Green Capital Requirements *

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Abstract

We study the effects of green capital requirements that give preferential capital treatment to clean loans. From a positive perspective, our analysis clarifies the differential effects of green supporting and brown penalizing factors. From a normative perspective, we contrast optimal capital requirements under a classic prudential mandate, which is affected by carbon emissions only through climate-related risks to the banking sector, with those under a broader green mandate that accounts for more general carbon externalities. While climate-related risks that affect bank stability can be optimally addressed by a combination of green supporting and brown penalizing factors, capital regulation is a less effective tool to address carbon externalities that manifest itself outside the banking sector.

Keywords: Green Finance, Bank Capital Regulation, Transition Risks, Physical Risks, European Green Deal, Capital Requirements, Green Supporting Factor, Brown Penalizing Factor, Climate Risk, Socially Responsible Investment.

JEL Classification: G21, G28

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Climate change has become a central topic for central banks and financial regulators. For example, the ECB strategy review dedicates a whole work stream to climate change. A recent report on the future of finance, commissioned by the Bank of England, devotes an entire chapter to the transition to a low-carbon economy.

From the perspective of bank regulators, climate change may be relevant along two potential dimensions. First, as a consequence of climate change, the banking sector could be exposed to financial risks that are not captured by the current regulatory framework. For example, transition risks, which include shifts in consumer preferences or repricing shocks due to future regulation, are forward looking, so that they may not be fully reflected in prudential capital requirements that are mainly based on historical default frequencies. Second, bank regulators may care about the consequences of climate change more broadly. In particular, in the absence of an effective carbon global carbon tax, some policymakers have argued that capital requirements could serve as a means to reduce carbon externalities generated by bank-funded firms. Both of these dimensions have been discussed controversially in the regulatory sphere and beyond.

To shed light on this ongoing debate, we develop a conceptual framework that embeds climate-related risks into an otherwise standard model of bank capital regulation. As a first step, our positive analysis clarifies the differential effects of lowering capital requirements for clean loans (a "green supporting factor") relative to raising capital requirements for dirty loans (a "brown penalizing factor") on equilibrium bank lending. We then provide a normative analysis that characterizes how optimal capital regulation depends on the nature of climate-related risks and the regulatory mandate. In particular, we distinguish between (1) climate-related risks that affect bank stability through their effect on cash flows of bank-funded firms and (2) climate-related risks that materialize outside the banking sector, as well as direct externalities imposed on agents in the economy, including future generations. A regulator with a *prudential mandate* only accounts for risks affecting the financial stability of the banking sector. There is an ongoing debate

regarding the size of climate-related risks for financial stability. In our policy analysis, we therefore consider both small and large climate-related risks. A regulator with a broader *green mandate* accounts for general carbon externalities in addition to financial stability in the banking sector.

Our main results are as follows. Regardless of the regulatory mandate, climate-related risks that affect bank stability can be optimally addressed by a combination of green supporting and brown penalizing factors, thereby rationalizing the proposed policy tools. Perhaps surprisingly, if climate risks are moderate in magnitude, optimal regulation can induce banks to reduce lending to clean firms rather than dirty firms, even if only dirty firms are negatively affected by climate risk. In contrast, optimal policies under the two regulatory mandates differ drastically with respect to carbon externalities imposed on firms or agents outside the banking sector. A prudential regulator does not change its policy in response to such externalities and views dirty lending as desirable as long as it generates surplus under the prudential mandate. In contrast, a regulator with a broader green mandate views lending to dirty firms as socially detrimental if the externalities caused by these loans are sufficiently large. Capital regulation then becomes a tool to incentivize clean and deter dirty lending. Our results show that such policies are generally costly from a prudential perspective and are limited in their effectiveness in deterring dirty lending. We conclude that capital regulation has limited power to address such externalities.

In our model, banks extend loans to a finite mass of two types of borrowers, dirty and clean. As in Oehmke and Opp (2019), there is a tension between financial profitability and social value. Dirty borrowers have access to a more profitable production technology, but they generate significant carbon emissions. Clean borrowers are financially less profitable, but produce lower carbon emissions.[1] Loans to both clean and dirty firms are risky. In particular, with some probability the cash flows generated by both the clean and the dirty

[1] Our framework can be easily adjusted to incorporate many more types, including clean types that also generate high cash flows. The main insights are robust, see Section 5.

firm are low, so that both of them default. In this case, banks cannot repay deposits in full, so that the deposit insurance must step in. Because deposit insurance is not fairly priced, a deposit insurance subsidy arises, distorting banks' investment incentives. Capital requirements then serve to reduce these distortions, a common feature in many models of bank capital regulation following Kareken and Wallace (1978).

We first provide a positive analysis based on exogenous policy interventions (i.e., the interventions are ad hoc, rather than arising from the maximization of a particular objective function). We focus on the two most commonly proposed interventions, the green supporting factor (lower capital requirements for clean loans) and the brown penalizing factor (higher capital requirements for dirty loans), starting from a benchmark with equal capital requirements for clean and dirty loans. Under the baseline of equal capital requirements, banks prefer financing dirty loans due to their higher profitability. Our analysis highlights that raising capital requirements for dirty loans is not equivalent to lowering capital requirements for clean loans. This can be seen by drawing an analogy to income and substitution effects. The substitution effect is similar for both policies as they both increase the *relative* profitability of clean versus dirty loans. Where the policies differ is with regards to the income effect (i.e., the ability to fund loans with a given amount of bank equity capital). A brown penalizing factor is associated with a negative income effect, because it tightens the bank's balance sheet constraints. In contrast, a green supporting factor relaxes the bank's balance sheet constraints and is therefore associated with a positive income effect. The net effect of green tilts to prudential capital requirements depends on the relative size of the income and substitution effects.

Our baseline model with only two types of borrowers isolates income and substitution effect particularly clearly. In this setting, small interventions only generate an income effect. If, from the bank's perspective, dirty loans are more profitable, then a small brown penalizing factor increases the balance sheet space taken up by (inframarginal) dirty loans and, hence, *crowds out* (marginal) clean lending via the income effect. In

contrast, a small green supporting factor generates an income effect in the opposite direction, *crowding in* marginal loans, thereby increasing the number of green projects that receive financing. The substitution effect is at work only once the change in capital requirements is large enough to reverse the banks' profitability ranking of clean versus dirty loans. The substitution effect leads to more funding for clean loans, irrespective of whether the intervention is in the form of a green supporting or brown penalizing factor. These forces carry over into a more general setting with many types, except that now the income and substitution effect will typically both be present.

Building on this analysis of exogenous policy changes, we characterize how optimal capital requirements are affected by (1) financial risks arising in the banking sector as a consequence of climate change and (2) carbon externalities caused by bank funding of dirty firms. We start with an analysis of how to account for climate-related financial risks in the banking sector under a classic prudential regulatory mandate. The prudential regulator's objective is to maximize the NPV generated by bank-funded firms net of deadweight costs arising from the deposit insurance put. The prudential regulator does not care about carbon emissions per se, so that emissions are reflected in capital requirements only insofar as they correlate with the prudential objective. We illustrate optimal prudential capital requirements in the face of climate risk by considering a reduction in profitability or an increase in risk for dirty firms. This scenario captures a broad set of transition risks, including shifts in consumer preferences away from dirty firms or future regulation of dirty technologies. The prudential regulator responds to such risks by increasing capital requirements for dirty loans, sometimes coupled with a decrease in the capital requirement for clean loans. Notably, when climate-related risks for dirty firms are small, it can be optimal for the prudential regulator to increase capital requirements for dirty loans even if this crowds out lending to marginal clean firms. In this case, the prudential regulator does not act to reduce lending to dirty firms, but simply finds it optimal to require more capital for these loans in order to reduce their deposit insurance

put. This prediction changes in the presence of large climate-related risks for banks, in which case the prudential regulator uses capital requirements to change bank's private ranking of loan types and induce them to fund clean firms first.

We then turn to the analysis of externalities imposed by bank-funded dirty firms on parties outside the banking sector. Such externalities include production externalities affecting non-bank funded firms (or otherwise outside the regulator's perimeter) and consumption externalities imposed on agents in the economy, including future generations. While externalities on parties outside the banking sector can be an important concern for overall economic policy, they are unaccounted for by the traditional prudential objective.

Hence, an active debate has arisen whether financial regulators should consider a broader (green) mandate that directly accounts for such externalities caused by carbon emissions. Under such a mandate, sufficiently large carbon externalities associated with dirty lending will cause the regulator to actively seek to deter funding of dirty firms, independent of financial stability considerations. Our analysis reveals that capital requirements are an imperfect tool to achieve this goal. In particular, even at capital requirements of 100%, banks may still find dirty loans financially profitable (at least as long as carbon taxes are not in place). As a result, the only way to reduce the funding of dirty firms is to also subsidize clean lending by lowering capital requirements for clean loans. In doing so, the regulator has to sacrifice financial stability. Therefore, whereas capital requirements are an effective tool protect the banking sector from climate-related risks under the prudential regulatory objective, they are much more limited in addressing carbon externalities caused by bank-funded firms. Reducing carbon emissions via capital requirements is even more difficult when firms have access to non-bank financing (e.g., via the bond market). In this case, it may simply be impossible for the green regulator to reduce carbon emissions. (In contrast, a prudential regulator would welcome substitution to the bond market, because it removes risk from the banking sector.)

Related literature. To the best of our knowledge, our paper is the first to study the

positive and normative implications of climate change for bank capital regulation. Our framework builds on the literature on prudential bank capital regulation.[2] This literature has focused on capital regulation in the presence of distortions introduced by deposit insurance, but has not considered how climate changes affects capital requirements, which is the central focus of our paper. Introducing climate change leads two major departures from this literature. First, climate-related risks (see, e.g., Giglio, Kelly and Stroebel (2021)) become relevant for prudential bank capital regulation insofar as they affect financial risks in the banking sector. Second, climate change may lead to a change in the regulatory objective function to include carbon externalities, in addition to prudential risks in the banking sector. In this respect, our model is related to Thakor (2021), who develops a model of bank capital regulation in which the regulator's objective includes political considerations.[3]

Two recent papers have investigated the positive effects of green capital requirements. Dafermos and Nikolaidi (2021) study green differentiated capital requirements in a dynamic macrofinance model. Thomä and Gibhardt (2019) estimate the effect of green supporting and brown penalizing factors on required bank capital, based on the current composition of bank balance sheets. Both papers do not consider optimal (green) capital requirements.

While our paper focuses on bank capital regulation, Papoutsi, Piazzesi and Schneider (2021) study the environmental impact of central bank asset purchases. While bond purchases affect mainly firms that rely disproportionately on the bond market, bank capital regulation has the strongest effect on bank-dependent firms.

Finally, our paper highlights limits to green capital requirements that arise from

[2] This literature includes, among others, Kareken and Wallace (1978), Rochet (1992), Repullo (2004), Pennacchi (2006), Plantin (2014), Allen et al. (2011), Admati et al. (2011), Bahaj and Malherbe (2020), Malherbe (2020), and Harris et al. (2020).

[3] More broadly, a change away from the classic prudential regulatory objective function is akin to models that study socially responsible investors, whose objective function differs from the classic investor objective of profit maximization. See, e.g., Heinkel et al. (2001), Hart and Zingales (2017), Chowdhry et al. (2018), Oehmke and Opp (2019), Hakenes and Schliephake (2021) and Landier and Lovo (2020).

banks' incentive constraints with respect to funding new (clean or dirty) investments. This effect is reinforced if banks are worried that new green investments will devalue dirty legacy assets on their balance sheets, as pointed out by Degryse, Roukny and Tielens (2020).

1 Model

We consider a model with two dates ($t = 0, 1$), universal risk-neutrality, and no time discounting. There are three types of agents: a continuum of firms with investment opportunities, a continuum of competitive banks, and a regulator who sets capital requirements.

Firms. There is a continuum of cashless firms with total mass normalized to 1. Each firm is of infinitesimal size and born to be one of two types, $q \in \{C, D\}$, which we will refer to as clean and dirty. Firm types are observable. The population fraction of type q is fixed and given by π_q. In Section 5 we discuss the possibility that firms can change their type, i.e., become cleaner, at some cost.

For each type, production requires an investment of fixed scale I at $t = 0$. At date $t = 1$, cash flows are realized. We assume that cash flows for each type q follow a log-normal distribution with parameters μ_q and σ_q, so that the firm's expected cash flow is given by:

$$\overline{X}_q = \exp\left[\mu_q + \sigma_q^2/2\right]. \tag{1}$$

In our baseline model, we assume equal variance parameters, $\sigma_q = \sigma$. Cash flows are perfectly correlated within each type and can have arbitrary correlation across types. Both firm types have positive NPV investment opportunities, $\text{NPV}_q := \overline{X}_q - I > 0$, but dirty firms are more profitable than clean firms,

$$\text{NPV}_D > \text{NPV}_C, \tag{2}$$

reflecting the additional costs incurred by clean firms (e.g., carbon capture). While dirty firms are financially more profitable, they produce higher carbon emissions than clean firms, $\phi_D > \phi_C = 0$ (where we normalize carbon emissions of clean firms to zero). This baseline scenario aims to capture that, at least historically, there has been a tension between profitability and sustainability. We follow Dewatripont and Tirole (2020) in assuming that Pigouvian taxation of carbon emissions is either absent or imperfect (see also Tirole (2012)). In the latter case, the firm's cash flows should be interpreted as already incorporating the effect of carbon taxes.[4]

Our framework is flexible enough to capture scenarios in which firms both cause emissions (thereby contributing to climate change) and are affected by climate risk (in the form of lower expected cash flows $\overline{X}_q$ or higher volatility σ_q).

We will discuss the detailed modeling of such climate risks in Section 4.2, in the context of optimal capital requirements.

Banks. Firms can raise funds for production by obtaining a loan from a continuum (also of mass one) of competitive and ex-ante identical banks. Each bank is endowed with inside equity $E < I$. Because there is a unit mass of banks, E also corresponds to the aggregate amount of equity in the banking sector.[5] To finance assets A, each bank can raise additional deposit funding D from competitive depositors, resulting in the balance sheet identity

$$A = E + D. \tag{3}$$

Bank capital structure matters because the model features two deviations from the Modigliani-Miller benchmark. First, we assume that outside equity issuance is subject to frictions. For ease of exposition, we assume that the associated issuance cost is pro-

[4] Irrespective of the feasibility of directly taxing carbon emissions, the issue of green capital requirement is of significant research interest given that regulators around the globe are actively considering such interventions.

