Searching for Hysteresis*

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Abstract

We search for hysteresis in aggregate post-WWII U.S. data. Defining hysteresis as the presence of aggregate demand shocks with a permanent impact on real GDP, we identify such shocks based on Bayesian VARs via a combination of zero long-run restrictions, and both short- and long-run sign restrictions. Evidence suggest that hysteresis effects have been virtually absent for the sample excluding the financial crisis and the Great Recession, and they only appear when including the period following the collapse of Lehman Brothers. Even then, however, these effects only account for at most 10 per cent of the long-run variance of GDP. We show via a DSGE model-based simulation analysis that there is a high probability of detecting hysteresis effects even when the data-generating process features none by construction. We account for this misspecification with a Monte Carlo-based correction to the 'simple' Bayesian estimates.

Keywords: Hysteresis; Bayesian methods; unit roots; permanent shocks.
JEL Classification: E2, E3.

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1 Introduction

The term hysteresis captures the notion that economic disturbances that are typically regarded as transitory, for instance, monetary policy shocks, can have very long-lived or even permanent effects. Hysteresis is most often associated with the labor market, where long and deep recessions can lead to long-term unemployment and could raise the natural (or equilibrium) unemployment rate by permanently changing job-finding prospects. Similarly, a broad array of demand shocks could have long-lasting effects on productivity and output through channels such as business formation or exit. The experience of the Great Recession and of the recession caused by the COVID-19 pandemic have rekindled interest in the possible presence of hysteresis in macroeconomic data. While the theoretical channels for the transmission of hysteresis shocks are well-understood, conclusive empirical evidence is scarce.

In this paper, we search for hysteresis, which we define as the presence of aggregate demand shocks with a permanent effect on output. We identify permanent aggregate demand and aggregate supply shocks by imposing a combination of zero restrictions on their long-run impact on GDP and sign restrictions on their short- and their long-run impacts on GDP and the price level. The identifying restrictions are derived from a standard macroeconomic model of aggregate demand and supply and are consistent with a wide range of theoretical frameworks, such as the DSGE models that are used in central banks. We then apply our identification scheme to Bayesian VARs estimated on post-WWII US data.

Our paper makes three main contributions. First, we find a non-negligible, but modest extent of hysteresis in the data, which explains roughly 20 percent of the frequency zero (i.e., long-run) variance of real GDP. However, these findings are characterized by a very large extent of uncertainty. Second, we show via Monte-Carlo analysis of a theoretical DSGE model with a hysteresis mechanism that reliably detecting hysteresis in macroeconomic data is a significant challenge. In particular, we demonstrate that model-consistent identification schemes spuriously detect hysteresis with non-negligible probability when there is none in the data-generating process. In addition, such identification schemes tend to have low power to discriminate between alternative extents of hysteresis.

Our third contribution lies in developing an approach to test for the presence of hysteresis and to estimate its extent. Our approach uses Monte-Carlo analysis to simulate specific statistics, such as the fraction of the long-run output variance that is due to hysteresis shocks, conditional on the extent of hysteresis imposed on the estimated Bayesian VAR.

We then estimate the extent of hysteresis in the data by comparing the thus obtained Monte-Carlo distributions to the corresponding distributions computed based on the actual data. Specifically, we use as a metric for the extent of hysteresis the closeness of the simulated and the actual distribution the Kolmogorov-Smirnov statistic. This also allows us to quantify the implied correction to misidentification issues. We then apply this approach to our dataset and find an even more modest extent of hysteresis, equal to about 10 percent of the long-run variance of GDP. When we exclude the Great Recession period from the sample, any evidence of hysteresis disappears.

The classic paper on hysteresis is Blanchard and Summers (1986), which introduced the concept to the economics profession and used it in order to explain the persistence of European unemployment in the 1980s. Although they provided some basic statistical evidence, their main argument was largely based on a theoretical model of the labor market. In a similar vein, Ljungqvist and Sargent (1998) provided a more microfounded theoretical approach to the same issue and offered corroborating evidence based on labor market data. More recently, theoretical frameworks based on the idea of endogenous TFP have also allowed for a possible role of hysteresis effects (see e.g., Anzoategui et al., 2019; or Jaimovich and Siu, 2020).

Empirical evidence in favor of or against hysteresis is somewhat sparse, arguably because of the difficulty in distinguishing permanent and highly persistent components in aggregate data. In fact, in his recent survey of the literature on the natural rate hypothesis, Blanchard (2018) argued that the evidence is not sufficiently clear-cut to reach strong conclusions. Cerra and Saxena (2008) showed that in a sample of 190 countries over the period 1960-2001 deep recessions permanently reduced the productive capacity of an economy. Ball (2009) used a simple Phillips curve framework to back out the effects of changes in inflation on unemployment, conditional on having observed large disinflations associated with deep recessions. Although he argued for the presence of hysteresis, in the end he did not distinguish between permanent and highly persistent effects. Galí (2015, 2020) took up Ball's analysis and integrated it within a New Keynesian model featuring an insider-outsider labor market framework as in Blanchard and Summers (1986). Based on a quantitative analysis of the model, he argued in favor of hysteresis as being an important driver of the European unemployment and wage/price inflation experience.

On the empirical side, Furlanetto et al. (2021) is the paper that is closest to our work. They also use a structural VAR framework that combines long-run zero restrictions with short- and long-run sign restrictions. In contrast with our results, they do detect an

important role for hysteresis effects in U.S. data. As we discuss in the next section, one of the differences between Furlanetto et al. (2021) and the present work is that they use a weaker set of restrictions in order to identify hysteresis shocks. Of note is also the recent contribution by Jordà et al. (2020), who estimate a dynamic panel with local projections and detect large (compared with most contributions in the literature) effects of monetary policy shocks on GDP in the very long run.

The paper is organized as follows. The next section discusses our identifying restrictions for hysteresis shocks. In section 3 we outline our empirical methodology and the VAR specification in detail. We present the empirical findings from a baseline specification and some robustness checks in section 4. Section 5 illustrates the difficulty of robust identification of hysteresis effects in macroeconomic data even when based on model-consistent identification schemes. To this end we perform a Monte-Carlo analysis of a DSGE model with embedded hysteresis. Section 6 discusses our proposed solution to this problem, based on a Monte Carlo-based correction to the 'simple' estimates produced by the Bayesian procedure. We also show and discuss estimates produced by the proposed Monte Carlo-based correction. Section 7 concludes. An extensive online appendix provides additional details on the methodology and on several aspects of our empirical work.

2 Identifying Hysteresis Shocks

It is often assumed in theoretical and empirical macroeconomics that permanent increases in GDP originate on the supply side of the economy, for instance, in form of permanent shocks to total factor productivity (TFP). The key idea underlying the notion of hysteresis is that some permanent output shocks could instead originate on the demand side. The challenge for empirical researchers is therefore to separate permanent supply from permanent demand shocks and also from persistent, but transitory moves in macroeconomic aggregates. We now discuss our identification assumptions on how to accomplish this.

We assume that the permanent component of real GDP is driven by two shocks that we associate with aggregate supply (AS) and aggregate demand (AD) disturbances. We label the two disturbances as 'BQ' (from Blanchard and Quah, 1989) and 'H' (for 'hysteresis'), respectively. The identifying restrictions we impose in order to disentangle the two disturbances are motivated by the standard AD-AS framework found in many macroeconomic textbooks. They are also consistent with a wide range of theoretical frameworks, such as medium- or large-scale DSGE models that are used in policy institutions. Specifically, we make the following assumptions:

(I) the AD curve is downward-sloping both in the short and in the long run;

(II) the AS curve is upward-sloping in the short run, but vertical in the long run;

(III) BQ shocks only affect the AS curve;

(IV) H shocks affect the AD curve and can affect the AS curve in the case of hysteresis, when a negative (positive) H shock has a negative (positive) permanent impact on the long-run AS curve.

The first three assumptions are standard and are consistent with a wide range of macroeconomic frameworks such as the textbook AD-AS model, simple New Keynesian models, and the large-scale macroeconomic models. The fourth assumption captures the essence of the notion of hysteresis: A negative (positive) AD shock has a negative (positive) permanent impact on long-run aggregate supply. As an example, deep recessions can permanently scar the economy by reducing its potential productive capacity, either by increasing the equilibrium unemployment rate or by reducing firm entry, and thereby long-run productivity. Alternatively, 'running the economy hot' could permanently attract people into the labor force, thereby increasing potential output. Figure 1 provides an illustration of our identification assumptions in output (Y)-price level (P) space.

In the first panel, BQ shocks in the short run shift the AS curve and therefore move the economy along the AD curve. This causes output and prices to move in opposite directions. Similarly, H shocks move the economy along the AS curve with output and prices moving in the same direction. Consequently, the first set of restrictions we impose in order to disentangle the two shocks is that on impact and up to h quarters, BQ shocks generate impulse responses for output and prices with *opposite signs*. In contrast, H shocks produce impulse responses with the *same sign*.[1] The second panel of Figure 1 illustrates assumption (III), namely that in the long run BQ shocks only affect the AS curve. This results in permanent effects on output and prices with *opposite signs*.

As for H shocks there are three possible cases. First, the case of no hysteresis in the third panel shows that the long-run impact on output is *zero*, while the *sign* of the long-run impact on prices is the *same* as the sign of the short-run impact. In the case of hysteresis (fourth and fifth panels) a negative (positive) H shock shifts the AD curve down and to the left (up and to the right) and the long-run AS curve to the left (right). Consequently, the

[1] These restrictions are the same as Canova and Paustian's (2011) DSGE-based 'robust sign restrictions' (see their Table 2, page 348). Their model features two demand shocks, namely a monetary and a preference shock. For both shocks, the impact on output and inflation are of the same sign.

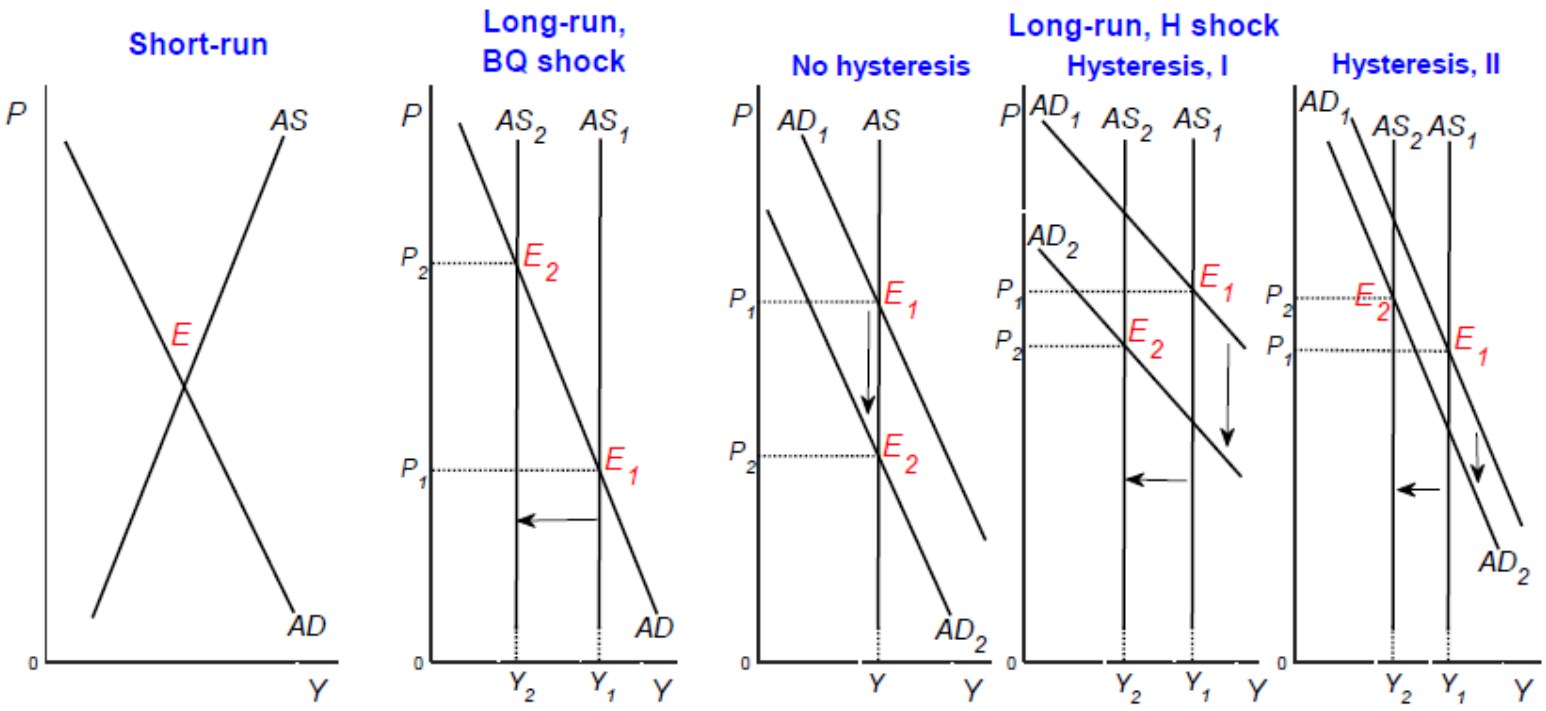


Figure 1 A simple illustration of the identifying assumptions

long-run effect on output is unambiguously negative (positive), so that its sign is the *same* as that of its short run response to the H shock. The long-run impact on prices, on the other hand, is ambiguous. It depends on the size of the hysteresis effect on output and the slope of the AD curve. In particular, the flatter the AD curve and the smaller the hysteresis effect on output, the greater the likelihood that a negative (positive) H shock causes a decrease (increase) in prices as in the fourth and fifth panels. In the limit case of no hysteresis on output in the long run (third panel), the long-run impact on prices of a negative (positive) H shock is unambiguously negative (positive).

In general, however, this pattern cannot be assumed. Some researchers may find the assumption that a negative (positive) H shock has a negative (positive) long-run impact not only on output, but also on prices more plausible. A key difference between our work and Furlanetto et al. (2021) is that they do not impose sign restrictions on the long-run impact of BQ and H shocks on output and the price level. Rather, they only impose the restriction that BQ and H shocks are the only two disturbances that have a permanent impact on output and employment (see their Table 1). In what follows, we therefore consider two sets of results, where we alternatively impose and not impose this restriction on prices. To anticipate the results, both assumptions lead to qualitatively the same outcomes, so that in practice imposing or not imposing this restriction does not appear to make a material difference.

We summarize the previously discussed restrictions in Table 1. We now turn to discussing our empirical specification, how we estimate the model via Bayesian methods, and our choice of priors, and how we implement the identifying restrictions outlined in Table 1.

Table 1: Identifying Restrictions

	Shock:				
Impact on:	BQ	H	BQ	H	
	Short run:		Long run:		
Output	+	+	+	0	+
Price level	−	+	−	+	?/+

3 Empirical Approach

Our baseline empirical specification includes the logarithms of real GDP, real consumption, real investment, and total hours worked per capita; a long- and a short-term nominal interest rate; and the logarithm of the PCE deflator. We collect these variables in the vector $Y_t = [y_t, c_t, i_t, R_t, r_t, p_t, h_t]'$, where y_t, c_t, i_t, and h_t are, respectively, the logarithms of GDP, consumption, investment, and total hours worked per capita; p_t is the logarithm of the PCE

deflator; and R_t and r_t are a short- and a long-term nominal interest rates. For the short-term rate we use the 'shadow rate' from Wu and Xia (2016) to capture the effective policy stance during the zero-lower bound period. The long-term rate is the 5-year government bond yield. The data and their sources are described in detail in Online Appendix A. We consider two sample periods, 1954Q3-2008Q4 and 1954Q3-2019Q4, whereby the latter sample includes the Zero Lower Bound (ZLB) period. As discussed in Online Appendix B, Elliot et al.'s (1996) tests of the null hypothesis of a unit root suggest that all of the series in Y_t are I(1) with the exception of inflation, $\pi_t = p_t - p_{t-1}$, for which a unit root is rejected.

We proceed with the empirical analysis as follows. Our baseline specification is a Bayesian cointegrated VAR given the findings from the unit root tests. Specifically, this VAR includes the logarithm of the price level. We also consider a second specification in terms of stationary variables. This is motivated by the argument that interest rates and hours per capita are likely not I(1) on conceptual grounds. Results from this specification are included in the Online Appendix, but they paint a similarly consistent picture as the baseline model.

Our approach to Bayesian cointegration is based on methods introduced by Strachan and Inder (2004) and Koop et al. (2010). We specify a VAR in form of a vector error correction model (VECM):

$$\Delta Y_t = B_0 + B_1 \Delta Y_{t-1} + ... + B_p \Delta Y_{t-p} + \alpha \beta' Y_{t-1} + u_t, \tag{1}$$

where β is the matrix of the cointegration vectors, α is the loading matrix, $E[u_t' u_t] = \Sigma$, and the rest of the notation is standard.

For the r cointegration vectors to be uniquely identified, the columns of the matrix β need to feature at least r restrictions (see Kleibergen and van Dijk, 1994, or Bauwens and Lubrano, 1996). The standard classical solution proposed by Johansen (1988, 1991) is to rely on the identification method for reduced-rank regression models introduced by Anderson (1951). Within a Bayesian setting, several methods have been proposed. Following Koop et al. (2010), we adopt the approach proposed by Strachan and Inder (2004), which is based on imposing the normalization that β is semi-orthogonal:

$$\beta' \beta = I_r, \tag{2}$$

where I_r is the $r \times r$ identity matrix.[2] This restriction is imposed by defining a semi-orthogonal matrix $H = H_g (H_g' H_g)^{-1/2}$, which centers the prior for the cointegration space

[2] We also consider the "linear normalization" approach proposed by Geweke (1996). The results are qualitatively similar, but we use Strachan and Inder (2004) because it is more general.

around the value that a researcher considers the most plausible. Since H and H_g span the same space, it can obtained by appropriately selecting the columns of H_g based on a priori information, for instance derived from economic theory.

As discussed in Online Appendix B, Johansen's tests strongly reject the null hypothesis of no cointegration for any of the three bivariate systems featuring GDP and consumption, GDP and investment, and a short- and a long-term nominal interest rate. Similarly, the same tests identify three cointegration vectors for the baseline 7-variables system in $Y_t = [y_t, c_t, i_t, R_t, r_t, p_t, h_t]'$ for the period 1954Q3-2019Q4 and two for the period 1948Q1-2008Q4. For the latter period, lack of identification of a third cointegration vector is marginal, with p-values for the trace and maximum eigenvalue tests both equal to 0.11. In the following, we therefore proceed under the assumption that the baseline system features three cointegration vectors for either sample.

In light of this evidence, we center the prior for β at the three cointegration relationships consistent with three bivariate systems. Specifically, this is obtained by setting H_g to:

$$H_g' = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -2/3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \end{bmatrix}. \tag{3}$$

The rows of H_g' correspond to the normalized cointegration vectors for GDP and consumption, GDP and investment, and the short and the long rate, respectively. This choice reflects the fact that in the long run GDP and consumption and short- and long-term nominal interest rates, respectively, tend to move essentially one-for-one in all countries and for all historical periods. On the other hand, investment typically tends to increase somewhat faster than GDP. For example, in the U.S. over the period 1947Q1-2019Q4 real GDP has increased by 1.97 on a log scale, whereas the corresponding increase for real investment has been 2.75. The increase for the former series is 71.6 per cent of the increase for the latter. We want to stress, however, that given the uninformativeness of the prior for β (discussed in Appendix A), the centering encoded in this choice for H_g imposes, in fact, very weak restrictions.

We estimate the cointegrated VAR using the Gibbs-sampling algorithm proposed by Koop et al. (2010). The algorithm iterates on four steps: (i) the loading matrix α in equation (1), (ii) the matrix of the cointegration vectors β, (iii) the VAR coefficient matrices B_j, and (iv) the covariance matrix Σ, which jointly describe a single pass of the Gibbs sampler. We run a burn-in pre-sample of 100,000 draws, which we discard. We then generate 1,000,000 draws that we thin by sampling every 100 draws in order to reduce their autocorrelation. This leaves us with 10,000 draws from the ergodic distribution, which we use for inference.

Our prior choice is informed by Geweke (1996) and Koop et al. (2010), from which we also take the conditional distributions to draw from the posterior described in steps (i)-(iv) above. The priors are generally not very informative. In particular we impose a flat prior on the matrix of the cointegration vectors β. We discuss the priors in more detail in Appendix A. Figure A.20 in the Online Appendix reports evidence on the convergence of the Markov chain, by showing Geweke's (1992) inefficiency factors of the draws for each individual parameter. For all parameters the factors are equal to at most 8, well below the values of 20-25 that are typically taken to indicate problems in the convergence of the Markov chain.

We jointly identify a BQ and an H shock using a combination of zero long-run restrictions on GDP and both short- and long-run sign restrictions on GDP and the price level; that is, we impose the restrictions summarized in Table 1 using the methodology proposed by Arias et al. (2018) and adapted for our Bayesian cointegrated VAR. Further details on the procedure are relegated to the Online Appendix. We impose the short-run sign restrictions both on impact and for the subsequent either four or eight quarters. Since the two sets of results are qualitatively very similar, we focus on those based on imposing the restrictions for eight quarters. For each draw from the posterior distribution of the reduced-form VAR we consider 100 random rotation matrices that we draw based on Arias et al.'s (2018) algorithm. We set the number of Gibbs-sampling iterations in the algorithm to 10.

4 Empirical Evidence on Hysteresis

We report three sets of empirical results. Figure 2 shows the impulse response functions (IRFs) of our baseline model to BQ and H shocks, while Figure 3 contains fractions of forecast error variance (FEV) of the variables explained by the shocks over the same horizon. Table 2 reports the posterior distributions of the fractions of the long-run variance of GDP and consumption explained by H shocks, which is a key metric in assessing the extent of hysteresis. As discussed above, the baseline specification is a cointegrated VAR estimated for the sample including the ZLB period and without imposing restrictions on the long-run impact of H shocks on the price level. Figures A.4 and A.5 in the Online Appendix show the corresponding results for the sample excluding the ZLB period. Figures A.6-A.9 report for either sample IRFs and fractions of FEV obtained by imposing the restriction that an H shock causing a permanent increase in GDP, consumption, and investment also causes a permanent increase in the price level.

