# Monetary-Based Asset Pricing: A Mixed-Frequency Structural Approach


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**Abstract**

We integrate a high-frequency monetary event study into a mixed-frequency macro-finance model and structural estimation. The model and estimation allow for jumps at Fed announcements in investor beliefs, providing granular detail on why markets react to central bank communications. We find that the reasons involve a mix of revisions in investor beliefs about the economic state and/or future regime change in the conduct of monetary policy, and subjective reassessments of financial market risk. However, the structural estimation also finds that much of the causal impact of monetary policy on markets occurs outside of tight windows around policy announcements.



JEL: E7, E27, E32, E17, G4.

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# 1 Introduction

It is practically a truism that the stock market is highly attuned to monetary news. Academic studies are generally consistent with this maxim, finding that the real values of long-term financial assets fluctuate sharply in response to the actions and announcements of central banks. Why?

A growing academic literature has offered a myriad of competing explanations. A classic view is that surprise central bank announcements proxy for shocks to a nominal interest rate rule of the type emphasized by Taylor (1993), which have short-run affects on the real economy in a manner consistent with canonical New Keynesian models (e.g., Christiano, Eichenbaum, and Evans (2005)). But other explanations abound, including the effects such announcements have on financial market risk premia, the information they impart about the state of the economy (the "Fed information effect"), or the role they play in revising the public's understanding of the central bank's reaction function and objectives. For the most part, the empirical facts of these asset market fluctuations have been established from high-frequency event studies in tight windows around Federal Reserve (Fed) communications, combined with estimations of reduced-form empirical specifications. By contrast, the interpretations of these facts largely follow from carefully calibrated theoretical models designed to show that one of the competing explanations fits with certain aspects of the reduced-form evidence.

Yet, as the mushrooming debate over how to interpret this evidence indicates, many questions about the interplay between markets and monetary policy remain unanswered. In this paper we consider three of them. First, theories focused on a single channel of monetary transmission are useful for elucidating its marginal effects, but may reveal only part of the overall picture. To what extent are several competing explanations or others entirely playing a role simultaneously? Second, monetary policy communications cover a range of topics, from interest rate policy, to forward guidance, to quantitative interventions, to the macroeconomic outlook. How do these varied communications affect market participants' perceptions of the primitive economic sources of risk hitting the economy? Third, by design, high frequency events studies only capture the causal effects of the surprise component of a monetary policy announcement. At best, this represents a binding lower bound on the overall causal impact of monetary policy on markets; at worst, it represents a gross underestimate. How much of the causal influence of shifting monetary policy occurs outside of tight windows around Fed communications, effects that are by construction impossible to observe from high-frequency event studies alone?

Our contribution to addressing these questions is to integrate a high-frequency event study into a mixed-frequency structural model and estimation. We examine Fed communications alongside both high- and lower-frequency data through the lens of a structural equilibrium asset pricing model with New Keynesian style macroeconomic dynamics. We use dozens of

series ranging from minutely financial market data to biannual survey forecast data in our estimation. The model and estimation allow for jumps in investor beliefs about the latent state of the economy, the perceived sources of economic risk, and the future conduct of monetary policy. The novelty of this approach allows us to investigate a variety of possible explanations for why markets respond strongly and swiftly to central bank actions and announcements, not merely by delineating which expectations are revised, but also by providing granular detail on why they are revised, with a decomposition of market responses into the primitive economic sources of risk responsible for observed forecast revisions. The mixed-frequency structural estimation further permits us to quantify the causal effects of shifts in monetary policy that may occur outside of tight windows surrounding Fed communications. Structural asset pricing models are especially valuable in this context because they place cross-equation restrictions on the type of news capable of moving the real values of extreme long-duration assets like the stock market, where expected payout accrues not just over the next business cycle or even the next decade, but indefinitely. The general approach can be applied in a wide variety of other structural and semi-structural settings, whenever a granular understanding of financial market responses to almost any type of news is desired.

In this paper, we apply the approach to a two-agent asset pricing model with New Keynesian style macroeconomic dynamics in which the two agents have heterogeneous beliefs, as in Bianchi, Lettau, and Ludvigson (2022). One agent is a representative "investor" who is forward-looking, reacts swiftly to news, and earns income solely from investments in two assets: the aggregate stock market and a one-period nominal bond. The representative investor takes macroeconomic dynamics as given. Macroeconomic dynamics are specified by a set of equations similar to those commonly featured in New Keynesian models, and are driven by a representative "household/worker" that supplies labor and has access to the nominal bond but holds no stock market wealth. The household/worker forms expectations in a backward-looking manner using adaptive learning rules.

An important feature of our model is that the conduct of monetary policy is not static over time, but is instead subject to infrequent nonrecurrent regime shifts, or "structural breaks." These take the form of shifts in the parameters of a nominal interest rate rule. Infrequent movements in the Fed's future reaction function and objectives give rise to endogenously persistent movements in real interests rates in the two-agent model and are an important reason for investors in the model to attend closely to what the central bank does and says. Such changes in what we refer to as the *conduct* of monetary policy give rise to movements in the nominal interest rate that are conceptually distinct from those generated by the monetary policy *shock*, an innovation in the nominal rate that is uncorrelated with inflation, economic growth, and shifts in the policy rule parameters.

We explicitly model investor beliefs about changes in the monetary policy rule. Investors in the model can estimate the current rule, but face uncertainty over how long the current

rule will remain in place and what will follow once the current regime ends. Central bank communications are closely monitored for information that would cause agents to revise the perceived likelihood of transitioning from the current policy regime to a perceived "Alternative" policy rule that they believe will come next. Investors are aware that they may change their minds subsequently in response to new information, and take that into account when forming expectations.

Importantly, however, investor reactions to central bank communications are not restricted to be only about the path of future short rates driven by shifts in the policy rule. A Fed announcement in our model is an actual news event to which investors may react by revising their nowcasts and forecasts of the current and future economic state, their beliefs about the future conduct of monetary policy, and their perceived risk in the stock market. To ensure that model expectations evolve in a manner that closely aligns with observed expectations, we map the theoretical implications for these beliefs into data on numerous forward-looking variables, including household and professional forecast surveys and financial market indicators from spot and futures markets, estimating all parameters and latent states.

The full structural framework is solved and estimated using Bayesian methods. The equilibrium solution illustrates the rich endogenous interactions between beliefs about central bank policy and the rest of the economy. Beliefs about the future conduct of monetary policy not only have direct effects on the real economy, they also amplify and propagate economic shocks that are entirely non-monetary in nature and cause the perceived quantity of stock market risk to vary with the expected future conduct of monetary policy.

Before undertaking the full structural model estimation, we establish model-free evidence of regime change in the conduct of monetary policy over our sample by documenting the existence of decades-long deviations of the real federal funds rate from a widely used measure of the neutral rate of interest, as in Bianchi, Lettau, and Ludvigson (2022). We refer to the difference between the two as the *monetary policy spread*, or *mps* for brevity. We estimate infrequent nonrecurrent regime shifts, i.e., "structural breaks," in the mean of *mps* that divide the sample from 1961:Q1 to 2020:Q1 into three distinct subperiods: a "Great Inflation" regime (1961:Q1-1978:Q3), a "Great Moderation" regime (1978:Q4-2001:Q1), and a "Post Millennial" regime (2001:Q2-2020:Q1). We use these estimates to pin down the timing of realized regime changes in monetary policy over our sample, while the structural model is used to assess whether estimated policy rules actually shifted across these exogenously identified subperiods.

Our main empirical results may be summarized as follows. First, the estimates imply that investors seldom learn only about conventional monetary policy shocks from central bank announcements. Instead, jumps in financial market variables are typically the result of a mix of factors, including revisions in investor beliefs about the economic state and/or about near-term regime change in monetary policy conduct. Indeed, the most quantitatively important FOMC announcements in our sample are associated with large high frequency revisions in professional

forecasts of inflation and GDP growth, as well as jumps in federal funds futures and the stock market. These fluctuations are themselves associated with announcement-driven revisions in the composition of primitive shocks that investors perceive are hitting the economy. For example, the FOMC announcement of January 22, 2008–at the height of the Great Financial Crisis– is associated with a 1.9% decline in the stock market. Our estimates imply that the market's nosedive in the minutes surrounding this announcement was driven by large downward revisions in investor nowcasts of aggregate demand and the earnings share of output, and an upward revision in the subjective risk premium. These forces outweighed other announcement-related factors that contributed to a rise in the market, such as the perception of an accommodative monetary policy shock and faster trend economic growth.

Second, a key aspect of the model for explaining stock market behavior is the evolution of investor beliefs about regime change in the monetary policy rule. We find that fluctuating beliefs about the future conduct of monetary policy can cause significant market volatility even if the current policy rule and target interest rate remain unchanged. Moreover, the estimates show that investor beliefs about regime change continuously evolve *out*side of tight windows around policy announcements and that most of the variation in these beliefs occurs at times that are not close to an FOMC announcement. An obvious explanation for this result is that most Fed announcements are not immediately associated with a change in the policy rule, but instead provide "forward guidance" in the form of a data-dependent recipe for what could trigger a change in the conduct of policy down the road. Overall, we find that a large fraction of the variation in the stock market and in the short-term real interest rate across time is explained by the combination of realized regime changes in the conduct of monetary policy, and fluctuating real-time beliefs about the possibility of future regime change. These results underscore the challenges with relying solely on high-frequency event studies for quantifying the channels of monetary transmission to markets and the real economy.

Finally, our results indicate that investor beliefs about a future regime change are especially important for the stock market because of their role in shaping perceptions of equity market risk. We find that the S&P 500 would have been 50% higher than it was in February of 2020, had investors counterfactually believed that the Fed was very likely to shift to a policy rule in the next month that featured greater activism in stabilizing the real economy.

**Related literature** The research in this paper touches on several strands of literature that connect monetary policy to movements in asset values. On the empirical side, we connect with a body of evidence that finds the values of long-term financial assets and expected return premia respond sharply to the announcements of central banks (Cochrane and Piazzesi (2002), Piazzesi (2005), Bernanke and Kuttner (2005), Krishnamurthy and Vissing-Jorgensen (2011), Hanson and Stein (2015), Gertler and Karadi (2015), Gilchrist, López-Salido, and Zakrajšek (2015), Boyarchenko, Haddad, and Plosser (2016), Brooks, Katz, and Lustig (2018), Kekre

and Lenel (2021), Cox, Greenwald, and Ludvigson (2020), Haddad, Moreira, and Muir (2020), Pflueger and Rinaldi (2020)).[1] We add to this literature by providing evidence that expected return premia vary in part because the perceived quantity of stock market risk fluctuates with beliefs about the future conduct of monetary policy.

A classic assumption in the extant literature is that high-frequency financial market reactions to Fed announcements proxy for conventional monetary policy "shocks," i.e., innovations in a Taylor (1993)-type nominal interest rate rule (e.g., Cochrane and Piazzesi (2002), Piazzesi (2005), Hanson and Stein (2015), Kekre and Lenel (2021); Pflueger and Rinaldi (2020)). By contrast, Jarocinski and Karadi (2020), Cieslak and Schrimpf (2019) and Hillenbrand (2021) argue that some of the fluctuations are likely driven by the revelation of private information by the Fed, a "Fed information effect" channel emphasized in earlier work by Romer and Romer (2000), Campbell, Evans, Fisher, Justiniano, Calomiris, and Woodford (2012), Melosi (2017), and Nakamura and Steinsson (2018). By contrast, Bauer and Swanson (2021) emphasize a "response to news" channel whereby markets are surprised by the response of the Fed to recent economic events. The mixed-frequency structural approach proposed in this paper can be used to empirically diagnose and distinguish among these types of alternative channels in the propagation of news shocks.

All of the papers cited above form their conclusions from reduced-form empirical event studies, a natural starting point. Yet the absence of a rich structural interpretation of these events makes it challenging to provide granular detail on why markets react so strongly to Fed news or to investigate whether multiple channels may be playing a role simultaneously, gaps our mixed-frequency structural approach is designed to fill.

Beyond event studies, contemporaneous work by Bauer, Pflueger, and Sundaram (2022) uses monthly survey data to estimate perceived policy rules, finding that they are subject to substantial time-variation. Their study differs from ours in that they do not investigate the joint determination of beliefs, the macroeconomy, financial markets, and the policy rule in a macro-finance model, or integrate a high-frequency event study into a structural framework, as is the focus of this paper.

Our work relates to a theoretical literature focused on the implications of monetary policy for asset prices. Piazzesi (2005) finds that accounting for monetary policy significantly improves the performance of traditional yield curve models with three latent factors. Kekre and Lenel (2021) and Pflueger and Rinaldi (2020) develop carefully calibrated theoretical models that imply stock market return premia vary in response to a monetary policy shock. These theories use different mechanisms but are all silent on the possible role of Fed announcement information

[1] These studies follow on earlier work finding a link between monetary policy surprises and short-term assets in high-frequency data (Cook and Hahn (1989), Gürkaynak, Sack, and Swanson (2005)). A separate literature studies the timing of when premia in the aggregate stock market are earned in weeks related to Federal Open Market Committee- (FOMC)-cycle time (Lucca and Moench (2015), Cieslak, Morse, and Vissing-Jorgensen (2019)).

effects or of changing policy rules in driving market fluctuations, features that are at the heart of our analysis.

The two-agent structural model of this paper builds on Bianchi, Lettau, and Ludvigson (2022) (BLL hereafter), who find that infrequent changes in the conduct of monetary policy generate large and persistent fluctuations in the real interest rate, in asset valuations, and in the equity premium. Nevertheless, this study differs substantively from BLL in a number of foundational ways. In comparison to BLL, this study uses a much larger dataset on forward-looking information, culls relevant information at different frequencies, and explicitly models high-frequency revisions in beliefs about monetary policy in the minutes surrounding Fed announcements, as well as at lower frequencies. The mixed-frequency structural approach of this paper offers a significant methodological advance over BLL and, to the best of our knowledge, the extant literature. Moreover, unlike BLL, we model regime changes in the conduct of monetary policy as nonrecurrent regimes, i.e., structural breaks, rather than recurrent regime-switching. We argue that structural breaks are a more plausible specification, since new policy regimes never exactly repeat old ones. Yet this requires a model of how expectations are formed in the presence of structural breaks. We show how forward looking variables, such as survey expectations and asset prices, can be used both to estimate the probability of a near-term policy rule regime change and to extract beliefs about the nature of future policy regimes. Thus, the model of this paper also innovates with respect to the literature on regime changes in general equilibrium models, which typically only considers recurrent regime-switching.

In contemporaneous work, Caballero and Simsek (2022) also study a two-agent, "two-speed" economy with investors and households similar in spirit to our framework. They postulate that the Fed directly controls aggregate asset prices in an attempt to steer the spending decisions of households. This differs from our study in that it is a purely theoretical investigation that studies asset pricing at an abstract level by thinking of the risky asset price as a broad-based financial conditions index. Our objective is instead to address the questions posed above by integrating a high-frequency monetary event study into a mixed-frequency asset pricing model and structural estimation, specifically modeling the risky asset as the stock market.

Finally, our mixed-frequency structural approach connects with a pre-existing econometric forecasting/nowcasting literature using mixed-frequency data in state space models (e.g., Giannone, Reichlin, and Small (2008), Ghysels and Wright (2009), Schorfheide and Song (2015)). The objective of these studies is to augment lower frequency prediction models with more timely high-frequency data. This is typically accomplished by specifying the state/transition equations at the highest frequency of data used. Our use of mixed-frequency data is designed for a very different purpose, namely as way of integrating a high-frequency event study into a structural model and estimation. Thus the standard reduced-form approach of specifying transition equations at highest frequency data sampling interval (minutely in our case) would be both impractical and inappropriate here, since the data sampling interval of the state/transition

equation is part of the structural model and needs to correspond to the optimizing decision intervals of agents. Instead, our approach uses forward-looking data available *within* the decision interval to infer revisions in the intraperiod beliefs of investors about the economic state to be realized at the *end* of the decision interval. This allows us to treat Fed announcements as bonafide news shocks (as perceived by investors) rather than as ultra high frequency primitive shocks. In the process, we preserve a cornerstone of high-frequency event study design, which is to measure the causal effect of the announcement itself, while plausibly holding fixed the current economic state.

The rest of this paper is organized as follows. The next section presents preliminary model-free empirical evidence that we use to pin down the timing of monetary regime changes in our sample. Section 3 describes the mixed-frequency structural macro-finance model and equilibrium solution. Section 4 describes the structural estimation, while Section 5 presents our empirical findings from the structural estimation. Section 6 concludes. A large amount of additional material on the model, estimation, and data has been placed in an Online Appendix.

## 2 Preliminary Evidence

Before getting into a detailed structural estimation, we investigate whether there is plausible model-free evidence of infrequently shifting monetary regimes over our sample. As in BLL, infrequent shifts in the stance of monetary policy generate persistent changes in real interest rates if aggregate inflation expectations are dominated by households who form beliefs using adaptive learning rules subject to substantial inertia, and forward-looking investors understand this. In contrast to canonical representative-agent New Keyenesian models, this generates persistent monetary non-neutrality and rationalizes how it is that central bank actions and announcements can have non-negligible effects on the real values of extreme long-duration assets, such as the stock market.

To that end, consider Figure 1, which plots the behavior over time of a key instrument of monetary policy, namely the federal funds rate, measured for the purposes of this plot in real terms as the nominal rate minus a four quarter moving average of inflation. The left panel plots this series along with an estimate of $r^*$ from Laubach and Williams (2003).[2] The data are quarterly and span the sample 1961:Q1-2020:Q1.[3]

The right panel plots the spread between the real funds rate and this measure of the neutral rate of interest, a variable we refer to as the *monetary policy spread*, and denote its time $t$ value as $mps_t$.[4] Since the Federal Reserve targets the federal funds rate but in theory has no

[2] In Laubach and Williams (2003) the neutral or natural rate is a purely empirical measure that amounts to estimates of the level of the real federal funds rate that consistent with no change in inflation.

[3] The 1961 start date is dictated by the availability of the natural rate of interest measure.

[4] $mps_t$ is computed as

$$FFR_t - (\text{Expected Inflation})_t - r_t^*,$$

control over the neutral rate, a non-zero value for $mps_t$ may be considered a measure of the stance of monetary policy, i.e., whether monetary policy is accommodative or restrictive, with spreads above zero indicative of restrictive monetary policy and those below zero indicative of accommodative monetary policy. According to this measure of the *mps*, monetary policy was accommodative over the sample up until about 1980, then sharply restrictive from about 1980 to about 2000, and subsequently mostly accommodative. While there is no secular trend downward in real interest rates over the full sample, there are large, low frequency movements in the real funds rate and a noticeable downward trend in both the real interest rate and *mps* since about 1980.

We allow for the possibility of regime changes in the mean of the *mps*:

$$mps_t = r_{\xi_t^P} + \epsilon_t^r, \tag{1}$$

where $\epsilon_t^r \sim N(0, \sigma_r^2)$, and the coefficient $r_{\xi_t^P}$ is an intercept governed by a discrete valued latent state variable, $\xi_t^P$, that is presumed to follow a $N_P$-state *nonrecurrent* regime-switching Markov process discussed below, with transition matrix $\mathbf{H}$. Let the vector $\boldsymbol{\theta}_r = \left(r_{\xi_t^P}, \sigma_r^2, vec(\mathbf{H})'\right)'$ denote the set of parameters to be estimated. Values for $r_{\xi_t^P} > 0$ are indicative of restrictive monetary policy, while values for $r_{\xi_t^P} < 0$ are indicative of accommodative policy.

We assume that the true data generating process for $\xi_t^P$ leads to infrequent regime changes in $r_{\xi_t}$ that are *nonrecurrent*. That is, when the stance of monetary policy shifts, there is no expectation that it must move to a regime that is exactly the same as one in the past (mathematically a probability zero event), though it could be quite similar. BLL estimate a similar specification using recurrent regime-switching with two latent states. Here, the estimation is free to choose $r_{\xi_t^P}$ across regimes that are arbitrarily close to those that have occurred in the past, without being identically equal. We view the specification of this paper as both more flexible and more general than a recurrent regime-switching model where parameters can only shift to one of a finite number of values that would necessarily have to recur in a long enough sample. The Online Appendix explains how the structural breaks can be modeled as nonrecurrent regime-switching with transition matrix $\mathbf{H}$ and $N_P$ nonrecurrent regimes ($N_P - 1$ structural breaks).

We use Bayesian methods with flat priors to estimate the model parameters in (1) over the period 1961:Q1-2020:Q1 and to estimate the most likely historical regime sequence $\xi_t^P$ over that sample. This procedure is described in the Online Appendix.

Figure 2 reports the results for the case of two structural breaks ($N_P = 3$) with the dates corresponding to the three regime subperiods reported in Table 1. We identify a first subperiod of accommodative monetary policy from 1961:Q1 to 1978:Q3, where $mps_t$ is persistently

where $FFR$ is the nominal federal funds rate and where expected inflation is a four quarter moving average of inflation. $r_t^*$ is the neutral rate of interest from Laubach and Williams. The quarterly nominal funds rate is the average of monthly values of the effective federal funds rate.

negative and its mean $r_{\xi_t^P} = -2.67\%$ at the posterior mode. This period coincides with the run up in inflation that began in the mid-1960s and with two oil shocks in the 1970s that were arguably exacerbated by a Fed that failed to react sufficiently proactively ((Clarida, Gali, and Gertler (2000); Lubik and Schorfheide (2004); Sims and Zha (2006); Bianchi (2013))). We refer to this first regime the "Great Inflation" regime. A second regime begins in 1978:Q4, when a structural break in the series drove an upward jump in the $mps_t$, leaving its mean $r_{\xi_t^P} = 1.38\%$ at the posterior mode. This period of restrictive monetary policy lasted until 2001:Q1 and covers the Volcker disinflation and moderation in economic volatility that followed. We label this second subperiod the "Great Moderation" regime. The third "Post Millennial" regime starts in 2001:Q2 and represents a new prolonged period of accommodative monetary policy, where $r_{\xi_t^P} = -1.27\%$ at the posterior mode. The beginning of this regime is labeled the "Greenspan Put," since it follows shortly after the inception of public narratives on the perceived attempt of Chair Greenspan to prop up securities markets in the wake of the IT bust, a recession, and the aftermath of 9/11, by lowering interest rates. The low *mps* subperiod at the end of the sample overlaps with the explicit forward guidance "low-for-long" policies under Chair Bernanke that repeated promised over several years to keep interest rates at ultra low levels for an extended period of time. Below we refer to the Great Inflation, the Great Moderation and the Post Millennial regimes in abbreviated terms as the GI, GM, and PM regimes.

Figure 2 shows that the low frequency deviations of the $mps_t$ from zero are quantitatively large and persistent across the three estimated regime subperiods. We argue that such evidence is strongly suggestive of structural change in the conduct of monetary policy over the course of our sample. In the next section we formally assess whether estimated monetary policy rules actually shifted across these subperiods. To accomplish this, we set the break dates for regime changes in the policy rule in the structural estimation to coincide with the regime sequence $\xi_t^P$ estimated using $mps_t$. We use Bayesian model comparison of different estimated structural models to decide on the appropriate number $N_P$ of policy regimes, and find $N_P = 3$ works well. With this, our structural estimation spans three different policy regimes across the Great Inflation, the Great Moderation, and the Post Millennial subperiods shown in Figure 2.

The preliminary evidence in this section allows us to build a structural model to fit these model-free empirical facts, rather than establishing evidence about the sequence of regimes that would be contingent on the details of the structural model. It should be emphasized, however, that the preliminary evidence of this section is used only to set the timing of any possible policy regime changes in the structural model. In particular, all regime-dependent parameters of the policy rule are freely estimated under flat priors, and in principle could show no change across the regime subperiods for $\xi_t^P$. We describe the structural model next.

# 3 A Mixed-Frequency Macro-Finance Model

This section presents a two-agent dynamic asset pricing model of monetary policy transmission. We work with a loglinear approximation to the model that can be solved analytically in which all random variables are conditionally lognormally distributed. Before getting into the details of this model, we provide a descriptive overview.

## 3.1 Overview

Risky asset prices are determined by the behavior of a representative investor who forms expectations in a forward-looking manner, reacts swiftly to news, and forms beliefs about future monetary policy. This agent earns all income from investments in two risky assets: the stock market and a risk-free nominal bond and takes macro dynamics as given. This agent is akin to a wealthy individual or large institution who owns the overwhelming majority of highly concentrated financial wealth in the U.S. but is small enough relative to the overall population that she takes macroeconomic dynamics as given. Households/workers supply labor, invest only in the bond, and form expectations using adaptive learning rules of the type documented in Malmendier and Nagel (2016). Their expectations predominate in aggregate inflation and output growth expectations. Monetary policy is characterized by a nominal interest rate rule subject to nonrecurrent regime changes, referred to as changes in the conduct of policy. Although there is no financial market trade between the two agents, that they are distinct agents who form expectations differently is fundamentally important for asset pricing. It is through such heterogeneity in their beliefs that regime changes in the conduct of monetary policy have large and protracted effects on real interest rates, despite the forward-looking, non-inertial nature of market participant expxectations.[5]

Both types of agents have a monthly decision intervals; hence $t$ denotes a month below. However, investors are attend to central bank communications, and we assume that their beliefs may exhibit jumps in response to those communications. Investors in the model are presumed to have enough information to estimate the current policy rule, and thus can observe when shifts in the policy rule occur. What they are uncertain about how long the current policy regime will last, and what the next policy rule will look like. This assumption can be motivated by noting that, in practice, Fed communications in the last 20 years have clearly promulgated the central bank's intention to change the stance of monetary policy, but have been comparatively vague about how long those changes will last and what is likely to come next.

Finally, we treat the policy rule parameters across nonrecurrent regimes as latent parameters to be estimated, an approach that side-steps the need to take a stand on why the Fed changes its policy rule. This is advantageous, since the reasons for such changes are likely to be difficult

[5] As in BLL, persistent monetary non-neutrality is an endogenous outcome of the inertia in household inflation expectations evident from household surveys, as discussed further below.

to plausibly parameterize as a function of historical data due to the degree of discretion the Fed has in interpreting its dual mandate and the possibility that distinct policy regimes are partly the result of a slow learning mechanism interacting with the bespoke perspectives of different central bank leaders across time.

## 3.2 Model Description

**Asset Pricing Block** There are a continuum of identical investors indexed by $i$ who derive utility from consumption at time $t$. Investors earn all income from trade in two assets: a one-period nominal risk-free bond and a stock market. In equilibrium, assets are priced by a representative investor who consumes per-capita aggregate shareholder payout, $D_t$. We therefore drop the $i$ index from here on and denote the consumption of the representative investor $D_t$.

The representative investor's intertemporal marginal rate of substitution in consumption is the stochastic discount factor (SDF) and takes the form:

$$M_{t+1} = \beta_{p,t} \left( D_{t+1}/D_t \right)^{-\sigma_p}, \tag{2}$$

where $\beta_{p,t} \equiv \beta_p \exp\left(\vartheta_{pt}\right)$ is a time-varying subjective time discount factor. The time discount factor is subject to an externality in the form of a patience shifter $\vartheta_{pt}$ that individual investors take as given, driven by the market as a whole. A time-varying specification for the subjective time-discount factor is essential for ensuring that, in equilibrium, investors are willing to hold the nominal bond at the interest rate set by the central bank's policy rule, specified below.

Let lowercase variables denote log variables, e.g., $\ln\left(D_t\right) = d_t$. We assume that aggregate payout is derived from a time-varying share $K_t$ of real output $Y_t$, implying $D_t = K_t Y_t$. Since in the model all earnings are paid out to shareholders, we refer to $K_t$ simply as the *earnings share* hereafter. The log payout to output ratio is $d_t - \ln\left(Y_t\right) = k_t$. Differencing this relation implies

$$\Delta d_t = \Delta k_t + \Delta \ln\left(Y_t\right). \tag{3}$$

Variation in the earnings share, $k_t$, is modeled as exogenous and latent following a specification given below.

The first-order-condition for optimal holdings of the one-period nominal risk-free bond with a face value equal to one nominal unit is

$$LP_t^{-1} Q_t = \mathbb{E}_t^b \left[ M_{t+1} \Pi_{t+1}^{-1} \right], \tag{4}$$

where $Q_t$ is the nominal bond price, $\mathbb{E}_t^b$ denotes the subjective expectations of the investor, and $\Pi_{t+1} = P_{t+1}/P_t$ is the gross rate of general price inflation. Investors' subjective beliefs, indicated with a "$b$" superscript on the expectation operator, play a central role in asset pricing and are discussed in detail below. We further assume that investors have a time-varying preference for

nominal risk-free assets over equity, accounted for by the term $LP_t > 1$ in (4), implying that the bond price $Q_t$ is higher than it would be absent these benefits, i.e., when $LP_t = 1$. We discuss the role of $LP_t$ further below.

Taking logs of (4) and using the properties of conditional lognormality delivers an expression for the real interest rate as perceived by the investor:

$$i_t - \mathbb{E}_t^b [\pi_{t+1}] = -\mathbb{E}_t^b [m_{t+1}] - .5\mathbb{V}_t^b [m_{t+1} - \pi_{t+1}] - lp_t \tag{5}$$

where the nominal interest rate $i_t = -\ln(Q_t)$, $\pi_{t+1} \equiv \ln(\Pi_{t+1})$ is net inflation, $\mathbb{V}_t^b[\cdot]$ is the conditional variance under the subjective beliefs of the investor, and $lp_t \equiv \ln(LP_t) > 0$.

Let $P_t^D$ denote total value of market equity, i.e., price per share times shares outstanding. The first-order-condition for optimal shareholder consumption obeys the following Euler equation:

$$\frac{P_t^D}{D_t} = \mathbb{E}_t^b \left[ M_{t+1} \left( \frac{P_{t+1}^D + D_{t+1}}{D_{t+1}} \right) \frac{D_{t+1}}{D_t} \right].$$

Taking logs on both sides of the above and using the properties of conditional lognormality, we obtain an expression for the log price-payout ratio $pd_t \equiv \ln\left(P_t^D/D_t\right)$:

$$\begin{aligned} pd_t &= \kappa_{pd,0} + \mathbb{E}_t^b [m_{t+1} + \Delta d_{t+1} + \kappa_{pd,1} pd_{t+1}] + \\ &\quad + .5\mathbb{V}_t^b [m_{t+1} + \Delta d_{t+1} + \kappa_{pd,1} pd_{t+1}]. \end{aligned}$$

The log equity return $r_{t+1}^D \equiv \ln\left(P_{t+1}^D + D_{t+1}\right) - \ln\left(P_t^D\right)$ obeys the following approximate identity (Campbell and Shiller (1989)):

$$r_{t+1}^D = \kappa_{pd,0} + \kappa_{pd,1} pd_{t+1} - pd_t + \Delta d_{t+1},$$

where $\kappa_{pd,1} = \exp(\overline{pd})/(1 + \exp(\overline{pd}))$, and $\kappa_{pd,0} = \log\left(\exp(\overline{pd}) + 1\right) - \kappa_{pd,1}\overline{pd}$. Combining all of the above, the log equity premium as perceived by the investor is:

$$\underbrace{\mathbb{E}_t^b \left[r_{t+1}^D\right] - \left(i_t - \mathbb{E}_t^b [\pi_{t+1}]\right)}_{\text{subj. equity premium}} = \underbrace{\left[ \begin{array}{c} -.5\mathbb{V}_t^b \left[r_{t+1}^D\right] - \mathbb{COV}_t^b \left[m_{t+1}, r_{t+1}^D\right] \\ +.5\mathbb{V}_t^b [\pi_{t+1}] - \mathbb{COV}_t^b [m_{t+1}, \pi_{t+1}] \end{array} \right]}_{\text{subj. risk premium}} + \underbrace{lp_t}_{\text{liquidity Premium}}, \tag{6}$$

where $\mathbb{COV}_t^b[\cdot]$ is the conditional covariance under the subjective beliefs of the investors. The equity premium has two components. The component labeled "subj. risk premium" is attributable to the agent's subjective perception of risk, which varies endogenously in the model with fluctuations in investor beliefs about the conduct of future monetary policy, as explained below. The term labeled "liquidity premium" represents a time-varying preference for risk-free nominal debt over equity and captures all sources of time-variation in the equity premium other than those attributable to subjective beliefs about the monetary policy rule. These include variation in the liquidity and safety attributes of nominal risk-free assets (e.g., Krishnamurthy and Vissing-Jorgensen (2012)), variation in risk aversion, flights to quality, or jumps in sentiment.

We refer to this catchall component simply as the *liquidity premium* hereafter. Variation in the liquidity premium, $lp_t$, is modeled as exogenous and latent following a specification given below.

