Global Natural Rates in the Long Run: Postwar Macro Trends and the Market-Implied r^* in 10 Advanced Economies*

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PRELIMINARY: DO NOT CITE OR CIRCULATE

Abstract

Benchmark finance and macroeconomic models deliver conflicting estimates of the natural rate and bond risk premia. This *natural rate puzzle* applies not only in the U.S. but across many advanced economies. We estimate a unified no-arbitrage macro-finance model with long-run trend factors to obtain a *market-implied natural rate* r^*. Our monthly natural rate estimates span 10 advanced economies over most of the postwar period, expanding coverage far further than previous studies, and drawing on new primary and secondary sources for bond yields and curves. This model improves the explanatory power of yield and return regressions. Most variation in yields is due to the macro trends r^* and π^*, and not bond risk premia. Our r^* differs from other estimates, and is typically lower, intensifying concerns about secular stagnation and the effective lower-bound.

Keywords: bond risk premia, natural rate of interest, inflation expectations, term structure, affine models

JEL classification codes: C13, C32, E43, E44, E47, G12

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1. Introduction

Like two trains running on different railroads, studies of the natural rate and bond risk premia in the macroeconomic and finance literatures have tended to follow their own line even if ostensibly headed for the same place. The destination is clear and important: rates of interest, across all maturities, matter for saving, investment, capital allocation, economic growth, and monetary policy. But passengers on each route see different landscapes: on the macro track, a panoramic debate about time-series patterns with slow-moving trends in natural rate Wicksellian models and their fundamental drivers (e.g, Holston, Laubach, and Williams, 2017; Rachel and Summers, 2019; Jordà and Taylor, 2019); on the asset pricing track, a more tightly-framed look at cross-section patterns with a no-arbitrage pricing approach using factors typically built from yields (e.g, Litterman and Scheinkman, 1991; Piazzesi, 2010; Adrian, Crump, and Moench, 2013).

But the two tracks converge and a collision has been unavoidable. Workhorse finance models of bond risk premia and inflation expectations generate a path for the natural rate dramatically at odds with the macro literature. Equivalently, workhorse macro models of the natural rate and inflation expectations generate a path for bond risk premia equally at odds with the finance literature. We call this the *natural rate puzzle*. To get on the same track, the two approaches must be somehow shunted together. A consensus unified model should not fail these consistency tests and this is a first-order challenge for macro-finance research. We build on a long literature and make new headway. We explore the international aspect of this problem with newly-constructed data from the U.S. and 9 other advanced economies and we advance a new empirical approach which jointly disciplines estimates of the natural rate and risk premia with both financial market *and* macro information.

We first document the puzzle, for both the U.S. and other countries. For clarity, we do nothing analytically new here: we rely only on off-the-shelf models and data. The analysis revolves around three trend estimates. For the U.S., we construct an estimate of the bond risk premium following the canonical model (Adrian, Crump, and Moench, 2013) used by academic and financial professionals, and also by the Federal Reserve. We estimate inflation expectations following recent research incorporating trend inflation into models of bond yields and risk premia (Cieslak and Povala, 2015). And we employ or construct an estimate of the natural rate following the seminal model in the macro literature (Laubach and Williams, 2003). We then use directly-observed long-dated forwards to show a contradiction. In the U.S., for example, bond premia, inflation, and forwards, the implied natural rate is nearly flat over six decades, inconsistent with the rise and fall seen in macro estimates (with the implication that changes in the bond risk premium mostly explain long-yield changes). Conversely, using natural rates, inflation, and forwards, the implied bond risk premium is nearly flat, inconsistent with the rise and fall seen in finance estimates (with the implication that changes in natural rates mostly explain long-yield changes). Both models cannot be true. The puzzle is not an artifact of these particular estimates, and obtains using other well-respected estimates of U.S. bond risk premia, trend inflation, and the natural rate from multiple credible sources. The same puzzle also exists internationally in data we have newly compiled from other advanced economies.

The puzzle matters because our understanding of recent history hinges on whether one or the other story is more accurate. One narrative (macro) is of steadily declining natural real rates from the 1980s to the present, arguably culminating in a global secular stagnation trap, with active debate over a wide range of causal factors including demography and aging, productivity growth, inequality, and safe-asset demand from emerging economies, and all the attendant problems (Caballero, Farhi, and Gourinchas, 2008; Summers, 2015; Carvalho, Ferrero, and Nechio, 2016; Holston, Laubach, and Williams, 2017; Rachel and Smith, 2017; Rachel and Summers, 2019). The other narrative (finance) is of a sweeping rise and fall in risk premia, from sometime in the 1970s to the 1980s, the backwash of the Great Inflation episode, accounting for most of the trajectory of nominal rate with little or no movement in the natural rate (Kim and Wright, 2005; Wright, 2011; Adrian, Crump, and Moench, 2013; Bernanke, 2015). But when adding these up we will find that something has to give.

At some level, it might be tempting to suggest that the puzzle can easily be brushed away with an argument to the effect that these various models from different traditions were never supposed to be merged together in this way. One might argue that the finance bond risk premium models were not designed to be used to infer the natural rate, or that a macro natural rate model is not intended as a tool for inferring the path of the bond risk premium. And one can certainly understand this objection. Yet, that is exactly our point—and the rationale for this paper. If these models are never unified and integrated into a fully consistent approach, different strands of the literature will be destined to keep on producing contradictory results, speaking to their own niche but unable to reach common ground with the other approach. More broadly, this leaves the wider audience in macro and finance research, and in markets and the policy world, confused at to what is ultimately the right historical accounting for important long-run trends in the bond markets.

To tackle this conflict we set out a unified macro-finance model to ground the empirical work that follows. We build on the idea that term structure models should allow all nominal rates to include a stochastic trend, as seen in early work by Campbell and Shiller (1987) and developed further in the seminal paper of Kozicki and Tinsley (2001). We follow the key contribution of Cieslak and Povala (2015) and allow two trend factors, inflation and the real rate, with yields and expected returns to bonds of different maturities derived under no-arbitrage constraints from a short-rate process linked to these two macroeconomic factors, r^* and π^*. The model crystallizes the uncontroversial view—among macroeconomists, at least—that nominal bond returns are not explained simply as a compensation for the compounded benchmark rate, as the failure of the expectations hypothesis shows. Rather, there must be extra compensation for macroeconomic risks linked to real factors and inflation (Ang and Piazzesi, 2003; Ludvigson and Ng, 2009; Cieslak and Povala, 2015).

Next, in the empirical core of the paper, we take the model to the data. We apply the model to the postwar data (as far back as the 1950s for some countries) for 10 advanced economies, an historical laboratory as large as any previously explored in the study of these questions as far as we know. We thus develop new estimates of the natural rate covering many more countries and several more decades than prior studies: for example Holston, Laubach, and Williams (2017) cover only 4 economies from 1961 to 2015.

We find strong support for the model. Trend inflation is treated as an observable, as in prior work, but the unobservable natural rate is treated as latent and estimated from a unified state-space model with the Kalman filter. However, we make a unifying link to natural rate research, by also including a macroeconomic growth driver in a Wicksellian $r^* = g + z$ state equation, in addition to yield and return measurement equations, so our model utilizes information from both macro and financial market data. We therefore refer to our r^* estimate as the *market-implied natural rate*.[1] Dropping either trend variable significantly worsens the model fit: the baseline R^2 statistics are relatively high, but fit worsens one or both trends are removed, especially in return forecast regressions. Indeed, the macroeconomic factors subsume much of the relevant information needed to predict returns as compared with benchmark yield-only term-structure models, leaving only detrended yields to play a role, amplifying the insight of Cieslak and Povala (2015), but now for two trends and more countries.

The main contribution of this paper is a unified model which bridges the methodological divide and exploits fully all the information used in previous finance and macro approaches. Summing up the model we connect to the literature and review the reasons why such an encompassing modeling approach is needed. Finance models of unobserved bond risk premia have utilized yield-based factors, Wicksellian macro models of the unobserved natural rate have utilized macroeconomic variables like growth. The two produce inconsistent results and we argue that a unified approach using both sets of information is necessary. To get there, our paper makes a number of specific points along the way, touching on questions that have emerged from distinct literatures. First, we document for many countries, over many decades, an important macro-finance puzzle which the separate paths of risk premia research and natural rate research have often skirted around. Second, to operationalize the model, we apply a joint estimation strategy; though novel, and computationally much more burdensome, this should be preferred to approaches which draw natural rate and risk premia estimates from disparate models, which can lead to inconsistency. Third, we present estimates from a long and wide sample of 10 advanced economies, where we find that this is not just a U.S. story and this allows us to identify diverse global trends. Fourth, we build a new, comprehensive database of zero-coupon bond yield time series for ten countries, a valuable data contribution in its own right for the use of future researchers.

By the end, we are in a position to assay the natural rate puzzle, and we get a clear answer: across advanced economies, most of the long-term variation in yields in recent decades has come from shifts in the natural rate and inflation trend components, not from shifts in bond risk premia. The key takeaway is that macro and finance models can go their separate ways no longer.

[1] A related paper is the contemporaneous, independent work by Bauer and Rudebusch (2020), with a modeling approach based on a single stochastic trend, a nominal natural rate factor i^*. Their estimation uses yield-based factors and restrictions in the model structure, but omits information on macroeconomic variables like growth in the state-space model (see their Appendix C). In contrast, as explained below, we enforce an $r^* = g + z$ equation; that is, we bring in GDP growth data, and tying into the canonical latent-variable natural rate formulation in the macro side of the literature. We explain these distinctions further below.

2. The natural rate puzzle

The natural rate puzzle is the observation that standard finance models of bond risk premia generate a natural rate path at odds with the macro models.

In a wide class of standard stationary affine asset pricing models, the term structures (of bond yields, prices, excess returns, and forwards) are affine functions of the model's vector of risk factors F which will be made precise in the next section. As we will formally demonstrate below, in a typical such model we can express the very-long maturity forward rate, in the limit, as

$$\text{limiting forward rate} \;=\; \text{natural rate trend} \;+\; \text{inflation trend} \;+\; \text{limiting risk premium}\,. \quad (1)$$

This expression is intuitive. Investors in long maturity forwards must be compensated by the sum of the natural rate and inflation, plus a term that is by definition the long bond risk premium. The seminal work of Kozicki and Tinsley (2001) captures this in a term structure model with a "shifting endpoint" and they found improved fit when the endpoint was inferred from long-dated forwards.

If extant models were consistent, we could empirically validate Equation 1 by taking proxies for each term and checking to see if they add up appropriately. We do this as follows. We naïvely take the limiting risk premium (denoted Γ) from benchmark models in the finance literature, we take the natural rate trend (denoted r^*) and the inflation trend (denoted π^*) from benchmark macro models, and take a proxy limiting forward rate (denoted f) from long-dated market data. Having constructed these four terms for multiple countries, we can show that the above equation fails to hold. This is what we term the natural rate puzzle.

This section sets out to document this fact across the advanced economies and the rest of the paper explores a hybrid macro-finance model which may offer a way out. As might be anticipated, Equation 1 offers only two escape routes. Given that the forward rate is an observed trending variable, and that the observed inflation trend π^* is not subject to large estimation error, or can be treated as quasi-observable, then either the trend in the natural rate r^* is mismeasured, or the trend in the risk premium Γ is mismeasured, or both.

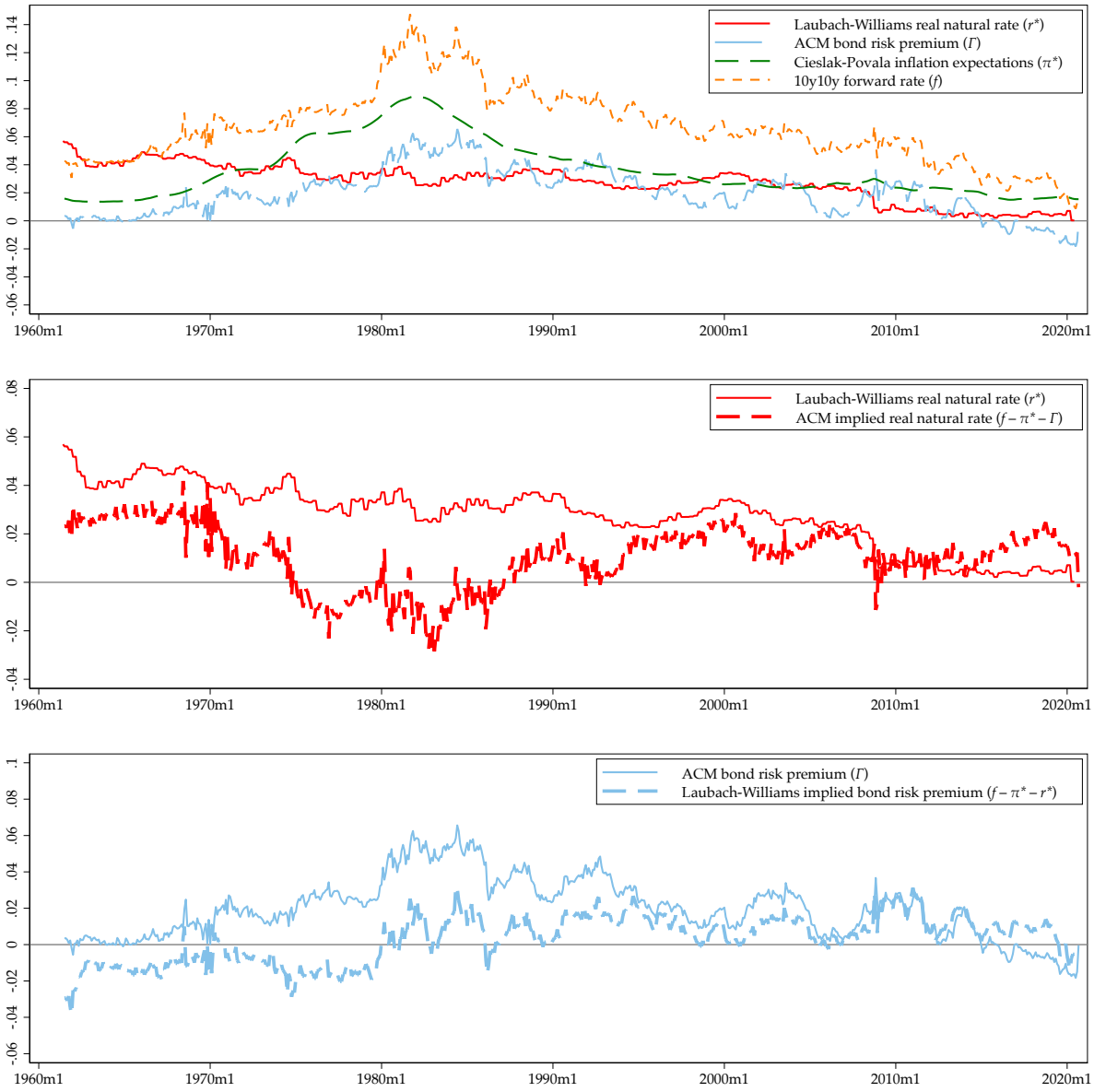
2.1. U.S. evidence

To see the puzzle, we take Equation 1 directly to the data. In Figure 1, Panel (a), the U.S. time-series estimates for each of the four terms are shown. We simply take these estimates from canonical models in the finance and macro literatures. The bond risk premium term Γ (two-sided) is constructed as in the baseline five-factor model of Adrian, Crump, and Moench (2013) [henceforth abbreviated ACM]; the inflation expectations term π^* as in Cieslak and Povala (2015) [CiP]; and the (two-sided) real natural rate term r^* as in Laubach and Williams (2003) [LW]. Finally, we have the long-dated 10-year, 10-year forward rate as a proxy (f) which is directly-observed raw data taken from Bloomberg, with $n = m = 120$ months here.