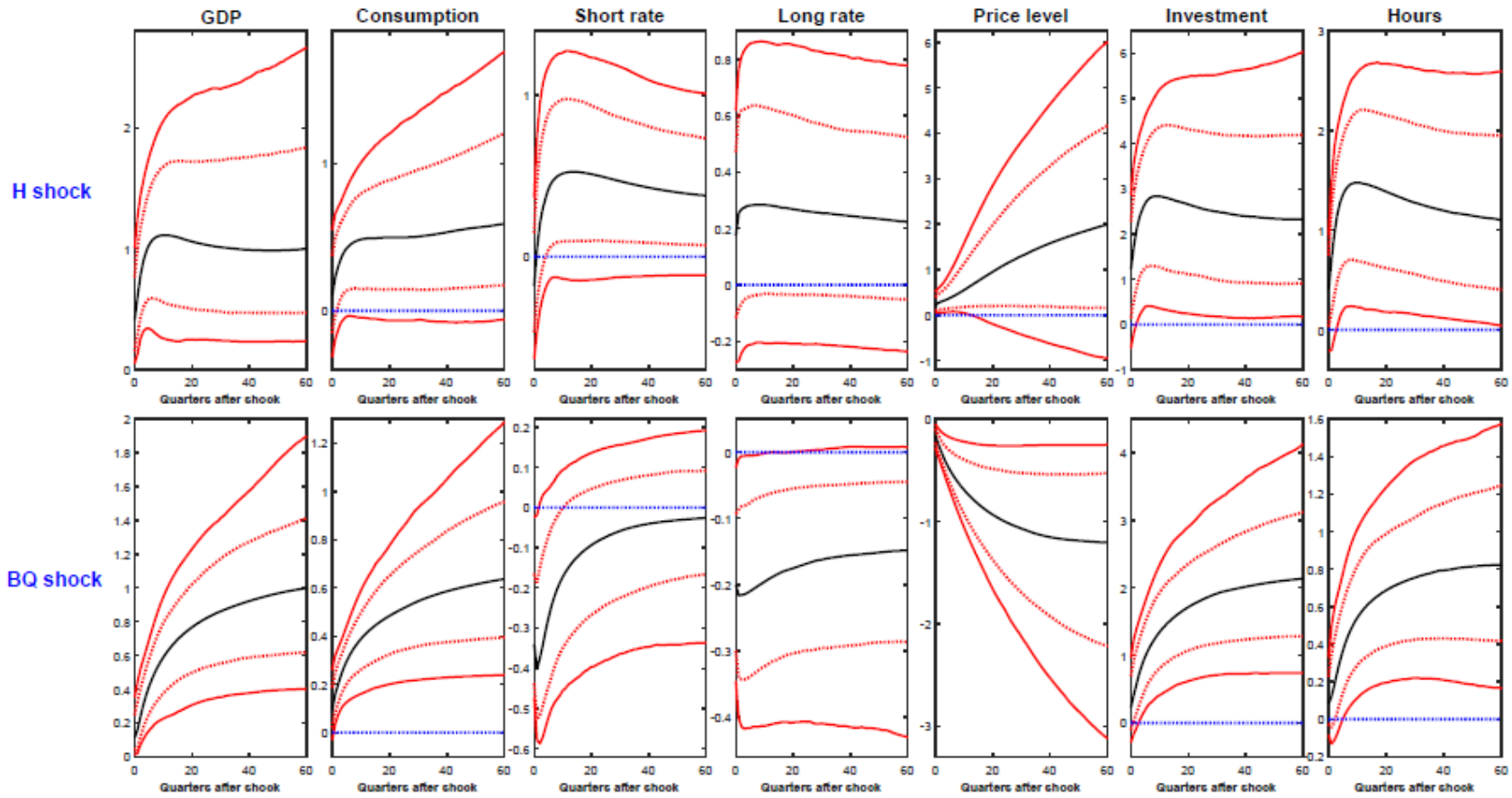


Figure 2 Impulse-response functions to H and BQ shocks, based on the 7-variables cointegrated VAR (including the Zero Lower Bound)

4.1 Impulse Response Functions

Figure 2 shows the median and the 16-84 and 5-95 percentiles of the posterior distributions of the IRFs to BQ and H shocks. The responses are essentially as expected and are, in fact, the result of the restrictions imposed on GDP, consumption, investment and the price level. Specifically, a BQ shock permanently increases the quantity variables and lowers the price level as would be consistent with a permanent TFP shock. A similar pattern applies to the IRFs to an H shock, with the exception of the price level. In contrast, the responses of the interest rates to either shock is insignificant at long horizons although the median responses are positive. The response of hours is estimated to be permanent for either shock, so that a positive BQ or H shock causes a long-run increase in hours worked.[3]

Turning to the price level, the IRFs for the two samples obtained without imposing restrictions on the long-run impact of H shocks reveal an interesting observation. In the full sample including the ZLB (Figure 2) a small, but non-negligible mass of the posterior distribution of the IRF to H shocks lies below zero. Moreover, the long-run impact of H shocks on the price level is estimated to be negative for 18.9 percent of the draws. This suggests that the data provide some support to the notion discussed in Section 2 (see the panel labelled as 'Hysteresis II' in Figure 1) that a positive (negative) H shock may counterintuitively have a negative (positive) long-run impact on the price level. In the sample that excludes the ZLB period (reported in Figure A.4 in the Online Appendix) 50 percent of the draws are associated with a negative impact of H shocks on prices at the 15-year horizon. In the long run, the corresponding fraction is equal to 65.6 per cent. Taken at face value these results imply that possibility illustrated in the panel labelled as 'Hysteresis II' in Figure 1 is significantly more likely than in the full sample. At the same time, the fallout from the financial crisis was a persistently high unemployment and a historically high long-term unemployment rate, which can be seen as a prime example of hysteresis. Including this time period in the estimation thus appears to produce prima facie evidence for the presence of hysteresis shocks, evidenced by an increase in the price level.

However, there is a second possible interpretation. It may very well be that in this sample H shocks are virtually absent. The result is then simply an artifact of the identifying assumption imposed on the data, namely that our model specified the existence of an H shock by brute force. Suppose that the true data-generating process (DGP) only features BQ shocks. Since we impose only short-run restrictions on the price level's response to H

[3] The IRFs produced by stationary VARs are closely in line with the evidence from the cointegration specification. The main difference is that by construction the responses of hours mean-revert to zero.

shocks the set of identified H shocks is obtained by retaining all of the models randomly generated by Arias et al.'s (2018) algorithm, for which one of the two permanent GDP shocks exhibits aggregate-demand features in the short run. At the same time, we do not impose any restriction on the long-run impact of H shocks on prices; moreover, all permanent GDP shocks cause a permanent decrease in the price level. Consequently, the response of prices to H shocks will tend to be negative in the long run. This insight offers a suggestion that there is the possibility of spuriously identifying hysteresis when the DGP features none. We investigate this issue in the next section.

4.2 Forecast Error Variance

Figure 3 shows the fractions of the forecast error variance that is explained by H and BQ shocks for our baseline specification. The two shocks jointly explain very large fractions of the FEV of GDP at long horizons, namely about 80 percent. A similar picture emerges from the other specifications (reported in the Online Appendix) for either sample period and any specification, whether cointegrated or stationary VARs, or either restricting or not restricting the long-run impact of H shocks on the price level. We also find that H shocks explain between 10 and 20 percent of the FEV for the other variables at the 15-year horizon, with hours worked at the upper end of this range.

It is noteworthy that the fraction of the consumption FEV is overall fairly small, ranging between about 4 and 10 percent, whereas the corresponding fraction explained by BQ shocks is between 60 and 65 percent. This is an important finding since the stochastic trend in GDP carries over closely to consumption (see Cochrane, 1994). It should therefore be informative about issues pertaining to the unit root component of output. The fact that H shocks are estimated to play such a small role for consumption suggests that if hysteresis effects are present in the data they are likely small. In addition, in the baseline specification over the full sample and without restricting the long-run impact of H shocks on prices, H shocks explain 14 percent of the overall fraction of the FEV of GDP at the 15-year horizon. The corresponding value for the sample excluding the ZLB period is 6 percent.

We now dig deeper into the contribution of H shocks by computing the frequency-zero, or long-run, variance of GDP and consumption. Table 2 reports the median and the 16-84 and 5-95 percentiles of the respective posterior distributions attributable to H shocks. We find that the estimates for consumption are consistently smaller than those for GDP, in line with the previous discussion. While the difference is small for the full sample, when we exclude the ZLB period it is non-negligible. Based on our preferred model, namely without

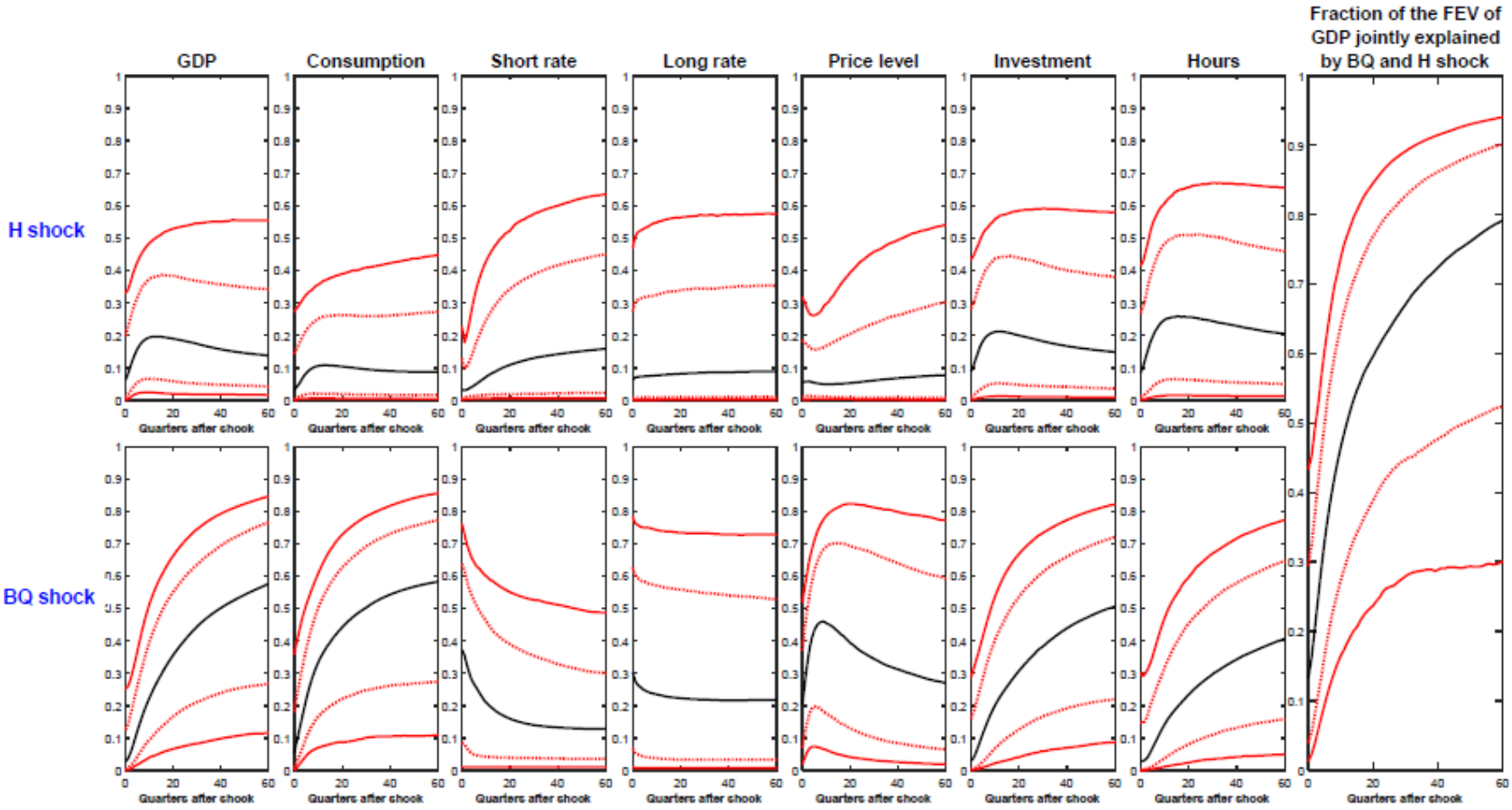


Figure 3 Fractions of forecast error variance explained by H and BQ shocks, based on the 7-variables cointegrated VAR (including the Zero Lower Bound)

restricting the long-run impact of H shocks on the price level, the median estimates are 22.1 percent for GDP and 11.9 percent for consumption.

When we focus on consumption, the evidence suggests that estimates for the sample excluding the ZLB are materially lower than those for the full sample. Specifically, median estimates for the samples including and excluding the ZLB period, respectively, are 0.175 and 0.119 when not imposing restrictions on the long-run impact of H shocks on the price level. When imposing such restrictions, the corresponding values are 0.218 and 0.056. This shows that for the sample excluding the ZLB period hysteresis effects are very small to negligible and that they only appear when also considering the years including the financial crisis and the Great Recession. In a sense, this is not surprising as it is that period that likely has seen hysteresis effects at play because of historically high long-term unemployment. But what also plays a role is the assumption on the price level response to H shocks. In particular, when we restrict the long-run impact of H shocks on prices as in Furlanetto et al. (2021), we typically find smaller estimates of the fraction of the unit root of either GDP or consumption explained by H shocks. This is especially apparent in the case of stationary VARs.

Overall, the evidence from Bayesian VARs points towards a modest, but non-negligible extent of hysteresis. In our baseline specification with a cointegrated VAR this amounts to 20 percent of the long-run variance of GDP for the sample including the ZLB period. When we exclude the ZLB period, the evidence is more mixed, but overall points towards a smaller role played by H shocks in driving long-horizon variation in GDP and consumption. There are two caveats to this result. First, the degree of uncertainty is fairly high as is evidence by the wide spread of the coverage regions in Figures 1 and 2. This is largely due to the nature of the problem econometricians are facing when attempting to extract information pertaining to either long horizons or in the limit the long run from finite data samples (see Faust and Leeper, 1997).

A second caveat is that our approach imposes on the data the presence of H shocks by construction. This raises the obvious question of whether the small, but non-negligible role of H shocks we have identified is a genuine result, or whether it is an artifact of our identification strategy. We tackle this issue in the following section. We show that the problem is indeed potentially serious by conducting a simulation exercise in a simple DSGE model featuring the possible presence of hysteresis. In particular, we show that model-consistent identification schemes detect a non-negligible, and possibly sizeable role of H shocks even when the DGP features no hysteresis by construction. We then propose a

simple Monte Carlo-based correction designed to remedy this issue.

5 Pitfalls in Identifying Hysteresis Shocks

In our baseline implementation we pursued the strategy of jointly identifying a BQ and an H shock, that is, two permanent shocks that affect GDP, the former through what may be deemed supply-side effects, and the latter via an aggregate demand channel. Arguably, this is the most natural way of imposing the restrictions in Table 1. However, this implementation suffers from the potential shortcoming that it may spuriously detects hysteresis when there is none in the underlying DGP. The reason for this is straightforward. Since this strategy jointly identifies both a BQ and H shock, it, in fact, imposes on the data the very existence of hysteresis by assumption. Ultimately, it is an empirical issue how problematic this is in an assessment of hysteresis. To that end, we conduct a simulation study that explores and quantifies the potential for misidentification.

5.1 The Data-Generating Process

The theoretical model that serves as the DGP for the Monte-Carlo study is a version of the model described in Galí (2015) augmented with habit-formation in consumption and a unit root in technology (and therefore in the natural level of output). Hysteresis is introduced in a somewhat reduced-form manner, designed to capture the essence of the mechanism without taking a precise stand on its origin. The model is described in detail in Online Appendix D. In the following we therefore only provide an outline of its structure.

Households solve the optimization problem:

$$U_0^* = \underset{C_t, N_t}{Max} E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t - bC_{t-1}) - \frac{N_t^{1+\phi}}{1+\phi} \right], \tag{4}$$

subject to:

$$P_t C_t + Q_t B_t = B_{t-1} + W_t N_t - T_t, \tag{5}$$

where C_t and N_t are real consumption and hours worked; $0 < b < 1$ is the habit-formation parameter; P_t is the price of consumption goods; B_t is the stock of nominal bonds; Q_t is the time-t price of a nominal bond paying one dollar at time $t+1$; W_t is the nominal wage; T_t are nominal lump-sum taxes; and the rest of the notation is standard.

We follow Galí (2015) in modeling the firms, the only difference being the process for technology. Firms produce output Y_t via the production function $Y_t = A_t N_t^{1-\alpha}$, with $0 < \alpha < 1$, A_t being technology, and the capital stock being constant and normalized to 1.

They maximize profits with respect to N_t. We assume that the logarithm of technology, $a_t = \ln(A_t)$ evolves according to:

$$a_t = a_{t-1} + \epsilon_t^a + \delta \tilde{y}_{t-1}, \tag{6}$$

where $\epsilon_t^a \sim N(0, \sigma_a^2)$ is the technology shock, $\tilde{y}_t$ the transitory component of output, and $\delta \geq 0$ captures the possible presence of hysteresis effects. If $\delta = 0$ there is no hysteresis, whereas if $\delta > 0$ positive (negative) transitory output fluctuations cause subsequent permanent increases (decreases) in a_t. This specification is conceptually the same as in Jordà et al. (2020). It is designed to capture the notion that positive (negative) deviations of GDP from potential (here, deviations of output from its stochastic trend A_t) may have a positive (negative) impact on potential GDP itself. Finally, monetary policy is described by a standard Taylor rule with interest-rate smoothing:

$$R_t = \rho R_{t-1} + (1-\rho)\phi_\pi \pi_t, \tag{7}$$

where the notation is standard.[4]

We calibrate the structural parameters as follows, based on standard values in the literature: $\beta = 0.99$, $\alpha = 1/3$, $b = 0.8$, $\phi = 1$, $\rho = 0.9$, $\phi_\pi = 1.5$, and $\sigma_a = 0.007$.[5] The model is augmented with three transitory AR(1) disturbances; two of them (v_t and u_t) can be thought of as supply shocks, while the third (e_t) is a demand shock. We also include an additive disturbance to the log of the production function, $z_t \sim N(0, \sigma_z^2)$, with $\sigma_z = 0.005$. The autoregressive parameters are calibrated as $\rho_v = \rho_u = \rho_e = 0.75$, while the standard deviations of the disturbances' innovations ϵ_t^v, ϵ_t^u, and ϵ_t^e (all zero-mean, and normally distributed) are set to $\sigma_u = 0.001$, $\sigma_v = 0.005$ and $\sigma_e = 0.045$.

Based on this calibration the permanent technology shock (ϵ_t^a) explains one-third of the forecast error variance of log GDP on impact. In the absence of hysteresis, when $\delta = 0$, it explains about 96 per cent of GDP's FEV 15 years ahead. These numbers are broadly in line with the evidence produced by the structural VAR literature (see for example Cochrane, 1994). Moreover, the demand innovation ϵ_t^e is essentially the only driver of the transitory component of output. In that sense, this validates the identifying restrictions in Table 1. In the Monte Carlo exercise we consider a grid of values for the hysteresis parameter δ,

[4] Online Appendix D reports the log-linearized equations of the stationarized model. Output and the real wage inherit the unit root in a_t, and therefore need to be stationarized for standard solutions methods for linear rational expectations models to be applicable. This is accomplished by defining the stationarized variables $\tilde{Y}_t \equiv Y_t/A_t$ and $\tilde{\Omega}_t \equiv (W_t/P_t)/A_t$, with $\tilde{y}_t$ and $\tilde{\omega}_t$ being their respective log-deviations from the steady-state. $\tilde{y}_t$ is thus the component of output that is driven by the transitory disturbances.

[5] The value for σ_a is based on Watson's (1986) estimate of the standard deviation of shocks to the stochastic trend of log real GDP. The rationale is that a_t is the random-walk component of log real GDP.

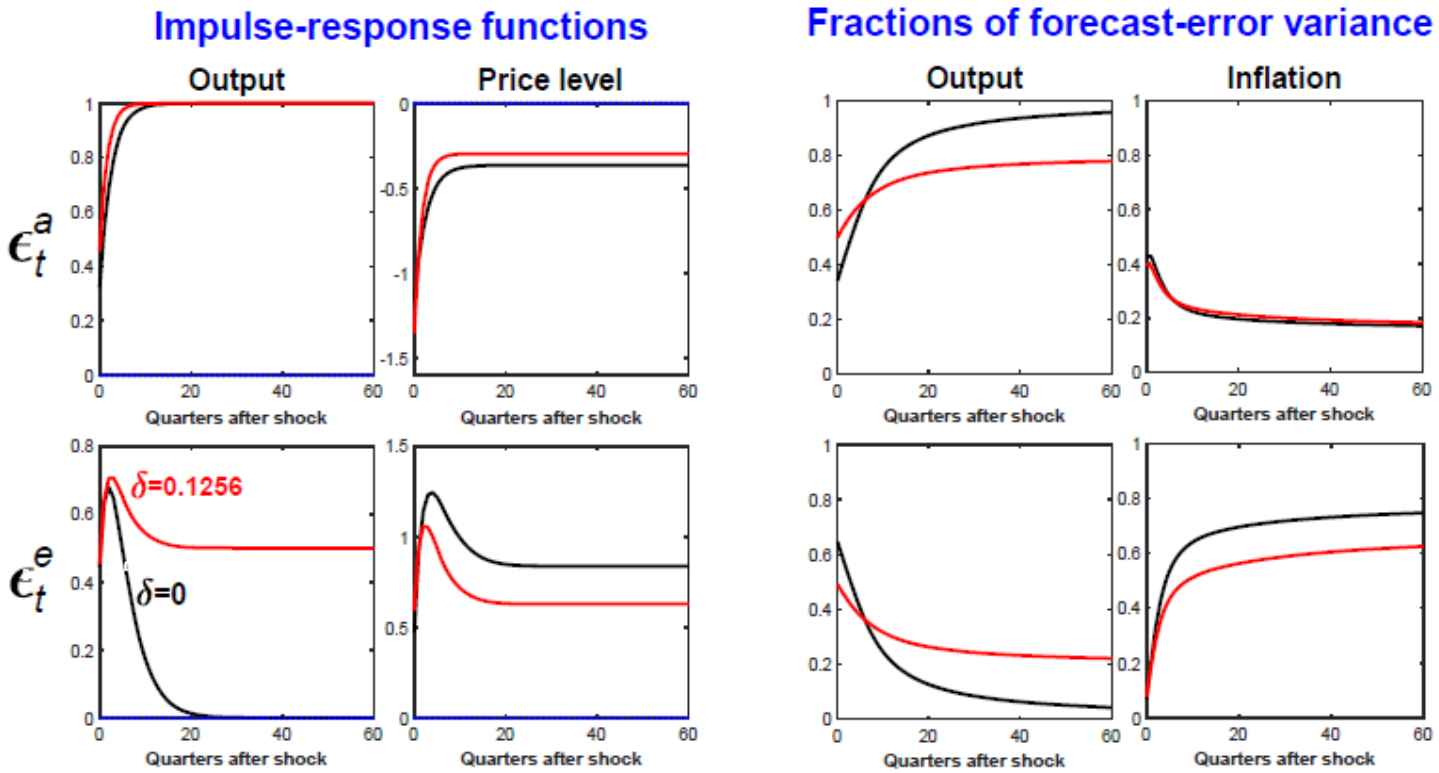


Figure 4 Theoretical impulse-response functions and fractions of forecast error variance for the RBC model for output and prices, for ϵ_t^a (BQ) and ϵ_t^e (H) shocks

ranging from $\delta = 0$ (no hysteresis) to $\delta = 0.1256$. In the latter case, the demand shock ϵ_t^e explains 20 per cent of the long-run variance of output, while at long horizons the other three shocks contribute essentially nothing to the variation in output.

