Demand Imbalances, Global Inflation and Monetary Policy Coordination *

Javier Bianchi
Federal Reserve Bank of Minneapolis

Louphou Coulibaly
Federal Reserve Bank of Minneapolis, University of Wisconsin-Madison & NBER

December 2022

PRELIMINARY AND INCOMPLETE

Link to Latest Draft

Abstract

This paper studies the scope for international monetary cooperation in an environment with global demand imbalances. We ask whether the Nash equilibrium features over or under-tightening relative to the optimal cooperative solution. We show that is possible that cooperation calls for higher nominal interest rates. This occurs in states where the economy is overheating, as central banks fail to internalize that depressing domestic demand helps to cool down the world economy. On the other hand, when there is slack in the global economy, cooperation calls for slowing down interest rate hikes.

Keywords: Global inflation, international monetary policy spillovers, coordination

JEL Classifications: E21, E23, E43, E44, E52, E62, F32

*The views expressed in this paper are entirely those of the authors. They do not necessarily represent the views of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

Central banks nearly everywhere feel accused of being on the back foot. The present danger, however, is not so much that current and planned moves will fail eventually to quell inflation. It is that they collectively go too far and drive the world economy into an unnecessarily harsh contraction...by simultaneously all going in the same direction, they risk reinforcing each other's policy impacts without taking that feedback loop into account.

Maurice Obstfeld "Uncoordinated monetary policies risk a historic global slowdown", Blog post, Peterson Institute, 09/12/2022

1 Introduction

Following a prolonged phase of expansionary monetary policy, central banks around the world have started tightening in an effort to quell inflation and cool down the economy. Concerns, however, have started to emerge about the fast pace of collective interest rate hikes and the extent to which it can trigger an unnecessarily sharp contraction in economic activity (Obstfeld, 2022).[1]

In this paper, we tackle the question of whether a cooperative monetary policy indeed calls for a slower pace of hikes or whether it is possible that even tighter policy could be socially optimal. To study this question, we consider a setup in which the scope for coordination emerges because central banks do not internalize how their monetary policy decisions alter monetary policy tradeoffs abroad, as in Fornaro and Romei (2022).[2] Our contribution is to establish that the extent of over-tightening or under-tightening can be framed entirely on the degree of slack in the economy. We show that if the economy is experiencing a recession, coordinated monetary policy calls for more expansionary monetary policy, while if the economy is overheating, coordinated monetary policy calls for more contractionary monetary policy.

We consider a model with a continuum of identical small open economies and a single final tradable consumption good. Each economy produces tradable and non-tradable goods under sticky wages. In the non-cooperative equilibrium, each central bank chooses its own monetary policy to maximize its own welfare. The tradeoff the central bank faces is between stabilizing inflation around a target and deviations from an output gap. In

[1] See the Peterson Institute blog post by Maurice Obstfeld "Uncoordinated Monetary Policies Risk a Historic Global Slowdown", from September 12, 2022, which has spearheaded extensive policy discussions.

[2] As we discuss below, we abstract from terms of trade externalities, which call naturally for cooperative policies and have been the subject of a large literature reviewed below.

equilibrium, the economy may feature a recession or overheating, depending on the type of shock. Because all countries are identical, in equilibrium there are no capital flows, and by interest parity, exchange rates remain constant.

The key externality at play is that an individual country does not internalize that by changing its monetary stance, it affects output, which in turn affects the demand for borrowing. For example, a contractionary monetary policy reduces tradable output, which in turn leads to an increase in the demand for borrowing. When all countries contract monetary policy, the lower demand for borrowing induces in general equilibrium an increase in the world real interest rate. Furthermore, the increase in the world real rate leads to a reduction in aggregate demand in all economies, which has a depressing effect on non-tradable output.

We then show that whether the cooperative outcome calls for a expansionary or a more tighter monetary policy depends on the sign of the labor wedge. When the Nash equilibrium features a positive labor wedge (or a recession), a coordinated monetary expansion can help alleviate the recession. This is because in this case, the monetary expansion lowers the world real interest rate and helps alleviate the recession in all countries, a result consistent with the example provided by Fornaro and Romei (2022). On the other hand, when the Nash equilibrium features a negative labor wedge (or overheating), a coordinated monetary contraction can help cool down the economy.

The intuition for the results can be understood as follows. When a country has an overheated economy, it perceives it can raise the demand from goods produced abroad to mitigate the excessive domestic demand. Thus, the country may choose not to tighten monetary policy in a situation in which it would find optimal to do so in the absence of an open capital account. However, by increasing goods imports, it raises demand for foreign goods. When the rest of the world is also in a situation of overheating, this generates negative spillovers. If all countries were to tighten further monetary policy, this would reduce overall demand and help cool down all economies. Because individual countries do not internalize these general equilibrium effects, they under-tighten relative to the global optimum.

When countries face a recession, the sign of the spillovers flips. Now, each country facing a recession perceives it can export goods abroad to mitigate the lack of insufficient demand. As a result, it fails to expand monetary policy enough, because it does not internalize that by increasing its exports, it increases the slack in output abroad. In this case, the cooperative outcome dictates a more expansive monetary policy, and the Nash equilibrium features overtightening.

Furthermore, we analytically trace the sign of the externality to the sign of households' Euler equation as an implementability constraint for the central bank problem. When the sign of this constraint is positive (indicating that households overborrow relative to the country optimum), an increase in the real rate has positive welfare effects. In turn, this implies that countries would benefit from tighter monetary policy in the world economy.

These results suggest the prevailing view that countries may be over-tightening relative to the socially optimal allocations is not warranted. By contrast, depending on the degree of slack and therefore the kind of disturbance that economies are facing, a more contractionary monetary policy may be desirable.

Related Literature. There is a vast literature on international monetary policy coordination going back to Hamada (1976) and Canzoneri and Henderson (1991). Two pioneer papers adopting a microfounded approach to cooperative monetary policy are Obstfeld and Rogoff (2002) and Corsetti and Pesenti (2005) (see also, Benigno and Benigno, 2006, and Devereux and Engel, 2003). A key theme in this literature is that individual countries have incentives to lower their own production to shift terms of trade in their own favor, at the expense of other countries. Because of this monopoly power effect, the overall implication in this literature is that central banks over-tighten monetary policy relative to the socially optimal level. We highlight instead a mechanism operating through capital markets that generates the possibility of under-tightening.

The paper that is closest to our work is Fornaro and Romei (2022) who also examine international spillovers operating through capital markets in an economy in which there are no terms of trade manipulation incentives. In contrast to their work, ours points to the possibility of under-tightening. In particular, we consider a model that admits the possibility of both overheating and unemployment and show analytically that whether countries individually over-tighten or under-tighten relative to the socially optimal level depends on the degree of slack in the economy. The key mechanism at play in our model is also related to the literature on aggregate demand externalities and pecuniary externalities.[3]

[3]See, for example, Bianchi (2011), Farhi and Werning (2016), Schmitt-Grohé and Uribe (2016), Fornaro and Romei (2019), Bianchi and Coulibaly (2021), Bengui and Coulibaly (2022).

2 Model

Time is discrete and infinite. We consider a world economy composed of a continuum of identical small open economies $k \in [0,1]$ with two types of goods: tradables and non-tradables. The environment is deterministic and features nominal rigidities.

2.1 Households

Each economy k is populated by a continuum of identical households of measure one.[4] Their preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[\log c_t - (h_t^T + h_t^N) + \frac{\chi}{2} (\pi_t - \bar{\pi})^2 \right], \tag{1}$$

Households have log utility over the consumption good c_t which is a composite of tradable consumption c_t^T and non-tradable consumption c_t^N, according to a Cobb-Douglass aggregator:

$$c_t = \left(c_t^T\right)^{\phi_t^T} \left(c_t^N\right)^{\phi_t^N}$$

where $\phi_t^T \in (0,1)$ captures the preference for tradables, $\phi_t^N = 1 - \phi_t^T$ is the preference for non-tradables. In addition, they have a linear disutility from working, where h_t^T denotes hours worked in the tradable sector and h_t^N denotes aggregate hours worked in the non-tradable sector. We assume that households face a cost of inflation which is quadratic on deviations of inflation $\pi_t \equiv \frac{P_t}{P_{t-1}} - 1$ from a target $\bar{\pi}$. The parameter $\beta \in (0,1)$ represents the discount factor and $\chi > 0$ is the disutility from inflation.

