Industry Linkages from Joint Production

Xiang Ding*
Georgetown University

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Abstract

Joint production is a source of aggregate increasing returns. In US manufacturing, greater demand in one industry of a firm increases output in its other industries that share *knowledge* inputs like R&D, software, and management. I develop a quantitative model where heterogeneous firms use shared inputs to produce output jointly across industries. I estimate that, different from other potentially shared inputs, knowledge inputs are scalable and partially non-rival within the firm. On average prices in one industry fall by 0.4 percent for every 10 percent increase in output in other industries. Such economies of scope account for one-quarter of aggregate increasing returns and manifest disproportionately among knowledge-proximate industries. The resulting industry linkages imply large spillover impacts of recent US-China trade policy on producer prices.

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Introduction

Multi-industry firms produce three-quarters of US manufacturing output. Is output jointly determined across the industries of these firms, and if so, in which industries? Despite much theoretical research on the microeconomics of joint production,[1] there is little systematic evidence on how output in one industry affects a firm's marginal costs in another industry. Cannonical models in trade and macroeconomics assume that firms operate nonjoint, industry-specific production functions and remain silent on whether joint production matters in the aggregate.[2]

This paper combines new evidence and theory to show that joint production within the firm is a source of aggregate increasing returns. On average, producer prices fall with respect to output in not only the same industry but also other industries. I estimate that such economies of scope are due to the scalability and partial non-rivalry of shared knowledge inputs such as R&D, software, and management. As firms scale up these shared inputs to generate more output in any one industry, the non-rivalrous nature of these inputs also increases firms' output in *other* industries. Economies of scope under joint production contribute to an aggregate elasticity of prices to output of -0.04, accounting for one quarter of typical estimates of aggregate increasing returns to scale.

I begin in Section 1 by testing and rejecting the usual assumption that a firm's production technology is *nonjoint* across industries. I assemble panel data from the US Economic Census between 1997 and 2007 on the sales and exports of all US firms in each of 206 manufacturing industries. I leverage variation in firms' exports by product and destination country along with changes in the size of these markets to construct plausibly exogenous demand shifters for each industry of the firm. If the production technology were nonjoint, a demand shock in one industry of a firm would have no impact on its sales in any another industry.

Instead, I find that a positive demand shock in one industry of a firm increases its sales in another the more that the two industries share *knowledge* inputs. I define knowledge inputs using the NAICS classification of headquarter services, professional and technical services, information, and the leasing of intangible assets, and I measure input-proximity using bilateral input-output expenditure data from the BEA. The cross-industry impact of demand shocks on sales increases with proximity in the use of knowledge inputs but not proximity in any other type of input, such as agricultural commodities, or manufactured intermediates. These findings suggest that knowledge inputs have distinct properties in production, consistent with recent evidence on their sharability and scalability within the firm (Atalay et al., 2014; Haskel and Westlake, 2017; Ding et al., 2022).

To rationalize these cross-elasticities, in Section 2, I develop a quantitative model of joint production that transforms multiple inputs to multiple outputs over two stages. First, the firm decides how much knowledge to accumulate in each of its industries. Accumulated knowledge is a proxy

[1] A production technology is nonjoint if and only if the cost function can be written as $C(q_1, ..., q_J) = \sum_{j=1,...,J} C_j(q_j)$ (see Shephard, 1953; Diewert, 1973; Lau, 1972; Hall, 1973; Baumol et al., 1982)

[2] This assumption is adopted in influential papers in international trade (Bernard et al., 2010; Mayer et al., 2014), macroeconomics (Long and Plosser, 1983; Klette and Kortum, 2004), as well as industrial organization (Foster et al., 2008; De Loecker et al., 2016).

for the combined contribution of any firm-wide inputs such as R&D software, management, or physical capital that are shared across the firm's industries. Second, conditional on accumulated knowledge, the firm uses a range of industry-specific inputs to generate final output facing CES demand under monopolistic competition in each industry. Whereas the firm's second-stage production decisions are separable by industry, knowledge accumulation in the first-stage causes marginal costs in one industry to depend on output in other industries. Modeling joint production using two stages allows me to retain tractability and nest the benchmark case of nonjoint, constant-returns production (e.g. Bernard et al., 2010) under knife-edge parameter values.

The direction of industry interdependence depends on two key properties of shared inputs: scalability and rivalry. The more scalable and the less rival are shared inputs, the more that a positive demand shock in one industry of a firm increases its sales in another. Consider a shared input like R&D within General Electric. In response to increased demand in the aerospace industry, GE would scale up its overall R&D expenditures, for example with the intention of improving the hot gas path of its aviation turbines. But the less rival is R&D within the firm, the more likely are the additional scientists to also create ideas (e.g., high-sensitivity scanning) that are used to improve productivity in GE's MRI scanners or even to start up entirely new industries.

My model parametrizes scalability and rivalry as continuous elasticities and allows them to differ across types of shared inputs. Industries that primarily share scalable and non-rival inputs are therefore likely to be complements under joint production. Expanding output in one industry lowers marginal costs and increases sales in another. In contrast, industries that primarily share non-scalable and rival inputs are likely substitutes under joint production. Take, for example, a shared input like physical capital, which enters intensively in the manufacture of glass and metal hardware. If physical capital not scalable and also rival within the firm, expanding output of glass would come at the expense of metal hardware.

In Section 3, I develop a nested fixed point algorithm that combines estimation of micro and macro model parameters. Conditional on micro parameters that specify the joint production technology and underlying heterogeneity in firm fundamentals, I solve for the macro aggregates in the model that exactly match industry-level data in each cross-section. Industry-level output in the data can be inverted to identify industry-level residual profitability in the model, which take on the role of industry-year fixed effects. Likewise, industry-level expenditures on different shared inputs in the data can be inverted to identify technology coefficients in the model, which parametrize differences across industries in their benefit from each type of shared input.

Conditional on the model's macro aggregates in each period, I leverage variation *within* firms over time to estimate the joint production technology. I express same- and cross-industry covariances of demand shocks and sales growth *within* the firm as micro moment conditions. In the same way that demand shocks identify the firm's supply curve in the single-industry case, in my multi-output setting demand shocks identify both own and cross-partial derivatives of the firm's cost function (and thus the scalability and rivalry of shared inputs). Given observed demand

shocks, a firm's overall sales growth identifies input scalability, whereas its *relative* sales growth across industries identifies input rivalry. In turn, differences in macro technology coefficients across industries allow me to separately identify the scalability and rivalry of knowledge-sector inputs from other shared inputs like physical capital.

I estimate that knowledge inputs are scalable as well as partially non-rival within the firm. My estimates of scalability are consistent with Aghion et al. (2019) and Lashkari et al. (2019), who find that French firms increase R&D and IT expenditures in response to positive demand shocks. Whereas this existing literature treats the entire firm as the unit of analysis (and models production at the level of the firm), my concurrent estimates of input non-rivalry imply that output is complementary across knowledge-proximate industries *within* the firm.[3] In comparison, other shared inputs are less scalable and more rival, which causes output across industries that do not share knowledge inputs to be potentially substitutes. The co-existence of both complementarities and substitutabilities imply both economies as well as diseconomies of scope under joint production.

In Section 4 I use these estimates and to quantify the macroeconomic implications of joint production within the firm. I compute the aggregate elasticity of producer price indices in any one industry to demand shocks in any other industry. In general equilibrium, the initial responses of firms to an industry demand shock affects price indices and competition in other industries, triggering subsequent responses among other firms. I show, however, that all of these percolation effects can be expressed by the inverse of a matrix whose elements are determined by parameters of the joint production technology: the scalability and rivalry of shared inputs in the first stage and same-industry production returns to scale in the second stage.

Under a calibration to US manufacturing data from 2017, I simulate a proportional increase in foreign demand in each industry and decompose the equilibrium change in the US producer price index into (i) same-industry increasing returns to scale and (ii) cross-industry economies of scope. I find that both same- and cross-industry elasticities of price to output are on net negative. My estimates of same-industry increasing returns to scale match those in recent papers such as Bartelme et al. (2019) and Lashkaripour and Lugovskyy (2019). However, negative *cross*-industry price elasticities of output are unique to my model of joint production where shared knowledge inputs are scalable and partially non-rival within the firm.

On net, economies of scope from joint production contribute to an aggregate elasticity of prices to output of -0.04, one-quarter of aggregate increasing returns to scale. Economies of scope manifest disproportionately among knowledge-proximate industries and overwhelm mild diseconomies of scope among industries that share less knowledge inputs. For example, a demand shock in the computer and peripherals industry that raises output by 10 percent lowers prices in other industries

[3]Numerous papers study the effects of specific knowledge inputs in isolation, for example R&D in Aw et al. (2011), marketing in Arkolakis (2010), management in Bloom et al. (2019), and ICT in Fort (2016). These papers all focus on the scalability of knowledge inputs in single-output production rather than their rivalry in use *across* multiple types of output. My results are also consistent with Hsieh and Rossi-Hansberg (2020), where firms can lower their marginal costs across all sectors by paying large fixed costs of ICT investment.

by on average 1.4 percent. In contrast, in the production of flavoring syrup, the industry with the highest diseconomies of scope, a demand shock that raises output by 10 percent only raises prices in other industries by on average 0.1 percent.

As a proof of concept, I show that joint production changes the consumer price impact of the recent US-China trade war compared to conventional quantitative models (with nonjoint, linear production) surveyed in Costinot and Rodríguez-Clare (2014). Endogenous producer price declines under joint production mutes the direct adverse CPI-impact of unilateral tariffs on Chinese imports. By expanding US firms' market access at home, protection triggers scale and scope economies that lower US producer prices by 0.37 percent. The US manufacturing CPI rises by only 0.5 percent in my model compared to 0.76 percent under linear production. However, while joint production mitigates the harms of domestic import protection, it amplifies the harms of retaliatory tariffs by China. Retaliatory tariffs restrict US exporters' market access in China and raise producer prices, offsetting more than half of the producer price decline triggered by unilateral US import tariffs. The endogeneity of producer prices under joint production suggest that tariff policy and market access are determinants of a country's comparative advantage, and that comparative advantage is likely positively correlated among knowledge-proximate industries.

Related Literature

This paper relates to a vast literature on multi-output firms. Related papers in international trade have modeled interdependence across a firm's products through demand-side cannibalization (e.g., Eckel and Neary, 2009; Feenstra and Ma, 2007; Dhingra, 2013) and span-of-control (e.g., Nocke and Yeaple, 2014; Bernard et al., 2018). Whereas these models predict negative and symmetric cross-product impacts of demand shocks on sales (or entry), I provide new evidence that cross-industry impacts increase with knowledge input proximity. A different strand of empirical research studies spillovers within internal firm networks (e.g., Giroud and Mueller, 2019) but does not focus on industry heterogeneity or consider joint production as the mechanism.[4] On the other hand, papers in industrial organization and agricultural economics provide more flexible estimates of joint production but in settings limited to only two or three types of outputs.[5]

I contribute to these strands of the literature by developing and estimating a tractable, quantitative model of joint production across 206 specific manufacturing industries. My model places no restrictions on the sign, magnitude, or symmetry of cross-industry price elasticities of supply, and endogenizes both extensive and intensive margin responses within heterogeneous firms. Whereas

[4]For example, mechanisms for intra-firm spillovers range from internal capital markets (Stein, 1997; Lamont, 1997), multinational knowledge transfer (Keller and Yeaple, 2013; Cravino and Levchenko, 2016; Bilir and Morales, 2019), vertical supply linkages (Desai et al., 2009; Boehm et al., 2019a), to distance (Giroud, 2013; Gumpert et al., 2019). None of these papers consider industry heterogeneity in the use of inputs.

[5]For example, Dhyne et al. (2017) estimate pairwise relationships between manufacturing industries, and Grieco and McDevitt (2016) estimate tradeoffs between the quality and quantity of dialysis care. The Cobb-Douglas functional form in these papers, however, presumes that different types of outputs are substitutes in production. Pokharel and Featherstone (2019) non-parametrically estimate the cost frontier but limit their analysis to four types of outputs among agricultural cooperatives.

Penrose (1959), Gort (1962), and Rubin (1973) have proposed that knowledge generates economies of scope within the firm, I provide the first quantitative framework where properties of these shared inputs (among others) can be estimated in the data.[6] Relative to existing stylized models cast under perfect competition (e.g., Jovanovic, 1993; Klette, 1996), I rationalize the heterogeneous responses of profit-maximizing firms under monopolistic competition. This particular structure of my model is advantageous for estimation. Firm sales, demand shocks and input-output tables suffice for identifying the joint production technology. My estimation strategy contrasts with traditional approaches that require real measures of inputs and outputs at the level of the firm (e.g., following Färe and Primont, 1995), where the high-dimensionality of cross-elasticities would be computationally infeasible in my empirical setting with 206 industries.

My estimates of the joint production technology imply quantitatively large aggregate economies of scale and scope. These results contribute to a literature in macroeconomics estimating returns to scale. Early research by Hall (1973) provides an econometric framework for testing joint production using macro data, but subsequent empirical findings have remained inconclusive (see, e.g., Burgess, 1976 and Kohli, 1981, who use data aggregated to the level of two or three sectors). I circumvent many of the challenges confronting these papers through a different micro-to-macro model and estimation strategy. Under my estimates, economies of scope from joint production resolve the 'aggregation puzzle' noted by Caballero and Lyons (1992) and Basu and Fernald (1997). Aggregate increasing returns are more than 33 percent higher than the sum of industry-level increasing returns to scale because industry-level estimates miss cross-industry price declines in equilibrium.

Besides aggregate price impacts, my quantitative model generates predictions for how a shock in any one particular industry affects outcomes in another. I offer a wholly technological explanation in contrast to other papers where mechanisms *external* to the firm shape industry linkages. For example, in the neoclassical trade literature that estimates similar aggregate supply functions (e.g. Harrigan, 1997), general equilibrium factor price movements determine cross-industry impacts. Other external mechanisms feature in research on agglomeration externalities (Ellison et al., 2010), innovation spillovers (Bloom et al., 2013), input complementarity (Jones, 2011), and production networks (Hulten, 1978).[7] These alternative mechanisms operate *across* firms and do not dilute the quantitative relevance of joint production. For example, embedding joint production within an input-output production structure à la Caliendo and Parro (2014) more than doubles the cross-industry price impact of demand shocks.

[6] Using data from Indian manufacturing, Boehm et al. (2019b) also emphasize industry heterogeneity, although they study the role of physical manufacturing inputs (instead of knowledge) and focus on the extensive margin (instead of extensive and intensive margins).

[7] Research on production networks (e.g. Gabaix, 2011; Acemoglu et al., 2012, 2016; di Giovanni et al., 2018; Baqaee and Farhi, 2019; Liu, 2019; Lim, 2018) has focused on how productivity shocks propagate across industries, typically under constant returns to scale, whereas my paper focuses on how demand shocks propagate under non-constant returns.

1 Data and Empirical Evidence

1.1 Multi-Industry Firms in US Manufacturing

I assemble data on the universe of US manufacturing firms' sales by industry, every five years between 1997 and 2012. First, I obtain sales by product line at each establishment of a firm using product trailer files in the US Census of Manufactures. Next, I aggregate sales across a firm's products and establishments to the level of 206 industries $j \in \mathcal{J}$ (roughly 5-digit NAICS), the most disaggregated level at which BEA input-output data are available.[8] Finally, I combine this dataset with the Longitudinal Foreign Trade Transaction Database (LFTTD), which contains data on each firm's exports and imports (if any), by product and country.

Table 1 highlights the prevalence of multi-industry firms as well as their persistence over time. One-fifth of all US manufacturers produce in two or more manufacturing industries, accounting for a disproportionate three-quarters of manufacturing sales, exports, imports, and employment in each year. The second and third rows of Table 1 reveal that this dominance stems not only from each firm's primary (highest sales) industry. Output in remaining industries within multi-industry firms still account for more than one-quarter of the entire manufacturing sector.

Table 1 also reveals considerable limits to firm scope. The median multi-industry firm produces in only two industries, and the two industries are sufficiently dissimilar that they belong in different *sectors* (3-digit NAICS). Given the already-coarse definition of an industry ($j \in \mathcal{J}$ is a 5-digit NAICS code), I interpret multi-industry firm activity in my data as reflecting the production of sufficiently distinct categories of goods (like MRI scanners versus jet engines in General Electric) rather than closely substitutable product varieties.[9]

1.2 An Empirical Test of Nonjoint Production

The cross-sectional data in Table 1 reveal nothing about whether output is jointly determined across a firm's industries. It would be far simpler to model the multi-industry firm as a collection of industry lines each operating independently (e.g., under nonjoint production, as in Bernard et al., 2010). I use within-firm variation over time to test and reject this assumption. Under the null hypothesis of nonjoint production, a firm's sales in a given industry would increase in response to a demand shock in that same industry but would be unaffected by demand shocks in any of its other industries.

[8] By aggregating over plants and products within an industry, I abstract from the plant-dimension and product-variety dimension of the firm, the subject of much existing research. Further, to avoid the possibility that sales in multi-plant firms would be over-counted (relative to that of a single-plant firm) if output is shipped from one plant for use as an input in another, I subtract intra-firm shipments from a plant's total shipments and use the resulting value as my measure of (external) sales. This matters little in practice because intra-firm shipments constitute a trivial fraction (between 1 and 2 percent) of aggregate shipments in my data, consistent with Atalay et al. (2014).

[9] In comparison, existing research (e.g., Feenstra and Ma, 2007; Arkolakis et al., 2019; Macedoni and Xu, 2019) has focused on interdependence within the firm at the level of disaggregated (often substitutable) products, highlighting a demand-side rather than supply-side mechanism.

Table 1: Summary Statistics on US Multi-Industry Manufacturing Firms

	1997	2002	2007	2012
Share of aggregate outcome (out of all manufacturing firms)				
Number of firms	.19	.20	.20	.20
External manufacturing sales	.74	.74	.74	.75
in firm's primary (highest grossing) industry	.47	.48	.51	.52
in firm's remaining industries	.27	.25	.23	.23
Manufacturing employment	.62	.63	.61	.60
Exports	.84	.80	.81	.76
Imports	.82	.79	.79	.77
Mean and median scope (among multi-industry firms)				
Mean number of industries	2.69	2.73	2.63	2.65
Median number of industries	2	2	2	2
Mean number of sectors	1.69	1.74	1.69	1.70
Median number of sectors	2	2	2	2

Notes: A firm is multi-industry if it manufactures goods under at least two distinct industry classifications in that year. An industry is defined at roughly the 5-digit NAICS level, of which there are 206 in manufacturing. A sector refers to a 3-digit NAICS code, of which there are 21 in manufacturing. External manufacturing sales is equal to the firm's gross manufacturing sales less its total inter-plant shipments reported. Manufacturing employment refers to the firm's employment at manufacturing establishments.

I carry out this test using variants of the reduced-form specification in equation (1). I regress each firm's sales growth in a given industry j ($\Delta \log X_{fjt}$) on demand shocks in the same industry j ($\Delta \log S_{fjt}$), demand shocks in its other industries $k \neq j$ ($\Delta \log S_{fjt}^{OTHER}$), and other controls:

$$\Delta \log X_{fjt} = \psi^{SAME} \Delta \log S_{fjt} + \psi^{CROSS} \Delta \log S_{fjt}^{OTHER} + \text{Controls}_{fj,t-1} + FE_{jt} + \epsilon_{fjt}, \quad (1)$$

where Δ is a five-year first-difference operator between t and $t-1$, and $t \in \{1, 2, 3\}$ maps to years 1997, 2002, and 2007 in the data.[10] Industry-year fixed effects (FE_{jt}) control for unobserved supply and demand shocks common to all firms within each given industry j, while $\text{Controls}_{fj,t-1}$ include initial-period firm and firm-industry-level characteristics (such as size and export intensity) that might explain non-parallel growth trends.

Constructing Export Demand Shocks

My identifying assumption requires demand shocks ($\Delta \log S_{fkt}$) in any other industry $k \neq j$ of the firm to be conditionally uncorrelated with the firm's industry-j residual ϵ_{fjt}, which contains (i) unobserved supply-side shocks (that shift firm costs) and (ii) unobserved demand-side shocks (that shift product demand).

[10] I do not use the time period 2007-2012 because (i) the financial crisis generated correlated shocks across countries, industries, as well as firms, jeopardizing independent variation in demand shocks, and (ii) variation in input expenditure shares from the input-output table (which I hold fixed to 1997) used to construct other-industry demand shocks would have become less relevant by 2012.

I use changes in foreign market size to construct plausibly exogenous variation in demand faced by US manufacturers. First, using the BACI Comtrade dataset, I measure each foreign destination n's import growth in each HS 6-digit product h (excluding imports from the US). I denote this five-year change by $\Delta \log IMP_{nht}$ and use this as a proxy for the change in market size (a destination n and product h pair).[11] Next, I average these market size shifters to the level of each industry of each US firm using the firm's pre-existing export shares across these markets as weights:

$$\Delta \log S_{fjt} = s^*_{fj,t-1} \sum_n \sum_{h \in H_j} s_{fjnh,t-1} \Delta \log IMP_{nht}, \tag{2}$$

where $s_{fjnh,t-1}$ is the firm's exports of HS product h to destination n as a share of its total exports in industry j (containing HS6 products $h \in H_j$), and the share $s^*_{fj,t-1}$ scales the demand shock by the firm's export intensity in industry j in year $t-1$ (so firms that sell predominantly at home receive appropriately smaller export demand shocks). I use data from the LFTTD on firm exports by destination and product to construct the export intensity and share variables.

Key to my test for joint production is that demand shocks vary across industries *within* the firm. A large literature has constructed *firm*-level export demand shocks by aggregating over variation across all products and destinations among a firms' exports (see Hummels et al., 2014; Mayer et al., 2016; Aghion et al., 2019; Garin and Silverio, 2018, who use firm-level data from various countries in Europe). I build on this approach by extracting variation at the level of different industries *within* a firm. Much to my advantage, in the US data, manufacturers have extensive export networks that vary by destination and product. The median number of destination-product (nh) export markets within a single industry of a firm in my sample is 6.2, and the mean is 24.1, from among over one million potential combinations of destination-product pairs.

Parametrizing the Cross-Industry Impact of Demand Shocks: Input Proximity

Under the null hypothesis that the production technology is nonjoint, a firm's output in an industry j would be unaffected by a demand shock in *any* other industry $k \neq j$. While in principle I can test whether all 206×205 pairwise cross-elasticities ψ^{CROSS}_{jk} are zero (by setting $\Delta \log S^{OTHER}_{fjt} = \Delta \log S_{fkt}$ in equation 1), I lack statistical power given my limited sample size and sparsity in firms' industry presence. Instead, I test two simpler, necessary conditions for the null to hold: whether (i) cross-elasticities are zero on average, and (ii) cross-elasticities do not vary with industries' input

[11] Changes in import growth $\Delta \log IMP_{nht}$ could reflect both (i) changes in the level of demand in that market, and (ii) changes in the degree of foreign versus home producer competition in that market that affect the price index. Both sources of variation are relevant shifters of a US firm's residual demand, though they would move residual demand in opposite directions. Column (1) of Table 2 shows that empirically, the first force dominates. Nevertheless, one threat to my identification assumption is that measured changes in foreign market size could reflect idiosyncratic supply-side shocks within a sufficiently large US exporter. For example, if GE, a major exporter, became more productive as a firm and exported more MRI scanners to India, Indian imports of MRI scanners from *non-US* countries could fall . At the same time, the firm-wide productivity shock would raise GE's output in other industries. To mitigate this possibility, I construct export demand shocks in equation (2) using only variation from export markets where the firm's market share is below 10%. None of my results hinge on this choice or on the value of the threshold.

proximity. A rejection of either condition in the data is evidence to reject the null hypothesis of nonjoint production.

First, to test condition (i), I average demand shocks in each of the firm's other industries $k \neq j$ using the industry's relative share of sales as weights:

$$\Delta \log S_{fjt}^{OTHER} \equiv \sum_{k \neq j} \left(\frac{X_{fk,t-1}}{\sum_{k' \neq j} X_{fk',t-1}} \right) \Delta \log S_{fkt}. \tag{3}$$

The coefficient on $\Delta \log S_{fjt}^{OTHER}$ identifies the average cross-elasticity of sales to demand shocks. A regression coefficient different from zero would imply that this necessary condition for nonjoint production does not hold.

However, condition (i) is not sufficient for production to be nonjoint. Joint production can yield positive cross-elasticities for some industry pairs and negative cross-elasticities for other industry pairs, such that the average effect measured above will wash out to zero. Such heterogeneity could come from properties of different *types* of shared inputs under joint production. For example, suppose real estate is a rival input in fixed supply within the firm. To expand output in a real estate-intensive industry, the firm may have to reallocate real estate resources and decrease output in its other real estate-intensive industries. On the other hand, suppose another input, like information technology (IT), is non-rival and can be scaled to meet increased demand. When expanding output in an IT-intensive industry, the firm can purchase incremental IT resources and the non-rival nature of IT would increase output in its other IT-intensive industries.

Condition (ii) tests for such heterogeneity by stipulating that cross-elasticities are uncorrelated with industries' input proximity. Using data on input expenditures by industry from the BEA's input-output (I/O) and capital flow tables, I define input-proximity, $Prox_{fjkm}$ to measure how an industry-k shock might affect industry-j output through the firm's use of input m:

$$Prox_{fjkm} \equiv \beta_{jm} \left(\frac{\beta_{km} X_{fk}}{\sum_{k' \neq j} \beta_{k'm} X_{fk'}} \right), \tag{4}$$

where β_{jm} is the share of industry j expenditures on input m, and X_{fj} is the firm's output in industry j. Input proximity is the product of two share terms and is intentionally asymmetric and firm-specific. $Prox_{fjkm,t-1}$ is increasing in both share terms: (i) industry j's expenditures on input m (relative to other inputs m'), and (ii) expenditures on input m by shocked industry k (relative to the firm's other industries k'). The first share reflects how much industry j output might increase from a marginal change in input m, and the second share reflects how the firm's overall use of inputs m might change with respect to a demand shock in k.

To test condition (ii), I interact the firm's demand shock in each other industry k with its

proximity to industry j in relation to various sets of inputs $m \in \mathcal{M}$:

$$\Delta \log S_{fjt}^{OTHER \times \mathcal{M}} \equiv \sum_{k \neq j} \left(\sum_{m \in \mathcal{M}} Prox_{fjkm,t-1} \right) \Delta \log S_{fkt}. \tag{5}$$

A regression coefficient of zero on this interaction variable is consistent with inputs $\mathcal{M}$ being used separately in the production processes of j and k, i.e., nonjoint production. But a regression coefficient different from zero for *any* set of inputs $\mathcal{M}$ would imply that this necessary condition for nonjoint production does not hold. A negative coefficient on $\Delta \log S_{fjt}^{OTHER \times \mathcal{M}}$ is consistent with shared inputs $\mathcal{M}$ being scarce and rival within the firm (like real estate in the example above), while a positive coefficient would imply that shared inputs $\mathcal{M}$ are scalable and non-rival within the firm (like IT in the example above).

I group inputs m in the BEA input-output data (roughly 5-digit NAICS) into sets $\mathcal{M}$ according to the following root codes: agriculture (NAICS 1); construction, mining, and utilities (NAICS 2); manufacturing (NAICS 3); transportation, wholesale, and retail (NAICS 4); finance, insurance, and real estate (NAICS 52, 531, 532); information, intellectual property, management, and professional, scientific and technical services (NAICS 51, 533, 54, 55); administrative services (NAICS 56); other service inputs (NAICS 6, 7, 8, and 9); labor; and capital. Like the examples above, the different groupings of $\mathcal{M}$ allow me to estimate whether properties of one category of inputs, such as manufacturing, differ from that of another category, such as professional services.

1.3 Cross-Industry Impact of Demand Shocks on Output

Equipped with these different parametrizations of other-industry shocks, I estimate variants of equation (1) on a regression sample of all exporting multi-industry firms in each base year, $t-1$. An observation is a continuing industry of one of these firms over a five-year period from $t-1$ to t. This regression sample of roughly 5000 multi-industry firms per year accounts for over half of all US manufacturing gross output. Appendix Table A.2 provides summary statistics on regression variables and other attributes of firms in the sample.

Table 2 presents estimates of the same- and cross-industry impacts of demand shocks on firm sales. First, column (1) reveals that the same-industry impact is positive and statistically significant, suggesting the constructed shocks are empirically relevant as shifters of demand.[12] Next, column (2) tests and does not reject condition (i) for nonjoint production. Cross-elasticities are on average statistically insignificant from zero.

In the remaining columns, I test condition (ii)—that cross-elasticities do not vary with input-

[12]Appendix Table A.3 shows that this effect is not driven by any correlation between the export intensity variable ($s^*_{fj,t-1}$) embedded in the shock and unobserved pre-trends in growth rates (e.g., if more export-intensive industries of the firm grow faster). Results are robust to controlling for a full set of industry-year dummy variables interacted with the export intensity variable, following Borusyak et al. (2018)'s recommendation for specifications with 'incomplete' shares.

Table 2: Same- and Cross-Industry Impacts of Demand Shocks on Sales within the Firm

Change in sales, $\Delta \log X_{fjt}$	(1)	(2)	(3)	(4)	(5)
Same-industry demand shock	0.45***	0.46***	0.46***	0.51***	0.37*
$\Delta \log S_{fjt}$	(0.10)	(0.10)	(0.09)	(0.10)	(0.19)
Other-industry demand shocks					
(i) Average effect		-0.08	-0.83***	-0.81***	-1.67***
$\Delta \log S_{fjt}^{OTHER}$		(0.12)	(0.24)	(0.26)	(0.52)
(ii) × knowledge input-proximity			8.00***	8.26***	13.31***
$\Delta \log S_{fjt}^{OTHER \times KLG}$			(2.25)	(2.22)	(3.58)
Industry-year fixed effects	✓	✓	✓	✓	✓
Firm-wide and firm-industry characteristics				✓	
Firm-year fixed effects					✓
Observations	21,500	21,500	21,500	21,500	17,500
R^2	0.06	0.06	0.06	0.12	0.39

Notes: This table estimates regression equation (1): the response of sales in one industry of the firm to demand shocks in the same industry and also other industries, in 5-year differences over the period 1997-2007. Two measures of other-industry demand shocks are considered. Measure (i) is a sales-weighted average demand shock (equation 3). Measure (ii) interacts other-industry k demand shocks with knowledge input-proximity to industry j (equation 5 where m is within NAICS 51, 533, 54, and 55). Firm-wide and firm-industry level characteristics include: initial period firm size, firm export intensity, firm-industry size, firm-industry export status, firm-industry export intensity, as other-industry characteristics, and other-industry characteristics interacted with knowledge input-proximity. Standard errors are clustered at the firm level, with asterisks indicating p-values below 0.1, 0.05, and 0.01 respectively.

proximity. To allow for differences in cross-elasticities across input categories, I estimate a triple-differences specification separately for each category $\mathcal{M}$. I regress the firm's sales growth in industry j on other-industry demand shocks interacted by input-m proximity ($\Delta \log S_{fjt}^{OTHER \times \mathcal{M}}$), controlling for the average (common) impact of other-industry demand shocks ($\Delta \log S_{fjt}^{OTHER}$) and the same-industry demand shock ($\Delta \log S_{fjt}$). The table does not show separate coefficients on the input-proximity interaction terms because they sum to industry j's expenditure share on inputs $m \in \mathcal{M}$ and are absorbed by industry-year fixed effects ($\sum_{k \neq j} \sum_{m \in \mathcal{M}} Prox_{fjkm,t-1} = \beta_{j\mathcal{M}}$).

