A theory of front-line management

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Preliminary and incomplete.

Abstract

Front-line managers are essential to the success of large organizations, yet they must often find a way to incentivize their workers without altering their wage. We propose a model of the interaction between a front-line manager and a worker in a large organization, and show that different "managerial styles" arise endogenously as a function of the information structure and the players' relative patience. In addition to improving the understanding of the interaction between workers and their immediate managers, the theory alludes to novel connections between an organization's personnel policies and the output that front-line managers induce from their workers.

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1 Introduction

A significant restructuring of large organizations was observed toward the end of the 20th century. While there was a decline in mid-level management, front-line managers became increasingly important and began receiving more discretion and managerial roles in addition to their traditional supervisory functions.[1] The managerial task of the front-line managers is special. On the one hand, their position and expertise improve their ability to monitor their subordinates and effectively adapt their tasks to changing operational needs. On the other hand, front-line managers are rarely the ones who design the labor contracts of their subordinates and typically are unable to adjust their wages in response to work fluctuations.

If the labor contracts were complete, the role of the front-line manager would mainly consist of monitoring activities. However, such contracts are extremely complex and unrealistic.[2] Workers may sometimes need to stay beyond their usual work hours to prepare urgent presentations or comply with an unexpected demand from a major client, deal with unexpected breakdowns, or take on temporary additional duties when a coworker is on sick leave. More generally, while it is hard to fully anticipate all possible events requiring special effort on the part of the worker, it is not controversial that such occasions form a substantial component of the dynamics in many organizations.[3] In such cases, the front-line manager can fulfill another fundamental role within a hierarchical organization beyond the supervisory function discussed in, for example, Williamson (1967), Calvo and Wellisz (1978,1978), Tirole (1986), and Laffont (1988). Namely, if given sufficient discretion, a front-line manager can be an effective means to *complement* the incomplete labor contract of the worker. Rather than specifying a long and detailed list of if-then circumstances within the worker's labor contract, the front-line managers can use their informational advantage and discretion to take opportune managerial decisions and provide additional incentives when needed.

Workers value not only the end-of-month payroll but also care about their work

[1] See, e.g., Cameron, Freeman and Mishra (1991) and Thomas and Dunkerley (1999).

[2] Indeed, Williamson (1975) and Hart (1995) argue that incomplete contracting is necessary to have a cogent theory of the firm.

[3] This has been noted by Williamson (1967) who writes "...the firm is required to adapt to circumstances which are predictable in the sense that although they occur with stochastic regularity, precise advance knowledge of them is unavailable. ... Coordination in these circumstances is thus essential. (p. 125)."

atmosphere and work-life balance.[4] Moreover, while the worker's monthly wage is typically outside the control of the front-line manager, it is not uncommon for such managers to have discretion over these aspects of the workplace. In particular, they often have discretion over their managerial practices and the allocation of certain perks to their team members (e.g., flexible work schedule, work from home, training, exemption from some duties, conferences and events, etc.). The centrality of such incentives may increase if, for example, owing to regulations or collective agreements, the amount paid to the worker at the end of the month is not responsive to the output or quality of work provided.

We argue that these incentivization tools have a substantially different structure compared to the traditional quasilinear model of monetary incentives. In particular, the per-period incentive budget is typically *small* (e.g., allowing work from home for only one day may not be enough compensation for substantial effort), and also such incentives are usually *perishable* (e.g., the possibility to allow work from home on a given day is wasted if not utilized that day).

We propose a tractable model to study incentive provision by the front-line manager. In our model, the (front-line) manager can only offer small and perishable per-period compensation, which he uses to incentivize worker's "extra" effort which is needed on random occasions. The interpretation is that, occasionally, there are peaks at work or events that require special attention and effort, beyond the regular work requirements. We refer to such events as "opportunities." In some cases, the manager observes the arrival of opportunities (e.g., when he receives an urgent task from his superiors), while in other cases, opportunities may be observed privately by the worker (e.g., creative ideas) who then decides whether or not to conceal them from the manager.

The inability of the manager to instantaneously compensate the worker for his effort captures the manager's restricted access to incentivization tools. The induced structural asynchronicity between work and compensation gives rise to a dynamic spillover between opportunities since new opportunities may arrive while the worker is still receiving compensation for his previous effort. Thus, the front-line manager faces a rather complex dynamic problem where the optimal way to resolve tradeoffs depends on the players' relative patience. While most papers focus on the case where the players discount future using the same discount

[4]For example, Mas and Pallais (2017) find that the average worker in their experiment is willing to give up 20 percent of his wages to avoid a schedule set by an employer on short notice.

factor, there is no inherent reason to assume that, within their specific interaction, the worker and the manager consider the future in the same way. For example, a manager who plans to lead the team for only a short while might be impatient (or a short-termist) compared to a worker who expects to stay in the same position for a very long time. In general, each party can be more or less patient than the other, and to an extent, this can be endogenously determined by the organizational promotion patterns and layoff policies.

We characterize the optimal contracts for any pair of the players' discount rates, both for the case where the arrival of opportunities is observable and for the case where opportunities can be concealed by the worker. The optimal contracts generate distinct qualitative features for different specifications of the model that naturally correspond to various "managerial styles." For example, what degree of flexibility at work do the workers enjoy and what affects this flexibility? How do the worker's workload and perks change over time? Do managers generate artificial ranks within their teams or do they maintain equal status between similar workers? If certain perks are granted, are they permanent or temporary?

We begin by considering the case in which the arrival of opportunities is observed by the manager. When the manager is *patient* relative to the worker, the manager treats the worker identically, regardless of past work or tenure. When an opportunity arrives, the manager requires the worker's effort and promises all perks within his discretion for a given amount of time. These promises, however, are "conditional" and do not accumulate: each promise will be nullified upon the arrival of the next opportunity. It is therefore a question not of *how much* work has been done but *whether* some work was done in the recent past.

When the manager is *impatient*, on the other hand, the interaction features completely different dynamics. There are three distinct phases. In the beginning, the worker is a *junior* and expected to exert maximal effort on every opportunity that arrives, without enjoying any perks. At some point, the worker moves to an *intermediate* status, in which he still needs to work whenever an opportunity arrives, but now he also enjoys all the perks at the manager's discretion. Finally, he is "promoted" to a *senior* status where he enjoys the maximal level of perks without exerting any effort (beyond his unmodeled baseline duties). Moreover, the promotion times to the intermediate and senior statuses are fixed (up to the arrival of the first opportunity) and do not depend on the actual amount of effort the worker provided over time. Hence, the manager generates a "*tenure-based seniority sys-*

tem"—a substantially different managerial style from that of the patient manager.

Note that the above contracts (both for the patient and impatient manager) feature very little correlation between work and compensation. In particular, the arrival of an opportunity is typically "bad news" for the worker. Hence, such contracts cannot be used to incentivize *concealable* opportunities. For the worker to reveal that an opportunity has arrived, the promise for future compensation must increase by at least the cost of the required effort. Hence, the optimal contracts for the case of concealable opportunities will have the *perfect bookkeeping* property: On every path of play, the discounted value (using the worker's discount rate) of granted perks is equal to the discounted cost of the work provided.

As in the case of observable opportunities, the relative patience of the players affects managerial style. However, the transition between the different managerial styles is more nuanced in the case of concealable opportunities. A seniority system still becomes more likely as the manager becomes more impatient. As a consequence of the perfect bookkeeping property, however, the seniority system that may arise in this case is *performance-based* rather than *tenure-based*. The *junior* and *senior* positions under the latter system are similar to those arising under the former: at the beginning, the worker exerts effort without receiving perks, and eventually, he enjoys all the perks without exerting any effort when opportunities arrive. The *intermediate* position, however, has a different structure: when opportunities are concealable, the worker enjoys some *partial* level of perks as a guaranteed baseline, and whenever he exerts effort he enjoys an immediate *temporary* increase in perks.[5]The seniority system is performance-based in that, unlike before, promotion is related directly to the realized amount of work over time.

Another substantial difference is that if opportunities are concealable, the seniority system arises only if the manager is *sufficiently* impatient relative to the worker. If the manager is only slightly impatient (or if he is patient), then the worker will enjoy perks immediately whenever he exerts effort. Due to the perfect bookkeeping property, these promises for future compensation accumulate. The slightly impatient manager will continue to incentivize maximum effort until he runs out of incentives. On the other hand, a manager who is more patient than the worker will choose to forgo available opportunities when the accumulated promise to the worker is sufficiently high.

Table 1 summarizes the qualitative dynamics of incentive provision for varying

[5]This intermediate position exists so long as the manager is not extremely impatient.

information structures and relative patience.

Opportunities	*Manager*		
	Patient	**Slightly Impatient**	**Very Impatient**
Observable	Conditional promises (Finite)	Tenure-based seniority system	
Concealable	Accumulating promises (Finite)	Accumulating promises (Infinite)	Performance-based seniority system

Table 1: Incentive Dynamics.

Based on the above analysis, we can now take the perspective of the organization in which the manager-worker interaction takes place and draw broader observations regarding the impact of the endogenous managerial style at the bottom of the hierarchical ladder. While the managerial style under concealable opportunities is stable when players have similar discount rates, when opportunities are observable the managerial style changes dramatically with the identity of the more patient player. From the perspective of the manager, the value from the interaction changes continuously with the players' relative patience. However, these changes may have significant implications for the organization. Assuming that both the front-line manager and the worker are less patient than the organization (arguably, a natural assumption for large organizations), we show that the value from the organization's perspective drops discontinuously when the manager becomes slightly less patient than the worker. That is, there is a discontinuity point at the point that is typically assumed in the literature. To illustrate that the drop in value from the organization's perspective can be significant, we show that, perhaps surprisingly, when the manager is slightly impatient relative to the worker, the organization may sometimes be better off under *concealable* rather than the *observable* opportunities case.

This observation regarding incentive provision in hierarchical organizations is interesting in its own right as it suggests that the organization may benefit from increasing information frictions down the organizational hierarchy. This may suggest that an organization might prefer to hire a professional manager who lacks relevant technical expertise, even if he does not have exceptional managerial skills.

Related Literature

This paper contributes to the substantial literature that has provided various explanations for the prevalence of hierarchical structures in large organizations. Williamson (1967) and Calvo and Wellisz (1978,1979) argue that hierarchies arise due to limitations on the number of employees that a manager can effective control and monitor.[6] Rosen (1982) suggests that hierarchies enable highly-talented senior managers to increase the productivity of their subordinates. Garicano (2000) and Harris and Raviv (2002) propose the idea that hierarchies enable the efficient utilization of expert knowledge within the firm, whereas Rajan and Zingales (2001) argue that hierarchies can also prevent employees from stealing a firm's core knowledge. Hart and Moore (2005) show that hierarchies can be an efficient method by which to allocate resources within the firm. See Mookherjee (2013) for an extensive review of this literature.[7]

Two papers that, like us, assume that the front-line manager is responsible for determining the responsibilities of his workers are McAfee and McMillan (1995) and Melumad, Mookherjee and Reichelstein (1995). These papers differs from us in that they consider a static problem in which there is no stohcasticity in the tasks that the worker must perform, and the manager can offer monetary incentives to the worker. Moreover, they focus on deriving conditions under which adding an intermediate level between the principal and worker is beneficial.

Our paper also complements works that study the optimal timing of compensation (e.g., Lazear 1981; Carmichael 1983) and, in particular, those that analyze the mixture between short- and long-term incentives in settings where information changes over time (e.g., Sannikov 2008; Garrett and Pavan 2012, 2015). We contribute to this literature by analyzing the efficient use of limited and perishable incentives in a dynamically changing environment.

Our work also contributes to the recent literature that studies optimal contracting under different discount factors. Opp and Zhu (2015) study relational contracting in a repeated moral hazard setting, Frankel (2016) studies dynamic dele-

[6] Tirole (1986) and Laffont (1988) show that, in this case, hierarchical structures are susceptible to collusion between the workers and the low-level managers that monitor them. More recently, Letina, Liu and Netzer (2020), show that if low-level managers are lenient towards workers, but do not collude with them outright, this problem can be overcome by an appropriately designed contest.

[7] In the broader theme of organizational design, Rantakari (2008) studies how the need to coordinate a firm's activities affects the optimal allocation of decision rights within a two tier hierarchy.

gation, Hoffmann, Inderst and Opp (2021) study a one-shot moral hazard problem in which there is delay in the arrival of information, and Krasikov, Lamba and Schacherer (2020) analyze a canonical adverse selection problem.