We denote by P_t^N and P_t^T respectively, the price of non-tradables and tradables (in terms of the domestic currency). Given the assumption on unitary elasticity of substitution between tradables and non-tradables, the consumer price index P_t satisfies

$$P_t = \frac{1}{(\phi_t^T)^{\phi_t^T} (\phi_t^N)^{\phi_t^N}} (P_t^T)^{\phi_t^T} (P_t^N)^{\phi_t^N}. \tag{2}$$

We assume that the law of one price holds for the tradable good; that is for any country pair k and j, $P_{kt}^T = e_{kt}^j P_{jt}^T$, where e_{kt}^j is the nominal exchange rate defined as the price of the country j currency in terms of the domestic currency, and P_{jt}^T is the price of the tradable good denominated in term of country j's currency.

[4] The notation does not index variables in each country by k to avoid clutter.

We will assume that in the initial period, wages are sticky at $\bar{W}$, and for $t > 0$, wages become flexible. In particular, we assume that households are off their labor supply in the initial period and their hours are determined by firms' labor demand, as we will see below.

Each period households receive their labor income, $W_t(h_t^T + h_t^N)$, where $h_t^T + h_t^N$ denotes the hours they work in each sector. In addition, households collect firms' profits, T_t, and trade two types of one-period non-state-contingent bonds in credit markets: a real international bond that pays a constant net return of R units of tradables, and a nominal domestic bond that pays $\tilde{R}_t$ in units of domestic currency. The budget constraint of the representative household is therefore given by

$$P_t^T c_t^T + P_t^N c_t^N + \frac{b_{t+1}}{R_t} + \frac{P_t^T b_{t+1}^*}{R_t^*} = W_t(h_t^T + h_t^N) + T_t + b_t + P_t^T b_t^*. \tag{3}$$

where b_t and b_t^* denote respectively the amount of nominal bond and real bond assumed in period $t-1$ and due in period t. The left-hand side represents total expenditures in tradable and non-tradable goods and purchases of bonds, while the right-hand side represents total income, including the returns from bond issuance.

The households' problem consists of choosing sequences of $\{c_t^N, c_t^T\}_{t=0}^{\infty}$, asset positions $\{b_{t+1}, b_{t+1}^*\}_{t=0}^{\infty}$, and hours $\{h_t^T, h_t^N\}_{t=1}^{\infty}$, to maximize the expected present discounted value of utility (1), subject to (3) and taking as given profits $\{T_t\}$, and prices $\{W_t, P_t^N, P_t^T, R_t, R_t^*\}_{t=0}^{\infty}$.

The optimality with respect to c_t^T and c_t^N equate the marginal rate of substitution between the two goods to the relative price. Owing to the homotheticity of preference and unitary elasticity of substitution, this implies that the ratio of expenditures on tradable consumption relative to expenditures on non-tradable consumption equals the relative preference for the two goods:

$$\frac{P_t^N c_t^N}{P_t^T c_t^N} = \frac{\phi_t^N}{\phi_t^T}. \tag{4}$$

For $t \geq 1$, households are on their labor supply. Given that disutility of working is linear in the sum of total hours, optimality implies that

$$\frac{\phi_t^N}{c_t^N} = \frac{W_t}{P_t^N}, \tag{5}$$

Finally, the first-order conditions for the nominal and real bond holdings yield the follow-

ing Euler equation and interest parity condition:

$$\frac{\phi_t^T}{c_t^T} = \beta R_t^* \frac{\phi_{t+1}}{c_{t+1}^T} \tag{6}$$

$$R_t^* = R_t \frac{P_t^T}{P_{t+1}^T}, \tag{7}$$

The Euler equation (6) equates the marginal cost of purchasing a real bond with its marginal benefit. Condition (7) is a non-arbitrage condition that relates the return on real international bonds to the return on nominal domestic bonds and the expected inflation in the tradable sector.

2.2 Firms

There is a continuum of firms producing tradable goods and non-tradable goods, both of measure one. Each firm produces the final good using a production function that uses labor as the sole input. We assume that the production functions have decreasing returns $y_t^N = (h_t^N)^{\alpha^N}$ and $y_t^T = (h_t^T)^{\alpha^T}$. Firms' optimality then imply that

$$P_t^N \alpha^N (h_t^N)^{\alpha^N - 1} = W_t. \tag{8}$$

$$P_t^T \alpha^T (h_t^T)^{\alpha^T - 1} = W_t. \tag{9}$$

That is, firms equate the value of the marginal product of labor to the nominal wage in the two sectors.

As we mentioned before, we assume that the nominal wage is fully rigid at date $t = 0$ and that hours are determined by labor demand. That is, given prices, hours in equilibrium in each sector will be determined at $t = 0$ by (8) and (9) evaluated at $W_0 = \bar{W}$.

2.3 Monetary Policy

The policy instrument is the sequence of nominal interest rates $\{R_t\}$. Because of the assumption that prices are flexible for $t > 0$, we assume monetary policy targets inflation equal to $\bar{\pi}$ for $t > 0$. This implies that the nominal rate follows $R_t = R_t^*(1 + \bar{\pi})$ for $t > 0$. For $t = 0$, we will evaluate the optimal monetary policy, comparing the cooperative and non-cooperative outcomes.

2.4 Market Clearing Conditions

Market clearing for the non-tradable good requires that output be equal to the demand for non-tradables in each country:

$$c_t^N = h_t^N. \tag{10}$$

We assume without loss of generality that the bond denominated in domestic currency is traded only domestically. Market clearing therefore implies

$$b_{t+1} = 0. \tag{11}$$

Combining the budget constraints of households, firms, and the central bank, as well as market clearing conditions, we arrive at the resource constraint for tradables, or the balance of payment condition:

$$c_t^T - (h_t^T)^{\alpha^T} = b_t^* - \frac{b_{t+1}^*}{R_t^*}, \tag{12}$$

which says that the trade balance in each country must be financed with net bond issuances.

Finally, at the world level, real bonds are in zero net supply. Therefore, aggregating across countries, we have

$$\int b_{kt+1}^* dk = 0 \tag{13}$$

2.5 Competitive Equilibrium

We now define a competitive equilibrium in the global economy. With full capital mobility, the real rate must be the same across countries, and the market for the real bond must clear globally. In addition, we must have that agents in each of the k economies optimize and markets for non-tradable goods clear in each country. We use the following notation: a variable of the form $\{x_t\}$ denotes a vector of values, one for each country k. An equilibrium is then defined as follows.

Definition 1 (Competitive Equilibrium)**.** Given initial positions for each country b_{k0}^* and a sticky wage $\bar{W}$, an equilibrium is a sequence of government policies $\{R_t\}_{t=0}^{\infty}$ for each country, world real rate $\{R_t^*\}_{t=0}^{\infty}$, and prices $\{P_t^T, P_t^N, W_t\}_{t=0}^{\infty}$ and allocations $\{c_t^T, c_t^N, h_t^T, h_t^N, b_{t+1}, b_{t+1}^*\}_{t=0}^{\infty}$ in each country such that

(i) Households optimize, and hence the following conditions hold: (4), (5) (6), (7);

(ii) Firms optimize, implying (8) and (9);

(iii) The law of one price holds for tradables: $P_{kt}^T = e_t^j P_{jt}^T$ for any country-pair k and j,

(iv) The market for non-tradables (10) and domestic bonds (11) clears; moreover, the labor market clears for $t \geq 1$.

(v) Globally, the market for the real bond clears: implying (13)

Notice that by combining firms' optimality condition with households, optimality condition (4), we arrive at an equation determining the demand for hours in the non-tradable sector in each country as a function of the price of tradables P_t^T, the nominal wage W_t and the level of tradable consumption c_t^T:

$$h_t^N = \frac{\phi_t^N}{\phi_t^T} \frac{\alpha^N}{W_t} P_t^T c_t^T. \tag{14}$$

On the other hand, equilibrium hours in the tradable sector are independent of consumption of domestic households, a distinction that will play central role in the analysis.

We assume that countries have symmetric initial net foreign asset positions at $t = 0$.