Figure 4 shows impulse responses for output and prices in response to ϵ_t^a and ϵ_t^e, together with the fractions of FEV of output and inflation explained by the two shocks, for both $\delta = 0$ and $\delta = 0.1256$.[6] The dynamic behavior of the model suggests that it is an appropriate DGP for assessing the ability of the identifying restrictions in Table 1 to recover the extent of hysteresis in the data. First, for either of the two values of δ (as well as for any value inbetween) the two shocks explain sizeable, or even dominant, fractions of the FEV of either variable at least over some non-negligible horizon. For example, even for $\delta = 0$, ϵ_t^e is the dominant driver of output at short horizons and of prices at medium-to-long horizons. Second, it is apparent from the impulse responses how ϵ_t^a and ϵ_t^e satisfy the restrictions for BQ and H shocks. In particular, with $\delta = 0.1256$ (and in fact for any $\delta > 0$) the long-run impact of a positive H shock ϵ_t^e on the price level is positive instead of being ambiguous as in the last two panels in Figure 1. Accordingly, we impose this restriction for the identification of H shocks in the Monte Carlo exercises below.

5.2 Empirical Assessment

We simulate the model for samples of length equal to the actual U.S. sample 1954Q3-2019Q4, i.e., 261 observations. For each simulation we run a pre-sample of 100 observations that we then discard. We then estimate Bayesian VARs for output (y_t), the real wage (ω_t), hours (n_t), inflation (π_t), and technology (a_t) on each artificial sample. Whereas n_t and π_t are I(0), y_t and ω_t inherit the unit root in a_t. They are both cointegrated with log technology with cointegration vector $[1, -1]'$. In what follows we work with $Y_t = [\Delta y_t, (y_t - a_t), (\omega_t - a_t), \pi_t, n_t]'$, so that cointegration between a_t and either y_t and ω_t is imposed directly on the estimated system as in King et al. (1991).[7] For each Monte Carlo simulation we choose the lag order as the maximum of the lag orders chosen by the Schwartz and Hannan-Quinn criteria, based on VARs estimated by OLS. We estimate the Bayesian VARs as in Uhlig (2005). For each value of δ we perform 10,000 Monte Carlo simulations and then conduct inference by taking 10,000 draws from the posterior distribution.

[6] Figures A.1 and A.2 in the Online Appendix report the full set of responses and fractions of FEV for output, the price level, hours, real wage, and technology.

[7] See the discussion in Cochrane (1994) about alternative ways of estimating cointegrated systems.

5.2.1 Model-Consistent Identifying Assumptions

As a first exercise, we assume that there is no hysteresis in the model and impose the correct identification assumption on the VAR. We simulate the model for the case $\delta = 0$ and assess whether the VAR correctly recovers the key features of the DGP when we impose the correct identifying restrictions, i.e., that the DGP has a single shock (ϵ_t^a) with a permanent impact on output. In each Monte Carlo simulation and for each draw from the posterior distribution of the Bayesian VAR's reduced-form coefficients we identify the permanent shock as in Blanchard and Quah (1989), that is, as the only shock having a permanent impact on y_t. We then compute impulse responses and fractions of FEV and report the median and various percentiles of their posterior distributions to build up the Monte Carlo distributions of the medians and the percentiles of the IRFs and fractions of FEV. The results are reported in Figure A.3 in the Online Appendix. In brief, the evidence confirms that VARs recover the main features of the DGP in terms of both IRFs and fractions of FEV with great precision. This is, in particular, the case for the main series of interest within the present context, namely output.

5.2.2 Baseline Identifying Assumptions

In the next exercise we impose the identifying assumptions as summarized in Table 1 and discussed in section 2. We first impose the no-hysteresis specification and analyze whether imposing joint identification of two permanent shocks leads to a spurious detection of hysteresis. We then simulate the model under hysteresis and assess whether the identification schemes come to the correct conclusion and to what extent the identification schemes lead to a spurious detection.

No hysteresis in the theoretical model In our baseline we jointly identify a BQ and an H shock, which imposes the presence of hysteresis a priori. Given that the DGP features no hysteresis, we explore to what extent the restrictions in Table 1 tend to spuriously detect it. We jointly impose the zero long-run restrictions and the short- and long-run sign restrictions, implemented using methodology introduced by Arias et al. (2018). We impose the short-run sign restrictions both on impact and for the subsequent eight quarters. Since the long-run impact of a positive hysteresis shock ϵ_t^e on the price level is consistently positive for all values of δ, we impose this restriction in the identification of H shocks.

The first column of Figure 5 shows the results. The two panels show the averages of the medians and percentiles of the posterior distribution (top panel) and posterior cumulative

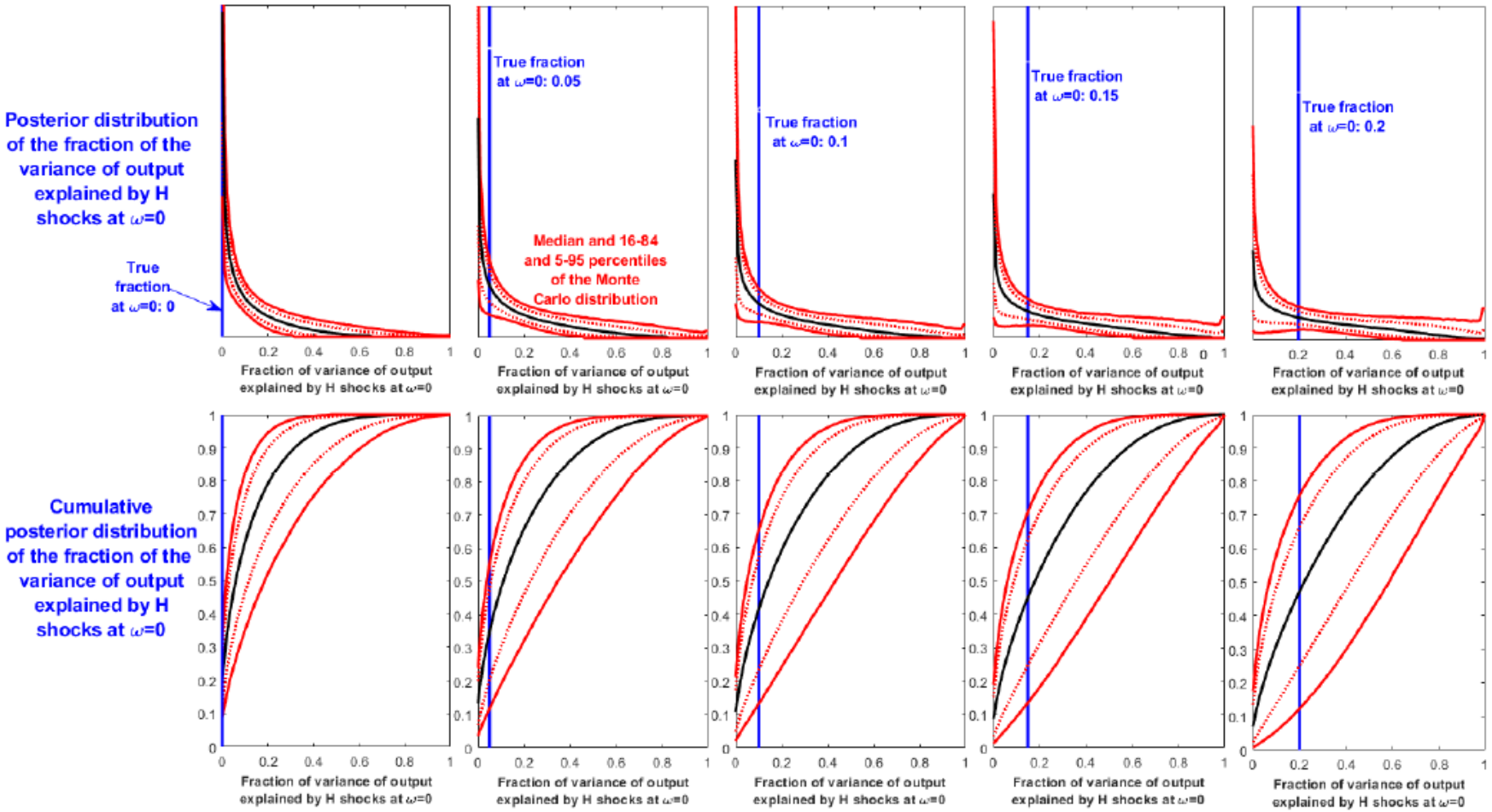


Figure 5 True fractions of the variance of output explained by ϵ_t^e (H) shocks at ω=0 in the RBC model, and medians and 16-84 and 5-95 percentiles of the Monte Carlo distributions of the estimated fractions, based on VARs in differences

distribution (bottom panel) of the statistics of interest, namely the fraction of the output variance explained by identified H shocks at $\omega = 0$, together with the corresponding true fractions of the output variance explained by ϵ_t^e in the theoretical model.[8] The conclusion drawn from this exercise seems fairly clear-cut. Our baseline identification approach tends to detect hysteresis when there is none in the DGP with non-negligible and quite sizeable probability. The likely reason for this is that by its very logic this scheme imposes on the data the very existence of both BQ *and* H shocks.

Hysteresis in the theoretical model Columns 2 to 4 of Figure 5 report the corresponding sets of results for four different values of the true fraction of the output variance explained by ϵ_t^e in the long run. What emerges from the figure is that our baseline identification approach has low power to discriminate between alternative extents of hysteresis. This is especially apparent from the evidence in the top row, which shows the posterior distributions of the fraction of the output variance explained by H shocks. These are consistently spread out, with materially different values being associated with quite similar probabilities. For example, when the fraction is 10 percent, the mode of the posterior in the upper right-hand panel still has a clear peak at 0, but values in excess of 20, 30, or even 40 percent are associated with non-negligible probabilities.

In summary, the analysis in this section provides evidence that our baseline identification scheme (and likely others, such as Furlanetto et al., 2021) are prone to spuriously detect hysteresis when, in fact, there is none. Moreover, they have low power in discriminating between alternative extents of hysteresis. Given these misclassification issues, which we can think of as Type-I and Type-II errors in a frequentist context, we develop a Monte-Carlo based approach to correct for these errors in the next section.

6 A Monte Carlo-Based Approach to Identifying Hysteresis

The identifying restrictions in Table 1 produce a posterior distribution for the fraction of the frequency-zero (or long-run) variance of GDP that is explained by H shocks, which we label Φ. We denote as $F^*(\Phi)$ the posterior distribution obtained by imposing the identifying restrictions upon the Bayesian VAR estimated based on the actual data. Similarly, $F_C^*(\Phi)$ is the corresponding cumulative distribution. In what follows we describe a simulation-

[8] The Bayesian SVAR produces a posterior distribution of the fraction of the variance of output explained by H shocks at $\omega = 0$. For each simulation we extract the median and the percentiles of this distributionand thus build up the Monte Carlo distribution of these objects. Figure 5 shows the means of these objects taken across the Monte Carlo distributions.

based approach that corrects for the misclassification errors inherent in empirical research of hysteresis. We first give a broad outline of the general idea, then discuss the simulation algorithm in more detail, and finally present empirical results.

6.1 General Idea

The basic idea behind our proposed approach is that we simulate estimated Bayesian VARs upon which we impose specific extents of hysteresis. We take Φ as the relevant statistic. We consider a grid of K values for Φ from 0 (no hysteresis) to $\Phi_{Max} = 0.5$, for which H shocks explain half of the long-run variance of output. The individual values of Φ in the grid are labelled as Φ_k, with $k = 1, 2, ..., K$. For each simulated, artificial sample s, with $s = 1, 2, ..., S$, we then estimate the same Bayesian VAR as in the baseline specification for the actual data. We consider this VAR as conditional on Φ_k. In the estimation, we impose the identifying restrictions from Table 1, which results in a posterior distribution $F^s(\Phi_k)$ and a corresponding cumulative posterior distribution $F^s_C(\Phi_k)$ of the fraction of the long-run variance of output explained by H shocks.

Repeating this process for all s and for each Φ_k builds up the Monte Carlo distribution of the $F^s_C(\Phi_k)$. The evidence presented in Figure 5 suggests that the larger Φ_k, the more the Monte Carlo distributions of the $F^s_C(\Phi_k)$ shifts to the right. We then compute the Kolmogorov-Smirnov (henceforth, KS) statistic from each simulated sample s. This provides a measure of the statistical distance between (i) the cumulative distribution of the fraction of the long-run variance of GDP explained by H shocks estimated on actual data, $F^*_C(\Phi)$, and (ii) the corresponding Monte-Carlo cumulative distribution computed for sample s conditional on the imposed value Φ_k. We thus build up a Monte Carlo distribution of the KS statistic. This statistic serves as a statistical assessment of the true extent of hysteresis in the data when correcting for the distortions documented in the previous Section. We now discuss our approach in more detail.

6.2 The Monte-Carlo Approach to Hysteresis in More Detail

Recall the cointegrated structural VAR representation of the model in equation (1):

$$\Delta Y_t = B_0 + B_1 \Delta Y_{t-1} + ... + B_p \Delta Y_{t-p} + \alpha \beta' Y_{t-1} + A_0 \epsilon_t. \tag{8}$$

A_0 is the impact matrix of the structural shocks ϵ_t, with $\epsilon_t = A_0^{-1} u_t$, where the u_t are the reduced-form shocks from equation (1). We define the matrix of the long-run impacts of the structural shocks as $L = C \times A_0$, where $C = \beta_{OC}[\alpha'_{OC}(I_N - B)^{-1}\beta_{OC}]\alpha'_{OC}$; $B =$

$B_1 + B_2 + ... + B_p$; and α_{OC} and β_{OC} are the orthogonal complements of α and β. Finally, let D be the number of draws from the posterior distribution. We now describe how we impose specific values of Φ in the estimated Bayesian structural VARs and how we use the thus-obtained models as DGPs in the Monte Carlo approach.

The first step is to select among the D models the one that is closest to the 'median model'. Let $\bar{A}_0$, $\bar{B}$, and $\bar{L}$ be the medians of the posterior distributions of A_0, B, and L. Our selection criterion minimizes:

$$\Gamma_d = \sum_{i=1}^{N}\sum_{j=1}^{N}\left[(A_{0,d}^{ij} - \bar{A}_0^{ij})^2 + (L_d^{ij} - \bar{L}^{ij})^2\right] + \sum_{i=1}^{N}\sum_{j=1}^{Np}(B^{ij} - \bar{B}^{ij})^2, \text{ for } d = 1, 2, ..., D, \quad (9)$$

namely the sum of the squared deviations of the individual elements of A_0, $B_0, ..., B_p$, and L from the corresponding elements of $\bar{A}_0$, $\bar{B}$, and $\bar{L}$, where N is the number of variables in the VAR and p the lag order. We thus obtain the model

$$\Delta Y_t = \tilde{B}_0 + \tilde{B}_1 \Delta Y_{t-1} + ... + \tilde{B}_p \Delta Y_{t-p} + \tilde{\alpha}\tilde{\beta}' Y_{t-1} + \tilde{A}_0 \epsilon_t, \quad (10)$$

for 'optimal' parameters $\tilde{A}_0$, $\tilde{B}_0, ..., \tilde{B}_p$, and $\widetilde{L}$. Finally, we impose each value of $\Phi_k \geq 0$ in the grid on model (10) by appropriately rescaling the column of $\tilde{A}_0$ associated with H shocks.[9] We then use the resulting model as DGP in the Monte Carlo exercise, thus building up the Monte Carlo distribution of the KS statistic for each value of Φ_k.

$F_C^s(\Phi)$ is the cumulative distribution of Φ, which we obtain by estimating a VAR and imposing the identifying restrictions in Table 1. This is done repeatedly for each artificial sample of Monte Carlo simulations, where we set the total number of simulations to $S = 1000$. The KS statistic for $F_C^*(\Phi)$ and $F_C^s(\Phi)$ is defined as:

$$KS = \sup_{\Phi} \left| F_C^*(\Phi) - F_C^s(\Phi) \right|, \quad (11)$$

with $\Phi \in [0, 1]$, i.e. as the maximum absolute value of the difference between the two cumulative distributions. We compute univariate KS statistics for GDP, consumption, and investment. We extend the univariate KS statistic in equation (11) to the bivariate case as follows. Let $F_C^*(\Phi_y, \Phi_c)$ and $F_C^s(\Phi_y, \Phi_c)$ be the joint cumulative posterior distributions of the fractions of long-run variance of GDP (Φ_y) and consumption (Φ_c) explained by H shocks. The joint KS statistic for Φ_y and Φ_c is:

$$KS_J = \sup_{\Phi_y, \Phi_c} \left| F_C^*(\Phi_y, \Phi_c) - F_C^s(\Phi_y, \Phi_c) \right|, \quad (12)$$

[9] As a side note, if we imposed $\Phi_1 = 0$ the model would become stochastically singular and therefore impossible to estimate the reduced-form model (1). We therefore impose $\Phi_1 = 10^{-6}$ instead.

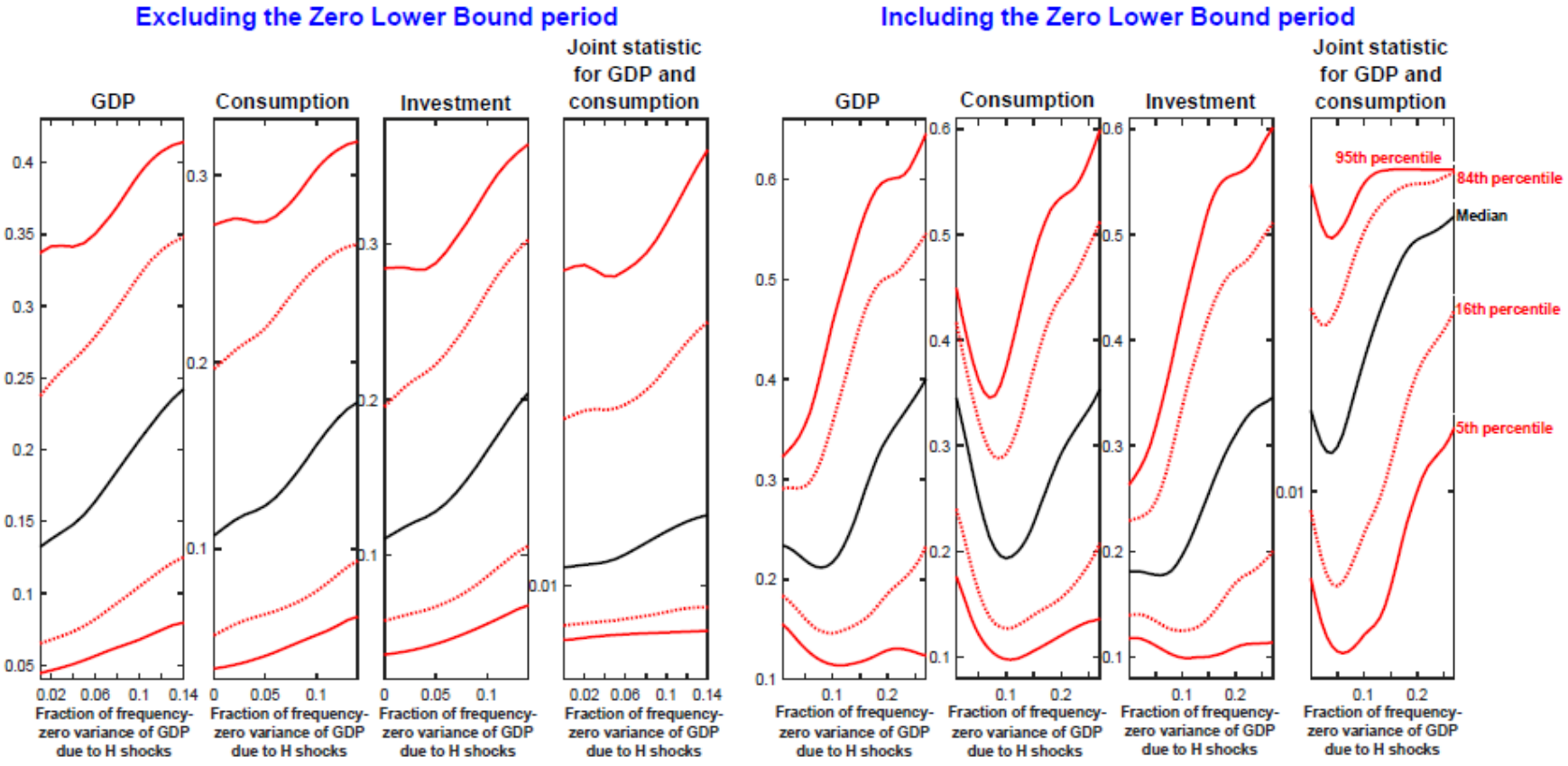


Figure 6 Median and selected percentiles of the Monte Carlo distribution of the Kolgomorov-Smirnov statistic

namely the maximum absolute value of the difference between the two cumulative distributions. We note that both $F_C^*(\Phi_y, \Phi_c)$ and $F_C^s(\Phi_y, \Phi_c)$ are two-dimensional surfaces, rather than one-dimensional lines.

6.3 Results

We perform the Monte Carlo exercise for the full sample and for the sample excluding the ZLB period. Figure 6 shows the median and the 16-84 and 5-95 percentiles of the Monte Carlo distributions of the KS statistics for the two samples. We report the joint statistic for GDP and consumption and individual statistics for GDP, consumption, and investment. As in the previous exercises, there is a fairly high degree of uncertainty, which is intrinsic to the analysis we are performing. When we focus on the median estimates, two key observations emerge from the figure.