We approximate our nonlinear SDF (2) as

$$m_{t+1} \simeq \ln\left(\beta_p\right) + \vartheta_{pt} - \sigma_p\left(\Delta d_{t+1}\right). \tag{7}$$

Combining (5) and (7), we see that $\vartheta_{p,t}$ is implicitly defined as

$$\vartheta_t^p = -\left[i_t - \mathbb{E}_t^b\left[\pi_{t+1}\right]\right] + \mathbb{E}_t^b\left[\sigma_p \Delta d_{t+1}\right] - .5\mathbb{V}_t^b\left[-\sigma_p \Delta d_{p,t+1} - \pi_{t+1}\right] - lp_t - \ln\left(\beta_p\right). \tag{8}$$

Summarizing, the model implies the following asset pricing relations:

1. Log SDF:
$$m_{t+1} = \log\left(\beta_p\right) + \vartheta_{pt} - \sigma_p\left(\Delta d_{t+1}\right) \tag{9}$$

2. Log price-payout ratio:
$$\begin{aligned} pd_t &= \kappa_{pd,0} + \mu + \mathbb{E}_t^b\left[m_{t+1} + \Delta d_{t+1} + \kappa_{pd,1} pd_{t+1}\right] + \\ &\quad + .5\mathbb{V}_t^b\left[m_{t+1} + \Delta d_{t+1} + \kappa_1 pd_{t+1}\right] \end{aligned} \tag{10}$$

3. Log Euler equation for bonds:
$$i_t - \mathbb{E}_t^b\left[\pi_{t+1}\right] = -\mathbb{E}_t^b\left[m_{t+1}\right] - .5\mathbb{V}_t^b\left[m_{t+1} + i_t - \pi_{t+1}\right] - lp_t \tag{11}$$

4. Log excess stock market return:
$$er_{t+1}^D = r_{t+1}^D - \left(i_t - \pi_{t+1}\right) = \kappa_{pd,0} + \kappa_{pd,1} pd_{t+1} - pd_t + \Delta d_{t+1} + \mu - \left(i_t - \pi_{t+1}\right) \tag{12}$$

5. Laws of motion for exogenous processes:
$$k_t - \overline{k} = \left(1 - \rho_k\right)\left(\lambda_{k,y}\widetilde{y}_t + \lambda_{k,\Delta y}\Delta y_t\right) + \rho_k\left(k_{t-1} - \overline{k}\right) + \sigma_k \varepsilon_{k,t} \tag{13}$$
$$lp_t - \overline{lp} = \rho_{lp}\left(lp_{t-1} - \overline{lp}\right) + \sigma_{lp}\varepsilon_{lp,t}. \tag{14}$$

   In the above, $\widetilde{y}_t$ is the output gap, defined below. Equation (13) allows the earnings share $k_t$ to vary with the output gap and economic growth, as well as an independent i.i.d. shock $\varepsilon_{kt} \sim N\left(0,1\right)$. The liquidity premium in equation (14) is specified to follow a first-order autoregressive (AR(1)) process subject to an i.i.d. shock $\varepsilon_{lp,t} \sim N\left(0,1\right)$.

**Macro Dynamics** Macroeconomic dynamics are driven by the behavior of households/workers (the "macro agent") and feature a set of equations similar to those commonly featured in New Keynesian models, with two distinctive features: adaptive learning, and regime changes in the conduct of monetary policy.[6] These distinctions are discussed below. We interpret equations (15) through (17) below as equilibrium dynamics and not a micro-founded structural model. We consider an equilibrium in which bonds are in zero-net-supply in both the macro and asset pricing blocks and thus there is no trade between the asset pricing agent and macro agent.[7]

Let $\ln(A_t/A_{t-1}) \equiv g_t$ represent the stochastic trend growth of the economy, which follows an AR(1) process $g_t = g + \rho_g(g_{t-1} - g) + \sigma_g \varepsilon_{g,t}$, $\varepsilon_{g,t} \sim N(0,1)$. Log of detrended output in the model is defined as $\ln(Y_t/A_t)$. As above, log variables are denoted in lower case, while log-detrended variables are denoted with a tilde, e.g., $\widetilde{y}_t = \ln(Y_t/A_t)$. This implies that $\widetilde{y}_t$ is positive when $y_t$ is above potential output, and negative when it is below; thus $\widetilde{y}_t \neq 0$ represents a standard New Keynesian output gap. In keeping with New Keynesian models, write most equations in the macro block in terms of detrended real variables.

As in prototypical New Keynesian models, macroeconomic dynamics satisfy a loglinear Euler or "IS" equation. In our setting this Euler equation is driven by the behavior of a representative household referred to as the "macro agent" that consumes a labor share $(1-K_t)$ of $Y_t$. This agent can be considered typical of a household in the general population who holds small amounts of wealth in the form of nominal bonds and no equity. The linearized Euler equation takes the form[8]

$$\widetilde{y}_t = \mathbb{E}_t^m(\widetilde{y}_{t+1}) - \sigma[i_t - \mathbb{E}_t^m(\pi_{t+1}) - \overline{r}] + f_t \tag{15}$$

where $i_t$ is the short-term nominal interest rate, $\mathbb{E}_t^m(\cdot)$ is the expectation under the subjective beliefs of the macro agent, $\overline{r}$ is the steady state real interest rate, and $f_t$ is a demand shock and also absorbs any variation in the macro agent's consumption attributable to movements in the labor share, $\ln(1-K_t)$. The demand shock follows an AR(1) process $f_t = \rho_f f_{t-1} + \sigma_f \varepsilon_f$, $\varepsilon_f \sim N(0,1)$. The coefficient $\sigma$ in (15) is a positive parameter.

We introduce two equations for inflation and the nominal interest rate rule. Inflation dynamics are described by the following equation, which takes the form of a New Keynesian

[6] Outside of these two distinctive features, macroeconomic dynamics are essentially the same as those that arise from the prototypical New Keynesian model of Galí (2015), Chapter 3.

[7] Heterogeneous agent macro models often specify equilibria with financial market trade, which allows for the study of distributional dynamics. Models with trade are computationally difficult and slow to solve and would present a significant challenge to the mixed-frequency structural estimation of this paper; hence we leave this to future research. We conjecture, however, that an empirically plausible version of our model with trade is unlikely to imply appreciably different findings for the aggregate dynamics that we focus on in this paper. See for example Chang, Chen, and Schorfheide (2021), who provide econometric evidence that spillovers between aggregate and distributional dynamics in heterogeneous agent models are generally small.

[8] We assume that the Euler equation (15) holds under nonrational expectations. Honkapohja, Mitra, and Evans (2013) provide microfoundations for such Euler equations with nonrational beliefs.

Phillips curve:

$$\begin{aligned}\pi_t - \overline{\pi}_t &= \beta\left(1-\lambda_{\pi,1}-\lambda_{\pi,2}\right)\mathbb{E}_t^m\left[\pi_{t+1}-\overline{\pi}_t\right]+\beta\lambda_{\pi,1}\left[\pi_{t-1}-\overline{\pi}_t\right]+\beta\lambda_{\pi,2}\left[\pi_{t-2}-\overline{\pi}_t\right] \\ &\quad +\kappa_0\widetilde{y}_t+\kappa_1\widetilde{y}_{t-1}+\sigma_\mu\varepsilon_{\mu,t}\end{aligned} \tag{16}$$

where $\overline{\pi}_t$ denotes the household's perceived trend inflation rate and $\varepsilon_{\mu,t} \sim N(0,1)$ is a markup shock. The specification in (16) implies that deviations of inflation from macro agent's perception of trend inflation are a function of the expected future value and lagged value of such deviations, as well as the current and lagged output gap. Lags beyond the current values of these variables are used to capture persistent inflation dynamics. The coefficients $\beta$, $\lambda_{\pi_1}$, $\lambda_{\pi_2}$, $\kappa_0$, and $\kappa_1$ are positive parameters.

The central bank obeys the following nominal interest rate rule subject to nonrecurrent regime changes in the policy rule parameters:

$$\begin{aligned}i_t - \left(\overline{r}+\pi^T_{\xi^p_t}\right) &= \left(1-\rho_{i,\xi^p_t}-\rho_{i_2,\xi^p_t}\right)\left[\psi_{\pi,\xi^p_t}\left(\pi_{t,t-3}-3\pi^T_{\xi^p_t}\right)+\psi_{\Delta y,\xi^p_t}\left(\Delta y_{t,t-3}\right)\right] \\ &\quad +\rho_{i_1,\xi^p_t}\left[i_{t-1}-\left(\overline{r}+\pi^T_{\xi^p_t}\right)\right]+\rho_{i_2,\xi^p_t}\left[i_{t-2}-\left(\overline{r}+\pi^T_{\xi^p_t}\right)\right]+\sigma_i\varepsilon_i,\end{aligned} \tag{17}$$

where $\pi_{t,t-3} \equiv \pi_t+\pi_{t-1}+\pi_{t-2}$ is quarterly inflation, $y_{t,t-3} \equiv y_t - y_{t-3} = \widetilde{y}_t-\widetilde{y}_{t-3}+3g+\widehat{g}_t+\widehat{g}_{t-1}+\widehat{g}_{t-2}$ is quarterly output growth (with $\widehat{g}_t \equiv g_t - g$, $\varepsilon_{i,t} \sim N(0,1)$ is a monetary policy shock, and where the parameters of the rule depend on the discrete-valued latent random variable $\xi^p_t$. In the above policy rule, the central bank reacts to quarterly data even though the baseline decision interval of agents is monthly. Lags of the left-hand-side variable appear in the rule to capture the observed smoothness in adjustments to the central bank's target interest rate.

An important feature of this interest rate policy rule, and a departure from the prototypical model, is that it allows for nonrecurrent regime changes in the conduct of monetary policy driven by $\xi^p_t$. The parameter $\pi^T_{\xi^P_t}$ plays the role of an *implicit* time-$t$ inflation target. In particular, it may periodically deviate from the central bank's stated long-term inflation objective when the central bank is actively trying to move inflation back toward that objective. There are also regime shifts in the activism coefficients $\psi_{\pi,\xi^P_t}$, and $\psi_{\Delta y,\xi^P_t}$ that govern how strongly the central bank responds to deviation from the implicit target and to economic growth, and in the autocorrelation coefficient $\rho_{i,\xi^P_t}$. As discussed, these coefficients are modeled as varying with the same discrete-valued random variable $\xi^P_t$ that determined the previously identified regime sequence for $r_{\xi^P_t}$, referred to above as delineating distinct accommodative and restrictive regimes. It is important to emphasize, however, that these labels do not imply that we impose any constraints on the estimated values of policy rule parameters across the previously estimated regimes. Since we freely estimate the policy rule parameters under flat priors, they could in principle show no shift across regimes, or shifts that go in the "wrong" direction with respect to the previously estimated *mps* regimes.

The macro agent's expectations about inflation are formed using an adaptive algorithm, following survey evidence in Malmendier and Nagel (2016) (MN). Specifically, macro agent

expectations about inflation are formed using an autoregressive process, $\pi_t = \alpha + \phi\pi_{t-1} + \eta_t$, where the agent must learn about the parameter $\alpha$. Each period, agents form a belief about $\alpha$, denoted $\alpha_t^m$, that is updated over time. Updating affects beliefs about next period inflation as well as beliefs about long-term trend inflation. Define *perceived trend inflation* to be the $\lim_{h\to\infty} \mathbb{E}_t^m [\pi_{t+h}]$ and denote it by $\overline{\pi}_t$. Given the presumed autoregressive process, it can be shown that $\overline{\pi}_t = (1-\phi)^{-1} \alpha_t^m$. This implies that expectations of one step ahead inflation are a weighted average of perceived trend inflation and current inflation:

$$\mathbb{E}_t^m [\pi_{t+1}] = \alpha_t^m + \phi\pi_t = (1-\phi)\overline{\pi}_t + \phi\pi_t. \tag{18}$$

We allow the evolution of beliefs about $\alpha_t^m$ and $\overline{\pi}_t$ to potentially reflect both an adaptive learning component as well as a signal about the central bank's inflation target. For the adaptive learning component, we follow evidence in MN that the University of Michigan Survey of Consumers (SOC) mean inflation forecast is well described by a constant gain learning algorithm. For the signal component, we assume that beliefs could be partly shaped by additional information the agent receives about the current inflation target. This signal could reflect the opinion of experts (as in MN) or a credible central bank announcement. Combining these two yields updating rules for $\alpha_t^m$ and $\overline{\pi}_t$ that are a weighted averages of two terms:

$$\alpha_t^m = \left(1-\gamma^T\right) \underbrace{\left[\alpha_{t-1}^m + \gamma\left(\pi_t - \phi\pi_{t-1} - \alpha_{t-1}^m\right)\right]}_{\alpha_t^{mCG}} + \gamma^T \left[(1-\phi)\,\pi_{\xi_t}^T\right] \tag{19}$$

$$\overline{\pi}_t = \left(1-\gamma^T\right) \underbrace{\left[\overline{\pi}_{t-1} + \gamma(1-\phi)^{-1}\left(\pi_t - \phi\pi_{t-1} - (1-\phi)\,\overline{\pi}_{t-1}\right)\right]}_{\overline{\pi}_t^{CG}} + \gamma^T \pi_{\xi_t}^T \tag{20}$$

The first terms in square brackets, $\alpha_t^{mCG}$ and $\overline{\pi}_t^{CG}$, are the recursive updating rules implied by constant gain learning, where $\gamma$ is the constant gain parameter that governs how much last period's beliefs $\alpha_{t-1}^m$ and $\overline{\pi}_{t-1}$ are updated given new information, $\pi_t$. The second term in square brackets captures the effect of the signal about the current inflation target $\pi_{\xi_t}^T$. If $\gamma^T = 1$, the signal is completely informative and the agent's belief about trend inflation is the same as the perceived inflation target. If $\gamma^T = 0$, the signal is completely uninformative and the agent's belief about trend inflation depends only on the adaptive learning algorithm. A weight of less than one on the target could arise either because the target is imperfectly observed, or because central bank announcements about the target are not viewed as fully informative or credible. Note that the parameter $\gamma^T$ is closely related to the speed with which the macro agent learns about a new inflation target as well as to the credibility of Fed announcements regarding changes in the target. Small values for $\gamma^T$ are indicative of slow learning and low credibility, since in that case the macro agent continues to base inflation expectations mostly on a backward looking rule even when there has been a shift in the inflation target. Since $\gamma^T$

is freely estimated, we can empirically assess the magnitude of this learning speed/credibility and its role in macroeconomic fluctuations.

The macro agent forms expectations about detrended output using a simple backward looking rule:

$$\mathbb{E}_t^m\left(\widetilde{y}_{t+1}\right) = \varrho_1\widetilde{y}_{t-1} + \varrho_2\widetilde{y}_{t-2} + \varrho_3\widetilde{y}_{t-3}. \tag{21}$$

Unlike inflation, agents do not perceive a moving mean for detrended output. This assumption is consistent with the equilibrium of the model, which implies that the central bank cannot have a permanent effect on real activity.

Using equations (18), (20), and (21), we substitute out $\mathbb{E}_t^m\left[\pi_{t+1}\right]$, $\overline{\pi}_t$, and $\mathbb{E}_t^m\left(\widetilde{y}_{t+1}\right)$ in the model equations (15), (16), and (17) to obtain the following system of equations that must hold in equilibrium:

1. Real activity

$$\widetilde{y}_t = \varrho_1\widetilde{y}_{t-1} + \varrho_2\widetilde{y}_{t-2} + \varrho_3\widetilde{y}_{t-3} - \sigma\left[i_t - \phi\pi_t - (1-\phi)\,\overline{\pi}_t - r_{ss}\right] + f_t. \tag{22}$$

2. Phillips curve:

$$\pi_t - \overline{\pi}_t = \widetilde{\phi}\beta\lambda_{\pi,1}\left[\pi_{t-1} - \overline{\pi}_t\right] + \widetilde{\phi}\beta\lambda_{\pi,2}\left[\pi_{t-2} - \overline{\pi}_t\right] + \widetilde{\phi}\kappa_0\widetilde{y}_t + \widetilde{\phi}\kappa_1\widetilde{y}_{t-1} + \widetilde{\phi}\sigma_\mu\varepsilon_{\mu,t}. \tag{23}$$

where $\widetilde{\phi} = \left[1 - \beta\left(1 - \lambda_{\pi,1} - \lambda_{\pi,2}\right)\phi\right]^{-1}$.

3. Monetary policy rule:

$$\begin{aligned} i_t - \left(\overline{r} + \pi^T_{\xi_t^p}\right) &= \left(1 - \rho_{i,\xi_t^p} - \rho_{i_2,\xi_t^p}\right)\left[\psi_{\pi,\xi_t^p}\left(\pi_{t,t-3} - 3\pi^T_{\xi_t^p}\right) + \psi_{\Delta y,\xi_t^p}\left(\Delta y_{t,t-3}\right)\right] \\ &\quad + \rho_{i_1,\xi_t^p}\left[i_{t-1} - \left(\overline{r} + \pi^T_{\xi_t^p}\right)\right] + \rho_{i_2,\xi_t^p}\left[i_{t-2} - \left(\overline{r} + \pi^T_{\xi_t^p}\right)\right] + \sigma_i\varepsilon_i, \end{aligned} \tag{24}$$

4. Law of motion for demand $f_t$:

$$f_t = \rho_f f_{t-1} + \sigma_f\varepsilon_{ft},\ \varepsilon_{ft} \sim N\left(0,1\right). \tag{25}$$

5. Law of motion for trend growth $g_t \equiv \ln\left(A_t/A_{t-1}\right)$:

$$g_t = g + \rho_g\left(g_{t-1} - g\right) + \sigma_g\varepsilon_g,\ \varepsilon_{gt} \sim N\left(0,1\right). \tag{26}$$

6. Perceived trend inflation:

$$\overline{\pi}_t = \left[1 - \gamma^T\right]\left[\overline{\pi}_{t-1} + \gamma\left(1-\phi\right)^{-1}\left(\pi_t - \phi\pi_{t-1} - (1-\phi)\,\overline{\pi}_{t-1}\right)\right] + \gamma^T\pi^T_{\xi_t}. \tag{27}$$

Investors know the structure of the macro block above. That is, they observe equations (22)-(27) and take those dynamics into account when forming expectations. But they must form beliefs about the future conduct of monetary policy, as described next.

### 3.3 Investor Beliefs

We now describe how investor beliefs about monetary policy regime changes evolve over time.

Investors understand that the true data generating process for the monetary policy rule is subject to infrequent, nonrecurrent regime changes. We further assume that investors closely follow central bank communications and are therefore capable of accurately estimating the current policy rule indexed by $\xi_t^P$. What they are uncertain about is how long the current regime will last, and what will come after the current regime ends.[9] These considerations require a model of how expectations are formed in the presence of structural breaks. Investors must contemplate a future with a central bank that could operate differently from the one today or any that has come before.

To model these ideas, we assume that, for each time $t$ policy rule regime indexed by $\xi_t^P$, investors hold in their minds an "Alternative policy rule" indexed by $\xi_t^A$ that they believe will come next, whenever the current policy regime ends. The Alternative policy rule is isomorphic to the current policy rule, except that it has different parameters, i.e.,

$$\begin{aligned} i_t - \left(\overline{r} + \pi^T_{\xi_t^A}\right) &= \left(1 - \rho_{i,\xi_t^A} - \rho_{i_2,\xi_t^A}\right)\left[\psi_{\pi,\xi_t^A}\left(\pi_{t,t-3} - 3\pi^T_{\xi_t^A}\right) + \psi_{\Delta y,\xi_t^A}\left(\Delta y_{t,t-3}\right)\right] \\ &\quad + \rho_{i_1,\xi_t^A}\left[i_{t-1} - \left(\overline{r} + \pi^T_{\xi_t^A}\right)\right] + \rho_{i_2,\xi_t^A}\left[i_{t-2} - \left(\overline{r} + \pi^T_{\xi_t^A}\right)\right] + \sigma_i \varepsilon_i, \end{aligned} \tag{28}$$

Investors form beliefs about the probability of staying in the current regime $\xi_t^P$ versus switching to the Alternative regime $\xi_t^A$. For each $\xi_t^P$, investors hold in their minds a "grid" of $B$ beliefs about the probability of remaining in $\xi_t^P$ versus changing to the Alternative $\xi_t^A$, and do not consider anything after that. This can be considered a form of bounded rationality.[10] In the nonrecurrent regime setup of the model, this implies that the pondered Alternative is treated as an absorbing state as of time $t$, since the probability of returning precisely to any previous policy rule must be zero by definition. When the current policy regime ends, the new policy regime that replaces it will never be exactly as previously imagined by the investor. Nevertheless, at that time investors update their understanding of the current policy rule and proceed to contemplate a new perceived Alternative for the next rule.

These ideas can be formalized by introducing the notion of a *belief regime* sequence governed by a discrete-valued variable $\xi_t^b \in \{1, 2, ...B, B+1\}$ with $B+1$ states. The overall policy regime process includes the regime in place, and investor beliefs about transitioning out of that regime and moving to the Alternative. Specifically, each *overall policy regime* $\xi_t = \left\{\xi_t^P, \xi_t^b\right\}$ is characterized by knowledge of the current policy regime $\xi_t^P$ and a belief about the probability of

[9] We argue that this is a plausible specification especially when regime changes are infrequent. The Fed clearly telegraphs when it seeks to change the stance of policy, but is comparatively vague about how long that will last and what will come after. Moreover, learning about Markov-switching parameters tends to be fast, so the specification here would closely approximate one with learning.

[10] In theory it is straightforward to consider multiple alternative policy rules, and multiple alternatives to the alternatives. In practice the number of parameters can quickly proliferate creating an intractable estimation problem. We consider a single alternative at each $t$ in order to keep the solution and estimation tractable.

staying in the current policy rule $\xi_t^P$ versus moving to $\xi_t^A$. To keep notation simple, we exclude $\xi_t^A$ in the set of arguments of $\xi_t$. It should be kept in mind, however, that each policy rule regime $\xi_t^P$ has associated with it a single perceived Alternative policy rule $\xi_t^A$. Thus if there are a total of $N_p$ true policy regimes over the course of the sample, there are also $N_p$ perceived Alternative policy regimes associated over the same time span.

The regimes $\xi_t^b = 1, 2, ...B$ represent a grid of beliefs taking the form of perceived probabilities that the current policy rule will still be in place next period, given that it is in place this period. The regime $\xi_t^b = B + 1$ is a belief regime capturing the perceived probability of staying in the Alternative regime once it is reached. We order these so that belief regime $\xi_t^b = 1$ is the lowest perceived probability that the current policy rule will remain in place and belief regime $\xi_t^b = B$ is the highest.

The perceived regimes are modeled with a perceived transition matrix taking the form:

$$\mathbf{H}^b = \begin{bmatrix} p_{b1}p_s & p_{b2}\left(1-p_s\right)/(B-1) & \cdots & p_{bB}\left(1-p_s\right)/(B-1) & 0 \\ p_{b1}\left(1-p_s\right)/(B-1) & p_{b2}p_s & & p_{bB}\left(1-p_s\right)/(B-1) & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ p_{b1}\left(1-p_s\right)/(B-1) & & & p_{bB}p_s & 0 \\ 1-p_{b1} & 1-p_{b2} & \cdots & 1-p_{bB} & p_{B+1,B+1}=1 \end{bmatrix}, \tag{29}$$

where $\mathbf{H}_{ij}^b \equiv p\left(\xi_t^b = i | \xi_{t-1}^b = j\right)$. In the above, $p_{b1}$ is the subjective probability of remaining in the current policy rule under belief 1. For example, belief 1 could mean that investors believe with $p_{b1} = 0.05$ that the current policy rule will still be in place next period; belief 2 could mean that investors believe with $p_{b2} = 0.10$ that the current rule will still be in place, and so on. The non-zero off diagonal elements in the upper left $B \times B$ submatrix allow for the possibility that investors might receive subsequent information that could change their beliefs, and take that into account when forming expectations. Thus, $\left(1-p_s\right)/(B-1)$ is the probability investors assign to possibility that they will change their beliefs tomorrow as the result of new information or sentiment, while $p_s$ is the probability investors assign to not changing their minds, i.e., to having the same beliefs tomorrow as today. To avoid overparameterization, we assume that investors assign an equal probability of having any of the other of the $(B-1)$ beliefs tomorrow. With this parameterization, $p_{b1}p_s$ measures the subjective probability of being in belief regime 1 tomorrow, conditional on having belief 1 today, while $p_{b1}\left(1-p_s\right)/(B-1)$ is the subjective probability of being in any of belief regimes 2,3,...,or $B$ tomorrow, conditional on having belief 1 today. Finally, $1-p_{bi}$ is the probability of having belief $i$ today but exiting to the Alternative regime tomorrow. The parameter $p_{B+1,B+1}$ is the perceived probability of remaining in the Alternative regime conditional on having moved there. With perceived nonrecurrent regimes and our bounded rationality assumption, this probability is unity by definition. As a result, the model of beliefs takes the form of a reducible Markov chain, implying that investors believe with probability 1 that they will eventually transition out of the current policy rule, to the

perceived Alternative rule.

**Equilibrium** An equilibrium is defined as a set of prices (bond prices, stock prices), macro quantities (inflation, output growth, inflation expectations), laws of motion, and investor beliefs such that equations (9)-(14) in the asset pricing block are satisfied, equations (22)-(27) in the macro block are satisfied, and investors beliefs about the persistence of policy regimes are characterized by the perceived Alternative policy rule (28) and the perceived belief regime sequence described above with transition matrix (29).

## 3.4 Model Solution

The asset pricing block of equations involves conditional subjective variance terms that are affected by Markov-switching random variables in the model. The subsection "Risk Adjustment with Lognormal Approximation," in the Online Appendix explains the approximation used to preserve lognormality of the entire system. This part uses the approach in Bianchi, Kung, and Tirskikh (2018) who in turn build on Bansal and Zhou (2002). We use the algorithm of Farmer, Waggoner, and Zha (2011) to solve the system of model equations that must hold in equilibrium, where agents form expectations taking into account the probability of regime change in the future. The solution of the model takes the form of a Markov-switching vector autoregression (MS-VAR) in the state vector

$$S_t = \left[S_t^M, m_t, pd_t, k_t, lp_t, \mathbb{E}_t^b\left(m_{t+1}\right), \mathbb{E}_t^b\left(pd_{t+1}\right)\right],$$

where $S_t^M$ is a vector of macro state variables given by $S_t^M \equiv [\widetilde{y}_t, g_t, \pi_t, i_t, \overline{\pi}_t, f_t]'$. The MS-VAR solution consists of a system of equations taking the form

$$S_t = \underbrace{C\left(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b\right)}_{\text{level}} + \underbrace{T(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b)}_{\text{propagation}} S_{t-1} + \underbrace{R(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b)}_{\text{amplification}} Q\varepsilon_t, \tag{30}$$

where $\varepsilon_t = (\varepsilon_{f,t}, \varepsilon_{i,t}, \varepsilon_{g,t}, \varepsilon_{k,t}, \varepsilon_{lp,t}, \varepsilon_{\mu,t})$ is the vector of primitive Gaussian shocks. To obtain this solution, we assume that both types of agents have a monthly decision interval. We further assume that the economic state $S_t$ is observed by the investor at the end of each month. With these assumptions, investor expectations multiple steps ahead maybe be computed for any variable. The Online Appendix explains how these are computed in the presence of nonrecurrent regime switching with the perceived Alternative policy rule.

The solution (30) depends on the realized policy rule in place ($\xi_t^P$), but also on the investor's subjective beliefs about staying in the current policy regime next period, which depend on $\xi_t^b$ and $\mathbf{H}^b$. Notice that the parameter vector $\theta_{\xi_t^P}$ includes the parameters of the Alternative policy rule $\xi_t^A$, since there is a single such Alternative for each realized policy rule indexed by $\xi_t^P$. Equation (30) thus shows that the realized policy regime $\xi_t^P$ and investor beliefs $\xi_t^b$ about future changes

in the policy rule amplify and propagate shocks in three ways. First, they have "level" effects, as captured by the coefficients $C\left(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b\right)$, that affect the economy absent shocks. These are driven by changes in the central bank's objectives such as the inflation target, as well as by the perceived risk of the stock market given by the risk-premium terms in (6). Second, they have "propagation" effects, as captured by the matrix coefficient $T(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b)$, that determine how today's economic state is related to tomorrow's. Third, they have "amplification" effects, governed by the matrix coefficient $R(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b)$, that generate endogenous heteroskedasticity of the primitive Gaussian shocks.

An implication of this endogenous heteroskedasticity is that *perceived quantity of risk* in the stock market varies with the expected conduct of future monetary policy. Indeed, it is only through $R(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b)$ that the subjective risk premium in (6) varies, which in turn varies only with (i) realized regime changes $\xi_t^P$ in the conduct of monetary policy, and (ii) time-varying beliefs $\xi_t^b$ regarding future policy. All other sources of variation in the equity premium (6) are attributable the liquidity shocks.

## 3.5 Investor Information and Updating

Let $\mathbb{I}_t$ denote the time $t$ information set of investors, which includes their current belief, $\xi_t^b$, the current policy regime $\xi_t^P$ and their perceived Alternative regime $\xi_t^A$, and additional data available at mixed frequencies that we don't explicitly specify. Since investors can observe the economic state $S_t$ only at the end of each month, we assume that any news event that the investor attends to *within* a month results in the updating of a *nowcast* of $S_t$, which they can produce by filtering the timely, high-frequency information in $\mathbb{I}_t$. Thus, $S_t$ is effectively latent within a month, though it is observed at the end of each month.

Investors use $\mathbb{I}_t$ in two ways. First, given a baseline monthly decision interval, they update their previous nowcasts and subjective expectations of $S_t$ on the basis of new information at the end of every month. Second, investors allocate additional attention to updating nowcasts of $S_t$–akin to forming "advance" estimates of $S_t$–and beliefs $\xi_t^b$ about the future conduct of monetary policy at specific times *within* a month when the central bank releases information. This higher-frequency attentiveness to Fed news echoes real-world "Fed watching" and is the mechanism through which the model accommodates swift market reactions to surprise central bank announcements. These updates in the immediate aftermath of a Fed announcement lead to endogenous jumps in subjective expectations, financial market returns, and in investor perceptions of stock market risk, driven by $\mathbb{COV}_t^b\left[m_{t+1}, r_{t+1}^D\right]$.

Importantly, the estimation approach described in the next section does not require the econometrician to take a stand on the information set $\mathbb{I}_t$ of investors or on the filtering algorithm investors use to update nowcasts of $S_t$ within a month. Instead, we rely on numerous forward-looking series embedded in an observation vector $X_t$ to *infer* investor updating of nowcasts and beliefs $\xi_t^b$ about future monetary policy, by combining a mixed-frequency filtering algorithm

with a structural estimation.

# 4 Structural Estimation

The solution to the model may be written in state-space form by combining the system of state equations (30) with an observation equation taking the form

$$\begin{aligned} X_t &= D_{\xi_t,t} + Z_{\xi_t,t}\left[S_t', \widetilde{y}_{t-1}\right]' + U_t v_t \\ v_t &\sim N\left(0, I\right), \end{aligned} \tag{31}$$

where $X_t$ denotes a vector of data, $v_t$ is a vector of observation errors, $U_t$ is a diagonal matrix with the standard deviations of the observation errors on the main diagonal, and $D_{\xi_t,t}$, and $Z_{\xi_t,t}$ are parameters mapping the model counterparts of $X_t$ into the latent discrete- and continuous-valued state variables $\xi_t$ and $S_t$, respectively, in the model. The matrices $Z_{\xi_t,t}$, $U_t$, and the vector $D_{\xi_t,t}$ depend on $t$ because some of our observable series are not available at all frequencies and/or over the full sample. As a result, the state-space estimation uses different measurement equations to include these series when the relevant data are available, and exclude them when they are missing.

We estimate the state-space representation using Bayesian methods, with the parameters of the monetary policy rule estimated under flat priors. As mentioned, since we are interested in understanding the connection between the previously estimated accommodative/restrictive regimes for $mps_t$ and the interest rate rule in the theoretical model, we force the regime sequence $\xi_t^P$ for the policy rule parameters to correspond to the estimated sequence for the mean of $mps_t$. This sets the timing of the structural breaks in the policy rule, but places no restrictions on the policy rule parameters across the previously estimated regimes.[11] The estimation approach uses Kim's (Kim (1994)) basic filter and approximation to the likelihood for Markov-switching state space models, and a random-walk metropolis Hastings MCMC algorithm to characterize uncertainty. Details on the estimation are provided in the Online Appendix.