2.6 Efficient Allocation

We conclude the description of the model by presenting the first-best allocation. We consider a benevolent social planner of the world economy who chooses allocations subject to a resource constraint. Because preferences are symmetric and all countries are identical, each country consumes its tradable output that is, there are no movement in the net foreign asset position of the countries. Using the resource constraint for non-tradables (10), the planner's problem can be written as

$$\max_{\{h_t^N, h_t^T\}} \sum_{t=0}^{\infty} \beta^t \left[\phi_t^T \alpha^T \log(h_t^T) + \phi_t^N \alpha^N \log(h_t^N) - (h_t^T + h_t^N) \right].$$

The first-best allocation equates the value of one additional employed unit of labor in each sector to the marginal cost of leisure:

$$\frac{\alpha^N \phi_t^N}{h_t^N} = 1, \qquad \frac{\alpha^T \phi_t^T}{h_t^T} = 1. \tag{15}$$

Given a linear disutility from labor, this implies that hours in each sector are proportional to the weight of each good in the consumption aggregate. We summarize these results in the following Lemma:

Lemma 1 (First-best). *The first-best allocation features* $h_t^N = \alpha^N \phi_t^N$ *and* $h_t^T = \alpha^T \phi_t^T$.

Proof. See Appendix A.1 □

It should be clear that the first-best allocation coincides with the allocations in a competitive equilibrium with flexible prices. This can be seen by noting that if the nominal wage could be adjusted, we would have from households' labor supply decision $c_t^N/(1-\phi) = W_t/P_t^N$ and $c_t^T/\phi = W_t/P_t^T$, which, combined with firms' demand for labor, would yield (15).

Output gaps. To simplify the description of the policy trade off that central banks face, we introduce a measure of "output gaps", which are defined as the gap between the current and the efficient level of employment

$$\tilde{h}_t^N \equiv \frac{h_t^N}{\alpha^N \phi_t^N} - 1, \qquad \tilde{h}_t^T \equiv \frac{h_t^T}{\alpha^T \phi_t^T} - 1 \tag{16}$$

Nominal rigidities will distort these ratios away from one, which would correspond to the first-best allocations. In our case, the assumption that prices are flexible and that labor is perfectly mobile across sectors, imply that output gaps are equated across sectors.

Lemma 2. *In any competitive equilibrium, the relative output gaps in the two sectors are equated. That is, the labor wedge for tradables and non-tradables are equalized* $\tilde{h}_0^T = \tilde{h}_0^N = \tilde{h}$.

Proof. See Appendix A.2 □

We note that this result applies to a competitive equilibrium given any monetary policy. Depending on aggregate conditions and the stance of monetary policy, this could imply a situation with a positive or negative output gap (i.e., $\tilde{h} > 0$ or $\tilde{h} < 0$).

2.7 Experiment

We assume that the economy starts at the stationary equilibrium where allocations and relative prices are constant. At the steady state, allocations are efficient and symmetric, and the inflation rate coincides with the central bank's target $\bar{\pi}$, which we assume to be zero for simplicity. We consider values for P_{-1} and $\bar{W}$ such that in the absence of any shocks, the central bank is able to implement the efficient allocation where all labor wedges equal

zero. Without loss of generality we normalize $\bar{W} = 1$ which implies

$$P_{-1} = \prod_{i \in \{T,N\}} (\alpha^i \phi^i)^{-\alpha^i \phi^i} \tag{17}$$

Moreover, by symmetry we have that $c^T = (h^T)^{\alpha^T}$ for all countries and $R^* = 1/\beta$.

We then assume that at $t = 0$, there is a shock. In particular, we will focus us on an increase in the demand for non-tradables ϕ_0^N and an increase in $\bar{\pi}$. We see in particular the former as a way to capture the rebalancing of consumption towards consumption of services post-Covid and the latter as capturing the tolerance for higher inflation.

Roadmap. In the next sections, we examine optimal monetary policy under cooperation and contrast with a Nash equilibrium where each country chooses its monetary policy taking as given policies abroad.

3 Cooperative Monetary Policy

We start by examining the optimal monetary policy under cooperation. The optimal policy problem consists of choosing a sequence of nominal interest rates, allocations, and prices for each country to maximize total welfare. Given that all countries are identical, the problem can be reduced to maximizing an arbitrary country's welfare under financial autarchy.

The planner chooses a sequence of nominal interest rates, prices and allocations to maximize (1) subject to firms' and houesholds' optimality conditions (4) and (8)-(9) and market clearing constraints for labor and consumption (10)-(12). Notice that because there is a single tradable good and countries are identical, we must have that $h_t^T = c_t^T$. The problem is thus static.[5] Replacing constraints in the objective, we arrive to the following problem:

$$\max_{h_0^T, h_0^N} \alpha^T \phi_0^T \log(h_0^T) + \alpha^N \phi_0^N \log(h_0^N) - (h_0^T + h_0^N) - \left[\frac{(h_0^T)^{\phi_0^T (1-\alpha^T)} (h_0^N)^{\phi_0^N (1-\alpha^N)}}{P_{-1} (\alpha^T \phi_0^T)^{\phi_0^T} (\alpha^N \phi_0^N)^{\phi_0^N}} - (1 + \bar{\pi}) \right]^2$$

[5] Recall also that prices are flexible for $t \geq 1$.

subject to

$$h_0^N = \frac{\alpha^N \phi_0^N}{\alpha^T \phi_0^T} h_0^T \tag{18}$$

The restriction (18) emerges from combining household optimal mix for tradable and non-tradable consumption and the firms' optimality condition for employment.[6] This condition reflects that the relative of hours employed in each sector is determined by the relative preference of households for the two goods as well as the labor intensity in each sector. As we will see, this implementability constraint will play a crucial role in the analysis.

Denoting by η the multiplier on (18), we can write the Lagrangian as follows:

$$\begin{aligned}\mathcal{L}(h_0^T, h_0^N, \eta) = & \sum_{i\in\{T,N\}} \left[\alpha^i \phi^i \log(h_0^i) - h^i\right] \\ & - \frac{\chi}{2}\left[\frac{1}{P_{-1}} \prod_{i\in\{T,N\}} \frac{(h_0^i)^{(1-\alpha^i)\phi_0^i}}{(\alpha^i \phi_0^i)^{\phi_0^i}} - (1+\bar{\pi})\right]^2 + \eta\left[\frac{\alpha^N \phi_0^N}{\alpha^T \phi_0^T} h_0^T - h_0^N\right]\end{aligned} \tag{19}$$

Notice here that a multiplier $\eta > 0$ would reflect the fact that the planner perceives that being able to reallocate hours towards the non-tradable sector would improves welfare. The opposite happens when $\eta < 0$. The optimality conditions with respect to h_0^T and h_0^N are respectively:

$$\frac{\phi_0^N \alpha^N}{h_0^N} = 1 + \frac{\phi_0^N}{h_0^N}(1-\alpha^N)\chi(1+\pi_0)(\pi_0 - \bar{\pi}) + \eta \tag{20}$$

$$\frac{\phi_0^T \alpha^T}{h_0^T} = 1 + \frac{\phi_0^T}{h_0^T}(1-\alpha^T)\chi(1+\pi_0)(\pi_0 - \bar{\pi}) - \frac{h_0^N}{h_0^T}\eta \tag{21}$$

Condition (20) equates the marginal utility benefits from raising one more unit of non-tradable employment and associated increase in output with the marginal utility costs. The latter is given by the leisure value of time, the costs of higher inflation, and the effect on the implementability constraint (18). The inflation cost emerges because raising employment in the non-tradable sector requires a higher price level to stimulate firms to hire more workers, a standard Phillips curve relationship. Condition (21) is the analogous condition for tradable employment. Notice that the last term in the two conditions have the opposite

[6] In particular, it uses (4), (8), (9) and (12).

sign as increasing h^T and h^N have the opposite effects on constraint (18).

Combining these conditions we arrive to

$$\left(\frac{\phi_0^N \alpha^N}{h_0^N} - 1\right)\left(1 + \frac{h_0^N}{h_0^T}\right) = \left[\frac{\phi_0^T}{h_0^T}(1-\alpha^T) + \frac{\phi_0^N}{h_0^T}(1-\alpha^N)\right]\chi(1+\pi_0)(\pi_0 - \bar{\pi}) \tag{22}$$

This condition reveals how depending on the nature of the shock, only one of the two scenarios can emerge: either the world economy is overheating, $\tilde{h}_0 > 0$ and inflation is below target or there is a recession $\tilde{h}_0 < 0$ and inflation is above target. To understand the intuition, consider the possibility that in a coordinated equilibrium, there is a negative output gap in the tradable sector (and the non-tradable) and inflation is below the target. In that case, by lowering the nominal interest rate and allow for higher prices, the central bank can narrow the output gap and the inflation gap. By the same token, if is a positive output gap and inflation is above the target, it would be optimal to raise the policy rate, as this would help lower inflation and take output closer to the efficient level. It is also clear from these conditions that if the inflation cost is zero $\chi = 0$, the planner can implement the first-best allocation for any shocks.

