j')=s} \omega_{jsj'} = \sum_i \lambda_{ij} = 1$. Producer cell j uses N_i units of labor supplied by consumer cell i in the same industry, $s(i) = s(j)$, with total j-specific labor income share γ_j^N.[20] Cell j uses $X_{j'j}$ units of intermediate goods, purchased from firm cell j', for each $j' \in \mathcal{J} \cup \{\mathcal{R}\}$. The elasticity of substitution between producer cells in the same industry is $\eta > 0$. The elasticity of substitution between labor supplied by different consumer cells is $\zeta > 0$. Total factor productivity is given by Z_j. We assume weakly decreasing returns to scale, $\gamma_j^N + \gamma_j^X \leq 1$, allowing for potentially fixed factors. Pretax profits are

$$\Pi_j^{pre} = P_j Q_j - \sum_{i \in \mathcal{I}} W_i N_{ij} - \sum_{j' \in \mathcal{J} \cup \{\mathcal{R}\}} P_{j'} X_{j'j}. \tag{8}$$

Profits are taxed at the corporate tax rate $\tau_j^{corp} \geq 0$. After-tax profits are then

$$\Pi_j = \left(1 - \tau_j^{corp}\right) \Pi_j^{pre}. \tag{9}$$

Rest of the world. Domestic consumers and producers buy foreign goods at price $P_\mathcal{R}$. Export demand for domestically produced goods is

$$x_j = \tilde{x}_j \cdot P_j^{-\gamma}, \tag{10}$$

where the elasticity of exports to the terms of trade is equal to $\gamma > 0$ and $\tilde{x}_j$ is a shifter for the rest of the world's preference for good j. Trade will be balanced in equilibrium.

[20] We set $\lambda_{ij} = 0$ if $s(i) \neq s(j)$.

Government. The government pays (nominal) transfers T_i, spends on domestically produced goods G_j, and imports goods $G_{\mathcal{R}}$, all financed by tax revenue. The government budget constraint is given by

$$\sum_{j\in\mathcal{J}\cup\{\mathcal{R}\}} P_j G_j + \sum_{i\in\mathcal{I}} T_i = \sum_{i\in\mathcal{I}} \tau_i Y_i + \sum_{j\in\mathcal{J}} \tau_j^{corp} \Pi_j^{pre} + \tau^{vat} \sum_{j\in\mathcal{J}\cup\{\mathcal{R}\}} P_j c_{ij}. \tag{11}$$

We assume that the government follows a fiscal rule that adjusts government purchases and transfers in line with revenues, that is

$$P_j G_j = g_j \cdot \left(\sum_{i\in\mathcal{I}} \tau_i Y_i + \sum_{j\in\mathcal{J}} \tau_j^{corp} \Pi_j^{pre} + \tau^{vat} \sum_{j\in\mathcal{J}\cup\{\mathcal{R}\}} P_j c_{ij} \right)$$
$$T_i = t_i \cdot \left(\sum_{i\in\mathcal{I}} \tau_i Y_i + \sum_{j\in\mathcal{J}} \tau_j^{corp} \Pi_j^{pre} + \tau^{vat} \sum_{j\in\mathcal{J}\cup\{\mathcal{R}\}} P_j c_{ij} \right)$$

such that $\sum_j g_j + \sum_i t_i = 1$. We vary the fiscal rule in Section VIII.B.

We assume further that goods purchased by the government maximize some homothetic aggregator $v\left(\{G_j\}\right)$, with price index $\mathcal{P}(\{P_j\})$, of which consumer cell i consumes a fixed share ν_i, $G_i = \nu_i \cdot v\left(\{G_j\}\right)$.[21] The weight χ_i on consumer cell i's consumption of government output is consistent with the Samuelson condition for government spending, $\chi_i = \nu_i \cdot \left(\sum_{j\in\mathcal{J}\cup\{\mathcal{R}\}} P_j G_j\right) / \sum_{j\in\mathcal{J}\cup\{\mathcal{R}\}} P_j c_{ij}$.

Equilibrium. We define equilibrium as in any competitive model with flexible prices.

Definition 1. A *competitive equilibrium* in the economy consists of prices and wages $\{P_j, W_i\}$ and an allocation $\{Q_j, N_{ij}, X_{j'j}, \Pi_j, T_i, G_j, Y_i, c_{ij}, x_j\}$ such that (a) income is given by (5); (b) all consumer cells maximize utility (4) subject to (6); (c) all producer cells maximize profits (9); (d) the government's budget (11) is balanced; (e) labor markets clear for each consumer cell,

$$N_i = \sum_j N_{ij}; \tag{12}$$

and (f) the goods market clears for each producer cell,

$$Q_j = \sum_{j'\in\mathcal{J}} X_{jj'} + x_j + \sum_i c_{ij} + G_j.$$

[21] The exact functional form of v will not affect our analysis.

VI.C Calibration

We calibrate the model to match the disaggregated economic accounts measured in Section IV, as summarized in Table A.VII. For the baseline calibrated equilibrium, we normalize all prices and wages to 1, that is, $P_j = W_i = P_{\mathcal{R}} = 1$. This is without loss since (a) the units of production can always be chosen to set $P_j = 1$ (by choosing Z_j); and (b) the units of labor supply can always be chosen to obtain $W_i = 1$ (by choosing N_i).[22] With this normalization, the average spending share of consumer cell i on producer cell j can be expressed as

$$\frac{P_j c_{ij}}{\sum_j P_j c_{ij}} = \begin{cases} \alpha_{i\mathcal{R}} & \text{if } j = \mathcal{R} \\ (1 - \alpha_{i\mathcal{R}})\, \alpha_{is}\alpha_{isj} & \text{if } j \neq \mathcal{R} \end{cases}.$$

We calibrate the average spending shares $\alpha_{i\mathcal{R}}, \alpha_{is}, \alpha_{isj}$ of consumer cell i to match the corresponding spending shares from the disaggregated consumer spending flows (both domestic and foreign spending).

Similarly, we measure the share of producer cell j's wage bill going to consumer cell i, $\frac{N_{ij}W_i}{\sum_i N_{ij}W_i} = \lambda_{ij}$, in the disaggregated labor compensation flows. The distribution of profit income κ_{ij} is calibrated by matching, for each producer cell j, the distribution of dividend, mixed income, and surplus of owner-occupied housing flows across consumer cells. We calibrate τ_i to match the ratio of consumer non-product taxes and social contributions to total income for each consumer cell i; and τ_i^{vat} to match the ratio of consumer product taxes to total consumer cell income. We calibrate τ_j^{corp} to match, as fraction of j's total output $P_j Q_j$, the sum of: dividends and surplus paid to the government, producer product and non-product taxes paid, less subsidies.

We choose cell-level government transfer shares t_i to match social benefits received by i plus adjustments for pension entitlements. We choose government spending shares $g_j, \tilde{g}$ to match government final consumption expenditure on producer cell j and government imports.

We set the labor share γ_j^N in line with the ratio of labor compensation paid by producer cell j to output by producer cell j, and γ_j^X in line with the ratio of intermediate input purchases to output. We set intermediate input shares $\omega_{js}\omega_{jsj'} = \frac{P_{j'}X_{j'j}}{\sum_{j'} P_{j'}X_{j'j}}$ in line with the disaggregated intermediates trade matrix. We choose the relative magnitudes of $\tilde{x}_j$ to match the distribution of exports across producer cells j. We choose the level of $\tilde{x}_j$ to match aggregate GDP.

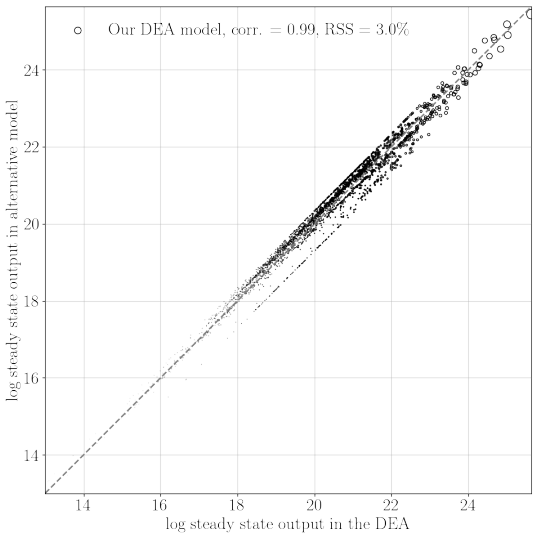
The calibrated model closely matches the Danish disaggregated economic accounts. The only major difference is the absence of a capital accumulation cell in the model. In Figure XI, we compare the measured values to the model's calibrated steady state. The correlation between model and data is close to 1 both for producer sales as well as for domestic consumer spending,

[22] Note that prices and wages will change in our comparative statics results below. These are mere normalizations and do not represent rigidities.

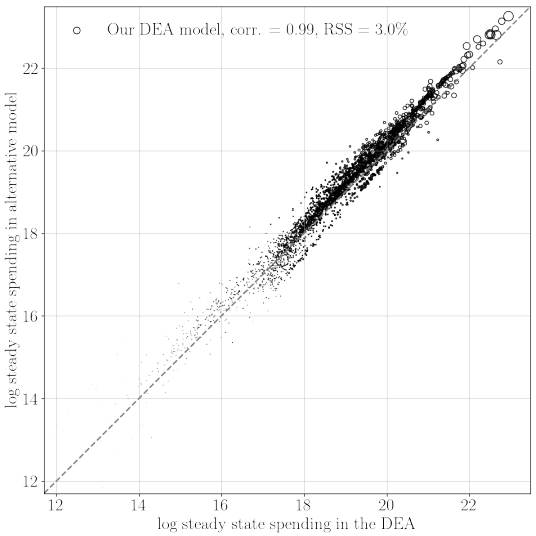
with a residual sum of squares of only 3%.

Figure XI: Match between model economy and disaggregated accounts

(a) Sales by producer cell



(b) Domestic consumer spending



Notes: Panel a shows a scatter plot of log producer cell output in the model relative to log producer cell in the disaggregated accounts (node size is proportional to producer cell output in the disaggregated accounts). Panel b shows a binned scatter plot of log consumer spending in the model versus the disaggregated accounts (node size is proportional to domestic spending in the disaggregated accounts). "corr" stands for correlation. "RSS" stands for the residual sum of squares. It captures the variance in the data not explained by the model, $\text{var}(y^{data} - y^{model})$, for any variable y, in percent of variance in the data $\text{var}(y^{data})$. If larger than 100%, this implies that the variation in the model is not useful to explain the data.

The only parameters that cannot be directly calibrated from the disaggregated accounts are the four elasticities σ, ζ, η, and γ. These elasticities are typically assumed to be small in the short run, often below 1 (e.g., Baqaee and Farhi 2022a; Gourinchas et al. 2021; Boehm et al. 2022) and large in the long run. Our baseline choice is Cobb-Douglas, $\sigma = \zeta = \gamma = \eta = 1$, which will make our theoretical results below tractable. We show in Section VIII.D that the results are quantitatively similar for $\sigma = \zeta = \gamma = \eta = 2$.

VI.D The Circular Flow Matrix

We capture the full set of disaggregated flows in a single matrix: the *circular flow matrix* $\mathbf{M}$, a $(\mathcal{J} + \mathcal{J} + \mathcal{I} + \mathcal{I} + 1) \times (\mathcal{J} + \mathcal{J} + \mathcal{I} + \mathcal{I} + 1)$ square matrix. It characterizes all the flows from the set $\mathcal{J}$ of producer cells to the set $\mathcal{I}$ of consumers, which travel via payments to capital (set $\mathcal{J}$) and labor (set $\mathcal{I}$), as well as the flows from consumer to producer cells through consumer spending.

The last dimension ("1") represents flows in and out of the government. Mathematically,

$$\mathbf{M} = \begin{pmatrix} \Omega \cdot \mathbf{D}(\gamma^X) & 0 & 0 & \mathbf{A} \cdot \mathbf{D}\left(\frac{1}{1+\tau^{vat}}\right) & g \\ \mathbf{D}(\gamma^K) & 0 & 0 & 0 & 0 \\ \Lambda \cdot \mathbf{D}(\gamma^N) & 0 & 0 & 0 & 0 \\ 0 & \mathbf{K} \cdot \mathbf{D}\left(1-\tau^{corp}\right) & \mathbf{D}\left(1-\tau\right) & 0 & t \\ 0 & \left(\tau^{corp}\right)' & \tau' & \mathbf{1}'\mathbf{A} \cdot \mathbf{D}\left(\frac{\tau^{vat}}{1+\tau^{vat}}\right) & 0 \end{pmatrix}. \tag{13}$$

We use notation $\mathbf{D}(\gamma)$ for a diagonal matrix with entries γ_j along the diagonal; $\Omega_{j'j} \equiv \omega_{js(j')}\omega_{js(j')j'}$ is the intermediates trade matrix; $\Lambda = \lambda_{ij}$ is the labor compensation matrix; $\mathbf{K}_{ij} = \kappa_{ij}$ is the matrix characterizing operating surplus and dividend payments flowing to consumers; $\mathbf{A}_{ji} = (1-\alpha_{i\mathcal{R}})\,\alpha_{is(j)}\alpha_{is(j)j}$ is the consumer spending matrix; $g, t, \frac{\tau^{vat}}{1+\tau^{vat}}, \tau^{corp}$ are vectors representing the flows in and out of the government cell. Details on model derivations and proofs are in Appendix P.

$\mathbf{M}$ represents the circular flow inherent in the model.[23] It can be interpreted as a Markov transition matrix. Each column gives the average destination of a krone paid to a given producer cell, capital owner, labor, consumer cell, or the government. The columns of $\mathbf{M}$ add to a number weakly less than 1 and strictly less than 1 when there are imports from the rest of the world.

VII Application I: Aggregate Effects of Export Demand Shocks

We view the model calibrated using disaggregated economic accounts as a general tool that allows researchers to understand the propagation of a range of shocks. In two specific applications, we now highlight that the calibrated model changes our understanding of the aggregate and distributional effects of export demand shocks.

In the first application, we study the effects of export demand shocks on aggregate welfare. We define aggregate welfare as the inverse marginal utility weighted sum of U_i in (4). This measure naturally incorporates redistributive concerns and its first-order change equals the marginal change in the economy's real gross national expenditure (GNE),

$$dGNE = \sum_{j' \in \mathcal{J} \cup \{\mathcal{R}\}} P_{j'} dG_{j'} + \sum_{j' \in \mathcal{J} \cup \{\mathcal{R}\}} P_{j'} dc_{ij'}. \tag{14}$$

We study producer cell-specific shocks to $\tilde{x}_j$, which change the exports of cell j to the rest of the world. In the model, $\tilde{x}_j$ captures foreign preferences for the output of cell j. In the real world, any shock to the demand for cell j's exports would affect $\tilde{x}_j$ (e.g., changes in demand for exports

[23] We could also include the rest of the world as an extra dimension of $\mathbf{M}$. However, since the rest of the world is large, it would be an "absorbing state" and therefore not change any of the circular flow in the rest of the economy.

due to technological growth or business cycles abroad). We will summarize the aggregate effects of a shock to $\tilde{x}_j$ in terms of *multiplier* μ_j: the krone increase in real GNE per krone of increased export demand. Mathematically,

$$\mu_j \equiv \frac{\sum_{j' \in \mathcal{J} \cup \{\mathcal{R}\}} P_{j'} dG_{j'} + \sum_{j' \in \mathcal{J} \cup \{\mathcal{R}\}} P_{j'} dc_{ij'}}{d\tilde{x}_j}. \tag{15}$$

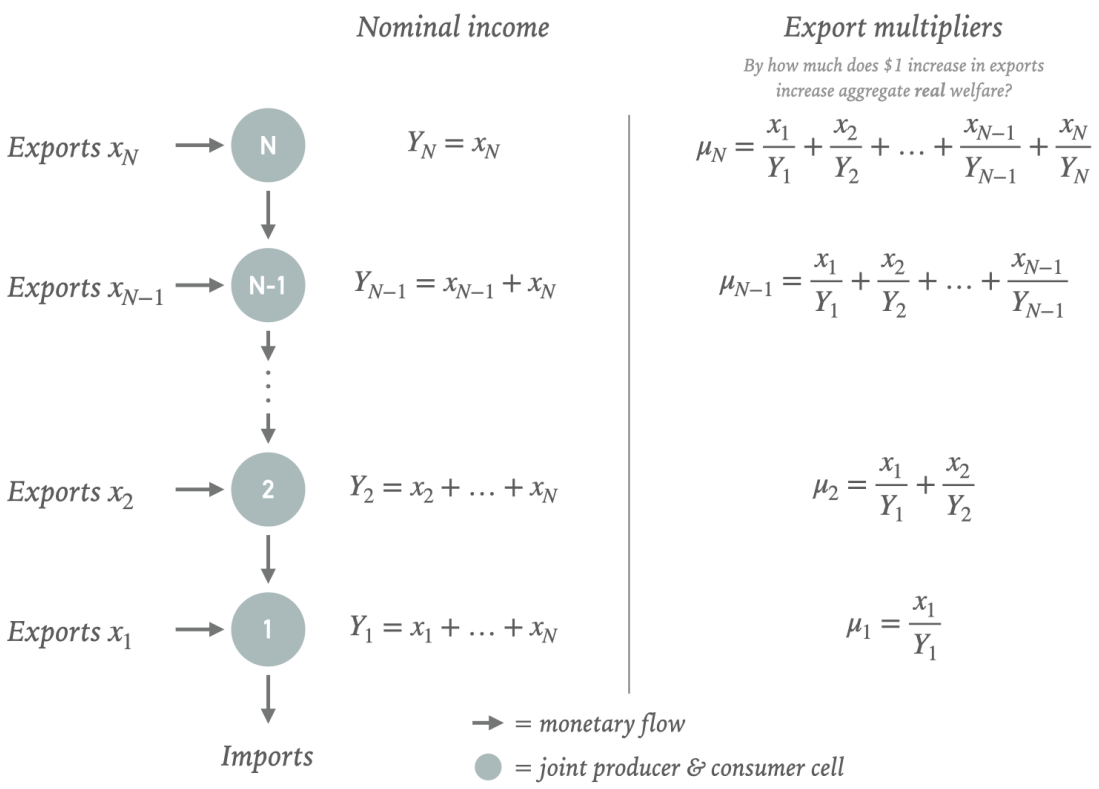
We will compute multipliers for all producer cells j and for all industries s.[24] Our focus will be on how cell j's position in the system of disaggregated economic accounts affects μ_j. Hulten's (1978) theorem does not apply to μ_j, even though we assume a neoclassical, frictionless economy. The reason is that our definition of the multiplier focuses on the small open economy's own welfare, not aggregate welfare of the entire world. Shocks to $\tilde{x}_j$ are neutral for world-wide real GNE, but not for the small open economy.

VII.A Export Demand Multipliers With a Vertical Circular Flow

We begin with a special case of our model to gain intuition for why a cell's position in the system of disaggregated economic accounts changes its multiplier μ_j. The special case is the vertical circular flow in Figure XII. This economy consists of N consumer and producer cells, labeled $i = 1, \ldots, N$, where each producer cell i converts labor supplied by consumer cell i into output. Figure XII therefore combines the corresponding producer and consumer cells into a single "consumer-producer" cell labeled i. Cell i sells exports $\tilde{x}_i$ to the rest of the world and also sells output to cell $i + 1$, for $i < N$. The first cell $i = N$ only exports and does not sell domestically. The final cell $i = 1$ imports from the rest of the world.

[24] Figure A.XVII shows the distribution of exports across industries in our circular flow.

Figure XII: Export shocks in a vertical circular flow



If there is only a single cell, $N = 1$, the multiplier is $\mu_1 = 1$, as consumption equals exports (i.e., $C_1 = Y_1 = x_1$). With two cells, an export demand shock to the final cell, $i = 1$, has a lower multiplier: raising x_1 by dx_1 still raises cells 1's income and real consumption by the same amount, $dC_1 = dx_1$. However, as the price of good 1 increases by $d \log P_1 = \frac{dx_1}{Y_1}$, there is a negative pecuniary externality on cell 2, and $dC_2 = -\frac{C_2}{Y_1} dx_1$. Taken together, the multiplier μ_1 is now

$$\mu_1 = \frac{dC_1}{dx_1} + \frac{dC_2}{dx_1} = 1 - \frac{C_2}{Y_1} = \frac{x_1}{Y_1}.$$

This multiplier μ_1 is equal to the "export exposure" of good 1. Compare μ_1 to the case where cell 2 experiences the export demand shock. Now, the income of cell 2 increases in line with the shock, $dY_2 = dx_2$, and so does cell 2's nominal spending, $d(P_1 C_2) = dx_2$. This mechanism drives up the price of good 1 by $d \log P_1 = \frac{d(P_1 C_2)}{Y_1}$. Real consumption C_2 then increases by only

$$dC_2 = dx_2 - \frac{C_2}{Y_1} dx_2 = \frac{x_1}{Y_1} dx_2.$$

Intuitively, real consumption of cell 2 increases proportionately to the export exposure of good 1.

As cell 1's income increases by dx_2, its consumption rises by $dC_1 = dx_2$. The multiplier is then

$$\mu_2 = \frac{dC_1}{dx_2} + \frac{dC_2}{dx_2} = 1 + \frac{x_1}{Y_1}.$$

This finding highlights that export demand shocks for more "upstream" producers (cell 2) have larger effects on aggregate welfare compared to shocks to "downstream" producers (cell 1).