[5] The restriction that $E < I$ therefore rules out the case in which all firms can receive funding even if capital requirements are set to 100% for all loans.

hibitively high, so that bank equity is fixed at E.[6] Second, deposit insurance (or an implicit or explicit bailout guarantee for debtholders) results in an effective subsidy for deposit financing. In our model, deposit insurance is not priced, so that total payouts to bank security holders are increasing in the deposit-to-asset ratio $\frac{D}{A}$. The results would be similar if deposit insurance were priced imperfectly, as in Chan et al. (1992).[7]

The banks' objective is to maximize the value of their equity

$$V = \max_{e,\mathbf{w}} E\left[1 + r_E\left(\mathbf{w},e\right)\right], \tag{4}$$

where we define $e := \frac{E}{A}$ as the bank's equity ratio and $r_E\left(\mathbf{w},e\right)$ as the bank's expected return on equity (ROE), and where $\mathbf{w} = (w_C, w_D)$ denotes the vector of bank loan portfolio weights for clean and dirty loans, respectively. Given that bank equity E is fixed, this objective function boils down to maximizing bank ROE, consistent with what bank managers commonly state as their objective in practice. (Note that in our risk-neutral setting, any ROE exceeding 0 should be interpreted as a scarcity rent rather than a risk premium.)

Bank Regulator. The bank regulator sets loan capital requirements $\underline{e}_q$ as a function of the (observable) firm type q.[8] Given a bank's loan portfolio weights w_q, the bank then faces an equity-ratio constraint of the form

$$e \geq e_{\min}\left(\mathbf{w}\right) := \sum_q w_q \cdot \underline{e}_q. \tag{5}$$

Capital requirements have two effects. First, higher capital requirements reduce trans-

[6] Banks could theoretically pay out part of their equity capital as dividends. However, as we will see below, this will never be optimal for banks under optimal regulation. Our results remain qualitatively unchanged if banks can issue additional outside equity at a positive but non-prohibitive marginal cost.

[7] One may wonder why we assume both a cost of outside equity and a (private) benefit of debt, given that either of these frictions would be sufficient to ensure that banks favor debt financing. The reason is that, in the absence of costly equity issuance, the regulator could simply eliminate bailout distortions by setting capital requirements to 100%. Such an analysis would neither be theoretically interesting nor would it appear to have particular practical relevance.

[8] It is not crucial that firm types are perfectly observable. The main results continue to hold if the regulator observes a noisy signal of firm type (see Section 5).

fers from the deposit insurance fund. When analyzing optimal capital requirements in Section 4, we will follow Farhi and Tirole (2020) by assuming that these transfers are associated with a deadweight cost (e.g., due to a positive shadow cost of public funds). Second, a higher capital requirement for a firm of type q affects banks' loan decisions and, therefore, the mass of funded firms, ω_q.

2 Equilibrium with Exogenous Capital Requirements

We start by characterizing the banking sector equilibrium for exogenously given capital requirements. This analysis will form the basis of our analysis of green tilts to capital requirements in Section 3 and optimal capital requirements in Section 4. The analysis in this subsection draws on Harris et al. (2020), and we therefore present the results in a heuristic fashion.

We first characterize optimal leverage decisions by individual banks and then turn to the equilibrium loan decisions of the banking sector as a whole.

Result 1 *Banks go to the regulatory leverage constraint* $e^* = e_{\min}(\mathbf{w}^*)$ *and find it optimal to either specialize in funding clean or dirty firms.*

Result 1 states that banks maximize the amount of deposit funding and choose specialized portfolios.[9] This is optimal because deposit insurance acts like a subsidy for deposit funding. Given competitive pricing of deposits, the ex-ante value of this deposit insurance put is passed on to bank equityholders.[10]

It is useful to frame the banking sector equilibrium in terms of aggregate bank equity. Bank equity E is the scarce resource in the economy. Therefore, drawing an analogy to demand theory, a firm borrowing from a bank is similar to a consumer, and the

[9] While this prediction is somewhat extreme, specialization is key for tractability (in particular, closed-form solutions). All main insights carry through to the case in which banks' portfolios are not specialized.

[10] Bank equityholders, in turn, may pass on some of this subsidy to firms via the pricing of loans.

relevant consumption good is units of bank equity (i.e., space on the bank's balance sheet). Specifically, when a firm of type q demands a loan of size I, this translates into demand for $I\underline{e}_q$ units of bank equity.

Given objective function (4), banks rank borrowers according to the maximum ROE associated with loan to a borrower, which is determined by the maximum interest rate the borrower is willing to pay. As in standard demand theory, the demand curve is then characterized by reservation prices, in the form of the maximum return on equity that a borrower can offer to a bank.

Result 2 *At the maximum interest rate that a borrower of type q is willing to pay, the bank equityholders' expected ROE is given by*

$$r_q^{\max}\left(\underline{e}_q\right) = \frac{NPV_q + PUT_q\left(\underline{e}_q\right)}{I\underline{e}_q}, \tag{6}$$

where $PUT_q\left(\underline{e}_q\right)$ denotes the contribution of the loan to the bank's deposit insurance put.

$$PUT_q\left(\underline{e}_q\right) = \mathbb{E}\left[\max\left\{I(1-\underline{e}_q) - X_q(s), 0\right\}\right]. \tag{7}$$

At the borrower's reservation interest rate, all expected surplus generated by the loan accrues to bank equityholders.[11] This surplus consists of the NPV of the firm's project and the value of the deposit insurance put associated with the loan under optimal (maximum) leverage, see Result 1. Given the log-normal cash flow distribution, the value of the deposit insurance put can be readily determined using the Black-Scholes formula (Merton (1977)).

Because banks behave competitively, they typically cannot extract all surplus from borrowers. Instead, the equilibrium return on bank equity r_E^* is pinned down by the

[11] If borrowers had access to non-bank financing, say via a competitive bond market, then this outside option would pin down the maximum interest rate the borrower is willing to pay for a bank loan (see Section 5 for a discussion). For simplicity, in our baseline model firms are bank-dependent. Therefore, the outside option is not to invest at all and therefore equal to zero.

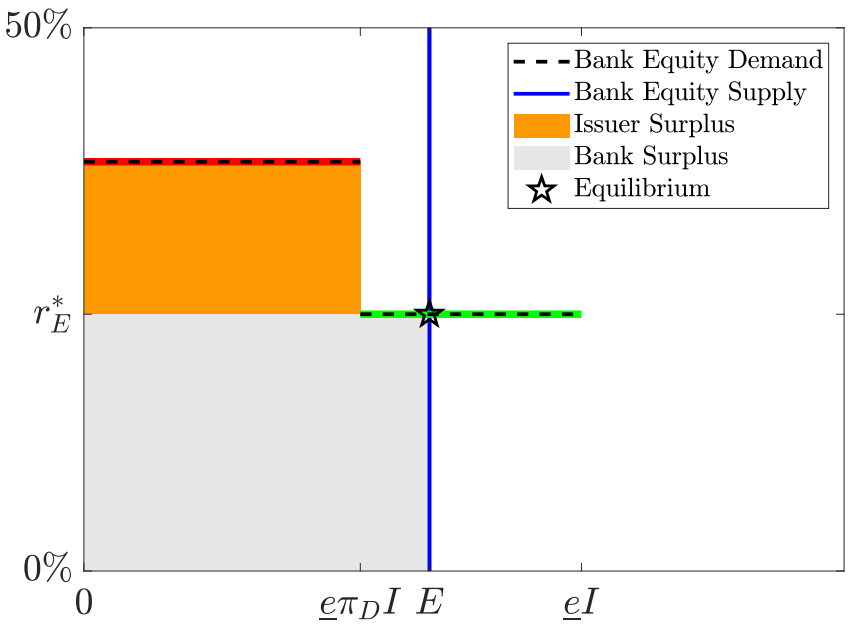


Figure 1. Banking Sector Equilibrium. This figure illustrates the banking sector equilibrium for equal capital requirements of $\underline{e} = 16\%$ for both firm types. The equilibrium ROE is denoted by r_E^*. Dirty firms are depicted in red, clean firms in green.

intersection of the aggregate demand for bank equity (from funded loans) and its (fixed) supply. The resulting equilibrium is illustrated in Figure 1. Given two borrower types with different reservation prices, the demand curve is a step function. In the illustrated equilibrium, dirty (red) borrowers are fully funded (they are inframarginal), whereas borrowers of the clean (green) type are only partially funded (they are marginal). Since both types are funded in equilibrium, Result 1 implies that a subset of banks will specialize in funding all dirty firms and the remaining banks will finance exclusively clean firms. The loan rate for the marginal green borrowers is set such that all surplus accrues to banks (i.e., there is no consumer surplus for marginal loans). Inframarginal borrowers, on the other hand, obtain some consumer (or "issuer") surplus, which ensures that banks are indifferent between funding either type. More generally, we obtain

Result 3 *All borrowers with $r_q^{max} > r_E^*$ are fully funded by banks. Marginal borrower types, satisfying $r_q^{\max} = r_E^*$, are partially funded. The banking sector's equilibrium ROE*

satisfies:

$$r_E\left(\mathbf{w}^*, e_{\min}\left(\mathbf{w}^*\right)\right) = r_E^*. \tag{8}$$

This result highlights the importance of the marginal borrower type, which pins down r_E^* and, hence, the funding terms and loan allocation to all inframarginal types $r_q^{\max} > r_E^*$. Which type is marginal depends not only on exogenous firm characteristics (such as the firm's NPV, and the capitalization of the banking sector) but also on the regulator's choice of capital requirements.

3 Green Tilts to Capital Requirements

In this section, we investigate the effects of (exogenous) green tilts to capital requirements. Green tilts can take the form of a reduction in the capital requirement for clean loans (a *green supporting factor*) or an increase in capital requirement for dirty loans (a *brown penalizing factor*.) For expositional clarity, we consider a benchmark policy regime with equal capital requirements for dirty and clean loans ($\underline{e}_C = \underline{e}_D = \underline{e} < 1$) and study the effects of green tilts relative to this benchmark. While policy interventions analyzed in this section are exogenous, the positive analysis of this section will help clarify the economic forces that are at play, and will therefore continue to be relevant when we derive optimal capital requirements in Section 4.

In the benchmark equilibrium with equal capital requirements, loans to dirty firms rank strictly higher in the aggregate demand curve (i.e., $r_D^{\max}(\underline{e}) > r_C^{\max}(\underline{e}) > 0$), because dirty firms are financially more profitable than clean firms, as illustrated in Figure 1. Accordingly, if bank capital is very scarce ($E < \pi_D \underline{e} I$), the marginal borrower type is dirty and no clean loans are funded. For intermediate levels of bank capital ($\pi_D \underline{e} I < E < \underline{e} I$), all dirty firms are funded, and the marginal loan is extended to a clean firm (the case presented in Figure 1). Finally, in the (somewhat less interesting) case of abundant bank capital, $E > \underline{e} I$, both dirty and clean firms are fully funded.

To analyze the effects of green tilts to capital requirements, it is instructive to interpret a change in the capital requirement for one type of loan as a change in the relative prices of bank balance sheet space for clean and dirty loans. Therefore, in analogy to standard demand theory, we can characterize the effects of green tilts in terms of *income* and *substitution effects*. The substitution effect captures that a green tilt reduces the *relative* price of providing green loans, irrespective of whether the green tilt consists of a green supporting or a brown penalizing factor. However, the two policies differ with respect to the income effect. Whereas a brown penalizing factor constrains bank balance sheet space, akin to a negative income effect, a green supporting factor frees up balance sheet space. In our setting, the relative magnitude of income and substitution effects is governed by the size of the intervention. Since dirty loans rank strictly higher under equal capital requirements, marginal changes to capital requirements do not alter the ranking of borrowers. Therefore, small interventions work solely through the income effect and only affect lending to the marginal borrower type. In contrast, large interventions that change the ranking of borrowers induce a substitution effect (in addition to the income effect).

3.1 Brown Penalizing Factor

To consider the effects of a brown penalizing factor, it is useful to start with a small intervention (i.e., a marginal increase in the capital requirement for dirty loans relative to the benchmark with equal capital requirements). The case of intermediate bank capital $E \in (\pi_D \underline{e} I, \underline{e} I)$, plotted in the left panel of Figure 2, is most interesting. In this case, a marginal increase in the cost of lending to dirty firms leads to crowding out of lending to clean firms. Relative to the benchmark (dashed red line), a higher capital requirement for dirty loans reduces their attractiveness and results in a downward shift in the dirty borrower's reservation price (from the dashed to the solid red line), without inducing a change in the ranking of types. In addition, because funding the same number of

dirty loans now requires more bank equity, the dirty-loan segment of the demand curve lengthens. As a result, less equity is left over to fund clean loans (a rightward shift of the dashed to the solid green line), resulting in a reduction in lending to clean firms.

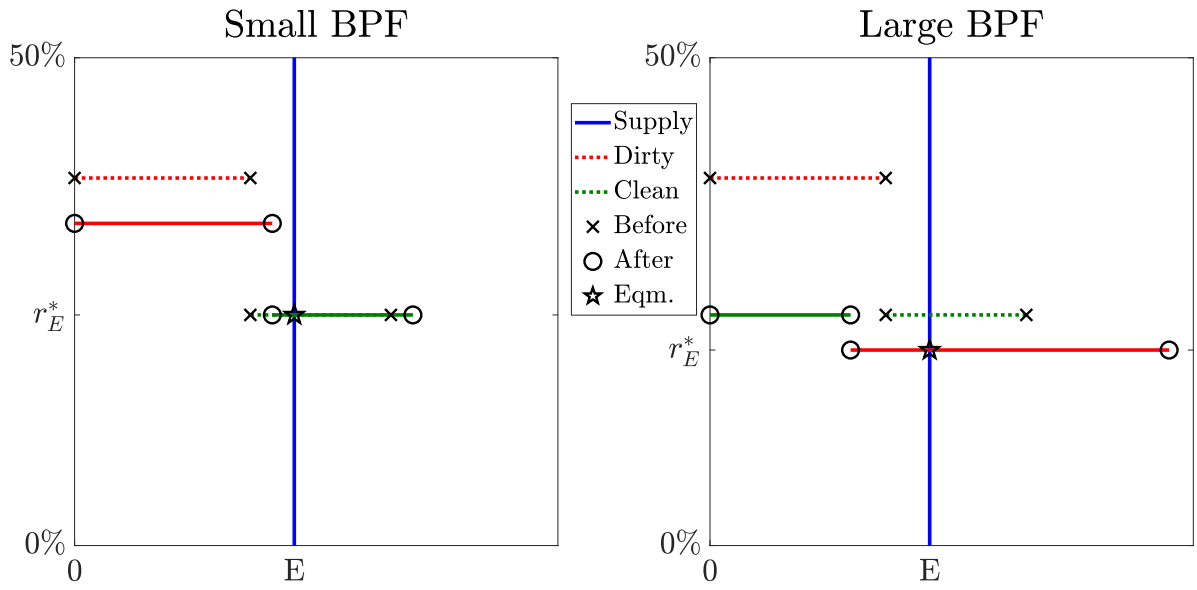


Figure 2. Brown penalizing factor. The figure illustrates the effect of introducing a brown penalizing factor starting from the benchmark of equal capital requirements of $\underline{e} = 16\%$. The left panel illustrates the equilibrium impact of a small brown penalizing factor that leaves the relative ranking of firm types unchanged. The right panel illustrates the equilibrium impact of a large brown penalizing factor that reverses the relative ranking of firm types. Dotted lines and segment endpoints marked $\circ$ denote the benchmark equilibrium. Solid lines and segment endpoints marked $\times$ denote the equilibrium after the introduction of the brown penalizing factor.