If we combine (21) and (20) we arrive to

$$\eta = \left[\frac{1-\alpha^T}{\alpha^T} - \frac{1-\alpha^N}{\alpha^N}\right]\frac{\alpha^T \phi_0^T}{h_0^T + h_0^N}\chi(1+\pi_0)(\pi_0 - \bar{\pi}) \tag{23}$$

From this condition, we can infer that the sign of the Lagrange multiplier on constraint (23) depends on the sign of $(\alpha^N - \alpha^T)$ and $(\pi_0 - \bar{\pi})$. When the two sector are equally labor intensive $\alpha^T = \alpha^N$, household optimal mix for tradable and non-tradable consumption is socially optimal. The global planner does not perceive any additional from reallocating consumption across sectors. However, when the non-tradable sector is more labor intensive than the tradable sector (i.e., $\alpha^N > \alpha^T$), the planner internalizes that a reallocation of consumption from the tradable sector toward the nontradable sector would reduce inflation. Therefore, if inflation is above its target and the non-tradable sector is relatively less labor intensive, the global planner finds it optimal to engineer a policy that would reallocate consumption toward the non-tradable sector, implying that $\eta > 0$. Conversely, if the non-tradable sector is less labor intensive than the tradable sector and inflation is above its target, it is optimal to reallocate consumption away from the non-tradable sector.

Proposition 1. *Consider the optimal monetary under cooperation. We have that employment is*

uniquely determined by

$$\tilde{h}_0^N = \tilde{h}_0^T = -\frac{(1-\alpha^T)\phi_0^T + (1-\alpha^N)\phi_0^N}{\alpha^T\phi_0^T + \alpha^N\phi_0^N}\chi\left[1+\pi_0(\tilde{h}_0^N)\right]\left[\pi_0(\tilde{h}_0^N) - \bar{\pi}\right], \tag{24}$$

where the inflation rate of the consumer price index $\pi_0(h_0^N)$ *satisfies*

$$\pi_0(\tilde{h}_0^N) = \left(1+\tilde{h}_0^N\right)^{(1-\alpha^T)\phi_0^T+(1-\alpha^N)\phi_0^N}\frac{\prod_i(\alpha^i\phi_0^i)^{-\alpha^i\phi_0^i}}{\prod_i(\alpha^i\phi^i)^{-\alpha^i\phi^i}} - 1. \tag{25}$$

Proof. See Appendix A.3 □

Condition (24) describes the standard output-inflation that arises in an environment in which price adjustments are costly. It states that in the cooperative solution, the global planner trade-offs the goals of eliminating output gaps and driving inflation back to its steady-state level. In this model, however, the weights given to the labor wedge and inflation is determined by the concavity of the production function, which as we will see, will be important for shaping the optimal policy and the international spillovers .

3.1 Two Targets, One Instrument

The policy trade-off faced by central banks following an increase in ϕ_0^N can be inferred from the analysis of the Philips curve (25). Suppose that following an increase in ϕ_0^N, central banks coordinate on ensuring full employment by setting nominal rates to close the output gap. Then, the inflation rate would be deviate from its target in all countries

$$\pi_0 = \frac{\prod_i(\alpha^i\phi_0^i)^{-\alpha^i\phi_0^i}}{\prod_i(\alpha^i\phi^i)^{-\alpha^i\phi^i}} - 1 \neq 0$$

Conversely, a coordinated action to fully stabilize prices would lead to excess demand for goods, $\tilde{h}_0^N \neq 0$. The optimal monetary policy thus requires an increase in interest rates that trade off the welfare benefit of reducing excess demand with the cost of inflation.

Figure 1 illustrates the workings of the optimal monetary policy. The upward sloping line represents the inflation-output tradeoff ("IO curve") of central banks given by (24). This curve illustrates the optimal output gap given arbitrary levels of inflation. The downward sloping line represents the short-run Philips curve (PC), given by (25). This curve illustrates the level of inflation for arbitrary levels of the output gap. The optimal

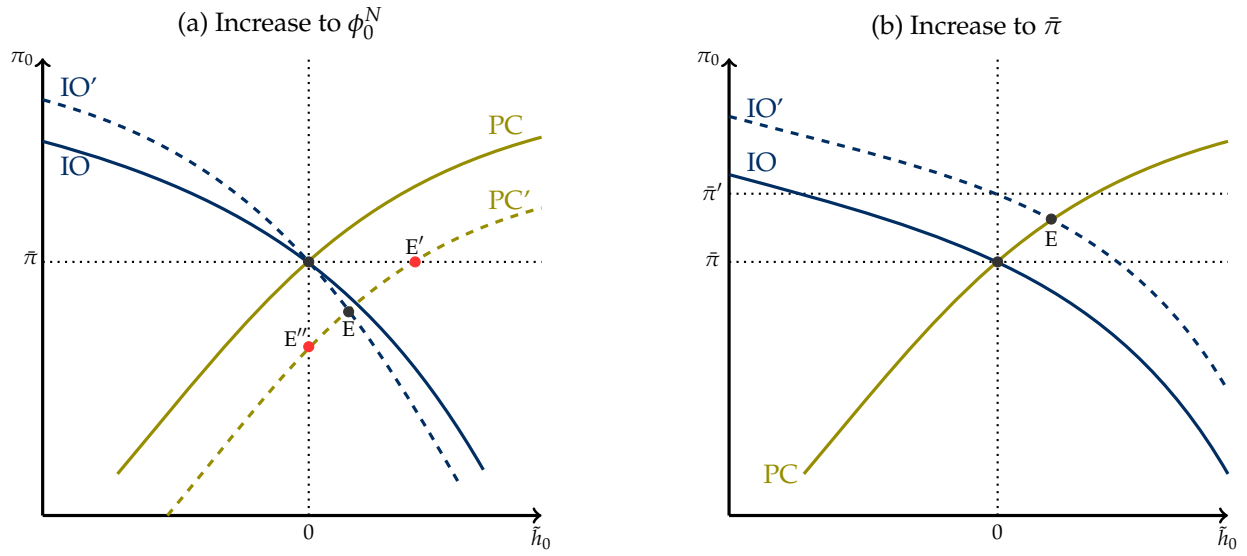


Figure 1: Optimal policy response for $\phi^N > 1/2$ and $\alpha^N > \alpha^T$.

IO is the inflation-output tradeoff curve and corresponds to (24). PC is the Philips curve and corresponds to (25). The grey line corresponds. In the left panel, IO′ and PC′ are the corresponding curves following a increase in ϕ_0^N. In the right panel, IO′ is the inflation-output tradeoff curve following an increase in $\bar{\pi}_0$

monetary policy given parameters is represented by the intersection between the two curves. The figure illustrates these two curves starting at the steady state (solid lines) and how they shift in response to a shock (dashed lines). The left panel considers an increase in ϕ_0^N and the right panel an increase in $\bar{\pi}$.

Notice that when starting at steady state, the two curves intersect at $(\bar{\pi}, 0)$. That is, the first-best allocation is feasible. In response to an increase in ϕ_0^N, the Philips curve shifts down. Point E_1' and point E_1'' represent equilibria where the global planner sets monetary policy to fully stabilize prices and close the output gap respectively. Starting from E_1', the global planner find it optimal to raise nominal interest rate in order to balance the welfare cost of inflation with the welfare benefit of reducing excess demand. The proposition below describes negative relationship between nominal output gap and nominal interest rate.

Proposition 2. *Given an allocation, the nominal interest rate is uniquely determined by*

$$R_0 = \left(1+\tilde{h}_0\right)^{-\left(\alpha^N\phi_0^N+\alpha^T\phi_0^T\right)} \frac{1}{\beta}\left[\prod_{i\in\{T,N\}}\left(\alpha^i\phi^i\right)^{\alpha^i\left(\phi^i-\phi_0^i\right)}\left(\frac{\phi^i}{\phi_0^i}\right)^{\alpha^i\phi_0^i}\right] \tag{26}$$

Proof. See Appendix A.4 □

4 Non-cooperative Monetary Policy

This section studies non-cooperative monetary policy. We model the non-cooperative game as a Nash equilibrium where central banks choose their monetary policy to maximize their own welfare, taking as given monetary policy abroad. We will show that the cooperative solution cannot be supported as a Nash equilibrium. We will characterize when central banks over-tighten or under-tighten relative to the optimal cooperative outcome.