It is noteworthy that cell 2's multiplier exceeds 1, again due to a pecuniary externality. In the world as a whole, pecuniary externalities from price increases of good 1 net out: buyers lose as much as sellers gain. Within the small open economy, however, pecuniary externalities do not net out. In Figure XII, the rest of the world is a net buyer of good 1. As a result, the small open economy experiences a net welfare gain when the price of good 1 increases. The welfare gain is precisely proportional to good 1's export exposure x_1/Y_1.[25] We generalize this intuition for a vertical circular flow of arbitrary length.

Proposition 1. *In the vertical circular flow, the export demand multiplier for producer cell i is*

$$\mu_i = \frac{x_1}{Y_1} + \ldots + \frac{x_i}{Y_i}. \tag{16}$$

Proposition 1 emphasizes that two forces generate high multipliers. First, any krone of additional export revenue that circulates domestically for longer (i.e., is spent more often) has a higher aggregate multiplier. If we assume that export exposures are equal to the same ratio x/Y for all producers $i < N$, we would have that $\mu_i = i \cdot x/Y$ for $i < N$, directly relating the length of circulation i to the multiplier of i. Second, any krone of additional export revenue that gets spent on goods with greater export exposure also generates higher export multipliers.

A useful result is that the multiplier is always 1 for an aggregate export demand shock in an economy without intermediate imports. Increasing all exports by 1% increases all incomes, prices, and nominal consumption by 1%. As a result, the quantity of domestic consumption must be unchanged, while the quantity of imported consumption rises by 1%. Using equation (15), the aggregate multiplier is then

$$\overline{\mu} = \frac{\sum x_i \mu_i}{\sum x_i} = 1. \tag{17}$$

Relation to intermediates trade networks. The example of Figure XII does not contain trade in intermediates. The heterogeneity in multipliers we document is therefore distinct from recent work on intermediates trade among producers. Instead, our analysis relates to recent studies

[25] This logic transcends our Cobb-Douglas unitary elasticities, as we show numerically below. For example, in the case of the vertical circular flow, allowing for a higher, but still finite, elasticity of export demand, $\gamma > 1$, would attenuate multipliers further up in the circular flow, but they would always exceed 1 at the top.

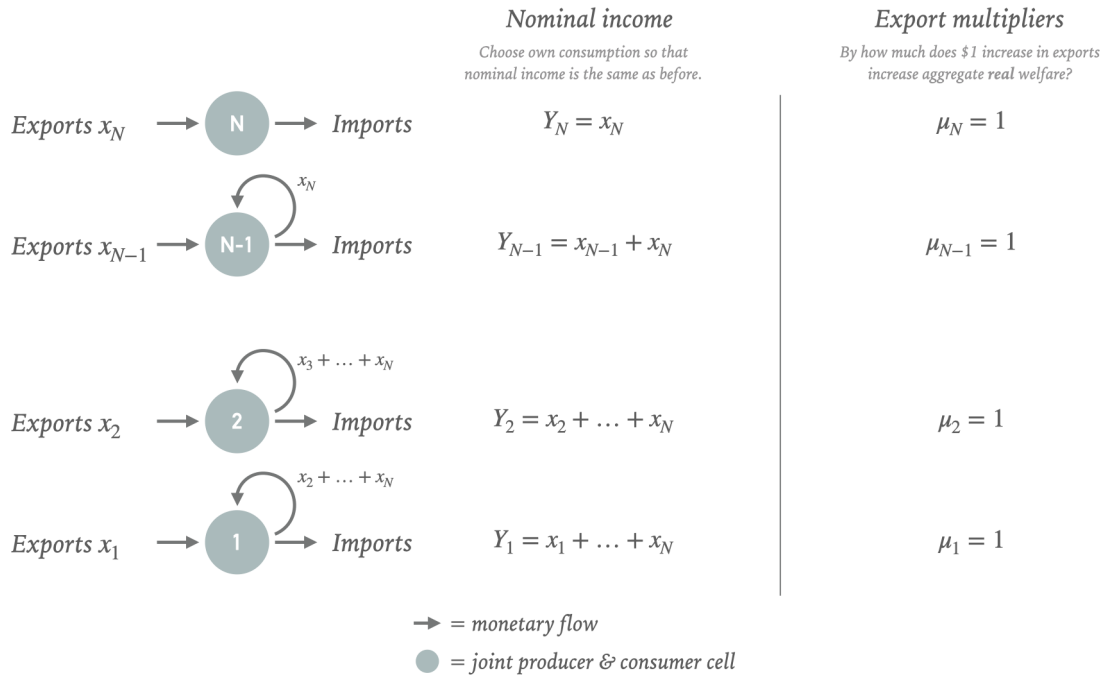
analyzing the propagation of shocks in a world with many consumers in different regions, such as Caliendo and Parro (2015), Antràs and Chor (2018), Kleinman et al. (2020), and Baqaee and Farhi (2022a). Different from these papers, our focus is on heterogeneous consumers within a country and the circular flow of spending between them. We show that this flow matters because it determines aggregate export demand multipliers.[26]

Role of disaggregated consumer spending. The example of Figure XII also illustrates that data on disaggregated consumer spending flows is crucial to understanding aggregate multipliers. Imagine, for example, that the spending pattern was such that consumer cell i spends entirely on producer cell i, as shown in Figure XIII. This implies that there are now N separate "islands" in the economy operating in parallel. Without information on spending patterns, this "island economy" is indistinguishable from the one in Figure XII. Yet, the multipliers are totally different. For each island i, the multiplier in the "island economy" is

$$\mu_i = \frac{dC_i}{dx_i} = 1.$$

To derive this multiplier, note that μ_i is the "aggregate" multiplier of island i and apply result (17).

Figure XIII: Export shocks in an island version of the vertical circular flow



[26] In the general economy with a single consumer cell introduced in Section VI, a producer cell's export multiplier is always equal to one minus the producer cell's network-adjusted intermediate import share. In particular, it can never be larger than 1.

VII.B Export Demand Multipliers With a Full System of DEA

We now move from the stylized vertical circular flow economy to our full model that was calibrated to match the Danish disaggregated economic accounts (DEA). We begin by describing *factor-level export exposures*. These exposures capture what percent of capital and labor income depends directly on export demand. They are the factor-level analog to the export exposures x_i/Y_i in the previous section. We construct them as in Adão et al. (2022). The export exposure of capital $\mathbf{x}^K \in \mathbb{R}^{\mathcal{J}}$ is

$$\mathbf{x}^K \equiv \mathbf{D}\left(\frac{1}{\Pi^{pre}}\right)\mathbf{D}\left(\gamma^K\right)\left(\mathbf{I}-\Omega\mathbf{D}\left(\gamma^X\right)\right)^{-1}\mathbf{x}, \tag{18}$$

where $\mathbf{x} \in \mathbb{R}^{\mathcal{J}}$ is the vector of exports across producer cells. The export exposure of labor $\mathbf{x}^N \in \mathbb{R}^{\mathcal{I}}$ is

$$\mathbf{x}^N \equiv \mathbf{D}\left(\frac{1}{NW}\right)\Lambda\mathbf{D}\left(\gamma^N\right)\left(\mathbf{I}-\Omega\mathbf{D}\left(\gamma^X\right)\right)^{-1}\mathbf{x}. \tag{19}$$

With these objects introduced, we can now state the main result of this section.

Proposition 2. *In the model of Section VI, the vector of export demand multipliers is*

$$\boldsymbol{\mu} = \begin{pmatrix} 0 & \mathbf{x}^K & \mathbf{x}^N & 0 & 0 \end{pmatrix} \cdot (\mathbf{I}-\mathbf{M})^{-1} \begin{pmatrix} \mathbf{I} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{20}$$

This result generalizes Proposition 1 to the full DEA economy. The intuition is similar. Imagine producer cell j receives one more krone due to higher export demand. The corresponding column of $\left[(\mathbf{I}-\mathbf{M})^{-1}\right]_{\cdot,j}$ gives the average number of times that any cell in the economy is visited before the additional krone exits the economy back to the rest of the world. This means that, for example,

$$\begin{pmatrix} 0 & \mathbf{1} & \mathbf{1} & 0 & 0 \end{pmatrix} \cdot \left[(\mathbf{I}-\mathbf{M})^{-1}\right]_{\cdot,j} \tag{21}$$

counts how many times the krone is earned as labor or capital income, before exiting the economy. This measure captures cell j's "upstreamness" in the circular flow.[27] In the vertical example of Figure XII, upstreamness equals $i-1$.[28]

The formula in (20) specifies that the export demand multipliers in the full DEA model equal the sum of downstream factor-level export exposures. Given the presence of intermediates trade

[27] We use the language of "upstream" and "downstream" in terms of monetary flows, not goods flows. See Antràs and Chor (2021) for similar measures of upstreamness in the context of world input-output tables.

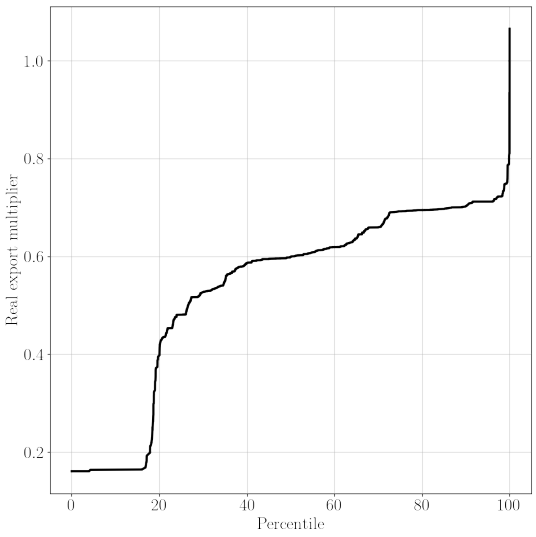
[28] Another way to view (21) is the multiplier of *nominal* GNE with respect to the export demand increase.

in the DEA model, it is important to compute export exposure at the factor, not product, level. Just like in the vertical economy, a factor's export exposure captures the positive pecuniary externality on the domestic economy of additional spending going toward that factor.

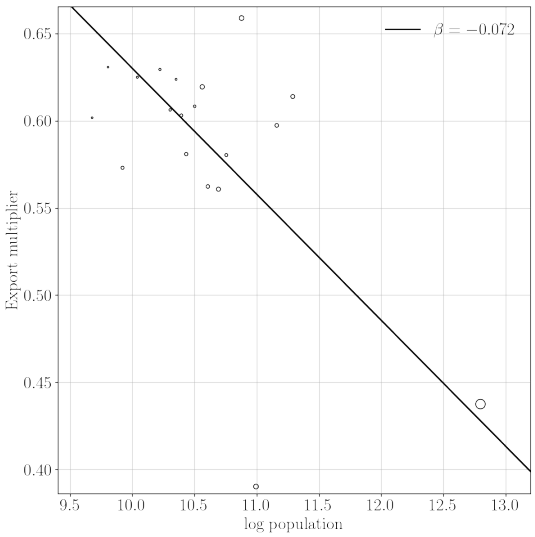
We plot the DEA model's export demand multipliers in Figure XIV. Panel (a) plots the distribution of multipliers. We see a wide range of multipliers across producer cells in Denmark, from below 0.2 to over 0.8. A significant fraction of that variation is geographical, as we show in Panel (b). Rural multipliers tend to exceed multipliers in large cities by about 50%.

Figure XIV: Export demand multipliers in the DEA

(a) Distribution of multipliers



(b) Multipliers by region population

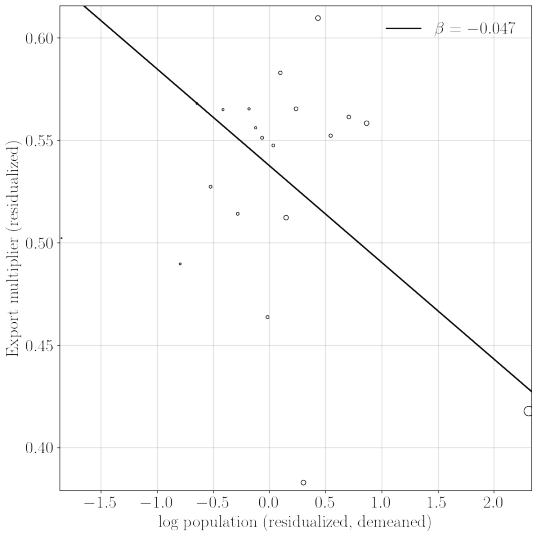


Notes: Panel (a) shows the distribution of export multipliers across producer cells, weighted by a producer cell's export share. Panel (b) shows export multipliers across producer cells by log population (20 bins), weighted by a producer cell's export share.

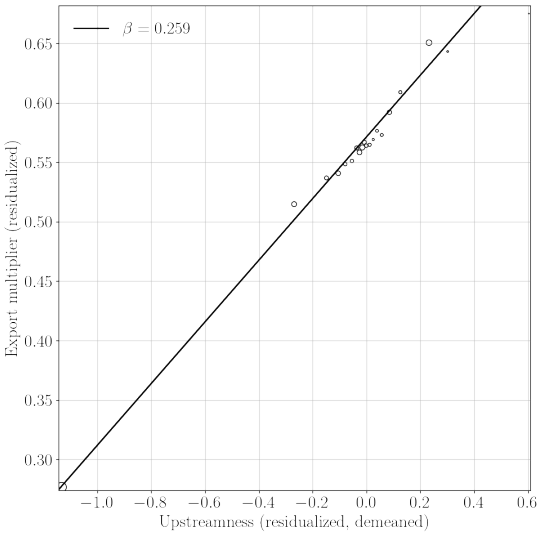
One factor that influences multipliers are regional differences in industrial composition. For example, the Danish shipping industry has high intermediate import shares and tends to be located in more urban regions. Panel (a) in Figure XV partials out industry fixed effects. The line flattens somewhat, but a significant portion of cross-regional variation remains. To focus on cross-regional variation, we report all remaining results in this section conditional on industry fixed effects, but the unconditional results are similar.

Figure XV: Within-industry variation in export demand multipliers

(a) Multipliers by population



(b) Multipliers and upstreamness



Notes: Panel (a) shows export multipliers across producer cells by log population (20 bins), controlling for industry dummies, weighted by a producer cell's export share. Panel (b) shows export multipliers across producer cells by "upstreamness" (20 bins), controlling for industry dummies, as defined in (21), weighted by a producer cell's export share.

We next investigate what accounts for within-industry heterogeneity in multipliers. As a starting point, we ask whether variation in multipliers is due to heterogeneity in export exposures $(\mathbf{x}^K, \mathbf{x}^N)$ or in upstreamness, as in (21), across producer cells. Panel (b) in Figure XV plots export demand multipliers versus upstreamness, again controlling for industry dummies. Evidently, the variation in multipliers is almost entirely driven by upstreamness and rural regions tend to be more upstream than urban ones.

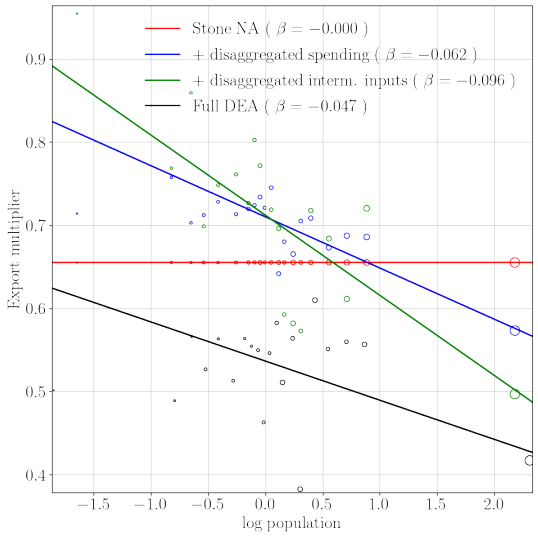
VII.C Export Demand Multipliers and the Structure of Disaggregated Flows

The facts presented in Section V show that consumer spending travels toward cities from rural regions and that urban consumers spend more abroad. We explore whether these facts contribute to heterogeneity in export demand multipliers.

In Figure XVI, we plot export demand multipliers by regional population. We compare the multipliers derived under different measurement systems. We begin with a benchmark system, which is a multi-region version of standard national accounts in the spirit of Richard Stone. The system contains a single representative consumer, many producer industries, a government, and the rest of the world. The inflows and outflows of each producer industry are heterogeneous and consistent with the flows as measured in the DEA. Since standard national accounts do not contain multiple regions and only multiple industries, we construct many regions that are scaled-down

versions of the aggregate economy, in proportion to the true size distribution of Danish regions. This benchmark economy aggregates exactly to the national accounts and allows us to build toward the DEA, as it has a regional dimension. Export demand multipliers are constant across regions in the benchmark economy, as all regions are replicas of another.

Figure XVI: Why do cities have lower multipliers?



Notes: This figure shows export multipliers across producer cells by log population (20 bins), controlling for industry dummies, weighted by a producer cell's export share. "Stone NA" is a benchmark economy in which each region is home to an (up to scale) identical circular flow equal to the national circular flow with only industry disaggregation on the producer side. "+ disaggregated spending" allows for disaggregated domestic and foreign spending. "+ disaggregated spending and I-O" in addition allows for intermediate input trade, domestically and with the rest of the world.

Next, we augment the benchmark economy with the correctly disaggregated measures of domestic and foreign consumer spending. This introduces a downward slope in multipliers, consistent with the facts on consumer spending from Section V. The disaggregated spending patterns make rural regions upstream, which raises rural multipliers, and cities downstream, which lowers urban multipliers. Taken together, these findings show that the nature of disaggregated consumer spending contributes to the geography of export multipliers in Denmark.

Incorporating the correctly disaggregated intermediates trade increases the slope further. The slope in the full DEA system remains negative but becomes slightly weaker because labor compensation, mixed income, surplus payments, and government transactions with consumers redistribute across regions, on net.

VII.D Discussion and Robustness

Role of region-by-industry classification. The results on heterogeneous multipliers hinge on the relatively fine region-by-industry classification of the DEA. If we did not have any regional variation, there would be no heterogeneity in multipliers conditional on industry. If we did not have any industry variation, we would generally overestimate regional multipliers, as illustrated in Figure A.XVIII. This bias arises from the fact that factors with high export exposure are generally employed in export-oriented industries, such as manufacturing and shipping, and therefore receive little consumer spending in the DEA. Without a disaggregated industry dimension, one would incorrectly assume that substantial consumer spending goes to these industries, increasing the multiplier.

Measurement error tests. One may worry that measuring disaggregated consumer spending using transaction data induces measurement error, as studied by Dingel and Tintelnot (2021). We confirm that such measurement error does not substantially affect the results in Figure A.XIX, where we replace the smallest 90% of consumer spending by imputed values from the standard gravity regression (2).

A separate source of measurement error may be the gravity approximation used to measure the disaggregated intermediates trade matrix. In Figure A.XX, we compare DEA export multipliers with those from a fully local matrix (all intermediates are sourced from the home region) and a uniform matrix (all producer cells purchase intermediate goods in the same proportion). While these changes are drastic, the results are similar.

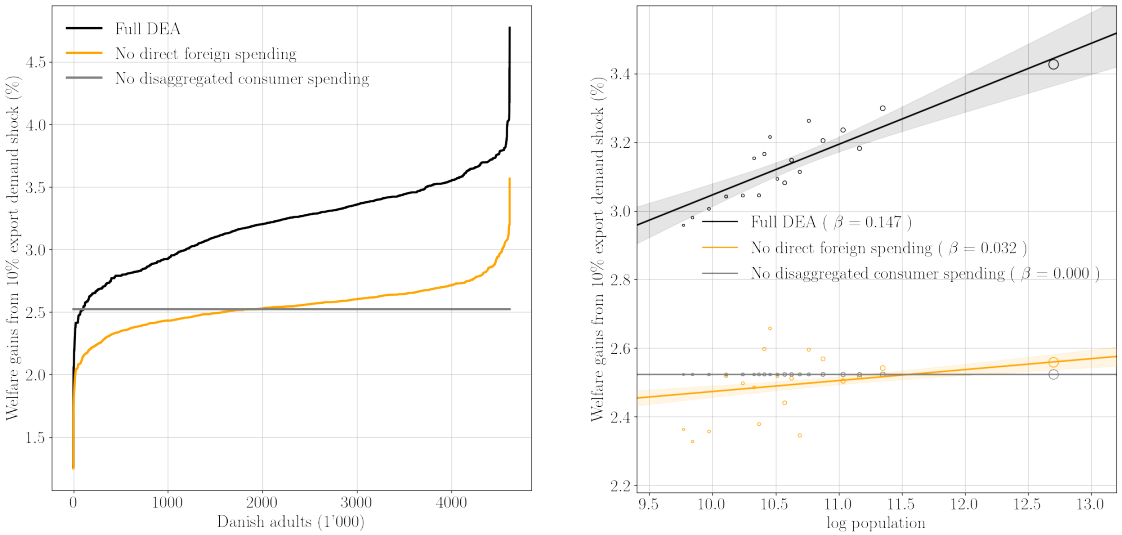
VIII Application II: Distributional Effects of Export Demand Shocks

In our second application, we show that disaggregated economic accounts (DEA) affect our understanding of the distributional effects of export demand shocks. We focus on welfare derived from consumer cell-specific market consumption C_i because we measure market consumption relatively more accurately than consumption of government.

