Can Deficits Finance Themselves?*

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Abstract

We ask how fiscal deficits are financed in environments with two key features: (i) fiscal adjustment is promised but delayed, and (ii) Ricardian equivalence is violated because households have finite horizons or face binding liquidity constraints. Fiscal deficits can contribute to their own financing by inducing a demand-driven Keynesian boom, raising prices (which erodes the value of real debt) and economic activity (which expands tax base and increases fiscal revenues). We show that, in a setting with our two features, fiscal deficits will in fact fully finance themselves, with nominal rigidities governing the split between nominal and real self-financing. The result is general—it is a feature of a large family of models with our two key features—as well as quantitatively and thus practically relevant—plausible delays in fiscal financing and departures from permanent-income behavior suffice for material self-financing of fiscal deficits.

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1 Introduction

How are fiscal deficits financed? The perhaps most obvious option is *fiscal adjustment*: the government cuts future expenditure or increases future taxes to increase primary surpluses. In this paper we investigate a different margin—what we will refer to as the *self-financing* of fiscal deficits. The idea is simple and familiar: fiscal deficits increase demand and thus are likely to at least partially contribute to its own financing, through equilibrium adjustments in either prices—as in the standard "fiscal theory of the price level" (FTPL)—or quantities—a boom in output automatically generates some additional tax revenue (DeLong et al., 2012).

The main contribution of the paper is the theoretical characterization of the conditions under which fiscal deficits will finance themselves through equilibrium feedback to prices and quantities. We show that self-financing robustly obtains in environments with the following two features. First, fiscal adjustment is delayed, in the sense that the government stands ready to adjust taxes in the (far but finite) future to return its fiscal debt to trend, if needed. Second, Ricardian Equivalence is violated because households have finite horizons (as in overlapping-generations models, Blanchard, 1985) or face binding liquidity constraints (as in heterogeneous-household models, Kaplan et al., 2018).

We begin by establishing the self-financing result in a textbook New Keynesian model, augmented to feature perpetual-youth households. Fiscal deficits take the form of a lump-sum transfer, and the monetary authority fixes the expected real rate of interest. In this environment, we prove that the fiscal deficits will invariably induce a boom precisely big enough to finance itself, with the split between prices and quantities depending on nominal rigidities. Key to our proof is that the initial fiscal deficit leads to a *short-lived* boom; as a result, the revenue required to finance the deficit is raised quickly, and so the promised delayed tax hike is never necessary in equilibrium. Households having short horizons—either due to death or due to binding liquidity constraints—is precisely enough to ensure this. A responsive monetary authority either even further accelerates this boom (if accommodative) or delays it (if contractionary); if the monetary response is too aggressive, then no equilibrium exists. We further establish that our theory is general—in the sense of applying to richer environments, including in particular models with investment—as well as quantitatively relevant—empirically plausible delays in fiscal adjustment and departures from permanent-income behavior suffice to lead to meaningful self-financing.

Baseline environment The economy is populated by a unit continuum of perpetual-youth households with survival probability $\omega \in (0,1)$, sticky-price retailers, and a government. Solving the consumption-savings problem of households and aggregating across cohorts, we obtain an aggregate demand-type relation that collapses to the familiar Euler equation if $\omega = 1$. Relative to this baseline, for $\omega < 1$, households on average spend their income more quickly and respond less to news of future income

and tax changes, violating Ricardian Equivalence and reflecting their finite horizons. Next, combining household labor supply with firm pricing, we obtain a standard aggregate Phillips curve (NKPC) as the supply block. The model is closed with a government, consisting of a fiscal authority and a central bank. The fiscal authority generates revenue through (i) a time-invariant distortionary tax τ_y on labor and dividend income and (ii) additional lump-sum taxes on households which can vary with public debt and deficit.[1] Our policy experiment is a date-0 increase in the fiscal deficit, paid out as lump-sum transfers to households. Lump-sum taxes are set according to a fiscal rule, with fiscal deficits today mapping into higher lump-sum taxes, though potentially with a delay. Finally, the monetary authority sets the nominal rate of interest; for much of our analysis, we consider a benchmark case in which the monetary authority neither accommodates nor leans against the initial deficit, with nominal rates set equal to expected inflation, thus fixing expected real rates.

How will the fiscal deficits be financed in equilibrium? Iterating the government budget constraint forward in time, we can see that there are three possible sources of financing. First, future lump-sum taxes may increase—i.e., *fiscal adjustment*. Second, the date-0 price level may jump, inflating away government debt. And third, real economic activity may boom, automatically expanding tax base y and generating additional tax revenue of $\tau_y \times y$, without any need for fiscal adjustment—a kind of "self-sustaining fiscal stimulus" discussed prominently in DeLong et al. (2012). The second and third sources of financing are what we refer to as *self-financing*, and we denote the share of financing through these channels by ν.

The self-financing result The central result of the paper is that, as the promised fiscal adjustment is delayed further and further, the overall share of self-financing ν converges to 1—that is, the initial fiscal deficit is entirely financed through a combination of date-0 price level jump and output boom. The split between sources of financing depends on the degree of nominal rigidities: if prices are approximately flexible then almost all financing occurs through p_0, while if prices are fully rigid then all financing occurs through adjustment in tax base: the cumulative output multiplier of fiscal stimulus is equal to $1/\tau_y$. The finite-horizon coefficient ω governs the speed of convergence to this self-financing outcome but does not change the essence of self-financing: as long as $\omega < 1$, a sufficient delay of fiscal adjustment suffices for full self financing.

The economic intuition for our self-financing result is best understood in two steps. First, prior to any general equilibrium price or quantity adjustments, as long as Ricardian equivalence is broken through $\omega < 1$, a lump-sum transfer today financed with higher taxes in the (far) future leads to a boom today and a bust in the future. Second, in general equilibrium, prices and quantities adjust in

[1] Our baseline analysis assumes that distortionary taxes are time-invariant and that lump-sum taxes change over time, for analytical clarity. We show later that our limiting self-financing results do not depend on this assumption.

response to this partial-equilibrium boom-bust impact. Consider first the initial boom, and suppose momentarily that prices were fully rigid. A standard static Keynesian cross would suggest that the response of output y to a fiscal deficit ϵ would satisfy

$$y = \frac{\text{MPC}}{1 - \text{MPC} \times (1 - \tau_y)} \times \epsilon$$

—for every dollar of income, $\text{MPC} \times (1 - \tau_y)$ dollars get spent, where *MPC* is the average household marginal propensity to consume. This simple relationship would suggest that self-financing requires an MPC of one, giving the required fiscal multiplier of $1/\tau_y$. In our dynamic environment, however, every dollar of income is necessarily spent *at some point*, so lifetime MPCs are always one and so the initial boom in fact invariably raises enough revenue to finance itself. The same logic, of course, also applies in reverse to the bust caused by the future tax hike. Our self-financing result thus additionally requires that the initial boom is *quick enough*, raising all required revenue before the promised future tax hike ever becomes necessary. In our environment, $\omega < 1$ ensures that households on average spend any income gain quickly, thus leading to a general equilibrium demand boom that plays out in finite time. Finally, even if prices are not perfectly rigid, the basic logic continues to apply; the only difference is that the "quick" Keynesian boom now translates partially into prices and partially into quantities.

Relation to FTPL Our results share the flavor of the classical "fiscal theory of the price level": deficits are financed not through outright fiscal adjustment but through equilibrium responses of prices and quantities. The fundamental difference between our results and the classical FTPL literature (Sims, 1994; Leeper, 1991; Woodford, 1995; Bassetto, 2008; Cochrane, 2023), however, is that our results are not subject to several controversies of that literature. First, we can relax the assumption of a fiscal authority that *never* adjusts to one that adjusts with a *delay*. Our fiscal authority satisfies its budget constraint on- and off-equilibrium, sidestepping a long theoretical debate (Woodford, 1995; Kocherlakota and Phelan, 1999; Bassetto, 2002). Second, because households have finite horizons or face binding liquidity constraints, fiscal deficits directly enter our economy's aggregate demand relationship. As a result, self-financing in our economy is not driven by equilibrium selection and is not sensitive to frictions in memory and coordination highlighted in Angeletos and Lian (2023).

Generality & extensions Our core result on the possibility of self-financing deficits is general, reflecting the simplicity and robustness of the underlying economic intuition.

Our first two extensions alter the assumptions on the conduct of policy. First, we show that nothing changes if the promised future tax hike is instead distortionary. The argument is trivial: under our assumptions this tax hike will never materialize in equilibrium, and so our original construction of a self-financing equilibrium applies without change. Second, we consider what happens if the mone-

tary authority moves away from our "neutral" benchmark, either accommodating or leaning against the deficit-induced boom. Intuitively, if the monetary authority accommodates (i.e., cuts expected real rates), then households are incentivized to front-load spending even further, so the Keynesian boom is even quicker, and convergence to the self-financing equilibrium is even faster. Conversely, if expected real rates are increased, then convergence is slower. In particular we show that, if the monetary response is sufficiently aggressive in leaning against the boom, then convergence becomes so slow that a self-financing equilibrium ceases to exist. Equivalently, for such a monetary policy, equilibrium existence requires that fiscal adjustment is sufficiently quick.

Our second set of extensions generalizes the model environment. First, we show that the self-financing result continues to apply for aggregate demand blocks beyond OLG. Theoretically, we provide a general set of sufficient conditions on MPCs out of current income, future income, and wealth that allow us to obtain our self-financing result. Second, we repeat our analysis in an environment with investment, and find that the self-financing result extends with little change. Key to the argument is that the "Keynesian cross"-type logic sketched above continues to apply, just now to total output less investment.

Quantitative relevance As the final step in our analysis we ascertain the quantitative and thus practical relevance of our self-financing result. Our theoretical analysis has revealed that the two model ingredients that will matter most are: (i) the delay in fiscal financing; and (ii) the deviation from permanent-income consumer behavior. Prior empirical work suggests that, even in classical "passive" fiscal regimes, fiscal adjustment tends to be very gradual (Galí et al., 2007; Bianchi and Melosi, 2017; Auclert and Rognlie, 2018). On the consumer side, we enrich our baseline OLG model to also feature a margin of hand-to-mouth spenders; this extension is simple enough to fit into our generalized aggregate demand block mentioned above, yet rich enough to very closely approximate aggregate consumption behavior in HANK-type environments (Auclert et al., 2018; Wolf, 2021a). The model is disciplined using empirical evidence on household consumption behavior in response to lump-sum income gains (Fagereng et al., 2021).

We then use our quantitative model to study the effects of a one-off, deficit-financed lump-sum transfer to households—a policy reminiscent of the various rounds of "stimulus checks" seen recently in the U.S. Our headline finding is that, in equilibrium, such policies will indeed turn out to be largely self-financed, inducing a large and persistent output boom together with meaningful aggregate inflation.[2] We note that those results are informative about the recent "excess savings" debate after deficit-driven stimulus checks: if households have finite horizons or face binding liquidity constraints, then

[2]For completeness, we show that our results extend with little change to a numerically solved HANK-type environment, confirming the findings of Auclert et al. (2018) and Wolf (2021a).

our theory predicts that such excess savings can induce long-lasting excess demand pressure. Depending on the extent of nominal rigidities this demand pressure will then feed through to either prices or quantities.

Literature Our analysis relates and contributes to several strands of literature.

First, we add to prior work that considers the possibility of "self-financing" fiscal deficits. In the classical FTPL, deficits self-finance through a jump in the price level (Leeper, 1991; Woodford, 1995; Bassetto, 2002; Cochrane, 2005). In the textbook version of the FTPL such "self-financing" requires (i) a fiscal authority committing not to balance its budget as well as (ii) monetary accommodation. These assumptions have been criticized. First, it may not be credible to allow the government to commit to not raise tax in response to debt and deficits (Kocherlakota and Phelan, 1999). Second, in the conventional FTPL equilibrium, debt and deficits only matter through equilibrium selection. With infinitely-lived households without liquidity constraints, debt and deficits do not enter aggregate demand. Such non-fundamental equilibrium is not robust to frictions in intertemporal coordination (Angeletos and Lian, 2023).

Our environment with finite-horizon or liquidity-constrained households and delayed fiscal adjustment allows us to sidestep these concerns. Our New-Keynesian environment also allows us to emphasize a complementary, "real" source of self-financing: an expansion in the tax base. The theoretical possibility of fiscal deficits paying for themselves in this way is discussed prominently in DeLong et al. (2012). We show that such self-financing emerges naturally through the combination of two plausible model ingredients: delays in fiscal financing and finite household horizons.

Second, we contribute to recent work on the aggregate effects of stimulus check policies (Auclert et al., 2018; Wolf, 2021a,b). Relative to this prior work, our key contribution is to explicitly characterize the *limiting* effects of such stimulus checks as fiscal financing is delayed further and further. Our results are thus particularly relevant for the "excess savings" debate surrounding recent stimulus check policy experiments in the U.S. (Bilbiie et al., 2021): those checks were sent out without any commitment to medium-term financing, so our theory predicts—qualitatively consistent with recent empirical evidence—protracted booms in inflation and output.

Third, we add to a fast-growing literature on the macroeconomic consequence of non-PIH household behavior (e.g., Kaplan et al., 2018; Auclert, 2019; Bilbiie, 2020). Our key contribution is to identify a *qualitative* difference between HANK- or OLG-type models and spender-saver environments: only the former feature consumers spending their income quick enough to allow equilibrium self-financing. Intuitively, with permanent-income households, the initial boom is never quick enough—such households bring the future tax hike to the present and delay spending to the infinite future, so our self-financing argument breaks down.

Outline Section 2 begins by presenting the model and characterizing its equilibrium. Section 3 then presents our core self-financing result. Further extensions and the quantitative analysis follow in Sections 4 and 5, respectively. Finally Section 6 concludes.

2 Model

For our main analysis we consider a perpetual-youth, overlapping-generations version of the standard textbook New Keynesian model. The key role of mortality risk—and thus finite horizons in household consumption-savings decisions—is to break Ricardian Equivalence, with debt and deficits exerting a direct (or partial equilibrium) effect on aggregate demand. As in Del Negro et al. (2015), Farhi and Werning (2019), and Angeletos and Huo (2021), finite horizons serve as analytically convenient proxies for liquidity constraints. We return to more general aggregate demand functions and in particular to richer heterogeneous-agent models in Sections 4 and 5.

The section proceeds as follows. Section 2.1 describes the government, and Section 2.2 then introduces the private sector. We characterize the equilibrium in Section 2.3. Throughout, we study the economy's log-linearized dynamics in response to a surprise increase in fiscal deficits. We use uppercase variables to indicate levels; unless indicated otherwise, lowercase variables denote log-deviations from the economy's deterministic steady state. Time is discrete.

2.1 Government

The government consists of two blocks: a fiscal authority issuing debt and setting taxes, and a monetary authority setting nominal interest rates.

Fiscal authority The government issues nominal debt and collects tax revenue. Letting B_t denote the total amount of nominal public debt outstanding at the beginning of period t and writing total real tax revenue as T_t, the nominal government budget constraint is

$$\frac{1}{I_t} B_{t+1} = B_t - P_t T_t. \tag{1}$$

where P_t is the price level and I_t is the gross nominal interest rate; furthermore, for future reference, we will let $D_t \equiv B_t / P_t$ denote the real value of total public debt outstanding at the beginning of period t. We log-linearize around a deterministic zero-inflation steady state with $P^{ss} = 1$ and $I^{ss} = \frac{1}{\beta}$, where we anticipate that the steady state real rate of interest will be given by the inverse of the household discount factor β.[3] We then get $T^{ss} = (1-\beta)D^{ss} = (1-\beta)B^{ss}$. To allow for zero steady state government debt, we will for the fiscal variables $\{B, D, T\}$ use lowercases to denote deviations (rather

[3] This guarantees that the steady state real rate of interest $\frac{1}{\beta} > 1$ and we focus on the "$r > g$" environment.

than log-deviations) from steady state, scaled by total output; that is, we define $d_t \equiv (D_t - D^{ss})/Y^{ss}$, $b_t \equiv (B_t - B^{ss})/Y^{ss}$, and $t_t \equiv T_t/Y^{ss}$.[4] Re-writing (1) in real terms and linearizing, we obtain

$$d_{t+1} = \frac{1}{\beta}\left[d_t - t_t + \beta\frac{D^{ss}}{Y^{ss}}(i_t - \pi_{t+1})\right] \tag{2}$$

where $\pi_{t+1} \equiv p_{t+1} - p_t$ is the rate of inflation from t to $t+1$.

The fiscal rule prescribes how taxes evolve over time. We will assume that taxes consist of two components. First, for every dollar of real activity in the economy, tax revenue mechanically increases by τ_y; as we will see in Section 2.2, such revenue is generated if the government imposes a time-invariant distortionary tax on household labor and dividend income. Second, the government adjusts lump-sum taxes to actively balance its budget in the long run. The fiscal rule thus specifies the mapping from fiscal deficits into this lump-sum tax component. We will consider two examples of such fiscal rules.

1. *Baseline fiscal policy.* The first rule—our baseline fiscal policy—sets total taxes as

$$T_t = \tau_d(D_t + \mathscr{E}_t) + \tau_y Y_t + \bar{T} - \mathscr{E}_t, \tag{3}$$

where $\bar{T}$ guarantees budget balance at the steady state, $\mathscr{E}_t$ is a mean-zero, i.i.d., stochastic deficit shock, $\tau_d \in [0,1]$, and $\tau_y \geq 0$. After (log-)linearization (3) becomes

$$t_t = \underbrace{\tau_d \cdot (d_t + \epsilon_t)}_{\text{fiscal adjustment}} + \underbrace{\tau_y y_t}_{\text{tax base adjustment}} - \underbrace{\epsilon_t}_{\text{deficit shock}}, \tag{4}$$

where $\epsilon_t \equiv \mathscr{E}_t/Y^{ss}$. (4) is a natural generalization of the canonical fiscal rule in Leeper (1991). We highlight two features. First, the coefficient τ_d captures the strength and speed of fiscal adjustment. In particular, if $\tau_d = 0$, then taxes never adjust (an "active" fiscal policy, in the language of Leeper), while if $\tau_d = 1$ then taxes adjust to immediately balance the budget and thus return government debt to steady state. For intermediate τ_d taxes adjust gradually and a lower τ_d captures the delay in fiscal adjustment. If $\tau_d \in (1-(I^{ss})^{-1}, 1]$, then by combining (4) and (2) we see that public debt will always return to steady state—a "passive" fiscal policy inLeeper (1991). Much of our analysis will be concerned with the question of how the equilibrium financing of fiscal deficits depends on τ_d and how self financing ($\tau_y > 0$) can return public debt to its steady-state value even if $\tau_d \in [0, 1-(I^{ss})^{-1}]$. Second, changes in the tax base automatically generates additional tax revenue, without any change to tax rates. This realistic feature of our model will allow us to speak to the possibility of "self-financing fiscal deficits" through a real boom, as notably discussed in DeLong et al. (2012).

2. *Alternative fiscal policy.* Our second fiscal rule is a time-dependent generalization of (3) in which

[4] This is standard. See Woodford (2011).

the response coefficient τ_d is allowed to vary over time. For notational simplicity written in date-0 sequence-space notation (see e.g., Wolf, 2021a), this rule sets[5]

$$\tau_{d,t} = \begin{cases} 0 & t < H, \quad \text{initially no fiscal adjustment,} \\ 1 & t \geq H, \quad \text{eventually full fiscal adjustment.} \end{cases} \tag{5}$$

In words, after a deficit shock at date 0, there is no fiscal response prior to horizon H, and then full fiscal adjustment returning government debt to steady state at and after horizon H. Just like a smaller τ_d, a higher H captures delays in fiscal adjustment. While less standard, (5) has the benefit of offering a somewhat more transparent illustration of the idea of "delayed" fiscal adjustment. This rule will thus in particular play an important role for some of our numerical illustrations in Section 3.

Our assumption of fiscal adjustment through lump-sum taxes is purely for notational convenience. We return to the case of fiscal adjustment through time-varying distortionary taxation $\tau_{y,t}$ in Section 4.4.

Central bank The monetary authority sets the rate of interest on nominal bonds as

$$i_t = \mathbb{E}_t[\pi_{t+1}] \tag{6}$$

so the expected real rate of interest $r_t \equiv i_t - \mathbb{E}_t[\pi_{t+1}]$ is kept constant. This rule captures a lack of monetary accommodation: the monetary authority does not allow fiscal financing through low real rates, but neither does it aggressively lean against booms. This is the same "baseline" monetary rule as in Woodford (2011), allowing us to cleanly isolate how the interaction of fiscal variables and private spending behavior shape the scope for fiscal self-financing. We will return to more general monetary rules in Section 4.3.

2.2 Private sector

The economy is populated by a unit continuum of overlapping generations of households (as in Blanchard, 1985) and sticky-price retailers (as in the canonical New Keynesian model of Woodford, 2003b and Galí, 2008) . Households consume and supply labor; firms use labor to produce and return their profits to the household sector.

Households & aggregate demand We index households by $i \in [0,1]$. A household survives from one period to the next with probability $\omega \in (0,1]$; if a household dies then it is replaced by a new household.

[5]The rule (5) should be interpreted as pertaining to the impulse responses induced by a date-0 deficit shock ϵ_0. See Wolf (2021a) for a discussion of the (notationally somewhat involved) mapping from policy rules in sequence-space to their state-space analogues.