First, evidence uniformly suggests that the most likely value of the extent of hysteresis is zero in the sample excluding the ZLB period. The second insight is that the evidence for the full sample period is less clear-cut. It points towards a non-zero, but modest extent of hysteresis. This is the case based on both the joint KS_J statistic and the individual KS statistics for consumption. In particular, the minimum of the KS_J statistic corresponds to a value of hysteresis of about 5 percent of the long-run variance of GDP, while the individual statistic for consumption suggests around 10 percent. Evidence from the individual statistic for GDP is broadly consistent. The statistic for investment, on the other hand, is essentially flat until about 7-8 per cent and then starts to increase rapidly, which suggests that larger values are less plausible. The comparison between the shapes of the median estimates for GDP and consumption points towards a natural interpretation. Since consumption is very close to the permanent component of GDP (Cochrane, 1994), it is not necessarily surprising that evidence for consumption is sharper than for GDP.

7 Conclusion

We have gone on a search for hysteresis, looking for aggregate demand shocks that have a permanent impact on real GDP. Methodologically, our search is primarily empirical and statistical, and it is mainly based on cointegrated structural VARs that identified such shocks via a combination of zero long-run restrictions and both short- and long-run sign restrictions. These restrictions are consistent with a wide range of theoretical models and are broad enough to leave room for identifying hysteresis shocks if they exist. Conceptually, we propose a simple approach to test for the presence and extent of hysteresis. It is based on

simulating specific statistics (e.g., the fraction of long-run variance of GDP due to hysteresis shocks) conditional on alternative extents of hysteresis imposed on the estimated VAR. The hysteresis statistics from the resulting Monte Carlo distributions are then compared to the corresponding distributions computed from the actual data via the Kolgomorov-Smirnov statistic.

Evidence suggests that in the post-WWII U.S. hysteresis effects have been virtually absent for samples excluding the financial crisis and the Great Recession. They only appear when including the period following the collapse of Lehman Brothers. Even then, however, these effects only account for at most 10 percent of the frequency-zero variance of GDP. An important role in obtaining our results is played by a Monte Carlo-based correction to the 'simple' Bayesian estimates, designed to control for the fact that, as we show via DSGE models, there is a high probability of detecting hysteresis effects even when the DGP features none by construction.

While not as large as other estimates in the literature such as Jordà et al. (2020), our estimate nevertheless points towards the potential presence of a mechanism that converts economic demand disturbances into permanent outcomes for output. The difference between the results for the various sample periods also suggest that hysteresis shocks are operational in times of extreme economic distress, such as the Great Recession, which came with a dramatic increase in the long-term unemployment as one possible hysteresis channel.

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Appendix

A Computational Details for the Bayesian Cointegrated VAR

A.1 Choice of Priors

We follow Koop et al. (2010) in the choice of the priors for α, β, and Σ. The prior for α is a shrinkage prior with zero mean:

$$\alpha \mid \beta, B_0, B_1, ..., B_p, \Sigma, \tau, \nu \sim MN\left(0, \nu(\beta' P_{1/\tau}\beta)^{-1} \otimes G\right), \tag{C.1}$$

where MN denotes the matricvariate-normal distribution; $P_{1/\tau} = HH' + \tau^{-1}H_\perp H_\perp'$; and G is a square matrix that we set to $G = \Sigma$ following Strachan and Inder (2004). In addition, τ is a scalar between 0 and 1, and ν is a scalar that controls the extent of shrinkage.We set $\tau = 1$, which corresponds to a flat, non-informative prior on β, and $\nu = 1000$, corresponding to a very weakly informative prior for α (the standard non-informative prior would be obtained by setting $1/\nu = 0$).

The prior for β is based on a matrix angular central Gaussian distribution:

$$p(\beta) \propto |P_\tau|^{-r/2}|\beta'(P_\tau)^{-1}\beta|^{-N/2}, \tag{C.2}$$

where $P_\tau = HH' + \tau H_\perp H_\perp'$, and N is the number of variables in the system. The prior for the covariance matrix Σ is the non-informative prior:

$$p(\Sigma) \propto |\Sigma|^{-(N+1)/2}. \tag{C.3}$$

As for the coefficient matrices B_j, we follow Geweke (1996) and choose a multivariate normal prior, which implies that the posterior is also multivariate normal.[10]

A.2 Drawing from the Posterior

The conditional posterior distribution for $G = \Sigma$ is inverse-Wishart:

$$\Sigma \mid \alpha, \beta, B_0, B_1, ..., B_p, Y_t \sim IW\left(u_t' u_t + \nu\alpha(\beta' P_{1/\tau}\beta)^{-1}\alpha', T + r\right), \tag{C.4}$$

where T is the sample length. The conditional posterior for the coefficient matrices B_j is given by (see Geweke, 1996):

$$vec(A) \mid \alpha, \beta, \Sigma, \tau, \nu, Y_t \sim N\left\{(\Sigma^{-1} \otimes Z'Z + \lambda^2 I)^{-1}(\Sigma^{-1} \otimes Z'Z)vec(A'), (\Sigma^{-1} \otimes Z'Z + \lambda^2 I)^{-1}\right\}, \tag{C.5}$$

[10] We thank Rodney Strachan for confirming to us that taking step (*iii*) of the Gibbs-sampling algorithm discussed in the main text of the paper from Geweke (1996) and the remaining steps from Koop et al. (2010) is indeed appropriate.

with $A' \equiv [B_0 \ B_1 \ B_2 \ ... \ B_p]$ and $\hat{A} = (Z'Z)^{-1}Z'(\Delta Y - \tilde{Y}\beta\alpha')$. ΔY and $\tilde{Y}$ are $T \times N$ matrices, whose t-th rows are equal to $\Delta Y_t'$ and Y_{t-1}', respectively. Z is a $T \times 1 + Np$ matrix, whose t-th row is $Z_t \equiv [1 \ \Delta Y_{t-1}' \ \Delta Y_{t-2}' \ ... \ \Delta Y_{t-p}']$. Following Geweke (1996) we set λ^2=10.

We derive the posterior distributions for α and β as in Koop et al. (2010). Using the transformation $\beta\alpha' = (\beta\kappa)(\alpha\kappa^{-1})' = \left[\beta(\alpha'\alpha)^{1/2}\right]\left[\alpha(\alpha'\alpha)^{-1/2}\right]' \equiv ba'$, with $\kappa = (\alpha'\alpha)^{1/2}$, $a = \alpha\kappa^{-1}$, $b = \beta(\alpha'\alpha)^{1/2}$, $\beta = b(b'b)^{-1/2}$, and $b'b = \alpha'\alpha$, the priors for α and β imply the following priors for a and b:

$$b \mid a, B_0, B_1, ..., B_p, \Sigma, \tau, \nu \sim MN\left(0, (a'G^{-1}a)^{-1} \otimes \nu P_\tau\right), \tag{C.6}$$

$$p(a) \propto |G|^{-r/2}|a'G^{-1}a|^{-N/2}. \tag{C.7}$$

The conditional draws for α and β, that is, steps (i) and (ii) of the Gibbs-sampling algorithm discussed in the main text of the paper, can then be obtained via the following two steps:

(i') draw $\alpha^{(*)}$ from $p(\alpha|\beta, B_0, B_1, ..., B_p, \Sigma, \tau, \nu, Y_t)$ and transform it to obtain a draw $a^{(*)} = \alpha^{(*)}(\alpha^{(*)\prime}\alpha^{(*)})^{-1/2}$; and

(ii') draw b from $p(b|a^{(*)}, B_0, B_1, ..., B_p, \Sigma, \tau, \nu, Y_t)$ and transform it to obtain draws $\beta = b(b'b)^{-1/2}$ and $\alpha = a^{(*)}(b'b)^{1/2}$.

Figure A.15 of the Online Appendix report evidence on the convergence of the Markov chain, by showing Geweke's (1992) inefficiency factors (IFs) of the draws for each individual parameter. The inefficiency factors are defined as the inverse of the relative numerical efficiency measure of Geweke (1992),

$$RNE \ = (2\pi)^{-1}\frac{1}{S(0)}\int_{-\pi}^{\pi} S(\omega)d\omega, \tag{C.8}$$

where $S(\omega)$ is the spectral density of the sequence of draws from the Gibbs sampler for the quantity of interest at frequency ω. We estimate the spectral densities via the lag-window estimator as described in chapter 10 of Hamilton (1994). We also considered an estimator based on the fast-Fourier transform, and results were very close. For all parameters the IFs are equal to at most 8, that is, well below the values of 20-25 which are typically taken to indicate problems in the convergence of the Markov chain.

Online Appendix for: Searching for Hysteresis

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A The Data

Quarterly seasonally adjusted series for real GDP ('GDPC1: Real Gross Domestic Product, Billions of Chained 2012 Dollars, Quarterly, Seasonally Adjusted Annual Rate') are from the U.S. Department of Commerce, Bureau of Economic Analysis. Quarterly seasonally adjusted series for real chain-weighted investment, real chain-weighted consumption, and the chain-weighted consumption deflator have been computed based on the data found in Tables 1.1.6, 1.1.6B, 1.1.6C, and 1.1.6D of the National Income and Product Accounts. Whereas consumption pertains to non-durables and services, investment has been computed by chain-weighting the relevant series pertaining to durable goods; private investment in structures, equipment, and residential investment; Federal national defense and non-defense gross investment; and State and local gross investment. Inflation has been computed as the log-difference of the personal consumption expenditures (PCE) deflator. A quarterly seasonally adjusted series for total hours worked by all persons in the nonfarm business sector ('HOANBS: Nonfarm Business Sector: Hours of All Persons, Index 2009=100') is from the U.S. Department of Labor, Bureau of Labor Statistics.

A quarterly seasonally adjusted series for working age population (i.e., aged 15-64) has been constructed by linking the series from Francis and Ramey (2009) and that from FRED II, the Federal Reserve Bank of St. Louis' internet data portal ('LFWA64TTUSQ647S: Working Age Population, Aged 15-64, Noninstitutional, non-armed forces Population for the United States, Persons, Quarterly, Seasonally Adjusted'). Over the period of overlapping the two series are identical, which justifies their linking.

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Monthly series for the Federal Funds rate ('FEDFUNDS: Effective Federal Funds Rate, Percent, Monthly, Not Seasonally Adjusted') and the 5-year government bond yield ('GS5: 5-Year Treasury Constant Maturity Rate') are from the Board of Governors of the Federal Reserve System. A monthly series for Wu and Xia's 'shadow rate' is from Cynthia Wu's website, at: https://sites.google.com/view/jingcynthiawu. All monthly interest rate series have been converted to the quarterly frequency by taking averages within the quarter.

We consider two sample periods, one excluding the Zero Lower Bound (ZLB) period, 1954Q3-2008Q4, and one including it, 1954Q3-2019Q4. We end the latter period in 2019Q4 in order to exclude the impact of COVID. As discussed in the main text of the paper, the starting date for the two samples is dictated by the fact that the Federal Funds rate is available since July 1954 (the 5-year goverment bond yield is available since April 1953).

B Unit Root and Cointegration Properties of the Data

B.1 Evidence from unit root tests

Table B.1 reports results from Elliot *et al.* (1996) unit root tests for the (log) levels of the series for the two sample periods 1954Q3-2008Q4 and 1954Q3-2019Q4, and for three alternative lag orders, $p = 2$, 4, and 6.

The null of a unit root cannot (near) uniformly be rejected for either sample for GDP, consumption, investment, long- and short-term nominal interest rates, hours, and the PCE deflator. On the other hand, as expected based on economic theory, it is uniformly rejected for either sample for the spread between long- and short-term rates.

Table B.1 Bootstrapped *p*-values for Elliot, Rothenberg, and Stock unit root tests[a]

	1954Q3-2008Q4			1954Q3-2019Q4		
Series	p=2	p=4	p=6	p=2	p=4	p=6
Log real GDP *per capita*	0.304	0.459	0.472	0.252	0.446	0.448
Log real consumption *per capita*	0.456	0.466	0.620	0.373	0.469	0.601
Log real investment *per capita*	0.344	0.255	0.171	0.235	0.172	0.110
Log total hours *per capita*	0.140	0.235	0.195	0.070	0.133	0.102
Long rate	0.501	0.429	0.545	0.603	0.544	0.668
Short rate	0.175	0.119	0.083	0.249	0.174	0.139
Log PCE deflator	0.233	0.104	0.150	0.993	0.977	0.936
PCE deflator inflation	0.095	0.098	0.179	0.006	0.025	0.085
Spread (long rate minus short rate)	0.000	0.000	0.000	0.000	0.001	0.000

[a] Based on 10,000 bootstrap replications. For details, see Appendix B.

Finally, turning to PCE deflator inflation, a unit root is strongly rejected for the period

including the ZLB, whereas for the period 1954Q3-2008Q4 the rejection at the 10% level is weaker, with the p-value for p=6 being equal to 0.179. It is to be noticed, however, that for the full post-WWII sample period 1947Q2-2019Q4 rejection of a unit root is very strong, with the p-values equal to 0.000, 0.000, 0.002. In the paper we therefore work under the assumption that inflation is I(0), and that, conceptually in line with Benati (2008), the weak rejection of the null of a unit root for the period 1954Q3-2008Q4 is simply the figment of a comparatively short sample largely dominated by the experience of the Great Inflation.

B.2 Evidence from cointegration analysis

Turning to the cointegration properties of the data, basic economic theory suggests that, within the present context, we should expect at least three cointegration relationships: one between the short- and the long-term nominal interest rates, and two between GDP, consumption, and investment.

Table B.2*a* provides evidence that this is indeed the case. The table reports bootstrapped p-values[1] for Johansen's trace and maximum eigenvalue tests for three bivariate systems featuring GDP and consumption, GDP and investment, and a short- and a long-term nominal rate. For all systems evidence of cointegration is very strong, with the largest p-value across all systems being equal to just 0.0044.

Table B.2*a* Bootstrapped p-values for Johansen's cointegration tests[a]

	Trace tests:
Log real GDP *per capita* and	
log real consumption *per capita*[b]	3.0e-4
log real investment *per capita*[b]	0.0021
Federal Funds rate / Wu-Xia shadow rate and 5-year	
government bond yield[c]	0.0014
	Maximum eigenvalue tests:
Log real GDP *per capita* and	
log real consumption *per capita*[b]	3.0e-4
log real investment *per capita*[b]	0.0044
Federal Funds rate / Wu-Xia shadow rate and 5-year	
government bond yield[c]	0.0011

[a] Based on 10,000 bootstrap replications of Cavaliere *et al.*'s (2012) procedure. [b] 1948Q1-2019Q4. [c] 1954Q3-2019Q4.

[1] Bootstrapping has been implemented as in Cavaliere *et al.* (2012). Cavaliere *et al.* (2012), Benati (2015), and especially the Online Appendix of Benati *et al.* (2021) provide extensive Monte Carlo evidence on the excellent performance of this bootstrapping procedure.

Turning to the 7-variables systems, Table B.2*b* reports bootstrapped *p*-values for the corresponding tests for the two samples.[2] For the period 1954Q3-2008Q4 both the trace and the maximum eigenvalue tests point towards three cointegration vectors, whereas for the period 1954Q3-2019Q4 they point respectively towards four and three cointegration vectors. Accordingly, in the paper we proceed under the assumption that in both samples the 7-variables system features three cointegration vectors, and, as discussed there, we center the prior for the matrix encoding the cointegration vectors around the three cointegration relationships suggested by economic theory.

Table B.2*b* Bootstrapped *p*-values for Johansen's cointegration tests for the 7-variables systems[a]

	Trace tests of the null of no cointegration against the alternative of h or more cointegration vectors:				
Period	*h = 1*	*h = 2*	*h = 3*	*h = 4*	*h = 5*
1954Q3-2008Q4	0.040	0.034	0.052	0.600	
1954Q3-2019Q4	0.002	0.001	0.002	0.069	0.342
	Maximum eigenvalue tests of h versus h+1 cointegration vectors:				
	0 versus 1	*1 versus 2*	*2 versus 3*	*3 versus 4*	*4 versus 5*
1954Q3-2008Q4	0.142	0.057	0.030	0.782	
1954Q3-2019Q4	0.066	0.014	0.009	0.349	

[a] Based on 10,000 bootstrap replications of Cavaliere et al.'s (2012) procedure.

C The Real Business Cycle Model Used in the Monte Carlo Exercise of Section 5.2

Here follows a detailed description of the real business cycle (RBC) model we use as data-generation process (DGP) in the Monte Carlo exercise of Section 5. In essence, the model is that described in Galí's (2015) Chapter 2, augmented with habit-formation in consumption and a unit root in technology (and therefore in the natural level of output), and featuring the possible presence of hysteresis effects (as discussed below). Whenever possible we follow Galí's own notation. Households solve the following problem

$$U_0^* \equiv \underset{C_t, N_t}{\text{Max}} \; E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t - bC_{t-1}) - \frac{N_t^{1+\phi}}{1+\phi} \right] \tag{D.1}$$

subject to

$$P_t C_t + Q_t B_t = B_{t-1} + W_t N_t - T_t \tag{D.2}$$

[2] Again, bootstrapping has been implemented based on Cavaliere *et al.*'s (2012) methodology.

where C_t and N_t are real consumption and hours worked, respectively; $0<b<1$ is the habit-formation parameter; P_t is the price of consumption goods; B_t is the stock of nominal bonds; Q_t is the price, at time t, of a nominal bond paying 1 dollar at time $t+1$; W_t is the nominal wage; T_t are nominal lump-sum taxes; and the rest of the notation is obvious. The two first-order conditions (FOCs) are

$$N_t^{\phi} = \frac{W_t}{P_t} E_t \left[\frac{1}{C_t - bC_{t-1}} - \beta \frac{b}{C_{t+1} - bC_t} \right] \tag{D.3}$$

and

$$E_t \left[\frac{Q_t}{C_t - bC_{t-1}} \frac{P_{t+1}}{P_t} \right] = \beta E_t \left[\frac{1 + bQ_t}{C_{t+1} - bC_t} - \beta \frac{b}{C_{t+2} - bC_{t+1}} \right] \tag{D.4}$$

With no habit-formation (i.e., with $b=0$) the two FOCs collapse to those found in Galí (2015), i.e.

$$N_t^{\phi} = \frac{W_t}{P_t} \frac{1}{C_t} \quad \text{and} \quad 1 = \beta E_t \left[\frac{P_t C_t}{Q_t P_{t+1} C_{t+1}} \right] \tag{D.5}$$

As for the firms we exactly follow Galí (2015), with the only difference pertaining to the process for technology. Firms produce output (Y_t) via the production function $Y_t = A_t N_t^{1-\alpha}$, with $0<\alpha<1$ being the Cobb-Douglas parameter, A_t being technology, and the capital stock being constant and normalized to 1. Maximization of profits, $P_t Y_t - W_t N_t$, produces the FOC

$$\frac{W_t}{P_t} = A_t (1 - \alpha) N_t^{1-\alpha} \tag{D.6}$$

As for the logarithm of technology, $a_t = \ln(A_t)$, we postulate that it evolves according to

$$a_t = a_{t-1} + \epsilon_t^a + \delta \tilde{y}_{t-1} \tag{D.7}$$

with $\epsilon_t^a \sim N(0, \sigma_a^2)$, $\tilde{y}_t$ being the transitory component of output (to be discussed below), and $\delta \geq 0$ capturing the possible presence of hysteresis effects. If $\delta = 0$ there is no hysteresis, whereas if $\delta > 0$ positive (negative) transitory fluctuations of output cause subsequent permanent increases (decreases) in the level of technology. This specification, which is conceptually the same as the one used by Jordà et al. (2020), is designed to capture, in a very simple and stripped-down fashion, the notion that positive (negative) deviations of GDP from potential (here, deviations of output from its stochastic trend A_t) may have a positive (negative) impact on potential GDP itself. Although we could have considered more complex formalizations of the notion of hysteresis, we chose to use (D.7) because for our own purposes (i.e., performing Monte Carlo simulations) it is perfectly appropriate. Finally, since there is no investment, no government expenditure, and no foreign sector, $C_t = Y_t$.

Since a_t is I(1), log-linearization of the FOCs and of the production function requires the preliminary stationarizazion of the relevant variables. We define the stationarized variables

$$\tilde{Y}_t \equiv \frac{Y_t}{A_t} \quad \text{and} \quad \tilde{\Omega}_t \equiv \frac{\left(\frac{W_t}{P_t}\right)}{A_t}, \tag{D.8}$$

with $\tilde{y}_t$ and $\tilde{\omega}_t$ being the log-deviations of $\tilde{Y}_t$ and $\tilde{\Omega}_t$ from the steady-state, so that $\tilde{y}_t$ is the component of output that is driven by the transitory disturbances (discussed below). With $\tilde{Y}_t$ and $\tilde{\Omega}_t$ defined as in (D.8), the production function and the three FOCs can be trivially stationarized. Then, log-linearization of the stationarized production function and of the three stationarized FOCs produces the following four log-linear relationships characterizing the dynamics of the economy in a neighborhood of the steady-state,

$$\tilde{y}_t - (1-\alpha)n_t = 0 \tag{D.9}$$

$$\tilde{\omega}_t + \alpha n_t - v_t = 0 \tag{D.10}$$

$$-R_t + \pi_{t+1|t} + u_t + \phi n_t - \tilde{\omega}_t - \phi n_{t+1|t} + \tilde{\omega}_{t+1|t} = 0 \tag{D.11}$$

$$\phi n_t - \tilde{\omega}_t - e_t + \frac{1}{(1\text{-}b\beta)(1\text{-}b)}\left[(1+\beta b^2)\tilde{y}_t - b\beta\tilde{y}_{t+1|t} - b\tilde{y}_{t-1} + b\Delta a_t\right] = 0 \tag{D.12}$$

where $n_t \sim I(0)$ is the log-deviation of N_t from the steady-state, $\pi_t \equiv p_t\text{-}p_{t-1} = \ln(P_t)\text{-}\ln(P_{t-1})$ is inflation, e_t is a transitory demand disturbance, and v_t and u_t are two transitory supply disturbances, with

$$v_t = \rho_v v_{t-1} + \epsilon_t^v \tag{D.13}$$

$$u_t = \rho_u u_{t-1} + \epsilon_t^u \tag{D.14}$$

$$e_t = \rho_e e_{t-1} + \epsilon_t^e \tag{D.15}$$

with $\epsilon_t^v \sim N(0,\sigma_v^2)$, $\epsilon_t^u \sim N(0,\sigma_u^2)$, $\epsilon_t^e \sim N(0,\sigma_e^2)$, $|\rho_v, \rho_u, \rho_e| < 1$, and where the rest of the notation is obvious. The logarithm of technology, a_t, evolves according to (D.7). Monetary policy is characterized by a standard Taylor rule with smoothing,

$$R_t = \rho R_{t-1} + (1-\rho)\phi_\pi \pi_t \tag{D.16}$$

where the notation is obvious. Finally, we augment the model with an additive disturbance to the log of the production function, $z_t \sim N(0,\sigma_z^2)$, with σ_z=0.005.