4.1 Optimal Monetary Policy for a Single Country

We study the individual problem of a central bank that takes as given R^* and policies conducted in other countries. Recall that because prices are flexible for $t \geq 1$, we can focus on a situation where the government implements a flexible price allocation with $\pi_t = \bar{\pi}$ for all $t \geq 1$.[7] Starting from $t \geq 1$, the lifetime welfare for a government that starts with debt b_1^* is given by the following lemma

Lemma 3. *Consider a government that starts with b_1^* at $t = 1$. Then, the value is given by*

$$V(b_1^*) = \frac{1}{1-\beta}\left[\phi^T \log c_1^T - h_1^T + \phi^N \alpha^N (\log(\alpha^N \phi^N) - 1)\right], \tag{27}$$

where policies c_1^T and h_1^T are uniquely determined by

$$c_1^T = (h_1^T)^{\alpha^T} + (1-\beta)b_1^*, \quad \text{and} \quad (h_1^T)^{1-\alpha^T} c_1^T = \alpha^T \phi^T \tag{28}$$

Proof. See Appendix A.5 □

We now turn to the problem at $t = 0$. The objective of the government is to choose the domestic nominal interest rate R_0, to maximize the utility flow in period 0 plus the continuation value (27) characterized above. The problem consists of choosing prices and allocations to maximize the flow utility in period 0 and the continuation value subject to households' budget constraint (3), optimality conditions (4), (6), (7), firms' optimality conditions (8)-(9) and market clearing constraints (10)-(12). The key difference relative to the problem for the planner under coordination is that the individual central bank takes as given R^* and can choose allocations that lead to deviate from a zero trade balance (even though in the Nash equilibrium, with identical countries, the trade balance would be

[7] Notice there are no time inconsistency issues associated with the optimal monetary policy.

zero).[8]

Replacing constraints (8)-(10) in the objective, and using the fact that given an allocation (7) can be used to back out the policy rate of the central bank, we arrive to the following problem for the national planner

$$\max_{b_1^*, h_0^N, h_0^T, \pi_0} \phi_0^T \log(c_0^T) + \phi_0^N \alpha^N \log(h_0^N) - (h_0^T + h_0^N) - \frac{\chi}{2}(\pi_0 - \bar{\pi})^2 + \beta V(b_1^*) \tag{29}$$

$$\text{s.t.} \quad c_0^T = (h_0^T)^{\alpha^T} - \frac{b_1^*}{R_0^*} \tag{30}$$

$$h_0^N = \frac{\alpha^N \phi_0^N}{\alpha^T \phi_0^T} (h_0^T)^{1-\alpha^T} c_0^T \tag{31}$$

$$\frac{\phi_0^T}{c_0^T} = \beta R_0^* \frac{\phi^T}{\mathcal{C}_1^T(b_1^*)} \tag{32}$$

$$1 + \pi_0 = \frac{1}{P_{-1}} \prod_{i \in \{T,N\}} \frac{(h_0^i)^{(1-\alpha^i)\phi_0^i}}{(\alpha^i \phi_0^i)^{\phi_0^i}} \tag{33}$$

where $\mathcal{C}_1^T(b_1^*)$ is defined in (28). The key difference between the optimization problem of the national planners and the global planner is that the national planners do not internalize the effects of a change in their nominal interest rate on the equilibrium world interest rate. Substituting out (30) and (33), and denoting by η and ϑ the multiplier on (31) and (32) respectively, the Lagrangian can be written as follows:

$$\begin{aligned}
\mathcal{L} = {} & \phi_0^T \log\left[(h_0^T)^{\alpha^T} - \frac{b_1^*}{R_0^*}\right] + \phi_0^N \alpha^N \log(h_0^N) - (h_0^T + h_0^N) \\
& - \frac{\chi}{2}\left[\frac{1}{P_{-1}} \prod_{i \in \{T,N\}} \frac{(h_0^i)^{(1-\alpha^i)\phi_0^i}}{(\alpha^i \phi_0^i)^{\phi_0^i}} - \bar{\pi}\right]^2 + \beta V(b_1^*) \\
& + \vartheta \left[\frac{\phi_0^T}{(h_0^T)^{\alpha^T} - \frac{b_1^*}{R_0^*}} - \beta R_0^* \frac{\phi^T}{\mathcal{C}_1^T(b_1^*)}\right] + \eta \left[\frac{\alpha^N \phi_0^N}{\alpha^T \phi_0^T} (h_0^T)^{1-\alpha^T} \left((h_0^T)^{\alpha^T} - \frac{b_1^*}{R_0^*}\right) - h_0^N\right]
\end{aligned}$$

A multiplier $\vartheta > 0$ would reflect the fact that the national planner perceives that borrowing from abroad would improve welfare, while a multiplier $\eta > 0$, similar to the global planner problem, would reflect the fact that the national planner perceives that being able to reallocate hours towards the non-tradable sector would improves welfare. The

[8] Owing to the possibility of non-zero zero trade balance, the output gaps need not to be equated across sectors from the individual central bank's perspective. However, they are equated in the Nash equilibrium as shown in Lemma 2.

first-order conditions with respect to h^N, h^T, b_1^*

$$\frac{\phi_0^N \alpha^N}{h_0^N} = 1 + \frac{\phi_0^N}{h_0^N}(1-\alpha^N)\chi\,(1+\pi_0)\,(\pi_0-\bar{\pi}) + \eta \tag{34}$$

$$\left(1-\frac{\vartheta}{c_0^T}\right)\frac{(h_0^T)^{\alpha^T}}{c_0^T}\frac{\phi_0^T \alpha^T}{h_0^T} = 1 + \frac{\phi_0^T}{h_0^T}(1-\alpha^T)\chi\,(1+\pi_0)\,(\pi_0-\bar{\pi}) - \left[(1-\alpha^T)+\alpha^T\frac{(h^T)^{\alpha^T}}{c_0^T}\right]\frac{h_0^N}{h_0^T}\eta \tag{35}$$

$$\frac{\phi_0^T}{c_0^T}\vartheta = \left[1+\frac{(1-\alpha^T)\,\phi_0^T}{h_1^T+(1-\alpha^T)\phi^T}\frac{1-\beta}{\beta}\right]^{-1} h_0^N \eta \tag{36}$$

The first equation is identical to the corresponding equation for the global planner. The second equation now differs in that tradable consumption may not equate tradable output for an individual country, thus altering marginal utilities of tradable goods.

In contrast to the global planner, an individual central bank can in principle change its borrowing from abroad. By doing so, it affects the amount of tradable resources, as reflected in equation (36) Remarkably this equation reveals that the Lagrange multipliers on (31) and (32). have the same sign. In other words, when the planner would rather reallocate more hours towards non-tradable, it is also the case that it would like to households to borrow more. Combining conditions (34)-(36), an expression for η can be derived

$$\eta = \left[\frac{1-\alpha^T}{\alpha^T} - \frac{1-\alpha^N}{\alpha^N}\frac{(h_0^T)^{\alpha^T}}{c_0^T}\right]\frac{\alpha^T\phi_0^T}{h_0^T+\left[(1-\alpha^T)+\alpha^T(1-\xi)\frac{(h_0^T)^{\alpha^T}}{c_0^T}\right]h_0^N}\chi(1+\pi_0)(\pi_0-\bar{\pi}) \tag{37}$$

where $\xi \equiv \left[1+\frac{(1-\alpha^T)\phi_0^T}{h_1^T+(1-\alpha^T)\phi^T}\frac{1-\beta}{\beta}\right]^{-1}$. Condition (37) describes the domestic central bank's welfare benefit schedule from reallocating consumption toward non-tradables for given equilibrium world interest rate R_0^*. Notice from (37) that when R_0^* is such that the trade balance is zero and $\alpha^T = \alpha^N$, we arrive to $\eta = 0$, just like in the coordinated solution. In this case, this also implies that the Euler equation for borrowing is not binding, $\vartheta = 0$.

