Labor Market Structure, Wages and Inequality*

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Abstract

How does local labor market competition affect the wage distribution and job flows? Using administrative Norwegian data on job flows, employment, wages and vacancies, we document that in less competitive markets, firms pay lower wages, post fewer vacancies and have lower job flows. Motivated by this evidence, we develop a dynamic general equilibrium search-theoretic model of the labor market that includes a finite number of firms, strategic wage and vacancy posting, and on-the-job search. We show that our framework is capable of replicating the observed relationships between market concentration and both job flows and wages. We use the model to measure the effects of labor market competition on the wage distribution and welfare, finding that making labor markets more competitive would lower within market wage inequality and the unemployment rate, and improve labor productivity.

JEL codes: E2, J2, J42

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1 Introduction

How does local labor market competition affect wages and job flows? What are the distributional consequences of strategic wage setting? There is a growing consensus that imperfect competition in the labor market is pervasive.[1] Few firms dominate many local labor markets giving them the ability to set wages and pay workers less than their marginal product. In this paper, we develop a theory of dynamic oligopsony in the labor market use it to measure the effects of labor market power on the wage distribution and welfare.

While prior work has shown that a higher level of market concentration is associated with lower wages (e.g Benmelech, Bergman, and Kim, 2018; Azar, Marinescu, and Steinbaum, 2017; Schubert, Stansbury, and Taska, 2022), our first contribution is to document that highly concentrated labor markets also exhibit lower job-to-job mobility. Using detailed data about workplace locations and line of work, the left panel of Figure 1 compares workers in the same career across local labor markets in Norway. We see a clear negative relationship between wages and the concentration of employers. Similar to existing evidence from the US, the relationship is economically strong. A one-standard-deviation increase in concentration is associated with almost 3 percent lower wages. Turning to the right panel, we show that on-the-job search is a key source of wage gains: a standard deviation increase in concentration is associated with 10 percent lower employer-to-employer mobility relative to the average transition rate. This evidence strongly suggests that on-the-job search is a key mechanism through which market structure affects wage inequality and motivates our theoretical framework.

We develop a general equilibrium model of dynamic oligopsony in the labor market and use it to investigate further how market structure affects wage inequality and welfare. The model features a finite number of firms, on-the-job search and strategic wage and vacancy posting. These first two features are crucial to capturing the fact that both employer-to-employer (E-to-E) flows and wages are lower in more concentrated markets. It is well-known that job-to-job transitions are a key source of wage growth. In more concentrated markets, there are fewer firms competing for workers directly leading employed workers in these markets to have lower outside options and hence lower wages because there are fewer opportunities to receive outside offers and move to higher paying competitors. Thus our model easily matches the fact that both E-to-E flows and wages are lower in more concentrated markets. The third modeling feature, strategic wage and vacancy posting is crucial to closing our model and allowing us to conduct counterfactuals. While the previous literature assumed contact rates are proportional to firm size, by modeling oligopsonistic vacancy posting we instead have a fully internally consistent general equilibrium framework.

We make three main contributions. First, we contribute a tractable and general equilibrium dynamic theory of how monopsony power affects wages and job flows. Our framework significantly extends the

[1] See Manning (2003) for a summary of the literature as well as and recent papers Azar, Marinescu, and Steinbaum (2017), Arindrajit Dube (2019), Benmelech, Bergman, and Kim (2018), Card, Cardoso, Heining, and Kline (2018), Lamadon, Mogstad, and Setzler (2018).



Figure 1: Average market wage and employment-to-employment job flows by market concentration

Note: These figures display the empirical plots of average monthly wages at the market level (where a market is defined to be a 4-digit occupation by commuting zone), the monthly employment-to-employment job transition rate against residualized employer concentration. The regressions control for fully interacted occupation and time fixed effects and a flexible specification for number of firms per worker in a market (20 dummies). The sample includes occupations that are distributed across all CZs over the period 2006-2016 and excludes the top and bottom 2 percent of the HHI. The average NOK/USD in 2021 is 9.

literature by developing a framework that incorporates on the job search in frictional labor markets, with finitely many strategic firms *and* strategic vacancy posting. While some recent papers include the first two ingredients, we are the first equilibrium theory of dynamic oligopsony, which allows us to conduct meaningful counterfactuals. Second, we use the universe of Norwegian administrative data to document systematically how job flows and wage inequality vary with local labor market concentration. We find that E-to-E flows and inequality are significantly lower in concentrated markets. To the best of our knowledge we are the first to document systematically how wage inequality and labor market flows vary with market structure. Third, we calibrate our model using this rich, administrative data and conduct multiple counterfactuals in order to understand the welfare consequences of strategic interaction in concentrated labor markets. We find that monopsony lowers inequality moderately relative to a competitive labor market, but at a cost of lower efficiency. Overall, both our empirical and model results suggest that addressing market power would significantly affect wage inequality.

Our framework substantially extends Postel-Vinay and Robin (2002) to accommodate a finite number of firms in each labor market. The number of firms in each market is given by $N < \infty$ and firms differ in their idiosyncratic, but fixed productivity levels. Unemployed workers randomly meet the N firms within their labor market, whereas employed workers randomly meet the remaining $N-1$ firms, excluding their current employer. When an unemployed worker meets a firm, they Nash-bargain over surplus. When an employed worker meets a firm, the incumbent and poaching firms compete via alternating offers (e.g. Cahuc, Postel-Vinay, and Robin, 2006), yielding equilibrium values in which outside options are

determined by Bertrand competition and the remaining surplus is split by Nash-bargaining. This surplus sharing protocol yields the first form of strategic interactions between the incumbent and poaching firms. Bertrand competition generates strategic complementarity. A second form of strategic interaction emerges from firms being large in the vacancy posting decision. We assume that firms Cournot compete over vacancies. Modeling firm's vacancy posting decisions explicitly generates endogenous contact rates which both closes the model and delivers a meaningful firm size distribution.

We characterize the interactions between two firms in a single market in order to highlight how firms engage in a strategic oligopoly. We characterize the oligopsonistic vacancy posting decisions and we contrast those with the competitive vacancy posting decisions. This example can (almost) be solved in closed form. Assume that firm 1 is more productive than firm 2. We show that if firms are playing an oligopoly vacancy posting game, firm 1's strategy is universally strategic complements with firm 2's vacancies. That is, if the less productive firm posts more vacancies, the best response of the most productive firm is to post more. What drives this extreme complementarity is that the more productive firm is free riding on the less productive firms hiring. Because hiring is costly and because of on-the-job search, firm 1 knows that it can always poach workers from firm 2 with certainty if it wants to in the strategic vacancy posting game. This free ridership implies that the most productive firm, firm 1, posts too few vacancies in the strategic equilibrium relative to the competitive equilibrium, choosing to restrict their vacancy postings to lower implicit prices. As is typical for monopsonists, the most productive firm in the labor market is inefficiently small, leading to efficiency losses.

We use linked employer-employee data from Norway to discipline the quantitative model. The administrative data offers several advantages over other data sources. First, it includes information about the type of work each employee is hired to do and the workplace location of every employment contract from 2006 to 2018. These bibliographic records allow us to count the number of employers within each market and accurately track wage changes of individual workers when they move between employers. Our market definition is a combination of a commuting zone and a four-digit occupation. The second advantage is that the data includes information about job vacancies from the employment agency's database. Relative to the frequently used Burning Glass data in the U.S., our vacancy information comprises the universe of formal employment and includes information on the number of vacancies posted. Combined, the data allows us to calibrate the model and test key predictions of the model on untargeted data moments.

We estimate the model to match key moments in 2012 in Norway, including aggregate employment concentration, E-to-E rates, and U-to-E rates. Despite the model's parsimony, the quantitative model fits the data well. Consistent with our empirical evidence, our model generates lower wages in more concentrated markets because of lower E-to-E flows. Job finding rates (U-to-E flows) are declining in labor market concentration, while E-to-U flows exhibit no significant relationship with market concentration.

We use the model to conduct a number of counterfactuals to highlight the importance of labor market competition and strategic interaction for welfare and wage inequality. Our baseline focuses on markets

with 10 or fewer firms and we start by discussing the role of strategic vacancy posting. Our main result is that when firms strategically compete for workers, within market wage inequality and unemployment is higher and labor productivity is lower, though consumption (welfare) taking into account hiring costs is effectively unchanged. The reason that strategic competition worsens inequality is that it allows productive, large firms to free-ride on the hiring of smaller firms leading to fewer job-to-job transitions. If we increase the total number of firms in a market to 20, strategic vacancy posting matters much less because in that case most firms are essentially atomistic already. This increase in competition raises labor productivity by 3% and welfare by 1%. Since search frictions are a key source of monopsony power in our model, we explore the consequences of lessening them on welfare and inequality by varying the frequency of job offers and the productivity of match efficiency. Lowering search frictions increases labor productivity as workers quickly move to the best, largest firms, however, this also leads to a sharp increase in measured concentration as the most productive firms employ a larger share of the workforce within a market. This suggests, as in our previous work (Berger, Herkenhoff, and Mongey, 2019), that there is not a one-to-one correspondence between productivity and measures of concentration like HHIs.

We review the literature and then proceed as follows. Section 2 describes our Norwegian administrative data and offers motivating evidence. Section 3 lays out the single-market model and characterizes the equilibrium. Section 4 highlights intuition in a simpler, two firm example with and without on the job search. Section 5 extends the model to include multiple markets. Section 6 describes how we calibrate the model and explores the model's fit relative to the data. Section 7 then conducts the main counterfactual exercises, including measuring the output, consumption, and employment responses to various competitive limits.

Related literature. Our key innovation is to substantially extend Postel-Vinay and Robin (2002) to accommodate a finite number of firms in each labor market. Our framework is distinct from existing search models with finitely many agents: Zhu (2012), Bagga (2022), and Jarosch, Nimcsik, and Sorkin (2019). Unlike these existing frameworks, our model simultaneously features (1) worker reallocation through on-the-job search and and (2) strategic vacancy posting. By modeling vacancy posting, our model links employer concentration to both prices and quantities. It also allows us to discuss welfare and conduct normative counterfactuals.

Our model is truly dynamic because forward looking behavior is an inherent feature of a search-and-matching model. While, the general problem of N forward looking oligopolist firms who maintain long-run matches with workers is intractable, by restricting the oligopsonistic interactions to one-on-one interactions between incumbent and poaching firms and overall vacancy posting of each firm, we greatly simplify the computation of the model. This simplification allows us both to solve them model with enough flexibility to be able to solve the model quickly despite it being in general equilibrium and for large market sizes. In his 2022 *AEA Presidential Address* David Card argued that developing a tractable framework combining preference heterogeneity (in the *McFadden-IO* tradition) and search-and-matching frictions with job ladders (in the Postel-Vinay and Robin, 2002, tradition) was key to understanding the importance of market power in the labor market. In future iterations, the flexibility our framework

allows will hopefully allow us to answer David Card's charge by integrating other sources of monopsony power due to worker preferences.[2]

Our work is related to the large literature that models monopsony in labor markets. We depart from benchmark models of monopsony described in Burdett and Mortensen (1998); Manning (2003); Card, Cardoso, Heining, and Kline (2018); Lamadon, Mogstad, and Setzler (2018); Kroft, Luo, Mogstad, and Setzler (2020); Hurst, Kehoe, Pastorino, and Winberry (2022) by focusing (for now) on search frictions as the source of monopsony power (rather than worker preferences) and by explicitly modeling an oligopsonistic equilibrium, where a finite set of firms strategically compete workers.

A growing literature that explores the implications of market power on local labor markets. Many studies have documented the cross-sectional relationships between standard measures of concentration (Herfindahls) and wages/employment (Benmelech, Bergman, and Kim, 2018; Rinz, 2018; Hershbein, Macaluso, and Yeh, 2020) and vacancies (Azar, Marinescu, Steinbaum, and Taska, 2018; Azar, Marinescu, and Steinbaum, 2017). Our contributions to this literature are i) we are the first to document strong cross-market relationships between Herfindahls and job flows and wage inequality ii) because we have access to high quality information on the quantity of jobs posted in each job post, we are able to construct a measure of concentration using vacancy data and we explore it's relationship with labor market variables.

There are relatively few studies of the distributional consequences of monopoly (let alone monopsony). Most studies regarding the distributional consequences of monopoly focus on specific industries, such as housing, healthcare, or manufacturing (see Schmitz (2016) for a summary). We are using the sequential auction framework of Postel-Vinay and Robin (2002), and combining it with a finite number of firms, e.g. Atkeson and Burstein (2008), Zhu (2012), Jarosch, Nimcsik, and Sorkin (2019). In terms of theory, Herkenhoff (2019) nests a Bewley economy in a job ladder framework, and Luo and Mongey (2019) studies the implications for job search for the trade-off between wages and non-wage amenities in an environment with consumption-savings but where wages and amenities are exogenous. The wage-amenity trade-off is the key source of monopsony power in a 'classical' setting.

Our models incorporate strategic interaction through wage setting. In product market settings, strategic interactions have been tested and found to be significant by Amiti, Itskhoki, and Konings (2014) and Amiti, Itskhoki, and Konings (2016). Early studies such as Staiger, Spetz, and Phibbs (2010) study wage spillovers from monopsony in a particular industry, but there is no existing work on strategic wage setting across a broad set of industries, or in response to a broad set of shocks, to our knowledge.

[2] A preliminary version of this framework with match-specific amenities is available upon request.

2 Data and Evidence

This section presents new evidence on market structure, job mobility, and wage-setting behavior from Norwegian labor markets.

2.1 Data and Measurement

Our empirical contribution is made possible by linked employer-employee data covering every firm, its establishments and employees. The key advantage of the Norwegian data is that employers must record the type of work each employee is hired to do and the workplace location of every employment contract from 2006 to 2018. These bibliographic records allow us to count the number of employers within each market and track wage changes of individual workers when they move between employers.

The data collection consists of three steps. In the first step, we use the information about the work contract that the employer submits to the employment agency (NAV). To comply with labor laws, the employer must enter a specific position from a list of more than 6,000 possible job titles, the workplace location, wage income, and work hours.[3] The job titles are grouped into 354 four-digit occupational categories by Statistics Norway based on the similarity of the type of work. For example, the occupational code for "economics and business" includes consultants, controllers, junior and senior credit analysts, and research and chief economists, to name a few. The occupational codes also cluster unskilled positions, such as maintenance workers and janitors, and different levels of management positions into distinct 4-digit groups. In the second step, we combine the linked employer-employee data with socio-economic variables from longitudinal population registers. These registers include demographic information, such as sex, age, residential municipality, and education, and permit the measurement of the local labor market by aggregate commuting statistics between residential homes and workplaces. Importantly, these details allow local labor markets to cross the administrative borders of municipalities and counties. See the Data Appendix G for a more detailed description of the sources and variables, and Bhuller (2009) for commuting patterns. In the final step, we use data on job openings from the Norwegian employment agency's database (NAV) to measure the available jobs workers can apply for. The vacancy database is available from 2001 to 2016 and is by caseworkers to help match unemployed workers to job openings. Caseworkers at the employment agency collects information about the number of vacant jobs, the establishment, and uses information from the advertisement to classify the position into a four-digit occupational code. Since 2012, we have linked information from the major online job board Finn.no,

[3]The Norwegian labor law stipulates that firms undergoing a mass layoff, defined as laying off more than ten workers over 30 days, must follow the last-in, first-out principle. The ordering is typically defined within position and establishment, which provides firms an incentive to report consistently. The employment agency uses information about occupation and the workplace location for targeted job search assistance. In practice, employers report positions by a 7-digit system (see, https://www.ssb.no/klass/klassifikasjoner/145/, in Norwegian), with new job titles being added at regular intervals. The 4-digit occupational classification is based on the International Standard Classification of Occupations (ISCO) adapted to Norwegian labor markets by Statistics Norway and the employment agency.

which includes additional information about the number of applications each vacancy receives. [4]

2.1.1 Institutional Details

Norway has a population of 5 million and Oslo, the capital, accounts for about one-fifth. The labor force in the age range from 25 to 66 equals about 2 million, and the labor share of income is about 0.7.[5] In 2016, the unemployment rate was about 4.5. There were 176,019 firms and 234,941 establishments with workers on their payrolls.

Firms can hire employees on either fixed-term or permanent contracts and dismiss workers who underperform relative to their peers or operate at a loss. Employment protection in Norway ranks about the median among OECD countries and is comparable to France and Sweden.[6] Wages and typical working hours, in turn, are typically set by collective bargaining at the industry level, after which wages are supplemented by local adjustments or wage drift, which is bargained over at the firm level. This two-tier framework gives rise to a relatively compressed wage structure. The Norwegian safety net covers lost income from unemployment. The primary insurance source is unemployment benefits, which kick in after a three-day waiting period, replacing about two-thirds of workers' past net of tax earnings (see e.g. Røed and Zhang, 2003).[7]

2.2 Empirical Analysis

Our empirical analysis focuses on residents from ages 25 to age 66 to avoid issues of entry and exit from the labor force. We define a *local labor market* as Bhuller's (2009) measure of a commuting zone (CZ).[8] While it is well known that workers sometimes move to new regions (e.g., Jarosch, Nimcsik, and Sorkin, 2019), Panel A of Table 4 shows that workers rarely move out of their local labor market when they

[4] In Appendix 12, we compare the trend and level of vacancies with survey evidence from 2010 to 2016. The survey is administered quarterly and asks a representative sample of around 5 percent of the total population of establishments about vacant positions that it can start within 30 days. The definition of a vacancy is consistent with that of the JOLTS, where recruitment must be made from outside the firm, including full-time, part-time, permanent, temporary, and short-term job openings. Job openings are recorded using the ISCO08 version. However, to combine the employment register with information about job openings, the two versions must be cross-walked. This many-to-many crosswalk is detailed in Appendix G.3 and reduces the number of unique, non-overlapping occupations from 403 and 354 to 259.