Cross-industry Impacts of Demand Shocks Increase with Knowledge Input-Proximity

Figure 1 plots the estimated interaction coefficients on $\Delta \log S_{fjt}^{OTHER \times \mathcal{M}}$ by input category $\mathcal{M}$, and Appendix Table A.4 provides the results in tabular form. Cross-elasticities are higher among industries that are more proximate in their use of inputs from the information, intellectual property, management, and professional, scientific, and technical services sectors (NAICS 51, 533, 55, 54). I abbreviate these 21 input industries as *knowledge* inputs in the rest of the paper, and summarize their use in manufacturing production in Appendix Table A.1.[13] Besides from knowledge, cross-elasticities do not correlate with proximity in the use of inputs from any other category. The estimated coefficients relating to all other input categories are insignificant from zero, suggesting that these inputs may indeed be industry-specific as commonly assumed.[14]

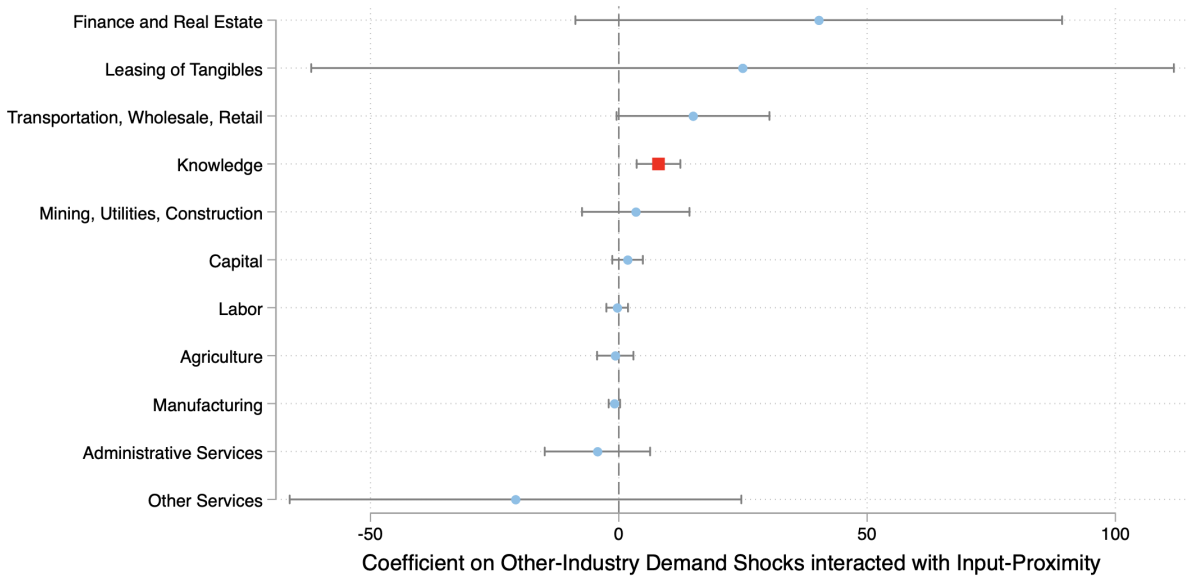
These results suggest knowledge inputs have distinct properties under joint production compared to other inputs. This is not surprising. Knowledge inputs produce much of the intangible capital hypothesized to be sharable within the boundary of the firm (Atalay et al., 2014). It is also plausible that inputs like IT services and R&D are less rival in production than other sharable inputs such as real estate or physical capital (Haskel and Westlake, 2017). Altogether, as of 1997, knowledge industries constitute 15 percent of US GDP and are used intensively by manufacturing firms. Manufacturing firms' expenditures on knowledge inputs constitute 9 percent of their gross output as a whole, and vary greatly by the input-output industry pair. For example, organic chemical production (NAICS 325190) has the largest expenditure share (2.4 percent of gross output) on architectural, engineering and related services (NAICS 541300), while semiconductor manufacturing has the largest expenditure share (1 percent of gross output) on computer systems design services (NAICS 541512).

Column (3) of Table 2 reports this key finding from Figure 1: cross-elasticities increase with knowledge input-proximity. The exact coefficient estimates suggest that cross-elasticities are negative for industry-pairs that do not use any knowledge inputs in common, while positive for

[13]Examples of such input industries include data processing services, scientific R&D, engineering, consulting, architectural, advertising, and legal services. Knowledge input industries in my model relate closely to the classification of 'professional and technical services' in Ding et al. (2022), 'skilled scalable services' in Eckert et al. (2020) and 'tradable services' in Gervais and Jensen (2019) and Eckert (2019).

[14]The results do suggest a mildly significant (at the 10 percent level) positive cross-elasticity across industries proximate in their use of inputs from the 'transportation, wholesale, and retail' sector, which is consistent with cost savings from shared warehousing and distribution.

Figure 1: Cross-industry Impact of Demand Shocks: Heterogeneity by Input-Proximity



Notes: This figure displays point estimates and 95% confidence intervals of cross-industry elasticities $\psi^{CROSS \times \mathcal{M}}$, where $\mathcal{M}$ references a different input category (in each row of the figure) in the triple-differences regression specification (3) of Table 2. See Appendix Table A.4 for the corresponding regression table and the exact industry codes of these input categories.

industry-pairs that have high expenditure shares on the same knowledge inputs.[15] The existence of both positive and negative cross-elasticities are consistent with the estimate of a zero average cross-elasticity in column (2).

The economic magnitudes of these reduced-form elasticities are hard to assess because the regressions are unweighted and fixed effects absorb any correlated changes across firms. While the rest of the paper develops and structurally estimates a theoretical model to quantify the impact of joint production, I first provide a back-of-the-envelope calculation. I find sizable impacts using the estimates in column (3). Consider two different industries j and j' within a firm: industry j has knowledge input-proximity relative to the firm's other industries equal to 0.06, one standard deviation below the mean, while industry j' has knowledge input-proximity equal to 0.12, one standard deviation above the mean. Suppose this firm receives a uniform demand shock in all of its industries equal to 10 log points, roughly one standard deviation in the sample ($\Delta \log S_{fjt} = 0.1 \ \forall j$). The demand shock in industry j alone would increase sales in the same industry by 4.6 log points ($= 0.1 \times 0.46$), while the demand shocks in other industries would decrease sales in industry j by 3.5 log points ($= 0.1 \times -0.83 + 0.1 \times 0.06 \times 8.00$). Combining the direct and cross-industry impacts,

[15] I also estimate specifications with multiple interactions of input proximity with other-industry demand shocks jointly. In all specifications the coefficient on the knowledge input-proximity weighted demand shock, $\psi^{CROSS \times KLG}$, is positive and significant. See, for example, specification (4) of Appendix Table A.5, which simultaneously estimates coefficients on three measures of other-industry demand shocks: (i) average, (ii) interaction with knowledge-input proximity, and (iii) interaction with other input-proximity.

sales in industry j would increase on net by only 1.1 log points.

In comparison, the same demand shocks would increase output by more in industry j', which is more knowledge input-proximate to the firm's other industries. The same-industry demand shock would still increase sales in industry j' by the same 4.6 log points, but now other-industry demand shocks would additionally increase (rather than decrease) sales in industry j' by 1.3 log points ($= 0.1 \times -0.83 + 0.1 \times 0.12 \times 8.00$). Combining the direct and cross-industry impacts, output in industry j would increase on net by 5.9 log points, of which more than one-fifth comes from the cross-industry impact.

Robustness to Additional Covariates

The estimates in Table 2 are robust to controlling for an exhaustive set of initial-period firm-industry-level characteristics that explain subsequent growth in a firm's industries. These controls mitigate omitted variables bias whenever demand shocks are non-randomly assigned and correlated with covariates. I construct both same-industry and *other-industry* measures of these covariates, just as I do for shocks. For example, an other-industry control for size represents the average log sales among the firm's other industries. I also interact these other-industry characteristics with the same knowledge input-proximity measures that are interacted with demand shocks. For example, the control variable for any characteristic $Y_{fk,t-1}$ in the firm's other-industries interacted with knowledge input-proximity is:

$$Control_{fjt}^{OTHER \times KLG}(Y) \equiv \sum_{k \neq j} \left(\sum_{m \in KLG} Prox_{fjkm,t-1} \right) Y_{fk,t-1}.$$

These cross-industry controls address the potential for omitted variables bias whenever a firm's growth in one industry j is correlated with pre-existing covariates $Y_{fk,t-1}$ its other knowledge input-proximate industries.

In column (4) of Table 2, I saturate the triple-differences specification with an exhaustive list of controls for pre-period characteristics: firm log sales, firm export intensity, firm-industry-level log sales, firm-industry export intensity, firm-industry export status, the value of these characteristics in the other industries of the firm ($Control_{fjt}^{OTHER}(Y)$), as well as these other-industry covariates interacted with knowledge input proximity ($Control_{fjt}^{OTHER \times KLG}(Y)$). Controlling for all these covariates affects neither the significance nor magnitude of the key regression coefficients ψ^{CROSS} and $\psi^{CROSS \times KLG}$. Since the same proximity measures, sales shares, and export intensities used to construct other-industry demand shocks are also used to construct these other-industry covariates, this specification also provides reassurance that the cross-elasticities are identified off of changes in foreign market size.

Finally, column (5) of Table 2 shows that results remain robust to controlling for firm-year fixed effects, which soak up any unobserved supply and demand changes common to each industry of

the firm.[16]

1.4 Mechanisms and Discussion of Results

Table 2 provides evidence that knowledge inputs are shared in a firm's multi-industry production technology. When a positive demand shock in one industry causes the firm to increase its use of knowledge inputs, the non-rival property of these inputs also increases output in the firm's other industries. In comparison, other potential mechanisms for explaining interdependence within the firm, such as demand cannibalization or credit constraints, would have resulted in cross-elasticities that are on average negative rather than increasing with knowledge input-proximity. The null result in column (2) rules out these alternative mechanisms as the primary driver of cross-industry impacts in my empirical setting.[17]

Evidence that Knowledge Inputs are Scalable within the Firm

Next, I provide evidence that firms' knowledge input expenditures increase with a shock to demand, a necessary condition behind my proposed mechanism. While each firm's total knowledge input expenditures (which include, for example, expenditures on in-house knowledge inputs) are hard to measure, the CMF provides data on a particular subset: firms' purchases of professional services, which comprise software, data processing, management, and advertising services. Table 3 estimates the elasticity of these knowledge input expenditures with respect to firm-level demand shocks, and compares the elasticity to that of other firm-level outcomes Y_{ft}: capital expenditures, payroll, and sales. I run the following firm-level regressions:

$$\Delta \log Y_{ft} = \tilde{\nu} \sum_{k} \eta_{fk,t-1} \Delta \log S_{fkt} + \epsilon_{ft}, \tag{6}$$

where I use weights $\eta_{fk,t-1}$ (the share of industry k in the firm's initial-period outcome $Y_{f,t-1}$) to construct the relevant firm-wide average demand shock.[18]

I estimate these regressions at the firm-level because unlike sales, there is no data on firms' input expenditures by industry of use. Of course, this data limitation arises naturally in the context of joint production when inputs are shared. Both the reduced-form evidence in this section and model estimation in Section 3 rely only on industry-level rather than firm-level input expenditure data. There is an econometric advantage to doing so. Input proximity constructed from aggregate expenditure shares β_{jm} is unlikely to be correlated with firm-specific unobservables (e.g., unequal

[16] Results from the triple-differences specification are also robust to further including firm-industry fixed effects, which limits identifying variation to *changes* in growth rates and demand shocks between the period 1997-2002 and the period 2002-2007.

[17] This null result is consistent with Borusyak and Okubo (2016), who also do not find average intra-firm, cross-segment impacts of demand shocks in Japanese firm-level data.

[18] Data Appendix A.3.4 provides the precise definitions. Results are robust to using simple averages or any other common type of weight across all four outcome variables in Table 3.

Table 3: The Impact of Demand Shocks on Firm-level Input Expenditures

	(1) Purchased Prof. Services	(2) Capex	(3) Payroll	(4) Sales
Outcome-relevant demand shock	0.65***	0.47	0.25**	0.37***
$\sum_k \eta_{fk,t-1} \Delta \log S_{fkt}$	(0.22)	(0.37)	(0.11)	(0.15)
Year-FE	✓	✓	✓	✓
Observations	3,900	3,900	3,900	3,900
R^2	0.02	0.04	0.01	0.05

Notes: This regression table estimates the impact of demand shocks (averaged across the firm's industries) on firm-level variables, in 5-year differences over the period 1997-2007. Weights used for the average, $\eta_{fk,t-1}$, are defined in Data Appendix A.3.4. Standard errors are clustered at the firm level, with asterisks indicating p-values below 0.1, 0.05, and 0.01 respectively. Number of observations are rounded for disclosure avoidance. The sample of firms is limited to those reporting non-zero purchased professional services.

access to input markets), therefore mitigating a potential source of endogeneity bias.

Column (1) of Table 3 shows that firms increase their expenditures on these professional services in response to positive firm-wide demand shocks. The coefficient of 0.65 comprises the product of two elasticities: (i) a "first-stage" elasticity of marginal revenue to the empirically measured demand shocks, and (ii) the elasticity of knowledge input expenditures to the said shift in marginal revenue. The remaining columns of Table 3 repeat the analysis for other firm-wide outcomes. While each coefficient shares the same "first-stage" elasticity, the second elasticity is different. Columns (2) and (3) show that the elasticity of expenditures on capital and payroll with respect to the same shift in marginal revenue is lower, consistent with knowledge inputs being more scalable than these other inputs. Finally, column (4) estimates the firm-level elasticity of sales to demand shocks at 0.37. This coefficient lies in between the elasticity of various input expenditures in the prior columns, consistent with the assumption of constant markups in the model in Section 2.

Other Interpretations and Threats to Identification

My identification assumption requires that demand shocks in a firm's *other* industries $k \neq j$ are conditionally uncorrelated with unobserved demand and supply shifters in a given industry j of the firm.[19] I entertain and rule out various potential threats to this identification assumption.

First, the lack of statistical significance in column (2) of Table 2 rules out the possibility of a simple, symmetric correlation structure between export demand shocks and unobserved shifters across industries. Therefore, correlations between demand shocks in industry k and unobservable shifters in industry j would be problematic for the main results in columns (3)-(5) *only if* the

[19]Demand shocks can be arbitrarily correlated with own-industry unobserved shocks without affecting the use of cross-industry coefficients ψ^{CROSS} to test for nonjoint production. Demand shocks also do not need to be unanticipated. The possibility that a particular shock in k is anticipatable t years ahead of time simply changes the interpretation of the relevant time horizon for a supply-side response in j to materialize.

correlation happens to be precisely stronger among knowledge-proximate industries.

To entertain the possibility of such a correlation, suppose that import demand in each foreign country co-moves more among more knowledge-proximate industries. Under this scenario, the cross-elasticities in Table 2 could reflect firms receiving correlated shocks across industries *within* their export markets and taking advantage of 'demand-scope' complementarities (Bernard et al., 2018) or shared market access costs (Arkolakis et al., 2019). A related threat occurs if knowledge-intensive industries are disproportionately demand-complementary among common buyers. Under any of these scenarios, a positive demand shock in one industry would raise the firm's sales in its other knowledge-proximate industries.

I address these concerns in three ways. First, in Appendix Section A.3.5 I find no evidence that import growth within a foreign destination is positively correlated among knowledge-proximate industries. Second, the regressions in Table 2 control directly for same-industry demand shocks ($\Delta \log S_{fjt}$), which would contain any correlated shocks in a given export destination as long as the firm already exports in industry j to that destination n. Third, my results are robust to excluding from the definition of industry sales $\Delta \log X_{fjt}$ any exports of j to destination countries where demand shocks in *other* industries k originated.[20]

A different threat to identification comes from selection on correlated supply-side shocks. If such shocks were anticipatable by the firm and, again, happen to be positively correlated in knowledge input-proximate industries, selection on industry and export market entry could generate spurious positive cross-industry impacts of demand shocks specifically among knowledge input-proximate industries. However, this hypothesis does not survive the following placebo exercise. I re-assign firm-industry exporters in each industry k different export demand shocks drawn from the empirical distribution of shocks received by other firms in that same industry. Keeping all remaining firm variables (e.g. firm-industry sales weights, and other controls) unchanged in these placebo regressions, I do not find a statistically significant share of positive cross-industry coefficients.[21]

Finally, I use data on intra-firm shipments of goods to reject other vertical-production-related mechanisms. In Appendix Section A.3.6, I find no evidence that intra-firm shipments in an upstream industry respond to demand shocks in a downstream industry (or vice versa).

[20] As a fourth step, I find that results are also robust to controlling for *latent* demand shocks—a measure of demand for industry j of the firm not from where it is *currently* exporting its products in industry j (which is $\Delta \log S_{fjt}$) but from any other destinations in which it currently exports products in other industries k. These results are undisclosed but can be provided upon request.

[21] These results are undisclosed but can be provided upon request. This placebo exercise also provides further reassurance that pre-existing variation in Bartik weights or bilateral industry characteristics such as knowledge input-proximity are not picking up correlated industry trends in the error terms.

2 Model of Joint Production

I develop a theory of joint production that rationalizes the within-firm, cross-industry transmission of demand shocks in the data. Under joint production, inputs are potentially shared across multiple industries, so that marginal cost in a given industry depends on the firm's output in not only the same industry but also other industries. Two key properties of shared inputs—their *scalability* within the firm and their *rivalry* in use across industries—parametrize this interdependence in costs and allow for arbitrary returns to scale and scope.

I embed this joint production technology within a conventional monopolistic competition setting featuring CES industry demand and endogenous entry and exit of firms across industries. The model relaxes the assumption of constant-returns and nonjoint production found in workhorse models of heterogeneous firms (e.g., Melitz, 2003; Bernard et al., 2010, nesting their general equilibrium predictions as special cases). Under joint production, different parameter values of input scalability and rivalry generate within-firm, cross-industry elasticities of sales to demand shocks that are heterogeneous and unrestricted in sign.

2.1 Production Technology and Market Structure

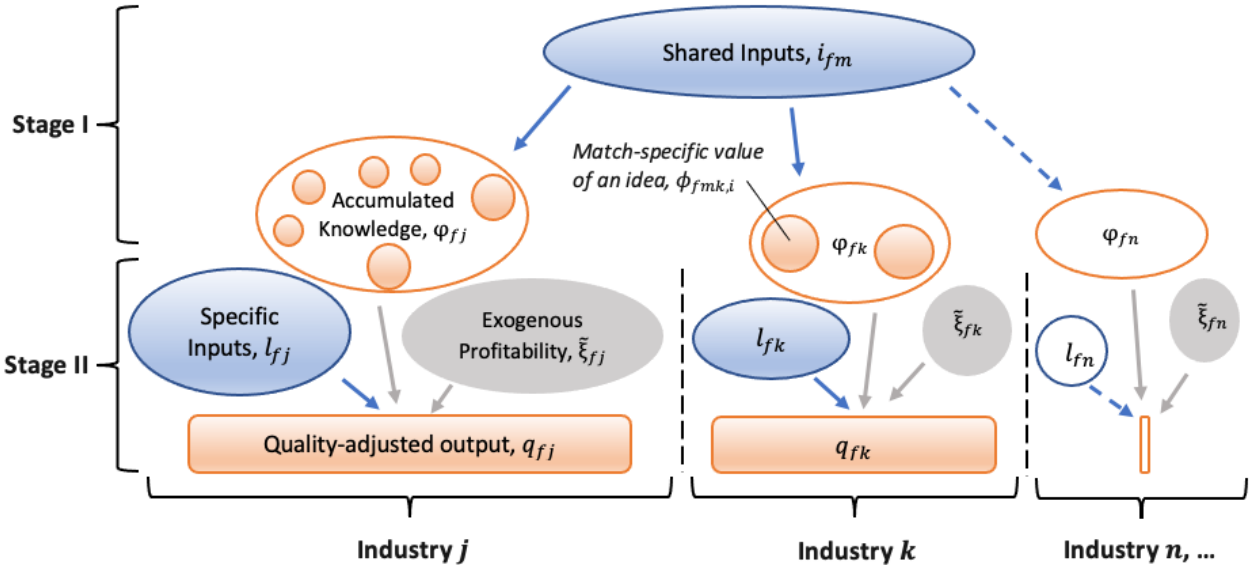
Figure 2 illustrates the static joint production technology of the firm. I model production as taking place sequentially over two stages, where inputs are denoted in blue and outputs are denoted in orange. While the mapping between inputs and outputs in the model is general, I use specific labels motivated by the empirical evidence to aid exposition.

I interpret the first stage as the firm's production of knowledge (or intangible) capital φ_{fj} across its industries using shared inputs ι_{fm} like information technology, intellectual property, and professional services from the knowledge-producing sector. Knowledge capital accumulated in this ex-post form is proprietary to the firm and cannot be bought or sold on the market. By the second stage, accumulated knowledge φ_{fj} acts as a revenue productivity shifter. The firm takes φ_{fj} as given and combines it with industry-specific inputs l_{fj} like assembly-line labor, materials, and energy to produce its differentiated variety q_{fj} in each industry.

Using this technology, a continuum of firms compete across a set of industries $j \in \mathcal{J}$ under monopolistic competition facing CES demand (with elasticity σ_j) within each industry. In both stages of production, firms face constant input prices and know their own fundamental (exogenous) profitability shifters $\tilde{\xi}_{fj}$, which reflect, for example, idiosyncratic differences in product appeal, production know-how, or access to foreign output and input markets.

In stage I, the firm chooses quantities of each type of shared input m (e.g., engineering services, software) and the industries in which to accumulate knowledge capital. The firm's decisions in stage I also determine its extensive margin—the set of industries $j \in \mathcal{J}$ in which it sells final output. In stage II, the firm chooses quantities of each type of industry-specific input (e.g., production labor and materials) to maximize total profits. I describe the two production stages in reverse order.

Figure 2: An Illustration of the Firm's Joint Production Technology



Notes: This figure illustrates the firm's two-stage joint production technology that transforms inputs (in blue) to quality-adjusted outputs across multiple industries, $j, k, n, ...$ (in orange). Inputs used in stage I are shared across the firm's industries, whereas inputs used in stage II are industry-specific. The firm uses shared inputs to develop discrete ideas (displayed by the small orange circles) that, when adapted to an industry j, increases the value of knowledge capital φ_{fj}. Profitability shifters (shaded grey) are exogenous and observable by the firm in both stages. In the example, the firm does not produce output (does not enter) in industry n because it has not accumulated any knowledge capital in that industry ($\varphi_{fn} = 0$).

Stage II: Production with Industry-specific Inputs

By this latter stage, production of final, quality-adjusted output $\{q_{fj}\}_{j \in J}$ is independent across industries given that stage II inputs are industry-specific (and available at constant unit prices). I assume that standard Cobb-Douglas production functions given by equation (7) transform inputs into outputs.

Assumption 1 (Stage II Industry Production Functions) *In each industry j, quality-adjusted output q_{fj} is a Cobb-Douglas function over (i) a homothetic index l_{fj} of industry-specific inputs, (ii) an index φ_{fj} of accumulated knowledge determined in stage I, and (iii) an exogenous profitability shifter, $\tilde{\xi}_{fj}$:*

$$q_{fj} = l_{fj}^{\gamma_j} \, \varphi_{fj} \, \tilde{\xi}_{fj}, \qquad \forall j \in \mathcal{J}, \tag{7}$$

where $\gamma_j \in [0, \frac{\sigma_j}{\sigma_j - 1})$ is the elasticity of final output with respect to stage-II inputs l_{fj}.

My production technology is a generalization of that found in standard models of heterogeneous firms. For example, when stage-II returns to scale are constant ($\gamma_j = 1$) and stage I knowledge accumulation is exogenous (so the combined revenue productivity term $\varphi_{fj}\tilde{\xi}_{fj}$ is exogenous), equation (7) yields the constant-returns, nonjoint-production benchmark of Bernard et al. (2010). Relative to this benchmark, my production technology provides more flexibility in two

dimensions. First, properties of stage-II inputs allow for arbitrary increasing ($\gamma_j > 1$) as well as decreasing ($\gamma_j < 1$) *within*-industry returns to scale. Second, properties of shared inputs in stage I, described below, lead to interdependence *across* industries between marginal costs and output and therefore arbitrary economies or diseconomies of scope.

Stage I: Production with Shared Inputs

Assumption 2 completes the description of the firm's production technology. The firm uses shared knowledge-producing inputs (indexed by type $m \in \mathcal{M}$) to develop ideas and adapt them across different industries to improve knowledge capital (i.e., enhance revenue-productivity in stage II). For example, inputs like scientists and managers contribute to knowledge by developing automation techniques, configuring factory floor space, improving assembly-line productivity, or raising brand awareness. The combined value of all these ideas adapted in an industry make up the index of accumulated knowledge, φ_{fj}.

Assumption 2 (Stage I Stochastic Accumulation of Knowledge) *Shared inputs ι_{fm} contribute to the firm's Poisson rate of development of ideas of each type m:*

$$A_{fm} \sim Poisson\left(Z\left(\frac{\rho_m}{\rho_m - 1}\iota_{fm}\right)^{\frac{\rho_m - 1}{\rho_m}}\right), \qquad \forall m \in \mathcal{M}, \tag{8}$$

where parameter Z governs the average arrival rate of ideas, and $\rho_m \in (1, \infty)$ measures input m's scalability. Each idea $i \in \{1, ..., A_{fm}\}_m$ has match-specific value $\phi_{fmi,j}$ when adapted in an industry j. Match-specific values are drawn i.i.d. from a Fréchet distribution with shape parameter $\theta_m \in (1, \infty)$:

$$Pr(\phi_{fmi,j} \leq x) = e^{-x^{-\theta_m}}, \qquad \forall j \in \mathcal{J}. \tag{9}$$

The firm chooses the industry j in which to adapt each idea (denoted by indicator $\mathbf{1}_{fmi,j}$) after observing the idea's match-specific values in each industry. Total accumulated knowledge φ_{fj} in an industry is a power sum over the value of all ideas adapted in that industry:

$$\varphi_{fj} = \left(\sum_{m \in \mathcal{M}} \sum_{i=1}^{A_{fm}} \tilde{\alpha}_{mj} \phi_{fmi,j} \mathbf{1}_{fmi,j}\right)^{\frac{\sigma_j}{\sigma_j - 1} - \gamma_j}, \qquad \forall j \in \mathcal{J}, \tag{10}$$

where technology coefficients $\{\tilde{\alpha}_{mj}\}_{m,j}$ denote the average value of type-m ideas when adapted in industry j.

I model the development of ideas within the firm as an endogenous Poisson process, given by equation (8). The more shared inputs ι_{fm} the firm uses, the greater the number A_{fm} of type-m ideas the firm expects to develop. Input scalability, $\rho_m \in (1, \infty)$, parametrizes the elasticity of the arrival rate to the quantity of inputs used. The lower is ρ_m, the less responsive are the firm's

input m expenditures to changes in demand (profitability) conditions. In the limit as $\rho_m \to 1$, the arrival rate of ideas is inelastic to input use, and knowledge accumulation becomes exogenous: $A_{fm} \sim \text{Poisson}(Z)$.

Each idea $i_m = 1, ..., A_{fm}$ that the firm develops has an idiosyncratic i.i.d. value $\phi_{fmi,j}$ when adapted to improve knowledge capital in a given industry j. The firm observes $\{\phi_{fmi,j}\}_{j \in \mathcal{J}}$ and chooses the most suitable industry j in which to adapt that idea. Equation (9) parametrizes variation in match-specific values using a Fréchet distribution with shape parameter θ_m. This variability is intrinsic to knowledge creation. Consider, for example, General Electric, which employs ceramics scientists to develop R&D ideas. A particular idea represents an invention like gemstone scintillators, which are more valuable when adapted in GE's CT medical scanners than in GE's aviation turbines. Whereas some shared inputs like scientists may generate more variable ideas (indicated by a lower θ_m), other shared inputs like legal and accounting services could generate very predictable ideas (indicated by a higher θ_m).

Finally, equation (10) combines the value of all ideas adapted in an industry j into a single index of accumulated knowledge, φ_{fj}. The functional form assumes that the marginal profit contribution of each idea is additively separable from that of other ideas.[22] In addition, exogenous technology coefficients $\tilde{\alpha}_{mj}$ allow the ideas of a given type m to be either more or less valuable on average when adapted in a given industry j. Similar to conventional input-output coefficients, $\{\tilde{\alpha}_{mj}\}_{m \in \mathcal{M}, j \in \mathcal{J}}$ generate variation across industries in the use of stage-I shared inputs m and facilitate the quantitative mapping between the model and the data. For example, software inputs are intensively used in computer manufacturing while management consulting inputs are intensively used in petrochemicals production. Because the two industries are not proximate in the types m of shared inputs used, the cross-industry transmission of demand shocks from petrochemicals to computers will be close to zero even under joint production.

2.2 Solution of the Firm

Given the production technology described by Assumptions 1 and 2, it is easy to solve for the firm's profit-maximizing decisions in reverse order.

In stage II, conditional on accumulated knowledge φ_{fj}, the firm's gross profit maximization problem is separable by industry. Equation (11) describes the well-known solution under monopolistic competition and CES industry demand. The firm's revenues X_{fj} and gross profits π_{fj} (revenues less stage-II industry-specific input costs) in each industry can be expressed in terms

[22] Additive separability in profits does not imply that shared inputs are perfect substitutes in production. While estimating input substitutability is beyond of the focus of this paper, Appendix xxx derives what the parametric assumptions made here imply for the elasticities of substitution across shared inputs. In addition to generating additive separability, the index $\sigma_j/(\sigma_j - 1) - \gamma_j$ in equation (10) serves as a normalization that, when combined with equation (7), limits the overall returns to scale in production to be smaller than $\sigma_j/(\sigma_j - 1)$. This normalization ensures that the firm's profit-maximization problem is well-defined, i.e., the supply curve is less steeply downward sloping than the demand curve. Note that since stage-I inputs have arbitrary scalability γ_j and stage-II inputs have arbitrary scalability ρ_m, the firm's supply curve is not dependent on demand elasticities σ_j.

of accumulated knowledge (φ_{fj}), an exogenous profitability shifter (ξ_{fj}), and an industry-level profitability index (B_j) common to all firms:

$$\pi_{fj} = (1-\varsigma_j)X_{fj} = B_j \xi_{fj} \varphi_{fj}^{\frac{\sigma_j - 1}{\sigma_j(1-\varsigma_j)}}, \tag{11}$$

where $\xi_{fj} \equiv \tilde{\xi}_{fj}^{\frac{\sigma_j-1}{\sigma_j(1-\varsigma_j)}}$ is a convenient re-normalization and $\varsigma_j \equiv \gamma_j \frac{\sigma_j - 1}{\sigma_j} < 1$ is an industry-level parameter equal to the share of sales expensed on stage-II inputs l_{fj}. Industry-wide profitability B_j is an equilibrium object that depends on gross profit margins $1-\varsigma_j$, the unit cost c_j of the industry-specific input composite l_{fj}, and two shifters of residual demand: P_j (the CES price index), and Y_j (total expenditures):

$$B_j = (1-\varsigma_j)\left(\frac{\varsigma_j}{c_j}\right)^{\frac{\varsigma_j}{1-\varsigma_j}} \left(P_j^{\sigma_j - 1} Y_j\right)^{\frac{1}{\sigma_j(1-\varsigma_j)}}. \tag{12}$$

Having optimized over the firm's stage-II input use, the only remaining endogenous variable is the index value for accumulated knowledge (φ_{fj}), the outcome of the firm's decisions in stage I.

In stage I, the firm faces a multi-dimensional profit maximization problem. It chooses (i) the industry j in which to adapt each idea $\{\mathbf{1}_{fmi,j}\}_{\{i=1,\ldots,A_{fm}\}_m}$, (ii) overall quantities of shared inputs $\{\iota_{fm}\}_m$, and (iii) its "extensive margin"—the set of industries $j \in \mathcal{J}$ to produce output in. Replacing knowledge capital φ_{fj} in the firm's stage-II gross profit function (11) with its definition in equation (10) yields firm gross profits as an additively separable function of the number and value of ideas adapted to that industry:

$$\pi_{fj} = \sum_{m \in \mathcal{M}} \sum_{i=1}^{A_{fm}} B_j \xi_{fj} \tilde{\alpha}_{mj} \phi_{fmi,j} \mathbf{1}_{fmi,j}, \qquad \forall j \in \mathcal{J}.$$

Given this additive separability, the firm's decisions in Stage I can be solved for in isolation. First, the choice of which industry j in which to adapt a given idea $\{i_m = 1, \ldots, A_{fm}\}_m$ becomes a repeated discrete choice problem. The firm observes the Fréchet-distributed industry match-specific values $\{\phi_{fmi,j}\}_{j \in \mathcal{J}}$ and chooses the industry where adapting that idea would yield the greatest (additive) increase in profits. The firm adapts each type-m idea to industry j with probability μ_{fmj}:

$$\mu_{fmj} \equiv \frac{\delta_{fmj}^{\theta_m}}{\sum_{k \in \mathcal{J}} \delta_{fmk}^{\theta_m}}; \qquad \delta_{fmj} \equiv B_j \xi_{fj} \alpha_{mj} Z, \tag{13}$$

which is increasing in δ_{fmj}, an index of input-by-industry-level exogenous profitability terms: the firm's stage-II profit shifters ($B_j \xi_{fj}$ in equation 11), stage I technology coefficients (renormalized as $\alpha_{mj} \equiv \tilde{\alpha}_{mj} \Gamma(1 - 1/\theta_m)$), and the exogenous rate of arrival of ideas (Z).