If the brown penalizing factor is sufficiently large, the ranking of clean and dirty loans in terms of the borrower reservation price is reversed, so that $r_C^{\max} > r_D^{\max}$ post intervention. This case is illustrated in the right panel of Figure 2. In this case, the substitution effect implies that banks first exhaust all clean lending opportunities before they start funding of dirty firms. Therefore, clean lending increases and dirty lending decreases. This result is driven by both the substitution effect (clean loans get funded first) and the income effect (the lengthening of the dirty-loan segment of the demand curve).

We summarize the effects of the introduction of a brown penalizing factor in Proposition 1. Part 1 of the proposition characterizes the effects of a marginal brown penalizing

factor (including the cases of very scarce or abundant bank capital, which we omitted for brevity in the discussion above). Part 2 of the proposition characterizes the effects of a large brown penalizing factor.[12]

Proposition 1 (Brown Penalizing Factor) *Relative to a benchmark with equal capital requirements for clean and dirty loans:*

1. *The effect of a marginal increase in capital requirements for dirty loans (marginal BPF) depends on the capitalization of the banking sector:*

   (a) *If* $E < \pi_D \underline{e} I$*, a marginal BPF only reduces lending to dirty firms.*

   (b) *If* $E \in (\pi_D \underline{e} I, \underline{e} I)$*, a marginal BPF only reduces lending to clean firms.*

   (c) *If* $E > \underline{e} I$*, a marginal BPF does not affect lending.*

2. *If the increase in capital requirements for dirty firms exceeds a cut-off* $\Delta_{BPF} > 0$*, characterized by* $r_D^{\max}(\underline{e} + \Delta_{BPF}) = r_C^{\max}(\underline{e})$*, lending to clean firms increases whereas lending to dirty firms decreases.*

3.2 Green Supporting Factor

We now turn to the introduction of a green supporting factor. Mirroring the analysis of the brown penalizing factor, we initially consider the introduction of a small green supporting factor.

As before, the intermediate equity case, $E \in (\pi_D \underline{e} I, \underline{e} I)$, is the most interesting. In this case, a small decrease in the capital requirement for clean loans unambiguously increases funding of clean firms, as illustrated in the left panel of Figure 3. While the dirty-loan segment of the demand curve is unchanged, the clean-loan segment shifts

[12] To reduce the number of cases in part 2 of the proposition, we assume that the reservation price for clean loans exceeds the reservation price for dirty loans when the capital requirement for dirty loans is raised to 100%, i.e., $r_D^{\max}(1) < r_C^{\max}(\underline{e})$. This assumption is satisfied if the mean productivity of dirty firms μ_D is sufficiently close to that of clean firms μ_C.

upward (albeit without changing the relative ranking of borrower reservation prices) and shortens. The upward shift reflects that clean loans become relatively more attractive in terms of their reservation price, whereas the shortening of the clean-loan segment reflects that each clean loan now requires less capital. As long as the ranking of reservation prices does not change, the equilibrium effect is driven entirely by the income effect (i.e., the shortening of the clean-loan segment). Therefore, some previously unfunded clean firms are now able to obtain funding. The funding of dirty firms is unaffected.

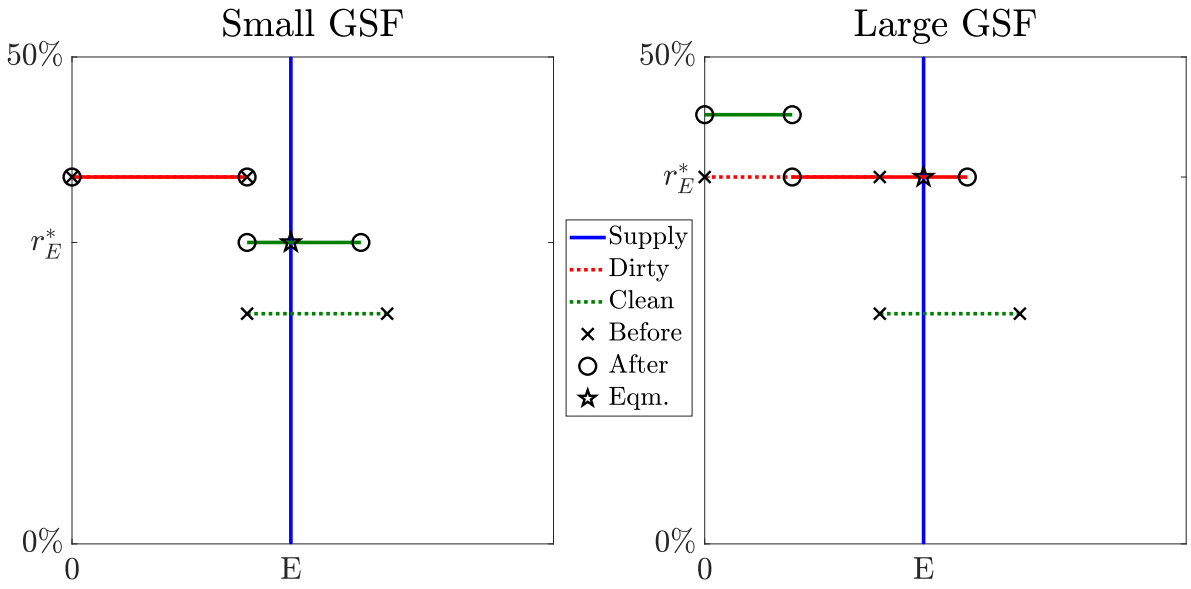


Figure 3. Green supporting factor. The figure illustrates the effect of introducing a green supporting factor starting from the benchmark of equal capital requirements $\underline{e} = 16\%$. The left panel illustrates the equilibrium impact of a small green supporting factor that leaves the relative ranking of firm types unchanged. The right panel illustrates the equilibrium impact of a large green supporting factor that reverses the relative ranking of firm types. Dotted lines and segment endpoints marked $\circ$ denote the benchmark equilibrium. Solid lines and segment endpoints marked $\times$ denote the equilibrium after the introduction of the green supporting factor.

Consider now a green supporting factor that is large enough to reverse the relative ranking of clean and dirty loans, so that $r_C^{\max} > r_D^{\max}$ post intervention. This case is illustrated in the right panel of Figure 3. For clean firms, the income and the substitution effect both push towards strictly more financing. The clean-loan segment of the demand curve shifts upward and shortens, so that after the intervention all clean loans are funded. For dirty loans, the income and substitution effects work in opposite directions. The

dirty-loan segment shifts to the right (substitution effect) but this shift is attenuated by the shortening of the clean-loan segment (income effect). After introduction of the large green supporting factor, dirty loans take up the balance sheet space that remains after all clean loans have been funded.

We summarize the effects of a green supporting factor, including the cases of very scarce or abundant bank capital, in Proposition 2.

Proposition 2 (Green Supporting Factor) *Relative to a benchmark with equal capital requirements for clean and dirty loans:*

1. *The effect of a marginal decrease in capital requirements for clean loans (marginal GSF) depends on the capitalization of the banking sector:*

   (a) *If* $E \in (\pi_D \underline{e} I, \underline{e} I)$*, a marginal GSF increases lending to clean firms.*

   (b) *A marginal GSF has no effect on bank lending otherwise.*

2. *If the decrease in capital requirements for clean firms exceeds a cut-off* $\Delta_{GSF} > 0$*, characterized by* $r_D^{\max}(\underline{e}) = r_C^{\max}(\underline{e} - \Delta_{GSF})$*, lending to clean firms increases whereas lending to dirty firms decreases, strictly so if not all firms are financed post intervention.*

In sum, Propositions 1 and 2 show that, for small interventions, brown penalizing and green supporting factors have quite different effects, driven by opposite income effects. For large interventions, on the other hand, the substitution effect becomes dominant, so that their effects are qualitatively similar. However, quantitative differences remain. Comparing lending to dirty firms in the respective right panels of Figures 2 and 3 reveals that a much larger fraction of dirty firms is funded in the case of a large green supporting factor. This is a consequence of the positive income effect associated with the green supporting factor.

Many types. The above analysis has focused on a framework with only two types of homogeneous firms. This discrete setup is helpful in isolating the relevant economic forces in a particularly tractable manner. However, our framework can be adapted to account for any number of types. As in our two-type setup, the banking sector would then rank firm types q according to their reservation price $r_q^{\max}$. The key difference is that income and substitution effects will now typically both be present. The key message remains that changes to capital requirements of infra-marginal borrower types will crowd out lending to the marginal borrower type. For example, if the economy is populated by clean and dirty firms, each with a distribution of productivity levels, a small change to capital requirements will affect both a fraction of clean and a fraction of dirty borrowers. In our two-type setup, we obtain the extreme (binary) corner cases, in which crowding out either affects only clean or only dirty firms.

Figure 4 illustrates a setting with a finite (but large) number of types, where the productivity of clean and dirty firms μ_q is drawn from a normal distribution, retaining the assumption that dirty firms are more profitable on average. Note that, given random productivity draws, some firm ROE's are negative. These firms are never financed, irrespective of the amount of equity in the banking system.

The upper panel plots the resulting equilibrium for equal capital requirements. In contrast to the baseline model, the demand function is now (approximately) continuous. Because dirty firms are on average more productive, under equal capital requirements they are located mainly on the high-ROE segment of the demand curve. The lower panel illustrates the effect of introducing a brown penalizing factor. Relative to the original equilibrium (retained in black for comparison), the demand curve rotates and lengthens. Moreover, dirty firms look less attractive on average, allowing more clean firms to move into the funded region to the left of the supply (this can be seen from the fraction of dirty firms in each firm decile, displayed on the x-axis). In the illustrated example, the substitution effect dominates, so that the brown penalizing factor increases credit to clean

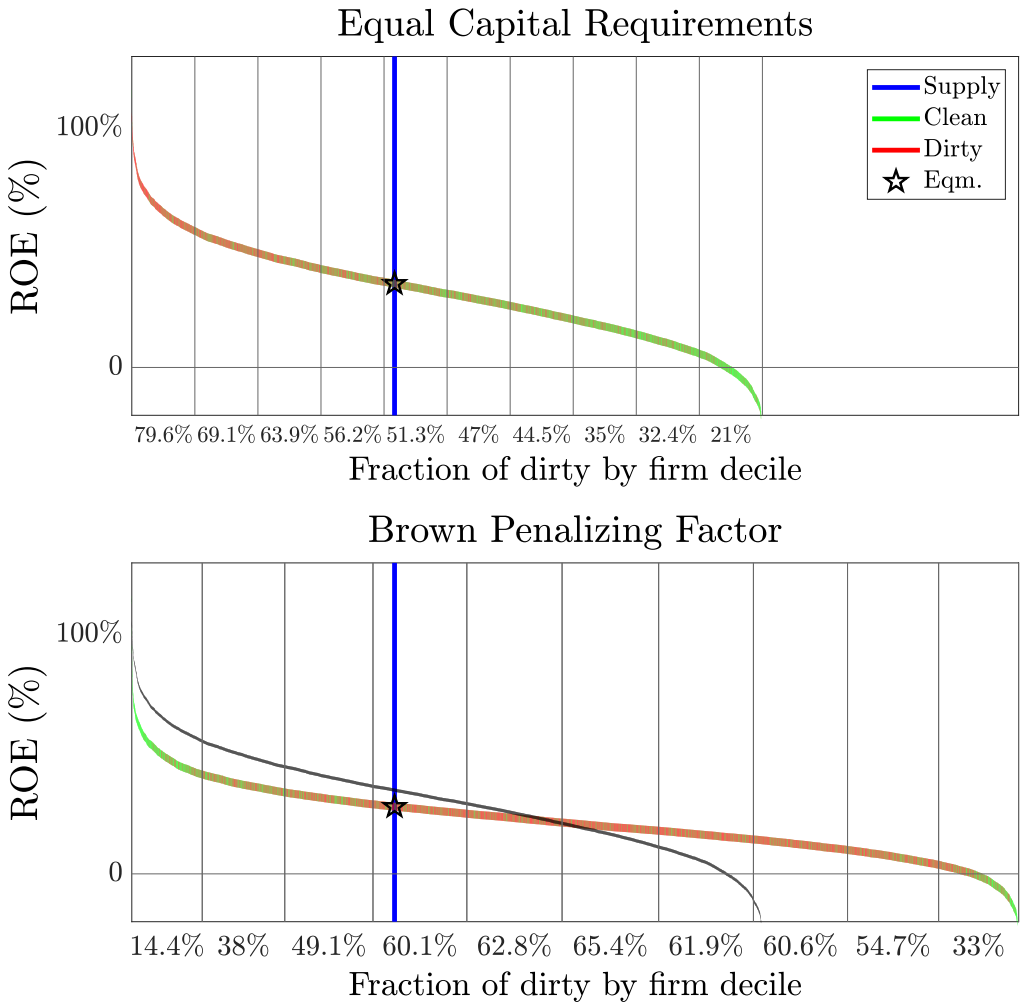


Figure 4. Many types. This figure illustrates an extension of our baseline model, in which the productivity of clean and dirty firms μ_q is drawn from a normal distribution, retaining the assumption that dirty firms are more profitable on average. The top panel illustrates the equilibrium for equal capital requirements of 16% for clean and dirty firms. The bottom panel illustrates the effect of introducing a brown penalizing factor, raising the capital requirement for dirty loans to 29% (this corresponds to the large BPF in Figure 2). In this example, the substitution effect dominates, so that the brown penalizing factor increases credit to clean firms and reduces credit to dirty firms.

firms and reduces credit to dirty firms.