VIII.A Uniform Export Demand Shock

Figure XVII: Welfare gains from a uniform export demand shock

(a) Distribution of the gains from export demand shocks(b) Gains from export demand shocks by log population



Notes: Panel (a) shows the distribution of the welfare gains from a 10% increase in overall export demand uniform, across consumer cells. Panel (b) plots the welfare gains from a 10% increase in export demand uniform across all industries in a binned scatter plot with 20 bins, across log region population. Shaded areas reflect 95% robust confidence intervals. The plots compare the full DEA model with a model in which direct foreign spending is set to zero and with a model in which direct foreign spending is zero and all consumers are assumed to have the same spending pattern. Scatter weights are number of bank customers by cell.

We begin our analysis with a uniform export demand shock. Figure XVII feeds a 10% increase in demand for all Danish exports into the calibrated model. As Panel (a) shows, the uniform shock generates large heterogeneity in welfare gains in the DEA model. Aside from outliers, the gains range from 2.5 percentage points to 4 percentage points at the top, a difference of 60 percent.

Panel (b) illustrates that the welfare gains experienced by consumers in the largest (most urban) 20% of regions are about 20 percent larger than those experienced in the smallest (most rural) 20%. Compared with the literature on the gains from trade, this is a large difference (e.g., Borusyak and Jaravel 2021). The distribution of welfare gains across regions is at odds with the direct incidence of export shocks, as rural regions tend to produce exports, as shown in Section V.

The discrepancy between direct and general equilibrium gains is driven by the structure of disaggregated flows. The additional export revenue flowing into rural regions raises rural incomes and prices. The price increases partially offset the income gains, implying that real consumption of rural consumers rises by less than the initial income shock. Part of the additional export revenue also flows into cities, due to the urban bias of consumer spending and intermediates trade, pushing up urban incomes and prices. Urban consumers benefit from the income gains to a larger extent

than rural consumers because urban consumers spend a relatively large share abroad and foreign prices have not increased. As a result, the real consumption gains of urban consumers are larger than those of rural consumers.

This logic is a special case of the famous Lerner symmetry (see Costinot and Werning 2019 for a modern treatment). An increase in export demand has the same welfare consequences as a reduction in import prices, benefiting consumers with higher foreign spending. We formalize the welfare effects, in the context of our DEA model, in the following proposition:

Proposition 3. *A first-order uniform export demand shock, increasing* $\log \tilde{x}_j$ *by* $d \log \tilde{x}$*, affects market consumption* C_i *by*

$$\frac{d \log \mathbf{C}}{d \log \tilde{x}} = \underbrace{(\mathbf{1} - \mathbf{A}'\mathbf{1})}_{\textit{direct foreign spending share}} + \underbrace{\mathbf{A}' \left(\mathbf{I} - \mathbf{D}(\gamma^X)\Omega'\right)^{-1} \mathbf{D}(\gamma^X) \left(\mathbf{1} - \Omega'\mathbf{1}\right)}_{\textit{indirect foreign spending share}}. \tag{22}$$

Proposition 3 emphasizes that direct and indirect foreign spending shares matter for the welfare gains from a uniform export demand shock. Figure XVII illustrates the relative role of direct and indirect foreign spending shares. The orange line is from an economy in which direct foreign spending is set to zero, $\alpha_{i\mathcal{R}} = 0$, but otherwise the model calibration is kept the same. This adjustment nearly equalizes the welfare gains between rural and urban consumers. Hence, measuring direct foreign spending as part of the DEA is crucial for our understanding of the distributional effects of export demand shocks.

VIII.B Role of the Government in Redistributing the Gains from Export Demand Shocks

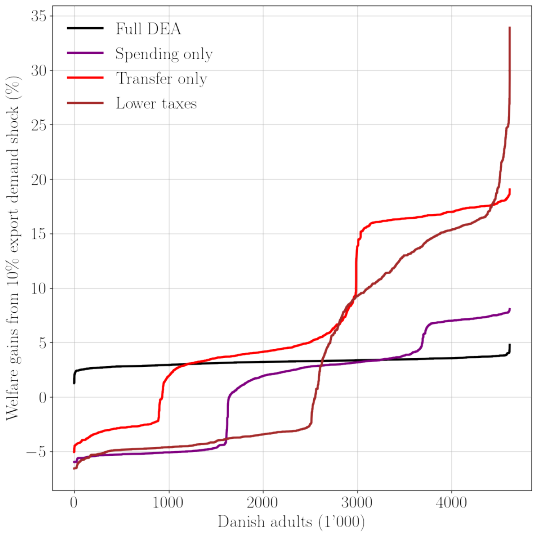
We explore whether redistribution and spending by the Danish government change the distributional effects of export demand shocks. In the full DEA model, the government responds to changes in tax revenue due to exogenous shocks by raising transfers and spending in proportion to their existing distribution across cells. We compare our DEA model to three alternatives: the "transfers only" model, where the government uses any additional revenue only to increase transfers; the "spending only" model, where the government uses any additional revenue only to increase spending; and the "lower taxes" model, where the government reduces taxes so that its revenue remains stable.[29]

The results in Figure XVIII highlight that the fiscal response of the Danish government is central to distributing the gains from export shocks. When only transfers are adjusted, rural consumers tend to benefit more; when only spending is adjusted, urban consumers benefit nearly twice as much; when taxes are reduced, effects are more uniform across space.

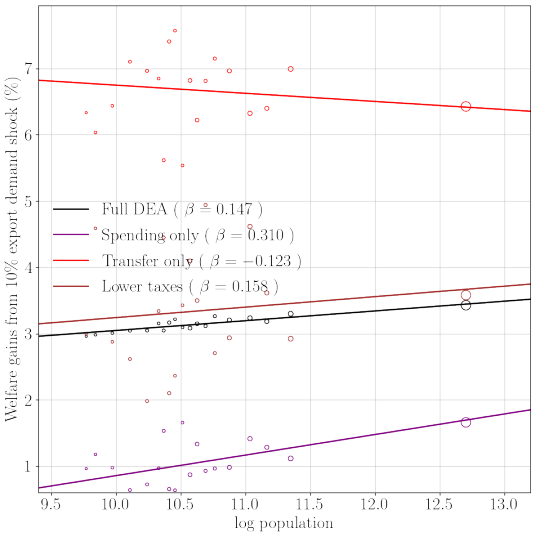
[29] We assume this is done separately for each of the revenue sources in 11; for example, if income of consumer cell i increases, it is assumed that τ_i falls in order to keep $\tau_i Y_i$ unchanged.

Figure XVIII: Gains from export demand shocks depend on the fiscal response

(a) Distribution of the welfare gains



(b) Welfare gains by log population



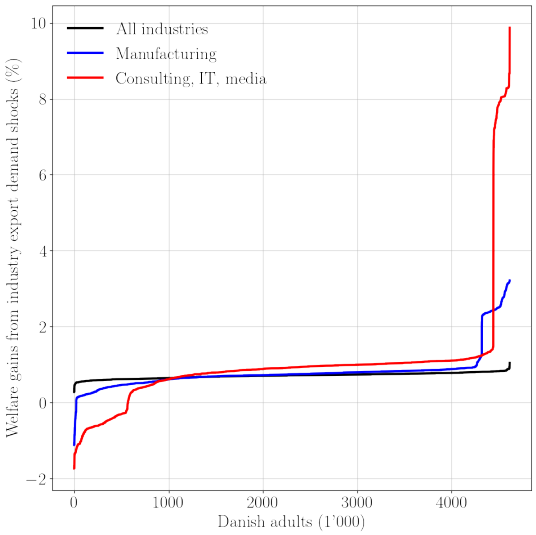
Notes: Panel (a) compares the distribution of the welfare gains from a 10% increase in overall export demand across four models: the full DEA model; one in which any government revenue increase is spent; one in which it is used for transfer payments; and one in which taxes are being lowered. Panel (b) shows a binned scatter plot of welfare gains across log region population. Scatter weights are number of bank customers by cell.

VIII.C Industry-Specific Export Demand Shocks

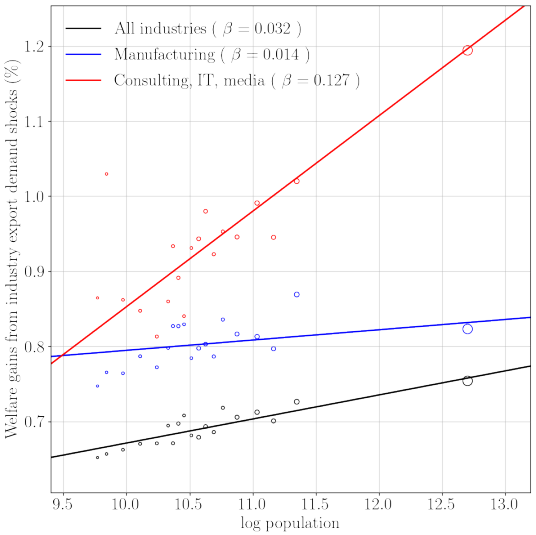
We next consider industry-specific, rather than uniform, export demand shocks. In Panel (a) of Figure XIX, we compare an increase in export demand for manufacturing goods, by 1% of Danish GDP, with an increase in demand for consulting and IT services, also by 1% of Danish GDP. We find that the gains from the manufacturing shock are significantly more equally distributed than the gains from the consulting and IT shock.

Figure XIX: The distributional gains from industry-specific export shocks

(a) Distribution of the welfare gains



(b) Welfare gains by log population



Notes: Panel (a) and (b) compare the distribution as well as the population gradient of the welfare gains from an increase in exports across two industries: Manufacturing and consulting, IT, and media. The shock is normalized to 1% of GDP. Scatter weights are number of bank customers by cell.

Panel (b) suggests that the structure of disaggregated flows across regions contributes to this finding. Manufacturing exports are predominantly produced in rural regions, while consulting and IT services are mostly produced in cities. Due to the upstream location of manufacturing in the circular flow, its revenues tend to flow downstream, toward cities, benefiting consumers along the way. In contrast, consulting and IT revenues mostly benefit city consumers, whose spending bids up prices in cities, hurting consumers of city-produced services all over Denmark. Again, the disaggregated circular flow reveals why some shocks have more unequal effects than others.

VIII.D Discussion and Robustness

We test whether the results in this section may be due to measurement error, following the methods of Section VII.D. In Figure A.XXI, we replace the smallest 90% of consumer spending flows by values imputed using the standard gravity regression (2) and compute distributional welfare gains from a uniform export demand shock. We find the gains to be similar to the ones in the DEA model. Figure A.XXII computes the distributional gains in two models with alternative intermediates trade matrices, as described in Section VII.D. Again, the gains are similar to the ones in the DEA model. Finally, the welfare gains derived under Cobb-Douglas are quantitatively close to the ones derived under a model with larger elasticities, for example to $\sigma = \zeta = \gamma = \eta = 2$ in Figure A.XXIII and to various combinations of other elasticities (e.g., raising γ to 5 or 6).

IX Conclusion

The idea of a comprehensively disaggregated circular flow of money goes back to at least 18th century France. Richard Cantillon and François Quesnay envisioned systems of measurement that showed how money cycles across small subgroups in the economy through consumption, income, and production links. Conceptual and measurement challenges meant that this idea lay dormant for almost 200 years, until Wassily Leontief developed ways to measure cross-industry production links and Richard Stone integrated these flows into modern national accounts.

Despite these seminal contributions, the core vision of Cantillon and Quesnay remains unfulfilled. There exists no system of measurement that records flows between small subgroups of consumers and producers, so that (i) bilateral transactions add up to national aggregates, (ii) accounting identities across different levels of aggregation are satisfied, (iii) rich heterogeneity in bilateral flows across different consumer and producer groups is accurately recorded, and (iv) the full circular flow of consumption, income, and production links is measured.

The advent of detailed microdata on consumption and income transactions allows us to take a step toward realizing the vision of a fully disaggregated circular flow. We describe general methods and implement the measurement of a system of disaggregated economic accounts in Denmark. The new data allow us to present facts on how money circulates across regions and industries. We document, for instance, disproportionate consumer spending flows from rural regions into urban service industries, larger spending abroad by urban consumers, and the role of the government in transferring resources across regions.

We show that these facts have practical bite by combining a disaggregated model with the new data. We show that the aggregate effects of cell-specific export demand shocks depend on the structure of disaggregated economic accounts. Moreover, we find that the welfare gains from export shocks are larger for city residents, in contrast to the direct incidence of the shocks, because of the distribution of disaggregated consumer spending flows.

Overall, we underscore that disaggregated economic accounts enrich our understanding of the aggregated and distributional effects of shocks. Given the potential benefits of disaggregated economic accounts to researchers and policymakers, the system developed in this paper may provide a starting point for similar measurement efforts in other countries.

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Appendix for "Disaggregated Economic Accounts"

Appendix A Additional Tables and Figures

Appendix A.A Data on the Spending Matrix

Table A.I: Classification of industries

	Producer industry	Produces output	Sells directly to consumers	Pays labor compensation to consumers ("work industry")
1	Food away from home	Yes	Yes	Yes
2	Entertainment	Yes	Yes	Yes
3	Groceries	Yes	Yes	Yes
4	Personal services, pharmacies	Yes	Yes	Yes
5	Vehicles, fuel, vehicle repair, public transport	Yes	Yes	Yes
6	Hotels, rental cars	Yes	Yes	Yes
7	Airlines	Yes	Yes	Yes
8	Telecommunication, insurance	Yes	Yes	Yes
9	Online stores	Yes	Yes	Yes
10	Utilities	Yes	Yes	Yes
11	Specialized retail stores	Yes	Yes	Yes
12	Home improvement	Yes	Yes	Yes
13	Consulting, information technology, media	Yes	No	Yes
14	Business administration and janitorial services	Yes	No	Yes
15	Manufacturing	Yes	No	Yes
16	Wholesale	Yes	No	Yes
17	Finance, real estate	Yes	Yes	Yes
18	Cultural and social organizations	Yes	Yes	Yes
19	Agriculture, mining	Yes	No	Yes
20	Construction	Yes	No	Yes
21	Shipping, transport	Yes	No	Yes
22	Out of workforce and others	No	No	No
23	Retired	No	No	No
24	Health	Yes	No	Yes
25	Students	No	No	No
26	Education	Yes	No	Yes
27	Public administration	Yes	No	Yes
28	Unemployed	No	No	No
29	Private landlords	Yes	Yes	No
30	Owner-occupied housing	Yes	Yes	No
31	Government-owned housing	Yes	Yes	No

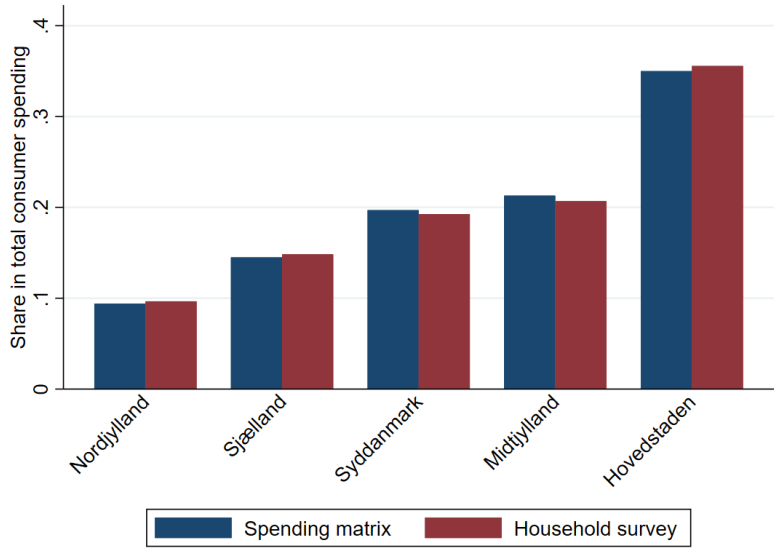
Notes: The table lists the industry classification used throughout the paper. Specialized retail stores include shops selling a specialized set of goods not listed in another industry, e.g., books, computers, shoes, clothing.

Table A.II: Summary statistics on the population and Danske Bank sample

	Full population	Danske Bank sample
Number of adults	4,367,226	858,409
Mean age	48.56	49.97
Mean income (DKK)	298,834	281,039
Age distribution		
18-39	0.35	0.34
40-59	0.35	0.32
60+	0.30	0.34
Income distribution		
Quintile 1	0.20	0.22
Quintile 2	0.20	0.23
Quintile 3	0.20	0.21
Quintile 4	0.20	0.18
Quintile 5	0.20	0.17
Ratio of liquid assets to income distribution		
Quintile 1	0.20	0.20
Quintile 2	0.20	0.21
Quintile 3	0.20	0.19
Quintile 4	0.20	0.20
Quintile 5	0.20	0.20

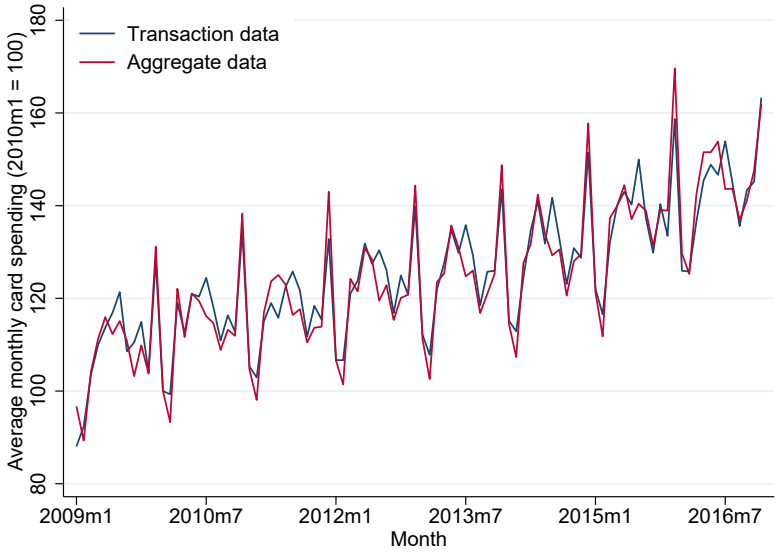
Notes: The table compares summary statistics for the Danish population from administrative registers with our sample of Danske Bank customers.

Figure A.I: Spending shares of regions in the spending matrix and household survey



Notes: The figure compares spending shares of consumer regions from the spending matrix with the Danish household budget survey accounts consumption data.

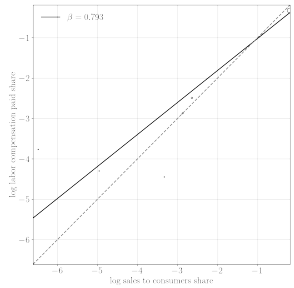
Figure A.II: Card spending over time



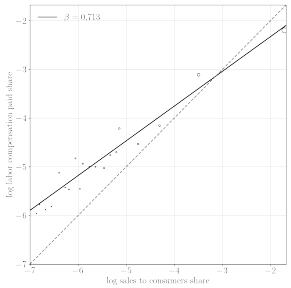
Notes: The figure compares aggregated card spending over time in data from Danske Bank and Statistics Denmark (statistikbanken.dk/MPK60).