Second, the firm chooses its overall level of each shared input ι_{fm} such that the marginal benefit of that input equals its marginal cost (a constant). The marginal benefit is the product of

two terms: (i) the effect of the marginal input towards increasing the Poisson arrival rate of ideas, and (ii) the expected profit contribution of a given idea that arrives. The first term is decreasing in ι_{fm} given concavity in the production of ideas ($(\rho_m - 1)/\rho_m < 1$ in equation 8). The second term, the expected profit contribution of a given type-m idea, is a constant and given by:

$$\Delta_{fm} \equiv \mathbb{E}\left[\max_j B_j \xi_{fj} \tilde{\alpha}_{mj}\, \phi_{fmi,j}\right] = \left(\sum_{j \in \mathcal{J}} \delta_{fmj}^{\theta_m}\right)^{\frac{1}{\theta_m}}, \tag{14}$$

a power sum of the exogenous profitability terms δ_{fmj} in each industry. Even though each idea is only adapted in a single industry, the power sum captures the firm's *option value* from being able to observe $\{\phi_{fmi,j}\}_j$ and then choose the most profitable industry to adapt that idea. The power sum expression is consistent with option value being higher whenever variation in match-specific productivities is high (whenever θ_m is low). Altogether, trading off a constant marginal cost against a diminishing marginal benefit yields a unique interior solution for shared inputs $\{\iota_{fm}\}_m$.

Lastly, the industry entry decision of the firm is simple in the absence of fixed costs. Each firm can adapt ideas and produce output in any of the $\mathcal{J}$ industries. But because industry knowledge capital φ_{fj} is essential for production in stage II, the choice of whether to adapt the first idea to an industry (which causes $\varphi_{fj} > 0$) *is in fact* the choice of whether to "enter" that industry. The first idea adapted to an industry marks entry, while adaptations of subsequent ideas to the same industry improve knowledge capital (and thus output) on the intensive margin. Given their additive separability in profits, each idea is adapted to industry j with probability μ_{fmj}, regardless of whether it is the first or subsequent idea.[23]

Lemma 1 puts these results together and derives closed-form expressions for the firm's *ex-ante* expected industry sales, probability of industry entry, and net profits.

Lemma 1 (The Firm's Solution) *Let w denote the (normalized) constant unit cost of each shared input. The firm's expected gross profits in each industry $j \in \mathcal{J}$ is a constant fraction $(1 - \varsigma_j)$ of expected sales:*

$$\mathbb{E}[\pi_{fj}] = (1 - \varsigma_j)\, \mathbb{E}[X_{fj}] = \sum_m \delta_{fmj}^{\theta_m} \Delta_{fm}^{\rho_m - \theta_m} w^{1-\rho_m}, \tag{15}$$

and the probability of industry entry (denoted $\chi_{fj} = 1$), is one minus the probability the firm does not adapt

[23] The use of discrete stochastic processes to explain 'zeros' (the absence of firm entry) is inspired by Klette and Kortum (2004), Eaton et al. (2013), and Armenter and Koren (2014), and presents theoretical and computational advantages over settings with literal fixed costs. Fixed costs generate non-convexities from the point of view of not just firms but also the aggregate economy. Recent work by Jia (2008), Antràs et al. (2017), and Arkolakis and Eckert (2017) provide algorithms that reduce the computational burden of fixed-cost models but operate under a partial equilibrium framework where industry profitability is fixed. Instead, in my stochastic setting each individual firm's profit maximization problem is convex, which guarantees a unique solution for industry profitability $\{B_j\}_j$ in multi-industry equilibrium.

any idea to industry j:

$$Pr(\chi_{fj} = 1) = 1 - \exp\left(-Z \sum_{m \in \mathcal{M}} \delta_{fmj}^{\theta_m} \Delta_{fm}^{\rho_m - 1 - \theta_m} w^{1-\rho_m}\right). \tag{16}$$

The firm's ex-ante expected net profit (revenues less stage-I and stage-II input costs) is given by:

$$\mathbb{E}[\Pi_f] = \sum_j \mathbb{E}[\pi_{fj}] - \sum_{m \in \mathcal{M}} w \iota_{fm} = \sum_{m \in \mathcal{M}} \frac{1}{\rho_m} \Delta_{fm}^{\rho_m} w^{1-\rho_m}. \tag{17}$$

2.3 Interdependence from the Scalability and Rivalry of Shared Inputs

In Lemma 1, the firm's expected sales in a given industry j (both the intensive and extensive margin) depends on profitability shifters in not only the same industry but also other industries $k \neq j$ (contained in the Δ_{fm} terms). The precise direction of interdependence is governed by parameters ρ_m and θ_m of shared inputs. I define ρ_m as input scalability and θ_m as input rivalry.

These two properties of shared inputs generate scale and rivalry effects that have opposing effects on cross-elasticities. The more scalable are shared inputs (the higher is ρ_m), the more that the firm increases their overall use in response to an industry-k-specific demand shock. As long as inputs are not fully rival, this increase in shared inputs benefits knowledge accumulation (and thus sales) in other industries j of the firm. But the more rival are shared inputs (the higher is θ_m), the more that the firm will optimally substitute its use of shared inputs away from other industries j to meet the increase in demand in industry-k. As long as inputs are not fully scalable, this substitution comes at the expense of knowledge accumulation (and sales) in its other industries j.

I describe these two effects analytically before combining them in Proposition 1. First, the firm's sales in each industry j can be re-written as

$$\mathbb{E}[X_{fj}] = \frac{B_j \xi_{fj}}{1 - \varsigma_j} \mathbb{E}\left[\varphi_{fj}^{\frac{\sigma_j - 1}{\sigma_j - \gamma_j(\sigma_j - 1)}}\right] = \frac{B_j \xi_{fj}}{1 - \varsigma_j} \sum_m \underbrace{\mathbb{E}[A_{fm}]}_{\text{scale effect}} \underbrace{\mu_{fmj}\, \mathbb{E}[\phi_{fmi,j} \,|\, \mathbf{1}_{fmi,j} = 1]}_{\text{rivalry effect}}, \tag{18}$$

where the last equality decomposes expected knowledge φ_{fj} into a term determined by input scalability (the total number of ideas of each type m developed), and a term determined by input rivalry (the share and expected value of the ideas that the firm adapts in industry j). A demand shock in another industry k can raise industry-j output through the scale effect but also lower it through the rivalry effect.

Rivalry effect is parametrized by θ_m. The rivalry effect consists of two terms. First, in response to an increase in demand in industry k, the firm will find it more profitable to adapt a greater share of ideas to industry k and a smaller share μ_{fmj} to industry j. From equation (13), the higher is θ_m, the more that the share of ideas adapted in industry j will fall. Besides from μ_{fmj}, the second

term contributing to input rivalry measures the expected value of an idea *conditional* on the firm adapting it in industry j. This second term is slightly offsetting due to selection: when demand is higher in industry k, any ideas that the firm still chooses to adapt to industry j must have on average a higher idiosyncratic match-specific value in industry j. Overall the rivalry effect can be expressed as:

$$\mu_{fmj}\ \mathbb{E}[\phi_{fmi,j} \mid \mathbf{1}_{fmi,j} = 1] = \left(\frac{\delta_{fmj}}{\Delta_{fm}}\right)^{\theta_m - 1}, \qquad \forall m \in \mathcal{M},$$

which is decreasing in other-industry demand shocks (Δ_{fm}) with elasticity $\theta_m - 1$.

In the limit as $\theta_m \to 1$, shared inputs m become fully non-rival within the firm. In response to a demand shock in industry k, the slight decline in the share of ideas μ_{fmj} adapted in industry j is fully offset by the increase in the expected value of ideas still being adapted in industry j, and the rivalry effect in equation (18) disappears. In the other limit as $\theta_m \to \infty$, relative adaptation shares are so sensitive that a slight increase in profitability in another industry k can cause virtually all ideas to be adapted in that industry, therefore shutting down production in all industries $j \neq k$. Values of θ_m between 1 and ∞ therefore flexibly parametrize input rivalry in my model.

Scale effect is parametrized by ρ_m. In response to an increase in demand in industry k, the firm will also find it more profitable to increase its overall arrival rate of ideas, $\mathbb{E}[A_{fm}]$. From equation (8), the higher is ρ_m, the more elastic is the arrival rate of ideas to the firm's use of shared inputs. The firm chooses quantities of shared inputs ι_{fm} to equate marginal expected gross profits with its marginal input cost. The firm's expected number of adaptable ideas can be expressed as:

$$\mathbb{E}[A_{fm}] = \left(\frac{\rho_m}{\rho_m - 1}\right)^{\frac{\rho_m - 1}{\rho_m}} \Delta_{fm}^{\rho_m - 1},$$

which is increasing in other-industry demand shifters (Δ_{fm}) with elasticity $\rho_m - 1$.

In the limit as $\rho_m \to 1$, shared inputs are not scalable and knowledge accumulation within the firm is an exogenous process. Firms do not adjust their use of shared inputs, and the scale effect in equation (18) disappears. In the other limit as $\rho_m \to \infty$, shared inputs are so scalable that the slightest increase in profitability in industry k causes the firm's profits in each industry to increase infinitely (this virtuous cycle is possible in partial equilibrium as the firm moves down its cost curve and lower prices invite even more residual demand). Values of ρ_m between 1 and ∞ therefore flexibly parametrize input scalability in my model.

Net effect on cross-elasticities. Proposition 1 combines these two effects to derive the net cross-industry elasticities of sales ($\mathbb{E}[X_{fj}]$) with respect to demand shocks ($\xi_{fk}B_k$).[24] Cross-elasticities are

[24]The elasticities in Proposition 1 include both extensive and intensive margin responses in expectation. In the Supplementary Online Notes, I use equation (16) to decompose the two margins and show that the total elasticity of sales to demand shocks occurs mostly on the extensive margin for smaller firms and mostly on the intensive margin for larger firms (such as those in my regression sample).

increasing in the scalability (ρ_m) and decreasing in the rivalry (θ_m) of proximate shared inputs m, such that the net effect can be either positive or negative. The model-relevant measure of shared-input-m proximity is $\lambda_{fjm}\mu_{fmk}$, the theoretical counterpart to $Prox_{fjkm}$ in Section 1. Finally, the last term in equation (19) includes a strictly positive term whenever the outcome industry is the same as the shocked industry ($k = j$). A positive demand shock always increases same-industry sales because scale and rivalry effects push in the same direction.

Proposition 1 (Cross-Industry Elasticities within the Firm) *The elasticity of expected firm sales in any industry j, $\mathbb{E}[X_{fj}]$, to a change in profitability in any industry k, $\xi_{fk}B_k$, is given by:*

$$\psi_{fjk} \equiv \frac{\mathrm{d}\log \mathbb{E}[X_{fj}]}{\mathrm{d}\log \xi_{fk}B_k} = \sum_{m\in\mathcal{M}} (\rho_m - \theta_m)\,\lambda_{fjm}\,\mu_{fmk} \;+\; \mathbf{1}_{j=k} \sum_{m\in\mathcal{M}} \theta_m\,\lambda_{fjm}, \tag{19}$$

where industry adaptation shares μ_{fmk} are given by equation (13) and input utilization shares λ_{fjm} denote the share of industry j gross profits attributable to shared input m:

$$\lambda_{fjm} \equiv \frac{\mu_{fmj}\Delta_{fm}^{\rho_m} w^{1-\rho_m}}{\sum_{m'} \mu_{fm'j}\Delta_{fm'}^{\rho_{m'}} w^{1-\rho_{m'}}}.$$

The theory nests two edge cases where the null hypothesis of nonjoint production would hold ($\psi_{fjk} = 0$). The first, trivial, case occurs when all shared inputs in stage-I are in fact industry-specific: each input m is only ever useful when adapted to improve knowledge in a given industry j, so $\alpha_{mk} = 0$ for all $k \neq j$. In this case $\lambda_{fjm}\mu_{fmk} = 0$, so the cross-elasticity is zero. The second case occurs on a knife's edge when scale and rivalry effects perfectly offset each other ($\rho_m = \theta_m \;\forall m$). For example, if all proximate shared inputs (e.g., brand capital) were completely unscalable and also perfectly non-rival ($\rho_m = \theta_m = 1$), knowledge accumulation is fixed, and the model is isomorphic to the firm receiving exogenous firm-industry 'productivity draws' of φ_{fj}.

Outside of these edge cases, cross-elasticities are asymmetric and heterogeneous across firms and industry-pairs. They depend flexibly on the scalability and rivalry of shared inputs as well as the technology coefficients α_{mj} that parametrize input-proximity, allowing me to estimate these parameters from the observed elasticities of sales to demand shocks in the data.

3 Model Estimation

This section connects the empirical evidence in Section 1 with the theory in Section 2. I leverage the conditional exogeneity of demand shocks at the firm-industry level to estimate the firm's joint production technology. I base inference on exact model-implied moment conditions, allowing demand shocks and other general equilibrium controls to affect firm outcomes non-linearly. The moment estimator allows me to leverage variation in the data from not only the intensive margin

(e.g., decrease in sales) but also the extensive margin (e.g., closure of an industry), consistent with the model.

3.1 Overview and Assumptions

Notationally, I use variables in boldface to refer to vectors and matrices, for example $\boldsymbol{B} \equiv \{B_{jt}\}_{j,t}$.

I develop a nested fixed-point algorithm to jointly estimate the model's micro and macro parameters. First, conditional on micro parameters (scalability $\boldsymbol{\rho}$ and rivalry $\boldsymbol{\theta}$), I set the model's macro parameters (industry profitability levels $\boldsymbol{B}$ and technology coefficients $\boldsymbol{\alpha}$) to exactly match BEA industry-level data on output and input expenditures. Second, conditional on macro parameters, I compute the model's structural residuals—the difference between the model and the data in each firm's output growth in each industry, ΔX_{fj}. I exploit the orthogonality of these structural residuals with respect to same- and cross-industry demand shocks to identify micro parameters.

Input Taxonomy

Estimation requires taking a stance on which inputs in the data are shared inputs used in stage I of joint production. The reduced-form evidence in Section 1 is consistent with knowledge inputs being shared within the firm. I classify inputs from the knowledge sector into three categories of shared inputs in the model: (i) leasing of intangibles (NAICS 533), (ii) headquarters services (NAICS 55), and (iii) information and professional services (NAICS 51, 54). I specify a pair of scalability and rivalry parameters $(\rho^{KLG}, \theta^{KLG})$ common to these shared knowledge-sector inputs.

In addition, I create a fourth residual shared input in the model to accommodate regression evidence of negative cross-elasticities among industries that are not knowledge-proximate. I map this residual shared input to capital as well as inputs from finance and real estate (NAICS 52), the leasing of tangibles (NAICS 531, 532), administrative services (NAICS 56), other services (NAICS 6, 7, 8, and 9) for which uncertainty around interaction effects in Figure 1 is high. I aggregate all these inputs in the data into one composite residual input in the model to speed up computation (leaving only $4 \times |\mathcal{J}|$ technology coefficients to identify). I let the scalability and rivalry of this residual shared input $(\rho^{RES}, \theta^{RES})$ differ from those of knowledge-sector shared inputs. In practice, this residual shared input allows the model to quantitatively account for any other mechanism that generates interdependence, including, for example, span-of-control or internal capital markets.

I assume that production in stage II uses inputs from all remaining BEA sectors: agriculture, mining, construction, utilities, manufactures, wholesale, retail and transportation industries, as well as labor value added. Since these inputs are industry-specific by assumption, their impact on firms' production decisions are absorbed in the estimation of industry profitability $\boldsymbol{B}$.

Altogether $\boldsymbol{\Theta} \equiv \{\rho^{KLG}, \theta^{KLG}, \rho^{RES}, \theta^{RES}\}$ represents the key micro parameters to be estimated.

Firm Profitability Shifters and Demand Shocks

Similar to the reduced-form regressions, I exploit variation within the firm over time for identification. Firms compete under a separate static equilibrium in each period $t \in \{1, 2, 3\}$ in the model (corresponding to years 1997, 2002, and 2007 in the data). Firms optimize their input expenditures, knowledge accumulation, and final output in each period t after observing profitability conditions $\boldsymbol{B}_t, \boldsymbol{\xi}_{ft}$.[25]

In the first period, firms draw their exogenous profitability shifters $\{\xi_{fj}\}_j$ in each industry from a joint lognormal distribution specified by Assumption 3. Profitability shifters $\boldsymbol{\xi}_{ft}$ stand in for any firm-specific demand and supply conditions (e.g., differences in product appeal, non-depreciating capital stocks) unobservable to the econometrician. I assume that the firm-industry-specific demand shocks constructed in Section 1 shift $\boldsymbol{\xi}_{ft}$ over time,[26] thereby triggering changes in firm sales in the model (as a function of $\boldsymbol{\Theta}$). Besides from the impact of idiosyncratic demand shocks, I assume that firms retain their initial exogenous profitability shifters over time, allowing the model to explain persistence in firm size and industry specialization in the data.

Assumption 3 (Demand Shocks as Profitability Shifters) *Each firm's fundamental profitability shifters are distributed joint lognormal in period $t = 1$ according to:*

$$\xi_{fj,t=1} = \zeta_{fj}\,\zeta_f, \quad \log \zeta_{fj} \sim_{i.i.d.} \mathcal{N}(0, \gamma_0), \quad \log \zeta_f \sim_{i.i.d.} \mathcal{N}(0, \gamma_1), \quad \forall f \in \mathcal{F}, j \in \mathcal{J}.$$

In years $t = \{2, 3\}$, a measure-zero set of firms $\mathcal{F}_t^D$ (corresponding to the regression sample) receive demand shocks $\{\Delta \log S_{fjt}\}_{j \in \mathcal{J}}$ as constructed in Section 1, which affect their profitability shifters according to:

$$\Delta \log \xi_{fjt} = \nu\, \Delta \log S_{fjt}, \qquad \forall f \in \mathcal{F}_t^D,\ j \in \mathcal{J},\ t = \{2, 3\}. \tag{20}$$

Other firms retain their initial-period profitability shifters over time.

The variance parameters $\boldsymbol{\gamma} \equiv \{\gamma_0, \gamma_1\}$ control firm-level comparative and absolute advantage respectively. First, the higher is γ_0, the more dispersed is profitability across industries *within* a firm, and the more persistent is a firm's pattern of specialization over time. This occurs as the firm is more likely to repeatedly accumulate knowledge in industries with very high ξ_{fj}. I estimate γ_0 by matching the share of industries in multi-industry firms that survive over 5-year intervals to that in the data, equal to 0.42. Second, the higher is γ_1, the more dispersed is size *across* firms. I estimate γ_1 by matching the aggregate share of sales by multi-industry firms in 1997 to that in the

[25] In other words, I assume that accumulated knowledge completely depreciates across periods. Using the BEA's estimates of knowledge capital depreciation rates of 0.33, only 14 percent of accumulated knowledge would remain across five-year intervals, so this assumption is not far off. In addition, by allowing profitability shifters $\boldsymbol{\xi}_f$ to persist over time, I account for the impact of any (unobserved) capital that does not depreciate across periods.

[26] This assumption can be explicitly micro-founded in a multi-destination export setting in which firms draw different latent initial taste shifters across destinations. These initial shifters inform pre-existing patterns of exporting, and subsequent changes in foreign market size across destinations will manifest in changes in firms' profitability shifters in the model.

data, equal to 0.74. I normalize the means of the lognormal distributions to zero because they are isomorphic to shifters of industry profitability $\boldsymbol{B}$.

I estimate the 'first-stage' elasticity ν in equation (20) by leveraging the model's log-linear relationship between a firm's expenditures on professional services M_{ft}^{PROF} (one of the four categories of shared inputs) and the firm's average demand shock:

$$\Delta \log M_{ft}^{PROF} = \nu \rho^{KLG} \sum_{k \in \mathcal{J}} \eta_{fk,t-1}^{PROF} \Delta \log S_{fkt},$$

where model-consistent expenditure shares $\eta_{fk,t-1}^{PROF}$ are approximated using BEA industry-level expenditure shares on professional services $\beta_{k,PROF}$:

$$\eta_{fk,t-1}^{PROF} \equiv \frac{\mu_{fmk,t-1} \Delta_{fm,t-1}^{\rho_m}}{\Delta_{fm,t-1}^{\rho_m}} \approx \frac{\beta_{k,PROF} X_{fk,t-1}}{\sum_k \beta_{k,PROF} X_{fk,t-1}}, \qquad \text{for } m = PROF.$$

Intuitively, the elasticity of professional service input expenditures with respect to demand shocks depends on the product of two elasticities (i) ν, the elasticity of firm profitability with respect to demand shocks, and (ii) ρ^{KLG}, the elasticity of professional service input expenditures with respect to firm profitability. Column (1) of Table 3 provides a regression estimate of the combined elasticity $\nu \rho^{KLG} = 0.65$, allowing identification of ν conditional on knowledge input scalability ρ^{KLG}. Before turning to identification of micro scalability and rivalry parameters $\boldsymbol{\Theta}$, I describe identification of macro variables conditional on $\boldsymbol{\Theta}$.

3.2 Identification of Macro Variables

The first half of Table 4 summarizes the sources of identification of macro variables. First, I set the mass of potential entrants at $N = 318000$, the total number of firms (including administrative and inactive records) in the 1997 Census of Manufactures.[27] Second, $\varsigma_j = \gamma_j(\sigma_j - 1)/\sigma_j$ in the model is equal to the share of gross output expensed on stage-II inputs, which is readily available in BEA input-output data.[28] Third, I calibrate the average arrival rate of ideas, Z_t, by matching the share of multi-industry firms in the model to that in the data in each year (0.2).

Finally, I calibrate industry profitability $\boldsymbol{B}_t$ and technology coefficients $\boldsymbol{\alpha} \equiv \{\alpha_{mj}\}_{m,j}$ so that the model exactly matches BEA data on gross output $\boldsymbol{X}_t$ and expenditures on shared (stage-I) inputs $\boldsymbol{M}_t$ by industry. In the model, firms' expenditures on stage-I inputs are shared across industries, but these expenditures (e.g., on R&D) are reported in BEA data separately by industry. I assume that an equivalent statistical agency in the model registers the entire firm's expenditures on a given

[27] Firms in my model move in and out of active status due to stochasticity in knowledge accumulation. An inactive firm is any firm that, despite positive stage-I input expenditures, has accumulated zero knowledge. In any period in which this happens, the firm will register zero sales and fall out of the observed sample, i.e., become an inactive record.

[28] Estimation is invariant to values of actual production returns to scale γ_j or demand elasticities σ_j. Under monopolistic competition, the sufficient equilibrium parameter is ς_j, a combination of the two.

Table 4: Overview of Model Parameters and Sources of Identification

Variable and Description		Source of Identification
Macro Variables		
N	Mass of all firms $\mathcal{F}$	All active and inactive firms (318,000)
ς_j	Stage-II input expenditures as share of output	Corresponding shares in BEA I/O Table
Z_t	Average arrival rate of ideas	Share of multi-industry firms (0.2)
B_{jt}	Industry profitability	BEA Industry Gross Output X_{jt}
α_{mj}	Average value of shared input m in industry j	BEA Input-by-industry Expenditures M_{mj}
Micro Parameters		
γ_0	Variance in ξ_f across industries within the firm	Share of industries that continue (0.42)
γ_1	Variance in ξ_f across firms	Share of sales by multi-industry firms (0.74)
ν	Responsiveness of ξ_f to demand shocks	Assumption 3 and Table 3 ($\nu\rho^{KLG} = 0.65$)
$\boldsymbol{\Theta}$	Scalability and rivalry of shared inputs	Proposition 2

shared input m under the industry j where the firm adapts its first type-m idea. Given a continuum of firms, assuming that each firm registers shared input expenditures under their first industry of use is equivalent under aggregation to apportioning each firm's expenditures on input m across industries according to adaptation probabilities μ_{fmj}.

Equation (21) provides the national accounting identities that define industry-level output (X_{jt}) and input expenditures (M_{mjt}) as a sum over respective firm-level variables in the model:

$$\begin{aligned} X_{jt} &= \frac{1}{1-\varsigma_j} N \int \sum_{m \in \mathcal{M}} \delta_{fmjt}^{\theta_m} \Delta_{fmt}^{\rho_m - \theta_m} \, \mathrm{d}G(\boldsymbol{\xi}), \qquad \forall j \in \mathcal{J}, \;\; t \in \{1,2,3\}, \\ M_{mjt} &= \frac{\rho_m - 1}{\rho_m} N \int \delta_{fmjt}^{\theta_m} \Delta_{fmt}^{\rho_m - \theta_m} \, \mathrm{d}G(\boldsymbol{\xi}), \qquad \forall j \in \mathcal{J}, \;\; t = 1, \;\; m \in \mathcal{M}, \end{aligned} \tag{21}$$

where $G(\boldsymbol{\xi}; \boldsymbol{\gamma})$ is the joint-lognormal distribution parametrized by Assumption 3.[29] I normalize the price of each type of shared input w_t to equal one in each year (differences across types of shared inputs m are absorbed by $\boldsymbol{\alpha}$) and deflate data on X_{jt} and M_{mjt} in each year by wage inflation. I calibrate $\boldsymbol{\alpha}$ to match input-by-industry expenditure data from the first cross-section, 1997, and assume that $\boldsymbol{\alpha}$ is time-invariant. Equation (21) is a system of $|\mathcal{J}| \times 3 + |\mathcal{J}| \times |\mathcal{M}|$ equations with as many unknowns. I develop a fast recursive computational algorithm that inverts the system of equations to solve for $\boldsymbol{B}_t$ and $\boldsymbol{\alpha}$ (contained in $\boldsymbol{\delta}$ and $\boldsymbol{\Delta}$) given data on $\boldsymbol{X}_t$ and $\boldsymbol{M}$, taking as given other macro and micro parameters. I provide more details in Quantitative Appendix C.1.

Notice that identification of these macro variables does not require specifying other general equilibrium details of the model (such as trade or vertical input-output linkages) as long as such features affect all firms equally. Industry-level profitability $\boldsymbol{B}_t$ encapsulates the combined effect of export market access, import market competition, as well as prices of industry-specific intermediate inputs in each year, as long as they are common to all firms.

[29] With an abuse of notation, a variable subscripted with f indicates that it is dependent on ξ_f.

3.3 Identification of Scalability and Rivalry Parameters

Lastly, I exploit within-firm variation over time and use the vector of demand shocks $\Delta \log \boldsymbol{S}_{ft}$ as instruments to identify scalability and rivalry parameters $\boldsymbol{\Theta}$. By Assumption 3, demand shocks shift firms' profitability $\boldsymbol{\xi}_{ft}$ between t and $t-1$, allowing me to identify Θ from changes in firms' sales across its industries. Just as demand shocks trace out the firm's marginal cost curve in the textbook single-product case, in my multi-industry setting, same- and cross-industry demand shocks identify the *matrix* of same- and cross-industry elasticities of marginal cost to output—which is parametrized by $\boldsymbol{\Theta}$.

An immediate challenge for mapping the model to the data is the non-random assignment of demand shocks to firms. Export demand shocks $\Delta \log \boldsymbol{S}_{ft}$ from Section 1 are constructed only for firms that are already selling in a given industry j. Firms active in industry j would have a higher-than-average fundamental profitability $\boldsymbol{\xi}_{fj,t-1}$ in that industry, and comparing the outcomes of such a firm in the data against a those of a *randomly* drawn firm in the model would lead to selection bias.

I address this potential selection bias using Assumption 4. I assume that demand shocks are uncorrelated with initial-period unobserved profitability $\boldsymbol{\xi}_{f,t-1}$ and sales $\boldsymbol{X}_{f,t-1}$ *conditional on* the firm's initial industry presence, $\boldsymbol{\chi}_{f,t-1}$. In other words, identification only requires that shocks are as good as randomly assigned among firms with identical initial-period extensive margins.

Assumption 4 (Conditional Independence) *Demand shocks are randomly assigned to firms conditional on pre-existing industry presence:*

$$\Delta \log \boldsymbol{S}_{ft} \perp \boldsymbol{\xi}_{f,t-1}, \boldsymbol{X}_{f,t-1} \mid \boldsymbol{\chi}_{f,t-1}, \qquad t \in \{2,3\}.$$

Proposition 2 describes the moment conditions I use to estimate $\boldsymbol{\Theta}$. Equation (22) stipulates that under true values of $\boldsymbol{\Theta}$, conditional covariances between demand shocks and sales growth in the data should equate that in the model for any pair of industries j, k. This yields a $|\mathcal{J}|^2$ matrix of moment conditions for each of the two time-differenced periods $t = 2, 3$. In the sample analogs of each moment condition, I include firms with industry activity in j and k in the initial period, $t-1$ and include endogenous exit as an outcome of the firm (whereby $X_{fjt} = 0$). By conditioning model predictions on the firm's initial-period extensive margin $\boldsymbol{\chi}_{f,t-1}$, I am able to correct for potential selection bias arising from a firm having higher-than-average ξ_{fk} and ξ_{fj} whenever it is observed to be jointly active in those industries.

Proposition 2 (Identification of Scalability and Rivalry) *Define structural residuals $\Delta\epsilon_{fjt}$ as:*

$$\Delta\epsilon_{fjt} \equiv \left(X_{fjt} - X_{fj,t-1}\right) - \left(\mathbb{E}_t[X_{fjt} \mid \boldsymbol{\xi}_{ft}] - \mathbb{E}_{t-1}[X_{fj,t-1} \mid \boldsymbol{\xi}_{f,t-1}]\right),$$

the difference between a firm's change in sales in a given industry j in the data (the first bracketed term) and its expected change in sales in the model (the second bracketed term, where $\mathbb{E}_t$ is an expectation operator

expressing the firm's ex-ante expected sales conditional on Θ *as well as macro parameters* $\boldsymbol{B}_t, Z_t, \boldsymbol{\alpha}, \boldsymbol{\varsigma}$*, as in equation 15). Under Assumptions 3 and 4, the following moments hold in expectation (for any* j, k*):*

$$\mathbf{E}_f \left[\Delta \epsilon_{fjt} \; \Delta \log S_{fkt} \;\middle|\; \boldsymbol{\chi}_{f,t-1} \right] = 0, \qquad \forall t = \{2, 3\}, \qquad \forall j, k \in \mathcal{J}. \tag{22}$$

In Appendix xxx, I derive analytical sample analogs for these micro moment conditions as functions of the data and the micro and macro parameters in Table 4. By estimating micro and macro parameters jointly, I account for the effect of changes in industry demand and supply conditions on firm sales growth. Macro parameters $\boldsymbol{B}_t, Z_t, \boldsymbol{\alpha}, \boldsymbol{\varsigma}$ behave in my micro moment conditions as non-linear fixed effects. For example, any changes in industry-wide demand (or supply) conditions between t and $t-1$ are reflected in differences between $\boldsymbol{B}_{jt}$ and $\boldsymbol{B}_{j,t-1}$, which affect output growth $\mathbb{E}_t[X_{fjt} \mid \xi_{ft}] - \mathbb{E}_{t-1}[X_{fj,t-1} \mid \xi_{f,t-1}]$ in the model.

Given the sparsity in firms' extensive margins in the data, I create four groupings of moments in each year $t = \{2, 3\}$. Each grouping contains an average over the following elements from the $\mathcal{J} \times \mathcal{J}$ matrix of moments: (i) main-diagonals $j = k$ for industries j with higher-than-average expenditure shares on knowledge inputs, (ii) remaining main-diagonals, (iii) off-diagonals $j \neq k$ for industry pairs j, k with higher-than-average knowledge-input proximity, and (iv) remaining off-diagonals.

Grouping moments according to same- versus cross-industry covariances helps identify scalability (ρ^{KLG}, ρ^{RES}) separately from rivalry ($\theta^{KLG}, \theta^{RES}$). Recall from Proposition 1 that scale and rivalry effects push in opposite directions for the response of sales to cross-industry shocks, but push in the same direction for the response of sales to same-industry shocks. For example, if a positive demand shock in industry k increases sales in another industry j of the firm, shared inputs used by k and j can be either more scalable or less rival (i.e., $\rho_m > \theta_m$) to match this covariance in the data. If, however, the same demand shock also raises sales in the same industry k of the firm, shared inputs must be sufficiently scalable on an absolute basis (i.e., ρ_m is high).