4 Optimal Capital Requirements

Up to now, our analysis has focused on two specific interventions, the brown penalizing and the green supporting factor, starting from a benchmark equilibrium with exogenous

symmetric capital requirements for clean and dirty loans. In this section, we first characterize optimal prudential capital requirements in the absence of climate-related risks. This benchmark can be interpreted as regulation that is calibrated using backward-looking data, during a time period in which climate-related risks have not played a relevant role.

We then analyze how different categories of climate-related risks affect optimal capital requirements under the *prudential mandate*. Given that climate change has broader effects (beyond the banking sector) we will also consider a regulator with a broader *green mandate* that additionally accounts for effects of climate change outside the banking system. Whether such a green mandate is warranted is currently discussed controversially in the regulatory sphere.

4.1 The Principles of Prudential Regulation

The prudential regulator trades off the financial value (or NPV) created by bank lending against the deadweight costs generated by deposit insurance. For simplicity, we assume that the deadweight cost of the deposit insurance put is linear in the size of the fiscal transfer to the banking sector, reflecting a constant marginal cost of public funds λ. The regulator's objective function is therefore

$$\max_{\underline{\mathbf{e}}} \Omega_P = \max_{\underline{\mathbf{e}}} \sum \omega_q \left(\underline{\mathbf{e}}\right) \left[\mathrm{NPV}_q - \lambda \cdot \mathrm{PUT}_q(\underline{e}_q)\right], \tag{9}$$

where the notation $\omega_q\left(\underline{\mathbf{e}}\right)$ and $\mathrm{PUT}_q(\underline{e}_q)$ highlights the dependence of bank funding decisions and the deposit insurance put on the capital requirements for clean and dirty firms, $\underline{\mathbf{e}} = (\underline{e}_C, \underline{e}_D)$.

To characterize optimal prudential capital requirements, it is instructive to rewrite

the regulator's objective function as

$$\max_{\underline{\mathbf{e}}} \Omega_P = E \max_{\underline{\mathbf{e}}} \sum \tilde{\omega}_q \left(\underline{\mathbf{e}}\right) \mathrm{PPI}_q(\underline{e}_q), \tag{10}$$

where $\tilde{\omega}_q \in [0,1]$ reflects the fraction of total bank equity that the banking sector allocates to funding type q, and where $\mathrm{PPI}_q(\underline{e}_q)$ denotes the *prudential profitability index*. In analogy to the banker's maximal ROE given in equation (6), the PPI reflects the surplus created per unit of bank equity as seen from the prudential regulator's perspective,

$$\mathrm{PPI}_q(\underline{e}_q) = \frac{\mathrm{NPV}_q - \lambda \cdot \mathrm{PUT}_q(\underline{e}_q)}{I\underline{e}_q}. \tag{11}$$

Comparing equations (6) and (11), we see that there are two main differences between the regulator's PPI and the bankers' maximal ROE. First, the deposit insurance put enters with opposite sign, reflecting the wedge between prudential preferences and those of the banking sector. Second, while banks take ROEs as given, the regulator internalizes that the PPIs for each type are affected by the chosen capital requirement.

We assume that the cost of public funds is sufficiently high, $\lambda > \max_q \frac{\mathrm{NPV}_q}{\mathrm{PUT}_q(0)}$, which ensures that the PPI is bounded above for each type.[13] Then the capital requirement that maximizes the PPI for type q, $\underline{e}_q^{\mathrm{PPI}}$, satisfies the first-order condition

$$\frac{1}{\underline{e}_q}\left[\mathrm{NPV}_q - \lambda \mathrm{PUT}_q\right] = \lambda \left|\frac{\partial \mathrm{PUT}_q}{\partial \underline{e}_q}\right|. \tag{12}$$

The left-hand side captures the marginal benefit of lowering capital requirements. More firms can be financed, resulting in additional prudential surplus. The right-hand side captures the marginal (social) cost of lower capital requirements, in the form of a higher deposit insurance put.

From the regulator's perspective, a borrower with a higher PPI delivers more "bang

[13] If $\lambda < \frac{\mathrm{NPV}_q}{\mathrm{PUT}_q(0)}$, the PPI approaches positive infinity as the capital requirements approach zero.

for the buck" (prudential value per unit of equity capital) and is therefore preferred. As shown in Proposition 3 below, the PPI plays an important role for the characterization of optimal prudential capital regulation.

Definition 1 *The regulator's preferred type is the one that achieves the highest possible PPI, i.e.,* $\max_q PPI_q(\underline{e}_q^{PPI})$.

In the baseline case, $\sigma_C = \sigma_D = \sigma$ and $\mu_D > \mu_C$, the prudential regulator's preferred type (highest PPI) is the dirty type D. This happens for two reasons. First, dirty loans create larger financial surplus than clean loans, $\text{NPV}_D > \text{NPV}_C$. Second, given equal variance parameters σ, for any capital requirement $\underline{e}$ dirty loans induce a smaller expected transfer from the deposit insurance fund, $\text{PUT}_D(\underline{e}) < \text{PUT}_C(\underline{e})$.

Proposition 3 (Principles of Optimal Prudential Regulation) *Optimal prudential regulation is characterized by the following four principles.*

P1: All bank equity is used to fund loans,

$$\sum_q \omega_q(\underline{\mathbf{e}})\,\underline{e}_q I = E. \tag{13}$$

P2: For sufficiently low levels of bank equity, $E \leq \pi_D \underline{e}_D^{PPI} I$*, the regulator induces banks to lend to its preferred type* D*.*

P3: If firm type q *is partially funded (there is at most one such type), its capital requirement maximizes* PPI_q*,*

$$e_q^* = \underline{e}_q^{PPI}. \tag{14}$$

P4: If both firm types are funded, marginal deposit-insurance puts are equalized across types,

$$\frac{\partial PUT_D}{\partial \underline{e}_D} = \frac{\partial PUT_C}{\partial \underline{e}_C}. \tag{15}$$

Principle P1 states that the equity of the banking sector is fully exhausted. This means that, under optimal prudential regulation, banks do not find it optimal to pay out dividends. Intuitively, this principle helps mitigate the deadweight costs arising from insured deposit financing. Principle P2 says that the first funded type is the regulator's preferred type D. This can always be made incentive compatible for banks by setting a sufficiently high capital requirement for the (unfunded) clean type. Principle P3 states that the optimal capital requirement for the marginally funded type maximizes the PPI, $e_q^* = \underline{e}_q^{\text{PPI}}$. Finally, Principle P4 links capital requirements of inframarginal borrowers with those of marginal borrowers. Specifically, optimality condition (15) requires that the marginal reductions in the deposit insurance put resulting from higher capital requirements are equalized across funded types.

Based on these four principles, Figure 5 highlights four distinct regions linking optimum prudential capital regulation to the capitalization of the banking sector E. The lower panel plots optimal prudential capital requirements and the upper panel plots the corresponding funding decisions ω of the banking sector. As can be seen, capital requirements are generally increasing in the capitalization of the banking sector E, which can be interpreted as procyclical capital requirements or, equivalently, countercyclical capital buffers.

For sufficiently scarce equity, $E < E_1 := \pi_D \underline{e}_D^{\text{PPI}} I$, only the regulator's preferred (dirty) type is funded. Since the dirty type is partially funded, $\omega_D < \pi_D$ (see upper panel), Principle P3 applies and the optimum prudential capital requirement maximizes PPI_D, $\underline{e}_D^* = \underline{e}_D^{\text{PPI}}$. To ensure that banks find it incentive compatible to exclusively lend to dirty firms, capital requirements for the (unfunded) clean type must be set sufficiently high so as to ensure that $r_C^{\max}(\underline{e}_C^*) \leq r_D^{\max}(\underline{e}_D^*)$. This incentive constraint imposes a lower bound on the capital requirement for clean loans, which is illustrated by the dotted green line. As equity in the banking sector increases, the mass of funded dirty firms increases accordingly, as illustrated in the upper panel of Figure 5. Once $E = E_1$, all dirty firms

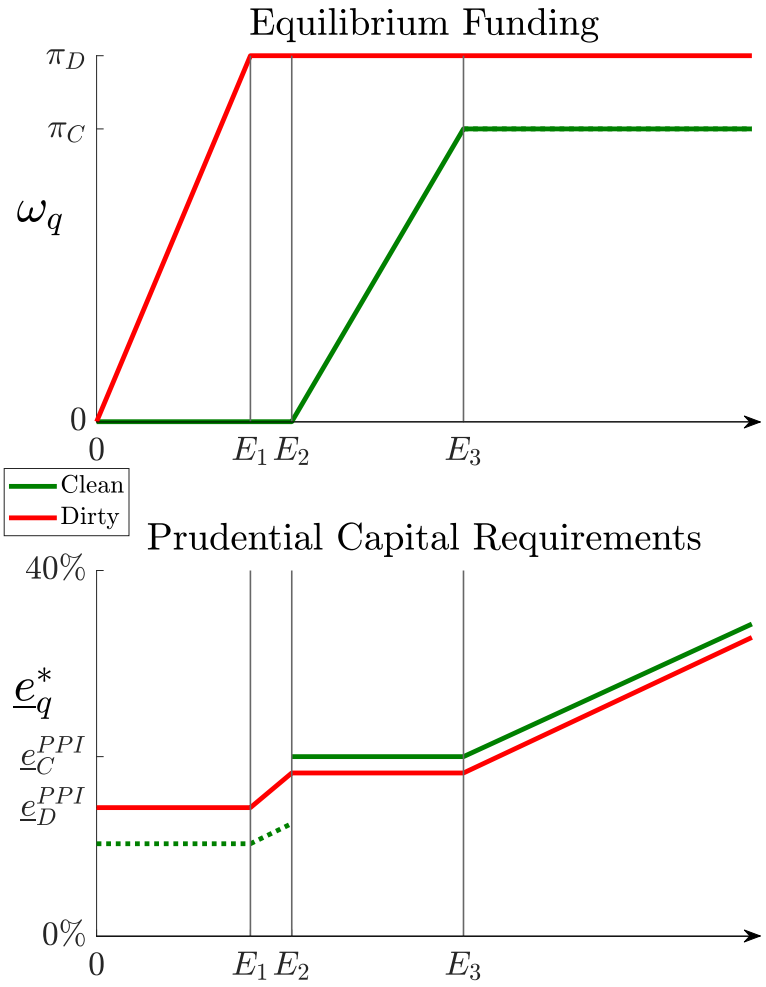


Figure 5. Optimal prudential capital regulation. This figure plots equilibrium funding decisions (upper panel) and capital requirements (lower panel) under optimal prudential bank capital regulation. Clean firms and their capital requirements are plotted in green, dirty in red. The dotted green line indicates the lower bound on the capital requirement for (unfunded) clean firms in the region in which only dirty loans are funded and the capital requirement for dirty loans is set to $\underline{e}_D^*$. This lower bounds satisfies $r_C^{\max}(\underline{e}_C) = r_D^{\max}(\underline{e}_D^*)$.

are funded, $\omega_D = \pi_D$.

In the second region, $E \in (E_1, E_2)$, dirty firms are fully funded, $\omega_D = \pi_D$. However, clean firms remain unfunded (see upper panel). Intuitively, once all dirty lending opportunities in the economy have been exhausted, the marginal benefit of funding the next best investment opportunity, the clean type, is lower by a discrete amount. Therefore, it is initially optimal to use additional equity in the banking sector to reduce the deposit insurance put associated with funding dirty firms, rather than inducing banks

to fund clean firms. In this region, Principle P1 pins down optimal prudential capital requirements for the dirty type, $\underline{e}_D^* = \frac{\pi_D E}{I}$ (see lower panel).[14]

Once the capitalization of the banking sector reaches $E = E_2$, the marginal reduction in the deadweight cost associated with the deposit insurance put is equal to the marginal value of funding a clean firm. Therefore, in region 3, $E \in (E_2, E_3)$, it becomes optimal to induce banks to fund clean firms, which then become the marginal type. The clean capital requirement is, therefore, pinned down by Principle P3, so that $\underline{e}_C^* = \underline{e}_C^{PPI}$. The capital requirement for the inframarginal dirty type is determined by Principle P4, the equalization of marginal puts.

Finally, in region 4, $E > E_3$, both types are fully funded $\omega_q = \pi_q$ (see upper panel). In this region, banking sector equity does not affect production decisions in the economy. Therefore, any additional equity is used to phase out the deadweight costs arising from deposit insurance. Principles P1 and P4 jointly pin down the optimal capital requirements for clean and dirty types.

4.2 The Effect of Climate-Related Risks

We now adapt our baseline model to account for the most relevant effects of climate change (physical risks, transition risks, legal risks, etc.). However, from the perspective of the financial regulator, it is not only important to gauge the effects of climate change, but also their cause, to the extent that carbon emissions (the cause) are endogenous to financial regulation. Figure 6 illustrates the resulting matrix classification of climate-related risks based on cause (row) and effect (column).

The category "*Financial-Financial*" (**FF**) considers the effects of emissions caused by bank-funded firms on risks in the banking sector. For example, the resulting physical risks (e.g., floods or droughts) lower profitability $\overline{X}_q$ and/or increase risk σ_q for both types

[14] Optimum clean capital requirements are again set to ensure that $r_C^{\max}(\underline{e}_C^*) < r_D^{\max}(\underline{e}_D^*)$, see green dotted line in the lower panel. Because dirty capital requirements are increasing in E in this region, the lower bound for clean capital requirement is increasing too to satisfy the bankers' IC constraint.

		EFFECT ON	
		Firms funded by financial sector	Other
CAUSED BY	Firms funded by financial sector	Financial-Financial (*TBD*)	Financial-Other (✓)
	Other	Other-Financial (✓)	Other-Other (✗)

Figure 6. Climate risk categorization

of firms. The category "*Financial-Other*" (**FO**) captures risks or externalities generated by the emissions of bank-funded firms which then affect firms or consumers outside the regulator's jurisdiction (e.g., via physical risks). The category "*Other-Financial*" (**OF**) captures effects on bank-funded firms caused by agents outside the banking system. For example, shifts in consumer tastes or transition risks (e.g., the introduction of carbon taxes) lower $\overline{X}_q$ and/or increase σ_q. Finally, the category "*Other-Other*" (**OO**) covers risks that are caused by firms or consumers outside the banking sector and affect firms or consumer outside the bank regulator's sphere.