Finally, our work is related to the growing literature that studies principal agent interactions with randomly arriving "projects." Forand and Zápal (2020) and Bird and Frug (2021) derive the optimal contract under symmetric information: Forand and Zápal (2020) study a model with no transfers in which projects of different types arrive randomly over time, whereas Bird and Frug (2021) study a canonical employment model in which the agent's productivity of effort varies over time. Li, Matouschek and Powell (2017), Bird and Frug (2019), and Lipnowski and Ramos (2020) consider transfer-free environments with asymmetric information. More specifically, Li, Matouschek and Powell (2017) derive the optimal relational contract when the agent has private information on project availability. Bird and Frug (2019) derive the optimal contract under full commitment in a setting where the agent privately observes the stochastic arrival of different types of compensation opportunities. Lipnowski and Ramos (2020) characterize efficient equilibria when the agent has private information on project payoffs.

The paper proceeds as follows. In Section 2 we present the model. In Sections 3 and 4 we analyze, respectively, the case where opportunities are observable and concealable. In Section 5 we present some organizational implications of our analysis and discuss the role of selected modeling assumptions. All proofs are relegated to the Appendix.

2 Model

We consider an infinite-horizon continuous-time interaction between a manager and a worker, in which opportunities arrive stochastically over time according to a Poisson process with an arrival rate $\mu > 0$. The no-effort action, $a = 0$, is always available to the worker. When an opportunity arrives, and only then, in addition to the no-effort action, the worker can exert effort[8] $a \in (0, 1]$. The worker's effort $a \in [0, 1]$ induces a benefit of $a \cdot B$ to the manager and a cost of $a \cdot A$ to the worker, where $B > A > 0$. At each instant, the manager chooses a flow compensation $\varphi \in [0, 1]$. We assume that both the worker's marginal utility from compensation and the manager's marginal cost of compensation are constant and equal to 1.

[8]We discuss the case of storable opportunities in Section 5.3.

The players maximize expected discounted payoffs. We denote the worker's discount factor by r_w and focus on the case where there is no fundamental shortage of incentives. That is, we assume that the worker's discounted payoff from setting $\varphi = 1$ indefinitely exceeds his expected discounted cost of full-intensity work on all opportunities that arrive, even if one is currently available. Formally, we assume that[9]

$$A + \frac{\mu A}{r_w} < \frac{1}{r_w}.$$

We denote the manager's discount factor by r_m and refer to the manager as *patient* if $r_m < r_w$ and as *impatient* if $r_m > r_w$.

Our objective is to capture formal as well as informal arrangements between the manager and the worker, and also the asymmetry in their commitment abilities. Our solution concept will be the manager's optimal contract (assuming the manager has full commitment power and the worker has none). This approach captures the asymmetry between the manager and the worker more than the relational contracts approach (e.g., MacLeod and Malcomson 1989, Ray 2002, and Levin 2003).[10] It is worth empathizing that due to the upper bound on flow compensation the manager's compensation budget is limited. Hence, the assumption that the manager can fully commit to a contact is weaker than it is in models that allow for (unlimited) monetary compensation and can easily be justified via standard arguments about his reputational concerns.

Throughout the paper we assume that the worker's effort is perfectly observed by the manager, but we vary our assumptions about whether or not the manager observes the arrival of opportunities. If the manager does observe the arrival of opportunities ("*observable opportunities*"), then a public history h_t specifies for every $s < t$ whether or not an opportunity was available and the worker's choice of effort.[11] If the manager does not observe the arrival of opportunities ("*concealable opportunities*"), then a public history h_t contains only the worker's effort choices. Given the Poisson arrival of opportunities, any private information that the worker has about the availability of opportunities in the past is irrelevant, and so there is no need to keep track of his private information. Hence, to reduce notation and terminology we refer to a public history as a history under both information

[9] Allowing for the opposite inequality would add trivial cases with corner solutions that would not add much of substance but would needlessly impede the exposition.

[10] We discuss relational contracting in Section 5.2.

[11] In this case, whether or not the worker observes the arrival of opportunities is immaterial.

structures we consider. We denote the set of all histories of length t by H_t and the set of all finite histories by $H = \cup_{t \in \mathbb{R}_+} H_t$.

At the beginning of the interaction, the manager specifies a *work schedule*

$$\alpha : H \to [0, 1],$$

which assigns a required effort to every history should an opportunity arrive at that history, and he commits to a *compensation policy*

$$\varphi : H \to [0, 1],$$

which maps histories into a flow compensation. A pair $\langle \alpha(\cdot), \varphi(\cdot) \rangle$ is referred to as a *contract*. Without loss of generality, we assume that after the manager *detects* a deviation from the work schedule, he provides no compensation indefinitely.

We say that the contract $\langle \alpha(\cdot), \varphi(\cdot) \rangle$ is *measurable* if, at every history, the worker's continuation utility and the manager's continuation value are well defined. That is, the expectations

$$\mathbb{E}\left(\int_{s=t}^{\infty} e^{-r_w(s-t)} \left(\varphi(h_s) - \mu \alpha(h_s) A \right) ds | h_t \right),$$
$$\mathbb{E}\left(\int_{s=t}^{\infty} e^{-r_m(s-t)} \left(\mu \alpha(h_s) B - \varphi(h_s) \right) ds | h_t \right)$$

exist for every $h_t \in H$.

We say that the contract $\langle \alpha(\cdot), \varphi(\cdot) \rangle$ is *incentive compatible* if it is measurable and, for every $h_t \in H$, it is optimal for the worker to choose $\alpha = \alpha(h_t)$ (conditional on the availability of an opportunity), given the continuation of the contract. Note that if a deviation to a positive effort level is profitable at a given history, then so is a deviation to no effort.[12] Hence, a contract is incentive compatible if and only if the worker (weakly) prefers to follow $\alpha(\cdot)$ than to deviate to $\alpha = 0$ at some history. Since the worker can guarantee himself a payoff of zero by never exerting effort, there is no need to impose an explicit individual rationality constraint. In what follows, we restrict attention to incentive-compatible contracts. We state the

[12] A deviation to a non-zero level of effort is detected by the manager under both information structures, and so following such a deviation the worker will receive a continuation utility of zero. Since exerting effort is costly for the worker, such a deviation provides the worker with a strictly lower discounted payoff than exerting no effort on the current opportunity and then receiving a nonnegative continuation payoff.

incentive compatibility constraints formally in the following sections.

In our analysis and discussion of the results, we often compare contracts in terms of the timing of effort/compensation. We use the following relations. A work schedule $\alpha(\cdot)$ *postpones* effort relative to $\alpha'(\cdot)$ at a history h_t if, for all $\tau > t$,

$$\mathbb{E}\left(\int_{s=t}^{\tau} e^{-r_w(s-t)}\alpha(h_s)|h_t\right) \leq \mathbb{E}\left(\int_{s=t}^{\tau} e^{-r_w(s-t)}\alpha'(h_s)|h_t\right) \tag{1}$$

with an equality for $\tau = \infty$ and a strict inequality for some[13] τ. Similarly, a compensation policy $\varphi(\cdot)$ *postpones* compensation relative to $\varphi'(\cdot)$ at h_t if, for all $\tau > t$,

$$\mathbb{E}\left(\int_{s=t}^{\tau} e^{-r_w(s-t)}\varphi(h_s)|h_t\right) \leq \mathbb{E}\left(\int_{s=t}^{\tau} e^{-r_w(s-t)}\varphi'(h_s)|h_t\right) \tag{2}$$

with an equality for $\tau = \infty$ and a strict inequality for some τ. Analogous definitions for *expediting* effort and compensation are obtained by reversing the inequalities in (1) and (2).

Note that the above definitions use the worker's discount factor. In addition, observe that the manager-discounted marginal cost of providing the worker with a worker-discounted util t units of time from now is $e^{-r_m t}\frac{1}{e^{-r_w t}} = e^{(r_w - r_m)t}$ and, similarly, the manager-discounted marginal benefit from the worker exerting one worker-discounted util t units of time from now is $e^{-r_m t}\frac{B}{A}\frac{1}{e^{-r_w t}} = \frac{B}{A}e^{(r_w - r_m)t}$. Whether these expressions increase or decrease in t is fully pinned down by whether the manager is patient or impatient. The following observation is implied.

Observation 1.

1. *Expediting compensation and postponing effort are both profitable for a patient manager.*

2. *Postponing compensation and expediting effort are both profitable for an impatient manager.*

3 Observable Opportunities

We begin our analysis by studying the case where the manager observes the arrival of opportunities (e.g., assignments allocated to the front-line manager by his

[13]The latter requirement implies that a work schedule does not postpone effort relative to itself.

superiors). In this case, any deviation by the worker is detected, and since the worker can guarantee himself a continuation payoff of zero at any point in time, the incentive compatibility constraints are given by

$$-\alpha(h_t)A + \mathbb{E}\left(\int_{s=t}^{\infty} e^{-r_w(s-t)}\left(\varphi(h_s) - \mu\alpha(h_s)A\right)ds|(h_t, O, \alpha(h_t))\right) \geq 0 \quad \forall h_t \in H, \tag{IC_{obs}}$$

where $(h_t, O, \alpha(h_t))$ is the event in which, before time t, play proceeds according to h_t, and, at time t, an opportunity arrives and the worker exerts an effort of $\alpha(h_t)$ on that opportunity.

Patient manager: ***"Have you done anything for me lately?"***

When the manager is patient ($r_m < r_w$), increasing the lag between compensation and effort in a manner that keeps the worker indifferent is detrimental for the manager. It is therefore easy to see that to maximize the gain from the *first* opportunity that arrives, the manager would have to provide the worker with maximal compensation $\varphi = 1$ immediately after his work, for an interval of time that is just long enough to compensate the worker for his cost of effort. However, were the manager to do so, it would not be possible for him to provide prompt compensation for any additional opportunities that might arrive within that time interval. Hence, the cheapest form of compensation for the very first opportunity reduces the potential gain from further opportunities. In fact, a patient manager faces a complex optimization problem where he endeavors to provide timely compensation for the worker's effort on each of the randomly arriving opportunities. The main result of this section shows that the solution to this problem is simple and qualitatively appealing: under the optimal contract, the worker exerts the same effort α^* on all opportunities, and, along the path of play, he receives a flow compensation of $\varphi = 1$ if an opportunity has been available in the last S^* units of time.

A useful interpretation of this form of compensation is that of "conditional promises"; i.e., following the worker's effort on a given opportunity, the manager promises a fixed periodic compensation for a given time interval, but this promise is nullified upon the arrival of the next opportunity. The complete nullification of the manager's obligations to the worker upon the arrival of a new opportunity frees incentivization resources precisely when they are needed, and enables the manager to incentivize effort on the currently available opportunity via a new

conditional promise.

Our assumption that $A + \frac{\mu A}{r_w} < \frac{1}{r_w}$ implies that there exist incentive-compatible contracts that induce full effort on all opportunities that arrive. However, such contracts need not be optimal for the manager. To see why this is the case, recall that the manager's cost of providing the worker with one worker-discounted util t units of time in the future is $e^{(r_w - r_m)t}$. As the manager's profit from a util worth of effort exerted by the worker is $\frac{B}{A}$, it follows that the *maximal profitable lag* between compensation and work is T^*, where T^* is defined implicitly by

$$e^{T^*(r_w - r_m)} = \frac{B}{A}$$

if $r_m < r_w$, and is given by $T^* = \infty$ if $r_w \leq r_m$. If a conditional promise of length T^* is insufficient to incentivize the worker to exert full effort, then $\alpha^* < 1$; i.e., the manager instructs the worker to forgo part of each opportunity.

To formally characterize the optimal contract, let $\sigma_{-1}(h_t)$ denote the supremum of opportunity arrival times along the history h_t, and let $\sigma_{-1}(h_t) = -\infty$ for histories in which no opportunity have arrived. In all subsequent results, equalities and uniqueness statements should be interpreted as holding almost surely.

Proposition 1. *Assume that the manager observes the arrival of opportunities. If $r_m \leq r_w$, then there exist $\alpha^* \in (0, 1]$ and $S^* \in (0, T^*]$ such that*

$$\alpha(h_t) = \alpha^* \quad ; \quad \varphi(h_t) = \begin{cases} 1 & \text{if } t - \sigma_{-1}(h_t) \leq S^* \\ 0 & \text{if } t - \sigma_{-1}(h_t) > S^* \end{cases}$$

is an optimal contract. Moreover, this is the unique optimal contract if $r_m < r_w$.

When $r_w = r_m$ there are multiple optimal contracts. Intuitively, in this case, postponing compensation does not alter the manager's value or violate any of the worker's incentive compatibility constraints. Thus, any contract that results from postponing compensation relative to the optimal contract characterized in Proposition 1 is also an optimal contract. The assumption that there is no shortage of resources implies that if $r_w = r_m$, then under the optimal contract described in Proposition 1, $\alpha^* = 1$ and $S^* < \infty$. Hence, postponing compensation is feasible.