[5] The petroleum sector accounts for a large fraction of income but is excluded when calculating the labor share.

[6] Union membership in Norway is relatively high compared to other countries in the OECD and the US, but has fallen from 58 to 53 percent from 1992 to 2013 (OECD Statistics Trade Union Statistics). Unions play an important role in ensuring that firms comply with labor law, stating, for example, that downsizing requires a one-month notification to employees, and dismissal time varying from one to six months, depending on age and tenure. Wrongful discharge can end with a lawsuit where firms must compensate dismissed employees for lost income.

[7] Payroll taxes finance the UI system, and there is no experience rating on the firm. The potential benefit period is 52 weeks for workers who have earned less than twice the National Insurance basic amount for the last three years. The National Insurance basic amount is currently set at approximately USD 12,000 p.a., and benefits are capped at a maximum level of previous earnings at six times the basic amount. To remain eligible for the cash benefits, work hours must have fallen by at least 50%, and recipients must be actively looking for work and be willing to take any employment.

[8] The academic track leads to a bachelor's degree after three years and is usually followed by a two-year master's degree, usually completed by age 24. The normal retirement age is age 67.

switch employers. Using a panel of employer to employer (EE) transitions between two employers with no time gap and no spells of unemployment, we find only 0.6 percent of the 840 thousand EE transitions involve moving from the current to one of the other 45 commuting zones.

We define a market as four-digit occupation and commuting zone. This gives us 9,904 markets, with approximately 10 firms per market. This market definition performs relatively well in capturing moves in the data: conditional on being employed in a given month, a worker has a 99.5% chance of remaining in a market each period. This suggests that workers do not move across markets very often. Because our definition of occupation is quite granular, conditional on undergoing a job-to-job transition, the probability of switching occupations is over 60% (see Appendix G.4 for full details). However, EE moves are quite rare (0.4%) and even conditional on an EE move, a worker is 173 more likely to move to the same occupation than to any other four-digit occupation.

2.2.1 Empirical Framework

Our goal is to estimate the effects of employer concentration on job-to-job transitions and the wage distribution. The empirical framework we implement to achieve this goal compares workers in the same career trajectory (e.g., a 4-digit occupation) but who face different firm sizes across commuting zones. To do so, we estimate the following equation using ordinary least squares (OLS)

$$y_{z,c,t} = \beta \text{HHI}_{z,c,t} + \sum_t \sum_c \gamma_{c,t} \text{Career}_{ct} + \sum_s \delta_s \text{Size}_{z,c,t} + r_{z,c,t} \tag{1}$$

where z denotes the commuting zone, c denotes a 4-digit occupation, and t is a calendar year and month. We include dummies for each 4-digit occupation per time period, Career_{ct}, with paramater $\gamma_{c,t}$, and hold the average number of firms per worker constant by including a series of dummies for different firm-per-worker ratios, δ_s. The variable HHI measures the concentration of employers using the Herfindahl index, which varies from 0 to 1, where one means that the market is dominated by one firm. Our target parameter β captures how the EE rate changes with an increase in the concentration index from 0 to 1.

Our main identifying assumption is that after controlling for the average number of firms a worker faces in a local labor market, the distribution of firm sizes is exogenous. To facilitate this interpretation, we restrict the sample to occupations who are present in every commuting zone, and flexibly control for the number of firms per worker. This leaves us with 93 occupations, 46 regions and 175,306 market-time observations. The average HH index is 0.162 and the standard deviation is .175. After residualizing the HHI measure using the control variables in equation (1), the standard deviation deviation shrinks to 0.121. We normalize the HHI variable by the standard deviation to better interpret the economic magnitude of the parameter and perform inference using clustered standard errors at the market level.

2.2.2 Graphical Evidence

We turn to a graphical presentation of our regression evidence in Figure 2.[9] Panel A plots the monthly employment-to-employment job transition rate (E-to-E rate). The graph exhibits a clear negative and statistically significant relationship between the employment Herfindahl and the E-to-E rate. A one standard deviation increase in the market employment Herfindahl is associated with a 0.05 percentage point reduction in the E-to-E rate, which corresponds to roughly 10% of the sample average E-to-E rate.

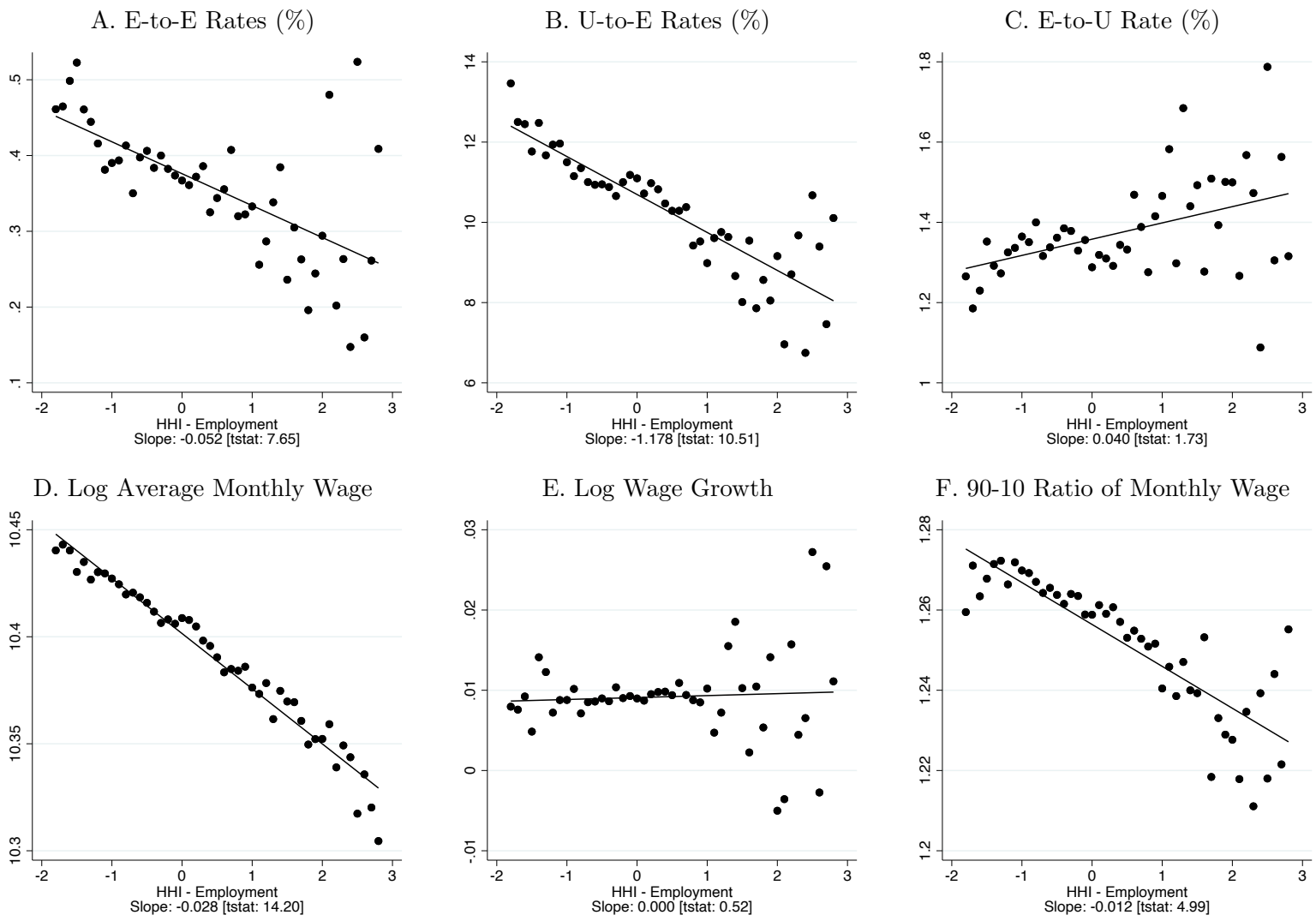
Panel B shows a negative relationship between U-to-E rates and concentration; Panel C shows an insignificant relationship between job separations into unemployment and concentration; Panel D shows a negative relationship between log average monthly wages in a market and concentration; Panel E shows a negative relationship between log wage growth and concentration; lastly, Panel F shows a negative relationship between p90-p10 wage ratios within a market and concentration.

Figure 3 extends our analysis to labor market tightness. Panel A plots the monthly vacancy rate, normalized by the labor force, against the measure of market structure. There is a moderately strong, negative relationship between residualized employment Herfindahls and the vacancy rate. Panel B presents the relationship between the HHI and the average number of applications measured from Finn.no over the total number of vacancies. We observe a very strong positive relationship between applications and market concentration. In Panel C, we explore how unemployment covaries with the HHI. There is a strong, positive relationship between the unemployment rate and concentration - more concentrated markets have higher unemployment rates. All three of these relationships are statistically significant at conventional levels.

Together, Figures 2 and 3 suggest that there is strong evidence in the data that job flows and market tightness vary with labor market concentration. They motivate the structure of the model that we describe in the next section and serve as empirical targets for our model (see Section 6.2).

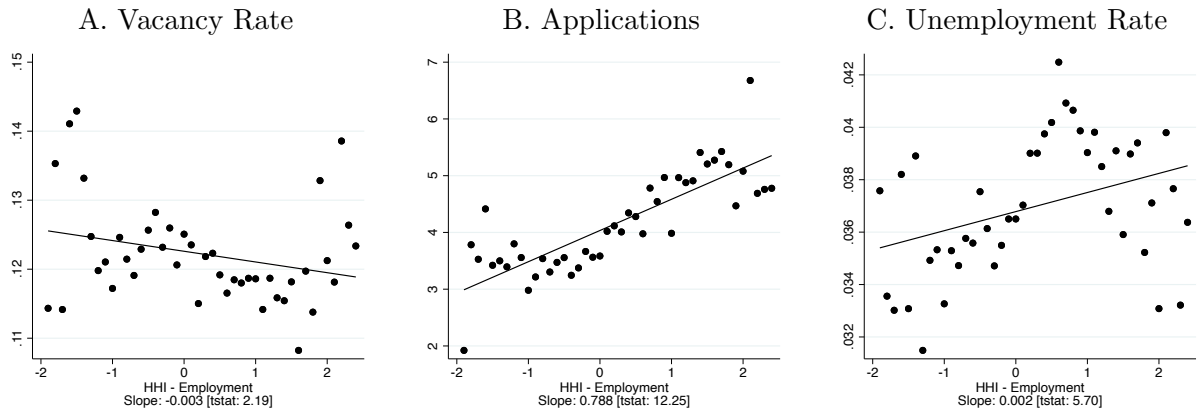
[9] The findings are robust to also including fixed effects for commuting zones, that would capture common effects across all occupations, such as agglomeration effects.

Figure 2: Market Structure, Labor Market Flows, and Wages



Note: These figures display the empirical plots of (A) employment-to-employment (E-to-E), (B) unemployment-to-employment (U-to-E), (C) employment-to-unemployment (E-to-U) , (D) average monthly wages, (E) monthly wage growth, and (F) the p90-p10 ratio of monthly wages against residualized employer concentration. The regressions control for fully interacted occupation and time fixed effects and a flexible specification for number of firms per worker in a market (20 dummies). The sample includes occupations that are distributed across all CZs over the period 2006-2016 and excludes the top and bottom 2 percent of the HHI. The average NOK/USD in 2021 is 9.

Figure 3: Market Structure and Labor Market Tightness



Note: These figures display the empirical plots of vacancies, applications per vacancy and unemployment rates against residualized employer concentration. Unemployment and vacancy rates are normalized by the labor force.. The data is annual, winsorized at 95th. The regressions control for fully interacted occupation and time fixed effects and a flexible specification for number of firms per worker in a market (20 dummies). The sample includes occupations that are distributed across all CZs over the period 2012-2016 and excludes the top and bottom 2 percent of the HHI. The average NOK/USD in 2021 is 9.

3 Single Market Model

Motivated by these facts, we develop a model in which concentration and job flows interact to determine both prices and quanitites of labor. We begin by exposting the simplest single-market version of our model, and we demonstrate the mechanisms of the framework in a two-firm example in Section 4. In Section 5, we extend the model to have multiple labor markets.

Consider a single labor market which is populated by a measure $\overline{n}$ of workers, and a finite number of firms M. Firms are indexed by $i \in \{1, \ldots, M\}$. Time is discrete and runs forever. Both workers and firms discount the future at a rate β.

Workers. Workers have linear utility and maximize the net present value of discounted utility. A worker's per-period utility is equal to her income. Workers are either employed or unemployed. Among the employed, a measure n_i are employed at firm i. Total employment in a labor market is therefore $n = \sum_{i=1}^{M} n_i$, and thus the measure of unemployed individuals in a labor market is $u = \overline{n} - n$.

A fraction ϕ of individuals who are unemployed randomly search for job openings each period. Unemployed individuals receive home production b. Employed individuals randomly search except that that do not apply to jobs at their existing firm. That is, individuals who are employed at firm i search for jobs at all other firms $j \neq i$. The probability with which they engage in on-the-job search is exogenous and given by $\xi\phi$.

Firms. Firms post job openings v_i at a cost $c(v_i)$ in order to maximize their expected net present value of profits. We let z_i denote the productivity of firm i.

Meeting rates. When workers apply for jobs, only a fraction of those applications actually result in a meeting with a prospective employer. A match occurs if the worker moves to the prospective employer. Matches end through on-the-job-search as well as at an exogenous rate δ.

From the worker's perspective, let λ_{ik} denote the rate at which workers from firm i meet firm k vacancies. From the firm's perspective, let λ_{ik}^f denote the rate at which workers from firm i meet vacancies posted by firm k. Our convention with notation for meeting rates is that the first index i is the origin and the second index k is the destination.

In order to describe the meeting process, we must keep track of the origin and destination of job applicants. Let x_{ik} denote the measure of workers from firm i who randomly apply for jobs at firm k. Likewise, let x_{uk} note the measure of workers who are unemployed and randomly apply for jobs at firm k. Let $x_i = \sum_{k \neq i} x_{ki} + x_{ui}$ denote the total measure of workers who randomly apply for jobs at firm i.

Throughout the paper, we assume that meetings *at each firm* are governed by a constant returns to scale meeting function $\overline{m}(v_i, x_i) \leq \min\{v_i, x_i\}$. Let A denote match efficiency. In practice, we use the matching function of Den Haan et al. (Haan, Ramey, and Watson (1997)):

$$\overline{m}(v, x) = A \frac{vx}{(v^\alpha + x^\alpha)^{1/\alpha}}.$$

We let $f(\theta_i)$ denote the job finding rate at firm i, where tightness at firm i is defined to be $\theta_i = \frac{v_i}{x_i}$. Constant returns to scale implies that $\overline{m}(v_i, x_i) = x_i f(\theta_i)$.

At each firm i there is a measure $\xi\phi n_i$ of workers who engage in on-the-job search. Because employed workers only randomly apply for vacancies that are posted by firms other than where they are currently employed, the measure of workers at firm i who apply to firm k is given by

$$x_{ik} = \frac{v_k}{\sum_{j \neq i} v_j} \xi\phi n_i.$$

A fraction ϕ of unemployed individuals apply to all M firms randomly. Therefore the measure of workers who are unemployed and apply to firm k is given by

$$x_{uk} = \frac{v_k}{\sum_j v_j} \phi u.$$

To derive the probability that a worker at firm i meets firm k, we split the meeting rate into three terms. The first term is the probability a worker searches, $\xi\phi$. Conditional on searching, the second term is the probability that the worker applies to a vacancy at firm k, $\frac{v_k}{\sum_{j \neq i} v_j}$. Conditional on applying to a vacancy at firm k, the third term describes the probability that a meeting occurs, $\frac{\overline{m}(v_k, x_k)}{x_k}$. Combining

these terms yields the worker meeting rate:

$$\lambda_{ik} = \xi\phi \times \left(\frac{v_k}{\sum_{j\neq i} v_j}\right) \times \left(\frac{\overline{m}\left(v_k, x_k\right)}{x_k}\right) = \xi\phi \times \left(\frac{v_k}{\sum_{j\neq i} v_j}\right) \times f\left(\theta_k\right).$$

The contact rate of the unemployed is defined similarly, except the unemployed may meet any firm, $\lambda_{uk} = \phi \times \left(\frac{v_k}{\sum_j v_j}\right) \times f\left(\theta_k\right)$.

We similarly divide the probability that firm k's vacancy meets a worker at firm i into two terms. The first term is the probability that a vacancy meets a worker $\frac{\overline{m}(v_k,x_k)}{v_k}$. The second term is the probability that the worker originated at firm i, $\frac{x_{ik}}{x_k}$. Combining these terms yields the firm meeting rate:

$$\lambda^f_{ik} = \frac{\overline{m}\left(v_k, x_k\right)}{v_k} \times \left(\frac{x_{ik}}{x_k}\right).$$

The probability that firm k meets an unemployed worker is given by $\lambda^f_{uk} = \frac{\overline{m}(v_k,x_k)}{v_k} \times \left(\frac{x_{uk}}{x_k}\right)$.

Using the definition's of x_{ik} and x_k, it is possible to show that $n_i\lambda_{ik} = v_k\lambda^f_{ik}$. This simply states that the number of workers at firm i hired by firm k equals the number of vacancies at firm k filled by workers at firm i.