## 4.1 Mixed-Frequency Filtering Algorithm

This section discusses the mixed-frequency filtering algorithm we use to infer jumps in investor expectations in response to Fed news. In reduced-form filtering applications that use mixed-frequency data, it is common to specify the state transition equation at the highest frequency, relying on the filter to fill in missing observations at lower frequencies. In the structural setting of this paper, such an approach would be both impractical and unrealistic, since the data

[11] We use the regime sequence $\hat{\xi}^{PT} = \{\hat{\xi}_1^P, ..., \hat{\xi}_T^P\}$ that is most likely to have occurred, given our estimated posterior mode parameter values. See the Online Appendix for details.

sampling interval of the state/transition equation is part of the structural model and needs to correspond to the optimizing decision intervals of agents. Because our highest frequency data consists of minutely financial market observations, the standard approach would necessitate modeling minutely decision intervals and ultra high-frequency structural shocks, which is unlikely to be reasonable. We instead assume that investors have monthly decision/forecasting intervals, but update their perception of what the current, i.e., end-of-month, economic state will be on the basis of any new information or news within the month, a form of nowcasting. Since investors in the model can observe the full state vector at the end of each month, these perceptions are then supplanted by their observed values the end of each $t$. The filtering algorithm described in this section is consistent with these modeling assumptions.

Suppose we have information up through the end of month $t-1$ and new high-frequency information arrives at $t-1+\delta_i$. Here $\delta_i \in (0,1)$ represents the number of time units that have passed during month $t$ up to point $t-1+\delta_i$.[12] The full estimation and filtering procedure refers to the state space equations (30) and (31) and involves iterating on the following steps, which are described in greater detail in the Appendix.

**(i) Kalman Filter:** Conditional on $\xi_{t-1}^b = i$ and $\xi_t^b = j$ run the Kalman filter for $i, j = 1, 2, ..., B$ to produce $S_{t|t-1}^{(i,j)}$ and its mean squared error (MSE) $P_{t|t-1}^{(i,j)}$. At $t-1+\delta_i$, compute updated errors $e_{t|t-1+\delta_i,t-1}^{(i,j)} = X_{t-1+\delta_i}^{\delta_i} - X_{t|t-1}^{\delta_i}$ for the subset of series $X^{\delta_i}$ available at $t-1+\delta_i$. Fixing $S_{t|t-1}^{(i,j)}$ and $P_{t|t-1}^{(i,j)}$ from $t-1$, use $e_{t|t-1+\delta_i,t-1}^{(i,j)}$ to re-run the filter and update to $S_{t|t-1+\delta_i}^{(i,j)}$ and $P_{t|t-1+\delta_i}^{(i,j)}$.

**(ii) Hamilton Filter:** With $e_{t|t-1+\delta_i,t-1}^{(i,j)}$ in hand, re-run the Hamilton filter to calculate new regime probabilities $\Pr\left(\xi_t^b, \xi_{t-1}^b | X_{t-1+\delta_i}, X^{t-1}\right)$, $\Pr\left(\xi_t^b | X_{t-1+\delta_i}, X^{t-1}\right)$ for $i, j = 1, 2, ..., B$.

**(iii) Approximations**: Collapse the $B \times B$ values of $S_{t|t-1+\delta_i}^{(i,j)}$ and $P_{t|t-1+\delta_i}^{(i,j)}$ into $B$ values $S_{t|t-1+\delta_i}^{(j)}$ and $P_{t|t-1+\delta_i}^{(j)}$ using Kim's (Kim (1994)) approximation.

**(iv) Store or Iterate**: If $t-1+\delta_i = t$ iterate forward by setting $t-1 = t$ and return to step (i). Otherwise store the updates $S_{t|t-1+\delta_i}^{(j)}$, $P_{t|t-1+\delta_i}^{(j)}$, $\Pr\left(\xi_t^b, \xi_{t-1}^b | X_{t-1+\delta_i}, X^{t-1}\right)$, and $\Pr\left(\xi_t^b | X_{t-1+\delta_i}, X^{t-1}\right)$ and return to step (i) at the subsequent intramonth time unit, keeping $t-1$ fixed.

Several points about the above algorithm bear noting. First, because intramonth updates of $S_t$ and $\Pr\left(\xi_t^b | X_{t-1+\delta_i}, X^{t-1}\right)$ are based on filtering numerous forward-looking series, the procedure can be run pre- and post-announcement to infer how investors in the model revise their beliefs in response to Fed communications, without having to take a stand on their unobservable forecasting models or information sets. Second, the notation $e_{t|t-1+\delta_i,t-1}^{(i,j)}$ explicitly denotes that

[12] For example, if we have minutely data, $\delta_i$ could correspond to the number of time units that have passed when we are at 10 minutes before or 20 minutes after an FOMC announcement.

we use a subset of data available at $t-1+\delta_i$ to estimate how intraperiod, high-frequency news affects the structural shocks investors *perceive* will be realized at the end of $t$, conditional on $t-1$ information. This is distinct from estimating true time $t$ shocks conditional on $t-1+\delta_i$ information, which would require a structural model of ultra high frequency primitive shocks. Instead, the approach here treats Fed announcements as bonafide news shocks (as perceived by investors), in alignment with the high-frequency event study literature that analyzes market movements in very narrow windows around news events with the express purpose of measuring the causal effect of the news *per se*, holding fixed the economic state. Third, the filters can be rerun as frequently as desired without iterating forward to the next period. This allows for repeated updates on the perceived $S_t$ and $\Pr\left(\xi_t^b | X_{t-1+\delta_i}, X^{t-1}\right)$ at any point within a month even as the transition dynamics are still specified across months. This is also helpful when number of news events during the sampling interval is non-uniform over time, as when the number of FOMC meetings during a month varies over the sample. Fourth, by embedding this into a structural estimation, we can reestimate the entire perceived state vector $S_t$ at any point within a month, provided only that a subset of data are available at frequencies higher than a month. For example, we can infer revisions to investor nowcasts of aggregate demand or of the earnings share from the information encoded in more timely financial market data, even if data on output, earnings, inflation, etc., are only available once per month.

## 4.2 Data and Measurement

This section describes the data $X_t$ used in our structural estimation, which spans January 1961 through February 2020. Our full sample of FOMC announcements consists of 220 FOMC press releases spanning February 4th, 1994 to January 29th, 2020.

The complete estimation relies on data at different frequencies. Lower frequency (monthly, quarterly, biannual) macro data inform estimates of the policy rule and structural equations driving macroeconomic and stock market dynamics over the full sample. High frequency (daily and minutely) data use information on forward-looking variables from financial markets and surveys in response to FOMC announcements, allowing us to estimate jumps in the perceived state vector at high frequencies.

We now summarize the observations we use in $X_t$. Observations on every series listed below are fed in at a monthly sampling interval. Series with observations available less frequently than monthly have missing values that are filled in by the filtering algorithm. A subset of series available at higher frequency are also used intramonth in the minutes or days surrounding FOMC press releases. An explicit description of the mapping between our observables and their model counterparts, as well as a complete description of each data series and our sources is given in the Online Appendix.

Among the observations available at monthly, quarterly, or biannual sampling intervals but not at higher frequency, we use a monthly 12-month GDP growth estimate (see the Online

Appendix), 12-month CPI inflation, the University of Michigan Survey of Consumers (SOC) 12- and 60-month ahead mean inflation forecast, the Bluechip (BC) survey, Survey of Professional Forecasters (SPF) and Livingston (LIV) surveys' mean 12-month- and 120-month-ahead CPI inflation forecasts, the SPF mean 12-month ahead GDP deflator inflation forecast, the means of the BC and SPF 12-month ahead GDP growth forecasts, and the ratio of S&P 500 (SP500) earnings relative to last month's GDP observation, a variable we refer to as the earnings-lagged GDP ratio. The SPF survey is available quarterly while the LIV survey is biannual. All other series listed above are monthly.

Among data available at daily sampling intervals, we use the mean of the Bloomberg (BBG) consensus 12-month ahead inflation and GDP growth forecasts and the effective federal funds rate (FFR). We also use the Moody's Baa 20-year bond return minus the 20-year U.S. Treasury bond (referred to hereafter as the "Baa spread"). Although FFR is available daily, we use observations on this variable only at the end of each month, instead relying on the current contract fed funds futures rate to measure jumps in the funds rate following an FOMC announcement, since these are available on a minutely basis. At the end of the month, FFR and the current contract fed funds futures rate coincide.

Among data available at minutely sampling intervals, we use the ratio of SP500 market capitalization relative to lagged GDP, which we refer to as the SP500-lagged GDP ratio. At minutely frequency we also use the current contract and the 6, 10, 20, and 35 month contracts of the federal funds futures (FFF) prices.

Our motivation for these choices is as follows.

Our use of high-frequency pre- and post-FOMC observations on survey expectations of inflation, GDP growth, the Baa credit spread, several federal funds rate futures contracts, and the stock market is important for two reasons. First, it allows us to measure the causal effect of Fed announcements on the stock market and other variables at high frequency, which is of interest in its own right. Second, the use of these high frequency data allow us to control for news reflected in inflation and GDP growth forecasts, Fed fund futures, credit spreads, and the stock market that may have arrived between the end of the month immediately preceding an FOMC announcement-month and the intramonth announcement itself. This is important because the arrival of economic news within this particular window can lead to revisions in monthly survey forecast data (e.g., the monthly BC survey) around FOMC announcements that appear to support a Fed information effect, when in reality markets may have been surprised by the reaction of the Fed to known economic news that pre-dated the FOMC announcement but arrived after last month's BC survey was taken (Bauer and Swanson (2021)). By conditioning on close-range, pre- and post-announcement observations for inflation and GDP growth expectations and credit spreads (the day before and day after), interest rate futures, and the stock market (10 minutes before and 20 minutes after), we explicitly control for any economic news reflected in these forward looking variables that came out in the weeks between the last monthly BC

survey and the FOMC announcement. It follows that jumps in the post-announcement jumps in expectations and forward-looking variables cannot be readily attributed to stale economic news that came out earlier in the announcement month.

A second motivation for these data series is the ability to use multiple observables on a single variable of interest, especially on expectations. We use the household-level SOC to discipline household expectations and professional forecaster surveys to discipline investor expectations. We measure investor expectations of inflation and GDP growth using four different professional surveys (BBG, BC, LIV, SPF) and treat each of these as a noisy signal on the true underlying investor expectations process.

A number of series are used because they have obvious model counterparts. Data for GDP growth and inflation are mapped into the model implications for output growth and inflation, while data on SOC inflation forecasts are mapped into the model's implications for household inflation forecasts. Likewise, data on the current effective federal funds rate are mapped into the model's implications for the current nominal interest rate, while data on the FFF market are mapped into the model's implications for investor expectations of the future federal funds rate, as is the mean of the BC survey measure of the federal funds rate 12 months-ahead.[13] Importantly, the model of investor beliefs given in (29) takes the form of a reducible Markov chain, implying that investors believe with probability 1 that they will eventually exit the current policy rule and thus their longer-run forecasts of the funds rate are dominated by the perceived Alternative rule. The inclusion of data on long-dated FFF contracts and survey forecasts of the funds rate a year or more out are therefore especially helpful for identifying the parameters of the Alternative policy rule.

We discipline observations on $D_t$ and the earnings share of output $K_t$ with data on S&P 500 earnings and its ratio with GDP. Recall that output $Y_t$ in the model is divided between shareholder cash-flow $D_t = K_t Y_t$ and that all earnings are paid out to shareholders in the model. To account for the fact that earnings in the data differs from the payout shareholders actually receive, the theoretical concepts for $d_t$ and $k_t$ are mapped into their respective data series using specifications with freely estimated constant and slope coefficients, while also allowing for measurement error in both observation equations.

Finally, data on the Baa spread are mapped into the model's implications for the liquidity premium, $lp_t$. This premium is a catchall for factors outside of the model that could effect the equity premium, such as changes in the liquidity and safety attributes of Treasuries, default risk, flights to quality, and/or sentiment. We use the Baa spread as an observable likely to be correlated with many of these factors, but our measurement equation allows for both a constant and a slope coefficient on the Baa spread along with measurement error, in order to soak up

[13] In principle, fed funds futures market rates may contain a risk premium that varies over time. If such variation exists, it is absorbed in the estimation by the measurement error for these equations. In practice, risk premia variation in fed funds futures is known to be small when that variation is measured over the short 30-minute windows surrounding FOMC announcements that we analyze (Piazzesi and Swanson (2008)).

variation in this latent component of the equity premium that may not move identically with the spread.

### 4.3 Estimating Beliefs

Beliefs are modeled with $B+1$ belief regimes governed by the perceived transition matrix $\mathbf{H}^b$ given in (29). In the applied estimation, we set $B = 11$ and take the parameters $p_{bi}$ from a discretized estimated beta distribution, where the mean and variance of the beta distribution are estimated along with the rest of the model parameters. In our model simulations we use the modal estimates of these parameters.

We use estimates of $\mathbf{H}^b$ to compute investors' perceived probabilities of a change in the policy rule multiple steps ahead. Let $T$ be the sample size used in the estimation and let the vector of observations as of time $t$ be denoted $X_t$. Let $P\left(\xi_t^b = i | X_T; \boldsymbol{\theta}\right) \equiv \pi_{t|T}^i$ denote the probability that $\xi_t^b = i$, for $i = 1, 2, ..., B+1$, based on information that can be extracted from the whole sample and knowledge of the parameters $\boldsymbol{\theta}$, while $\pi_{t|T}$ is a $(B+1) \times 1$ vector containing the elements $\left\{\pi_{t|T}^i\right\}_{i=1}^{B+1}$. We refer to these as the smoothed regime probabilities. The time $t$ perceived probability of exiting the current policy rule, i.e., of transitioning in the next period to the Alternative policy regime $\xi_t^A$, is given by $\overline{P}_t^{bE} \equiv \sum_{i=1}^{B} \pi_{t|T}^i (1 - p_{bi})$. The time $t$ perceived probability of exiting the current policy rule and transitioning in $h$ periods to $\xi_t^A$ is $\overline{P}_{t+h,t}^{bE} = \mathbf{1}_{B+1}' \left(\mathbf{H}^b\right)^h \pi_{t|T}$, where $\mathbf{1}_{B+1}'$ is an indicator vector with 1 in the $(B+1)$th position and zeros elsewhere. We use these estimated regime probabilities to compute the most likely belief regime at each point in time and track how it changes around Fed announcements and the whole sample.

Structural estimates of expectations play a crucial role in determining asset prices in the model. For a given policy rule $\xi_t^P$ in place, the model implies that forward-looking variables depend both on the Alternative policy rule indexed by $\xi_t^A$, and on the probability assigned to visiting that alternative. The Online Appendix provides a description of how expectations are computed in this setting with structural breaks and a perceived alternative policy rule.

## 5 Structural Estimation Results

This section presents results from the structural estimation. The first subsection discusses the parameter and latent state estimates. The next three subsections discuss the model implications for investor anticipation of realized policy rule regime changes, high frequency analysis around FOMC announcements, and the connection between markets and monetary policy changes both inside and outside of tight windows around FOMC announcements.

Before getting into these results, its worth pointing out that the estimated model-implied series track their empirical counterparts quite well, as shown in Figure 3.[14] In the estimation,

[14]The model-implied counterparts are based on smoothed estimates $S_{t|T}$ of $S_t$, using observations through

we allow for observation errors on all variables except for inflation, GDP growth, the FFR, and the SP500-lagged GDP ratio. For professional forecasters, we have multiple measures of expectations, which we treat as noisy signals on the latent "market" expectation.

## 5.1 Parameter and Latent State Estimates

We begin with parameter estimates for the monetary policy rule. Table 2 reports the posterior distributions for the policy rule parameters $\pi^T_{\xi^P_t}$, $\psi_{\pi,\xi^P_t}$, $\psi_{\Delta y,\xi^P_t}$ and $\rho_{i,\xi^P_t}$, where we use flat priors. A key finding is that the previously estimated regime subperiods (given in Table 1) are associated with quantitatively large changes in the estimated policy rule, as well as in the associated Alternative policy rules that we estimate investors perceived would come after the current rule of each regime subperiod ended. We report the values of the activism coefficients $\psi_{\pi,\xi^P_t}$ and $\psi_{\Delta y,\xi^P_t}$ separately, as well as the ratio $\psi_{\pi,\xi^P_t}/\psi_{\Delta y,\xi^P_t}$. When output fluctuations are dominated by demand shocks (as in our sample according to parameter estimates below), the ratio $\psi_{\pi,\xi^P_t}/\psi_{\Delta y,\xi^P_t}$ is also relevant for the central bank's commitment to stabilizing the real economy around potential, since below target inflation tends to coincide with output below potential, and vice versa for above target inflation.

Table 2 shows that the Great Inflation (GI) regime (1961:Q1-1978:Q3) is characterized by a high estimated inflation target and a modest level of inflation activism ($\psi_{\pi,\xi^P_t}$) relative to output activism ($\psi_{\Delta y,\xi^P_t}$). The perceived Alternative policy rule for this subperiod also has a high inflation target, but differs in terms of the focus on inflation and output growth, with inflation stabilization perceived as the main objective. The anticipation of a heightened focus on inflation stabilization is in fact a defining feature of the realized policy rule during the Great Moderation (GM) regime that began in 1978:Q4, when the activism coefficient $\psi_{\pi,\xi^P_t}$ on inflation is estimated to be far larger than the coefficient on output growth. The GM regime also features a much lower estimated inflation target than the GI regime, indicative of the more hawkish monetary policy that characterizes the GM regime. This aspect of the realized GM regime was not well anticipated by investors during the GI regime according to the estimates of the Alternative rule in the GI subperiod. Moving to the Post-Millennial (PM) regime, we find that policy rule parameters then shifted back to accommodative values with far less activism: the PM rule has both a higher inflation target compared to the GM regime, virtually no activism on inflation ($\psi_{\pi,\xi^P_t} = 0$) and very little activism on output ($\psi_{\Delta y,\xi^P_t} = 0.08$). The PM regime is also characterized a large increase in the persistence of the federal funds rate, consistent with the forward guidance policies implemented at the zero lower bound (ZLB) that promised to keep interest rates low for a prolonged period of time.

Investors' perceived Alternative policy rules also show marked differences across the three

then end of the sample at date $T$, which exploit the mapping to observables in (31) using the modal parameter estimates. The difference between the model-implied series and the observed counterpart is attributable to observation errors.

regime subperiods. In the GM regime, the perceived Alternative rule indicates that investors expected the next rule to have an inflation target that was lower than what was actively in place at the time, along with greater activism in stabilizing both inflation and economic growth. Likewise, investors' perceived Alternative rule in the PM period also implies that they expected a much lower inflation target and a more active Federal Reserve relative to the lackluster activism of the realized rule during the PM subperiod. Thus both the GM and PM periods are characterized by expectations that the next policy rule would be both more hawkish and more active than the realized rules of the times. Since a more active rule is associated with more aggressive action to stabilize the real economy, these features of the perceived Alternative rules are closely related to perceived risk in the stock market, as discussed below.

A comment is in order about the estimated magnitudes for $\pi^T_{\xi^P_t}$ shown in Table 2. Although this parameter plays the role of an "inflation target" in the interest rate rule, unlike traditional New Keynesian models with a time invariant inflation target, $\pi^T_{\xi_t}$ is not a value toward which true inflation and inflation expectations in the model necessarily tend in the long-run. In this setting, $\pi^T_{\xi^P_t}$ is more appropriately thought of as an implicit time $t$ target rather than an explicit long-run objective. To understand why consider the PM period for example. Here, the structural estimation implies that to achieve observed average CPI inflation of roughly 1.96% over this period, $\pi^T_{\xi^P_t}$ needed to be 2.5%, well above what ultimately became the explicitly stated long-run objective of 2% in 2012. Such higher implicit objectives are especially important when the economy has been subject to a sequence of adverse shocks and the central bank operates at or close to the ZLB, as it did over much of the PM period. Forward guidance and quantitative easing, two tools that were employed at the ZLB, are channels that manifest in the model as a higher values for $\pi^T_{\xi^P_t}$, since, as long as $\gamma^T > 0$, these tools should generate higher perceived trend inflation by households even as nominal interest rates remain unchanged at the ZLB (see equation (20)).

Table 3 presents estimation results for key model parameters other than those of the policy rule.[15] It is worth emphasizing that the estimates imply a very high level of inertia in household inflation expectations. The constant gain parameter $\gamma$ controlling the speed with which beliefs about inflation are updated with new information on inflation is estimated to be quite low ($\gamma = 0.0001$). Furthermore, the parameter $\gamma^T$ controlling the speed with which household perceived trend inflation is influenced by shifts in the implicit inflation target is, though positive, also estimated to be small ($\gamma^T = 0.005$). Taken together, these findings imply that households revise their beliefs about trend inflation only very slowly over time and mostly based on past realizations of inflation rather than on changes in the inflation target.

We estimate a moderate level of risk aversion for the investor ($\sigma_P = 6.0$). In terms of the magnitude of the primitive economic shocks, monthly demand shocks are estimated to be the

[15]The model has a large number of additional auxiliary parameters that are used to map observables into their model counterparts. To conserve space, these additional parameters are reported in the Online Appendix.

largest quantitatively ($\sigma_f = 17$), compared to "supply side" shocks to trend growth ($\sigma_g = 1.9$) or the markup shock ($\sigma_\mu = 0.13$). Finally, the parameter $p_s$ is estimated to be 0.9875, indicating that investors maintain very firmly held beliefs, rarely contemplating the possibility that they may change their minds in the future on the basis of new information.

Before leaving this section we report the model implications for basic asset pricing moments. Table 4 shows the annualized mean and standard deviation of the log excess return on equity, as measured by the log difference in the S&P 500 stock market value, the real interest rate, as measured by the difference between the annualized FFR and the average of the one-year-ahead forecast of inflation averaged across the SPF, BC, SOC, and Livingston surveys,[16] and the log difference in real, per capita S&P 500 earnings growth. The model based moments for these series are based on the modal parameter and latent state estimates and match their data counterparts closely.

## 5.2 Investor Beliefs About Monetary Policy Over the Sample

Figure 4 plots the estimated perceived probability that investors assign to being in a new policy rule regime in one year's time. Specifically, the figure reports the end-of-the-month value for $\overline{P}^{bE}_{t+12,t} \equiv \pi^{B+1}_{t+h,t|T} = \mathbf{1}'_{B+1}\left(\mathbf{H}^b\right)^{12}\pi_{t|T}$, where $\mathbf{1}'_{B+1}$ is an indicator vector with 1 in the $(B+1)$th position and zeros elsewhere and $\pi_{t|T}$ is the smoothed estimate of the time $t$ belief regime probabilities. The vertical lines mark the timing of the two realized policy regime changes in our sample.

Figure 4 shows that the perceived probability of a policy rule regime change fluctuates strongly over the sample and typically increases before a realized policy change, suggesting that financial markets have some ability to anticipate the realized shifts in the conduct of policy. This occurs despite the fact that investors do not perfectly predict what the new policy rule will look like. The perceived probability of a policy rule change occasionally shoots up at times during which no actual change subsequently occurs over the next year, though these movements in beliefs are typically short-lasting. Particularly noticeable in this regard is the sharp increase in the estimated perceived probability of a policy rule change during the 2008/9 financial crisis. The GM regime is associated with sharp increases in the perceived probability of a regime change at both the beginning and the end of that subperiod. These fluctuations in investor beliefs drive expectations about future central bank conduct and thus movements in asset prices in the model, as we discuss below.

An important feature of the findings displayed in Figure 4 is that investor beliefs about the probability of a regime change in the Fed's policy rule continuously evolve *out*side of tight windows surrounding policy announcements. Indeed, most of the variation in investor beliefs

[16] We interpolate the biannual Livingston survey observations to obtain monthly values, and only average in the observations for the quarterly SPF with the monthly BC, SOC, and interpolated-to-monthly Livingston surveys when observations on the SPF are not missing.

about the future conduct of monetary policy occurs at times over the sample that are not close temporally to an FOMC announcement. This indicates that the causal effect of central bank policy on investor beliefs and therefore on markets is substantially more far reaching than what can be observed from market reactions in tight windows surrounding Fed announcements. An obvious explanation for this result is that most Fed announcements are not immediately associated with a change in the rule. Instead, they provide forward guidance on what is likely to trigger a change in the policy stance down the road. As new data is revealed in between Fed communications, investor beliefs about monetary policy are shaped by what was previously communicated, having consequences for markets. Because high frequency event studies surrounding Fed communications only capture the causal effects of the surprise component of any announcement, they are by construction incapable of accommodating these additional channels of influence outside of tight windows around events. The estimates portrayed in Figure 4 are key inputs into our estimated overall causal impact of the Fed on markets over the sample (discussed below in Section **??**), pointing to the challenges with relying solely on high-frequency event studies for quantifying the effects of monetary policy on markets.[17]

To underscore this point, Figure 5 shows the *change* in the estimated perceived probability of a monetary policy regime change within the next year this time in tight windows around every FOMC announcement in our sample. For this figure we focus on the post 1994 period, when we have data for FOMC announcements. We see that most FOMC announcements result in little if any change in the perceived probability of a regime change in monetary policy, again implying that financial markets do not learn about the possibility of policy regime change only from the surprise component of a policy announcement. Naturally, many FOMC announcements carry little news of any kind, consistent with the majority of points lining up along the horizontal line at zero and the idea that significant changes in the policy rule are infrequent.

With that in mind, we find that some announcements are associated with sizable changes in the perceived probability of exiting the current policy regime. The largest declines occur in the aftermath of the financial crisis, namely on June 24th, 2009, and October 29th, 2008, where in each case the perceived probability of a regime change in the next year declined by more than 1% in the 30 minutes surrounding the FOMC press release. The largest increase in the perceived probability of a policy regime change occurs on April 18th, 2001, with the probability increasing more than 1%. For the first two, a likely relevant aspect of these specific announcements is that they repeated the statement that the FOMC committee "anticipates that economic conditions are likely to warrant exceptionally low levels of the federal funds rate for an extended period," indicating a quick return to the ultra low real rates that began early in the PM period under Greenspan. The FOMC press release for April 18, 2001 announced the

[17]The findings of Brooks, Katz, and Lustig (2018) have a similar flavor, but for a different reason. They document evidence of persistent post-FOMC announcement drift in longer term Treasury yields, implying that monetary policy has a long-lasting influence on markets outside of tight windows around FOMC announcements.

decision to lower its target for the federal funds rate by 50 basis points.

## 5.3 High-Frequency Analysis

In this subsection, we present estimation results relevant for the question of why markets respond strongly to central bank announcements. Figure 6 displays, for each FOMC announcement in our sample, the log change in pre-/post- announcement values of variables we measure at high frequency, where the pre-FOMC value is either 10 minutes before or the day before the FOMC press release time, depending on data availability (daily versus minutely), and the post-FOMC value is either 20 minutes after or the day after the release. The figure shows that some FOMC announcements have large effects on these forward looking variables, with jumps that are especially pronounced around the 2000/01 recession and tech bust in the stock market, and the 2008/9 financial crisis. Some of these announcements are associated with declines within 30 minutes surrounding the FOMC press release in the stock market that exceed 2% in absolute terms or increases above 4%, as when the FOMC met off-cycle on January 3, 2001 and decided to lower its target for the federal funds rate by an unusual (for the time) 50 basis points.

The mixed-frequency structural approach developed in this paper allows us to investigate a variety of possible explanations for these large market reactions. We briefly discuss how we use the filtering algorithm described above to obtain these results. The complete description and all technical details on the algorithm are relegated to the Online Appendix.

Consider an FOMC announcement in month $t$. As above, let $\delta_i \in (0,1)$ represent the number of time units that have passed during month $t$ up to point $t-1+\delta_i$. Let $S^j_{t|t-1+\delta_i}$ denote a filtered estimate of the perceived economic state that will be revealed at the end of time $t$ from data up to time $t-1+\delta_i$, conditional on $\xi^b_t = j$. We use the filtering algorithm described above along with high-frequency, forward-looking data on investor expectations and financial markets to obtain estimates of the pre- and post-FOMC announcement values of $S^j_{t|t-1+\delta_i}$, and the associated regime probabilities $\pi^j_{t|t-1+\delta_i}$ for the belief regimes, where $\delta_i$ here assumes distinct values $d_{pre}$ and $d_{post}$ that denote the time right before and right after the FOMC meeting. We compute announcement-related revisions in $S$ and in the belief regime probabilities $\pi^j$ by taking the difference between the estimated values for these variables pre- and post-announcement. These differences represent our estimates of the market's revised nowcasts for $S$ and beliefs about the future conduct of monetary policy that are attributable to the FOMC announcement.

Recall that the state vector $S_t = \left[S^M_t, m_t, pd_t, k_t, lp_t, \mathbb{E}^b_t\left(m_{t+1}\right), \mathbb{E}^b_t\left(pd_{t+1}\right)\right]$ where, $S^M_t$ is a vector of macro state variables with $S^M_t \equiv \left[\widetilde{y}_t, g_t, \pi_t, i_t, \overline{\pi}_t, f_t\right]'$. Figure 7 displays the percent changes in pre-/post- announcement values of different elements of $S_t$ for every FOMC announcement in our sample, providing an estimate of how investor perceptions about the current state of the economy shifted in the minutes surrounding a Fed announcement. The figure shows that FOMC meetings during the financial crisis led to frequent and large changes

in investor perceptions about trend growth $g_t$, detrended output, $\widetilde{y}_t$, inflation, current demand $f_t$, the earnings share $k_t$, and the liquidity premium $lp_t$. This evidence implies that FOMC announcements occasionally convey substantive information that causes investors to significantly revise their beliefs about the state of the economy and its core driving forces.

To make further progress of our understanding of what markets learn from FOMC announcements, we select the most relevant FOMC announcements for various series and decompose movements in several high-frequency variables into revisions in beliefs about the future conduct of monetary policy and about the primitive shocks affecting the economy. This decomposition is computed as follows. First, we filter high-frequency around announcements to obtain estimates of the perceived state vector $S^j_{t|t-1+\delta_i}$ and the belief regimes $\pi^j_{t|t-1+\delta_i}$ in the minutes and days surrounding an FOMC meeting. Second, we use these estimates to observe changes in the primitive shocks that investors *perceive* must have hit the economy in order to explain these movements in the perceived state vector:

$$S^j_{t|t-1+\delta_i} = C\left(\theta_{\xi^P_t}, \xi^b_t = j, \mathbf{H}^b\right) + T(\theta_{\xi^P_t}, \xi^b_t = j, \mathbf{H}^b)S_{t-1} + R(\theta_{\xi^P_t}, \xi^b_t = j, \mathbf{H}^b)Q\varepsilon^j_{t|t-1+\delta_i},$$

where $\varepsilon^j_{t|t-1+\delta_i}$ denotes the perceived Gaussian shocks estimated on the basis of data available at time $t-1+\delta_i$, conditional on being in belief regime $\xi^b_t = j$. For each FOMC announcement, we compute the contribution of one particular shock in the perceived shock vector $\varepsilon^j_{t|t-1+\delta_i}$ by setting all other shocks to zero and integrating out the belief regimes. Thus, the contribution of perceived shock $k$ is measured by:

$$S^{\cdot,k}_{t|t-1+\delta_i} = \textstyle\sum_{j=1}^{B} \pi^j_{t|t-1+\delta_i} R(\theta_{\xi^P_t}, \xi^b_t = j, \mathbf{H}^b)Q\varepsilon^{j,k}_{t|t-1+\delta_i} \tag{32}$$

The contribution of the belief regime is the remaining part:

$$S^{\cdot,b}_{t|t-1+\delta_i} = \textstyle\sum_{j=1}^{B} \pi^j_{t|t-1+\delta_i} \left[C\left(\theta_{\xi^P_t}, \xi^b_t = j, \mathbf{H}^b\right) + T(\theta_{\xi^P_t}, \xi^b_t = j, \mathbf{H}^b)S_{t-1}\right]. \tag{33}$$

We can then compute the contribution of revisions in investors perceptions of the shocks and/or about regime shifts in the policy rule to jumps in observed variables by taking the difference between the pre- and post-announcement values of $S^{\cdot,k}_{t|t-1+\delta_i}$ and $S^{\cdot,b}_{t|t-1+\delta_i}$.

The next several figures display the decomposition above for four different high-frequency observable variables in $X_t$: the BBG consensus forecasts of inflation and GDP growth 12-months ahead, the 6-month FFF contract rate, and the SP500 stock market value. To keep the figures manageable, we report the decomposition for the 10 most quantitatively important announcements according to the absolute value of the pre-/post-announcement change in a particular variable. For the four variables of interest, the figures report black dots to indicate the observed change in the series, and red triangles to indicate the model implied change. For the stock market, the black dot and red triangles coincide as we do not allow for observation error in that series.