By defining the state vector as

$$\xi_t = [R_t, \pi_t, \tilde{y}_t, n_t, \tilde{\omega}_t, \Delta a_t, v_t, u_t, e_t, \pi_{t+1|t}, \tilde{y}_{t+1|t}, n_{t+1|t}, \tilde{\omega}_{t+1|t}]' \tag{D.17}$$

and augmenting the system with the definition of the four rational expectations forecast errors

$$\pi_t = \pi_{t|t-1} + \eta_t^{\pi} \tag{D.18}$$

$$\tilde{y}_t = \tilde{y}_{t|t-1} + \eta_t^{\tilde{y}} \tag{D.19}$$

$$n_t = n_{t|t-1} + \eta_t^{n} \tag{D.20}$$

$$\tilde{\omega}_t = \tilde{\omega}_{t|t-1} + \eta_t^{\tilde{\omega}} \tag{D.21}$$

the system can be put into the 'Sims canonical form' (see Sims, 2002) and solved.

We calibrate the structural parameters as follows: β=0.99, α=1/3, b=0.8, ϕ=1, ρ=0.9, ϕ_π=1.5, and σ_a=0.007.[3] Of the three transitory AR(1) disturbances, two of them (v_t and u_t) are supply-side, and one (e_t) is demand-side. The AR parameters are calibrated to ρ_v=ρ_u=ρ_e=0.75, whereas the standard deviations of the disturbances' innovations ϵ_t^v, ϵ_t^u, and ϵ_t^e (all zero-mean, and normally distributed) are set to σ_u=0.001, σ_v=0.005 and σ_e=0.045. Based on this calibration the permanent technology shock (ϵ_t^a) explains exactly 1/3 of the forecast error variance (FEV) of log GDP on impact, and with δ=0 it explains slightly more than 96 per cent of GDP's FEV 15 years ahead. These figures are broadly in line with the evidence produced by the structural VAR literature: for example, as for the fraction of the FEV of GDP explained by permanent shocks on impact see Table I.2 of Cochrane (1994). Further, based on these values of the structural parameters the demand-side disturbance ϵ_t^e is quite close to being the only driver of the transitory component of output, so that the identifying restrictions in Table 1 of the paper are, for practical purposes, correct. Finally, as for δ in the Monte Carlo exercise we consider a grid of values, from δ=0 (no hysteresis) to δ=0.1256, for which the technology shock ϵ_t^a and the demand-side shock ϵ_t^e both explain virtually half of the frequency-zero variance of output.

D Evidence Based on Stationary VARs

As mentioned in Section 3 in the main text of the paper, in spite of the results from unit root tests, the notion that interest rates and hours *per capita* are I(1) may be questioned on conceptual grounds. In this Appendix we therefore discuss evidence from stationary Bayesian VARs for either $Y_{1,t} = [\Delta y_t, R_t, r_t\text{-}R_t, \pi_t, h_t, CR_t^{y,c}, CR_t^{y,i}]'$ or $Y_{2,t} = [\Delta c_t, R_t, r_t\text{-}R_t, \pi_t, h_t, CR_t^{c,y}, CR_t^{c,i}]'$, where the notation is the same as in the main text; Δy_t and Δc_t are

[3] The value for σ_a is close to Watson's (1986, p. 60) estimate of the standard deviation of shocks to the stochastic trend of log real GDP (0.0057). The rationale is that, within the present context, a_t is the random-walk component of log real GDP.

the log-differences of real GDP and real consumption *per capita*; r_t-R_t is the spread between long- and short-term nominal interest rates; and $CR_t^{x,z}$ is the cointegration residual between x_t and z_t. To anticipate, these results are in line with those based on cointegrated SVARs in the main text of the paper, with hysteresis effects estimated to have been small to negligible for the sample including the ZLB period, and possibly *nil* for the sample excluding it.

Before proceeding it is worth clarifying the following issue. For the reasons discussed, e.g., by Cochrane (1994)—i.e., by the Permanent Income Hypothesis (PIH) consumption is very close to the unit root component of GDP—in what follows we pay special attention to the role played by H shocks in driving the long-horizon variation of consumption. Since the cointegrated system features both y_t and c_t, based on the estimated VECMs it is possible the explore the role played by H shocks in driving long-horizon variation in both variables. Based on the stationary VARs, on the other hand, this is not possible. In particular, based on the VAR for $Y_{1,t}$ it is only possible to explore this issue for y_t, since at the frequency zero c_t is obtained by rescaling y_t via the cointegration residual $CR_t^{y,c}$. This is why, within the present context, we are compelled to consider two alternative systems for $Y_{1,t}$ and $Y_{2,t}$. It is also a key reason why, overall, our preference goes to the evidence produced by the cointegrated SVARs.

For either $Y_{1,t}$ or $Y_{2,t}$ we estimate the following stationary VAR

$$Y_{i,t} = B_0 + B_1 Y_{i,t-1} + ... + B_p Y_{i,t-p} + u_t, \tag{D.1}$$

where the notation is obvious, and $i = 1, 2$. We estimate the Bayesian VARs as in Uhlig (1998, 2005). Specifically, we exactly follow Uhlig's approach in terms of both distributional assumptions (the distributions for the VAR's coefficients and its covariance matrix are postulated to belong to the Normal-Wishart family) and of priors. For estimation details the reader is therefore referred to either the Appendix of Uhlig (1998), or to Appendix B of Uhlig (2005).

We jointly impose the zero long-run restrictions, and the short- and long-run sign restrictions, based on the methodology proposed by Arias et al. (2018). We impose the short-run sign restrictions both on impact and for the subsequent either four or eight quarters. Since the two sets of results are qualitatively the same, in what follows we only report and discuss those based on imposing the restrictions for eight quarters, but the alternative set of results is available upon request. For each draw from the posterior distribution of the reduced-form VAR we consider 100 random rotation matrices that we draw based on Arias et al.'s (2018) algorithm. We set the number of Gibbs-sampling iterations in the algorithm to 10.

D.1 'Simple' Evidence from Stationary VARs

Figures A.10 and A.11 show the median and the 16-84 and 5-95 percentiles of the posterior distributions of the impulse-response functions (IRFs) to BQ and H shocks, and the fractions of forecast error variance (FEV) of the variables explained by the shocks, for the sample including the ZLB period, and without imposing restrictions on the long-run impact of H shocks on the price level. Figures A.12 and A.13 show results for the sample excluding the ZLB period. Finally, Figures A.14-A.17 show the corresponding set of results obtained by imposing restrictions on the long-run impact of H shocks on the price level. Exactly as for the results based on the cointegrated SVARs, a common feature that uniformly pertains to all Figures A.10-A.17 is a large extent of uncertainty.

The IRFs require little comment, since they are near-uniformly as expected (for GDP, consumption, investment, and the price level this is obviously the product of the restrictions that are being imposed). For example, at essentially all horizons the response of either the short rate, or the long-short spread to either shock is uniformly insignificant at the two, and often even at the one standard deviations level. Different from the evidence produced by cointegrated SVARs, the response of hours is estimated to be transitory for either shock. Focusing on the price level, the IRFs for the two samples obtained without imposing restrictions on the long-run impact of H shocks are in line with the corresponding evidence produced by cointegrated SVARs. In particular,

first, for the sample including the ZLB (Figure A.10) a non-negligible mass of the posterior distribution of the IRF to H shocks lies below zero. Further, at the frequency zero, the long-run impact of H shocks on the price level is estimated to be negative for ?? per cent of the draws. As discussed in the main text, this suggests that the data provide some support to the notion, discussed in Section 2 (see the panel labelled as 'Hysteresis II' in Figure 1), that a positive (negative) H shock may counterintuitively have a negative (positive) long-run impact on the price level.

Second, for the sample excluding the ZLB (Figure A.12), at the 15 years horizon ?? per cent of the draws are associated with a negative impact of H shocks on prices, whereas at the frequency zero the corresponding fraction is equal to ?? per cent. As discussed in the main text, a possible interpretation is that in this sample H shocks are (virtually) absent, and that this result is simply the figment of the fact that our identifying restrictions impose upon the data their very existence by 'brute force' (see the discussion there). This explanation is compatible with the fact that, as we show in Section 5 in the main text, spuriously identifying hysteresis when the DGP features none by construction is, in fact,

quite likely.

Table D.1 Fractions of frequency-zero variance of GDP and consumption explained by H shocks (median and 16-84 and 5-95 percentiles)		
	Restricting the long-run impact of H shocks on the price level:	
	No	Yes
1954Q3-2019Q4		
GDP	0.325 [0.070 0.759] [0.013 0.944]	0.249 [0.038 0.740] [0.005 0.947]
consumption	0.280 [0.052 0.723] [0.011 0.929]	0.240 [0.033 0.735] [0.005 0.945]
1954Q3-2008Q4		
GDP	0.140 [0.024 0.474] [0.004 0.768]	0.025 [0.003 0.116] [0.000 0.267]
consumption	0.094 [0.015 0.333] [0.002 0.624]	0.024 [0.003 0.109] [0.000 0.249]

Turning to the frequency zero, Table D.1 reports the median and the 16-84 and 5-95 percentiles of the posterior distributions of the fractions of long-run variance of GDP and consumption explained by H shocks. Once again, estimates are characterized by a remarkable extent of uncertainty, which, as mentioned in the man text, is intrinsic to the investigation we are performing. Ignoring this issue, and focusing on the median estimates, the following facts emerge from Table D.1:

first, in line with the evidence from cointegrated SVARs, the estimates for consumption are most of the time smaller than those for GDP. The exception is the period excluding the ZLB period when restricting the long-run impact of H shocks on the price level, for which the estimates for GDP and consumption are virtually identical.

Second, focusing on consumption, which by the PIH and the evidence in Cochrane (1994) should be regarded as likely producing more reliable and informative results, evidence clearly suggests that estimates for the sample excluding the ZLB are dramatically lower than those for the full sample. E.g., the median estimates for the samples including and excluding the ZLB period are equal to 0.280 and 0.094, respectively, when not imposing restrictions on the long-run impact of H shocks on the price level, whereas they are equal to 0.240 and 0.024 when imposing such restrictions. This shows that for the sample excluding the ZLB period hysteresis effects are very small to negligible, and they only appear when also considering the years including the financial crisis and the Great Recession.

Third, restricting the long-run impact of H shocks on prices consistently produces smaller estimates of the fraction of the unit root of either GDP or consumption explained by H shocks.

Overall, the evidence produced by stationary VARs points towards a non-negligible extent of hysteresis, equal to 24 or 28 per cent of the frequency-zero variance of GDP for

the sample including the ZLB period. For the sample excluding the ZLB period estimates are significantly smaller.

D.2 Evidence From the Monte Carlo-Based Correction

Figure A.18 shows the median and the 16-84 and 5-95 percentiles of the Monte Carlo distributions of the Kolgomorov-Smirnov (KS) statistics for the two samples excluding and including the ZLB, respectively. Based on the stationary VARs featuring Δy_t and Δc_t we report individual statistics for GDP and consumption, respectively, whereas we eschew investment because, by construction, its frequency-zero behavior is (up to a scale factor) identical to that of y_t and c_t, respectively. Once again, evidence is characterized by a significant extent of uncertainty. Abstracting from this, and focusing on the median estimates, two facts emerge from the two figures:

first, for the sample excluding the ZLB period evidence uniformly suggests that the most likely value of the extent of hysteresis is zero. This is starkly apparent based on consumption, which, for the reasons discussed in the main text, should be regarded as the most reliable.

Second, for the full sample period evidence clearly points towards a non-negligible extent of hysteresis, equal to about 10 per cent of the frequency-zero variance of GDP based on the VAR featuring Δy_t, and to nearly 30 per cent based on the VAR featuring Δc_t.

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Figures for the Online Appendix

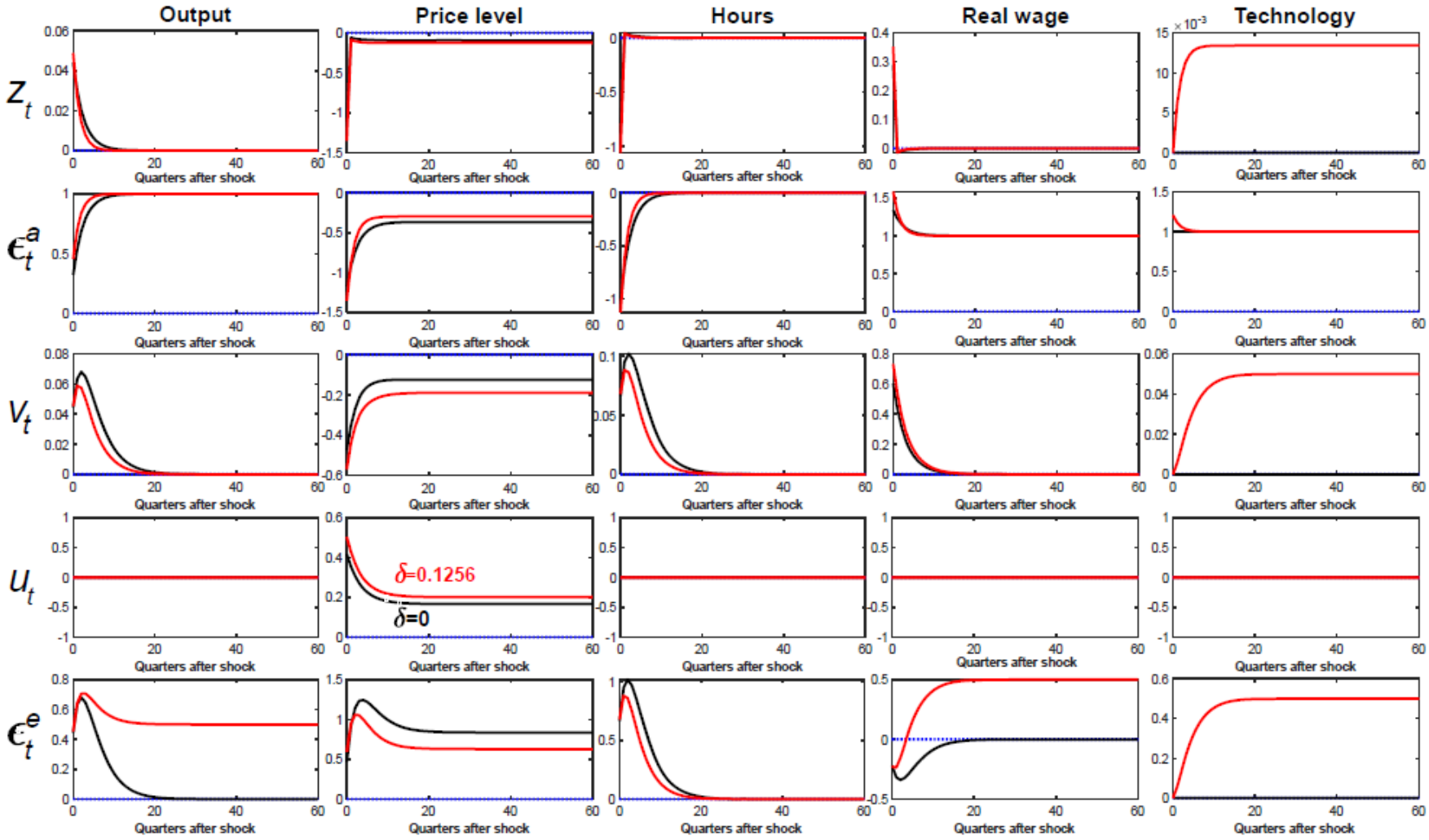


Figure A.1 Theoretical impulse-response functions for the RBC model

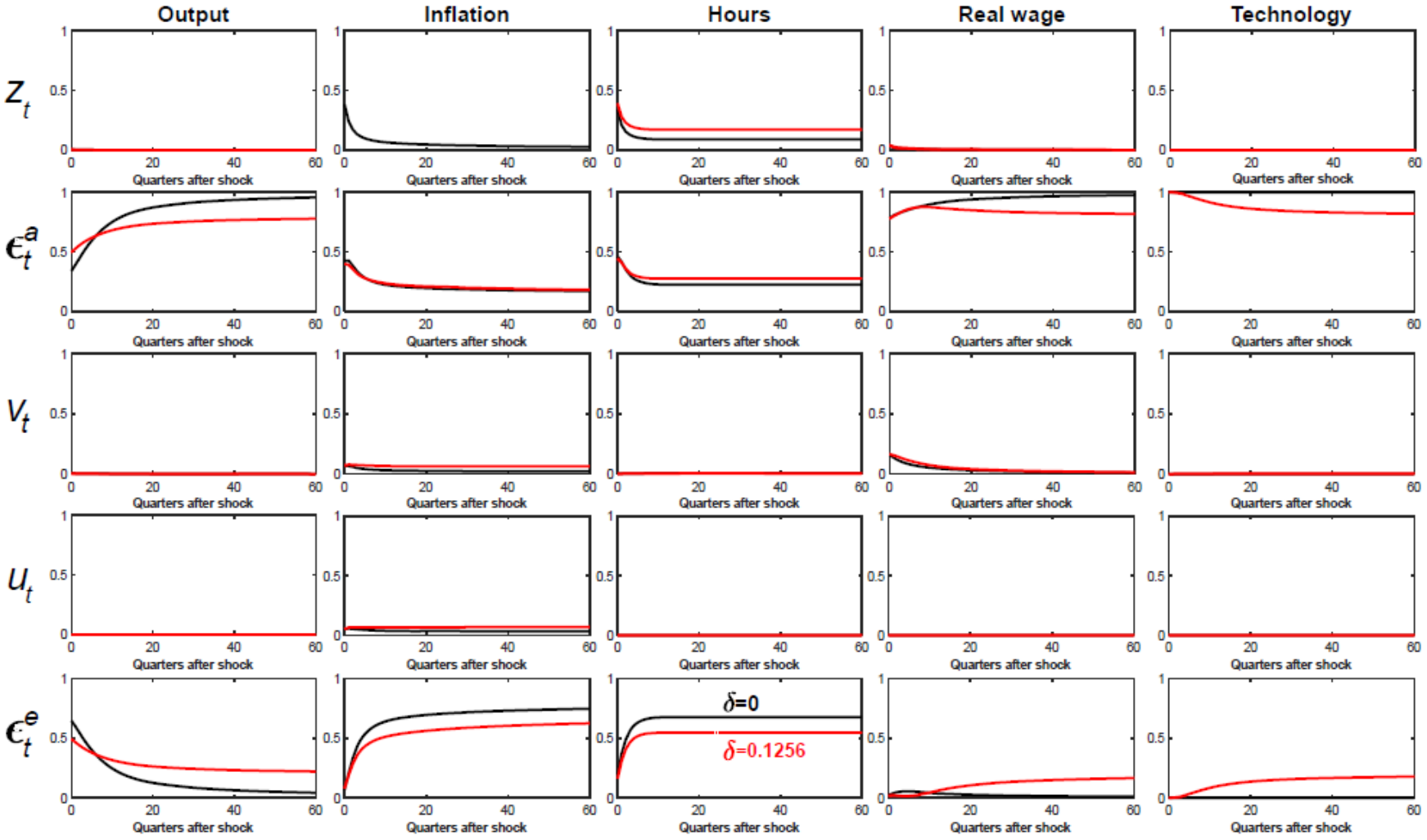


Figure A.2 Theoretical fractions of forecast error variance for the RBC model

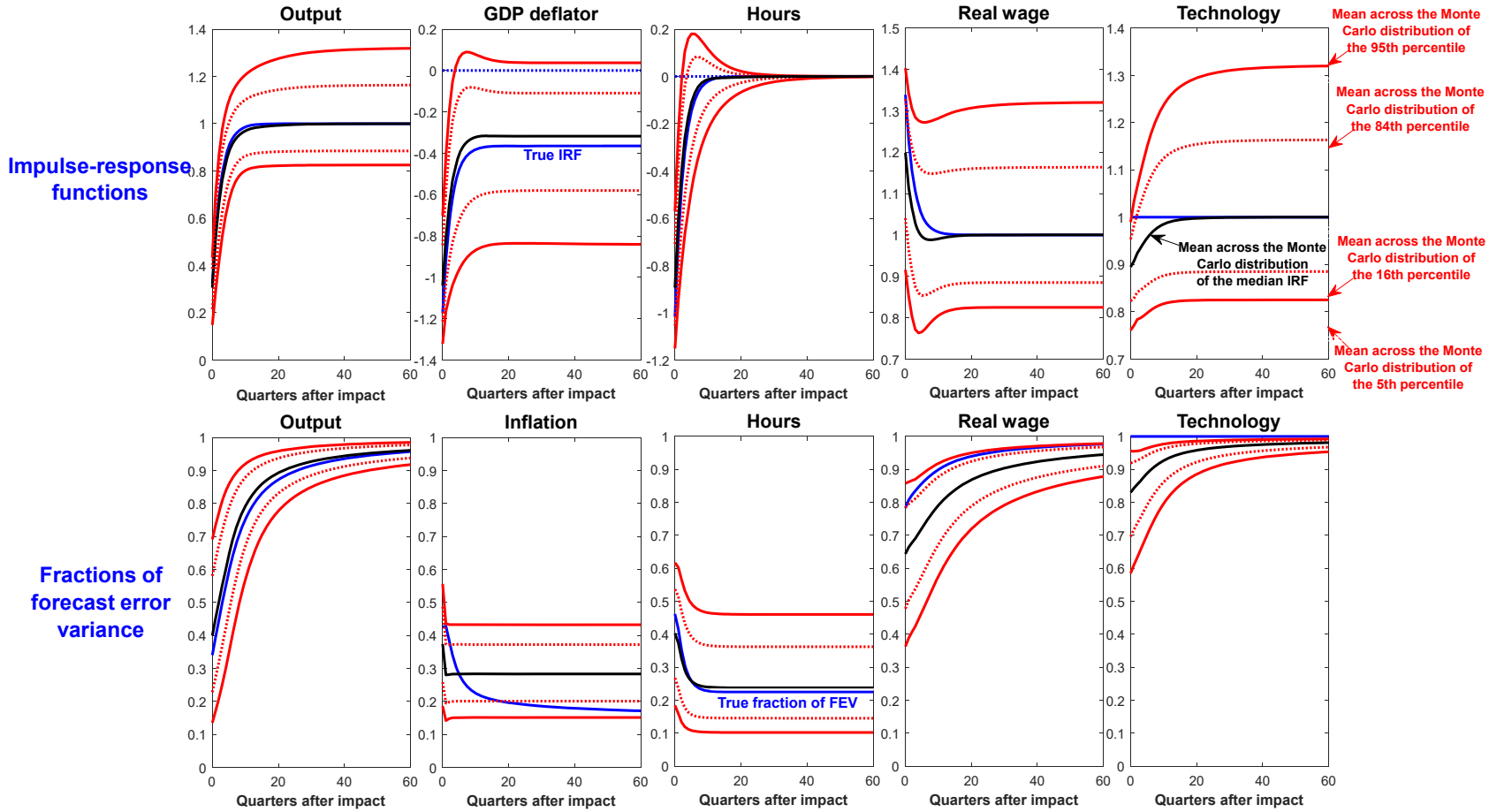


Figure A.3 Monte Carlo evidence on recovering the RBC model's true IRFs and fractions of forecast error variance with no hysteresis, conditional on the correct identification scheme