Impatient manager: ***"Tenure-based senioirity"***

By Observation 1, postponing compensation and expediting effort (according to the worker's discount factor) are both profitable for an impatient manager. The first observation underlying the characterization in this section is that neither postponing compensation nor expediting effort violates incentive compatibility. A direct implication of this is that *within* each history (i.e., along a given sequence of realized arrival times of opportunities), the worker will exert full effort on opportunities that arrive before some given (history-dependent) date and enjoy compensation from some other date onward. However, since the arrival of an opportunity is not informative about the future, it turns out that the manager is able to postpone compensation and expedite effort, in an incentive compatible manner, *across* histories as well. This is beneficial to the manager since, for example, relative to beginning compensating the worker on history-dependent dates, starting the compensation on a fixed date across all histories postpones compensation from the ex-ante perspective.

We now provide a slightly more detailed intuition for our next result: An impatient manager uses a tenure-based seniority system in which the clock begins at the arrival time of the first opportunity. Assume for a moment that an opportunity is always available at the beginning of interaction. Clearly, it is optimal for the impatient manager (and feasible by Assumption 1) to incentivize maximal effort on that opportunity. Consider an arbitrary incentive compatible contract $\mathcal{C}$ that does so, and denote by X and Y the worker's future discounted expected effort and compensation, respectively. Note that incentive compatibility of $\mathcal{C}$ implies $Y \geq X + A$. Define τ_X^O implicitly as

$$\int_0^{\tau_X^O} \mu A e^{-r_w t} dt = X,$$

and similarly define τ_Y^C such that

$$\int_{\tau_Y^C}^{\infty} e^{-r_w t} dt = Y.$$

Note that, after working on the first opportunity, the worker is indifferent between the continuation of $\mathcal{C}$ and the modified continuation contract where he is required to exert full effort on all opportunities that arrive before time τ_X^O and no effort afterwards, and he will receive no compensation until τ_Y^C and maximal compensation

thereafter. Moreover, the modified continuation contract is incentive compatible (irrespective of the realized arrival times of opportunities) due to the Poisson arrival of opportunities and the fact that $Y \geq X + A$.

Since this modification of the contract expedites effort and postpones compensation relative to C, such a modification is profitable for the manager. Hence, if the interaction begins with an available opportunity, the optimal contract must have a threshold structure as described above. If the interaction does not begin with an opportunity, it is straightforward that providing compensation to the worker before he exerts effort for the first time is suboptimal. Hence, in the optimal contract, "the clock is set to zero" at the arrival of the first opportunity, and thresholds relative to that arrival time determine the optimal contact.

The relation between the two thresholds under the optimal contact, τ^O and τ^C, can be identified by two simple optimality conditions. First, because compensation is postponed and effort is expedited, the only relevant incentive-compatibility constraint is the one at the arrival time of the first opportunity, which we denote σ_1. As compensation is costly to the manager, in optimum, this constraint will bind

$$A + \int_{\sigma_1}^{\tau^O+\sigma_1} \mu A e^{-r_w(t-\sigma_1)} dt = \int_{\tau^C+\sigma_1}^{\infty} e^{-r_w(t-\sigma_1)} dt. \tag{3}$$

Second, by marginally decreasing τ^C and increasing τ^O, the manager can incentivize the worker to exert effort $\tau^O - \tau^C$ after the time at which compensation is provided. Thus, in optimum, $\tau^O - \tau^C$ is such that the net surplus generated by effort is offset by the manager's relative impatience over $\tau^O - \tau^C$ units of time,

$$\frac{B}{A} = e^{(r_m - r_w)(\tau^O - \tau^C)}, \tag{4}$$

which, in particular, implies that $\tau^O > \tau^C$ for any $r_m > r_w$.

To characterize the optimal contract formally, let $\sigma_1(h_t)$ denote the infimum of the arrival times of opportunities along the history h_t, and let $\sigma_1(h_t) = \infty$ for histories in which no opportunity has arrived.

Proposition 2. *Assume that the manager observes the arrival of opportunities. If $r_m > r_w$, then the unique optimal contract is*

$$\alpha(h_t) = \begin{cases} 1 \text{ if } t \leq \sigma_1(h_t) + \tau^O \\ 0 \text{ if } t > \sigma_1(h_t) + \tau^O \end{cases} \quad \text{and } \varphi(h_t) = \begin{cases} 0 \text{ if } t \leq \sigma_1(h_t) + \tau^C \\ 1 \text{ if } t > \sigma_1(h_t) + \tau^C \end{cases},$$

where τ^C, τ^O *are the unique solution to* (3) *and* (4).

Proposition 2 suggests that tenure-based seniority systems arise naturally if the front-line manager is impatient. Under the contract characterized in the proposition, initially, the worker exerts effort but does not receive compensation; at some point ($\sigma_1 + \tau^C$), he begins to receive compensation but still has to work whenever an opportunity arrives; and finally (at $\sigma_1 + \tau^O$), he receives compensation without being required to exert further effort. Furthermore, as the manager becomes more impatient relative to the worker, seniority is attained more quickly and the worker's expected effort decreases. To see this, note that the optimality condition (4) can be written as

$$\frac{B}{A} e^{(r_w - r_m)\tau^O} = e^{(r_w - r_m)\tau^C}, \tag{5}$$

and since $\tau^O > \tau^C$, the derivative, with respect to r_m, of the LHS of (5) is less than that of the RHS. As (3) implies that τ^C is decreasing in τ^O, it follows that increasing r_m will lead to a decrease in τ^O (and an increase in τ^C).

Propositions 1 and 2 are visualized in Figure 1 for a typical sequence of opportunity arrivals. The upper panel depicts the worker's effort (in red) and compensation (in blue) when the manager is patient, and the lower panel depicts the same outcomes when the manager is impatient.

The defining feature of the problem faced by the front-line manager (regardless of the relative impatience) is that he uses *frequent but low-value* compensation in order to incentivize the worker to exert effort on *rare but difficult* opportunities. Thus, it is natural to compare settings where the frequency and magnitude of opportunities vary, while keeping the total value of expected opportunities constant. Formally, we say that the opportunities represented by (A_1, B_1, μ_1) are lumpier than those represented by (A_0, B_0, μ_0) if $A_1 = \lambda A_0, B_1 = \lambda B_0$, and $\mu_1 = \frac{\mu_0}{\lambda}$ for some[14] $\lambda > 1$.

When $r_w = r_m$, it turns out, that making opportunities lumpier does not impact the manager's value so long as the assumption that $A + \frac{\mu A}{r_w} < \frac{1}{r_w}$ continues to hold. This follows from three observations. First, since $r_w = r_m$ the assumption that $A + \frac{\mu A}{r_w} < \frac{1}{r_w}$ implies that under the contract characterized in Proposition 1 the worker exerts full effort on all opportunities. Second, the worker's expected utility from that contract is zero. Finally, if $r_w = r_m$, then the timing of compensation does not affect the manager's cost of providing compensation. However, even though

[14] Analogously, one could define *smoother* opportunities by considering $\lambda \in (0, 1)$.

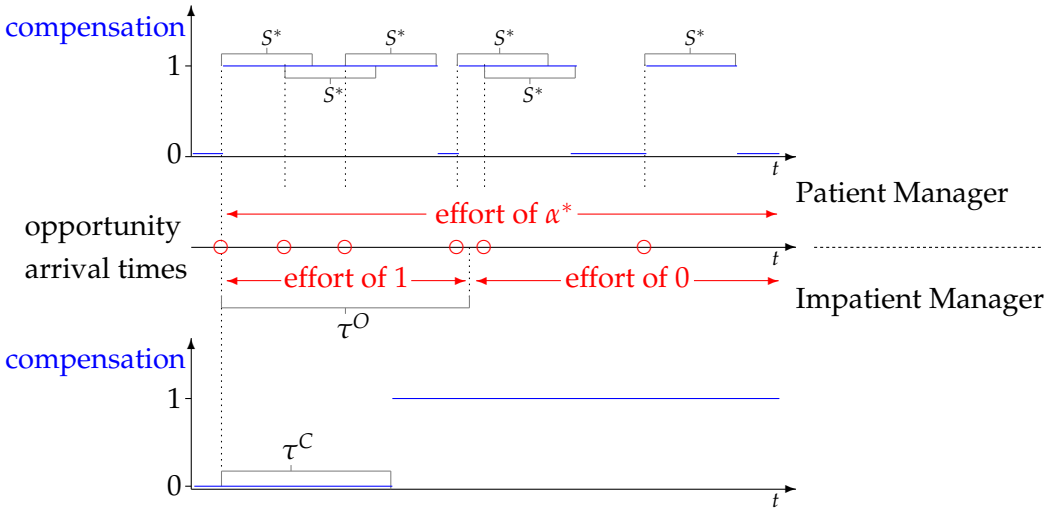


Figure 1: Qualitative dynamics of effort and compensation for patient and impatient managers for a representative history of opportunity arrivals.

common discounting is the standard assumption in the literature, the irrelevance of the degree of lumpiness in this case is a knife-edge result. In general, lumpiness is detrimental to the manager due to the asynchronicity of compensation and effort.

Proposition 3. *Assume that opportunities are observable and that $r_m \neq r_w$. The manager's value strictly decreases when opportunities become lumpier.*

4 Concealable Opportunities

In the optimal contracts derived in the previous section, the arrival of opportunities typically leads to an immediate decrease in the worker's continuation utility. In settings where the arrival of an opportunity is observed only by the worker, such contracts provide incentives for the worker to conceal opportunities from the manager. In order to provide incentives for the worker to reveal when opportunities become available, the arrival of opportunities must never be "bad news" for the worker. Hence, the incentive compatibility constraints when opportunities are concealable are

$$-\alpha(h_t)A + \mathbb{E}\left(\int_{s=t}^{\infty} e^{-r_w(s-t)}\left(\varphi(h_s) - \mu\alpha(h_s)A\right)ds|(h_t, O, \alpha(h_t))\right) \geq$$

$$\mathbb{E}\left(\int_{s=t}^{\infty} e^{-r_w(s-t)}\left(\varphi(h_s) - \mu\alpha(h_s)A\right)ds|(h_t, N)\right) \quad \forall h_t \in H, \qquad (IC_{conc})$$

where (h_t, N) denotes the event in which, before time t, play proceeds according to h_t, and, at time t, an opportunity does not arrive; and $(h_t, O, \alpha(h_t))$ denotes the event in which before time t, play proceeds according to h_t, and, at time t, an opportunity arrives and the worker exerts an effort of $\alpha(h_t)$ on that opportunity.

The need to make the arrival of opportunities not bad news for the worker suggests that when opportunities are concealable, the manager must keep track of both the worker's compensation and his effort. This, in turn, suggests that the optimal contact should depend on the worker's continuation utility. It is well known in the dynamic contracting literature that if the environment is stationary, then the agent's continuation utility can be used as a state variable for deriving the optimal contract (see Spear and Srivastava 1987 and Thomas and Worrall 1990). The argument behind this useful result relies on the property that the continuation payoffs of an efficient contract must always lie on the constrained Pareto frontier. This, in turn, follows from two simple observations: first, if the agent receives a continuation utility u via an inefficient continuation contract, then the principal can increase his value by replacing that continuation contract with a different contract that provides the agent with the same agent-discounted continuation utility u; second, since the agent is indifferent between the original and modified continuations of the contract, this change has no impact on earlier incentive compatibility constraints. Notice that these observations do not depend on the assumption that the players share the same discount factor and, thus, they are valid in our model where the players use different discount factors.

We denote, respectively, by $\alpha(u)$, $\varphi(u)$, and $V(u)$ the Markovian work schedule, the Markovian compensation policy, and the manager's value as a function of the worker's continuation utility. Notice that $u \in [0, \frac{1}{r_w}]$ as, from any point in time onward, the worker can guarantee himself a nonnegative payoff by exerting no effort, and the worker's value from receiving the maximal compensation indefinitely is $\int_0^{\infty} 1 \cdot e^{-r_w t} dt = \frac{1}{r_w}$. The Markovain value function satisfies two properties that help derive the optimal contract:

Lemma 1. *$V(u)$ is strictly decreasing and weakly concave.*

Lemma 1 has two important consequences. First, it directly implies that the worker's expected utility from an optimal contract is zero. Second, it implies that if opportunities are concealable, then the incentive compatibility constraint at every history is binding regardless of the relative patience of the players.

Corollary 1. *Assume that opportunities are concealable. Under an optimal contract:*

1. *The worker's expected utility is zero.*

2. *All the incentive compatibility constraints are binding.*

Corollary 1 provides an important prediction about the managerial style a front-line manager that does not observe the arrival of opportunities: he engages in perfect bookkeeping wherein, path by path, compensation and effort discounted according to the worker's discount rate are equal. This is in stark contrast to the style of a manager that observes the arrival of opportunities who induces a low correlation between work and compensation.