Figure 1: *The natural rate puzzle in U.S. data*

This figure displays market data (f) and existing trend data (other variables) based on other studies in Panel (a), and then displays the puzzle in the form of the difference between existing trend data and implied data in Panels (b) and (c). The presentation is based on Equation 1, which we can rewrite in simplified form, omitting subscripts and expectations, and taking them as understood, with the notation $f = r^* + \pi + \Gamma$. The puzzle is that existing benchmark estimates violate this equation.

In Panel (a), the four terms are shown: the bond risk premium Γ from Adrian, Crump, and Moench (2013); inflation expectations π from Cieslak and Povala (2015); and the real natural rate r^* from Laubach and Williams (2003). We also show the 10-year, 10-year forward rate (f) from from Bloomberg. The sample period is June 1961–December 2020. In Panel (b), we compare the real natural rate r^* (two-sided) from Laubach and Williams (2003) to that implied by $r^* = f - \pi^* - \Gamma$. There is a large difference between these two series. In Panel (c), we compare the bond risk premium Γ (two-sided) from Adrian, Crump, and Moench (2013) to that implied by $\Gamma = f - r^* - \pi^*$. There is the same large difference between these two series.



The first version of the consistency test rearranges Equation 1 to obtain a formula for the real natural rate $r^* = f - \pi^* - \Gamma$, and Panel (b) plots both sides of this expression using the above data sources: the left-hand side is taken from an LW model and the right-hand side is the implied value using an ACM model. The equality is violated, and the disparity is often quite large. The ACM-implied r^* does not match the LW r^*. The ACM series starts around +2% in the 1960, displays a sharp decline to a level below -2% during the Great Inflation period of the 1970s, returns to +2% in the 1990s, drops to near zero after the financial crisis, and then shows a consistent increase after 2013 to a level close to 2% in 2019. In contrast, the familiar LW estimate of r^* has fallen gradually from a +4% level in the 1960s and 1970s, with the sharpest decline occurring after the mid-2000s, and since 2010 it has sat in the 0.5%–1.0% range, and never turned negative. The difference between the two series, before the last decade, is often large, between 100 and 600 basis points (bps), with the LW r^* much higher than the ACM r^*, on average. Around 2012 the two series intersected and then the difference inverted to about -100 bps in the other direction.

A second, equivalent, version of the test is shown in Panel (c). We rearrange again to obtain a formula for the bond risk premium $\Gamma = f - r^* - \pi^*$, and Panel (b) plots both sides of this expression using the aforementioned data sources. Now the left-hand side is from an ACM model and the right-hand side is the implied value in an LW model. This equality is, of course, also violated, and the same large disparity is seen. The ACM bond risk premium starts near zero in the 1960s, rises sharply in the Great Inflation period of the 1970s to about 6%, then gradually falls back, reaching zero again in the mid-2010s. The LW bond risk premium behaves very differently, and is almost flat by comparison. It actually starts at a negative level in the 1960s, rises much later, but only to a modest 2% by the early 1980s, then declines by a small amount up to the mid 2000s. After that the two series cross, with LW signaling a small positive bond risk premium, but ACM turning negative.

The puzzle is vividly apparent in these charts. Persistent inconsistencies of several hundred basis points are quantitatively just too large to ignore. Both approaches cannot be simultaneously right. A substantial contradiction thus emerges from the heart of benchmark macro and finance models once they are studied in unison. The rest of this paper is devoted to building theory and empirics to help resolve the puzzle.

2.2. Alternative trend measures

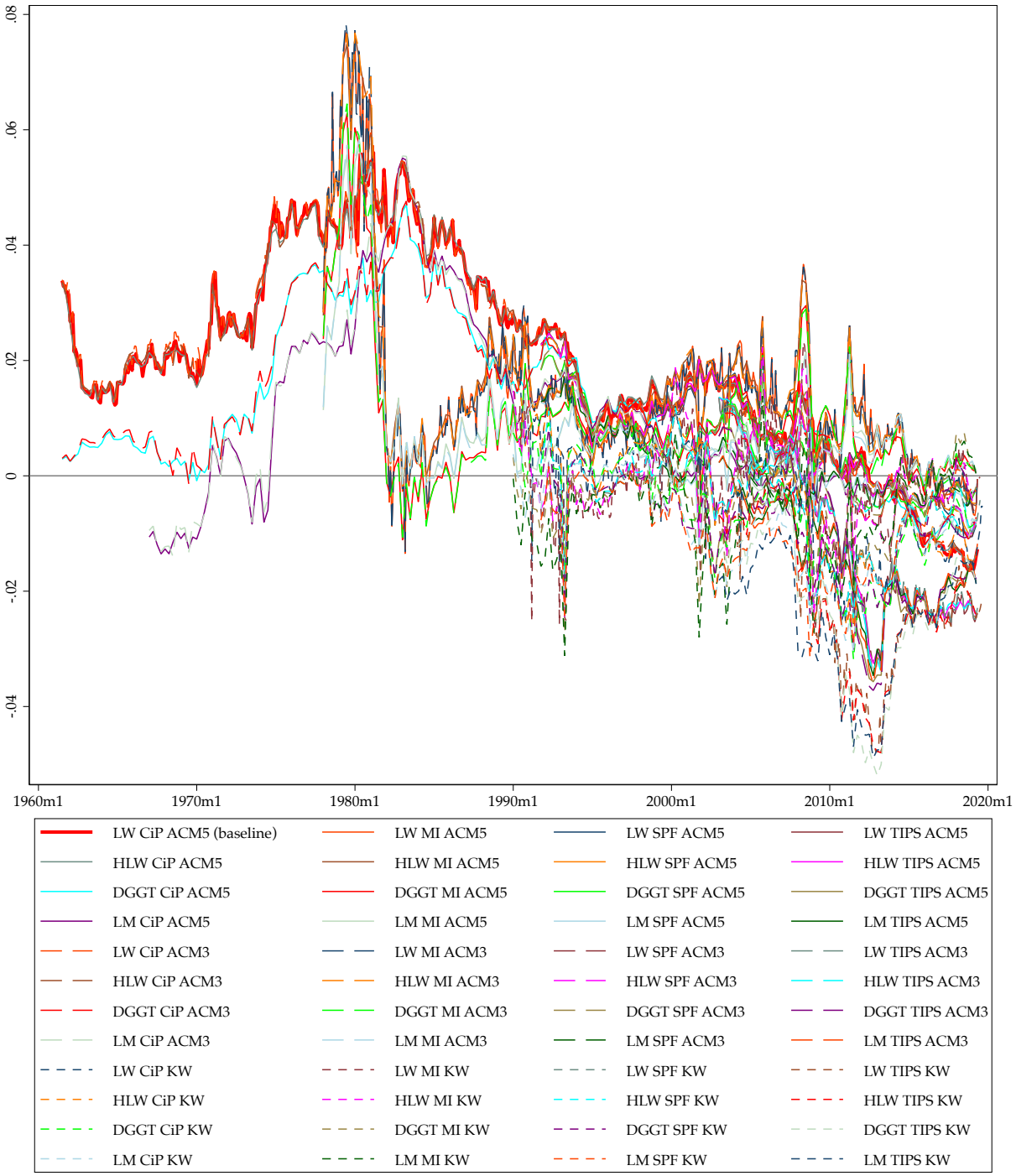
As a robustness check, Figure 2 examines whether the existence of the puzzle for the U.S. is sensitive to the source data used. For a variety of widely used and respected sources we compute the discrepancy in Equation 1 as $discrepancy = r^* - f + \pi^* + \Gamma$, and plot the series over time.

The same forward rate data f from Bloomberg are used in all cases. The sources of the other three series rotate through all possible combinations, with the sources are abbreviated as follows:

- Natural rate estimates r^*: Laubach and Williams (2003) [LW]; Holston, Laubach, and Williams (2017) [HLW]; Del Negro, Giannone, Giannoni, and Tambalotti (2017) [DGGT]; and Lubik and Matthes (2015) [LM].

Figure 2: *The natural rate puzzle in U.S. data using alternative trend measures*

This chart displays the discrepancy in Equation 1 for the United States. The presentation is based on Equation 1 and the series computed is *discrepancy* $= r^* - f + \pi^* + \Gamma$. See text.



- Inflation estimates π^*: Cieslak and Povala (2015) [CiP]; the University of Michigan Inflation Expectations from FRED [MI]; the Survey of Professional Forecasters from the Federal Reserve Bank of Philadelphia [SPF]; and the TIPS 10-Year Breakeven Inflation Rate from FRED [TIPS].

- Bond risk premium estimates Γ: Adrian, Crump, and Moench (2013), 5-factor model [ACM5]; the same authors' 3-factor model [ACM3]; and Kim and Wright (2005), 3-factor model [KW].

Note that because quite a few of these series (e.g., TIPS, KW) are only available for a shorter span of recent years, full-sample comparisons across all trend estimates are not always possible.

The figure reveals that the natural rate puzzle is a quite robust phenomenon in recent U.S. data. A discrepancy arises in all cases. It is often more than 100 bps, and at certain times it exceeds 500 bps. It is present in a wide variety of trend estimates currently used in the macro-finance literatures. The figure shows that, as in the baseline variant above, the extent of the puzzle varies from year to year, and over decades. Most series combinations make errors in one direction, but a few go the other way. The discrepancies are large in the 1970s, and often surge to their highest levels around 1980. The discrepancies are smaller by the late 1990s and early 2000s, but they open up again for some series, in the opposite direction to the vicinity of -400 bps, after the global financial crisis.

2.3. International evidence

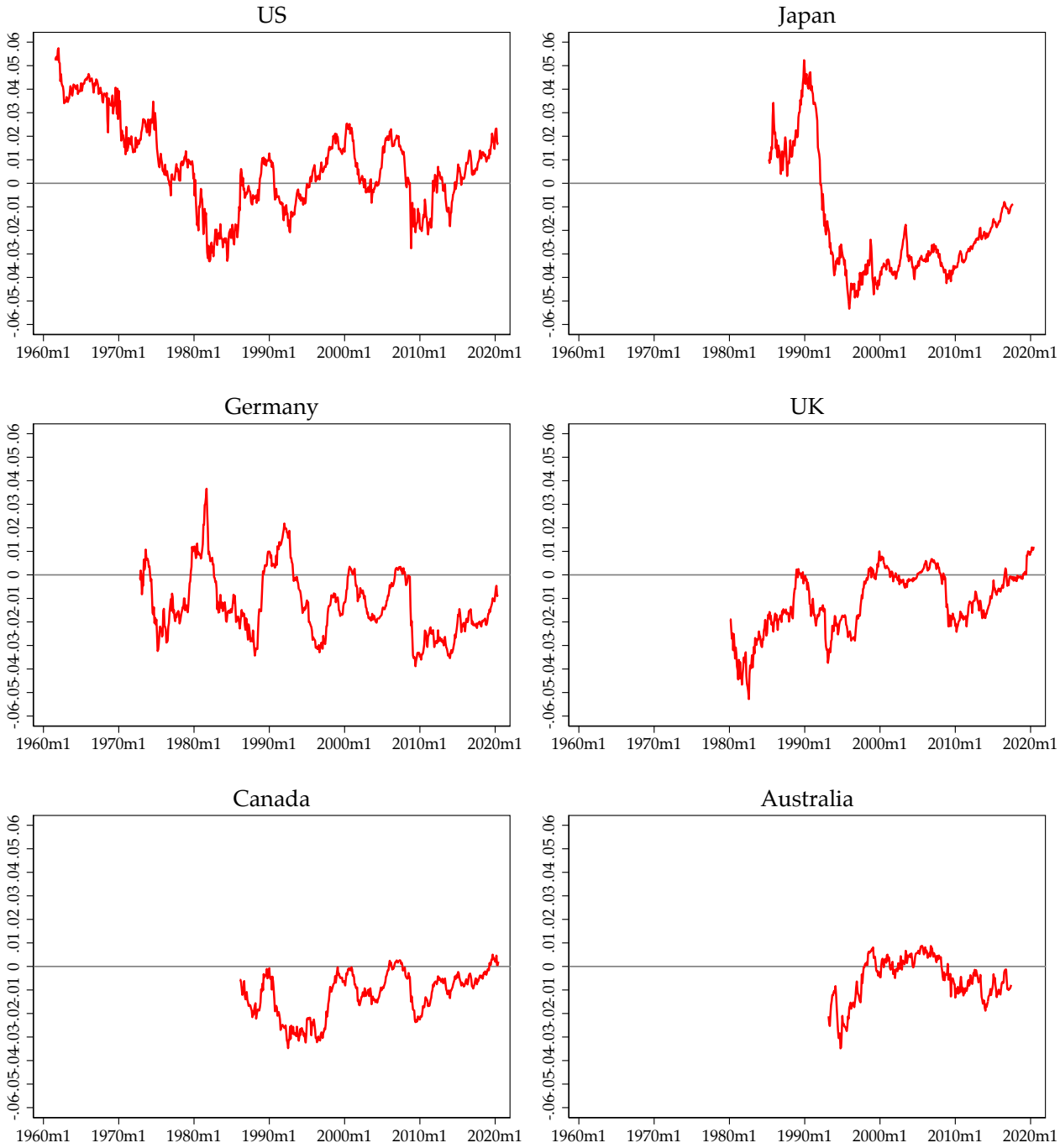
We also sought evidence for or against the natural rate puzzle in 5 other advanced economies: Japan, Germany, the U.K., Canada, and Australia. Figure 3 presents these findings. [Will be expanded to 10 advanced economies, adding CHE SWE ESP FRA. This work is in progress.]