Bargaining over promised values. When a worker meets a new firm, the incumbent and poaching firm propose sequential offers (Cahuc, Postel-Vinay, and Robin (2006)). We assume that firms propose promised values to workers and that they commit to those promised values. There are three cases (i) the worker meets a firm that can only deliver a maximum promised value which is less than the value which is currently promised to the worker by the incumbent firm, (ii) the worker meets a firm that can deliver a maximum promised value which is greater than the value currently promised to the worker by the incumbent firm, but less than the maximum promised value that the incumbent firm can offer, and (iii) the worker meets a firm that can offer a promised value greater than the maximum promised value that the incumbent firm can offer. In case (i), we assume that there is no change to the worker's wage and the worker remains with the incumbent firm. In case (ii) we assume that the incumbent firm offers a new promised value that delivers the full joint value of the match with the poaching firm. And in case (iii) the worker moves to the poaching firm and then Nash bargains over the remaining surplus, where the full joint value of the match with the incumbent firm becomes the worker's outside option. Let θ denote the worker's Nash bargaining weight. We denote σ as the worker's share of match surplus relative to unemployment. Because of cases (ii) and (iii), the worker's share σ may differ from a worker's Nash bargaining weight θ.

Wage determination. While the bargaining protocol pins down promised values, the way the promised value is delivered is indeterminate. We assume that firms deliver promised values to workers as a constant wage, unless the worker receives an outside option. In that case, the firm and worker renegotiate the promised value. The new promised value is delivered in the same way.

3.1 Forward looking decisions

Let the continuation value of a worker with bargained surplus share σ be given by $W_i(\sigma)$. Likewise, let the continuation value of a firm with bargained surplus share σ be given by $J_i(\sigma)$. The continuation value of an unemployed individual is given by U. We assume that once workers and firms separate, the job position is destroyed. In order to hire again, then, a firm will need to post new vacancies. We will frequently work with both the joint value of a match, $P_i \equiv W_i(\sigma) + J_i(\sigma)$, as well as match surplus $S_i \equiv W_i(\sigma) - U + J_i(\sigma) \equiv P_i - U$. Because firms commit to promised values and both workers and firms have linear utility, the match surplus and joint value are independent of the division of surplus.

Unemployed. The unemployed enjoy home production b, and with probability λ_{uk}, they meet firm k. They receive a share θ of surplus. The continuation value of an unemployed agent can be written as follows

$$U = b + \beta \left[U + \theta \sum_k \lambda_{uk} \max\{S_k, 0\} \right].$$

Employed. To facilitate the description of an employed worker's continuation value, we define σ to be the surplus share of the worker. The worker value is the value of unemployment plus some share σ of the match surplus. In other words, σ is defined to be the number that satisfies the following equation:

$$W_i(\sigma) = U + \sigma [P_i - U].$$

The Cahuc, Postel-Vinay, and Robin (2006) bargaining protocol implies that when a worker at firm i contacts a vacancy at firm k, there are three possible outcomes:

1. If $P_i > W_i(\sigma) > P_k$, the worker stays at firm i with promised value $W_i(\sigma)$.
2. If $P_i > P_k > W_i(\sigma)$, the worker stays at firm i but is now delivered a promised value $W_i(\sigma') = P_k$, where $\sigma' = \frac{S_k}{S_i}$.
3. If $P_k > P_i$, then the worker moves to firm k and Nash bargains over the gains from trade $(P_k - P_i)$, using the full joint value at their prior employer P_i as their outside option.

The worker's continuation value maximizes the following objective where we have defined the worker's bargaining weight as θ:

$$\max_{\widehat{W}} \left(P_k - \widehat{W} \right)^{1-\theta} \left(\widehat{W} - P_i \right)^{\theta}$$

The resulting promised value is equal to the entire joint value between the worker and firm i, plus a

fraction θ of the gains from trade,

$$\widehat{W} = P_i + \theta \left[P_k - P_i \right].$$

It is be convenient to express the promised value as a fraction of surplus relative to unemployment,

$$W_k(\sigma') = \widehat{W} = U + \sigma'[P_k - U], \quad \text{where} \quad \sigma' = \theta + (1-\theta)\frac{S_i}{S_k}.$$

Combining cases (1) through (3), the worker's share of surplus evolves according to the following law of motion:

$$\sigma' = \begin{cases} \left(\frac{\theta S_k + (1-\theta) S_i}{S_k} \right) & \text{if } S_k > S_i \\ \max\left\{ \sigma, \frac{S_k}{S_i} \right\} & \text{if } S_k < S_i \end{cases}$$

As discussed above, we assume that promised values are delivered as a constant wage $w_i(\sigma)$, unless they contact another employer with a sufficiently high productivity to trigger renegotiation. Employed workers at firm i contact firm k with probability λ_{ik}. Under the bargaining protocol of Cahuc, Postel-Vinay, and Robin (2006), the continuation value of the worker can be written as:

$$W_i(\sigma) = w_i(\sigma) + \beta \left[W_i(\sigma) + \sum_{k \neq i} \lambda_{ik} \max\left\{ 0, \min\left\{ \theta \left[S_k - S_i \right], S_k - S_i \right\} + (1-\sigma) S_i \right\} - \delta \sigma S_i \right]. \quad (2)$$

Joint value. Rather than exposit the continuation value of a firm directly, we will focus on the joint value of a worker and firm i. They jointly produce z_i unless (i) the worker receives an outside offer at a firm which generates greater surplus, or the match exogenously dissolves. Thus, the joint value takes into account both the worker value from a new match and the worker value in unemployment. The joint value can therefore be written as:

$$P_i = z_i + \beta \left[P_i + \theta \sum_{k \neq i} \lambda_{ik} \max\left\{ S_k - S_i, 0 \right\} - \delta S_i \right].$$

Surplus. Lastly, the surplus of a match can be expressed in a similar way. Surplus reflects the worker's outside option today, b, as well as the option value they forfeited to join the match, $\theta \int \sum_k \lambda_{uk} \max\{S_k, 0\}$, as well as the potential additional surplus generated from on-the-job search, which is given by $\theta \int \sum_{k \neq i} \lambda_{ik} \max\{S_k - S_i, 0\}$. The resulting expression for surplus is,

$$S_i = z_i - b + \beta \left[S_i + \theta \sum_{k \neq i} \lambda_{ik} \max\left\{ S_k - S_i, 0 \right\} - \delta S_i - \theta \sum_k \lambda_{uk} \max\left\{ S_k, 0 \right\} \right]. \quad (3)$$

Wage equation. Combing the worker's value (2) and surplus (3), we can compute the wage function

based on surplus values alone. The wage function $w_i(\sigma)$ is wage rate delivered by a surplus share σ at firm i: follows,

$$w_i(\sigma) = \sigma z_i + (1-\sigma)\left[b + \beta\theta \sum_k \lambda_{uk} \max\{S_k, 0\}\right] - \beta \sum_{k \neq i} \lambda_{ik} \max\{0, \min\{(1-\sigma)\theta(S_k - S_i), (S_k - S_i)\} + (1-\sigma) S_i\}\Bigg].$$

The wage equation includes three terms: (1) workers obtain σ of production, (2) workers obtain $(1-\sigma)$ of their outside option, and lastly (3) there is backloading since firms that offer greater future pay prospects through future contacts initially pay less.

Firms. Firms post vacancies subject to a convex vacancy posting cost $c(v_i)$, where v_i is the number of vacancies posted. We follow a similar strategy to Burdett and Mortensen (1998), allowing firms to maximize the steady state value of their matches (instead of solving the full transition dynamics, in which case only flow values matter).[10] The value of firm i when its competitors are posting vacancies $v_{-i} = (v_1, \ldots, v_{i-1}, v_{i+1}, \ldots, v_M)$ is given by:

$$\mathbf{V}_i(v_i, v_{-i}) = \int \underbrace{(1-\sigma)S_i}_{\text{Firm value}} n_i(\sigma) d\sigma + \frac{1}{1-\beta}\Bigg[- c(v_i) + \underbrace{(1-\theta) v_i \lambda^f_{ui} \max\left\{S_i, 0\right\}}_{\text{Hire from unemployment}} + \underbrace{(1-\theta) v_i \sum_{k \neq i} \lambda^f_{ki} \max\left\{S_i - S_k, 0\right\}}_{\text{Hire from employment}}\Bigg] \tag{4}$$

We consider two market structures for the vacancy posting decision of firms: (1) an oligopolistic market structure in which firms internalize the impact of their vacancies on contact rates, surplus, and measures of workers, and (2) a competitive market in which firms take as given contact rates, surplus, and measures of workers.

3.2 Single Market Equilibrium Definition

We can then define a *stationary competitive equilibrium*. A *stationary competitive equilibrium* is a set of vacancies and employment $\{v_i, n_i\}_{i=1}^M$, surplus values $\{S_i\}_{i=1}^M$, matching rates $\left\{\lambda^f_{ui}\right\}_{i=1}^M$ and $\left\{\left\{\lambda^f_{ki}, \lambda_{ik}\right\}_{k \neq i}\right\}_{i=1}^M$ such that

[10] Burdett and Mortensen (1998) motivate this with the limit $r \to 0$ in their setting. We do not take that limit here, but we maintain the assumption that firms maximize the steady state value of all of their existing matches, as well as the NPV of all future match surplus.

1. **Vacancy optimality -** Given surplus values and meeting rates, each firm i takes meeting rates λ_{ui}^f and $\left\{\lambda_{ki}^f, \lambda_{ik}\right\}_{k\neq i}$ as given and chooses v_i to solve (4).

   A. **Competitive:** Firms take first order conditions of (4), taking contact rates as given. Thus, the optimal vacancy choice is given by,

$$v_i = c'^{,-1}\left((1-\theta)\lambda_{ui}^f \max\left\{S_i, 0\right\} + (1-\theta)v_i \sum_{k\neq i} \lambda_{ki}^f \max\left\{S_i - S_k, 0\right\}\right) \tag{5}$$

   B. **Oligopoly:** Firms maximize (4) understanding the impact of their vacancies on contact rates, surplus, and measures of workers. The optimal vacancy posting decision cannot be expressed in closed form and will be characterized numerically.

2. **Surplus** - The surplus values S_i are consistent with the meeting rates via (3)

3. **Employment** - The distribution of unemployment is the stationary distribution implied by the meeting rates and policies

4. **Meeting rates** - The meeting rates are consistent with vacancies and employment

   A. **Worker meeting rates**

$$\lambda_{ik} = \xi\phi \times \left(\frac{v_k}{\sum_{j\neq i} v_j}\right) \times \left(\frac{\overline{m}\left(v_k, x_k\right)}{x_k}\right)$$

   B. **Firm meeting rates**

$$\lambda_{ik}^f = \frac{\overline{m}\left(v_k, x_k\right)}{v_k} \times \left(\frac{x_{ik}}{x_k}\right)$$

4 Single Market Characterization: Two Firm Example

To build intuition we now present a simpler, two firm example which we can almost completely characterize analytically. The goal is to illustrates how strategic interaction works in our framework.

Suppose that there are two firms with $z_1 > z_2$ such that $S_1 > 0$ and $S_2 > 0$ in a single market. We set $\theta = 0$ so that firms have complete bargaining power. Under these assumptions, the surpluses of the firms can be written in closed form, and they do not depend on contact rates:

$$S_1 = \frac{z_1 - b}{1 - \beta\left(1-\delta\right)}, \qquad S_2 = \frac{z_2 - b}{1 - \beta\left(1-\delta\right)}.$$

Because there are only a finite number of firms in the market, how the surplus is divided is also solvable in closed form. In this simple example, there are two surplus shares that a worker can achieve.

If an unemployed worker has a meeting with either firm 1 or firm 2 (U-to-1 and U-to-2, respectively), the new employment relationship yields a surplus share of $\sigma_{u1} = \sigma_{u2} = 0$, where, in this section only, we subscript the σ term by contact histories (the first subscript is the 'origin' and the second subscript is the 'destination'), respectively. Meetings from 2-to-1 and 1-to-2 yield a surplus share $\sigma_{12} = \sigma_{21} = \frac{S_1}{S_2}$, where the later meeting does not result in a job transition because firm 2 is less productive than firm 1.

To compute the firms' best response functions, we must characterize the objective function of the firm. The objective function depends directly on the steady state measures of workers across surplus shares. Under our assumptions, no worker will move from firm 1 to firm 2 ('down the ladder'), which greatly simplifies the laws of motion for employment.[11] In this section only, we index employment by contact histories. n_{u2} denotes the measure of workers who were hired at firm 2 out of unemployment, similarly n_{u1} denotes the measure of workers who were hired at firm 1 out of unemployment. n_{12} is the measure of workers at firm 1 who met a firm 2 (but don't move). n_{21} is the measure of workers at firm 1 who were hired from firm 2. The laws of motion for each group are given by:[12]

$$
\begin{aligned}
n'_{u2} &= u\lambda_{u2} + (1 - \delta - \lambda_{21})\, n_{u2}, \qquad & n'_{u1} &= u\lambda_{u1} + (1 - \delta - \lambda_{12})\, n_{u1}, \\
n'_{12} &= n_{u1}\lambda_{12} + (1 - \delta)\, n_{12}, & n'_{21} &= n_{u2}\lambda_{21} + (1 - \delta)\, n_{21}, \\
u' &= u + \delta\,(n_{u2} + n_{u1} + n_{12} + n_{21}) - (\lambda_{u1} + \lambda_{u2})\, u.
\end{aligned}
$$

Aggregating across surplus shares, we have that total employment for each firm is $n_2 = n_{u2}$ and $n_1 = n_{u1} + n_{12} + n_{21}$: the least productive firm hires only from unemployment while the most productive firm hires both unemployed workers and workers who used to be employed at firm 2. The measure of searchers at firm 1 is given by $x_1 = \xi\phi n_2 + \frac{v_1}{v_1+v_2}\phi u$. Workers at firm 2 only search for jobs at firm 1 (a central feature shaping best response curves), and a fraction $\frac{v_1}{v_1+v_2}$ of unemployed searchers meet firm 1 vacancies. The opposite is true for the measure searchers at firm 2: $x_2 = \xi\phi n_1 + \frac{v_2}{v_1+v_2}\phi u$. Given the measures of workers and searchers, the contact rates are given by,

$$
\begin{aligned}
\lambda_{u1} &= \phi\left[\frac{v_1}{v_1+v_2}\frac{m(x_1,v_1)}{x_1}\right], \qquad & \lambda_{u2} &= \phi\left[\frac{v_2}{v_1+v_2}\frac{m(x_2,v_2)}{x_2}\right], \\
\lambda_{21} &= \xi\phi\frac{m(x_1,v_1)}{x_1}, & \lambda_{12} &= \xi\phi\frac{m(x_2,v_2)}{x_2}.
\end{aligned}
$$

We can use the relationship $\lambda_{ij} \times n_i = \lambda^f_{ij} \times v_j$ to determine contact rates from the firm's perspective. Firms then maximize their steady value of both new and existing matches, taking the vacancy postings

[11] This is not true with amenities. A preliminary version of our model with amenities is available upon request.

[12] This can be solved in closed form: $n_{u2} = \frac{\delta\overline{n}}{(\delta+\lambda_{21})(\lambda_{u1}+\lambda_{u2}+\delta)}\lambda_{u2}$, $n_{u1} = \frac{\delta\overline{n}}{(\delta+\lambda_{12})(\lambda_{u1}+\lambda_{u2}+\delta)}\lambda_{u1}$, $n_{12} = \frac{\overline{n}}{(\delta+\lambda_{12})(\lambda_{u1}+\lambda_{u2}+\delta)}\lambda_{u1}\lambda_{12}$, $n_{21} = \frac{\overline{n}}{(\delta+\lambda_{21})(\lambda_{u1}+\lambda_{u2}+\delta)}\lambda_{21}$, $u = \frac{\delta\overline{n}}{(\lambda_{u1}+\lambda_{u2}+\delta)}$.

of the other firm as given. The objective functions of the firms are given by,

$$V_2(v_1, v_2) = \max_{v_2} n_{u2}\left(1-\sigma_{u2}\right) S_2 + \frac{1}{1-\beta}\left[-c\left(v_2\right) + v_2 \lambda_{u2}^f \left(1-\sigma_{u2}\right) S_2\right] \tag{6}$$

$$V_1(v_1, v_2) = \max_{v_1} n_{u1}\left(1-\sigma_{u1}\right) S_1 + n_{12}\left(1-\sigma_{12}\right) S_1 + n_{21}\left(1-\sigma_{21}\right) S_1 \tag{7}$$
$$+ \frac{1}{1-\beta}\left[-c\left(v_1\right) + v_1\left[\lambda_{u1}^f \left(1-\sigma_{u1}\right) S_1 + \lambda_{21}^f \left(1-\sigma_{21}\right) S_1\right]\right].$$

While the competitive vacancy posting version of the two-firm example is tractable analtyically, the oligopoly equilibrium is not. We are able to characterize the law of motion of employment and contact rates in near-closed form, but maximization of (6) by the oligopolist is involves complicated expressions. We proceed to characterize the problem numerically. We provide the full numeric solution algorithm in Section E.

4.1 Best response curves

Panel A of Figure 4 plots the vacancy best response functions for firm 1 and firm 2. Firm 1's vacancies are on the x-axis, and firm 2's vacancies are on the y-axis. The solid line traces out the best response of firm 2 for each level of firm 1's vacancies on the x-axis. Likewise, the dashed line traces out the best response of firm 1 for each level of firm 2's vacancies on the y-axis. Where the best response curves intersect defines the point $v_1 = v_1^*\left(v_2^*(v_1)\right)$ and thus yields the equilibrium in the oligopoly vacancy posting game (we interchangeably refer to the oligopoly equilibrium as the *strategic* equilibrium). The strategic equilibrium level of vacancies is depicted by the red square in Panel A. The strategic equilibrium level of employment is also depicted by a red square in Panel B.