When opportunities are concealable, it is convenient to describe the optimal contract via its Markovian representation. By Corollary 1, the worker's continuation utility at the beginning of the interaction is zero and after he exerts effort $\alpha(u)$ his continuation utility increases by $\alpha(u)A$. Moreover, the drift in the worker's continuation utility while no opportunities arrive is

$$du = r_w u - \varphi(u). \tag{6}$$

Hence, the optimal contract is characterized by the solution of the HJB equation:

$$\begin{aligned}\sup_{\varphi(u),\alpha(u)\in[0,1]} \{&-r_m V(u) + V'(u)[r_w u - \varphi(u)] - \varphi(u) \\ &+\mu\Big(\alpha(u)B + V\left(u + \alpha(u)A\right) - V(u)\Big)\} = 0, \quad (HJB)\end{aligned}$$

subject to (6), where $V'(u)$ exists almost everywhere since $V(\cdot)$ is concave (Lemma 1). The following is the main result of this section.

Proposition 4. *If opportunities are concealable, then the optimal contract is generically unique. Moreover, there exist thresholds $u^C, u^O \in [0, \frac{1}{r_w}]$, such that the optimal contract*

is given by

$$\alpha(u) = \min\{1, \frac{u^O - u}{A}\} \quad ; \quad \varphi(u) = \begin{cases} 1 & \text{if } u > u^C \\ r_w u^C & \text{if } u = u^C \\ 0 & \text{if } u < u^C \end{cases}.$$

The threshold u^O dictates the dynamics of work. The threshold value $u^O = \frac{1}{r_w}$ corresponds to a work schedule in which the manager instructs the worker to fully exploit every opportunity that arrives until all of his compensation budget is exhausted. For lower values of u^O, the manager will initially instruct the worker to fully exploit every opportunity that arrives; however, once the worker's continuation utility reaches u^O this policy will change. In particular, if $u^C \leq u^O - A$, then the worker will temporarily exert low effort on opportunities and then revert to exerting high effort; if $u^C \in (u^O - A, u^O)$, then the worker will exert a low effort on all future opportunities; and if $u^C \geq u^O$, then the worker will cease exerting effort indefinitely.

The threshold u^C dictates the dynamics of compensation. In particular, it captures the degree of back/front-loading of compensation. So long as the worker's continuation utility is below u^C, compensation is deferred to the future. Setting the compensation threshold at the maximal possible value, $u^C = \frac{1}{r_w}$, corresponds to full back-loading: when the worker's continuation utility reaches that level, an indefinite provision of the maximal flow compensation, $\varphi = 1$, is necessary to provide the worker with his promised continuation utility. At the other extreme, the compensation threshold $u^C = 0$ corresponds to full front-loading because, in this case, the manager provides the maximal compensation whenever the worker's promised continuation utility is positive.

For $u^C \in (0, \frac{1}{r_w})$, the compensation dynamics consists of two phases. In the beginning, the *back-loading phase* takes place. So long as $u < u^C$, the worker exerts effort and accumulates promises for future compensation but does not receive any compensation. When his continuation utility attains (or exceeds) the threshold u^C, the *front-loading phase* begins. In this phase, the worker receives a permanent base compensation of $r_w u^C$—which can be thought of as compensation for the effort effort that was exerted in the back-loading phase—and a temporary additional compensation of $1 - r_w u^C$ whenever $u > u^C$—which can be thought of as compensation for the effort that was exerted in the initial front-loading phase. Observe that

if the worker's continuation utility exactly equals u^C, then the base compensation of $r_w u^C$ maintains the worker's continuation utility constant at that level. Hence, once the worker's continuation utility reaches u^C, it never drops below this level again, which, in turn, establishes that the optimal arrangement of compensation begins with the back-loading of compensation and transitions to the front-loading of compensation.

The relative patience of the players determines the values of both thresholds. In the next two propositions, we study the connection between the thresholds and relative patience. Moreover, we show that even though the manager engages in perfect bookkeeping, the aspects of his managerial styles that are related to relative patience are reminiscent of those aspects when opportunities are observable. Denote by $u(T^*) = \int_0^{T^*} e^{-r_w t} dt$ the maximal compensation that can be promised without exceeding the maximal profitable lag between effort and compensation.

Proposition 5. *If the manager is patient, then there is a unique optimal contract under which $u^O \in (0, u(T^*))$ and $u^C = 0$.*

A useful interpretation for the managerial style that corresponds to these thresholds is that compensation is provided via "accumulating promises"; i.e., following the worker's effort on a given opportunity, the manager promises the maximal compensation for a given time interval; in order to compensate the worker for additional effort that he exerts, the length of the existing promise is extended. Hence, after the worker has performed a lot of work in a short time interval, requiring additional effort necessitates providing compensation in the distant future. Since for a patient manager the net profit for incnetivizng effort is decreasing in the lag between effort and compensation, it follows that, in such cases, the manager will instruct the worker to refrain from exerting effort even though he could incentivize the worker to do so ($u^O < \frac{1}{r_w}$).

This feature of the optimal contract crates a distinct observable difference between the case in which opportunities are observable and the case in which they are concealable. In the former case, the front-line manager will require the worker to exert a constant effort, whereas in the latter case he will require fluctuating levels of effort. In fact, in the latter case the worker will oscillate between fully exploiting opportunities and exerting minimal effort on opportunities. To see this, observe then once the worker stops exerting full effort he will begin to gradually cash out his accumulated promises. Hence, after a long span of time in which only a few op-

portunities arrive, it will again be profitable for the manager to instruct the worker exert full effort.[15]

Next, we consider a (weakly) impatient manager. The threshold u^C is determined by the manager's benefit from postponing compensation. Postponing compensation impacts an impatient manager in two ways. First, it reduces his discounted cost of providing compensation, and second, it reduces his ability to incentivize the worker to exert effort in the future. Intuitively, the threshold u^C balances the cost and benefit from postponing compensation. Since the manager's benefit from postponing compensation is increasing in r_m and his cost from doing so is decreasing in r_m, it follows that full back-loading occurs only if the manager is "very" impatient, and that a slightly impatient manager will fully front-load compensation.

Proposition 6. *Fix A, B, μ, and r_w and suppose that $r_m \geq r_w$. Under an optimal contract*

- $u^C = \frac{1}{r_w}$.
- *There exists $r_m'' > r_m' > r_w$ such that* $u^C \in \begin{cases} \{0\} & \text{if } r_m < r_m' \\ (0, \frac{1}{r_w}) & \text{if } r_m \in (r_m', r_m'') \\ \{\frac{1}{r_w}\} & \text{if } r_m > r_m'' \end{cases}$.

The optimal contract offered by a impatient manager requires the worker to exert effort for as long as it is possible to incentivize him to do so. Due to the need for perfect booking, the worker will eventually stop exerting effort on all opportunities. Moreover, as was the case when opportunities are observable, a sufficiently impatient manager will establish a system of seniority, albeit a more nuanced one than the tenure based seniority system described in Section 3. Consider the case where $r_m \in (r_m', r_m'')$. In this case, the manager will institute a seniority system with three levels: juniors that work and don't receive compensation, intermediates that work and receive compensation, and seniors the receive compensation but don't work. However, this seniority system is performance based in the sense that "promotion" times depend on the entire history. In particular, the worker's attains intermediate status when his continuation utility reaches u^C for the first time, and

[15] It can be shown that u^O converges monotonically to $\frac{1}{r_w}$ as r_m increases toward r_w (proof available upon request). This suggests that as the manager's relative patience decreases (i.e., r_m increases), these low-effort intervals become longer and the average effort exerted on opportunities within these intervals becomes lower.

he reaches senior status when his continuation utility reaches the absorbing state of $\frac{1}{r_w}$. Moreover, the worker's compensation while he is in the intermediate status oscillates between $r_w w^C$ and 1 according to the random arrival of opportunities.[16]

The dependence of the work and compensation thresholds on the players' relative patience is visualized in Figure 2.

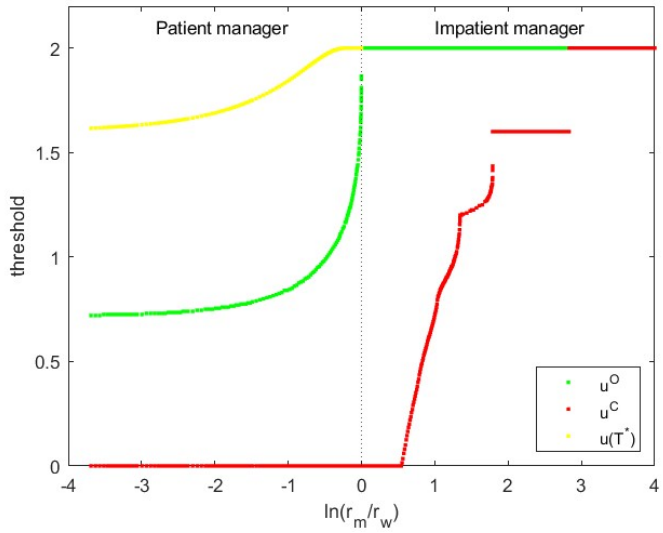


Figure 2: u^O (green) and u^C (red) as a function of $ln(\frac{r_m}{r_w})$, for $A = \frac{2}{5}, B = 2, r_w = \frac{1}{2}$, and $\mu = 2$. Note that the middle of the x-axis represents the case where[18] $r_m = r_w$.

Figure 2 shows that there are discontinuities in the optimal contract when the manager is sufficiently impatient relative to the worker. To understand the intuition for these discontinuity points suppose that the current state is $u = u^C = \frac{1}{r_w} - A$ (for the example in the figure this means $u = u^C = 1.6$). In this state the worker receives compensation, will exert full effort on the next opportunity to arrive, and will exert no effort on every subsequent opportunity. Thus, in essence, the manager is "paying" now for effort that will be exerted on the next opportunity to arrive. For this choice to be optimal, it must be the case that i) it is profitable to pay now for effort on the next opportunity to arrive (otherwise the manager could increase the compensation threshold), and ii) it is not profitable to pay now for

[16]The cases where $r_m \notin (r'_m, r''_m)$ induces a more degenerate seniority system, in which one of the three phases does not exist.

[18]The optimal thresholds were derived via Monte Carlo simulations. Note that on the extreme right of the figure the green dots are obscured by the red ones as $u^C = u^O = \frac{1}{r_w}$ when r_m is sufficiently large.

effort on the second opportunity to arrive (otherwise the manager could decrease the compensation threshold).

It is straightforward to verify that i) is violated if $r_m > r_w + (\frac{B}{A} - 1)\mu$. Hence, for such discount rates, the manager will not compensate the worker unless he must (i.e., when $u = \frac{1}{r_w}$). It follows that for $r_m > r_w + (\frac{B}{A} - 1)\mu$ the manager will set $u^c = \frac{1}{r_w}$ whereas for $r_m < r_w + (\frac{B}{A} - 1)\mu$ the manager will set $u^c \leq \frac{1}{r_w} - A$. Similarly, ii) is violated if $r_m < r_w + (\sqrt{\frac{B}{A}} - 1)\mu$. Hence, once r_m drops below this threshold the manager wants to set u^C below $\frac{1}{r_w} - A$. Note that were the manager to set u^C marginally below this level, then the worker's expected effort on the second opportunity to arrive would be negligible. Hence, for $r_m < r_w + (\sqrt{\frac{B}{A}} - 1)\mu$ the optimal compensation threshold is bounded away from $\frac{1}{r_w} - A$ (from below).

We conclude this section by revisiting the question of how the lumpiness of opportunities impacts the manager's value from the interaction. Intuitively, due to the perfect bookkeeping, when opportunities become lumpier, the asynchronicity of compensation and effort increases, which, in turn, reduces the manager's ability to use compensation resources effectively (regardless of whether he is more or less patient than the worker).

Proposition 7. *Assume that opportunities are concealable. The manager's value strictly decreases when opportunities become lumpier.*

5 Discussion

5.1 Organizational Implications

Both the front-line manager and the worker are part of a larger organization, and so the former's managerial choices, which depend on his relative patience and information structure, have implications for the organization at large. Our analysis can be used to derive the organization's preferences over the manager's information structure—which may depend on his qualifications—and relative patience—which can be influenced by promotion and layoff patterns within the organization. In particular, our model shows that, under some circumstances, the organization may prefer to hire managers that lack the technical expertise needed to observe the arrival of opportunities, even if they do not possess superior managerial skills.

The reason why the organization may prefer to deteriorate a front-line manager's information structure is related to how his managerial style changes around the focal point of same discounting. If the arrival of opportunities is observable, managerial style changes discontinuously around that point: a slightly patient manager front-loads compensation and back-loads effort, whereas a slightly impatient manager back-loads compensation and front-loads effort. In contrast, if opportunities are concealable, the managerial style of a slightly patient manager is similar to that of a slightly impatient one: both managers front-load compensation and use a similar work threshold.