While, in reality, all of these risks are at work simultaneously, we will analyze them separately for conceptual clarity. A regulator with a prudential mandate is only concerned about climate-related financial risks to the extent that they affect banks (via the firms that are funded by the banking sector), i.e., the prudential regulator cares about FF-risks and OF-risks. A regulator with a broader green mandate additionally internalizes FO risks, i.e., the contribution of bank-funded investments to climate change and its effects outside the banking system. Neither regulator can address OO-risks (which is why we omit this category going forward).

4.2.1 Exogenous climate-related financial risks (Type OF)

We start by analyzing the climate-related financial risks which affect bank-funded firms in the economy, but are caused by others. For example, regulatory transition risk (e.g., future environmental regulation, such as a ban of gasoline cars or the introduction of significantly higher carbon taxes) are (at least to a large extent) exogenous to bank funding decisions.[15] When transition risk materializes, it lowers the cash flows and increases the downside risk of dirty firms relative to clean firms. Technological risks (e.g., technological obsolescence), stakeholder risks (e.g., changes in consumer or employer preferences), and legal risks (e.g., legal exposures due to climate change) have similar financial ramifications.

In terms of our model, these effects can be understood as a comparative statics analysis relating to the parameters of the cash-flow distribution. In particular, given our assumption of a lognormal cash-flow distribution, we capture transition risks as decreases in the expected cash flow of dirty firms $\overline{X}_D$ and/or increases in their volatility σ_D, holding $\overline{X}_D$ constant.[16] There is a debate about the magnitude of these risks and over which horizon they realize. Following the structure of Section 3, we initially consider small changes, now relative to the prudential optimum.

Proposition 4 (Optimal Marginal Policy Adjustments: OF risks) *A marginal increase in the cash-flow volatility of dirty firm type σ_D or a marginal reduction in their expected cash flow $\overline{X}_D$*

1. *increases $\underline{e}^*_D$;*

2. *has no effect on $\underline{e}^*_C$ when clean is marginal (Region 3) and decreases $\underline{e}^*_C$ when both types are fully funded (Region 4).*

[15] To the extent that transition risks are partially related to funding decisions of banks, one could decompose the total transition risk into an OF component and an additional FF component.

[16] Since $\overline{X}_D$ is increasing in volatility, this means that we compensate μ_D so that $\overline{X}_D$ remains constant.

Intuitively, higher cash-flow volatility increases the put value associated with the dirty type and, hence, raises the marginal social cost of funding dirty projects. For reductions in $\overline{X}_D$, this effect on the put value is also at play, and, from the regulator's perspective, is reinforced by the reduction in NPV. In both cases, the regulator optimally responds by increasing capital requirements for dirty projects. The resultant increase in optimal capital requirements in response to moderate OF risks is illustrated in the left panel of Figure 7 (comparing the solid and dotted red lines).[17]

Part 2 of Proposition 4 investigates the spill-over effects on clean firms originating from climate-related risks that affect only dirty firms. In the region where clean firms are the marginal type, clean capital requirements are optimally set to maximize the PPI, $\underline{e}_C^* = \underline{e}_C^{PPI}$, so optimal prudential capital requirements for clean firms remain unaffected. However, in the region where both types are fully funded (Region 4), the optimal balancing of marginal puts (Principle P4) requires that capital requirements for clean firms are adjusted downward (comparing the solid and dotted green lines in the left panel of Figure 7).

In sum, the optimal regulatory response to moderate climate-related risks of type OF can be implemented with a brown penalizing factor for dirty firms (relative to the prudential baseline without climate risk) and, in some instances, a green supporting factor. As before, the exact calibration of these adjustments depends on the capitalization of the banking sector, as characterized by Proposition 3. This optimal regulatory response has the following allocative consequences.

Corollary 1 (Real Effects of Optimal Marginal Policy Adjustments: OF risks) *The optimal policy response to a marginal increase in the cash-flow volatility of the dirty firms σ_D and/or a marginal reduction in their expected cash flow $\overline{X}_D$*

1. *crowds out lending to dirty firms if bank equity is low, $E < E_1$;*

[17] As can be seen in Figure 7, in Region 2, this prediction holds weakly, in the sense that capital requirements. for dirty projects remain constant.

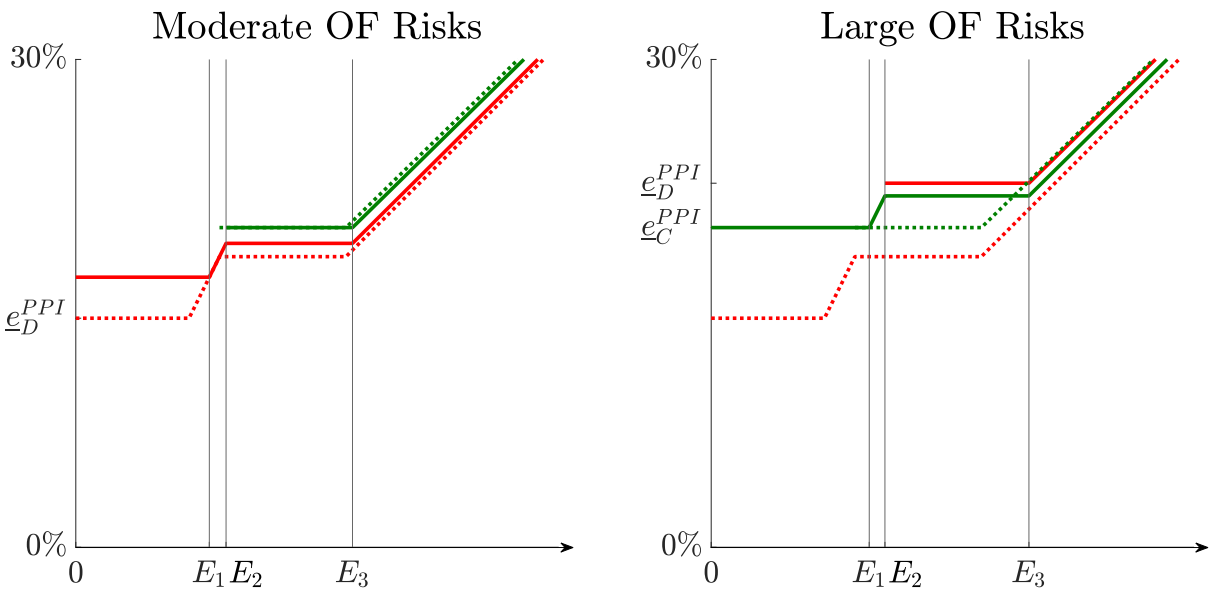


Figure 7. Effects of OF climate risks on optimal prudential capital regulation. The figure plots the effect of other-financial (OF) risks on optimal prudential capital requirements for clean and dirty types. In this example OF risks take the form of a reduction in the expected profitability of dirty firms $\overline{X}_D$, for example, due to transition risk. The left panel illustrates the effect of moderate OF risks, which do not affect the regulator's ranking of firm types. The right panels illustrates the effect of large OF risks that reverse the regulator's ranking of firm types. Solid lines depict optimal capital requirements that take into account OF risks. Dashed lines depict prudential capital requirements that do not take into account OF risks. Note that changes in capital requirements due to OF risks shift the region boundaries relative to Figure 5.

2. crowds out lending to clean firms if bank equity is intermediate $E \in (E_2, E_3)$ *.*

The second part of the corollary states that, perhaps surprisingly, moderate transition risks can make it optimal for a prudential regulator to sacrifice lending to clean firms. To see why this is the case, recall that a prudential regulator only cares about climate-related risks through their effect on firm cash flows and, in turn, financial stability in the banking sector. As long as these effects are small in magnitude, the prudential regulator still prefers the dirty type and therefore finds it optimal, at the margin, to allow the rationing of clean lending in the name of financial stability.

This conclusion changes for sufficiently large changes in the relative prospects of clean and dirty types. Such large OF risks result in a more drastic change in optimal prudential policy, even under a purely prudential regulatory objective, as illustrated in

the right panel of Figure 7.

Proposition 5 (Policy Adjustments in Response to Large OF risks) *A sufficiently large decrease in the expected cash-flow of dirty firms $\overline{X}_D$, or a sufficiently large increase in their cash-flow volatility σ_D, lead to a switch in the regulator's preferred type to clean. Clean types are funded first so that their funding is increased.*

Intuitively, once climate-related effects on relative cash flows are sufficiently large, clean firms deliver the highest PPI (see Definition 1). This change in the regulator's preferred type implies that prudential capital requirements are optimally set in to reverse banks' ranking of types accordingly. This is illustrated in the right panel of Figure 7. The optimal prudential capital requirements in the presence of large OF risks ensure that clean firms are funded first (up to the cutoff E_2), and dirty firms receive funding only after clean lending opportunities have been exhausted (once aggregate bank equity exceeds E_2).

In summary, the presence of OF risks that lower the prospects of dirty firms relative to clean firms rationalizes the use of the ad-hoc policy tools we analyzed in Section 3. In particular, brown penalizing and green supporting factors become part of the prudential toolkit, and their magnitudes are directly linked to the magnitude of the effect of climate risk on relative cash flow prospects of clean and dirty firms. Our results suggest that the directional effects of these policy adjustments on banks' lending decisions depend crucially on the severity of these risks (comparing Corollary 1 with Proposition 5).

4.2.2 Externalities on others (Type FO)

We now consider climate-related risks that are endogenous to the funding decisions of the banking sector and, hence, bank regulation, but manifest itself outside the banking sector. These could be pure consumption externalities (e.g., carbon emissions directly affect the utility of agents in the economy), or production externalities outside the bank

regulator's jurisdiction (e.g., floods in Bangladesh primarily affect local firms rather than the cash flows of firms funded by European banks). Because the prudential regulator's objective function (9) does not account for these externalities on others, we readily obtain

Observation 1 *The optimal prudential policy is unaffected by FO risks, regardless of their size.*

Taken together with the findings from the previous section, this observation implies that, even in the presence of substantial externalities on others, (i.e., high FO risks, and moderate OF risks), a prudential regulator may find it optimal to tolerate the crowding out of clean lending, in order to be able to fund an unchanged number of dirty firms at higher capital requirements (see Corollary 1).

Because mitigating carbon externalities has become an important global policy objective, it is natural to also consider a broader mandate for financial regulators that explicitly accounts for carbon externalities, in addition to the usual prudential objective.[18] We will refer to this broader objective function as a "green mandate" and the associated regulator as a "green regulator". Assuming, for simplicity, that the aggregate negative externality caused by dirty firms is linear in the mass of dirty firms that obtain financing (or, equivalently, linear in carbon emissions), the green regulator's objective function is given by

$$\max_{\underline{\mathbf{e}}} \Omega_G = \max_{\underline{\mathbf{e}}} \sum \omega_q \left(\underline{\mathbf{e}}\right) \left[\mathrm{NPV}_q - \phi_q - \lambda \cdot \mathrm{PUT}_q(\underline{e}_q)\right]. \tag{16}$$

In analogy to the PPI, we can then define the green regulator's social profitability index, which captures "bang for buck" including associated carbon externalities ϕ_q,

$$\mathrm{SPI}(\underline{e}_q) = \frac{\mathrm{NPV}_q - \phi_q - \lambda \cdot \mathrm{PUT}_q(\underline{e}_q)}{I\underline{e}_q}. \tag{17}$$

[18] In fact, the ECB's secondary mandate is to support economic policy in the eurozone. To the extent that a reduction in carbon emissions is part of general economic policy, this broader objective function would therefore seem to be within the scope of the ECB's mandate.

The green regulator's preferred type is the one that achieves the maximal SPI (not maximal PPI). Small carbon externalities ϕ_D do not lead to a change of the regulator's preferred type relative to the prudential regulator. In this case, the characterization of optimal capital requirements is, therefore, similar to the prudential regulator's case under moderate transition risk. The most interesting and novel case is the one where externalities large enough, so that the SPI becomes negative for the dirty firm:

Assumption 1 $\phi_D > NPV_D$.

Under this assumption, carbon emissions are so significant that funding dirty firms reduces social welfare, even when capital requirements for dirty loans are set to 100%, i.e., $\text{SPI}_D(1) = \frac{\text{NPV}_D - \phi_D}{I} < 0$. However, banks still find it privately profitable to finance dirty types, even at a 100% capital requirement, because $r_D^{\max}(1) = \frac{\text{NPV}_D}{I} > 0$.

Observation 2 *Even under maximum capital requirements of 100%, financing dirty firms is financially profitable for banks.*

This observation illustrates the limited power of capital requirements to address FO risks. Distortions generated by the deposit-insurance put can always be dealt with using capital requirements. In particular, if a firm generates negative prudential value for all levels of $\underline{e}_q$,[19] the prudential regulator can always deter banks from funding such a type by imposing sufficiently high capital requirements. This is not the case in the presence of externalities, because, even at capital requirements of 100%, dirty firms with negative social value can attract funding, since

$$r_D^{\max}(1) = \frac{\text{NPV}_D}{I} > 0 > \frac{\text{NPV}_D - \phi_D}{I} = \text{SPI}_D(1). \tag{18}$$

The fact that funding dirty projects is profitable even at a capital requirement of 100% constrains the green regulator's optimal policy. To see this, let us first consider

[19] Since the deposit insurance put can always be fully eliminated with a capital requirement of 100%, this is the case if and only if $\text{NPV}_q < 0$, since $\text{PPI}_q(1) = \text{NPV}_q / I$.