Next, grouping moments according to their knowledge-proximity helps identify parameters associated with shared knowledge inputs (ρ^{KLG}, θ^{KLG}) separately from that of residual shared inputs (ρ^{RES}, θ^{RES}). If industries with greater knowledge-proximity (parametrized by technology coefficients $\boldsymbol{\alpha}$ in the model) exhibit greater covariances of sales growth to demand shocks, it must be that the shared inputs used intensively by those industries (i.e., knowledge inputs) are more scalable and less rival than the residual shared input. Altogether these four groupings of moments provide sufficient identifying variation to estimate the four micro parameters ($\boldsymbol{\Theta} = \rho^{KLG}, \theta^{KLG}, \rho^{RES}, \theta^{RES}$) behind the joint production technology.

Table 5 presents estimates of input scalability and rivalry $\boldsymbol{\Theta}$ as well as the variances $\boldsymbol{\gamma}$ of fundamental firm profitability shifters. I use ten moment conditions: the four grouped moments above repeated for each of two years $t = 2, 3$, and two remaining cross-sectional moments (from the initial year $t = 1$, shown in Table 4), which identify the variances γ_0, γ_1 in the joint-lognormal

Table 5: Estimates of Scalability, Rivalry, and Firm Heterogeneity Parameters

Parameter	Description	Estimate	S.E.
ρ^{KLG}	Scalability of shared knowledge inputs	12.64	(0.39)
θ^{KLG}	Rivalry of shared knowledge inputs	3.61	(0.07)
ρ^{RES}	Scalability of residual shared inputs	2.63	(0.05)
θ^{RES}	Rivalry of residual shared inputs	4.06	(0.12)
γ_0	Degree of comparative advantage within the firm	0.85	(0.03)
γ_1	Variation in absolute advantage across firms	0.99	(0.05)
Test of Over-identifying Restrictions: $7.28 \sim \chi^2_4$			$p = 0.12$

Notes: This table reports estimates of micro parameters in the model. The six parameters are estimated on the sample of all multi-industry firms and all pairwise industries in which they are initially active, over years 2002-2007 and 1997-2002, using 10 moments. There are 13,000 (rounded) firm-year observations used in the sample. Standard errors of estimates are computed based on results from 21 bootstrap samples, where I re-draw over both the data and the simulated ξ samples.

distribution. Consistent with evidence in Section 1, the ranking $\rho^{KLG} > \theta^{RES} > \theta^{KLG} > \rho^{RES}$ implies that knowledge inputs induce stronger scale effects and weaker rivalry effects within the firm. In contrast, residual shared inputs induce weaker scale effects and stronger rivalry effects, which is consistent with negative cross-elasticities for industries that are not knowledge-proximate. For these baseline estimates I give each of the ten sample moments equal weight. Estimates do not change by much when using the optimal weighting matrix under two-step GMM. In the last row, I show that a test of over-identifying restrictions does not reject the null that the identifying moment conditions are jointly valid, suggesting the estimated parameters provide a good fit to these micro moments.

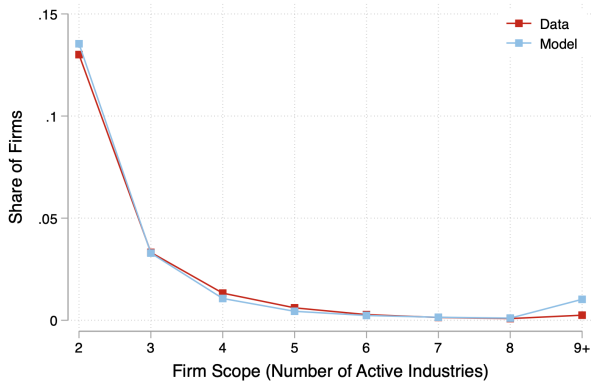
3.4 External Validation: Scale, Scope, and Industry Joint Production

Despite its limited number of (six) micro parameters, the estimated model reproduces other extensive-margin moments in the data not targeted in estimation. First, the model matches the distribution of the number of firms and their sales by firm scope in the data, shown in Figure 3. Both the data and the model attribute a significant size premium to the right tail of the firm scope distribution, though, for firms with nine or more industries, the model somewhat undershoots the data because it cannot account for the existence of true conglomerates and holding companies. Overall, the close fit between the model and data validate the Poisson and Fréchet functional form assumptions.

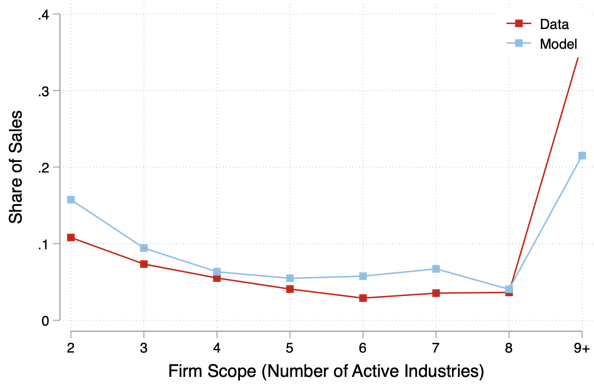
Next, I conduct a sharper test by comparing the model's predictions for *which* industries firms enter to their counterpart in the data. I create an asymmetric jk-level measure of co-production as

Figure 3: Model versus Data: Distribution of Firms and Sales by Scope in 1997

(a) Share of firms by scope



(b) Share of sales by scope



Notes: Panel (a) plots the distribution of firms by scope. Panel (b) plots the share of total sales accounted for by firms across the scope distribution. Data in 1997 is shown in green and model outcomes (computed using 1997 macro aggregates $\boldsymbol{B}_t$, $\boldsymbol{\alpha}$) are shown in orange.

the share of industry j sales by firms that also produce in k:

$$CoProd_{jk} \equiv \frac{\sum_f X_{fj} \mathbf{1}_{(X_{fk}>0)}}{\sum_f X_{fj}},$$

and I measure an industry-pair's knowledge-proximity similarly to equation (5) but with industry-level output X_j in place of firm-level output X_{fj}:

$$Prox_{jk}^{KLG} \equiv \sum_{m \in \mathcal{M}^{KLG}} \beta_{jm} \frac{\beta_{km} X_k}{\sum_{k' \in \mathcal{J}} \beta_{k'm} X_{k'}}. \tag{23}$$

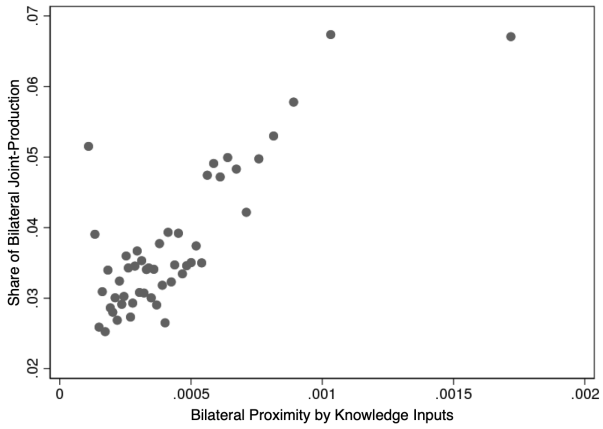
Panel (a) of Figure 4 finds that, in the data, co-production increases with knowledge input-proximity. In panel (b), I show that the estimated model reproduces this strong bilateral positive correlation. Co-production is more prevalent across these industries because knowledge inputs are more scalable (resulting in a higher overall arrival rate of ideas) and less rival (resulting in ideas being adapted to improve knowledge in a wider set of industries). These results provide external validation to the model estimates given that the cross-sectional patterns of co-production were not targeted during estimation.

4 The Macroeconomic Implications of Joint Production

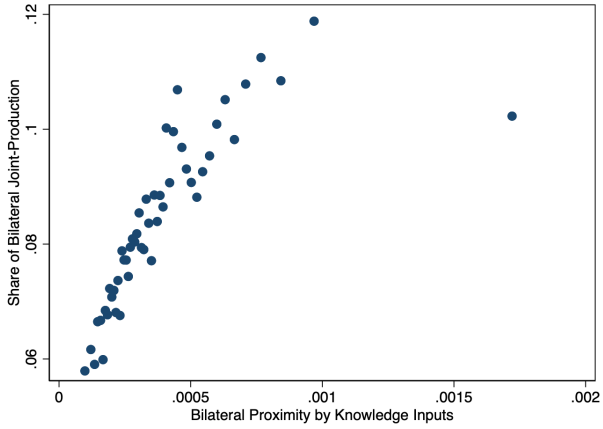
Under the estimated model parameters, joint production generates aggregate increasing returns to scale in the US manufacturing sector. Given the partial non-rivalry of shared inputs within the firm, increasing output in one industry will on average lower prices in not only the same industry

Figure 4: External Validity: Co-production in the Data and the Model in 1997

(a) Co-production in the Data



(b) Co-production in the Estimated Model



Notes: These panels correlate bilateral industry co-production (share of industry j sales by firms with activities in k) with bilateral knowledge-proximity, in (a) the data, and (b) the model.

but also others. I analytically characterize these industry linkages in general equilibrium using a matrix of same- and cross-industry elasticities of the producer price index to demand shocks. I use these macro elasticities to decompose aggregate increasing returns into same and cross-industry components and re-assess the implications of trade policy.

4.1 Joint Production in General Equilibrium

To highlight the impact of joint production, I close the model under general equilibrium assumptions that rule out any other cross-industry interdependence. Definition 1 in Quantitative Appendix C.5 lays out the equilibrium conditions. Consumer demand is Cobb-Douglas across industries, which shuts down demand-side linkages. The stage-II, industry-specific input composite l_{fj} consists of only labor, which shuts down conventional input-output linkages.[30] The US economy (denoted u) trades with a set of foreign partners (denoted $d \in \mathcal{D}^F$) each with exogenous macro aggregates (demand levels and foreign firm price indices), which shuts down cross-country linkages. I pin down wages by assuming that there is a large-enough non-manufacturing sector in which US exporters face infinitely elastic foreign demand. This assumption keeps wages fixed across counterfactuals, since overall trade can balance via changes in the non-manufacturing sector's net exports. I fix the total number of firms at N (which still allows for firm entry and exit across industries, as well as in and out of active status). Finally, all firm profits and tariff revenues

[30] In Appendix xxx, I prove a more general version of Proposition 3 that accommodates arbitrary stage-II input-output linkages across manufacturing industries. I make this simplifying assumption here only to isolate the quantitative impact of joint production, and doing so is not inconsistent with prior sections of the paper. Recall that estimation of model parameters in Section 3 does not require taking a stance on the input-output structure of the economy. Price effects from changes in stage-II input costs are absorbed by the industry profitability shifter $\boldsymbol{B}_t$.

are spent on the non-manufacturing sector, which shuts down expenditure-driven feedback effects.

The key endogenous macro aggregates are domestic industry-level producer price indices:

$$\mathcal{P}_j^{1-\sigma_j} \equiv N \int \mathbb{E}\left[p_{fj}^{1-\sigma_j}\right] \mathrm{d}G(\boldsymbol{\xi}), \qquad \forall j \in \mathcal{J},$$

which depend on joint production decisions undertaken by individual firms f. Proposition 3 derives the general equilibrium elasticity of industry PPI and output with respect to any exogenous shifter of industry market size $\boldsymbol{S}$ (e.g., from shocks to population size, foreign demand, or or foreign prices).

Proposition 3 (Industry Linkages from Joint Production) *Under the open economy general equilibrium conditions provided in Definition 1, domestic producer price indices* $\mathrm{d}\log \boldsymbol{\mathcal{P}}$ *and output* $\mathrm{d}\log \boldsymbol{X}$ *respond to exogenous shocks to industry market size* $\mathrm{d}\log \boldsymbol{S}$ *(defined in equation 36) according to:*

$$\mathrm{d}\log \boldsymbol{\mathcal{P}} = diag\left(\frac{1}{\boldsymbol{\sigma}-1}\right)\left(\mathbb{I} + \boldsymbol{\Psi}\ diag(\boldsymbol{\lambda}^{cpt})\right)^{-1}\boldsymbol{\Psi}\,\mathrm{d}\log \boldsymbol{S}, \tag{24}$$

$$\mathrm{d}\log \boldsymbol{X} = \mathrm{d}\log \boldsymbol{S} + diag(\boldsymbol{\lambda}^{cpt}(1-\boldsymbol{\sigma}))\,\mathrm{d}\log \boldsymbol{\mathcal{P}} \tag{25}$$

where (i) $\mathbb{I}$ *is the identity matrix, (ii)* $\boldsymbol{\Psi}$ *is a macro joint production matrix containing inverse cross-industry supply-side elasticities* $\boldsymbol{\Upsilon}$ *for the 'average' firm:*

$$\begin{aligned}
[\boldsymbol{\Psi}]_{jk} &\equiv \sigma_j(1-\varsigma_j)[\boldsymbol{\Upsilon}^{-1}]_{jk} - \mathbf{1}_{j=k}, \\
[\boldsymbol{\Upsilon}]_{jk} &\equiv \sum_{m\in\mathcal{M}} (\rho_m - \theta_m)\,\bar{\lambda}_{jm}\,\bar{\mu}_{jmk} + \mathbf{1}_{j=k}\sum_{m\in\mathcal{M}} \theta_m\,\bar{\lambda}_{jm}, \qquad \forall j,k \in \mathcal{J},
\end{aligned}$$

where industry choice shares $\bar{\mu}_{jmk}$ *indicate the average propensity for type m-ideas to be adapted in industry k (relative to other industries k') among firms that produce in j, and input utilization shares* $\bar{\lambda}_{jm}$ *indicate the average profit-contribution to industry j of shared input m (relative to other shared inputs m'):*

$$\bar{\mu}_{jmk} \equiv \int \frac{\mathbb{E}[X_{fj}]\lambda_{fjm}}{\int \mathbb{E}[X_{fj}]\lambda_{fjm}\,\mathrm{d}G(\boldsymbol{\xi})}\mu_{fmk}\,\mathrm{d}G(\boldsymbol{\xi}), \qquad \bar{\lambda}_{jm} \equiv \int \frac{\mathbb{E}[X_{fj}]}{\int \mathbb{E}[X_{fj}]\,\mathrm{d}G(\boldsymbol{\xi})}\lambda_{fjm}\,\mathrm{d}G(\boldsymbol{\xi}),$$

and (iii) $\boldsymbol{\lambda}^{cpt}$ *reflects the potential for US firms to gain share from foreign competitors in each market d:*

$$\lambda_j^{cpt} \equiv \sum_{d\in\{u,\mathcal{D}^F\}} \lambda_{dj}^X(1-\lambda_{dj}^M) \qquad \forall j \in \mathcal{J},$$

where λ_{dj}^M *is the share of country d's industry j consumption originating from US firms, and* λ_{dj}^X *is the share of US firms' industry j sales exported to d.*

The joint production matrix $\boldsymbol{\Psi}$ encapsulates the equilibrium impact of demand shocks on

industry-level PPI and highlights two within-firm sources of aggregate increasing returns to scale.[31] First, economies of scale (same-industry elasticities of cost with respect to output) are governed by the scalability of stage-II industry-specific inputs (γ) and stage I shared inputs (ρ). Second, economies of scope (cross-industry elasticities of cost with respect to output) are governed by the relative scalability and rivalry of shared inputs ($\rho - \theta$).

For intuition on how these forces manifest in equilibrium, first consider an economy under autarky, so $\lambda^{cpt} = 0$ and equation (24) simplifies to:

$$\mathrm{d}\log \boldsymbol{\mathcal{P}} = \mathrm{diag}\left(\frac{1}{\boldsymbol{\sigma}-1}\right)\boldsymbol{\Psi}\,\mathrm{d}\log \boldsymbol{S}.$$

The relative magnitude of economies of scale and scope depend on own-diagonal versus off-diagonal elements of the matrix $\boldsymbol{\Psi}$. I analyze each in turn.

Economies of Scale. When off-diagonal elements of $\boldsymbol{\Psi}$ are zero, the only force present is same-industry economies of scale. This occurs when either (i) shared inputs are in fact industry-specific (so that $\bar{\lambda}_{jm}\bar{\mu}_{jmk} = 0\ \forall m$), or (ii) scale and rivalry effects offset each other on a knife's edge ($\rho_m = \theta_m\ \forall m$). Whereas producer price indices in each industry are unaffected by demand shocks in any other industry, the main-diagonals of $\boldsymbol{\Psi}$ still allow for arbitrary same-industry returns to scale. Equation (24) simplifies further to:

$$\mathrm{d}\log \mathcal{P}_j = \frac{1}{\sigma_j - 1}\left(\frac{\sigma_j(1-\varsigma_j)}{\sum_m \bar{\lambda}_{jm}\rho_m} - 1\right)\mathrm{d}\log S_j, \qquad \forall j \in \mathcal{J},$$

with an elasticity bounded between $(-\frac{1}{\sigma_j - 1}, 1)$, nesting the range of elasticities in Kucheryavyy et al. (2019). The more scalable are stage I inputs (ρ_m) and stage II inputs (recall $\gamma_j = \varsigma_j \frac{\sigma_j}{\sigma_j - 1}$), the more negative is this elasticity, and the stronger are industry-level economies of scale.

Two limit cases are worth highlighting. First, when $\rho_m \to 1$, knowledge accumulation in stage I is exogenous and non-responsive to changes in industry profitability. The response of industry prices to demand shocks depends only on the scalability γ_j of stage II industry-specific inputs. Under, for example, constant returns to scale in stage II production ($\gamma_j = 1$), the matrix $\boldsymbol{\Psi}$ is element-wise 0 (since $\sigma_j(1-\varsigma_j) = \sigma_j - \gamma_j(\sigma_j - 1) = 1$), and prices are affected by neither same-industry nor cross-industry demand shocks. Outside of this knife-edge value of γ_j, higher values generate economies of scale ($\gamma_j > 1$), and lower values ($\gamma_j < 1$) generate diseconomies of scale.

Industry-level economies of scale also increase with the scalability of shared inputs in stage I, ρ_m. When stage-I shared inputs are scalable, firms easily respond to an increase in market size by accumulating more knowledge capital and lowering marginal costs of production, without

[31] While Proposition 3 suffices for the analysis in the rest of the section (evaluating the impact of counterfactual shocks to foreign demand), I provide more theoretical results in the Supplementary Online Notes. I show how the joint production matrix $\boldsymbol{\Psi}$ characterizes (i) partial-equilibrium aggregate cross-price elasticities of *supply*, and (ii) how supply-side rather than demand side shocks (e.g., industry-level TFP) propagate.

having to run into potential decreasing returns to scale in stage II. For any value of γ_j, in the limit as $\rho_m \to \infty$ for any m, industry-level economies of scale reaches its maximum strength (at $-1/(\sigma_j - 1)$). This limit reproduces the same elasticity of industry prices with respect to output as that in a standard multi-industry Krugman (1980) model. One difference, however, is that scale economies in my model are internal to the firm. They come from (quality-adjusted) cost improvements rather than new varieties by entrants.

Economies of Scope. On the other hand, economies of scope increase with the scalability and non-rivalry of stage-I shared inputs, $\rho_m - \theta_m$, which shape the off-diagonal elements of $\boldsymbol{\Psi}$. To understand this force, consider an economy with two symmetric industries, ex-ante identical firms, identical demand elasticities ($\sigma = 5$), constant returns to scale in stage-II production ($\gamma = 1$), and a single type of shared input that is scalable and partially non-rival ($\rho = 7$ and $\theta = 3$). In this case, using the fact that $\sigma(1 - \varsigma) = 1$, $\bar{\lambda} = 1$, and $\bar{\mu} = 0.5$, equation (24) reduces to:

$$\mathrm{d}\log \boldsymbol{\mathcal{P}} = \underbrace{\begin{pmatrix} 0.25 & 0 \\ 0 & 0.25 \end{pmatrix}}_{\mathrm{diag}\left(\frac{1}{\sigma-1}\right)} \underbrace{\left[\frac{1}{21}\begin{pmatrix} 5 & -2 \\ -2 & 5 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]}_{\boldsymbol{\Psi}=\boldsymbol{\Upsilon}^{-1}-\mathbb{I}} \mathrm{d}\log \boldsymbol{S} = \begin{pmatrix} -0.191 & -0.024 \\ -0.024 & -0.191 \end{pmatrix} \mathrm{d}\log \boldsymbol{S}.$$

A demand shock that raises profitability in one industry lowers firms' marginal costs in other industries the more that shared inputs are scalable and non-rival ($\rho > \theta$). This is reflected in positive cross-elasticities in $\boldsymbol{\Upsilon}$, which corresponds to the *firm-level* elasticities of sales with respect to demand shocks from Proposition 1. In equilibrium, these changes within each firm affect output price indices (PPI) and therefore competition in other industries, triggering additional changes in firms' production decisions. Similar to the Leontief inverse, $\boldsymbol{\Psi} = \boldsymbol{\Upsilon}^{-1} - \mathbb{I}$ captures the equilibrium impact of an industry demand shock on the price index as it percolates across industries and firms. In the parametrization above, a 1% increase in demand in the first industry lowers the PPI in the same industry by 0.19% and lowers the PPI in the other industry by 0.02%, one-eight the size of the own-industry effect. Outside of this toy setting, of course, cross-elasticities are asymmetric and unrestricted in sign. Cross-elasticities can even be positive if shared inputs were rival and non-scalable, indicating *diseconomies* of scope.

Open Economy Effects. Finally, under the presence of trade, a decline in producer prices allow domestic firms to win market share against foreign producers (in both the home market and abroad). The equilibrium impact of demand shocks on producer prices (equation 24) depends on an additional term, $\left(\mathbb{I} + \boldsymbol{\Psi}\ \mathrm{diag}(\boldsymbol{\lambda}^{cpt})\right)^{-1}$. Once again, the matrix inverse captures how domestic firms' market share changes against foreign firms generate additional changes in firm scale that in turn trigger further rounds of changes in the PPI.

4.2 Quantifying the Impact of Joint Production

I use Proposition 3 to decompose aggregate increasing returns in the US manufacturing sector into that contributed by economies of scale (same-industry elasticities) versus economies of scope (cross-industry elasticities). Whereas many different models can generate increasing returns at the industry-level (e.g., a Krugman model with free entry, or a perfectly competitive model with external economies of scale), economies of scope represent a new within-firm channel specific to joint production.

Making headway on this question requires knowledge of CES industry demand elasticities σ. Up until now, estimation of supply-side parameters ρ, θ *conditioned* on observable general equilibrium changes and firm input expenditure shares ς (a combination of σ and γ), absolving the need for σ. However, Proposition 3 illustrates that *counterfactual* changes in the economy in response to shocks depend separately on values of σ and γ. Under monopolistic competition, σ mediates firms' profit incentives and therefore the extent to which they increase production in response to a demand shock.

In my baseline estimates, I calibrate demand elasticities σ so that my model generates the same sector-level increasing returns to scale as estimates in Bartelme et al. (2019). In Appendix xxx, I show that the contribution of economies of scope towards aggregate increasing returns (in level terms) is not sensitive to using other direct estimates of σ in the literature or to alternative calibration strategies. I calibrate remaining aggregate parameters of the model to fit data on the US economy trading with two foreign regions: China, and the rest of the world. I calibrate foreign price indices and expenditure levels so the model's equilibrium exactly matches industry-level production and trade data from 2017, as detailed in Quantitative Appendix C.8.

Under this calibration, I simulate a proportional change in foreign demand in each industry k (so $\mathrm{d}\log S_k = 1 - \lambda^X_{uk}\ \forall k$, the share of industry sales that are exported), and use Proposition 3 to compute the aggregate scale elasticity, defined as the elasticity of overall manufacturing PPI with respect to output:

$$\begin{aligned}\left.\frac{\mathrm{d}\log PPI}{\mathrm{d}\log X}\right|_{\mathrm{d}\log \boldsymbol{S}} &= \left.\frac{\sum_{j\in\mathcal{J}}\lambda^X_j\,\mathrm{d}\log\mathcal{P}_j}{\sum_{j\in\mathcal{J}}\lambda^X_j\,\mathrm{d}\log X_j}\right|_{\mathrm{d}\log \boldsymbol{S}} \\ &= \underbrace{\left.\frac{\sum_{j\in\mathcal{J}}\lambda^X_j\,\mathrm{d}\log\mathcal{P}^{(\text{same})}_j}{\mathrm{d}\log X}\right|_{\mathrm{d}\log \boldsymbol{S}}}_{\text{economies of scale}_k} + \underbrace{\left.\frac{\sum_{j\in\mathcal{J}}\lambda^X_j\,\mathrm{d}\log\mathcal{P}^{(\text{cross})}_j}{\mathrm{d}\log X}\right|_{\mathrm{d}\log \boldsymbol{S}}}_{\text{economies of scope}_k},\end{aligned} \tag{26}$$

where weights λ^X_j represent industry j's share of sales within manufacturing. The second line decomposes the overall change in prices into those due to same-industry elasticities versus cross-industry elasticities (i.e., main and off-diagonals of the transmission matrix in equation 24). In my general equilibrium setting, cross-elasticities reflect the impact of any economies of scope.

The top panel of Figure 5 illustrates the relative contribution of economies of scope versus scale towards aggregate increasing returns. I estimate that economies of scope from joint production constitute one-quarter of aggregate increasing returns in US manufacturing. Aggregate manufacturing prices fall with respect to output with an elasticity of -0.16, with -0.04 (the darker bar) resulting from cross-elasticities and -0.12 (the lighter bar) coming from same-industry elasticities. In other words, economies of scope cause aggregate US producer prices to fall by 0.4 percent for every 10 percent increase in output induced by foreign demand. This quantitatively large spillover would be absent in a model that does not take into account joint production.

Moreover, I find that economies of scope are disproportionately concentrated among clusters of knowledge-proximate industries. In the bottom panel of Figure 5, I provide a disaggregated visualization of how demand shocks impact price indices. I display cross- and same-sector elasticities for *sectors* defined at the NAICS 4-digit level (there are 86 such sectors in manufacturing, each containing one or more *industries*). Each cell in the matrix represents the contribution to overall increasing returns from price changes in the row sector m induced by demand shocks in a column sector n. For example, the mn cell in the matrix in Figure 5 measures

$$\left.\frac{\sum_{j\in m}\lambda_j^X \,\mathrm{d}\log \mathcal{P}_j}{\mathrm{d}\log X}\right|_{\mathrm{d}\log \boldsymbol{S_n}},$$

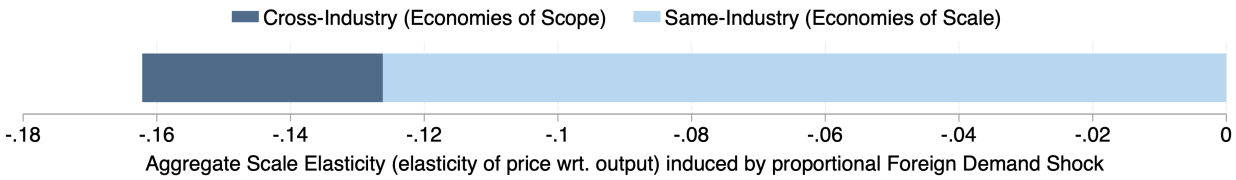
where $\mathrm{d}\log \boldsymbol{S_n}$ is the vector containing $(1-\lambda_{uk}^X)$ for all industries $k \in n$ and 0 everywhere else. Down each column n (a given demand-shocked sector), off-diagonal cells over rows $m \neq n$ sum to the net effect of economies of scope that manifest across sectors, while the main diagonal cell $m = n$ consists of both same-industry economies of scale and economies of scope that manifest across industries *within* that sector n.

While the main diagonals are strongly negative and reflect the contribution of within-sector economies of scale and scope, a substantial amount of economies of scope manifest even *across* four-digit manufacturing sectors. Cross-industry elasticities, indicated by the off-diagonal values, are heterogeneous and asymmetric. Sectors such as computers are strong contributors to aggregate increasing returns via economies of scope, while other sectors such as aerospace products are strong beneficiaries. For example, a demand shock to computers and peripherals (NAICS 3341) that raises manufacturing output by 10 percent trigger price declines in other sectors that lower the PPI by a total of 1 percent. This is indicated by the strong negative *column* for NAICS 3341 in the matrix. On the other hand, prices changes in just the aerospace products (NAICS 3354) sector lower the PPI by 2.4 percent for every 10 percent increase in manufacturing output caused by demand shocks in other sectors. This is indicated by the strong negative *row* for NAICS 3354 in the matrix.

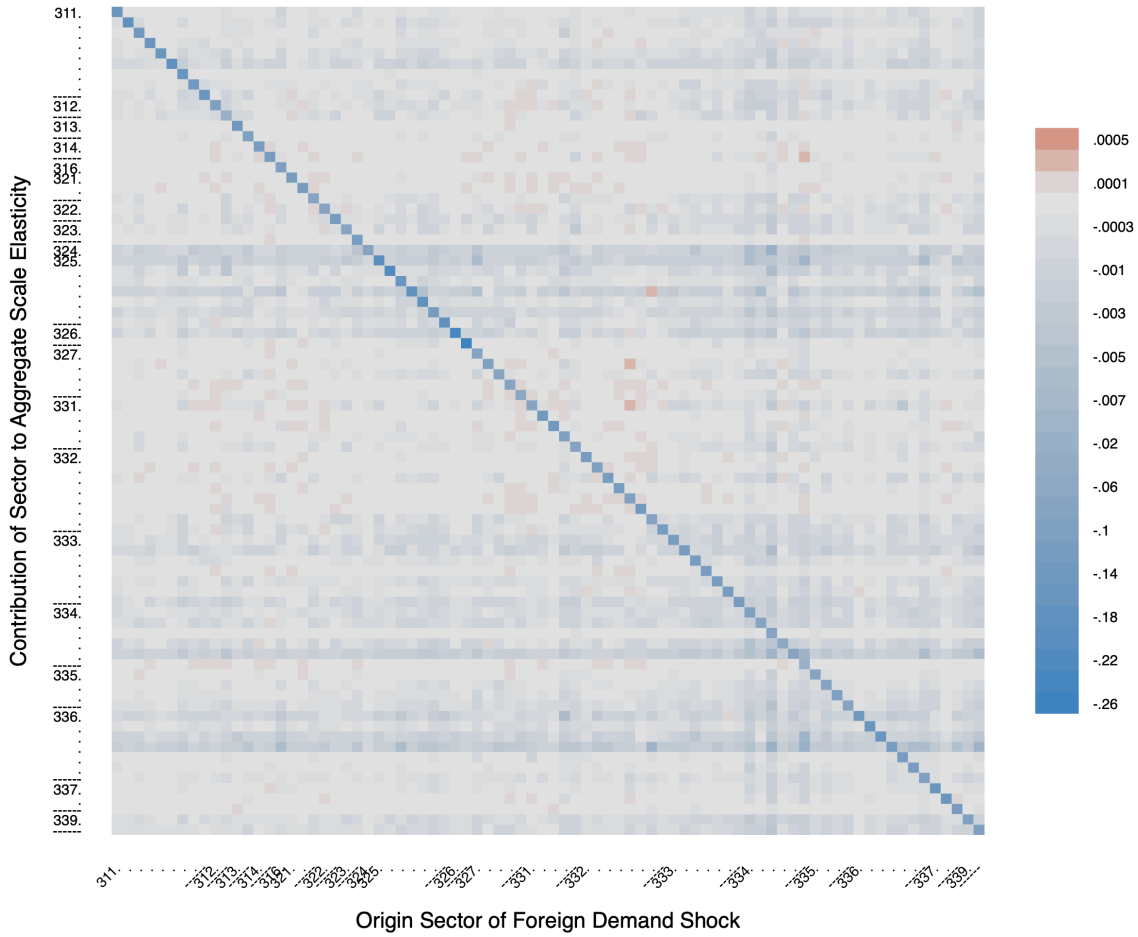
Other sectors that are less knowledge-proximate have cross-elasticities that are much smaller in magnitude. In fact, cross-elasticities are mildly positive for 264 sector pairs (four percent of all

Figure 5: Decomposition of Aggregate Scale Elasticity under Joint Production

(a) Economies of Scale versus Scope



(b) Contribution of row-industry PPI change in response to column-industry demand shock



Notes: Panel (a) decomposes the aggregate scale elasticity (the total elasticity of the manufacturing PPI with respect to manufacturing output in the US) caused by a proportional increase in foreign demand in all industries into same-industry versus net cross-industry impacts. Panel (b) illustrates shock propagation at the bilateral sector (NAICS 4-digit) level. Each cell highlights the contribution to the aggregate scale elasticity by industries in the row sector given a proportional increase in foreign demand in all industries in the column sector.

pairs), indicating *diseconomies* of scope, which I shade in red. For example, glass manufactures (NAICS 3272) and metal hardware (NAICS 3325)—two industries with the strongest diseconomies of scope—share more residual inputs such as capital and administrative services than they do knowledge. Prices of glass manufactures are predicted to *rise* in response to a demand shock for metal hardware as firms reallocate scarce and rivalrous residual shared inputs m ($\rho_m < \theta_m$) away from glass products and towards metal hardware.