Evidence Based on the 7-Variables Bayesian Cointegrated VARs

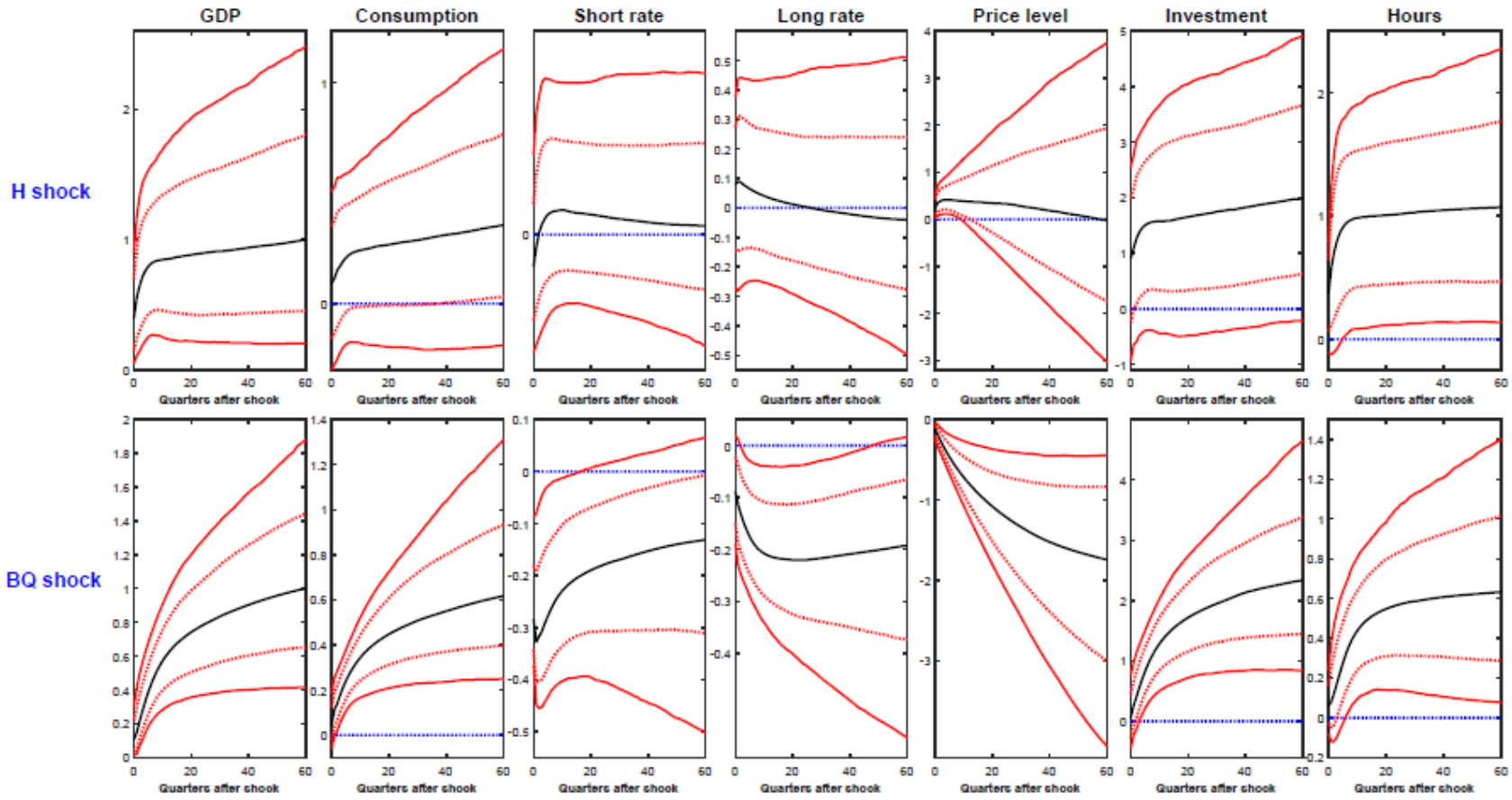


Figure A.4 Impulse-response functions to H and BQ shocks, based on the 7-variables cointegrated VAR (excluding the ZLB period)

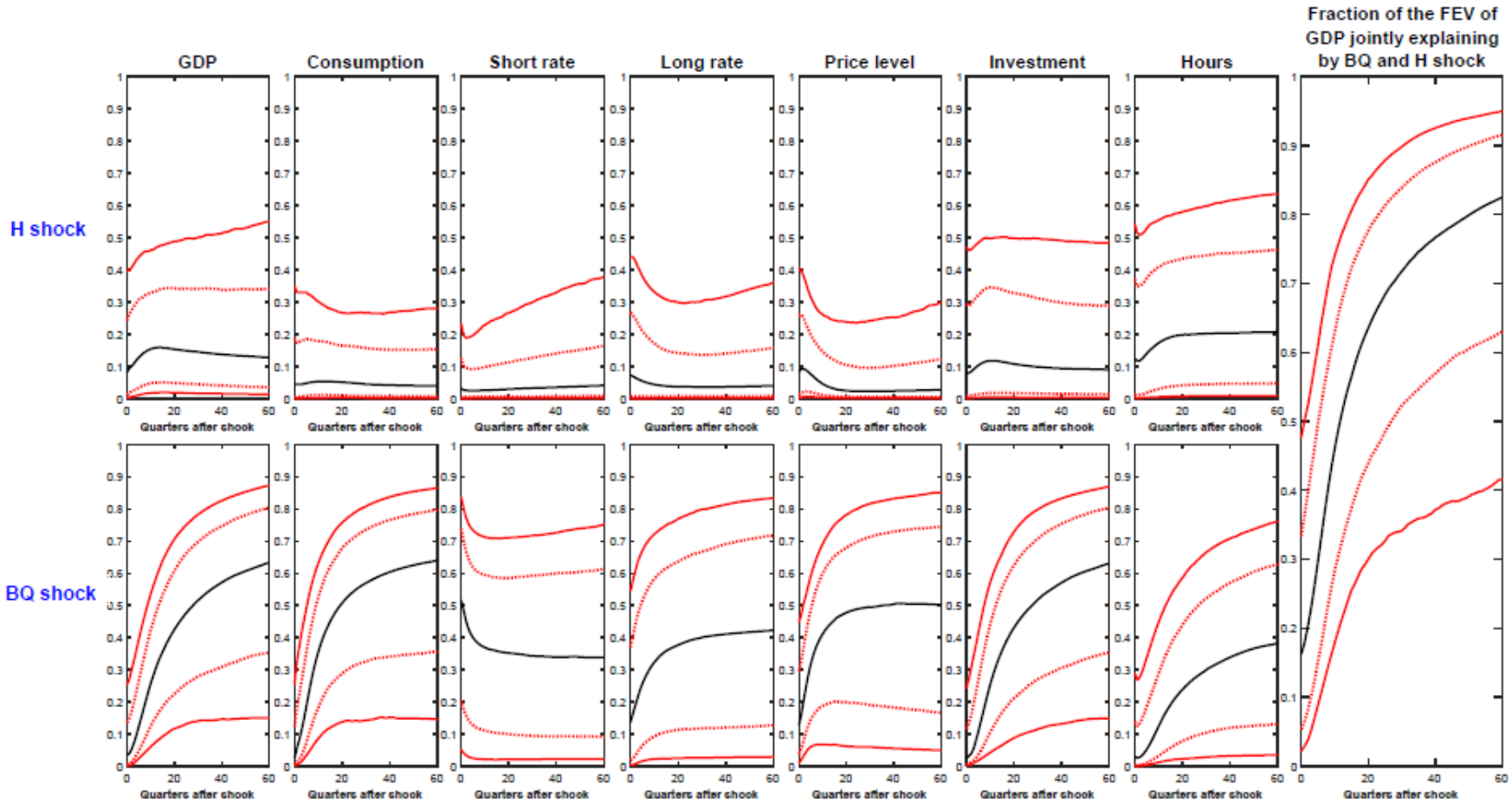


Figure A.5 Fractions of forecast error variance explained by H and BQ shocks, based on the 7-variables cointegrated VAR (excluding the ZLB period)

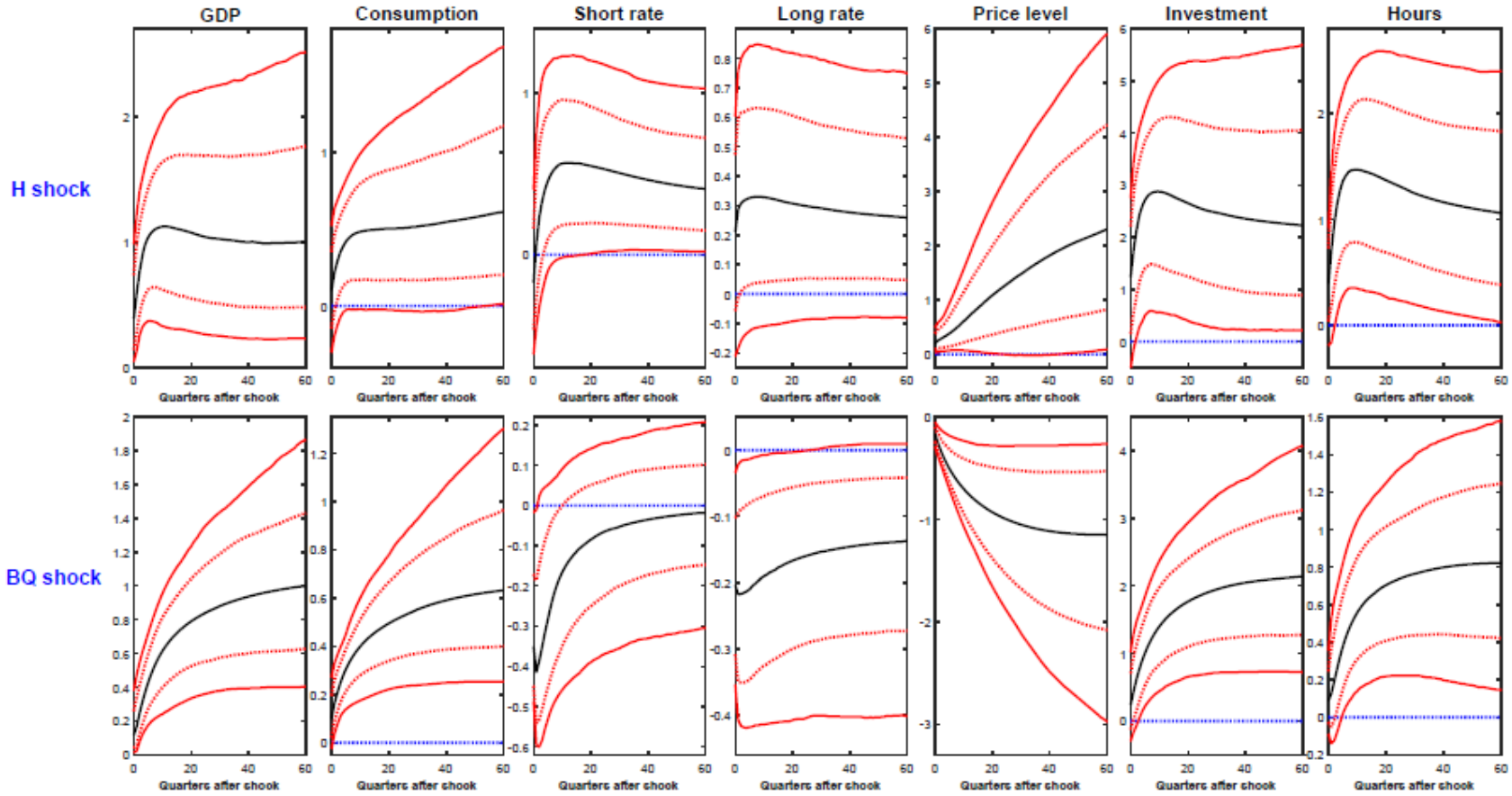


Figure A.6 Impulse-response functions to H and BQ shocks, based on the 7-variables cointegrated VAR, imposing restrictions on the long-run impact on prices (including the ZLB)

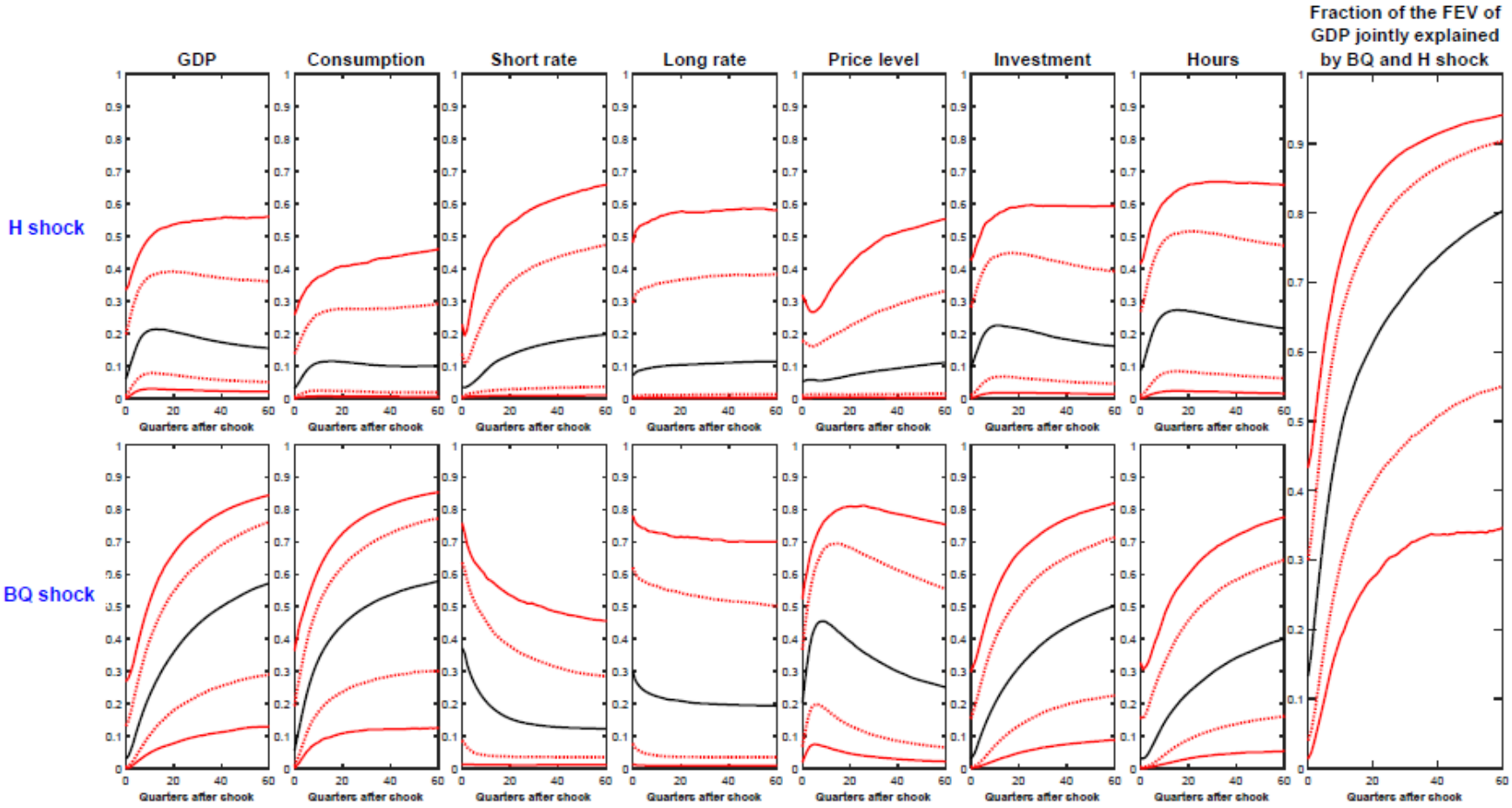


Figure A.7 Fractions of forecast error variance explained by H and BQ shocks, based on the 7-variables cointegrated VAR, imposing restrictions on the long-run impact on prices (including the ZLB)

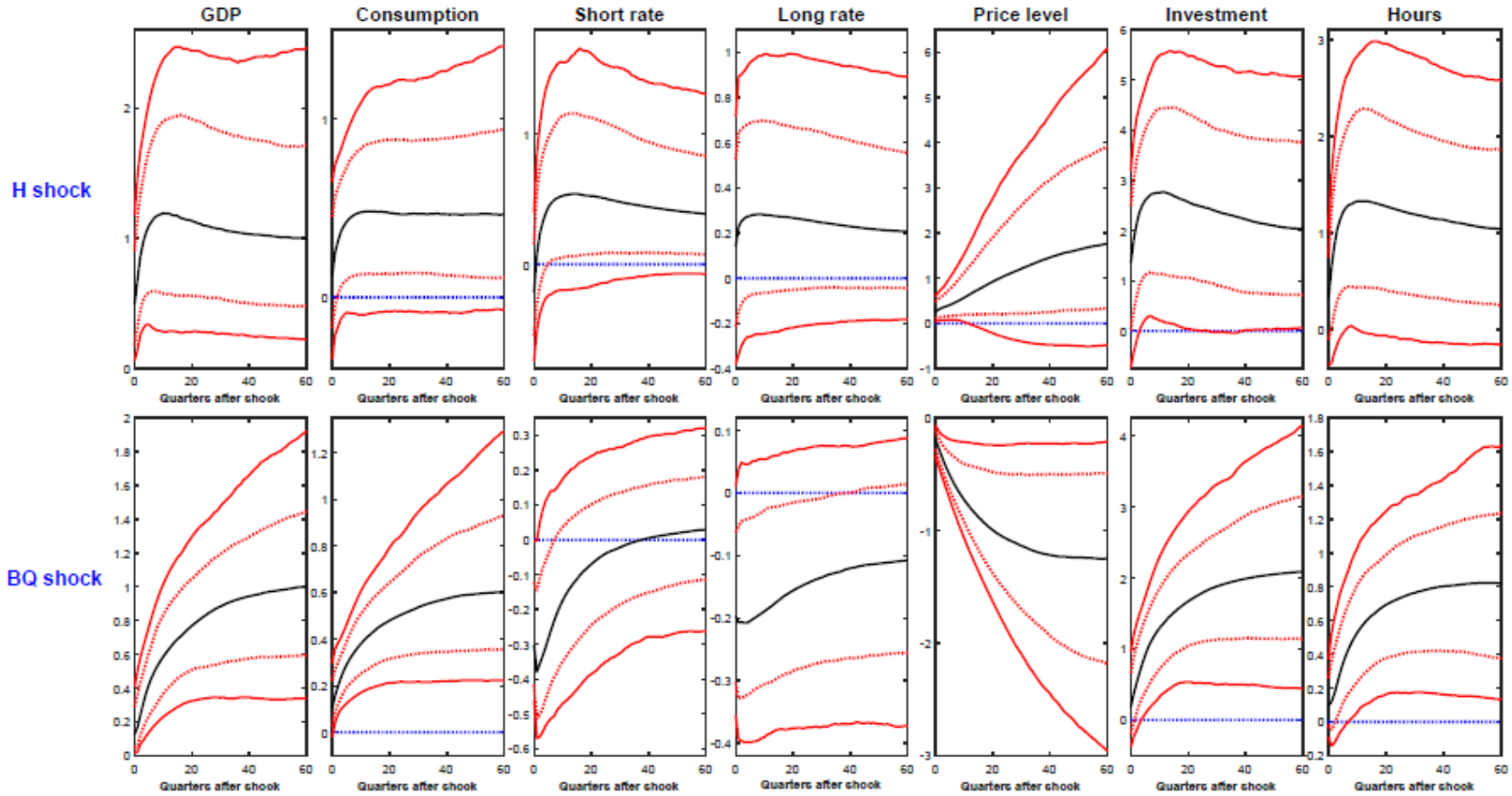


Figure A.8 Impulse-response functions to H and BQ shocks, based on the 7-variables cointegrated VAR, imposing restrictions on the long-run impact on prices (excluding the ZLB period)