Proposition 3. *Consider R_0^* such that the trade balance is zero. We have that $\vartheta = 0$ if and only if $\alpha^T = \alpha^N$.*

If the world interest rate R_0^* such that the trade balance is zero and the labor intensity differs across sectors $\alpha^T \neq \alpha^N$, relative to the global planner, the domestic central bank perceives a larger welfare benefit from reallocating consumption toward nontradables. This implies that the Euler equation for borrowing binds $\vartheta \neq 0$. An natural implication of this finding is that the cooperative solution cannot be supported as a Nash equilibrium. In other words, starting from the cooperative solution where the trade balance is zero, each

individual central bank perceives additional gains from borrowing or lending to the rest of the world and thus have incentives to deviate.

Intuitively, assume that the nontradable sector is relatively more labor intensive and consider that countries are overheating at R_0^* in the cooperative solution. From a single central bank's perspective, a reduction of one unit of borrowing reduces the relative expenditures in nontradables by the relative weight on nontradable consumption ϕ_0^N/ϕ_0^T, and reduces output in the non-tradable but more than by the relative weight on nontradable consumption $\frac{\alpha^N}{\alpha^T}\phi_0^N/\phi_0^T$, which in turns help cooling down the economy.

4.2 Nash Equilibrium

Because all countries are identical, in the competitive equilibrium where all central banks are optimizing, there are no capital flows and exchange rates are constant. Central banks' best responses functions (34)-(36) can then be combined to derive the following optimal inflation-output tradeoff analogous to (22),

$$\left(\frac{\phi_0^N\alpha^N}{h_0^N}-1\right)\left[1+(1-\alpha^T\xi)\frac{h_0^N}{h_0^T}\right]=\left[\frac{\phi_0^T}{h_0^T}(1-\alpha^T)+\frac{\phi_0^N}{h_0^T}(1-\alpha^N)(1-\alpha^T\xi)\right]\chi\,(1+\pi_0)\,(\pi_0-\bar{\pi}) \quad (38)$$

As in the cooperative case, the world economy in the competitive equilibrium where individual central banks respond optimally to shock can never experience both overheating with inflation above or be in a recession with inflation below its target. Comparing (38) with its counterpart (22) it follows that when $\alpha^T=\alpha^N$ competitive central banks and the global planner would put the same weight on fighting inflation ((38) and (22) coincide). If $\alpha^N>\alpha^T$, competitive central banks would put relatively less weight on fighting inflation and more weight on closing output gaps. Conversely, if $\alpha^T>\alpha^N$ competitive central banks would put relatively more weight on fighting inflation.

The next proposition provides a formal characterization of output-inflation that arises in the Nash equilibrium.

Proposition 4. *In the Nash equilibrium, employment in each country is uniquely determined by*

$$\tilde{h}_0^T=\tilde{h}_0^N=-\frac{(1-\alpha^T)\phi_0^T+(1-\alpha^T\xi)(1-\alpha^N)\phi_0^N}{\alpha^T\phi_0^T+(1-\alpha^T\xi)\alpha^N\phi_0^N}\chi\,(1+\pi_0)\,(\pi_0-\bar{\pi})\,, \quad (39)$$

where the inflation rate of the consumer price index $\pi_0(h_0^N)$ *satisfies*

$$\pi_0(\tilde{h}_0^N) = \left(1+\tilde{h}_0^N\right)^{(1-\alpha^T)\phi_0^T+(1-\alpha^N)\phi_0^N} \frac{\prod_i(\alpha^i\phi_0^i)^{-\alpha^i\phi_0^i}}{\prod_i(\alpha^i\phi^i)^{-\alpha^i\phi^i}} - 1. \tag{40}$$

Proof. See Appendix A.6 □

Figure 2 uses the same parameter values and shocks considered in Figure 1 to illustrate the workings of the central banks' monetary policy response in the competitive equilibrium. The upward sloping represents the inflation-output tradeoff ("IO curve") of central banks given by (39). The short-run Philips curve (PC) given by (40) is represented by the downward sloping line. At the steady, the two solid curves intersect at $(0, \bar{\pi})$.

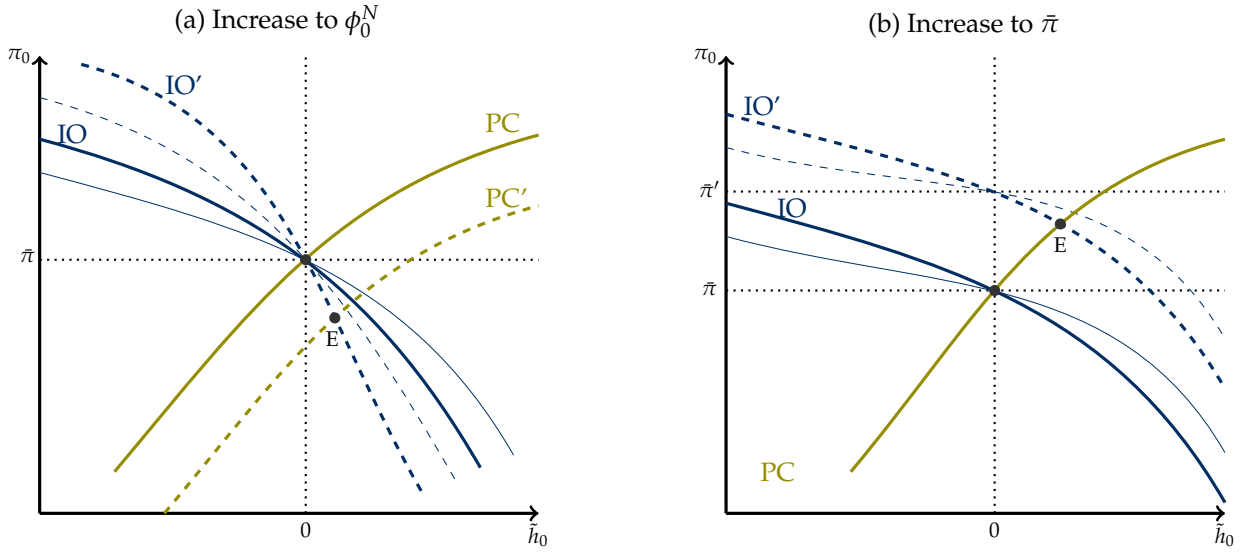


Figure 2: Policy response in the Nash equilbirum for $\phi^N > 1/2$ and $\alpha^N > \alpha^T$.

IO is the inflation-output tradeoff curve and corresponds to (24). PC is the Philips curve and corresponds to (25). The grey line corresponds. In the left panel, IO′ and PC′ are the corresponding curves following a increase in ϕ_0^N. In the right panel, IO′ is the inflation-output tradeoff curve following an increase in $\bar{\pi}_0$

Following an increase in ϕ_0^N (left panel) or an increase in $\bar{\pi}$ (right panel), the curves shift (dashed lines) and intersect in the right bottom of $(0,\bar{\pi}')$ where the economy experiences overheating, as in the cooperative case. The monetary policy response differs, however, quantitatively from the one under cooperation owing to the difference in the steepness of IO curve. In particular, as discussed in the next section, the nature of the shocks and the relative labor intensity between the two sectors are crucial sufficient statistics that determine whether central banks over- or under-tighten in the competitive equilibrium.

5 Over-tightening or Under-tightening?

Having characterized the cooperative monetary policy and the Nash equilibrium, we now turn to analyze the question of whether coordination calls for tighter or looser monetary policy relative to the Nash equilibrium.

Proposition 5. *We have under-tightening, i.e.* $R_0^{NE} < R_0^{GP}$ *(and* $\pi_0^{NE} > \pi_0^{GP}$*)*

(i) if $\alpha^N > \alpha^T$ *and the labor market faces overheating in the Nash equilibrium;*

(ii) if $\alpha^N < \alpha^T$ *and the labor market faces slack in the Nash equilibrium*

Proof. See Appendix A.7 □

The proposition establishes that when all economies are experiencing a recession, cooperation calls for lower interest rates. On the other hand, when the economies are experiencing overheating, cooperation calls for a higher interest rate.

Proposition 5 provides a sharp characterization on when a collective increase or decrease in nominal interest rates is desirable. The proposition therefore has implications for what kind of shocks may end up calling for higher or lower rates depending on what the effects are on the labor wedge in the non-cooperative outcome. We next consider different configuration of shock.