This suggests that if the manager is slightly less patient than the worker, who, in turn, is slightly less patient than the organization, the organization faces a dilemma in choosing the manager's information structure. By making opportunities observable, the organization enhances the manager's ability to utilize his limited capacity to incentivize the worker, however, by doing so, it also missaligns the manager's incentives regarding the timing of compensation and effort. Example 1 demonstrates that when the profitability of effort is low and the limitation on the manager's capacity to compensate the worker is not too severe, the organization may prioritize the alignment of the manager's inter-temporal incentives.[19]

Example 1. *Assume that* $A = 1, B = 1.2, \mu = 0.5, r_m = 0.11, r_w = 0.1$ *and that the organization discounts future payoffs at a rate of* 0.09*. The organization's value is* 0.477 *if the arrival of opportunities is observable, and* 1.031 *if opportunities are concealable.*

In fact, it is easy to construct examples in which giving the front-line manager managerial discretion is profitable for the organization if and only if he does not observe the arrival of opportunities.[20] Note that these examples also demonstrate how small differences in the discount factors used by the front-line manager and the worker, may have a drastic impact on the organization.

5.2 Relational Contracting

Due to the inherent asymmetry between workers and managers, we assumed that the manager can fully commit to a contract. However, our main insights would re-

[19]The value for the organization in Example 1 is calculated by solving the manager's problem numerically, and evaluating the resulting streams of effort and compensation according to the organization's discount factor.

[20]For example, This will occur if $A = 0.1, B = 0.15, \mu = 0.5, r_w = 0.5, r_m = 0.51$ and the organization uses a discount factor of 0.4.

main valid were we to use a relational contracting approach instead. First, consider the case in which opportunities are observable. If the manager is patient, then the optimal contract derived in Proposition 1 is an equilibrium if players are patient enough and the relative gain from exerting effort on opportunities ($\frac{B}{A}$) is not too low. If that contract does not constitute an equilibrium, then the manager-preferred equilibrium will take the same form as that contract, but with a lower level of effort a^*. On the other hand, if the manager is impatient, then the tenure-based promotion system characterized in Proposition 2 that enables senior workers to stop exerting effort cannot be part of an equilibrium. However, the manager-preferred equilibrium would still consist of a three tier tenure-based promotion system, albeit, one in which senior workers are required to exert the minimal (constant) level of effort that makes the manager willing to provide them with full compensation indefinitely. Note that if players are patient and $\frac{B}{A}$ is high, then the required level of effort will be small.

Next, consider the case in which opportunities are concealable. In this case, the manager engages in perfect bookkeeping regardless of his commitment power. By Proposition 5, a patient manager always preserves some of his capacity to compensate the worker in order to incentivize effort on future opportunities. Hence, the optimal contract offered by such a manager constitutes an equilibrium if players are patient enough and $\frac{B}{A}$ is not too low. If that contract does not constitute an equilibrium, then the manager-preferred equilibrium will take the same form as that contract, but with a lower work threshold. In particular, the equilibrium work threshold is pinned down by the restriction that the manager's continuation value conditional on $u = u^O$ is zero. On the other hand, if the manager is impatient, the performance-based seniority system characterized by Proposition 6 cannot be part of an equilibrium as the manager would deviate from that contact once the worker becomes a senior. Nevertheless, the manager-preferred equilibrium would still induce a performance-based promotion system, albeit, one with only two levels: a junior level that is identical to the one under that optimal contract, and a non-junior level in which workers are compensated in the same manner as intermediate workers are under that contract. The threshold u^O is pinned down by the restriction that the manager's continuation value at $u = u^O$ is zero.

5.3 Storable Opportunities

The focus of our analysis was on applications in which opportunities are wasted if they are not acted upon immediately. However, our analysis is also relevant for some applications where it is possible to store opportunities (and instruct the worker to work on them in the future). If the manager is impatient, expediting effort is profitable. Hence, storing opportunities is suboptimal for an impatient manager. In contrast, if the manager is patient, it is profitable for him to reduce the time lag between effort and compensation, even if doing so delays the time at which the worker exerts effort. To see this, consider the manager's benefit from requiring effort (at different points in time) in return for a util of compensation that will be provided in t units of time. If the worker is required to exert effort immediately, the manager's benefit is $e^{-r_w t}\frac{B}{A}$, whereas if the worker is required to exert effort after t units of time, the (patient) manager's discounted benefit is $e^{-r_m t}\frac{B}{A} > e^{-r_w t}\frac{B}{A}$. That is, by requiring immediate effort (rather then storing the opportunity) the manager is, in essence, using the worker's higher discount factor to discount future benefits. This suggests that if opportunities are perfectly strorable, a patient manager would slice arriving opportunities into infinitesimal bits, and have the worker exert a constant flow effort in return for immediate compensation.

The previous observation relies on the implicit assumption that opportunities do not become obsolete while they are stored. However, this assumption is arguably inappropriate in many applications in which opportunities represent random events that change the organization's operational needs. Indeed, if stored opportunities depreciate at a rate that is greater than $r_w - r_m$, then the cost of deprecation outweighs a patient manager's gain from reducing the time lag between effort and compensation. Hence, our analysis is also relevant for some applications in which there is depreciation of stored opportunities.

References

Bird, Daniel, and Alexander Frug. 2019. "Dynamic nonmonetary incentives." *American Economic Journal: Microeconomics*, 11(4): 111–150.

Bird, Daniel, and Alexander Frug. 2021. "Optimal contracts with randomly arriving tasks." *Economic Journal*, 131: 1905–1918.

Calvo, Guillermo A., and Stanislaw Wellisz. 1978. "Supervision, loss of control, and the optimum size of the firm." *Journal of Political Economy*, 86(5): 943–952.

Calvo, Guillermo A, and Stanislaw Wellisz. 1979. "Hierarchy, ability, and income distribution." *Journal of Political Economy*, 87(5): 991–1010.

Cameron, Kim S., Sarah J. Freeman, and Aneil K. Mishra. 1991. "Best practices in white-collar downsizing: managing contradictions." *Academy of Management Perspectives*, 5(3): 57–73.

Carmichael, Lorne. 1983. "Firm-specific human capital and promotion ladders." *The Bell Journal of Economics*, 14(1): 251–258.

Forand, Jean Guillaume, and Jan Zápal. 2020. "Production priorities in dynamic relationships." *Theoretical Economics*, 15(3): 861–889.

Frankel, Alexander. 2016. "Discounted quotas." *Journal of Economic Theory*, 166: 396–444.

Garicano, Luis. 2000. "Hierarchies and the organization of knowledge in production." *Journal of Political Economy*, 108(5): 874–904.

Garrett, Daniel F., and Alessandro Pavan. 2012. "Managerial turnover in a changing world." *Journal of Political Economy*, 120(5): 879–925.

Garrett, Daniel F., and Alessandro Pavan. 2015. "Dynamic managerial compensation: A variational approach." *Journal of Economic Theory*, 159: 775–818.

Harris, Milton, and Artur Raviv. 2002. "Organization design." *Management Science*, 48(7): 852–865.

Hart, Oliver. 1995. *Firms, contracts, and financial structure.* Oxford and New York:Oxford University Press and Clarendon Press.

Hart, Oliver, and John Moore. 2005. "On the design of hierarchies: Coordination versus specialization." *Journal of Political Economy*, 113(4): 675–702.

Hoffmann, Florian, Roman Inderst, and Marcus M. Opp. 2021. "Only time will tell: A theory of deferred compensation." *Review of Economic Studies*, 88(3): 1253–1278.

Krasikov, Ilia, Rohit Lamba, and Thomas Schacherer. 2020. "Implications of unequal discounting in dynamic contracting." *SSRN Electronic Journal*.

Laffont, Jean-Jacques. 1988. "Hidden gaming in hierarchies: Facts and models." *Economic Record*, 64(4): 295–306.

Lazear, Edward P. 1981. "Agency, earnings profiles, productivity, and hours restrictions." *American Economic Review*, 71(4): 606–620.

Letina, Igor, Shuo Liu, and Nick Netzer. 2020. "Delegating performance evaluation." *Theoretical Economics*, 15(2): 477–509.

Levin, Jonathan. 2003. "Relational incentive contracts." *American Economic Review*, 93(835–857): 24.

Li, Jin, Niko Matouschek, and Michael Powell. 2017. "Power dynamics in organizations." *American Economic Journal: Microeconomics*, 9(1): 217–241.

Lipnowski, Elliot, and João Ramos. 2020. "Repeated delegation." *Journal of Economic Theory*, 188: 105040.

MacLeod, W. Bentley, and James M. Malcomson. 1989. "Implicit contracts, incentive compatibility, and involuntary unemployment." *Econometrica*, 57(2): 447–480.

Mas, Alexandre, and Amanda Pallais. 2017. "Valuing alternative work arrangements." *American Economic Review*, 107(12): 3722–3759.

McAfee, R. Preston, and John McMillan. 1995. "Organizational diseconomies of scale." *Journal of Economics & Management Strategy*, 4(3): 399–426.

Melumad, Nahum D., Dilip Mookherjee, and Stefan Reichelstein. 1995. "Hierarchical decentralization of incentive contracts." *RAND Journal of Economics*, 26(4): 654–672.

Mookherjee, Dilip. 2013. "Incentives in hierarchies." In *The handbook of organizational economics*. , ed. Robert Gibbons and Roberts John. Princeton University Press.

Opp, Marcus M., and John Y. Zhu. 2015. "Impatience versus incentives." *Econometrica*, 83(4): 1601–1617.

Rajan, Raghuram G., and Luigi Zingales. 2001. "The firm as a dedicated hierarchy: A theory of the origins and growth of firms." *Quarterly Journal of Economics*, 116(3): 805–851.

Rantakari, Heikki. 2008. "Governing adaptation." *Review of Economic Studies*, 75(4): 1257–1285.

Ray, Debraj. 2002. "The time structure of self-enforcing agreements." *Econometrica*, 70(2): 547–582.

Rosen, Sherwin. 1982. "Authority, control, and the distribution of earnings." *Bell Journal of Economics*, 13(2): 311–323.

Sannikov, Yuliy. 2008. "A continuous-time version of the principal-agent problem." *Review of Economic Studies*, 75(3): 957–984.

Spear, Stephen E., and Sanjay Srivastava. 1987. "On repeated moral hazard with discounting." *Review of Economic Studies*, 54(4): 599–617.

Thomas, Jonathan, and Tim Worrall. 1990. "Income fluctuation and asymmetric information: An example of a repeated principal-agent problem." *Journal of Economic Theory*, 51(2): 367–390.

Thomas, Robyn, and David Dunkerley. 1999. "Careering downwards? Middle managers' experiences in the downsized organization." *British Journal of Management*, 10(2): 157–169.

Tirole, Jean. 1986. "Hierarchies and bureaucracies: On the role of collusion in organizations." *Journal of Law, Economics, and Organization*, 2(2): 181–214.

Williamson, Oliver E. 1967. "Hierarchical control and optimum firm size." *Journal of Political Economy*, 75(2): 123–138.

Williamson, Oliver E. 1975. *Markets and hierarchies: Analysis and antitrust implications.* New York:Free press.

A Appendix: Proofs

Proof of Proposition 1.

Providing the worker with compensation before he exerted effort is suboptimal. Moreover, by the definition of T^*, the worker will not receive compensation if he has not exerted effort in the last T^* units of time. Otherwise, the manager would benefit from reducing compensation at t and decreasing the required effort on the last opportunity along h_t on which effort was exerted. Hence, for the rest of this proof we focus only on contracts under which $\varphi(h_t) = 0$ if $t - \sigma_{-1}(h_t) \geq T^*$.

Assume that an opportunity is currently available and denote by σ the random amount of time that will pass until the arrival of the next opportunity. Denote by

$$\alpha^* \equiv \min\{\frac{1}{A}\mathbb{E}\left(\int_0^{\min\{T^*,\sigma\}} e^{-r_w t}dt\right), 1\}$$

the maximal effort that the worker is willing to exert on an opportunity in exchange for a conditional promise of length T^* (i.e., setting $\varphi = 1$ until either T^* units of time have passed or an opportunity arrives).

Assume by way of contradiction that under an optimal contract $\alpha(h_t) > \alpha^*$ for a set of histories with positive measure. Since $\alpha^* = 1$ if $r_w = r_m$, for this part of the proof we consider only the case where[21] $r_m < r_w$. Denote by ν the set of finite histories h (of various lengths) for which $\alpha(h) > \alpha^*$ and $\alpha(h') \leq \alpha^*$ for every h' that is a prefix of h. Note that the contract reaches a history in ν with positive probability and that if $h, \tilde{h} \in \nu$, then neither history is a prefix of the other. Thus, it is sufficient to construct a profitable modification of the continuation contract conditional on an opportunity arriving at every $h_t \in \nu$.