Again, we use the real natural rate r^* from an LW-type and the risk premium from an ACM-type estimation, inflation expectations from a CiP-type estimation, and the forward rate. For the LW-type natural rate estimates we use LW itself for the U.S. as above, Holston, Laubach, and Williams (2017) (two-sided estimates) for the Germany, U.K., Canada, Okazaki and Sudo (2018) (two-sided estimates) for Japan, and McCririck and Rees (2017) (two-sided estimates) for Australia. We then replicate the ACM and CiP methodologies and construct forward rates from zero-coupon bonds, as described later in this paper, and compute the discrepancy for all the countries to complete the analysis, with $discrepancy = r^* - f + \pi^* + \Gamma$, as above.

The discrepancy can be visualized over time in Figure 3, which presents the time-series data for each natural rate estimate. Here again, discrepancies of several hundred basis points are not uncommon. In short, the natural rate puzzle is not simply a U.S. puzzle. It applies to many advanced economies, suggesting a deeper and more general pattern of conflict between widely-used standard estimates of the natural rate and bond risk premia.

Figure 3: *The natural rate puzzle in international data*

This chart displays the discrepancy in Equation 1 for the set of 6 countries. The presentation is based on Equation 1 and the series computed is *discrepancy* $= r^* - f + \pi^* + \Gamma$. See text.



3. Term-structure models with two macro factors

A large literature over 20 years has built a foundation of affine term-structure models with a role for macroeconomic trends in inflation and the natural rate. Building on that we ask: first, can theory frame the natural rate puzzle seen above? Second, can joint estimation of the natural rate and the term-structure model deliver new insights and better forecasts? Third, can the approach work not just in U.S. data but also in other advanced economies? We will answer yes to these questions based on the approach described in this section.

Stationary term-structure model We use a standard setup as in the seminal paper of Cieslak and Povala (2015) which features two trends for inflation and the real rate, building on the earlier insights of Kozicki and Tinsley (2001). At time t, we denote the nominal yield on an n-period Treasury bond by $y_t^{(n)}$, the current trend inflation by π_t, and the current trend real natural rate by r_t. Nominal yields across all maturities are driven by the two trends and other factors contained in a price-of-risk factor x_t vector, so the full set of factors is $F_t = (\pi_t, r_t, x_t)'$. Building on, Cieslak and Povala (2015) as a baseline we will set x_t equal to the average yield across the curve, or, equivalently (as we see below) its residual component after projection onto the two macro trends (π_t, r_t).

The core of the model is the specification of the short-rate process and the stochastic discount factor, from which all other pricing relationships follow. The short-rate process is assumed to depend on the factors, which in turn follow independent AR(1) processes, with

$$y_t^{(1)} = \delta_0 + \delta_\pi \pi_t + \delta_r r_t \,, \tag{2}$$

$$r_t = \mu_r + \phi_r r_{t-1} + \sigma_r \epsilon_t^r \,, \tag{3}$$

$$\pi_t = \mu_\pi + \phi_\pi \pi_{t-1} + \sigma_\pi \epsilon_t^\pi \,, \tag{4}$$

where $\delta_\pi > 0, \delta_r > 0$, with $\delta_x = 0$, as shown, and $\epsilon_t^\pi, \epsilon_t^r$ are standard normal, i.i.d.

At Equation 2, a natural benchmark noted by Cieslak and Povala (2015) is $\delta_0 = 0, \delta_\pi = \delta_r = 1$, i.e., the Fisher constraints. At Equation 3 and Equation 4, with persistent inflation we expect $\phi_\pi < 1$ but close to unity. The natural rate also tends to be persistent, with $\phi_r < 1$ and close to unity. In very long run historical data (100+ years) both series are seen to be stationary. Cieslak and Povala (2015) estimate annual $\hat{\phi}_\pi = 0.975$, $\hat{\phi}_r = 0.75$ for their U.S. sample.

The *endpoints* for the trends are the long-run limits (Kozicki and Tinsley, 2001). Allowing for possibly time-varying parameters the unconditional means of the endpoints are $r_\infty^{(t)} = \mu_r^{(t)} / (1 - \phi_r^{(t)})$ and $\pi_\infty^{(t)} = \mu_\pi^{(t)} / (1 - \phi_\pi^{(t)})$. These values prevail in the long run when all shocks have died out.

Finally, the price-of-risk factor follows an AR(1) process with i.i.d. normal shocks,

$$x_t = \mu_x + \phi_x x_{t-1} + \sigma_x \epsilon_t^x \,. \tag{5}$$

Cieslak and Povala (2015) estimate $\hat{\phi}_x = 0.392$ for the U.S. when detrending with the inflation trend.

The model economy is then compactly described by the equations

$$F_t = \mu + \Phi F_{t-1} + \Sigma \epsilon_t \,, \tag{6}$$

$$y_t^{(1)} = \delta_0 + \delta_1' F_t \,, \tag{7}$$

with Φ and Σ diagonal, $\delta_1 = (\delta_\pi, \delta_r, 0)'$, and $\epsilon_t = (\epsilon_t^\pi, \epsilon_t^r, \epsilon_t^x)'$.

As is common, the simplest single-yield-factor formulation would define $x_t = \bar{y}_t$, so the price-of-risk factor is the average level of yields, $\bar{y}_t = \frac{1}{N}\sum_1^N y_t^{(n)}$.[2] Clearly then, in light of our earlier discussion, this is (see Duffee, 2011) a "yields-plus" model and not a "yields-only" model. That is, we are interested in whether the two macro trends r_t and π_t add useful information over and above a model using just a set of yield-based factors.

Our choice of $F_t = (\pi_t, r_t, x_t)^\top$ follows Cieslak and Povala (2015) and assumes that $\epsilon_t^\pi, \epsilon_t^r$ are uncorrelated. Thus, the natural rate factor r_t is intended to capture variations in the benchmark real rate due to factors other than shifts in trend inflation. Equation 2 specifies in a stylized way how monetary policy, through its control of the short end of the curve, may react to both changes in price dynamics and real-economy-driven natural rate forces. A large stream of literature has related these forces to demographic factors, global supply and demand equilibrium in the safe-assets market, changes in international capital flows, income and wealth inequality, among others.

Imposing standard restrictions on the model SDF Using a standard no-arbitrage affine term-structure model as in Cieslak and Povala (2015), the log nominal stochastic discount factor is exponentially affine in the risk factors,

$$m_{t+1} = -y_t^{(1)} - \frac{1}{2}\Lambda_t'\Lambda_t - \Lambda_t'\epsilon_{t+1} \,, \tag{8}$$

where Λ_t is the compensation for risk of shock ϵ_{t+1}, with $\Lambda_t = \Sigma^{-1}(\lambda_0 + \Lambda_1 F_t)$.

We need more structure to make progress. Suppose x_t is taken to be a single yield-based factor, and the loadings in Λ_t are assumed to take the form

$$\lambda_0 = \begin{pmatrix} \lambda_{0\pi} \\ \lambda_{0r} \\ 0 \end{pmatrix}, \qquad \Lambda_1 = \begin{pmatrix} 0 & 0 & \lambda_{\pi x} \\ 0 & 0 & \lambda_{rx} \\ 0 & 0 & 0 \end{pmatrix}. \tag{9}$$

As Cieslak and Povala (2015) note, this setup fits with the empirical finding of Cochrane and Piazzesi (2005) that bond risk premiums move on a single mean-revering factor that is largely unexplained by the level, slope, and curvature movements. In this spirit, as is seen in a wide range

[2] This baseline setup is motivated by the Cochrane and Piazzesi (2005) finding that a single-factor can explain bond pricing quite well, but x could be expanded to a vector as in three-factor yield models (Nelson and Siegel, 1987; Litterman and Scheinkman, 1991; Joslin, Singleton, and Zhu, 2011) or even five-factor yield models (Adrian, Crump, and Moench, 2015).

of established term structure models, the price of risk is assumed to follow a univariate process that is independent of other state variables. Cieslak and Povala (2015) estimate $\hat{\lambda}_{\pi x} = -0.47$ and $\hat{\lambda}_{rx} = 0.16$ for their U.S. sample.

Solving the model The model can then be solved (e.g., Duffee, 2013) as a set of affine equations for log bond prices, yields, forwards, and excess returns, in terms of the factors:

$$p_t^{(n)} = \mathcal{A}_n + \mathcal{B}_n' F_t \,, \tag{10}$$

$$y_t^{(n)} = A_n + B_n' F_t \,, \tag{11}$$

$$f_t^{(n,m)} = (\mathcal{A}_n - \mathcal{A}_{n+m}) + (\mathcal{B}_n - \mathcal{B}_{n+m})' F_t \,, \tag{12}$$

$$rx_{t+1}^{(n)} = \mathfrak{B}_n' F_t + v_t^n \,, \tag{13}$$

where $A_n = -\frac{1}{n}\mathcal{A}_n$, $B_n = -\frac{1}{n}\mathcal{B}_n$, $v_t^n = \mathcal{B}_{n-1}'\Sigma\epsilon_{t+1}$.

Solutions are derived from Riccati equations, where the recursions are

$$\mathcal{A}_{n+1} = -\delta_0 + \mathcal{A}_n + \mathcal{B}_n'\mu^q + \frac{1}{2}\mathcal{B}_n'\Sigma\Sigma'\mathcal{B}_n \,, \tag{14}$$

$$\mathcal{B}_{n+1} = -\delta_1 + (\Phi^q)'\mathcal{B}_n \,, \tag{15}$$

with $\mathcal{A}_0 = 0$, $\mathcal{B}_0 = 0$, and risk-neutral dynamics governed by $\mu^q = \mu - \lambda_0$ and $\Phi^q = \Phi - \lambda_1$.[3]

Concretely, in the baseline model used here, factor loadings of bond prices can be derived as

$$\mathcal{B}_n^{\pi} = -\delta_{\pi}\frac{1-\phi_{\pi}^n}{1-\phi_{\pi}} \,, \tag{16}$$

$$\mathcal{B}_n^{r} = -\delta_{r}\frac{1-\phi_{r}^n}{1-\phi_{r}} \,, \tag{17}$$

$$\mathcal{B}_n^{x} = -\mathcal{B}_{n-1}^{\pi}\lambda_{\pi x} - \mathcal{B}_{n-1}^{r}\lambda_{rx} + \mathcal{B}_{n-1}^{x}\phi_x \,, \tag{18}$$

and the factor loadings of excess returns attach only to the price-of-risk factor, with

$$\mathfrak{B}_n = \mathcal{B}_{n-1}'(\lambda_0 + \Lambda_1 \mathbf{1}_3)x_t - \frac{1}{2}\mathcal{B}_{n-1}'\Sigma\Sigma'\mathcal{B}_{n-1} \,. \tag{19}$$

Equivalent formulation using detrended yields As above, we might set $x_t = \bar{y}_t$, so the price-of-risk factor is the average level of yields. But to better describe the role of the trends we use the key innovation in Cieslak and Povala (2015), and reformulate the model using yields which have been *detrended* to orthogonalize them relative to the trends. We apply this idea to *both* trends, and define the detrended yield by $c_t^{(n)} = y_t^{(n)} - \hat{A}_n - \hat{B}_n^r r_t - \hat{B}_n^{\pi}\pi_t$, which is the residual from the regression defined by Equation 11 with the yield factor x suppressed. Now let the average of this detrended yield be $\bar{c}_t = \frac{1}{N}\sum_1^N c_t^{(n)}$.

[3]The closed form solution for $\mathcal{B}_n$ is $\mathcal{B}_n = -\left[\sum_{j=0}^{n-1}(\Phi^q)^j\right]'\delta_1$.

The model can then be expressed in our preferred form in terms of $x_t = \bar{c}_t$ and the full set of factors consists of the two trends and the detrended average yield, $F_t = (\pi_t, r_t, \bar{c}_t)'$. Due to detrending, in this setup the unconditional mean of F_t is now $\mu = E(F_t) = (\pi_\infty^{(t)}, r_\infty^{(t)}, 0)$. To be clear, this reformulation leaves the model unchanged, but this attribution exercise parses out those shifts in yield factors that are ultimately driven by the macro trends.

Statistical identification of the natural rate of interest A crucial step in affine models with trends is to obtain the right trends. Ever since Kozicki and Tinsley (2001) and Cieslak and Povala (2015), the literature takes inflation as observable $\pi_t = \pi_t^*$, e.g., from surveys or a learning model. However, the natural rate is unobserved or latent. One of our contributions is to focus on the joint problem of estimating both the affine model and the natural rate in a consistent framework.

We propose a way to bridge the statistical objects specified in Equation 2 through Equation 4 to the economic object of interest, the natural rate, consistent with the Wicksellian notion of r^*. To do this we link the natural rate to variations in trend growth g_t plus an additional component denominated the "headwinds" factor z_t, as in canonical state-space models of the natural rate. Thus, we aim to straddle the yields-plus approach of finance models and the Wicksellian estimation of macro models in the tradition of Laubach and Williams (2003), and this entails adding the equation:

$$r_t = r_t^* = z_t + g_t \,. \tag{20}$$

The headwinds factor is intended to capture all variations in the natural rate due to structural, slow-moving factors such as demographics, structural shifts in international capital flows, global income inequality, among others.

We also assume that the headwinds factor follows an AR(1) process, so that

$$z_{t+1} = \rho_z z_t + e_{t+1}^z \,, \qquad e_{t+1}^z \sim N\left(0, \sigma_z^2\right) \,. \tag{21}$$

Without any constraints, the headwinds factors would absorb all the high frequency variation in rates. We argue that the frequency and persistence of the headwinds factor should be more like the process followed by trend growth than, say, yields.

In order to discipline this process, and to make its frequency compatible with the fundamental growth trends we observe, we impose a prior on the value of ρ_z and σ_z. This prior forces the headwinds process to follow a lower frequency than financial data (yields, etc.), without forcing a link from the natural rate to particular economic data, other than the Wicksellian link to output growth.

Trend output growth g_t is assumed to be a stationary process and observable. We take it as an exogenous process that we can estimate separately by typical filtering procedures. To avoid forward-looking bias, we estimate the as-of-date (one-sided) Hodrick-Prescott filter on country real GDP growth (to be conservative, when calculating the growth rate at t, we use information available

at time $t-1$, namely growth rates up to $t-2$).

The above steps harmonize our state-space model with the macro approach to interest rate trends epitomized by Laubach and Williams (2003). Full details of the state-space model follow in the next section and, in detail, in the Appendix. For now we cover some further ground to explain how this approach connects to the natural rate puzzle presented earlier, and how it contributes to the literature.

Corollary 1: Long-dated forwards, macro trends, and the bond risk premium Using the baseline model we can provide justification for the decomposition at Equation 1 to introduce the natural rate puzzle.