Households do not altruistically value the utility of the future cohorts that replace them. Taking into account this mortality risk, the expected lifetime utility of a household i alive at some given period $t \in \{0, 1, \cdots\}$ is given by

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\omega)^k \left[u(C_{i,t+k}) - v(L_{i,t+k}) \right] \right], \tag{7}$$

where $C_{i,t+k}$ and $L_{i,t+k}$ denote household i's consumption and labor supply in period $t+k$ (conditional on survival), and preferences take the standard form $u(C) \equiv \frac{C^{1-1/\sigma}-1}{1-\frac{1}{\sigma}}$ and $v(L) = \chi \frac{L^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$. Households receive real wage income $W_t L_{i,t}$ and real dividend income $Q_{i,t}$. They consume and they can trade actuarially fair, risk-free nominal annuities; the nominal return to saving conditional on survival is thus given as I_t/ω, and the steady state interest rate is $I^{ss} = \frac{1}{\beta}$, as already previewed above. Overall we can write the date-t budget constraint of household i as

$$A_{i,t+1} = \underbrace{\frac{I_t}{\omega}}_{\text{annuity}} \left(A_{i,t} + P_t \cdot \left(\underbrace{W_t L_{i,t} + Q_{i,t}}_{Y_{i,t}} - C_{i,t} - T_{i,t} \right) \right) - \underbrace{\frac{(1-\omega) \cdot P_{t+1} \cdot D^{ss}}{\omega}}_{\text{transfer to newborn}}, \tag{8}$$

where: $A_{i,t}$ denotes household i's savings in nominal annuities at the beginning of date t; $Y_{i,t}$ is total real income; and $T_{i,t}$ denotes real tax payments. The last term in (8) captures a transfer to newborn households (which is fixed in its real value), with $D^{ss} \equiv A^{ss}/P^{ss}$ equal to the steady-state real value of total household wealth.[6] The sole purpose of this transfer is to ensure that all cohorts—in particular including newborns—share the same steady-state level of consumption.

The consumption-savings problem of a household i is to choose sequences of consumption and asset holdings to maximize (7) subject to (8). Aggregating across all households i, we obtain total consumption $C_t = \int_0^1 C_{i,t} di$ and total asset holdings $A_t \equiv \int_0^1 A_{i,t} di$. We will discuss household labor supply $L_{i,t}$ (and, relatedly, the composition of taxes $T_{i,t}$) together with the remainder of the supply side of the economy; for the purposes of our demand-side discussion here, it suffices to note that all households i will earn the same income $Y_{i,t} = Y_t$ and pay the same taxes $T_{i,t} = T_t$. We can then follow the same steps as in Angeletos and Huo (2021) to arrive at the following linearized aggregate demand relationship:

$$c_t = (1-\beta\omega) \left(d_t + \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\omega)^k (y_{t+k} - t_{t+k}) \right] \right) - \left[\sigma\beta\omega - (1-\beta\omega)\beta \frac{D^{ss}}{Y^{ss}} \right] \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\omega)^k (i_{t+k} - \pi_{t+k+1}) \right]. \tag{9}$$

(9) differs from conventional permanent-income aggregate demand relations since households discount future income and future changes in real interest rates at a rate $(\beta\omega)^k$ higher than the real rate

[6] Note that, by our assumptions on this transfer, the initial savings of a household i born at date $t+1$ are given as $A_{i,t+1} = P_{t+1} \cdot D^{ss}$.

of interest β^k faced by the government. Ricardian equivalence thus fails and marginal propensities to consume out of current income are elevated, equal to $1-\beta\omega$. If instead we had that $\omega = 1$ then (9) would collapse to the familiar standard Euler equation.[7]

Price-setting & aggregate supply Households supply their labor on a competitive spot market, earning real wage $(1-\tau_y)W_t$, where τ_y is a time-invariant, distortionary tax. To facilitate aggregation—similarly to our assumptions on transfers to newborns—we assume that labor supply is intermediated by unions.[8] Those unions supply labor to equalize the post-tax real wage and the average marginal rate of substitution between labor supply and consumption; linearizing (and using the fact that τ_y is time-invariant), we obtain the standard labor supply relation

$$\frac{1}{\varphi}\ell_t = w_t - \frac{1}{\sigma}c_t \tag{10}$$

We assume that unions demand an equal amount of labor from all households. Households also all receive identical shares of aggregate dividends, so all households in fact earn the same total income equal to aggregate income $Y_{i,t} = Y_t$. Dividends are similarly subject to the time-invariant distortionary tax τ_y, so the government indeed automatically increases its revenue by τ_y dollars for every additional dollar of aggregate income. Finally, we assume that the lump-sum tax component determined by the fiscal rule (4) or (5) is also equalized across households, so all households pay the same amount of taxes $T_{i,t} = T_t$.

A unit continuum of monopolistically competitive retailers hires labor on a spot market, sets prices subject to nominal rigidities, and pays out their profits as dividends back to households (exactly as in Galí, 2008). Combining the labor supply optimality condition (10) with output market-clearing ($y_t = c_t$) and the firm pricing relationship, we obtain a standard aggregate New Keynesian Phillips Curve (NKPC):

$$\pi_t = \kappa y_t + \beta\mathbb{E}_t[\pi_{t+1}]. \tag{11}$$

Note that, since the distortionary labor tax τ_y is time-invariant, there are no additional tax-related wedges entering the linearized NKPC (11). We will return to the case with time-varying distortionary taxes in Section 4.4.

2.3 Equilibrium

We now lay the groundwork for our self-financing result in Section 3, collapsing our economy's equilibrium relationships to a small number of equations. Our solution concept is that of Blanchard and

[7] Using market clearing $c_t = y_t$ and government budget (2), (9) collapses into the standard Euler equation: $c_t = -\sigma\mathbb{E}_t[i_t - \pi_{t+1}] + \mathbb{E}_t[c_{t+1}]$.

[8] See Appendix (A.1) for details.

Kahn (1980): we look for bounded solutions to the linearized equilibrium system. The analysis in this section will be presented for our baseline fiscal policy rule (4); equilibrium characterizations for the alternative rule (5) are similar and fully fleshed out in Appendix A.3.

A compact equilibrium system We consider an economy with initial conditions $B_0 = B^{ss}$ and $P_{-1} = P^{ss} = 1$.[9] We begin by reducing the equilibrium relationships of our economy to three key equations: an aggregate demand block; an aggregate supply block; and the law of motion for government debt.

1. *Aggregate demand.* We begin with consumer spending behavior as summarized in the demand relation (9). Imposing the baseline monetary policy rule (6) we see that the term in the second line of (9) vanishes. Furthermore using the government budget constraint and fiscal rule (4) to eliminate the net present value of taxes, and finally imposing output market-clearing (i.e., $y_t = c_t$), we arrive at the following simplified aggregate demand relationship:

$$y_t = \mathscr{F}_1 \cdot (d_t + \epsilon_t) + \mathscr{F}_2 \cdot \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\omega)^k y_{t+k} \right], \tag{12}$$

where $\mathscr{F}_1 \equiv \frac{(1-\beta\omega)(1-\omega)(1-\tau_d)}{1-\omega(1-\tau_d)}$ and $\mathscr{F}_2 = (1-\beta\omega)\left(1 - \frac{(1-\omega)\tau_y}{1-\omega(1-\tau_d)}\right)$. Equation (12) is the first of three key relations summarizing the equilibrium of our economy. The first term in (12) captures the direct PE effect of debt and deficit on aggregate demand; if $\omega < 1$ (no Ricardian equivalence) and $\tau_d < 1$ (no *immediate* financing), then this term is positive—government bonds are net wealth for the living households and thus stimulate demand (Barro, 1974). Intuitively, deficits today are financed with taxes in the future and thus re-distribute from future generations to the currently living cohorts. The second term is the dynamic (or intertemporal) Keynesian cross: elevated output in the future affects demand today, though additionally discounted at rate ω. This second term will govern how an initial demand impulse is propagated in general equilibrium. On the other hand, in the textbook case (Woodford, 2003b; Galí, 2008) of infinitely-lived households ($\omega = 1$), $\mathscr{F}_1$ is equal to zero and deficits exert no direct PE effect on aggregate demand.

2. *Aggregate supply.* Labor supply together with optimal firm price-setting gives an equilibrium relationship between inflation and output, already stated above. We only repeat this standard NKPC here for easy reference:

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t [\pi_{t+1}]. \tag{13}$$

3. *Government debt.* Combining the fiscal policy rule (4) and the monetary policy rule (6) with the

[9] Consistent with the transfer to newborn in (8), the initial conditions for individual savings at the start of period is given by $A_{i,0} = \frac{1-(1-\omega)P_0}{\omega} D^{ss}$ and $A_{i,0} = P_0 \cdot D^{ss}$.

government budget constraint (2), we obtain the law of motion for real government debt,

$$d_{t+1} = \beta^{-1}\left(d_t + \varepsilon_t - \underbrace{\tau_d \cdot (d_t + \epsilon_t)}_{\text{fiscal adjustment}} - \underbrace{\tau_y y_t}_{\text{tax base}}\right) - \underbrace{\frac{D^{ss}}{Y^{ss}}(\pi_{t+1} - \mathbb{E}_t[\pi_{t+1}])}_{\text{price level}}, \tag{14}$$

with real debt at date 0 given as

$$d_0 = b_0 - \frac{D^{ss}}{Y^{ss}}(\pi_0 - E_{-1}[\pi_0]) = -\frac{D^{ss}}{Y^{ss}}\pi_0. \tag{15}$$

We see that, in our environment, debt can be stabilized through (lump-sum) fiscal adjustment, equilibrium movements in output and thus the tax base, and surprise inflation.

To summarize, an equilibrium of our economy is a set of bounded sequences of output y_t, inflation π_t, and aggregate government debt d_t that satisfies the three above relations: aggregate demand (12), aggregate supply (13), and the debt evolution relation (14).

Equilibrium existence & uniqueness Our first auxiliary result is one on equilibrium existence and uniqueness.

Proposition 1. *Suppose that $\omega < 1$, $\tau_y > 0$, and $\tau_d \in [0,1)$. There exists a unique bounded equilibrium satisfying equations (12), (13), and (14). This equilibrium takes the following form:*

$$y_t = \chi(d_t + \epsilon_t), \quad \mathbb{E}_t[d_{t+1}] = \rho_d(d_t + \epsilon_t), \quad \textit{with} \quad \chi > 0, \ \rho_d \in (0,1). \tag{16}$$

χ is the impact equilibrium response of output to the deficit shock, and ρ_d is the equilibrium expected persistence of government debt and thus output. Our constructive proof of Proposition 1 reveals that (χ, ρ_d) solve the following fixed-point relationship. First, given χ, ρ_d by (14) satisfies

$$\rho_d = \frac{1}{\beta}\left(1 - \tau_d - \tau_y \chi\right) \tag{17}$$

A larger Keynesian boom raises more tax revenue and thus reduces the persistence of real debt. Second, given ρ_d, the aggregate demand relationship (12) gives χ as

$$\chi = \frac{\mathscr{F}_1}{1 - \frac{\mathscr{F}_2}{1-\beta\omega\rho_d}} \tag{18}$$

More persistent government debt implies more persistent output, leading to a stronger intertemporal Keynesian cross to feed back to y_t and thus χ. We conclude from (18) that, as long as $\omega < 1$ and $\tau_d < 1$ (ensuring that $\mathscr{F}_1 > 0$), our equilibrium will feature the "self-financing" channels discussed in prior work: output booms, thus automatically raising more tax revenue (as in DeLong et al., 2012); this boom will also translate to higher inflation (from (13), thus eroding the real value of government debt in (15), as in the FTPL. Our analysis in Section 3 asks just how important these self-financing channels are likely to be in equilibrium.

The equilibrium uniqueness in Proposition 1 with $\omega < 1$ contrasts with the textbook case with $\omega = 1$. In that case, it is well known that the textbook NK model features infinitely number of equilibria with fixed expected real rates in (6). Why uniqueness with $\omega < 1$? In our case, since debt and deficits matter directly for aggregate demand in (12), aggregate output are "anchored" towards the fundamental state variable, debt and deficit $d_t + \epsilon_t$. This leads to equilibrium determinacy.

Comparison with $\omega = 1$ & classical FTPL We already emphasize here that the assumption of $\omega < 1$—that is, the violation of Ricardian Equivalence from household demand—is essential to our subsequent analysis and in particular distinguishes it from the classical FTPL. It follows from our discussion above that, as $\omega \to 1$, we get $\mathscr{F}_1 \to 0$ and thus $\chi \to 0$; that is, the Keynesian boom becomes vanishingly small. In the case with infinitely-lived households ($\omega = 1$), (12) becomes

$$y_t = \mathbb{E}_t \left[y_{t+1} \right]. \tag{19}$$

$\mathscr{F}_1$ is equal to zero and deficits d_t do not directly enter this economy's aggregate demand relation. This $\omega = 1$ is the case focused by traditional FTPL (Leeper, 1991; Woodford, 1995; Bassetto, 2002; Cochrane, 2005). In traditional FTPL, deficits instead matter through *equilibrium selection*: if $\omega = 1$ then under our assumptions on monetary policy there is a continuum of equilibria; the fiscal authority then simply *selects* the equilibrium that requires no fiscal adjustment. Our $\omega < 1$ analysis is conceptually very different: there is a unique equilibrium (as revealed by Proposition 1), and we study the composition of fiscal financing in this particular, unique equilibrium. Importantly, as our analysis does not rely on equilibrium selection, it is by design not vulnerable to frictions in intertemporal coordination discussed in Angeletos and Lian (2023). We return to issues of equilibrium existence and uniqueness under alternative assumptions on monetary policy in Section 4.3, in particular allowing us to relate our findings to the famous determinacy regions of Leeper (1991).

3 Self-financing of fiscal deficits

This section presents our headline theoretical result on the possibility of self-financing fiscal deficits. We begin in Section 3.1 by using the government budget constraint to delineate the various possible sources of fiscal financing. Section 3.2 then provides sharp characterizations of the importance of self-financing relative to classical fiscal adjustment.

3.1 Sources of financing

Iterating the debt relations (14) and (15) forward and using $\lim_{t\to\infty}\mathbb{E}_0\left[\beta^t d_t\right]=0$ (since $\rho_d\in(0,1)$), we can re-write the government budget constraint in present-value form:

$$\underbrace{\epsilon_0}_{\text{deficit}} = \underbrace{\tau_d\left(\varepsilon_0+\sum_{k=0}^{\infty}\beta^k\mathbb{E}_0\left[d_k\right]\right)}_{\text{fiscal adjustment}} + \underbrace{\sum_{k=0}^{\infty}\tau_y\beta^k\mathbb{E}_0\left[y_k\right]+\frac{D^{ss}}{Y^{ss}}\left(\pi_0-\mathbb{E}_{-1}\left[\pi_0\right]\right)}_{\text{“self financing”}}. \tag{20}$$

The left-hand side of (20) is the initial fiscal deficit—ϵ_0—while the right-hand side contains the three possible sources of fiscal financing in our economy. First, lump-sum taxes may adjust in response to the initial deficit as well as the implied movements in outstanding government debt. This term captures conventional fiscal adjustment—the government actively adjusts its primary surplus to stabilize outstanding government debt. Second, equilibrium movements in output may generate additional tax revenue without the fiscal authority adjusting any of its fiscal instruments. And third, date-0 prices may jump in equilibrium, partially eroding the real value of nominal government debt. We refer to the combination of these two sources of financing—which reflect equilibrium movements in prices and quantities—as “self-financing.” Finally, we emphasize that, because of our assumptions on monetary policy, there is no additional fiscal financing through monetary accommodation.

For future reference, we will write the *share* of equilibrium financing that comes from tax base changes as

$$\nu_y\cdot\epsilon_0\equiv\sum_{k=0}^{\infty}\tau_y\beta^k E_0\left[y_k\right] \tag{21}$$

and similarly for the date-0 price level jump

$$\nu_p\cdot\epsilon_0\equiv\frac{D^{ss}}{Y^{ss}}\left(\pi_0-E_{-1}\left[\pi_0\right]\right) \tag{22}$$

We denote the total share of self-financing of fiscal deficits by

$$\nu\equiv\nu_y+\nu_p \tag{23}$$

We are interested in the determinants of ν; in particular, we wish to understand the conditions under which $\nu=1$—that is, in equilibrium, fiscal deficits finance themselves, without any need for on-equilibrium adjustments from the fiscal authority, as in either the classical FTPL or the “self-sustaining stimulus” discussion of DeLong et al. (2012).

3.2 The self-financing result

This section states our main theoretical result on the possibility of self-financing deficits. An accompanying visual illustration is provided in Figure 1. The remainder of the section then elaborates on

the economic intuition and digs deeper into the role of ω—the deviation from permanent-income behavior—and κ—the degree of nominal rigidity.

Theorem 1. *Consider our OLG-NK environment with $\omega < 1$, $\tau_y > 0$, $\tau_d \in [0,1)$, and the fiscal policy (4) or (5). The self-financing share ν has the following properties.*

1. *It is increasing in the delay of fiscal adjustment (i.e., ν is decreasing in τ_d for the fiscal rule (4) and increasing in H for the fiscal rule (5)).*

2. *There is complete self-financing, $\nu \to 1$—if the tax response is infinitely delayed, i.e., $\tau_d \to 0$ or $H \to \infty$. That is, fiscal adjustment is never necessary in equilibrium. These two limits induce the same equilibrium paths $\{y_t, \pi_t, d_t\}_{t=0}^{\infty}$, and in this common limit self-financing is sufficiently strong to return real government debt to steady state (i.e., $\rho_d \in (0,1)$).*

Theorem 1 is the main result of this paper. Its main implication is that "finite horizons" among households—here encapsulated in the restriction that $\omega < 1$—open the door for fiscal deficits to finance themselves. First, as fiscal adjustment is delayed, the initial fiscal deficit induces a larger and larger Keynesian boom (χ), thus increasing the share of self-financing through higher tax revenue and a larger date-0 price level jump. Second, the limit as either $\tau_d \to 0$ or $H \to \infty$ is one of complete self-financing: output boom and price level jump are precisely large enough to cover the cost of the initial fiscal outlay ϵ_0.

We provide a visual illustration of Theorem 1 in Figure 1.[10] The figure shows the aggregate effects of a deficit shock ϵ_0 under different assumptions on fiscal behavior. The left and middle panels in the top half of the figure begin by showing impulse responses of output y_t and government debt d_t as a function of the fiscal adjustment parameter τ_d in our baseline fiscal rule (4). Consistently with Theorem 1 we see that smaller fiscal response coefficients correspond to larger impact output booms (χ) and more persistent deviations of output and government debt from steady state (larger ρ_d). This boom contributes to financing of the initial deficit ϵ_0 through our two self-financing channels: a tax base expansion and a jump in p_0. The top right panel of the figure then reports this degree of self-financing ν—as well as the split into ν_y and ν_p—as a function of τ_d. We see that ν is decreasing in the strength of fiscal adjustment τ_d, i.e., increasing in the delay of fiscal adjustment. In particular, as τ_d declines towards zero, the degree of self-financing converges to one.

The bottom half of Figure 1 provides a different perspective on the same logic, using instead our non-Markovian fiscal rule (5) in which taxes adjust after some (finite) horizon H to perfectly balance

[10] We only emphasize *qualitative* features of our results here, so we do not in detail discuss the model parameterization; for the purposes of our analysis here, it suffices to note that we set $\omega \approx 0.7$—a meaningful departure from Ricardian equivalence. A more serious quantitative investigation of the likely importance of self-financing is relegated to Section 5.

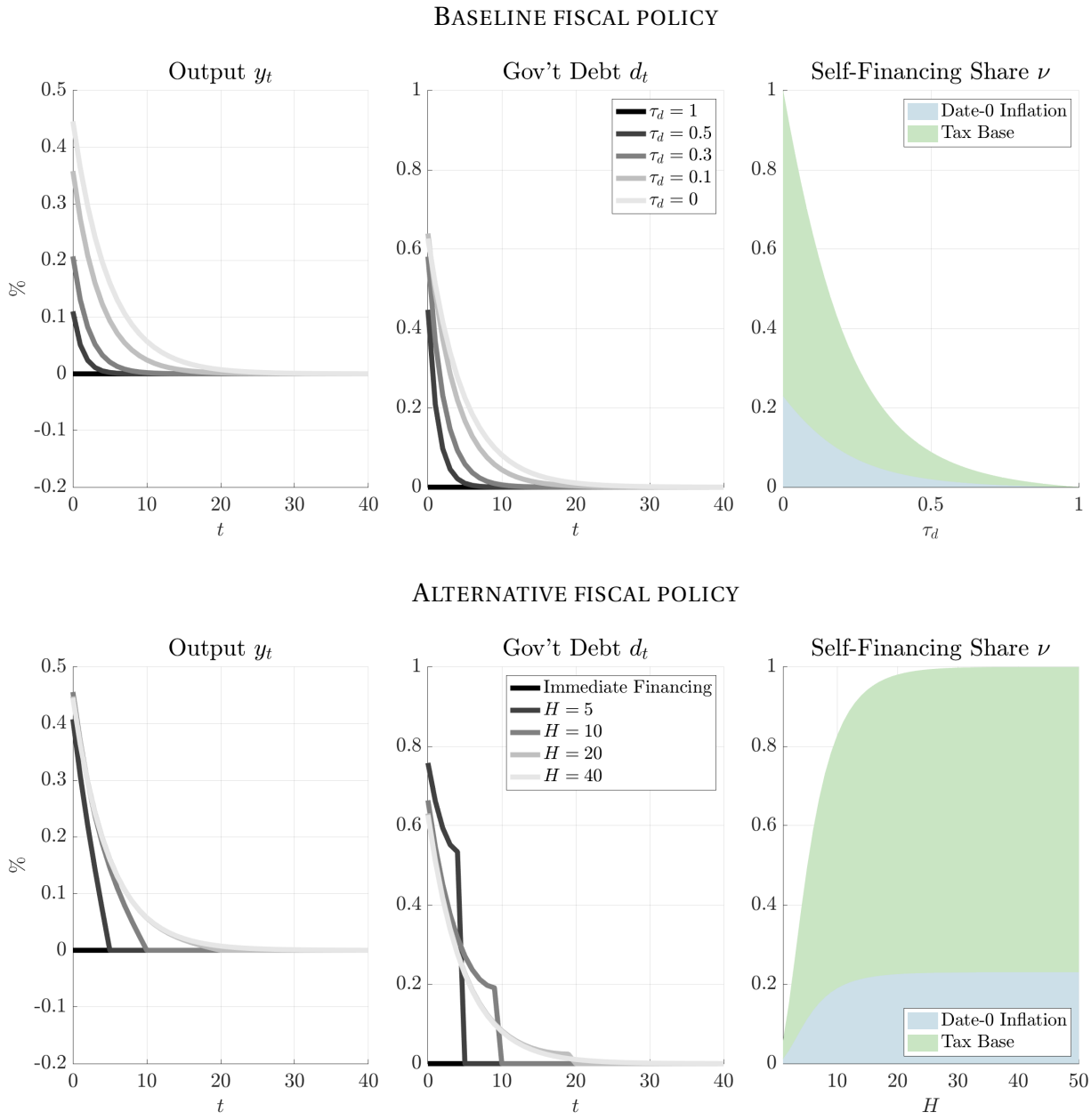


Figure 1: Top panel: impulse responses of output y_t, government debt d_t, and the total self-financing share ν to the deficit shock ϵ_0 as a function of τ_d. Bottom panel: same as above, but as a function of H.

the budget. We see again that ν is increasing in the delay of fiscal adjustment, and in particular again converges to one as fiscal adjustment is delayed further and further ($H \to +\infty$).