Figure A.III: Labor income shares and spending shares by industry

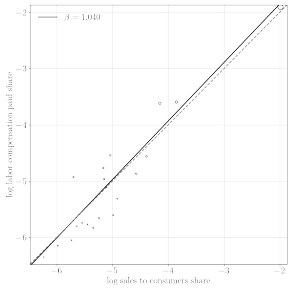
(a) Airlines



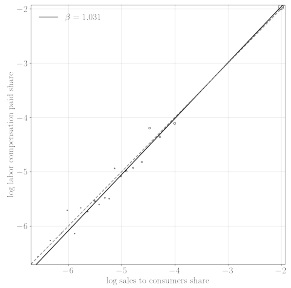
(b) Entertainment



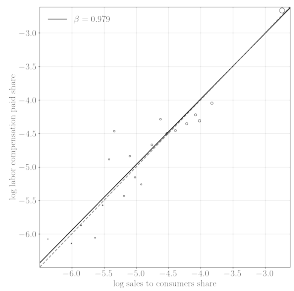
(c) Finance



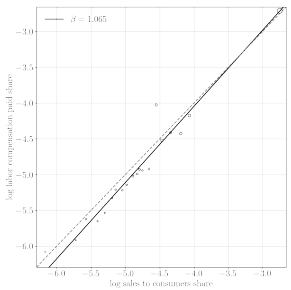
(d) Food away from home



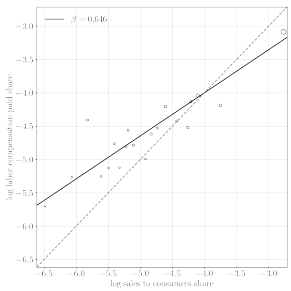
(e) Fuel, auto repair



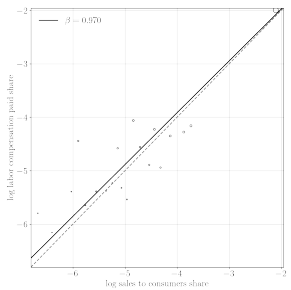
(f) Groceries



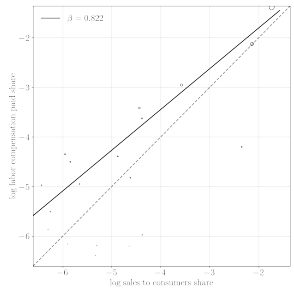
(g) Home improvement



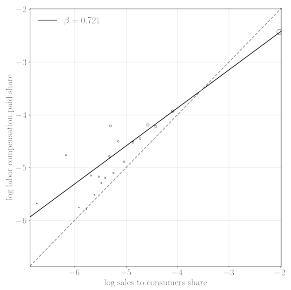
(h) Hotels, rental cars



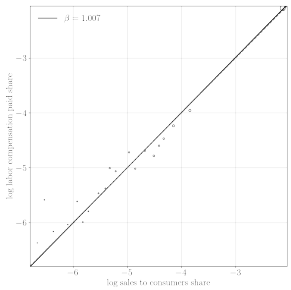
(i) Online stores



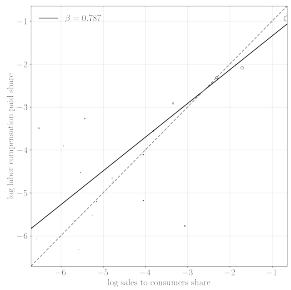
(j) Pers. services, pharmacies



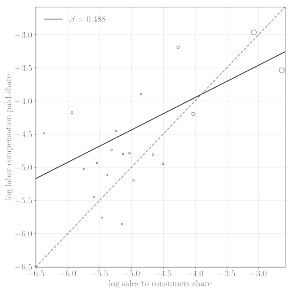
(k) Specialized retail stores



(l) Telecom., insurance



(m) Utilities

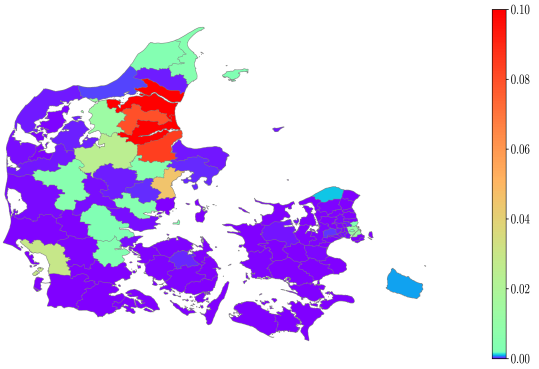


Notes: The sales-weighted binned scatters with 20 bins show the industry-specific relationships between log labor compensation paid (as share of total labor compensation in an industry) and log consumer spending received (as share of total consumer spending in an industry), across regions.

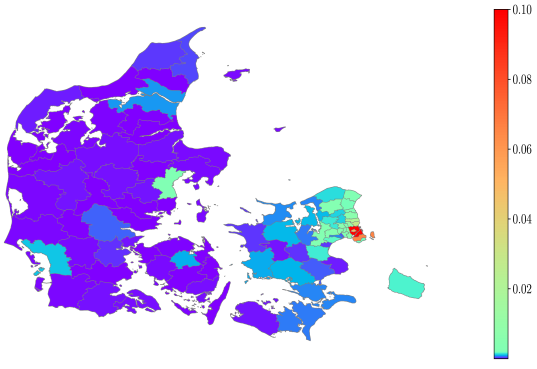
Appendix A.B Results on Regional Concentration, Sparsity, and Spending Into Cities

Figure A.IV: Labor compensation received across regions: Examples

(a) Restaurants in Mariagerfjord



(b) Restaurants in Copenhagen



Note: We plot the fraction of labor compensation received by consumers in each region, from restaurants in Mariagerfjord (Panel a) and Copenhagen (Panel b). The scale is truncated at 0.10.

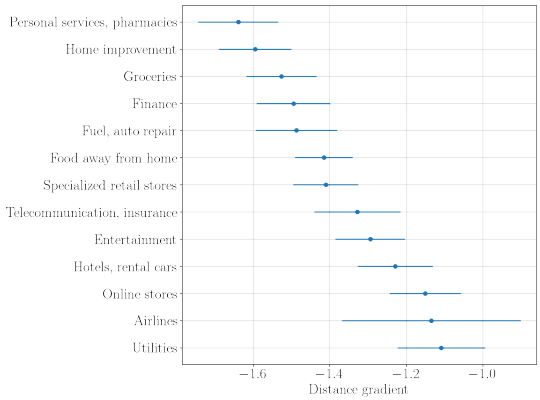
Table A.III: Gravity regressions

	log spending	log labor comp.	log intermediates	log mixed inc., surplus
	(1)	*(2)*	*(3)*	*(4)*
log distance	-1.463^{***}	-1.482^{***}	-0.557^{***}	-0.069^{***}
	(0.028)	(0.013)	(0.005)	(0.011)
Origin FE	Yes	Yes	Yes	Yes
Destination FE	Yes	Yes	Yes	Yes
Observations	2561036	1654401	5544292	5468081
R^2	0.332	0.279	0.059	0.002
F stat.	1268558.6	637191.3	348752.8	12746.1

$^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

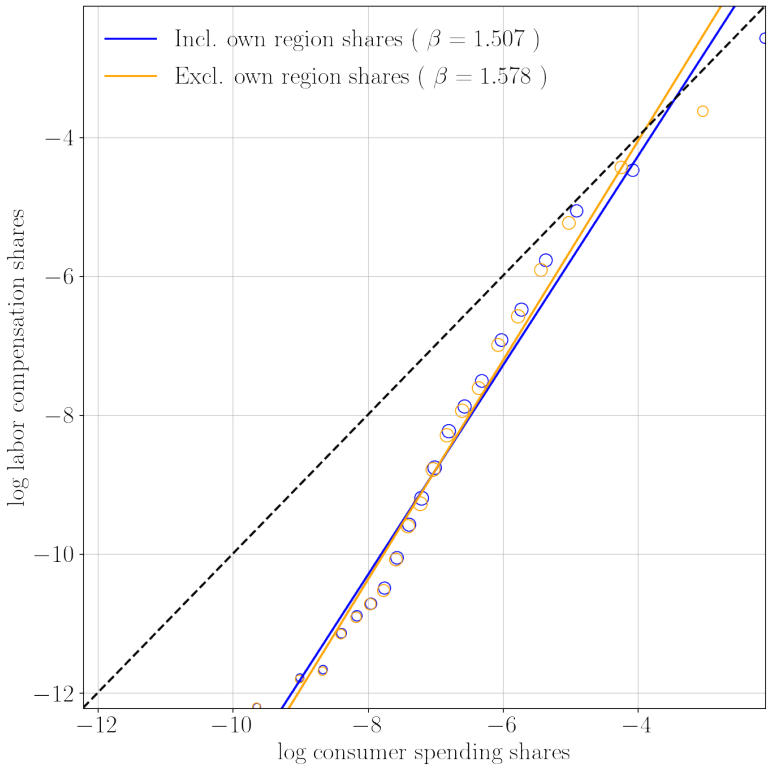
Notes: Distances are measured as driving distances between region centers.

Figure A.V: Gravity coefficients of labor compensation flows, by industry



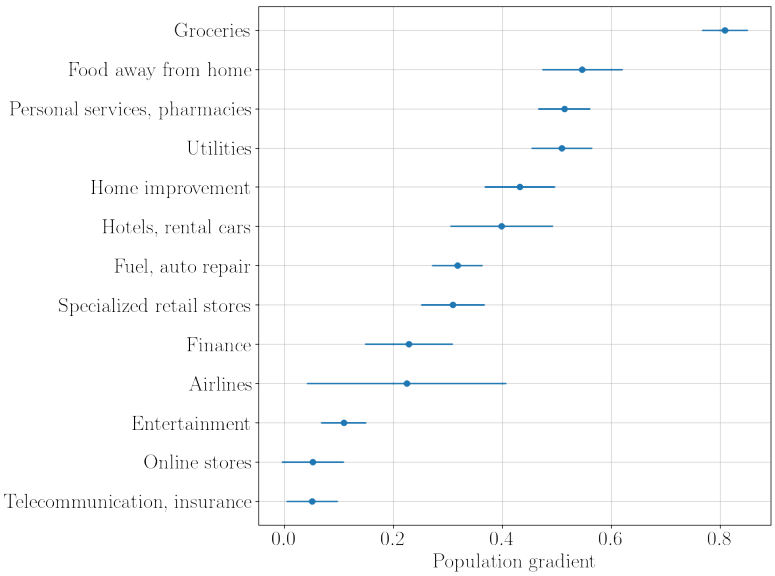
Notes: This figure shows regression coefficients for β using equation (2) for labor compensation flows by paying industry. We exclude observations with zero distance or zero flow. Distance is measured using driving distance between region centers. The solid line is the line of best fit, estimated using the cell-level data. The lines show 95% confidence intervals. All standard errors are clustered by region and industry.

Figure A.VI: Correlation between labor compensation shares and domestic consumer spending shares



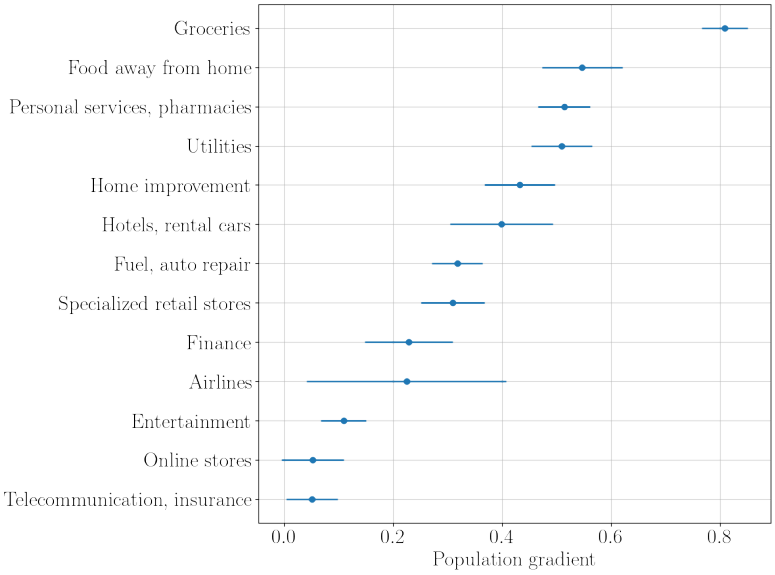
Notes: This figure shows a binned scatter plot with 20 bins between log labor compensation shares and log consumer spending shares. For each consumer cell i, we compute the distribution of labor compensation received across regions, and the distribution of consumer spending across regions. We either include or exclude own region flows. Weights are proportional to the number of bank customers in a consumer cell. The plot shows that the geographical distributions of labor compensation and consumer spending are correlated.

Figure A.VII: Population of home and receiving region: all industries



Notes: This plot shows the estimated coefficient of a regression of the average log population of regions receiving consumer spending within a specific spending category on the log population of the spending consumer cell. The error bars represent 95% robust confidence intervals.

Figure A.VIII: Population of home and receiving region: only in-person spending



Notes: This plot replicates Figure A.VII using data from a consumer spending matrix for only in-person spending.

Appendix A.C Results on Direct Foreign Spending Shares

Table A.V: Direct foreign spending shares

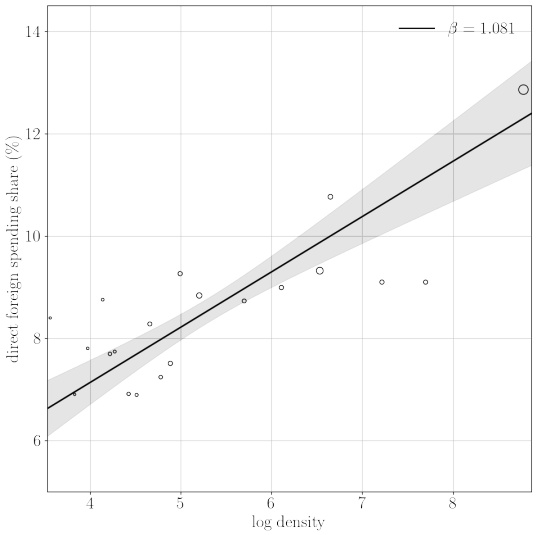
	direct foreign spending share (%)							
	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*
log population	1.622^{***}		1.766^{***}			1.664^{***}	1.616^{***}	1.648^{***}
	(0.264)		(0.244)			(0.229)	(0.225)	(0.218)
log spending per capita		3.683^{***}	4.724^{***}			3.737^{***}	3.887^{***}	5.009^{***}
		(0.595)	(0.576)			(0.641)	(0.618)	(0.960)
distance to border				-1.871^{***}		-1.329^{***}		
				(0.319)		(0.256)		
duration to border					-2.746^{***}		-1.897^{***}	-1.861^{***}
					(0.429)		(0.320)	(0.313)
log pop. $\times$ log spending p.c.								2.755^{**}
								(1.139)
const	-8.362^{***}	-34.926^{***}	-67.494^{***}	17.997^{***}	21.324^{***}	-48.573^{***}	-47.713^{***}	-61.803^{***}
	(2.820)	(7.271)	(7.333)	(1.527)	(1.931)	(7.888)	(7.731)	(11.796)
Observations	2741	2741	2741	2741	2741	2741	2741	2741
R^2	0.226	0.060	0.323	0.125	0.142	0.382	0.386	0.412
F stat.	37.7	38.3	64.0	34.3	41.0	47.1	48.5	37.3

$^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

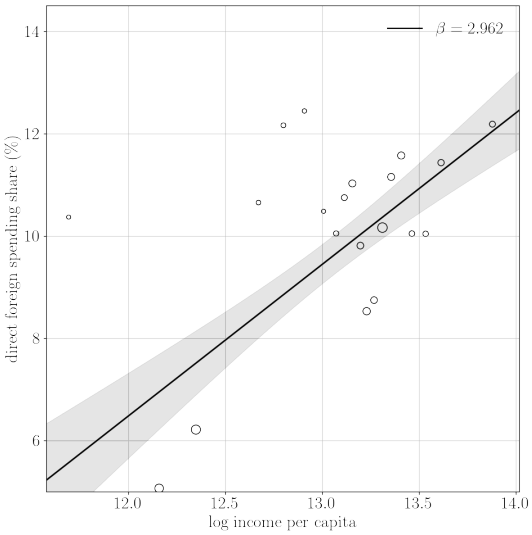
Notes: Foreign spending shares are in percent. For the log population $\times$ log income per capita interaction, the two individual variables are demeaned. We weight observations using the number of bank customers per cell. Standard errors are robust. We measure driving distance and driving duration from region centers to the nearest of eight foreign addresses (Malmø, Helsingborg, Rostock, Puttgarden, and four large border shopping centers along the Jutland-Germany border), using Google Maps API services.

Figure A.IX: Foreign spending by population density and total income

(a) By population density

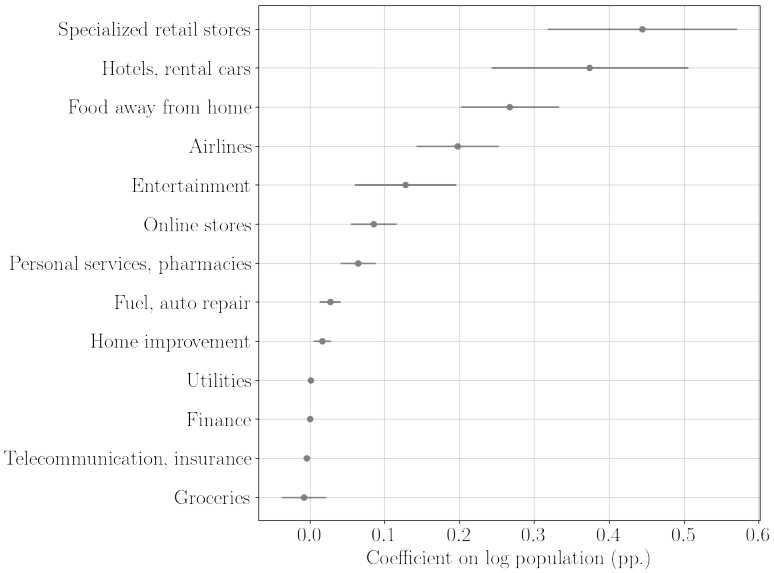


(b) By total income per capita



Notes: Panel (a) shows the direct foreign spending share of consumer cells by their home region's log population density, defined as log of population divided by area. Panel (b) shows the direct foreign spending share by log income per capita. Panel (b) excludes consumers that are "not in the workforce" as their income is likely mismeasured due to the absence of within-household transfer flows in the current iteration of the DEA. Both plots are binned scatter plots with 20 bins mirroring the number of regions in our analysis. Weights are proportional to the number of bank customers in a municipality. The shaded area are 95% robust confidence intervals.

Figure A.X: Log population slopes by foreign spending category



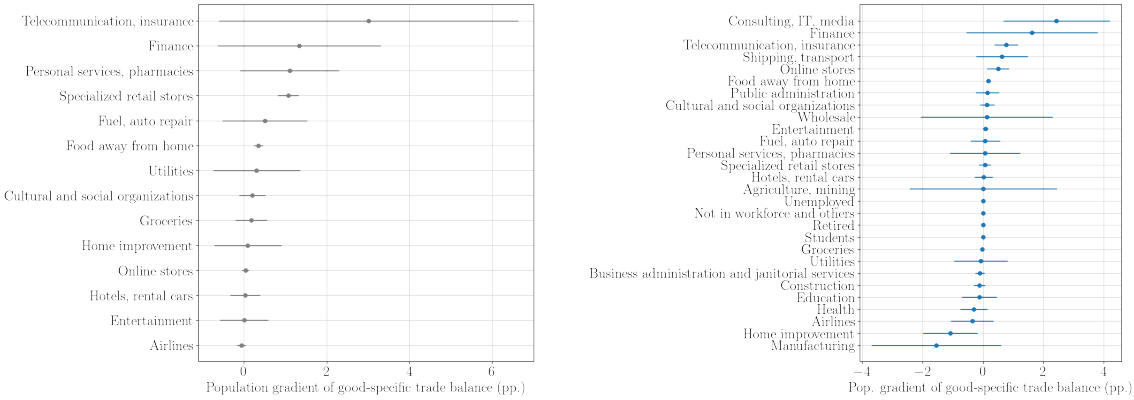
Notes: This figure shows estimated coefficients of a regression of direct foreign spending in a given category on log population. Weights are the number of bank customers in the spending cell. Coefficients are expressed in percentage points. The sum of all coefficients yields the estimated coefficient in Panel (a) of Figure VI. The error bars are 95% robust confidence intervals.

Appendix A.D Goods-Level Trade Balances

Figure A.XI: Goods-level regional trade balances and population size

(a) Consumers spending flows

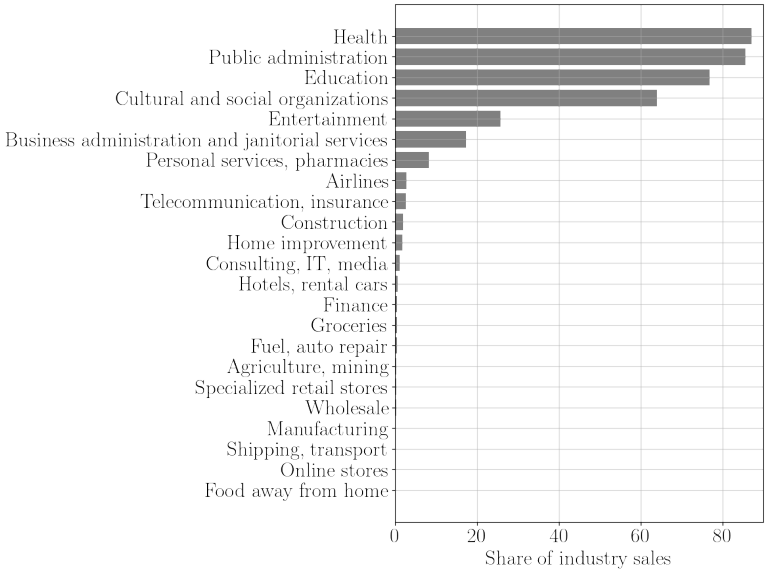
(b) Intermediate input trade flows



Notes: This plot shows estimated coefficients of a regression of goods-level trade balances (sales minus purchases by region residents elsewhere in Denmark) on log population, across regions. Weights are region GDP. Error bars are 95% robust confidence intervals.