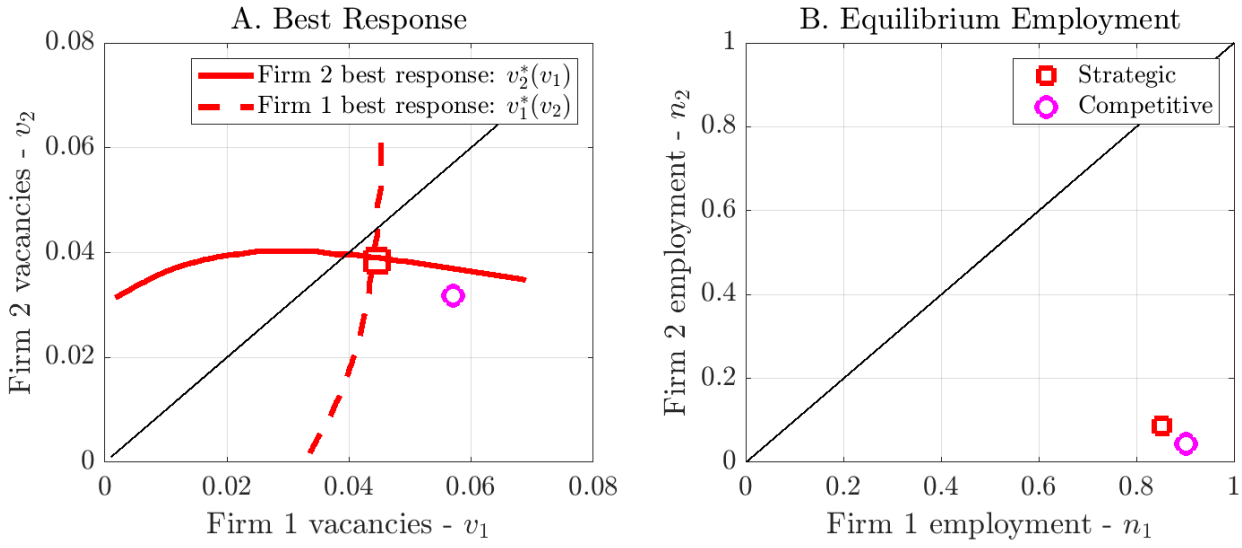


Figure 4: Two firm example, vacancy best response curves

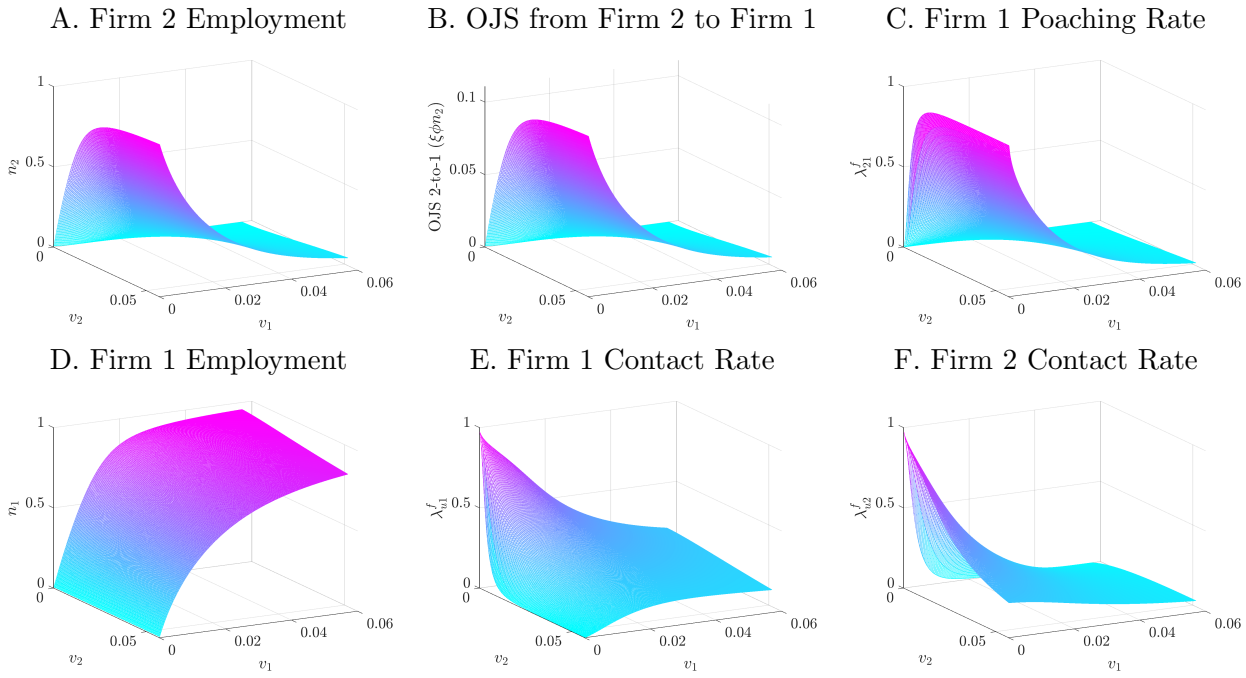


Figure 5: Two firm example, components of firm 1 and firm 2's Payoffs

There are two important features of the best response curves in Panel A of Figure 4. First, firm 1's strategy is universally strategic complements with firm 2's vacancies. In other words, if firm 2 posts more, firm 1's best response is to post more; if firm 2 posts less, firm 1's best response is to post less. We explore the mechanisms behind this result in Panels A through F of Figure 5.

Shape of firm 1 best response curve. What drives the extreme complementarity of firm 1's vacancies? Firm 1's free ridership on firm 2's size. Panel A of Figure 5 plots employment at firm 2 as a function of all (v_1, v_2) combinations. As firm 2 posts more vacancies, firm 2's employment grows. Since workers employed at firm 2 only search at firm 1 (recall, the measure of searchers at firm 1 is given by $x_1 = \xi\phi n_2 + \frac{v_1}{v_1+v_2}\phi u$), a corollary of Panel A is that the measure of on-the-job search intensity originating from firm 2 and being directed to firm 1 grows as firm 2 posts more vacancies. Panel B depicts the units of on-the-job search intensity being directed to firm 1, illustrating the strong positive effect of firm 2's size on the units of search efficiency being directed to firm 1. In the strategic vacancy posting game, firm 1 internalizes this relationship and free-rides on firm 2's size. As firm 2 gets larger, workers at firm 2 will always direct their search to the more productive firm 1, and conditional on meeting firm 1, they will leave firm 2 with certainty. Panel C shows that the poaching rate from firm 2 to firm 1 is increasing in firm 2's vacancies (and thus firm 2's size). Quantitatively the poaching term $\lambda_{21}^f (1 - \sigma_{21}) S_1$ in firm 1's objective function (equation (6)) dominates payoffs. The poaching rate λ_{21}^f strictly increases in v_2 and S_1 remains constant. Each vacancy firm 1 posts is more likely to meet a worker employed at firm 2 (since unemployment is also falling as firm 2 expands), and less likely to meet an unemployed agent.

For small values of firm 2's vacancies — when there is no free-ridership or OJS because the majority of workers remain unemployed — the behavior of unemployed contact rates (e.g. λ_{u1}) dominate the best-response curve. From firm 1's perspective, the contact rate is given by $\lambda_{u1} = \phi\left[\frac{v_1}{v_1+v_2}\frac{m(x_1,v_1)}{x_1}\right]$. Colloquially, there is a *tussle* between firm 1 and firm 2 over the unemployed that is governed by the vacancy ratio, $\frac{v_1}{v_1+v_2}$. As a result, the cross-partial derivatives of $\frac{v_1}{v_1+v_2}$ determine the best responses. To demonstrate the strategic complementarity and substitutability generated by this vacancy ratio, Figure 6 plots the cross-partial derivative of this term $\partial^2\left(\frac{v_1}{v_1+v_2}\right)/\partial v_1 \partial v_2$. When the cross-partial is positive, there is *strategic complementarity*, and when the cross-partial is negative, there is *strategic substitutability*. Figure 6 shows that there is extreme complementarity between firm 1's vacancy posting and firm 2's vacancy posting whenever firm 1 is larger (which is generally true if firm 1 is more productive, which it is in our example because it is more productive. Therefore firm 1 has strong incentives to post more vacancies when firm 2 posts more vacancies, generating strong complementarity near the origin, $(v_1 = 0, v_2 = 0)$.

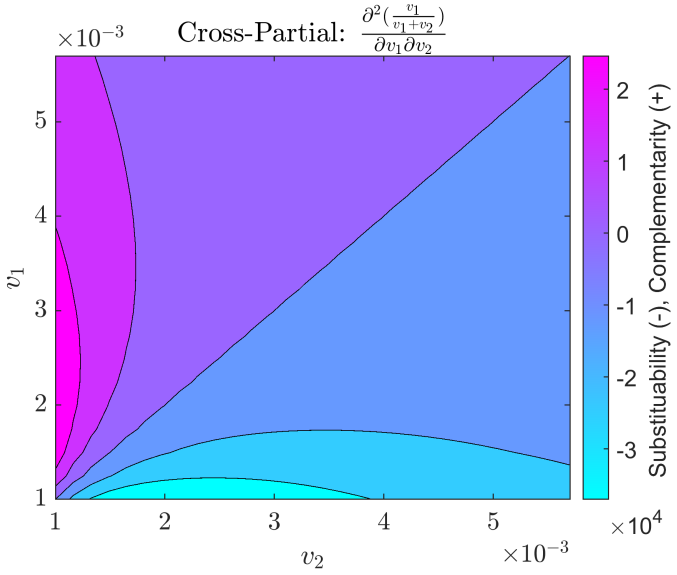


Figure 6: Two firm example, cross-partial derivative of $\frac{v_1}{v_1+v_2}$

Shape of firm 2 best response curve. The best response curve of firm 2 is largely determined by firm 2's contact rate out of unemployment (λ^f_{u2}), which is depicted in Panel F of Figure 5. Initially there is an extreme complementarity when firm 1 is small and firm 2 can post more vacancies (and thus enters the region of complementarity shown in Figure 6[13]. As firm 1 becomes ever-larger, the returns to firm 2 from posting additional vacancies decline. Firm 2 becomes smaller than firm 1, entering the region of substitutability shown in Figure 6. It is in this sense that firm 1 loses the tussle over the unemployed with firm 1.

[13]Notice that the cross-partial of $\partial^2\left(\frac{v_2}{v_1+v_2}\right)/\partial v_1 \partial v_2$ is symmetric).

Competitive equilibrium. Firm 1's free ridership implies that firm 1 posts too few vacancies in the strategic equilibrium relative to the competitive equilibrium. Firm 1, as is typical of monopsonists, restricts their vacancy postings to lower implicit prices. The result is that when we compute the competitive vacancy posting equilibrium, firm 1 expands significantly. Since firm 2's vacancies are strategic substitutes with firm 1's vacancies (since firm 1 is larger), they post fewer vacancies. The competitive equilibrium is depicted by the pink circle in Panel A of Figure 4. Vacancy concentration increases. Likewise, in Panel B of Figure 4, it is also the case that employment concentration increases. The most productive firm in the labor market is inefficiently small.[14]

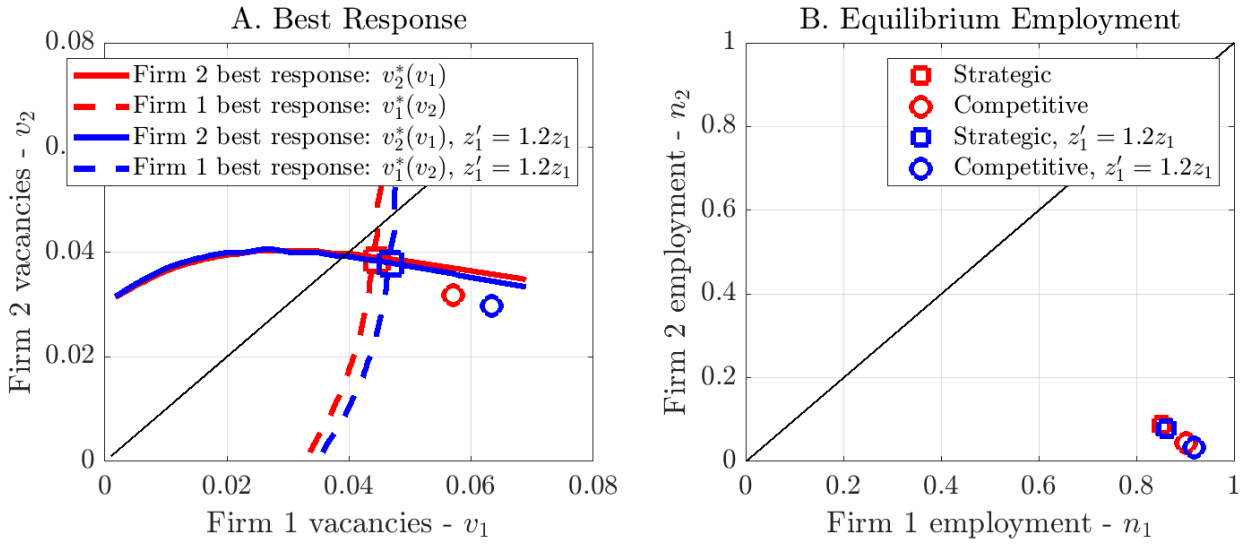


Figure 7: Two firm example, comparative statics: **Firm 1** productivity increases 20%

4.2 Two firm example: Comparative statics

We next explore some comparative statics of the two firm model. These exercises illuminate the mechanisms present in our counterfactual experiments while also providing strong testable implications of the oligopoly versus competitive market structures.

Firm 1 productivity improves. We begin with the simplest case which is when firm 1's productivity improves by 20%, in $z_1' = 1.2z_1$. Figure 7 plots the new best response curves in blue. The best response curve of firm 1 shifts outward since their surplus is increasing in z_1 and thus payoffs to vacancy posting increase when $z_1' = 1.2z_1$. This implies that for every vacancy posting by firm 2, firm 1's optimal strategy is to increase their postings relative to the baseline. Due to firm 2 being in a region of strategic

[14]Note that the first welfare theorems do not hold in this simple example since there are still search frictions. We do not characterize the planner's problem, but in our counterfactual examples we take competitive limits that eliminate search frictions (in which the welfare theorems hold), and, in that setting, it becomes clear that the most productive firms expand in the efficient allocation.

substitutability, they post fewer vacancies in the strategic oligopoly equilibrium, depicted by the blue square. The same is true in the competitive equilibrium, depicted by the blue circle. However, free-ridership is still present. The expansion of firm 1 vacancy postings in the oligopoly equilibrium is much less than the expansion of vacancy postings in the competitive equilibrium. In the oligopoly equilibrium, firm 1 understands that when they post more vacancies, they reduce firm 2's size; reducing firm 2's size reduces the 'directedness' of search in the economy (i.e. firm 1 hires more inefficiently out of unemployment, in which worker search effort is random and governed by $v_1/(v_1+v_2)$). As a result, firm 1 expands by less in the strategic market structure relative to the competitive market structure.

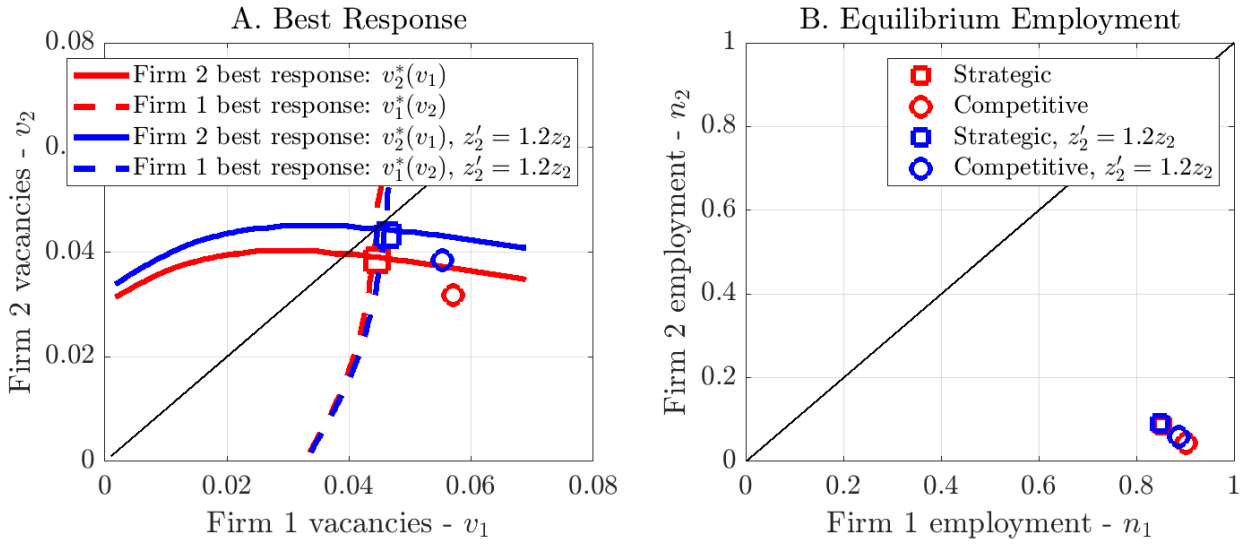


Figure 8: Two firm example, comparative statics: **Firm 2** productivity increases 20%

Firm 2 productivity improves. Next, we assume firm 2's productivity improves to $z_2' = 1.2z_2$ but is still smaller than firm 1. Figure 8 plots the new best response curves and equilibrium. Interestingly, firm 2 expands their vacancy postings, *and firm 1 also expands their vacancy postings.* In the strategic oligopoly equilibrium, firm 1's vacancy postings are complementary with firm 2 – firm 1 understands that search effort is even more directed toward firm 1 (the set of 'captured workers' $\xi\phi n_2$ increases has n_2 increases), meaning that firm 1's payoff to posting more vacancies increases. The opposite is true in the competitive equilibrium. Taking contact rates as given, firm 1's vacancy postings decline. As firm 2 expands, firm 1 takes 'residual demand' (i.e., the 'residual supply' of workers in this case) as given and posts fewer vacancies. Since vacancies translate to employment, we see a similar reversal of firm 1's employment response in the competitive vacancy posting equilibrium. Thus, the employment and vacancy posting responses of the market leader to the market follower uniquely identify the nature of competitive in the vacancy market.