Fix $h_t \in \nu$ and assume that an opportunity is available. A conditional promise of length T^* is not enough to compensate the worker for exerting an effort of more than α^* at h_t. Hence, with positive probability some of the compensation for the effort exerted at h_t must be provided after the arrival of the next opportunity. Formally, there exists a set $\nu_1(h_t)$ of continuation histories of h_t (of various lengths) with positive measure such that for every $h_s \in \nu_1(h_t)$ opportunities do not arrive in (t, s), and the worker's continuation utility conditional on an opportunity arriving at h_s is strictly positive (prior to exerting effort).

[21] If $r_w = r_m$, then $T^* = \infty$, and so the assumption that $A + \frac{\mu A}{r_w} < \frac{1}{r_w}$ implies that it is possible to incentivize full effort on all opportunities via an infinite conditional promise.

If there exists $\tilde{\nu} \subset \nu_1(h_t)$ with positive measure (conditional on reaching h_t) such that $\alpha(h_s) < 1$ for every $h_s \in \tilde{\nu}$, then postponing effort (according to the worker's discount factor) from h_t to the histories in $\tilde{\nu}$ is strictly profitable (Observation 1) and does not violate incentive compatibility.

If, on the other hand, $\alpha(h_s) = 1$ for almost all $h_s \in \nu_1(h_t)$, then for every such h_s there exists a set of continuation histories (of various lengths) with positive measure (conditional on reaching h_s) $\nu_2(h_s)$ such that for every $h_{s'} \in \nu_2(h_s)$: 1) no opportunities arrive in (s, s'), and 2) the worker's continuation utility if an opportunity arrives at $h_{s'}$ is greater than his continuation utility at h_s by at least $(1-\alpha^*)A$. To see why such histories exist, recall that $\varphi = 0$ if no opportunity arrived in the last T^* units of time, and so the maximal worker-discounted expected compensation that can be provided between two successive opportunities is $\alpha^* A$. Hence, to compensate the worker for exerting an effort of 1 on the opportunity at h_s, it must be the case that at least $(1-\alpha^*)A$ of this compensation is provided after the next opportunity arrives with positive probability.

If there exists $\tilde{\nu} \subset \nu_2(h_s)$ with positive measure (conditional on reaching h_s) such that $\alpha(h_{s'}) < 1$ for every $h_{s'} \in \tilde{\nu}$, then postponing effort from h_s to the histories in $\tilde{\nu}$ does not violate incentive compatibility. Moreover, if effort can be postponed in this manner from a subset of $\nu_1(h_t)$ with positive measure, then doing so increases the manager's value at h_t. Otherwise, for almost all $h_s \in \nu_1(h_t)$, it holds that $\alpha(h_{s'}) = 1$ for almost all $h_{s'} \in \nu_2(h_s)$. In this case, with strictly positive probability the worker's continuation utility at $h_{s'}$ is greater than his continuation utility at h_t by at least $(1-\alpha^*)2A$. Continuing in an iterative manner shows that the manager's value can be increased by postponing effort, as otherwise the worker's continuation utility increases without bound with positive probability (which cannot be the case as it is bounded from above by $\frac{1}{r_w}$).

For the rest of the proof we restrict attention to contracts for which $\alpha(h_t) \leq \alpha^*$ for all h_t. Denote by S^* the length of the conditional promise needed to provide the worker with $\alpha^* A$ discounted utils, i.e.,

$$\mathbb{E}\left(\int_0^{\min\{S^*,\sigma_1\}} e^{-r_w t} dt\right) = \alpha^* A.$$

Note that $S^* \leq T^*$ due to the definition of α^*.

Assume by way of contradiction that under an optimal contract $\alpha(h_t) < \alpha^*$ on a set of histories with positive measure. Let n be the minimal element of $\mathbb{N}$ for

which the worker's effort on the n^{th} opportunity to arrive is strictly less than α^* with positive probability. Denote by ν the set of histories (of various lengths) after which the worker's effort on the n^{th} opportunity to arrive is strictly less than α^*.

Suppose that an opportunity is available at h_t that is a prefix of a history in ν. The *No-Opportunity-Continuation of length S^* at (h_t, O)*, denoted by NOC-$S^*(h_t, O)$, is the continuation of (h_t, O) of length S^* along which opportunities do not arrive.[22]

First, consider the case where $\varphi = 1$ in almost all of NOC-$S^*(h_t, O)$ for every (h_t, O) that is a (proper or improper) prefix of a history that belongs in ν. Setting $\varphi = 1$ in NOC-$S^*(h_t, O)$ of the first n opportunities to arrive is enough to compensate the worker for exerting an effort of α^* on each of these opportunities. Thus, in this case, it is possible to increase the worker's effort on opportunities that arrive at every $h_t \in \nu$ without altering the compensation plan.

Second, consider the complementary case where there exists a maximal $m \in \{1, 2, \ldots, n\}$ for which there exists a set of histories (of various lengths) with strictly positive measure, ν', such that for every $h_t \in \nu'$: 1) $m - 1$ opportunities arrived along h_t, 2) (h_t, O) is a prefix of a history in ν, and 3) $\varphi < 1$ with positive probability in NOC-$S^*(h_t, O)$. Fix $h_t \in \nu'$ and assume an opportunity is available. By a similar argument to the one used above, the incentive compatibility constraint is not binding at any history that is both a continuation of (h_t, O) and a prefix of a history in ν. Hence, at h_t, the manager can increase his continuation value by increasing the worker's compensation in NOC-$S^*(h_t, O)$ and increasing $\alpha(h_s)$ at every $h_s \in \nu$ such that h_s is a continuation of (h_t, O). This is profitable, as the NOC-S^* of the m^{th} opportunity to arrive ends before S^* units of time have passed after the arrival of the m^{th} opportunity. Performing this modification at every $h_t \in \nu'$ increases the manager's value at time zero.

To conclude the proof, note that the contract described in the proposition is the only incentive compatible contract under which $\alpha(\cdot) \equiv \alpha$ and the worker does not receive compensation if he has not exerted effort in the last S^* units of time. ■

Proof of Proposition 2. When opportunities are observable, the manager's problem can be solved separately for each possible arrival time of the first opportunity. This is because the manager will not provide compensation prior to the first oppor-

[22] Recall that (h_t, O) denotes the event that play proceeds according to h_t and an opportunity arrives at h_t.

tunity, and the worker must have a nonnegative continuation utility at all times.

Consider an arbitrary first arrival time σ_1. We assumed that $A + \frac{\mu A}{r_w} < \frac{1}{r_w}$ and so the manager can incentivize the worker to exert maximal effort on the opportunity at σ_1. As the manager is impatient, he will do so in an optimal contract. Moreover, as we established in the main text, the manager will use a threshold structure. Hence, the manager's objective function (conditional on σ_1) is

$$\max_{\tau^O(\sigma_1),\tau^C(\sigma_1)} e^{-r_m\sigma_1}B + \int_{\sigma_1}^{\sigma_1+\tau^O(\sigma_1)} \mu B e^{-r_m t}dt - \int_{\sigma_1+\tau^C(\sigma_1)}^{\infty} e^{-r_m t}dt$$

$$\text{s.t. } A + \int_{\sigma_1}^{\sigma_1+\tau^O(\sigma_1)} \mu A e^{-r_w t}dt = \int_{\sigma_1+\tau^C(\sigma_1)}^{\infty} e^{-r_w t}dt. \tag{7}$$

This constraint (7) states that the worker's continuation utility at σ_1 is zero.

The assumption that $A + \frac{\mu A}{r_w} < \frac{1}{r_w}$ implies that, in optimum, both $\tau^O(\sigma_1)$ and $\tau^C(\sigma_1)$ are interior. To see this, note that the constraint (7) is violated if $\tau^C(\sigma_1) = \infty$ or $\tau^C(\sigma_1) = 0$. Furthermore, setting $\tau^O(\sigma_1) = 0$ implies that $\tau^C(\sigma_1) > \tau^O(\sigma_1)$, and so by slightly increasing $\tau^O(\sigma_1)$ (and decreasing $\tau^C(\sigma_1)$ to maintain incentive compatibility) the worker will exert more effort on opportunities for which he will receive compensation after he has exerted effort. As the manager is impatient, this change is profitable. Finally, setting $\tau^O(\sigma_1) = \infty$ implies that the worker continues exerting effort for an arbitrarily long period of time after he begins receiving compensation. Because the manager is impatient, slightly increasing $\tau^C(\sigma_1)$ (and decreasing $\tau^O(\sigma_1)$ to maintain incentive compatibility) is profitable.

The above discussion implies that the optimal thresholds are given by the FOC of the Lagrangian that corresponds to the above (concave) maximization problem. The first-order conditions with respect to $\tau^O(\sigma_1)$ and $\tau^C(\sigma_1)$ are, respectively,

$$\begin{aligned} &\mu B e^{-r_m\tau^O(\sigma_1)} - \gamma(\sigma_1)e^{-r_w\tau^O(\sigma_1)}\mu A = 0 \\ &e^{-r_m\tau^C(\sigma_1)} - \gamma(\sigma_1)e^{-r_w\tau^C(\sigma_1)} = 0, \end{aligned}$$

where γ is the Lagrange multiplier. It follows that $\frac{B}{A} = e^{(r_m-r_w)(\tau^O(\sigma_1)-\tau^C(\sigma_1))}$. Hence, $\tau^O(\sigma_1) = \tau^C(\sigma_1) + K$, where $K > 0$ is a constant that does not depend on σ_1. This relation implies that the LHS of (7) is increasing in $\tau^O(\sigma_1)$ while the RHS is decreasing in $\tau^O(\sigma_1)$. Hence, there is a unique optimal solution that does not depend on σ_1. ■

Proof of Proposition 3. We establish this proposition separately for the case where the manager is patient and the case for which he is impatient.

Case 1: patient manager. We start this proof by establishing the comparative statics of α^* with respect to λ. If the worker's expected utility from a conditional promise of length T^* is strictly greater than A, then the worker exerts full effort on all opportunities. Moreover, this will remain the case if opportunities become marginally lumpier. Thus, we focus on the case where the worker's expected utility from such a promise is at most A.

The worker's expected utility from a conditional promise of length T^*, as a function of the parameter λ, is $\frac{1}{r_w+\mu/\lambda}(1-e^{-T^*(r_w+\mu/\lambda)})$. The marginal increase in the value of such a promise from making opportunities lumpier is $\frac{\mu}{(\mu+r_w)^2}(1-e^{-T^*(\mu+r_w)}T^*(\mu+r_w))$. Thus, to establish that the α^* is decreasing in λ it is enough to show that making opportunities marginally lumpier has a larger impact on the cost of exerting the required effort than on the value of a conditional promise of length T^*.

Under the assumption that a conditional promise of length T^* does not provide excess compensation, it holds that $A \geq \frac{1-e^{-T^*(\mu+r_w)}}{\mu+r_w}$. Hence, it is sufficient to show that

$$\frac{\mu}{(\mu+r_w)^2}(1-e^{-T^*(\mu+r_w)}T^*(\mu+r_w)) < \frac{1-e^{-T^*(\mu+r_w)}}{\mu+r_w}.$$

When $\mu+r_w$ is kept constant, this inequality is harder to satisfy for higher values of μ. Thus, it is sufficient to show that it holds for $r_w=0$ i.e., to show that

$$1-e^{-\mu T^*}(1+\mu T^*) < 1-e^{-\mu T^*},$$

which is an inequality that is true for any $\mu T^* > 0$. Note that the above calculation does not depend on the value of T^*. Hence, the same calculation shows that when it is possible to induce full effort, S^* increases when opportunities become marginally lumpier.

Next, we show that making opportunities lumpier is detrimental for the manager. If $\alpha^* = 1$, this is an immediate consequence of S^* being increasing in λ. Assume that $\alpha^* < 1$ and let $f(r,\lambda) = \frac{1-e^{-T^*(r+\frac{\mu}{\lambda})}}{r\lambda+\mu}$ denote the $r-$discounted compensation that is provided via a conditional promise of length T^* as a function of λ. Note that the average cost of providing a util of compensation is $\frac{f(r_m,\lambda)}{f(r_w,\lambda)}$.

The cross-derivative of $f(r,\lambda)$ evaluated at $\lambda = 1$ is equal to

$$\frac{\partial^2 f(r,\lambda)}{\partial r \partial \lambda}|_{\lambda=1} = \frac{\mu e^{-T(\mu+r)}\left(T^*(\mu+r)(T^*(\mu+r)+2) - 2e^{T^*(\mu+r)}+2\right)}{(\mu+r)^3}.$$

The sign of this cross-derivative is the sign of $x(x+2)+2-2e^x$, where $x = T^*(r+\mu)$. As this sign is negative, the cross-derivative is negative. As $f(r,\lambda)$ is positive and decreasing in λ, it follows that the average cost of compensating the worker for his effort is increasing in λ (recall that $r_w > r_m$). As the worker's total effort is also decreasing in λ, we can conclude that making opportunities lumpier reduces the manager's value.