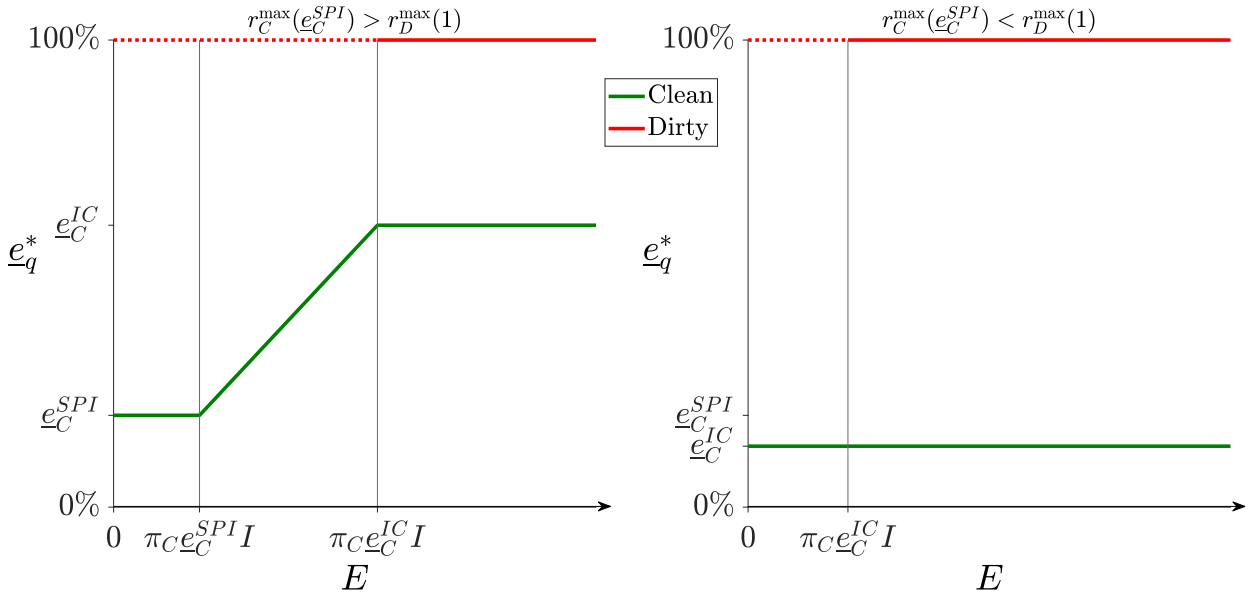


Figure 8. Capital regulation under a green mandate. This figure plots optimal capital requirements under the green regulator's objective function (16). The left panel illustrates the case, in which banks' IC constraint is not binding. Hence, it is initially possible to set $\underline{e}_C^* = \underline{e}_C^{SPI}$ and to incentivize banks to lend to clean firms by setting capital requirements for dirty firms to 100%. In the right panel, setting $\underline{e}_C^* = \underline{e}_C^{SPI}$ would violate banks' IC constraint and they would therefore lend to dirty firms first. Therefore, the green regulator is forced to lower the capital requirement for clean loans to $\underline{e}_C^* = \underline{e}_C^{IC} < \underline{e}_C^{SPI}$.

the case $r_C^{\max}\left(\underline{e}_C^{SPI}\right) > r_D^{\max}(1)$, as illustrated in the left panel of Figure 8. In this case, the regulator can set the capital requirement for clean loans to maximize their SPI and ensure that clean loans are funded first by setting the capital requirement for dirty loans to 100%. For small values of aggregate bank equity (region 1), $E \leq \pi_C \underline{e}_C^{SPI} I$, only clean firms are funded with the clean capital requirement optimally set to $\underline{e}_C^{SPI}$. Once all clean firms have been funded (region 2), the regulator then raises the capital requirements for clean types to avoid that additional equity is used to fund dirty firms and to lower the deposit-insurance put for clean loans. This increase of clean capital requirement is optimal up to the point at which banks are indifferent between funding clean and dirty loans, $r_C^{\max}\left(\underline{e}_C^{IC}\right) = r_D^{\max}(1)$. If the regulator raised capital requirements for clean projects beyond $\underline{e}_C^{IC}$, banks would prefer to fund dirty rather than clean funding, which explains why capital requirements on clean firms are initially capped at $\underline{e}_C^{IC}$. As a result, once aggregate bank equity exceeds the amount required to finance all clean firms at a

capital requirement of $\underline{e}_C^{IC}$, $E > \pi_C \underline{e}_C^{IC} I$, banks start to finance dirty firms at $e_D^* = 1$ (as indicated by the solid region for e_D^*), revealing that the regulator is eventually powerless to deter dirty funding.[20]

In the second case, $r_C^{\max}\left(\underline{e}_C^{SPI}\right) < r_D^{\max}(1)$, the regulator cannot set the capital requirements for cleans loans to the level that maximizes the SPI, because banks would then prefer to fund dirty loans. For low levels of bank equity (region 1), the green regulator is therefore forced to lower capital requirements below $\underline{e}_C^{SPI}$ in order to ensure that $r_C^{\max}\left(\underline{e}_C^{IC}\right) = r_D^{\max}(1)$. Intuitively, it is not possible to induce banks to make clean loans by solely raising the capital requirement for dirty loans. In addition, the regulator needs to subsidize clean loans by lowering their capital requirements below the level that would maximize the SPI, up to the point where banks are willing to fund clean loans.[21] As before, once aggregate bank equity exceeds the amount required to finance all clean firms at a capital requirement of $\underline{e}_C^{IC}$, $E > \pi_C \underline{e}_C^{IC} I$, the regulator is powerless to prevent the funding of dirty firms.

In sum, externalities that materialize outside the banking sector are not accounted for by the prudential mandate. A broader mandate would be required to account for such externalities. However, our analysis suggests that capital requirements are not the most natural tool to address these externalities: Even at maximal requirements, banks may find dirty loans still financially profitable. This wedge between welfare and bankers' profit motive may require the green regulator to make significant sacrifices with respect to financial stability in order to promote clean lending and deter dirty lending.

Such trade-offs between financial stability and negative consumption externalities would not arise if Pigouvian taxes are available to the regulator: These would ensure that the externalities (through Pigouvian taxation) are directly reflected in the bankers'

[20] For very high levels of equity, e.g., $E = I$, (outside the plotted region in Figure 8), it is again optimal to equalize marginal puts under full funding for both types, as in Region 4 of Figure 5.

[21] Clearly, for this course of action to be optimal, it also has to be the case that $\frac{\text{NPV}_C - \lambda \cdot \text{PUT}_C(\underline{e}_C^{IC})}{\underline{e}_C^{IC}} > \frac{\text{NPV}_D - \phi_D}{I}$, which we assume in the figure.

maximal ROE, rendering projects with negative SPI also unprofitable from a private perspective. We, thus, conclude that capital requirements are, at best, a second-best tool to address broader climate externalities.

4.2.3 Endogenous climate-related financial risks (Type FF)

If carbon emissions by bank-funded firms affect the payoffs of these firms themselves, then these externalities are accounted for by the prudential regulator. For example, physical risks caused by floods and droughts would likely impose negative production externalities in the form lower cash flows for both clean and dirty types in the economy. While competitive banks take these externalities as given and thus fail to account for their effect on the overall productivity in the economy, the prudential regulator has an incentive to account for the tragedy of the commons via imposing higher capital requirements for dirty firms. (The next draft of this paper will include a more thorough analysis of FF risks.)

5 Extensions

Non-bank financing. In our model, all firms are bank-dependent. This assumption gives a lot of power to a financial regulator controlling bank capital requirements: Emissions by dirty firms can be prevented by ensuring that banks do not fund these firms (thereby giving the green regulator maximum leverage). If instead dirty firms have access to public markets (or another alternative source of financing), financial regulation via capital requirements is powerless to reduce emissions, as emissions would occur regardless of the availability of bank funding. In the language of our risk classification (see Figure 6), these emissions are then no longer "caused" by bank funding, as they would materialize anyway. Carbon taxes would not suffer from this leakage concern, because they would reduce the profitability of lending to dirty firms for both banks and

non-banks. Whether substitution to non-bank financing is a concern for the regulator depends on the mandate. While the green regulator would like to prevent such substitution, the prudential regulator welcomes it, because it removes risk from the banking sector.

Imperfect observability of firm types by regulator. For expositional clarity, we assumed that the financial regulator can perfectly observe both the riskiness and emissions of a firm. If the regulator only observed a noisy signal of firm quality, the main results would be qualitatively similar. There are, however, some additional results regarding potential unintended consequences. If, for example, clean firms consisted both of risky clean firms and safe clean firms, a blunt green supporting factor for all clean firms would disproportionately benefit the risky ones (because of the larger increase in the put value). This could incentivize banks to take on excessive "green risks" (akin to subprime structured securities that were subject to lenient capital charges.)

Firms' choice of production technology. For ease of exposition, we assumed that firm types are exogenously given, which should be interpreted as firms operating either in a clean or a dirty sector. In this baseline model, green tilts to capital regulation would affect emissions via the banking sector's asset allocation across sectors. If, in addition, firms within a given sector had access to a costly pollution-reducing technology, as in Oehmke and Opp (2019), they may have an incentive to invest in these technologies to the extent that capital requirements reward such investments. The incentives to invest in such technologies would be determined by the increase in the reservation price post investment.[22]

[22] To the extent that marginal types remain "marginal", there would be no incentive to make such investments since banks would reap all the surplus.

6 Conclusion

This paper provides a flexible modeling framework to analyze the green capital requirements. Such a framework is needed because central banks and bank regulators around the world are grappling with the question of whether to adjust capital requirements to reflect carbon emissions.

Our analysis has two major takeaways. First, from a positive perspective, it clarifies that the most commonly suggested policy tools, the brown penalizing and green supporting factors, are not equivalent and have potentially subtle effects. Second, from a normative perspective, we characterize how optimal capital requirements depend on the regulatory objective (a prudential or a broader green mandate) and the nature of climate-related risks. While climate-related risks that affect bank stability can be optimally addressed by a combination of green supporting and brown penalizing factors, capital regulation is a less effective tool to address carbon externalities that manifest itself outside the banking sector.

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A Proofs

Proof of Result 1: Let $y_q \geq 0$ denote the interest rate that a borrower of type q pays on the loan of size I. (This yield will be endogenized in equilibrium, see Results 2 and 3). Then, if a bank were to only lend to borrowers of type q at their, i.e., $w_q = 1$, and chooses a feasible equity ratio $e \geq \underline{e}_q$ its expected return on equity can be written as:

$$r_E = \frac{\mathbb{E}\left[\max\left\{\min\left\{I\left(1+y_q\right), X_q(s)\right\} - \left(1-e\right)I, 0\right\}\right] - eI}{eI} \tag{A.1}$$

$$= \frac{\mathbb{E}\left[\max\left\{\min\left\{Iy_q, X_q(s) - I\right\}, -eI\right\}\right]}{eI}. \tag{A.2}$$

Equation (A.1) reflects that the bank receives (from each borrower of type q) the minimum of the promised loan repayment $I\left(1+y_q\right)$ and the actual cash flows of the borrower, i.e., $\min\left\{I\left(1+y_q\right), X_q(s)\right\}$. Given an equity ratio of e, the amount of debt financing (per borrower) is $\left(1-e\right)I$. Since depositors require zero percent interest on their deposits (due to bailouts / deposit insurance), the bank also needs to repay depositors $\left(1-e\right)I$. Since bank shareholders are protected by limited liability, their gross-payoff is bounded below by zero. The numerator, thus, reflects the expected payoff for bank shareholders net of their coinvestment eI. The denominator simply reflects this coinvestment. We can now decompose the numerator to write (A.2) as follows

$$r_E = \frac{\mathbb{E}\left[\min\left\{Iy_q, X_q(s) - I\right\}\right] + \mathbb{E}\left[\max\left\{-eI - \min\left\{Iy_q, X_q(s) - I\right\}, 0\right\}\right]}{eI}, \tag{A.3}$$

which follows from the general result that $\max\left\{a, b\right\} = a + \max\left\{b - a, 0\right\}$ and setting $a = \min\left\{Iy_q, X_q(s) - I\right\}$ and $b = -eI$. Here, $\mathbb{E}\left[\max\left\{-eI - \min\left\{Iy_q, X_q(s) - I\right\}, 0\right\}\right]$ can be interpreted as the value of the bank put (derived from lending to a firm of type q). This put value can be further simplified, since $\max\left\{-eI - \min\left\{Iy_q, X_q(s) - I\right\}, 0\right\} = 0$ in all states in which the borrower can repay, i.e., for all states s where $Iy_q > X_q(s) - I$. Therefore, we obtain:

$$r_E = \frac{\mathbb{E}\left[\min\left\{Iy_q, X_q(s) - I\right\}\right] + \mathbb{E}\left[\max\left\{I\left(1-e\right) - X_q(s), 0\right\}\right]}{eI}. \tag{A.4}$$

(A.4) now shows that the ROE is strictly decreasing in e, i.e., the bank optimally chooses the minimum equity co-financing $e = \underline{e}_q$. Moreover, mixing two borrower types is strictly suboptimal because diversification lowers the put value.[23] ■

Proof of Result 2: Let $y_q^{\max}$ denote the *maximum interest rate* (yield) that a borrower is willing to pay, then the maximum achievable ROE upon lending to a borrower of type q is achieved under maximal leverage, $e = \underline{e}_q$, so that (A.4) becomes

$$r_q^{\max}\left(\underline{e}_q\right) = \frac{\mathbb{E}\left[\min\left\{Iy_q^{\max}, X_q(s) - I\right\}\right] + \mathrm{PUT}_q\left(\underline{e}_q\right)}{I\underline{e}_q}, \tag{A.5}$$

[23] This follows from the general insight that the option value of a portfolio is (weakly) lower than the portfolio of options.

where we use the definition of the $\text{PUT}_q\left(\underline{e}_q\right) := \mathbb{E}\left[\max\left\{I(1-\underline{e}_q) - X_q(s), 0\right\}\right]$. (A.5) covers both the case where the firm type is bank bank-dependent (as in our baseline model) and where the firm has access to an outside option (as in Section 5).

Case 1: If the firm is bank-dependent (and thus, lacks an outside financing option) it is willing to pledge the entire NPV to the bank,[24] so that $\mathbb{E}\left[\min\left\{Iy_q^{\max}, X_q(s) - I\right\}\right] = \mathbb{E}\left[X_q(s) - I\right] = \text{NPV}_q$. Then (A.5) simplifies to (6).