In the rest of this section we proceed as follows. First, we provide an economic intuition for how the interaction of finite household horizons (small ω) and delayed fiscal adjustment (small τ_d or large H) generates on-equilibrium self-financing as established in Theorem 1. Second, we elaborate on the role of price stickiness κ, focusing in particular on how it determines the split between nominal and real self-financing. Third, we dig deeper into the role of ω for the speed of convergence to our limiting self-financing equilibria, in particular contrasting our results with the classical FTPL (which assumes $\omega = 1$).

Intuition To develop the intuition underlying Theorem 1, we begin by considering the special case of fully rigid prices ($\kappa = 0$). We return to the general case ($\kappa > 0$) in the next paragraph.

1. *Step 1: PE demand.* We begin by considering the effect of fiscal policy on household demand, prior to any subsequent equilibrium adjustment in prices and quantities. To make the importance of delays in fiscal financing as transparent as possible we consider a fiscal authority following our alternative fiscal rule (5); the discussion for (4) is largely analogous. The fiscal authority pays out a lump-sum transfer to households at date 0 and promises to hike taxes—if needed—at date H to return debt to steady state. If households were Ricardian ($\omega = 1$), then this policy would not at all affect aggregate demand; if households have finite horizons ($\omega < 1$), on the other hand, then such a policy redistributes from future to current generations. We thus in partial equilibrium see positive net excess demand in the short run (at and after date 0) and negative excess demand in the future (around the tax hike date H).

2. *Step 2: GE boom.* How do the partial equilibrium boom-bust dynamics induced by the fiscal experiment translate to general equilibrium outcomes? We begin by considering the initial boom in isolation, and ask how much tax revenue it will generate through a Keynesian cross-style general equilibrium amplification. A simple static Keynesian cross would suggest that

$$y = \frac{\text{MPC}}{1 - \text{MPC} \times (1 - \tau_y)} \times \epsilon \tag{24}$$

   The initial deficit increases demand by MPC$\times\epsilon$; this demand increase then gets amplified through a standard Keynesian multiplier, with each dollar of income generating MPC $\times$ $(1-\tau_y)$ additional dollars of demand. We thus see that, for the boom to be large enough to finance the initial deficit, we would require that MPC = 1, giving $y = \frac{1}{\tau_y} \times \epsilon$. In our dynamic economy, what matters are not such static (or impact) MPCs, but rather cumulative lifetime MPCs: the initial income gain ϵ_0 is invariably spent *at some point*, and similarly every dollar of post-tax income generated in general equilibrium is also ultimately spent again. We thus obtain the following

dynamic generalization of (24):

$$\text{NPV}(\{y_t\}_{t=0}^{\infty}) \equiv \sum_{k=0}^{\infty} \beta^k E_0\left[y_k\right] = \frac{\text{cumulative lifetime MPC}}{1 - \text{cumulative lifetime MPC} \times (1-\tau_y)} \times \epsilon_0 = \frac{1}{\tau_y} \times \epsilon_0 \tag{25}$$

where the second equality follows because cumulative lifetime MPC = 1 necessarily, as every dollar of income is always spent at some point.[11] It thus follows that, after all general equilibrium amplification, the initial income ϵ_0 inevitably generates an output response with discounted present value of $\frac{1}{\tau_y} \times \epsilon_0$—precisely enough to finance the initial fiscal outlay.

The same logic, however, also necessarily applies in reverse: the future tax hike leads to a partial equilibrium demand bust, and this bust will necessarily induce an output contraction with net present value equal to $-\frac{1}{\tau_y} \times \epsilon_0$. The key twist, however, is that this future tax hike only happens in equilibrium if necessary, i.e., government debt d_H remains elevated at date H. This opens the door for our self-financing result: if the initial Keynesian boom is *quick enough* to generate the required tax revenue of prior to date H, then the future tax hike turns out to not be necessary in equilibrium. In our OLG environment this condition on the speed of the Keynesian boom is indeed satisfied for any $\omega < 1$: by the Keynesian cross relationship (12), households spend the initial income quickly and do not respond to any possible future tax movements. We see this visually in the bottom panel of Figure 1: for $H = 40$, the Keynesian boom has already completely played out and debt has returned to steady state before the future tax hike was ever necessary.

Our analysis in Section 4.1 generalizes the self-financing result to richer aggregate demand structures. This generalization will identify sufficient conditions in line with the intuition offered here—conditions that ensure (i) deficits inducing a partial equilibrium boom, and (ii) this boom in general equilibrium being large enough and fast enough to raise the required tax revenue prior to the promised future tax hike.

The role of nominal rigidity The intuition offered in the previous paragraph considered the special case of rigid prices, but extends with very little change to the case of partially sticky prices. If prices are only somewhat sticky, then—through the NKPC (13)—any output boom also invariably increases prices. If $D^{ss}/Y^{ss} > 0$, such a date-0 price level jump redistributes from households to the governments, thus helping self-finance the deficit shock ϵ_0 (as ν_p defined in (22)) and dampening the Keynesian demand boom by reducing household wealth. In fact, through NKPC (13), two self-financing channels (tax base adjustment ν_y in (21) and date-0 price level jump ν_p in (22)) are proportional to each other :

$$\frac{D^{ss}}{Y^{ss}}\left(\pi_0 - E_{-1}\left[\pi_0\right]\right) = \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y} \sum_{k=0}^{+\infty} \tau_y \beta^k E_0\left[y_k\right] \Longrightarrow \nu_p = \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y} \nu_y. \tag{26}$$

[11] This property comes from consumer's intertemporal budget. That is, the first row of the Intertemporal Keynesian Cross matrix in Auclert et al. (2018) is always 1.

As a result, moving from $\kappa = 0$ to $\kappa > 0$ simply rescales the economy: the output path is scaled down while the implied date-0 price jump contributes to financing of the fiscal deficit.

Proposition 2 summarizes the implications of these observations for the total amount of fiscal deficits self-financing ν as well as its split into ν_p and ν_y.

Proposition 2. *Consider our OLG-NK environment with $\omega < 1$, $\tau_y > 0$, $\tau_d \in [0,1)$, and the fiscal policy (4) or (5). The self-financing share ν has the following properties.*

1. *The relative shares of self-financing coming through tax base expansion and the date-0 price level jump are given as*
$$\nu_y = \frac{\tau_y}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \times \nu \quad \textit{and} \quad \nu_p = \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \times \nu. \tag{27}$$
2. *The total self-financing share ν is increasing in the degree of price flexibility, with $\lim_{\kappa \to \infty} \nu = 1$ for any given τ_d or H. Real self-financing ν_y is decreasing in κ, while nominal self-financing ν_p is increasing in κ.*

Price stickiness does not affect our limiting self-financing result; rather, its chief effect is to govern the *split* between nominal and real sources of self-financing. If prices are fully rigid, then all self-financing occurs through an expansion in the tax base, with a cumulative fiscal multiplier of $1/\tau_y$; if instead are prices are approximately flexible, then financing occurs almost fully through a date-0 jump in the price level. In this case, an infinitesimal Keynesian boom leads to large enough adjustment in date-0 price level to finance the deficit shock. This is observationally equivalent to the traditional FTPL, but based on a different logic. The self-financing comes from the deficit-driven Keynesian boom instead of equilibrium selection. In general, for intermediate values of κ, the deficit is financed through both real and nominal channels.

A visual illustration of all of these effects is provided in the top panel of Figure 2—a variant of Figure 1 but with materially less rigid prices. First, we see that the share of self-financing still converges to one, just now with a different split between ν_p and ν_y. Second, for any given fiscal financing delay (here H), ν is strictly larger than before, because date-0 price level jump also helping self-finance the deficit.

The role of ω Finally we consider the importance of deviations from permanent-income behavior—i.e., the role of $\omega < 1$. Proposition 3 presents comparative statics of ν in ω, paying particular attention to what happens as household behavior become increasingly close to the Ricardian Equivalence/Permanent Income Hypothesis benchmark (i.e., $\omega \to 1$).

Proposition 3. *Consider our OLG-NK environment with $\omega < 1$, $\tau_y > 0$, $\tau_d \in [0,1)$, and the fiscal policy (4) or (5). The self-financing share ν has the following properties.*

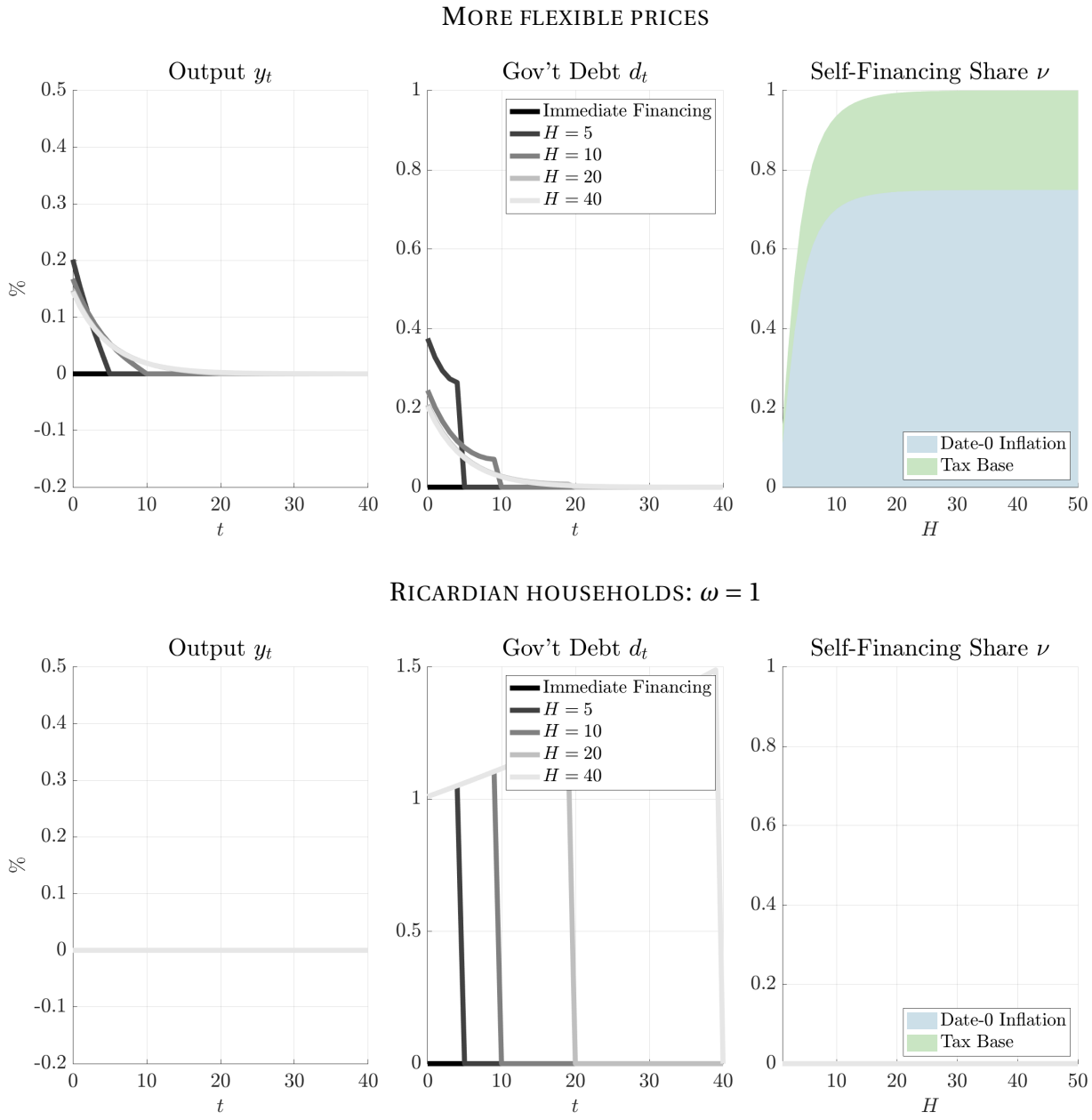


Figure 2: Top panel: impulse responses of output y_t, government debt d_t, and the total self-financing share ν to the deficit shock ϵ_0 as a function of H in an economy with relatively flexible prices. Bottom panel: same as above, but with $\omega = 1$.

1. *It is decreases in the horizon of households (i.e., decreasing in ω).*

2. *The persistence of government debt ρ_d in the complete self-financing equilibrium (with $\tau_d \to 0$ or $H \to \infty$) is increasing in ω, with $\lim_{\omega \to 1} \rho_d = 1$.*

The intuition for the comparative statics of Proposition 3 is straightforward: the smaller ω, the quicker and larger the initial Keynesian boom, and so the larger v (for any τ_d or H) and the smaller ρ_d in the limiting self-financing equilibrium.

A special case of particular interest is the limit as $\omega \to 1$. For any $\omega < 1$ a self-financing equilibrium still exists (by Theorem 1), but the implied ρ_d gets larger and larger, and so self-financing requires more and more delay in fiscal adjustment. The bottom panel of Figure 2 provides a visual illustration of the limit case of $\omega = 1$—that is, infinite-lived households, as notably assumed in the conventional FTPL. For any *finite* H and with $\omega = 1$, output never responds and the share of self-financing is zero.[12] Intuitively, for any finite H, the PE step discussed following Theorem 1 never gets off the ground, and so equilibrium output and inflation never respond. The conventional FTPL opens the door of self-financing by committing to never have fiscal adjustment ($\tau_d = 0$, or equivalently, fiscal adjustment that is infinitely delayed $H = \infty$) and selecting a non-fundamental equilibrium. Moving from the conventional FTPL ($\omega = 1$) to our case ($\omega < 1$) thus has important conceptual advantages: there is no doubt that government budget constraint holds on- and off-equilibrium (Kocherlakota and Phelan, 1999), and the possibility of self-financing ceases to be related to questions of equilibrium selection (Angeletos and Lian, 2023).

The discussion of the limiting case $\omega \to 1$ suggests that the practical relevance of our theory is ultimately a quantitative question. For ω below but close to one, meaningful self-financing may obtain only with implausibly long delays in fiscal financing. We tackle this question in Section 5, where we compute the on-equilibrium degree of self-financing under empirically relevant assumptions on (i) delays in fiscal financing (τ_d and H) and (ii) household departures from permanent-income behavior (ω). Before doing so, however, we first in Section 4 further theoretically investigate the generality of our self-financing results.

4 Extensions

This section discusses five extensions of our self-financing result: the first two generalizing the model and the last three considering alternative specifications of fiscal and monetary policy.

[12] In constructing the bottom panel of Figure 2 we face a challenge: under our assumptions on fiscal and monetary policy there exist multiple bounded equilibria. We proceed with a particular equilibrium selection—requiring continuity at the determinacy boundary for a monetary policy rule $i_t = \phi_\pi \mathbb{E}_t[\pi_{t+1}]$ with $\phi_\pi \to 1^+$.

First, in Section 4.1, we consider a much more general aggregate demand relation that nests—but goes materially beyond—our baseline OLG environment. Second, in Section 4.2, we discuss a model extension featuring investment. Third, in Section 4.3, we consider more general monetary policy rules, allowing the monetary authority to either accommodate or lean against the deficit-induced boom. Finally, in Section 4.4, we allow for distortionary tax financing. Finally, in Section 4.5, we show that our results extend without any change to fiscal stimulus in the form of government purchases rather than lump-sum transfers.

4.1 A more general aggregate demand relation

In this section we extend our self-financing result to much more general aggregate demand relations. The purpose of this extension is twofold. First, and more obviously, it illustrates the generality of our result. Second, our most general conditions offer additional important insights into the economics of self-financing.

Generalizing the demand block Recall that, in our baseline OLG environment, aggregate consumer demand as a function of current wealth d_t as well as current and future income and taxes $y_t - t_t$ was given as[13]

$$c_t = (1-\beta\omega)\left(d_t + \mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k\left(y_{t+k}-t_{t+k}\right)\right]\right). \tag{28}$$

This form of the aggregate demand relation embeds economically meaningful restrictions on consumer behavior: the MPC out of current income and wealth is the same (equal to $1-\beta\omega$), and MPCs out of the discounted presented value of future income decline at a constant rate ω. We consider a generalized aggregate demand relationship that relaxes these constraints:

$$c_t = M_d \cdot d_t + M_y \cdot \left((y_t - t_t) + \mathbb{E}_t\left[\sum_{k=1}^{\infty}\delta_k\beta^k\left(y_{t+k}-t_{t+k}\right)\right]\right). \tag{29}$$

(29) allows for different MPCs out of current income ($M_y \in (0,1]$) and wealth ($M_d \leq M_y \in [0,1]$) as well as general discounting ($\delta_k \in [0,1]$) for MPCs out of future income gains. We will give examples of several familiar models that are consistent with this general structure.

A general sufficient condition for self-financing We will now revisit our self-financing result in the baseline model of Section 2 but with the generalized demand relation (29) replacing our simpler OLG demand block. It turns out that self-financing obtains under two key restrictions on (29). We first state the assumptions, then present the generalized self-financing result, and finally discuss the economic content of the assumptions.

[13] (28) just re-states (9) but without the expected interest rate term, as we impose (6).

Assumption 1. *There exists* $\bar{\delta} \in (0,1)$ *such that* $0 \leq \max\left\{\delta_k, \frac{M_y}{M_d}\delta_k\right\} < \bar{\delta}$ *for all* $k \geq 1$.[14]

In words, MPCs out of future income relative to MPCs out of current income and wealth decline strictly faster than the rate of interest β. This condition guarantees that Ricardian Equivalence fails and debt and deficit exert positive PE effect on aggregate demand.

Assumption 2. *MPCs out of current and future income as well as wealth are large enough, in the precise sense that*

$$M_d + \frac{1-\beta}{\tau_y}\left(1-\tau_y\right) M_y \left(1 + \sum_{k=1}^{\infty} \delta_k \beta^k\right) > \frac{1-\beta}{\tau_y}. \tag{30}$$

(30) is a restriction on the various MPCs in (29), and will tend to be satisfied if those MPCs are "large." This assumption makes sure that the deficit-driven Keynesian Boom is large enough to play out in finite time.

Theorem 2. *Consider the model of Section 2, with the consumer demand block taking the generalized form (29). Suppose that Assumptions 1 and 1 hold, and consider a fiscal policy that delays fiscal adjustment (i.e.,* $\tau_d \to 0$ *or* $H \to \infty$*). Then:*

1. *There is complete self-financing and fiscal adjustment is never necessary in equilibrium (i.e.,* $\nu \to 1$*). Government debt automatically returns to steady state (*$d_t \to 0$*).*

2. *The financing composition of the limiting self-financing equilibrium is exactly as in Section 3:*

$$\nu_y = \frac{\tau_y}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}}, \quad \nu_p = \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \tag{31}$$

Theorem 2 reveals that, under Assumptions 1 and 2, our self-financing result extends, with the exact same split between nominal and real self-financing as before. The two assumptions on consumer spending behavior play different roles in ensuring that the result extends, echoing the two steps of our intuition in Section 3.2. First, Assumption 1—additional discounting of future income and thus future taxes relative to current income and wealth—ensures that the "partial equilibrium" demand step gets off the ground. Prior to any general equilibrium price adjustment, households receive income today and pay more taxes in the future; with the demand structure (29) such a policy stimulates demand on impact if and only if households additionally discount the future tax burden. Second, Assumption 2 ensures that the Keynesian boom happens quickly enough, echoing our "general equilibrium" discussion in Section 3.2. If the condition is satisfied, then the initial boom raises the required revenue ϵ_0 quickly enough so that $d_t \to 0$ on its own; if so, and if τ_d is small enough or H is large enough, then

[14] Since we restrict $M_y \geq M_d$, Assumption (1) reduces to $0 \leq \frac{M_y}{M_d}\delta_k < \bar{\delta}$ for all k.

fiscal adjustment turns out to never be necessary in equilibrium, thus delivering Theorem 2.[15] In the remainder of this section we will discuss examples of specific structural models of household demand that fit into the general form (29) and either satisfy or violate our two key conditions in Assumptions 1 and 2.

What environments are consistent with self-financing? Our generalized aggregate demand block (29) is consistent with many familiar models of household consumption-savings decisions.

To begin, the canonical permanent-income model readily fits into our generalized aggregate demand structure with $M_d = M_y = 1-\beta$ and $\delta_k = 1$ for all k. It is immediate that both Assumptions 1 and 2 are violated: a transfer-and-tax policy induces no partial equilibrium demand boom, and any general equilibrium boom is not "quick" because permanent-income households postpone part of their spending into the infinite future, violating (30).

A second important model consistent with (29) is the classical spender-saver model (Campbell and Mankiw, 1989), with $M_y = \mu + (1-\mu)(1-\beta)$, $M_d = (1-\beta)$ and $\delta_k = \frac{(1-\mu)(1-\beta)}{\mu+(1-\mu)(1-\beta)}$, where $\mu \in (0,1)$ denotes the fraction of spenders. This model satisfies Assumption 1: at date 0 spenders do not respond to the future tax, so in partial equilibrium the fiscal stimulus policy does indeed lead to an initial boom. Assumption 2, however, is violated: exactly as in the permanent-income model the presence of savers means that self-financing through the Keynesian boom is never fast enough. The top panel of Figure 3 provides a graphical illustration: in a spender-saver model, for any finite H, the self-financing share is zero, with the initial boom exactly offset (in net present value terms) by an increasingly large subsequent bust. The second panel of Figure 3 provides a different perspective on the same logic, showing results in a hybrid populated by spenders (as in Campbell and Mankiw, 1989) and savers that are subject to mortality risk, as in our OLG baseline. Qualitatively we see the same pattern as before: an initial boom followed by a subsequent bust. Crucially, however, as the financing horizon is increased, the subsequent bust becomes smaller and smaller—intuitively, with savers having finite horizons, the Keynesian boom is again quick, so our self-financing logic again works out, with the delayed on-equilibrium tax hike getting smaller and smaller.