Figure 6 offers systematic evidence that joint production among more knowledge-intensive industries generates stronger economies of scope. To illustrate this heterogeneity, I decompose the equilibrium scale elasticity induced by foreign demand shocks into same and cross-industry components for one shocked industry k at a time. Panel (a) of Figure 6 plots the contribution of cross-elasticities (economies of scope) towards the aggregate scale elasticity (y-axis) against the shocked industry's expenditure share on knowledge inputs (x-axis). The magnitude of economies of scope is strongly increasing in knowledge intensity. Panel (b) plots the same estimates of economies of scope on the y-axis against economies of scale on the x-axis. Interestingly, industries that induce strong economies of scope (computers, electrical equipment, machinery) tend to induce lower economies of scale, and vice versa for industries like chemicals and plastics. Focusing only on within-industry returns to scale would overweigh the contribution toward aggregate increasing returns of the chemicals and plastics industries relative to the computer electronics or medical equipment (part of Misc. sector) industries.

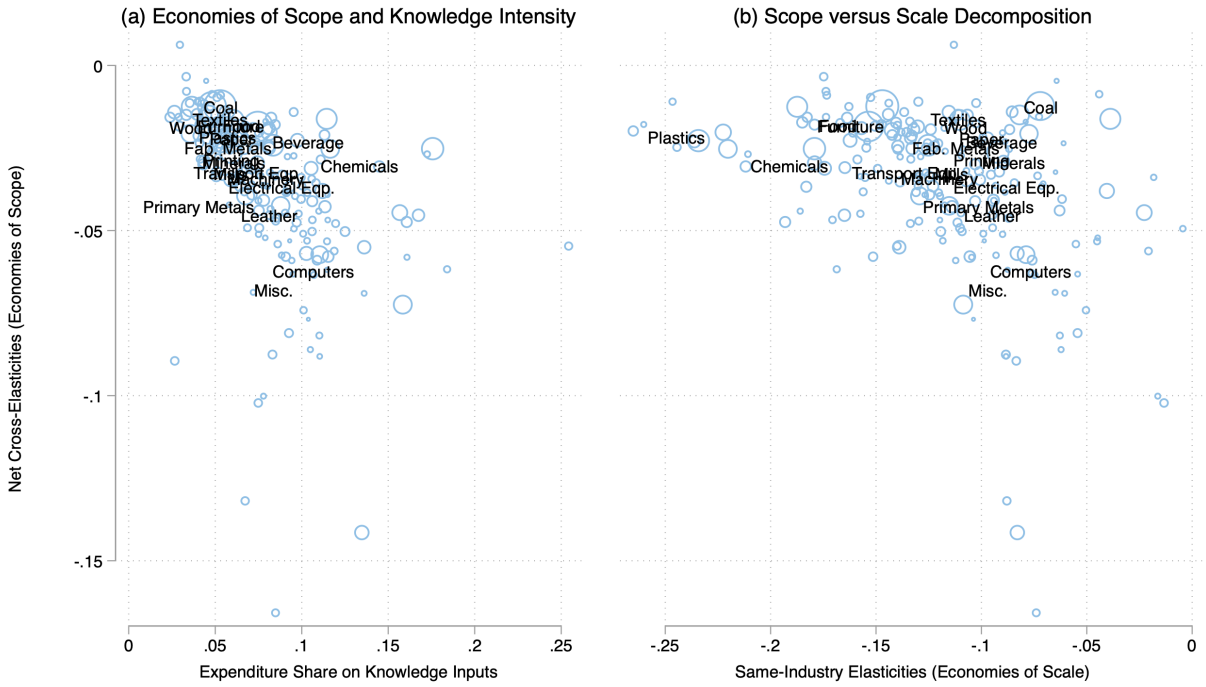
4.3 Joint Production in the US-China Trade War

Finally, I demonstrate that the endogenous responses of producer prices to market size in my model are quantitatively important in light of a real shock to manufacturing. I analyze the impact of bilateral import tariffs applied since the ongoing US-China "trade war" since 2018.

I compare the predictions of my model against an alternative production-side assumption where firms operate linear, nonjoint, constant-returns production functions. Under joint production, US domestic producer prices respond to market size changes due to economies of scale and scope. US tariffs on imports from China protect US firms from competition and expand their market access, while Chinese tariffs on imports from the US restrict US firms and reduce their market access. In comparison, under linear production, domestic producer prices do not change absent other general equilibrium forces.

I find that the difference in the producer price response between these two models are large in light of actual tariff changes applied during the US-China trade war. I average data on applied tariffs by commodity (HS10 for US imports and HS8 for US exports) from Falgelbaum et al. to the level of the 206 industries used in my paper (using, respectively, US imports and exports to China by product as weights). I calibrate the initial equilibrium of the model to fit data in 2017, before the onset of the trade war. I then solve for counterfactual changes in equilibrium price indices (both US consumption price indices and producer price indices by industry) in response to these tariffs,

Figure 6: Estimates of Economies of Scope, Scale, and Knowledge Intensity by Industry



Notes: This graph plots on the y-axis, for a demand shock in each given industry, the net contribution to the aggregate scale elasticity due to changes in *other* industries' PPI. Panel (a) plots on the x-axis the shocked industry's knowledge input expenditures as a share of output. Panel (b) plots on the x-axis the contribution to the aggregate scale elasticity due to changes in *same, shocked* industry' PPI. The size of each blue circle is proportional to the industry's gross output, and a list of top and bottom industries on the y-axis can be found in Table C.8. Overlaid in black text are additional scatterplots of the same y and x statistics but aggregated at the level of broad NAICS 3-digit sectors.

Table 6: Impact of the US-China Trade War on US Manufacturing

	Linear (CRS) Production		Joint Production	
Import tariffs by:	US only	US + China	US only	US + China
Change (%) in U.S. Manuf. Sector Outcome				
CPI	0.76	0.76	0.50	0.61
PPI	0	0	-0.37	-0.17
Imports from China	-37.23	-37.23	-38.23	-38.09
Imports from RoW	6.29	6.29	4.91	5.52
Exports to China	0	-36.76	1.59	-36.82
Exports to RoW	0	0	2.01	0.63
Output	1.53	0.92	2.46	1.34
Manufacturing Sector Trade Deficit	-12.32	-7.39	-19.72	-10.75
US Tariff Revenues as share of initial Manuf. Output	0.57	0.57	0.55	0.55

Notes: This table presents estimates of the impact of two different sets of tariffs on outcomes in the US economy, under two different model settings calibrated to match US industry-level aggregates in 2017. Column (1) estimates the impact of a 20% increase in tariffs on Chinese imports across all industries, under a neoclassical model with nonjoint production, constant returns to scale and perfect competition. Column (2) compares the impact of the same set of tariffs to my model with joint production. Column (3) presents results from alternative tariffs identified by my model that minimize the CPI impact while achieving the same reduction in total imports from China. See Definition 1 for a characterization of the equilibrium and Appendix Section C.10.3 for details on how I compute alternative tariffs.

holding all else equal. I develop and apply an "exact hat" system of equations to solve for changes in endogenous variables (industry price indices) in response to any set of exogenous shocks under the equilibrium given by Definition 1. More details are provided in Appendix C.10.

Table 6 compares the impact of US-China tariffs on various economy-wide aggregates. I first compute the impact of unilateral US tariffs on imports from China, before computing the full impact after retaliatory tariffs by China on imports from the US. In the first two columns, under linear production, changes in market size have no effect on producer prices (recall that wages are pinned down by a non-manufacturing sector and there is no entry and exit of firms). The only effect of US tariffs is to raise consumer prices by 0.76 percent, while expanding manufacturing output by 1.5 percent. Retaliatory tariffs have no additional impact on the US CPI but of course reduces US manufacturing output by 0.6 percent, so that on net output increases by only 0.9 percent.

In the last two columns of Table 6, I find that the price impacts of tariffs are substantially different under joint production. Given the sizable estimates of economies of scale and scope, US producer prices fall by 0.4 percent as unilateral import protection expands US firms' market size. Each industry-level import tariff causes prices of US goods to fall not only in that same industry but also in other, knowledge-proximate industries. Altogether these producer price declines offset about one third of the CPI impact under linear production, so that the consumption price index rises by only 0.5 percent. Other margins of the US economy also improve compared to the case of linear production. As US producer prices fall, manufacturing exports rise, the deficit shrinks, and import substitution towards the rest of the world is less pronounced.

However, while joint production mitigates the harms of domestic import protection, it also amplifies the harms of foreign import protection. In the last column, retaliatory tariffs by China

restrict the foreign market access of US firms and push up producer prices, offsetting a majority of the aggregate producer price decline. US producer prices decrease by only 0.17 percent compared to 0.37 percent under unilateral tariffs, leading to an overall CPI increase of 0.61 percent.

While this brings the aggregate CPI change closer to the alternative model of linear production, the aggregate impact masks substantial heterogeneity at the industry level. Because the industries facing restricted market access due to Chinese tariffs are different from those facing import protection due to US tariffs, the distribution of the changes in industry prices are actually wider after Chinese retaliation. Producer prices rise by more than one percent in optical instruments, pulp, computers, broadcast and wireless communications equipment and small electrical appliances. Producer prices fall by more than three percent in lighting fixtures, furniture, textiles, and printing ink. These results illustrate the potential for tariffs to alter a nation's comparative advantage and trade structure—not only in directly affected industries, but also in other industries linked under joint production.

While the general equilibrium assumptions used in these counterfactuals are stark, it is straightforward to extend the model to feature firm entry, exit, endogenous input price changes, or input-output linkages. These additional details introduce further interactions but all preserve the intuition conveyed by the quantitative results thus far. Joint production represents a sizable mechanism through which shocks propagate across industries. Economies of scope generate large, negative cross-elasticities of price with respect to output in knowledge-proximate industries. On the whole, joint production represents an economically significant and new channel for aggregate responses of the economy to shocks.

Conclusion

Much of the existing literature in international trade and macroeconomics assumes that firms operate independently across industries. I show this assumption is inconsistent with the behavior of manufacturing firms in US micro-data. A demand shock in one industry of a firm increases its sales in another industry the more that the two industries share knowledge inputs.

This paper develops a model of joint production to explain and quantify such interdependence within the firm. Whereas solving for a firm's decisions under interdependence is typically a hard computational problem, I provide a micro-foundation where firms use shared inputs to accumulate knowledge in a firm's industries under an endogenous stochastic process. This convexifies the firm's *ex-ante* decisions and yields analytical expressions for the firm's extensive and intensive margins in each industry. I estimate that knowledge inputs stand out from other shared inputs in terms of their scalability and non-rivalry in joint production. Output among knowledge-proximate industries are complements in production.

Joint production within the firm generates a new dimension of cross-industry linkages in the aggregate. Firms derive economies of scope from the scalability and rivalry of shared knowledge

inputs. I find that this intra-firm mechanism is quantitatively important in the aggregate. On average a demand shock that raises output by 10 percent would lower prices in other industries by 0.4 percent. This accounts for more than one quarter of conventional values of aggregate increasing returns in US manufacturing. Endogenous price responses under joint production suggest that trade policy and market size are determinants of comparative advantage across countries. Moreover, the concentration of economies of scope among knowledge-proximate industries highlight sensitive industry clusters that could particularly benefit from unilateral import protection as well as be harmed by retaliatory tariffs.

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ONLINE APPENDICES - IN PROGRESS AND SUBJECT TO CHANGE

A Data Appendix

A.1 Data Construction and Details

Firms, Plants, and Products. I assemble data from the Economic Censuses (EC), the Longitudinal Business Database (LBD), and the Longitudinal Firm Trade Transactions Database (LFTTD) from 1997 to 2012. The Censuses are conducted quinquennially in years ending with '2' and '7'. Data on product shipments made by establishments come from the product trailer (PT) files which are attached to the Census of Manufactures (CMF). These trailer files contain responses of establishments that are sent a CMF 'Long Form'. The long form is sent to all establishments belonging to multi-establishment firms as well as a sample of single-establishment firms. The long form elicits shipments made by the establishment at a disaggregated level (varying from 6 to 10 digit NAICS).[32]

Using firm identifiers in the LBD, I match establishments to their parent firms and aggregate industry-level shipments to the level of the firm. The firm identifier in the LBD comes from information the Census collects from the Company Organization Survey and from tax identifier and plant identifier information in the Business Register. An establishment is a physical location where business activity occurs. The firm is defined (by the Census) as the highest level entity that controls more than 50% of each of the establishments assigned to the firm. I drop establishments that are administrative records (for which sales data is imputed).

External Sales. The CMF contains data on the shipments of a plant made to other plants within the same firm. However, this data is not broken down at the product-line level. For plants that produce in multiple industries, I apportion this inter-plant shipment data into industry-level intra-firm shipments using shares taken from the plant's total sales across industries. I then define the external sales of a firm in each industry as its total sales in that industry minus its intra-firm shipments. I drop external sales computed in this way in any industries of the firm that (i) account for less than 0.5% of firm-wide external shipments and (ii) are never the main produced industry of any plant the firm owns. This is conservative and allows product shipments in very small industries of the firm to be entirely intra-firm. This also prevents the spurious adding / dropping of products simply because of changes to the PT forms over the years.

Firm Trade Data. I use two sources. First, the LFTTD contains the value of all import and export transactions, by trading country and by HS10 product, that each firm entity (a set of EIN tax codes) is a counter-party to. Second, the CMF also contains data on plant-level shipments that are ultimately destined for export markets (whether directly or indirectly through an intermediary). If the plant is a multi-industry plant, I apportion this plant-level shipment across the plant's industries using product trailer product shipment shares. I use both LFTTD and CMF data on exports to construct the export demand shock, detailed below. Data on firm exports and imports reported in Table 1 come from the LFTTD.

Country-level Trade Data. I use data from BACI and Comtrade (bilateral country-level trade flows at the HS6 level) to generate the five-year growth rates in imports of a destination n in product h used in the analysis, $\Delta \log IMP_{nht}$.

Knowledge Inputs. I use BEA data from 1997 to collect input expenditure data by industry. Table

[32] This procedure is likely to underestimate the significance of multi-product activity in the US economy for two reasons. First, the long-form elicits questions about product sales over a pre-specified list of products (specific to the plant's classified industry). Although there is space for the firm to report shipments in products not covered by that pre-specified list, in practice firms rarely do. Second, the long-forms do not cover all single-establishment firms in the economy. A single-establishment firm could be selling in multiple industries but would not report the breakdown of its sales over these industries unless it was sent a long-form.

Table A.1: Definition of Knowledge Inputs and their Use in Manufacturing in 1997

		Expenditures as Share of Gross Output (%)		
Code	Description	Mean	25th pctile	75th pctile
550000	Management of companies and enterprises	3.54	2.60	4.94
541700	Scientific research and development services	0.62	0.25	0.96
541300	Architectural, engineering, and related services†	0.62	0.31	0.96
5419A0	All other professional, scientific, and technical services	0.61	0.61	0.63
541511	Custom computer programming services†	0.58	0.21	0.86
541800	Advertising, public relations, and related services	0.48	0.14	0.62
541610	Management consulting services	0.28	0.28	0.30
541100	Legal services	0.28	0.09	0.30
541200	Accounting, tax prep., bookkeeping, & payroll services	0.15	0.08	0.21
541400	Specialized design services	0.09	0.01	0.02
541512	Computer systems design services†	0.07	0.02	0.06
54151A	Other computer related services	0.04	0.02	0.04
5416A0	Environmental and other technical consulting services	0.04	0.01	0.02
541940	Veterinary services	0.00	0.00	0.00
541920	Photographic services	0.00	0.00	0.00
533000	Lessors of nonfinancial intangible assets	0.69	0.06	0.34
5111A0	Wired telecommunications carriers	0.34	0.17	0.37
511200	Software publishers†	0.33	0.05	0.19
518200	Data processing, hosting, and related services	0.20	0.17	0.26
512100	Motion picture and video industries	0.03	0.00	0.03
512200	Sound recording industries	0.00	0.00	0.00
	All Knowledge Inputs	9.01	6.38	11.48

Notes: Mean refers to the weighted average across all 206 BEAX manufacturing industries, with industry gross output as weights. 25th and 75th pctile refers to expenditure shares of the corresponding percentiles (unweighted) across the 206 manufacturing industries. Codes in the first column refer to BEAX codes that are hand-developed; they roughly correspond to codes available in BEA I/O tables but are aggregated to ensure consistency over time.
Source: BEA Input-Output & Capital Flow Tables, 1997.
† Indicates industries where data on capitalized investments from the capital flow tables are used to compute expenditures. Capitalized investments make up only 0.64% of gross manufacturing output.

A.1 lists the input industries from BEA input-output and capital flow tables that I classify as knowledge inputs. These fall under NAICS sectors 55, 54, 51, and 533. Although results are robust to including finance, insurance, real estate, and other rental leasing (NAICS 52, 531, and 532), I exclude these inputs because of the separate way that financial inputs affect businesses. Instead, I am able to separately account for these mechanisms using the $\Delta \log S_{fj}^{OTHER,SYM}$ control variable in Section 1, and using residual shared inputs in my quantitative framework.

In the three columns of Table A.1, I compute aggregate expenditures by manufacturing firms on these input industries. The input-output tables record only expenses on inputs whose accounting value fully depreciates within one year. Given the arbitrary depreciation rates of many intangible assets and idiosyncratic rules around which inputs are expensed versus capitalized, I incorporate data from the capital flow tables on capitalized investments made by firms in manufacturing industries on knowledge input industries (for example, a shoemaker investing in software capital). I count both capitalized investments and expensed investments as knowledge input expenditures. In practice, capitalized expenditures on intangibles in 1997 are so small (0.64% of output) that it makes no difference to the results in the paper if I exclude data from

the capital flow tables. Most knowledge input expenditures circa 1997 (like R&D) were still expensed under national accounting rules. I do not use data on knowledge input expenditures after 1997 because of subsequent changes to accounting rules that generate a lot of time variation in the data series, and because the capital flow tables are no longer published.

The largest category of knowledge input expenditures is NAICS 55, 'Management of companies and enterprises', at 3.54% of gross output. To my understanding, this reflects the BEA's best estimates of the value of professional services (the categories under NAICS 54) produced internally by the firm's headquarters for use by the firm's other manufacturing plants. By comparison, expenses over the remaining delineated professional services industries (NAICS 54) are outsourced.

Industry Definition. I construct a unified industry nomenclature, BEAX, that is time-invariant over the period 1997 and 2012 and concordable with HS, NAICS, and BEA industry codes in each year. There are 206 BEAX industries in manufacturing. I use the HS-NAICS concordance in US Census Bureau data provided by Schott (2008) and Pierce and Schott (2012) to convert import and export HS codes (at the 10-digit and 6-digit levels) in each year to NAICS. I use the concordances provided by US Census Bureau and BEA to go between NAICS codes and BEA codes in each year. I use an iterative algorithm to aggregate over m:m splits over years and in each cross section so that in any given year, each NAICS code and HS10 code is entirely contained within a BEAX code.

A.2 Export Demand Shocks

I leverage both the LFTTD and CMF sources of data on firm-industry exports to construct demand shocks, $\Delta \log S_{fjt}$. First, among LFTTD data, I compute export shares of each industry of each firm across destinations n and HS6 products h. I exclude destination-product markets whenever the firm's exports in those markets exceed 10% of the market's imports from the rest of the world. I use these shares as $s_{fjnh,t-1}$ in equation (2). Next, I use data from the CMF on export shipments to compute export intensity, $s^*_{fj,t-1}$. If a firm reports no exports in an industry from among its manufacturing plants that produce in that industry, it is likely that its exports in the customs data is an instance of carry-along trade, made by the firm's wholesale / retail arm. In this case customs-data-derived demand shocks would be uninformative: they are as likely to affect this firm as they are to affect any other exporter in the industry. Export intensity from the CMF thus helps to discipline the export demand shocks derived from the LFTTD. I also set export intensity to zero for instances where carry-along trade of the firm (customs exports less census exports) in an industry exceeds its total external shipments in the CMF. After purging these edge cases, I am left with two measures of export intensity: (i) census exports divided by census sales in an industry, and (ii) customs exports divided by census sales in an industry. I take the average of these two measures as my measure of $s^*_{fj,t-1}$.

A.3 Regression Analysis

A.3.1 Summary Statistics

Table A.2 displays summary statistics on common variables that appear in regression Table 2. The regression sample consists of all continuing firm-industries (across 5-year periods) of firms that have at least one industry with a non-zero export demand shock. For example, suppose a firm f produces in industries A and B in 1997 but only produces in A in 2002. As long as the firm received a demand shock in either industry A or B in 1997, I include the firm in the sample (where it takes up a single observation). However, if the firm had switched to producing industries C and D in 2002, there is no intensive margin overlap and this firm would not be included in my sample.

Table A.2: Summary Statistics on Key Regression Sample

Statistics by firm-industry:	Variable	Mean	Std. Dev.
Change in sales	$\Delta \log X_{fjt}$	0.15	0.99
Has export demand shock?	-	0.68	0.47
Export intensity	$s^*_{fj,t-1}$	0.06	0.10
Same-industry demand shock	$\Delta \log S_{fjt}$	0.028	0.082
Other-industry demand shocks			
(i) Average effect	$\Delta \log S^{OTHER}_{fjt}$	0.025	0.063
(ii) × knowledge input-proximity	$\Delta \log S^{OTHER \times KLG}_{fjt}$	0.002	0.007
Initial Period Sales (millions)	$X_{fj,t-1}$	165	1225
Initial Period Employment	-	522	2245

Other Statistics	Value
Number of manuf. firms from 1997-2002	5000
Number of manuf. firms from 2002-2007	4700
Share of U.S. manuf. sales accounted for by sample	0.51
Share of U.S. manuf. employment accounted for by sample	0.37

Notes: This table reports sample statistics for the particular sample of multi-industry firms and their continuing industries used in the regression in Table 2. Number of firms are rounded for disclosure avoidance. The selection criteria is any firm-industry with continuing sales over a 5-year period, and belonging to a firm with at least one industry exporting in the initial period.

A.3.2 Export Demand Shock Relevance

I verify that demand shocks are indeed able to shift firm sales in the same industry by running the following regression for the sub-sample of firm-industries that have non-zero same-industry export demand shocks:

$$\Delta \log X_{fjt} = \alpha \Delta \log S_{fjt} + Controls_{jt}(s^*_{fj,t-1}) + FE_{jt} + \epsilon_{fjt},$$

where $Controls_{jt}(s^*_{fj,t-1})$ refers to various ways of controlling for the export intensity scaling variable to ensure that the estimated impact of the export demand shock is not driven by firms with different export intensities being on different growth trends. Results are presented in Table A.3. Across all three columns (that vary in terms of the control for export intensity used), the coefficient on the shock variable is positive and ranges from 0.32 to 0.59. The impact of the demand shock is higher without controlling for export intensity (column 1), consistent with selection on export intensity.

In undisclosed results I also run a placebo test where for each firm-industry fj, I compute $\Delta \log S^{placebo}_{fjt} = s^*_{fj,t-1} \frac{\Delta \log S_{ijt}}{s^*_{ij,t-1}}$ where, for each fj, i references a randomly selected firm from the set of firms with non-zero demand shocks in j. The placebo tests return false positives in column (1) but not columns (2) and (3). This suggests that linear controls for export intensity control adequately for selection on export intensity.

A.3.3 Cross-Industry Impacts and the Potential for Input-Sharing

Table A.4 displays the regression table counterpart to coefficients shown in Figure 1. Each row of the regression refers to a specification that where demand shocks in other industries of the firm are interacted with proximity to j with respect to a specific category of inputs. The numbers next to the description in parentheses display the NAICS subroot (1, 2 or 3 digits) of the input category. Taxes, government sector inputs, and the two types of value-added (labor and gross operating surplus) are specific BEA categories that

Table A.3: Relevance of Export Demand Shocks for Predicting Change in Sales

Change in sales, $\Delta \log X_{fjt}$	(1)	(2)	(3)	(4)
Same-industry demand shock	0.59***	0.36***	0.34***	0.32***
$\Delta \log S_{fjt}$	(0.10)	(0.12)	(0.11)	(0.11)
Industry-year-FE	✓	✓	✓	✓
$s^*_{fj,t-1}\times$ year-FE		✓	✓	
$s^*_{fj,t-1}\times$ Industry-year-FE				✓
Control for pre-period sales, $\log X_{fj,t-1}$			✓	✓
Observations	14,500	14,500	14,500	14,500
R^2	0.08	0.08	0.12	0.15

Notes: This table displays responses of firm-industry sales to same-industry demand shocks, in 5-year differences over the period 1997-2007. Standard errors in parentheses are clustered at the firm level, with asterisks indicating p-values below 0.1, 0.05, and 0.01 respectively. Number of observations are rounded for disclosure avoidance. The control $s^*_{fj,t-1}$ is the firm's export intensity (exports over sales) in industry j in the initial census year.

have no corresponding numeric NAICS code. The first three rows of the table break out the knowledge input category into finer constituent subcategories: the leasing of intangibles (NAICS 533), headquarter services (NAICS 55), and professional services and information (NAICS 51, 54) and show that cross-industry impacts increase with proximity with respect to each constituent subcategory.

Table A.4: Cross-industry Impact of Demand Shocks: Heterogeneity by Input-Proximity

Change in sales, $\Delta \log X_{fjt}$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Same-industry demand shock $\Delta \log S_{fjt}$	0.48***	0.45***	0.46***	0.46***	0.46***	0.45***	0.45***	0.45***	0.45***	0.46***	0.45***	0.45***	0.45***	0.45***
Other-industry demand shocks														
(i) Average effect $\Delta \log S^{OTHER,SYM}_{fjt}$	-0.26**	-0.68**	-0.49**	-0.82*	-0.18	-0.45*	-0.12	-0.15	-0.29	-0.00	-0.07	0.25	-0.05	0.19
(ii) × proximity by use of inputs in $\mathcal{M}^{BLK}$: $\Delta \log S^{OTHER,BLK}_{fjt}$														
Leasing of Intangibles (533)	27.73***													
Headquarter Services (55)		15.82**												
Professional Services & Information (54, 51)			8.262*											
Finance, Insurance, and Real Estate (52, 531)				40.26										
Leasing of Tangibles (532)					24.92									
Transportation, Wholesale, and Retail (4)						14.94*								
Taxes and Government							3.603							
Utilities and Construction (2)								3.413						
Gross Operating Surplus									1.766					
Labor Value-Added										-0.325				
Agriculture (1)											-0.715			
Manufacturing (3)												-0.889		
Administrative Services (56)													-4.305	
All Other Services (6, 7, 8, 9)														-20.8
Industry-year-FE	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Observations	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500
R^2	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06

Notes: This table reproduces the specification in column (3) of Table 2 but with other-industry demand shocks interacted with proximity in the use of various categories of inputs (given by the rows of the table). Standard errors are clustered at the firm level (omitted for brevity), with asterisks indicating p-values below 0.1 (*), 0.05 (**), and 0.01 (***) respectively. Number of observations are rounded for disclosure avoidance.

Table A.5: Same and Cross-Industry Impacts of Demand Shocks: Additional Specifications

Change in Sales, $\Delta \log X_{fjt}$	(1)	(2)	(3)	(4)
Same-industry demand shock			0.47***	0.47***
$\Delta \log S_{fjt}$			(0.09)	(0.09)
Other-industry demand shocks				
(i) Average effect	-0.74***			-0.48
$\Delta \log S_{fjt}^{OTHER}$	(0.24)			(0.39)
(ii) × knowledge input-proximity	7.51***	6.54***	7.02***	8.14***
$\Delta \log S_{fjt}^{OTHER \times KLG}$	(2.22)	(2.08)	(2.11)	(2.22)
(iii) × remaining input-proximity		-0.75***	-0.86***	-0.44
$\Delta \log S_{fjt}^{OTHER \times REM}$		(0.26)	(0.26)	(0.44)
Industry-year-FE	✓	✓	✓	✓
Observations	21,500	21,500	21,500	21,500
R^2	0.12	0.05	0.06	0.06

Notes: This table displays additional specifications using the same sample of firms as regression Table 2. Standard errors are clustered at the firm level, with asterisks indicating p-values below 0.1, 0.05, and 0.01 respectively. Number of observations are rounded for disclosure avoidance.

Table A.5 estimates variants of the main regression equation (1) and finds that cross-industry elasticities of sales with respect to demand shocks are robust to alternative specifications. Column (1) finds that the impact of other-industry demand shocks are robust to dropping controls for same-industry demand shocks, providing reassurance that same-industry demand shocks have independent variation with respect to other-industry demand shocks in the data. In columns (2)-(4), instead of using average other-industry demand shocks $\Delta \log S_{fjt}^{OTHER}$, I focus only on the input-sharing mechanism and separate out knowledge inputs from the remaining inputs in the BEA I/O tables. I denote the remaining set of inputs by $\mathcal{M}_f^{REM}$ and construct $\Delta \log S_{fjt}^{OTHER,REM}$ using the same equation (5) as $\Delta \log S_{fjt}^{OTHER,KLG}$. Column (2) shows that they pull in opposite directions within the firm. Cross-industry impacts increase with the sharing of knowledge inputs, and decrease with the sharing of remaining inputs. Column (3) adds the same-industry shock back to the regression, and column (4) includes both $\Delta \log S_{fjt}^{OTHER,REM}$ and $\Delta \log S_{fjt}^{OTHER}$. Across specifications (2)-(4), the effect of other-industry demand shocks interacted with knowledge input proximity is always positive and statistically significant.

A.3.4 Impact of Demand Shocks at the Firm-level

Weights η_{fkt} used in the firm-level regression equation (6) are given by:

$$\eta_{fkt} = \frac{\beta_{k,y} X_{fkt}}{\sum_{k'} \beta_{k',y} X_{fk't}},$$

where X_{fkt} is firm sales in industry k and $\beta_{k,y}$ is defined depending on the outcome of interest y:

(i) Purchased professional services: $\beta_{k,y} = \beta_{k,PROF}$, professional services expenses as a share of gross output in industry k,

(ii) Sales: $\beta_{k,y} = 1$ (so η are simply sales shares),

(iii) Capex: $\beta_{k,y} = \beta_{k,CAP}$, gross operating surplus as a share of gross output in industry k,

(iv) Payroll: $\beta_{k,y} = \beta_{k,LAB}$, labor value added as a share of gross output in industry k.

Data on purchased professional services at the firm level come from aggregating responses of plants of the firm to the following questions in the Annual Survey of Manufactures (ASM): expenses on legal, accounting, management, communication, advertising, and computer software and data processing services. Firms that do not have plants respond to these questions in the ASM and firms that more than 5% of sales outside of the manufacturing sector are dropped for this particular regression. Data on firm-wide capex come from summing up plant-level capital expenditures, and data on payroll come from summing up plant-level production worker payroll. Both variables come from the CMF.

A.3.5 Threats to Identification

Related to the discussion on threats to identification in Section 1.4, I directly test and reject the hypothesis that import growth patterns across industries within a destination are positively correlated among knowledge-intensive manufacturing industries. I aggregate imports of each destination to the industry level, $IMP^{\sim US}_{nk,t-1}$ for $k \in \mathcal{J}$, and interact industry-level import growth with knowledge proximity corresponding to the intra-firm equation (5) used in the main firm-industry regressions:

$$\Delta \log IMP^{\sim US,OTHER \times KLG}_{njt} \equiv \sum_{k \neq j} \sum_{m \in \mathcal{M}^{KLG}} \beta_{jm} \left(\frac{\beta_{km} IMP^{\sim US}_{nk,t-1}}{\sum_{k \neq j} \beta_{km} IMP^{\sim US}_{nk,t-1}} \right) \Delta \log IMP^{\sim US}_{nkt},$$

for $BLK = \{KLG, REM\}$ to separate out knowledge inputs from remaining inputs.

$$\Delta \log IMP^{\sim US,OTHER}_{njt} \equiv \sum_{k \neq j} \left(\frac{X_k}{\sum_{k' \neq j} X_{k'}} \right) \Delta \log IMP^{\sim US}_{nkt},$$

I then run the following regression, at the level of destination-industries, over the same time period (in 5-year differences):

$$\Delta \log IMP^{\sim US}_{njt} = \psi^{REM} \Delta \log IMP^{\sim US,OTHER,REM}_{njt} + \psi^{KLG} \Delta \log IMP^{\sim US,OTHER,KLG}_{njt} + FE_{jt} + FE_{nt} + \epsilon_{njt}.$$

I do not find that ψ^{KLG} is positive, either with or without destination-year fixed effects.