Case 2: If the firm has access to a competitive outside option, the interest rate on that outside option corresponds to the reservation interest rate $y_q^{\max}$. The value of $y_q^{\max}$ is set such that the competitive outside investor breaks even on the investment, i.e., $y_q^{\max}$ satisfies:

$$\mathbb{E}\left[\min\left\{I\left(1+y_q^{\max}\right), X_q(s)\right\}\right] = I, \tag{A.6}$$

which implies that $\mathbb{E}\left[\min\left\{Iy_q^{\max}, X_q(s) - I\right\}\right] = 0$. Therefore, the maximal ROE for bank shareholders in (A.5) becomes:

$$r_q^{\max}\left(\underline{e}_q\right) = \frac{\text{PUT}_q\left(\underline{e}_q\right)}{\underline{e}_q I}. \tag{A.7}$$

Intuitively, the only comparative advantage of banks relative to competitive outside investors results from access to deposit insurance. ■

Proof of Result 3: Given that equity is the scarce resource, the banking sectors prioritizes borrowers according to the maximal expected return on equity (acting akin to reservation prices). We need to distinguish between two cases.

Scarce equity: If not all firms can be financed with the equity E, then the marginal borrower type q_M pays the maximal interest rate $y_{q_M}^{\max}$ on its loan. A fraction of marginal firms with $r_{q_M}^{\max}\left(\underline{e}_{q_M}\right) = r_E^*$ is rationed. Even though banks are competitive, banks earn a scarcity rent of $r_E^* = r_{q_M}^{\max}\left(\underline{e}_{q_M}\right) > 0$. All borrower types with $r_q^{\max}\left(\underline{e}_q\right) > r_E^*$ are inframarginal and will be fully financed. The interest rate on their loan $y_q < y_q^{\max}$ is set below their reservation interest rate which ensures that banks also earn a ROE of r_E^* on loans to inframarginal borrower types (who, thus, obtain some surplus from their projects).

Non-scarce equity: In this case, banks finance all firms in the economy with $r_{q^*}^{\max}\left(\underline{e}_{q^*}\right) > 0$. Since banks are competitive and equity is not scarce, loan interest rates are such that banks earn a ROE of $r_E^* = 0$ on all loans. All surplus (including the put value) is passed on to borrowers. ■

Proof of Proposition 1: Given equal capital requirements, we obtain that $r_D^{\max}(\underline{e}) > r_C^{\max}(\underline{e})$. This follows from our (baseline) assumption that the cash flow distribution of dirty firms first-order stochastically dominates the cash flow distribution of clean firms. It is easiest to see this using (A.1) and $e = \underline{e}$ (see Result 1).

1 (a) If $E < \pi_D \underline{e} I$ the dirty firm type is marginal, where the threshold $\pi_D \underline{e} I$ is the amount of equity needed to fund all dirty firms with population fraction π_D and

[24] Given that the log-normal cash-flow distribution is unbounded above, this corresponds to $y_q^{\max} = \infty$.

investment size I. A marginal increase in the capital requirements for dirty firms will, not reverse the banks' ranking based on $r_q^{\max}$, and, thus only crowd out dirty lending.

1 (b) If $E \in (\pi_D \underline{e} I, \underline{e} I)$, all dirty firms are inframarginal and the clean firm type is marginal (not all clean types can be financed). Therefore, an increase in the capital requirement for (inframarginal) dirty firms will reduce the equity that can be allocated to clean firms. Lending to clean firms is crowded out.

1 (c) if $E > \underline{e} I$, the banking sector has excess equity relative to the funding needs of all firms, $\underline{e} I$. Marginal changes in capital requirements will have no effect.

2 For any Δ_{BPF} that satisfies $r_D^{\max}(\underline{e} + \Delta_{BPF}) < r_C^{\max}(\underline{e})$, clean firms rank above dirty firms.[25] As a result, dirty firms will only get bank funding after lending opportunities to clean firms are exhausted. This increases lending to clean firms (strictly so if they were rationed before) and decreases lending to dirty firms (strictly so if bank capital is scarce after the increase in capital requirements).

■

Proof of Proposition 2: The proof is analogous to the Proof of Proposition 1. ■

Lemma A.1 *If cash flows are lognormally distributed with* $\ln(X_q) - \ln(I) \sim N(\mu_q, \sigma)$, *we obtain a mean cash flow of*

$$\overline{X}_q = I e^{\mu_q + \frac{\sigma^2}{2}}, \tag{A.8}$$

so that $NPV_q = \overline{X}_q - I$. *The put value is given by:*

$$PUT_q\left(\underline{e}_q\right) = N(-d_2)\left(1 - \underline{e}_q\right) I - N(-d_1)\,\overline{X}_q \tag{A.9}$$

$$d_1 = \frac{\ln\left(\overline{X}_q\right) - \ln\left(I\left(1 - \underline{e}_q\right)\right)}{\sigma} + \frac{\sigma}{2} \tag{A.10}$$

$$d_2 = \frac{\ln\left(\overline{X}_q\right) - \ln\left(I\left(1 - \underline{e}_q\right)\right)}{\sigma} - \frac{\sigma}{2} \tag{A.11}$$

where N *denotes the standard normal cumulative distribution function.*

Proof of Lemma A.1: (A.8) follows directly from standard properties of the log-normal distribution. The textbook Black-Scholes formula, see e.g., Hull (2003), for an asset with price S, volatility σ, strike price K, maturity T and risk-free rate r states that:

$$P = e^{-rT} K N(-d_2) - S N(-d_1) \tag{A.12}$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}} = d_2 + \sigma\sqrt{T} \tag{A.13}$$

[25] Such a BPF always exists by our assumption that the productivity of clean and dirty firms is sufficiently close.

By universal risk-neutrality and zero discounting the project asset value is $S = \overline{X}_q$. The strike price of the option is $K = I\left(1 - \underline{e}_q\right)$. This results in (A.9), (A.10) and (A.11). ■

Lemma A.2 *The following comparative statics of the put value apply:*

$$\frac{\partial PUT_q}{\partial \sigma} > 0 \tag{A.14}$$

$$\frac{\partial PUT_q}{\partial \overline{X}_q} = -N\left(-d_1\right) < 0 \tag{A.15}$$

$$\frac{\partial PUT_q}{\partial \bar{e}_q} = -I \cdot \left(1 - N\left(d_2\right)\right) < 0 \tag{A.16}$$

$$\frac{\partial^2 PUT_q}{\partial \bar{e}_q^2} = If\left(d_2\right)\frac{1}{\sigma}\frac{1}{1-\underline{e}_q} > 0 \tag{A.17}$$

$$\frac{\partial^2 PUT_q}{\partial \bar{e}_q \partial \overline{X}_q} = If\left(d_2\right)\frac{1}{\sigma}\frac{1}{\overline{X}_q} > 0 \tag{A.18}$$

$$\frac{\partial^2 PUT_q}{\partial \bar{e}_q \partial \sigma} = -If\left(d_2\right)\left(\frac{\ln\left(\overline{X}_q\right) - \ln\left(I\left(1-\underline{e}_q\right)\right)}{\sigma^2} + \frac{1}{2}\right) < 0 \tag{A.19}$$

Proof: The first three results are standard textbook results, see, e.g., Hull (2003). It is useful to write $\frac{\partial PUT_q}{\partial \bar{e}_q}$ as

$$\frac{\partial PUT_q}{\partial \bar{e}_q} = -I + IN\left(d_2\right). \tag{A.20}$$

Since $d_2 = \frac{\ln\left(\overline{X}_q\right) - \ln\left(I\left(1-\underline{e}_q\right)\right)}{\sigma} - \frac{\sigma}{2}$, see (A.11), we obtain

$$\frac{\partial d_2}{\partial \bar{e}_q} = \frac{1}{\sigma}\frac{1}{1-\underline{e}_q} > 0 \tag{A.21}$$

$$\frac{\partial d_2}{\partial \overline{X}_q} = \frac{1}{\sigma}\frac{1}{\overline{X}_q} > 0 \tag{A.22}$$

$$\frac{\partial d_2}{\partial \sigma} = -\left(\frac{\ln\left(\frac{\overline{X}_q}{I\left(1-\underline{e}_q\right)}\right)}{\sigma^2} + \frac{1}{2}\right) < 0 \tag{A.23}$$

Using $\frac{\partial^2 PUT_q}{\partial \bar{e}_q^2} = If\left(d_2\right)\frac{\partial d_2}{\partial \bar{e}_q}$ and (A.21), we obtain (A.17) and, analogously, (A.18) and (A.19). Note that (A.23) is unambiguously negative because both projects are, by assumption, positive NPV from a financial perspective, i.e., $\overline{X}_q > I > I\left(1-\underline{e}_q\right)$ and $\ln\left(x\right) > 0$ for any $x > 1$. ■

Lemma A.3 *If* $\lambda > \frac{NPV_q}{PUT_q(0)}$, *the maximizer of* $PPI_q(\underline{e}_q) = \frac{NPV_q - \lambda \cdot PUT_q(\underline{e}_q)}{I\underline{e}_q}$ *is finite and uniquely determined by the first-order condition*

$$IPPI_q(\underline{e}_q) = -\lambda \frac{\partial PUT_q(\underline{e}_q)}{\partial \underline{e}_q}. \tag{A.24}$$

Proof of Lemma A.3: The first-order condition $\frac{\partial PPI_q(\underline{e}_q)}{\partial \underline{e}_q} = 0$ implies

$$\frac{I\underline{e}_q\left(-\lambda \cdot \frac{\partial \mathrm{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q}\right) - \left(\mathrm{NPV}_q - \lambda \cdot \mathrm{PUT}_q(\underline{e}_q)\right) I}{I^2 \underline{e}_q^2} = 0. \tag{A.25}$$

Rearranging yields (A.24). To prove uniqueness, it is useful to rewrite (A.25) as follows

$$G\left(\underline{e}_q\right) = \mathrm{NPV}_q, \tag{A.26}$$

where the function

$$G\left(\underline{e}_q\right) := \lambda\left[\mathrm{PUT}_q(\underline{e}_q) - \underline{e}_q \frac{\partial \mathrm{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q}\right] \tag{A.27}$$

is defined on the domain $[0,1]$. It is now easy to verify that the function G takes on its maximum value at 0 with $G(0) = \lambda \mathrm{PUT}_q(0) > 0$ and the minimum value at 1 with $G(1) = 0$. Moreover, it is differentiable and strictly decreasing with slope:

$$G'\left(\underline{e}_q\right) = \lambda\left[\frac{\partial PUT}{\partial \underline{e}} - \left[\frac{\partial PUT}{\partial \underline{e}} + \underline{e}\frac{\partial^2 PUT}{\partial \underline{e}^2}\right]\right] = -\lambda \underline{e}\frac{\partial^2 PUT}{\partial \underline{e}^2} < 0 \tag{A.28}$$

as $\frac{\partial^2 PUT}{\partial \underline{e}^2} > 0$, see (A.17). Since G is strictly decreasing and $G(1) = 0 <$NPV$_q$, (A.26) will have a solution if and only if $G(0) >$NPV$_q$, which is equivalent to $\lambda > \frac{\mathrm{NPV}_q}{PUT_q(0)}$. By continuity of G, the solution for $\underline{e}_q$ is unique. ■

Proof of Proposition 3: We prove each claim separately.

P1 We first prove that under optimal prudential regulation it is without loss of generality to restrict dividends to zero.

First, suppose that at the optimal capital requirements $\underline{\mathbf{e}}^*$, banks earn a scarcity rent, i.e., $r_E^* > 0$. In this case, banks strictly prefer not to pay out dividends since they can earn an excess return, i.e., dividends are endogenously zero.

Second, consider the case where bank equity is not scarce so that all types are funded, $\omega_q\left(\underline{\mathbf{e}}^*\right) = \pi_q$, and banks earn no scarcity rent, i.e., $r_E^* = 0$. In this case the regulator's payoff is given by:

$$\sum \pi_q \left[\mathrm{NPV}_q - \lambda \cdot \mathrm{PUT}_q(\underline{e}_q^*)\right]$$

Now suppose (by contradiction) that under optimal prudential regulation not all equity is used, $E - \sum \pi_q \underline{e}_q^* I > 0$, so that the banking sector as a whole finds it

optimal to pay out the excess equity as dividends (as to ensure maximal leverage, see Result 1). Then the regulator could increase capital requirements of both types to $\underline{\mathbf{e}}^{**} > \underline{\mathbf{e}}^{*}$ (where the inequality is strict for at least one type) until all equity is exhausted, i.e., $E = \sum \pi_q \underline{e}_q^{**} I$. By construction, this would leave firm funding unaffected and strictly reduce the put value, thereby increasing the regulator's payoff. Hence, $\underline{\mathbf{e}}^{*}$ could not have been optimal.

We now turn to the remaining claims. It is useful to phrase the regulator's problem in terms of the $\text{PPI}_q(\underline{e}_q) = \frac{\text{NPV}_q - \lambda \cdot \text{PUT}_q(\underline{e}_q)}{I\underline{e}_q}$, see Lemma A.3, and using the definition of $\tilde{\omega}_q$, the fraction of equity allocated to type q.

Problem 1 *The prudential regulator solves:*

$$\max_{\underline{\mathbf{e}}} \Omega_P = E \max_{\underline{\mathbf{e}}} \sum \tilde{\omega}_q \left(\underline{\mathbf{e}}\right) PPI_q(\underline{e}_q), \tag{A.29}$$

s.t. to the short-selling constraint, i.e., the equity allocated to each type is non-negative

$$\tilde{\omega}_q \left(\underline{\mathbf{e}}\right) \geq 0, \tag{A.30}$$

and the mass of funded firms cannot exceed the supply of each type π_q:

$$\tilde{\omega}_q \left(\underline{\mathbf{e}}\right) E \leq \pi_q \underline{e}_q I, \tag{A.31}$$

and, finally, the banking sector's privately optimal allocation of equity

$$\tilde{\omega}_q \left(\underline{\mathbf{e}}\right) = \frac{\min \left\{ \max \left\{ E - \sum_{\breve{q}: r_{\breve{q}}^{\max} > r_q^{\max}} \pi_{\breve{q}} \underline{e}_{\breve{q}} I, 0 \right\}, \pi_q \underline{e}_q I \right\}}{E}. \tag{IC}$$

(IC) fully determines the funding decisions of the banking sector based on the ranking implied by $r_q^{\max}$.[26] For any given type q, the equity that's left from funding all types with higher ROE is given by $\max \left\{ E - \sum_{\breve{q}: r_{\breve{q}}^{\max} > r_q^{\max}} \pi_{\breve{q}} \underline{e}_{\breve{q}} I, 0 \right\}$. The actual amount allocated to a given type is then the minimum of the "left-over" equity for this type, $\max \left\{ E - \sum_{\breve{q}: r_{\breve{q}}^{\max} > r_q^{\max}} \pi_{\breve{q}} \underline{e}_{\breve{q}} I, 0 \right\}$, and the amount of equity needed to fund all firms of type q, $\pi_q \underline{e}_q I$. As is clear, banks' optimal decisions in (IC) automatically ensure that the constraints (A.30) and (A.31) are satisfied. It is still useful to add these constraints to prove the claims of principles 2) to 4).