Figure 8 reports the decomposition for a selection of FOMC announcements based on 10 most important absolute changes in the 6-month FFF rate. For all such events the model is able to match the direction of the jump in the observed series and in most cases the magnitude is also in line with the data. Many of these jumps are associated with times of important economic change, the largest of which occurs during the financial crisis on January 22, 2008 when the FOMC announced the lowering of the target for the FFR by an unusually large 75 basis point increment. From panel (c) we observe that most of the selected FOMC announcements are associated with a downward revision in the 6-month FFF rate, implying that markets were surprised by monetary policy that was more accommodative than anticipated, consistent with evidence in Cieslak (2018) and Schmeling, Schrimpf, and Steffensen (2020) who argue that markets systematically underestimated the Fed's response to large adverse economic shocks, and more generally with the arguments of Bauer and Swanson (2021), who argue that markets are often surprised by the Fed's response to economic events. Importantly, however, these announcements are rarely estimated to be solely attributable to a perceived monetary policy shock. Indeed, most announcements convey information about non-monetary shocks as well. For example, the January 22, 2008 announcement during the height of the financial crisis caused an upward revision in the perceived markup shock, resulting in a jump upward in the BBG expected inflation measure. This event is also associated with a jump up in the BBG forecast of GDP growth over the next year, driven mostly by an revision upward in perceived trend growth but also by a revision downward in perceived demand, which causes survey respondents to expect faster future growth from a lower current nowcast (the base effect). Meanwhile, the stock market declined by more than 1.9% in the 30 minutes surrounding the January 22, 2008 announcement, dragged down by a lower nowcast for the earnings share, a higher perceived liquidity premium, and a lower perceived demand shock.[18] Investor perceptions of a surprisingly accommodative monetary policy shock on this date helped to support the stock market, as did the upwardly revised nowcast for trend growth, but these factors were outweighed by revisions in beliefs about the state of the economy that were overwhelmingly in a pessimistic direction. This finding speaks to the importance of "information effects" as emphasized by Romer and Romer (2000), Campbell, Evans, Fisher, Justiniano, Calomiris, and Woodford (2012), and Nakamura and Steinsson (2018). The structural approach here adds to this literature by providing a granular decomposition of market reactions into the perceived economic sources of risk responsible for jumps in the stock market and other forward-looking variables. Other authors, notably Jarocinski and Karadi (2020) and Cieslak and Schrimpf (2019), have used a positive stock price response to a Fed tightening to identify instances where a Fed information effect was particularly strong, since under standard economic theory

[18] The negative contribution made by the revision in the perceived demand is not necessarily logically inconsistent with the upward revision in GDP growth expectations. A higher growth rate over the next year–from a lower current level of output–can coincide with persistently sluggish growth beyond the next 12 months.

a surprise monetary policy tightening should cause stock prices to fall rather than rise. The mixed-frequency approach of this paper compliments these findings by using a structural model to identify information effects and shows that they can be present even if the funds rate and the stock market comove in the direction standard economic theory predicts.

Figures 9-12 report the same decomposition for a selection of FOMC announcements based on 10 largest absolute changes in the BBG consensus inflation forecast, the stock market, the BBG consensus GDP growth forecast, and in the perceived probability of exiting the current policy rule in the next year, respectively. Different perceived shocks have different effects on different variables. For inflation forecasts (Figure 9), we again find that the January 22, 2008 FOMC announcement is an important surprise event, with markup shocks driving almost all of the jump upward in inflation expectations. For forecasts of output growth (Figure 11), we find that the FOMC press release of March 11, 2008–when the Fed announced an expansion of its securities lending programs in coordination with similar efforts at other central banks–is associated with both a negative FFF rate surprise and a downward revision in GDP growth forecasts, a positive comovement between the two that Nakamura and Steinsson (2018) argue is consistent with a strong Fed information effect. Monetary policy shocks played essentially no role in the downward revision to the GDP growth forecast for this event, which was instead driven by a downward revision in the nowcast for trend growth.

Figure 10 shows that the most quantitatively important FOMC announcement in our sample for the stock market was the one on January 3, 2001 discussed above, when the market increased 4.2% in the 30 minutes surrounding the news. The estimates imply that this jump was driven by a lower perceived nowcast for the liquidity premium component of the subjective equity premium, higher perceived aggregate demand and earnings share shocks, as well as a more accommodative perceived monetary policy shock. Although not evident from the figure, this event–in which the Fed surprised the market by lowering the funds rate by an unusually large 50 basis points–was also associated with a downward revision in the perceived trend growth rate of the economy, which may be interpreted as an upward revision in the perceived probability of a future characterized secular stagnation. But since detrended output growth didn't fall, this shows up as an upward revision in the perceived output gap and thus a higher perceived demand shock. The higher perceived demand shock in turn explains the jump upward in expected inflation observed in panel (a). The second and third most important FOMC events for the stock market were those on April 18, 2001 and October 29, 2008, respectively, when the market increased 2.5% and declined 2%, respectively, in the 30 minutes surrounding those press releases. For these latter two events, investor beliefs about the probability of monetary policy regime change played large quantitative roles.

If we focus on announcements that were most important for revisions in investor beliefs about the probability of regime change, the events of the April 18, 2001 and October 29, 2008 are the second and third most important according to that cut. This is exhibited in Figure 12, although

the most important FOMC announcement for belief revisions is the announcement on June 24, 2009 in the aftermath of the Great Recession and financial crisis. In this announcement the Fed promulgated the continued expansion of its balance sheet, the maintenance of the target range for the federal funds rate at 0 to 0.25%, and the statement that it "anticipates that economic conditions are likely to warrant exceptionally low levels of the federal funds rate for an extended period." The estimates imply that this announcement is associated with a large decline in the perceived probability of exiting the current policy rule in the next year that exceeded 2.9% in absolute terms, visible as the lowest dot in Figure 5. Panel (d) of Figure 12 shows that this jump in beliefs made a large negative contribution to the stock market's slightly negative overall response to this announcement, more than fully offsetting the positive contributions from upward revisions in the nowcasts for demand, trend growth, and the earnings share. The estimates for this event also show that changing beliefs about the policy rule around Fed announcements do not occur in a vacuum and often coincide with changing perceptions about the economic state that can have offsetting effects on the market, underscoring the empirical relevance of multiple channels of monetary transmission operating simultaneously in response to Fed communications.

Two other events bear noting. First, the event of October 29, 2008 is the event associated with second largest *decline* in the probability of exiting the policy rule, visible as the second lowest dot in Figure 5. This shift in investor beliefs about monetary policy regime change is the largest contributor to the market's 2% decline in the 20 minutes following that announcement. Second, the event of April 18, 2001 is the event associated with largest *increase* in the perceived probability of exiting the policy rule, visible as the highest dot in Figure 5. This shift in beliefs was the largest contributor to the market's 2.5% increase in the 20 minutes following that announcement. The event of October 28, 2008 (like June 24, 2009) is associated with a decline in the market because it lowered the perceived probability that central bank would soon shift to a policy rule capable of more actively stabilizing the economy. Since the lack of active stabilization increases expected volatility it increases the perceived quantity of risk in the stock market, lowering stock market valuations. By contrast, the event of April 18, 2001 is associated with a rise in the market because it increased the perceived probability that the central bank would soon shift to a policy rule with more active stabilization, thereby lowering expected volatility and the perceived quantity of risk.

## 5.4 Markets and Monetary Policy Over the Sample

We now zoom out from the announcements to study the role of monetary policy in driving financial market fluctuations over our entire sample. To do so, we first decompose the stock price to lagged output ratio into components driven by the representative investor's subjective beliefs about future earnings, future return premia, and future real interest rates. The price-

lagged output ratio is

$$\frac{P_t}{Y_{t-1}} = \frac{P_t}{D_t}\frac{D_t}{Y_t}\frac{Y_t}{Y_{t-1}}$$

or in logs

$$pgdp_t = pd_t + k_t + \Delta y_t,$$

where $pgdp_t \equiv \ln(P_t/Y_{t-1})$ and $pd_t \equiv \ln(P_t/D_t)$. Let $r^{ex}$ denote the log return on the stock market in excess of the log real interest rate, and let $rir$ denote the log real interest rate. We decompose $pd_t$ as in Campbell and Shiller (1989) into the sum of three forward-looking terms:

$$pd_t = \frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + pdv_t(\Delta d) - pdv_t(r^{ex}) - pdv_t(rir) \tag{34}$$

where the first term is a constant, $pdv_t(x) \equiv \sum_{h=0}^{\infty}\beta_p^h \mathbb{E}_t^b[x_{t+1+h}]$, and $rir_{t+1} \equiv \left(i_{t+1} - \mathbb{E}_t^b[\pi_{t+1}]\right)$ is the expected real interest rate from the perspective of the investor.[19] The subjective expectations of the investor $\mathbb{E}_t^b[\cdot]$ are computed from the structural estimates and depend on the beliefs about the future conduct of monetary policy as well as the expected paths of Gaussian variables. Subjective equity market return premia embedded in $pdv_t(r^{ex})$ are driven in the model by just three factors: (i), realized regime change in monetary policy $\xi_t^P$, (ii) changing investor beliefs about the probability of future regime change $\xi_t^b$, and (iii) the liquidity premium $lp_t$. Subjective expectations of future real interest rates embedded in $pdv_t(rir)$ depend these factors, as well as expectations about inflation and output growth that enter the monetary policy rule. With (34) in hand, we can decompose $pgdp_t$ into the sum of four components:

$$pgdp_t = \underbrace{ey_t}_{\text{earning share}} + \underbrace{pdv_t(\Delta d)}_{\text{earnings}} - \underbrace{pdv_t(r^{ex})}_{\text{premia}} - \underbrace{pdv_t(rir)}_{\text{real int rate}}, \tag{35}$$

where $ey_t \equiv \frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + k_t + \Delta y_t$ is the earnings to lagged output ratio. We refer to $ey_t$ as the "earnings share" for simplicity, though the reader is reminded that this variable depends on both $k_t$ and on output growth $\Delta y_t$, and is shifted up by a constant.

Figure 13 reports the empirical decomposition of $pgdp_t$ into the estimated components of (35). The solid (blue) line in each panel plots the data for $pgdp_t$ (the S&P 500-lagged GDP ratio) over our sample. The red lines in panels (a)-(d) successively cumulate the right hand side components in (35) so that they add to the observed $pgdp_t$ as we move from panel (a) to panel (d).

Panel (a) of Figure 13 shows the data for $pgdp_t$ (in blue) plotted along with the $ey_t$ component alone (in red).[20] This panel shows that the earnings share plays little role in fluctuations in $pgdp_t$ up to about the year 2000. The $ey_t$ component then declines sharply during the financial crisis of 2008/09 contributing to the sharp drop in the stock market (blue line) during the crisis. Subsequently, the earnings share recovers and increases sharply, helping to boost the market

[19] The derivation of this decomposition is given in the Online Appendix.

[20] Note that the $ey_t$ term includes the constant $\frac{\kappa_{pd,0}}{1-\kappa_{pd,1}}$ so it can be greater than $pgdp_t$.

in the years after the financial crisis, similar to results in Greenwald, Lettau, and Ludvigson (2019).

Moving to panel (b) of Figure 13, the model components (red) line adds $-pdv_t\left(r^{ex}\right)$ to $ey_t$. A comparison of panels (a) and (b) shows that adding $-pdv_t\left(r^{ex}\right)$ to $ey_t$ brings the red (dashed) line much closer to the observed $pgdp_t$ data series (blue line) especially in the PM regime. Panel (b) also plots a counterfactual for the component $-pdv_t\left(r^{ex}\right)+ey_t$ (green line) in which we turn off the liquidity premium shocks $lp_t$, implying that the only factor causing fluctuations in $pdv_t\left(r^{ex}\right)$ for that counterfactual case are realized policy rule regime changes and changing investor beliefs about the probability of a regime change. The green counterfactual line is quite close to the baseline estimate over most of the sample, implying that much of the variation in the estimated subjective return premium is driven by fluctuating monetary policy rules and beliefs about future policy rule shifts, rather than by fluctuations in the liquidity premium. Indeed, panel (b) shows that $pgdp_t$ rises sharply at both the beginning and the end of the GM regime primarily because these were times when the perceived probability of a monetary policy regime change rose sharply (Figure 4). Given the estimates of the GM perceived Alternative policy rule, these were then times when investors perceived a greater likelihood that the central bank would move to a policy rule with greater activism to stabilize the economy, especially on the real side (Table 2). These shifts in beliefs thus drove down the perceived quantity of risk in the stock market and drove up valuations.

Panel (c) of Figure 13 adds $-pdv_t\left(rir\right)$ to the components $ey_t-pdv_t\left(r^{ex}\right)$ plotted in panel (b), so that the differences between panels (b) and (c) isolate the role of subjectively expected real interest rates in stock market fluctuations. Expectations of persistently low future real rates helped support the stock market in the GI regime from 1961:Q1-1978:Q3, but by contrast expectations of persistently higher future real rates pulled down the market in the early part of the GM regime, with the shift to a hawkish policy rule during the Volcker disinflation. Comparing panels (b) and (c) we see that expectations of persistently higher future real interest rates largely explain the low stock market valuations between 1978:Q3 to about 1990. Taken together, these results imply that the Volcker disinflation and the Great Moderation that followed set the stage for the high valuations in 1990s by reducing perceived volatility and lowering subjective return premia, but initially it dragged the market down through the shift to a more hawkish policy rule with persistently high real interest rates.

Finally, panel (d) of Figure 13 adds $pdv_t\left(\Delta d\right)$ to $ey_t-pdv_t\left(r^{ex}\right)-pdv_t\left(rir\right)$. A comparison of panel (d) with panel (c) shows that expected future cash flow growth plays a small role in these stock market fluctuations.

To further explore the role of investor beliefs about future monetary regime change in driving stock market fluctuations over the sample, Figure 13 exhibits the results of a counterfactual analysis for the PM regime subperiod. We again report a decomposition of $pgdp_t$ into different components, but this time adding only one of the $pdv\left(\cdot\right)$ terms in (35) at a time to $ey_t$. Denote

these components as

$$\begin{aligned} pgdp_{r^{ex},t} &\equiv ey_t - pdv_t\left(r^{ex}\right) \\ pgdp_{rir,t} &\equiv ey_t - pdv_t\left(rir\right) \\ pgdp_{\Delta d,t} &\equiv ey_t + pdv_t\left(\Delta d\right). \end{aligned}$$

The solid (blue) line in each panel of Figure 13 plots our baseline estimate of the component series named in the subpanel. For panel (a), which plots $pgdp_t$, our baseline model estimate and the data series coincide by construction. Panels (b)-(c) plot the components $pgdp_{r^{ex},t}$, $pgdp_{rir,t}$, and $pgdp_{\Delta d,t}$, respectively. The red (dashed) line in each panel plots a counterfactual in which investors believe throughout the PM subperiod that the probability of exiting the policy rule was the highest value that they would ever entertain given our estimates on the grid. The purple (dashed-dotted) line in each panel plots a counterfactual in which investors believe that the probability of exiting the policy rule was the lowest value they would entertain.[21]

Figure 13 conveys two main findings. First, it shows that investor beliefs about the conduct of future monetary policy play an outsized role in stock market fluctuations. This can be observed from the quantitatively large gap between the red and purple lines in panel (a). Had investors counterfactually maintained the belief that the central bank was very likely to exit the PM policy rule, the stock market would have been much higher than it actually was over most of this period (red dashed line), and substantially higher than if they had counterfactually held the opposite belief that regime change was very *un*likely (purple dashed-dotted line). Second, panels (b)-(d) show that the reason for this large discrepancy has to do with the affect of beliefs on investors' subjective expectations for future equity return premia, rather than with their effect on subjective expectations of future real rates or future payout growth. This can be observed by noting that the red/blue line discrepancy is largest for $pgdp_{r^{ex},t}$ in panel (b), small for $pgdp_{rir,t}$ in panel (c), and non-existent for $pgdp_{\Delta d,t}$ in panel (d). In short, had investors counterfactually believed throughout the PM period that monetary policy regime change was highly likely, the market would have been higher because subjective equity risk premia would have been lower.

We close this section with Figure 15, which examines these same forces at high frequency around FOMC announcements. The figure decomposes the announcement-related jumps in $pd_t$ into fluctuations driven by the $pdv_t(\cdot)$ components on the right-hand-side of (34) for the 5 most relevant FOMC announcements sorted on the basis of jumps in the estimated perceived

[21] Recall that $P\left(\xi_t^b = i | X_T; \boldsymbol{\theta}\right) \equiv \pi_{t|T}^i$ is the estimated probability that $\xi_t^b = i$, for $i = 1, 2, ..., B+1$, while $\pi_{t|T}$ is a $(B+1)\times 1$ vector containing the elements $\left\{\pi_{t|T}^i\right\}_{i=1}^{B+1}$. The regime $\xi_t^b = 1$ is the belief regime corresponding to the lowest perceived probability that the central bank will stay with the current policy rule, i.e., the highest perceived probability of exiting. The first counterfactual replaces the estimated belief regime probabilities $\pi_{t|T}$ with a vector that has unity as the first element and zeros elsewhere. The second counterfactual replaces $\pi_{t|T}$ with a vector that has unity as the $B$th element and zeros elsewhere.

probability of a regime change in the conduct of monetary policy over the next year. Panel (a) shows the change in the perceived probability of a regime change for each of these 5 events in 30 minute windows, while panel (b) shows the decomposition of the resulting jump in $pd_t$ into its $pdv_t(\cdot)$ components. The FOMC announcement of June 24, 2009 is associated with the largest absolute revision (downward in this case) in the perceived probability of a regime change in monetary policy. The FOMC communication on this date occurred in the aftermath of the financial crisis and Great Recession and announced the maintenance of the funds rate in the 0 to 0.25% range, the continued expansion of the Fed balance sheet, and the promise to keep interest rates ultra low for an extended period of time. Panel (b) shows that this news is associated with a large jump upward in subjective expected return premia, as measured by $pdv_t(r^{ex})$, and a modest jump downward in the present discounted value of subjectively expected real interest rates, as measure by $pdv_t(rir)$. Subjective perceptions of risk rise because of the sharp decline in the perceived probability that the central bank will transition the PM Alternative policy rule with greater activism to stabilize the economy. The dovish tone of the announcement helped to support the market by generating expectations of persistently lower future real interest rates, but this was not enough to offset the rise in subjective return premia, and $pd_t$ fell.

## 6 Conclusion

We integrate a high-frequency monetary event study into a mixed-frequency macro-finance model and structural estimation. The approach allows for jumps at Fed announcements in investor beliefs, providing granular detail on why markets react strongly to central bank announcements. We also provide a methodology for modeling expectations in the presence of structural breaks, and show how forward-looking data can be used to infer what agents expect from the next policy regime. The methodology can be used in a variety of other settings to provide an richer understanding of the role of news shocks of almost any category in driving financial market volatility.

Why do financial markets react strongly to central bank communications? In this study we find that the reasons involve a mix of factors, including revisions in investor beliefs about the latent state of the economy ("Fed information effects"), uncertainty over the future conduct of monetary policy, and subjective reassessments of risk in the stock market. This occurs for three main reasons. First, we beliefs about the conduct of future policy react to Fed news even if current policy is unchanged, affecting the perceived quantity of risk in the stock market. Second, realized shifts in the central bank policy rule over our sample have had a persistent influence on short rates, affecting valuations. Third, occasionally we find big announcement-driven revisions in investor perceptions of the economic state that include shifts in the composition of perceived shocks hitting the economy such as those attributable to demand versus supply factors, to distributional dynamics and/or pricing power, as well as to perceptions about monetary policy

shocks.

At the same time, the mixed-frequency structural approach permits us to estimate the effects of monetary policy over an extended sample, not merely in tight windows around Fed announcements. Most Fed announcements are not associated with a change in the policy rule, but instead provide forward guidance on the likely triggers of a future change in policy conduct. We find that beliefs about the future conduct of monetary policy continuously evolve over the sample, implying that event studies understate the impact of monetary policy on financial markets.

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Table 1: **Regime Subperiods and Parameter Estimates**

| | 1961:Q1-1978:Q3 | 1978:Q4-2001:Q1 | 2001:Q2-2020:Q1 |
|---|---|---|---|
| Regime | Great Inflation | Great Moderation | Post-Millennial |
| $r_{\xi_t^P}$ | $-2.67\%$ | $1.38\%$ | $-1.27\%$ |

Notes: Table reports the most likely regime sequence based on the posterior mode estimates. The second row reports the model estimate of the mean of *mps* $r_{\xi_t^P}$ at the posterior mode. The estimation sample spans 1961:Q1-2020:Q1.

Table 2: **Taylor Rule Parameters**

| | Great Inflation Regime | | Great Moderation Regime | | Post-Millennial Regime | |
|---|---|---|---|---|---|---|
| | Realized | Alternative | Realized | Alternative | Realized | Alternative |
| $\pi_\xi^T$ | 12.5288 | 11.8454 | 1.9142 | 0.8244 | 2.4878 | 0.0597 |
| $\psi_\pi$ | 1.4831 | 2.0735 | 3.0071 | 3.6075 | 0.0000 | 0.6783 |
| $\psi_y$ | 1.1985 | 0.0349 | 0.0005 | 0.6763 | 0.0751 | 0.5270 |
| $\psi_\pi/\psi_y$ | 1.2375 | 59.4126 | 6014.2 | 5.3342 | 0.0000 | 1.2871 |
| $x = \rho_{i,1} + \rho_{i,2}$ | 0.9960 | 0.8189 | 0.9905 | 0.9852 | 0.9972 | 0.9444 |

Notes: For each realized policy regime, the table reports the posterior mode values of the parameters for the current and alternative policy rules. The estimation sample spans 1961:Q1-2020:Q1.

Table 3: **Other Key Parameters**

| Parameter | Mode | Parameter | Mode | Parameter | Mode | Parameter | Mode |
|---|---|---|---|---|---|---|---|
| $\sigma$ | 0.0522 | $\gamma^T$ | 0.0050 | $\sigma_f$ | 17.2460 | $\sigma_{lp}$ | 0.6211 |
| $\beta$ | 0.7529 | $\sigma_p$ | 6.0097 | $\sigma_i$ | 0.0344 | $\sigma_g$ | 1.9079 |
| $\phi$ | 0.7424 | $\beta_p$ | 0.9919 | $\sigma_\mu$ | 0.1348 | | |
| $\gamma$ | 0.0001 | $p_s$ | 0.9875 | $\sigma_k$ | 6.1267 | | |

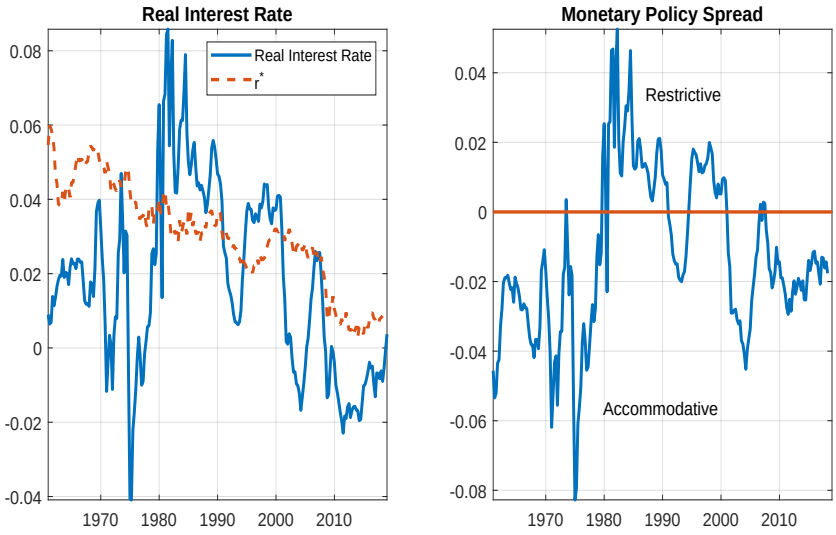
Notes: The table reports the posterior mode values of the parameters named in the row. The estimation sample spans 1961:Q1-2020:Q1.

Table 4: **Asset Pricing Moments**

| Moments | Model | | Data | |
|---|---|---|---|---|
| | Mean | StD | Mean | StD |
| Log Excess Return | 7.71 | 14.92 | 7.42 | 14.85 |
| Real Interest Rate | 1.63 | 2.58 | 1.72 | 2.53 |
| Log Real Earning Growth | 1.97 | 16.60 | 1.96 | 17.24 |

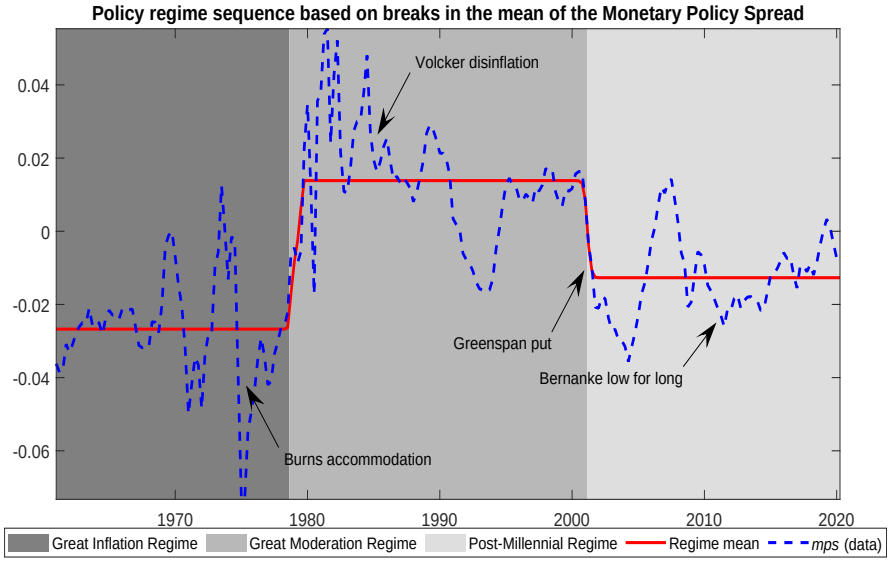
Notes: All reported statistics are annualized monthly statistics (means are multiplied by 12 and standard deviations by $\sqrt{12}$) and reported in units of percent. Excess returns are computed as the log difference in SP500 market capitalization minus FFR. The real interest rate is computed as the difference between FFR and average of the one-year ahead forecast of inflation across different surveys including BC, SPF, SOC, and Livingston. SP500 Earnings is deflated using GDP deflator and divided by population. The sample is 1961:M1 - 2020:M2.

## Figure 1: Real Interest Rate



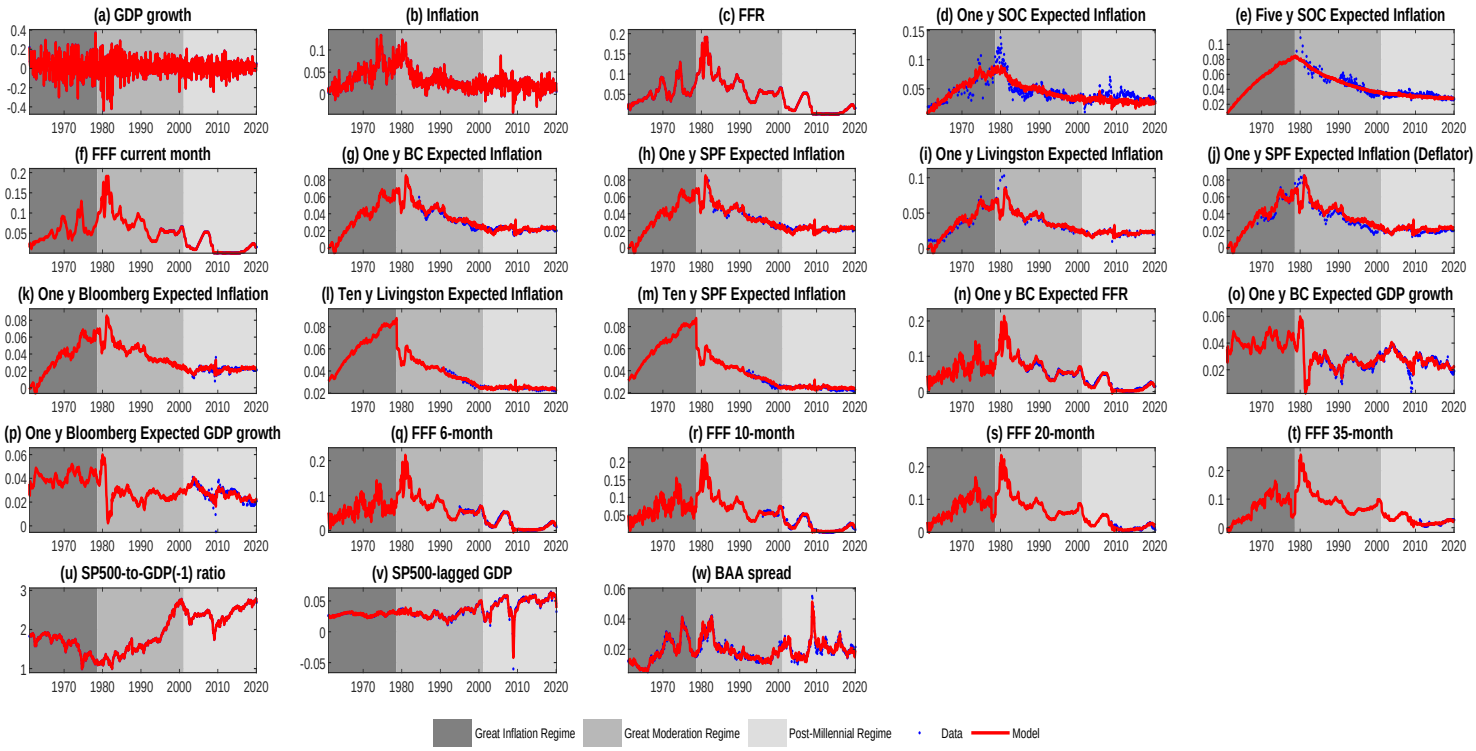

Notes: The real interest rate is measured as the federal funds rate minus a four quarter moving average of inflation. The left panel plots this observed series along with an estimate of $r^*$ from Laubach and Williams (2003). The right panel plots the monetary policy spread, i.e., the spread between the real funds rate and the Laubach and Williams (2003) natural rate of interest. The sample spans 1961:Q1-2020:Q1.

## Figure 2: Breaks in Monetary Policy



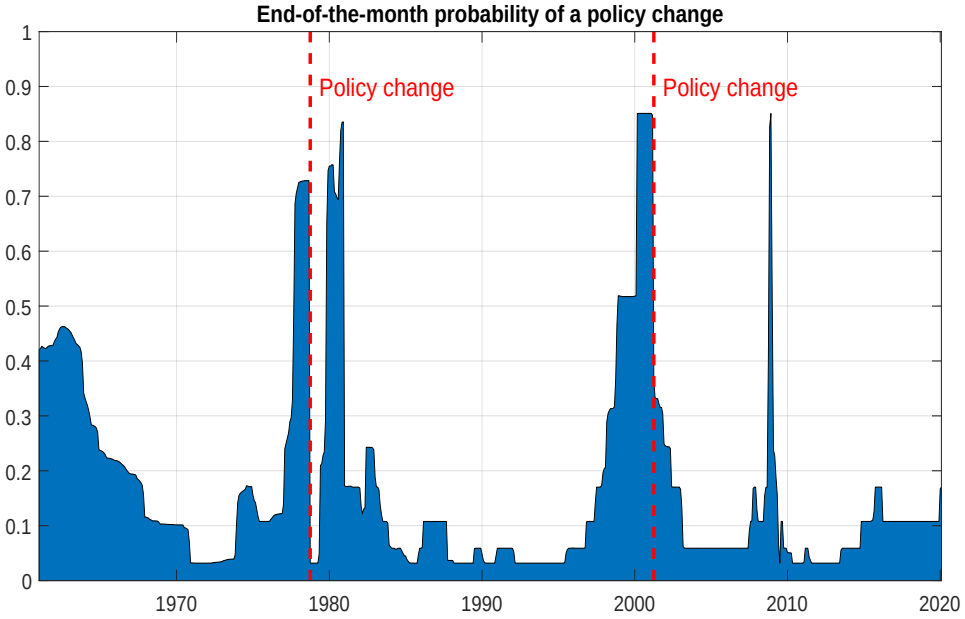

Notes: Monetary policy spread $mps_t \equiv FFR_t - \text{Expected Inflation}_t - r_t^*$. $r^*$ is from Laubach and Williams (2003). The red (dashed) line represents the data. The blue (solid) line is the estimated regime mean. Accomdative regimes have $mps_t < 0$; restrictive regimes have $mps_t > 0$. The sample spans 1961:Q1-2020:Q1.