It is straightforward to see that the argument in the previous paragraph in fact extends beyond spender-saver models to any environment that contains a margin of classical permanent-income savers. The intuition is simple: those savers provide the "connection at infinity" that we need to break for our self-financing result to go through. In our view this is a feature and not a bug of our model: classical permanent-income savers correspond to an unrealistic infinite-horizon, infinite-liquidity,

[15] Appendix A.4—which discusses our results from a sequence-space perspective—provides a different perspective on Assumption 2. This assumption implies that the off-diagonal entries of the intertemporal MPC matrix of our model decay to zero sufficiently quickly—that is, income gained at some period i is not spent at period j if i and j are sufficiently far apart. This condition is precisely what is needed to ensure a "quick" Keynesian boom.

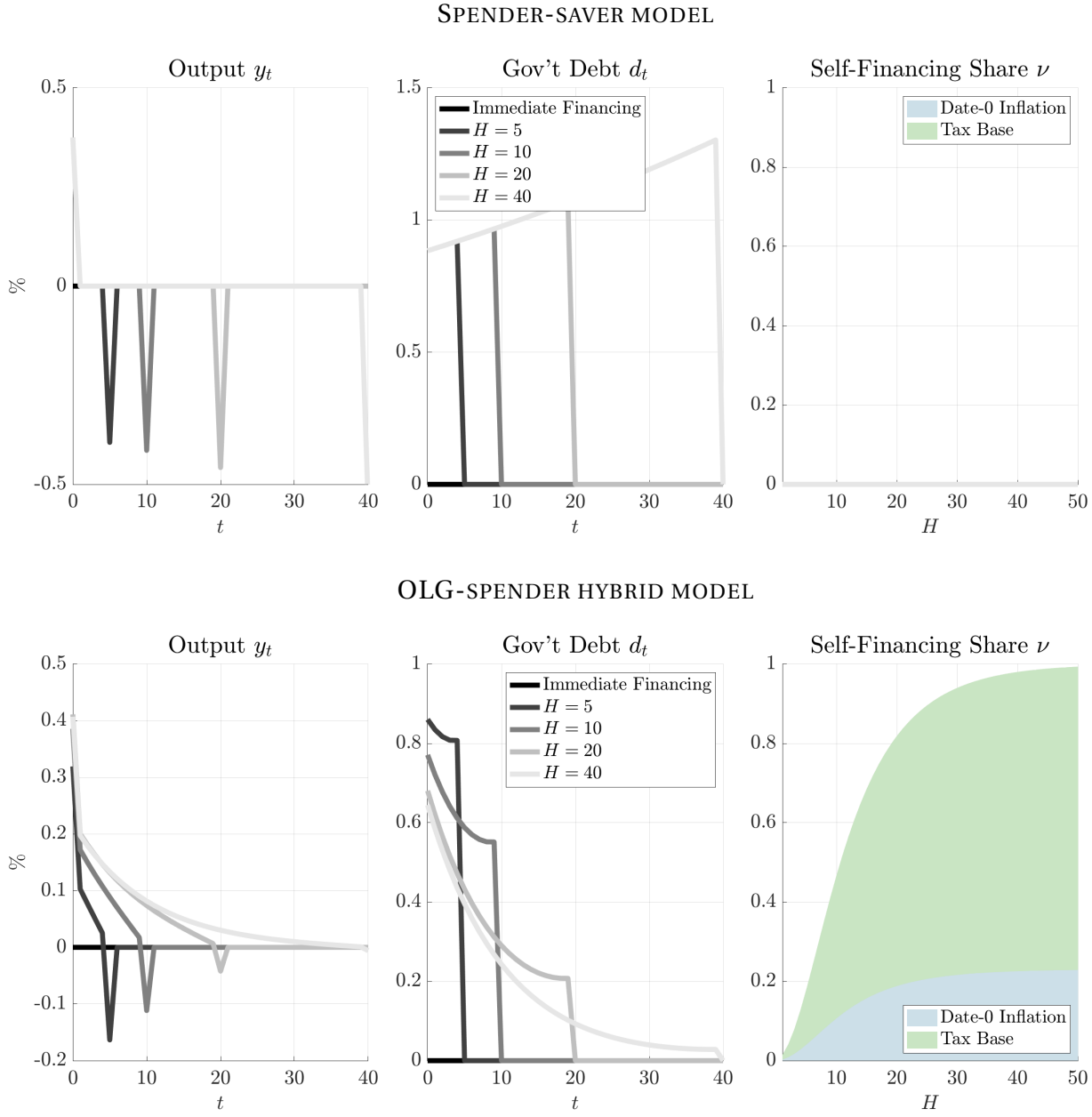


Figure 3: Top panel: impulse responses of output y_t, government debt d_t, and the total self-financing share ν to the deficit shock ϵ_0 as a function of H in a spender-saver economy. Bottom panel: same as above, but for a spender-OLG hybrid economy.

and infinite-rationality limit, implying for example that the elasticity of household asset demand is infinite—a prediction of the model that is clearly at odds with data (Moll et al., 2022). Finite horizons (as in our baseline model), wealth in the utility function of savers (Michaillat and Saez, 2021), or binding household liquidity constraints (in HANK) all break this unrealistic model feature and return us to our self-financing result. We provide a further numerical illustration of this observation in Appendix C.4, where we solve a quantitative HANK model and show that $\nu \to 1$ as fiscal financing is delayed further and further.[16]

Finally, we emphasize that our general demand block (29) also nests popular behavioral models of consumer spending behavior (see Appendix B.1), including in particular models with cognitive discounting (Gabaix, 2020). Behavioral frictions of this sort lead to additional discounting of future income changes, thus lowering the δ_k's in our general relation (29). This has two interesting and partially offsetting effects. First, the initial partial equilibrium demand boom will be amplified: the future tax hike is discounted even further, so the initial boom is larger. Second, the intertemporal Keynesian cross is weakened, as future gains in income do not feed back as strongly to today's consumer spending. We can see this in Assumption 2: smaller δ_k's decrease the right-hand side, reflecting a Keynesian boom that is less front-loaded. We provide a quantitative investigation of these offsetting effects in Section 5.2 and Appendix C.5.

4.2 Investment

For our second extension we consider a substantially generalized model that also features firm investment. Our main result is that our self-financing logic extends almost without change to this case. We provide a sketch of the argument here, with details provided in Appendix B.2.

We enrich the supply side of our model. Firms now produce using labor and capital, owned and accumulated by the firm itself. As before firms are subject to standard nominal rigidities and pay out dividends to households. For our baseline results we assume that policy is specified exactly as in Section 2: the monetary authority fixes the expected real rate of interest while the fiscal authority taxes labor income and dividends, pays out transfers, and adjusts lump-sum taxes to balance the government budget as needed.

Our main insight is that the self-financing result of Theorem 1 extends with almost no change to this generalized environment. The logic of the argument is again seen easiest in the rigid-price case. Households consume out of labor plus dividend income less taxes, and the automatic fiscal tax adjustment τ_y is by assumption levied only on exactly this income. Furthermore, in equilibrium labor

[16] The argument for bond-in-utility models is straightforward in light of the equivalence between such models and OLG models proved in Wolf (2021a). The detailed analysis is omitted for brevity.

plus dividend income equals total consumption. The aggregate household demand relation (12) and the government debt equation (14) thus still constitute a bivariate system, just now in $\{c_t, d_t\}_{t=0}^{\infty}$ rather than $\{y_t, d_t\}_{t=0}^{\infty}$. Intuitively, with expected real interest rates fixed, the consumer and fiscal blocks of the model still induce a Keynesian cross relation, and so our equilibrium characterization in Section 3.2 applies completely unchanged, including in particular Theorem 1. Finally, partially sticky prices only somewhat complicate matters: the production block of the model now implies that the mapping from consumption as implied by the Keynesian cross into date-0 inflation is more complicated; conditional on this mapping, however, the same fixed-point logic as discussed in Section 3.2 applies, with inflation affecting the split into nominal and real self-financing, but without any effect on the overall self-financing limit.

4.3 Monetary accommodation

We now return to the environment of Section 2 but generalize our assumptions on policy. In this section we allow for more general monetary policy rules, taking the form

$$i_t = \mathbb{E}_t[\pi_{t+1}] + \phi y_t \tag{32}$$

Recall that our baseline monetary policy rule corresponds to the special case $\phi = 0$; we will now consider what happens if either $\phi < 0$ (i.e., monetary accommodation) or $\phi > 0$ (i.e., a monetary authority leaning against the deficit-induced boom). Intuitively, we would expect $\phi \neq 0$ to interact with the general equilibrium step underlying our self-financing intuition: $\phi < 0$ pulls spending forward in time and thus accelerates the deficit-driven boom even further, while $\phi > 0$ delays the boom and self-financing logic.[17] The remainder of this section formalizes these observations, proceeding in two steps. First, we provide a discussion of equilibrium determinacy, closely echoing and generalizing Leeper (1991). Second, we present a generalized self-financing result.

Determinacy regions We begin by providing a visual illustration of equilibrium determinacy—i.e., the famous Leeper (1991) regions—in our OLG model. The two panels in Figure 4 show whether a bounded equilibrium (i.e., the standard solution concept of Blanchard and Kahn (1980)) exists and is unique under different assumptions on monetary policy (ϕ) and fiscal policy (τ_d in (4)), for a standard permanent-income model (left panel) and our OLG economy (right panel).

The figure reveals that the determinacy properties of the two economies are materially different. Results for the permanent-income model are well-known and require little exploration: equilibrium

[17] We work with a "real" Taylor rule in (32) for expositional simplicity, but we find that results here hold for more traditional "nominal" Taylor rule too, i.e., $i_t = \phi_\pi \pi_t$. The traditional "active" monetary policy $\phi_\pi > 1$ corresponds to $\phi > 0$ in (32), i.e., pro-cyclical expected real rates. The traditional "passive" monetary policy $\phi_\pi < 1$ corresponds to $\phi < 0$ in (32), i.e., counter-cyclical expected real rates.

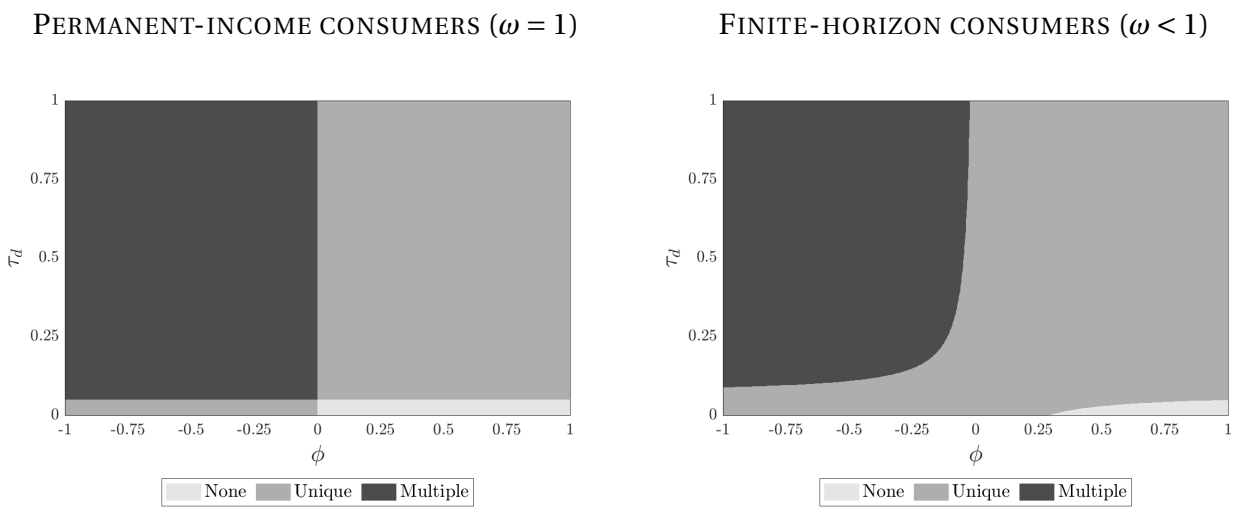


Figure 4: Determinacy regions for $\omega = 1$ (left panel) and $\omega < 1$ (right panel) as a function of the fiscal and monetary policy rule coefficients τ_d (y-axis) and ϕ (x-axis).

uniqueness requires that fiscal policy is passive ($\tau_d > 1 - \beta$) and monetary policy is active ($\phi > 0$), or vice-versa; if both rules are active then no bounded equilibrium exists, and if both are passive then there are multiple bounded equilibria. With finite horizons on the household side (i.e., $\omega < 1$), the regions of equilibrium determinacy look rather different. Perhaps most importantly, the benchmark monetary rule of $\phi = 0$ now induces determinate equilibria for any τ_d, consistent with Proposition 1. Intuitively, since deficits with $\omega < 1$ directly enter the aggregate demand relation, self-financing is now strong enough to pull debt as well as spending towards zero, even if interest rates do not provide any further Euler equation tilting. This automatic stabilization of government debt also shrinks the equilibrium non-existence region in the bottom right corner of the figure.

In the remainder of this section we will mostly focus on the bottom of the figure, studying equilibria in the case of a weak fiscal response—i.e., the case relevant for our focus on self-financing equilibria. A more detailed discussion of the equilibrium determinacy regions is provided in Appendix B.3.

On the possibility of self-financing equilibria Our main result is that self-financing continues to be possible in equilibrium if the monetary authority does not lean against the fiscal "too aggressively." We first state the formal result and then discuss its intuition.

Theorem 3. *Consider our OLG-NK environment with $\omega < 1$, $\tau_y > 1$, $\tau_d \in [0, 1)$, the fiscal policy (4), and the monetary policy rule (32). There exists a $\bar{\phi} > 0$ such that:*

1. *If $\phi < \bar{\phi}$, there exists a bounded, complete self-financing equilibrium of the form (16) with $\tau_d \to 0$, $\nu \to 1$, and $\rho_d \in (0, 1)$.*

2. *If $\phi > \bar{\phi}$, there does not exist a complete, self-financing bounded equilibrium. Furthermore, when and only when τ_d is sufficiently large (i.e., fiscal adjustment needs to be sufficiently fast), any kind of bounded equilibrium to exist.*

The economic intuition underlying Theorem 3 is straightforward. Consider first the case of monetary accommodation (i.e., $\phi < 0$). As before, the fiscal deficit induces a demand boom. In response to this boom the monetary authority now cuts expected real rates, bringing household spending forward in time even further. The Keynesian boom is thus even quicker and debt is even less persistent—i.e., a smaller ρ_d than in our baseline self-financing equilibrium of Theorem 1. It follows that, with monetary accommodation, even less of a delay in the promised fiscal adjustment is needed to ensure material self-financing. Next consider a monetary authority leaning against the deficit-induced boom (i.e., $\phi > 0$). In that case, higher real rates result in households postponing spending, thus delaying the Keynesian boom—i.e., a larger ρ_d. The cutoff $\bar{\phi}$ is exactly the point where the monetary policy-induced delay exactly offsets the "quick" boom implied by finite household horizons, delivering $\rho_d(\bar{\phi}) = 1$. For any strictly more aggressive monetary policy the equilibrium explodes, and so no self-financing equilibrium exists.

The previous discussion suggests an intimate connection between our results here and those for a more general aggregate demand relation in Section 4.1. Intuitively, both an aggressive monetary authority as well as low MPCs can result in demand being delayed sufficiently to not deliver a Keynesian cross that generates the required fiscal revenue quickly enough. This intuition turns out to be correct: as we show formally in Appendix B.3, the cutoff $\bar{\phi}$ can equivalently be interpreted as being the value that delivers spending sufficiently postponed to violate our most general "GE" condition in Assumption 2.

4.4 Distortionary taxation

Our final model variant is one in which fiscal adjustment instead relies on distortionary taxation. Our results turn out to extend with almost no change to this case; we only sketch the argument here, with details relegated to Appendix B.4.

Model sketch We consider an extended environment in which the deficit shock ϵ_t is still paid out as a lump-sum transfer to households ("stimulus checks"), but then the distortionary tax τ_y adjusts over time to balance the government budget. Specifically, total taxes are now given as

$$T_t = \tau_{y,t} Y_t + \bar{T} - \mathscr{E}_t \tag{33}$$

where the distortionary tax rate $\tau_{y,t}$ is increasing in the fiscal deficit,

$$\tau_{y,t} = \tau_y + \tau_{d,t}(D_t - D^{ss}). \tag{34}$$

Note that the response coefficient $\tau_{d,t}$ is allowed to depend on time, nesting both kinds of fiscal rules considered in our original analysis. Relative to our baseline model, the *only* effect of this alternative fiscal financing rule is to introduce a time-varying wedge in the log-linearized aggregate NKPC:

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t[\pi_{t+1}] + \zeta_t d_t \tag{35}$$

where ζ_t is a function of model primitives and such that $\zeta_t > 0$ if $\tau_{d,t} > 0$—intuitively, higher deficits map into higher distortionary taxes, introducing a labor wedge.

A generalized self-financing result Our limiting self-financing results turn out to extend with no change to this environment. The underlying logic is closely echoes the proof of Theorem 1: as financing is delayed, the initial boom generates enough revenue to finance the initial deficit ϵ_0, so no on-equilibrium tax adjustment is needed. It thus in particular does not matter whether this fiscal adjustment (which never happens) *would* have been distortionary or lump-sum.

4.5 Government spending

While we have so far considered lump-sum transfers to households as an example of a stimulative fiscal deficit policy, we emphasize that our results extend with very little change to deficit-financed government purchases. We sketch our self-financing results for government purchases here, with details as well as a numerical illustration relegated to Appendix B.5.

Policy experiment The only change relative to our baseline economy is that the government now also itself consumes some amount of the final good G_t. We assume that $G_t = \mathcal{G}_t$, where $\mathcal{G}_t$ is a stochastic spending shock. The linearized government budget constraint becomes

$$d_{t+1} = \frac{1}{\beta}\left[d_t - t_t + g_t + \beta \frac{D^{ss}}{Y^{ss}}(i_t - \pi_{t+1})\right] \tag{36}$$

while we consider linearized fiscal rules of the form

$$t_t = \tau_{d,t} \cdot (d_t + (1-\tau_y)g_t) + \tau_y y_t. \tag{37}$$

The presence of $(1-\tau_y)$ in the adjustment term ensures that $\tau_d = 1$ corresponds to immediate and complete tax financing. Note that, since $\tau_{d,t}$ here is allowed to depend on time, this specification again nests both of our fiscal adjustment rules. Finally, the aggregate output market-clearing condition is replaced by

$$y_t = c_t + g_t \tag{38}$$

Self-financing government purchases We obtain a result analogous to Theorem 1: as fiscal financing is delayed it turns out to not be necessary in equilibrium. In particular, if prices are rigid, then the cumulative fiscal multiplier is again equal to $1/\tau_y$. The only difference is that, even for an immediately tax-financed spending increase, aggregate output responds, and thus the share of self-financing is not zero. This immediate tax-financed fiscal multiplier is in fact already well-known to be equal to one (Woodford, 2011; Auclert et al., 2018), so in that case the share of tax base self-financing is $\nu_y = \tau_y$.

5 Quantitative analysis

Having established the generality of our theory of self-financing deficits, we now ascertain its empirical relevance. Specifically, we in this section ask: for empirically realistic delays in fiscal financing and for plausible departures from permanent-income behavior among households, how important is the self-financing margin likely to be? Our analysis proceeds in two steps. First, in Section 5.1, we first present and then calibrate a quantitatively relevant extension of our baseline model—a model rich enough to speak to empirical evidence on household consumption behavior, yet simple enough to still be consistent with our general demand block studied in Section 4.1. Second, in Section 5.2, we use the model to compute the self-financing share ν under different assumptions on delays on fiscal financing.

5.1 Model and calibration strategy

Our analysis in Section 3 has revealed that the equilibrium self-financing share is largely governed by two model features: household consumption behavior and the fiscal financing rule. In this section we review empirical evidence on these two key ingredients and present a model that is consistent with that evidence.

Environment: spender-OLG hybrid We begin with a sketch of the model environment. We consider a variant of our model in Section 2 with the baseline fiscal rule (4), but with one twist: we generalize the household block to consist of a mix of overlapping-generations households (as before) and a residual margin $\mu \in (0,1)$ of fully hand-to-mouth spenders, as already very briefly discussed in Section 4.1. Both groups receive labor and dividend income and pay taxes, but only the OLG block holds government bonds. Further details are presented in Appendix C.1.

A hybrid spender-OLG model is the ideal environment for our quantitative analysis, for two reasons. First, it remains simple enough to fit into our generalized aggregate demand relationship discussed in Section 4.1. As a result, we will later be able to invoke Theorem 2 to verify whether or not our calibrated model admits (limiting) full self-financing. Second, it is still rich enough to very closely

Parameter	Description	Value	Target	Value
Consumer spending				
μ	Share of HtM	0.17	Average MPC	0.30
ω	OLG survival rate	0.85	MPC slope	0.85
Fiscal adjustment				
τ_d	Tax feedback	$\{0.09, 0.04, 0.03\}$	Literature, see text	
Rest of the model				
σ	EIS	1	Standard	
β	Discount factor	0.99	Real rate	0.01
τ_y	Tax rate	0.3	Average Labor Tax	0.30
D^{ss}/Y^{ss}	Gov't debt	1.04	Liq. wealth holdings	1.04
κ	NKPC slope	0.1	Standard	

Table 1: Hybrid OLG-spender model, calibration.

agree with quantitative HANK-type models on their implied intertemporal marginal propensities to consume (Auclert et al., 2018; Wolf, 2021a). Since those MPCs are all that matters for aggregate equilibrium dynamics (see Appendix A.4 for the formal discussion), it follows that our model is quantitatively relevant in the particular sense of agreeing with state-of-the-art quantitative models of the household consumption-savings problem.[18]

All parameter values for our spender-OLG hybrid model are displayed in Table 1. The calibration strategy used to arrive at those parameter values is discussed in the next subsection.

Calibration We discuss the model calibration in three steps: evidence on household consumption behavior to pin down the spender share μ and the OLG coefficient ω; evidence on fiscal adjustment to pin down the tax response coefficient τ_d; and all other parts of the model.