5 Multi-Market Model

We assume that there are J markets indexed by $j \in \{1, \ldots, J\}$. Within each market there are $\overline{n}_j$ workers and M_j firms. The measure of firms is exogenous, and workers are assumed to be immobile across markets.

We note that unemployment rates vary relatively little across markets that vary widely in terms of firms-per-worker. In order for the model to scale and achieve this stylized fact, we remove variation in firms-per-worker by scaling the vacancy costs accordingly. Anticipating the calibration, we will assume the vacancy cost is given by $c(M, \overline{n})\overline{c}\frac{1}{1+\gamma}v^{1+\gamma}$ where $\overline{c}$ is a constant and $c(M, \overline{n})$ is an arbitrary function of the market number of firms and labor force. With this functional form, to achieve neutrality of the unemployment rate with respect to firms-per-worker, we assume the vacancy cost is given by $c(M, \overline{n}) = \left(\frac{M}{\overline{n}}\right)^{\gamma}$. Appendix F details the derivation.

Our convention is to superscript all equilibrium objects with the market identity j. With this notation and our parametric cost function, the objective of firm i in market j becomes:

$$\mathbf{V}_i^j(v_i, v_{-i}) = \int (1-\sigma) S_i^j n_i^j(\sigma) d\sigma + \frac{1}{1-\beta}\Bigg[(-1)\overline{c}\frac{1}{1+\gamma}v_i^{1+\gamma}\left(\frac{M_j}{\overline{n}_j}\right)^{\gamma} + (1-\theta)v_i\lambda_{ui}^{f,j}\max\left\{S_i^j, 0\right\} + (1-\theta)v_i\sum_{k\neq i}\lambda_{ki}^{f,j}\max\left\{S_i^j - S_k^j, 0\right\}\Bigg].$$

6 Calibration and Model Fit

This section describes our calibration approach. We then explore the model fit relative to the data moments in Section 2. We find that the model does well at matching the relationship between employment Herfindahls and both job flows and wages.

6.1 Calibration

Markets. We discipline the number of markets J, firms-per-market M_j, and workers-per-market $\bar{n}_j$ under the assumption that a market is a 4-digit occupation by commuting zone. We assume that there are $J = 100$ markets, and in each market, we draw the number of firms $M_j \sim^{iid} G(M_j)$ where $G(\cdot)$ is the empirical CDF of firms-per-market truncated at $M_j = 20$.[15] In the data, we estimate the average number of workers in markets with $M_j \in \{1, \ldots, 20\}$, yielding a function $\bar{n}(M_j)$. We impose this linear relationship to determine the number of workers in a market with M_j firms.

[15] This top-codes roughly 30% of markets in which $M_j > 20$. However, these markets with $M_j = 20$ behave approximately competitively and thus we believe this top-coding will have little impact on our results.

Parameter		Value	Moment	Data	Model
Home production	b	0.480	Replacement rate	0.61	0.72
OJS search intensity	ξ	0.117	EE	0.006	0.009
Bargaining power	θ	0.071	Avg Log Wage Growth	0.006	0.006
Productivity dispersion	$\log z \sim N\left(-\frac{1}{2}\sigma_z^2, \sigma_z\right)$	0.208	HHI - Employment	0.111	0.094
			$Cov(HHI, logwages)$	-0.060	-0.214
Vacancy cost power	γ	2.59	$Cov(HHI, EE)$	-0.004	-0.004
Vacancy cost constant	$\overline{c}$	1667.77	UE	0.156	0.151

Table 1: Summary of Parameters

Preferences and technology. On an annual basis, the discount rate is 4 percent ($\beta = 0.96$). We use Haan, Ramey, and Watson (1997)'s matching function, with an elasticity 0.5. We use an isoelastic vacancy cost function, $c(v) = \overline{c}v^{\gamma}$. We have six parameters that we match with a variety of cross-market moments. They are, the value of home production (b), the constant and the convexity of the the vacancy cost function ($\overline{c}$ and γ), the frequency that agents bargain on the job, ξ, worker's bargain power, θ, and the dispersion of firm productivity, $\log z \sim N\left(-\frac{1}{2}\sigma_z^2, \sigma_z\right)$.

The results are shown in Table 1 along with the corresponding data moment that identifies them. Overall, the fit is quite good. As is standard, the level of home production, b, sharply identifies the replacement rate. Second, the intensity of on-the-job search, ξ, directly effects the job-to-job transition rate (EE). While the current calibration struggles to match the empirical level of quarterly EE flows, adding in idiosyncratic, preference based reasons to move (in the next draft), will improve the fit. Third, there is a strong connection between the level of worker's bargain power, θ, and the amount of average log wage growth. Fourth, high dispersion in firm productivity within and across markets implies higher HHIs within a market because it is the existence of a very productive, large firm in a market leads to higher levels of concentration. Higher concentration also means more market power and fewer EE flows. This generates a negative relationship between HHIs and log wages in the model. Finally, the two parameters that affect how costly vacancy posting pin down the job finding rate (UE) and determine how sensitive EE flows are to changes in concentration since they affect how strong dominant firms free riding incentives are.

6.2 Model fit

Using model-simulated data, Panels (A) through (F) of figure 9 replicate their data counterparts in Figure 2. In this section, and in our counterfactuals, we restrict our attention to concentrated markets with less than 10 employers. The model is able to qualitatively capture key relationships between market concentration (measured as the employment Herfindahl in the market, HHI_j^n) and both flows and wages in the data: (1) the model successfully captures the negative relationships between E-to-E job flows, U-to-E job flows, and market-level employment concentration, and (2) the model successfully captures the

negative relationships between average market wage levels, wage growth, and market-level employment concentration.

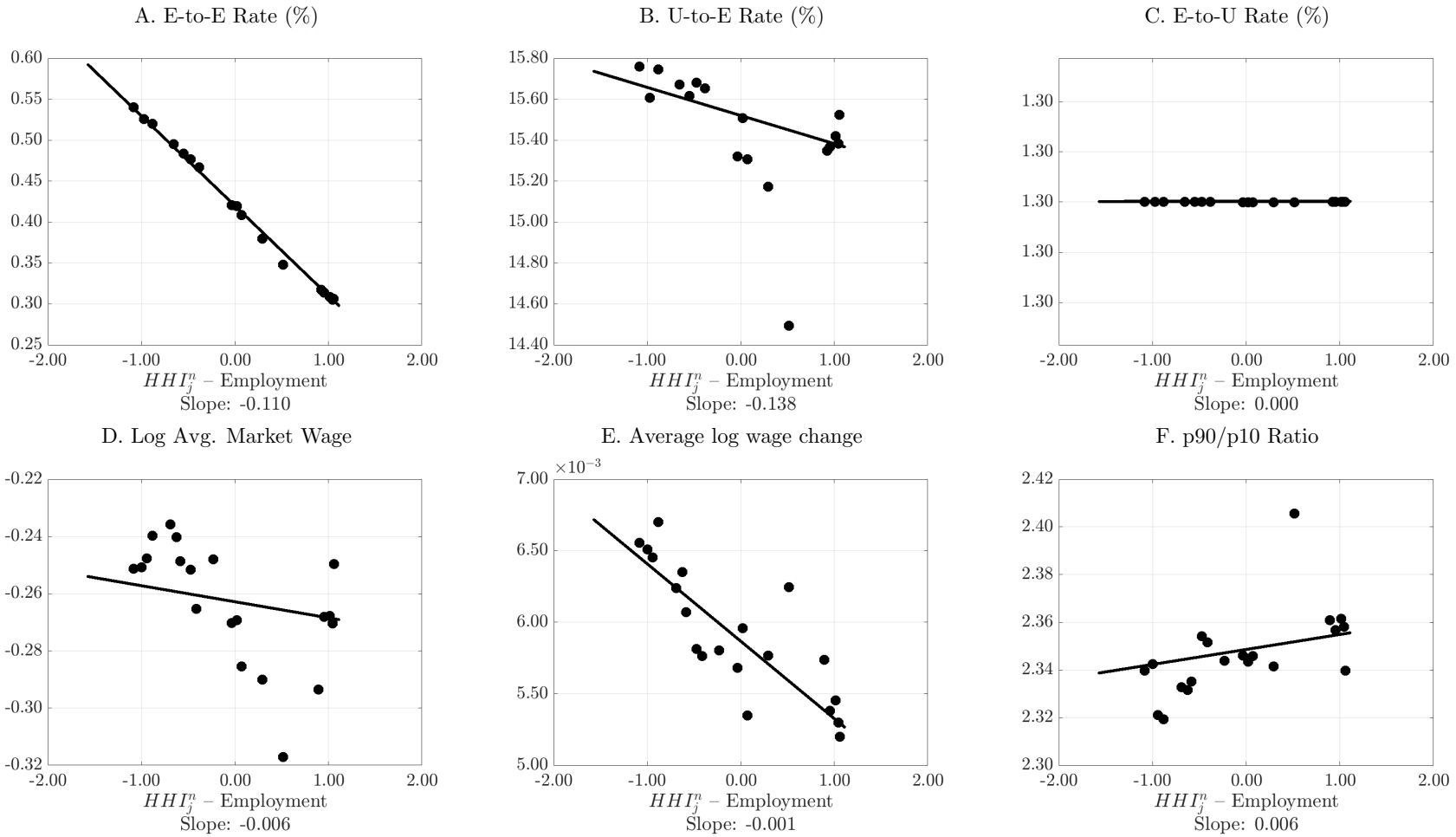
Panel A shows that in both the model and data, the employment-to-employment job transition rate is strongly decreasing in the degree on market concentration. In the data, a one standard deviation increase in market concentration is associated with a reduction in the EE rate of 0.052 percentage points, whereas in the model, a one standard deviation increase in market concentration is associated with a reduction in the EE rate of 0.11 percentage points. Thus, the model's elasticity of employment-to-employment transition rates in concentrated markets is twice the data.

Panel B shows that the model also qualitatively captures the negative relationship between market concentration and job finding rates (U-to-E rate). While the qualitative fit matches the data, the model's elasticity of U-to-E rates with respect to HHI_j^n is roughly one tenth of the data. Panel C plots the job separation rate into unemployment. In the data, the relationship between E-to-U rates and concentration is insignificant at 5% levels. In the model, the job separation rate into unemployment is constant and exogenous, and so we view the data as providing strong support for this assumption.

Panel D plots the wages in both the model and data. The log of the average wage in the market is negatively related to concentration (with a slope of -0.006 in the model, meaning a 1std deviation increase in concentration is associated with a .006 percent lower log wage), although this relationship is somewhat weaker than in the data (a slope of -0.028 in the data). Panel E plots the average log wage change in the market. We find that more concentrated markets exhibit marginally weaker individual wage growth (with a slope of -.001, meaning a 1std deviation increase in concentration is associated with a .001 percent lower log wage growth rate). In the data, there is zero correlation between individual wage growth and the employment Herfindahl shown in Panel E in 2. Lastly, in Panel F, we find an economically insignificant relationship between p90/p10 wage ratios within the market and the employment Herfindahl. In the data, the relationship between the p90/p10 wage ratio and market concentration is negative; however, we note that many calibrations of the present framework are capable of generating such a downward sloping relationship.

Why are the model's predictions for wages not as pronounced as the data? In the current calibration, the smallest market size is $M_j = 2$ firms. As a consequence, the most concentrated markets are markets in which the leader firm is extremely productive and has effectively taken over the labor market, resulting in an employment Herfindahl of one. In these markets, wages are higher (due to selection on productivity) and this generates the non-linearities seen in Panel D of Figure 9 (i.e. wage levels initially fall in the employment Herfindahl and then rise again as the Herfindahl approaches one).

Figure 9: Model relationship between concentration (HHI_j^n), job flows and wages



7 Counterfactuals

We now use the model to conduct a number of counterfactuals to highlight the importance of labor market competition and strategic interaction for welfare and wage inequality. In our first counterfactual we consider two market structures for the vacancy posting decision of firms: (1) an oligopolistic market structure in which firms internalize the impact of their vacancies on contact rates, surplus, and measures of workers, and (2) a competitive market in which firms take as given contact rates, surplus, and measures of workers. In order to highlight the importance of strategic interaction, our baseline counterfactual focuses on markets with 10 or fewer firms.

Atomistic vacancy posting and market structure. We begin by exploring the effects of atomistic vacancy posting in Figure 10 and column B of Table 2. Panels A through F of Figure 10 contrasts atomistic vacancy posting with our baseline model's covariances of job flows and wages with employment Herfindahls. Panel A shows that the covariance of E-to-E job flows and employment concentration becomes significantly more negative under atomistic vacancy posting. What drives the decline in E-to-E rates in concentrated markets is that the most productive firm strategically under-posts vacancies in the oligopoly equilibrium as they free-ride on the less productive competitor (recall from Section 4, the more productive player benefits from their less productive competitor being larger since employees at the less productive firm direct their search solely toward the more productive player). The expansion of the most productive firms with atomistic vacancy posting results in employment concentration rising from 0.388 in our baseline to 0.429 with atomistic vacancy posting. As the more productive firm expands, E-to-E rates fall since workers never leave the most productive firm via an E-to-E transition.

Panel B of Figure 10 shows that the relationship between job finding rates and market concentration reverses sign under atomistic vacancy posting. The ex-ante highly concentrated markets are concentrated because of the presence of a very productive firm. The presence of a very productive firm implies significant monopsony power and severely under-posted vacancies. As a result, in these concentrated markets, the U-to-E rate increases significantly. Lastly, in terms of flows, the exogenous layoff rate (E-to-U) is unaffected by market structure.

Panel D of Figure 10 shows that market wages increase as markets become more concentrated when vacancy posting is atomistic. Similar to the rising U-to-E rate, these concentrated markets feature a highly productive firm. As they expand and become a large share of the market's total employment, the average market wage increases. Panel E shows that wage growth falls in the Herfindahl when vacacny posting is atomistic. As the large firm becomes a greater share of total employment, there are fewer and fewer wage renegotiations. Workers go straight to the most productive firm, receive a high wage, but limited future wage growth (only receiving wage gains when they meet the less productive firm and renegotiate their pay). Lastly, panel F shows that the p90/p10 wage ratio is decreasing in labor market concentration when vacancy posting is atomistic. As more and more workers are employed at the large firm, wages become compressed.

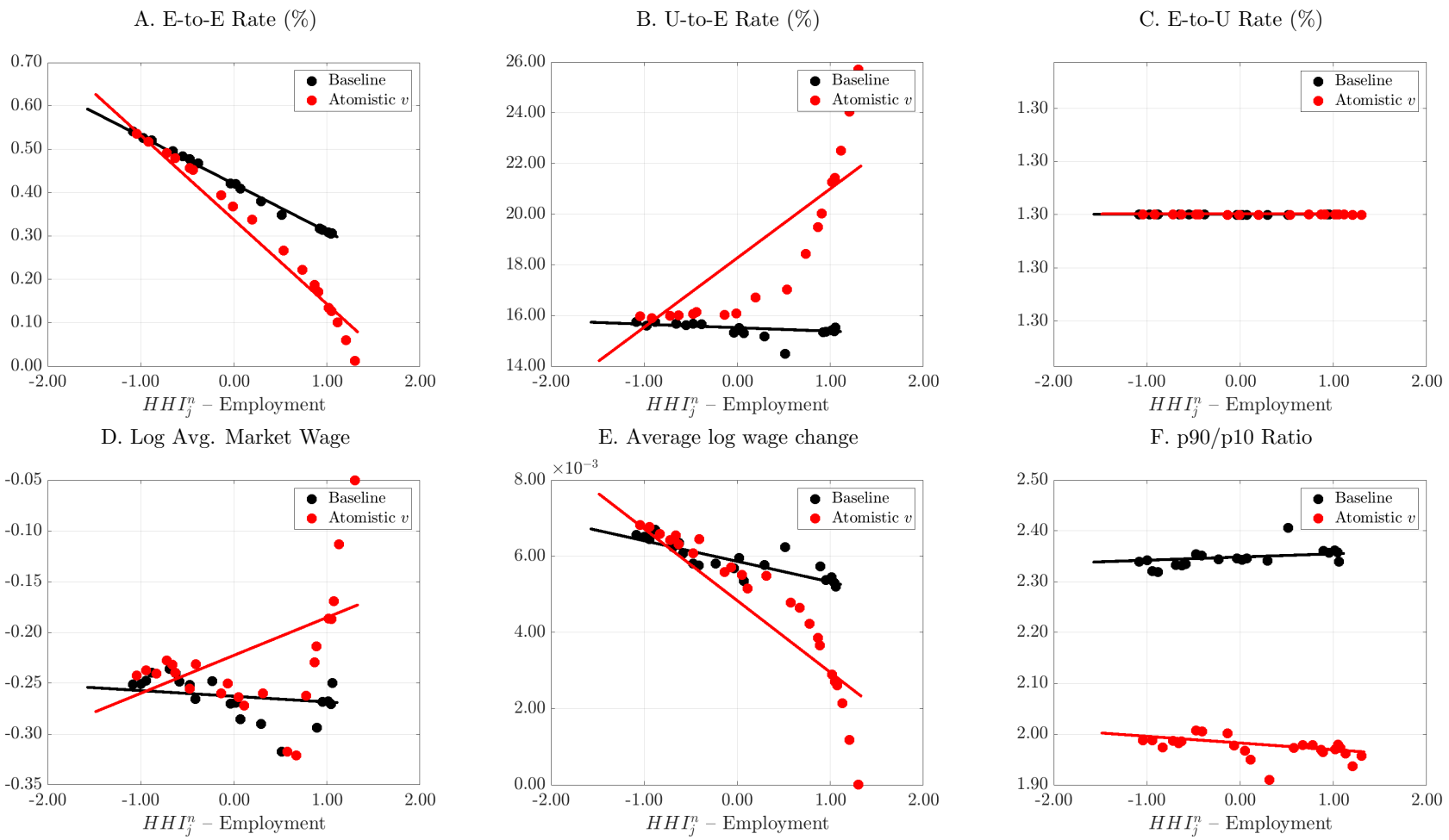
Atomistic vacancy posting, welfare, and output. Column B of Table 2 further explores the real implications of atomistic vacancy posting. As discussed in the prior section concentration in both employment and vacancies increases because atomistic vacancy posting shuts down the free-ridership of the most productive firms. The most productive firms grow larger, and this is true across all markets as shown in Panel A of Figure 11. In addition to rising concentration, labor's share of income increases from 68.1% to 69.0% as wages improve and workers find jobs at more productive firms. Once employed at a more productive firm, they obtain outside offers and capture a greater share of surplus.