For a 1-period forward at horizon n, $f_t^{(n,1)} = (\mathcal{A}_n - \mathcal{A}_{n+1}) + (\mathcal{B}_n - \mathcal{B}_{n+1})' F_t$ and from the recursions (e.g., Duffee, 2013) we obtain

$$f_t^{(n,1)} = \underbrace{-\frac{1}{2}\mathcal{B}_{n-1}'\Sigma\Sigma'\mathcal{B}_{n-1}}_{\text{negative convexity term}} + \underbrace{\delta_0 - \mathcal{B}_n'\mu^q + \delta_1'(\Phi^q)^n F_t}_{\text{expected short rate under q}}. \tag{22}$$

Let the benchmark Fisher constraints hold, $\delta_0 = 0, \delta_\pi = \delta_r = 1$, substitute for μ_i, and we have

$$\lim_{n\to\infty} f_t^{(n,1)} = \underbrace{r_\infty^{(t)}}_{\substack{\text{endpoint for} \\ \text{natural rate } (r^*)}} + \underbrace{\pi_\infty^{(t)}}_{\substack{\text{endpoint for} \\ \text{inflation } (\pi^*)}} + \underbrace{\mathcal{B}_\infty^\pi \lambda_{0\pi} + \mathcal{B}_\infty^r \lambda_{0r} - \frac{1}{2}\mathcal{B}_\infty'\Sigma\Sigma'\mathcal{B}_\infty}_{\text{bond risk premium as } n\to\infty}. \tag{23}$$

Thus the limiting forward rate equals the sum of the natural rate and inflation endpoints and the limiting bond risk premium.[4] Intuitively, at shorter maturities the difference between the forward and the sum of the two endpoint will also be affected by transitory variation in short-rate expectations (i.e., due to the term $\delta_1'(\Phi^q)^n F_t$); in the long-horizon limit we get the non-transitory limiting bond risk premium, and short-term expectation deviations fade away, as Equation 23 shows.

Thus, in the limit at large maturities, or in a large sample limit where the mean of x_t is zero, we obtain Equation 1,

$$\text{forward rate} = \text{natural rate trend} + \text{inflation trend} + \text{bond risk premium},$$

where we associate the endpoints with the natural rate trend and inflation trend. We now see how the limit result using forwards frames the puzzle in a clean way. This approach can be complemented by an examination of the direct implications of different specifications of affine term structure models on observables such as yields, forward rates, and excess bond returns.

[4] As $n\to\infty$, $\mathcal{B}_n^\pi \to -\pi_\infty^{(t)}/\mu_\pi$, $\mathcal{B}_n^r \to -r_\infty^{(t)}/\mu_r$, and $(1-\phi_x)\mathcal{B}_n^x \to \lambda_{\pi x}\pi_\infty^{(t)}/\mu_\pi + \lambda_{rx} r_\infty^{(t)}/\mu_r$, with $\mu_c = 0$.

Corollary 2: Expected returns and the cyclical factor We can also solve (e.g., Duffee, 2013) for the factor loadings of excess returns,

$$rx_{t+1}^{(n)} = \mathcal{B}_{n-1}'(\lambda_0 + \Lambda_1 F_t) - \frac{1}{2}\mathcal{B}_{n-1}'\Sigma\Sigma'\mathcal{B}_{n-1} + \mathcal{B}_{n-1}'\Sigma\epsilon_{t+1}\,. \tag{24}$$

Setting $x_t = \bar{c}_t$, the expected return for the average bond across all maturities is

$$E_t(\overline{rx}_{t+1}) \;=\; \underbrace{(\overline{\mathcal{B}}^{\pi}, \overline{\mathcal{B}}^{r})'(\lambda_{0\pi} + \Lambda_{\pi x}\bar{c}_t, \lambda_{0r} + \Lambda_{rx}\bar{c}_t) \;-\; \text{convexity terms}}_{\text{bond risk premium for average bond}}\,. \tag{25}$$

As n grows large the average risk premia in Equation 23 and Equation 25 are the same. Note that, with detrending, i.e., having conditioned on trends, only the cyclical factor $x_t = \bar{c}_t$ predicts bond excess returns, and not the trends themselves r_t and π_t. Of course, this does *not* mean than shifts in r_t and π_t play no role: they will of course change the cyclical deviation $x_t = \bar{c}_t$ via detrending.

The nonstationary case Recent contemporaneous work by Bauer and Rudebusch (2017, 2020) studies the trends in the U.S. case and allows the trends to be nonstationary. Bauer and Rudebusch (2017) extend Cieslak and Povala (2015), and allow the factor process to include unit roots in r and π, as well as short run AR(1) disturbance terms, with 5 state variables instead of 3. The two corollaries noted above are preserved exactly, and the endpoints are the current values of the r and π, the expected values as $t \to \infty$. The loadings on the permanent components of r and π are unity at all horizons, so the Fisher constraints hold mechanically. In Bauer and Rudebusch (2020, see Appendix C) a different theoretical framework is used as a backdrop. In the real-world measure there is a unit root in a single nominal natural rate trend (i^*) and three stationary cyclical yield factors. The model could be extended to two trends and/or fewer yield factors. The model is restricted to be stationary under the risk-neutral measure, and thus threads a new path between widely-used stationary models (Cieslak and Povala, 2015) and models with unit roots under both measures (Kozicki and Tinsley, 2001).[5] The corollaries stated in the previous section no longer hold exactly, but empirically there is very close alignment.[6] In this sense, the key empirical lessons still remain intact for this case when compared with the baseline stationary setting.

Bridging three interest rate traditions The modeling of interest rates is one of the oldest questions in the field of economics. To sum up, we clarify how this paper falls between two distinct approaches, the purely macro and purely finance approaches, with some important precursors.

[5]The model is unspanned, and the yield curve does not contain all relevant information for predicting future interest rates. The Fisher constraints hold on the short rate, but forwards and excess returns now load on the trend in a different way. Unit loadings down the yield and forward curves are not implied here, a prediciton that might better fit the data.

[6]This is demonstrated by Bauer and Rudebusch (2020, see Appendix C): for forwards, at *practically relevant* horizons in the data (e.g., up to 15 years), the loadings on the trends are very close to one (and simulated confidence intervals are wide); and for excess returns, loadings on non-cyclical factors are close to zero, and second order.

The pure *finance approach* uses information purely from financial markets, with no macroeconomic variables. Here, the yields-only model is the starting point of all affine term structure models (Duffie and Kan, 1996; Piazzesi, 2010). The simplest model uses a single factor, the average level of yields, or a tent-shape weighted average (Cochrane and Piazzesi, 2005). More refined models may include 3 or 5 yield factors, using level-slope-curvature or principal components (Litterman and Scheinkman, 1991; Kim and Wright, 2005; Adrian, Crump, and Moench, 2015). In the pure form of this approach no macroeconomic information is used.

The pure *macro approach* focuses on a latent trend in the natural rate. These state-space models are grounded in the Wicksellian equation $r^* = g + z$, where the latent trend is disciplined by information on the real growth rate of the economy, and no financial market information is used (Holston, Laubach, and Williams, 2017; Laubach and Williams, 2003). To extend from this estimation approach to a latent nominal natural rate trend, one might then call upon the Fisher conditions, to solve for a nominal yield level $y = \pi^* + r^*$. Here survey expectations or a backward-looking learning model can be used to treat π^* as an observable, not latent variable.

A third way in the literature is a hybrid *macro-finance* approach and into which our paper fits. This large tradition has many different ingredients, so we must situate our paper properly, and our approach using inflation and natural rate trends overlaps with some but by no means all antecedents in this literature. Since at least Campbell and Shiller (1987), the literature has considered stochastic trends. One strand includes Ang and Piazzesi (2003) and Ludvigson and Ng (2009) who brought macro factors and latent factors into an affine term structure model, but without specifically the Wicksellian equation $r^* = g + z$. Kozicki and Tinsley (2001) argued that a "shifting endpoint" inferred from long-dated forwards delivered a marked improvement when added to a term structure model, while Cieslak and Povala (2015) located the inflation trend as one such factor. Both left a Wicksellian estimation equation of the real rate trend to one side. So too did Bauer and Rudebusch (2020), who used either external observed trend estimates of the natural rate from the macro literature, or an internally estimated natural rate not governed by the Wicksellian equation.[7]

Now our point of departure comes into focus: without *joint* estimation of a macro Wicksellian equation $r^* = g + z$ *and* the affine model of yields, other approaches must take sides. On the one hand, a yields-only finance model has an implied endpoint, but ignores information in the Wicksellian condition $r^* = g + z$ to guide its estimate. On the other hand, macro models (e.g., Holston, Laubach, and Williams, 2017; Laubach and Williams, 2003) use the Wicksellian equation but information in yields play no role. The two types of natural rate estimates must, in general, differ; symmetrically, the same disparity will manifest in implied bond risk premia too, given observable forwards and the inflation trend, as we saw earlier. As we see it, this is the heart of the natural rate puzzle, and it is this tension that we set out to resolve.

[7]Their estimated-shifting-endpoint model has no Wicksellian equation, one trend $\tau = i^* = r^* + \pi^*$, with trend estimation disciplined with priors similar to Del Negro, Giannone, Giannoni, and Tambalotti (2017).

4. Estimation and model evaluation

Formally, we proceed as follows, where from now on, we restore the stars, and denote by r_t^* the trend natural rate, and by π_t^* trend inflation. We test extracting r_t^* from average bond yields by using state-space estimation with an affine measurement equation of the form:

$$\overline{y}_t = a_y + b_\pi \pi_t^* + b_r r_t^* + \epsilon_t^{cyc}\,, \tag{26}$$

where π_t^* is trend inflation, a variable which is treated as an observable, and for each country is set equal to the Cieslak and Povala (2015) measure $\pi_t^* = (1-\nu)\sum_{i=0}^{t-1}\nu^i \pi_{t-i}$, where π_t denotes year-on-year CPI inflation reported in month t.

We further assume that the error term ϵ_{t+1}^{cyc} follows AR(1) processes of the form

$$\epsilon_{t+1}^{cyc} = \rho_y \epsilon_t^{cyc} + e_{t+1}^{cyc}\,, \qquad e_{t+1}^{cyc} \sim N(0, \sigma_{cyc}^2)\,. \tag{27}$$

Now let g_t denote trend GDP growth. We also treat this variable as observable conditional on time t, set equal to the as-of-date exogenously detrended rate of GDP growth using a Hodrick-Prescott filter. To avoid forward-looking bias, the filter is recalculated for each date in which the natural rate model is computed, so the only r_t^* estimate that contains the full sample information about realized output growth is the last point in the sample.

We then define the state variable z_t as a "headwinds" factor related to the natural rate through the state transition equation

$$r_t^* = z_t + g_t\,, \tag{28}$$

as is standard in state-space models of the natural rate, such as Laubach and Williams (2003).

Including an equation linking r_t^* to bond market data, Equation 26, and one linking r_t^* to growth, Equation 28, is a distinctive feature of our unified empirical macro-finance model, as we bring information from both financial and macroeconomic data to bear on estimating the natural rate.

Finally, we also assume that the headwinds factor follows an AR(1) process, so that

$$z_{t+1} = \rho_z z_t + e_{t+1}^z\,, \qquad e_{t+1}^z \sim N(0, \sigma_z^2)\,. \tag{29}$$

The headwinds factor is conjectured to be determined by structural phenomena with a frequency similar to that of macroeconomic series. In Appendix A we provide details on how explicit identification assumptions about ρ_z and σ_z prevent the headwinds variable from taking a high frequency similar to that of financial prices. We use priors to impose this view on variable volatility, without explicitly linking the headwinds factor to a particular set of low frequency structural data.

The Kalman system is thus defined by the following state equation,

$$\begin{pmatrix} z_t \\ \epsilon_t^{cyc} \end{pmatrix} = \begin{pmatrix} \rho_z & 0 \\ 0 & \rho_{cyc} \end{pmatrix} \begin{pmatrix} z_{t-1} \\ \epsilon_{t-1}^{cyc} \end{pmatrix} + \begin{pmatrix} e_t^z \\ e_t^{cyc} \end{pmatrix}\,. \tag{30}$$

The associated measurement equation is

$$\overline{y}_t = a_y + b_\pi \pi_t^* + b_{r^*} g_t + b_{r^*} z_t + \epsilon_t^{cyc} . \tag{31}$$

This fully describes the state-space model, which has then to be estimated. The estimation algorithm is described in Appendix A.

We experimented with the use of various sets of observation equations to extract long-run trends. In state-space models of the natural rate typically only a single measurement equation on bond yields is employed to estimate the single latent trend. This was our baseline. As a second choice, forward-looking excess bond returns might provide a second measurement equation to discipline the estimation of latent trends. We found that this variant provided little additional information and could not outperform the single bond-yield equation. Thus, in the main results below we use only the measurement equation on bond yields at Equation 26 above. [TBC]

5. Data for estimation and evaluation

Estimation and model evaluation lead us to collect bond data for 10 advanced countries for a maximal sample over the postwar period. The data requirements are as follows, and lead us to collect data over two different windows, a broad window and a narrow window.

Estimation and evaluation: narrow window using full zero-coupon curves Estimation uses the measurement affine equation $\overline{y}_t = a_y + b_\pi \pi_t^* + b_{r^*} g_t + b_{r^*} z_t + \epsilon_t^{cyc}$. In samples where we have complete information on the zero-coupon yield curve we can construct a true average yield based on 1 to 15 year maturity zero-coupon yields. We also need the more easily sourced data on the rate of output growth (g_t) and the inflation trend (π_t^*, computed via constant-gain learning),.

We refer to these samples as the *narrow window*, and to evaluate the model we calculate the fit of affine equations for yields and returns (or equations derived therefrom), $y_t^{(n)} = A_n + B_n' F_t$ and $rx_{t+1}^{(n)} = \mathfrak{B}_n' F_t + v_t^n$. Specifically we examine yields (at various maturities, 5, 10, and 15 years) as well as average excess returns (averaged over maturities 2 to 180 months, and computed with inverse-maturity weights).

Estimation and evaluation: broad window using proxy average yields Estimation uses the measurement affine equation $\overline{y}_t = a_y + b_\pi \pi_t^* + b_{r^*} g_t + b_{r^*} z_t + \epsilon_t^{cyc}$. Given data on the rate of output growth and the inflation trend, this step can still be carried through in samples where we have an acceptable proxy of average yields, and we lack the zero-coupon yield curve.

We refer to these samples as the *broad window*, where we achieve an estimate of the latent natural rate r_t^*. Also in this case to evaluate the model we can still examine the fit of the affine equations for excess returns, $rx_{t+1}^{(n)} = \mathfrak{B}_n' F_t + v_t^n$, but without the zero-coupon yield curve we need to rely on a proxy measure of excess returns as detailed below.

Whilst this broad window method may give less precise estimates of the natural rate, we find strong correlations between our proxies for yields and returns and their values computed using complete information on the zero-coupon yield curve where the methods overlap in the narrow window. Still, one virtue is that we can compute natural rates often for an extra 10 or 20 years going back to the 1950s and 1960s for the countries in our sample. A second important virtue is that the longer samples provided by this proxy method allow us to achieve more stable estimates of the latent variables by the time the starting point of the narrow window is reached, pushing back the period of initialization where the filter can produce unstable and unreliable estimates (we typically drop the first 2 years a a burn-in period in results shown below).