A.3.6 Vertical Explanations

There are four general reasons a demand shock in industry k may increase sales in industry j within the firm: (i) j supplies k, (ii) k supplies j, (iii) k, j use similar inputs, and (iv) k, j are demand-complementary and have similar buyers. My focus is on mechanism (iii). The discussion in Section 1.4 rules out (iv), demand-complementarity. I also rule out the first two, vertical mechanisms:

(i) This is unlikely to explain the main regression results, which show *external* sales of the firm changing. However, it could still be the case that external sales growth is driven by productivity effects (i.e. increasing returns to scale) induced by intra-firm sales growth. I find, though, that intra-firm sales growth in j does not respond to demand shocks in k (even among only the tiny fraction of j industries that have any inter-plant shipments at all).

(ii) For this to occur there must first be an increase in internal shipments in the shocked industry k. Then the story would be that increased quality of shipments (as measured by increased internal sales) drives productivity growth in industry j. I use growth in inter-plant (intra-firm) shipments as an outcome

variable across the specifications in Table A.3. I find that they do not respond (even among the tiny fraction of k industries that have any inter-plant shipments at all).

A.3.7 Deflating

Even though the main regressions specifications all include industry-year fixed effects (which absorb industry price deflators), whether variables are nominal or deflated (with industry-deflators) could still make a difference in terms of the relative sizes of export shares and expenditure shares used in weights. All the reduced-form results are virtually unchanged when the following variables are deflated with industry-level price deflators from the NBER-CES manufacturing database: demand shocks (import growth at destinations), outcomes (external shipments of a firm-industry), as well as 'initial-period' variables, for example the proximity weights behind $\Delta \log S_{fjt}^{OTHER,KLG}$.

B Theory Appendix

B.1 Proof of Lemma ??: Option Value from Mutability

Combine the assumption of additive separability (equation 10) with the expression for firm-industry profits in equation (11) to derive the expected impact on firm gross profits from an additional opportunity i to allocate capital of type m:

$$\frac{\Delta_{fm}}{Z} \equiv \mathbb{E}\left[\max_j \tilde{\alpha}_{mj} B_j \xi_{fj} \phi_{fmi,j}\right],$$

where $\phi_{fmi,j}$ is an independent random draw from a Fréchet distribution. The expected impact is the change in gross profits in the industry in which the capital (conditional on the match-specific draws of $\phi_{fmi,j}$ in different industries) generates the highest change in profitability. The remainder of this proof simply relies on properties of the Fréchet distribution popularized by Eaton and Kortum (2002). I can re-express the profit contribution as:

$$\frac{\Delta_{fm}}{Z} = \mathbb{E}\left[\max_j \tilde{\phi}_{fmi,j}\right],$$

where $\tilde{\phi}_{fmi,j}$ is an independent random draw from a different Fréchet distribution that absorbs the multiplicative shifters:

$$Pr(\tilde{\phi}_{fmi,j} \leq x) = e^{-\left(\tilde{\alpha}_{mj} B_j \xi_{fj}\right)^{\theta_m} x^{-\theta_m}}, \qquad \forall j \in \mathcal{J},$$

and it follows that

$$\Delta_{fm} = \left(\sum_j \left(Z \tilde{\alpha}_{mj} B_j \xi_{fj}\right)^{\theta_m}\right)^{\frac{1}{\theta_m}} \Gamma(1 - 1/\theta_m) = \left(\sum_j \delta_{fmj}^{\theta_m}\right)^{\frac{1}{\theta_m}}.$$

where Γ is the gamma function and $\delta_{fmj} \equiv \xi_{fj} \alpha_{mj} B_j Z$.

B.2 Proof of Lemma 1: The Firm's Solution

In stage II, the firm decides its expenditures on production inputs given its accumulated capital levels, $\{\varphi_{fj}\}_{j \in \mathcal{J}}$. This problem is separable by industry. Under monopolistic competition, the solution for the firm's gross profits (sales less production input expenses) and sales is given by equation (11).

At the beginning of stage I, the firm decides its expenditures on capital inputs. Throughout stage I, the firm allocates new capital to a chosen industry whenever it has an opportunity to do so. Given the additive

separability assumption in equation (10), expected firm net profits Π_f can be written as:

$$\mathbb{E}[\Pi_f] = \max_{\{\iota_{fm}\}_{m\in\mathcal{M}}} \mathbb{E}\left[\sum_j \sum_m B_j \xi_{fj} \tilde{\alpha}_{mj} \sum_i^{A_{fm}} \phi_{fmi,j} \mathbf{1}_{fmi,j}\right] - \sum_m w\iota_{fm}.$$

The first half of the expression denotes the expected gross profits of the firm given how its choices of capital inputs ι_{fm} affect the Poisson-distributed number of opportunities A_{fm} to allocate capital and its allocation decisions $\mathbf{1}_{fmi,j}$. (Note the expectation operator is taken over $A_{fm}, \phi_{fmj,j}$, as well as the firm's allocation decisions $\mathbf{1}_{fmi,j}$). The second half of the expression relates to the costs of capital inputs, which I normalize at w. Any differences in unit prices across types of capital inputs are isomorphic to capital coefficients α_{fm}.

Given the linearity of this problem and the independence of the Poisson and Fréchet distributions, capital allocation in each instance i is independent of past and future decisions. The allocation decision $\mathbf{1}_{fmi,j}$ has the following expectational properties inherited from Fréchet (Section B.1):

$$Pr(\mathbf{1}_{fmi,j} = 1) = Pr(j = \arg\max_{k\in\mathcal{J}} \tilde{\phi}_{fmi,k}) = \frac{\delta_{fmj}^{\theta_m}}{\Delta_{fm}^{\theta_m}} \equiv \mu_{fmj},$$

where μ_{fmj} are industry allocation probabilities for any given instance of new capital of type m and

$$\mathbb{E}[\tilde{\alpha}_{mj} B_j \xi_{fj} \phi_{fmi,j} \mid \mathbf{1}_{fmi,j} = 1] = \mathbb{E}\left[\max_j \tilde{\alpha}_{mj} B_j \xi_{fj}\, \phi_{fmi,j}\right] = \frac{\Delta_{fm}}{Z},$$

by Lemma **??**. Recalling that A_{fm} is distributed independently with Poisson mean $Z\left(\frac{\rho_m}{\rho_m - 1}\iota_{fm}\right)^{\frac{\rho_m - 1}{\rho_m}}$, expected firm net profits Π_f can be re-written as

$$\begin{aligned}\mathbb{E}[\Pi_f] &= \max_{\{\iota_{fm}\}_{m\in\mathcal{M}}} \sum_m \sum_j \mathbb{E}[A_{fm} \mid \iota_{fm}]\mathbb{E}[\tilde{\alpha}_{mj} B_j \xi_{fj} \phi_{fmi,j} \mid \mathbf{1}_{fmi,j} = 1] Pr(\mathbf{1}_{fmi,j} = 1) - \sum_m w\iota_{fm}. \\ &= \max_{\{\iota_{fm}\}_{m\in\mathcal{M}}} \sum_m \left(\frac{\rho_m}{\rho_m - 1}\iota_{fm}\right)^{\frac{\rho_m - 1}{\rho_m}} \Delta_{fm} - \sum_m w\iota_{fm}.\end{aligned}$$

This is a convex optimization problem separable across capital input types m, with optimal inputs given by:

$$\iota_{fm} = \frac{\rho_m - 1}{\rho_m} \Delta_{fm}^{\rho_m} w^{-\rho_m}, \qquad \forall m,$$

and thus net profits are equal to

$$\mathbb{E}[\Pi_f] = \sum_m \Delta_{fm}^{\rho_m} w^{1-\rho_m} - \sum_m \frac{\rho_m - 1}{\rho_m} \Delta_{fm}^{\rho_m} w^{1-\rho_m} = \sum_m \frac{1}{\rho_m} \Delta_{fm}^{\rho_m} w^{1-\rho_m}.$$

Likewise, expected gross profits in a single industry j are given by

$$\mathbb{E}[\pi_{fj}] = \sum_m \mu_{fmj} \Delta_{fm}^{\rho_m} w^{1-\rho_m}.$$

The probability that a firm is active in industry j, denoted $\chi_{fj} = 1$, is one minus the probability that no capital (of any type) is allocated to that industry. Since capital allocation is independent across opportunities, and

the total arrival rate of capital of any type m is a Poisson process with rate A_{fm}, the arrival of *allocated capital* in j is also a Poisson process, with rate $A_{fm}\mu_{fmj}$. The probability of industry entry is thus one minus the probability that there are no arrivals from the joint Poisson processes of allocated capital to industry j over all capital types $m \in \mathcal{M}$:

$$Pr(\chi_{fj} = 1) = 1 - \exp\left(\sum_m \mu_{fmj} A_{fm}\right) = 1 - \exp\left(-Z \sum_m \delta_{fmj}^{\theta_m} \Delta_{fm}^{\rho_m - 1 - \theta_m} w^{1-\rho_m}\right),$$

and is independent across industries due to Poissonization. Similarly, an inactive firm is a firm with no opportunities to allocate capital. The probability that a firm is inactive is thus given by

$$Pr(\chi_f = 1) = 1 - \exp\left(\sum_m A_{fm}\right) = 1 - \exp\left(-Z \sum_m \Delta_{fm}^{\rho_m - 1} w^{1-\rho_m}\right).$$

B.3 Proof of Proposition 1: Cross-Industry Elasticities within the Firm

Log-differentiating equation (15) with respect to shifters of firm profitability in industries k, holding factor prices w constant, yields

$$\mathrm{d}\log \mathbb{E}[X_{fj}] = \mathrm{d}\log \mathbb{E}[\pi_{fj}] = \sum_m \lambda_{fjm} \left(\theta_m \mathbf{1}_{k=j}\, \mathrm{d}\log\left(\xi_{fk} B_k\right) + (\rho_m - \theta_m) \sum_k \mu_{fmk}\, \mathrm{d}\log\left(\xi_{fk} B_k\right)\right),$$

where μ_{fmj} are industry allocation shares given in Lemma 1, and λ_{fjm} denote input utilization shares: the share of gross profits of industry j attributable to capital contributions of type m (relative to m'):

$$\lambda_{fjm} \equiv \frac{\mu_{fmj} \Delta_{fm}^{\rho_m} w^{1-\rho_m}}{\sum_{m'} \mu_{fm'j} \Delta_{fm'}^{\rho_{m'}} w^{1-\rho_{m'}}}.$$

B.4 Connecting Firm-level Elasticities in the Model and Reduced-Form

The firm-level cross-industry elasticity from Proposition 1 combines responses on both intensive and extensive margins ($\mathbb{E}[X_{fj}]$ includes the non-trivial probability of zero sales). But for sufficiently large firms (high in ξ_f), all the responses load on the intensive margin. The intuition is that the largest firms choose a level of capital expenditures so high to start with that the likelihood of cross-industry shocks affecting the extensive margin vanishes. For example, a demand shock for General Electric's MRI machines might affect GE's intensive margin sales of jet engines but is unlikely to affect whether the company is active at all in the jet engine business. With a high enough arrival rate, the expectation operator becomes exact due to the law of large numbers. (This large firm limit corresponds to the framework pioneered in Tintelnot (2016) and Antràs et al. (2017), whereby outcomes are smoothed across a continuum within the firm instead of being granular.) The following Lemma clarifies this point and motivates the focus of the reduced-form regressions on the intensive margin (given that the regression sample comprises large firms):

Lemma 2 (Intensive Margin Spillovers for Large Firms) *Cross-industry elasticities between j and k characterized by Proposition 1 load completely onto the intensive margin as ξ_{fj} and ξ_{fk} become arbitrarily high:*

$$\lim_{\min(\xi_{fj}, \xi_{fk}) \to \infty} \frac{\mathrm{d}\log \mathbb{E}[X_{fj}]}{\mathrm{d}\log \xi_{fk} B_k} = \frac{\mathrm{d}\log \mathbb{E}[X_{fj} | X_{fj} > 0]}{\mathrm{d}\log \xi_{fk} B_k}.$$

As a corollary, the share of the cross-industry elasticity in Proposition 1 explained by extensive margin changes within the firm ranges from 1 (for the lowest ξ firms) to 0 (for the highest ξ firms).

Proof. Decompose the expected gross sales into intensive margin and extensive margins:

$$\log \mathbb{E}[X_{fj}] = \log \mathbb{E}[X_{fj}|X_{fj} > 0] + \log Pr(X_{fj} > 0).$$

Differentiate the extensive margin:

$$\frac{\mathrm{d} \log Pr(X_{fj} > 0)}{\mathrm{d} \log \xi_{fk} B_k} = \frac{\exp(-\Sigma_{fj})\Sigma_{fj}}{1 - \exp(-\Sigma_{fj})} \sum_m s_{mj} \left(\theta_m \mathbf{1}_{k=j} + (\rho_m - \theta_m)\mu_{fmk}\right),$$

where s_{mj} are weights bounded between 0 and 1:

$$s_{mj} \equiv \frac{Z \mu_{fmj} \Delta_{fm}^{\rho_m - 1} w^{1-\rho_m}}{\Sigma_{fj}},$$

and $\Sigma_{fj} \equiv Z \sum_m \mu_{fmj} \Delta_{fm}^{\rho_m - 1} w^{1-\rho_m}$. Because the term $s_{mj} \left(\theta_m \mathbf{1}_{k=j} + (\rho_m - \theta_m)\mu_{fmk}\right)$ in the derivative of the extensive margin is bounded (weighted average of elasticities),

$$\lim_{\min(\xi_{fj}, \xi_{fk}) \to \infty} \frac{\mathrm{d} \log Pr(X_{fj} > 0)}{\mathrm{d} \log \xi_{fk} B_k} = \lim_{\Sigma_{fj} \to \infty} \frac{\exp(-\Sigma_{fj})\Sigma_{fj}}{1 - \exp(-\Sigma_{fj})} = 0,$$

where the last equality makes use of L'hopital's rule. ■

B.5 Substitutability in the Production Function

xxx respond to Ref 1's comments about substitutability.

C Quantitative Appendix

C.1 Identification of Macro Variables

Conditional on micro parameters Θ, γ, I identify macro variables—capital coefficients $\boldsymbol{\alpha}$, industry profitability, $\boldsymbol{B}_t$, and the technology coefficient Z_t—by relating the aggregate predictions of the model to their counterparts in the data. I do so in a block-recursive manner.

First, I solve the second line of equation (21) separately for each of the three types of knowledge capital inputs $m \in \mathcal{M}^{KLG}$. For each type of knowledge input m, given data on capital expenditures $\{M_{mj}\}_{j \in \mathcal{J}}$ in the base period ($t = 1$), I invert a separate system of $|\mathcal{J}|$ equations for $|\mathcal{J}|$ model variables $\{\alpha_{mj} B_{j,t=1} Z_t\}_{j \in \mathcal{J}}$ (with the three terms grouped together).

For each industry j, the mean of α_{mj} across m is isomorphic to a constant term in $B_{j,t=1}$. Thus, I am free to normalize the capital coefficient of the last capital input type $\alpha_{RES,j} = 1$. As a second step, I subtract knowledge expenditures (the second line of equation 21 for all $m \in \mathcal{M}^{KLG}$) from gross profits (the first line of equation 21) to yield (for $t = 1$):

$$\frac{\pi_j - \sum_{m \in \mathcal{M}^{KLG}} \frac{\rho^{KLG}}{\rho^{KLG}-1} M_{mj}}{N} = \int \delta_{f,RES,j}^{\theta^{RES}} \Delta_{f,RES}^{\rho^{RES} - \theta^{RES}} \, \mathrm{d}G(\xi), \quad \forall j \in \mathcal{J},$$

where the left-hand-side is data contained in BEA input-output tables, and the right-hand side contains a $|\mathcal{J}|$ vector of unknowns $\{B_{j,t=1}Z\}_{j\in\mathcal{J}}$ (since $\alpha_{RES,j}=1$).[33] This represents the forth system of $|\mathcal{J}|$ equations for $|\mathcal{J}|$ model variables that I invert. (The other three being each of the three types of knowledge inputs, described in the first step).

Third, given values of $\{B_{j,t=1}Z_{t=1}\}_{j\in\mathcal{J}}$ from step 2, and $\{\alpha_{mj}B_{j,t=1}Z_{t=1}\}_{j\in\mathcal{J},m\in\mathcal{M}^{KLG}}$ from step 1, I can directly back out capital coefficients $\{\alpha_{mj}\}_{j\in\mathcal{J},m\in\mathcal{M}^{KLG}}$. I hold $\boldsymbol{\alpha}$ constant over all three time periods due to lack of expenditure data on knowledge inputs in subsequent years.

Forth, I use the expression for gross output $\boldsymbol{X}_t$ in equation (21) in years $t=2,3$ to find future-period industry profitability $\{B_{j,t=2}Z_{t=2}, B_{j,t=3}Z_{t=3}\}_{j\in\mathcal{J}}$. In each year, given values of α_{mj}, I can invert a system of $|\mathcal{J}|$ equations for $|\mathcal{J}|$ model variables $\{B_{jt}Z_t\}_{j\in\mathcal{J}}$.

Fifth, given the full set of $\{B_{jt}Z_t\}_{j\in\mathcal{J},t=1,2,3}$ (from steps 2 and 4) and capital coefficients $\boldsymbol{\alpha}$, I solve for Z_t such that the closed-form expression for the share of single-industry firms in the model matches 0.8 in the data:

$$\frac{\int \sum_j \left(Pr(\chi_{fjt}=1)\prod_{k\neq j}(1-Pr(\chi_{fkt}=1))\right) \mathrm{d}G(\boldsymbol{\xi})}{\int Pr(\chi_{ft}=1)\,\mathrm{d}G(\boldsymbol{\xi})} = 0.8. \tag{27}$$

where entry probabilities by industry (χ_{fjt}) and firm-wide (χ_{ft}) are given in Theory Appendix B.2.

Finally, given $\{B_{jt}Z_t\}_{j\in\mathcal{J},t=1,2,3}$ and Z_t, I directly back out $\boldsymbol{B}_t$.

C.2 Proof of Proposition 2: Inference of Scalability and Mutability

Notationally, many functions described below depend on macro variables (i.e. $\boldsymbol{B}_t, \boldsymbol{\alpha}, Z$), which I suppress into a time subscript t for ease of exposition.

First, I show that at true parameter values $\Theta, \boldsymbol{\gamma}$, the following $J\times J$ structural moment conditions hold true for any pair of industries j,k:

$$\mathbf{E}_f\left[(\epsilon_{fjt}-\mathring{\epsilon}_{fjt-1})\,\widetilde{\Delta\log S_{fkt}}\;\middle|\;\boldsymbol{\chi}_{f,t-1}\right]=0, \qquad \forall t=\{2,3\}, \tag{28}$$

where $\widetilde{\Delta\log S_{fkt}}$ is the de-meaned shock among shocks received by all firms that are active in industry k, and $\mathring{\epsilon}_{fj,t-1}$ is a modified structural error conditional on the firm's extensive margin:

$$\mathring{\epsilon}_{fj,t-1} \equiv X_{fj,t-1} - \mathbb{E}[X_{fj,t-1} \mid \boldsymbol{\xi}_{f,t-1}, \boldsymbol{\chi}_{f,t-1}].$$

By the law of iterated expectations, the moment condition for any pair of industries j,k in any year $t=\{2,3\}$ can be written as

$$\mathbf{E}_{\boldsymbol{\xi}_{f,t-1},\Delta\log \boldsymbol{s}_{ft}}\left[\widetilde{\Delta\log S_{fkt}}\;\mathbf{E}_f\left[(\epsilon_{fjt}-\mathring{\epsilon}_{fjt-1})\;\middle|\;\boldsymbol{\chi}_{f,t-1},\boldsymbol{\xi}_{f,t-1},\Delta\log \boldsymbol{S}_{ft}\right]\;\middle|\;\boldsymbol{\chi}_{f,t-1}\right],$$

[33] Note that no expenditure data on the residual capital input category is needed. The residual category is set up to also absorb payments to latent factors (e.g. venture capital, sweat equity). This equation imposes a non-negativity restriction which manifests as a lower bound on the value of ρ^{KLG} according to the model:

$$\begin{aligned}\pi_j > \sum_m \frac{\rho^{KLG}}{\rho^{KLG}-1}M_{mj} \iff (1-\varsigma_j) &> \frac{\rho^{KLG}}{\rho^{KLG}-1}\sum_{m\in KLG}\beta_{jm}, \qquad \forall j\\ \iff \frac{\rho^{KLG}-1}{\rho^{KLG}} &> \max_j \frac{\beta_{j,KLG}}{1-\varsigma_j} = \max_j \frac{\beta_{j,KLG}}{\beta_{j,KLG}+\beta_{j,RES}},\end{aligned}$$

for BEA expenditure shares β_{jm}. In the data, this restriction corresponds roughly to imposing that $\rho^{KLG}>3$.

where the $(\epsilon_{fjt} - \mathring{\epsilon}_{fjt-1})$ terms inside the inner expectation are zero in expectation because (i) Assumption 3 (relevance) implies that $\xi_{f,t}$ can be computed from $(\xi_{f,t-1}, \Delta \log S_{ft})$, so:

$$\mathbf{E}_f \left[X_{fjt} \mid \chi_{f,t-1}, \xi_{f,t-1}, \Delta \log S_{ft} \right] = \mathbb{E} \left[X_{fjt} \mid \xi_{f,t} \right],$$

and (ii) Assumption 4 (conditional independence) implies that $\Delta \log S_{ft}$ is independent of outcomes $X_{fj,t-1}$ conditional on the industry presence χ and unobserved profitability shifters $\xi_{f,t-1}$, so:

$$\mathbf{E}_f \left[X_{fj,t-1} \mid \chi_{f,t-1}, \xi_{f,t-1}, \Delta \log S_{ft} \right] = \mathbb{E} \left[X_{fj,t-1} \mid \chi_{f,t-1}, \xi_{f,t-1} \right].$$

Next, I construct sample analogs of the moment conditions in equation (28). Since the moment conditions are valid conditional on χ, I am free to limit attention to the set of firms that are active in each pair of industries j, k in year $t-1$. I label this set of firms by $\mathcal{F}^D_{jk,t-1}$. I break out the terms inside structural residuals into two parts. The first part is pure data—involving the interaction of realized sales growth $\boldsymbol{X}_{ft} - \boldsymbol{X}_{f,t-1}$ and demand shocks:

$$\Xi^o_{jkt} \equiv \frac{1}{|\mathcal{F}^D_{jk,t-1}|} \sum_{f \in \mathcal{F}^D_{jk,t-1}} \Delta X_{fjt} \; \widetilde{\Delta \log S_{fkt}}, \quad \forall j, k, \; \forall t = \{2, 3\}.$$

The second part of the moment conditions involve the model-based counterpart, given by

$$\mathbf{E}_f \left[\left(\mathbb{E}[X_{fjt} | \xi_{ft}] - \mathbb{E}[X_{fj,t-1} | \xi_{f,t-1}, \chi_{f,t-1}] \right) \widetilde{\Delta \log S_{fkt}} \;\middle|\; \chi_{f,t-1} \right].$$

Closed-form expressions for sales $\mathbb{E}[X_{fjt}|\xi_{ft}]$ and sales conditional on entry $\mathbb{E}[X_{fj,t-1}|\xi_{f,t-1}, \chi_{f,t-1}]$ require knowledge of ξ_{ft} and $\xi_{f,t-1}$. A naive approach would have been to integrate over the unconditional distribution $G(\xi)$, but both demand shocks and the firm's extensive margin may be correlated with underlying firm shifters $\xi_{f,t-1}$. Instead, I integrate over the conditional distribution $Pr(\xi|\chi_{f,t-1}, \Delta \log S_{ft})$. I exploit the model's closed-form solutions for the extensive margin probability of entry to express this likelihood analytically using Bayes' rule. I show this over a series of steps. First, in step (i), I define a closed-form analytical object g_{jk} which is a function of three terms: demand shocks $\Delta \log S_{ft}$ and extensive margin presence $\chi_{f,t-1}$ which are observable in the data, as well as unobservable profitability shifters $\xi_{f,t-1}$:

$$g_{jk}(\xi_{f,t-1}, \Delta \log S_{ft}, \chi_{f,t-1}) \equiv \mathbf{E}_f \left[\left(\mathbb{E}[X_{fjt}|\xi_{ft}] - \mathbb{E}[X_{fj,t-1}|\xi_{f,t-1}, \chi_{f,t-1}] \right) \widetilde{\Delta \log S_{fkt}} \;\middle|\; \xi_{f,t-1}, \Delta \log S_{ft}, \chi_{f,t-1} \right].$$

This is true because under Assumption 3 (relevance), $\xi_{f,t}$ can be computed from $(\xi_{f,t-1}, \Delta \log S_{ft})$. Over the next series of steps I manipulate the moment condition line by line as follows:

$$\begin{aligned}
&\mathbf{E}_f \left[\left(\mathbb{E}[X_{fjt} \mid \xi_{ft}] - \mathbb{E}[X_{fj,t-1} \mid \xi_{f,t-1}, \chi_{f,t-1}] \right) \widetilde{\Delta \log S_{fkt}} \;\middle|\; \chi_{f,t-1} \right] \\
&\quad = \mathbf{E}_{\Delta \log S_{ft}, \xi_{f,t-1}} \left[g_{jk}(\xi_{f,t-1}, \Delta \log S_{ft}, \chi_{f,t-1}) \mid \chi_{f,t-1} \right] \\
&\quad = \mathbf{E}_{\Delta \log S_{ft}} \left[\int_\xi g_{jk}(\xi, \Delta \log S_{ft}, \chi_{f,t-1}) \; Pr(\xi \mid \chi_{f,t-1}) \, \mathrm{d}\xi \;\middle|\; \chi_{f,t-1} \right] \\
&\quad = \mathbf{E}_{\Delta \log S_{ft}} \left[\int_\xi g_{jk}(\xi, \Delta \log S_{ft}, \chi_{f,t-1}) \; \frac{Pr(\chi_{f,t-1} \mid \xi)}{\int_\xi Pr(\chi_{f,t-1} \mid \xi) \, \mathrm{d}G(\xi)} \, \mathrm{d}G(\xi) \;\middle|\; \chi_{f,t-1} \right] \\
&\quad = \mathbf{E}_{\Delta \log S_{ft}} \left[\int_\xi g_{jk}(\xi, \Delta \log S_{ft}, \chi_{f,t-1}) \; \frac{\prod_{j \in \mathcal{J}} Pr(\chi_{fj,t-1} \mid \xi)}{\int_\xi \prod_{j \in \mathcal{J}} Pr(\chi_{fj,t-1} \mid \xi) \, \mathrm{d}G(\xi)} \, \mathrm{d}G(\xi) \;\middle|\; \chi_{f,t-1} \right],
\end{aligned}$$

where step (ii) applies the law of iterated expectations and replaces the inner expectation term with g_{jk}, step (iii) breaks up the expectation over the joint probability distribution of $\Delta \log \mathbf{S}_{ft}, \boldsymbol{\xi}_{f,t-1}$ in terms of a conditional $Pr(\boldsymbol{\xi} \mid \Delta \log \mathbf{S}_{ft}, \boldsymbol{\chi}_{f,t-1}) = Pr(\boldsymbol{\xi} \mid \boldsymbol{\chi}_{f,t-1})$ (given the conditional independence Assumption 4) and a marginal $Pr(\Delta \log \mathbf{S}_{ft} \mid \boldsymbol{\chi}_{f,t-1})$, left with the expectation operator $\mathbf{E}_{\Delta \log s_{ft}}$. Step (iv) applies Bayes' rule to transform $Pr(\boldsymbol{\xi} \mid \boldsymbol{\chi}_{f,t-1})$ into known analytical extensive margin probabilities. Finally, step (v) exploits known properties of the Poisson arrival process where the probability of industry entry in $j \in \mathcal{J}$ is independent of any $j' \in \mathcal{J}$ conditional on shifters $\boldsymbol{\xi}$. The sample analog of the last line can be constructed as

$$\Xi^m_{jkt} \equiv \frac{1}{|\mathcal{F}^D_{jk,t-1}|} \sum_{f \in \mathcal{F}^D_{jk,t-1}} \sum_{i \in \mathcal{F}^S} \omega_{i,t-1}(\boldsymbol{\chi}_{f,t-1})\ \Delta \hat{X}_{jt}(\boldsymbol{\chi}_{f,t-1}, \boldsymbol{\xi}_i, \Delta \log \mathbf{S}_{ft})\ \widetilde{\Delta \log S_{fkt}},$$

where the summation of $\Delta \hat{X}_{jt} \widetilde{\Delta \log S_{fkt}}$ over simulated draws $\boldsymbol{\xi}_i$ from $G(\boldsymbol{\xi})$ with probability weights $\omega_{i,t-1}$ defined in the main text of Proposition 2 approximates the integral.

Lastly, by the law of large numbers, $m_{jkt} \equiv \Xi^o_{jkt} - \Xi^m_{jkt}$ in Proposition 2 can be written as

$$\lim_{|\mathcal{F}^D_{jk,t-1}| \to \infty} \lim_{|\mathcal{F}^S| \to \infty} \Xi^o_{jkt} - \Xi^m_{jkt} = \mathbf{E}_f \left[(\epsilon_{fjt} - \mathring{\epsilon}_{fjt-1})\ \widetilde{\Delta \log S_{fkt}} \;\middle|\; \boldsymbol{\chi}_{f,t-1} \right] = 0.$$

C.3 More Details on Nested Fixed Point Estimation Algorithm

This weaker identification assumption creates a challenge for inference. Fundamental profitability $\boldsymbol{\xi}_{ft}$ is unobservable to the econometrician and potentially correlated with demand shocks. If demand shocks were simply unconditionally randomly assigned, inference could proceed by integrating over the primitive data generating process ($G(\boldsymbol{\xi})$ in Assumption 3). Demand shocks would serve as observable mean-shifters of the distribution of $\boldsymbol{\xi}_{ft}$ relative to $\boldsymbol{\xi}_{f,t-1}$ for each individual firm. In my setting, however, correlation between demand shocks and $\boldsymbol{\xi}_{f,t-1}$ would generate bias.

Proposition 2 develops a similar procedure for inference under *conditional* random assignment of demand shocks. I integrate model-implied moments over the conditional distribution $Pr(\boldsymbol{\xi}_{t-1} \mid \boldsymbol{\chi}_{f,t-1})$—the likelihood that a firm f in the data had shifters $\boldsymbol{\xi}_{t-1}$ conditional on its extensive margin $\boldsymbol{\chi}_{f,t-1}$. Using Bayes rule, I express this otherwise-intractable likelihood function in terms of its posterior distribution $Pr(\boldsymbol{\chi}_{f,t-1} \mid \boldsymbol{\xi}_{t-1})$—which are closed-form industry entry probabilities given by equation (16). Finally, I approximate the integral by summing over closed-form posterior probabilities computed from draws of $G(\boldsymbol{\xi}_i)$ representing a fixed set of simulated firms $i \in \mathcal{F}^S$. Intuitively, I assign probabilities that a firm in the data with extensive margin $\boldsymbol{\chi}_{f,t-1}$ has its unobserved profitability $\boldsymbol{\xi}_{f,t-1}$ equal to $\boldsymbol{\xi}_i$ relative to all candidate values of $\{\boldsymbol{\xi}_i\}_{i \in \mathcal{F}^S}$. I then use these probabilities as weights to compute an average model-implied sales growth for firm f assuming it had $\boldsymbol{\xi}_{f,t-1} = \boldsymbol{\xi}_i$ for each $i \in \mathcal{F}^S$.