The overall p90/p10 ratio does not change, however, within markets, the p90/p10 ratio compresses enormously. It falls from 2.349 in our baseline oligopoly economy to 1.982 in our atomistic vacancy posting economy. Panel B of Figure 11 shows that the p90/p10 ratio in the atomistic vacancy economy becomes more compressed in every market. This compression is driven by the rising employment share of the most productive firms. Average wages are higher, putting upward pressure on the p10 of the wage distribution in each market. But, there are still very unproductive markets and so economy-wide p90/p10 ratios are effectively unchanged.

As labor is reallocated to more productive firms, labor productivity increases. Under atomistic vacancy posting, labor productivity increases by roughly 0.8% and output increases by 1.2%. However, most of these additional output gains are absorbed by convex vacancy posting costs. Consumption (output minus vacancy posting costs) is effectively unchanged under atomistic vacancy posting. Since agents have linear utility, consumption alone is sufficient to measure welfare, and thus welfare is effectively unchanged under atomistic vacancy posting.

Panel C of Figure 10 demonstrates the role reallocation patterns in the model play in driving the above results. Since markets may only include 2 firms, we are constrained to show reallocation from below-median-productivity firms to above-median-productivity firms. Focusing on the blue bar (employment) and light blue bar (vacancies), we find that employment at the below-median productivity firms declines by 35% (i.e. with two firms where $z_1 > z_2$ in a market $n_2'/n_2 = 0.65$). Employment at the most productive firm increases by nearly 6% (i.e. with two firms where $z_1 > z_2$ in a market $n_1'/n_1 = 1.06$). Vacancies follow a similar pattern of reallocation with more productive firms expanding their vacancy postings by 20%.

Figure 10: Counterfactuals: Atomistic vacancy posting



Increased Entry $M_j' > M_j$. Our second counterfactual explores increased entry in the labor market. Holding the employee-to-firm ratio fixed (through our choice of the scaling of the vacancy cost), we simulate an increase in entry from our baseline set of markets in which $M_j \leq 10$ firms to $M_j' = 20$ firms. Under the maintained assumption of oligopoly in the labor market, column C of Table 2 illustrates the results. Concentration declines uniformly, and sharply, from 0.388 in the baseline to 0.098 with 20 firms. As a result of the larger job ladder and the presence of more productive firms (i.e. the maximal productivity from a larger number of iid draws increases, but so does the min), labor productivity increases by 3.2%. Since large firms free-ride and under-post vacancies, smaller less productive firms post disproportionately more vacancies. They pay lower wages with the promise of future wage gains through renegotiation and job mobility. As a results labor's share of income falls. Overall, we find productivity gains of 3.2%, output gains of 3.4%, and consumption gains of 1.1%.

We then assume the $M_j' = 20$ firms compete atomistically instead of strategically in column D of Table 2. We find productivity gains of 3.2%, output gains of 3.3%, and consumption gains of 1.4%. It is important to note that these gains are very similar to the oligopoly economy suggesting that even with only 20 firms in the market, 20 firms generate enough competition to mitigate any free-riding through strategic vacancy posting behavior.

Search frictions and monopsony. In Table 3, we report results from two additional counterfactuals aimed at measure the interaction between search frictions and oligopoly: (1) we double the on-the-job search efficiency parameter $\xi' = 2\xi$, and (2) we increase overall match efficiency by 10%, $A' = 1.1A$. In both experiments, we move the economy toward the "competitive limit" with no search frictions.

Column A of Table 3 shows that with greater on-the-job search, concentration rises sharply relative to our baseline economy. Workers find their way to the most productive firms faster, and they never leave those firms except through exogeneous separations. This lowers the incentives of the less productive firms to post vacancies, and labor's share of income falls. The E-to-E rate rises, as expected, and the overall p90/p10 ratio roughly doubles. The larger firm free-rides even more, resulting in greater unemployment, lower consumption, and lower output. The yellow and red bars of Panel D of Figure 11 illustrate the reallocation patterns when on-the-job search intensity doubles. Employment at below-median-productivity firms declines by more than 50%, while the most productive firms grow by 7%. Above-median-productivity firms achieve thse employment gains by posting 20% more vacancies. Despite these output gains from reallocation, aggregate employment and vacancy postings decline which results in a welfare loss.

Column C of Table 3 simulates greater match efficiency. Again, concentration rises sharply relative to our baseline economy; however, unlike the prior counterfactual, unemployment falls. Despite the larger size and greater free-ridership of the most productive firm in the market, unemployed workers are more likely to meet firms. This results in a greater labor share of income (69.1% with greater match efficiency versus 68.1% in our baseline). We find significant output gains of 2.9% and welfare gains of

Variable	A. Baseline	B. Atomistic v	C. $M_j = 20$	D. Atomistic v & $M_j = 20$
HHI_n	0.388	0.429	0.098	0.098
HHI_v	0.228	0.260	0.058	0.058
Vacancy rate	0.082	0.083	0.087	0.086
Labor Share	0.681	0.690	0.671	0.669
EE Rate	0.0049	0.0046	0.0062	0.0062
p90/p10 Wage Ratio	2.558	2.558	2.566	2.568
Avg. p90/p10 within market	2.349	1.982	2.548	2.544
Unemployment Rate	0.077	0.073	0.076	0.077
Labor Productivity vs. Baseline(=1)	1	1.008	1.032	1.032
Cons. Equivalent vs. Baseline(=1)	1	1.000	1.011	1.014
Output vs Baseline(=1)	1	1.012	1.034	1.033

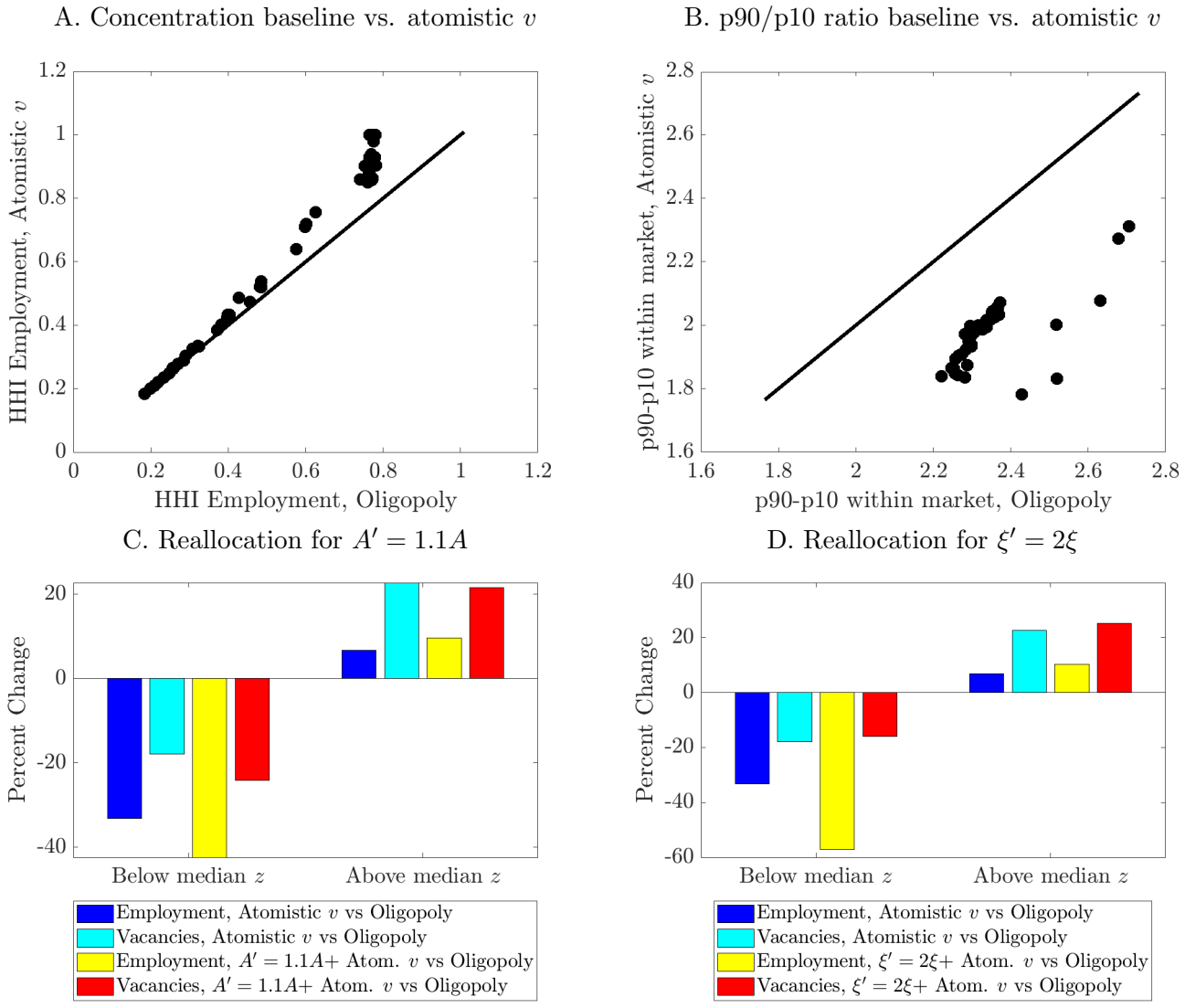
Table 2: Counterfactuals: Comparing competitive versus strategic vacancy posting

Variable	A. $\xi' = 2\xi$	B. Atomistic v & $\xi' = 2\xi$	C. $A' = 1.1 \cdot A$	D. Atomistic v & $A' = 1.1 \cdot A$
HHI_n	0.473	0.518	0.410	0.459
HHI_v	0.223	0.259	0.229	0.272
Vacancy rate	0.088	0.088	0.080	0.080
Labor Share	0.651	0.663	0.691	0.701
EE Rate	0.0059	0.0054	0.0052	0.0047
p90/p10 Wage Ratio	6.172	6.175	2.516	2.520
Avg. p90/p10 within market	3.120	2.175	2.349	1.905
Unemployment Rate	0.099	0.092	0.070	0.066
Labor Productivity vs. Baseline(=1)	1.019	1.026	1.007	1.016
Cons. Equivalent vs. Baseline(=1)	0.988	0.990	1.019	1.019
Output vs Baseline(=1)	0.995	1.010	1.015	1.029

Table 3: Counterfactuals: Competitive limits

1.9%. The yellow and red bars of Panel C of Figure 11 illustrate how grater match efficiency reallocates both employment and vacancies, respectively, to the most productive firms. Relative to our baseline oligopoly economy, employment at above-median-productivity firms increases by 8%, and their vacancy postings increase by 25%.

Figure 11: Counterfactuals: Reallocation patterns versus baseline



8 Conclusions

In this paper, we develop the first general equilibrium theory of dynamic oligopsony. Workers and firms are forward looking, firms are strategic, wages are back-loaded, and worker reallocation occurs through job-to-job transitions. We build on a long tradition of theories of monopsonistic competition with atomistic firms pioneered by Burdett and Mortensen (1998), and we advance the relatively new literature on partial equilibrium models of dynamic oligopsony (e.g. Jarosch, Nimcsik, and Sorkin (2019)). Our closure of the vacancy posting decisions means that our framework has predictions for both quantities (vacancies, employment, job flows) and wages, and it can be used for normative analysis. We show that the model is not only novel, but may also be useful for policy analysis. Using administrative data from Norway, we show that the model is able to match new facts on the relationship between labor market concentration (measured by the employment Herfindahl in 4-digit×commuting zone markets) and both job transitions and wage dynamics. We document significant and robust negative relationships between concentration and job flows, as well as negative relationships between concentration, wage levels, and wage dispersion (measure by the within-market p90/p10 wage differential).

In addition to the questions we explore in this paper, our framework lets researchers ask questions related to the *distributional* consequences of large, strategic, oligopsonistic players in the labor market. As Schmitz (2016) and Herkenhoff and Raveendranathan (2019) emphasize, the dead-weight costs of monopoly are sizable and borne primarily by low income households. We leave these questions to future research.

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Appendix For Online Publication

A Derivation of Value Functions

Nash bargaining when moving to firm k, which happens when $P_k > P_i$, where outside option if staying is $W_i = P_i$:

$$\max_{W_k} (P_k - W_k)^{1-\theta} (W_k - P_i)^{\theta}$$
$$W_k = P_i + \theta [P_k - P_i]$$

We think of the worker value as the value of unemployment plus some share σ of the match surplus $S = P - U$

$$W_k(\sigma) = U + \sigma [P_k - U]$$

Using this

$$\begin{aligned} W_k(\sigma) &= P_i + \theta [P_k - P_i] \\ U + \sigma [P_k - U] &= P_i + \theta [P_k - P_i] \\ \sigma(P_i, P_k) &= \frac{\theta [P_k - U] + (1-\theta)[P_i - U]}{[P_k - U]} \\ \sigma(S_i, S_k) &= \theta + (1-\theta)\frac{S_i}{S_k} \end{aligned}$$

The joint value of worker and firm is as follows, which takes into account the worker value from a new match and the worker value in unemployment:

$$P_i = z_i + \beta \left[P_i + \sum_{k \neq i} \lambda_{ik} \mathbf{1}[P_k > P_i] [W_k(\sigma(P_i, P_k)) - P_i] + \delta (U - P_i) \right]$$
$$P_i = z_i + \beta \left[P_i + \theta \sum_{k \neq i} \lambda_{ik} \max\{S_k - S_i, 0\} - \delta S_i \right]$$

When coming from unemployment the outside option of the worker is U, so $W_i = U + \theta [P_i - U]$, which implies directly that $\sigma = \theta$. The value of unemployment is

$$U = b + \beta \left[U + \sum_k \lambda_{uk} \mathbf{1} [W_k(\theta) > U] [W_k(\theta) - U] \right]$$
$$U = b + \beta \left[U + \theta \sum_k \lambda_{uk} \mathbf{1} [P_k > U] [P_k - U] \right]$$
$$U = b + \beta \left[U + \theta \sum_k \lambda_{uk} \max \{S_k, 0\} \right]$$

We can write this in terms of surplus

$$P_i - U = \left\{ z_i + \beta \left[P_i + \theta \sum_{k \neq i} \lambda_{ik} \max \{S_k - S_i, 0\} - \delta S_i \right] \right\} - \left\{ b + \beta \left[U + \theta \sum_k \lambda_{uk} \max \{S_k, 0\} \right] \right\}$$
$$S_i = z_i - b + \beta \left[S_i + \theta \sum_{k \neq i} \lambda_{ik} \max \{S_k - S_i, 0\} - \delta S_i - \theta \sum_k \lambda_{uk} \max \{S_k, 0\} \right]$$

The worker value

$$W_i(\sigma) = w_i(\sigma) + \beta \Big[W_i(\sigma) + \sum_{k \neq i} \lambda_{ik} \Big\{ \mathbf{1} [P_k > P_i] [W_k(\sigma(P_i, P_k)) - W_i(\sigma)] + \mathbf{1} [W_i(\sigma) < P_k < P_i] [W_i(\sigma(P_i, P_k)) - W_i(\sigma)]$$
$$+ \mathbf{1} [P_k < W_i(\sigma)] [W_i(\sigma) - W_i(\sigma)] \Big\} + \delta [U_i - W_i(\sigma)] \Big]$$
$$W_i(\sigma) = w_i(\sigma) + \beta \Big[W_i(\sigma) + \sum_{k \neq i} \lambda_{ik} \{ \mathbf{1} [P_k > P_i] [\theta P_k + (1-\theta) P_i - W_i(\sigma)] + \mathbf{1} [W_i(\sigma) < P_k < P_i] [P_k - W_i(\sigma)] \}$$
$$+ \delta [U_i - W_i(\sigma)] \Big]$$