Data: narrow window In the narrow window samples we need complete information on the zero-coupon yield curve. We use yields at all monthly maturities from 1 to 180 months in all countries. We rely here on existing datasets by official institutions and other researchers.

We employ a Svensson (1994) model of the zero-coupon yield curve. Here a set of time-varying parameters $\beta_{0,t}, \beta_{1,t}, \beta_{2,t}, \beta_{3,t}, \tau_{1,t}$, and $\tau_{2,t}$ are estimated to express the smoothly-approximated yield $y_t^{(n)}$, at any given time t, of a maturity n zero-coupon bond as

$$y_t^{(n)} = \beta_{0,t} + \beta_{1,t}\frac{1-e^{-n/\tau_{1,t}}}{n/\tau_{1,t}} + \beta_{2,t}\left(\frac{1-e^{-n/\tau_{1,t}}}{n/\tau_{1,t}} - e^{-n/\tau_{1,t}}\right) + \beta_{3,t}\left(\frac{1-e^{-n/\tau_{2,t}}}{n/\tau_{2,t}} - e^{-n/\tau_{2,t}}\right). \quad (32)$$

Obtaining the parameters of a Svensson model lets us generate zero-coupon yields for all maturities at each point in time, circumventing the problem of data sparsity in some parts of the curve. For some countries, the datasets by official institutions and other researchers already provide the above parameters; we can then directly compute the zero-coupon yields at each date as above without any further step. In other cases, the datasets available to us consist of zero-coupon yields at a large number (but maybe not all) maturities, and possibly build from alternative models, and in these cases we simply fit a Svensson curve as a preliminary step at each date.

For the 10 countries our sources for these fitted Svensson zero-coupon yield curves are as follows:

- For the U.S., we use Svensson parameters estimated by Gürkaynak, Sack, and Wright (2007) from June 1961 to the present.
- For the U.K., we use Bank of England data on yields; this allows us to recover complete yield curves from January 1970 to the present.
- For Japan, yields come from the Ministry of Finance starting in September 1974, and comprising maturities from 1 to 40 years, in yearly maturity increments.
- For Germany, we use Svensson parameters estimated by the Bundesbank from October 1972 to the present.
- For Canada, we use of Bank of Canada data comprising estimates of yield curves for maturities ranging from 0.25 years to 30 years from January 1986 to the present.

- For Australia, we employ data from the Reserve Bank of Australia dating from July 1992 to the present, where yields are available from 0 to 10 years in quarterly maturity increments.
- For France, up to 2004 yield curves are kindly supplied by Cadorel (2022), comprising maturities from 1 to 40 years; from 2005 to the present yield curves, comprising maturities from 1 to 50 years, by Grishchenko, Moraux, and Pakulyak (2020) with subsequent updates.
- For Spain, from January 1991 to June 2019 we use Svensson parameters kindly supplied by the BIS to construct yield curves (with permission granted to use and share these derived curves).
- For Sweden, we use Svensson parameters estimated by the Riksbank from December 1992 to the present.
- For Switzerland, we use Svensson parameters estimated by the Swiss National Bank from January 1988 to the present.

To the best of our knowledge, these estimations provide a new and unique set of zero-coupon data unmatched in the literature by extending the Gürkaynak, Sack, and Wright (2007) methodology consistently to other developed markets and over many more years.[8]

Data: broad window As we have seen above, the availability of complete zero-coupon yield curves varies from country to country. We have full coverage for all 10 countries in the last three decades, since roughly the early 1990s. For most the 1970s and 1980s a handful of countries can be covered. For the 1960s, only the U.S. is available.

How then can we estimate the model and, thus, the natural rate outside these windows? As noted, we can use proxies ($\tilde{y}_t$ and $\tilde{y}_t^{(1)}$) to approximate average yields and short rates, and given those proxies, and without a curve, we can approximate average excess bond returns ($\widetilde{rx}_t = \tilde{y}_t - k\tilde{y}_{t+1} - (1-k)y_t^{(1)}$), where the average is over bonds of maturities from 2 to 15 years.[9]

We therefore rely on an array of secondary sources for proxy average yields and short rates. These will typically reference an average built from a vaguely defined basket of long bonds, or will present a time series of multiple long bond yields from which we can mechanically construct an average. Source used here include technical documents from central banks and finance ministries, or from the OECD, and aggregators such as Global Financial Data and Haver. A full description of these sources can be found in the online appendix [TBD].

[8]The closest prior work was a decade ago. Wright (2011) compiled a 10-country panel of zero-coupon yields with the data series ending in 2009, using Svensson, Nelson-Siegel, and spline models.

[9]Consider an inverse-duration weight basket of bonds at annual maturity increments $n = 2, \ldots, 15$. The forward excess return of the maturity n bond is $ny_t^{(n)} - (n-1)y_{t+1}^{(n-1)} - y_t^{(1)}$. Applying a weight $1/n$ and summing over all 14 bonds we obtain a weighted average return $\overline{rx}_t = \frac{1}{14}\sum_{n=2}^{15} y_t^{(n)} - \frac{1}{14}\sum_{n=2}^{15}\frac{(n-1)}{n}y_{t+1}^{(n-1)} - \frac{1}{14}\sum_{n=2}^{15}\frac{1}{n}y_t^{(1)}$. We take the approximation to this given by $\widetilde{rx}_t = \tilde{y}_t - \left(\frac{1}{14}\sum_{n=2}^{15}\frac{(n-1)}{n}\right)\tilde{y}_{t+1} - \left(\frac{1}{14}\sum_{n=2}^{15}\frac{1}{n}\right)y_t^{(1)}$. Now define $k \equiv \left(\frac{1}{14}\sum_{n=2}^{15}\frac{(n-1)}{n}\right)$ and we can write $\widetilde{rx}_t = \tilde{y}_t - k\tilde{y}_{t+1} - (1-k)y_t^{(1)}$.

6. Estimates and fit of yield and return equations

6.1. Bond yield regressions

In this subsection, we now apply the affine bond pricing model with OLS regressions based on Equation 11, $y_t^{(n)} = \mathcal{A}_n + \mathcal{B}_n^r r_t^* + \mathcal{B}_n^\pi \pi_t^* + \mathcal{B}^{\bar{c}} \bar{c}_t$. This is the specification in Cieslak and Povala (2015), but with a second trend for the estimated natural rate. The detrended average yield then represents a cyclical fluctuation of bond yields (or prices) about the long run trends. We could also run this regression in equivalent form with $\bar{c}_t$ replaced by $\bar{y}_t$, but this would only produce different coefficients requiring a different interpretation, but the fit would be the same.

Our findings confirm and extend those of Cieslak and Povala (2015), in two ways. First, the inclusion of the inflation trend improves the fit of the yield model relative to a specification with only one yield factor and no trends. Second, this is true for all 10 advanced economies and not just the U.S., and notably, the fit improvement is most marked at the longer end of the curve. Third, the inclusion of the natural rate trend improves the fit even more, confirming that both trends are relevant for bond pricing, as in Bauer and Rudebusch (2020).

Table 1 presents full-sample OLS yield regressions at the 5-year maturity point. Panel (a) uses the average yield as the only factor. Panel (b) uses the average yield and the inflation trend only, where the average yield is first detrended. Panel (c) uses the average yield and both trends, where the average yield is first detrended. In this case, at the shorter end of the curve, the improvements in fit are small as we move from the first specification to the second and then the third. And, as is generally seen in the literature, the fit is always very close to an R^2 of 1.

Table 2 presents OLS yield regressions at the 10-year maturity point and Table 3 presents OLS yield regressions at the 15-year maturity point. Here, the improvements in fit are a little more evident, but we start from a very high baseline fit in the yields-only specification so these increments can only be quite small.

Of greater interest, as highlighted by Cieslak and Povala (2015), is the role of the trends versus the cyclical detrended yield term in accounting for the fit. This is shown in Table 4 and the result is striking. Here the R^2 measures are typically around 90% of those seen in the previous tables, indicating that when it comes to explaining the level of yields, the macro trends are doing almost all of the work, and that the orthogonal component of yields $\bar{c}$ only accounts for around 10% of the fit. Indeed, complementary regressions (not shown) including only the detrended yields $\bar{c}$ show this to be the case.

Thus, what Cieslak and Povala (2015) and Bauer and Rudebusch (2020) found for the U.S. alone is here confirmed around the world in a sample of 10 major advanced economies, and the level of yields is primarily attributable to the slow moving macro trends in inflation and the natural rate.

Table 1: *International yields, 5-year maturity*

The tables reports OLS estimates on international monthly data of yields $y_t^{(n)} = \tilde{\mathcal{A}}_n + \tilde{\mathcal{B}}^{\bar{c}} \bar{c}_t$. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. The sample varies by country.

(a) Narrow window. Detrending: none.

	(1) AUS	(2) CAN	(3) CHE	(4) DEU	(5) ESP	(6) FRA	(7) GBR	(8) JPN	(9) SWE	(10) USA
$\bar{y}$	1.058***	1.010***	1.023***	1.041***	1.062***	0.989***	1.011***	1.024***	1.011***	1.043***
	(0.003)	(0.002)	(0.003)	(0.002)	(0.003)	(0.005)	(0.003)	(0.002)	(0.003)	(0.002)
Constant	-0.003***	-0.003***	-0.003***	-0.004***	-0.006***	-0.003***	-0.002***	-0.003***	-0.003***	-0.005***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
N	360	438	414	597	342	721	630	559	355	640
R^2	0.997	0.998	0.997	0.997	0.998	0.983	0.993	0.997	0.998	0.997
RSS	0.0007	0.0009	0.0005	0.0020	0.0008	0.0136	0.0071	0.0016	0.0007	0.0022

(b) Narrow window. Detrending: inflation only, $\bar{c}$ equal to projection of $\bar{y}$ on π.

	(1) AUS	(2) CAN	(3) CHE	(4) DEU	(5) ESP	(6) FRA	(7) GBR	(8) JPN	(9) SWE	(10) USA
π	2.267***	1.688***	1.671***	1.979***	1.809***	0.997***	0.939***	0.903***	1.626***	1.467***
	(0.009)	(0.005)	(0.005)	(0.006)	(0.005)	(0.005)	(0.003)	(0.002)	(0.004)	(0.004)
$\bar{c}$	1.059***	1.019***	1.042***	1.039***	1.052***	1.006***	1.045***	1.003***	1.054***	1.068***
	(0.005)	(0.004)	(0.007)	(0.004)	(0.009)	(0.011)	(0.006)	(0.005)	(0.004)	(0.005)
Constant	-0.015***	0.000*	-0.002***	-0.003***	-0.015***	0.010***	0.018***	0.010***	0.001***	-0.000*
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
N	360	438	414	597	342	721	630	559	355	640
R^2	0.997	0.998	0.997	0.997	0.998	0.984	0.994	0.997	0.998	0.997
RSS	0.0007	0.0009	0.0005	0.0020	0.0008	0.0135	0.0067	0.0016	0.0004	0.0021

(c) Narrow window. Detrending: inflation and natural rate, $\bar{c}$ equal to projection of $\bar{y}$ on π^* and r^* .

	(1) AUS	(2) CAN	(3) CHE	(4) DEU	(5) ESP	(6) FRA	(7) GBR	(8) JPN	(9) SWE	(10) USA
π^*	1.074***	1.051***	1.405***	1.001***	2.003***	0.267***	0.625***	0.408***	1.151***	1.118***
	(0.011)	(0.005)	(0.005)	(0.007)	(0.007)	(0.010)	(0.004)	(0.004)	(0.005)	(0.004)
r^*	1.366***	1.237***	1.003***	1.432***	-0.316***	1.525***	1.392***	0.905***	1.299***	1.128***
	(0.007)	(0.006)	(0.011)	(0.006)	(0.009)	(0.018)	(0.010)	(0.005)	(0.006)	(0.008)
$\bar{c}$	1.023***	1.098***	1.092***	1.104***	1.057***	1.064***	0.991***	1.043***	1.078***	1.083***
	(0.010)	(0.009)	(0.009)	(0.008)	(0.009)	(0.024)	(0.011)	(0.009)	(0.007)	(0.008)
Constant	-0.008***	-0.006***	-0.005***	-0.004***	-0.015***	-0.006***	-0.008***	-0.007***	-0.005***	-0.006***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
N	360	438	414	597	342	721	630	559	355	640
R^2	0.997	0.998	0.998	0.997	0.998	0.984	0.994	0.997	0.998	0.997
RSS	0.0007	0.0008	0.0004	0.0018	0.0008	0.0134	0.0064	0.0015	0.0004	0.0021

Table 2: *International yields, 10-year maturity*

The tables reports OLS estimates on international monthly data of yields $y_t^{(n)} = \tilde{\mathcal{A}}_n + \tilde{\mathcal{B}}^{\bar{c}} \bar{c}_t$. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. The sample varies by country.

(a) Narrow window. Detrending: none.

	(1) AUS	(2) CAN	(3) CHE	(4) DEU	(5) ESP	(6) FRA	(7) GBR	(8) JPN	(9) SWE	(10) USA
$\bar{y}$	1.000***	0.989***	0.966***	0.985***	0.949***	0.991***	0.997***	0.986***	1.006***	0.960***
	(0.004)	(0.003)	(0.004)	(0.003)	(0.004)	(0.004)	(0.003)	(0.002)	(0.003)	(0.003)
Constant	0.002***	0.003***	0.003***	0.004***	0.006***	0.003***	0.002***	0.003***	0.003***	0.005***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
N	360	438	414	597	342	721	630	559	355	640
R^2	0.995	0.995	0.992	0.994	0.995	0.986	0.995	0.996	0.997	0.993
RSS	0.0009	0.0020	0.0014	0.0036	0.0015	0.0114	0.0057	0.0017	0.0008	0.0040

(b) Narrow window. Detrending: inflation only, $\bar{c}$ equal to projection of $\bar{y}$ on π.

	(1) AUS	(2) CAN	(3) CHE	(4) DEU	(5) ESP	(6) FRA	(7) GBR	(8) JPN	(9) SWE	(10) USA
π^*	2.188***	1.646***	1.592***	1.889***	1.598***	1.010***	0.950***	0.861***	1.656***	1.365***
	(0.009)	(0.007)	(0.008)	(0.008)	(0.006)	(0.005)	(0.003)	(0.002)	(0.005)	(0.005)
$\bar{c}$	0.971***	1.009***	0.928***	0.968***	1.032***	0.968***	0.955***	0.994***	0.996***	0.933***
	(0.005)	(0.006)	(0.012)	(0.005)	(0.011)	(0.010)	(0.006)	(0.005)	(0.005)	(0.007)
Constant	-0.010***	0.006***	0.004***	0.004***	-0.002***	0.016***	0.021***	0.015***	0.005***	0.008***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
N	360	438	414	597	342	721	630	559	355	640
R^2	0.996	0.995	0.992	0.994	0.996	0.986	0.995	0.997	0.997	0.993
RSS	0.0008	0.0019	0.0013	0.0035	0.0012	0.0113	0.0051	0.0017	0.0007	0.0039

(c) Narrow window. Detrending: inflation and natural rate, $\bar{c}$ equal to projection of $\bar{y}$ on π^* and r^*.