1. *Consumer spending behavior.* Empirical evidence on the household-level consumption response to income gains suggests two salient features of consumer spending behavior (Fagereng et al., 2021). First, the average MPC out of income gains is elevated, with a standard quarterly value of around 0.30. Second, the income gain is spent gradually. Our baseline OLG environment of Section 2 provides a tight joint restriction on the level of the MPC and its dynamics: the level MPC is $1-\beta\omega$ and slope of the spending profile is ω. This model-implied connection between

[18] We note that the same is not true for our baseline OLG model. In that model, the coefficient ω governs both the average MPC as well as the time profile of how quickly income is spent. This restriction is inconsistent with empirical evidence on consumer spending behavior (see the discussion on calibration below). Allowing for spenders disentangles MPCs and dynamic spending profiles.

level and slope is, unfortunately, inconsistent with the data; in particular, relative to the initial impact MPC, income in the data is subsequently spent more quickly than predicted by the theoretical OLG-implied relationship. The spender-OLG hybrid model instead allows us disentangle the level of the MPC and its slope in a way that is consistent with empirical evidence on household-level spending behavior. Our empirical targets as well as the implied model parameters are displayed in the first part of Table 1, and a visual illustration of the implied intertemporal marginal propensities to consume is provided in Appendix C.1.[19]

Current and future MPCs out of today's income gains—i.e., the estimand of Fagereng et al. (2021)—is of course not all that matters for our theory; in particular, for our general equilibrium Keynesian cross, it is similarly important how anticipated income changes in the (far) future affect spending today. Given the lack of empirical evidence on such responses to (far-away) income news shocks, our baseline exercise simply extrapolates these news shock spending responses through the structure of the model. Encouragingly, as discussed in Wolf (2021a), our spender-OLG model extrapolates in a way very similar to state-of-the-art quantitative HANK-type models.[20] Nevertheless, in Section 5.2 and in particular Appendix C.5 we also consider how results change with additional cognitive discounting among households, consistent with suggestive evidence from Ganong and Noel (2019).

2. *Delays in fiscal financing.* For our quantitative analysis we restrict attention to the fiscal rule (4), consistent with the seminal analysis of Leeper (1991). Prior work has estimated fiscal rules of this sort and thus in particular is informative about empirically relevant values of the speed of fiscal adjustment τ_d. For our quantitative analysis we consider three such studies (Galí et al., 2007; Bianchi and Melosi, 2017; Auclert and Rognlie, 2018) with three implied values for τ_d, displayed in the middle part of Table 1. These three values imply half lives of government debt between two and ten years.[21]

3. *Rest of the model.* The remaining model parameters—displayed in the bottom part of Table 1 are set to standard values. First, we set $\sigma = 1$, giving log preferences. Since all quantitative exercises in this section fix the expected real rate of interest this parameter is in fact immaterial. Second, we set the discount factor to hit a steady-state real rate of interest of one per cent. Third, we

[19] Application C.2 considers a very different calibration strategy based on empirical evidence on long-run elasticities of household asset demand. Interestingly, even this very different (and much less direct) approach suggests very similar overall conclusions on the plausibility of equilibrium self-financing.

[20] C.1 reports the matrix of intertemporal marginal propensities in our spender-OLG hybrid model. This matrix looks extremely similar to the HANK-implied spending matrices reported in Auclert et al. (2018) and Wolf (2021a).

[21] We note that these half lives come from fiscal adjustment alone, without any equilibrium movements in prices and output. With those movements the half-life would be strictly smaller. Since empirical evidence arguably also contains these additional general equilibrium effects our quantitative analysis is conservative.

set $\tau_y = 0.3$, in line with the discussion in DeLong et al. (2012): for every dollar of additional output, we assume that the primary surplus automatically rises by 30 cents. Fourth, we set the slope of the NKPC to 0.1, a value at the large end of recent empirical estimates (e.g., Barnichon and Mesters, 2020) but arguably relevant for our application in Section 5.2—stimulus checks, as notably seen in the inflationary post-covid environment.

5.2 Are stimulus checks plausibly self-financed?

It is straightforward to verify numerically that our calibrated spender-OLG model satisfies the general sufficient conditions identified in Theorem 2 required for a limiting self-financing equilibrium to exist. It thus remains to ascertain how close we are to this limit for the empirically relevant fiscal rules discussed in Section 5.1. Results are displayed in Figure 5.

The figure shows output (left panel) and inflation (middle panel) impulse responses to a date-0 deficit shock for the three fiscal adjustment coefficients τ_d estimated in prior work (shades of grey); the right panel furthermore shows the degree of self-financing ν as a function of τ_d over the entire unit interval. As expected the share of self-financing ν is decreasing in τ_d and approaches one as $\tau_d \to 0$, consistent with Theorem 2. For the purposes of the quantitative analysis here, the key takeaway is that we are already quite close to this limit for the values of τ_d estimated in prior work, with ν around 0.5 - 0.8. Importantly, in all of these cases, the date-0 stimulus check policy is predicted to lead to a meaningful *and protracted* output and inflation boom. Intuitively, upon income receipt, households spend but also save; crucially, in our model, these savings then still translate to additional spending relatively quickly, thus leading to a gradual, persistent boom. This contrasts markedly with the behavior of Ricardian households who would postpone additional spending into the *infinite* future. We emphasize that our results are informative about the recent "excess savings" debate (Bilbiie et al., 2021): if households violate Ricardian Equivalence because of reasonably long but finite horizons, then fiscal deficits without any quick tax offset are predicted to lead to a long-lived, inflationary boom.

Self-financing in alternative models As discussed in detail in A.4, the household block of the model affects our conclusions on self-financing *only* through the model-implied matrix of intertemporal marginal propensities to consume—i.e., the mapping from current and expected future income sequences to current and expected future aggregate consumption demand. Our model is disciplined to be consistent with empirical evidence on consumer spending behavior in response to income shocks *today*; the similarly important consumption behavior in response to income *news* shocks, on the other hand, is extrapolated through the structure of the model. We here briefly report the results from two alternative models that extrapolate somewhat differently, with details offered in Appendices C.4 and C.5.

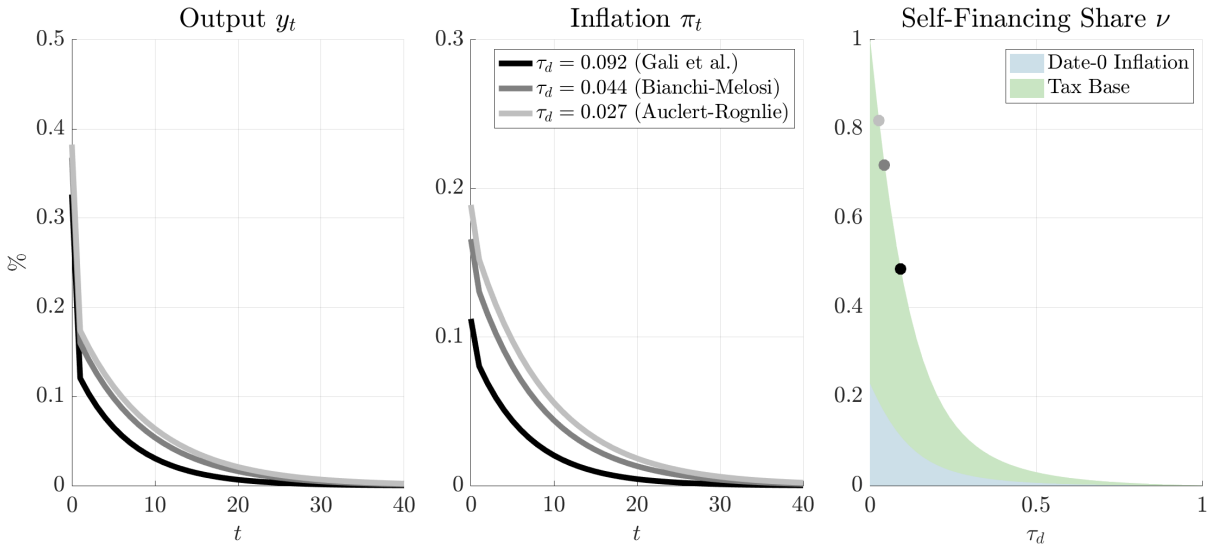


Figure 5: Impulse responses of output y_t, inflation π_t, and the total self-financing share ν to the deficit shock ϵ_0 as a function of τ_d. The left and middle panels show the impulse responses for the three particular values of τ_d discussed in Section 5.1. In the right panel these three points are marked with circles.

First, in Appendix C.4, we consider a quantitative HANK model. Consistent with the results in Auclert et al. (2018) and Wolf (2021a) we find that this model extrapolates MPCs across horizons in almost the same way as our reduced-form spender-OLG hybrid model. It is thus unsurprising that our conclusions on self-financing are largely unchanged: self-financing equilibria still exist and the share of self-financing is large for empirically relevant values of τ_d.

Second, we consider a variant of our spender-OLG model with household cognitive discounting. As discussed in Section 4.1, we expect such behavioral discounting to introduce two offsetting effects on ν. On the one hand, the future tax hike is discounted by more, pushing up fiscal multipliers and thus the degree of self-financing. On the other hand, higher income in the future does not feed back to spending today, so it takes longer for the self-financing Keynesian boom to play out in equilibrium. Our model simulations confirm this intuition: we find that ν is higher than in our baseline model for intermediate values of τ_d, but converges to the full self-financing limit somewhat more gradually. Overall, however, the basic conclusions from our full-information rational-expectations analysis in Figure 5 extend with relatively little change.

6 Conclusion

The central contribution of this paper is to clarify the conditions under which fiscal deficits can finance itself. Our results have important conceptual as well as practical implications. *Conceptually*,

they suggest that the spirit of the classical fiscal theory of the price level—that fiscal adjustment is not necessary in equilibrium—is more robust than commonly believed: it does not rely on a government violating its budget constraint off-equilibrium, nor is it vulnerable to debate regarding equilibrium selection. Rather, it merely requires delays in fiscal financing together with empirically plausibly departures from permanent-income behavior. *Practically*, our theory predicts that deficit-financed stimulus will invariably induce a meaningful and persistent boom, with its effect on prices versus quantities governed by the degree of nominal rigidities and thus slack in the economy. These insights are particularly relevant for recent U.S. fiscal stimulus experiments.

We emphasize that our analysis has been entirely positive, not normative. We leave an investigation of the implications of our results for *optimal* fiscal and monetary policy to future work.

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Online Appendix for:
Can Deficits Finance Themselves?

This online appendix contains supplemental material for the article "Can Fiscal Stimulus Finance Itself?". We provide: (i) supplementary discussion of our baseline model Sections 2 and 3; (ii) details for the model extensions considered in Section 4; (iii) additional analysis and alternative results for our quantitative investigation in Section 5. The end of this appendix contains all proofs.

Any references to equations, figures, tables, assumptions, propositions, lemmas, or sections that are not preceded "A."—"D." refer to the main article.

A Supplementary model details

We here provide some additional discussion of our baseline structural model in Section 2. Appendix A.1 considers household labor supply, and Appendix A.2 then combines this labor supply relationship with firm pricing decisions to derive our NKPC supply block (11). Appendix A.3 explains how we characterize equilibria under the alternative fiscal policy rule (5). Finally, in Appendix A.4, we briefly discuss our self-financing results from a sequence-space perspective.

A.1 Labor supply

Unions equalize the post-tax real wage and the average marginal rate of substitution between labor supply and consumption. The aggregate optimal labor supply relation is thus

$$(1-\tau_y)W_t = \frac{\chi \int_0^1 L_{i,t}^{\frac{1}{\varphi}} di}{\int_0^1 C_{i,t}^{-1/\sigma} di} \tag{A.1}$$

Log-linearizing, we obtain (10).

A.2 Price-setting and the NKPC

Optimal firm pricing decisions as usual give inflation as a function of real marginal costs. With a standard constant-returns-to-scale, labor-only production function this gives, e.g., see the textbook derivations in Woodford (2003a) and Galí (2008).

$$\pi_t = \tilde{\kappa} w_t + \beta \mathbb{E}_t [\pi_{t+1}], \tag{A.2}$$

where $\tilde{\kappa}$ is a function of firm-side primitives, including in particular the stickiness of prices. Combining (A.2) with (10) and imposing that $c_t = \ell_t = y_t$ we obtain

$$\pi_t = \underbrace{\tilde{\kappa}\left(\frac{1}{\varphi}+\frac{1}{\sigma}\right)}_{\equiv \kappa} y_t + \beta \mathbb{E}_t [\pi_{t+1}], \tag{A.3}$$

as required.

A.3 Equilibrium characterization with the alternative fiscal rule

We characterize the equilibrium in our OLG-NK environment with $\omega < 1$, $\tau_y > 0$, and the alternative fiscal rule (5) here. The AD in (9) together with monetary policy (6) and market clearing $y_t = c_t$ lead to

the following recursive AD curve[22]

$$y_t = \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(d_t - t_t) + \mathbb{E}_t\left[y_{t+1}\right], \tag{A.4}$$

where I use $\mathbb{E}_t[d_{t+1}] = \frac{1}{\beta}(d_t - t_t)$ from (2).

We characterize the unique bounded equilibrium path through backward induction. Given the alternative fiscal rule (5), we know that, for $t \geq H$,

$$d_t - t_t = -\tau_y y_t \Longrightarrow y_t = \frac{1}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}\mathbb{E}_t\left[y_{t+1}\right].$$

Since $\omega < 1$ and $\tau_y > 0$, we know the unique bounded equilibrium features $y_t = 0$ for $t \geq H$. Now we use (A.4) to find the unique bounded equilibrium path of $\{y_t, d_t\}_{t=0}^{H-1}$ through backward induction starting from

$$y_H = \chi_0(d_H + \epsilon_H) \quad \text{with} \quad \chi_0 = 0.$$

For $t \leq H-1$,substitute the alternative fiscal rule (5) into (A.4),

$$y_t = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}(d_t + \epsilon_t) + \frac{1}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}\mathbb{E}_t\left[y_{t+1}\right].$$

As a result, for $t \leq H-1$,

$$y_t = \chi_{H-t}(d_t + \epsilon_t) \quad \text{with} \quad \chi_{H-t} = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y} + \frac{\frac{1}{\beta}\left(1-\tau_y\chi_{H-t}\right)}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}\chi_{H-t-1}. \tag{A.5}$$

Rearranging terms, we find the recursive formula for χs,

$$\chi_{H-t} = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega} + \frac{\chi_{H-t-1}}{\beta}}{1+\left(\frac{(1-\beta\omega)(1-\omega)}{\beta\omega} + \frac{\chi_{H-t-1}}{\beta}\right)\tau_y} = g\left(\chi_{H-t-1}\right), \tag{A.6}$$

where

$$g(\chi) = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega} + \frac{\chi}{\beta}}{1+\left(\frac{(1-\beta\omega)(1-\omega)}{\beta\omega} + \frac{\chi}{\beta}\right)\tau_y} \quad \text{and} \quad g'(\chi) = \frac{1}{\beta}\frac{1}{\left(1+\tau_y\left(\frac{(1-\beta\omega)(1-\omega)}{\beta\omega} + \frac{\chi}{\beta}\right)\right)^2} \geq 0 \quad \forall \chi \geq 0. \tag{A.7}$$

We know that

$$\chi_k \in (0, \frac{1}{\tau_y}) \quad \forall k \geq 1 \quad \text{and} \quad \chi_k \text{ increases in } k. \tag{A.8}$$

From (2) and (A.5), we know that

$$\mathbb{E}_0[d_t] = \frac{1}{\beta^t}\Pi_{j=0}^{t-1}\left(1-\tau_y\chi_{H-j}\right)(d_0 + \epsilon_0) \tag{A.9}$$

[22]Note that (12) does not apply here, since its derivation uses the baseline fiscal policy (4).

Now consider an alternative economy with rigid price ($\kappa = 0$) but otherwise same as the baseline economy. Let ν' denote the self financing share in this alternative economy, similar to (21)

$$\nu' \cdot \epsilon_0 = \nu'_y \cdot \epsilon_0 \equiv \sum_{k=0}^{\infty} \tau_y \beta^k \mathbb{E}_0 \left[y_k \right],$$

noting that in this alternative economy all self-financing comes from tax base changes. In this economy, there is no $t = 0$ price level jump and the real value of public outstanding at $t = 0$, $d_0 = b_0 = 0$ is pre-determined. From (A.5) and (A.9), we know

$$\nu' = \sum_{t=0}^{H-1} \Pi_{j=0}^{t-1} \left(1 - \tau_y \chi_{H-j}\right) \tau_y \chi_{H-t} = 1 - \Pi_{j=0}^{H-1} \left(1 - \tau_y \chi_{H-j}\right). \tag{A.10}$$

Now, consider the general case with $\kappa \geq 0$. From NKPC (13) and the definitions in (21) – (23), we have

$$\nu_p = \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y} \nu_y = \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu \tag{A.11}$$

From the formula of d_0 (15), we know

$$d_0 = -\nu_p \epsilon_0 \quad \text{and} \quad \nu_y = \left(1 - \nu_p\right) \nu'.$$

Together, we have

$$\nu = \frac{\nu'}{\frac{\tau_y}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} + \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu'} \quad \nu_y = \frac{\frac{\tau_y}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu'}{\frac{\tau_y}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} + \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu'}, \quad \text{and} \quad \nu_p = \frac{\frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu'}{\frac{\tau_y}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} + \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu'}. \tag{A.12}$$

A.4 A sequence-space perspective

All results in the main part of this paper are stated and proved using a standard state-space approach to equilibrium characterization. We can, however, develop some additional insights by instead adopting a sequence-space perspective. In the context of the paper as a whole the purpose of the sequence-space analysis in this section is twofold. First, by adopting this sequence-space perspective, we will be able to very easily substantiate a claim made in Section 5—that intertemporal MPCs fully characterize limiting self-financing equilibria. Second, we provide a different perspective on Assumption 2, re-phrasing it as a sufficient condition ensuring that intertemporal MPCs decay "sufficiently quickly."

Equilibrium For the analysis in this section we again consider the baseline model of Sections 2 and 3, but substantially generalize the aggregate demand relation (9) to the following sequence-space relation:

$$\boldsymbol{c} = \mathscr{C}_y \times \left(\boldsymbol{y} - \boldsymbol{t}\right) + \mathscr{C}_i \times \boldsymbol{i} + \mathscr{C}_\pi \times \boldsymbol{\pi} \tag{A.13}$$

where boldface denotes sequences. Our objective in this section is to characterize self-financing equilibria, so we impose the baseline monetary policy rule (6), throughout consider the limiting case of $\tau_d = 0$, and for simplicity also assume that prices are rigid ($\kappa = 0$).[23] Imposing market-clearing as well as the monetary policy rule, the demand relation (A.13) becomes

$$\boldsymbol{y} = \mathscr{C}_y \times \left(\boldsymbol{y} - \boldsymbol{t}\right) \tag{A.14}$$

Now note that, under our assumptions on fiscal policy, taxes are given

$$\boldsymbol{t} = \tau_y \times \boldsymbol{y} - \boldsymbol{\epsilon} \tag{A.15}$$

where $\boldsymbol{\epsilon}$ denotes the fiscal policy stimulus. Combining (A.14) and (A.15), we find that output in the limiting self-financing equilibrium is completely characterized through the following system of dynamic equations:

$$\boldsymbol{y} = (1 - \tau_y)\mathscr{C}_y \times \boldsymbol{y} + \mathscr{C}_y \times \boldsymbol{\epsilon} \tag{A.16}$$

(A.16) is a variant of the intertemporal Keynesian cross studied previously in Auclert et al. (2018), but with a crucial difference: automatic tax financing is embedded in the tax revenue term $\tau_y \times \boldsymbol{y}$, rather than being specified directly as part of the policy intervention (here $\boldsymbol{\epsilon}$). This seemingly subtle distinction has important implications and in particular connects tightly with our self-financing results in Sections 3 and 4.1.

Discussion The previous discussion first of all immediately substantiates the claim made in Section 5: for a large family of models (including in particular our spender-OLG hybrid), intertemporal MPCs fully characterize the dynamics of output in the limiting self-financing equilibrium. It remains to further characterize the solution of (A.16), allowing us to connect with the economic intuitions offered in Sections 3 and 4.1. First, we emphasize that it is immediate from the properties of $\mathscr{C}_y$ that any solution $\boldsymbol{y}$ of (A.16) necessarily has net present value equal to $\frac{1}{\tau_y}$ times the net present value of the fiscal stimulus. Formally, letting $\boldsymbol{r} \equiv (1, \frac{1}{1+\bar{r}}, \frac{1}{(1+\bar{r})^2}, \ldots)$, we see that

$$\boldsymbol{r}'\boldsymbol{y} = (1 - \tau_y)\boldsymbol{r}'\mathscr{C}_y \times \boldsymbol{y} + \boldsymbol{r}'\mathscr{C}_y \times \boldsymbol{\epsilon}$$

and so from the properties of $\mathscr{C}_y$—i.e., that every dollar of income is spent at some point—we obtain that indeed

$$\tau_y \times \boldsymbol{r}'\boldsymbol{y} = \boldsymbol{r}'\boldsymbol{\epsilon}, \tag{A.17}$$

as claimed. These derivations mirror the first part of our general equilibrium step in Section 3.2.

[23] By an argument analogous to our discussion surrounding 1 the extension to the partially sticky price case is conceptually straightforward.

Second, we note that the solution of (A.16) takes the simple form

$$\boldsymbol{y} = \left[I - (1-\tau_y)\mathscr{C}_y\right]^{-1} \times \mathscr{C}_y \times \boldsymbol{\epsilon} \tag{A.18}$$

where the inverse exists by the properties of $\mathscr{C}_y$ together with the fact that we have $\tau_y > 0$. Our self-financing results in Theorems 1 and 2 concern the question of whether, as fiscal financing is gradually delayed further and further, we indeed converge to the general self-financing equilibrium characterized by (A.18). As discussed following Theorem 1, the condition required for such convergence to occur is that the Keynesian boom happens sufficiently quickly, raising all required revenue before fiscal adjustment is ever actually necessary. In (A.18), the "quickness" of the Keynesian boom is entirely governed by the properties of $\left[I - (1-\tau_y)\mathscr{C}_y\right]^{-1}$: if the off-diagonal entries of $\mathscr{C}_y$ decay to zero sufficiently quickly, then the same is true for the off-diagonal entries of $\left[I - (1-\tau_y)\mathscr{C}_y\right]^{-1}$ (e.g., see Bickel and Lindner, 2012). This then ensures that the solution $\boldsymbol{y}$ and thus the debt path $\boldsymbol{d}$ converge to zero, which in turn is precisely what is needed for self-financing to obtain as fiscal adjustment is delayed further and further. For our general aggregate demand relation (29), the condition stated in Assumption 2 is simply what is needed to ensure that indeed the off-diagonal entries of $\mathscr{C}_y$ and thus $\left[I - (1-\tau_y)\mathscr{C}_y\right]^{-1}$ decay to zero sufficiently quickly.