P2 Consider the regulator's relaxed problem where (A.31) and (IC) are ignored. This

[26] Our assumptions ensure that $r_q^{\max} > 0$ for all types q. If this weren't the case, then we would obtain $\tilde{\omega}_q \left(\underline{\mathbf{e}}\right) = 0$ for all types with $r_q^{\max} < 0$.

relaxed problem provides an upper bound on the regulator's payoff. In this relaxed problem the regulator simply maximizes the convex combination of prudential profitability indices, i.e., $\sum \tilde{\omega}_q(\underline{\mathbf{e}})\, PPI_q(\underline{e}_q)$, see (A.29). The optimal choice is given by allocating all equity to the regulator's preferred type $\hat{q}$ (see Definition 1), which offers the maximal PPI, $\max_q \text{PPI}_q(\underline{e}_q^{PPI})$ yielding an overall payoff $E \max_q \text{PPI}_q(\underline{e}_q^{PPI}) = E \cdot \text{PPI}_{\hat{q}}(\underline{e}_{\hat{q}}^{PPI})$.[27] In our baseline model, the dirty type is the preferred type, i.e., $\hat{q} = D$, since dirty firms have a higher NPV and a lower put (for any given level of capital requirements). We now prove that the regulator can achieve this upper bound payoff in the full problem – considering (IC) and (A.31) – if and only if the equity needed to fund all firms of the preferred type, $\pi_{\hat{q}} \underline{e}_{\hat{q}}^{PPI} I$, is greater than the supply of bank equity, i.e., $\pi_{\hat{q}} \underline{e}_{\hat{q}}^{PPI} I > E$. To see this, set $\underline{e}_{\hat{q}} = \underline{e}_{\hat{q}}^{PPI}$ (i.e., $\underline{e}_D = \underline{e}_D^{PPI}$ in our baseline setting) and $\underline{e}_q = 1$ for all types with $q \neq \hat{q}$ (i.e., $\underline{e}_C = 1$ in our baseline setting). Given these capital requirements, banks rank quality $\hat{q}$ highest, i.e., $r_{\hat{q}}^{\max}\left(\underline{e}_{\hat{q}}^{PPI}\right) > r_q^{\max}(1)$ so that (IC) implies that banks optimally allocate all equity to firm type $\hat{q}$, i.e., $\tilde{\omega}_{\hat{q}} = \frac{\min\left\{E, \pi_{\hat{q}} \underline{e}_{\hat{q}}^{PPI} I\right\}}{E} = 1$. To see why banks rank type $\hat{q}$ highest note that

$$\begin{aligned} r_{\hat{q}}^{\max}\left(\underline{e}_{\hat{q}}^{PPI}\right) &= \frac{\text{NPV}_{\hat{q}} + \text{PUT}_{\hat{q}}\left(\underline{e}_{\hat{q}}\right)}{I \underline{e}_{\hat{q}}} \\ &> \text{PPI}_{\hat{q}}(\underline{e}_{\hat{q}}) = \frac{\text{NPV}_{\hat{q}} - \lambda \text{PUT}_{\hat{q}}\left(\underline{e}_{\hat{q}}\right)}{I \underline{e}_{\hat{q}}} \\ &> \max_{\underline{e}_q} \text{PPI}_q(\underline{e}_q) \\ &\geq \text{PPI}_q(1) = \frac{\text{NPV}_q}{I} = r_q^{\max}(1) \end{aligned}$$

where line 2 follows from the fact that the put is positive, line 3 follows from the fact that $\hat{q}$ (rather than q) maximizes the PPI. Line 4 follows from the fact that the maximized value of the PPI must exceed $\text{PPI}_q(1) = \frac{\text{NPV}_q}{I}$, which is also the maximal ROE of the banking sector for type q if $\underline{e}_q = 1$. As a result, $r_{\hat{q}}^{\max}\left(\underline{e}_{\hat{q}}^{PPI}\right) > r_q^{\max}(1)$ and (A.31) is slack.

P3 Suppose that type q_M is marginal, i.e., $0 < \tilde{\omega}_{q_M}(\underline{\mathbf{e}}) < \frac{\pi_{q_M} \underline{e}_{q_M} I}{E}$ then (A.29) and (IC) imply that welfare is given by:

$$\sum_{q: r_q^{\max} > r_{q_M}^{\max}} \pi_q \left(\text{NPV}_q - \lambda \cdot \text{PUT}_q(\underline{e}_q)\right) + \left(E - \sum_{q: r_q^{\max} > r_{q_M}^{\max}} \pi_q \underline{e}_q I\right) \text{PPI}_{q_M}(\underline{e}_{q_M}) \tag{A.32}$$

It is now easy to see that optimality of $\underline{e}_{q_M}$ requires that $\underline{e}_{q_M}^* = \arg\max_{\underline{e}_{q_M}} \text{PPI}_{q_M}\left(\underline{e}_{q_M}\right)$ since all other terms are independent of $\underline{e}_{q_M}$. This maximizer exists by Lemma A.3 and satisfies the first-order condition (A.24).

P4 We have to consider two cases. First, consider the case where all profitable types

[27] Our assumptions ensure that $\max_q \text{PPI}_q(\underline{e}_q^{PPI})$ is finite, see proof of Lemma A.3.

are financed. Then, the objective of the regulator is

$$\sum_{q:r_q^{\max}>0} \pi_q \left(\text{NPV}_q - \lambda \cdot \text{PUT}_q(\underline{e}_q)\right) \tag{A.33}$$

s.t. to the (binding) equity capacity constraint (by Principle P1)

$$E - \sum \pi_q \underline{e}_q I = 0 \tag{A.34}$$

(A.33) is a concave objective subject to a linear constraint (A.34). Denoting the associated Lagrange multiplier as κ we obtain the necessary and sufficient condition

$$-\lambda \pi_q \frac{\partial \text{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q} = \kappa \pi_q I,$$

which, means that the marginal put value for all types is a constant

$$-\frac{\partial \text{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q} = \frac{\kappa I}{\lambda},$$

which implies (15).
Next suppose that not all types are fully financed, i.e., there is a marginal firm type $0 < \tilde{\omega}_{q_M}(\underline{\mathbf{e}}) < \frac{\pi_{q_M} \underline{e}_{q_M} I}{E}$. Then for all inframarginal types q, the first-order condition of (A.32) satisfies:

$$-\pi_q I \text{PPI}_{q_M}(\underline{e}_{q_M}) = \lambda \pi_q \frac{\partial \text{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q} \tag{A.35}$$

Since the marginal type's capital requirement maximizes its PPI, we obtain by (A.24) that

$$I\text{PPI}_{q_M}(\underline{e}_{q_M}) = -\lambda \frac{\partial \text{PUT}_{q_M}(\underline{e}_{q_M})}{\partial \underline{e}_{q_M}} \tag{A.36}$$

Taken together, the two first-order conditions (A.35) and (A.36) imply that the marginal puts are equalized

$$\frac{\partial \text{PUT}_{q_M}(\underline{e}_{q_M})}{\partial \underline{e}_{q_M}} = \frac{\partial \text{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q}. \tag{A.37}$$

■

Proof of Proposition 4: Marginal changes in the cash flow distribution will not affect the ranking. In particular, this implies that dirty firms still rank above clean firms. The proof is written so that its claims can be applied to changes in the cash flow distribution of either type.

1. Suppose a firm type q is marginal, then its capital requirement is only a function of its own cash flow distribution characteristics $\left(\bar{X}_q, \sigma_q\right)$ and satisfies by principle

P3 (see Proposition 3) and Proof of Lemma A.3 the first-order condition

$$\bar{X}_q - I - G\left(\underline{e}_q\right) = 0, \tag{A.38}$$

where

$$G\left(\underline{e}_q\right) := \lambda \left[\mathrm{PUT}_q(\underline{e}_q) - \underline{e}_q \frac{\partial \mathrm{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q}\right]. \tag{A.39}$$

Since $G'\left(\underline{e}_q\right) < 0$ (see Proof of Lemma A.3), we obtain that $\left|G'\left(\underline{e}_q\right)\right| = -G'\left(\underline{e}_q\right)$. The comparative statics of the marginal type now follow from the implicit function theorem on (A.38)

$$\frac{\partial \underline{e}_q}{\partial \bar{X}_q} = -\frac{1 - \frac{\partial G\left(\underline{e}_q\right)}{\partial \bar{X}_q}}{\left|G'\left(\underline{e}_q\right)\right|} < 0, \tag{A.40}$$

$$\frac{\partial \underline{e}_q}{\partial \sigma_q} = \frac{\frac{\partial G\left(\underline{e}_q\right)}{\partial \sigma_q}}{\left|G'\left(\underline{e}_q\right)\right|} > 0, \tag{A.41}$$

where

$$\frac{\partial G\left(\underline{e}_q\right)}{\partial \bar{X}_q} = \lambda \left[\frac{\partial \mathrm{PUT}_q(\underline{e}_q)}{\partial \bar{X}_q} - \underline{e}_q \frac{\partial^2 \mathrm{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q \partial \bar{X}_q}\right] < 0 \tag{A.42}$$

$$\frac{\partial G\left(\underline{e}_q\right)}{\partial \sigma_q} = \lambda \left[\frac{\partial PUT_q}{\partial \sigma} - \underline{e}_q \frac{\partial^2 \mathrm{PUT}_q(\underline{e}_q)}{\partial \underline{e}_q \partial \sigma_q}\right] > 0 \tag{A.43}$$

The respective signs follow directly from the comparative statics in Lemma A.2.

2. In region 2, see Figure 5, where one type is fully financed and the other type is not at all financed, the capital requirement is just a function of the supply constraint, i.e.,

$$\underline{e}_q = \frac{E}{\pi_q I}, \tag{A.44}$$

which is independent of $\bar{X}_q$ and σ_q.

3. If the capital requirements of the types are equalized (see e.g., regions 3 or 4 in Figure 5, the first-order condition applies

$$\frac{\partial PUT_q}{\partial \underline{e}_q} - \frac{\partial PUT_{\tilde{q}}}{\partial \underline{e}_{\tilde{q}}} = 0. \tag{A.45}$$

Since $\frac{\partial^2 PUT_q}{\partial \underline{e}_q^2} > 0$ (by Lemma A.2), we obtain the following comparative statics

with respect to changes in the own cash flow characteristics:

$$\frac{\partial \underline{e}_q}{\partial \bar{X}_q} = -\frac{\frac{\partial^2 PUT_q}{\partial \underline{e}_q \partial X_q}}{\frac{\partial^2 PUT_q}{\partial \underline{e}_q^2}} < 0, \tag{A.46}$$

$$\frac{\partial \underline{e}_q}{\partial \sigma_q} = -\frac{\frac{\partial^2 PUT_q}{\partial \underline{e}_q \partial \sigma_q}}{\frac{\partial^2 PUT_q}{\partial \underline{e}_q^2}} > 0. \tag{A.47}$$

where $\frac{\partial^2 PUT_q}{\partial \underline{e}_q \partial X_q} > 0$ (by Lemma A.2) and $\frac{\partial^2 PUT_q}{\partial \underline{e}_q \partial \sigma_q} < 0$ (by Lemma A.2). The comparative statics regarding changes in the cash flow distribution of the other type $\tilde{q}$ satisfy:

$$\frac{\partial \underline{e}_q}{\partial \bar{X}_{\tilde{q}}} = \frac{\frac{\partial^2 PUT_{\tilde{q}}}{\partial \underline{e}_{\tilde{q}} \partial X_{\tilde{q}}}}{\frac{\partial^2 PUT_q}{\partial \underline{e}_q^2}} > 0, \tag{A.48}$$

$$\frac{\partial \underline{e}_q}{\partial \sigma_{\tilde{q}}} = \frac{\frac{\partial^2 PUT_{\tilde{q}}}{\partial \underline{e}_{\tilde{q}} \partial \sigma_{\tilde{q}}}}{\frac{\partial^2 PUT_q}{\partial \underline{e}_q^2}} < 0. \tag{A.49}$$

Taken together all cases, we obtain the claims in Proposition 4.

■

Proof of Proposition 5: Recall the definition of the PPI, i.e.,

$$\text{PPI}_q(\underline{e}_q^{PPI}) = \max_{\underline{e}_q} \frac{\bar{X}_q - I - \lambda \cdot \text{PUT}_q(\underline{e}_q)}{I\underline{e}_q}.$$

We first consider changes in the mean payoff $\bar{X}_q$. As is immediate, the PPI is strictly increasing in $\bar{X}_q$ (both because of lower financial NPV and higher put). Moreover, if $\bar{X}_q < I$, its associated $\text{PPI}_q(\underline{e}_q^{PPI})$ is below 0. As a result, there exists a threshold for $\bar{X}_D$ such that for any $\bar{X}_D$ below that threshold, the PPI of the dirty technology is below the clean technology satisfying $\text{PPI}_C(\underline{e}_C^{PPI}) > 0$. That is, the regulator's preferred types switches.

We now consider changes in the volatility (holding the mean payoff $\bar{X}_q$ constant). Similarly, the PPI is strictly decreasing in σ_q. The lowest possible PPI is achieved, as volatility approaches infinity (which is optimally countered by $\lim_{\sigma_q \to \infty} \underline{e}_q^{PPI} = 1$). Hence,

$$\begin{aligned}
\lim_{\sigma_q \to \infty} \text{PPI}_q(\underline{e}_q^{PPI}) &= \lim_{\sigma_q \to \infty} \max_{\underline{e}_q} \frac{\bar{X}_q - I - \lambda \cdot \text{PUT}_q(\underline{e}_q)}{I\underline{e}_q} \\
&= \frac{\bar{X}_q - I - \lambda \cdot \text{PUT}_q(1)}{I} \\
&= \frac{\bar{X}_q - I}{I} = \frac{\text{NPV}_q}{I}
\end{aligned}$$

Then, as long as $\frac{\text{NPV}_D}{I} < \text{PPI}_C(\underline{e}_C^{PPI})$, a change in the regulator's preferred type occurs if σ_D becomes sufficiently large. ■