Lemma 3 (Inference of Scalability and Rivalry) *Let $\widetilde{\Delta \log S_{fkt}}$ denote demeaned demand shocks by industry-year. Define the following analytical sample moment conditions for every pair of industries j, k and year $t = \{2, 3\}$:*

$$m_{jkt} \equiv \frac{1}{|\mathcal{F}^D_{jk,t-1}|} \sum_{f \in \mathcal{F}^D_{jk,t-1}} \left(\Delta X_{fjt} - \sum_{i \in \mathcal{F}^S} \omega_{i,t-1}(\boldsymbol{\chi}_{f,t-1})\ \Delta \hat{X}_{jt}\left(\boldsymbol{\chi}_{f,t-1}, \boldsymbol{\xi}_i, \Delta \log \mathbf{S}_{ft}\right) \right) \widetilde{\Delta \log S_{fkt}},$$

where $\mathcal{F}^D_{jk,t-1}$ denotes firms in the regression sample active in both j and k in year $t-1$, $\mathcal{F}^S$ is a sample of simulated firms with profitability shifters $\boldsymbol{\xi}_i$ drawn from distribution $G(\boldsymbol{\xi}_i)$ according to Assumption 3, $\omega_{i,t-1}$ computes the

probability that a firm f in the data with extensive margin $\boldsymbol{\chi}_{f,t-1}$ has shifters equal to $\boldsymbol{\xi}_i$ of simulated firm i relative to that of other simulated firms $i' \in \mathcal{F}^S$:

$$\omega_{i,t-1}(\boldsymbol{\chi}_{f,t-1}) \equiv \frac{\prod_{j \in \mathcal{J}} Pr(\chi_{ij,t-1} = \chi_{fj,t-1} \mid \boldsymbol{\xi}_i)}{\sum_{i' \in \mathcal{F}^S} \prod_{j \in \mathcal{J}} Pr(\chi_{i'j,t-1} = \chi_{fj,t-1} \mid \boldsymbol{\xi}_{i'})},$$

and $\Delta \hat{X}_{jt}$ is model-implied expected sales growth of a firm conditional on prior-period extensive margin $\boldsymbol{\chi}_{f,t-1}$, fundamental profitability $\boldsymbol{\xi}_{f,t-1} = \boldsymbol{\xi}_i$ and demand shocks $\Delta \log \boldsymbol{S}_{ft}$:

$$\Delta \hat{X}_{jt}(\boldsymbol{\chi}_{f,t-1}, \boldsymbol{\xi}_i, \Delta \log \boldsymbol{S}_{ft}) \equiv \mathbb{E}[X_{fjt} \mid \boldsymbol{\xi}_i'] - \mathbb{E}[X_{fj,t-1} \mid \boldsymbol{\xi}_i, \boldsymbol{\chi}_{f,t-1}],$$

where next-period profitability $\boldsymbol{\xi}_i'$ evolves conditional on $\boldsymbol{\xi}_i$ and empirical demand shocks $\Delta \log \boldsymbol{S}_{ft}$ according to Assumption 3.

At true parameter values Θ, as the data and simulation samples become large, $|\mathcal{F}^D|, |\mathcal{F}^S| \to \infty$, the sample moment $m_{jkt} = 0$ for any j, k and $t \in \{2, 3\}$.

Estimation proceeds over five steps:

1. Simulate a set of 2000 firms $i \in \mathcal{F}^S$ with fixed draws of ζ_{ij}, ζ_i from standard normal distributions. I use stratified sampling to over-weigh firms with higher ζ_i.
2. Guess a starting $\hat{\Theta}, \hat{\gamma}_0, \hat{\gamma}_1$, then repeat Steps 3-5 until convergence.
3. Compute $\{\boldsymbol{\xi}_i\}_{i \in \mathcal{F}^S}$ given $\hat{\gamma}_0, \hat{\gamma}_1$ from Assumption 3 and baseline draws ζ_{ij}, ζ_i.
4. Use $\{\boldsymbol{\xi}_i\}_{i \in \mathcal{F}^S}$ and $\hat{\Theta}$ to compute $\boldsymbol{\alpha}, \boldsymbol{B}_t, Z_t$ via equation (21) and Table 4.
5. Compute the sample moment conditions in Proposition 2, stack the moments according to the four groups as described above, and use a bounded Nelder-Mead simplex search algorithm to adjust the guess of $\hat{\Theta}, \hat{\gamma}_0, \hat{\gamma}_1$ given the change in the objective value.

C.4 External Validity

Table C.6 displays the distribution of firms and sales over firm scope behind Figure 3. Unlike estimation, which requires only simulated values of firm profitability shifters $\boldsymbol{\xi}_i$, these outcomes are computed by simulating the actual *outcomes* of firms in the model.

C.5 General Equilibrium Definition

I introduce some more notation used to characterize the open economy equilibrium. Take the perspective of US as the domestic economy trading with a set $\mathcal{D}^F$ of foreign countries. Let D denote US net exports of non-manufacturing goods vis-à-vis the rest of the world (also equal to the manufacturing trade deficit). Let $\bar{Y}_{d,j}$ denote the total market size faced by US firms in each industry j in a foreign destination $d \in \mathcal{D}^F$, and suppose that all firms are common exporters.[34] Let $\bar{PX}_{dj}$ represent indices of price competitiveness in

[34]My paper does not consider the selection-into-exporting margin. But in fact, the common-exporter assumption is not extreme. It suffices that firms have common ex-ante *expectations* of exporting. One micro-foundation, for example, would be if each capital allocated to industry j has a probability of being used at the same time for export market production. A firm then enters into exporting if and only if it has a non-zero amount of capital adaptable for export markets. Despite this common probability of exporting, empirically larger firms would be more likely to export because of a higher chance of having at least some capital be adapted for export markets.

Table C.6: Distribution of Outcomes by Firm Scope in the Data and Model, 1997

Number of Industries	Share of Firms (%) Data	Model	Share of Sales (%) Data	Model
1	80.99	80.15	26.13	25.00
2	13.01	13.54	10.80	15.74
3	3.32	3.29	7.34	9.43
4	1.33	1.07	5.51	6.33
5	0.61	0.44	4.08	5.48
6	0.28	0.23	2.89	5.76
7	0.14	0.15	3.54	6.71
8	0.08	0.10	3.63	4.05
9 +	0.25	1.03	36.05	21.49

Notes: The distribution of outcomes by firm scope, in the data and in the model (with the six estimated parameters, $\Theta, \gamma_0, \gamma_1$). Sales of firms with 9 or more industries could not be simulated via brute force due to memory issues when simulating the discrete Poisson process. Instead, it is backed out from the fact that the share of sales by firms with one industry was set to equal 25% in the estimation.

foreign market d by all non-US firms. Let $P\bar{M}_{dj}$ represent indices of price competitiveness in the US market by foreign firms from d. For example, an increase in $P\bar{M}_{china,j}$ indicates that prices of Chinese goods in the US have been lowered (become more competitive).

The equilibrium set-up in Definition 1 accommodates both the case of flexible and fixed wages, w. First, if the foreign residual demand curve for US exports in non-manufacturing is completely inelastic, then D is fixed and wages of US workers (and prices of the non-manufacturing good) adjust to balance overall trade. Second, if instead the foreign residual demand curve for US exports in non-manufacturing is completely elastic, then wages are pinned down by world prices of the non-manufacturing good and all adjustment loads on D. I solve for equilibrium in the paper under this second assumption so as to prevent wage effects from contaminating cross-industry impacts.

Definition 1 (General Equilibrium) *Let PD_j denote domestic price competitiveness in an industry $j \in \mathcal{J}$:*

$$PD_j \equiv \mathcal{P}^{1-\sigma_j} \equiv N \int \mathbb{E}\left[p_{fj}^{1-\sigma_j}\right] \mathrm{d}G(\boldsymbol{\xi}).$$

Given total labor L, a mass of firms N, exogenous foreign price competitiveness abroad and at home, $\{P\bar{X}_{dj}, P\bar{M}_{dj}\}_{j\in\mathcal{J},d\in\mathcal{D}^F}$, foreign expenditures $\{\bar{Y}_{dj}\}_{j\in\mathcal{J},d\in\mathcal{D}^F}$, and other parameters of the model, general equilibrium is described by either the tuple $\{w, \eta_M, \boldsymbol{PD}\}$ (when foreign demand for non-manufacturing D is fixed) or $\{D, \eta_M, \boldsymbol{PD}\}$ (when foreign demand for non-manufacturing D is perfectly elastic) such that the following equilibrium conditions and related definitions hold:

(i) Total industry expenditures in the US is given by

$$Y_j = \sum_{k\in\mathcal{J}} \beta_{kj} X_k + \beta_{F,j} wL, \quad \forall j \in \mathcal{J},$$

where X_j stands for domestic industry gross output, β_{kj} is the share of gross output of industry k expensed on inputs from industry j, and $\beta_{F,j}$ is the share of final consumption spent on industry j.

(ii) Goods market clearing yields a system of $|\mathcal{J}|$ equations in $|\mathcal{J}|$ industry profitability levels $\boldsymbol{B}$: output produced over all firms has to equal total domestic industry output, which has to equal output consumed at home plus

output exported to foreign markets:

$$\begin{aligned} X_j &= N \int \mathbb{E}[X_{fj}\,;\,\boldsymbol{B}]\,\mathrm{d}G(\boldsymbol{\xi}) \\ &= Y_j \frac{PD_j}{PD_j + \sum_{d\in\mathcal{D}^F} P\bar{M}_{dj}} + \sum_{d\in\mathcal{D}^F} \bar{Y}_{dj} \frac{PD_j}{PD_j + P\bar{X}_{dj}}, \quad \forall j \in \mathcal{J}, \end{aligned} \tag{29}$$

(iii) Domestic competitiveness $\boldsymbol{PD}$ *can be related to industry profitability* $\boldsymbol{B}$ *by combining equation (29) with an open-economy version of equation (12):*

$$B_j = (1-\varsigma_j)\left(\frac{c_j}{\varsigma_j}\right)^{\frac{\varsigma_j}{\varsigma_j-1}} \left(\frac{X_j}{PD_j}\right)^{\frac{1}{\sigma_j(1-\varsigma_j)}}, \quad \forall j \in \mathcal{J}, \tag{30}$$

where c_j *stands for the unit price index of a bundle of production inputs assembled using a homothetic Cobb-Douglas technology:*

$$c_j \equiv w^{\tilde{\beta}_{jl}} \prod_{k\in J} P_k^{\tilde{\beta}_{jk}},$$

where $\tilde{\beta}_{jl}, \tilde{\beta}_{jk}$ *denote expenditures of industry* j *on labor value-added* l *or input* $k \in J$ *as a share of total expenditures on production inputs (not to be confused with* $\beta_{jk} = \tilde{\beta}_{jk}\varsigma_j$*, which are shares over gross output), and the domestic consumption price index* P_j *(for both final and intermediate consumption) is:*

$$P_j^{1-\sigma_j} \equiv PD_j + \sum_{d\in\mathcal{D}^F} P\bar{M}_{dj}. \tag{31}$$

(iv) Balance in overall trade requires that the consumption value of manufacturing imports (less any tariffs T *collected) equal manufacturing exports plus net exports in the non-manufacturing sector, denoted* D *(also the manufacturing trade deficit):*

$$\sum_{j\in\mathcal{J}} Y_j \frac{\sum_{d\in\mathcal{D}^F} P\bar{M}_{dj}}{PD_j + \sum_{d\in\mathcal{D}^F} P\bar{M}_{dj}} - T = D + \sum_{j\in\mathcal{J}}\sum_{d\in\mathcal{D}^F} \bar{Y}_{dj} \frac{PD_j}{PD_j + P\bar{X}_{dj}}. \tag{32}$$

(v) The residual non-manufacturing sector is produced with constant returns to scale using labor under perfect competition. Domestic value-added and output in the residual sector is given by

$$(1-\eta_M)wL = D + \Pi + T + \beta_{F,NM} wL,$$

$\beta_{F,NM}$ *is the share of final consumption by private households on the non-manufacturing sector, and* Π *is net profits in the manufacturing sector given by equation (17).*

(vi) Manufacturing sector payroll is the sum of factor payments in stage II production and stage I capital accumulation:

$$\eta_M wL = \sum_{k\in\mathcal{J}} \left(1 - \sum_{j\in\mathcal{J}} \beta_{kj}\right) X_k - \Pi.$$

C.6 Proof of Proposition 3: Aggregate Economies of Scale and Scope

[xxx] I provide and prove a more general version of Proposition **??** in the main text. This more general version includes input-output linkages.

Proposition 4 (Aggregate Economies of Scale and Scope) *In the open economy equilibrium in Definition 1, domestic producer price indices* $\mathrm{d}\log \boldsymbol{\mathcal{P}}$ *respond to exogenous shocks to market size* $\mathrm{d}\log \boldsymbol{S}$ *according to:*

$$\mathrm{d}\log \boldsymbol{\mathcal{P}} = diag\left(\frac{1}{\sigma-1}\right)\left(\mathbb{I} - \boldsymbol{\Omega}^S diag(\boldsymbol{\lambda}_u^M) + \boldsymbol{\Psi}\left(\mathbb{I}-\boldsymbol{\Omega}^D\right)^{-1} diag(\boldsymbol{\lambda}^{cpt})\right)^{-1} \times \boldsymbol{\Psi}\left(\mathbb{I}-\boldsymbol{\Omega}^D\right)^{-1} \mathrm{d}\log \boldsymbol{S}, \tag{33}$$

where (i) $\mathbb{I}$ *is the identity matrix, (ii)* $\boldsymbol{\Psi}$ *is a macro joint production matrix containing inverse cross-industry supply-side elasticities* $\boldsymbol{\Upsilon}$ *for the 'average' firm:*

$$\begin{aligned}
[\boldsymbol{\Psi}]_{jk} &\equiv \sigma_j(1-\varsigma_j)[\boldsymbol{\Upsilon}^{-1}]_{jk} - \mathbf{1}_{j=k}, \\
[\boldsymbol{\Upsilon}]_{jk} &\equiv \sum_{m\in\mathcal{M}} (\rho_m - \theta_m)\,\bar{\lambda}_{jm}\,\bar{\mu}_{jmk} + \mathbf{1}_{j=k}\sum_{m\in\mathcal{M}} \theta_m\,\bar{\lambda}_{jm}, \qquad \forall j,k \in \mathcal{J},
\end{aligned} \tag{34}$$

where industry allocation shares $\bar{\mu}_{jmk}$ *indicate the average propensity for capital of type* m *to be allocated to industry* k *(relative to other industries* k'*) among firms that produce in* j*, and input utilization shares* $\bar{\lambda}_{jm}$ *indicate the average profit-contribution to industry* j *of capital* m *(relative to other capital types* m'*):*

$$\bar{\mu}_{jmk} \equiv \int \frac{\mathbb{E}[X_{fj}]\lambda_{fjm}}{\int \mathbb{E}[X_{fj}]\lambda_{fjm}\,\mathrm{d}G(\boldsymbol{\xi})}\mu_{fmk}\,\mathrm{d}G(\boldsymbol{\xi}), \qquad \bar{\lambda}_{jm} \equiv \int \frac{\mathbb{E}[X_{fj}]}{\int \mathbb{E}[X_{fj}]\,\mathrm{d}G(\boldsymbol{\xi})}\lambda_{fjm}\,\mathrm{d}G(\boldsymbol{\xi}),$$

(iii) $\boldsymbol{\Omega}^S$*,* $\boldsymbol{\Omega}^D$ *are matrices containing external input-output coefficients* β_{jk} *that reflect the share of industry* j *gross output expensed on production inputs from industry* k*:*

$$[\boldsymbol{\Omega}^S]_{jk} \equiv \beta_{jk}\frac{\sigma_j}{\sigma_k - 1}, \qquad [\boldsymbol{\Omega}^D]_{jk} \equiv \lambda_{uj}^X(1-\lambda_{j,F}^X)\frac{\beta_{kj}X_k}{\sum_{k'\in\mathcal{J}}\beta_{k'j}X_{k'}},$$

where $\lambda_{j,F}^X$ *is the share of final use among all expenditures on industry* j*,* λ_{dj}^X *is the share of US firms' sales exported to* d*, and (iv)* $\boldsymbol{\lambda}^{cpt}$ *reflects the potential for US firms to gain market share from foreign competitors:*

$$\lambda_j^{cpt} \equiv \sum_{d\in\{u,\mathcal{D}^F\}} \lambda_{dj}^X(1-\lambda_{dj}^M) \qquad \forall j \in \mathcal{J},$$

where λ_{dj}^M *is the share of country* d*'s consumption originating from US firms.*

Notation: denote the home country (the US) by u, and foreign countries by $d \in \mathcal{D}^F$.

I log-differentiate the system of equations in Definition 1 to express endogenous equilibrium variables (domestic price competitiveness, sales, etc) as a function of changes in exogenous variables (changes to domestic scale L, foreign demand $\tilde{\boldsymbol{Y}}_d$, and foreign price competitiveness $\boldsymbol{PX}$, $\boldsymbol{PM}$).

It is convenient to solve for the equilibrium impact on endogenous variables through their effect on domestic producer price competitiveness $\boldsymbol{PD}$ (an inverse price term introduced in Definition 1). Equation (29) is a market clearing condition that equates supply with demand. The first line describes the supply-side relationship between domestic producer price competitiveness $\boldsymbol{PD}$ and market output $\boldsymbol{X}$ such that firm production incentives are sustained under monopolistic competition. The second line describes a downward-sloping industry demand-curve: the higher is domestic price competitiveness PD_j (the lower

are producer prices), the greater is the value of market output. In autarky, this demand curve would be unit-elastic because both final demand and intermediate demand is Cobb-Douglas. In the open economy setup assumed here, demand is more than unit-elastic due to an additional foreign-market-share-stealing effect.

Demand-side. Log-differentiating the second line of equation (29) yields the following demand-side equilibrium relationship between sales $\boldsymbol{X}$ and domestic competitiveness $\boldsymbol{PD}$, in matrix algebra:

$$\mathrm{d}\log \boldsymbol{X} = \left(\mathbb{I} - \boldsymbol{\Omega}^D\right)^{-1} \left(\mathrm{diag}(\boldsymbol{\lambda}^{cpt})\,\mathrm{d}\log \boldsymbol{PD} + \mathrm{d}\log \boldsymbol{S} + \mathrm{diag}(\boldsymbol{\lambda}^X_u)\,\mathrm{d}\log \boldsymbol{PS}\right), \tag{35}$$

where λ^{cpt}_j measures the potential for US firms to gain market share from foreign competitors in each market d:

$$\lambda^{cpt}_j \equiv \sum_{d\in\{u,\mathcal{D}^F\}} \lambda^X_{dj}(1-\lambda^M_{dj}),$$

λ^X_{dj} is the share of the home country's sales going to d, λ^M_{dj} is the share of country d's consumption originating from the home country, the matrix $\boldsymbol{\Omega}^D$ contains external input-output coefficients denoting the extent to which changes in gross output in other industries k' affect gross output in j:

$$[\boldsymbol{\Omega}^D]_{jk} \equiv \lambda^X_{uj}(1-\lambda^X_{j,F})\frac{\beta_{kj}X_k}{\sum_{k'\in\mathcal{J}}\beta_{k'j}X_{k'}},$$

where $\lambda^X_{j,F}$ is the share of final use among all domestic consumption of industry j.

I group together the different types of exogenous shocks into two terms: (i) changes to market size faced by US producers, $\mathrm{d}\log \boldsymbol{S}$, and (ii) changes to prices of foreign goods in the US, $\mathrm{d}\log \boldsymbol{PS}$ (while this is also a market size shifter, I single it out here because import prices are a cost shifter in the supply-side equation):

$$\mathrm{d}\log S_j \equiv \lambda^X_{uj}\lambda^X_{j,F}\,\mathrm{d}\log L + \sum_{d\in\mathcal{D}^F}\lambda^X_{dj}\,\mathrm{d}\log \bar{Y}_{dj} - \sum_{d\in\mathcal{D}^F}\lambda^X_{dj}(1-\lambda^M_{dj})\,\mathrm{d}\log P\bar{X}_{dj}, \tag{36}$$

$$\mathrm{d}\log PS_j \equiv -\sum_{d\in\mathcal{D}^F}\lambda^{UM}_{dj}\,\mathrm{d}\log P\bar{M}_j,$$

where λ^{UM}_{dj} is the share of home country's consumption originating from country d.

Note that when the home country is in autarky, $\boldsymbol{\lambda}^{cpt} = 0$, there is no demand-side adjustment of industry output with respect to prices (given the unit-elastic demand curve).

Supply-side. I next turn to the supply-side relationship between market size (industry profitability) and prices. The first line of equation (29) can be log-differentiated (switching the order of summation across inputs, industries, and firms) to yield

$$\boldsymbol{\Upsilon}^{-1}\,\mathrm{d}\log \boldsymbol{X} = \mathrm{d}\log \boldsymbol{B},$$

where $\boldsymbol{\Upsilon}$ is the aggregate matrix of supply-side elasticities given by equation (34). I solve out for $\mathrm{d}\log \boldsymbol{B}$ in this expression. I log-differentiate the expression for industry profitability B_j in equation (30), open up the production input cost index c_j to reflect intermediate input purchases from manufacturing industries to yield,

$$\mathrm{d}\log B_j = \frac{\varsigma_j}{\varsigma_j - 1}\sum_{k\in\mathcal{J}}\tilde{\beta}_{jk}\,\mathrm{d}\log P_k + \frac{1}{\sigma_j(1-\varsigma_j)}(\mathrm{d}\log X_j - \mathrm{d}\log PD_j), \tag{37}$$

and using equation (31) to replace P_k with PD_k and exogenous foreign cost shocks $P\bar{M}_{dk}$ and combining

the previous two equations yields

$$\boldsymbol{\Psi}\,\mathrm{d}\log \boldsymbol{X} = -\left(\mathbb{I} - \boldsymbol{\Omega}^S\ \mathrm{diag}\left(\lambda_u^X\right)\right)\,\mathrm{d}\log \boldsymbol{PD} - \boldsymbol{\Omega}^S\,\mathrm{d}\log \boldsymbol{PS}, \tag{38}$$

where $\boldsymbol{\Psi}$ is an inverse matrix of supply elasticities with terms given by

$$[\boldsymbol{\Psi}]_{jk} \equiv \sigma_j(1-\varsigma_j)[\boldsymbol{\Upsilon}^{-1}]_{jk} - \mathbf{1}_{j=k},$$

and $\boldsymbol{\Omega}^S$ is matrix of external input-output price-related coefficients given by

$$[\boldsymbol{\Omega}^S]_{jk} \equiv \beta_{jk}\frac{\sigma_j}{\sigma_k - 1}.$$

Equations (38) and (35) represent two systems of equations in two vectors of unknowns ($\boldsymbol{X}$ and $\boldsymbol{PD}$)—aggregate industry-level demand and supply curves. I combine them to express the impact on equilibrium domestic price competitiveness $\boldsymbol{PD}$ changes in terms of arbitrary external shocks collected in $\mathrm{d}\log \boldsymbol{PS}$ and $\mathrm{d}\log \boldsymbol{S}$:

$$\begin{aligned}\mathrm{d}\log \boldsymbol{PD} = -\left(\mathbb{I} - \boldsymbol{\Omega}^S\mathrm{diag}(\lambda_u^M) + \boldsymbol{\Psi}\left(\mathbb{I} - \boldsymbol{\Omega}^D\right)^{-1}\ \mathrm{diag}(\lambda^{cpt})\right)^{-1} \times \\ \times\left(\boldsymbol{\Psi}\left(\mathbb{I} - \boldsymbol{\Omega}^D\right)^{-1}\,\mathrm{d}\log \boldsymbol{S} + \left(\boldsymbol{\Omega}^S + \boldsymbol{\Psi}\left(\mathbb{I} - \boldsymbol{\Omega}^D\right)^{-1}\mathrm{diag}(\lambda_u^X)\right)\,\mathrm{d}\log \boldsymbol{PS}\right).\end{aligned} \tag{39}$$

Proposition 3 follows the fact that the PPI is defined as $(1-\sigma_j)\,\mathrm{d}\log \mathcal{P}_j = \mathrm{d}\log PD_j$ and from setting $\mathrm{d}\log PS = 0$ in the above expression (so that the only exogenous shock is to market size).

C.7 Other Results in General Equilibrium

Proposition 3 describes how domestic producer prices $\mathrm{d}\log \boldsymbol{\mathcal{P}}$ respond to exogenous shifters of market size $\mathrm{d}\log \boldsymbol{S}$. This relationship depends on both demand-side and supply-side elasticities. I focus on the relationship between $\mathrm{d}\log \boldsymbol{\mathcal{P}}$ and $\mathrm{d}\log \boldsymbol{S}$ because they are useful for directly evaluating the impact of a range of counterfactual shocks. For example, market size shifters $\mathrm{d}\log \boldsymbol{S}$ include not only demand shocks such as changes in the labor force (a conventional scale shock), but also other shocks that shift the residual demand curve of the firm in an open economy, such as changes in foreign competitiveness.

Note that this relationship in Proposition 3 is generally neither (i) the elasticity of the PPI to gross output nor (ii) the elasticity of the PPI to aggregate quantities (price-elasticity of supply). Only in the special case under autarky when industry demand is unit-elastic is condition (i) true (that $\mathrm{d}\log \boldsymbol{S} = \mathrm{d}\log \boldsymbol{X}$). However, the proof laid out in the prior subsection is more than sufficient for computing other elasticities of interest. For example, the elasticity of output $\mathrm{d}\log \boldsymbol{X}$ with respect to exogenous shocks $\mathrm{d}\log \boldsymbol{S}$ can be computed by combining equations (35) and (38) to solve out for $\mathrm{d}\log \boldsymbol{PD}$.

I present two additional Corollaries of Proposition 3 that are of interest. For the sake of brevity I focus on the economy under autarky (and, in the case of Corollary 1, without input-output linkages), although the open-economy and input-output versions are straightforward to derive.

Corollary 1 focuses on the supply-side relationship and characterizes aggregate price-elasticities of supply in the economy. Suppose that there is an industry-wide composite good, Q_j, defined as a homothetic CES aggregator over individual quality-adjusted quantities q_{fj} provided by monopolistically competitive

firms (who operate joint production functions as described in our model):

$$Q_j \equiv \left(N \int q_{fj}^{\frac{\sigma_j - 1}{\sigma_j}} \, \mathrm{d}G(\boldsymbol{\xi}) \right)^{\frac{\sigma_j}{\sigma_j - 1}}, \quad \forall j \in \mathcal{J}.$$

The price index dual to this aggregator is the domestic PPI, $\mathcal{P}_j$ in each industry j.

With this representation I define aggregate economies of scale and scope in terms of own and cross-industry elasticities of prices $\boldsymbol{\mathcal{P}}$ with respect to composite quantities $\boldsymbol{Q}$. Locally, there are industry-level economies of scale if own-price elasticities are negative, and pairwise economies of scope (cost-complementarities) if cross-price elasticities are negative between j, k. To derive these partial-equilibrium supply-side elasticities I take equation (38) and replace $\mathrm{d}\log \boldsymbol{X} = \mathrm{d}\log \boldsymbol{Q} + \mathrm{d}\log \boldsymbol{\mathcal{P}}$. Rearranging terms yields the following result.

Corollary 1 (Aggregate Price Elasticities of Supply) *Under autarky without input-output linkages, the partial-equilibrium supply-side elasticity of prices to composite quantities is given by*

$$\mathrm{d}\log \boldsymbol{\mathcal{P}} = -\left(\boldsymbol{\Psi} - diag(\boldsymbol{\sigma} - 1)\right)^{-1} \boldsymbol{\Psi} \, \mathrm{d}\log \boldsymbol{Q}.$$

In general, off-diagonals in the joint production matrix $\boldsymbol{\Psi}$ generate non-zero cross-price elasticities. Note, however, that because $\boldsymbol{\Psi}$ appears twice and contain the inverse cross-industry matrix of sales responses to demand shocks for the average firm, $\boldsymbol{\Upsilon}$, the sign of pairwise industry responses within the firm (as measured in Section 1) is neither sufficient nor necessary for inferring economies of scope. In general equilibrium percolation effects across all industries need to be considered.

Under the special case of nonjoint production (i.e. $\rho_m = \theta_m$ for all capital inputs m), the off-diagonals of $\boldsymbol{\Psi}$ are zero and we recover certain well-known cases. When there are constant returns to scale in stage II production $\gamma_j = 1$, it is easy to check that as $\rho_m = \rho \to \infty$, we reach the limit where $\mathrm{d}\log \mathcal{P}_j = -\frac{1}{\sigma_j} \mathrm{d}\log Q_j$, so that (replacing Q_j with X_j/P_j) industry-level returns to scale reaches its maximum, $\mathrm{d}\log \mathcal{P}_j = -\frac{1}{\sigma_j - 1} \mathrm{d}\log X_j$. On the other hand as $\rho_m = \rho \to 1$, there are overall constant returns to scale over both stages I and II and so $\mathrm{d}\log \mathcal{P}_j = 0 \, \mathrm{d}\log Q_j$.

Next, in Corollary 2 I show that productivity (TFP) shocks operate differently from demand shocks in a monopolistically competitive environment. Define TFP shocks as industry-wide shifts to the $\tilde{\xi}_{fj}$ terms in the stage II physical production function (equation 1). Under the special case of autarky and Cobb-Douglas demand, Corollary 2 shows that joint production parameters do not affect the propagation of industry-wide cost shocks in the economy. Intuitively, firms are driven by profit incentives and industry-level changes in the cost structure do not affect profits at all in monopolistically competitive equilibrium. Cost savings are passed-through fully to the consumer, and due to unit-elastic industry-level demand there is no adjustment in industry-level expenditures. In the absence of input-output linkages (when $\boldsymbol{\Omega}^S = 0$) industry-level TFP shocks are contained within the industry of origin and do not propagate.

Corollary 2 (Propagation of Cost Shocks under Joint Production) *Under autarky, the general equilibrium elasticity of prices with respect to profitability shocks* $\mathrm{d}\log \tilde{\boldsymbol{\xi}}$ *in the firm's physical production function (in equation 1) is given by*

$$\mathrm{d}\log \boldsymbol{\mathcal{P}} = diag\left(\frac{1}{\boldsymbol{\sigma} - 1}\right) \left(\mathbb{I} - \boldsymbol{\Omega}^S\right)^{-1} diag\left(\boldsymbol{\sigma} - 1\right) \, \mathrm{d}\log \tilde{\boldsymbol{\xi}}.$$

Proof. The proof operates in similar fashion to the supply-side part of the proof of Proposition 3. Starting,

again, with the supply-side equation (29) but this time accounting for industry-level changes in $\tilde{\xi}_{fj}$ yields

$$\Upsilon^{-1} \, \mathrm{d} \log \boldsymbol{X} = \mathrm{d} \log \boldsymbol{B} + \operatorname{diag}\left(\frac{\sigma - 1}{\sigma(1-\varsigma)}\right) \mathrm{d} \log \tilde{\xi}.$$

Substituting in for $\mathrm{d} \log \boldsymbol{B}$ using equation (37) and rearranging terms and noting that $\mathrm{d} \log \boldsymbol{X} = 0$ on the demand side in autarky (unit-elastic industry demand curve) yields the result. ■

A final remark is that Corollary 2 is close to the result in Hulten (1978) with the exception of wedges σ_j created by monopolistic competition (in the outer sandwich diagonal matrices and also $\sigma_j/(\sigma_k - 1)$ in the input-output matrix Ω). In the limit as $\sigma_j = \sigma \to \infty$, Hulten's theorem holds for evaluating the impact of industry-level TFP shocks. I leave the evaluation of the impact of firm-level TFP shocks to future work.

C.8 Calibration to US Manufacturing Sector

Notation: $\mathcal{D}^F = \{c, r\}$ refer to China and the rest-of-the-world composite respectively.

Data in the Initial Equilibrium. These equilibrium definitions allow me to impute consumption expenditure shares β_j, the manufacturing deficit D, all price competitiveness indices, and foreign expenditures $\bar{Y}_{c,j}, \bar{Y}_{r,j}$ given US and world trade and industry level data in 2017. I use the following publicly available data in 2017:

1. Data on gross output by manufacturing industry, X_j come from the BEA in 2017.
2. I hold the number of total manufacturing firms, N, fixed, at 318,000.
3. Data on β_{jk} and ς_k come from the 2012 BEA I/O tables (the 2017 tables are not yet available).[35]
4. Trade data in 2017 on US imports and exports by country and industry (after mapping HS10 to BEAX) come from the US Census Bureau (made available by Schott (2008)).[36]
5. World trade data in 2017 by industry and country come from BACI Comtrade.

Variables in the Model. Using the trade data, I compute λ^X_{dj} as the share of US firms' total sales in industry j going to destination $d \in \{u, r, c\}$, and λ^M_{dj} as the share of consumption in destination $d \in \{u, r, c\}$'s in industry j on goods sold by the US. I express all the ratios of price competitiveness in Definition 1 as functions of these observable trade shares. I compute industry gross expenditures as $Y_j = \frac{X_j \lambda^X_{uj}}{\lambda^M_{uj}}$.

Using the estimated micro parameters, I repeat the same macro inversion steps as in the structural estimation to estimate macro variables $\boldsymbol{\alpha}, \boldsymbol{B}, Z$ in 2017. I use 1997 expenditure shares on knowledge inputs categories $m \in \mathcal{M}$ by each industry j combined with 2017 output data to impute expenses on knowledge inputs M_{jm} used in the inversion. With these macro variables on hand I compute net profits in the manufacturing sector Π integrating equation (17) over $G(\boldsymbol{\xi})$.