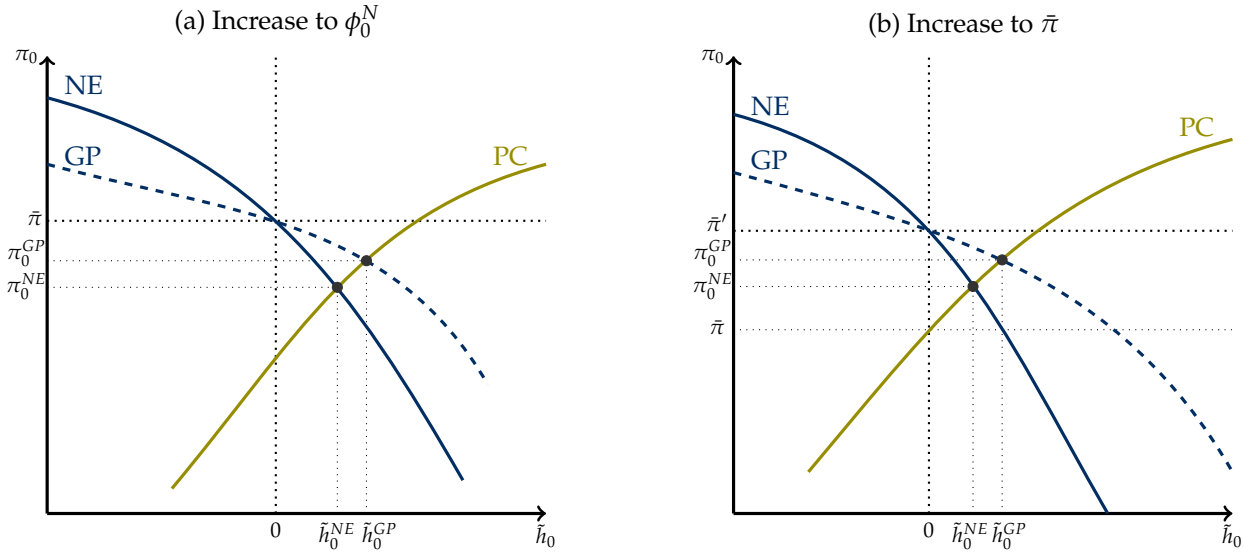


Figure 3: Policy response in the Nash equilibrium vs under cooperation for $\alpha^N > \alpha^T$.

GP and NE are respectively is the inflation-output tradeoff curve in the cooperative and Nash equilibrium.

6 Conclusion

We studied the scope for international monetary cooperation in an environment with global demand imbalances. We show that the Nash equilibrium mau feature under-tightening relative to the optimal cooperative solution. This occurs in states where the economy is overheating, as central banks fail to internalize that depressing domestic demand helps to cool down the world economy. On the other hand, when there is slack in the global economy, cooperation calls for slowing down interest rate hikes.

□

References

Bengui, Julien and Louphou Coulibaly, "Stagflation and Topsy-Turvy Capital Flows," 2022. University of Wisconsin Mimeo.

Benigno, Gianluca and Pierpaolo Benigno, "Designing targeting rules for international monetary policy cooperation," *Journal of Monetary Economics*, 2006, *53* (3), 473–506.

Bianchi, Javier, "Overborrowing and Systemic Externalities in the Business Cycle," *American Economic Review*, 2011, *101* (7), 3400–3426.

_ **and Louphou Coulibaly**, "Liquidity Traps, Prudential Policies and International Spillovers," 2021. Fed Working Paper.

Canzoneri, Matthew B and Dale W Henderson, *Monetary policy in interdependent economies: A game-theoretic approach*, MIT press, 1991.

Corsetti, Giancarlo and Paolo Pesenti, "International dimensions of optimal monetary policy," *Journal of Monetary economics*, 2005, *52* (2), 281–305.

Devereux, Michael B and Charles Engel, "Monetary policy in the open economy revisited: Price setting and exchange-rate flexibility," *The review of economic studies*, 2003, *70* (4), 765–783.

Farhi, Emmanuel and Iván Werning, "A Theory of Macroprudential Policies in the Presence of Nominal Rigidities," *Econometrica*, 2016, *84* (5), 1645–1704.

Fornaro, Luca and Federica Romei, "The Paradox of Global Thrift," *American Economic Review*, 2019, *109* (11), 3745–3779.

_ **and** _ , "Monetary policy during unbalanced global recoveries," 2022.

Hamada, Koichi, "A strategic analysis of monetary interdependence," *Journal of Political Economy*, 1976, *84* (4, Part 1), 677–700.

Obstfeld, "Uncoordinated monetary policies risk a historic," 2022. Blog post, Peterson Institute.

Obstfeld, Maurice and Kenneth Rogoff, "Global implications of self-oriented national monetary rules," *The Quarterly journal of economics*, 2002, *117* (2), 503–535.

Schmitt-Grohé, Stephanie and Martin Uribe, "Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment," *Journal of Political Economy*, 2016, *124* (5), 1466–1514.

APPENDIX

A Proofs

A.1 Proof of Lemma 1

The proof follows directly from rearranging (15). □

A.2 Proof of Lemma 2

Using (8), (9), and that $c_t^T = (h_t^T)^{\alpha^T}$ we obtain

$$\frac{P_t^N}{P_t^T} = \frac{\alpha^T (h^T)^{\alpha_T - 1}}{\alpha^N (h^N)^{\alpha_N - 1}}. \tag{A.1}$$

Combining this with (4)

$$\frac{\phi^T \alpha^T}{h_T^T} = \frac{\phi^N \alpha^N}{h_t^N}. \tag{A.2}$$

Rearranging and using Lemma (1), we obtain

$$\frac{h_t^T}{\tilde{h}_t^T} = \frac{h_t^N}{\tilde{h}_t^N}. \tag{A.3}$$

□

A.3 Proof of Proposition 1

The proof follows directly from (23) and (18). Rearranging (23) using the definition of the output gap (16), we get

$$-\frac{\phi_0^T \alpha^T}{h_0^T} \tilde{h}_0^T \left(1 + \frac{h_0^N}{h_0^T}\right) = \frac{1}{h_0^T} \left[(1-\alpha^T)\phi_0^T + (1-\alpha^N)\phi_0^N\right] \chi (1+\pi_0)(\pi_0 - \bar{\pi}) \tag{A.4}$$

Then, we use (18) to obtain

$$\tilde{h}_0^T = -\frac{(1-\alpha^T)\phi_0^T + (1-\alpha^N)\phi_0^N}{\alpha^T \phi_0^T + \alpha^N \phi_0^N} \chi \left[1 + \pi_0(\tilde{h}_0^N)\right] \left[\pi_0(\tilde{h}_0^N) - \bar{\pi}\right] \tag{A.5}$$

To obtain (25), we plug directly (18) into

$$\pi_0 = \frac{P_0}{P_{-1}} - 1 = \frac{\prod_i (\phi_0^i)^{(1-\alpha^i)\phi_0^i}}{\prod_i (\phi^i)^{(1-\alpha^i)\phi^i}} \left(1 + \tilde{h}_0^N\right)^{(1-\alpha^T)\phi_0^T + (1-\alpha^N)\phi_0^N} - 1 \tag{A.6}$$

A.4 Proof of Proposition 2

Using (2) and (4) along with market clearing $c_t^T = (h_t^T)^{\alpha^T}$ and $c_t^N = (h_t^N)^{\alpha^N}$, we obtain

$$\frac{P_1}{P_0} = \frac{(\phi_0^T)^{\phi_0^T}(\phi_0^N)^{\phi_0^N}}{(\phi^T)^{\phi^T}(\phi^N)^{\phi^N}} \left(\frac{\phi_0^N}{\phi_0^T}\frac{(h_0^T)^{\alpha^T}}{(h_0^N)^{\alpha^N}}\right)^{-\phi_0^N} \left(\frac{\phi^N}{\phi^T}\frac{(h_1^T)^{\alpha^T}}{(h_1^N)^{\alpha^N}}\right)^{\phi^N} \frac{P_1^T}{P_0^T}$$

Next, using $h_1^T = \alpha^T \phi^T$, $h_1^N = \alpha^N \phi^N$ and $P_1 = P_0$ since prices are flexible at $t = 1$ and monetary policy stabilizes prices we get

$$1 = \frac{P_1^T}{P_0^T} \left(\frac{(h_0^T)^{\alpha^T}}{(h_0^N)^{\alpha^N}}\right)^{-\phi_0^N} \frac{\phi_0^T}{\phi^T} \frac{(\alpha^T \phi^T)^{\alpha^T \phi^N}}{(\alpha^N \phi^N)^{\alpha^N \phi^N}} \tag{A.7}$$

then plugging (18) into (A.7) we arrive at

$$\frac{P_1^T}{P_0^T} = \left(h_0^T\right)^{\phi_0^N(\alpha^T - \alpha^N)} \Lambda \tag{A.8}$$

with

$$\Lambda \equiv \left(\frac{\alpha^N}{\alpha^T}\frac{\phi_0^N}{\phi_0^T}\right)^{\alpha^N(\phi^N - \phi_0^N)} \left(\frac{\phi^N}{\phi_0^N}\frac{\phi_0^T}{\phi^T}\right)^{\alpha^N \phi^N} \frac{\phi^T}{\phi_0^T} (\phi^T \alpha^T)^{(\alpha^N - \alpha^T)\phi^N}$$