Using $W_i(\sigma) = U + \sigma [P_i - U]$ and noting that $W_i(\sigma) - U = \sigma [P_i - U]$

$$W_i(\sigma) = w_i(\sigma) + \beta \left[W_i(\sigma) + \sum_{k \neq i} \lambda_{ik} \max \{0, \min \{\theta P_k + (1-\theta) P_i, P_k\} - W_i(\sigma)\} + \delta [U - W_i(\sigma)] \right]$$
$$W_i(\sigma) = w_i(\sigma) + \beta \left[W_i(\sigma) + \sum_{k \neq i} \lambda_{ik} \max \{0, \min \{\theta [P_k - P_i], P_k - P_i\} + (1-\sigma) [P_i - U]\} - \delta \sigma S_i \right]$$

We can write this in terms of surplus

$$W_i(\sigma) = w_i(\sigma) + \beta \left[W_i(\sigma) + \sum_{k \neq i} \lambda_{ik} \max \{0, \min \{\theta [S_k - S_i], S_k - S_i\} + (1-\sigma) S_i\} - \delta \sigma S_i \right]$$

We also have

$$
\begin{aligned}
W_i(\sigma) &= U + \sigma S_i \\
(1-\beta) W_i(\sigma) &= (1-\beta) U + \sigma (1-\beta) S_i \\
(1-\beta) W_i(\sigma) &= \sigma z_i + (1-\sigma) b + \beta \left[\sigma\theta \sum_{k \neq i} \lambda_{ik} \max\{S_k - S_i, 0\} - \sigma\delta S_i + (1-\sigma)\theta \sum_k \lambda_{uk} \max\{S_k, 0\} \right]
\end{aligned}
$$

Equating these to obtain the wage

$$
\begin{aligned}
& w_i(\sigma) + \beta \left[\sum_{k \neq i} \lambda_{ik} \max\{0, \min\{\theta [S_k - S_i], S_k - S_i\} + (1-\sigma) S_i\} - \delta\sigma S_i \right] \\
& = \sigma z_i + (1-\sigma) b + \beta \left[\sigma\theta \sum_{k \neq i} \lambda_{ik} \max\{S_k - S_i, 0\} - \sigma\delta S_i + (1-\sigma)\theta \sum_k \lambda_{uk} \max\{S_k, 0\} \right]
\end{aligned}
$$

Thus

$$
\begin{aligned}
w_i(\sigma) = \sigma z_i + (1-\sigma) b & \\
& + \beta \Bigg[(1-\sigma)\theta \sum_k \lambda_{uk} \max\{S_k, 0\} \\
& - \sum_{k \neq i} \lambda_{ik} \max\{0, \min\{(1-\sigma)\theta (S_k - S_i), (S_k - S_i)\} + (1-\sigma) S_i\} \Bigg]
\end{aligned}
$$

B Vectorization

Consider the surplus equation in the case that the value of the vacancy is zero if not constituted:

$$
\begin{aligned}
S_i &= z_i - b + \beta \left[S_i + \theta \sum_{k \neq i} \lambda_{ik} \max\{S_k - S_i, 0\} - \delta S_i - \theta \sum_k \lambda_{uk} \max\{S_k, 0\} \right] \\
(1 - \beta(1-\delta)) S_i &= \tilde{z}_i + \beta\theta \sum_{k \neq i} \lambda_{ik} \max\{S_k - S_i, 0\} - \beta\theta \sum_k \lambda_{uk} \max\{S_k, 0\}
\end{aligned}
$$

where we let $\tilde{z}_i = z_i - b$

In the case of $N = 3$ firms

$$\left(\boldsymbol{I}_N - \beta\left(1-\delta\right)\boldsymbol{I}_N\right)\left[\begin{array}{c} S_1 \\ S_2 \\ S_3 \end{array}\right] = \left[\begin{array}{c} \tilde{z}_1 \\ \tilde{z}_2 \\ \tilde{z}_3 \end{array}\right] + \beta\theta\left[\begin{array}{ccc} 0 & 0 & 0 \\ \lambda_{21}\left(S_1 - S_2\right) & 0 & 0 \\ \lambda_{31}\left(S_1 - S_3\right) & \lambda_{32}\left(S_2 - S_3\right) & 0 \end{array}\right]\left[\begin{array}{c} 1 \\ 1 \\ 1 \end{array}\right]$$

$$- \beta\theta\left[\begin{array}{ccc} \lambda_{u1} & \lambda_{u2} & \lambda_{u3} \\ \lambda_{u1} & \lambda_{u2} & \lambda_{u3} \\ \lambda_{u1} & \lambda_{u2} & \lambda_{u3} \\ \lambda_{u1} & \lambda_{u2} & \lambda_{u3} \end{array}\right]\left[\begin{array}{c} S_1 \\ S_2 \\ S_3 \end{array}\right]$$

$$\left(\boldsymbol{I}_N - \beta\left(1-\delta\right)\boldsymbol{I}_N\right)\boldsymbol{s} = \tilde{\boldsymbol{z}} + \beta\theta\left[\begin{array}{c} 0 \\ \lambda_{21}\left(S_1 - S_2\right) \\ \lambda_{31}\left(S_1 - S_3\right) + \lambda_{32}\left(S_2 - S_3\right) \end{array}\right] - \beta\theta\left(\boldsymbol{i}_N \otimes \boldsymbol{\lambda}_u'\right)\boldsymbol{s}$$

$$\left(\boldsymbol{I}_N - \beta\left(1-\delta\right)\boldsymbol{I}_N + \beta\theta\left(\boldsymbol{i}_N \otimes \boldsymbol{\lambda}_u'\right)\right)\boldsymbol{s} = \tilde{\boldsymbol{z}} + \beta\theta\left(\left[\begin{array}{ccc} 0 & 0 & 0 \\ \lambda_{21} & 0 & 0 \\ \lambda_{31} & \lambda_{32} & 0 \end{array}\right] - \left[\begin{array}{ccc} 0 & 0 & 0 \\ 0 & \lambda_{21} & 0 \\ 0 & 0 & \lambda_{31}+\lambda_{32} \end{array}\right]\right)\left[\begin{array}{c} S_1 \\ S_2 \\ S_3 \end{array}\right]$$

The first matrix is Ψ, the second matrix is $diag\left(\Psi \times \boldsymbol{i}_N\right)$:

$$\Psi \times \boldsymbol{i}_N = \begin{bmatrix} 0 & 0 & 0 \\ \lambda_{21} & 0 & 0 \\ \lambda_{31} & \lambda_{32} & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \lambda_{21} \\ \lambda_{31}+\lambda_{32} \end{bmatrix}$$

We can generalize this to N firms as follows:

$$\left(\boldsymbol{I}_N - \beta\left(1-\delta\right)\boldsymbol{I}_N + \beta\theta\left(\boldsymbol{i}_N \otimes \boldsymbol{\lambda}_u'\right)\right)\boldsymbol{s} = \tilde{\boldsymbol{z}} + \beta\theta\left(\Psi - diag\left(\Psi \times \boldsymbol{i}_N\right)\right)\boldsymbol{s}$$

$$\left(\boldsymbol{I}_N - \beta\left(1-\delta\right)\boldsymbol{I}_N + \beta\theta\left(\boldsymbol{i}_N \otimes \boldsymbol{\lambda}_u' - \Psi + diag\left(\Psi \times \boldsymbol{i}_N\right)\right)\right)\boldsymbol{s} = \tilde{\boldsymbol{z}}$$

$$\boldsymbol{s} = \left(\boldsymbol{I}_N - \beta\left(1-\delta\right)\boldsymbol{I}_N + \beta\theta\left(\boldsymbol{i}_N \otimes \boldsymbol{\lambda}_u' - \Psi + diag\left(\Psi \times \boldsymbol{i}_N\right)\right)\right)^{-1} \times \tilde{\boldsymbol{z}}$$

C Competitive Vacancy Posting Algorithm

- The algorithm proceeds as follows
  1. Guess $\boldsymbol{v}_0$
  2. Given $\boldsymbol{v}_0$ guess a stationary $\boldsymbol{n}_0^{(0)}$

(a) Compute meeting rates

$$\begin{aligned}
\tilde{\boldsymbol{v}} &= \left[\left(\boldsymbol{i}_N \otimes \boldsymbol{i}_N'\right) - \boldsymbol{I}_N\right] \boldsymbol{v} \\
\tilde{\boldsymbol{n}} &= \xi \boldsymbol{n} \odot (1/\tilde{\boldsymbol{v}}) \\
\tilde{u} &= \left(1 - \boldsymbol{i}_N' \boldsymbol{n}\right) / \left(\boldsymbol{i}_N' \boldsymbol{v}\right) \\
\boldsymbol{\lambda}_u &= \left[diag\left(\left[\left(\boldsymbol{i}_N \otimes \boldsymbol{i}_N'\right) - \boldsymbol{I}_N\right] \times \tilde{\boldsymbol{n}}\right) + \tilde{u} \times \boldsymbol{I}_N\right]^{-1} \times \left(\boldsymbol{v} ./ \left(\boldsymbol{i}_N' \boldsymbol{v}\right)\right) \\
vec\left(\Lambda\right) &= \left[diag\left(\left[\left(\boldsymbol{i}_N \otimes \boldsymbol{i}_N'\right) - \boldsymbol{I}_N\right] \times \tilde{\boldsymbol{n}}\right) \otimes \boldsymbol{I}_N + \left(\boldsymbol{I}_N \otimes \boldsymbol{I}_N\right) \times \tilde{u}\right]^{-1} \left[\xi \left(\boldsymbol{v} \otimes \boldsymbol{i}_N\right) \oslash \left(\boldsymbol{i}_N \otimes \tilde{\boldsymbol{v}}\right)\right] \\
&\qquad \odot vec\left(\left(\boldsymbol{i}_N \otimes \boldsymbol{i}_N'\right) - \boldsymbol{I}_N\right)
\end{aligned}$$

where we let

$$\begin{aligned}
\boldsymbol{M} &= \left(\boldsymbol{i}_N \otimes \boldsymbol{i}_N'\right) - \boldsymbol{I}_N \\
\Gamma &= diag\left(\left[\left(\boldsymbol{i}_N \otimes \boldsymbol{i}_N'\right) - \boldsymbol{I}_N\right] \times \tilde{\boldsymbol{n}}\right) = diag\left(\boldsymbol{M} \times \tilde{\boldsymbol{n}}\right)
\end{aligned}$$

(b) We then apply aggregate match efficiency

$$\begin{aligned}
\boldsymbol{\lambda}_u &= \Phi \times \boldsymbol{\lambda}_u \\
\Lambda &= \Phi \times \Lambda
\end{aligned}$$

(c) Given Λ and λ_u^f we can compute the stationary distribution of employment by finding the unit Eigen vector of the matrix Π which is computed by the following steps

$$\begin{aligned}
\Psi &= tril\left(\Lambda\right) - diag\left(diag\left(\Lambda\right)\right) \\
\Pi_{in} &= \begin{bmatrix} \Psi' & \boldsymbol{\lambda}_u \\ \delta \times \boldsymbol{i}_N' & 0 \end{bmatrix} \\
\Pi_{out} &= \begin{bmatrix} diag\left(\delta + \left(\Psi \times \boldsymbol{i}_N\right)\right) & 0 \times \boldsymbol{i}_N \\ 0 \times \boldsymbol{i}_N' & \left(\boldsymbol{i}_N' \times \boldsymbol{\lambda}_u^f\right) \end{bmatrix} \\
\Pi &= \boldsymbol{I}_{N+1} + \Pi_{in}\left(\boldsymbol{\lambda}\right) - \Pi_{out}\left(\boldsymbol{\lambda}\right)
\end{aligned}$$

(d) Compute the eigen vector associated with the unit Eigen value of Π. Normalize this such that it sums to 1. Call this $\boldsymbol{n}_0^{(1)}$

(e) Iterate until $\left|\boldsymbol{n}_0^{(k)} - \boldsymbol{n}_0^{(k-1)}\right| < \varepsilon_n$

(f) Compute the consistent $\boldsymbol{\lambda}_u^f$ and Λ^f from the following using

$$\begin{aligned}
vec\left(\Lambda^f\right) &= vec\left(\Lambda\right) \odot \left(\left(\boldsymbol{i}_N \otimes \boldsymbol{n}\right) \oslash \left(\boldsymbol{v} \oslash \boldsymbol{i}_N\right)\right) \\
\boldsymbol{\lambda}_u^f &= \boldsymbol{\lambda}_u \odot \left(u \oslash \boldsymbol{v}\right)
\end{aligned}$$

3. Compute surplus

$$\boldsymbol{s} = \left(\boldsymbol{I}_N - \beta\left(1-\delta\right)\boldsymbol{I}_N + \beta\theta\left(\boldsymbol{i}_N \otimes \boldsymbol{\lambda}_u' - \Psi + diag\left(\Psi \times \boldsymbol{i}_N\right)\right)\right)^{-1} \times \tilde{\boldsymbol{z}}$$

and then set any values of $\boldsymbol{s}$ that are negative to zero: $\boldsymbol{s}\left(\boldsymbol{s} < 0\right) = 0$.

4. Then compute implied vacancies

$$\Psi^f = tril\left(\Lambda^f\right) - diag\left(diag\left(\Lambda^f\right)\right)$$
$$\boldsymbol{v}^* = \left\langle \left[diag\left(\boldsymbol{\lambda}_u^f\right) + \left(diag\left(\left(\Psi^f\right)' \times \boldsymbol{i}_N\right) - \left(\Psi^f\right)'\right)\right]\boldsymbol{s}\right\rangle^{\frac{1}{\gamma-1}}$$

noting that this will give 0 vacancies for the negative surplus firms

5. Update $\boldsymbol{v}^{(1)} = \boldsymbol{v}^{(0)} + \Upsilon_v\left(\boldsymbol{v}^{(1)} - \boldsymbol{v}^{(0)}\right)$
6. Iterate until $\left|\boldsymbol{v}_0^{(k)} - \boldsymbol{v}_0^{(k-1)}\right| < \varepsilon_v$

D Oligopoly Algorithm

- Guess a vector of vacancies: $\mathbf{v}^{*(0)} = \left(v_k^{*(0)}, v_{-k}^{*(0)}\right)$
- Solve for the best-response $v_k^{*(1)}$
  1. For $k = 1$, conjecture $v_k^{*(1)}$
  2. Solve for equilibrium meeting rates
     a Guess stationary distribution of employment $\mathbf{n}_{in} = \left(n_k, n_{-k}\right)$
     b Given $\mathbf{v}$ and $\mathbf{n}$, compute implied meeting rates $\rightarrow \left\{\lambda_{jk},\, \lambda_{jk}^f,\, \lambda_{uk},\, \lambda_{uk}^f\right\} \rightarrow$ surplus S and policy functions
     c Compute implied stationary distribution of employment $\mathbf{n}_{out}$. Iterate until $\mathbf{n}$ converges.
  3. Compute firm objective value using equation (4).
  4. Update guess for $v_k^{*(1)}$. Iterate until maximize firm value.
- Iterate over firms, $k = 2, ..., N$, to obtain $\mathbf{v}^{*(1)}$. Iterate until $\mathbf{v}$ converges.

E Two Firm Example

The solution algorithm for the two firm case is as follows.

1. Compute surplus and surplus shares:

$$S_1 = \frac{z_1 - b}{1 - \beta(1-\delta)}, \qquad \sigma_{u1} = \sigma_{u2} = 0$$
$$S_2 = \frac{z_2 - b}{1 - \beta(1-\delta)}, \qquad \sigma_{12} = \sigma_{21} = \frac{S_2}{S_1}$$

2. Fix (v_1, v_2) on a lattice. For each pair of (v_1, v_2):

   (A.) Guess (n_1, n_2, u) and then compute the following set of equations:

$$x_1 = \xi\phi n_2 + \frac{v_1}{v_1 + v_2}\phi u$$
$$x_2 = \xi\phi n_1 + \frac{v_2}{v_1 + v_2}\phi u$$

$$\lambda_{u1} = \phi\left[\frac{v_1}{v_1 + v_2}\frac{m(x_1, v_1)}{x_1}\right], \qquad \lambda_{12}^f = \lambda_{12}\left(\frac{n_1}{v_2}\right), \qquad n_{u2} = \frac{\delta\lambda_{u2}}{(\delta + \lambda_{21})(\lambda_{u1} + \lambda_{u2} + \delta)}$$
$$\lambda_{u2} = \phi\left[\frac{v_2}{v_1 + v_2}\frac{m(x_2, v_2)}{x_2}\right], \qquad \lambda_{21}^f = \lambda_{21}\left(\frac{n_2}{v_1}\right), \qquad n_{u1} = \frac{\delta\lambda_{u1}}{(\delta + \lambda_{12})(\lambda_{u1} + \lambda_{u2} + \delta)}$$
$$\lambda_{21} = \xi\phi\frac{m(x_1, v_1)}{x_1}, \qquad \lambda_{u2}^f = \lambda_{u2}\left(\frac{u}{v_2}\right), \qquad n_{12} = \frac{\lambda_{u1}\lambda_{12}}{(\delta + \lambda_{12})(\lambda_{u1} + \lambda_{u2} + \delta)}$$
$$\lambda_{12} = \xi\phi\frac{m(x_2, v_2)}{x_2}, \qquad \lambda_{u1}^f = \lambda_{u1}\left(\frac{u}{v_1}\right), \qquad n_{21} = \frac{\lambda_{u2}\lambda_{21}}{(\delta + \lambda_{21})(\lambda_{u1} + \lambda_{u2} + \delta)}$$

   (B.) Update (n_1, n_2, u)

$$u^{update} = \frac{\delta}{(\lambda_{u1} + \lambda_{u2} + \delta)}$$
$$n_2^{update} = n_{u2}$$
$$n_1^{update} = n_{u1} + n_{12} + n_{21}$$

3. Repeat (A.) and (B.) until convergence. This yields a lattice of equilibrium employment and contact rates for each (v_1, v_2).