## Figure 3: Smoothed Series



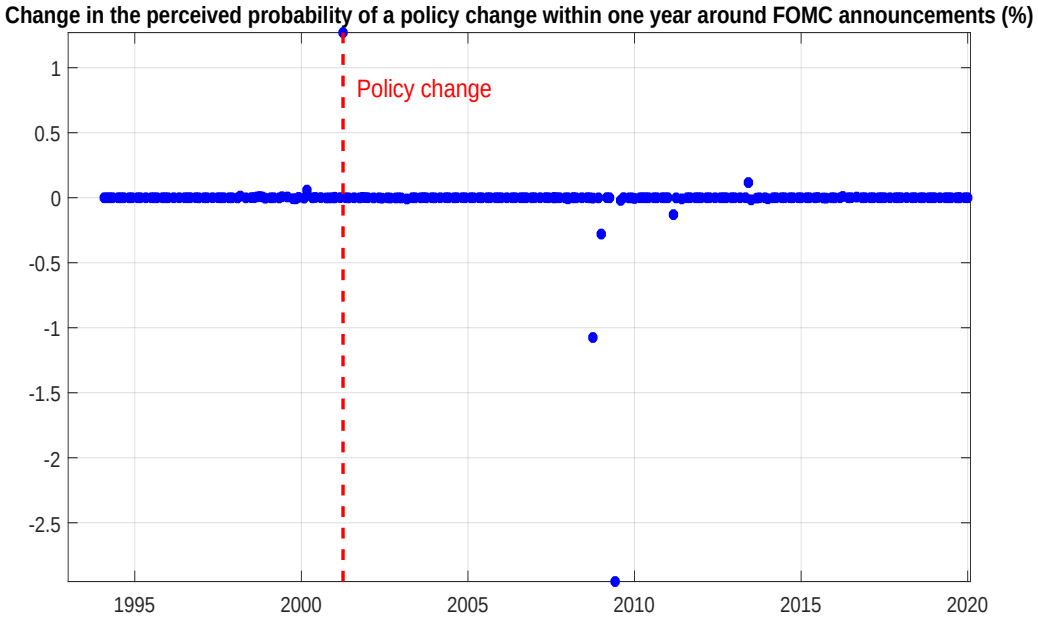

Notes: The figure displays the model-implied series (red, solid line) and the actual series (blue dotted line). The model-implied series are based on smoothed estimates $S_{t|T}$ of $S_t$, using observations through then end of the sample at date $T$, and exploit the mapping to observables in (31) using the modal parameter estimates. The difference between the model-implied series and the observed counterpart is attributable to observation error. We allow for observation errors on all variables except for GDP growth, inflation, the FFR, and the SP500 capitalization to GDP ratio. The sample is 1961:M1-2020:M2.

**Figure 4: Perceived Probability of Monetary Policy Regime Change**



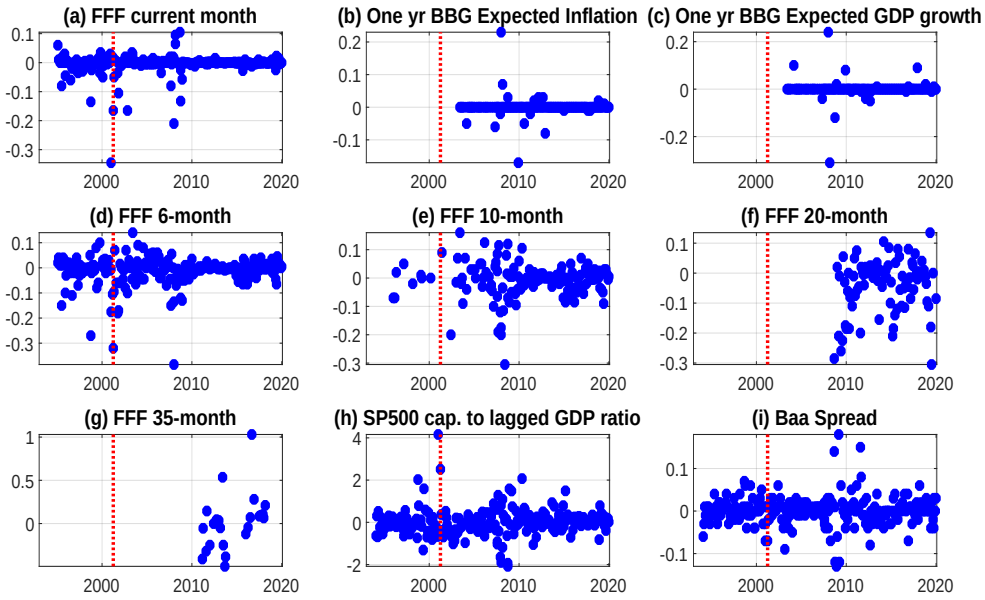

Notes: The figure displays the estimated end-of-month perceived probability investors assign to exiting the current monetary policy rule within one year, computed as the estimated perceived transition probability of being in the Alternative rule at $t + 12$ under each $\xi_t^b = i$, weighted by the smoothed regime probabilities $\Pr(\xi_t^b = i | X_T; \boldsymbol{\theta})$. The sample spans 1961:M1-2020:M2.

**Figure 5: Change in the probability of a policy switch around FOMC announcements**



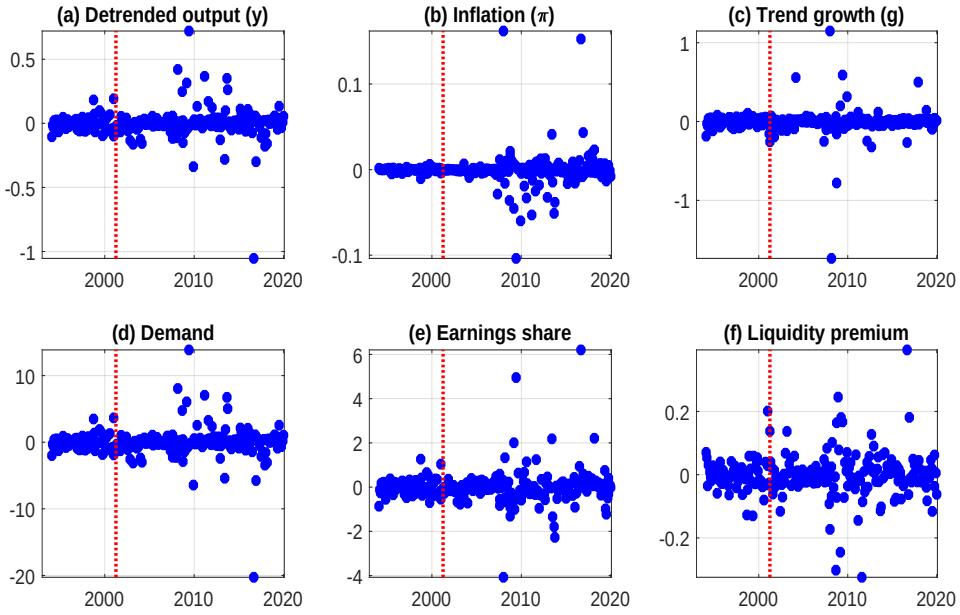

Notes: The figure displays, for each FOMC announcement in our sample, the pre-/post- FOMC announcement log change (10 minutes before/20 minutes after) in the probability that financial markets assign to a switch in the monetary policy rule occurring within one year. The full sample has 220 announcements spanning February 4th, 1994 to February 28th, 2020. The sample reported in the figure is 1993:M1-2020:M2.

## Figure 6: HF Changes in Prices and Expectations



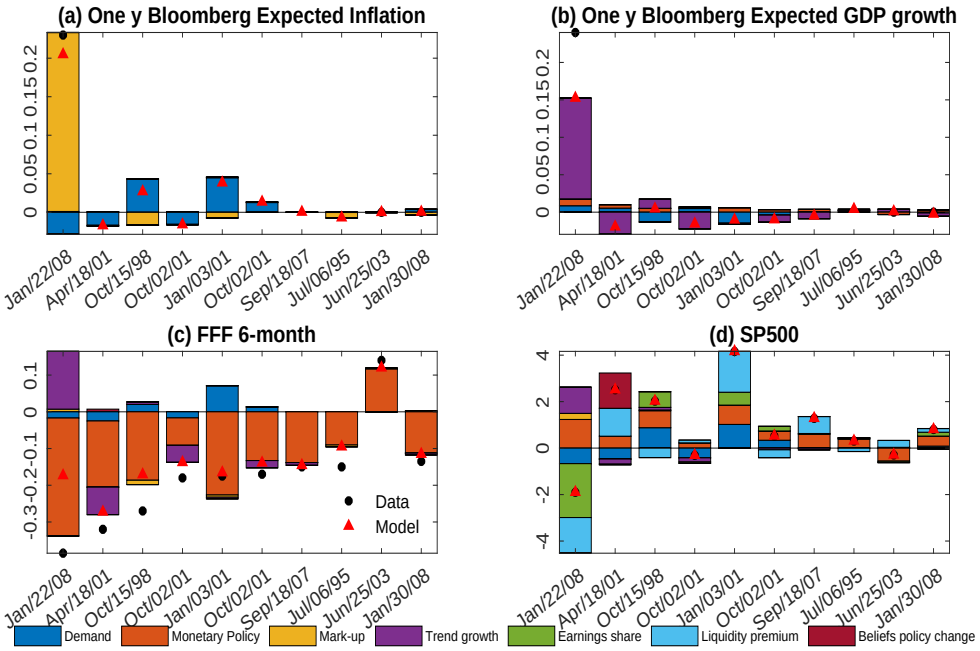

Notes: The figure displays, for each FOMC meeting in our sample, the log change in the observed variables in a short time-window around FOMC meetings. For all but panels (b) and (c), this corresponds to a change measured from 10 minutes before to 20 minutes after an FOMC statement is released. For panels (b) and (c), this corresponds to one day before to one day after the FOMC statement is released. The full sample has 220 FOMC announcements spanning February 4th, 1994 to February 28th, 2020. The sample reported in the figure is 1993:M1-2020:M2.

## Figure 7: HF Changes in State Variables



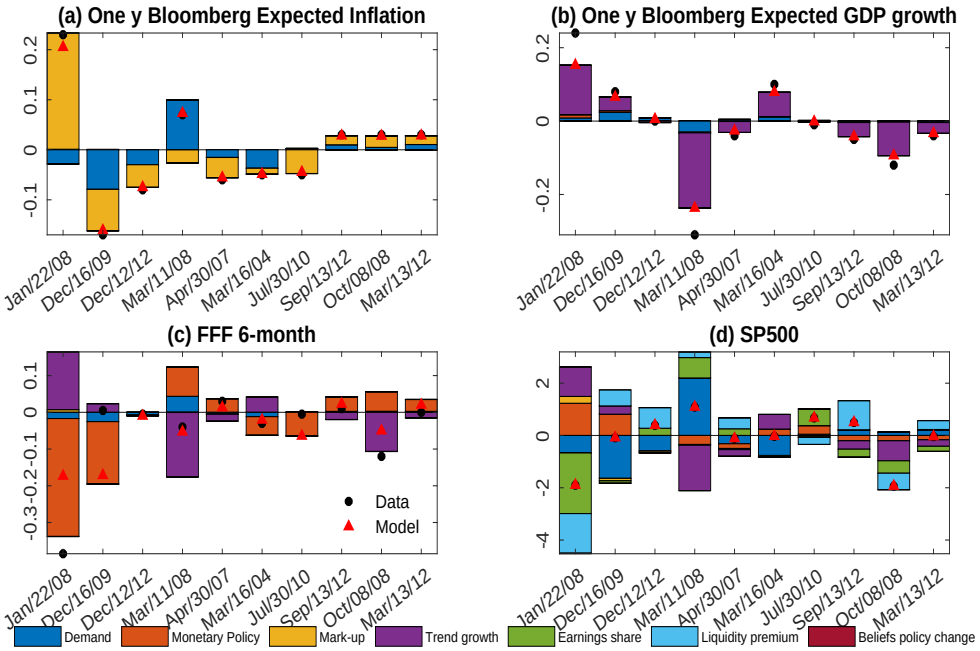

Notes: The figure displays, for each FOMC announcement in our sample, the change in the perceived state of the economy from 10 minutes before to 20 minutes after an FOMC statement is released. The full sample has 220 FOMC announcements spanning February 4th, 1994 to February 28th, 2020. The sample reported in the figure is 1993:M1-2020:M2.

## Figure 8: Top Ten FOMC: 6-month FFF rate



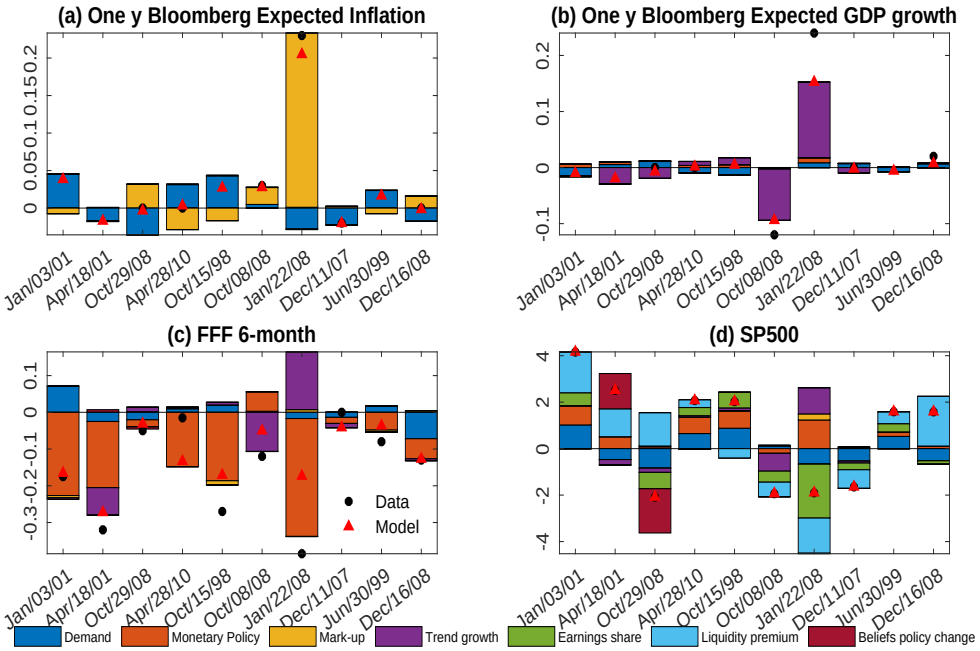

Notes: The figure reports the decomposition of movements in Bloomberg expected inflation, Bloomberg expected GDP growth, the 6-month FFF rates, and the stock market attributable to revisions in the perceived shocks hitting the economy and in the belief regimes for the 10 most relevant FOMC announcements based on changes in the 6-month FFF rate. For panel (d), because we do not have measurement error in the equations for the SP500 to lagged GDP ratio, the black dot (data) and the red triangles (model) lie on top of each other, so the black dot is obscured. The sample is 1961:M1-2020:M2.

## Figure 9: Top Ten FOMC: Bloomberg Expected Inflation



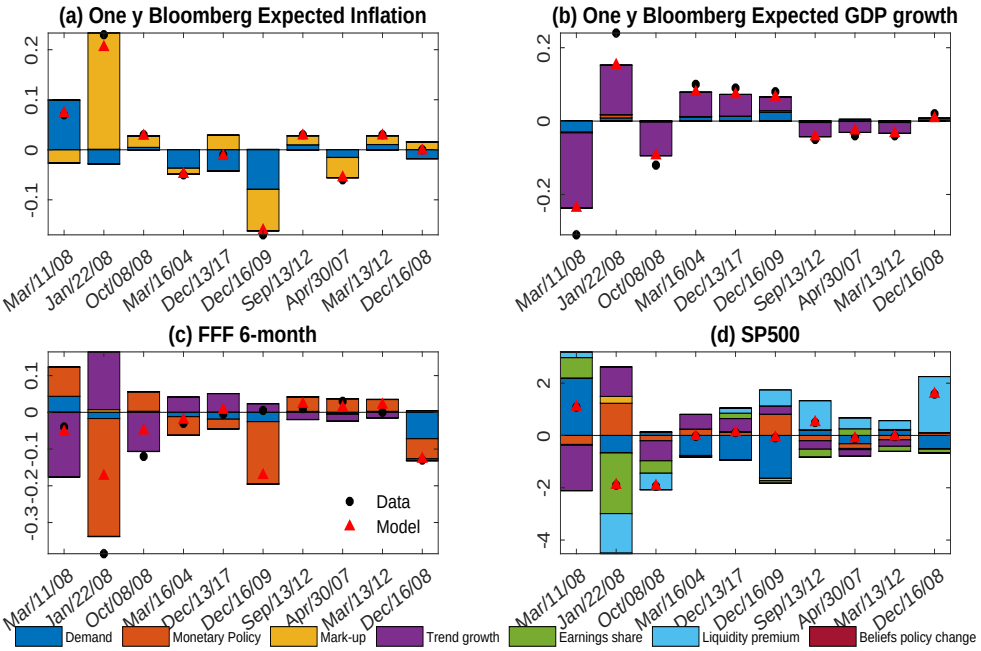

Notes: See Figure 8. The figure reports the decomposition of movements in Bloomberg expected inflation, Bloomberg expected GDP growth, the 6-month FFF rates, and the stock market attributable to revisions in the perceived shocks hitting the economy and in the belief regimes for the 10 most relevant FOMC announcements based on changes in the Bloomberg one-year inflation expectations. The sample is 1961:M1-2020:M2.

## Figure 10: Top Ten FOMC: SP500



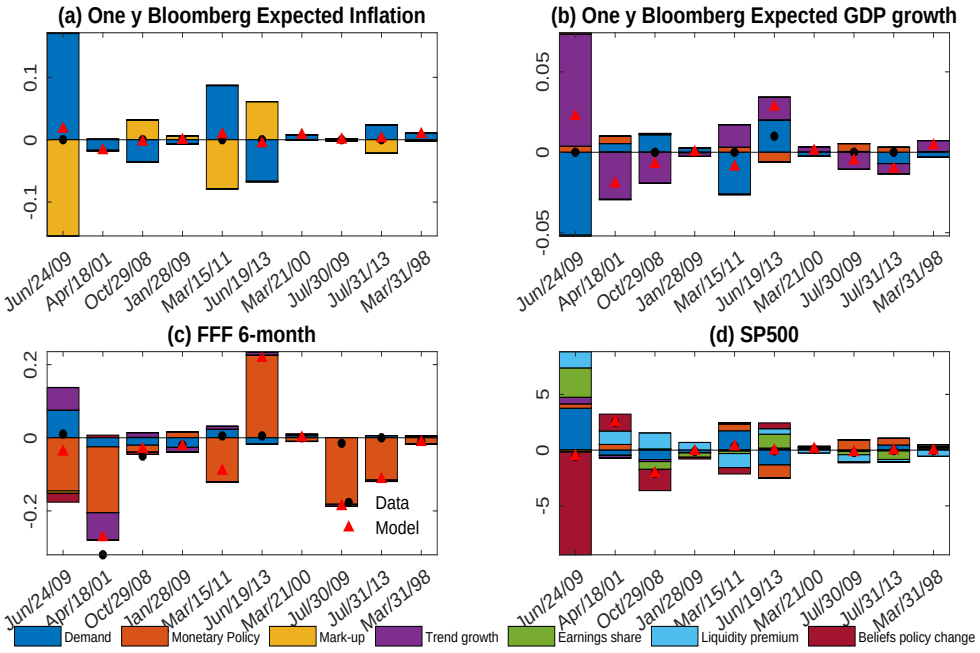

Notes: See Figure 8. The figure reports the decomposition of movements in Bloomberg expected inflation, Bloomberg expected GDP growth, the 6-month FFF rates, and the stock market attributable to revisions in the perceived shocks hitting the economy and in the belief regimes for the 10 most relevant FOMC announcements based on changes in the SP500-lagged GDP ratio. The sample is 1961:M1-2020:M2.

## Figure 11: Top Ten FOMC: Bloomberg Expected GDP growth



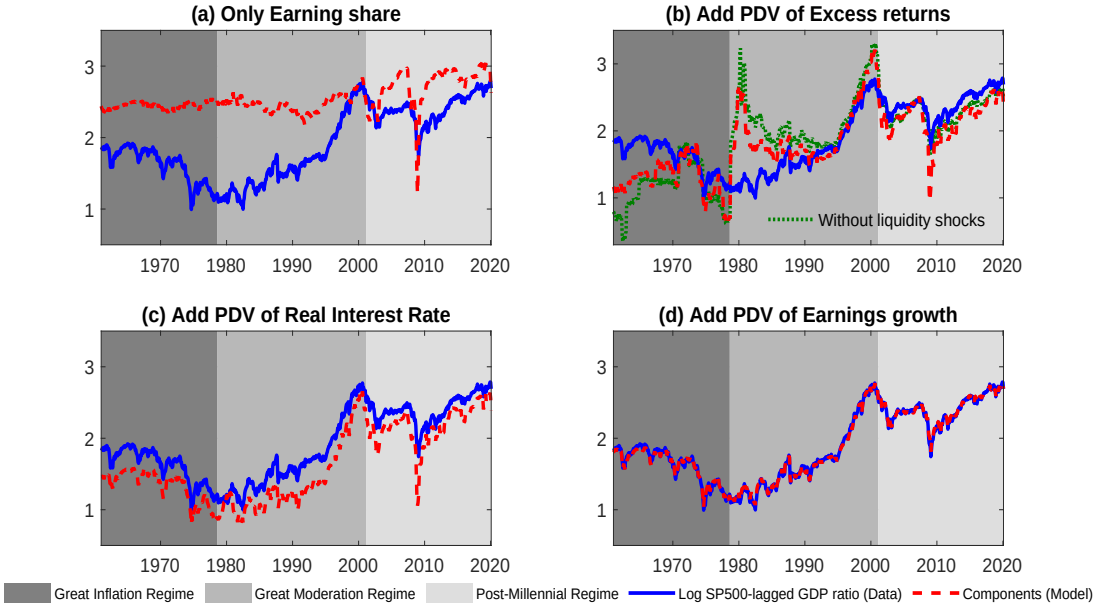

Notes: See Figure 8. The figure reports the decomposition of movements in Bloomberg expected inflation, Bloomberg expected GDP growth, the 6-month FFF rates, and the stock market attributable to revisions in the perceived shocks hitting the economy and in the belief regimes for the 10 most relevant FOMC announcements based on changes in the Bloomberg one-year GDP growth expectations. The sample is 1961:M1-2020:M2.

**Figure 12: Top Ten FOMC: Probability of Exiting Policy Rule over the Next Year**



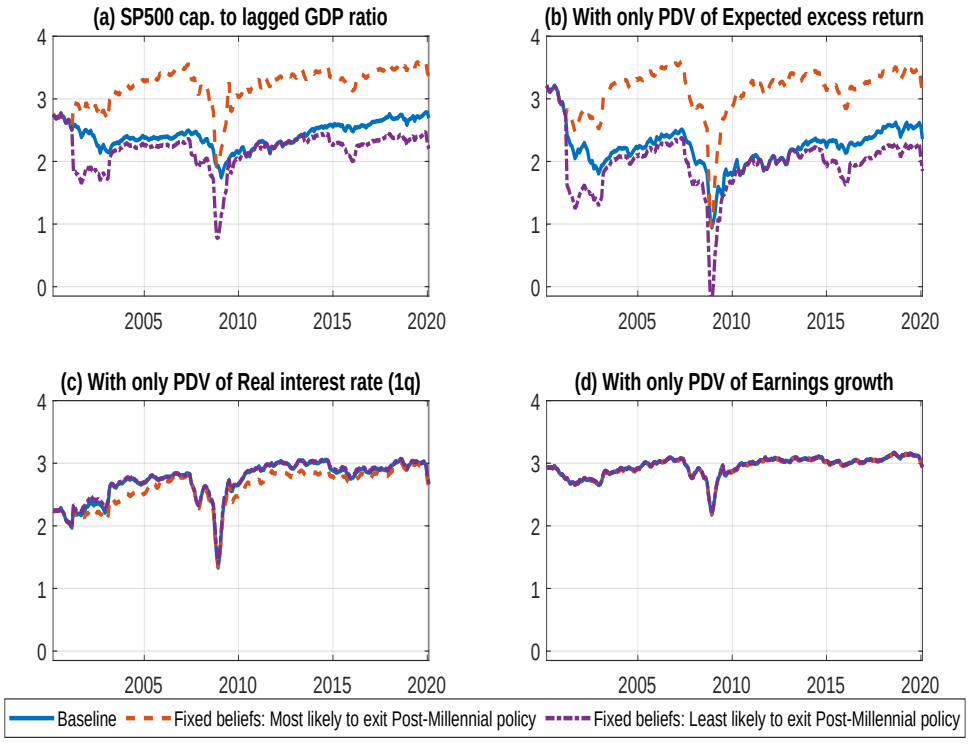

Notes: See Figure 8. The figure reports the decomposition of movements in Bloomberg expected inflation, Bloomberg expected GDP growth, the 6-month FFF rates, and the stock market attributable to revisions in the perceived shocks hitting the economy and in the belief regimes for the 10 most relevant FOMC announcements based on changes in the beliefs about the probability of exiting the policy rule over the next 12 months. The sample is 1961:M1-2020:M2.

## Figure 13: SP500-to-GDP decomposition



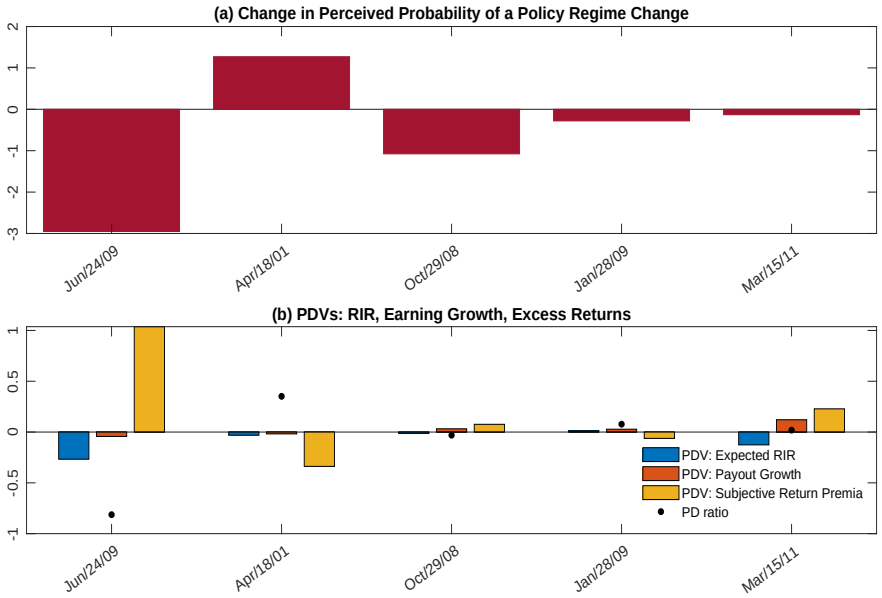

Notes: The figure displays a decomposition of the log SP500-to-lagged GDP ratio. The blue (solid) line represents the data. The dashed (red) lines represent component in the model. The log ratio in the model may be decomposed as $pgdp_t = ey_t + pdv_t(\Delta d) - pdv_t(r^{ex}) - pdv_t(rir)$, where $pdv_t(x) \equiv \sum_{h=0}^{\infty} \beta_p^h \mathbb{E}_t^b [x_{t+1+h}]$ and $ey_t$ is the earnings-lagged output ratio plus linearization constant. Panel (a) plots $pgdp_t$ along with $ey_t$. Panel (b) plots $pgdp_t$ with $ey_t - pdv_t(r^{ex})$. Panel (c) plots $pgdp_t$ with $ey_t - pdv_t(r^{ex}) - pdv_t(rir)$. Panel (d) plots $pgdp_t$ in the data along with $ey_t + pdv_t(\Delta d) - pdv_t(r^{ex}) - pdv_t(rir)$. The sample spans 1961:M1 - 2020:M2

**Figure 14: Counterfactual simulations: The Post-Millennial period**




Notes: The figure displays counterfactual simulations for the post-Millennial period. The red (dashed) line corresponds to a counterfactual simulation in which agents' beliefs are set assuming that the $(B+1)$-dimensional belief regime probability vector $\pi_{t|T}$ is replaced by a counterfactual regime probability vector equal to $(1, ..., 0, 0)\prime$ at each $t$. The purple (dashed-dotted) line corresponds to a counterfactual simulation in which agents' beliefs are set assuming that $\pi_{t|T}$ is replaced by a counterfactual regime probability vector equal to $(0, ..., 1, 0)\prime$ at each $t$. Panel (a) plots the model implications for the price-lagged output ratio $pgdp_t$. This series perfectly matches our observed series for the SP500-lagged GDP ratio. Panel (b) plots $pgdp_{r^{ex},t}$. Panel (c) plots $pgdp_{rir,t}$. Panel (d) plots $pgdp_{\Delta d,t}$. The sample for the counterfactual spans 2000:M3 to 2020:M2.

**Figure 15: Jumps in risk perceptions, short rates, and earnings expectations**




Notes: The table reports jumps in subjective expectations of risk, future short rates, and future earnings growth within tight windows around an FOMC announcement. Panel (a) shows the pre-/post-FOMC announcement change (10 minutes before/20 minutes after) in the perceived probability that financial markets assign to a switch in the monetary policy rule occurring within one year, for the 10 most quantitatively important FOMC announcements based on changes in investor beliefs about the future conduct of monetary policy. Panel (b) shows a decomposition of the model's fluctuations in the log price-payout ratio $pd = pdv_t(\Delta d) - pdv_t(r^{ex}) - pdv_t(rir)$ in 30 minute windows around these 10 announcements that are driven by subjective equity risk premium variation, as measured by $pdv_t(r^{ex})$ (yellow bar), subjective expected future real interest rate fluctuations, as measured by $pdv_t(RIR)$ (blue bar), and subjective expected earnings growth, as measured by $pdv_t(\Delta d)$ (red bar). PD ratio is $pdv_t(\Delta d) - pdv_t(r^{ex}) - pdv_t(rir)$. The sample is 1961:M1-2020:M2.

# Online Appendix

Table A.1: **Other Parameters**

| Parameter | Mode | Parameter | Mode | Parameter | Mode | Parameter | Mode |
|---|---|---|---|---|---|---|---|
| $\sigma$ | 0.0650 | $\phi$ | 0.7436 | $\rho_k$ | 0.9980 | scale Baa | 0.3998 |
| $\varrho_1$ | 0.5372 | $\bar{r}$ | 0.0000 | $\lambda_{k,y}$ | 26.9629 | $\sigma_f$ | 23.4733 |
| $\beta$ | 0.7161 | $\gamma^T$ | 0.0051 | $\rho_{lp}$ | 0.8407 | $\sigma_i$ | 0.0331 |
| $\kappa_1$ | 0.0036 | $e^{\bar{k}}$ | 0.0507 | $\varrho_2$ | 0.2338 | $\sigma_\mu$ | 0.1379 |
| $\gamma$ | 0.0001 | $\sigma_p$ | 5.8542 | $\varrho_3$ | 0.1887 | $\sigma_k$ | 6.2614 |
| $\rho_g$ | 0.0914 | $\beta_p$ | 0.9936 | $\lambda_{k,2}$ | 10.7499 | $\sigma_{lp}$ | 0.5699 |
| $\kappa_0$ | 0.0026 | $\overline{lp}$ | -0.0130 | $p_s$ | 0.9876 | $\sigma_g$ | 1.7200 |
| $\beta_a$ | 0.3905 | $\lambda_{\pi,1}$ | 0.4244 | mean beta be1 | 0.9286 | | |
| $\gamma_\pi$ | 0.0000 | $\lambda_{\pi,2}$ | 0.3139 | std beta be1 | 0.1090 | | |
| $\rho_f$ | 0.5010 | $\sigma_2$ | 0.0383 | int Baa | 0.0140 | | |

Note: For each realized policy regime, the table reports the posterior mode values of the parameters for the current and alternative policy rules.