B Details on model extensions

We elaborate on the various extensions considered in Section 4: the richer aggregate demand block in Appendix B.1; a model with investment in Appendix B.2; equilibria under more general monetary rules in Appendix B.3; fiscal financing through distortionary taxes in Appendix B.4; and fiscal stimulus in the form of government purchases in Appendix B.5.

B.1 A more general aggregate demand relation

In Section 4.1 we showed explicitly how several popular models of the household consumption-savings problem can be written in our general form (29). The only model for which we stated but did not prove such nesting was the model of cognitive discounting of Gabaix (2020). We provide those missing details here.

Under cognitive discounting, a shock h periods in the future is additionally by a factor of θ, with $\theta = 1$ corresponding to the standard full-information, rational-expectations model and $\theta = 0$ corresponding to myopic households. It is immediate that cognitive discounting added to our baseline OLG model gives the adjusted aggregate demand relation

$$c_t = (1-\beta\omega)\left(d_t + \mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega\theta)^k\left(y_{t+k} - t_{t+k}\right)\right]\right). \tag{B.1}$$

This fits into our general demand structure (29) with $M_y = M_d = 1-\beta\omega$ and $\delta_k = (\omega\theta)^k$. It is immediate that, for $\omega < 1$ and $\theta < 1$, Assumption 1 holds. Differently from the baseline OLG case, however, Assumption 2 does not hold automatically; plugging in to (30) and re-arranging we find that we need

$$\tau_y > \frac{\omega(1-\theta)}{1-\omega\theta}\frac{1-\beta}{1-\beta\omega} \tag{B.2}$$

This relation holds automatically for $\theta = 1$, but need not hold for $\theta < 1$; intuitively, as already discussed in the main text, $\theta < 1$ dampens demand spillovers from the future to the present and thus slow down the Keynesian boom. (B.2) is, however, a very mild condition: even for $\theta = 0$, as long as β is close to one and for values of ω as considered in Section 5, Assumption 2 holds even for small τ_y.

B.2 Investment

This section provides the missing details on the extension to models with investments discussed in Section 4.2. We begin by stating the (linearized) equations of the extended model before then characterizing its equilibrium.

Model equations The household block changes very little. Households still receive labor income and dividends; we now denote this total household income by e_t (which in equilibrium will be equal to

total household consumption rather than total aggregate income). The linearized household demand relation is now

$$c_t = (1-\beta\omega)\left(d_t + \mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k(e_{t+k}-t_{t+k})\right]\right) - \left[\sigma\beta\omega - (1-\beta\omega)\beta\frac{D^{ss}}{Y^{ss}}\right]\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k(i_{t+k}-\pi_{t+k+1})\right], \quad \text{(B.3)}$$

while labor supply still satisfies

$$\frac{1}{\varphi}\ell_t = w_t - \frac{1}{\sigma}c_t \quad \text{(B.4)}$$

The firm block on the other hand changes materially relative to our baseline model. Since this production side is entirely standard our discussion here will be brief and only present linearized optimality conditions; a detailed discussion of an almost identical model is offered in Wolf (2021b). The production sector consists of three parts: perfectly competitive intermediate goods producers who accumulate capital and hire labor on spot markets; monopolistically competitive retailers who purchase the intermediate good and costlessly differentiate it, subject to nominal rigidities; and a competitive final goods aggregator. Profits of the corporate sector as a whole are returned to households, subject to the time-invariant tax τ_y. The relevant equilibrium relations follow from the behavior of the intermediate goods producers and the retailers.

1. *Intermediate goods producers.* The production function takes a standard Cobb-Douglas form with capital share α, and capital depreciates at rate δ. We let p_t^I denote the real relative price of the intermediate good. Optimal labor demand gives the static relation

$$w_t = p_t^I + \alpha k_{t-1} - \alpha\ell_t \quad \text{(B.5)}$$

while optimal capital accumulation gives[24]

$$\frac{1}{\beta}(i_t - \mathbb{E}_t[\pi_{t+1}]) = \left(\frac{1}{\beta} - 1 + \delta\right) \times \mathbb{E}_t\left[p_{t+1}^I + (\alpha-1)k_t + (1-\alpha)\ell_{t+1}\right] \quad \text{(B.6)}$$

By our assumptions on the production function total output is given as

$$y_t = \alpha k_{t-1} + (1-\alpha)\ell_t \quad \text{(B.7)}$$

and finally investment x_t satisfies

$$x_t = \frac{1}{\delta}(k_t - (1-\delta)k_{t-1}) \quad \text{(B.8)}$$

2. *Retailers.* Optimal price-setting as usual relates real marginal costs—here the relative price of

[24] Adjustment costs on the capital stock or investment flows would complicate this relation but not affect any of our subsequent arguments.

the intermediate good, p_t^I—to aggregate inflation:

$$\pi_t = \kappa p_t^I + \beta \mathbb{E}_t [\pi_{t+1}] \tag{B.9}$$

Aggregating dividend payments from intermediate goods producers and retailers, we obtain

$$Q_t = Y_t - W_t L_t - X_t \tag{B.10}$$

which implies that total household income E_t (in levels) is given as

$$E_t = W_t L_t + Q_t = Y_t - X_t \tag{B.11}$$

Aggregate output market-clearing dictates that

$$y_t = \frac{C^{ss}}{Y^{ss}} c_t + \frac{X^{ss}}{Y^{ss}} x_t \tag{B.12}$$

Finally we return to the government. The monetary rule (6) and the government budget constraint (2) are unchanged. The fiscal policy rules (4) or (5) are also unchanged up to the definition of the tax base revenue term: since the government taxes labor and dividend income, this term is now equal to $\tau_y \times E_t$.

Equilibrium characterization Our key building block result is that we can reduce the equilibrium of this extended model to a system of equations almost as simple as that of our baseline model in Section 2. First, combining market-clearing and the policy rules with the private-sector demand equation we obtain

$$c_t = \mathscr{F}_1 \cdot (d_t + \varepsilon_t) + \mathscr{F}_2 \cdot \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\omega)^k c_{t+k} \right]. \tag{B.13}$$

Relative to our baseline model, the only change is that this equilibrium demand relationship is in aggregate consumption c_t rather than aggregate output y_t. We emphasize that this is possible precisely because the government taxes dividend and labor income, which as discussed above in equilibrium is equal to total consumption. Second, the law of motion for aggregate debt is now

$$d_{t+1} = \beta^{-1} \left(d_t + \varepsilon_t - \underbrace{\tau_d \cdot (d_t + \varepsilon_t)}_{\text{fiscal adjustment}} - \underbrace{\tau_y c_t}_{\text{tax base}} \right) - \underbrace{\frac{D^{ss}}{Y^{ss}} (\pi_{t+1} - \mathbb{E}_t [\pi_{t+1}])}_{\text{price level}}, \tag{B.14}$$

with real debt at date 0 given as

$$d_0 = b_0 - \frac{D^{ss}}{Y^{ss}} (\pi_0 - E_{-1} [\pi_0]) = -\frac{D^{ss}}{Y^{ss}} \pi_0. \tag{B.15}$$

Again, relative to the baseline model, the only change is that now it is aggregate consumption rather than aggregate output appearing in (B.14).

We note that (B.13) - (B.14) is a system in $\{c_t, d_t\}_{t=0}^{\infty}$ that depends on the rest of the economy—and

so in particular the investment block—*only* through the presence of π_0. π_0 on the other hand can be obtained as a function of the consumption path $\{c_t\}_{t=0}^{\infty}$ by solving the system (B.4), (B.5), (B.6), (B.7), (B.8), (B.9) and (B.12) given consumption, and with the monetary policy rule (6) imposed. We write this function as

$$\pi_0 = \Pi_0\left(\{c_t\}_{t=0}^{\infty}\right) \tag{B.16}$$

The equilibrium described by equations (B.13) - (B.16) is straightforward to characterize given our earlier analysis of the model without investment in Sections 2 and 3. We begin with the case of perfectly rigid prices ($\kappa = 0$), and for simplicity restrict attention to the limiting self-financing case ($\tau_d \to 0$ or $H \to \infty$). In that case $\pi_0 = 0$, so we can focus on the bivariate system (B.13) - (B.14) in $\{c_t, d_t\}_{t=0}^{\infty}$. Crucially, this system is *exactly* the same as that covered in Theorem 1, so the equilibrium characterization underlying that result applies unchanged, with c_t replacing y_t.

We now turn to the case of general κ. To this end let $c_{t,0}$ denote the solution of the rigid-price system, and furthermore let $p^I_{t,0}$ denote the corresponding equilibrium intermediate goods price obtained by solving the system (B.4), (B.5), (B.6), (B.7), (B.8), and (B.12) for p^I given $\{c_{t,0}\}_{t=0}^{\infty}$. Proceeding analogously to the proof of Theorem 1, we will now construct the equilibrium for general κ by simply scaling the $\kappa = 0$ equilibrium. To this end conjecture that equilibrium consumption satisfies $a \times c_{t,0}$, for some scalar a. It is then immediate that then we would also have $p^I_t = a \times p^I_{t,0}$. But then, from (B.9), we have that

$$\pi_0 = a \times \kappa \times \sum_{t=0}^{\infty} \beta^t p^I_{t,0} \tag{B.17}$$

Finally it follows from the government budget constraint that—in our limiting self-financing equilibrium—we must have

$$\epsilon_0 = a \times \tau_y \times \sum_{t=0}^{\infty} \beta^t c_{t,0} + a \times \frac{D^{ss}}{Y^{ss}} \times \kappa \times \sum_{t=0}^{\infty} \beta^t p^I_{t,0}$$

Solving this equation for a we obtain consumption and thus inflation as well as government debt in the general sticky-price equilibrium. In particular we see that self-financing yet again obtains exactly as in our baseline economy. We summarize these observations in the following corollary.

Corollary B.1. *Consider the extended OLG-NK environment with investment. Fiscal adjustment is never necessary in equilibrium—that is, $\nu \to 1$—if the tax response is infinitely delayed, i.e., $\tau_d \to 0$ or $H \to \infty$. These two limits induce the same equilibrium paths $\{c_t, \pi_t, d_t\}_{t=0}^{\infty}$, and in this common limit self-financing is sufficient to return government debt to steady state (i.e., $\rho_d \in (0,1)$).*

B.3 Monetary accommodation

First, in (D.18) in the proof of Theorem 3, we characterize the equilibrium in the economy with monetary policy (32) and fiscal (4) policy recursively. Figure 4 is based on the number of eigenvalues of (D.18) within the unit cycle.

Second, as discussed in the main text, the cutoff $\bar{\phi}$ plays the same role as Assumption 2. They both make sure that deficit-driven Keynesian Boom is large enough to play out in finite time. In fact, when $\phi = \bar{\phi}$ or when (30) holds with equality, the expected persistence of real value of debt (ρ_d in (16)) becomes 1 when fiscal adjustment is infinitely delayed ($\tau_d \to 0$). As a result, this prevents the existence of a bounded complete self-financing equilibrium. In fact, when $\frac{D^{ss}}{Y^{ss}} = 0$, one can re-write the aggregate demand with general MP in Section 4.3 in a way similar to (29) in Section 4.1.

Specifically, the AD in (9) together with monetary policy (32) and $\frac{D^{ss}}{Y^{ss}} = 0$ can be written as

$$c_t = \left(1-\beta\omega\right) d_t + \left(1-\beta\omega-\phi\sigma\beta\omega\right) E_t\left[\sum_{k=0}^{+\infty}\left(\beta\omega\right)^k y_{t+k}\right] - \left(1-\beta\omega\right) E_t\left[\sum_{k=0}^{+\infty}\left(\beta\omega\right)^k t_{t+k}\right], \tag{B.18}$$

which can be nested by a general aggregate demand equation:

$$c_t = M_y\left(y_t + E_t\left[\sum_{k=1}^{+\infty}\delta_k\beta^k y_{t+k}\right]\right) - M_t\left(t_t + E_t\left[\sum_{k=1}^{+\infty}\delta_k\beta^k t_{t+k}\right]\right) + M_d d_t, \tag{B.19}$$

with the only difference from (29) is that the MPC out of income M_y is now allowed to differ the MPC out of tax M_t. However, one can easily generalize the proof of Theorem 2 based on a more general Assumption 2

$$M_d + \frac{1-\beta}{\tau_y}\left(M_y - \tau_y M_t\right)\left(1+\sum_{k=1}^{+\infty}\delta_k\beta^k\right) > \frac{1-\beta}{\tau_y}. \tag{B.20}$$

(B.19) nests (B.18) with $M_y = 1-\beta\omega-\phi\sigma\beta\omega$, $M_d = M_t = 1-\beta\omega$, and $\delta_k = \omega^k$. One can see that, when $\phi = \bar{\phi}$ in (D.20), (B.20) holds with equality. This formalizes how the cutoff $\bar{\phi}$ plays the same role as Assumption 2.

B.4 Distortionary taxation

We begin by showing how the equilibrium relations of the model change with time-varying distortionary taxes. We then discuss implications for our limiting self-financing equilibria.

Environment With time-varying distortionary taxes the optimal labor supply relation becomes

$$(1-\tau_{y,t})W_t = \frac{\chi\int_0^1 L_{i,t}^{\frac{1}{\varphi}}di}{\int_0^1 C_{i,t}^{-1/\sigma}di} \tag{B.21}$$

Log-linearizing, we find that

$$w_t - \frac{1}{\sigma} c_t - \frac{1}{1-\tau_y} \widehat{\tau}_{y,t} = \frac{1}{\varphi} \ell_t \tag{B.22}$$

where $\widehat{\tau}_t \equiv \tau_{y,t} - \tau_y$. Next note that the firm optimal pricing relationship is still

$$\pi_t = \tilde{\kappa} w_t + \beta \mathbb{E}_t [\pi_{t+1}] \tag{B.23}$$

Combining (B.22), (B.23), and the adjusted fiscal rule (34) we obtain

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t [\pi_{t+1}] + \underbrace{\frac{\tilde{\kappa}}{1-\tau_y} \tau_{d,t}}_{\equiv \zeta_t} \times d_t \tag{B.24}$$

All other equilibrium relations of the model are unaffected.

Self-financing result Note that, as either $\tau_d \to 0$ or $H \to \infty$, we obtain that $\zeta_t \to 0$ for all t. It thus follows that the equilibrium characterization of Theorem 1 for the self-financing limit applies without change to the alternative economy in which fiscal adjustment is distortionary. Intuitively, since the adjustment is not necessary in equilibrium, it is immaterial whether the adjustment *would have been* distortionary or lump-sum.

B.5 Government spending

The government spending policy experiment was already described fully in Section 4.5. Since the analytics of the self-financing result are exactly analogous to our baseline "stimulus checks" case, we do not repeat those derivations here and instead just provide a visual illustration of the self-financing result.

We summarize our results in Figure B.1—the government spending analogue of Figure 1. We emphasize two main takeaways. First, as $\tau_d \to 0$ or $H \to \infty$, we indeed again converge to a self-financing limit. Second, even immediately tax-financed fiscal purchases have a positive spending multiplier, and thus the share of self-financing ν for $\tau_d = 1$ (top panel) and $H = 0$ (bottom panel) is already strictly positive.[25]

[25] As discussed in Section 4.5, if prices were rigid, then in this immediately tax-financed case $\nu = \nu_y = \tau_y$. With partially sticky prices the initial inflation further increases the degree of self-financing.

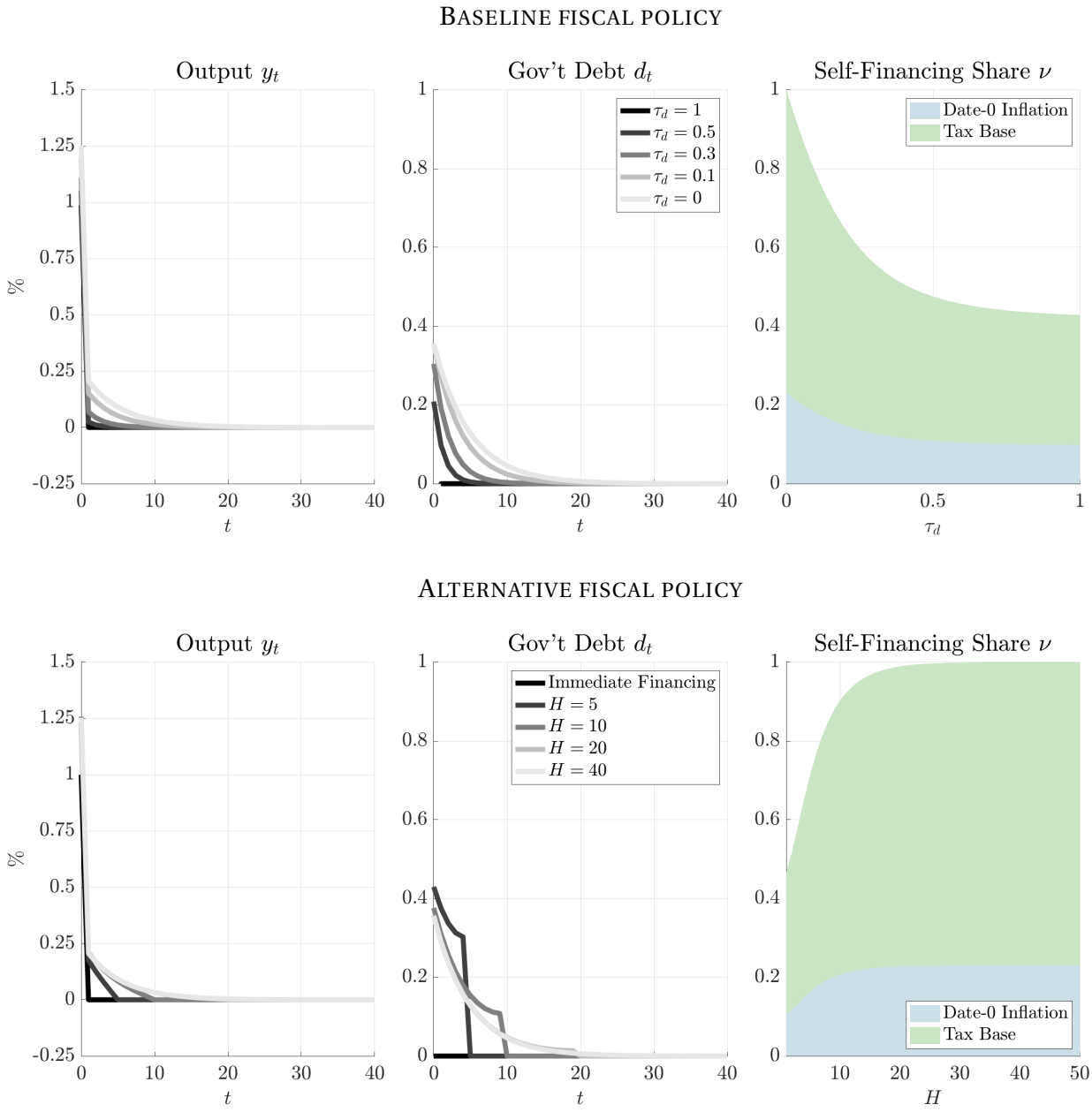


Figure B.1: Top panel: impulse responses of output y_t, government debt d_t, and the total self-financing share ν to the deficit shock ϵ_0 as a function of τ_d. Bottom panel: same as above, but as a function of H.

C Quantitative analysis

This section supplements our quantitative analysis in Section 5. We: provide some missing details on the specification of our spender-OLG model in Appendix C.1; consider an alternative for calibration of that model in Appendix C.2; review empirical evidence on fiscal adjustment in Appendix C.3; and present detailed results from a HANK model and a model with cognitive discounting in Appendices C.4 and C.5, respectively.

C.1 Further details on the hybrid model

We first elaborate on the model environment and discuss in greater detail the model's implications for household consumption behavior, contrasting it in particular with the predictions of quantitative HANK-type models.

Model The only change relative to our baseline model of Section 2 is that we generalize the household block to also feature a margin μ of spenders—that is, households who do not hold any assets and immediately spend any income they receive. The remaining fraction $1-\mu$ of households are exactly as described in Section 2.2. Both groups of households receive labor income as well as dividends and pay taxes, but only the OLG block holds government bonds.

We will make assumptions ensuring that both groups of households receive the same labor and dividend income, pay the same taxes (up to a between-group steady-state transfer), and have identical steady-state consumption. First, we assume that unions assigns identical hours worked to both groups, and that dividends accrue equally to both. Second, we assume that the government in lump-sum fashion redistributes between the two groups to ensure identical steady-state consumption; given that government bonds are held by the OLG block, this will generally require lump-sum transfers to spenders. Under those assumptions, it is first of all immediate that the supply block of the economy—notably (11)—is unchanged. Next, the demand block of the economy generalized (28) as follows:

$$c_t = (1-\beta\omega)\cdot d_t + \left[\mu + (1-\mu)(1-\beta\omega)\right]\cdot\left((y_t - t_t) + (1-\mu)\mathbb{E}_t\left[\sum_{k=1}^{\infty}(\omega\beta)^k\left(y_{t+k}-t_{t+k}\right)\right]\right). \tag{C.1}$$

Replacing (28) by (C.1) is the only difference between our baseline OLG economy and the generalized hybrid model. Relative to (28), the key change in (C.1) is that we allow the MPC out of income to be larger than that out of wealth. As we discuss next, this minimal departure from our baseline OLG model is all that is needed to ensure consistency with consumption-savings behavior even in quantitative HANK-type models.