Using market clearing for tradables and plugging $h_1^T = \alpha^T \phi^T$ into (6), we obtain

$$R^* = \frac{1}{\beta}\frac{\phi_0^T}{\phi^T}\frac{(\alpha^T \phi^T)^{\alpha^T}}{(h_0^T)^{\alpha^T}} \tag{A.9}$$

Finally, substituting (A.7) and (A.9) into the IP condition (7), we get

$$\begin{aligned} R_0 &= \left(h_0^T\right)^{-\alpha^N \phi_0^N - \alpha^T \phi_0^T} \frac{1}{\beta}\frac{\phi_0^T}{\phi^T} \left(\alpha^T \phi^T\right)^{\alpha^T} \Lambda \\ &= \left(1 + \tilde{h}_0\right)^{-\alpha^N \phi_0^N - \alpha^T \phi_0^T} \frac{1}{\beta} \left[\prod_{i \in \{T,N\}} \left(\alpha^i \phi^i\right)^{\alpha^i(\phi^i - \phi_0^i)} \left(\frac{\phi^i}{\phi_0^i}\right)^{\alpha^i \phi_0^i}\right] \end{aligned}$$

where the last equality uses $\tilde{h}_0 = h_0^T/(\alpha^T \phi_0^T)$ □

A.5 Proof of Lemma 3

Because for $t \geq 1$ the economy is in a stationary equilibrium, the continuation value is given by

$$V(b_1^*) = \frac{1}{1-\beta}\left[\phi^T \log(c_1^T) - h_1^T + \phi^N \log(c_1^N) - h_1^N\right] \tag{A.10}$$

By the market clearing condition for tradables and international bond market clearing $b_{t+1}^* = 0$ for $t \geq 1$, we have $\beta R_1^* = 1$, which implies $c_1^T = (h_1^T)^{\alpha^T} + (1-\beta)b_1^*$. Moreover, since wages are flexible for $t \geq 1$, households are on their labor supply. We then combine households' optimality conditions for labor, $1 = \frac{\phi^i}{c_1^i}\frac{W_1}{P_1^i}$, with firms' optimality conditions (9) to get

$$c_1^i (h_1^i)^{1-\alpha^i} = \phi^i \alpha^i \tag{A.11}$$

Using market clearing for non-tradables $c_1^N = (h_1^N)^{\alpha^N}$ we arrive at $h_1^N = \alpha^N \phi^N$. Substituting it into the value function (A.10) we obtain (27). □

A.6 Proof of Proposition 4

The proof follows directly from (23) and (18). Rearranging (23) using the definition of the output gap (16), we get

$$-\frac{\phi_0^T \alpha^T}{h_0^T}\tilde{h}_0^T\left[1+(1-\alpha^T\xi)\frac{h_0^N}{h_0^T}\right] = \left[\frac{\phi_0^T}{h_0^T}(1-\alpha^T)+\frac{\phi_0^N}{h_0^T}(1-\alpha^N)(1-\alpha^T\xi)\right]\chi\,(1+\pi_0)\,(\pi_0-\bar{\pi}) \tag{A.12}$$

Then, we use (18) to obtain

$$\tilde{h}_0 = -\frac{(1-\alpha^T)\phi_0^T + (1-\alpha^T\xi)(1-\alpha^N)\phi_0^N}{\alpha^T\phi_0^T + (1-\alpha^T\xi)\alpha^N\phi_0^N}\chi\,(1+\pi_0)\,(\pi_0-\bar{\pi}) \tag{A.13}$$

To obtain (25), we plug directly (18) into

$$\frac{P_0}{P_{-1}} - 1 = \frac{\prod_i (\phi_0^i)^{(1-\alpha^i)\phi_0^i}}{\prod_i (\phi^i)^{(1-\alpha^i)\phi^i}}\left(1+\tilde{h}_0^N\right)^{(1-\alpha^T)\phi_0^T+(1-\alpha^N)\phi_0^N} - 1 \tag{A.14}$$

A.7 Proof of Proposition 5

Let define

$$F_N(\tilde{h}_0) \equiv \tilde{h}_0 + \frac{(1-\alpha^T)\phi_0^T + (1-\alpha^T\xi)(1-\alpha^N)\phi_0^N}{\alpha^T\phi_0^T + (1-\alpha^T\xi)\alpha^N\phi_0^N}\chi\prod_i \frac{(\alpha^i\phi_0^i)^{-\alpha^i\phi_0^i}}{(\alpha^i\phi^i)^{-\alpha^i\phi^i}} \times \left(1+\tilde{h}_0\right)^{\sum_i(1-\alpha^i)\phi_0^i}\left[\frac{\prod_i(\phi_0^i)^{(1-\alpha^i)\phi_0^i}}{\prod_i(\phi^i)^{(1-\alpha^i)\phi^i}}(1+\tilde{h}_0)^{\sum_i(1-\alpha^i)\phi_0^i} - (1+\bar{\pi})\right] \quad \text{(A.15)}$$

and

$$F_G(\tilde{h}_0) \equiv \tilde{h}_0 + \frac{(1-\alpha^T)\phi_0^T + (1-\alpha^N)\phi_0^N}{\alpha^T\phi_0^T + \alpha^N\phi_0^N}\chi\prod_i \frac{(\alpha^i\phi_0^i)^{-\alpha^i\phi_0^i}}{(\alpha^i\phi^i)^{-\alpha^i\phi^i}} \times \left(1+\tilde{h}_0\right)^{\sum_i(1-\alpha^i)\phi_0^i}\left[\frac{\prod_i(\phi_0^i)^{(1-\alpha^i)\phi_0^i}}{\prod_i(\phi^i)^{(1-\alpha^i)\phi^i}}(1+\tilde{h}_0)^{\sum_i(1-\alpha^i)\phi_0^i} - (1+\bar{\pi})\right] \quad \text{(A.16)}$$

Notice that $F_N(\tilde{h}_0) = 0$ is the implicit function that determines the output gap in the Nash equilibrium. It combines (39) and (40). Similarly $F_G(\tilde{h}_0) = 0$ is the one that determines the output gap under cooperation by combining (24) and (25). Let $\tilde{h}_0^{NE}$ denote the solution to $F_N(\tilde{h}_0) = 0$. We have

$$F_G(\tilde{h}_0^{NE}) = \underbrace{F_N(\tilde{h}_0^{NE})}_{=0} + \frac{(\alpha^N - \alpha^T)\,\alpha^T\phi_0^T\phi_0^N\xi}{[(1-\alpha^T)\phi_0^T + (1-\alpha^T\xi)(1-\alpha^N)\phi_0^N][\alpha^T\phi_0^T + \alpha^N\phi_0^N]}\tilde{h}_0^{NE} \quad \text{(A.17)}$$

Moreover, we have

$$F_G'(\tilde{h}_0) = 1 + \frac{\left(\sum_i(1-\alpha^i)\phi_0^i\right)^2}{\sum_i\alpha^i\phi_0^i}\frac{\chi}{1+\tilde{h}_0}(1+\pi_0)\,(1+2\pi_0-\bar{\pi}) > 0. \quad \text{(A.18)}$$

Let $\tilde{h}_0^{GP}$ denote the solution to $F_G(\tilde{h}_0) = 0$. It follows from and that if $\alpha^N > \alpha^T$ and $\tilde{h}_0^{NE} > 0$ or if $\alpha^N < \alpha^T$ and $\tilde{h}_0^{NE} < 0$, then $\tilde{h}_0^{GP} < \tilde{h}_0^{NE}$ and by the Phillips curve (33), $\pi_0^{GP} < \pi_0^{NE}$. Using equation (26) in Proposition 2, $\tilde{h}_0^{GP} < \tilde{h}_0^{NE}$ implies that $R_0^{GP} > R_0^{NE}$