**Table A.2: FED Announcements**

| **Date** | **Day** | **Hour** | **Event** | **Information** |
|---|---|---|---|---|
| | | | **Top Ten FOMC: 6-month FFF rate** | |
| 1/22/2008 | Tuesday | 8:30 AM | Conference Call | • The Federal Open Market Committee has decided to lower its target for the federal funds rate 75 basis points to 3-1/2 percent.<br>• The Committee took this action in view of a weakening of the economic outlook and increasing downside risks to growth<br>• The Committee expects inflation to moderate in coming quarters, but it will be necessary to continue to monitor inflation developments carefully. Appreciable downside risks to growth remain. |
| 4/18/2001 | Wednesday | 10:00 AM | Conference Call | • The Federal Open Market Committee decided today to lower its target for the federal funds rate by 50 basis points to 4-1/2 percent.<br>• Capital investment has continued to soften and the persistent erosion in current and expected profitability, in combination with rising uncertainty about the business outlook, seems poised to dampen capital spending going forward. |

## Table A.2: FED Announcements (Cont'd)

| Date | Day | Hour | Event | Information |
|---|---|---|---|---|
| 10/15/1998 | Thursday | 3:15 PM | Conference Call | • The Board of Governors approved a reduction in the discount rate by 25 basis points from 5 percent to 4-3/4 percent.<br>• The federal funds rate is expected to fall 25 basis points from around 5-1/4 percent to around 5 percent.<br>• Growing caution by lenders and unsettled conditions in financial markets more generally are likely to be restraining aggregate demand in the future. Against this backdrop, further easing of the stance of monetary policy was judged to be warranted to sustain economic growth in the context of contained inflation. |
| 1/3/2001 | Wednesday | 1:15 PM | Conference Call | • The Federal Open Market Committee decided today to lower its target for the federal funds rate by 50 basis points to 6 percent.<br>• These actions were taken in light of further weakening of sales and production, and in the context of lower consumer confidence...Moreover, inflation pressures remain contained. Nonetheless, to date there is little evidence to suggest that longer-term advances in technology and associated gains in productivity are abating. |
| 10/2/2001 | Tuesday | 1:15 PM | FOMC Meeting | • The Federal Open Market Committee decided today to lower its target for the federal funds rate by 50 basis points to 2-1/2 percent<br>• The terrorist attacks have significantly heightened uncertainty in an economy that was already weak...Nonetheless, the long-term prospects for productivity growth and the economy remain favorable and should become evident once the unusual forces restraining demand abate.<br>• The Committee continues to believe that...the risks are weighted mainly toward conditions that may generate economic weakness in the foreseeable future. |
| 9/18/2007 | Tuesday | 2:15 PM | FOMC Meeting | • The Federal Open Market Committee decided today to lower its target for the federal funds rate 50 basis points to 4-3/4 percent.<br>• Economic growth was moderate during the first half of the year, but the tightening of credit conditions has the potential to intensify the housing correction and to restrain economic growth more generally. |
| 7/6/1995 | Thursday | 2:15 PM | FOMC Meeting | • Today's action will be reflected in a 25 basis point decline in the federal funds rate from about 6 percent to about 5-3/4 percent.<br>• As a result of the monetary tightening initiated in early 1994, inflationary pressures have receded enough to accommodate a modest adjustment in monetary conditions. |

## Table A.2: FED Announcements (Cont'd)

| Date | Day | Hour | Event | Information |
|---|---|---|---|---|
| 6/25/2003 | Wednesday | 1:30 PM | FOMC Meeting | • The Federal Open Market Committee decided today to lower its target for the federal funds rate by 25 basis points to 1 percent.<br><br>• Recent signs point to a firming in spending, markedly improved financial conditions, and labor and product markets that are stabilizing...With inflationary expectations subdued, the Committee judged that a slightly more expansive monetary policy would add further support for an economy which it expects to improve over time.<br><br>• The Committee perceives that the upside and downside risks to the attainment of sustainable growth for the next few quarters are roughly equal. |
| 1/30/2008 | Wednesday | 2:15 PM | FOMC Meeting | • The Federal Open Market Committee decided today to lower its target for the federal funds rate 50 basis points to 3 percent.<br><br>• TodayŠs policy action, combined with those taken earlier, should help to promote moderate growth over time and to mitigate the risks to economic activity. However, downside risks to growth remain. The Committee will continue to assess the effects of financial and other developments on economic prospects and will act in a timely manner as needed to address those risks. |

## Table A.2: FED Announcements (Cont'd)

| Date | Day | Hour | Event | Information |
|---|---|---|---|---|
| **Top Ten FOMC: BBG Expected Inflation** | | | | |
| 1/22/2008 | Tuesday | 8:30 AM | Conference Call | • The Federal Open Market Committee has decided to lower its target for the federal funds rate 75 basis points to 3-1/2 percent.<br>• The Committee took this action in view of a weakening of the economic outlook and increasing downside risks to growth<br>• The Committee expects inflation to moderate in coming quarters, but it will be necessary to continue to monitor inflation developments carefully. Appreciable downside risks to growth remain. |
| 12/16/2009 | Wednesday | 2:15 PM | FOMC Meeting | • The Committee will maintain the target range for the federal funds rate at 0 to 1/4 percent and continues to anticipate that economic conditions...are likely to warrant exceptionally low levels of the federal funds rate for an extended period.<br>• The Federal Reserve is in the process of purchasing $1.25 trillion of agency mortgage-backed securities and about $175 billion of agency debt...the Committee is gradually slowing the pace of these purchases, and it anticipates that these transactions will be executed by the end of the first quarter of 2010.<br>• In light of ongoing improvements in the functioning of financial markets, the Committee and the Board of Governors anticipate that most of the Federal ReserveŠs special liquidity facilities will expire on February 1, 2010, consistent with the Federal ReserveŠs announcement of June 25, 2009. |
| 12/12/2012 | Wednesday | 12:30 PM | FOMC Meeting | • The Committee will continue purchasing additional agency mortgage-backed securities at a pace of $40 billion per month. The Committee also will purchase longer-term Treasury securities after its program to extend the average maturity of its holdings of Treasury securities is completed at the end of the year, initially at a pace of $45 billion per month.<br>• To support continued progress toward maximum employment and price stability, the Committee expects that a highly accommodative stance of monetary policy will remain appropriate for a considerable time after the asset purchase program ends and the economic recovery strengthens. In particular, the Committee decided to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that this exceptionally low range for the federal funds rate will be appropriate at least as long as the unemployment rate remains above 6-1/2 percent, inflation between one and two years ahead is projected to be no more than a half percentage point above the CommitteeŠs 2 percent longer-run goal, and longer-term inflation expectations continue to be well anchored. |

## Table A.2: FED Announcements (Cont'd)

| Date | Day | Hour | Event | Information |
|---|---|---|---|---|
| 3/11/2008 | Tuesday | 8:30 AM | Conference Call | • The Federal Reserve announced today an expansion of its securities lending program. Under this new Term Securities Lending Facility (TSLF), the Federal Reserve will lend up to $200 billion of Treasury securities to primary dealers secured for a term of 28 days.<br><br>• In addition, the Federal Open Market Committee has authorized increases in its existing temporary reciprocal currency arrangements (swap lines) with the European Central Bank (ECB) and the Swiss National Bank (SNB). The FOMC extended the term of these swap lines through September 30, 2008. |
| 5/9/2007 | Wednesday | 2:15 PM | FOMC Meeting | • Economic growth slowed in the first part of this year and the adjustment in the housing sector is ongoing. Nevertheless, the economy seems likely to expand at a moderate pace over coming quarters.<br><br>• Core inflation remains somewhat elevated. Although inflation pressures seem likely to moderate over time, the high level of resource utilization has the potential to sustain those pressures. |
| 3/16/2004 | Tuesday | 1:15 PM | FOMC Meeting | • The Committee perceives the upside and downside risks to the attainment of sustainable growth for the next few quarters are roughly equal...With inflation quite low and resource use slack, the Committee believes that it can be patient in removing its policy accommodation. |
| 8/10/2010 | Tuesday | 2:15 PM | FOMC Meeting | • Information received since the Federal Open Market Committee met in June indicates that the pace of recovery in output and employment has slowed in recent months.<br><br>• The Committee will maintain the target range for the federal funds rate at 0 to 1/4 percent and continues to anticipate that economic conditions...are likely to warrant exceptionally low levels of the federal funds rate for an extended period.<br><br>• To help support the economic recovery in a context of price stability, the Committee will keep constant the Federal Reserve's holdings of securities at their current level by reinvesting principal payments from agency debt and agency mortgage-backed securities in longer-term Treasury securities. The Committee will continue to roll over the Federal Reserve's holdings of Treasury securities as they mature. |

## Table A.2: FED Announcements (Cont'd)

| Date | Day | Hour | Event | Information |
|---|---|---|---|---|
| 9/13/2012 | Thursday | 12:30 PM | FOMC Meeting | • The Committee agreed today to increase policy accommodation by purchasing additional agency mortgage-backed securities at a pace of $40 billion per month. The Committee also will continue through the end of the year its program to extend the average maturity of its holdings of securities...and it is maintaining its existing policy of reinvesting principal payments from its holdings of agency debt and agency mortgage-backed securities in agency mortgage-backed securities.<br>• If the outlook for the labor market does not improve substantially, the Committee will continue its purchases of agency mortgage-backed securities, undertake additional asset purchases, and employ its other policy tools as appropriate until such improvement is achieved in a context of price stability.<br>• In particular, the Committee also decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015. |
| 10/8/2008 | Wednesday | 7:00 AM | Conference Call | • The Federal Open Market Committee has decided to lower its target for the federal funds rate 50 basis points to 1-1/2 percent. The Committee took this action in light of evidence pointing to a weakening of economic activity and a reduction in inflationary pressures.<br>• Inflation expectations are diminishing and remain anchored to price stability. The recent intensification of the financial crisis has augmented the downside risks to growth and thus has diminished further the upside risks to price stability. |
| 3/13/2012 | Tuesday | 2:15 PM | FOMC Meeting | • The Committee decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that economic conditions...are likely to warrant exceptionally low levels for the federal funds rate at least through late 2014.<br>• The Committee expects moderate economic growth over coming quarters and consequently anticipates that the unemployment rate will decline gradually toward levels that the Committee judges to be consistent with its dual mandate.<br>• The Committee also decided to continue its program to extend the average maturity of its holdings of securities as announced in September. The Committee is maintaining its existing policies of reinvesting principal payments from its holdings of agency debt and agency mortgage-backed securities in agency mortgage-backed securities and of rolling over maturing Treasury securities at auction. |

## Table A.2: FED Announcements (Cont'd)

| Date | Day | Hour | Event | Information |
|---|---|---|---|---|
| **Top Ten FOMC: SP-to-GDP ratio** | | | | |
| 1/22/2008 | Tuesday | 8:30 AM | Conference Call | • The Federal Open Market Committee has decided to lower its target for the federal funds rate 75 basis points to 3-1/2 percent.<br>• The Committee took this action in view of a weakening of the economic outlook and increasing downside risks to growth<br>• The Committee expects inflation to moderate in coming quarters, but it will be necessary to continue to monitor inflation developments carefully. Appreciable downside risks to growth remain. |
| 4/28/2010 | Wednesday | 2:15 PM | FOMC Meeting | • The Committee will maintain the target range for the federal funds rate at 0 to 1/4 percent and continues to anticipate that economic conditions...are likely to warrant exceptionally low levels of the federal funds rate for an extended period.<br>• The Federal Reserve has closed all but one of the special liquidity facilities...The only remaining such program, the Term Asset-Backed Securities Loan Facility, is scheduled to close on June 30 for loans backed by new-issue commercial mortgage-backed securities; it closed on March 31 for loans backed by all other types of collateral. |
| 3/11/2008 | Tuesday | 8:30 AM | Conference Call | • The Federal Reserve announced today an expansion of its securities lending program. Under this new Term Securities Lending Facility (TSLF), the Federal Reserve will lend up to $200 billion of Treasury securities to primary dealers secured for a term of 28 days.<br>• In addition, the Federal Open Market Committee has authorized increases in its existing temporary reciprocal currency arrangements (swap lines) with the European Central Bank (ECB) and the Swiss National Bank (SNB). The FOMC extended the term of these swap lines through September 30, 2008. |
| 10/8/2008 | Wednesday | 7:00 AM | Conference Call | • The Federal Open Market Committee has decided to lower its target for the federal funds rate 50 basis points to 1-1/2 percent. The Committee took this action in light of evidence pointing to a weakening of economic activity and a reduction in inflationary pressures.<br>• Inflation expectations are diminishing and remain anchored to price stability. The recent intensification of the financial crisis has augmented the downside risks to growth and thus has diminished further the upside risks to price stability. |

## Table A.2: FED Announcements (Cont'd)

| Date | Day | Hour | Event | Information |
|---|---|---|---|---|
| 12/16/2008 | Tuesday | 2:15 PM | FOMC Meeting | • The Federal Open Market Committee decided today to establish a target range for the federal funds rate of 0 to 1/4 percent...The Committee anticipates that weak economic conditions are likely to warrant exceptionally low levels of the federal funds rate for some time.<br>• Since the Committee's last meeting, labor market conditions have deteriorated...Financial markets remain quite strained and credit conditions tight. Overall, the outlook for economic activity has weakened further.<br>• The Committee is also evaluating the potential benefits of purchasing longer-term Treasury securities. Early next year, the Federal Reserve will also implement the Term Asset-Backed Securities Loan Facility to facilitate the extension of credit to households and small businesses. |
| 8/7/2007 | Tuesday | 2:15 PM | FOMC Meeting | • Economic growth was moderate during the first half of the year...The economy seems likely to continue to expand at a moderate pace over coming quarters, supported by solid growth in employment and incomes and a robust global economy.<br>• Although the downside risks to growth have increased somewhat, the Committee's predominant policy concern remains the risk that inflation will fail to moderate as expected. |
| 10/29/2008 | Wednesday | 2:15 PM | FOMC Meeting | • The Federal Open Market Committee decided today to lower its target for the federal funds rate 50 basis points to 1 percent.<br>• In light of the declines in the prices of energy and other commodities and the weaker prospects for economic activity, the Committee expects inflation to moderate in coming quarters to levels consistent with price stability. |
| 8/9/2011 | Tuesday | 2:15 PM | FOMC Meeting | • The Committee now expects a somewhat slower pace of recovery over coming quarters than it did at the time of the previous meeting...Moreover, downside risks to the economic outlook have increased. The Committee also anticipates that inflation will settle, over coming quarters, at levels at or below those consistent with the Committee's dual mandate.<br>• The Committee currently anticipates that economic conditions...are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013. The Committee also will maintain its existing policy of reinvesting principal payments from its securities holdings. |

## Table A.2: FED Announcements (Cont'd)

| Date | Day | Hour | Event | Information |
|---|---|---|---|---|
| 12/19/2018 | Wednesday | 2:00 PM | FOMC Meeting | • The Committee judges that some further gradual increases in the target range for the federal funds rate will be consistent with sustained expansion of economic activity, strong labor market conditions, and inflation near the Committee's symmetric 2 percent objective over the medium term.<br>• In view of realized and expected labor market conditions and inflation, the Committee decided to raise the target range for the federal funds rate to 2-1/4 to 2-1/2 percent. |
| 2/5/1997 | Wednesday | 12:00 PM | FOMC Meeting | • The Committee at this meeting established ranges for growth of M2 and M3 of 1 to 5 percent and 2 to 6 percent respectively, measured from the fourth quarter of 1996 to the fourth quarter of 1997. The monitoring range for growth of total domestic nonfinancial debt was set at 3 to 7 percent for the year.<br>• In the context of the Committee's long-run objectives for price stability and sustainable economic growth...somewhat greater reserve restraint would or slightly lesser reserve restraint might be acceptable in the intermeeting period. |
| **Top Ten FOMC: BBG Expected GDP Growth** | | | | |
| 3/11/2008 | Tuesday | 8:30 AM | Conference Call | • The Federal Reserve announced today an expansion of its securities lending program. Under this new Term Securities Lending Facility (TSLF), the Federal Reserve will lend up to $200 billion of Treasury securities to primary dealers secured for a term of 28 days.<br>• In addition, the Federal Open Market Committee has authorized increases in its existing temporary reciprocal currency arrangements (swap lines) with the European Central Bank (ECB) and the Swiss National Bank (SNB). The FOMC extended the term of these swap lines through September 30, 2008. |
| 1/22/2008 | Tuesday | 8:30 AM | Conference Call | • The Federal Open Market Committee has decided to lower its target for the federal funds rate 75 basis points to 3-1/2 percent.<br>• The Committee took this action in view of a weakening of the economic outlook and increasing downside risks to growth<br>• The Committee expects inflation to moderate in coming quarters, but it will be necessary to continue to monitor inflation developments carefully. Appreciable downside risks to growth remain. |
| 10/8/2008 | Wednesday | 7:00 AM | Conference Call | • The Federal Open Market Committee has decided to lower its target for the federal funds rate 50 basis points to 1-1/2 percent. The Committee took this action in light of evidence pointing to a weakening of economic activity and a reduction in inflationary pressures.<br>• Inflation expectations are diminishing and remain anchored to price stability. The recent intensification of the financial crisis has augmented the downside risks to growth and thus has diminished further the upside risks to price stability. |

## Table A.2: FED Announcements (Cont'd)

| Date | Day | Hour | Event | Information |
|---|---|---|---|---|
| 3/16/2004 | Tuesday | 1:15 PM | FOMC Meeting | • The Committee perceives the upside and downside risks to the attainment of sustainable growth for the next few quarters are roughly equal...With inflation quite low and resource use slack, the Committee believes that it can be patient in removing its policy accommodation. |
| 12/13/2017 | Wednesday | 2:00 PM | FOMC Meeting | • In view of realized and expected labor market conditions and inflation, the Committee decided to raise the target range for the federal funds rate to 1-1/4 to 1-1/2 percent.<br>• The Committee expects that economic conditions will evolve in a manner that will warrant gradual increases in the federal funds rate; the federal funds rate is likely to remain, for some time, below levels that are expected to prevail in the longer run.<br>• Information received since the Federal Open Market Committee met in November indicates that the labor market has continued to strengthen and that economic activity has been rising at a solid rate. |
| 12/16/2009 | Wednesday | 2:15 PM | FOMC Meeting | • The Committee will maintain the target range for the federal funds rate at 0 to 1/4 percent and continues to anticipate that economic conditions...are likely to warrant exceptionally low levels of the federal funds rate for an extended period.<br>• The Federal Reserve is in the process of purchasing $1.25 trillion of agency mortgage-backed securities and about $175 billion of agency debt...the Committee is gradually slowing the pace of these purchases, and it anticipates that these transactions will be executed by the end of the first quarter of 2010.<br>• In light of ongoing improvements in the functioning of financial markets, the Committee and the Board of Governors anticipate that most of the Federal ReserveŠs special liquidity facilities will expire on February 1, 2010, consistent with the Federal ReserveŠs announcement of June 25, 2009. |
| 9/13/2012 | Thursday | 12:30 PM | FOMC Meeting | • The Committee agreed today to increase policy accommodation by purchasing additional agency mortgage-backed securities at a pace of $40 billion per month. The Committee also will continue through the end of the year its program to extend the average maturity of its holdings of securities...and it is maintaining its existing policy of reinvesting principal payments from its holdings of agency debt and agency mortgage-backed securities in agency mortgage-backed securities.<br>• If the outlook for the labor market does not improve substantially, the Committee will continue its purchases of agency mortgage-backed securities, undertake additional asset purchases, and employ its other policy tools as appropriate until such improvement is achieved in a context of price stability.<br>• In particular, the Committee also decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015. |

**Table A.2: FED Announcements (Cont'd)**

| Date | Day | Hour | Event | Information |
|---|---|---|---|---|
| 5/9/2007 | Wednesday | 2:15 PM | FOMC Meeting | • Economic growth slowed in the first part of this year and the adjustment in the housing sector is ongoing. Nevertheless, the economy seems likely to expand at a moderate pace over coming quarters.<br>• Core inflation remains somewhat elevated. Although inflation pressures seem likely to moderate over time, the high level of resource utilization has the potential to sustain those pressures. |
| 3/13/2012 | Tuesday | 2:15 PM | FOMC Meeting | • The Committee decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that economic conditions...are likely to warrant exceptionally low levels for the federal funds rate at least through late 2014.<br>• The Committee expects moderate economic growth over coming quarters and consequently anticipates that the unemployment rate will decline gradually toward levels that the Committee judges to be consistent with its dual mandate.<br>• The Committee also decided to continue its program to extend the average maturity of its holdings of securities as announced in September. The Committee is maintaining its existing policies of reinvesting principal payments from its holdings of agency debt and agency mortgage-backed securities in agency mortgage-backed securities and of rolling over maturing Treasury securities at auction. |
| 12/16/2008 | Tuesday | 2:15 PM | FOMC Meeting | • The Federal Open Market Committee decided today to establish a target range for the federal funds rate of 0 to 1/4 percent...The Committee anticipates that weak economic conditions are likely to warrant exceptionally low levels of the federal funds rate for some time.<br>• Since the Committee's last meeting, labor market conditions have deteriorated...Financial markets remain quite strained and credit conditions tight. Overall, the outlook for economic activity has weakened further.<br>• The Committee is also evaluating the potential benefits of purchasing longer-term Treasury securities. Early next year, the Federal Reserve will also implement the Term Asset-Backed Securities Loan Facility to facilitate the extension of credit to households and small businesses. |

# Data

## Real GDP

The real Gross Domestic Product is obtained from the US Bureau of Economic Analysis. It is in billions of chained 2012 dollars, quarterly frequency, seasonally adjusted, and at annual rate. The source is from Bureau of Economic Analysis (BEA code: A191RX). The sample spans 1959:Q1 to 2021:Q2. The series was interpolated to monthly frequency using the method in

Stock and Watson (2010). The quarterly series was downloaded on August 20th, 2021.

**GDP price deflator**

The Gross Domestic Product: implicit price deflator is obtained from the US Bureau of Economic Analysis. Index base is 2012=100, quarterly frequency, and seasonally adjusted. The source is from Bureau of Economic Analysis (BEA code: A191RD). The sample spans 1959:Q1 to 2021:Q2. The series was interpolated to monthly frequency using the method in Stock and Watson (2010). The quarterly series was downloaded on August 20th, 2021.

**Earnings Share $K_t$**

The earnings share $K_t$ is defined as $1 - LS_t$ where $LS_t$ is the nonfarm business sector labor share. Labor share is measured as labor compensation divided by value added. The labor compensation is defined as Compensation of Employees - Government Wages and Salaries-Compensation of Employees of Nonprofit Institutions - Private Compensation (Households) - Farm Compensation of Employees - Housing Compensation of Employees - Imputed Labor Compensation of Self-Employed. The value added is defined as Compensation of Employees + Corporate Profits + Rental Income + Net Interest Income + Proprietors' Income + Indirect Taxes Less Subsidies + Depreciation. The quarterly, seasonally adjusted data spans from 1959:Q1 to 2021:Q2. The source is from Bureau of Labor Statistics. The labor share index is available at http://research.stlouisfed.org/fred2/series/PRS85006173 and the quarterly LS level can be found from the dataset at https://www.bls.gov/lpc/special_requests/msp_dataset.zip. The series was interpolated to monthly frequency using the method in Stock and Watson (2010). The quarterly series was downloaded on September 21th, 2021.

**Federal funds rate (FFR)**

The Effective Federal Funds Rate is obtained from the Board of Governors of the Federal Reserve System. It is in percentage points, quarterly frequency, and not seasonally adjusted. The sample spans 1960:02 to 2021:06. The series was downloaded on August 20th, 2021

**SP500 and SP500 futures**

We use tick-by-tick data on SP500 index obtained from tickdata.com. The series was downloaded on September 22th, 2021 from `https://www.tickdata.com/` .We create the minutely data using the close price within each minute. Our sample spans January 2nd 1986 to September 17th, 2021. Within trading hours, we construct SP500 market capitalization by multiplying the SP500 index by the SP500 Divisor. The SP500 Divisor is available at the URL: `https://ycharts.com/indicators/sp\_500\_divisor`. We supplement SP500 index using SP500 futures for events that occur in off-market hours. We use the current-quarter contract futures.

We purchased the SP500 futures from CME group at URL: https://datamine.cmegroup.com/. The SP500 futures data were downloaded on October 6, 2021.

**SP500 Earnings and Market Capitalization**

We obtained monthly S&P earnings from multpl.com at URL: `https://www.multpl.com/shiller-pe`. For S&P market cap, we obtain the series from Ycharts.com available at `https://ycharts.com/indicators/sp%\_500\_market\_cap`. Both series were downloaded on December 22nd, 2021.

**Baa Spread, 20-yr T-bond, Long-term US government securities**

We obtained daily Moody's Baa Corporate Bond Yield from FRED (series ID: DBAA) at URL: `https://fred.stlouisfed.org/series/BAA`, US Treasury securities at 20-year constant maturity from FRED (series ID: DGS20) at URL: `https://fred.stlouisfed.org/series/DGS20`, and long-term US government securities from FRED (series ID: LTGOVTBD) at URL: `https://fred.stlouisfed.org/series/LTGOVTBD`. To construct the long term bond yields, we use LTGOVTBD before 2000 and use DGS20 after 2000. The Baa spread is the difference between the Moody's Corporate bond yield and the 20-year US government yield. The excess bond premium is obtained at URL: `https://www.federalreserve.gov/econresdata/notes/feds-notes/2016/recession-risk-and-the-excess-bond-premium-20160408.html`. All series were downloaded on Feb 21, 2022.

**Bloomberg Consensus Inflation and GDP forecasts**

We obtain the Bloomberg (BBG) US GDP (id: EXGDUS) and inflation (id: ECPIUS) consensus mean forecast from the Bloomberg Terminal available on a daily basis up to a few days before the release of GDP and inflation data. The Bloomberg (BBG) US consensus forecasts are updated daily (except for weekends and holidays) and reports daily quarter-over-quarter real GDP growth and CPI forecasts from 2003:Q1. These forecasts provide more high-frequency information on the professional outlook for economic indicators. Both forecast series were downloaded on October 21, 2021.

**Livingston Survey Inflation Forecast**

We obtained the Livingston Survey mean 1-year and 10-year CPI inflation forecast from the Federal Reserve Bank of Philadelphia, URL: https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/livingston-historical-data. The forecast series were downloaded on September 20, 2021.

**Michigan Survey of Consumers Inflation Forecasts**

We construct MS forecasts of annual inflation of respondents answering at time $t$. Each month, the SOC contains approximately 50 core questions, and a minimum of 500 interviews are conducted by telephone over the course of the entire month, each month. We use two questions from the monthly survey for which the time series begins in January 1978.

1. Annual CPI inflation: To get a point forecast, we combine the information in the survey responses to questions A12 and A12b.

   - Question A12 asks (emphasis in original): *During the next 12 months, do you think that prices in general will go up, or go down, or stay where they are now?*
   - A12b asks (emphasis in original): *By about what percent do you expect prices to go (up/down) on the average, during the next 12 months?*

2. Long-run CPI inflation: To get a point forecast, we combine the information in the survey responses to questions A13 and A13b.

   - Question A13 asks (emphasis in original): *What about the outlook for prices over the next 5 to 10 years? Do you think prices will be higher, about the same, or lower, 5 to 10 years from now?*
   - A13b asks (emphasis in original): *By about what percent per year do you expect prices to go (up/down) on the average, during the next 5 to 10 years?*

All series were downloaded on September 17th, 2021.

**Bluechip Inflation and GDP Forecasts**

We obtain Blue Chip expectation data from Blue Chip Financial Forecasts. The surveys are conducted each month by sending out surveys to forecasters in around 50 financial firms such as Bank of America, Goldman Sachs & Co., Swiss Re, Loomis, Sayles & Company, and J.P. Morgan Chase. The participants are surveyed around the 25th of each month and the results published a few days later on the 1st of the following month. The forecasters are asked to forecast the average of the level of U.S. interest rates over a particular calendar quarter, e.g. the federal funds rate and the set of H.15 Constant Maturity Treasuries (CMT) of the following maturities: 3-month, 6-month, 1-year, 2-year, 5-year and 10-year, and the quarter over quarter percentage changes in Real GDP, the GDP Price Index and the Consumer Price Index, beginning with the current quarter and extending 4 to 5 quarters into the future.

In this study, we look at a subset of the forecasted variables. Specifically, we use the Blue Chip micro data on individual forecasts of the quarter-over-quarter (Q/Q) percentage change in the Real GDP, the GDP Price Index and the CPI, and convert to quarterly observations as explained below.

1. CPI inflation: We use quarter-over-quarter percentage change in the consumer price index, which is defined as

   *"Forecasts for the quarter-over-quarter percentage change in the CPI (consumer prices for all urban consumers). Seasonally adjusted, annual rate."*

   Quarterly and annual CPI inflation are constructed the same way as for PGDP inflation, except CPI replaces PGDP.

2. For real GDP growth, We use quarter-over-quarter percentage change in the Real GDP, which is defined as

   *"Forecasts for the quarter-over-quarter percentage change in the level of chain-weighted real GDP. Seasonally adjusted, annual rate. Prior to 1992, Q/Q % change (SAAR) in real GNP."*

The surveys are conducted right before the publication of the newsletter. Each issue is always dated the 1st of the month and the actual survey conducted over a two-day period almost always between 24th and 28th of the month. The major exception is the January issue when the survey is conducted a few days earlier to avoid conflict with the Christmas holiday. Therefore, we assume that the end of the last month (equivalently beginning of current month) is when the forecast is made. For example, for the report in 2008 Feb, we assume that the forecast is made on Feb 1, 2008.

**Survey of Professional Forecasters (SPF)**

The SPF is conducted each quarter by sending out surveys to professional forecasters, defined as forecasters. The number of surveys sent varies over time, but recent waves sent around 50 surveys each quarter according to officials at the Federal Reserve Bank of Philadelphia. Only forecasters with sufficient academic training and experience as macroeconomic forecasters are eligible to participate. Over the course of our sample, the number of respondents ranges from a minimum of 9, to a maximum of 83, and the mean number of respondents is 37. The surveys are sent out at the end of the first month of each quarter, and they are collected in the second or third week of the middle month of each quarter. Each survey asks respondents to provide nowcasts and quarterly forecasts from one to four quarters ahead for a variety of variables. Specifically, we use the SPF micro data on individual forecasts of the price level, long-run inflation, and real GDP.[1] Below we provide the exact definitions of these variables as well as our method for constructing nowcasts and forecasts of quarterly and annual inflation for each respondent.[2]

The following variables are used on either the right- or left-hand-sides of forecasting models:

[1]Individual forecasts for all variables can be downloaded at https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/historical-data/individual-forecasts.

[2]The SPF documentation file can be found at https://www.philadelphiafed.org/-/media/research-and-data/real-time-center/survey-of-professional-forecasters/spf-documentation.pdf?la=en.

1. Quarterly and annual inflation (1968:Q4 - present): We use survey responses for the level of the GDP price index (PGDP), defined as

   *"Forecasts for the quarterly and annual level of the chain-weighted GDP price index. Seasonally adjusted, index, base year varies. 1992-1995, GDP implicit deflator. Prior to 1992, GNP implicit deflator. Annual forecasts are for the annual average of the quarterly levels."*

   Since advance BEA estimates of these variables for the current quarter are unavailable at the time SPF respondents turn in their forecasts, four quarter-ahead inflation and GDP growth forecasts are constructed by dividing the forecasted level by the survey respondent-type's nowcast. Let $\mathbb{F}_t^{(i)}[P_{t+h}]$ be forecaster $i$'s prediction of PGDP $h$ quarters ahead and $\mathbb{N}_t^{(i)}[P_t]$ be forecaster $i$'s nowcast of PGDP for the current quarter. Annualized inflation forecasts for forecaster $i$ are

   $$\mathbb{F}_t^{(i)}[\pi_{t+h,t}] = (400/h) \times \ln\left(\frac{\mathbb{F}_t^{(i)}[P_{t+h}]}{\mathbb{N}_t^{(i)}[P_t]}\right),$$

   where $h = 1$ for quarterly inflation and $h = 4$ for annual inflation. Similarly, we construct quarterly and annual nowcasts of inflation as

   $$\mathbb{N}_t^{(i)}[\pi_{t,t-h}] = (400/h) \times \ln\left(\frac{\mathbb{N}_t^{(i)}[P_t]}{P_{t-h}}\right),$$

   where $h = 1$ for quarterly inflation and $h = 4$ for annual inflation, and where $P_{t-1}$ is the BEA's advance estimate of PGDP in the previous quarter observed by the respondent in time $t$, and $P_{t-4}$ is the BEA's most accurate estimate of PGDP four quarters back. After computing inflation for each survey respondent, we calculate the 5th through the 95th percentiles as well as the average, variance, and skewness of inflation forecasts across respondents.