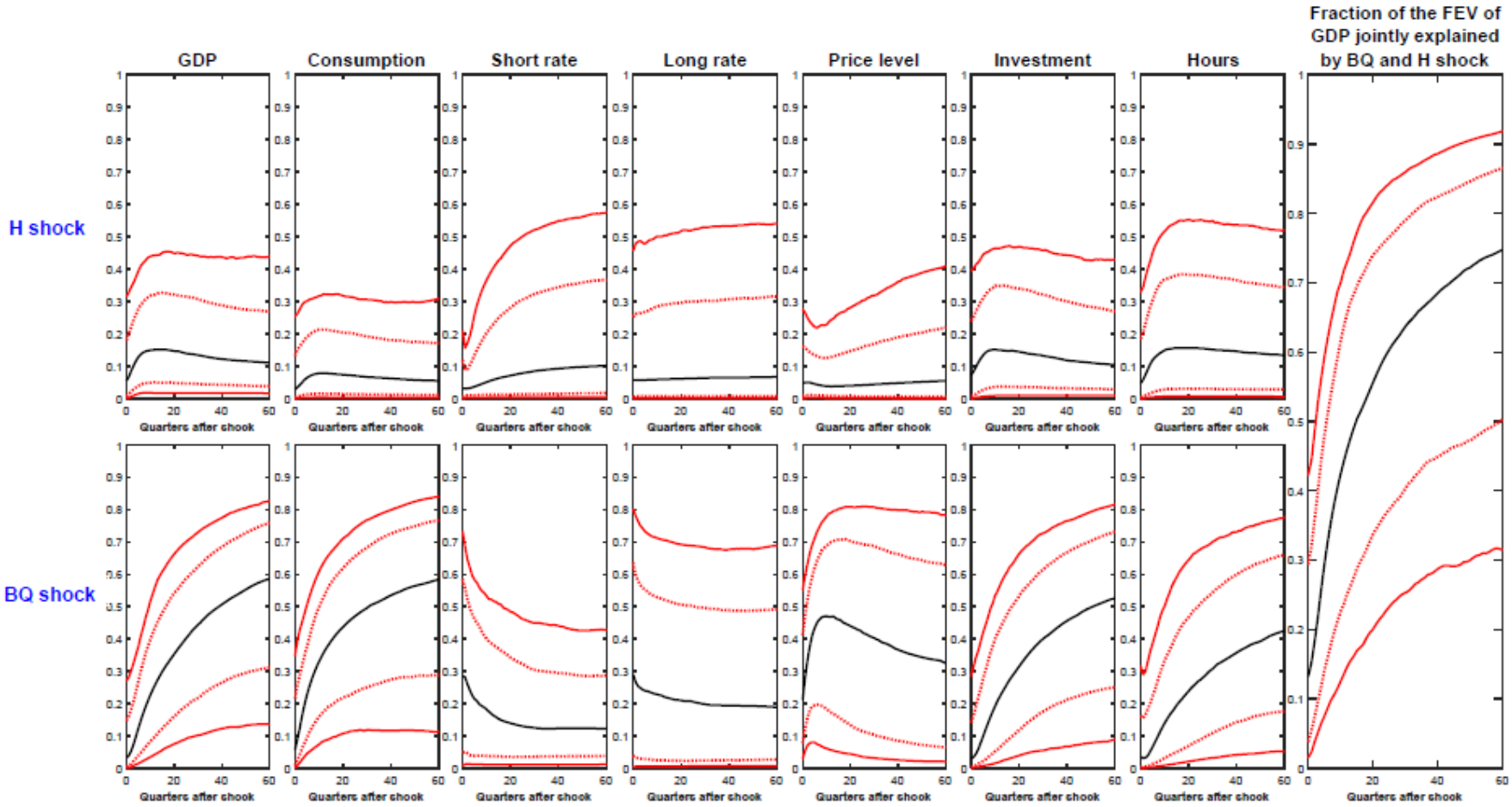


Figure A.9 Fractions of forecast error variance explained by H and BQ shocks, based on the 7-variables cointegrated VAR, imposing restrictions on the long-run impact on prices (excluding the ZLB period)

Evidence Based on the 7-Variables Stationary Bayesian VARs

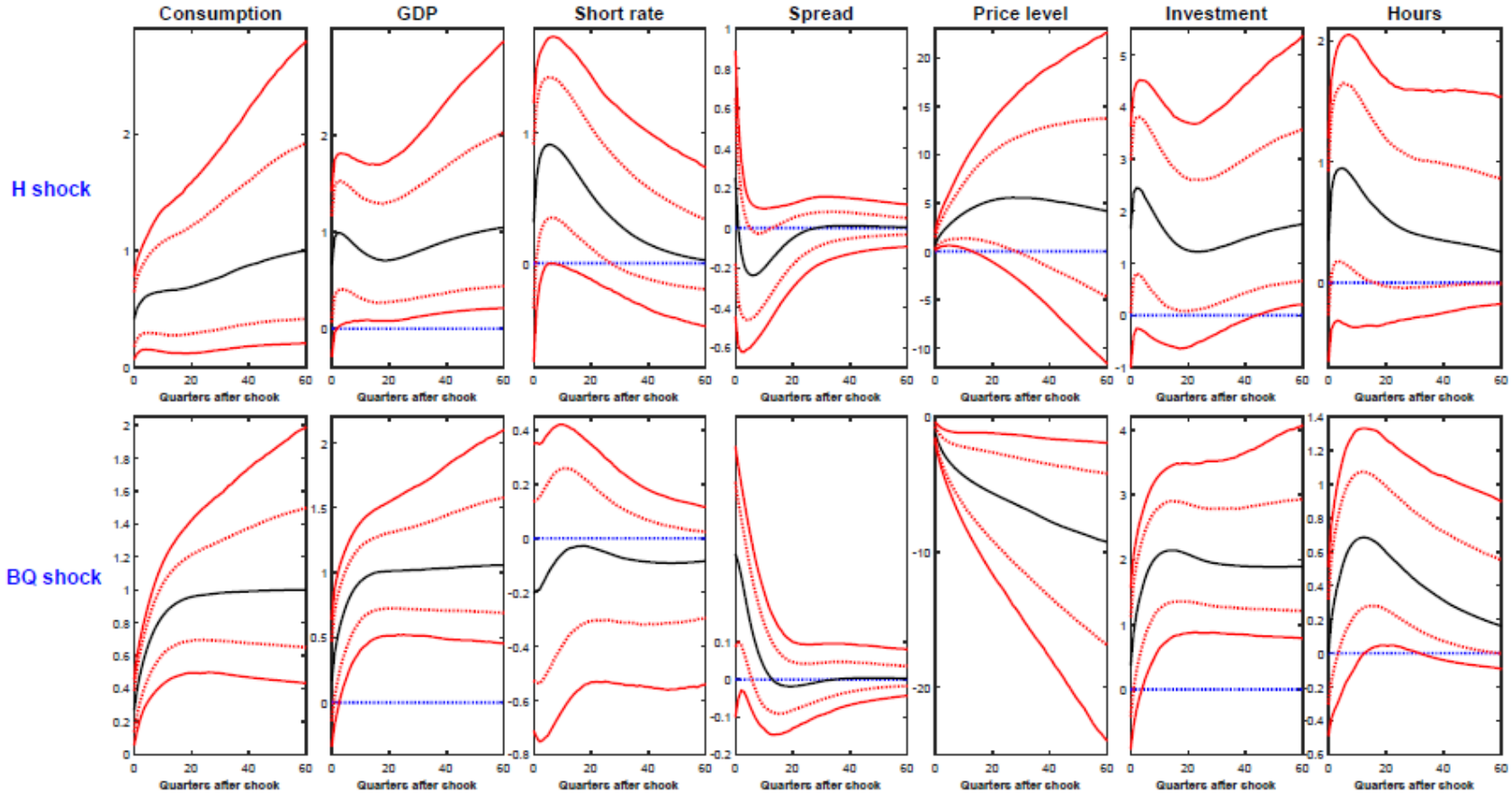


Figure A.10 Impulse-response functions to H and BQ shocks, based on the 7-variables stationary Bayesian VAR (including the ZLB period)

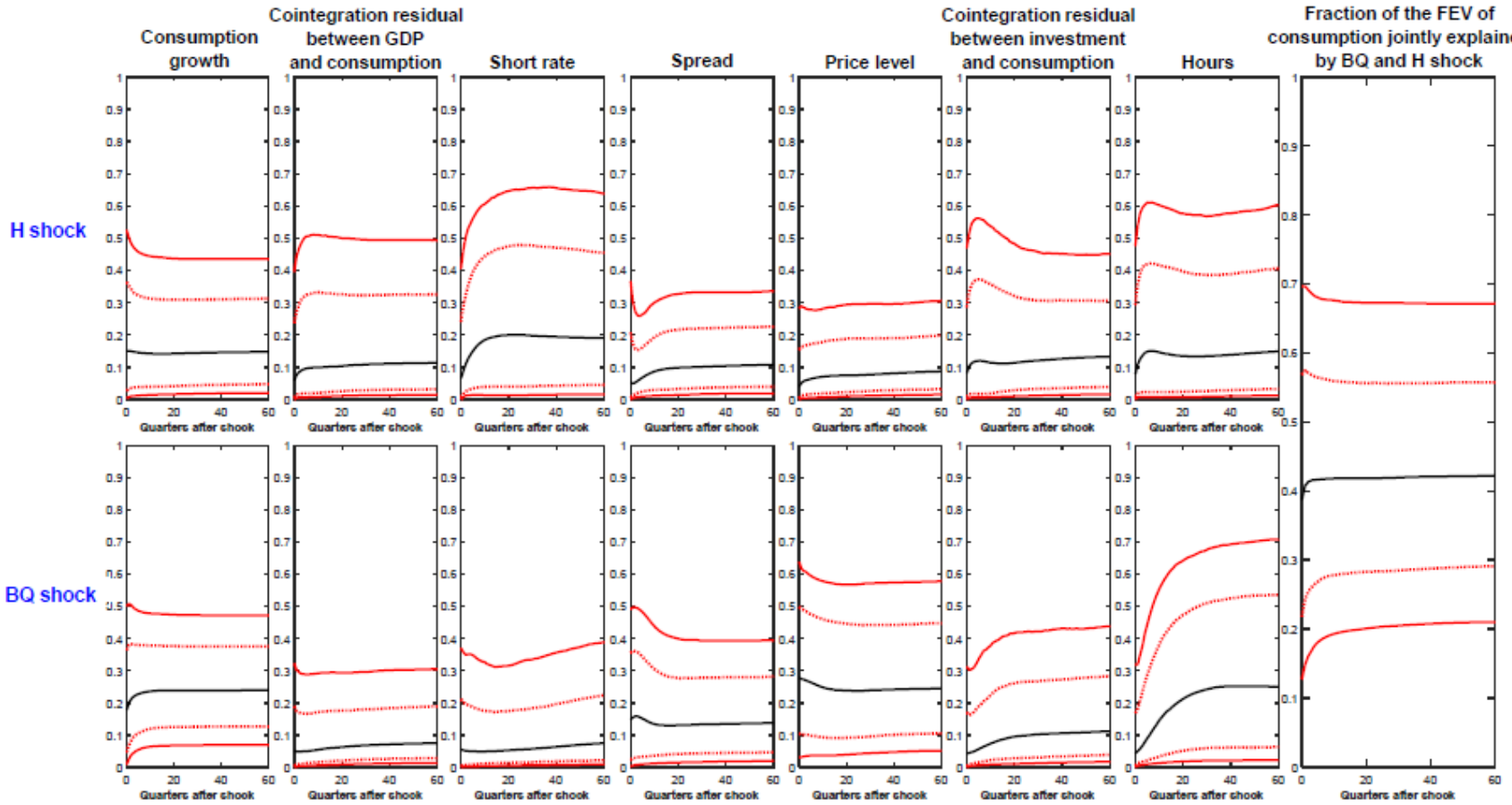


Figure A.11 Fractions of forecast error variance explained by H and BQ shocks, based on the 7-variables stationary Bayesian VAR (including the ZLB period)

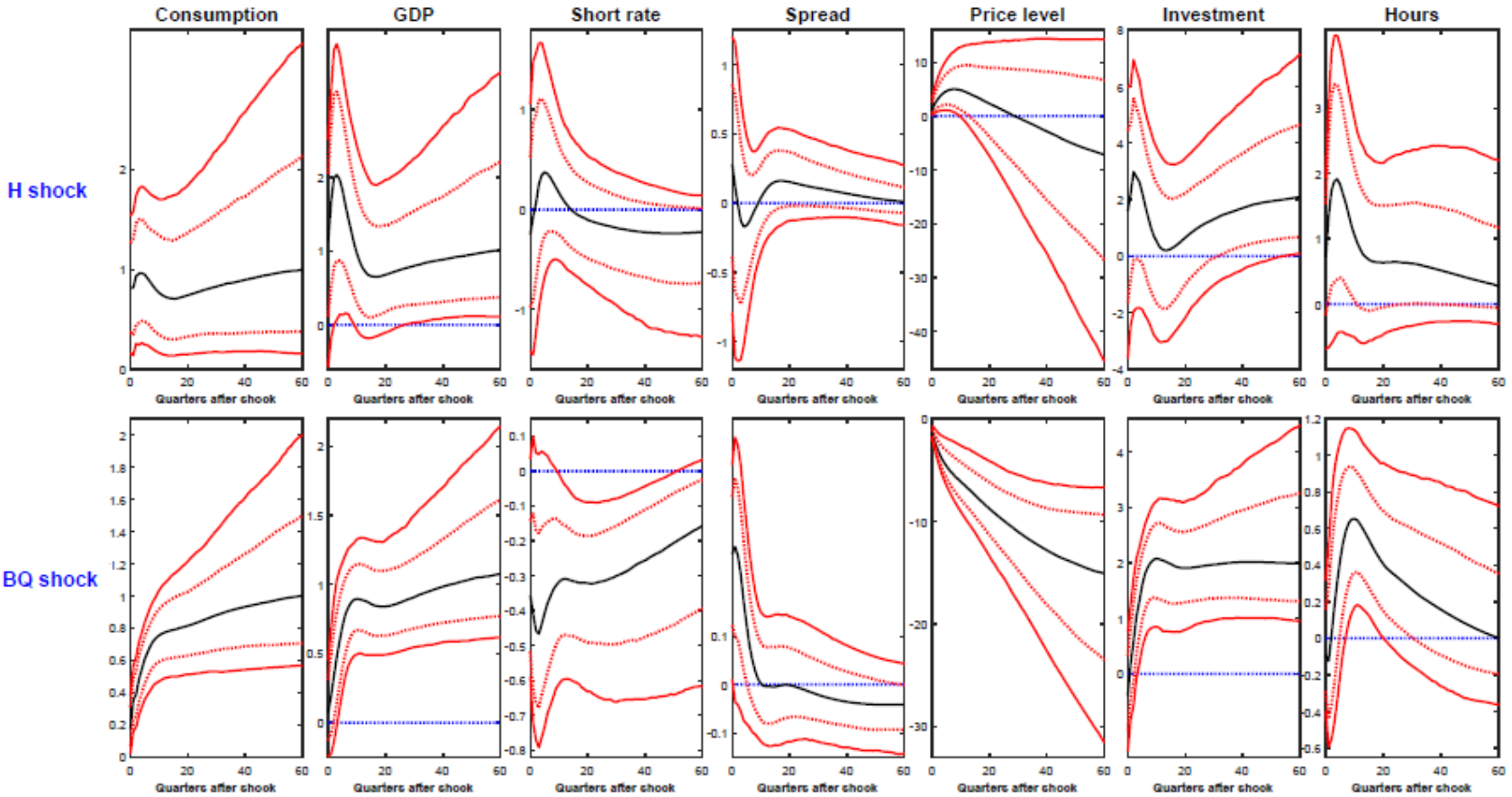


Figure A.12 Impulse-response functions to H and BQ shocks, based on the 7-variables stationary Bayesian VAR (excluding the ZLB period)

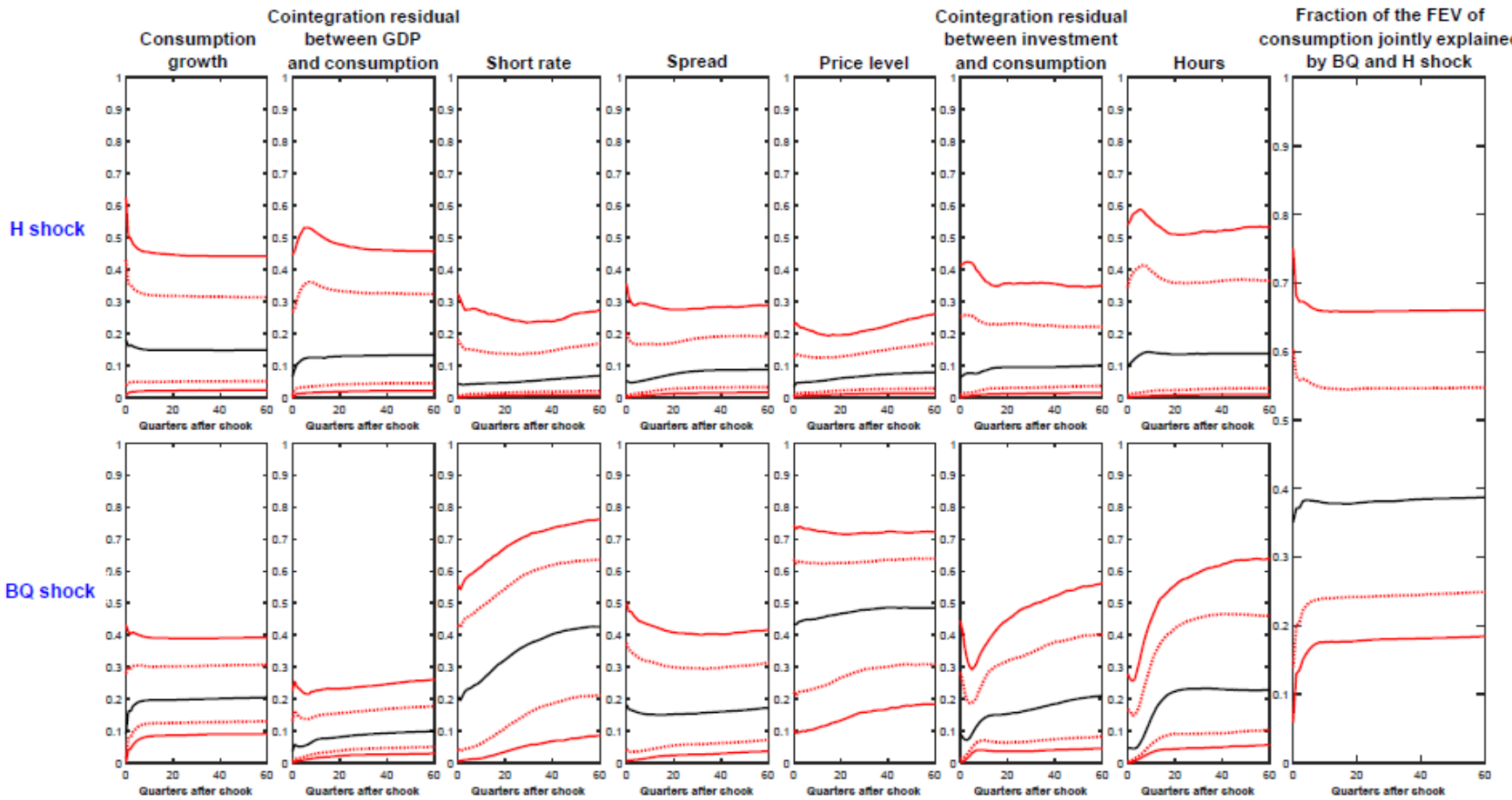


Figure A.13 Fractions of forecast error variance explained by H and BQ shocks, based on the 7-variables stationary Bayesian VAR (excluding the ZLB period)

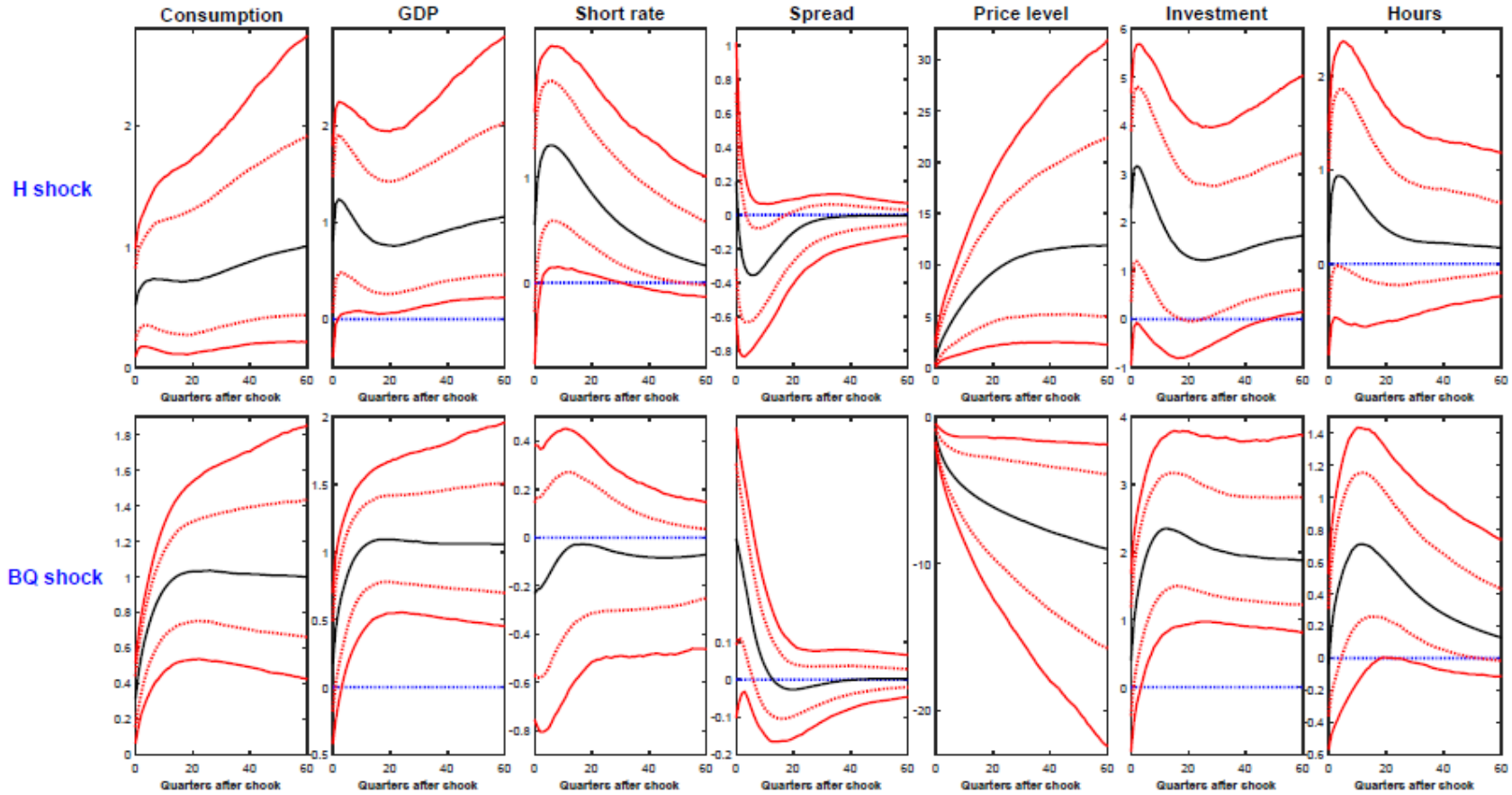


Figure A.14 Impulse-response functions to H and BQ shocks, based on the 7-variables stationary Bayesian VAR, imposing restrictions on the long-run impact on prices (including the ZLB period)