Case 2: Impatient manager. Solving the optimal thresholds for the contact characterized in Proposition 2, τ^O, τ^W, as a function of λ gives

$$\tau^O(\lambda) = \frac{\ln\left(A\mu\left(\frac{B}{A}\right)^{\frac{r_w}{r_w-r_m}}+1\right) - \ln(A(\mu+\lambda r_w))}{r_w} + \frac{\ln\left(\frac{B}{A}\right)}{r_m - r_w},$$
$$\tau^C(\lambda) = \frac{\ln\left(A\mu\left(\frac{B}{A}\right)^{\frac{r_w}{r_w-r_m}}+1\right) - \ln(A(\mu+\lambda r_w))}{r_w}.$$

Recall that the manager's value is

$$\mathbb{E}\left(e^{-r_m\sigma_1}\left(B\lambda + \frac{B\mu(1-e^{-r_m\tau^O(\lambda)})}{r_m} - \frac{e^{-r_m\tau^C(\lambda)}}{r_m}\right)\right).$$

Plugging in the expressions for the optimal thresholds, differentiating with respect to λ, and evaluating at $\lambda = 1$ shows making opportunities marginally lumpier changes the manager value by

$$(r_w - r_m)\frac{\mu\left(\frac{B}{A}\right)^{-\frac{r_m}{r_w-r_m}}\left(B\mu\left(\frac{B}{A}\right)^{\frac{r_m}{r_w-r_m}}+1\right)e^{-r_m\left(\frac{\ln\left(A\mu\left(\frac{B}{A}\right)^{\frac{r_w}{r_w-r_m}}+1\right)-\ln(A(\mu+r_w))}{r_w}+\frac{\ln\left(\frac{B}{A}\right)}{r_m-r_w}\right)}}{(\mu+r_w)(\mu+r_m)^2}.$$

This expression is negative as $(r_w - r_m) < 0$ and the ratio is positive. ■

In the analysis that follows we use a technical lemma that states that for ev-

ery incentive-compatible contract for which $u > 0$, there exists another incentive-compatible contract that implements the same work schedule via a compensation policy that is (pointwise) weakly lower.

Lemma A.1. *Assume that opportunities are concealable. Moreover, assume that under an incentive-compatible contract the continuation contract at h_t, $\langle \alpha(\cdot), \varphi(\cdot) \rangle$, is such that the worker's continuation utility is $u > 0$. There exists $\tilde{u} < u$ such that for every $u' \in (\tilde{u}, u)$ there exists an incentive-compatible contract $\langle \alpha'(\cdot), \varphi'(\cdot) \rangle$ that provides the worker with a continuation value of u', and for which $\varphi'(h_s) \leq \varphi(h_s)$ and $\alpha'(h_s) = \alpha(h_s)$ at every h_s that is a continuation of h_t.*

Proof of Lemma A.1. Consider an incentive-compatible contract $\langle \alpha(\cdot), \varphi(\cdot) \rangle$ under which the worker's continuation utility is $u > 0$ and normalize the current time to zero. If the worker's expected discounted compensation along the histories in which there are no binding incentive compatibility constraints is positive, then the worker's continuation utility at time zero can be decreased by reducing his compensation along those histories. If this is not the case, then the worker's compensation is almost surely zero prior to a binding incentive compatibility constraint. Hence, concealing all opportunities with probability one is a best response for the worker. However, as $\varphi = 0$ before the worker exerts effort, this best response provides a payoff of $0 < u$. ■

Proof of Lemma 1. Let $\langle \hat{\alpha}(\cdot), \hat{\varphi}(\cdot) \rangle$ be an incentive-compatible (continuation) contract under which the worker's expected discounted payoff is $u > 0$. From Lemma A.1 it follows that there exists $\tilde{u} < u$ such that if the worker's continuation utility is in $(\tilde{u}, u)$, then the manager can induce the same work schedule for a lower compensation. Thus, there is an open neighborhood to the left of u for which the manager can obtain a value strictly greater than $V(u)$. The strict monotonicity of $V(\cdot)$ follows from the fact that the choice of u is arbitrary.

Next, we show that $V(u)$ is weakly concave. Let $u_1 < u_2$ such that $u_1, u_2 \in [0, \frac{1}{r_w}]$. One (unnatural) way the manager can deliver a promise of $\frac{u_1+u_2}{2}$ is to fictitiously split all opportunities and compensation in half and create two (perfectly correlated) fictitious worlds, each of which contains half of the compensation flow and half of each opportunity. Observe that scaling all payoffs by $\frac{1}{2}$ multiplies the players' discounted payoffs by half in any contract, and so any optimal contract in the original non-scaled world is also an optimal contract in each fictitious world.

The manager can then use the continuation contract that supports $V(u_1)$ in the non-scaled world to provide the worker with a continuation utility of $\frac{u_1}{2}$ in fictitious world 1, and the continuation contract that supports $V(u_2)$ in the non-scaled world to provide the worker with a continuation utility of $\frac{u_2}{2}$ in fictitious world 2. Since using these continuation contracts cannot increase the manager's payoff, $V(\frac{1}{2}(u_1+u_2)) \geq \frac{1}{2}V(u_1)+\frac{1}{2}V(u_2)$, which establishes the concavity of $V(\cdot)$. ■

Proof of Proposition 4. We establish this proposition separately for the case where the manager is weakly patient and the case where he is impatient. In each case, we first derive one part of the optimal contract (the work schedule when the manager is impatient, and the compensation policy when he is impatient), and then use the HJB equation to derive the optimal contract and show that it is, generically, unique.

Case 1: impatient manager ($r_m > r_w$). The first step of the proof is to show that under any optimal contract the work schedule is $\overline{\alpha}(u) = \min\{1, \frac{1/r_w - u}{A}\}$.

Assume by way of contradiction that $\alpha(\hat{u}) < \min\{1, \frac{1/r_w-\hat{u}}{A}\}$ for some $\hat{u} \in [0, \frac{1}{r_w}]$. Suppose that the current state is $\hat{u}$ and that an opportunity is currently available. If the worker's expected discounted future effort is zero, then it is both possible and profitable to increase α and increase the worker's compensation in the future without changing his continuation utility. If, on the other hand, the worker's expected discounted future effort is positive, then the manager can expedite effort (in the non-Markovian representation of the contract) without altering the compensation policy. By Observation 1 it is profitable for the manager to expedite effort, and, since he does so according to the worker's discount factor, it also relaxes all incentive-compatibility constraints.

The above claim enables us to simplify the HJB equation to

$$(HJB_{imp}) \qquad \sup_{\varphi(u)\in[0,1]} \{-r_m V(u) + V'(u)[r_w u - \varphi(u)] - \varphi(u) + \mu\Big(\overline{\alpha}(u)B + V\left(u + \overline{\alpha}(u)A\right) - V(u)\Big)\} = 0.$$

From the FOC of (HJB_{imp}) it follows that $\varphi(u) = 1$ if $V'(u) < -1$ and that $\varphi(u) = 0$ if $V'(u) > -1$. Since $V(\cdot)$ is weakly concave (Lemma 1), there is a (possibly degenerate) interval $I \subset [0, \frac{1}{r_w}]$ over which $V'(u) = -1$. Note that for

any $u^\dagger \in I$ the compensation policy given by

$$\varphi_{u^\dagger}(u) = \begin{cases} 1 & \text{if } u > u^\dagger \\ r_w u^\dagger & \text{if } u = u^\dagger \\ 0 & \text{if } u < u^\dagger \end{cases}$$

is an optimal compensation policy.

Next, we show that, generically, I is degenerate. Fix A, B, μ, and r_w, and let $I(r_m)$ denote the interval (or point) for which $V'(\cdot) = 1$ for a manager with discount rate r_m. To establish the generic uniqueness of optimal contracts we will show that if there exist $\tilde{r}_m < \hat{r}_m$ such that both $I(\tilde{r}_m)$ and $I(\hat{r}_m)$ have a positive measure, then these intervals have a disjoint interior. The result then follows from a standard argument about the density of rational numbers.

Assume by way of contradiction that for some $\tilde{r}_m < \hat{r}_m$, the set $I^* \equiv I(\tilde{r}_m) \cap I(\hat{r}_m)$ has a nonempty interior. Select u^* and $\epsilon > 0$ such that $u^*, u^* - \epsilon \in int(I^*)$.

Fix the optimal compensation policy $\varphi_{u^*}(\cdot)$, and let $\Delta\varphi_s = \mathbb{E}(\varphi_s | u_0 = u^* - \epsilon) - \mathbb{E}(\varphi_s | u_0 = u^*)$ and $\Delta\alpha_s = E(\alpha_s | u_0 = u^* - \epsilon) - \mathbb{E}(\alpha_s | u_0 = u^*)$. Since the chosen compensation policy, $\varphi_{u^*}(\cdot)$, is optimal, we have

$$V(u^* - \epsilon) - V(u^*) = \int_0^\infty e^{-r_m s}(\mu B \Delta\alpha_s - \Delta\varphi_s) ds. \tag{8}$$

As path by path u_s is monotone in u_0, and the work schedule and compensation policies are threshold policies, it follows that $\mu B \Delta\alpha_s - \Delta\varphi_s \geq 0$ for all s, with a strict inequality on a set of times with strictly positive measure. Hence, differentiating the RHS of (8) with respect to r_m shows that the RHS of (8) is decreasing in r_m. However, as $V'(u) = -1$ for all $u \in I^*$ it holds that $V(u^* - \epsilon) - V(u^*) = \epsilon$. Hence, (8) can be satisfied for at most one r_m and so the interior of I^* is empty.

It follows that if the interior of $I(r_m)$ is nonempty, then the compensation policies corresponding to elements of $int(I(r_m))$ are suboptimal for any $r'_m \neq r_m$. Thus, we can index every r_m for which the optimal contract is not unique by a rational number from the interior of $I(r_m)$. Hence, the set of manager-discount factors for which the optimal contract is not unique is (at most) countable.

Case 2: weakly patient manager ($r_m \leq r_w$). We begin by showing that the optimal

compensation policy is

$$\overline{\varphi}(u) = \begin{cases} 1 & \text{if } u > 0 \\ 0 & \text{if } u = 0. \end{cases}$$

To do so, we show that if $u(h_t) > 0$ then in an optimal contract $\varphi(h_t) = 1$ in the next dt units of time conditional on no opportunity arriving in that interval. If $u(h_t) = \frac{1}{r_w}$, this is immediate. Assume by way of contradiction that $u(h_t) \in (0, \frac{1}{r_w})$, and that the worker does not receive the maximal compensation with probability 1 in the next dt units of time conditional on no opportunity arriving in that interval. By arguments analogous to those used in the proof of Lemma A.1, it is possible to expedite compensation into the interval $[t, t + dt]$ (conditional on no opportunity arriving) without violating the incentive compatibility constraints in any history that is a continuation of h_t. If $r_m < r_w$, then expediting compensation is profitable (Observation 1). If, on the other hand, $r_m = r_w$, expediting compensation is profitable as it enables the manager to require more effort in the future.

The above claim enables us to simplify the HJB equation to

$$(HJB_p) \quad V(u) = \sup_{\alpha(u) \in [0, \min\{1, \frac{\frac{1}{r_w} - u}{A}\}]} \{-r_m V(u) + V'(u)[r_w u - \overline{\varphi}(u)] - \overline{\varphi}(u) + \mu \Big(\alpha(u) B + V\left(u + \alpha(u) A\right) - V(u) \Big) \} = 0.$$

The FOC of HJB_p with respect to $\alpha(u)$ is $B + V'\left(u + \alpha(u)A\right) A = 0$. Thus, to show that there is a unique optimal contract, it is sufficient to show that $V(\cdot)$ is strictly concave. To do so, we return to the construction used to establish the weak concavity in Lemma 1 and further the analysis by utilizing the structure of $\bar{c}(\cdot)$.

In the event with strictly positive probably where no opportunity arrives for T units of time, where T solves $u_1 = \frac{1 - e^{-r_w T}}{r_w}$, the worker's continuation utility in fictitious world 1 is zero while his continuation utility in fictitious world 2 is strictly positive. At this point, the manager can temporarily merge the two fictitious worlds and expedite compensation in world 2 by using the compensation from world 1. By Observation 1 this modification is profitable for a strictly patient manager and, hence, $V(u)$ is strictly concave if $r_m < r_w$. If $r_w = r_m$, then merging the fictitious worlds increases the discounted effort the worker can be incentivized to exert in the future, which also strictly increases the manager's profit. ■

Proof of Proposition 5.