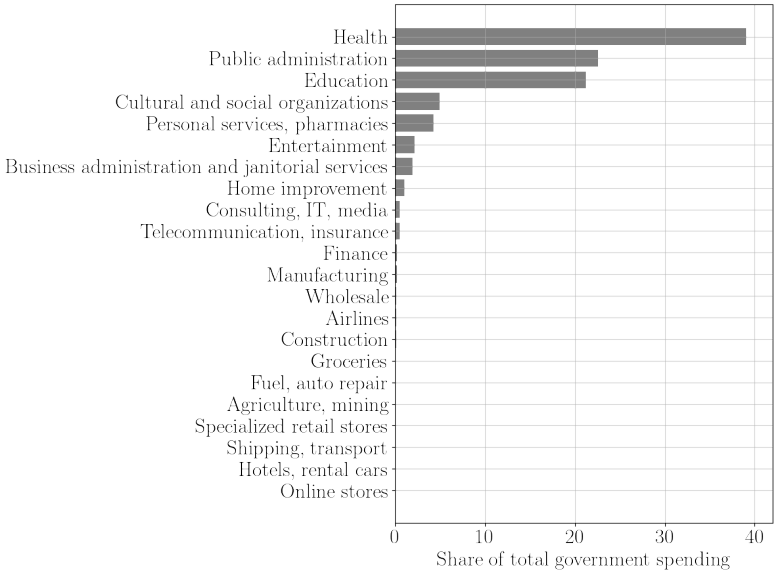
Appendix A.E Results on the Role of the Government in Denmark

Figure A.XII: Share of industry sales accounted for by government spending



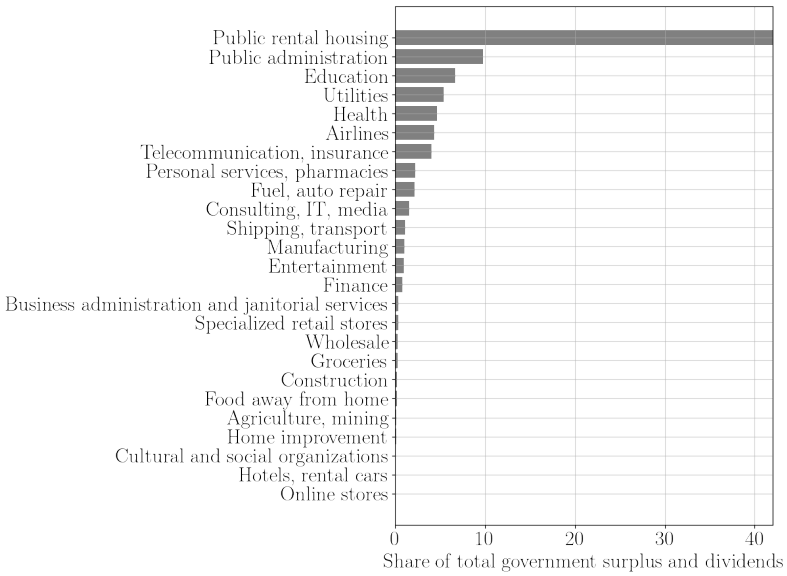
Notes: This figure shows the share of an industry's total sales accounted for by government spending.

Figure A.XIII: Distribution of government spending across industries



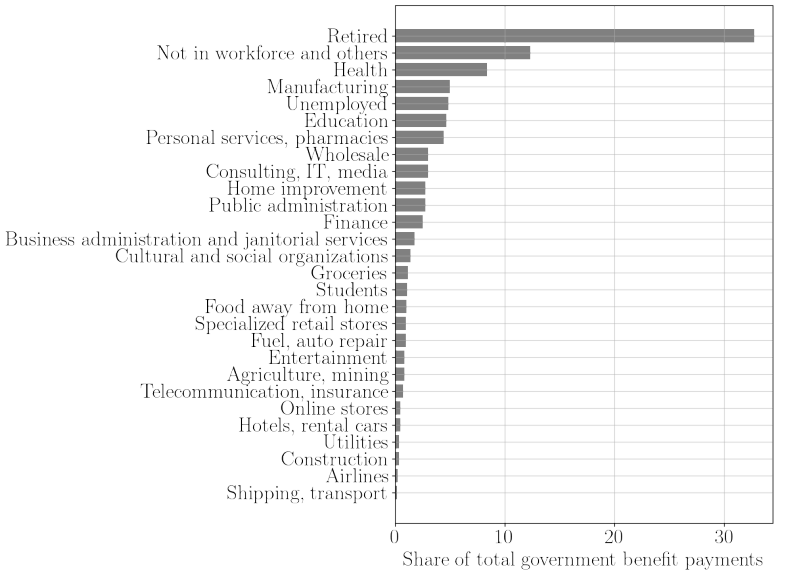
Notes: This figure shows the share of total government spending an industry accounts for.

Figure A.XIV: Shares of government surplus and dividends by industry



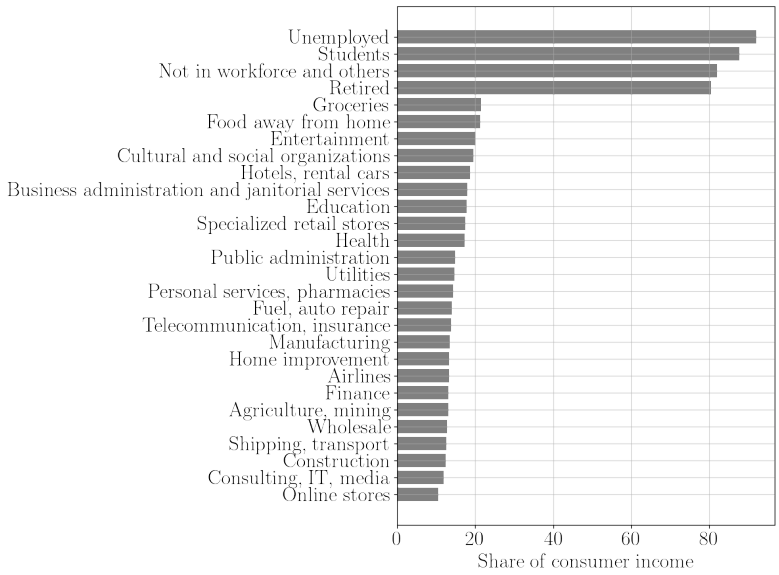
Notes: This figure shows which industries account for what share of total government surplus and dividends.

Figure A.XV: Shares of total government social benefits received by consumer industry



Notes: This figure shows which consumer industries account for what share of total government social benefits received.

Figure A.XVI: Government benefits received as shares of total consumer income



Notes: This figure shows the share of total consumer income that is accounted for by government social benefits received across consumer industries.

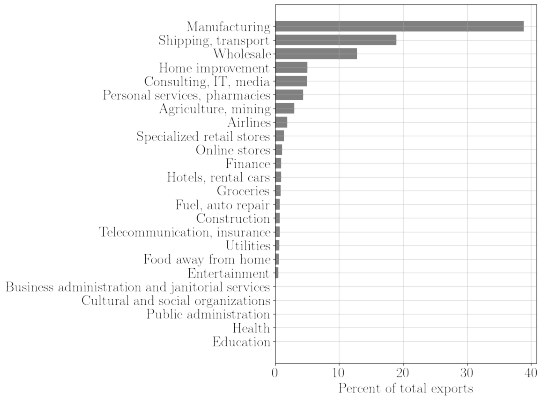
Appendix A.F Details on Calibration

Table A.VII: Calibration overview

Parameter(s)	Symbol	Calibration target	Flow numbers from Table II
Spending shares	$\alpha_{i\mathcal{R}}, \alpha_{is}, \alpha_{isj}$	Disaggregated consumer spending flows	1, 2
Labor compensation shares	λ_{ij}, γ_j^N	Disaggregated labor compensation flows	10
Profit income shares	κ_{ij}, γ_j^K	Mixed income, surplus to consumers	11, 12, 13
Income tax rate	τ_i	Consumer non-product taxes and social contributions	4, 5
Value-added tax rate	τ_i^{vat}	Consumer product taxes paid	3
Intermediate input shares, imports	$\omega_{js}, \omega_{jsj'}, \gamma^X$	Trade flows in domestic intermediates, producer imports	20, 25
Corporate tax rate	τ_j^{corp}	Dividends and surplus from gov. enterprises, taxes	21, 22, 23
Relative distribution of exports	$\tilde{x}_j/\tilde{x}_{j'}$	Producer exports	28
Gov. spending share of gov. revenue	g_j	Domestic government spending	26
Gov. import share of gov. revenue	$\tilde{g}$	Government imports	29
Gov. transfer share of gov. revenue	t_i	Consumer social benefits received, pension adjustment	14, 15
Aggregate export flows	$\tilde{x}_j$	Aggregate GDP	
Within-industry elasticity of sub. of consumers	σ	Cobb-Douglas (1) as baseline, 2 later	
Within-industry elasticity of sub. b/w labor	ζ	Cobb-Douglas (1) as baseline, 2 later	
Within-industry elasticity of sub. b/w intermediates	η	Cobb-Douglas (1) as baseline, 2 later	
Elasticity of export demand	γ	Cobb-Douglas (1) as baseline, 2 later	

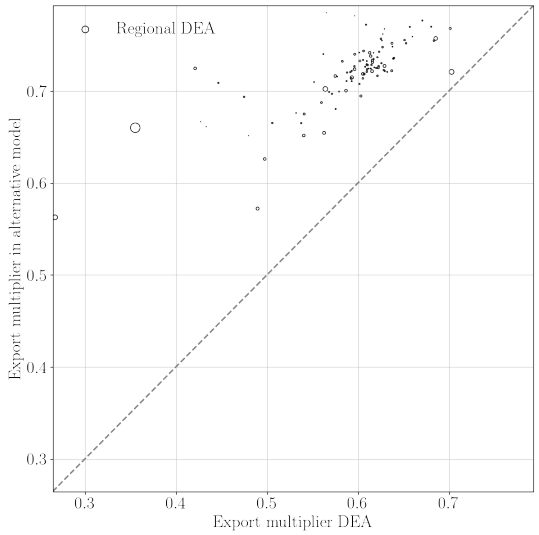
Appendix A.G Results for Application I

Figure A.XVII: Distribution of exports across industries



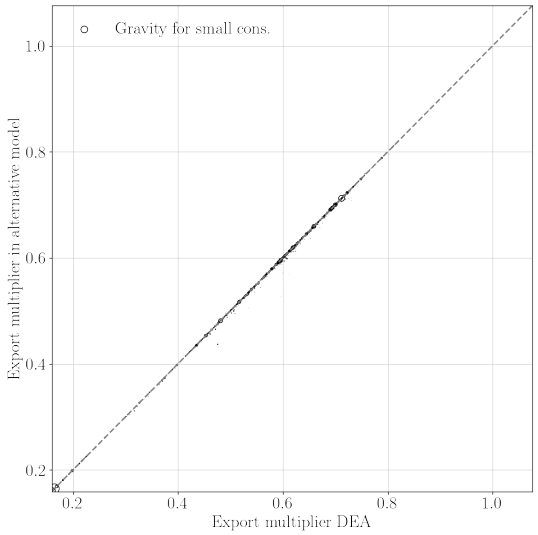
Notes: This figure shows the distribution of total exports across industries.

Figure A.XVIII: Full DEA vs regional DEA: Export multipliers across regions



Notes: This figure shows a scatter plot of export multipliers in a regional DEA model vs the full DEA model we introduced and calibrated in Section VI. The regional model is equal in all regards, and calibrated exactly as outlined in Section VI, except that it does not have an industry dimension. Node size is proportional to exports.

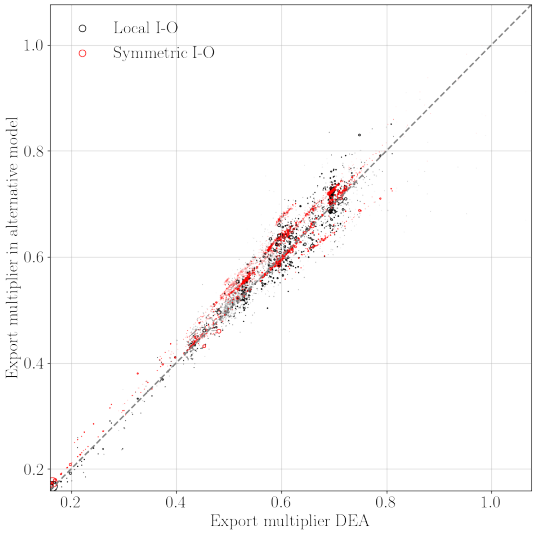
Figure A.XIX: Testing for overfitting of disaggregated consumer spending: Export multipliers



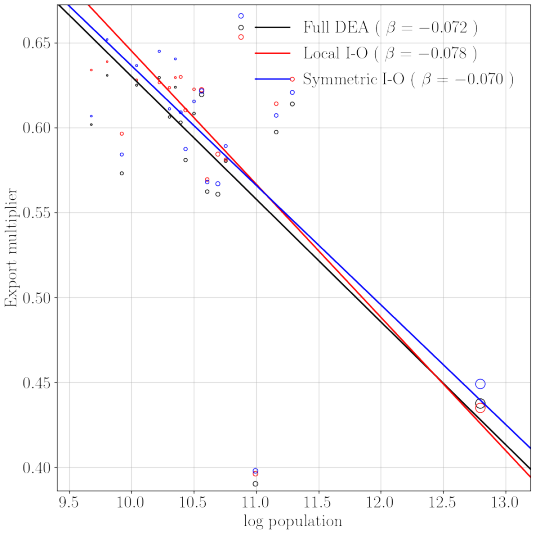
Notes: This figure shows a scatter plot of export multipliers in a modified DEA model vs the full DEA model we introduced and calibrated in Section VI. The modified model is equal in all regards, and calibrated exactly as outlined in Section VI, except that the smallest 90% of the bilateral consumer spending flows by imputed values from the standard gravity regression (2). Node size is proportional to exports.

Figure A.XX: Testing for measurement error in the disaggregated intermediates trade flows: Export multipliers

(a) Scatter plot of all export multipliers



(b) Binned scatter against log population

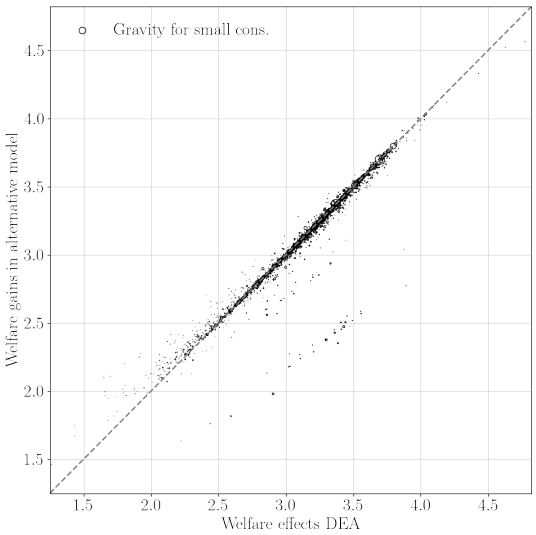


Notes: Panel (a) shows a scatter plot of export multipliers in two modified DEA models vs the full DEA model we introduced and calibrated in Section VI. Both modified models are equal in all regards, and calibrated exactly as outlined in Section VI, except that: the first modified model assumes that all intermediates trade flows are local and do not cross region borders; the second modified model assumes that all intermediates trade flows are symmetric across purchases, that is, Ω is replaced by the unique rank-1 matrix that has the same column sums and same row sums as the correct Ω matrix. Node size is proportional to exports. Panel (b) regresses the export multipliers of the three models against log population in a 20-bin scatter plot.

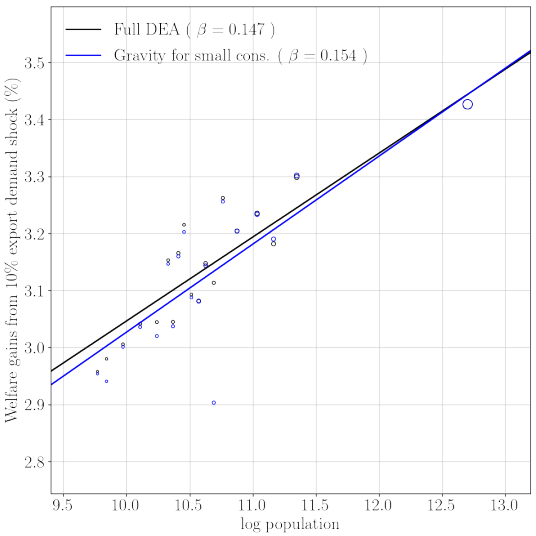
Appendix A.H Results for Application II

Figure A.XXI: Testing for overfitting of disaggregated consumer spending: Distributional gains from uniform export shock

(a) Comparing welfare gains with the DEA



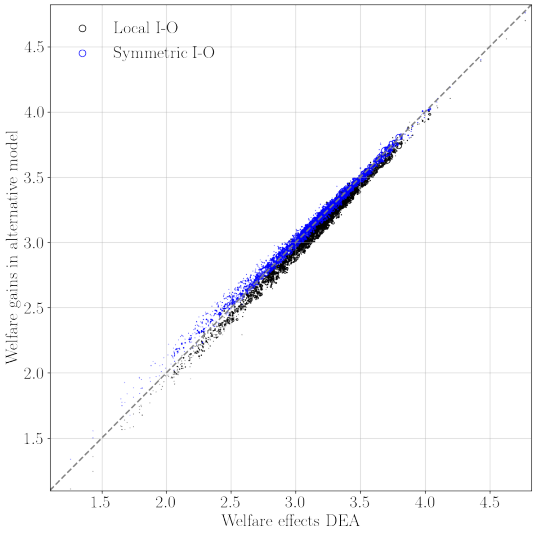
(b) Welfare gains by log population



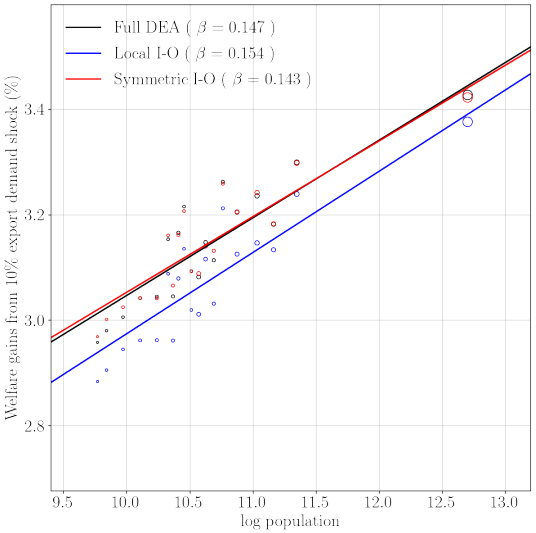
Notes: Panel (a) compares the distribution of the welfare gains from a 10% increase in overall export demand across two models: the full DEA model; and a model that is equal in all regards, and calibrated exactly as outlined in Section VI, except that the smallest 90% of the bilateral consumer spending flows by imputed values from the standard gravity regression (2). Panel (b) shows a binned scatter plot of welfare gains across log region population across the models. Scatter weights are number of bank customers by cell.

Figure A.XXII: Testing for measurement error in the disaggregated intermediates trade flows: Distributional gains from uniform export shock

(a) Comparing welfare gains with the DEA



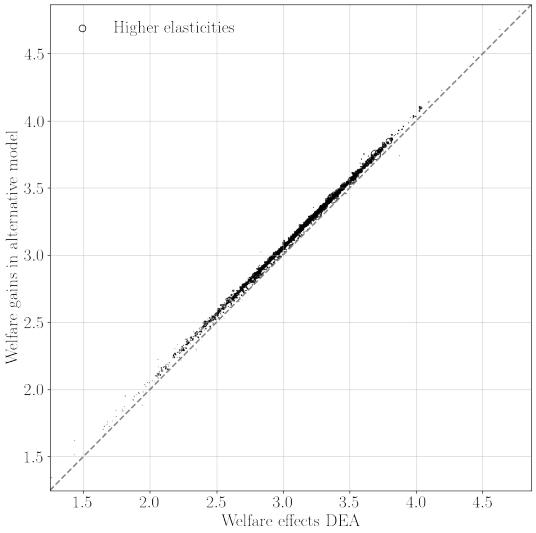
(b) Welfare gains by log population



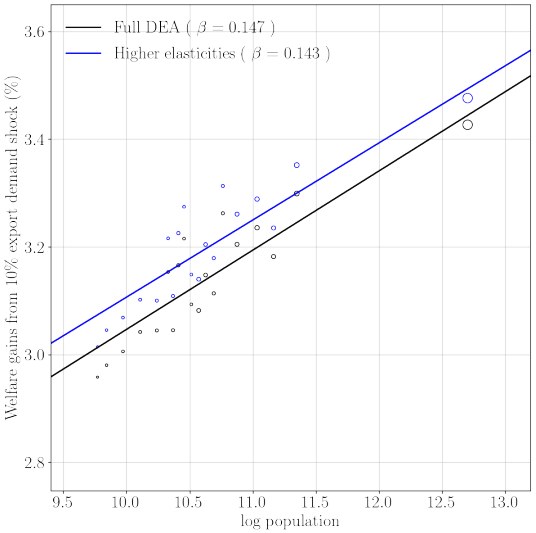
Notes: Panel (a) compares the distribution of the welfare gains from a 10% increase in overall export demand across two models: the full DEA model and two modified models. Both modified models are equal in all regards, and calibrated exactly as outlined in Section VI, except that: the first modified model assumes that all intermediates trade flows are local and do not cross region borders; the second modified model assumes that all intermediates trade flows are symmetric across purchases, that is, Ω is replaced by the unique rank-1 matrix that has the same column sums and same row sums as the correct Ω matrix. Panel (b) shows a binned scatter plot of welfare gains across log region population across the models. Scatter weights are number of bank customers by cell.

Figure A.XXIII: Comparison of distributional welfare gains under Cobb-Douglas with ones for higher elasticities

(a) Comparing welfare gains with the DEA



(b) Welfare gains by log population



Notes: Panel (a) compares the distribution of the welfare gains from a 10% increase in overall export demand across two models: the DEA model with unitary elasticity of substitution and one with higher elasticities of substitution, $\sigma = \zeta = \gamma = \eta = 2$. Panel (b) shows a binned scatter plot of welfare gains across log region population across the models. Scatter weights are number of bank customers by cell.