2. Long-run inflation (1991:Q4 - present): We use survey responses for 10-year-ahead CPI inflation (CPI10), which is defined as

   *"Forecasts for the annual average rate of headline CPI inflation over the next 10 years. Seasonally adjusted, annualized percentage points. The "next 10 years" includes the year in which we conducted the survey and the following nine years. Conceptually, the calculation of inflation is one that runs from the fourth quarter of the year before the survey to the fourth quarter of the year that is ten years beyond the survey year, representing a total of 40 quarters or 10 years. The fourth-quarter level is the quarterly average of the underlying monthly levels."*

   Only the median response is provided for CPI10, and it is already reported as an inflation rate, so we do not make any adjustments and cannot compute other moments or percentiles.

3. Real GDP growth (1968:Q4 - present): We use the level of real GDP (RGDP), which is defined as

   *"Forecasts for the quarterly and annual level of chain-weighted real GDP. Seasonally adjusted, annual rate, base year varies. 1992-1995, fixed-weighted real GDP. Prior to 1992, fixed-weighted real GNP. Annual forecasts are for the annual average of the quarterly levels. Prior to 1981:Q3, RGDP is computed by using the formula NGDP / PGDP * 100."*

All series were downloaded on September 17th, 2021.

**Fed Funds Futures and Eurodollar Futures**

We use tick-by-tick data on Fed funds futures (FFF) and Eurodollar futures obtained from the CME Group. Our sample spans January 3, 1995 to June 2, 2020. FFF contracts settle based on the average federal funds rate that prevails over a given calendar month. Fed funds futures are priced at $100 - f_t^{(n)}$, where $f_t^{(n)}$ is the time-$t$ contracted federal funds futures market rate that investors lock in. Contracts are monthly and expire at month-end, with maturities ranging up to 60 months. For the buyer of the futures contract, the amount of $\left(f_t^{(n)} - r_{t+n}\right) \times$ \$D, where $r_{t+n}$ is the ex post realized value of the federal funds rate for month $t+n$ calculated as the average of the daily Fed funds rates in month $t+n$, and \$D is a dollar "deposit", represents the payoff of a zero-cost portfolio.

Eurodollar futures contracts are quarterly, expiring two business days before the third Wednesday in the last month of the quarter. Eurodollar futures are similarly quoted, where $f_t^{(q)}$ is the average 3-month LIBOR in quarter $q$ of contract expiry. Maturities range up to 40 quarters. For both types of contracts, the implied contract rate is recovered by subtracting 100 from the price and multiplying by $-1$.

Both types of contracts are cleaned following the same procedure following communication with the CME Group. First, trades with zero volume, which indicate a canceled order, are excluded. Floor trades, which do not require a volume on record, are included. Next, trades with a recorded expiry (in YYMM format) of 9900 indicate bad data and are excluded (Only 1390 trades, or less than 0.01% of the raw Fed funds data, have contract delivery dates of 9900). For trades time stamped to the same second, we follow Bianchi, Kind, and Kung (2019) and keep the trade with the lowest sequence number, corresponding to the first trade that second.

Fed funds futures data require additional cleaning. Trade prices were quoted in different units prior to August 2008. To standardize units across our sample, we start by noting that Fed funds futures are priced to the average effective Fed funds rate realized in the contract month. And in our sample, we expect a reasonable effective Fed funds rate to correspond to prices in the 90 to 100 range. As such, we rescale prices to be less than 100 in the pre-August 2008 subsample.[3] After rescaling, a small number of trades still appear to have prices that are far

[3]For trades with prices significantly greater than 100, we repeatedly divide by 10 until prices are in the range

away from the effective Fed funds rates at both trade day and contract expiry, along with trades in the immediate transactions. The CME Group could not explain this data issue, so following Bianchi, Kind, and Kung (2019) and others in the high frequency equity literature(Brownlees and Gallo 2006, Barndorff-Nielsen, Hansen, Lunde, and Shephard 2008, Andersen, Bollerslev, and Meddahi 2005), we apply an additional filter to exclude trades with such non-sensible prices. Specifically, for each maturity contract, we only keep trades where

$$|p_t - \overline{p}_t(k, \delta)| < 3\sigma_t(k, \delta) + \gamma,$$

where $p_t$ denotes the trade price (where $t$ corresponds to a second), and $\overline{p}_t(k, \delta)$ and $\sigma_t(k, \delta)$ denote the average price and standard deviation, respectively, centered with $k/2$ observations on each side of $t$ excluding $\delta k/2$ trades with highest price and excluding $\delta k/2$ trades with lowest price. Finally, $\gamma$ is a positive constant to account for the cases where prices are constant within the window. Our main specification uses $k = 30$, $\delta = 0.05$ and $\gamma = 0.4$, and alternative parameters produce similar results.

**High Frequency Changes Around FOMC Meetings**

We follow Guraynak, Sack, and Swanson (2005) and Nakamura and Steinsson (2018) among others in constructing high frequency changes around FOMC meetings. Although we do not use these changes directly in the structural model estimation, we constructed these changes as a cross-check on the construction of our high-frequency FFF data around meetings.

First, we compile dates and times of FOMC meetings from 1994 to 2004 from Guraynak, Sack, and Swanson (2005). The dates of the remaining FOMC meetings are collected from the Federal Reserve Board website. The times of statement releases were coalesced in the following priority: the Federal Reserve Board calendar, the Federal Reserve Board minutes, Bloomberg's FOMC page, and the first news article to appear on Bloomberg. We only include scheduled meetings and unscheduled meetings where a statement was released.

Next, we calculate changes in implied futures rates in a tight window around each FOMC statement release. Our main specification uses an inner window of 30 minutes, from 10 minutes before the FOMC announcement to 20 minutes after it, along with an outer window from 12am to noon the next day. Specifically, on the left side of the window, we use the first trade at 10 minutes before the FOMC announcement, or the nearest trade before 10 minutes if there is no trade at 10 minutes exactly, but not before 12am. Similarly, on the right side of the window, we use the first trade at 20 minutes after the announcement, or nearest trade after 20 minutes otherwise, but not after noon the next day. In other words, we use the nearest trades on or outside the inner window, but inside of the outer window.

For example, suppose the FOMC announcement is at 2:15pm. Then the inner window is from 2:05pm to 2:35pm. On the left side, we take the first trade at 2:05 or earlier, but not

of 90 to 100. We exclude all trades otherwise.

before 12am. On the right side, we take the first trade at 2:35pm or later, but not after noon the next day. Then we subtract the two implied rates.

As a robustness check, we also consider an inner window of 60 minutes (15 minutes before the FOMC announcement and 45 minutes after), along with outer windows of 12am to 1 hour after the statement release, and 12am to 2 hours after the statement release.

In addition to calculating the change in implied rates from Fed funds futures and Eurodollar futures, we also calculate the surprise component of Fed funds futures. We follow Kuttner (2001) in unwinding the average rate into a surprise measure.

To make notation consistent, for a variable $X_t^j$ let the superscript $j$ index the current or future FOMC meetings ($j = 0$ is the current meeting), and let the subscript $t$ index the "real-time" of when the statement is released ($t - \Delta t$ and $t + \Delta t$ are the inner window before and after the statement, respectively). Let $d^j$ be the day of the $j$th FOMC meeting and $m^j$ denote the number of calendar days in the month of the FOMC meeting. Let $r^j$ denote the target Fed funds rate prevailing after the $j$th meeting. And let $f^j_{t-\Delta t}$ and $f^j_{t+\Delta t}$ denote the implied rate from the Fed funds futures contract expiring in the month of the $j$th meeting, before and after the current meeting. Finally, $\mathbb{E}_{t-\Delta t} \equiv \mathbb{E}[\cdot|\mathcal{I}_{t-\Delta t}]$ is the conditional expectation using information up to an inner window before the FOMC meeting at $t$.

The implied rate from the Fed funds futures in an inner window around the current FOMC can be written as

$$\begin{aligned} f^0_{t-\Delta t} =& \frac{d^0}{m^0} r^{-1} + \frac{m^0 - d^0}{m^0} \mathbb{E}_{t-\Delta t}(r^0) + \mu^0_{t-\Delta t} \\ f^0_{t+\Delta t} =& \frac{d^0}{m^0} r^{-1} + \frac{m^0 - d^0}{m^0} (r^0) + \mu^0_{t+\Delta t}. \end{aligned}$$

Here we make three assumptions. First, the effective Fed funds rate equals the target rate; if not, then replace $r^{-1}$ by the average effective rate realized so far in the month. Second, $r^0$ only changes from the FOMC meeting, and is constant for the remainder of the month after the FOMC meeting. In other words, $\mathbb{E}_{t+\Delta t}(r^0) = r^0$. Third, high frequency changes around the term premium $\mu^0$ are negligible. Piazzesi and Swanson (2008) argue the narrow daily window largely "differences out" risk premia that are moving primarily at lower, business cycle frequencies.

With these three assumptions, we can then calculate the current FOMC surprise as a scaled change in the current Fed funds implied rates,

$$e^0_{t+\Delta t} \equiv \frac{m^0}{m^0 - d^0} \left[ f^0_{t+\Delta t} - f^0_{t-\Delta t} \right], \tag{A.1}$$

where the scaling is proportional to when in the month the FOMC meeting occurs. And the change in implied rate equals the expected component plus the surprise component.

We also calculate longer horizon surprises around the $j$th meeting, after the current meeting,

as

$$e^j_{t+\Delta t} \equiv \frac{m^j}{m^j - d^j}\left[\left(f^j_{t+\Delta t} - f^j_{t-\Delta t}\right) - \frac{d^j}{m^j} e^{j-1}_t\right]. \tag{A.2}$$

Lastly, the scale factor can get large if the meeting is at the end of the month and Fed funds futures only trade in half a basis point increments. Therefore if a meeting is in the last 3 to 7 days of the month, then we use the current change in next month's Fed funds futures implied rate.

## Structural Breaks as Nonrecurrent Regime-Switching

Let $T$ be the sample size used in the estimation and let the vector of observations as of time $t$ be denoted $z_{r,t}$, here $z_{r,t} = mps_t$. The sequence $\xi^P_t = \{\xi^P_1, ..., \xi^P_T\}$ of regimes in place at each point is unobservable and needs to be inferred jointly with the other parameters of the model. We use the Hamilton filter (Hamilton (1994)) to estimate the smoothed regime probabilities $P\left(\xi^P_t = i | z_{r,T}; \boldsymbol{\theta}_r\right)$, where $i = 1, ..., N_P$. We then use these regime probabilities to estimate the most likely historical regime sequence $\xi^P_t$ over our sample as described in the next subsection.

To capture the phenomenon of nonrecurrent regimes, we suppose that $\xi^P_t$ follows a Markov-switching process in which new regimes can arise but do not repeat exactly as before. This is modeled by specifying the transition matrix over nonrecurrent states, or "structural breaks." If the historical sample has $N_P$ nonrecurrent regimes (implying $N_P - 1$ structural breaks), the transition matrix for the Markov process takes the form

$$\mathbf{H} = \begin{bmatrix} p_{11} & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ 1-p_{11} & p_{22} & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & 1-p_{22} & p_{33} & 0 & \cdots & \cdots & \vdots \\ \vdots & 0 & 1-p_{33} & \ddots & & & \\ \vdots & \vdots & 0 & \vdots & \ddots & & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & p_{N_P,N_P} & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & 1-p_{N_P,N_P} & 1 \end{bmatrix}, \tag{A.3}$$

where $\mathbf{H}_{ij} \equiv p\left(\xi^P_t = i | \xi^P_{t-1} = j\right)$. For example, if there were $N_P = 2$ nonrecurrent regimes in the sample, we would have

$$\mathbf{H} = \begin{bmatrix} p_{11} & 0 \\ 1-p_{11} & 1 \end{bmatrix}.$$

The above process implies that, if you are currently in regime 1, you will remain there next period with probability $p_{11}$ or exit to regime 2 with probability $1 - p_{11}$. Upon exiting to regime 2, since there are only two regimes in the sample and the probability $p_{12}$ of returning exactly to the previous regime 1 is zero, $p_{22} = 1$.

## Most Likely Regime Sequence

In this section we explain how to compute the most likely regime sequence. This most likely regime sequence is the particular regime sequence $\xi^{P,T} = \{\hat{\xi}_1^P, ..., \hat{\xi}_T^P\}$ that is most likely to have occurred, given our estimated posterior mode parameter values for $\boldsymbol{\theta}$ . This sequence is computed as follows. Let $P\left(\xi_t^P = i | z_{t-1}; \boldsymbol{\theta}_r\right) \equiv \pi_{t|t-1}^i$. First, we run Hamilton's filter to get the vector of filtered regime probabilities $\pi_{t|t}$, $t = 1, 2, ..., T$. The Hamilton filter can be expressed iteratively as

$$\begin{aligned} \pi_{t|t} &= \frac{\pi_{t|t-1} \odot \eta_t}{\mathbf{1}'\left(\pi_{t|t-1} \odot \eta_t\right)} \\ \pi_{t+1|t} &= \mathbf{H}\pi_{t|t} \end{aligned}$$

where $\eta_t$ is a vector whose $j$-$th$ element contains the conditional density $p(mps_t | \xi_t^P = j; \boldsymbol{\theta}_r)$, the symbol $\odot$ denotes element by element multiplication, and $\mathbf{1}$ is a vector with all elements equal to 1. The final term, $\pi_{T|T}$ is returned with the final step of the filtering algorithm. Then, a recursive algorithm can be implemented to derive the other smoothed probabilities:

$$\pi_{t|T} = \pi_{t|t} \odot \left[\mathbf{H}'\left(\pi_{t+1|T} \left(\div\right) \pi_{t+1|t}\right)\right]$$

where $(\div)$ denotes element by element division. To choose the regime sequence most likely to have occurred given our parameter estimates, consider the recursion in the next to last period $t = T - 1$:

$$\pi_{T-1|T} = \pi_{T-1|T-1} \odot \left[\mathbf{H}'\left(\pi_{T|T} \left(\div\right) \pi_{T|T-1}\right)\right].$$

We first take $\pi_{T|T}$ from the Hamilton filter and choose the regime that is associated with the largest probability, i.e., if $\pi_{T|T} = (.9, .1)$, where the first element corresponds to the probability of regime 1, we select $\hat{\xi}_T^P = 1$, indicating that we are in regime 1 in period $T$. We now update $\pi_{T|T} = (1, 0)$ and plug into the right-hand-side above along with the estimated filtered probabilities for $\pi_{T-1|T-1}$, $\pi_{T|T-1}$ and estimated transition matrix $\mathbf{H}$ to get $\pi_{T-1|T}$ on the left-hand-side. Now we repeat the same procedure by choosing the regime for $T - 1$ that has the largest probability at $T - 1$, e.g., if $\pi_{T-1|T} = (.2, .8)$ we select $\hat{\xi}_{T-1} = 2$, indicating that we are in regime 2 in period $T - 1$, we then update to $\pi_{T-1|T} = (0, 1)$, which is used again on the right-hand-side now

$$\pi_{T-2|T} = \pi_{T-2|T-2} \odot \left[\mathbf{H}'\left(\pi_{T-1|T} \left(\div\right) \pi_{T-1|T-2}\right)\right].$$

We proceed in this manner until we have a most likely regime sequence $\xi^{P,T}$ for the entire sample $t = 1, 2, ..., T$. Two aspects of this procedure are worth noting. First, it fails if the updated probabilities are exactly $(.5, .5)$. Mathematically this is virtually a zero probability event. Second, note that this procedure allows us to choose the most likely regime sequence by using the recursive formula above to update the filtered probabilities sequentially working backwards from $t = T$ to $t = 1$. This allows us to take into account the time dependence in the regime sequence as dictated by the transition probabilities.

## Price-Output Decompositions

Mapping from price to output (measured as $GDP_t$) is

$$\begin{aligned}\frac{P_t}{GDP_{t-1}} &= \frac{P_t}{D_t}\frac{D_t}{GDP_t}\frac{GDP_t}{GDP_{t-1}} \\ pgdp_t &= pd_t + k_t + \widetilde{y}_t + g_t - \widetilde{y}_{t-1}\end{aligned}$$

Below we decompose $pd_t$ to write:

$$\begin{aligned}pgdp_t &= \underbrace{\frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + k_t + y_t + g_t - \widetilde{y}_{t-1}}_{\text{earning share component}} + \underbrace{pdv_t\left(\Delta d\right)}_{\text{earnings}} - \underbrace{pdv_t\left(r^{ex}\right)}_{\text{premia}} - \underbrace{pdv_t\left(rir\right)}_{\text{RIR}} \\ pgdp_{r^{ex},t} &= \underbrace{\frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + k_t + \widetilde{y}_t + g_t - \widetilde{y}_{t-1}}_{\text{earning share component}} - \underbrace{pdv_t\left(r^{ex}\right)}_{\text{premia}} \\ pgdp_{rir,t} &= \underbrace{\frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + k_t + \widetilde{y}_t + g_t - \widetilde{y}_{t-1}}_{\text{earning share component}} - \underbrace{pdv_t\left(rir\right)}_{\text{RIR}} \\ pgdp_{\Delta d,t} &= \underbrace{\frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + k_t + \widetilde{y}_t + g_t - \widetilde{y}_{t-1}}_{\text{earning share component}} + \underbrace{pdv_t\left(\Delta d\right)}_{\text{earnings}}\end{aligned}$$

where

$$\begin{aligned}pd_t &= \kappa_{pd,0} + \mathbb{E}_t^b\left[m_{t+1} + \Delta d_{t+1} + \kappa_{pd,1} pd_{t+1}\right] + \\ &\quad + .5\mathbb{V}_t^b\left[m_{t+1} + \Delta d_{t+1} + \kappa_{pd,1} pd_{t+1}\right].\end{aligned}$$

The solution approximates around the balanced growth path with $\frac{D_{t+1}}{D_t} = G$, where $G$ is the gross growth rate of the economy. The Euler equation under the balanced growth path is

$$\begin{aligned}1 &= \left[M_{t+1}\left(\frac{P_{t+1}/D_{t+1}+1}{P_t/D_t}\right)\frac{D_{t+1}}{D_t}\right] \\ &= \left[\beta_p\left(\frac{D_{t+1}}{D_t}\right)^{-\sigma_p}\left(\frac{P_{t+1}/D_{t+1}+1}{P_t/D_t}\right)\frac{D_{t+1}}{D_t}\right] \\ &= \left[\underbrace{\beta_p G^{1-\sigma_p}}_{\widetilde{\beta}_p}\left(\frac{P/D+1}{P/D}\right)\right] => \\ \frac{1}{\widetilde{\beta}_p} &= \left(\frac{P/D+1}{P/D}\right) => \\ P/D &= \frac{\widetilde{\beta}_p}{1-\widetilde{\beta}_p}.\end{aligned}$$

Denote the log steady state price-payout ratio as $\ln(P/D) = \overline{pd}$, thus we have

$$\overline{pd} = \ln\left(\frac{\widetilde{\beta}_p}{1-\widetilde{\beta}_p}\right).$$

$$\begin{aligned}
\kappa_{pd,1} &= \exp(\overline{pd})/(1+\exp(\overline{pd})) = \frac{\widetilde{\beta}_p}{1-\widetilde{\beta}_p}\left[1+\frac{\widetilde{\beta}_p}{1-\widetilde{\beta}_p}\right]^{-1} = \widetilde{\beta}_p \\
\kappa_{pd,0} &= \ln(\exp(\overline{pd})+1) - \kappa_{pd,1}pd = \ln\left(\frac{1}{1-\widetilde{\beta}_p}\right) - \widetilde{\beta}_p \ln\frac{\widetilde{\beta}_p}{1-\widetilde{\beta}_p} \\
&= -\widetilde{\beta}_p \ln\widetilde{\beta}_p - \left(1-\widetilde{\beta}_p\right)\ln\left(1-\widetilde{\beta}_p\right)
\end{aligned}$$

The log return obeys the following approximate identity (Campbell and Shiller (1989)):

$$r^D_{t+1} = \kappa_{pd,0} + \kappa_{pd,1}pd_{t+1} - pd_t + \Delta d_{t+1},$$

where $\kappa_{pd,1} = \exp(\overline{pd})/(1+\exp(\overline{pd}))$, and $\kappa_{pd,0} = \log\left(\exp(\overline{pd})+1\right) - \kappa_{pd,1}\overline{pd}$. Combining all of the above, the log equity premium is

$$\underbrace{\mathbb{E}^b_t\left[r^D_{t+1}\right] - \left(i_t - \mathbb{E}^b_t\left[\pi_{t+1}\right]\right)}_{\text{Equity Premium}} = \underbrace{\left[\begin{array}{c} -.5\mathbb{V}^b_t\left[r^D_{t+1}\right] - \mathbb{COV}^b_t\left[m_{t+1}, r^D_{t+1}\right] \\ +.5\mathbb{V}^b_t\left[\pi_{t+1}\right] - \mathbb{COV}^b_t\left[m_{t+1}, \pi_{t+1}\right] \end{array}\right]}_{\text{Risk Premium}} + \underbrace{\overline{lp}_t}_{\text{Liquidity Premium}},$$

Then

$$\begin{aligned}
pd_t &= \kappa_{pd,0} + \mathbb{E}^b_t\left[\Delta d_{t+1} - r^D_{t+1} + \kappa_{pd,1}pd_{t+1}\right] \\
pd_t &= \kappa_{pd,0} + \mathbb{E}^b_t\left[\Delta d_{t+1} - \left(r^{ex}_{t+1} - rir_{t+1}\right) + \kappa_{pd,1}pd_{t+1}\right]
\end{aligned}$$

where $\mathbb{E}^b_t\left[r^{ex}_{t+1}\right] = \mathbb{E}^b_t\left[r^D_{t+1}\right] - rir_{t+1}$, where $rir_{t+1} \equiv \left(i_{t+1} - \mathbb{E}^b_t\left[\pi_{t+1}\right]\right)$.

Solving forward:

$$\begin{aligned}
pd_t &= \kappa_{pd,0} + \mathbb{E}^b_t\left[\Delta d_{t+1} - r^{ex}_{t+1} - rir_{t+1}\right] + \\
&\quad + \kappa_{pd,1}\mathbb{E}^b_t\left[\kappa_{pd,0} + \mathbb{E}^b_t\left[\Delta d_{t+2} - r^{ex}_{t+2} - rir_{t+1} + \kappa_{pd,1}pd_{t+2}\right]\right]
\end{aligned}$$

Thus:

$$pd_t = \frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + \left(1_{\Delta d} - 1_{\mathbb{E}(r^{ex})} - 1_{rir}\right)\sum_{h=0}^{\infty}\kappa^h_{pd,1}\mathbb{E}^b_t\left[S_{t+1+h}\right]$$

where $1_x$ is a vector of all zeros except for a 1 in the $x$th position. This can be written as:

$$pd_t = \frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + pdv_t\left(\Delta d\right) - pdv_t\left(r^{ex}\right) - pdv_t\left(rir\right)$$

Using the solution:

$$pd_t = \frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + \left(1_{\Delta d} - 1_{\mathbb{E}(r^{ex})} - 1_{r^b}\right)\left(\mathbf{I} - \kappa_{pd,1}T_{\xi_t}\right)^{-1}\left[T_{\xi_t}S_t + \left(\mathbf{I} - \kappa_{pd,1}\right)^{-1}C_{\xi_t}\right].$$

Thus, we can decompose movements in the $pd_t$ into those attributable to expected dividends, equity premia, and expected real interest rates:

$$pgdp_t = \underbrace{\frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + k_t + y_t + g_t - y_{t-1}}_{\text{earning share component}} + \underbrace{pdv_t\left(\Delta d\right)}_{\text{earnings}} - \underbrace{pdv_t\left(r^{ex}\right)}_{\text{premia}} - \underbrace{pdv_t\left(rir\right)}_{\text{RIR}}.$$

## Solution and Estimation Details

### Computing Expectations with Regime Switching and Alternative Policy Rules

In what follows, we explain how to use expectations to infer what alternative regimes agents have in mind. Expectations about inflation, FFR, and GDP growth depend on the regime currently in place, the alternative regime, and the probability of moving to such regime. This note is based on "Methods for measuring expectations and uncertainty in Bianchi (2016). That paper explains how to computed expected values in presence of regime changes. In the models described above, for each policy rule in place, agents would have different beliefs about alternative future policy rules. This would lead to changes in expected values for the endogenous variables of the model.

Consider a MS model:

$$S_t = C_{\xi_t} + T_{\xi_t} S_{t-1} + R_{\xi_t} Q \varepsilon_t \tag{A.4}$$

where $\xi_t = \left\{\xi_t^P, \xi_t^b\right\}$ controls the policy regime $\xi_t^P$ controls the policy rule currently in place and the alternative policy rule, while the belief regime $\xi_t^b$ controls agents' beliefs about the possibility of moving to the alternative policy rule.

Let $n$ be the number of variables in $S_t$. Let $m = B + 1$ be the number of Markov-switching states and define

$$\xi_t = i \equiv \left\{\xi_t^P, \xi_t^b = i\right\}, \quad i = 1, ..., B+1.$$

Define the $mn \times 1$ column vector $q_t$ as:

$$\underset{mn\times 1}{q_t} = \left[q_t^{1\prime}, ..., q_t^{m\prime}\right]'$$

where the individual $n \times 1$ vectors $q_t^i = \mathbb{E}_0\left(S_t 1_{\xi_t=i}\right) \equiv \mathbb{E}\left(S_t 1_{\xi_t=i} | \mathbb{I}_0\right)$ and $1_{\xi_t=i}$ is an indicator variable that is one when belief regime $i$ is in place and zero otherwise. Note that:

$$q_t^i = \mathbb{E}_0\left(S_t 1_{\xi_t=i}\right) = \mathbb{E}_0\left(S_t | \xi_t = i\right) \pi_t^i$$

where $\pi_t^i = P_0\left(\xi_t = i\right) = P\left(\xi_t = i | \mathbb{I}_0\right)$. Therefore we can express $\mu_t = \mathbb{E}_0\left(S_t\right)$ as:

$$\mu_t = \mathbb{E}_0\left(S_t\right) = \textstyle\sum_{i=1}^m q_t^i = w q_t$$

where the matrix $\underset{n\times mn}{w} = [I_n, ..., I_n]$ is obtained placing side by side $m$ $n$-dimensional identity matrices. Then the following proposition holds:

PROPOSITION 1: *Consider a Markov-switching model whose law of motion can be described by (A.4) and define* $q_t^i = \mathbb{E}_0\left(S_t 1_{\xi_t=i}\right)$ *for* $i = 1...m$. *Then* $q_t^j = C_j \pi_t^j + \sum_{i=1}^m T_j q_{t-1}^i p_{ji}$.

It is then straightforward to compute expectations conditional on the information available at a particular point in time. Suppose we are interested in $\mu_{t+s|t} \equiv \mathbb{E}_t^b\left(S_{t+s}\right)$, i.e. the expected value for the vector $S_{t+s}$ conditional on the information set available at time $t$. If we define:

$$q_{t+s|t} = \left[q_{t+s|t}^{1\prime}, ..., q_{t+s|t}^{m\prime}\right]'$$

where $q_{t+s|t}^i = \mathbb{E}_t^b\left(S_{t+s} 1_{\xi_t=i}\right) = \mathbb{E}_t^b\left(S_{t+s}|\xi_t = i\right)\pi_{t+s|t}^i$, where $\pi_{t+s|t}^i \equiv P\left(\xi_{t+s} = i|\mathbb{I}_t\right)$, we have

$$\mu_{t+s|t} = \mathbb{E}_t^b\left(S_{t+s}\right) = w q_{t+s|t}, \tag{A.5}$$

where for $s \geq 1$, $q_{t+s|t}$ evolves as:

$$\begin{aligned} q_{t+s|t} &= C\pi_{t+s|t} + \Omega q_{t+s-1|t} && \text{(A.6)} \\ \pi_{t+s|t} &= \mathbf{H}^b \pi_{t+s-1|t} && \text{(A.7)} \end{aligned}$$

with $\pi_{t+s|t} = \left[\pi_{t+s|t}^1, ..., \pi_{t+s|t}^m\right]'$, $\Omega = bdiag\left(T_1, ..., T_m\right)\left(\mathbf{H}^b \otimes I_n\right)$, and $\underset{mn\times m}{C} = bdiag\left(C_1, ..., C_m\right)$, where e.g., $C_1$ is the $n \times 1$ vector of constants in regime 1, $\otimes$ represents the Kronecker product and *bdiag* is a matrix operator that takes a sequence of matrices and use them to construct a block diagonal matrix.

The formulas above are used to compute expectations conditional on each belief regime $\xi_t^b$ and policy rule regime $\xi_t^P$. For each composite regime $\xi_t = \left\{\xi_t^P, \xi_t^b\right\}$, we can obtain a forecast for each of the variables of the model. For example, conditional on $\xi_t^P$ and $\xi_t^b = j$ in place we have

$$q_{t,\xi_t=j} = e_j \otimes S_t$$

where $e_j$ is a variable that has elements equal to zero except for the one in position $\xi_t^b$. For example, with $B = 5$ belief regimes and $\xi_t^b = 3$ we have

$$q_{t,\xi_t=3} = \left[\mathbf{0}', \mathbf{0}', S_t', \mathbf{0}', \mathbf{0}', \mathbf{0}'\right]'.$$

where $\mathbf{0}$ and $S_t$ are column vectors with $n$ rows. We have $B+1$ subvectors in $q_{t,\xi_t=j}$ to take into account the alternative policy mix. The fact that all subvectors are zero except for the one corresponding to the belief regime $b = 3$ reflects the assumption that agents can observe the current state $S_t$ and, by definition, their own beliefs (while the econometrician cannot observe any of the two and she uses macro data and survey expectations to estimate both $S_t$ and agents' beliefs).

Thus, suppose we want to compute the expected value for a variable $x$ over the next year under the assumption that agents' beliefs are $\xi_t^b = j$. With monthly data, we have:

$$\begin{aligned} \mathbb{E}_t^b\left(x_{t,t+s}|\xi_t = j\right) &= \sum_{s=1}^{12} \mathbb{E}_t^b\left(x_{t+s}|\xi_t = j\right) \\ &= e_x \sum_{s=1}^{12} \mu_{t+s|t,\xi_t=j} \\ &= e_x w \sum_{s=1}^{12} q_{t+s|t,\xi_t=j} \end{aligned}$$

where for $s \geq 1$, $q_{t+s|t}$ evolves as:

$$q_{t+s|t,\xi_t=j} = C\pi_{t+s|t} + \Omega q_{t+s-1|t,\xi_t=j} \tag{A.8}$$

$$\pi_{t+s|t,\xi_t=j} = \mathbf{H}^b \pi_{t+s-1|t,\xi_t=j} \tag{A.9}$$

with $\pi_{t+s|t} = \left[\pi^1_{t+s|t}, ..., \pi^m_{t+s|t}\right]'$, $\Omega = bdiag\left(T_1, ..., T_m\right)\left(\mathbf{H}^b \otimes I_n\right)$, and $\underset{mn\times m}{C} = bdiag\left(C_1, ..., C_m\right)$, where e.g., $C_1$ is the $n \times 1$ vector of constants in regime 1, $\otimes$ represents the Kronecker product and *bdiag* is a matrix operator that takes a sequence of matrices and use them to construct a block diagonal matrix. The recursive algorithm is initialized with $\pi_{t|t,\xi_t=j} = 1_{\xi_t=j}$ and $q_{t,\xi_t=j} = e_j \otimes S_t$.