Household consumption-savings behavior By our discussion in Appendix A.4 we know that the role of the household consumption-savings decision in driving our self-financing result is fully governed

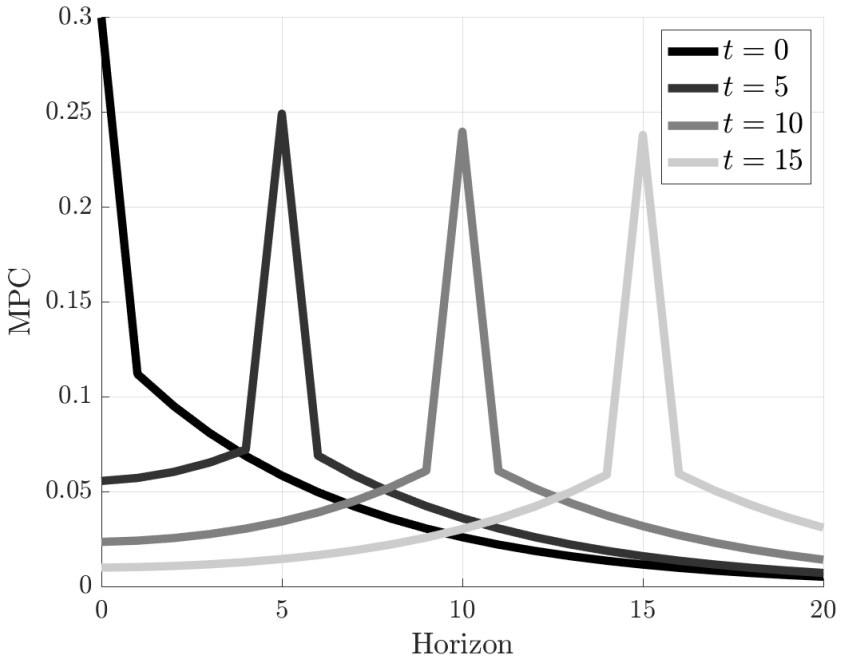


Figure C.1: Consumer spending responses to income gains at different dates (shades of grey) in our spender-OLG hybrid model. Model parameterization as in Table 1.

by the matrix of intertemporal marginal propensities to consume $\mathscr{C}_y$. Figure C.1 provides a visual illustration of this matrix in our spender-OLG hybrid model, as implied by the generalized demand block (C.1).

The figure plots the spending response over time to (anticipated) income gains at different dates. We emphasize two key takeaways. First, the response to a date-0 income gain—that is, the first column of $\mathscr{C}_y$—agrees closely with prior empirical evidence (Fagereng et al., 2021), as already discussed in Section 5.1. Second, the higher-order columns are qualitatively and quantitatively similar to those implied by quantitative HANK-type models. This observation has been made previously in Auclert et al. (2018) and Wolf (2021a). For our purposes, the important takeaway is that our analysis is indeed quantitatively relevant—as far as our question of self-financing is concerned, our model will have the exact same predictions as richer quantitative HANK-type models. We further illustrate this observation in Appendix C.4.

C.2 An alternative calibration approach for the household block

For our baseline analysis in Section 5 we discipline our model's departure from permanent-income behavior by requiring consistency with empirical evidence on the *level* and *slope* of household consumption behavior lump-sum income receipt, as in Auclert et al. (2018) and Wolf (2021a). We here

instead present results from a very different approach, combining evidence on level MPCs with long-run price elasticities of household asset demand. This calibration strategy is promising because models with permanent-income savers invariably imply a (counterfactual) infinite elasticity of household asset demand (e.g., Kaplan and Violante, 2018).

Our main building block result for this calibration approach is Proposition C.1 which gives the long-run elasticity of household asset demand as a function of model primitives.

Proposition C.1. *Consider the spender-OLG hybrid model. Let η denote the long-run interest rate elasticity of household asset demand—that is, the long-run response of asset demand to a permanent change in real interest rates. It is given as*

$$\eta \;=\; (1-\mu)\times\frac{\sigma}{1-\beta}\times\left(\frac{1}{1-\omega}-\frac{1}{1-\beta\omega}\right) \tag{C.2}$$

Empirical work suggests a range for η of around 1.25 to 35 (see Moll et al., 2022). Setting $\beta = 0.99$, $\sigma = 1$, and requiring the model to generate an impact MPC of 30 per cent (all as in our baseline calibration), we find $\omega \in [0.25, 0.86]$. Our baseline calibration lies at the upper end of this range and is thus conservative.

C.3 Empirical evidence on fiscal adjustment

Notable prior work that has estimated fiscal financing rules and thus in particular the speed of fiscal adjustment in response to deficits includes Galí et al. (2007), Bianchi and Melosi (2017), and Auclert and Rognlie (2018).

1. Galí et al. (2007). The authors estimate impulse responses to an identified government spending shock and in particular report impulse responses of government debt. Setting $\tau_d = 0.09$ implies a deficit half-life comparable to their estimates.

2. Bianchi and Melosi (2017). The authors estimate a structural macroeconomic model with policy uncertainty in rich fiscal-monetary interactions. The model features both fiscal- as well as monetary-led regimes; setting $\tau_d = 0.04$ implies a half-life of government of around five reads, comparable to the simulations reported by Bianchi and Melosi.

3. Auclert and Rognlie (2018). The authors estimate fiscal adjustments on data from a sample of OECD countries and find a half-life of deficits around 10 years. Translated to our notation this corresponds to around $\tau_d = 0.03$.

We emphasize that our calibration choices described above are conservative. We set τ_d so that, *even in the absence of self-financing fiscal feedback*, taxes adjust sufficiently quickly to deliver the desired debt

half-life. Taking into account this self-financing fiscal feedback, the half-life of fiscal deficit shocks becomes even smaller.

C.4 A full HANK model

This section provides a sketch of the quantitative HANK model that we use to numerically illustrate the generality of our self-financing result. The discussion is brief because the household block of the model is essentially borrowed from Wolf (2021a).

Model sketch & calibration The model economy is exactly as in Section 2, but with one twist: the OLG household block is replaced by a unit continuum of households $i \in [0,1]$ that face uninsurable income risk. Households have preferences

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} \beta^t \left[u(C_{i,t+k}) - v(L_{i,t+k}) \right] \right]$$

Households save and borrow (subject to a constraint) in a nominally risk-free bond, as in our baseline model. They receive labor and dividend income in proportion to their (stochastic) productivity, pay a proportional tax τ_y on that income, and finally pay additional lump-sum uniform taxes $\tilde{T}_t$. We can thus write the household budget constraint in real terms as

$$C_{i,t} + B_{i,t+1} = (1-\tau_y) e_{i,t} Y_t - \tilde{T}_t + \frac{I_{t-1}}{\Pi_t} B_{i,t}, \quad B_{i,t+1} \geq \underline{B}$$

Whenever possible we calibrate the model exactly as our baseline model. The remaining HANK-specific parameters are: the income risk process; the borrowing constraint; and the discount factor. The income risk process is taken from Kaplan et al. (2018), just ported to discrete time as in Wolf (2021b). The borrowing constraint $\underline{B}$ is set to zero, and the discount factor β is backed out residually to clear the asset market. Finally, we need to make one more change relative to our baseline model: in the model set-up as described so far, tax revenue $\tau_y \times Y^{ss}$ would far exceed debt servicing costs, so the government would make a substantial uniform transfer, thus materially dampening household MPCs. We instead set the steady-state transfer share as in the data We instead set the steady-state transfer share as in the data following Kaplan et al. (2018) and giving $\tilde{T}^{ss}/Y^{ss} = 0.06$, and then clear the government budget by additionally allowing for positive (and time-invariant) government purchases.

Results We use the quantitative HANK model to revisit our numerical exercises in Section 5.2. Exactly as done there, we here compute the aggregate effects of one-off fiscal stimulus for different assumptions on the delay in fiscal financing. Results are reported in Figure C.2.

Our results closely echo those of Section 5.2. We emphasize two main takeaways. First, Figure C.2 is *qualitatively* very similar to Figure 5: output and inflation responses as well as the share of self-financing ν are all increasing in the delay in fiscal adjustment (i.e., decreasing in τ_d). Furthermore,

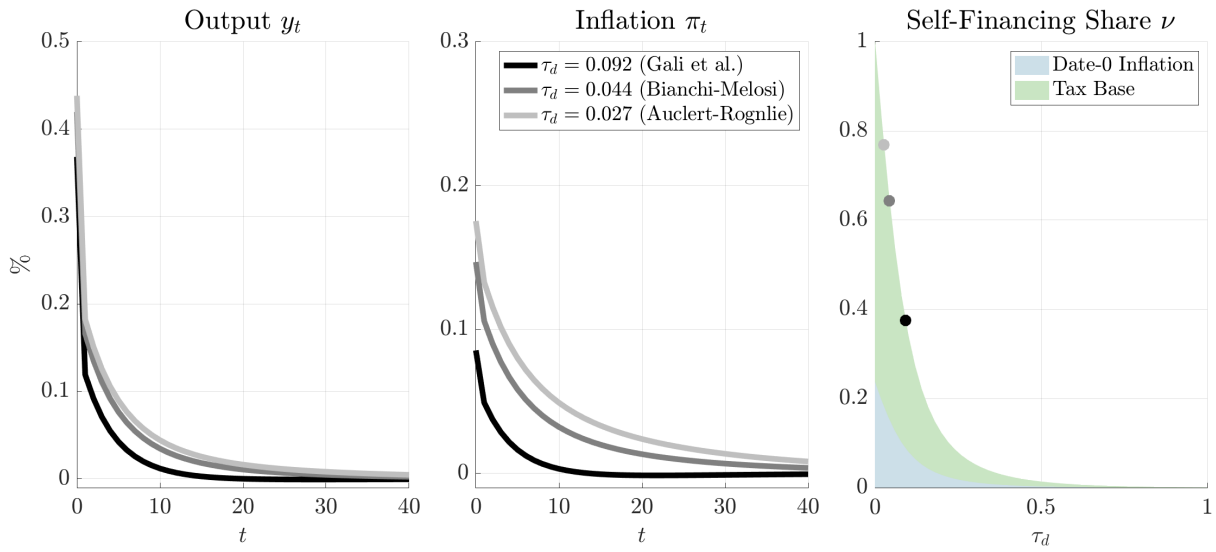


Figure C.2: Impulse responses of output y_t, inflation π_t, and the total self-financing share ν to the deficit shock ϵ_0 as a function of τ_d, for the quantitative HANK model. The left and middle panels show the impulse responses for the three particular values of τ_d discussed in Section 5.1. In the right panel these three points are marked with circles.

as $\tau_d \to 0$, we again converge to a full self-financing limit. We have also verified numerically that, for each $\tau_d \in [0,1]$ considered in construction of Figure C.2, the constructed equilibrium is (locally) unique, again echoing our baseline analysis. Second, the two figures are also *quantitatively* similar: for our three values of τ_d taken from prior work, the impulse responses of output and inflation as well as the share of self-financing ν are very similar to the spender-OLG hybrid model. This conclusion confirms prior work arguing that, as far the dynamics of macroeconomic aggregates are concerned, spender-OLG hybrid models and fully specified HANK models look extremely similar (Auclert et al., 2018; Wolf, 2021a)

C.5 The effects of cognitive discounting

Figure C.3 repeats our analysis of Section 5.2 in a variant of our spender-OLG hybrid model with cognitive discounting. To illustrate the effects of discounting as clearly as possible we consider the limit of fully myopic households (i.e., $\theta = 1$).

The figure illustrates the two effects described in Section 4.1. First, for τ_d close to one, the Keynesian boom and thus the share of self-financing ν are larger than in our baseline model. Intuitively, in this case, the strong discounting of the not-so-distant tax hike meaningfully amplifies the initial boom. Second, for τ_d close to one, the self-financing limit is approached more slowly, reflecting a weakening of the Keynesian cross. More empirically relevant values of θ give results somewhere in

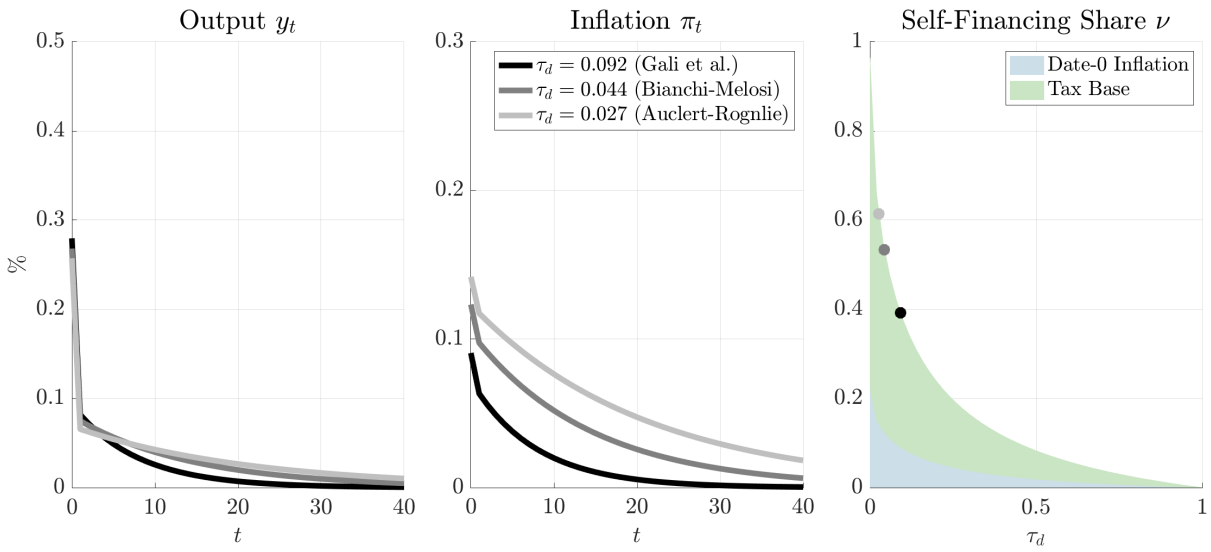


Figure C.3: Impulse responses of output y_t, inflation π_t, and the total self-financing share ν to the deficit shock ϵ_0 as a function of τ_d, for fully myopic households. The left and middle panels show the impulse responses for the three particular values of τ_d discussed in Section 5.1. In the right panel these three points are marked with circles.

between Figure C.3 and Figure 5.

D Proofs and auxiliary lemmas

D.1 Proof of Proposition 1

We restrict that $\omega \in (0,1)$, $\tau_y > 0$, and $\tau_d \in [0,1)$. We first wrote (12) recursively:

$$\begin{aligned} y_t - \mathscr{F}_1 \cdot (d_t + \epsilon_t) &= \mathscr{F}_2 \cdot y_t + \beta\omega \mathbb{E}_t \left[y_{t+1} - \mathscr{F}_1 \cdot (d_{t+1} + \epsilon_{t+1}) \right] \\ &= \mathscr{F}_2 \cdot y_t + \beta\omega \mathbb{E}_t \left[y_{t+1} - \mathscr{F}_1 \cdot \frac{1}{\beta} \left[(1-\tau_d)(d_t + \epsilon_t) - \tau_y y_t \right] \right]. \end{aligned}$$

After rearranging terms and using the formula of $\mathscr{F}_1$ and $\mathscr{F}_2$ after (12), we have

$$\begin{aligned} y_t &= \frac{(1-\omega(1-\tau_d))\mathscr{F}_1}{1-\omega\tau_y\mathscr{F}_1 - \mathscr{F}_2}(d_t + \epsilon_t) + \frac{\beta\omega}{1-\omega\tau_y\mathscr{F}_1 - \mathscr{F}_2}\mathbb{E}_t\left[y_{t+1}\right] \\ &= \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}(d_t + \epsilon_t) + \frac{1}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}\mathbb{E}_t\left[y_{t+1}\right]. \end{aligned}$$

Applying period-t expectation $\mathbb{E}_t[\cdot]$ to (14), we have

$$\begin{pmatrix} \mathbb{E}_t[d_{t+1}] \\ \mathbb{E}_t\left[y_{t+1}\right] \end{pmatrix} = \begin{pmatrix} \frac{1-\tau_d}{\beta} & -\frac{\tau_y}{\beta} \\ -\frac{(1-\beta\omega)(1-\omega)(1-\tau_d)}{\beta\omega} & 1+\frac{(1-\beta\omega)(1-\omega)\tau_y}{\beta\omega} \end{pmatrix} \begin{pmatrix} d_t + \epsilon_t \\ y_t \end{pmatrix} \tag{D.1}$$

Two eigenvalues of of the system are given by the solutions of

$$\lambda^2 - \lambda\left(\frac{1}{\beta}(1-\tau_d) + 1 + \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right) + \frac{1}{\beta}(1-\tau_d) = 0,$$

with

$$\begin{aligned} \lambda_1 &= \frac{\left(\frac{1}{\beta}(1-\tau_d) + 1 + \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right) + \sqrt{\left(1-\frac{1}{\beta}(1-\tau_d) - \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2 + 4\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)}}{2} \\ &> \frac{\left(\frac{1}{\beta}(1-\tau_d) + 1 + \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right) + \left|1-\frac{1}{\beta}(1-\tau_d) - \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right|}{2} \geq 1 \end{aligned} \tag{D.2}$$

and

$$\begin{aligned} \lambda_2 &= \frac{\left(\frac{1}{\beta}(1-\tau_d) + 1 + \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right) - \sqrt{\left(1-\frac{1}{\beta}(1-\tau_d) - \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2 + 4\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)}}{2} \\ &< \frac{\left(\frac{1}{\beta}(1-\tau_d) + 1 + \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right) - \left|1-\frac{1}{\beta}(1-\tau_d) - \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right|}{2} \leq 1, \end{aligned} \tag{D.3}$$

with $\lambda_2 > 0$ too since $\lambda_1 = \lambda_2 = \frac{1}{\beta}(1-\tau_d) > 0$. Let $\left(1,\chi_2\right)'$ denote the eigenvector associated with λ_2,where

$$\lambda_2 = \frac{1}{\beta}\left(1-\tau_d-\tau_y\chi_2\right) \quad \text{and} \quad \chi_2 = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y-\lambda_2} > 0. \tag{D.4}$$

This means, any bounded equilibrium path $\{d_t, y_t\}_{t=0}^{+\infty}$ of (D.1)[26] must take the form of

$$y_t = \chi(d_t+\varepsilon_t) \quad \text{and} \quad \mathbb{E}_t[d_{t+1}] = \rho_d(d_t+\epsilon_t),$$

where

$$\chi = \chi_2 > 0 \quad \text{and} \quad \rho_d = \lambda_2 \in (0,1). \tag{D.5}$$

In other words, the equilibrium takes the form of (16), which satisfy (17) and (18).[27]

To prove equilibrium uniqueness, note that the total amount of nominal public debt outstanding at the start of $t=0$, $B_0 = B^{ss}$, is given. From (13) and (15), we know d_0 is uniquely pinned down by

$$d_0 = -\frac{D^{ss}}{Y^{ss}}\pi_0 = -\kappa\frac{D^{ss}}{Y^{ss}}\sum_{k=0}^{+\infty}\beta^k\mathbb{E}_0\left[y_k\right] = -\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}(d_0+\epsilon_0) = -\frac{\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}{1+\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}\epsilon_0. \tag{D.6}$$

Similarly, for $t \geq 1$,

$$\begin{aligned} d_t - \mathbb{E}_{t-1}[d_t] &= -\frac{D^{ss}}{Y^{ss}}(\pi_t - \mathbb{E}_{t-1}[\pi_t]) = -\kappa\frac{D^{ss}}{Y^{ss}}\sum_{k=0}^{+\infty}\beta^k\left(\mathbb{E}_t\left[y_{t+k}\right] - \mathbb{E}_{t-1}\left[y_{t+k-1}\right]\right) \\ &= -\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}(d_t - \mathbb{E}_{t-1}[d_t] + \epsilon_t) = -\frac{\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}{1+\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}\epsilon_t. \end{aligned}$$

This pins down the unique path bounded equilibrium path of $\{\pi_t, d_t, y_t\}_{t=0}^{+\infty}$ with (13) and (16).

D.2 Proof of Theorem 1

We start with the case based on the baseline fiscal policy (4). Let ν' denote the self financing share in this alternative economy, similar to (21)

$$\nu'\cdot\epsilon_0 = \nu_y'\cdot\epsilon_0 \equiv \sum_{k=0}^{\infty}\tau_y\beta^k\mathbb{E}_0\left[y_k\right],$$

noting that in this alternative economy all self-financing comes from tax base changes. In this economy, there is no $t=0$ price level jump and the real value of public outstanding at $t=0$, $d_0 = b_0 = 0$ is pre-determined. From (16) and (21), we know

$$\nu' = \tau_y\frac{\chi}{1-\beta\rho_d}. \tag{D.7}$$

[26] This means that $\lim_{k\to+\infty}\mathbb{E}_t[d_{t+k}]$ and $\lim_{k\to+\infty}\mathbb{E}_t\left[y_{t+k}\right]$ are bounded for any t, $d_t+\epsilon_t$, and y_t, similar to Blanchard and Kahn (1980).

[27] To see (18), combine (12) with (16).

Now, consider the general case with $\kappa \geq 0$. From NKPC (13) and the definitions in (21) – (23), we have

$$\nu_p = \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y} \nu_y = \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu \tag{D.8}$$

From the formula of d_0 (15), we know

$$d_0 = -\nu_p \epsilon_0 \quad \text{and} \quad \nu_y = \left(1 - \nu_p\right) \nu'.$$

Together, we have

$$\nu = \frac{\nu'}{\frac{\tau_y}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} + \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu'} \quad \nu_y = \frac{\frac{\tau_y}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu'}{\frac{\tau_y}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} + \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu'}, \quad \text{and} \quad \nu_p = \frac{\frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu'}{\frac{\tau_y}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} + \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu'}. \tag{D.9}$$

From (17), we know that

$$\frac{\chi}{1 - \beta \rho_d} = \frac{\chi}{\tau_d + \tau_y \chi}. \tag{D.10}$$

From (D.3) and (D.5), we know

$$\rho_d = \lambda_2 = f(a, b) \equiv \frac{a + b + 1 - \sqrt{(a + b - 1)^2 + 4b}}{2} \tag{D.11}$$

where $f(a,b)$, $a = \frac{1}{\beta}(1 - \tau_d) > 0$, and $b = \frac{1-\beta\omega}{\beta\omega} \tau_y (1 - \omega) > 0$. Since $\frac{\partial f}{\partial a} = \frac{1}{2} - \frac{(a+b-1)}{2\sqrt{(a+b-1)^2+4b}} > 0$,we know that ρ_d decreases with τ_d. From (D.4) and (D.5), we then know $\chi = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y - \rho_d}$ also decreases in τ_d. From (D.10), we know $\frac{\chi}{1-\beta\rho_d}$ decreases in τ_d. Finally, from (D.7) and (D.9), we know ν decreases in τ_d. This finishes the proof of Part 1.