Appendix B Accounts Aggregated by Cell Type Based on Danish Disaggregated Economic Accounts

Table A.VIII: Aggregate consumer account

Outflows			Inflows		
Outflow	Outflow to	Value (bn DKK)	Inflow	Inflow from	Value (bn DKK)
Domestic consumer spending	Producers	771.9	Labor compensation	Producers	1141.81
Foreign consumer spending	Rest of the World	81.94	Mixed income from non-corporate producers	Producers	80.72
Consumer product taxes paid	Government	173.18	Surplus of corporate producers to consumers (dividends)	Producers	38.48
Consumer non-product taxes paid	Government	566.35	Surplus of owner-occupied housing to consumers	Producers	83.27
Consumer social contributions paid	Government	181.06	Consumer social benefits received	Government	422.2
Consumer interest paid	Capital accumulation	29.67	Consumer adjustment for pension entitlements received	Government	92.48
Consumer natural resource rents paid	Capital accumulation	3.44	Consumer interest received	Capital accumulation	5.32
Consumer other transfers paid	Capital accumulation	44.82	Consumer pension investment income	Capital accumulation	75.48
Consumer gross saving	Capital accumulation	130.05	Consumer natural resource rents received	Capital accumulation	3.44
			Consumer other transfers received	Capital accumulation	39.21
Total value outflows		1982.42	**Total value inflows**		1982.42

Table A.IX: Aggregate producer account

Outflows			**Inflows**		
Outflow	Outflow to	Value (bn DKK)	Inflow	Inflow from	Value (bn DKK)
Labor compensation	Consumers	1141.81	Domestic consumer spending	Consumers	771.9
Mixed income from non-corporate producers	Consumers	80.72	Domestic intermediates	Producers	1414.53
Surplus of corporate producers to consumers (dividends)	Consumers	38.48	Domestic government spending	Government	572.26
Surplus of owner-occupied housing to consumers	Consumers	83.27	Domestic capital accumulation spending	Capital accumulation	359.51
Domestic intermediates	Producers	1414.53	Producer exports	Rest of the World	1077.94
Dividends and surplus of government-owned/operated producers to government	Government	67.86			
Producer product taxes paid	Government	71.94			
Producer net non-product taxes paid and subsidies received	Government	82.79			
Producer net interest, transfers, and saving	Capital accumulation	422.41			
Producer imports	Rest of the World	792.33			
Total value outflows		4196.14	**Total value inflows**		4196.14

Table A.X: Aggregate government account

Outflows			Inflows		
Outflow	Outflow to	Value (bn DKK)	Inflow	Inflow from	Value (bn DKK)
Consumer social benefits received	Consumers	422.2	Consumer product taxes paid	Consumers	173.18
Consumer adjustment for pension entitlements received	Consumers	92.48	Consumer non-product taxes paid	Consumers	566.35
Domestic government spending	Producers	572.26	Consumer social contributions paid	Consumers	181.06
Government imports	Rest of the World	4.26	Dividends and surplus of government-owned/operated producers to government	Producers	67.86
Government net interest, transfers, and saving	Capital accumulation	51.99	Producer product taxes paid	Producers	71.94
			Producer net non-product taxes paid and subsidies received	Producers	82.79
Total value outflows		1143.19	**Total value inflows**		1143.19

Table A.XI: Aggregate rest of the world account

Outflows			Inflows		
Outflow	Outflow to	Value (bn DKK)	Inflow	Inflow from	Value (bn DKK)
Producer exports	Producers	1077.94	Trade surplus	Capital accumulation	100.55
			Foreign consumer spending	Consumers	81.94
			Producer imports	Producers	792.33
			Government imports	Government	4.26
			Capital accumulation imports	Capital accumulation	98.86
Total value outflows		1077.94	**Total value inflows**		1077.94

Table A.XII: Aggregate capital accumulation account

Outflows			**Inflows**		
Outflow	Outflow to	Value (bn DKK)	Inflow	Inflow from	Value (bn DKK)
Consumer interest received	Consumers	5.32	Consumer interest paid	Consumers	29.67
Consumer pension investment income	Consumers	75.48	Consumer natural resource rents paid	Consumers	3.44
Consumer natural resource rents received	Consumers	3.44	Consumer other transfers paid	Consumers	44.82
Consumer other transfers received	Consumers	39.21	Consumer gross saving	Consumers	130.05
Domestic capital accumulation spending	Producers	359.51	Producer net interest, transfers, and saving	Producers	422.41
Capital accumulation imports	Rest of the World	98.86	Government net interest, transfers, and saving	Government	51.99
Trade surplus	Rest of the World	100.55			
Total value outflows		682.38	**Total value inflows**		682.38

Appendix C Overview of Data Sources

We rely on several types of data to construct the disaggregated economic accounts. First, we use customer data from Danske Bank with information about individual customers and about their transactions such as payments and transfers. Second, we use administrative data from a range of government registers such as the population register and the income register with information about individuals and the employment register with information about employment relations. Third, we use publicly available tabulations of aggregate statistics, such as housing statistics, health statistics and securities statistics.

We use these different types of data to disaggregate the flows between consumers, producers and the government described in the national accounts to the level of region-by-industry cells. In the following appendices, we provide more details about the data sources and the various steps of the disaggregated measurement. Table A.XIII lists the disaggregated datasets, data sources, and the Appendix where laying out the details of the disaggregated measurement.

We strive to obtain consistency across all the disaggregated measurement by relying, whenever possible, on the same sample of individuals, the entire adult population in 2018, and a uniform assignment of individuals and firm establishments to cells based on government registers. The only instance where we need to deviate from these general rules is when we construct of the spending matrix. Here, we work with the sample of Danske Bank customers, roughly 20% of the national population, as we do not have data on spending for customers of other banks. Moreover, we need to assign individuals and firm establishments to cells using the bank's internal data, as confidentiality concerns prevent us from merging the bank data and government registers. Finally, we use data for both 2018 and 2019 to maximize sample size and then scale to match 2018 national accounts.

Table A.XIII: Overview of data sources

Disaggregated dataset	Microdata from Danske Bank	Microdata from government registers	Aggregate statistics	Details
Region-by-industry cells	Customer records, transaction data on incoming transfers	Population register (BEF), labor market register (AKM), income register (IND), employment registers (BFL, IDAN)		Appendix D
Consumer spending matrix	Customer records, transaction data on card payments, cash withdrawals, bill payments and mobile applications	Income register (IND), credit register (URTE), auto register (DMR)	National Housing Statistics (BOL101)	Appendix E
Consumer taxes (product)			Input-Output table (NIO3)	Appendix F
Consumer taxes (non-product)		Income register (IND), pension contribution register (INPI), population register (BEF)		Appendix G
Consumer interest, transfers, and saving (paid)		Population register (BEF), income register (IND)		Appendix H
Labor compensation matrix		Income register (IND), pension contribution register (INPI), employment registers (BFL)		Appendix I
Consumer dividend, mixed income, and surplus matrix		Income register (IND), employment register (IDAN)	General Firm Statistics (GF5), Registered Securities Statistics (DNVPDKS)	Appendix J
Government benefits to consumers		Income register (IND), pension contribution register (INPI), population register (BEF)		Appendix K
Consumer interest, transfers, and saving (received)		Income register (IND)		Appendix L
Intermediates trade matrix			Input-Output tables (NIO1, NIO2, NIO3), Cargo transport statistics (Tables NVG23, SKIB481)	Appendix M
Government dividend and surplus income		CVR register (hand-collected)		Appendix N
Consumption of government and NPISH output	Transaction data on payments	Population register (BEF), education register (UDD), income register (IND)	Public Expenditure Statistics (SYGU1), Health Statistics (INDAMP01), Education Statistics (UDDAKT20), Child Care Statistics (BOERN4), Culture and Leisure Statistics (BIB1)	Appendix O

Appendix D Defining Region-by-Industry Cells

The unit of our measurement is region-by-industry cells, the interaction of geographical regions and economic industries. Specifically, we define 99 regions (98 Danish municipalities and one foreign region) and 31 industries (Table A.I). We choose this definition of region and industries because we can observe it consistently across all underlying datasets and because it reveals a large degree of heterogeneity in disaggregated flows.

There are 24 industries that pay labor compensation to consumers. They include retail industries selling directly to consumers (e.g., food away from home, entertainment, grocery stores, drug stores), non-retail industries transacting mostly with firms (e.g., wholesale, agriculture, manufacturing, business services), and government-operated industries (e.g., public administration, health, education). We map six-digit NACE industry codes to the 24 industries with employment (the European NACE is based on the standard United Nations ISIC). Additionally, there are 4 separate industries for the non-working parts of the population (e.g., retired, students, unemployed, out of workforce) and 3 industries providing housing without employees.

In the government registers, we assign all adults to a region based on their home address at the start of 2018 observed in the administrative population register (BEF) and to an industry based on the NACE code of the firm establishment responsible for the largest part of their 2018 labor income in the labor market register (AKM). Individuals with no labor income are assigned to an industry based on their age, observed in the administrative population register (BEF), and other income sources, observed in the income register (IND). Specifically, individuals with no labor income are assigned to the industry "retired" if they are older than 65 years; to the industry "students" if they receive a government stipend (for which higher education students are almost universally eligible); to the industry "unemployed" if they receive unemployment or cash benefits; and otherwise to the industry "out of workforce." We assign firm establishments to producer region-by-industry cells using information from the employment registers (BFL and IDAN).

In the Danske Bank data, we identify consumers' industry by extracting the name of the employer from incoming salary payments and by identifying incoming government transfers (retired, students, government stipend, unemployed). We use the banks' customer records to identify their region of residence. The address register in the bank is linked to the government's address registers and updated on a monthly basis. To ensure that moves across regions do not distort the spending patterns, we update an individual's region every month when constructing the spending matrix. However, consistent with the assignment in the government registers, we define the industry of main employment on an annual basis, as the industry paying the largest share of consumer income.

To ensure anonymity, we censor information for the (very few) cells containing fewer than 3

individuals or establishments in all datasets. In the spending matrix, we censor information for cell-to-cell flows based on fewer than 10 transactions in total (in total, less than 0.1% of national spending). We instead impute these flows, setting them equal to the average per capita spending flows of all other cells in the same region.

Appendix E Measuring the Disaggregated Consumer Spending Matrix

Appendix E.A Data and Sample

To construct the spending matrix, we rely on data from Danske Bank for 2018 and 2019. We observe transaction-level information on consumers and merchants only for this period. Our sample consists of adult customers who conducted at least one transaction per month and registered their main bank account at Danske Bank throughout 2018 and 2019.[A1] For each customer, we observe all incoming and outgoing transactions. Card payments accounted for 47% of the total value of payment transactions. The most common card was Dakotan (debit cards issued by Nets A/S), followed by MasterCard (debit and credit cards) and Visa (debit and credit cards). Cash withdrawals accounted for 7% of transaction value, while bill payments (direct debits and bank transfers to merchants) accounted for 45%. Mobile applications, such as Apple Pay or the Denmark-specific MobilePay, made up 1%.

Appendix E.B Identifying the Merchant Region

We extract the address of the merchant establishment (i.e., the store) from the string that accompanies payment transactions in the bank's internal computer system. The information for card and mobile payments differs slightly by payment type.

- Dakotan statements include a unique ID number for each merchant establishment for transactions in Denmark and the country name for transactions abroad. We match the Danish merchant IDs to the exact merchant address using a table issued by Nets A/S.
- MasterCard includes a detailed merchant address directly in the transaction string in the following format: merchant ID, shop name, street name, house number, postal code, country.
- Visa reports the merchant ID, shop name, and town for each transaction. The merchant IDs used by Visa and MasterCard generally coincide. Based on the MasterCard data, we can therefore construct a table matching merchant ID and detailed address. In very few cases, a merchant ID gets used twice for a Danish merchant and a foreign merchant. In these cases, we assume that the transaction was with the Danish merchant.

[A1] All adults register one "main" account with the Danish government, through which they conduct all financial interactions with the government.

- Some transactions in MobilePay and Apple Pay contain merchant addresses, but many do not. There is no clear pattern in which transactions contain addresses, which is likely due to the recent development of these transactions.

For all card and mobile payments, we extract the merchant address and convert it to a consistent format using an API service provided by the government agency Styrelsen for Dataforsyning og Effektivisering (dawa.aws.dk/dok/api/adgangsadresse). This conversion identifies precise geocodes for each merchant while accounting for misspelled addresses and addresses that appear twice due to minor differences in formatting or spelling. The API compares the merchant address from a transaction with its database of official addresses. It iterates in a Levenshtein manner (i.e., it calculates the number of letters/digits that must be exchanged before one string is equal to another). We force the Levenshtein process to consider only addresses with exactly identical postcodes. Municipalities are combinations of several postcodes. By restricting to the same postcode, we ensure that the Levenshtein process cannot change the municipality information, the key information that we use to construct the spending matrix.

If the API cannot match the address unambiguously (so called C-match), we remove the first line of the address, which often combines abbreviations of merchant and street name, making it difficult to automatically recognize. We also check whether the address contains the name of a shopping mall, rather than an official address. If so, we replace the name of the mall with the mall's address and rerun the API process. Finally, we manually research the official address of the 100 most common unmatched addresses.

Using this procedure, we identify the official shop address for 95% of card and mobile spending. We assume that the remaining 5% go proportionally to the same regions as other card and mobile spending. These remaining 5% also include cases where mobile applications (e.g., MobilePay) and online services (e.g., PayPal) do not directly send the purchase amount to a sales terminal, but transfer to a central account first before paying the merchant.

For bill payments, we use a slightly different approach. We directly observe the merchant's postal code for recurrent bill payments, which make up 67% of all payments. These observed postal codes allow us to infer the merchant region for the majority of remaining bill payments. Specifically, we split merchants into 48 industries. We calculate the number of transactions from each consumer region going to each of these 48 industries. To minimize noise, we keep industries where at least 50% of incoming bill payments contain postal code information and where we observe at least 200 incoming transactions from every consumer region. (Industries receiving 80% of total bill transaction value satisfy these two requirements.) For these industries, we then assume that bill payments flow to the same postal codes as bill payments with observed postal codes from the same consumer region to the same industry. For the remaining industries (covering 20% of bill transaction value), we assume that bill payments flow to the same postal codes as card

payments from the same consumer region to the same industry.

We generally do not observe how cash withdrawals are spent, but we assign them to merchants in proportion to observed card spending in Denmark if the withdrawal was in Denmark and to observed card spending abroad otherwise. In a few cases, country information is missing, so we assume that withdrawals without decimal points (e.g., 100.00 DKK) are withdrawn and spent in Denmark and that other withdrawals (e.g., 100.76 DKK) are spent abroad.

Appendix E.C Identifying the Merchant Industry

We extract the merchant category code (MCC), a classification for the type of goods sold, for each merchant in the bank transaction data. To create a disaggregated system with consistently defined producer cells, we need to map MCCs to the industry codes used in the employment and production data. However, no such mapping exists so far. We therefore create a new cross-walk between MCCs and NACE industries.

First, we observe each merchant's Danish business identification number (CVR) and MCC in the bank's system. We link the CVR to the Danish Central Business Register where we can retrieve the merchant's industry (at the level of 741 NACE codes). Second, we manually assign MCCs to our 31 industries (only 14 of which are consumer-facing and are assigned MCCs). We then identify which of our industries appear most frequently among merchants in each of the 741 NACE codes. In very few cases, two industries appear equally often and we manually research the largest firms to identify the right match.

We create an alternative mapping between a merchant classification system called PCAT and our industries. The PCAT usually appears as part of the electronic transaction information for bill payments and can easily be mapped to our industries. MCC and PCAT are missing for some bill payments, amounting to 8% of total transaction value through bills. We assume that these payments go proportionally to the same industries as other bill payments by the same consumer cell.

Appendix E.D Online Spending

We identify whether card transactions took place in a physical store or online. Dakotan transactions include a straightforward indicator for online transactions. Mastercard and Visacard transactions contain ISO 8583 information, an internationally standardized message sent by a sales terminal in a transaction. If the POS7 code (input method) equals 1, 6, K, or L or if the POS5 (cardholder) code equals 5, the transaction is online. We treat payments using mobile applications (e.g., MobilePay) and online services (e.g., PayPal) that do not report a physical merchant ID and address as online transactions. For digital payment services, such as PayPal and DoorDash, we typically see the

correct MCC of the establishment receiving the final sales.

In constructing the spending matrix, we generally treat online spending identically to spending in a physical store. That means that we identify the region-industry cell of the merchant and assign the incoming spending to this cell. However, we adjust the regional (but not the industrial) distribution of online spending for industries where we know that consumption of the final good takes place entirely in a physical location. In these cases, we assign all online spending using the regional distribution of spending on physical merchants. The online spending on these industries often goes to the central payment terminal of a parent company before being assigned to the physical merchant. For instance, online purchases of cinema tickets are often booked through a central company terminal in Copenhagen, even though consumption happens in local movie theaters. The full list of industries where we adjust the regional distribution is: food away from home; entertainment; medical and other specialized merchants; commuting; vehicle repair; hotels; rental cars; home improvement services. (These industries are generally sub-categories of our 31 final industries.)

We also adjust spending on airlines flowing into Copenhagen because we know that the airline establishments receiving the spending are actually located in the neighboring Tårnby region, which also contains the airport. Specifically, we reassign a share of each consumer cell's spending on airlines flowing into Copenhagen to airlines in Tårnby, so that Copenhagen's share in airline spending received equals its share in airline employment (see also Figure A.III).

Appendix E.E Improving the Spending Matrix with Government Data

Consumer spending on four types of goods are not captured well in bank transaction data: housing, financial services, vehicles, and water and waste services. We replace the transaction-based values for these four goods in the spending matrix with adjusted values derived from combining Danske Bank data with government registers.

Appendix E.E.1 Consumption of Housing

We use different methods to assign consumption of owner-occupied housing and rented housing. First, owner-occupied rents are notoriously difficult to measure because they involve no financial transaction. However, the administrative income register (IND) contains the imputed rental value of owner-occupied housing for every individual. We thus allocate the national consumption of owner-occupied housing to consumer cells in proportion to the imputed rental value of their owner-occupied housing. Expenditure on owner-occupied housing flows to the producer cell for owner-occupied housing (our industry number 30) located in the same region as the home owners. The operating surplus of an owner-occupied housing producer cell in a region (SNA B.2G) then

goes back to the consumer cells owning homes in that same region.

Second, we do not observe all rental payments in the bank transaction data. We instead distribute the national consumption of rented housing across consumer cells in proportion to their estimated rental costs. We observe some rental payments in the bank data, which we use to estimate:

$$rental\ payment_p = \alpha + \vartheta\, region_p + \phi\, industry_p + \psi_1\, age_p + \psi_2\, age_p^2 + \varepsilon_p, \tag{A.1}$$

where p is an individual renter. Using the estimated fixed effects ϑ and ϕ, we predict the average rental cost in each consumer cell. We observe ownership of real estate in the administrative income register (IND) and assume that all consumers who do not own real estate are renters, which allows use to calculate the number of renters by consumer cell. Combining estimated rental payments with the number of renters allows us to estimate total rental costs by consumer cell, which we use to allocate national consumption of rented housing across cells.

Finally, we use aggregate statistics to assign rental payments to producer cells. In the National Housing Statistics (Table BOL101), we observe the number of rental housing units in each region owned by different owner types: individuals, non-profit building societies, limited liability companies, housing societies, and public authorities. The surplus of individual owners accrues to the "private landlord" industry (our industry number 29) and those of corporate owners accrue to the "finance, real estate" industry (our industry number 17). As there is no information about the geographical location of the owners, we assume that the geographical distribution of individual owners of rental units in a given region follows the geographical distribution of mixed income in the region and that the geographical distribution of the individuals behind corporate owners of rental units in a given region follows the geographical distribution of dividend payments. The remaining owner types are public or non-profit organization. The surplus of these owners goes to the "government-owned housing" industry (our industry number 31) in the same region where the housing is located.

Appendix E.E.2 Consumption of Financial Services

The consumption of financial services in the national accounts is composed of the value of financial advice provided by financial firms and the interest rate spreads applied by financial firms. While we observe payments for financial advice in the bank transaction data, it is difficult to disentangle the interest rate spreads from the raw value of interest payments in transaction data. Instead, we allocate the national consumption of financial services across consumer cells in proportion to their interest expenses. The tax register (IND) contains interest expenses for every individual. We aggregate interest expenses at the level of each consumer cell.

Producer cells in our "finance, real estate" industry (number 17) receive consumers' expenditures on financial services. To identify which regional producer cells receive expenditures from which consumer cells, we use loan-level data from the administrative credit register (URTE). Specifically, for each bank loan and each interest payment, the credit register contains an identifier for the bank branch that has recorded the loan and the interest payment. There are around 3,000 bank branches in Denmark. We do not observe the location of branches directly, so we define the region of each branch as the most common region of consumers holding loans recorded at the branch. We then compute how the interest payments of each consumer cell are distributed across bank branches in different regions. Finally, we assume that a consumer cells' expenditure of financial services is distributed across producer cells in proportion to its distribution of interest payments.