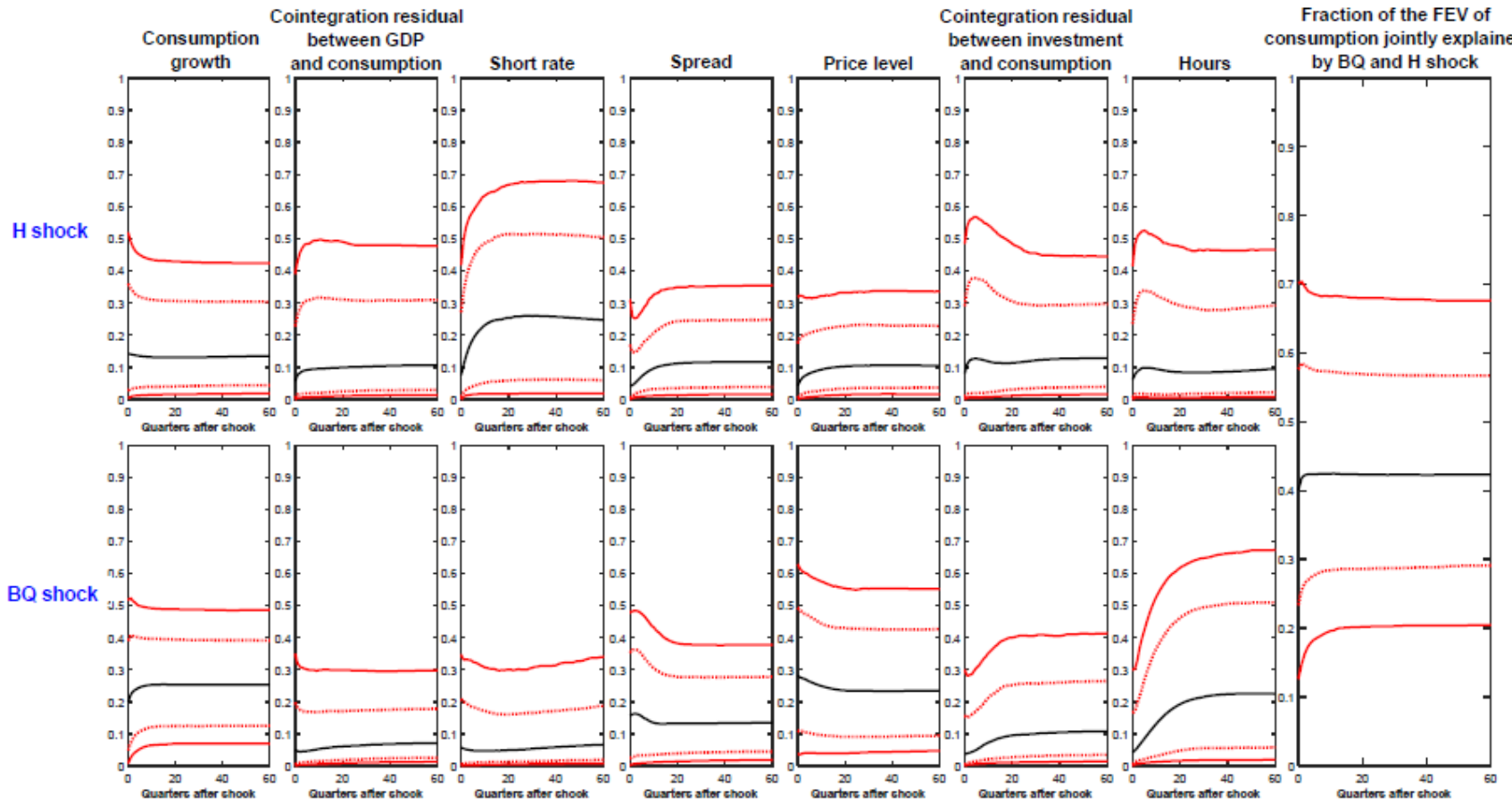


Figure A.15 Fractions of forecast error variance explained by H and BQ shocks, based on the 7-variables stationary Bayesian VAR, imposing restrictions on the long-run impact on prices (including the ZLB period)

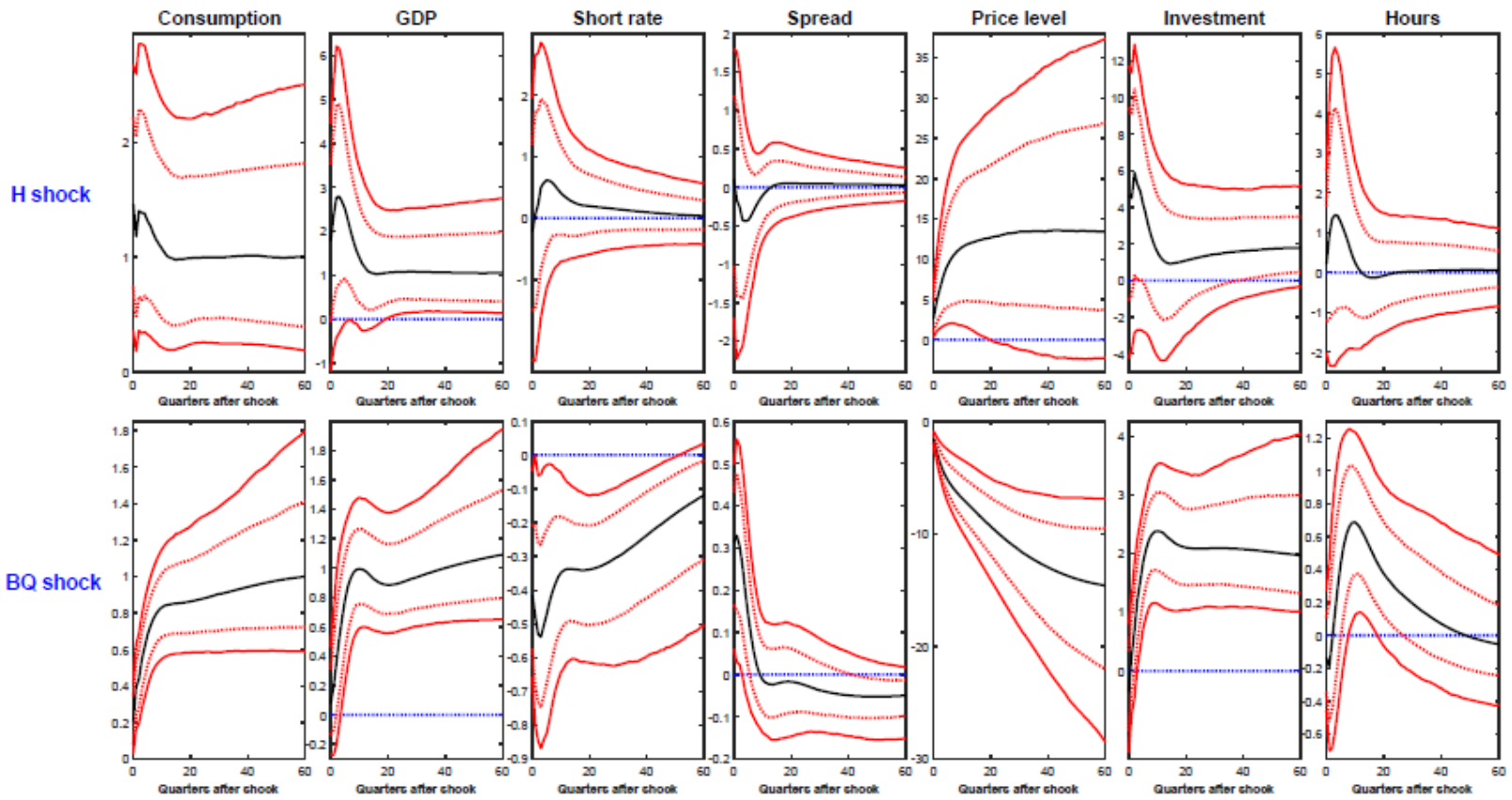


Figure A.16 Impulse-response functions to H and BQ shocks, based on the 7-variables stationary Bayesian VAR, imposing restrictions on the long-run impact on prices (excluding the ZLB period)

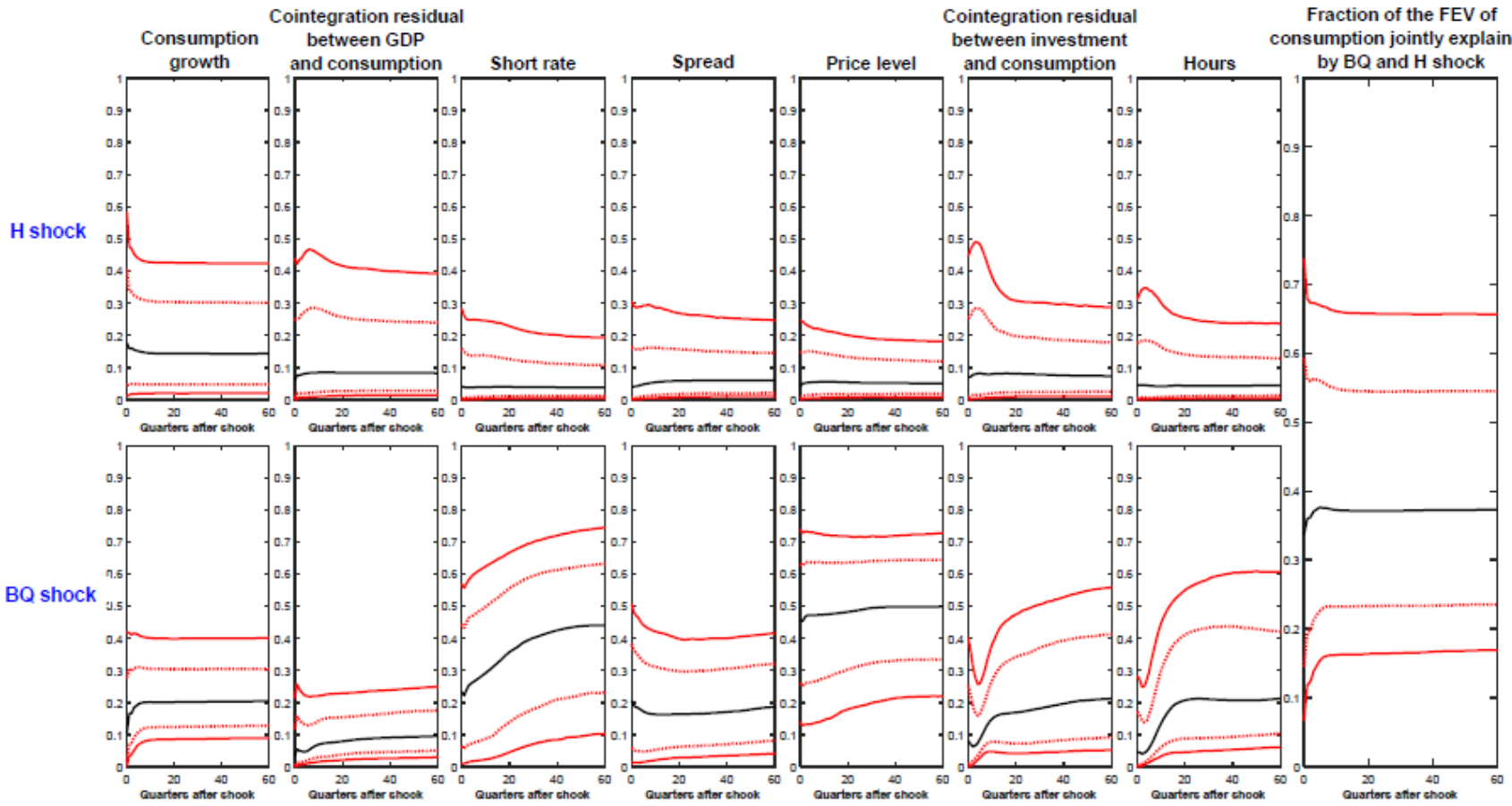


Figure A.17 Fractions of forecast error variance explained by H and BQ shocks, based on the 7-variables stationary Bayesian VAR, imposing restrictions on the long-run impact on prices (excluding the ZLB period)

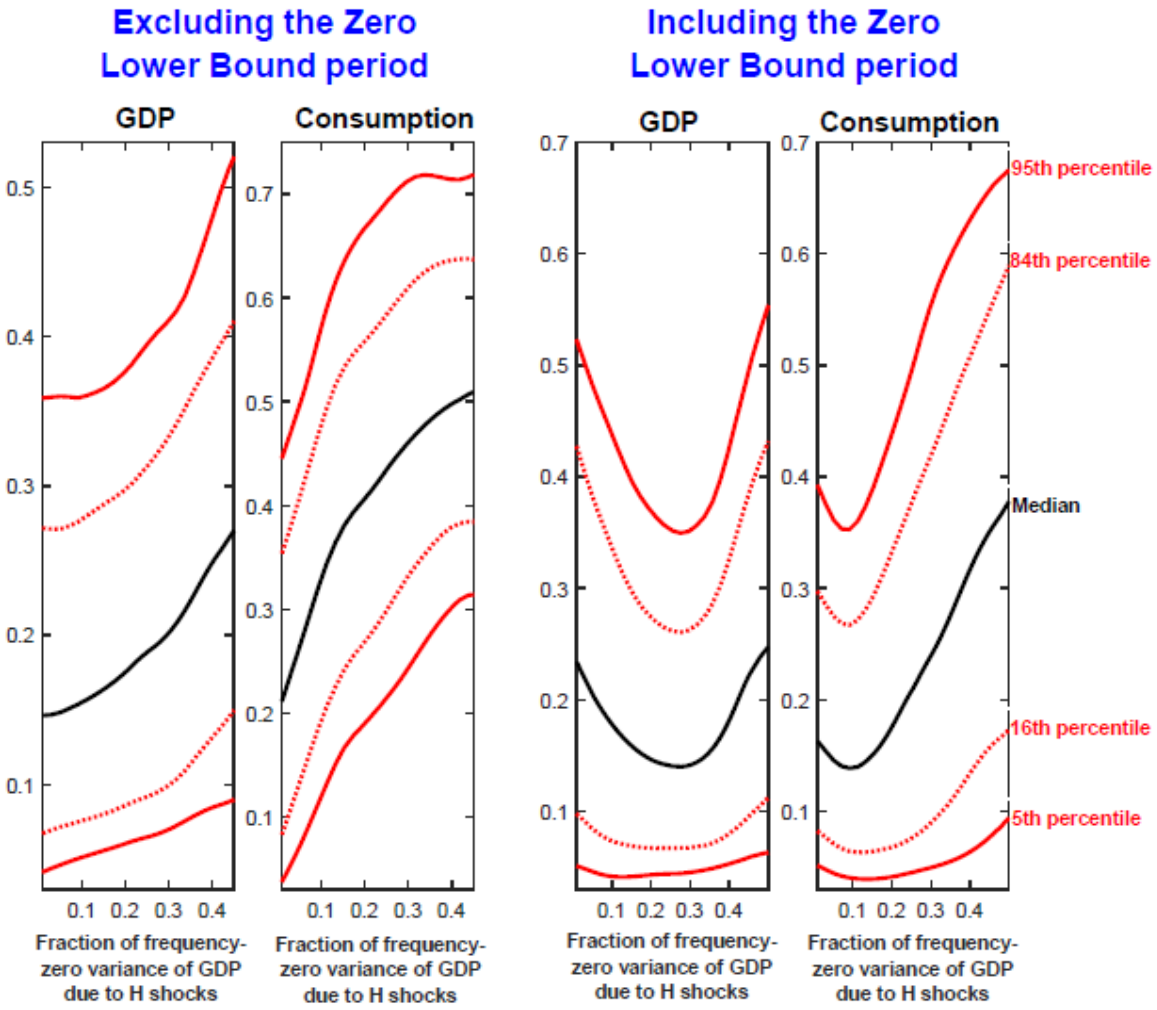


Figure A.18 Median and selected percentiles of the Monte Carlo distribution of the Kolgomorov-Smirnov statistic

Evidence on the Convergence of the Markov Chain

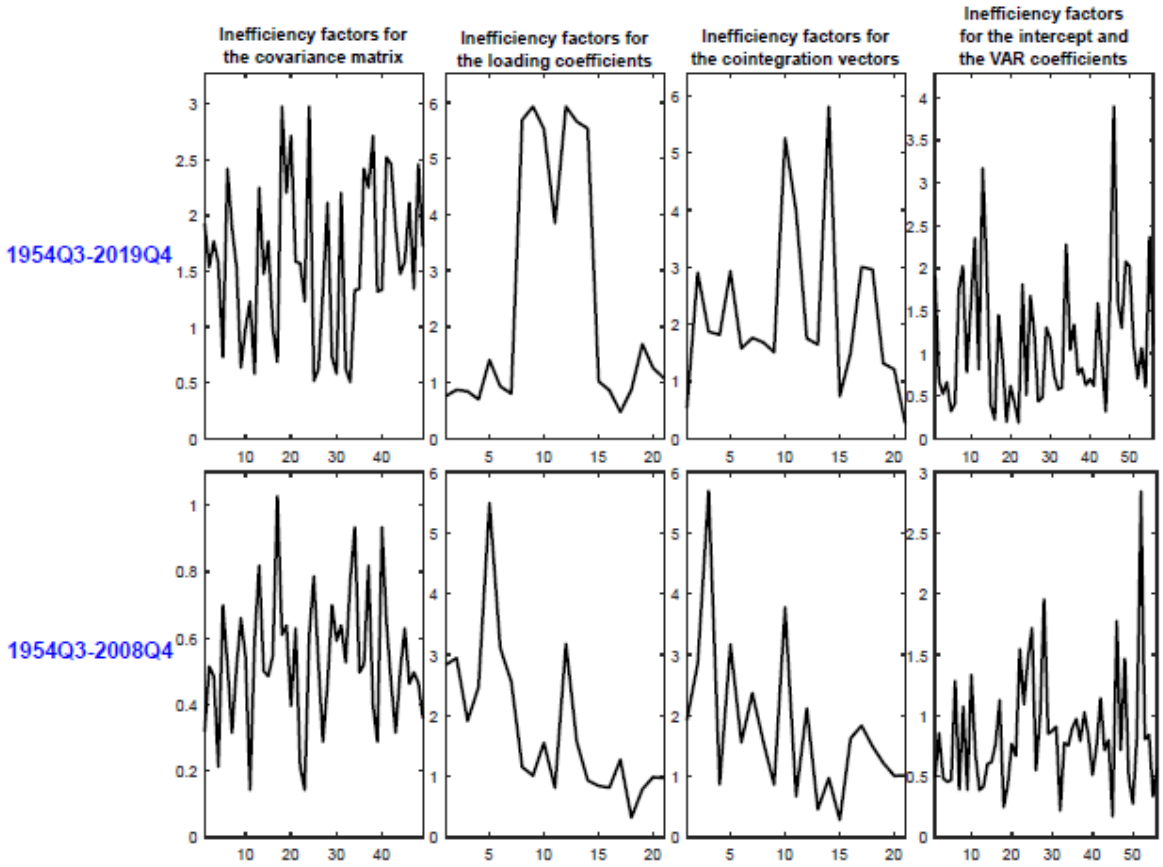


Figure A.19 Evidence on the convergence of the Markov chain for the 7-variables cointegrated VAR: Geweke's (1992) inefficiency factors of the draws for each individual parameter

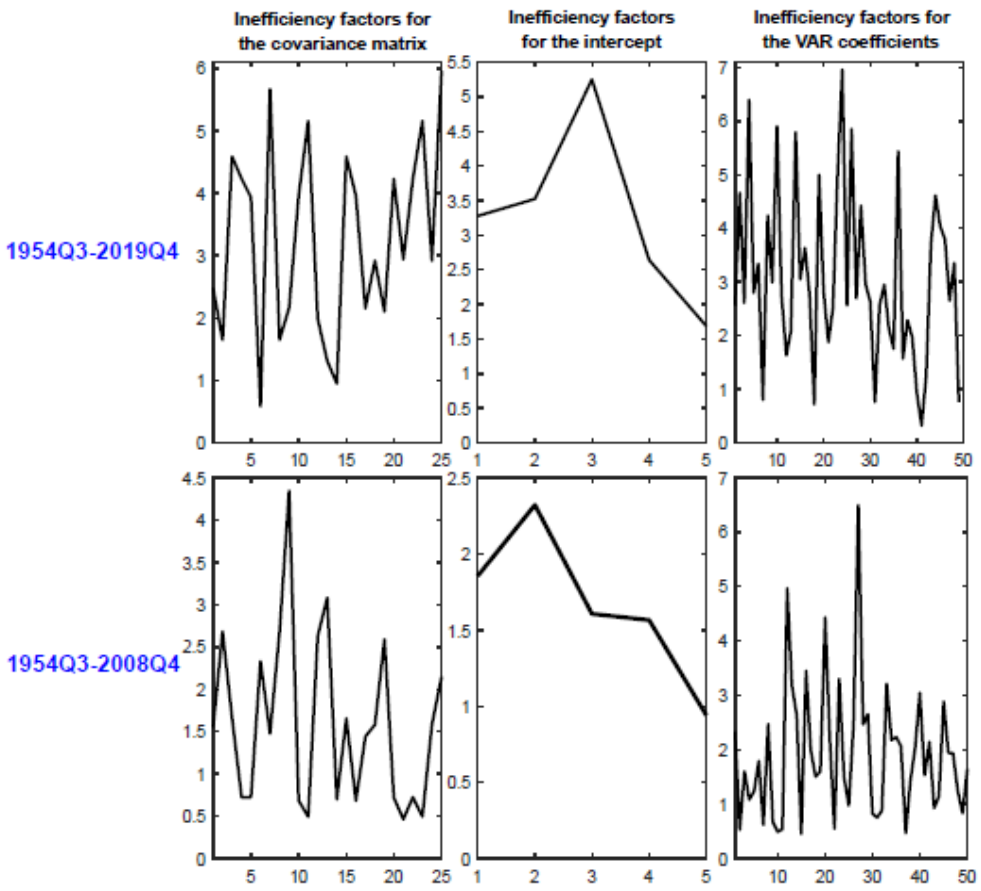


Figure A.20 Evidence on the autocorrelation of the draws for the 5-variables stationary Bayesian VAR: Geweke's (1992) inefficiency factors of the draws for each individual parameter