4. Compute and save the objective function of the firms:

$$V_2(v_1, v_2) = \max_{v_2} n_{u2}\left(1-\sigma_{u2}\right) S_2 + \frac{1}{1-\beta}\left[-c\left(v_2\right) + v_2 \lambda_{u2}^f \left(1-\sigma_{u2}\right) S_2\right]$$

$$V_1(v_1, v_2) = \max_{v_1} n_{u1}\left(1-\sigma_{u1}\right) S_1 + n_{12}\left(1-\sigma_{12}\right) S_1 + n_{21}\left(1-\sigma_{21}\right) S_1 + \frac{1}{1-\beta}\left[-c\left(v_1\right) + v_1\left[\lambda_{u1}^f \left(1-\sigma_{u1}\right) S_1 + \lambda_{21}^f \left(1-\sigma_{21}\right) S_1\right]\right]$$

5. With the lattice of equilibria for each (v_1, v_2), it is possible to compute $v_1^*(v_2)$, the point which optimizes $V_1(v_1, v_2)$ given v_2 as well as $v_2^*(v_1)$, the point which optimizes $V_2(v_1, v_2)$ given v_1.

6. The oligopoly equilibrium is defined by the fixed point $v_1 = v_1^*\left(v_2^*(v_1)\right)$.

F Scaling

To understand the scaling, consider a simple DMP model. Let $\overline{c}v^\gamma$ be the cost a vacancy on the margin, $q\left(\theta\right)$ the matching rate, J the value of the firm, V the measure of vacancies, U the measure of workers, M the measure of firms, and $\overline{N}$ the measure of workers in the labor force. We have the following equilibrium conditions that define a set of 4 equations in 4 unknowns $\{v, V, U, \theta\}$:

$$\begin{aligned}
\overline{c}v^\gamma &= q\left(\theta\right) J \\
V &= Mv \\
U &= \frac{\delta}{\delta + f\left(\theta\right)}\overline{N} \\
\theta &= \frac{V}{U}
\end{aligned}$$

We can write these in terms of the *unemployment rate* $\widetilde{u} = U/\overline{N}$, and the *vacancy rate* $\widetilde{v} = V/\overline{N}$. This yields a set of 4 equations in $\{v, \theta, \widetilde{v}, \widetilde{u}\}$:

$$\begin{aligned}
\overline{c}v^\gamma &= q\left(\theta\right) J \\
\widetilde{v} &= \left(\frac{M}{\overline{N}}\right) v \\
\widetilde{u} &= \frac{\delta}{\delta + f\left(\theta\right)} \\
\theta &= \frac{\widetilde{v}}{\widetilde{u}}
\end{aligned}$$

First note that $v = \overline{N}\widetilde{v}/M$, thus we have:

$$\overline{c}v^\gamma = \overline{c}\left(\frac{\overline{N}}{M}\right)^\gamma \widetilde{v}^\gamma = q\left(\theta\right) J.$$

Scaling Result- *Suppose that the constant of the cost function is*

$$c(v) = \overline{c} v^{\gamma} \quad , \quad \overline{c} = \left(\frac{M}{\overline{N}} \right)^{\gamma}$$

then the full set of equilibrium conditions is independent of M *and* $\overline{N}$*, such that in all markets the unemployment rate* $\widetilde{u} = M/\overline{N}$ *is constant and solves the intersection of the job creation and Beveridge curve conditions:*

$$\begin{aligned} \widetilde{v}^{\gamma} &= q(\theta) J \\ \widetilde{u} &= \frac{\delta}{\delta + f(\theta)} \\ \theta &= \frac{\widetilde{v}}{\widetilde{u}} \end{aligned}$$

G Data Appendix

This section describes the main data sources, key variables and offers some suggestive evidence of the quality of our measures of vacancies and local labor markets.

G.1 Data Sources

Linked Employer-Employee Register. Statistics Norway and NAV jointly maintain the Norwegian Matched Employer-Employee Register, a linked employer-employee database (LEED), covering workers' earnings and transitions between employers. The employer reports the data to tax authorities at the end of the year and includes separate identifiers for the firm and its establishments. The data serves multiple purposes, including third-party tax reporting, as the basis for pension contributions and eligibility for safety net programs.

Wages and Hours. There are some noteworthy changes in the data structure from 2014 to 2015. The reporting system was automated and based on monthly payments after 2015 and covers a slightly broader set of low-paying jobs and more detailed information on hours, bonus, overtime, fixed pay, and variable pay. Before 2015, contracts with fewer than 4 hours per week or below an annual NOK10,000 were not reported. Importantly, for the vast majority of jobs, we observe the dates of alterations to the contract and the corresponding wage, industry and occupational codes, geographic location, and tenure at the establishment in both data sources. Hours reported are reasonably well measured, at brackets from 4 to 19, 19 to 30, and above 30 hours per week as pension contributions depend on them. We classify workers as full-time employees if their weekly hours are at least 30.

Occupations and Workplace Locations. The employment registers report 5-digit occupations. There are about 6,000 different occupations, where some job descriptions have been adjusted from the EU version to meet Norwegian standards and occupational licensing rules. For some positions, the descriptions include information about the rank of the occupation in the hierarchy, e.g., assistants, mid-level managers, top-level management, or members of the executive board.[16]

We use the four-digit version. This version combines industry-variation of certain occupations, such as code 3114, machine engineers, which combines machine engineers in shipbuilding and construction. This version is also a natural definition of the career-ladder for several skilled occupations, such as the 2224 pharmacists, which includes over-the-counter shop assistant, the licensed occupation for handling prescriptions, and senior positions in private pharmacies and clinical and hospital pharmacists.

The tax registers also include the workplace location of the establishment – the municipality employees must be present to perform his or her tasks. There are about 400 municipalities and 19 counties in Norway, none of which represent the natural boundaries of a local labor market. We instead use the information about the residence and workplace to define a commuting zone, which is our preferred definition of a local labor market. We follow Bhuller (2009), who aggregate municipalities into 46 regions allowed to cross counties and combines commuting statistics and natural boundaries, such as mountains and fjords.

Population Registers. Demographic information on workers' comes from longitudinal administrative registers provided by Statistics Norway. This data covers every Norwegian resident from 2006 to 2018 and contains individual residential location, educational background, and demographic information (including sex, age, residential location, spouse, and children).

The national education database (NUDB) includes the highest obtained degree from 1983 to 2018. These files provide information on field (e.g., plumbing, mechanical engineering, nursing), the level (e.g., vocational track in high school, bachelor, master, PhD), and years of schooling. There are about 5,000 educational codes.

Job Seeker Data The job seeker database (SOFA) covers everyone who registers for a service at the employment agency, including part- and full-time unemployed. Job seekers can receive unemployment benefits as fully unemployed and *partially unemployed* which means they must have experienced a reduction in hours of at least 50 percent. Job seekers report their previous occupation and their desired occupation in a new job.

[16]The first digit gives the skill level. 1: CEO/manager/politicians, 2: master degree, 3: bachelor degree, 4: customer relations, 5: sales, health care, and service, 6: agriculture, 7: manual vocational occupations, 8: routine vocational, 9: no skill requirement, 10: military. Occupations 4-8 require 10-12 years of schooling. 1 and 9 and 10 have no formal educational requirements. The second digit gives the field.

Vacancy Data We use two sources of vacancy data. The first is from the employment agency (NAV), that collect individual vacancies are either manually from job boards and help-wanted ads, or are reported directly by employers. Using information from the advertisements or from the hiring firm, the vacancies are then classified by the number of positions the establishment is trying to fill, the workplace location (e.g., zip code) and the corresponding revised version of four-digit occupational code.

Since 2010, Statistics Norway has conducted quarterly surveys of establishments' vacancy posting behavior. These surveys are used primarily for statistical reporting purposes, and are designed as repeated cross-sections, with a sample of around 8,000 representative establishments drawn from the population of registered establishments. The definition of a vacant position in this survey is that it can start within 30 days, recruitment must be from outside the firm, and full-time, part-time, permanent, temporary, and short-term job openings are included.

G.2 Data Quality

We assess quality and representatives of the vacancy data from the employment agency in two ways. Frst, we convert vacancy flows into stocks using the duration of a vacancy, and include data from the online job board FINN.no as an additional comparison. The left panel of Figure 12 shows the aggregate trends of quarterly stocks of vacancies, starting in 2010, from when the survey data are available. The stock of vacancies is divided by the labor force of ages 25 to 66. The aggregate levels displays clear seasonality, where the survey has a distinct peak in the second quarter, while the two other sources have peaks that vary between the first two quarters of the year. Importantly, we see the three sources track each other well over time when the relative differences between the sources are stable over time.[17]

Next, we explore the comparability of the number of vacant positions by the size of an establishment. While 76 percent of establishments have zero postings in a given quarter, the right panel of Figure 12 shows a histogram of establishments with 1 to 25 vacant positions. This sample covers 31 % of all establishment-quarter observations but 92% of all actively recruiting establishments. The histogram shows substantial overlap and no evidence of systematic over- or under-counting from either data source.

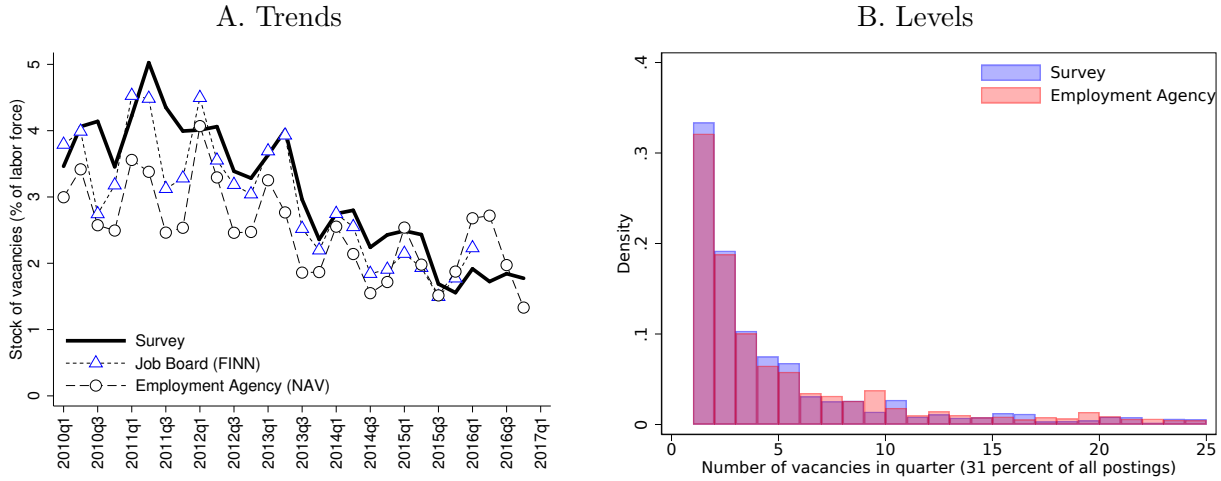
G.3 Combining Occupational Codes

The International Standard Classification of Occupations (ISCO) underwent a major revision due to a resolution at the Meeting of Experts on Labor Statistics in 2007.[18] Statistics Norway and the Norwegian Labour and Welfare Administration (NAV) jointly adopted the international versions, with 354 and 403 unique four-digit occupations from ISCO88 and ISCO08. The Norwegian versions are named STYRK98

[17] The aggregate numbers from the employment agency and FINN are approximately 20 percent below the survey at their respective peaks.

[18] See ILO, "ISCO–08 Volume I: International Standard Classification of Occupations, Structure, group definitions and correspondence tables."

Figure 12: Vacancies from Different Data Sources



Note: This left panel shows the aggregate trend of vacancy rates using three data sources on vacant jobs. The total number of vacancies is divided by the labor force. The right panel compares the quarterly number of positions reported by the employment agency (NAV) and the survey.

and STYRK08. STYRK98 is currently used in the employer-employee register, available from 2003, and STYRK08 is used in vacancy data from the employment agency/NAV and after 2011 in unemployment data. Several occupations were split into smaller groups during the revision, e.g., computer systems designers and computer programmers. Other codes were collapsed into a larger group; for example, many types of machine operators had been made obsolete by technological change. An official version of a two-way correspondence table from STYRK98 to STYRK08 is unavailable.

For our analysis, we need to be able to crosswalk both directions. Employees look for jobs classified in the new version. Similarly, employers must be able to see the potential candidates whose skills are classified by the old version. To create a consistent measure of occupations, we proceed in three steps. First, we identify occupations with an exact match in the two versions. This gives a match of 50 occupations. Second, we identify revised occupations using unemployment periods that overlap with both versions in 2011/2012. To reduce noise from case-worker reporting, we keep 1:1 and 1:many mappings but keep only those with at least 30 percent of each unique STYRK98 code's total. Third, we keep all occupations in the official ISCO correspondence table with 1:1, 1:2, and 2:1 mappings. The remaining occupations are manually identified using the available descriptions of STYRK codes and the ISCO crosswalk table.[19] A final set of consistent occupations depends on the clinked sets of occupations with 1:many and many:1 mappings. The resulting number of occupations is reduced from 403 unique versions to 259 consistent occupations. Our crosswalked occupations can be mapped from either STYRK08 or STYRK98 and are available from this link.

[19]Statistics Norway 1998 (NOS C521); Statistics Norway 2011 (Notater 17/2011)

Table 4: Summary Statistics: Employer to Employer Transitions Between Markets

	A. Commuting	B. Occupation			C. Industry	
	Zone (CZ)	7-digit	4-digit	2-digit	5-digit	2-digit
Moves (%)	0.6	75.5	59.1	42.5	70.9	58.8
Number of markets	46	6705	259	47	781	88
Number of CZ×markets		70,192	9,094	1,444	18,554	3,571
Avg. number of EE		11.5	90.6	578.1	44.1	233.1
Avg. number of employers		1.2	9.8	62.4	4.8	25.2

Note: This table describes the sample of EE transitions. A transition means moving from a full-time job to another full-time job in a new firm. There are 839,987 EE transitions over the period 2006 to 2016 among workers aged 25 to 66. Average number of employers measures all employers over the full period and divide by the number of unique markets.

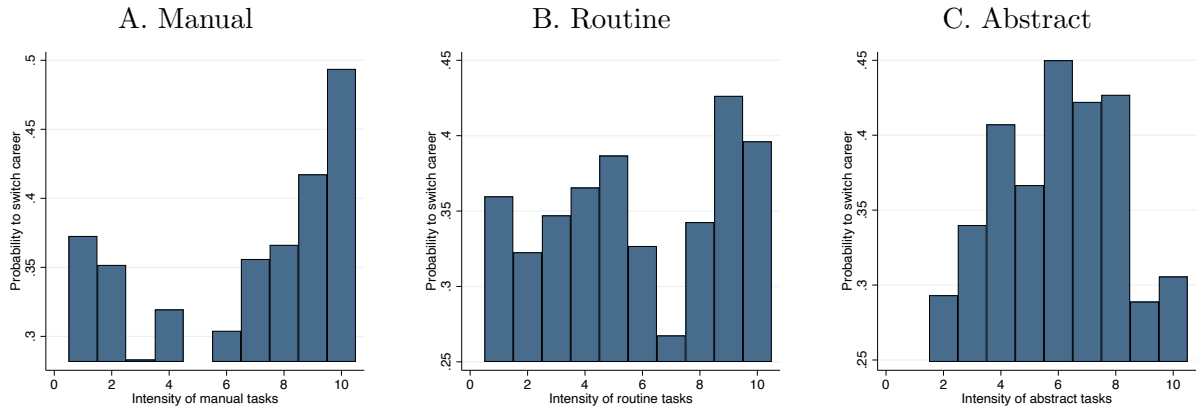
G.4 EE Flows by market

Next, we characterize the type of work a person does by occupation, which we will refer to as a career or occupation interchangeably. In the first column of Panel B, we show that people often switch between the 6,581 job titles. About three out of four moves involve starting a job with a new job title. The relative likelihood ratio of moving to the same job title vs. a random job title is still very high due to the many possible moves. Turning to the collection of jobs defined by a 4-digit occupation, such as "Economics and Business," we see that the probability of moving out of the occupation is lower than the more granular version. On average, 60 percent of the EE moves involves switching career. The relative likelihood of moving to the same occupation over any other occupation is 173.[20] The fourth row shows that the careers are quite well distributed across CZs. Of a maximum 258×46 possible combinations of commuting zone and occupation, 9,094 are observed in our EE transitions. By comparison, two-digit occupations capture a larger fraction of moves, but the likelihood ratio falls as there are only 47 unique occupations.

We choose the commuting zone and 4-digit version as our baseline market definition. This market definition performs relatively well in capturing moves in the data. The number of EE moves and employers per market is about twice as high as the 5-digit industry level and substantially larger than the 7-digit job titles. Using the task scores from Autor, Levy, and Murnane (2003) and Autor and Dorn (2013), Panels A to C in Figure 13 show that workers in more manual occupations are less attached and more likely to move out of their occupations. Consistent with work by Gathmann and Schonberg (2010), who find high-skilled workers are less likely to switch career, we find workers at the lowest and highest deciles of abstract cognitive tasks are more attached to their careers.

[20] The likelihood of moving to any other occupation is 0.6/(258-1) ≈ 0.23 percent.

Figure 13: Probability to Switch Career By Occupational Tasks



Note: These plots the probability of switching occupation when making a job to job transition against the task score from Autor and Dorn (2013). The sample consists of 840 thousand full-time moves from 2006 to 2016.