Appendix E.E.3 Vehicle Purchases

We do not observe all vehicle purchases in the bank transaction data because many purchases do not flow directly to the vehicle producer but through financial firms. We therefore use a top-down approach to assign national vehicle purchases to consumer cells in proportion to each cell's share of total spending on new cars. We estimate each consumer cell's total spending on new cars by combining bank transaction data on annual spending at vehicle dealers with information on vehicle registrations from the administrative auto register (DMR). We use data over the period 2014-2016, as this is the most recent period where we can combine transaction data from Danske Bank and administrative data from the auto register. We therefore assume that relative levels of vehicle spending are unchanged between 2014-2016 and 2018.

We first estimate the average vehicle price in each consumer cell. We use a sample of individuals where we observe just one new car registration in a given year and spending at vehicle dealers of at least 50,000 DKK in the bank transaction data in the same year. We then regress individual-level spending at vehicle merchants on industry-by-year and region-by-year fixed effects:

$$spending_{p,y}^{vehicle_merchants} = \theta_{i(p),y} + \eta_{r(p),y} + \epsilon_p.$$

where p is an individual in industry $i(p)$ and region $r(p)$ and year is y. We predict the average price of new cars in each consumer cell using the estimated fixed effects. We can directly calculate the number of newly registered vehicles in the government vehicle registers. Combining the estimated price with the number gives an estimate of total spending on new cars by consumer cell for each year in 2014-2016. We compute each cell's share of total spending on new cars in each year and then average across years. We use these shares to allocate national vehicle purchases in 2018 across consumer cells.

Producer cells in our "cars, fuel, car repair, public transport" industry (number 5) receive consumers' expenditures on vehicles. To identify which regional producer cells receive spending from which consumer cells, we use Danske Bank data on card transactions. Since vehicle purchases are relatively infrequent, we collapse to the regional level and assume that all consumers within the same region distribute their vehicle spending across regions in the same proportion.[A2]

Appendix E.E.4 Water and Waste Services

Rental payments often include consumption of water and waste services in Denmark, which implies that we cannot separately identify spending on water and waste in the transaction data. We therefore allocate the national consumption of water and waste services to consumer cells in proportion to their spending on other utilities. We assume that water and waste is produced locally, setting the region of the producer cell receiving the payments equal to region of the consumer cell.

Appendix F Measuring Consumer and Producer Product Taxes Paid

Product taxes are paid by buyers upon the purchase of a good. The most important product taxes are import taxes, product-specific taxes (e.g., on fuel, energy, cigarette, and alcohol), and valued added taxes (VAT).

We first focus on measuring product taxes paid by consumers. We observe import and product-specific taxes paid by consumers in the Danish national accounts table NIO3. It reports the total amount of import and product-specific taxes paid by consumers on purchases of goods produced by 117 distinct industries. These 117 industries do not map directly into our 31 industries. We therefore break down the 117 industries into the most granular grouping in the Danish national accounts (741 industries), assume that taxes paid are proportional to employment shares, and aggregate to our 31 industries. We then calculate the implied tax rate by combining information on total consumer spending on each industry from the consumer spending matrix with the tax data. The implied import and product-specific tax rates range from 0 percent (e.g., for "personal services, pharmacies" and "cultural and social organizations") to 50 percent in utilities (due to Denmark's very high energy tax rates).

The standard VAT rate in Denmark is 25 percent. We set this tax rate for consumer spending on all industries except a few industries whose products are VAT-exempt: airlines, finance, health, education, public administration, and all housing. In addition, spending on two of our industries

[A2] If there are less than 50 transactions for a region, we group that region with a neighboring region. This leads us to group Læsø with Frederikshavn; Langeland and Ærø with Svendborg; Fanø with Esbjerg; Ringkøbing-Skjern with Herning; Lemvig with Holstebro; Morsø with Thisted; and Samsø with Odder.

is partially exempt: insurance (part of our industry 8, "telecommunication and insurance") and culture (part of our industry 18 "cultural and social organizations"). We use data from NIO3 on total VAT paid by consumers by industry and calculate that the average VAT rate is 7 percent (for industry 8) and 5 percent (for industry 18).

We observe product taxes paid by Danish producers, the government and capital accumulation cells, and foreign countries in the Danish Input-Output table, described in detail in Appendix M. Notable facts are that VAT does not get paid on exports and that firms get reimbursed for any VAT they paid on intermediates.

Appendix G Measuring Disaggregated Consumer Non-Product Taxes

Consumer taxes comprise national accounting flows from consumer cells to the government. First, current taxes on income, wealth, etc., payable (SNA D.5) includes income taxes paid directly by consumers as well as a tax on pension wealth returns paid by pension funds on their behalf. We disaggregate each part separately. In the income register (IND), we observe total annual income tax liabilities, excluding pension returns taxes, for each individual in the population. We use this as the disaggregated measure of directly paid income taxes, scaling it by a factor **1.09** to match the national accounts value when aggregating across consumer cells. For the pension returns taxes paid by pension funds, we have no direct micro level measure. We therefore apply a top-down approach assuming that DKK pension wealth returns, and hence also returns taxes, are proportional to total accumulated pension contributions since 1995, which is the first year for which we have microdata on pension contributions in the pension contribution register (INPI).

Second, we use a bottom-up approach to disaggregate social contributions, payable (SNA D.61). In the income register (IND) and the pension contributions register (INPI), we extract information on total annual pension contributions, including contributions to a mandatory retirement savings program (ATP), as well as on membership fee payments to unemployment insurance funds. We aggregate these variables to the cell level and scale the cell totals by a factor **1.2** to make the national total match the value from national accounts.

Appendix H Measuring Disaggregated Consumer Interest and Transfers Paid

Consumer financial and rental expenditures include consumers' interest payments (SNA D.41) and payments related to renting of land and subsoil resources (SNA D.45). We disaggregate the former using a bottom-up approach. From the administrative income register (IND), we have information on each individual's interest payments on all financial liabilities. The sum of these interest payments exceeds the value of position D.41 in the national accounts because the micro-level measure includes the full nominal amounts paid by consumers to lenders, whereas the

national accounts value is net of Financial Services Indirectly Measured (FISIM). We handle this by **scaling** the micro-level variable so that it matches the national accounts value in the aggregate, the implicit assumption being that the ratio between FISIM and total nominal interest payment is the same across consumer cells. Since we have no microdata information about payments related to renting of land and subsoil resources, we use a top-down approach to disaggregate this national accounts flow. Each consumer cell is assigned a share of the aggregate value corresponding to its population share derived from the administrative population register (BEF). Other current transfers, payable (SNA D.7) includes non-life insurance premium payments and miscellaneous current transfers. We disaggregate it top-down, assigning to each cell a share of the national total equal to its population share derived from the administrative population register (BEF).

Appendix I Measuring the Disaggregated Labor Compensation Matrix

The labor compensation matrix records flows of labor income from producer cells to consumer cells. We use the income register (IND) and the pension contributions register (INPI) to measure total annual labor compensation, including employer-paid contributions to pension schemes, received by each individual. We define the producer cells based on the industry of the individual's main employer observed in the labor market register (AKM) and the region of the firm establishment where most of the payments come from observed in the employment register (BFL). We then aggregate these payments to a matrix at the level of consumer and producer cells. We scale all values by a factor of **1.02** to match the national accounts flow Compensation of employees, receivable (SNA D.1).

Appendix J Measuring the Disaggregated Dividend, Mixed Income, and Surplus Matrix

The matrix reveals dividends, mixed income, and surplus flowing from each producer cell to each consumer cell. We first calculate how each producer cell is split into a "corporate" and a "non-corporate" part and then, for each part separately, estimate the distribution of dividends, mixed income, and surplus across consumer cells.

In the first step, we use aggregate statistics to split firms in each industry into a corporate and a non-corporate part (Table GF5 in "General Firm Statistics"). We categorize sole proprietorships as "non-corporate," corporations with limited liability and partnerships as "corporate," and associations, funds and other rare legal forms as "other." For each industry, we are thus able to compute the share of total sales attributable to "non-corporate" and "corporate" firms, respectively. The sole exception is the financial industry where we compute the split based on employment, as

sales are not reported. We convert the industries reported in Table GF5 to our industries using employment shares as weights.

In the next step, we estimate how corporate ownership is distributed across consumer cells. We measure foreign ownership using aggregate statistics (Table "DNVPDKS" in the "Registered Securities Statistics"). The share of Danish stocks held by foreigners was 50% in 2018. We assume that this fraction is constant across all producer cells. We then distribute the ownership of the remaining 50% across domestic consumer cells in proportion to their stock market wealth. To implement this distribution, we aggregate the market value of stocks from the administrative income register (IND) for each consumer cell. We thereby assume that all consumer cells hold a fully diversified portfolio of Danish corporations.

Next, we determine how non-corporate ownership is distributed across consumer cells. Here, we construct a self-employment matrix similar to the labor supply matrix discussed above. To construct the matrix, we link information about establishments operated by sole proprietorships from the employment register (IDAN), including the municipality where the establishments are located, to the business income of the self-employed individual operating the establishment.

We distribute the income of stock corporations and privately-owned firms using distinct methods. For stock corporations, we rely on data from the income register (IND) about individual stock dividend income, which we use to disaggregate distributed income of corporations, receivable (SNA D.42). We **scale** so that the total across consumer cells matches the national aggregate.

Finally, the national accounting flow operating surplus, gross (SNA B.2G) corresponds to operating surplus from owner-occupied housing. We aggregate individual-level imputed rental values of owner-occupied housing as reported in the income register (IND), which produces the industry's total output. We scale this output by a **factor 0.67** to match the national value for B.2G. The implicit assumption is that the ratio between gross operating surplus and output in the owner-occupied dwellings industry is constant across consumer cells.

Appendix K Measuring Disaggregated Government Benefits to Consumers

National accounts describe three types of transfers to consumers. First, we aggregate all government income transfers and private pension savings payouts from the income register (IND) to calculate the cell-level measure of social benefits other than social transfers in kind, receivable (SNA D.62). We scale by a factor **1.03** to match the national accounts value. Second, other current transfers, receivable (SNA D.7) consist of miscellaneous current transfers, for example disaster and accident relief. We disaggregate this position top-down using cell population shares obtained from the administrative population register (BEF). Third, adjustment for the change in pension entitlements, receivable (SNA D.8) represents an accounting adjustment to make net changes

in pension entitlements on which consumers have a definite claim appear in the calculation of consumer saving. We disaggregate it by combining data from the income register and the pension contributions register (INPI) to construct a micro-level measure of pension contributions net of payouts. We then scale this measure to match the national accounts aggregate value.

Appendix L Measuring Disaggregated Consumer Interest and Transfers Received

First, we disaggregate interest, receivable (SNA D.41) bottom-up by using individual-level information on interest income from the income register (IND) and scaling so that the total across consumer cells matches the national accounts aggregate. Second, other investment income, receivable (SNA D.44) includes investment income from insurance policies and pension entitlements. We disaggregate this using a top-down approach where each consumer cell is assigned a share of the national accounts value proportional to its pension contributions accumulated since 1995. Third, rent, receivable (SNA D.45) consists of income from renting land and subsoil resources. We disaggregate it top-down using population shares.

Appendix M Measuring the Disaggregated Intermediates Trade Matrix

The disaggregated intermediates matrix describes how producers in one cell are connected to producers in other region-industry cells through their demand for production inputs. The Disaggregated Government Consumption Production Vector describes how producer cells contribute to the production of goods and services used for government consumption. To construct them, we start from a standard input-output table for the Danish economy at the most disaggregated level with 117 industries (Tables NIO1, NIO2, and NIO3 at statbank.dk). The input-output table illustrates how the output produced in one industry is used as intermediate inputs in other industries or in final use categories such as private consumption, government consumption, capital formation, or exports. It also shows how output from a given industry is produced from intermediate inputs from other domestic industries and from abroad (net of product taxes and import duties, which are added as additional primary inputs), and gross value added.

To convert the standard input-output table to a format suitable for our purposes, we need to address four challenges. First, the 117 industries do not map directly into our industry classification. Second, the input-output table has no geographical dimension: It describes inputs from firms in industry s to firms in industry t at the national level, but not from firms in industry s and region i to firms in region j and industry t. Third, the national accounting convention of defining production in the retail industry as the net trade margins makes the table unsuitable for combining with the disaggregated consumer spending matrix (which reports actual money flows) in order to follow the flow of money from consumers to retailers and further on to upstream producers. Fourth,

the input-output table does not distinguish between residents' and non-residents' spending in the moneys flows from private consumers to domestic producers, bringing it at odds with the disaggregated consumer spending matrix. The following subsections describe how we handle each of these challenges.

Appendix M.A Adapting the IO table to DEA Industry Classification

To address the first challenge, we first disaggregate the input-output table based on the national accounts (NA) classification to a more granular subindustry grouping. We later re-aggregate to the industry classification used in our system of disaggregated economic accounts (DEA). For the initial disaggregation, we use that the industry codes in the microdata (which have 744 categories including the three zero-employment housing services industries) maps directly into both classifications.

Formally, let lower-case letters $\{a, b, c...\}$denote the NA industries in the standard input-output table and let upper-case letters $\{A, B, C...\}$denote the 27 output-producing DEA industries shown in Table A.I. Let j denote a granular industry at the level used in the microdata. Consider a particular NA industry $x \in \{a, b, c...\}$ and a particular DEA industry $Y \in \{A, B, C...\}$: We define subindustry x_Y as the set of granular-level industries that are subindustries of both x and Y, $x_Y = \{j | j \subset x, j \subset Y\}$. This produces 173 non-empty subindustries. These represent the highest level of industry aggregation compatible with both the NA classification and the DEA classification.

In the microdata, we aggregate salary payouts within each non-empty subindustryx_Y and compute the subindustry's salary share within NA industry x. We then disaggregate the flows reported in the original input-output table using these shares. For example, the original input-output table reports the value of flows from NA industry a to NA industry b. We assume that the flow stemming from subindustry a_Y is proportional to its share of total salary payouts to workers in NA industry a. Similarly, we assume that the flow to subindustry b_Z is proportional to its share of total salary payouts to workers in NA industry b. Concretely, we compute the flow from a_Y to b_Z as the total flow from a to b multiplied by the product of the salary shares of a_Y and b_Z within their respective NA industries.[A3]

The input-output table also reports money flows from final use categories (private consumption, government consumption exports, capital accumulation, etc) to domestic producers, and from domestic producers to tax collectors and foreign producers. In both cases, we disaggregate to the subindustry level on the domestic producer side. As a general rule, we again use salary shares for the disaggregation procedure, but with one important exception: The shipping industry (NA

[A3] We make an exception to this general rule for the three zero-employment housing services industries where we use production shares rather than salary shares for the disaggregation.

industry 500000) is large in Denmark, accounting for 16% of both imports and exports in 2018. It has two subindustries, one related to transport of passengers and one related to transport of goods, and the latter pays about twice as much in total salary payments as the former. Detailed foreign trade statistics show that trade values are about 100 times larger for goods than for passengers. On this background, we assign all imports and exports to/from the shipping industry to the goods transport subindustry.

It should be noted that the disaggregation procedure may introduce considerable measurement error if flows are not proportional to salary shares. But to the extent that granular industries that belong to the same industry in the NA classification also belong to the same industry in our DEA classification, which is very often the case, such errors will vanish when we later re-aggregate to the DEA industry level.

Appendix M.B Adding Geography

The next challenge is to disaggregate the national flows between subindustries and final use categories by pairs of regions. To do this, we use aggregate cargo transport statistics describing general trade patterns across localities, as well as micro-level data on the economic size of each region within a subindustry. Specifically, we assume that regions send and receive flows in proportion to their shares of total salary payouts in each subindustry, except in the three zero-employment housing services industries where we use regional spending shares from the disaggregated consumer spending matrix (see Appendix E). Further, we take the regions' geographical locations into account by imposing a gravity structure that assigns larger flows to pairs of regions that are close to each other than to those that are far apart.

The first step in this procedure is to obtain an estimate of the elasticity of intra-country trade with respect to distance. For that purpose, we use data from publicly available cargo transport statistics (Tables NVG23 and SKIB481) to estimate the following gravity model:

$$log(Goods_{ij}) = \alpha + \gamma log(Empl)_i + \eta log(Empl)_j + \beta log(Distance_{ij}) + \epsilon_{it} \qquad \text{(A.2)}$$

where $Goods_{ij}$ denotes the weight of goods transported between provinces i and j by either road or sea; $Empl_i$ and $Empl_j$denote employment in provinces i and j respectively; and $Distance_{ij}$denotes the average driving distance between regions within province i and regions within province j (where all intra-province distances are set to 1km). We conduct the analysis at the province-level as the cargo data is not available at the more disaggregated region-level. To accommodate zeroes, we estimate the equation on its multiplicative form using a Poisson pseudo-maximum-likelihood estimator (Silva and Tenreyro, 2006). The model yields an estimated elasticity of cargo transport with respect to distance of -0.61 (s.e. 0.03).

Equipped with this elasticity estimate, we disaggregate the national flows between subindustries to the regional level in the origin dimension as well as the destination dimension. Formally, let a_Y and b_Z denote subindustries and let i and j denote regions. We then assume that the flow from firms in subindustry a_Y, region i to firms in subindustry b_Z, region j is

$$Flow_{a_Y,b_Z,i,j} = Flow_{a_Y,b_Z} * \theta_{a_Y,b_Z,i} * \eta_{a_Y,b_Z,j} * Distance_{ij}^{-0.6} \tag{A.3}$$

where $Flow_{a_Y,b_Z}$is the flow from subindustry a_Y to subindustry b_Z at the national level and $Distance_{ij}$ is the distance between region i and region j.[A4] The parameters $\theta_{a_Y,b_Z,i}$ and $\eta_{a_Y,b_Z,j}$ are origin- and destination region fixed effects within the a_Y-b_Z subindustry pair. These are set so that *i*) region i's total share of national a_Y-b_Z flows (i.e.$\sum_j Flow_{a_Y,b_Z,i,j}/Flow_{a_Y,b_Z}$) matches its share of total salary payouts in subindustry a_Z, and *ii*) region j's total share of industry a_Y-b_Z flows (i.e. $\sum_i Flow_{a_Y,b_Z,i,j}/Flow_{a_Y,b_Z}$) matches its share of total salary payouts in subindustry b_Z. In practice, we accomplish this through an iterative numerical procedure: Starting from initial guesses for $\theta_{a_Y,b_Z,i}$ and $\eta_{a_Y,b_Z,j}$, we compute the implied value of each $Flow_{a_Y,b_Z,i,j}$. We then adjust the guesses by a multiplicative constant that ensures that these implied values add up to their national level counterpart, $Flow_{a_Y,b_Z}$. Next, we update the guesses by multiplying them with the ratios of the regions' salary shares to the implied shares of national a_Y-b_Z flows. We repeat this procedure iteratively until the implied share of national a_Y-b_Z flows converges toward the relevant salary share for each origin and destination region.

For money flows from final use categories (private consumption, non-profits, government consumption, capital formation, exports) to domestic subindustries, we add a geographic dimension on the destination side only. Here, we simply assign each region a share of the total national flow equal to its share of total subindustry salary payouts. Conversely, for flows going from domestic subindustries to tax collectors and foreign producers, we add a geographic dimension on the origin side only: Once again, each region is assigned a share of the national flow equal to its salary share within the subindustry.

After disaggregating to the subindustry-region level, we aggregate to the industry-region cell level described in Appendix D by summing over subindustries belonging to the same DEA industry within each region.

Appendix M.C Redirecting Flows From Consumers Through Retailers

National accounting conventions imply that production in retail industries is given by trade margins, i.e. sales net of acquisition costs. Thus, if a retailer buys a product from a non-retail

[A4]Distances are measured as the distance between the two regions' centroids. We use Google Maps API service to calculate actual travel distances. We define a region's distance to itself as 1.

producer at price p_1 and sells it to a consumer at price p_2, the input-output table will display two money flows: 1) a flow of $p_2 - p_1$from consumers to retail, and 2) a flow of p_1 from consumers to the non-retail producer's industry. This makes the standard input-output table unsuitable for tracing the circular flow of spending starting from the Disaggregated Consumer Spending Matrix. The reason is that the spending matrix shows actual cash-flows from consumers to producers. Thus, in the example above, the spending matrix would show a flow of p_2 from the consumer to the retailer, but no flow from the consumer to the non-retail producer. Tracing the circular flow of spending from consumers and through the production network therefore requires an adjustment to make the two matrices compatible.

We adjust the input-output table by re-coding all flows from consumers to non-consumer facing industries (defined as those not selling directly to consumers) and foreign producers (except flows related to Danish residents' tourist spending abroad, see subsection Appendix M.D). We replace each of these flows with two equal-sized sets of flows: One from a retail industry to the non-retail industry in question (or foreign producers in the case of imports), and one from private consumers to the same retail industry. Each set splits the value of the original flow across retailers' regions, using the geographic distribution of existing retail-to-non-retail flows of the same type. The choice of retail industry depends on the type of consumption good traded. Exploiting that the input-output table breaks flows from private consumers down on 72 consumption categories, we manually assign each category to the retail industry most likely to sell products within that category.[A5]

Note that this adjustment increases the total sum of flows in the table; for retail industries the total sum of inflows thus no longer corresponds to the total value of production as defined in national accounts. One way to think of this is that we apply a different definition of production for retailer's sales to consumers: The output value of such sales is now defined as the full amount of the sale (net of taxes such as VAT), while the retailer's acquisition cost is treated as an intermediate input from the non-retail producer of the traded product.