	(1) AUS	(2) CAN	(3) CHE	(4) DEU	(5) ESP	(6) FRA	(7) GBR	(8) JPN	(9) SWE	(10) USA
π^*	1.098***	0.986***	1.287***	0.936***	1.801***	0.290***	0.673***	0.348***	1.199***	1.043***
	(0.011)	(0.008)	(0.007)	(0.008)	(0.009)	(0.009)	(0.004)	(0.003)	(0.006)	(0.006)
r^*	1.248***	1.281***	1.149***	1.395***	-0.331***	1.504***	1.223***	0.939***	1.249***	1.039***
	(0.008)	(0.008)	(0.015)	(0.007)	(0.011)	(0.016)	(0.009)	(0.005)	(0.007)	(0.010)
$\bar{c}$	0.950***	0.915***	0.809***	0.851***	1.031***	0.926***	0.980***	0.908***	0.970***	0.884***
	(0.011)	(0.013)	(0.012)	(0.010)	(0.011)	(0.022)	(0.010)	(0.009)	(0.010)	(0.011)
Constant	-0.004***	-0.000	0.000	0.003***	-0.001***	-0.001	-0.002***	-0.003***	-0.000	0.003***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
N	360	438	414	597	342	721	630	559	355	640
R^2	0.996	0.996	0.995	0.995	0.996	0.986	0.995	0.997	0.997	0.994
RSS	0.0008	0.0017	0.0008	0.0027	0.0012	0.0112	0.0050	0.0014	0.0007	0.0037

Table 3: *International yields, 15-year maturity*

The tables reports OLS estimates on international monthly data of yields $y_t^{(n)} = \tilde{\mathcal{A}}_n + \tilde{\mathcal{B}}^{\bar{c}} \bar{c}_t$. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. The sample varies by country.

(a) Narrow window. Detrending: none.

	(1) AUS	(2) CAN	(3) CHE	(4) DEU	(5) ESP	(6) FRA	(7) GBR	(8) JPN	(9) SWE	(10) USA
$\bar{y}$	0.923***	0.966***	0.938***	0.936***	0.875***	0.982***	0.979***	0.949***	0.989***	0.918***
	(0.007)	(0.006)	(0.007)	(0.006)	(0.006)	(0.008)	(0.008)	(0.005)	(0.005)	(0.005)
Constant	0.006***	0.007***	0.006***	0.008***	0.013***	0.006***	0.005***	0.006***	0.006***	0.010***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.000)	(0.000)	(0.000)
N	360	438	414	597	342	721	630	559	355	640
R^2	0.983	0.984	0.974	0.978	0.987	0.950	0.963	0.982	0.990	0.979
RSS	0.0031	0.0058	0.0041	0.0114	0.0032	0.0416	0.0388	0.0082	0.0026	0.0114

(b) Narrow window. Detrending: inflation only, $\bar{c}$ equal to projection of $\bar{y}$ on π.

	(1) AUS	(2) CAN	(3) CHE	(4) DEU	(5) ESP	(6) FRA	(7) GBR	(8) JPN	(9) SWE	(10) USA
π^*	2.023***	1.613***	1.551***	1.801***	1.469***	1.001***	0.945***	0.819***	1.653***	1.316***
	(0.018)	(0.012)	(0.013)	(0.014)	(0.009)	(0.009)	(0.008)	(0.005)	(0.010)	(0.008)
$\bar{c}$	0.893***	0.975***	0.881***	0.917***	0.977***	0.957***	0.898***	0.995***	0.942***	0.857***
	(0.010)	(0.011)	(0.020)	(0.009)	(0.017)	(0.019)	(0.015)	(0.011)	(0.009)	(0.012)
Constant	-0.005***	0.010***	0.007***	0.009***	0.006***	0.019***	0.023***	0.018***	0.008***	0.013***
	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)
N	360	438	414	597	342	721	630	559	355	640
R^2	0.983	0.984	0.975	0.978	0.988	0.951	0.965	0.983	0.991	0.980
RSS	0.0029	0.0058	0.0040	0.0112	0.0029	0.0415	0.0366	0.0078	0.0024	0.0109

(c) Narrow window. Detrending: inflation and natural rate, $\bar{c}$ equal to projection of $\bar{y}$ on π^* and r^* .

	(1) AUS	(2) CAN	(3) CHE	(4) DEU	(5) ESP	(6) FRA	(7) GBR	(8) JPN	(9) SWE	(10) USA
π^*	1.046***	0.954***	1.217***	0.875***	1.671***	0.283***	0.696***	0.303***	1.219***	1.010***
	(0.022)	(0.012)	(0.012)	(0.015)	(0.014)	(0.018)	(0.010)	(0.008)	(0.011)	(0.010)
r^*	1.119***	1.279***	1.263***	1.356***	-0.329***	1.499***	1.101***	0.943***	1.186***	0.988***
	(0.014)	(0.014)	(0.025)	(0.013)	(0.017)	(0.032)	(0.024)	(0.011)	(0.013)	(0.017)
$\bar{c}$	0.953***	0.760***	0.657***	0.708***	0.970***	0.884***	0.997***	0.900***	0.909***	0.773***
	(0.021)	(0.022)	(0.019)	(0.018)	(0.017)	(0.042)	(0.026)	(0.021)	(0.017)	(0.018)
Constant	0.001	0.003***	0.003***	0.008***	0.006***	0.003***	0.002**	0.001*	0.003***	0.008***
	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.000)	(0.000)	(0.000)
N	360	438	414	597	342	721	630	559	355	640
R^2	0.984	0.988	0.986	0.983	0.988	0.951	0.966	0.984	0.991	0.981
RSS	0.0029	0.0045	0.0022	0.0088	0.0029	0.0413	0.0353	0.0075	0.0023	0.0103

Table 4: *International yields, contribution of macro trends alone*

The tables reports OLS estimates on international monthly data of yields $y_t^{(n)} = \tilde{\mathcal{A}}_n + \tilde{\mathcal{B}}^{\bar{c}} \bar{c}_t$. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. The sample varies by country.

(a) Narrow window. 5-year yields regressed on π^* and r^* alone

	(1) AUS	(2) CAN	(3) CHE	(4) DEU	(5) ESP	(6) FRA	(7) GBR	(8) JPN	(9) SWE	(10) USA
π^*	1.074***	1.051***	1.405***	1.001***	2.003***	0.267***	0.625***	0.408***	1.151***	1.118***
	(0.056)	(0.031)	(0.033)	(0.038)	(0.047)	(0.020)	(0.015)	(0.018)	(0.036)	(0.023)
r^*	1.366***	1.237***	1.003***	1.432***	-0.316***	1.525***	1.392***	0.905***	1.299***	1.128***
	(0.037)	(0.034)	(0.069)	(0.032)	(0.058)	(0.035)	(0.038)	(0.025)	(0.044)	(0.042)
Constant	-0.008***	-0.006***	-0.005***	-0.004***	-0.015***	-0.006***	-0.008***	-0.007***	-0.005***	-0.006***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
N	360	438	414	597	342	721	630	559	355	640
R^2	0.914	0.930	0.906	0.911	0.908	0.939	0.916	0.932	0.906	0.911
rss	0.0195	0.0275	0.0177	0.0552	0.0324	0.0504	0.0906	0.0358	0.0260	0.0607

(b) Narrow window. 10-year yields regressed on π^* and r^* alone

	(1) AUS	(2) CAN	(3) CHE	(4) DEU	(5) ESP	(6) FRA	(7) GBR	(8) JPN	(9) SWE	(10) USA
π^*	1.098***	0.986***	1.287***	0.936***	1.801***	0.290***	0.673***	0.348***	1.199***	1.043***
	(0.053)	(0.026)	(0.025)	(0.030)	(0.047)	(0.017)	(0.015)	(0.016)	(0.033)	(0.020)
r^*	1.248***	1.281***	1.149***	1.395***	-0.331***	1.504***	1.223***	0.939***	1.249***	1.039***
	(0.035)	(0.029)	(0.053)	(0.025)	(0.057)	(0.031)	(0.037)	(0.022)	(0.040)	(0.035)
Constant	-0.004**	-0.000	0.000	0.003***	-0.001	-0.001	-0.002	-0.003***	-0.000	0.003***
	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
N	360	438	414	597	342	721	630	559	355	640
R^2	0.916	0.946	0.939	0.938	0.890	0.953	0.916	0.943	0.922	0.926
rss	0.0171	0.0203	0.0103	0.0345	0.0312	0.0392	0.0874	0.0274	0.0215	0.0427

(c) Narrow window. 15-year yields regressed on π^* and r^* alone

	(1) AUS	(2) CAN	(3) CHE	(4) DEU	(5) ESP	(6) FRA	(7) GBR	(8) JPN	(9) SWE	(10) USA
π^*	1.046***	0.954***	1.217***	0.875***	1.671***	0.283***	0.696***	0.303***	1.219***	1.010***
	(0.056)	(0.024)	(0.023)	(0.028)	(0.045)	(0.023)	(0.018)	(0.017)	(0.032)	(0.019)
r^*	1.119***	1.279***	1.263***	1.356***	-0.329***	1.499***	1.101***	0.943***	1.186***	0.988***
	(0.037)	(0.027)	(0.048)	(0.024)	(0.055)	(0.040)	(0.043)	(0.024)	(0.039)	(0.034)
Constant	0.001	0.003***	0.003***	0.008***	0.006***	0.003***	0.002	0.001	0.003***	0.008***
	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
N	360	438	414	597	342	721	630	559	355	640
R^2	0.890	0.952	0.947	0.940	0.879	0.920	0.884	0.928	0.923	0.925
rss	0.0192	0.0174	0.0085	0.0308	0.0295	0.0669	0.1206	0.0330	0.0205	0.0401

6.2. Excess return regressions

We just saw that accounting for trends can make some improvements in modeling yields, but we now see how they matter a great deal for predicting bond returns. This was shown for the U.S. case in Cieslak and Povala (2015) with just an inflation trend extracted, and also in the contemporaneous work of Bauer and Rudebusch (2020) with trends for inflation and the real natural rate, or a nominal natural rate trend. We show that the same applies more generally at the international level.

The intuition is quite straightforward. The macro trend factors, being slow moving and near unit-root, are mainly priced in one-period ahead and, being so highly persistent, they contain little new useful information about short-run returns. In contrast, the cyclical factor, being the more volatile driver of the high-frequency error-correction part of the bond price process, is very much more informative about how bond prices revert to trend in the short run.

Formally, in this section we will be presenting estimates for the excess return Equation 13, $rx_{t+1}^{(n)} = \mathfrak{B}_n^\top F_t + v_t^n$. These one-step ahead predictions are noisy but we will see that their explanatory power is almost entirely due to the role of the detrended, or cyclical, yield factor $\bar{c}$, inline with our intuition.

Table 5 present results for the excess return forecasts in the narrow window sample. Here we have full zero-coupon yield curves and do not have to rely on proxy yields or proxy returns. Note that these are in-sample regressions and, whilst illustrative, they should not be judged the same way as real-time out-of-sample forecasts.

As is clear, when we move from a yields only model in panel (a), to the model with the inflation trend in panel (b) and then the two trends in panel (c), the fit of the excess return forecasts improves dramatically. For the U.S., the R^2 rises from 0.004, to 0.128, to 0.204. Similar large improvements are seen across all 10 countries, though from different baselines.

Table 6 recasts the regressions with two trends in panel (a), but here replacing the detrended yield $\bar{c}$ with the average yield itself $\bar{y}$. This allows a different interpretation and a sense check. Holding fixed the average level of yields we would expect increases in either macro trend (higher inflation or higher natural rates) to be associated with lower returns (bond prices adjusting down) going forward, and this intuition is confirmed, and all 20 trend coefficients are negative in this panel. Panel (b) allows for a different check, where we recasts the regressions with two trends but now exclude the average yield term. his amounts to omitting the detrending, and our intuition above was that this should destroy the fit of the forecast in the volatile cyclical term is the main source of predictive power, and this is indeed the case. The fit here is 80%-90% attenuated relative to the fit achieved when the cyclical term is present.

Table 7 now expands the sample to the broad window, to see if similar success can be achieved where we make use of yield and return proxies in periods when the full curve is not available. Despite the noisier data, the fit of the full model in panel (a) remains good even for countries where we have large spans of proxy information. And in panel (b) the intuition about detrending is preserved, and the macro trends alone account for only a small share of the explanatory power.

Table 5: *International excess returns, weighted-average portfolio*

The table reports OLS estimates on international data of the excess return equation $rx_{t+1}^{(n)} = \mathfrak{B}_n^\top F_t + v_t^n$. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. The sample varies by country.

(a) Narrow window. Detrending: none.

	(1) AUS	(2) CAN	(3) CHE	(4) DEU	(5) ESP	(6) FRA	(7) GBR	(8) JPN	(9) SWE	(10) USA
$\bar{y}$	0.112***	0.031**	0.013	0.025*	0.082***	-0.013	0.019	0.035***	0.097***	0.021
	(0.019)	(0.012)	(0.016)	(0.011)	(0.019)	(0.010)	(0.010)	(0.009)	(0.017)	(0.013)
Constant	-0.002*	0.002**	0.002***	0.002***	0.001	0.002*	0.001	0.002***	0.001	0.002
	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)
N	348	426	403	586	330	693	618	547	345	629
R^2	0.090	0.016	0.002	0.008	0.053	0.002	0.006	0.027	0.089	0.004
RSS	0.0241	0.0221	0.0172	0.0402	0.0349	0.0600	0.0636	0.0223	0.0256	0.0624

(b) Narrow window. Detrending: inflation only, $\bar{c}$ equal to projection of $\bar{y}$ on π.