For Part 2, from (D.3) and (17), we know that ρ_d and χ are continuous with $\tau_d \in [0, 1)$, and

$$\rho_d^{\text{full}} \equiv \lim_{\tau_d \to 0} \rho_d = \frac{\left(\frac{1}{\beta} + 1 + \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right) - \sqrt{\left(1 - \frac{1}{\beta} - \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2 + 4\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)}}{2} < 1 \tag{D.12}$$

$$\chi^{\text{full}} \equiv \lim_{\tau_d \to 0} \chi = \frac{1 - \beta \rho_d^{\text{full}}}{\tau_y} > 0 \tag{D.13}$$

From (D.10), we know $\lim_{\tau_d \to 0} \frac{\chi}{1-\beta\rho_d} = \frac{1}{\tau_y}$. From (D.7) and (D.9), we know $\lim_{\tau_d \to 0} \nu = 1$.

Now we turn to the alternative fiscal policy in (5), which we use the equilibrium characterization in Appendix A.3. For the case with rigid price ($\kappa = 0$), one can see from (A.10) that ν' increases in H, which proves Part I. For Part II and to find $\lim_{H \to +\infty} \nu'$, first note that, from (A.8), $\{\chi_k\}_{k=0}^{\infty}$ is a bounded, increasing sequence. As a result, there exists $\chi^{\text{full,NM}}$ such that $\lim_{H \to +\infty} \chi_k = \chi^{\text{full,NM}}$ and $\chi^{\text{full,NM}} = g\left(\chi^{\text{full,NM}}\right) \in \left(0, \frac{1}{\tau_y}\right)$. From (A.10), we know that $\lim_{H \to +\infty} \nu' = 1$.

From (D.4) and (D.5), we also know $g\left(\chi^{\text{full}}\right) = \chi^{\text{full}}$ where χ^{full} defined in (D.13) parametrizes the output response in the complete self-financing limit ($\tau_d \to 0$). From the definition of $g(\cdot)$ in (A.7),

we know that there is a unique $\chi > 0$ such that $g(\chi) = \chi$ when $\omega < 1$ and $\tau_d \in (0,1)$. As a result, $\chi^{\text{full,NM}} = \chi^{\text{full}} < \frac{1}{\tau_y}$ and $\lim_{H\to+\infty} \chi_k = \chi^{\text{full}}$. That is, these two limits ($\tau_d \to 0$ and $H \to 0$) shares the same equilibrium path.

For the general case with $\kappa \geq 0$, the Theorem follows directly from the rigid price case and the fact that two cases share the same (A.12) and (D.9).

D.3 Proof of Proposition 2

We start with the case based on the baseline fiscal policy (4). Part 1 follows from (D.9). For Part 2, first note that, from the proof of Proposition 1, χ, ρ_d, and ν' are independent from κ. As a result, $\lim_{\kappa\to\infty} \nu = 1$ follows from (D.9). From (D.9), we also know that ν_y decreases in κ and ν_p increases in κ.

Now we turn to the alternative fiscal policy in (5), which we use the equilibrium characterization in Appendix A.3. Part I directly follows from (A.11). Part II directly follows from (A.12) and the fact that ν' is independent of κ.

D.4 Proof of Proposition 3

We start with the case based on the baseline fiscal policy (4). From (D.11), we know

$$\rho_d = \frac{a+b+1-\sqrt{(a+b+1)^2-4a}}{2} = \frac{2a}{a+b+1+\sqrt{(a+b+1)^2-4a}},$$

which decreases in $b = \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)$. As a result, ρ_d increases in $\omega \in (0,1)$. From (17), we know χ decreases in ω. From, (D.7), (D.10) and (D.9), we know that ν decreases in ω. This proves Part I.

For Part II, from (D.12), we know that $\lim_{\omega\to 1} \rho_d^{\text{full}} = \frac{\left(\frac{1}{\beta}+1\right)-\sqrt{\left(1-\frac{1}{\beta}\right)^2}}{2} = 1$.

Now we turn to the alternative fiscal policy in (5), which we use the equilibrium characterization in Appendix A.3. For Part I, first note that each χ_k ($k \geq 1$) decreases in ω. From A.10, we know that the self-financing share ν' in the rigid-price $\kappa = 0$ decreases in ω. From (A.12), we know the self-financing share in the general $\kappa \geq 0$ case decreases in ω.

Part II follows from Part II of Theorem 1, which states that $\tau_d \to 0$ and $H \to \infty$ share the same complete self financing limit.

D.5 Proof of Theorem 2

Under Assumptions 1 and 2, we construct the following completely self-financing bounded equilibrium similar to (16) when fiscal adjustment is infinitely delayed (i.e., $\tau_d \to 0$ or $H \to \infty$).

$$y_t = \chi_d d_t + \chi_\epsilon \epsilon_t, \quad E_t[d_{t+1}] = \rho_d d_t + \rho_\epsilon \epsilon_t \quad \text{with} \quad \chi_d, \chi_\epsilon > 0, \ \rho_d \in (0,1). \tag{D.14}$$

For (D.14) to be a self-financing equilibrium, it needs to satisfy (14) and (29). For (D.14) to satisfy to the government budget (14), we need

$$\rho_d = \frac{1}{\beta}\left(1 - \tau_y \chi_d\right) \quad \text{and} \quad \rho_\epsilon = \frac{1}{\beta}\left(1 - \tau_y \chi_\epsilon\right). \tag{D.15}$$

For (D.14) to satisfy aggregate demand (29) (together with market clearing $c_t = y_t$), we need

$$\chi_d = M_d + M_y\left(1-\tau_y\right)\left(1 + \sum_{k=1}^{+\infty} \delta_k \left(\beta \rho_d\right)^k\right)\chi_d = \frac{M_d}{1 - M_y\left(1-\tau_y\right)\left(1 + \sum_{k=1}^{+\infty} \delta_k \left(\beta\rho_d\right)^k\right)}. \tag{D.16}$$

and

$$\chi_\epsilon = M_y\left(1 + \left(1-\tau_y\right)\left(\chi_\epsilon + \sum_{k=1}^{+\infty} \delta_k \beta^k \rho_d^{k-1} \rho_\epsilon \chi_d\right)\right) = \frac{M_y\left(1 + \left(1-\tau_y\right)\left(\chi_\epsilon + \sum_{k=1}^{+\infty} \delta_k \beta^k \rho_d^{k-1} \rho_\epsilon \chi_d\right)\right)}{1 - M_y\left(1-\tau_y\right)\left(1 - \sum_{k=1}^{+\infty} \delta_k \beta^{k-1} \rho_d^{k-1} \tau_y \chi_d\right)}. \tag{D.17}$$

(D.15) and (D.16) together mean that ρ_d needs to be the root of the following equation:

$$h\left(\rho_d\right) = \frac{1-\beta\rho_d}{\tau_y} - \frac{M_d}{1 - M_y\left(1-\tau_y\right)\left(1 + \sum_{k=1}^{+\infty} \delta_k \left(\beta\rho_d\right)^k\right)} = 0,$$

with $h(0) = \frac{1}{\tau_y} - \frac{M_d}{1 - M_y(1-\tau_y)} \geq 0$ because $\tau_y > 0$ and $M_y \in [0,1]$ and $M_y \geq M_d$. When Assumption 2 hold, there are two cases.

First, $\left(1-\tau_y\right) M_y \left(1 + \sum_{k=1}^{\infty} \delta_k \beta^k\right) > 1$. In this case, there exists $\bar{\rho} \in (0,1)$ such that

$$M_y\left(1-\tau_y\right)\left(1 + \sum_{k=1}^{+\infty} \delta_k \left(\beta\bar{\rho}\right)^k\right) = 1,$$

and $\lim_{\rho_d \uparrow \bar{\rho}} h\left(\rho_d\right) = -\infty$. As a result, there exists a unique $\rho_d \in (0, \bar{\rho})$ such that $f\left(\rho_d\right) = 0$. Given ρ_d, one can find ρ_ϵ and $\chi_d, \chi_\epsilon > 0$ from (D.15) – (D.17) to constitute a self-financing equilibrium in (D.14).

Second, $\left(1-\tau_y\right) M_y \left(1 + \sum_{k=1}^{\infty} \delta_k \beta^k\right) < 1$. In this case,

$$h(1) = \frac{1-\beta}{\tau_y} - \frac{M_d}{1 - M_y\left(1-\tau_y\right)\left(1 + \sum_{k=1}^{+\infty} \delta_k \beta^k\right)} < 0$$

from (2). As a result, there exists a unique $\rho_d \in (0, \bar{\rho})$ such that $f\left(\rho_d\right) = 0$. Given ρ_d, one can find ρ_ϵ and $\chi_d, \chi_\epsilon > 0$ from (D.15) – (D.17) to constitute a self-financing equilibrium in (D.14).

Part 2 directly follows from (D.9), of which the derivation does not depend on the exact form of aggregate demand curve.

D.6 Proof of Theorem 3

In this proof, I restrict $\phi \in [-1/\sigma, \frac{\tau_y}{\beta \frac{D^{ss}}{Y^{ss}}}]$. The AD in (9) together with monetary policy (32), market clearing $y_t = c_t$, and government budget (2) lead to a recursive AD curve:

$$y_t = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}}{1+\sigma\phi-\frac{(1-\beta\omega)(1-\omega)}{\omega}\phi\frac{D^{ss}}{Y^{ss}}}(d_t - t_t) + \frac{1}{1+\sigma\phi-\frac{(1-\beta\omega)(1-\omega)}{\omega}\phi\frac{D^{ss}}{Y^{ss}}}\mathbb{E}_t\left[y_{t+1}\right].$$

Together with the baseline fiscal policy (4), we arrive at the following recursive AD curve:

$$y_t = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}(d_t+\epsilon_t) + \frac{1}{1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}\mathbb{E}_t\left[y_{t+1}\right].$$

Applying period-t expectation $\mathbb{E}_t[\cdot]$ to (14), we have

$$\begin{pmatrix} \mathbb{E}_t[d_{t+1}] \\ \mathbb{E}_t\left[y_{t+1}\right] \end{pmatrix} = \begin{pmatrix} \frac{1-\tau_d}{\beta} & -\frac{\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}}{\beta} \\ -\frac{(1-\beta\omega)(1-\omega)(1-\tau_d)}{\beta\omega} & 1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}{\beta\omega} \end{pmatrix}\begin{pmatrix} d_t+\epsilon_t \\ y_t \end{pmatrix} \tag{D.18}$$

Two eigenvalues of of the system are given by the solutions of

$$\lambda^2 - \lambda\left(\frac{1-\tau_d}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)+\left(1+\sigma\phi\right)\frac{1-\tau_d}{\beta}=0. \tag{D.19}$$

From $\phi \in [-1/\sigma, \frac{\tau_y}{\beta\frac{D^{ss}}{Y^{ss}}}]$ and $\tau_d \in [0,1]$, we know that $\lambda_1+\lambda_2 \geq 0$ and $\lambda_1\lambda_2 \geq 0$, so $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$.

We first prove Part 1 of Theorem 3. That is, if

$$\phi < \bar{\phi} \equiv \frac{\frac{(1-\beta\omega)(1-\omega)}{\omega}\tau_y}{\sigma\left(1-\beta\right)+\frac{(1-\beta\omega)(1-\omega)}{\omega}\beta\frac{D^{ss}}{Y^{ss}}}, \tag{D.20}$$

there exists a bounded, complete self-financing equilibrium of the form (16) with $\rho_d \in (0,1)$, $\nu \to 1$, and where ρ_d is increasing in ϕ.

Consider the limit that $\tau_d \to 0$. Since the eigenvalue of (D.19) is continuous with τ_d at 0, we can directly look at the case with $\tau_d = 0$. Here, we show that, when (D.20) holds, one can find a complete self-financing bounded equilibrium in the form (16) with $\tau_d = 0$.

Specifically, when $\tau_d = 0$, the two eigenvalues of (D.19) are given by

$$\begin{aligned}\lambda_1 &= \frac{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)+\sqrt{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2-4\frac{1+\sigma\phi}{\beta}}}{2} \\ &= \frac{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)+\sqrt{\left(1+\sigma\phi-\frac{1}{\beta}-\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2+4\left(1+\sigma\phi\right)\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}}{2}\end{aligned}$$

and

$$\lambda_2 = \frac{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)-\sqrt{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2-4\frac{1+\sigma\phi}{\beta}}}{2}$$
$$= \frac{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)-\sqrt{\left(1+\sigma\phi-\frac{1}{\beta}-\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2+4\left(1+\sigma\phi\right)\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}}{2}. \tag{D.21}$$

When $\phi\in[-1/\sigma,0]$, from (D.21),

$$\lambda_2 \le \frac{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)-\left|1+\sigma\phi-\frac{1}{\beta}-\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right|}{2} \le 1+\sigma\phi<1$$

When $\phi\in\left(0,\bar{\phi}\right)$, from (D.20), we have

$$\frac{\left(1-\beta\omega\right)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)>\sigma\phi\left(\frac{1}{\beta}-1\right).$$

Hence

$$\lambda_2 = \frac{2\frac{1+\sigma\phi}{\beta}}{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)+\sqrt{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2-4\frac{1+\sigma\phi}{\beta}}}$$
$$< \frac{2\frac{1+\sigma\phi}{\beta}}{\frac{1}{\beta}+1+\frac{\sigma\phi}{\beta}+\sqrt{\left(1+\frac{1+\sigma\phi}{\beta}\right)^2-4\frac{1+\sigma\phi}{\beta}}} = \frac{2\frac{1+\sigma\phi}{\beta}}{1+\frac{1+\sigma\phi}{\beta}+\left|\frac{1+\sigma\phi}{\beta}-1\right|} = 1.$$

Together, as long as (D.20), there exists an eigenvalue of (D.18) (with $\tau_d=0$), $\lambda_2\in[0,1)$. As a result, bounded equilibrium in the form of (16) with $\rho_d=\lambda_2$ and $\chi=1-\beta\rho_d>0$ (from (17)) will be a solution of (D.18). Given (16), one can then back out $\{\pi_t, d_t-\mathbb{E}_{t-1}[d_t]\}$ from (13) – (15). Together, we find a complete self-financing bounded equilibrium $\{y_t,d_t,\pi_t\}_{t=0}$.

For Part 2 of Theorem 3, note that, when $\phi>\bar{\phi}$, from (D.20), we have

$$\frac{\left(1-\beta\omega\right)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)<\sigma\phi\left(\frac{1}{\beta}-1\right).$$

Hence

$$\lambda_2 > \frac{2\frac{1+\sigma\phi}{\beta}}{\frac{1}{\beta}+1+\frac{\sigma\phi}{\beta}+\sqrt{\left(1+\frac{1+\sigma\phi}{\beta}\right)^2-4\frac{1+\sigma\phi}{\beta}}} = 1.$$

As a result, there exists no bounded completely self-financing equilibrium ($\tau_d=0$).

Now we study the case with $\phi>\bar{\phi}$ and $\tau_d>0$. We prove that, only when τ_d is sufficiently large (i.e.,

fiscal adjustment is sufficiently fast),

$$\tau_d > \bar{\tau}_d(\phi) \equiv 1-\beta-\frac{1}{\sigma\phi}\frac{1-\beta\omega}{\omega}\left(\tau_y-\beta\phi\frac{B^{ss}}{Y^{ss}}\right)(1-\omega), \tag{D.22}$$

any kind of bounded equilibrium to exist. First, when (D.22) holds, we have

$$\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{B^{ss}}{Y^{ss}}\right) > \sigma\phi\left(\frac{1-\tau_d}{\beta}-1\right),$$

As a result,

$$\begin{aligned}
\lambda_2 &= \frac{\left(\frac{1-\tau_d}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{B^{ss}}{Y^{ss}}\right)\right)-\sqrt{\left(\frac{1-\tau_d}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{B^{ss}}{Y^{ss}}\right)\right)^2-4\left(1+\sigma\phi\right)\frac{1-\tau_d}{\beta}}}{2} \\
&= \frac{2\left(1+\sigma\phi\right)\frac{1-\tau_d}{\beta}}{\frac{1-\tau_d}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{B^{ss}}{Y^{ss}}\right)+\sqrt{\left(\frac{1-\tau_d}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{B^{ss}}{Y^{ss}}\right)\right)^2-4\left(1+\sigma\phi\right)\frac{1-\tau_d}{\beta}}} \\
&< \frac{2\left(1+\sigma\phi\right)\frac{1-\tau_d}{\beta}}{\frac{(1-\tau_d)(1+\sigma\phi)}{\beta}+1+\sqrt{\left(\frac{(1-\tau_d)(1+\sigma\phi)}{\beta}+1\right)^2-4\frac{(1-\tau_d)(1+\sigma\phi)}{\beta}}} \\
&= \frac{2\left(1+\sigma\phi\right)\frac{1-\tau_d}{\beta}}{\frac{(1-\tau_d)(1+\sigma\phi)}{\beta}+1+\left|\frac{(1-\tau_d)(1+\sigma\phi)}{\beta}-1\right|} \\
&\leq 1.
\end{aligned}$$

So a bounded equilibrium in the form (16) exists.

Second, when (D.22) does not hold, we have

$$\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{B^{ss}}{Y^{ss}}\right) \leq \sigma\phi\left(\frac{1-\tau_d}{\beta}-1\right),$$

which means

$$\sigma\phi\left(\frac{1-\tau_d}{\beta}-1\right) \geq 0 \Longrightarrow \left(1+\sigma\phi\right)\frac{1-\tau_d}{\beta} > 1,$$

and λ_2 (the smaller eigenvalue of (D.19)),

$$\begin{aligned}
\lambda_2 &= \frac{\left(\frac{1-\tau_d}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{B^{ss}}{Y^{ss}}\right)\right)-\sqrt{\left(\frac{1-\tau_d}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{B^{ss}}{Y^{ss}}\right)\right)^2-4\left(1+\sigma\phi\right)\frac{1-\tau_d}{\beta}}}{2} \\
&= \frac{2\left(1+\sigma\phi\right)\frac{1-\tau_d}{\beta}}{\frac{1-\tau_d}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{B^{ss}}{Y^{ss}}\right)+\sqrt{\left(\frac{1-\tau_d}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{B^{ss}}{Y^{ss}}\right)\right)^2-4\left(1+\sigma\phi\right)\frac{1-\tau_d}{\beta}}} \\
&\geq \frac{2\left(1+\sigma\phi\right)\frac{1-\tau_d}{\beta}}{\frac{(1-\tau_d)(1+\sigma\phi)}{\beta}+1+\sqrt{\left(\frac{(1-\tau_d)(1+\sigma\phi)}{\beta}+1\right)^2-4\left(1+\sigma\phi\right)\frac{1-\tau_d}{\beta}}} \\
&= \frac{2\left(1+\sigma\phi\right)\frac{1-\tau_d}{\beta}}{\frac{(1-\tau_d)(1+\sigma\phi)}{\beta}+1+\left|1-\frac{(1-\tau_d)(1+\sigma\phi)}{\beta}\right|} \\
&\geq 1.
\end{aligned}$$

As a result, there is no bounded equilibrium.

D.7 Proof of Proposition C.1

We note that the proof heavily leverages results from Wolf (2021a). Following that paper, all arguments are established using sequence-space notation, with boldface denoting time paths.

The sequence of wealth holdings associated with an interest rate sequence $\boldsymbol{r}$ (both in deviation from steady state) is given as

$$\boldsymbol{d}(\boldsymbol{r}) = \mathscr{D}_r \times \boldsymbol{r}$$

where $\mathscr{D}_r$ is the sequence-space Jacobian of wealth holdings with respect to interest rates. The desired long-run elasticity η is the long-run response of asset holdings to a permanent change in interest rates; that is, it is given as the limit (if it exists) of the sequence $\boldsymbol{d}(\mathbf{1})$.

It follows from the aggregate household budget constraint that the savings matrix $\mathscr{D}_r$ and the analogous consumption matrix $\mathscr{C}_r$ are related as[28]

$$\mathscr{C}_r + \frac{1}{1+\bar{r}}\mathscr{D}_r = \begin{pmatrix} \mathbf{0}' \\ \mathscr{D}_r \end{pmatrix} \tag{D.23}$$

where $1+\bar{r} = \beta^{-1}$. Since by definition

$$\eta = \lim_{H\to\infty} \mathscr{D}_r(H,\bullet)\times\mathbf{1}$$

it follows from (D.23) that we have

$$\eta = \frac{1+\bar{r}}{\bar{r}} \lim_{H\to\infty} \mathscr{C}_r(H,\bullet)\times\mathbf{1} \tag{D.24}$$

[28] Note that this construction removes income effects related to steady-state wealth holdings.

It thus remains to characterize $\mathscr{C}_r$. For this we momentarily assume that there are no spenders ($\mu = 0$); the extension to the full spender-OLG model is straightforward and will come at the end. It follows from the results in Wolf (2021a) that $\mathscr{C}_r$ has the following limiting properties:

$$\begin{aligned}\lim_{H\to\infty} \mathscr{C}_r(H,H) &= -\sigma\beta\omega\frac{1-\omega}{1-\beta\omega^2}\\ \lim_{H\to\infty} \mathscr{C}_r(H,H-1) &= \sigma\frac{\omega(1-\beta\omega)}{1-\beta\omega^2}\end{aligned}$$

as well as

$$\begin{aligned}\lim_{H\to\infty} \frac{\mathscr{C}_r(H,H-s)}{\mathscr{C}_r(H,H-s+1)} &= \omega\beta, \quad s\geq 2\\ \lim_{H\to\infty} \frac{\mathscr{C}_r(H,H+s)}{\mathscr{C}_r(H,H+s-1)} &= \omega, \quad s\geq 1\end{aligned}$$

Plugging those relations into (D.24) and simplifying, we find

$$\eta = \frac{1}{1-\beta}\sigma\left[\frac{1}{1-\theta} - \frac{1}{1-\beta\theta}\right] \tag{D.25}$$

Finally, if there is a margin of spenders, then the elasticity is simply scaled down to correspond to the margin of OLG households ($1-\mu$), thus giving (C.2).