Appendix M.D Foreign Residents' Spending in Denmark

In the original input-output matrix, consumers' cross-border spending is captured by two separate flows from consumers to foreign producers: a positive representing residents' spending abroad, and a negative representing non-residents' spending in Denmark. The latter adjusts for the fact that the flows from consumers to domestic producers include spending from foreign residents and

[A5]For example, if we observe a flow of 1000 from private consumers to manufacturers in region X as payment for cheese,we replace it with 1) flows from grocery retailers in each of the 98 regions to manufacturers in region X, where the size of each flow is 1000 times the origin region's share of total existing flows from grocery retailers to manufacturers in region X; 2) flows from private consumers to grocery retailers in each of the 98 regions, where the size of each flow matches the corresponding retailer-to-manufacturer flow.

thus ensures that the total sum of flows from private consumers matches the national accounts value of resident consumers' private consumption expenditure (SNA P.31). To ensure consistency with national accounts aggregates for imports (SNA P.7) and exports (SNA P.6), the matrix also contains a positive, same-sized flow from foreign consumers (i.e. exports) to foreign producers (i.e. imports).

For our purposes, it is important that the recorded flows from domestic consumers to domestic producers represent the spending of resident consumers only. If not, consistency with the disaggregated consumer spending matrix is compromised. Sales to foreign tourists, on the other hand, should be recorded as exports for the producer cells making the sales. To achieve this, we adjust the flows from private consumers to domestic producers downwards so that the total adjustment across producer cells matches the aforementioned negative flow from consumers to foreign producers. We then adjust the total flow from consumers to foreign producers upwards by the same amount. To distribute the total adjustment across producer cells, we draw on two data sources: First, we use the industry distribution of Danish residents' spending abroad from the disaggregated consumer spending matrix to compute a proxy for each industry's share of foreign tourist spending, thus assuming that Danish tourists' spending behavior in other countries is indicative of foreign tourists' spending in Denmark. Second, for the distribution across regions, we use regional data from visitdenmark.dk on foreigners' overnight stays at hotels and hostel to compute a proxy for each region's share of foreign tourist spending. In an initial round of adjustments, we then allocate the total amount to be adjusted on cells in proportion to the product of these two shares, but subject to a constraint that the adjusted flow must be non-negative. The non-negativity constraint implies that less than the required amount is allocated in the first round, so we distribute the residual amount on the cells with positive tourist spending shares in proportion to their remaining private consumption flows. For each producer cell, we also adjust the flow from foreign consumers (i.e. exports) upwards by the exact same amount so that total flows from consumers (domestic and foreign) to the cell are unchanged. To complete the adjustment process, we adjust the flow from foreign consumers to foreign producers downwards by the same total amount.

Combined, these steps ensure that total consumer spending and total exports are both unaffected, but the split between flows to domestic producers and flows to foreign producers has changed for both. Similarly, total imports are unchanged, and so is total output for each producer cell, but the split between flows from domestic consumers and flows from foreign consumers is different in both cases.

Appendix N Measuring Disaggregated Government Dividend and Surplus Income

The matrix describes the income that the government receives from each producer cell. We start with a list of firms that sell to consumers and producers at market prices are (full or partly) owned by the government.[A6] We manually collect annual turnover for every firm from annual reports. We also collect information on establishment-level employment from the Danish business register (CVR). We combine these two datasets and split annual turnover regionally using each firm's distribution of employment across regions. We finally aggregate across industries (using the industry-code of the parent firm) and regions to get to our level of producer cells. We assume that the share of surplus received by the government is equal to the share of turnover by government-owned establishments in each producer cell.

Appendix O Measuring Disaggregated Consumption of Government and NPISH Output

We measure which consumer cells use different types of government services. The government purchases these services from producer cells (specifically, government-operated establishments in each producer cell) and provides them to consumers free of charge. We assume a uniform per capita consumption for collective public goods (including police, national defense, and public administration). We use individual-level data on actual uses of public services to allocate individual public consumption (including education, healthcare, and social protection), as detailed below.

Appendix O.A Education

We assign the aggregate consumption of education services observed in the national accounts to consumer cells according to the number of students in primary, secondary, and tertiary education in a cell. The education register ("UDD") contains information about the education program in which each individual is currently enrolled (if any) as well as each individual's highest completed education. We categorize individuals as primary school students if they are currently enrolled in a program and have no completed education; as secondary school students if they are currently enrolled in a program and their highest completed education is primary school (10 years); as tertiary education students if they are currently enrolled in a program and their highest completed education is secondary school (13 years); and as non-students if they are currently not enrolled in a program.

[A6] See fm.dk/arbejdsomraader/statens-selskaber/organisering.

We aggregate the number of students in primary, secondary and tertiary education to the level of consumer cells. As the cells only include the adult population, we assign the education consumption of minors to adults in the same household before aggregating, drawing on the intra-household links in the population register (BEF). For instance, two parents with three children, two of which are in primary school and one is in secondary school, each consume the equivalent of one year of primary education and half a year of secondary education.

Finally, we allocate the aggregate government consumption of education to cells in proportion to their share of students in primary, secondary and tertiary education and total government expenditure on education at each of these levels. Specifically, the estimated consumption of education services in region-by-industry cell i is given by

$$C_i^{edu} = \sum_{q=p,s,t} \frac{\#students_{i,q}}{\#students_q} \times expenditure_q$$

where q is the level of education (with p, s and t indicating primary, secondary, and tertiary, respectively) and expenditure is government consumption of education of level q.

Appendix O.B Healthcare

Government consumption of healthcare falls into six categories: outpatient services; hospital services; medical products, appliances and equipment; public health services; research and development; and other. We allocate government healthcare consumption summed over all six categories, as reported in national accounts, to consumer cells using publicly available statistics for the first two categories, which make up around 85% of the total.

"Outpatient services" capture government consumption flowing to primary healthcare providers, like general practitioners, specialist doctors, psychiatrists, and dentists. We obtain average primary healthcare expenditures by age, gender, and municipality (Table SYGU1 in the Public Expenditure Statistics). Based on a regression of average primary healthcare expenditure on a set of indicators for age, gender and municipality, we predict primary healthcare expenses for each individual in the population. We then aggregate the predicted expenditures to the level of consumer cells. Since children account for a non-negligible part of healthcare spending, we include the full population by assigning minors to the same consumer cells as the adults cohabiting with the child. If parents live together but work in different industries, we split the child's predicted healthcare expenditure equally between the two cells.

"Hospital services" capture government expenditure related to hospital treatments, including emergency room and outpatient hospital treatments. Again, we obtain information on the average number of days spent in hospital by age, gender, and municipality (Table INDAMP01 in the Health

Statistics). Regressing average hospital days on a set of indicators for age, gender, and municipality, we predict the number of hospital days for each individual in the population. We then aggregate the predicted hospital days to the consumer cell level, again allocating the hospital days of minors to their parents' cells.

Finally, we combine the two indicators of healthcare usage to disaggregate total consumption of healthcare services of consumer cell i as:

$$C_i^{health} = \sum_{q=o,h} \left(\frac{usage_{i,q}}{\sum_i usage_{i,q}} \times \frac{exp_q}{exp_o + exp_h} \right) \times exp, \tag{A.4}$$

where q indexes the type of healthcare (with o and h indicating outpatient and hospital services, respectively), $usage_{i,q}$ denotes cell *i's* usage of type q (expenditure on primary care and the number of hospital days), exp_q is national government consumption of healthcare of type q, and exp is national government consumption of healthcare summed over all six categories.

Appendix O.C Social Protection

Government consumption of social protection falls into five categories: sickness and disability; old age; family and children; unemployment; and other. We allocate social protection services to consumer cells based on government microdata. Specifically, we allocate the category "sickness and disability" to cells in proportion to the number of individuals on sick leave or disability pension as observed in the income register (IND); the category "old age" in proportion to the number of individuals aged 80 or older as observed in the population register (BEF); the category "family and children" in proportion to the number of pre-school children as observed in the population register (BEF); the category "unemployment" in proportion to the number of unemployed; and the category "other" by population shares. The estimated consumption of social protection services in consumer cell i is thus:

$$C_i^{social} = \sum_{q=s,o,d,u,z} \frac{usage_{i,q}}{\sum_i usage_{i,q}} \times exp_q, \tag{A.5}$$

where q indexes the type of social protection (with s, o, d, u and z denoting sickness/disability, old age, family/children, unemployment and other, respectively), $usage_{i,q}$ denotes the relevant indicator for cell i's usage of type q (see above), and exp_q is national government consumption of social protection services of type q.

Appendix O.D Measuring Disaggregated Consumption of NPISH Output

We measure the consumption of output provided by non-profit organizations (NPISH) for different consumer cells. NPISH output falls into five categories: education; social work activities; libraries; museums and other cultural activities; sports activities (non-market); and activities of membership organizations.

We first disaggregate usage of the first four categories by consumer region. For education, we use regional data on the share of children attending private schools relatively to public schools (Table "UDDAKT20" in the Education Statistics). For social work, we use regional data on the share of privately-owned (as opposed to government-operated) daycare institutions (Table "BOERN4" in the Child Care Statistics). For libraries, we use regional data on library usage per capita (Table "BIB1" in the Culture and Leisure Statistics). For museums and other cultural activities, we use regional data on members of sports associations per capita.

To infer usage by industry of work, we rely on the Danske Bank data. For education, we proxy use of NPISH education with payments to private schools. For social work, we calculate payments to private child-care institutions. For libraries, we use payments to libraries. For sports activities, we use membership payments to sports associations. For all categories, we count the number of transactions relative to the number of bank customers in each industry. We thereby estimate how likely consumers in each industry are to consume a given type of NPISH output.

We combine the information on consumption shares of NPISH by consumer region and industry to calculate consumption of NPISH output by consumer cell:

$$npish^{q}_{r,i} = \sum_{q} \frac{npish_{q,r} \times npish_{q,i} \times pop_{r,i}}{\sum_{r,s} npish_{q,r} \times npish_{q,i} \times pop_{r,i}} \times expenditure_{q},$$

where q is the NPISH category, r is region, i is industry, $pop_{r,i}$ is the cell's population, and expenditure is national NPISH output of type q.

For the final type of NPISH consumption, activities of membership organizations, we rely on Danske Bank data. This category consists mostly of trade unions and a small component of political or religious organizations. We disaggregate national consumption using as weights the share of individuals making payments to trade unions in each cell multiplied by the cell's population.

Appendix P Details on Model Derivations

Appendix P.A Derivation of the Circular Flow Matrix $\mathbb{M}$

To derive (13), we proceed column by column.

The first $\mathcal{J}$ columns of $\mathbf{M}$ describe producer cells' cost shares:

- Producer cell j has an intermediate input share of γ_j^X, of which share $\Omega_{j'j}$ goes to producer cell j'. In matrix notation, this gives $\Omega \cdot \mathbf{D}(\gamma^X)$.
- Producer cell j has a capital share of γ_j^K. In matrix notation, this means the flows from all producer cells to all $\mathcal{J}$ capital stocks is given by $\mathbf{D}(\gamma^K)$.
- Producer cell j has a labor share of γ_j^N. Of that, a share λ_{ij} is going to consumer cell i's labor. Together, this gives $\Lambda \cdot \mathbf{D}(\gamma^N)$ in matrix notation.

The second $\mathcal{J}$ columns of $\mathbf{M}$ describe the beneficiaries of capital income accruing to the $\mathcal{J}$ types of capital.

- A fraction τ_j^{corp} is paid to the government.
- A fraction $\kappa_{ij} \cdot \left(1 - \tau_j^{corp}\right)$ is paid to consumer cell i.

The third $\mathcal{I}$ columns capture the beneficiaries of labor income accruing to the $\mathcal{I}$ types of labor in the economy.

- The government receives a share τ_i of that income.
- Everything else goes to consumer cell i.

The fourth $\mathcal{I}$ columns of $\mathbf{M}$ captures the spending behavior of consumer cells.

- Consumer cell i allocates a fraction $\mathbf{A}_{ji}$ of spending to producer cell j. Of that $\frac{1}{1+\tau_i^{vat}}$ actually reaches producer cell j.
- The remainder, $\sum_{j \in \mathcal{J}} \mathbf{A}_{ji} \cdot \frac{\tau_i^{vat}}{1+\tau_i^{vat}}$ goes to the government in terms of product tax revenue. In matrix notation, this is given by the row vector $\mathbf{1}'\mathbf{A}\mathbf{D}\left(\frac{\tau^{vat}}{1+\tau^{vat}}\right)$.

The fifth column of $\mathbf{M}$ captures the allocation of spending of the government.

- A fraction g_j is spent on producer cell j's goods.
- A fraction t_i is transferred to consumer cell i.

This concludes our derivation and description of $\mathbf{M}$.

Appendix P.B Formula for change in GNE (14)

With welfare weights equal to inverse marginal utilities, the first order change in welfare is given by

$$\begin{aligned}
\sum_{i\in\mathcal{I}} P_i C_i \cdot dU_i &= \sum_{i\in\mathcal{I}} P_i dC_i + \sum_{i\in\mathcal{I}} \frac{P_i C_i}{G_i}\chi_i dG_i \\
&= \sum_{i\in\mathcal{I}} P_i dC_i + \sum_{i\in\mathcal{I}} \frac{\nu_i \cdot \sum_{j\in\mathcal{J}\cup\{\mathcal{R}\}} P_j G_j}{G_i} dG_i \\
&= \sum_{i\in\mathcal{I}} P_i dC_i + \frac{\sum_{j\in\mathcal{J}\cup\{\mathcal{R}\}} P_j G_j}{v\left(\{G_j\}\right)} \sum_{i\in\mathcal{I}} dG_i \\
&= \sum_{i\in\mathcal{I}} P_i dC_i + \sum_{i\in\mathcal{I}} \mathcal{P}(\{P_j\}) dG_i \\
&= \sum_{j\in\mathcal{J}\cup\{\mathcal{R}\}} P_j dc_{ij} + \sum_{j\in\mathcal{J}\cup\{\mathcal{R}\}} P_j dG_j \\
&= dGNE.
\end{aligned}$$

Appendix P.C Details on the Vertical Economy in Section VII.A

The vertical economy has a set $\mathcal{I} = \{1, \ldots, N\}$ of consumers and a set $\mathcal{J} = \{1, \ldots, N\}$ of producers. The circular flow matrix for this economy is given by

$$\mathbf{M} = \begin{pmatrix} 0 & 0 & 0 & \mathbf{A} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \mathbf{I} & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{I} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{A.6}$$

where

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

(A.6) has this specific form because there is no government; no capital; all production in producer cell j is done using labor of consumer cell $i = j$; and because consumer cell i spends on producer cell $j = i - 1$, unless $i = 1$ in which case consumer cell i spends abroad.

Observe that the exposure of factor i to a shock to producer cell j is given by

$$(\mathbf{I}-\mathbf{M})^{-1}_{N+N+i,j} = 1_{\{i \leq j\}},$$

where $1_{\{...\}}$ is an indicator function. This implies that Proposition 1 is a special case of Proposition 2.

Appendix P.D Proof of Proposition 2

We consider the case of a general export demand shock $d\tilde{\mathbf{x}}$ hitting the economy. Since the world economy in our model is frictionless, the change in domestic welfare must be coming from an increase in the price of exports. That is,

$$dGNE = \mathbf{x}' d\log \mathbf{P}$$

where $\mathbf{x} = (x_j)_{j\in\mathcal{J}}$, $d\log \mathbf{P} = (d\log P_j)_{j\in\mathcal{J}}$. How do prices change? Linearizing the unit cost function associated with (7), we find that

$$d\log P_j = \gamma_j^X \sum_{j'} \Omega_{j'j} d\log P_{j'} + \gamma_j^N \sum_i \lambda_{ij} d\log W_i + \gamma_j^K d\log\left(P_j Q_j\right),$$

where $d\log\left(P_j Q_j\right)$ appears as the return to the fixed factor. In matrix and vector notation, this becomes

$$d\log \mathbf{P} = \mathbf{D}(\gamma^X)\Omega' d\log \mathbf{P} + \mathbf{D}(\gamma^N)\Lambda' d\log \mathbf{W} + \mathbf{D}(\gamma^K) d\log \mathbf{PQ}, \tag{A.7}$$

where we use the notation $d\log \mathbf{PQ} \equiv (d\log(P_j Q_j))$ for the vector of changes in nominal output across producer cells; similarly, $d\log \mathbf{W} = (d\log W_i)$. Solving (A.7) for $d\log \mathbf{P}$, we find

$$d\log \mathbf{P} = \left(\mathbf{I} - \mathbf{D}(\gamma^X)\Omega'\right)^{-1}\left(\mathbf{D}(\gamma^N)\Lambda' d\log \mathbf{W} + \mathbf{D}(\gamma^K) d\log \mathbf{PQ}\right). \tag{A.8}$$

Given the definitions for $\mathbf{x}^K$ and $\mathbf{x}^N$ in (18) and (19), we can then write

$$dGNE = \mathbf{x}' d\log \mathbf{P} = \left(\mathbf{x}^N\right)' \mathbf{D}\left(NW\right) d\log \mathbf{W} + \left(\mathbf{x}^K\right)' \mathbf{D}\left(\Pi^{pre}\right) d\log \mathbf{PQ}.$$

Given that $\Pi_j^{pre} = \gamma_j^K P_j Q_j$, this simplifies to

$$dGNE = \mathbf{x}' d\log \mathbf{P} = \left(\mathbf{x}^N\right)' d\mathbf{NW} + \left(\mathbf{x}^K\right)' d\mathbf{\Pi}^{pre}, \tag{A.9}$$

where $d\mathbf{NW} = d\left(N_i W_i\right)$ is the change in labor income paid to consumer cell i in response to the shock; and $d\mathbf{\Pi}^{pre}$ is the change in pre-tax profits in response to the shock. Observe that both can

be computed directly via the circular flow matrix:

$$\begin{pmatrix} * \\ d\mathbf{\Pi}^{pre} \\ d\mathbf{NW} \\ * \\ * \end{pmatrix} = (\mathbf{I} - \mathbf{M})^{-1} \begin{pmatrix} d\tilde{\mathbf{x}} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \tag{A.10}$$

where $*$ indicates some other entries which we are not further characterizing. Putting (A.10) together with (A.9), we find

$$dGNE = \begin{pmatrix} 0 & \mathbf{x}^K & \mathbf{x}^N & 0 & 0 \end{pmatrix} (\mathbf{I} - \mathbf{M})^{-1} \begin{pmatrix} d\tilde{\mathbf{x}} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

Since $d\tilde{\mathbf{x}}$ was arbitrary, this establishes (20) and proves Proposition 2.

Appendix P.E Proof of Proposition 3

The proof of this proposition is straightforward. A uniform export demand shock, increasing all exports by $d\log\tilde{x}$, scales up all nominal flows by $d\log\tilde{x}$. This implies that nominal consumption of consumer cell i is scaled up by $d\log\tilde{x}$,

$$\frac{d\log\left(\sum_j P_j c_{ij}\right)}{d\log\tilde{x}} = 1.$$

Real consumption therefore moves according to

$$\frac{d\log C_i}{d\log\tilde{x}} = 1 - \sum_j \mathbf{A}_{ji} \frac{d\log P_j}{d\log\tilde{x}}. \tag{A.11}$$

Moreover, nominal labor income and nominal output scale with the shock, $d\log W_i = d\log(P_j Q_j) = d\log\tilde{x}$. Applying (A.8) to the uniform shock, we find price responses

$$d\log\mathbf{P} = \left(\mathbf{I} - \mathbf{D}(\gamma^X)\Omega'\right)^{-1} \left(\mathbf{D}(\gamma^N)\Lambda'\mathbf{1} + \mathbf{D}(\gamma^K)\mathbf{1}\right).$$

Since Λ is column stochastic, $\sum_{i\in\mathcal{I}} \lambda_{ij} = 1$, this can be simplified further,

$$\begin{aligned} d\log \mathbf{P} &= \left(\mathbf{I} - \mathbf{D}(\gamma^X)\Omega'\right)^{-1}\left(\mathbf{1} - \mathbf{D}(\gamma^X)\mathbf{1}\right) \\ &= \left(\mathbf{I} - \mathbf{D}(\gamma^X)\Omega'\right)^{-1}\left(\mathbf{1} - \mathbf{D}(\gamma^X)\Omega'\mathbf{1} - \mathbf{D}(\gamma^X)\left(\mathbf{1} - \Omega'\mathbf{1}\right)\right) \\ &= \mathbf{1} - \left(\mathbf{I} - \mathbf{D}(\gamma^X)\Omega'\right)^{-1}\mathbf{D}(\gamma^X)\left(\mathbf{1} - \Omega'\mathbf{1}\right). \end{aligned} \tag{A.12}$$

Substituting (A.12) into (A.11), we find

$$\begin{aligned} \frac{d\log \mathbf{C}}{d\log \tilde{x}} &= \mathbf{1} - \mathbf{A}' d\log \mathbf{P} \\ &= \mathbf{1} - \mathbf{A}'\mathbf{1} + \mathbf{A}'\left(\mathbf{I} - \mathbf{D}(\gamma^X)\Omega'\right)^{-1}\mathbf{D}(\gamma^X)\left(\mathbf{1} - \Omega'\mathbf{1}\right), \end{aligned}$$

which is precisely (22).

Appendix References

Silva, J. S., and S. Tenreyro (2006): "The Log of Gravity," *Review of Economics and Statistics*, 88, 641–658.