The formulas (A.8) and (A.9) can be written in a more compact form. If we define $\widetilde{q}_{t|t} = [q'_{t|t}, \pi'_{t|t}]'$, with $\pi_{t|t}$ a vector with elements $\pi^i_{t|t} \equiv P\left(\xi_t = i|\mathbb{I}_t\right)$ we can compute the conditional expectations in one step:

$$\mu_{t+s|t} = \mathbb{E}^b_t\left(S_{t+s}\right) = \widetilde{w}\widetilde{\Omega}^s\widetilde{q}_{t|t} \tag{A.10}$$

where $\widetilde{w} = [w, 0_{n\times m}]$. The formula above can be used to compute the expected value from the point of view of the agent of the model with beliefs $\xi_t = j$:

$$\mathbb{E}^b_t\left(x_{t+s}|\xi_t = j\right) = e_x\mu_{t+s|t,\xi_t=j} = e_x\widetilde{w}\widetilde{\Omega}^s\widetilde{q}_{t|t,\xi^b_t=j} = \underbrace{\underbrace{e_x w\widetilde{\Omega}^s_{\{1,nm\},\{n(j-1)+1,nj\}}}_{Z_{\xi_t,x_{t+s}}}\underbrace{S_t}_{(n\times 1)}}_{} + \underbrace{e_x w\widetilde{\Omega}^s_{\{1,nm\},nm+j}}_{D_{\xi_t,x_{t+s}}} \tag{A.11}$$

where $D_{\xi_t,x_{t+s}}$ is a scalar, $Z_{\xi_t,x_{t+s}}$ is an $(1 \times n)$ vector, $\widetilde{\Omega}^s_{\{1,nm\},\{n(j-1)+1,nj\}}$ is the submatrix obtained taking the first $nm$ rows and the columns from $n(j-1)+1$ to $nj$ of $\widetilde{\Omega}^s$, while $\widetilde{\Omega}^s_{\{1,nm\},nm+j}$ is the submatrix obtained taking the first $nm$ rows and the $nm+j$ column of $\widetilde{\Omega}^s$. Thus, we have that conditional on one belief regime and a policy rule regime, we can map the current state of the economy $S_t$ into the expected value reported in the survey. The matrix algebra in (A.11) returns the same results of the recursion in (A.8) and (A.9).

To see what the formulas above do, consider a simple example with $B = 2$ and we are currently in belief regime $b = 2$:

$$\begin{aligned}
\mathbb{E}^b_t\left(x_{t+s}|\xi_t = 2\right) &= e_x\widetilde{w}\widetilde{\Omega}^s\widetilde{q}_{t|t,\xi_t=2} = e_x\widetilde{w}\widetilde{\Omega}^s\begin{bmatrix}\mathbf{0}\\ n\times 1\\ S_t\\ n\times 1\\ \mathbf{0}\\ n\times 1\\ 0\\ 1\\ 0\end{bmatrix} = e_x\widetilde{w}\begin{bmatrix}\widetilde{\Omega}^s_{11} & \widetilde{\Omega}^s_{12} & \widetilde{\Omega}^s_{13} & \widetilde{\Omega}^s_{14} & \widetilde{\Omega}^s_{15} & \widetilde{\Omega}^s_{16}\\ \widetilde{\Omega}^s_{21} & \widetilde{\Omega}^s_{22} & \widetilde{\Omega}^s_{23} & \widetilde{\Omega}^s_{24} & \widetilde{\Omega}^s_{25} & \widetilde{\Omega}^s_{26}\\ \widetilde{\Omega}^s_{31} & \widetilde{\Omega}^s_{32} & \widetilde{\Omega}^s_{33} & \widetilde{\Omega}^s_{34} & \widetilde{\Omega}^s_{35} & \widetilde{\Omega}^s_{36}\\ & & & \widetilde{\Omega}^s_{44} & \widetilde{\Omega}^s_{45} & \widetilde{\Omega}^s_{46}\\ & & & \widetilde{\Omega}^s_{54} & \widetilde{\Omega}^s_{55} & \widetilde{\Omega}^s_{56}\\ & & & \widetilde{\Omega}^s_{64} & \widetilde{\Omega}^s_{65} & \widetilde{\Omega}^s_{66}\end{bmatrix}\begin{bmatrix}\mathbf{0}\\ n\times 1\\ S_t\\ n\times 1\\ \mathbf{0}\\ n\times 1\\ 0\\ 1\\ 0\end{bmatrix}\\
&= e_x\widetilde{w}\begin{bmatrix}\widetilde{\Omega}^s_{12}S_t + \widetilde{\Omega}^s_{15}\\ \widetilde{\Omega}^s_{22}S_t + \widetilde{\Omega}^s_{25}\\ \widetilde{\Omega}^s_{32}S_t + \widetilde{\Omega}^s_{35}\\ \widetilde{\Omega}^s_{44}\\ \widetilde{\Omega}^s_{54}\\ \widetilde{\Omega}^s_{64}\end{bmatrix} = e_x\left(\widetilde{\Omega}^s_{12} + \widetilde{\Omega}^s_{22} + \widetilde{\Omega}^s_{32}\right)S_t + e_x\left(\widetilde{\Omega}^s_{15} + \widetilde{\Omega}^s_{25} + \widetilde{\Omega}^s_{35}\right)
\end{aligned}$$

Finally, suppose we are interested in the forecast $\mathbb{E}_t^b\left(x_{t,t+s}|\xi_t^b=j,\xi_t^p\right)$:

$$\mathbb{E}_t^b\left(x_{t,t+s}|\xi_t=j\right)=\underbrace{\left[e_x\sum_{s=1}^{12}w\widetilde{\Omega}^s_{\{1,nm\},\{n(j-1)+1,nj\}}\right]}_{Z_{\xi_t,x_{t,t+s}}}\underbrace{S_t}_{(n\times 1)}+\underbrace{e_x\sum_{s=1}^{12}w\widetilde{\Omega}^s_{\{1,nm\},nm+j}}_{D_{\xi_t,x_{t,t+s}}} \tag{A.12}$$

Thus, we can include $Z_{\xi_t,x_{t,t+s}}$ as a row in $Z_{\xi_t}$ and $D_{\xi_t,x_{t,t+s}}$ as a row in $D_{\xi_t}$ in the mapping from the model to the observables described in (A.13). Note that the matrix $Z$ and vector $D$ are now regime dependent.

For GDP growth, we are interested in the average growth over a certain horizon. Our state vector contains $\widetilde{y}_t$. Thus, we can use the following approach:

$$\begin{aligned}\mathbb{E}_t^b\left[\left(gdp_{t+h}-gdp_t\right)h^{-1}|\xi_t=j\right] &= \mathbb{E}_t^b\left[\left(\widetilde{y}_{t+h}-\widetilde{y}_t+h\mu\right)h^{-1}|\xi_t=j\right]\\ &= h^{-1}\mathbb{E}_t^b\left[\widetilde{y}_{t+h}|\xi_t=j\right]-h^{-1}\widetilde{y}_t+\mu\end{aligned}$$

where $\mu$ is the average growth rate in the economy and $\widetilde{y}_t$ is GDP in deviations from the trend. With deterministic growth we have $gdp_{t+h}-gdp_t-h\mu\equiv\widetilde{y}_{t+h}-\widetilde{y}_t$. We then have

$$\begin{aligned}\mathbb{E}_t^b\left[\left(gdp_{t+h}-gdp_t\right)h^{-1}|\xi_t=j\right] &= h^{-1}\mathbb{E}_t^b\left[\widetilde{y}_{t+h}|\xi_t=j\right]-h^{-1}\widetilde{y}_t+\mu\\ &= h^{-1}\left[\underbrace{e_{\widetilde{y}}w\widetilde{\Omega}^s_{\{1,nm\},\{n(j-1)+1,nj\}}}_{Z_{\xi_t,\widetilde{y}_{t+s}}}\underbrace{S_t}_{(n\times 1)}+\underbrace{e_{\widetilde{y}}w\widetilde{\Omega}^s_{\{1,nm\},nm+j}}_{D_{\xi_t,\widetilde{y}_{t+s}}}-e_{\widetilde{y}}S_t\right]+\mu\\ &= h^{-1}\underbrace{\left[e_{\widetilde{y}}w\widetilde{\Omega}^s_{\{1,nm\},\{n(j-1)+1,nj\}}-e_{\widetilde{y}}\right]}_{Z_{\xi_t,\widetilde{y}_{t+s}-\widetilde{y}_t}}\underbrace{S_t}_{(n\times 1)}+h^{-1}\underbrace{e_{\widetilde{y}}w\widetilde{\Omega}^s_{\{1,nm\},nm+j}}_{D_{\xi_t,\widetilde{y}_{t+s}}}+\mu\end{aligned}$$

The expected values for the endogenous variables depend on the perceived transition matrix $\mathbf{H}^b$ and the properties of the alternative regime. The latter can be seen by recalling that the regime $\xi_t=B+1$ applies to the perceived alternative regime. Thus, data on expectations provide information about the perceived probability of moving across belief regimes as well as the parameters of the alternative regime.

**Estimation**

The solution of the model takes the form of a Makrov-switching vector autoregression (MS-VAR) in the state vector $S_t=\left[S_t^M,m_t,pd_t,k_t,z_t,lp_t,\mathbb{E}_t^b\left(m_{t+1}\right),\mathbb{E}_t^b\left(pd_{t+1}\right)\right]$. Here, $S_t^M$ is a vector of macro block state variables given by $S_t^M\equiv\left[\widetilde{y}_t,g_t,\pi_t,i_t,\overline{\pi}_t,f_t\right]'$. The asset pricing block of equations involves conditional subjective variance terms that are affected by Markov-switching random variables in the model. The subsection "Risk Adjustment with Lognormal Approximation," below, explains the approximation used to preserve lognormality of the entire system.

The model solution in state space form is

$$
\begin{aligned}
X_t &= D_{\xi_t,t} + Z_{\xi_t,t}\left[S_t', \widetilde{y}_{t-1}\right]' + U_t v_t \\
S_t &= C\left(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b\right) + T(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b) S_{t-1} + R(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b) Q \varepsilon_t \\
Q &= diag\left(\sigma_{\varepsilon_1}, ..., \sigma_{\varepsilon_G}\right), \ \varepsilon_t \sim N(0, I) \\
U &= diag\left(\sigma_1, ..., \sigma_X\right), \ v_t \sim N(0, I) \\
\xi_t^P &= 1...N_P, \ \xi_t^b = 1, ...B+1, H_{i,j} = p\left(\xi_t^b = i | \xi_{t-1}^b = j\right).
\end{aligned}
$$

where $X_t$ is a $N_X \times 1$ vector of data, $v_t$ are a vector of observation errors, $U_t$ is a diagonal matrix with the standard deviations of the observation errors on the main diagonal, and $D_{\xi_t,t}$, and $Z_{\xi_t,t}$ are parameters mapping the model counterparts of $X_t$ into the latent discrete- and continuous-valued state variables $\xi_t$ and $S_t$, respectively, in the model. The vector $X_t$ of observables is explained below. Note that the parameters $D_{\xi_t,t}$, $Z_{\xi_t,t}$, and $U_t$ vary with $t$ independently of $\xi_t$ because not all variables are observed at each data sampling period. To reduce computation time, we calibrate rather than estimate the parameters in $U = diag\left(\sigma_1, ..., \sigma_X\right)$ such that the variance of the observation error is 0.05 times the sample variance of the corresponding variable in $X$. In addition, some of the parameters in the system are dependent on the current policy rule and the associated Alternative rule, $\xi_t^P$, and the unobserved, discrete-valued $(B+1)$-state Markov-switching variable $\xi_t^b$ $\left(\xi_t^b = 1, 2, ..., B+1\right)$ with perceived transition probabilities

$$
\mathbf{H}^b = \begin{bmatrix}
p_{11} & \cdots & p_{1B} & 0 \\
\vdots & \ddots & \vdots & \vdots \\
p_{B1} & \cdots & p_{BB} & 0 \\
1 - \sum_{i=1}^{B} p_{i1} & \cdots & 1 - \sum_{i=1}^{B} p_{iB} & p_{B+1,B+1} = 1
\end{bmatrix},
$$

where $\mathbf{H}_{ij}^b \equiv p\left(\xi_t^b = i | \xi_{t-1}^b = j\right)$. We use the following notation:

$$
\begin{aligned}
C_{\xi_t^P,j} &= C\left(\theta_{\xi_t^P}, \xi_t^b = j\right), \ T_{\xi_t^P,j} = T\left(\theta_{\xi_t^P}, \xi_t^b = j\right), \ R_{\xi_t^P,j} = R\left(\theta_{\xi_t^P}, \xi_t^b = j\right) \\
D_{j,t} &= D_{\xi_t,t|\xi_t^b=j}, \quad Z_{j,t} = Z_{\xi_t,t|\xi_t^b=j}.
\end{aligned}
$$

**Kim's Approximation to the Likelihood and Filtering** We use Kim's (Kim (1994)) basic filter and approximation to the likelihood.

First note that, from the econometricians viewpoint, only the first $B$ regimes can actually be realized, since the true alternative that arises after one regime ends is never exactly as previously conceived by the investor. Thus, the estimation algorithm involves estimation just the upper $B \times B$ submatrix of $\mathbf{H}^b$, rescaled so that the elements sum to unity. The filtering described here therefore loops over just $B$ states, rather than $B+1$.

The sample is divided into $N_P$ policy regime subperiods indexed by $\xi_t^P$. Denote the last observation of each regime subperiod of the sample $T_1, ..., T_{N_P}$. The algorithm for the basic filter is described as follows.

Initiate values $\widetilde{S}_{0|0}$, $P_{0|0}$, for the Kalman filter and $\Pr\left(\xi_0^b\right) = \pi_0$ for the Hamilton filter and initialize $\mathcal{L}(\theta) = 0$. Denote $X^{t-1} \equiv \{X_1, ..., X_{t-1}\}$ and $\xi^{PT} = \left\{\xi_1^P, ..., \xi_T^P\right\}$.

In the mixed-frequency estimation, we use intra-month data to provide "early" estimates of the state space, while "final" estimates are obtained using a more complete set of data available at the end of each month. Let $t$ denote a month. Let $delta_i$ denote the number of time units that have passed *within* a month when we have reached a particular point in time, and let $nd$ denote the total number of time units in the month. Then $0 \leq d_i/nd \leq 1$, and the end of month $t$ is denoted $t - 1 + \delta_i$ with $\delta_i \equiv d_i/nd$. For example, $\delta_{100}$ could denote the point within the month that is exactly 10 minutes before an FOMC meeting during the month, while $\delta_{130}$ could denote point in the month 20 minutes after the same meeting. Intra-month observations used just prior to an FOMC meeting will typically include the daily BBG consensus forecasts from the day before the meeting, and the 10-minutes before FFF, ED and stock market data. Intermonth observations for the point of the month right after the FOMC meeting will typically include the daily BBG consensus forecasts from the day after the meeting, and the 20-minutes after FFF, ED and stock market data.

- For $t = 1$ to $T_1$ and $\theta_{\xi_t^P}$ relevant when $\xi_t^P = 1$:

1. Suppose we have information up through month $t-1$ and new information arrives at $t - 1 + \delta_i$. Conditional on $\xi_{t-1}^b = i$ and $\xi_t^b = j$ run the Kalman filter given below for $i, j = 1, 2, ..., B$ to update estimates of the latent state:

$$\begin{aligned}
S_{t|t-1}^{(i,j)} &= C_{\xi_t^P,j} + T_{\xi_t^P,j} S_{t-1|t-1}^i \\
P_{t|t-1}^{(i,j)} &= T_{\xi_t^P,j} P_{t-1|t-1}^i T_{\xi_t^P,j}' + R_{\xi_t^P,j} Q^2 R_{\xi_t^P,j}' \text{ with } Q^2 \equiv QQ' \\
e_{t|t-1+\delta_i,t-1}^{(i,j)} &= X_{t-1+\delta_i} - D_{j,t-1+\delta_i} - Z_{j,t-1+\delta_i}\left[S_{t|t-1}^{(i,j)\prime}, \widetilde{y}_{t-1}\right] \\
f_{t|t-1+\delta_i,t-1}^{(i,j)} &= Z_{j,t-1+d_1/nd} P_{t|t-1}^{(i,j)} Z_{j,t-1+d_1/nd}' + U_{t-1+d_1/nd}^2 \quad \text{with } U_t^2 \equiv U_t U_t' \\
S_{t|t-1+\delta_i}^{(i,j)} &= S_{t|t-1}^{(i,j)} + P_{t|t-1}^{(i,j)} Z_{j,t-1+\delta_i}' \left(f_{t|t-1+\delta_i,t-1}^{(i,j)}\right)^{-1} e_{t|t-1+\delta_i,t-1}^{(i,j)} \\
P_{t|t-1+\delta_i}^{(i,j)} &= P_{t|t-1}^{(i,j)} - P_{t|t-1}^{(i,j)} Z_{j,t-1+\delta_i}' \left(f_{t|t-1+\delta_i,t-1}^{(i,j)}\right)^{-1} Z_{j,t-1+\delta_i} P_{t|t-1}^{(i,j)}
\end{aligned}$$

2. Run the Hamilton filter to calculate new regime probabilities $\Pr\left(\xi_t^b, \xi_{t-1}^b | X_{t-1+\delta_i}, X^{t-1}\right)$ and $\Pr\left(\xi_t^b | X_{t-1+\delta_i}, X^{t-1}\right)$, for $i, j = 1, 2, ..., B$

$$\begin{aligned}
\Pr\left(\xi_t^b, \xi_{t-1}^b | X^{t-1}\right) &= \Pr\left(\xi_t^b | \xi_{t-1}^b\right) \Pr\left(\xi_{t-1}^b | X^{t-1}\right) \\
\ell\left(X_{t-1+\delta_i} | X^{t-1}\right) &= \textstyle\sum_{j=1}^B \sum_{i=1}^B f\left(X_{t-1+\delta_i} | \xi_{t-1}^b = i, \xi_t^b = j, X^{t-1}\right) \Pr\left[\xi_{t-1}^b = i, \xi_t^b = j | X^{t-1}\right] \\
f\left(X_{t-1+\delta_i} | \xi_{t-1}^b = i, \xi_t^b = j, X^{t-1}\right) &= (2\pi)^{-N_X/2} |f_{t|t-1+\delta_i}^{(i,j)}|^{-1/2} \exp\left\{-\frac{1}{2} e_{t|t-1+\delta_i,t-1}^{(i,j)\prime} f_{t|t-1+\delta_i,t-1}^{(i,j)} e_{t|t-1+\delta_i,t-1}^{(i,j)}\right\} \\
\mathcal{L}(\theta) &= \mathcal{L}(\theta) + \ln\left(\ell\left(X_{t-1+\delta_i} | X^{t-1}\right)\right) \\
\Pr\left(\xi_t^b, \xi_{t-1}^b | X_{t-1+\delta_i}, X^{t-1}\right) &= \frac{f\left(X_{t-1+\delta_i} | \xi_t^b, \xi_{t-1}^b, X^{t-1}\right) \Pr\left(\xi_t^b, \xi_{t-1}^b | X^{t-1}\right)}{\ell\left(X_{t-1+\delta_i} | X^{t-1}\right)} \\
\Pr\left(\xi_t^b | X_{t-1+\delta_i}, X^{t-1}\right) &= \sum_{i=1}^B \Pr\left(\xi_t^b, \xi_{t-1}^b = i | X_{t-1+\delta_i}, X^{t-1}\right)
\end{aligned}$$

3. Using $\Pr\left(\xi_t^b, \xi_{t-1}^b | X_{t-1+\delta_i}, X^{t-1}\right)$ and $\Pr\left(\xi_t^b | X_{t-1+\delta_i}, X^{t-1}\right)$, collapse the $B \times B$ values of $S_{t|t-1+\delta_i}^{(i,j)}$ and $P_{t|t-1+\delta_i}^{(i,j)}$ into $B$ values represented by $S_{t|t-1+\delta_i}^{j}$ and $P_{t|t-1+\delta_i}^{j}$:

$$S_{t|t-1+\delta_i}^{j} = \frac{\sum_{i=1}^{B} \Pr\left[\xi_{t-1}^b = i, \xi_t^b = j | X_{t-1+\delta_i}, X^{t-1}\right] S_{t|t-1+\delta_i}^{(i,j)}}{\Pr\left[\xi_t^b = j | X_{t-1+\delta_i}, X^{t-1}\right]}$$

$$P_{t|t-1+\delta_i}^{j} = \frac{\sum_{i=1}^{B} \Pr\left[\xi_{t-1}^b = i, \xi_t^b = j | X_{t-1+\delta_i}, X^{t-1}\right] \left(P_{t|t-1+\delta_i}^{(i,j)} + \left(S_{t|t-1+\delta_i}^{j} - S_{t|t-1+\delta_i}^{(i,j)}\right)\left(S_{t|t-1+\delta_i}^{j} - S_{t|t-1+\delta_i}^{(i,j)}\right)'\right)}{\Pr\left[\xi_t^b = j | X_{t-1+\delta_i}, X^{t-1}\right]}$$

4. If $t-1+\delta_i = t$, move to the next period by setting $t-1=t$ and returning to step 1

5. else, store the updated $S_{t|t-1+\delta_i}^{(j)}$, $P_{t|t-1+\delta_i}^{(j)}$, $\Pr\left(\xi_t^b, \xi_{t-1}^b | X_{t-1+\delta_i}, X^{t-1}\right)$, and $\Pr\left(\xi_t^b | X_{t-1+\delta_i}, X^{t-1}\right)$, and repeat steps 1-5 keeping $t-1$ fixed.

- At $t = T_1 + 1$ use $\theta_{\xi_t^P}$ relevant when $\xi_t^P = 2$, set $t-1=t$, and repeat steps 1-5
- At $t = T_2 + 1$ use $\theta_{\xi_t^P}$ relevant when $\xi_t^P = 3$, set $t-1=t$, and repeat steps 1-5
- $\vdots$
- At $t = T_{N_P-1} + 1$ use $\theta_{\xi_t^P}$ relevant when $\xi_t^P = N_P$, set $t-1=t$ and repeat steps 1-5
- At $t = T_N = T$ stop. Obtain $\mathcal{L}(\theta) = \sum_{t=1}^{T} \ln\left(\ell\left(X_t | X^{t-1}\right)\right)$.

The algorithm above is described in general terms; in principle the intermonth loop could be repeated at every instant within a month for which we have new data. In application, we repeat steps 1-5 only at certain minutes or days pre- and post-FOMC meeting.

**Observation Equation** The mapping from the variables of the model to the observables in the data can be written using matrix algebra to obtain the observation equation $X_t = D_{\xi_t,t} + Z_{\xi_t,t}\left[S_t', \widetilde{y}_{t-1}\right]' + U_t v_t$. Denote $\widehat{g}_t \equiv g_t - g$, and $\widehat{lp}_t = lp_t - lp$. Using the definition of stochastically detrended output, we have $\widetilde{y}_t = \ln\left(Y_t/A_t\right)$, $\Delta\ln\left(A_t\right) \equiv g_t = g + \rho_g\left(g_{t-1} - g\right) + \sigma_g \varepsilon_{g,t} \Rightarrow \widetilde{y}_t - \widetilde{y}_{t-1} = \Delta\ln\left(Y_t\right) - g_t \Rightarrow \Delta\ln\left(Y_t\right) = \widetilde{y}_t - \widetilde{y}_{t-1} + g_t = \widetilde{y}_t - \widetilde{y}_{t-1} + \widehat{g}_t + g$. Annualizing the monthly growth rates to get annualized GDP growth we have $\Delta\ln\left(GDP_t\right) \equiv 12\Delta\ln\left(Y_t\right) = 12g + 12\left(\widetilde{y}_t + \widehat{g}_t - \widetilde{y}_{t-1}\right)$. For quarterly GDP growth interpoloate to monthly frequency using the method in Stock and Watson (2010). For our other quarterly variables (SPF survey measures) and our biannual Liv survey, we drop these from the observation vector in the months for which they aren't available. The observation equation when all variables in $X_t$ are available takes the form:

$$\begin{bmatrix} \Delta \ln(GDP_t) \\ Inflation \\ FFR \\ SOC(Inflation)_{12m} \\ SOC(Inflation)_{60m} \\ f_t^{(0)} \\ BC(Inflation)_{12m} \\ SPF(Inflation)_{12m} \\ Liv(Inflation)_{12m} \\ SPF(GDPDInfl)_{12m} \\ BBG(Inflation)_{12m} \\ Liv(Inflation)_{120m} \\ SPF(Inflation)_{120m} \\ BC(FFR)_{12m} \\ BC(\Delta GDP)_{12m} \\ BBG(\Delta GDP)_{12m} \\ SPF(\Delta GDP)_{12m} \\ f_t^{(n)} \\ Baa_t \\ pgdp_t \\ egdp_t \end{bmatrix} = \begin{bmatrix} 12g \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ D_{\pi_{t,t+12},\xi_t} \\ D_{\pi_{t,t+12},\xi_t} \\ D_{\pi_{t,t+12},\xi_t} \\ D_{\pi_{t,t+12},\xi_t} \\ D_{\pi_{t,t+12},\xi_t} \\ D_{\pi_{t,t+120},\xi_t} \\ D_{\pi_{t,t+120},\xi_t} \\ D_{i_{t,t+12},\xi_t^b} \\ D_{y_{t+s},\xi_t^b} \\ D_{y_{t+s},\xi_t^b} \\ D_{y_{t+s},\xi_t^b} \\ D_{i_{t+X},\xi_t} \\ C_{Baa} \\ \ln(K) + g \\ C_{egdp} \end{bmatrix} + \begin{bmatrix} 12(\widetilde{y}_t + \widehat{g}_t - \widetilde{y}_{t-1}) \\ 12\pi_t \\ 12i_t \\ \left[h + (h-1)\phi + (h-2)\phi^2 + ... + \phi^{11}\right](1-\phi)\overline{\pi}_t + \left[\phi + \phi^2 + ... + \phi^{12}\right]\pi_t \\ \left[h + (h-1)\phi + (h-2)\phi^2 + ... + \phi^{59}\right](1-\phi)\overline{\pi}_t + \left[\phi + \phi^2 + ... + \phi^{60}\right]\pi_t \\ 12i_t \\ Z_{\pi_{t,t+12},\xi_t} S_t \\ Z_{\pi_{t,t+12},\xi_t} S_t \\ Z_{\pi_{t,t+12},\xi_t} S_t \\ Z_{\pi_{t,t+12},\xi_t} S_t \\ Z_{\pi_{t,t+12},\xi_t} S_t \\ Z_{\pi_{t,t+120},\xi_t} S_t \\ Z_{\pi_{t,t+120},\xi_t} S_t \\ Z_{i_{t,t+12},\xi_t^b} S_t \\ Z_{\xi_t^b, y_{t+s}-y_t} S_t \\ Z_{\xi_t^b, y_{t+s}-y_t} S_t \\ Z_{\xi_t^b, y_{t+s}-y_t} S_t \\ Z_{i_{t+X},\xi_t^b} S_t \\ B\widetilde{lp}_t \\ \widetilde{k}_t + pd_t + \widehat{g}_t + \widetilde{y}_t - \widetilde{y}_{t-1} \\ K\widetilde{k}_t \end{bmatrix} + U_t v_t \tag{A.13}$$

where in the last row we have used the fact that expectations for the macro agent in the model is:

$$\begin{aligned} \mathbb{E}_t^m[\pi_{t,t+h}] &= \left[h + (h-1)\phi + (h-2)\phi^2 + ... + \phi^{h-1}\right]\alpha_t^m + \left[\phi + \phi^2 + ... + \phi^h\right]\pi_t \\ &= \left[h + (h-1)\phi + (h-2)\phi^2 + ... + \phi^{h-1}\right](1-\phi)\overline{\pi}_t + \left[\phi + \phi^2 + ... + \phi^h\right]\pi_t \end{aligned}$$

The term $Inflation$ in the above stands for CPI inflation; $GDPDInfl$ refers to GDP deflator inflation. The variable $f_t^{(n)}$ refers to the time-$t$ contracted federal funds futures market rate. Here we use $n = \{6, 10, 20, 35\}$. The variable $pgdp$ is the SP500 capitalization-to-lagged GDP ratio, i.e., $P_t/GDP_{t-1}$; $egdp_t$ is the SP 500 earnings-to-lagged GDP ratio, i.e., $E_t/GDP_{t-1}$; $Baa_t$ is the Baa spread described above, where $C_{Baa}$ and $B$ and $K$ are parameters. To allow for the fact that the true convenience yield is only a function of $Baa_t$, we add a constant $C_{Baa}$ to our model-implied convenience yield $lp_t$ and scale it by the parameter $B$ to be estimated. Unless otherwise indicated, all survey expectations are 12 month-ahead forecasts in annualized units.

The above uses multiple measures of observables on a single variable, e.g., investor expectations of inflation 12 months ahead are measured by four different surveys (BC, SPF, LIV, and BBG). In the filtering algorithm above, these provide four noisy signals on the same latent variable.

### Computing the Posterior

The likelihood is computed with the Kim's approximation to the likelihood, as explained above, and then combined with a prior distribution for the parameters to obtain the posterior. A

block algorithm is used to find the posterior mode as a first step. Draws from the posterior are obtained using a standard Metropolis-Hastings algorithm initialized around the posterior mode. Here are the key steps of the Metropolis-Hastings algorithm:

- Step 1: Draw a new set of parameters from the proposal distribution: $\vartheta \sim N\left(\theta_{n-1}, c\overline{\Sigma}\right)$
- Step 2: Compute $\alpha\left(\theta^m;\vartheta\right) = \min\left\{p\left(\vartheta\right)/p\left(\theta^{m-1}\right),1\right\}$ where $p\left(\theta\right)$ is the posterior evaluated at $\theta$.
- Step 3: Accept the new parameter and set $\theta^m = \vartheta$ if $u < \alpha\left(\theta^m;\vartheta\right)$ where $u \sim U\left([0,1]\right)$, otherwise set $\theta^m = \theta^{m-1}$
- Step 4: If $m \geq n^{sim}$, stop. Otherwise, go back to step 1

The matrix $\overline{\Sigma}$ corresponds to the inverse of the Hessian computed at the posterior mode $\overline{\theta}$. The parameter $c$ is set to obtain an acceptance rate of around 30%. We use four chains of $540,000$ draws each (1 of every 200 draws is saved). The four chains combined are used to form an estimate of the posterior distribution from which we make draws. Convergence is checked by using the Brooks-Gelman-Rubin potential reduction scale factor using within and between variances based on the four multiple chains used in the paper.

**Risk Adjustment with Lognormal Approximation**

The asset pricing block of equations involves conditional subjective variance terms that are affected by Markov-switching random variables in the model. We extend the approach in Bansal and Zhou (2002) of approximating a model with Markov-switching random variables using a risk-adjustment while maintaining conditional log-normality. Consider the forward looking relation for the price-payout ratio:

$$\begin{aligned} P_t^D &= \mathbb{E}_t^b\left[M_{t+1}\left(P_{t+1}^D + D_{t+1}\right)\right] \\ \frac{P_t^D}{D_t} &= \mathbb{E}_t^b\left[M_{t+1}\frac{D_{t+1}}{D_t}\left(\frac{P_{t+1}^D}{D_{t+1}}+1\right)\right]. \end{aligned}$$

Taking logs on both sides, we get:

$$pd_t = \log\left[\mathbb{E}_t^b\left[\exp\left(m_{t+1}+\Delta d_{t+1}+\kappa_{pd,0}+\kappa_{pd,1}pd_{t+1}\right)\right]\right].$$

Applying the approximation implied by conditional log-normality we have:

$$\begin{aligned} pd_t &= \kappa_0 + \mathbb{E}_t^b\left[m_{t+1}+\Delta d_{t+1}+\kappa_{pd,1}pd_{t+1}\right] + \\ &\quad +.5\mathbb{V}_t^b\left[m_{t+1}+\Delta d_{t+1}+\kappa_{pd,1}pd_{t+1}\right]. \end{aligned}$$

To implement the solution, we follow Bansal and Zhou (2002) and approximate the conditional variance as the weighted average of the objective variance across regimes, conditional on $\xi_t$. Using the simpler notation of the state equation,

$$S_t = C_{\xi_t} + T_{\xi_t}S_{t-1} + R_{\xi_t}Q\varepsilon_t,$$

the approximation takes the form

$$\mathbb{V}_t^b \left[ x_{t+1} \right] \approx e_x \mathbb{E}_t^b \left[ R_{\xi_{t+1}} Q Q' R'_{\xi_{t+1}} \right] e_x \tag{A.14}$$

where $e_x$ is a vector used to extract the desired linear combination of the variables in $S_t$. This approximation maintains conditional log-normality of the entire system. In the solution, $C_{\xi_t}$ depends on the risk adjustment term $\mathbb{V}_t^b \left[ m_{t+1} + \Delta d_{t+1} + \kappa_{pd,1} pd_{t+1} \right]$ which depends on $R_{\xi_t}$. To solve this fixed point problem, employ the iterative approach of Bianchi, Kung, and Tirskikh (2018). Specifically, we solve the model and get $S_t$ for a given risk adjustment. Then, given $S_t$, we compute a new risk adjustment, which gives us a new $R_{\xi_{t+1}}$. If the new $R_{\xi_{t+1}}$ is the same as the $R_{\xi_{t+1}}$ that was implicitly assumed in the given risk adjustment, stop. Otherwise, use it to resolve $S_t$ and repeat the procedure until convergence.