In Proposition 4 we established that if $r_m \leq r_w$, then $u^C = 0$ and there is a unique optimal contact. Setting $u^O = 0$ implies that the worker never exerts effort, which, in turn, implies that the manager's value is zero; an outcome that is clearly suboptimal. If the manager is patient, the setting $u^O > u(T^*)$ is suboptimal as when the worker exerts effort that increases his continuation utility to above $u(T^*)$, the manager will have promised compensation more than T^* units of time in the future. By the definition of T^*, this reduces the manager's value. Thus, to establish the proposition it remains to show that setting $u^O = u(T^*)$ is also suboptimal.

Assume towards a contradiction that under the optimal contract $u^O = u(T^*)$, and let τ be the first time at which the worker's continuation utility reaches u^O. Since $u^C = 0$, an opportunity must be available at τ. If the first opportunity to arrive after τ arrives sufficiently quickly, then the worker will not exert full effort on that opportunity. Choose $s \in (0, T^*)$ such that if the first opportunity to arrive after τ arrives at $\tau + s$, then the worker's effort on that opportunity is strictly less than 1, and denote the probability that the first opportunity after τ arrives in $[\tau + s/2, \tau + s]$ by $p > 0$. For $\epsilon > 0$ define the following (non-Makrovian) modification of the contract at time τ: reduce the worker's required effort at τ by $u(\epsilon) = \left(\int_{T^*-\epsilon}^{T^*} e^{-r_w t} dt\right) / A$ and refrain from promising compensation in $[\tau + T^* - \epsilon, \tau + T^*]$. Then, use the freed compensation in $[\tau + T^* - \epsilon, \tau + T^*]$ to increase the worker's effort on the first opportunity to arrive after τ, if it arrives in $[\tau + s/2, \tau + s]$. If it does not arrive in this interval, never provide compensation in $[\tau + T^* - \epsilon, \tau + T^*]$.

The manager's loss of value at time τ from this deviation is bounded from above by the manager's value from the worker exerting an effort of $u(\epsilon)$ and receiving compensation in $T^* - \epsilon$ units of time. This upper bound is given by

$$\frac{e^{-r_w T^*}\left(e^{r_w \epsilon} - 1\right)\left(\frac{B}{A} - e^{(r_w - r_m)(T^* - \epsilon)}\right)}{r_w} = e^{-r_w T^*}\left(\frac{B}{A} - e^{(r_w - r_m)T^*}\right)\epsilon + O\left(\epsilon^2\right) = O\left(\epsilon^2\right),$$

since $\left(\frac{B}{A} - e^{(r_w - r_m)T^*}\right) = 0$ by the definition of T^*.

If ϵ is small enough, then the worker will not exert full effort on the first opportunity in $[\tau + s/2, \tau + s]$ after this deviation. In this case, the gain from the

additional effort on that opportunity is bounded from below by the product of p and the manager's time τ discounted gain should the the next opportunity arrive at $\tau + s/2$ (Observation 1). This bound is given by

$$p\left(\frac{B\left(e^{r_w\epsilon}-1\right)e^{r_w(s/2-T^*)-r_m s/2}}{Ar_w}-\frac{e^{-r_m T^*}\left(e^{r_m\epsilon}-1\right)}{r_m}\right)$$
$$=\epsilon p\left(e^{(r_w-r_m)s/2}-1\right)\left(\frac{B}{A}\right)^{\frac{r_m}{r_m-r_w}}+O\left(\epsilon^2\right).$$

Since $r_w > r_m$ and $s > 0$, the linear term in the approximation is strictly positive. Hence, for sufficiently small ϵ, the modification is profitable. ■

Proof of Proposition 6. In Proposition 4 we established that $u^O = \frac{1}{r_w}$ if $r_m > r_w$. Furthermore, when $r_m = r_w$ the manager will use the same threshold, to avoid wasting his limited capacity to compensate the worker. To establish the second part of the proposition we begin by showing that $u^C = \frac{1}{r_w}$ is an optimal threshold if and only if $r_m \geq r_m'' \equiv r_w + (\frac{B}{A} - 1)\mu$. An upper bound on the manager's marginal net gain from providing the worker with a util at present is attained by the worker exerting a worker-discounted util on the first opportunity to arrive. Note that if $u^C > \frac{1}{r_w} - A$ and $u = u^C$, then this upper bound is attained. The value of this upper bound is given by

$$\int_0^\infty \mu e^{-\mu t}\frac{B}{A}e^{(r_w-r_m)t}dt - 1 = \frac{\mu}{\mu + r_m - r_w}\frac{B}{A} - 1.$$

It is straightforward to show that this expression is positive if and only if $r_m \leq r_m''$. It follows that if $r_m > r_m''$, then providing compensation while $u < \frac{1}{r_w}$ is suboptimal. On the other hand, if $r_m < r_m''$, then it is strictly suboptimal to set $u^C = \frac{1}{r_w}$ since for such discount rates it is profitable to compensate the worker when $u > \frac{1}{r_w} - A$.

Next, we establish that there exists $r_m' > r_w$ for which $u^C = 0$. Let τ denote the random arrival time of the first opportunity on which the worker will not exert full effort under the contract with $u^C = 0$. Note that under any other contract, the worker will not exert full effort (weakly) earlier. It follows that the marginal value from decreasing the worker's continuation utility is at least $E(e^{-r_m\tau})(\frac{B}{A} - 1) > 0$. If $r_w = r_m$ the timing of compensation does not affect the manager's cost of providing compensation. This, in turn, implies that when $r_w = r_m$ the manager's marginal

gain from providing compensation is at least $E(e^{-r_m\tau})(\frac{B}{A}-1) > 0$ for all u. By the continuity of payoffs in r_m, it follows that there exists $r'_m > r_w$ such that the manager strictly benefits from full front-loading of compensation if $r_m < r'_m$.

To conclude the proof we use the following lemma that states that u^C is increasing in r_m (proof below) to show that $r'_m < r''_m$

Lemma A.2. *Fix B, C, μ, and r_w and assume that the manager is impatient. If u^C is an optimal threshold for r_m and $\tilde{u}^C$ is an optimal threshold for $\tilde{r}_m > r_m$, then $\tilde{u}^C \geq u^C$.*

To see why this inequality follows from the lemma, observe that when $r_m = r''_m$ there exist histories for which under the optimal contact the worker exerts effort on only the first opportunity, whereas, when $r_m = r'_m$ in every history the worker's discounted cost of effort must equal his discounted compensation, which, by Assumption 1, is greater than the cost of effort on a single opportunity. ■

Proof of Lemma A.2. Consider two contracts, $\mathcal{C}_1$ and $\mathcal{C}_2$, that differ in their compensation threshold, $u_2^C > u_1^C$. Denote by $u_{t,i}$ the worker's continuation utility at time t under contract $\mathcal{C}_i$ and let

$$\overline{\tau} = \sup\{t : u_{t,2} \leq u_2^C\}.$$

That is, $\overline{\tau}$ is the latest time at which the worker's continuation utility is lower than u_2^C under $\mathcal{C}_2$. Note that $\overline{\tau}$ is finite (almost surely) by the Borel–Cantelli lemma.

Observe that since $u_2^C > u_1^C$, it holds that $u_{t,1} \leq u_{t,2}$ for all t. This implies that the worker exerts the same effort on every opportunity that arrives before $\overline{\tau}$ under both $\mathcal{C}_1$ and $\mathcal{C}_2$. In addition, it implies that the compensation for effort exerted on those opportunities is postponed under $\mathcal{C}_2$ relative to $\mathcal{C}_1$. Denote by $g(r_m)$ the gain from this postponement as a function of r_m. Moreover, from $\overline{\tau}$ onwards, under $\mathcal{C}_2$ the worker exerts weakly less effort than he does under $\mathcal{C}_1$, and he receives a compensation of $\varphi_t = 1$ at all times. Let $d(r_m)$ denote the difference in the time-zero discounted continuation value from $\overline{\tau}$ onward between $\mathcal{C}_1$ and $\mathcal{C}_2$. The net gain from replacing $\mathcal{C}_1$ with $\mathcal{C}_2$ is $g(r_m) - d(r_m)$. Note that $g(\cdot)$ is increasing and $d(\cdot)$ is decreasing. Hence, whenever $g(r_m) \geq d(r_m)$, we also have $g(r'_m) > d(r'_m)$ for all $r'_m > r_m$, which establishes the monotonicity of u^C. ■

Proof of Proposition 7. To establish this proposition, it is convenient to think of each opportunity as being composed of many "small opportunities." We will

show that making opportunities lumpier in the original model is equivalent to a certain change in the correlation structure of these small opportunities.

First, we consider the case where opportunities become lumpier by a rational factor. Assume that opportunities become lumpier by $\frac{N}{M} > 1$, where $N, M \in \mathbb{N}$. We analyze this change by considering an auxiliary representation of the model in which there are $M \times N$ Poisson processes, each with an arrival rate of $\frac{\mu}{N}$, that govern the arrival of the small opportunities. Moreover, we assume that the payoff from exerting full effort on each small opportunity is $(-\frac{A}{M}, \frac{B}{M})$. Both the original and the lumpy versions of the model correspond to appropriately defined correlation structures of the arrival processes in the auxiliary representation.

To map the auxiliary representation to the original model, divide the Poisson processes into N groups of M processes each, such that within a group the processes are perfectly correlated, and across groups the processes are independent. To see why this correlation structure represents the original model, note that when a group of opportunities is available the payoff vector from exerting full effort on all opportunities in the group is $M \times (-\frac{C}{M}, \frac{B}{M}) = (-A, B)$, which is exactly the payoff vector from exerting full effort on a single opportunity in the original model. Moreover, the probability that a given group arrives in an (infinitesimal) interval dt is $\frac{\mu}{N}dt$, and since the groups are independent, the probability that some group arrives in that interval is $\sum_{i=1}^{N} \frac{\mu}{N}dt = \mu dt$.

To map the auxiliary representation to the lumpy model, divide the processes into M groups of N processes each, such that within a group the processes are perfectly correlated, and across groups the processes are independent. For this correlation structure, the payoff from exerting full effort on all opportunities in a group is $N \times (-\frac{A}{M}, \frac{B}{M}) = (-\frac{N}{M}A, \frac{N}{M}B)$ and the probability that some group of opportunities is available in an interval dt is $\frac{\mu}{N/M}dt$.

Next, we construct a sequence of modifications that begins with the lumpy representation and ends with the original one, such that the first two modifications do not impact the manager's value, and the third modification strictly increases it.

Consider the lumpy representation. The first modification utilizes the idea of splitting the interaction into fictitious worlds introduced in Lemma 1. In particular, we create N fictitious worlds, denoted by $(1, \ldots, N)$, that each contain $\frac{1}{N}$ of the flow compensation and M arrival processes, one from each group. We denote the processes in fictitious world n by $(P_1^n, \ldots, P_M^n)$. Note that the arrival processes within each fictitious world are independent of one another, and so each fictitious

world is a scaled version of the lumpy representation. Hence, by the argument used in Lemma 1, the sum of the manager's values across all fictitious worlds is equal to his value in the lumpy representation.

The second modification is to the correlation structure of the processes across fictitious worlds. Changing the correlation structure of two arrival processes that are assigned to *different* fictitious worlds does no impact the manager's value in either fictitious world. Hence, so long as the processes within each fictitious world are independent of one another, the correlation across fictitious worlds is immaterial. Thus, we can replace the original correlation structure with the following correlation structure: P_m^n and $P_{m'}^{n'}$ are perfectly correlated if $m - n \overset{(\mathrm{mod}N)}{=} m' - n'$, and independent otherwise. This modification maintains the independence of the processes within each fictitious world. To see this note that for any $n \leq N$ and $m, m' \leq M$, such that $m' \neq m$, the fact that $M < N$ implies that $m - n \overset{(\mathrm{mod}N)}{\neq} m' - n$.

The third modification is to re-merge the fictitious worlds. Note that under the correlation structure created in the second modification, there are N groups of M processes each, such that within a group the processes are perfectly correlated, and across groups the processes are independent. Thus, merging these fictitious worlds creates the auxiliary representation of the original model. Regardless of the relative patience, there are instances in which the manager benefits from merging two fictitious worlds: if $r_m < r_w$ this occurs when in fictitious world i the worker's continuation is positive while in fictitious world j it is zero, whereas if $r_m \geq r_w$ this occurs when in fictitious world i an opportunity is (partially) forgone while in fictitious world j the worker's continuation utility is below its maximal level. It follows that the sum of the manager's values across all fictitious worlds is strictly less than his value in the original model.

Finally, consider the case where $\lambda \notin \mathbb{Q}$. The manager's value is continuous in λ as i) the distribution of arrival times is continuous in λ, and ii) if opportunities are made slightly lumpier then the manager can instruct the worker to incur the same *cost* of effort on every opportunity that arrives by using the same compensation policy. As the set of rational numbers is dense, this establishes the proposition. ■