	(1) AUS	(2) CAN	(3) CHE	(4) DEU	(5) ESP	(6) FRA	(7) GBR	(8) JPN	(9) SWE	(10) USA
π^*	0.211***	-0.045*	-0.015	0.042	0.025	-0.053***	0.008	0.005	0.059	-0.042*
	(0.053)	(0.021)	(0.027)	(0.027)	(0.028)	(0.011)	(0.011)	(0.009)	(0.031)	(0.019)
$\bar{c}$	0.132***	0.189***	0.152***	0.029	0.654***	0.150***	0.055**	0.132***	0.251***	0.263***
	(0.029)	(0.021)	(0.042)	(0.018)	(0.048)	(0.022)	(0.020)	(0.018)	(0.030)	(0.028)
Constant	-0.003	0.005***	0.002***	0.002**	0.005***	0.003***	0.002**	0.003***	0.003***	0.004***
	(0.002)	(0.001)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)
N	348	426	403	586	330	693	618	547	345	629
R^2	0.092	0.174	0.033	0.008	0.360	0.091	0.012	0.089	0.180	0.128
RSS	0.0241	0.0186	0.0166	0.0402	0.0236	0.0546	0.0632	0.0209	0.0231	0.0546

(c) Narrow window. Detrending: inflation and natural rate, $\bar{c}$ equal to projection of $\bar{y}$ on π^* and r^*.

	(1) AUS	(2) CAN	(3) CHE	(4) DEU	(5) ESP	(6) FRA	(7) GBR	(8) JPN	(9) SWE	(10) USA
π^*	0.230***	-0.112***	-0.041	0.072*	0.035	-0.080***	0.025*	-0.024	0.016	-0.068**
	(0.053)	(0.024)	(0.032)	(0.032)	(0.041)	(0.020)	(0.012)	(0.013)	(0.031)	(0.021)
r^*	-0.050	0.124***	0.090	-0.048	-0.013	0.053	-0.086**	0.052**	0.109**	0.079*
	(0.036)	(0.027)	(0.071)	(0.028)	(0.049)	(0.036)	(0.031)	(0.019)	(0.039)	(0.039)
$\bar{c}$	0.718***	0.503***	0.194***	0.262***	0.716***	0.591***	0.281***	0.330***	0.650***	0.491***
	(0.053)	(0.042)	(0.054)	(0.038)	(0.049)	(0.046)	(0.033)	(0.033)	(0.050)	(0.040)
Constant	-0.002	0.004***	0.002***	0.003***	0.005***	0.003***	0.004***	0.002***	0.003***	0.004***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)
N	348	426	403	586	330	693	618	547	345	629
R^2	0.371	0.295	0.036	0.082	0.392	0.221	0.116	0.163	0.352	0.204
RSS	0.0167	0.0158	0.0166	0.0372	0.0224	0.0468	0.0566	0.0192	0.0182	0.0499

Table 6: *International excess returns, additional results, narrow window*

The table reports OLS estimates on international data of the excess return equation $rx_{t+1}^{(n)} = \mathfrak{B}_n^\top F_t + v_t^n$. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. The sample varies by country.

(c) Narrow window. Detrending: inflation and natural rate, without projecting $\bar{y}$.

	(1) AUS	(2) CAN	(3) CHE	(4) DEU	(5) ESP	(6) FRA	(7) GBR	(8) JPN	(9) SWE	(10) USA
π^*	-0.510***	-0.635***	-0.305***	-0.175***	-1.322***	-0.244***	-0.156***	-0.148***	-0.754***	-0.599***
	(0.076)	(0.049)	(0.081)	(0.048)	(0.102)	(0.023)	(0.024)	(0.018)	(0.066)	(0.048)
r^*	-0.968***	-0.500***	-0.111	-0.414***	0.214***	-0.856***	-0.451***	-0.250***	-0.699***	-0.446***
	(0.078)	(0.059)	(0.092)	(0.060)	(0.052)	(0.080)	(0.053)	(0.036)	(0.073)	(0.058)
$\bar{y}$	0.718***	0.503***	0.194***	0.262***	0.716***	0.591***	0.281***	0.330***	0.650***	0.491***
	(0.053)	(0.042)	(0.054)	(0.038)	(0.049)	(0.046)	(0.033)	(0.033)	(0.050)	(0.040)
Constant	0.001	0.006***	0.002***	0.003***	0.010***	0.005***	0.005***	0.004***	0.004***	0.005***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
N	348	426	403	586	330	693	618	547	345	629
R^2	0.371	0.295	0.036	0.082	0.392	0.221	0.116	0.163	0.352	0.204
RSS	0.0167	0.0158	0.0166	0.0372	0.0224	0.0468	0.0566	0.0192	0.0182	0.0499

(b) Narrow window. Macro trends only.

	(1) AUS	(2) CAN	(3) CHE	(4) DEU	(5) ESP	(6) FRA	(7) GBR	(8) JPN	(9) SWE	(10) USA
pistar	0.230***	-0.115***	-0.041	0.071*	0.032	-0.083***	0.025	-0.025	0.014	-0.068**
	(0.065)	(0.028)	(0.033)	(0.033)	(0.052)	(0.022)	(0.013)	(0.014)	(0.038)	(0.024)
rstar	-0.032	0.132***	0.101	-0.046	-0.015	0.061	-0.082*	0.054**	0.117*	0.076
	(0.044)	(0.031)	(0.072)	(0.029)	(0.063)	(0.040)	(0.033)	(0.020)	(0.048)	(0.044)
_cons	-0.002	0.004***	0.002***	0.003***	0.005***	0.003***	0.004***	0.002***	0.003**	0.004***
	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
N	348	426	403	586	330	693	618	547	345	629
R^2	0.040	0.050	0.006	0.008	0.001	0.034	0.011	0.013	0.025	0.013
rss	0.0254	0.0213	0.0171	0.0402	0.0368	0.0580	0.0633	0.0226	0.0274	0.0619

Table 7: *International excess returns, additional results, broad window*

The table reports OLS estimates on international data of the excess return equation $rx_{t+1}^{(n)} = \mathfrak{B}_n^\top F_t + v_t^n$. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. The sample varies by country.

(c) Broad window. Detrending: inflation and natural rate, without projecting $\bar{y}$.

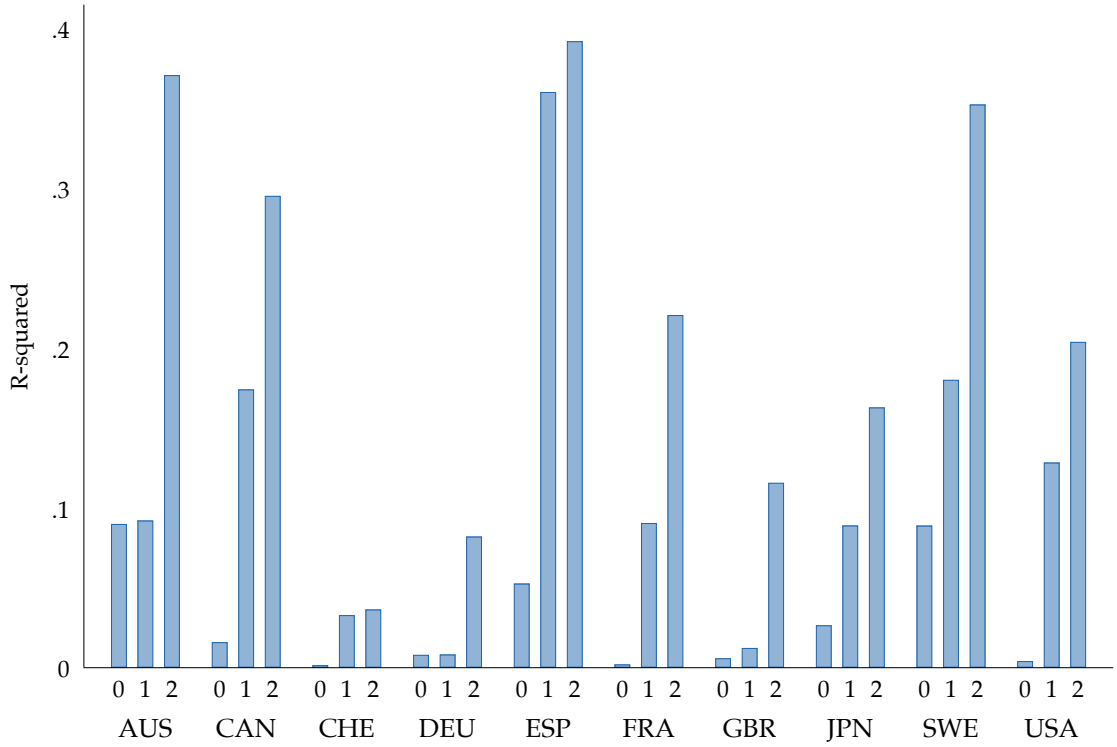
	(1) AUS	(2) CAN	(3) CHE	(4) DEU	(5) ESP	(6) FRA	(7) GBR	(8) JPN	(9) SWE	(10) USA
π^*	-0.414***	-0.503***	1.446***	-0.179***	-0.430***	-0.251***	-0.168***	-0.151***	1.147***	-0.614***
	(0.043)	(0.038)	(0.218)	(0.047)	(0.035)	(0.023)	(0.024)	(0.018)	(0.213)	(0.048)
r^*	-0.298***	-0.606***	-0.076	-0.525***	-0.154**	-0.850***	-0.452***	-0.258***	-0.798**	-0.467***
	(0.067)	(0.058)	(0.250)	(0.060)	(0.049)	(0.079)	(0.053)	(0.036)	(0.262)	(0.058)
ybaralt	0.338***	0.482***	-1.161***	0.298***	0.398***	0.593***	0.292***	0.338***	-0.631***	0.505***
	(0.037)	(0.036)	(0.160)	(0.038)	(0.034)	(0.046)	(0.033)	(0.034)	(0.159)	(0.040)
Constant	0.003*	0.004***	-0.010***	0.002**	0.004**	0.005***	0.005***	0.004***	-0.019***	0.005***
	(0.001)	(0.001)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.003)	(0.001)
N	628	736	628	628	508	712	628	547	628	628
R^2	0.154	0.205	0.212	0.111	0.231	0.223	0.119	0.168	0.199	0.214
rss	0.0673	0.0628	0.4253	0.0407	0.0635	0.0477	0.0570	0.0191	0.9022	0.0490

(b) Broad window. Macro trends only.

	(1) AUS	(2) CAN	(3) CHE	(4) DEU	(5) ESP	(6) FRA	(7) GBR	(8) JPN	(9) SWE	(10) USA
π^*	-0.041**	-0.034	-0.061	0.100**	-0.043**	-0.090***	0.019	-0.025	0.328***	-0.068**
	(0.015)	(0.018)	(0.070)	(0.033)	(0.013)	(0.022)	(0.013)	(0.014)	(0.050)	(0.024)
r^*	0.195***	0.028	-1.511***	-0.103***	0.115*	0.072	-0.064*	0.054**	-1.588***	0.072
	(0.042)	(0.038)	(0.159)	(0.027)	(0.048)	(0.039)	(0.032)	(0.020)	(0.173)	(0.044)
Constant	0.000	0.002**	-0.006**	0.002**	0.003**	0.002***	0.004***	0.002***	-0.017***	0.004***
	(0.001)	(0.001)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.003)	(0.001)
N	628	736	628	628	508	712	628	547	628	628
R^2	0.041	0.005	0.145	0.024	0.024	0.038	0.007	0.013	0.179	0.013
rss	0.0763	0.0785	0.4612	0.0446	0.0806	0.0591	0.0643	0.0226	0.9249	0.0615

Figure 4: *Measure of fit of excess return regressions with 0, 1, and 2 macro trends*

The chart displays R^2 values for the excess returns regressions by country. 0 denotes the model with no trend, 1 the model with π^* trend, and 2 the model with π^* and r^* trends.



6.3. Summary

We developed a macro-finance model with real and nominal factors in which bond yields and excess returns are explained by two slow-moving latent trend factors, the real natural rate r^* and inflation π^*, in an arbitrage-free affine term structure model. Empirically, we take the model to the data in 10 advanced economies using joint state-space estimation and the Kalman filter. By construction our estimates give model-consistent paths of bond risk premia, the natural rate, inflation, and forward rates, averting the natural rate puzzle.

The model succeeds on multiple dimensions. The pricing regressions for yields improve somewhat and estimates of excess returns are far more accurate than when one or both macro factors are excluded, in and out of sample, as shown in Figure 4. In all cases the model with 0 trends has a poor fit and adding the trends makes a substantial contribution. In only one case (CHE) is the inflation trend on its own enough to generate close to maximal fit, and this may just reflect the small trend in natural rates for that case alone. In the other 9 cases both the inflation and the natural rate trends make a significant impact. Without accounting for these trends, investors would be misguided as the path of rates.

7. Natural rate trends in 10 advanced economies

We now turn to the model estimates of the natural rate, the latent r^* time series which we obtain for all 10 countries. Associated with this trend, we can also plot the values of trend growth g, and the headwinds term z. These estimates are displayed in Figure 5.

Again we stress that prior work has not been able to generate a natural rate estimate for as many countries over as long a period in the postwar period, but we can still discuss the relevance and plausibility of these estimates as compared to prior consensus views and also by direct comparisons in places of overlap with other estimates in Figure 6.

The top panel in Figure 5 shows the natural rate estimates, and we can make several remarks here. First, the natural rate has in general been falling over time for all countries, although not monotonically, and some reversals are larger than others. This finding is consistent with previous historical works documenting the long-run decline of real rates over the long to very-long run (Jordà, Knoll, Kuvshinov, Schularick, and Taylor, 2019; Rogoff, Rossi, and Schmelzing, 2022). As recently as the 1990s, natural rates in these countries were between 100 and 600 bps. At the end of the sample 8 out of 10 countries have a negative r^*, and the other 2 are between 0 and 50 bps.

At higher frequency we do however see a general shift upwards in the natural rate across all countries in the 1970–1990 period, even if the timing varies from one country to the next. And perhaps the most dramatic coherence across countries is seen in the sharp downward trend in r^* that is set in motion around the year 2000. This was previously noted for one or two countries, starting with the U.S., and various explanations have been offered for that phenomenon, including not just slower growth but also demographics, the EM savings glut, the falling price of investment, among others (Rachel and Smith, 2017; Del Negro, Giannone, Giannoni, and Tambalotti, 2017; Rachel and Summers, 2019; Cesa-Bianchi, Harrison, and Sajedi, 2022). What we see here is how broadly that phenomenon was experienced across a wide swath of advanced economies.

Lastly, we see that the dispersion of natural rates across countries also falls over time At the start of each country's sample period, in the 1970s and 1980s, the natural rate estimates range widely between about 100 and 800 bps. But by the end of the sample in all 10 countries have a natural rate sit in a narrower interval between -100 and 50 bps. This would be consistent, mechanically, with convergence in either the g or z terms — which we discuss in a moment — and also aligns with the idea that under common forces (of demography or via globalization in technology or finance), in the long run, countries may be under the sway of a common global factor in r^* (Clarida, 2019).