I compute the manufacturing deficit as the difference between total consumption and total output: $D = \sum_{j \in \mathcal{J}} Y_j - \sum_{j \in \mathcal{J}} X_j$. I normalize the wage w to 1 by choosing an appropriate unit in which to measure efficiency-adjusted labor, so that

$$L = GDP - \Pi,$$

[35] There are a few industries where implied input-output use shares are so large that final use is predicted to be negative. I adjust input-output shares downward by a proportional factor for that industry until final use is at least 2% of gross consumption.

[36] There are a few industries where US exports is higher than measures of gross output in BEA data. I harmonize the two data sources by adjusting gross output, X_j, to be at least 1% higher than gross exports.

where GDP is 19.4 trillion in 2017. The share of consumption on non-manufacturing is then given by:

$$1 - \beta_{F,NM} = \frac{\sum_{j \in \mathcal{J}} \left(Y_j - \sum_k \beta_{kj} X_k \right)}{L}.$$

I compute final consumption shares in manufacturing, $\beta_{F,k}$, as:

$$\beta_{F,k} = \frac{Y_k - \sum_j \beta_{kj} X_k}{L}.$$

Foreign demand in the model is given by $\bar{Y}_{r,j} \lambda^M_{rj} = EX_{urj}$ where EX_{urj} is US exports to destination r in industry j. An identical expression pins down $\bar{Y}_{c,j}$.

C.9 Quantifying the Impact of Joint Production

I use the calibrated model and Proposition 3 to quantify the impact of small shocks on equilibrium producer price indices. I compute the proportional increase in export demand in each industry j as $\mathrm{d} \log S_j = \sum_{d \in \mathcal{D}^F} \lambda^X_{dj} \mathrm{d} \log \bar{Y}_{dj}$ and express the impact on prices as:

$$\mathrm{d} \log \boldsymbol{\mathcal{P}} = \boldsymbol{\Xi} \, \mathrm{d} \log \boldsymbol{S},$$

where $\boldsymbol{\Xi}$ is a transmission matrix defined below, taking on different values across the scenarios described in the main body of text.

$$\boldsymbol{\Xi} = \mathrm{diag}\left(\frac{1}{\sigma - 1} \right) \left(\mathbb{I} - \boldsymbol{\Omega}^S \mathrm{diag}(\boldsymbol{\lambda}^M_u) + \boldsymbol{\Psi} \left(\mathbb{I} - \boldsymbol{\Omega}^D \right)^{-1} \mathrm{diag}(\boldsymbol{\lambda}^{cpt}) \right)^{-1} \boldsymbol{\Psi} \left(\mathbb{I} - \boldsymbol{\Omega}^D \right)^{-1}$$

For each scenario (associated with a different transmission matrix $\boldsymbol{\Xi}$), I decompose $\boldsymbol{\Xi}$ into own-diagonal and cross-diagonal elements:

$$\boldsymbol{\Xi} = \boldsymbol{\Xi}^{OWN} + \boldsymbol{\Xi}^{CROSS}.$$

The values of $\lambda^X_j \equiv X_j / X$ are shares of industry j output among total manufacturing output X. I use these shares to average industry-level changes in PPI $\mathrm{d} \log \mathcal{P}_j$ into a change in the aggregate manufacturing PPI. I compute, for example, the own-industry impact of a proportional change in foreign market size across all industries $\mathcal{J}$ on the PPI as:

$$\mathrm{d} \log PPI^{OWN} = (\boldsymbol{\lambda}^X)' \times \boldsymbol{\Xi}^{OWN} \times (\boldsymbol{\lambda}^X_r + \boldsymbol{\lambda}^X_c), \tag{40}$$

and equivalently for $\mathrm{d} \log PPI^{CROSS}$ using the $\boldsymbol{\Xi}^{CROSS}$ matrix above instead of $\boldsymbol{\Xi}^{OWN}$, and where $\mathrm{d} \log PPI^{OWN} + \mathrm{d} \log PPI^{CROSS} = \mathrm{d} \log PPI$.

Table C.7 describes the numbers behind Figure 5. I compute the effects of a 1% proportional rise in foreign demand (across all industries) on the manufacturing PPI and gross output. To compute the effect on industry gross output, I solve out for the demand and supply equations (38) and (35) to express $\mathrm{d} \log \boldsymbol{X}$ in terms of exogenous shocks $\mathrm{d} \log \boldsymbol{S}$. The change in manufacturing sector gross output is computed using the same λ^X_j-weighted average over industry-level changes. In Table C.7 I show positive cross-industry impacts separately from negative impacts, to emphasize that the majority of gross cross-industry elasticities are negative, price-decreasing.

Table C.8 presents results for the industry-level counterfactual, scenario (b), when foreign demand shocks occur industry-by-industry rather than manufacturing sector-wide. I compute the net cross-industry

Table C.7: Effect of a 1% Increase in Foreign Demand on Manufacturing PPI and Output

Scenario	(a)	(b)	(c)	(d)
Change in PPI (%)	-0.066	-0.082	-0.636	-1.410
Own-Industry	-0.066	-0.066	-0.067	-0.134
Negative Cross-Industry	0	-0.018	-0.526	-1.275
Positive Cross-Industry	0	0.002	0	0.000
Change in Gross Output (%)	0.387	0.416	1.976	3.922
Own-Industry	0.387	0.392	0.515	0.581
Positive Cross-Industry	0	0.032	1.460	3.342
Negative Cross-Industry	0	-0.003	0	0.000
Elasticity of PPI with respect to Gross Output	-0.17	-0.20	-0.32	-0.36
Share due to joint production	/	0.20	/	0.55

Notes: This table depicts the change in manufacturing PPI and output in the US due to a proportional 1% foreign demand shock across all industries. The sub-rows decompose the total impact into those accruing due to own-industry versus cross-industry responses. The four columns depict four scenarios corresponding to different versions of the underlying economy: (a) nonjoint production, (b) joint production, (c) nonjoint production with external I/O linkages, (d) joint production with external I/O linkages. The bars displayed in Figure 5 correspond to own versus cross-industry components of the PPI impact scaled by the total change in gross output. Altogether they sum to the elasticity of total PPI with respect to gross output, the penultimate row. The last row, 'share due to joint production', operates across scenarios. I compute this as the difference in the PPI response between (b) and (a) divided by the output response in (b). I do the analogous computation for scenario (d) by comparing to (c).

PPI impact as a share of the total PPI impact. I list in the table the top ten and bottom ten industries in terms of this cross-industry share. Note that the top industries listed do not indicate the industries where demand shocks generate the highest total PPI change, nor industries where the total elasticity of PPI to gross output is highest. The top five industries by total aggregate returns to scale (greatest elasticity in the overall PPI with respect to change in output) are aircraft manufacturing, petroleum refineries, other motor vehicle parts, light truck and utility vehicles, and broadcast and wireless communications equipment. These results as well as the full ranking of industries are available upon request.

C.9.1 Robustness to Alternative values of σ

I explore the sensitivity of the results in Table C.7 to alternative values of σ_j, against the benchmark of $\sigma_j = 5 \; \forall j$. I consider three different exercises. First, I vary the absolute value of σ but continue to hold it constant across industries. Figure C.1 shows how the elasticity of the inverse PPI with respect to gross output varies with values of $\sigma \in (3, 10)$, decomposing the total PPI impact into own and cross-industry components. While the own-industry (and overall) effect decreases as expected with the value of σ_j, the cross-industry effect does so at a much slower pace, contributing between 3 and 5 percentage points to the elasticity of the PPI to gross output across the entire range of values of σ.

In the next two calibrations, I allow σ_j to vary across industries j by: (i) assuming that profit shares (gross operating profits) in each industry are equal to $\frac{1}{\sigma_j}$ (as would be true in a case with constant returns to scale, monopolistically competitive firms, and sunk entry costs paid in some pre-period), and (ii) targeting the same own-sector returns to scale as in Bartelme et al. (2019). While assumption (i) is ad-hoc (and not consistent with the model), the values of σ_j nevertheless serve as a useful benchmark as they appear in other papers. Assumption (ii) is, however, model-consistent. In this scenario, I first generate a mapping from BEAX to the two-digit manufacturing sectors in Table 1 of Bartelme et al. (2019), denoted by s. I allow for

Table C.8: List of top and bottom industries by level of economies of scope

Industry	Description	Cross-industry contribution to aggregate scale elasticity
334300	Audio and video equipment manufacturing	-0.17
334118	Computer terminals and other computer peripheral equipment manufacturing	-0.14
339910	Jewelry and silverware manufacturing	-0.13
33461X	Manufacturing and reproducing magnetic and optical media	-0.10
334514	Totalizing fluid meter and counting device manufacturing	-0.10
33141X	Nonferrous Metal (except Aluminum) Smelting and Refining	-0.09
336991	Motorcycle, bicycle, and parts manufacturing	-0.09
333242	Semiconductor machinery manufacturing	-0.09
33641A	Propulsion units and parts for space vehicles and guided missiles	-0.09
33451B	Watch, clock, and other measuring and controlling device manufacturing	-0.08
331313	Alumina refining and primary aluminum production	-0.01
314110	Carpet and rug mills	-0.01
326160	Plastics bottle manufacturing	-0.01
311514	Dry, condensed, and evaporated dairy product manufacturing	-0.01
337215	Showcase, partition, shelving, and locker manufacturing	-0.01
312140	Distilleries	-0.01
311221	Wet corn milling	-0.01
327992	Ground or treated mineral and earth manufacturing	-0.00
33142X	Copper rolling, drawing, extruding and alloying	-0.00
311930	Flavoring syrup and concentrate manufacturing	0.01

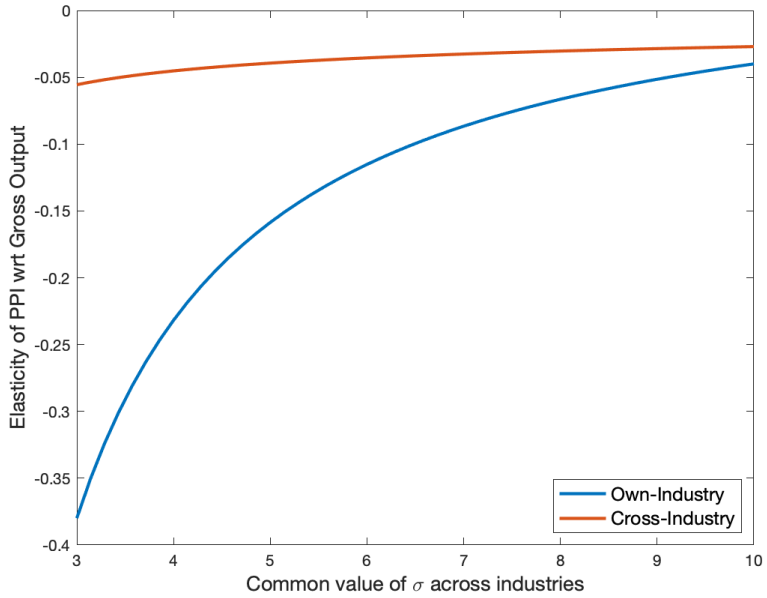
Notes: This table depicts the top and bottom ten industries in terms of the level of economies of scope induced by a marginal demand shock in that industry. Economies of scope (last column) are measured as the net cross-industry component of the manufacturing PPI change divided by the total change in manufacturing output. These effects are computed using equation (24); the numbers here correspond to y-axis values in the scatterplot in Figure 6.

Table C.9: Calibrated Demand Elasticities σ_j

BCDR sector	NAICS sectors	BCDR scale elasticity	σ under Joint Production
Food, Beverage, Tobacco	311, 312	0.16	5.7
Textiles	313, 314, 316	0.12	6.6
Wood Products	321	0.11	7.0
Paper Products	322, 323	0.11	6.5
Coke Petroleum	324	0.07	10.7
Chemicals	325	0.20	4.5
Rubber and Plastics	326	0.25	4.1
Mineral Products	327	0.10	6.1
Basic Metals	331	0.11	5.7
Fabricated Metals	332	0.13	5.8
Mach and Equipment	333	0.13	6.0
Computers	334	0.09	6.0
Electrical Machinery	335	0.09	7.4
Transport	336	0.15	6.1
All Other	337, 339	0.13	5.7

Notes: This table depicts the values of σ_j assigned to each BCDR sector in my baseline quantitative results. For example, industry 311930 and 311221 are both assigned $\sigma_j = 5.7$ as they belong to the same BCDR sector Food, Beverage, and Tobacco. These values of σ_j are calibrated so my joint production model yields the exact same within-sector increasing returns to scale as Bartelme et al. (2019). Under joint production own-sector scale elasticities are comprised of both within-industry and across-industry price declines among all industries that fall within each given BCDR sector.

Figure C.1: Elasticity of PPI to Gross Output under alternative common values of σ



Notes: This graph reports the sensitivity of the results in Figure 5 (where $\sigma_j = 5\ \forall j$) to alternative common values of σ across industries. It decomposes the elasticity of the PPI to output in scenario (b) (no I/O linkages) into own-industry impacts denoted in blue versus cross-industry impacts denoted in orange.

as many differences across σ_j as there are sectors s, so all σ_j is the same within a sector but different across sectors. I then solve for the values of σ_s that would generate the following relationship between sectoral price indices and sectoral size (from combining equations 40 and 38).

$$\frac{\mathrm{d}\log PPI_s}{\mathrm{d}\log X_s} = -\gamma_s^{BCDR} = -\sum_{j\in s}\frac{\lambda_j^X}{1-\sigma_s}\sum_{k\in s}\Psi_{jk}, \qquad \forall s,$$

where γ_s^{BCDR} are estimates of scale elasticities in Table 1 of Bartelme et al. (2019).[37]

Table C.10 displays how the baseline results in Table C.7 change with respect to the two different calibrations, both with and without I/O linkages. The different calibrations with heterogeneous σ_j do not alter the main quantitative message that cross-industry price impacts due to joint production are large. In the benchmark model set-up without input-output links (b), internal cross-industry elasticities arising from joint production accounts for between 19% (scenario (i), Profit Share) and 23% (scenario (ii), BCDR) of the total elasticity of PPI with respect to gross output. Whereas direct, own-industry effects under scenario (b) of the BCDR calibration generate an aggregate scale elasticity of 0.13 (=0.050/0.395), accounting for joint production raises this to 0.16 (=0.064/0.395).

[37]Note that this procedure does (correctly) attribute the cross-industry impacts *within* a sector to sectoral economies of scale. The only PPI impacts that would be missed in BCDR occur across sectors s.

Table C.10: Effect of a 1% Increase in Foreign Demand on Manufacturing PPI and Output: Robustness under different values of σ_j

Calibrated values of σ_j:	Baseline (=5)		(i) Profit Share		(ii) BCDR	
Scenario:	(b)	(d)	(b)	(d)	(b)	(d)
Change in PPI (%)	-0.082	-1.410	-0.085	-0.417	-0.064	-1.073
Own-Industry	-0.066	-0.134	-0.069	-0.094	-0.050	-0.101
Negative Cross-Industry	-0.018	-1.275	-0.018	-0.323	-0.016	-0.972
Positive Cross-Industry	0.002	0.000	0.002	0.000	0.002	0.000
Change in Gross Output (%)	0.416	3.922	0.408	1.639	0.395	3.299
Own-Industry	0.387	0.581	0.372	0.454	0.365	0.524
Positive Cross-Industry	0.032	3.342	0.039	1.185	0.032	2.775
Negative Cross-Industry	-0.003	0.000	-0.004	0.000	-0.003	0.000
Elasticity of PPI w.r.t. Gross Output	-0.20	-0.36	-0.21	-0.25	-0.16	-0.33
share due to joint production	0.20	0.55	0.19	0.43	0.23	0.52

Notes: This table explores the sensitivity of results in Table C.7 and Figure 5 to alternative calibrations of σ_j that vary across industries. Over the rows of each column I compute the change in PPI and output due to a proportional 1% foreign demand shock (across all industries). The sub-rows decompose the total change into own-industry versus cross-industry impacts. The last row, 'share due to joint production', operates across scenarios. I compute this as the difference in the PPI-change between (b) and (a) divided by the output-change in (b). I do the analogous computation for scenario (d) by comparing to (c). Across the columns, I alter the values of σ and the scenario. The first two columns reproduce key results in Table C.7: results obtained under the baseline assumption of $\sigma_j = 5\ \forall j$, for scenarios (b) (no I/O linkages) and (d) (incl. I/O linkages). The next set of two columns repeat the baseline calculations but using σ_j calibrated to match the profit share in each industry assuming constant returns to scale, and the final set of two columns repeat the baseline calculations but using σ_s calibrated (at the 2-digit manufacturing level) to match sectoral level returns to scale in Bartelme et al. (2019).

C.10 Counterfactuals: The Impact of Large Shocks

C.10.1 Tariff Shocks

The model accommodates different types of counterfactual shocks. I show how to evaluate the impact of new tariffs imposed by the US on imports from China, denoted by τ_{cuj}, as well as import tariffs imposed by China on imports from the US, denoted by τ_{ucj} (although the main exercise in the body of the paper considers only the former). I model tariffs $\tau \geq 1$ as ad-valorem, so that

1. The change in Chinese price competitiveness in the US is $\hat{\tilde{P}}\hat{M}_{cj} = \tau_{cuj}^{1-\sigma_j}$.
2. The change in US price competitiveness in China can be modeled as $\hat{\tilde{P}}\hat{X}_{cj} = \tau_{ucj}^{\sigma_j-1}$. Tariffs also cause take-home revenues of firms to fall to $\frac{1}{\tau_{ucj}}$ of tax-inclusive sales. This can be reflected by a change in $\hat{\tilde{Y}}_{cj} = \tau_{ucj}^{-1}$.
3. US import tariff revenues are given by

$$T' = \sum_j \frac{\tau_{cuj} - 1}{\tau_{cuj}} \tau_{cuj}^{1-\sigma_j} \lambda_{cj}^{UM} \hat{P}_j^{\sigma_j - 1},$$

I assume that pre-existing tariffs on Chinese imports are zero. If they are non-zero, the new tariffs change infra-marginal tariff revenues and the calculation needs to be revised. I assume that Chinese tariffs on US goods are taken out of the system and do not go towards increasing market demand $\bar{Y}_{c,j}$.

C.10.2 Solving for the Model's Variables in Exact Changes

For any set of counterfactual exogenous shocks, the system of equations admits a new solution for PD_j and w. I solve the system of equations in terms of exact hat changes. Specifically, for any guess of $\hat{PD}_j$ and $\hat{w}$, I can compute

$$\hat{B}_j = \hat{c}_j^{\frac{\varsigma_j}{\varsigma_j - 1}} \left(\frac{\hat{X}_j}{\hat{PD}_j} \right)^{\frac{1}{\sigma_j(1-\varsigma_j)}},$$

where $\hat{c}_j$ is given by

$$\hat{c}_j = \hat{w}^{\tilde{\beta}_{lj}} \prod_{k \in J} \hat{P}_k^{\tilde{\beta}_{jk}},$$

and $\hat{P}_j$ is the change in the domestic consumption price index given by

$$\hat{P}_j^{1-\sigma_j} = \hat{PD}_j \lambda_{uj}^{UM} + \hat{\tilde{P}}\hat{M}_{cj} \lambda_{cj}^{UM} + \hat{\tilde{P}}\hat{M}_{rj} \lambda_{rj}^{UM},$$

and $\hat{X}_j$ is given by

$$\hat{X}_j X_j = Y_j' \hat{PD}_j \lambda_{u,j}^{M} \hat{P}_j^{\sigma_j - 1} + \bar{Y}_{c,j} \hat{\tilde{Y}}_{c,j} \hat{PD}_j \lambda_{cj}^{M} \hat{P}_{chn,j}^{\sigma_j - 1} + \bar{Y}_{r,j} \hat{\tilde{Y}}_{r,j} \hat{PD}_j \lambda_{rj}^{M} \hat{P}_{row,j}^{\sigma_j - 1},$$

and $\hat{P}_{row,j}$ is the change in the rest-of-world consumption price index given by

$$\hat{P}_{row,j}^{1-\sigma_j} = \hat{PD}_j \lambda_{rj}^{M} + \hat{\tilde{P}}\hat{X}_{rj} (1 - \lambda_{rj}^{M}),$$

$\hat{P}_{chn,j}$ is the change in the consumption price index in China given by

$$\hat{P}_{chn,j}^{1-\sigma_j} = P\hat{D}_j \lambda_{cj}^M + P\hat{\tilde{X}}_{cj}(1-\lambda_{cj}^M),$$

and finally the new vector of gross expenditures Y_j' can be inverted from

$$Y_j' = \sum_k \beta_{kj}\left(\hat{X}_k X_k\right) + \beta_{F,j}\hat{w}L\hat{L},$$

where T' is tariff revenues defined above.

To evaluate the guess I use a system of $|\mathcal{J}|$ equations equal to deviations between industry sales as computed above, X_j', and the implied industry sales (by solving the firm's problem) given by equation 29 under the new B_j'. I also use the trade balance condition:

$$\sum_j Y_j' = D' + \sum_j \hat{X}_j X_j,$$

to either pin down D' when $\hat{w} = 1$ (foreign demand for non-manufacturing goods is assumed to be perfectly elastic), or to solve for $\hat{w}$ when D is held exogenous (as is more typical in trade counterfactuals). I find that a gradient based optimization algorithm works very well with this system of equations.

Equilibrium Changes. Throughout counterfactuals presented in Table 6, I compute several changes in macroeconomic variables of interest:

1. The change in the manufacturing CPI (consumer price index) is

$$\prod_j \hat{P}_j^{\beta_{F,j}}$$

2. The change in the manufacturing CPI excluding the domestic response of productivity is

$$\prod_j \left(\hat{P}_j^{CTF}\right)^{\beta_{F,j}},$$

where

$$\left(\hat{P}_j^{CTF}\right)^{1-\sigma_j} = \lambda_{uj}^{UM} + P\hat{\tilde{M}}_{cj}\lambda_{cj}^{UM} + P\hat{\tilde{M}}_{rj}\lambda_{rj}^{UM}.$$

3. Expressions for the change in imports, US output and US exports in each industry, tariff revenues and the deficit can also be computed directly given the equations above.

C.10.3 Alternative Tariffs

I compute the vector of alternative tariffs as follows. First, local effects of external shocks on the CPI can be computed by log-differentiating the price index equation (31):

$$\mathrm{d}\log CPI = \sum_j \beta_j \,\mathrm{d}\log P_j = \sum_j \frac{\beta_j}{1-\sigma_j}\left(\lambda_{uj}^M \,\mathrm{d}\log PD_j - \,\mathrm{d}\log PS_j\right),$$

where the change in foreign prices in the US is given by $\mathrm{d}\log PS_j = \lambda_{cj}^{UM}(\sigma_j - 1)\,\mathrm{d}\log\tau_j$. I use equation (39) to replace $\mathrm{d}\log PD_j$ above with its value in terms of the exogenous tariff shock, $\{\mathrm{d}\log\tau_j\}_j$.

Second, the change in imports from China is given by

$$\mathrm{d}\log IM_c = \sum_j \lambda_{cj}\left(-\lambda_{uj}^{UM}\,\mathrm{d}\log PD_j - (1-\lambda_{cj}^{UM})(\sigma_j-1)\,\mathrm{d}\log\tau_j + \mathrm{d}\log Y_j\right),$$

where λ_{cj} is the share among all US imports from China that come form industry j, and again $\mathrm{d}\log Y_j$ and $\mathrm{d}\log PD_j$ can be written in terms of $\mathrm{d}\log\tau_j$ using the equilibrium relationships surrounding equation (39).

A marginal tariff in each industry thus generates a cost (higher CPI) and a benefit (reducing imports from China, assuming that it is a policy objective). I manipulate the matrix of relationships above to yield

$$\mathring{\tau}_k = \frac{\frac{-\mathrm{d}\log IM_c}{\mathrm{d}\log\tau_k}}{\frac{\mathrm{d}\log CPI}{\mathrm{d}\log\tau_k}}, \quad \forall k \in \mathcal{J},$$

which expresses the effect of a marginal tariff in industry k towards achieving the policy target (reducing imports from China) per unit of increase in the CPI. Industries k with a higher $\mathring{\tau}_k$ should have higher tariffs (due to their higher benefit / cost ratio), but exactly how much higher is not clear given the non-linear structure of the model. I use this starting vector of alternative tariffs, $\mathring{\tau}$, as the basis for a search over parameter $\beta \geq 0$ that indicates the degree of bias away from uniform tariffs:

$$\tau_j = \max(1, \alpha\mathring{\tau}_j^{\beta}),$$

where the α is chosen so that the alternative tariffs τ_j reduce Chinese imports by 41%, the same amount as uniform tariffs of 20% would. Intuitively, values of β adjust the skew / bias relative to a baseline cost-benefit ratio per industry identified in $\mathring{\tau}_k$.[38]

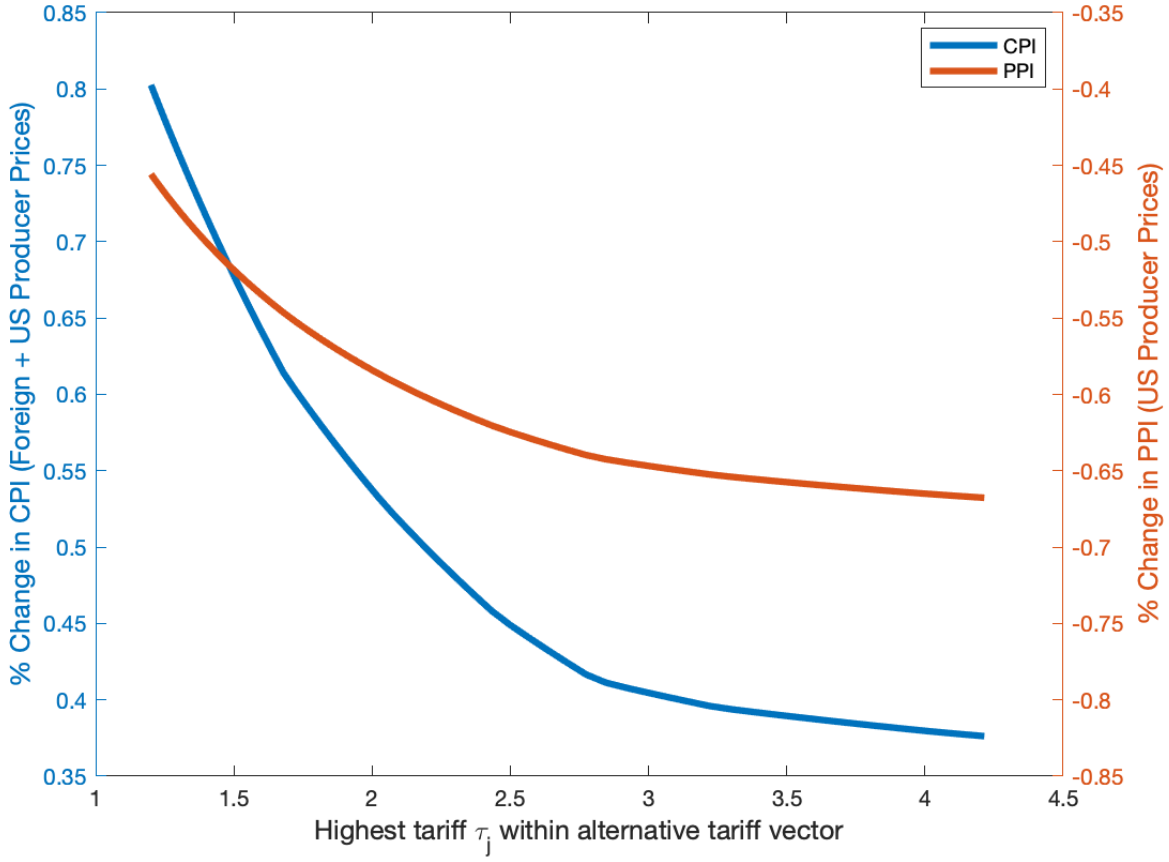
Figure C.2 displays the impact of alternative tariffs on the CPI and the (inverse, so they appear on the same axes) PPI, across varying degrees of industry bias in tariffs on the x-axis (induced by values of β). For each value of β, the parameter α is chosen to scale the set of tariffs so that the overall impact on reducing Chinese imports is 41%. The left-most point, where tariffs of $\tau = 1.2$ are symmetric across industries, corresponds to Column (2) of Table 6. Intuitively, as bias increases, more concentrated tariffs are required in order to achieve the same reduction in Chinese imports. Throughout these exercises I restrict tariffs to be weakly positive (so import subsidies are ruled out).

C.10.4 China Import Tariff Counterfactual: Model with Input-Output Linkages

Finally, in Table C.11, I repeat the China import tariff counterfactuals under the extension of the model with input-output linkages. First, in the neoclassical setting under constant returns to scale (Column 1), I find that input-output linkages generate higher increases in the CPI relative to the case without I/O linkages. Not only are prices of directly imported goods higher, but domestic goods prices also increase due to higher input costs. Column (2), which displays results from my model, highlights additional competing forces due to joint production. While import taxes raise the cost of foreign imported inputs, substitution towards the use of domestic intermediate goods generates a scale effect which lowers the cost of domestic inputs. These domestic price declines are amplified by further success in export markets (the foreign demand for US exports is elastic), so that the net effect on upstream input prices faced by US producers in any industry is theoretically ambiguous. I find, in this calibration, that the domestic scale effect dominates. The results

[38] While the values $\mathring{\tau}$ are identified using a local transmission matrix, the computed effects shown in Figure C.2 and Table 6 are fully non-linear and not local approximations.

Figure C.2: Impact of Alternative Tariff Policies on US Manufacturing CPI and PPI



Notes: This graph displays the impact on the manufacturing CPI (positive, in blue) and PPI (negative, in orange) from a range of tariff policies that are designed to achieve the same reduction in US imports from China (of 41%). Tariff policies range across the x-axis in terms of their degree of bias towards the vector $\hat{\tau}$ identified by the model's propagation matrix. The bias is summarized as the highest value of τ_j among the vector. A highest value of 1.2 (the initial point) reflects the benchmark counterfactual of blanket 20% tariffs.

Table C.11: Impact on the US Economy from US Tariffs on Chinese Imports: Extension of model with external I/O links

Model (with input-output linkages) Import tariffs on Chinese imports	(1) Neoclassical CRS 20% Uniform	(2) Joint Production 20% Uniform	(3) Joint Production Alternative
Change (%)			
Consumer Price Index (CPI)	1.69	0.11	-2.68
Producer Price Index (PPI)	0.58	-2.12	-6.10
Imports from China	-38.7	-40.8	-40.8
Imports from Rest of World	8.9	4.6	-4.2
US Output	0.6	7.4	20.2
US Exports	2.6	7.5	26.0
Manufacturing Trade Deficit	-3.0	-32.5	-86.3
As Share of Manufacturing Output (%)			
New Manufacturing Profits	0	0.28	0.72
New Tariff Revenues	0.89	0.86	0.15

Notes: This table extends the results in Table 6 to an environment with external, vertical input-output linkages. It presents estimates of the impact of two different sets of tariffs on outcomes in the US economy, under two different model settings calibrated to match US industry-level aggregates in 2017. Column (1) presents results from a 20% increase in tariffs on Chinese imports across all industries, under a neoclassical model with nonjoint production, constant returns to scale and perfect competition. Column (2) compares the impact of the same set of tariffs to my model with joint production. Column (3) presents results from alternative tariffs identified by my model that minimize the CPI impact while achieving the same reduction in total imports from China. See Definition 1 for a characterization of the equilibrium and Appendix Section C.10.3 for details on how I compute alternative tariffs.

in Column (2) under input-output linkages actually reduce welfare losses of tariffs compared to the main results presented in column (2) of Table 6 (without input-output linkages). The improved success of domestic producers is reflected in an increase in net profits (equal to 0.28% of gross manufacturing output) and a much larger reduction in the trade deficit. Finally, column (3) displays results from applying a similar process of searching for alternative import tariffs that mitigate the reduction in the CPI while achieving the same reduction in imports from China. The results demonstrate that there are pure gains from import protection in this environment with scale and scope economies, exogenous foreign prices, and wages pinned down by a residual (non-manufacturing) sector. These results are sensitive, however, to assumptions on the foreign import demand elasticity (implicit behind this setup is the ability of US producers to steal market share from foreign competitors in foreign markets). Inelastic foreign demand would curtail the amplification of domestic scale effects and would turn the impact on the CPI back to positive territory.