The middle panel in Figure 5 shows the component of the trend in natural rates attributable to the growth rate g. We can see two immediate implications from this chart. First, in line with common wisdom, trend growth rates have been declining inexorably during the postwar decades, with rare reversals. They have also tended to converge, but started to do so earlier than any convergence in natural rates. Thus, whilst some of the variation in the country-level natural rates r^* across space and time can be explained by g, this is by no means the entire story and so much work is left for the residual.

Figure 5: *Trends in r^*, g, and z in international data, our estimates*

The top chart displays our estimates of the natural rate r^*, the middle chart trend growth g, and the bottom chart the headwinds term z.

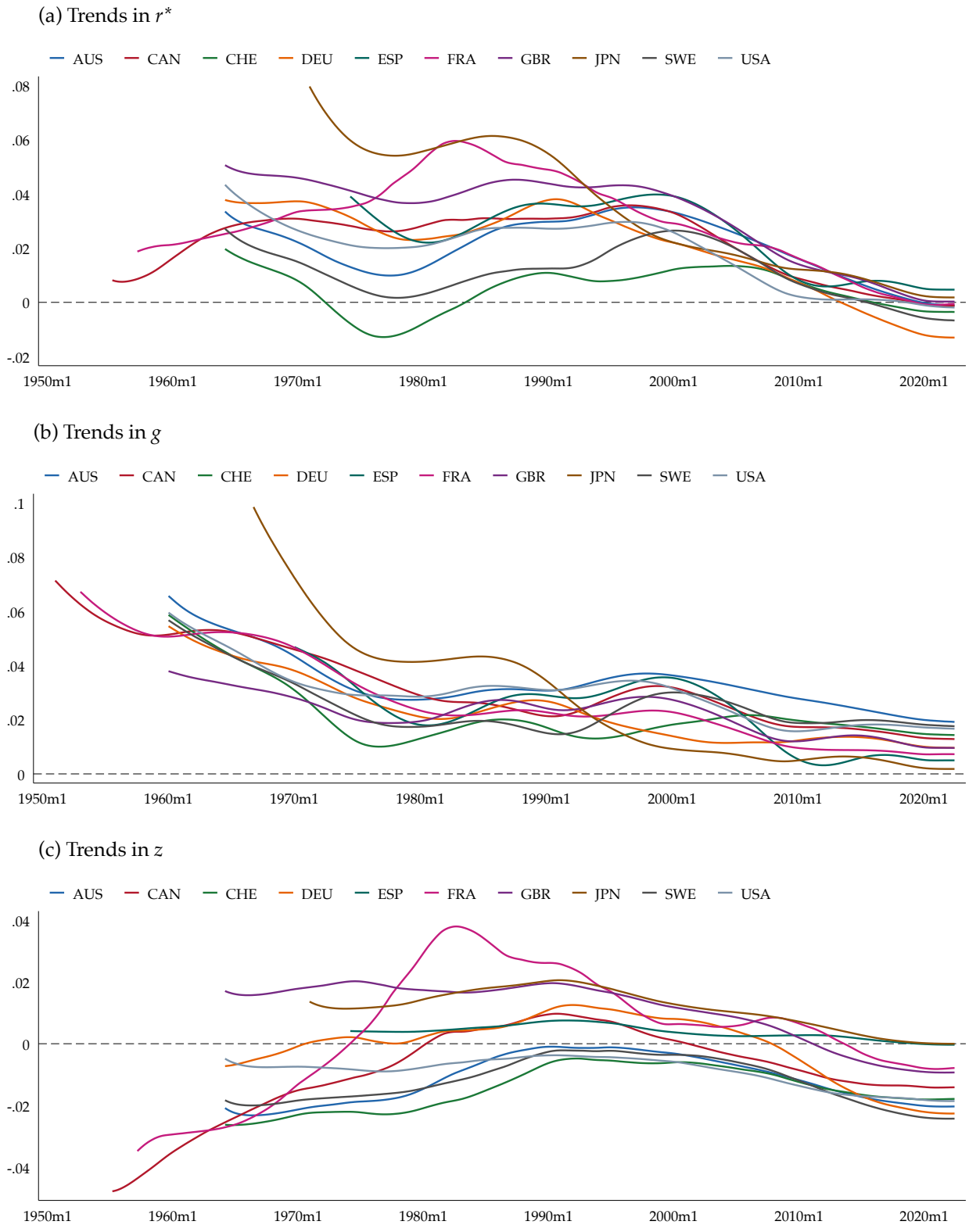
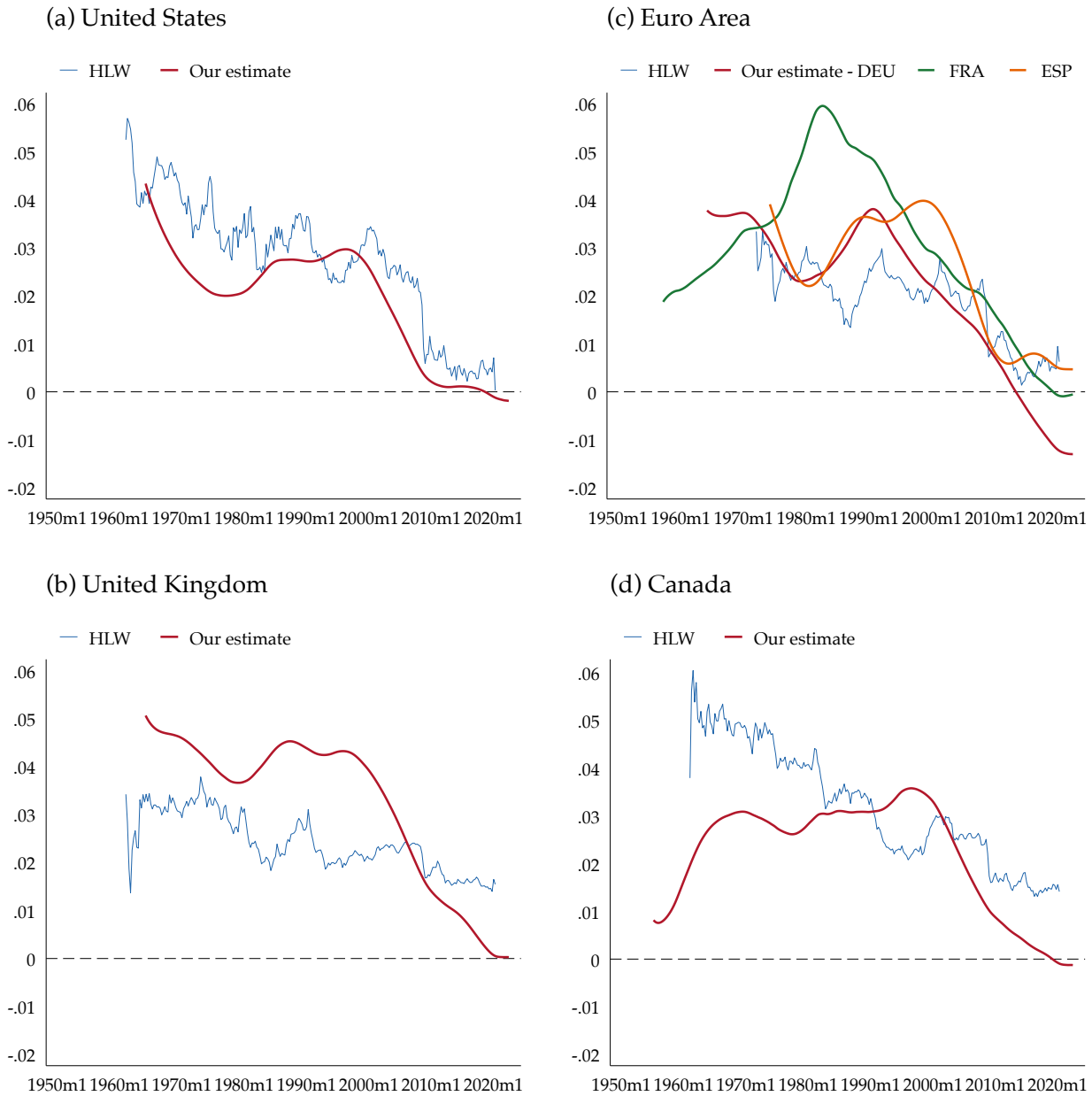


Figure 6: *Trends in r^*, our estimates versus HLW*

The charts display the four HLW estimates of the natural rate r^* and our estimates.



The final panel in Figure 5 shows the important role played by the non-growth headwinds term z in accounting for variations across time and space in natural rates. Most striking here is the sharp rise in this z term, often more than offsetting the decline in growth g during the 1970–1990 period, which is why natural rates tended to rise in that time frame. In contrast, the z term reaches a synchronized peak around 1990, and then enters a secular decline in all countries. This captures a global bond market "conundrum" — to use Greenspan's term — whereby the decline in real rates in the last thirty years takes on a pronounced turn beyond what can be explained by growth fundamentals alone. Seen here, the extra headwinds in the form of a fall in z were experienced not just in the U.S. but across all advanced economies, consistent with trends, like demographics and investment prices, that were witnessed in all of these countries.

Finally, in Figure 6 we compare our natural rate estimates to the best known cross country estimates in earlier reseacrh from (Holston, Laubach, and Williams, 2017) [HLW], for the U.S., U.K., Canada, and the Euro Area (which we compare to our series for Germany, France, and Spain). Our U.S. natural rate series is close the the HLW series in shape but is in general at a much lower level. In the 1970s and again in the 2000s, our series is about 100–200 bps lower and towards the end of the sample we find a negative natural rate when the HLW series is still positive. In the Euro Area, our series shows more of a rise and fall pattern, and generally finds a much steeper decline after the 1990s, with natural rates turning negative in France and especially Germany. For the U.K., we see an upswing and very large downswing in natural rates in the 1990s and afterwards, and the natural rate drops to zero at the end, about 200 bps below the HLW estimate. A similar decline is seen for the case of Canada, as well as much lower natural rates earlier in the sample. Our estimates differ from the HLW method by using information from the bond market at the longer end curve (via $\bar{y}$), so these different trends arise because substantial variations in the headwinds component z are apparent in longer-maturity bonds, with larger shifts in average yields than are implied from growth and short rates alone. [Will be expanded to 10 advanced economies, adding AUS CAN CHE SWE ESP FRA. This work is in progress.]

8. Conclusion

In sum, our results differ from both the prior macro and finance approaches, but they resolve the puzzle by coming down closer to the macro view. Our market-implied natural rate has trends and turning points much like consensus macro estimates, but differs in being typically somewhat lower in the most recent years—with bigger "headwinds" and going negative in all six countries in 2020—intensifying concerns about secular stagnation and proximity to the effective lower-bound on monetary policy in advanced economies.

The pure finance approach to bond pricing using term structure models excludes macro factors. Wicksellian pure macro models of the natural rate exclude term-structure modeling. Our findings suggest these tracks should merge to account for the powerful effects around the world of macro factors on the bond market.

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A. Appendix

Bayesian estimation of the state-space model

We estimate the model using Bayesian methods. We run a random-walk Metropolis-Hastings (RWMH) algorithm on the parameter vector of the state-space model defined for each country individually. The RWMH algorithm starts by drawing from a prior distribution as specified in the next section, and builds a Markov chain to approximate the distribution of the posterior. At each step i of this algorithm, conditional on the parameter vector θ^{i-1} calculated in the previous step, the set of observations Y and the proposal density $q(\cdot|\cdot)$, we accept a drawing ϑ with probability

$$\alpha\left(\vartheta|\theta\right) = \min\left\{\frac{p(Y|\vartheta)p(\vartheta)/q(\vartheta|\theta^{i-1})}{p(Y|\theta^{i-1})p(\theta^{i-1})/q(\theta^{i-1}|\vartheta)}\right\}, \tag{33}$$

where $p(Y|\vartheta)$ can be computed using the Kalman filter on the state-space representation of the model determined by equations Equation 30 and Equation 31. The RWMH version of the algorithm sets the proposal distribution to

$$q\left(\cdot|\theta^{i-1}\right) = N\left(\theta^{i-1}|\mathfrak{c}\cdot\widehat{\Sigma}\right). \tag{34}$$

Our algorithm sets the constant $\mathfrak{c} = 0.075$ and $\widehat{\Sigma}$ is estimated as the diagonal matrix with entries calculated from the empirical variance of the drawn parameters for the first 1000 draws. The target value of $\mathfrak{c}$ yields an acceptance rate of 23.4 percent, as it is the standard for the optimal acceptance target in Metropolis algorithms.

Our estimation is calculated with a number of simulations equal to 50,000, and assumes the same prior distribution for all countries. We run our RWMH in four parallel chains, which gives us a total of 200,000 iterations for the posterior distribution characterization.

For each country, we use as input information the inflation expectations measure π_t^* and the real GDP series. We obtain the trend GDP series by applying the HP filter over quarterly GDP data with a smoothing parameter equal to $25,600 = 1600 \cdot 16$. The series thus obtained is interpolated to monthly data, and the trend growth rate g_t is then calculated. Since a filter applied over the entire history of the series would contain forward-looking bias, we have been careful to recalculate the HP filter only with as-of-date information, allowing for the effect of late release in GDP data (that is, we do not assume that GDP figures are available at the end of the corresponding quarter, but at the usual dates at the end of January, April, July and October).

Our identification assumption is r^* cointegrates with the trend GDP series, and that the difference has mean reversion properties compatible with a half-life within business cycle frequency, and not higher. This avoids the problem of r^* acting as a residual term that would capture high-frequency oscillations in bond markets.

Prior specification

We specify tight priors for the parameters of the headwinds factor z_t, and relatively loose priors for other parameters in the model, as detailed in Table A.1. The prior distribution is common for all countries.

- The persistence parameter ρ_z is chosen to be close to unit root, but within the unitary circle to prevent an explosive solution. Its magnitude is taken to match a half life of around 60 months. This keeps the headwinds factor within business-cycle frequency and not higher. In a freer

setting, this term would act as a residual in the Kalman system and would tend to capture high-frequency variations that are hard to identify given the data.

- The volatility of the headwinds factor σ_z is set to match a relatively small range of variation for the frequency of the headwinds factors. While the drawing process allows for certain range around the 60 months business-cycle frequency, it imposes the view that this variable should not deviate largely from that horizon. Similarly, we impose relatively tight priors on σ_y.
- The model coefficients b_{r^*}, a_y, and b_π are set to have large variances with coefficients centered around values that calibrate the U.S. experience relatively well in a simple regression context.

Table A.1: *Prior specification for model parameters*

Parameter	Distribution	Mean	Variance
ρ_z	Beta	0.997	2.00×10^{-6}
ρ_y	Normal	0.9	0.025
σ_z	Log-Normal	1.00×10^{-4}	1.00×10^{-5}
σ_y	Log-Normal	1.00×10^{-3}	5.00×10^{-4}
a_y	Normal	0.00	7.50×10^{-4}
b_π	Normal	1.25	0.10
b_{r^*}	Normal	0.50	0.05