Understanding Post-Covid Inflation Dynamics*

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Abstract

We propose a macroeconomic model with a nonlinear Phillips curve that has a flat slope when inflationary pressures are subdued and steepens when inflationary pressures are elevated. The nonlinear Phillips curve in our model arises due to a quasi-kinked demand schedule for goods produced by firms. Our model can jointly account for the modest decline in inflation during the Great Recession and the surge in inflation during the Post-Covid period. Because our model implies a stronger transmission of shocks when inflation is high, it generates conditional heteroscedasticity in inflation and inflation risk. Hence, our model can generate more sizeable inflation surges due to cost-push and demand shocks than a standard linearized model. Finally, our model implies that the central bank faces a more severe trade-off between inflation and output stabilization when inflation is high.

JEL Classification: E30, E31, E32, E37, E44, E52

Keywords: Inflation dynamics, inflation risk, monetary policy, linearized model, nonlinear model, real rigidities.

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1 Introduction

After more than two decades of low and stable inflation in advanced economies, inflation has risen sharply during the Post-Covid period. In light of the substantial evidence of flat slope of the Phillips curve, the recent surge in inflation has caught many macroeconomists off guard. Hence, an intense debate has unfolded about how to explain the increase in inflation.[1] Has the Phillips curve steepened or are large exogenous demand and supply factors key to understand why inflation has risen so much? Our paper attempts to make a contribution to this debate.

Specifically, we propose a macroeconomic model which implies a nonlinear Phillips curve. The Phillips curve in our model is flat when inflationary pressures are subdued and steepens as inflationary pressures rise. The key feature of our model underpinning this nonlinear relationship stems from a quasi-kinked demand schedule for goods produced by firms. The flat part of the Phillips curve was used by Harding, Lindé and Trabandt (2022) to resolve the missing deflation puzzle during the Great Recession. In this paper, we argue that the very same model is very successful explaining Post-Covid inflation dynamics.

Our explanation rests on the steeper portion of the Phillips curve, which implies that all shocks in the model transmit stronger to inflation when inflation is surging. For demand and technology shocks, the relationship between the initial inflation level and how strongly shocks propagate to inflation is precise and increases monotonically when we vary these shocks according to their estimated standard deviation. However, cost-push shocks propagate even stronger to inflation than demand and technology shocks when inflation is high and rising above its steady state level. But they also can also transmit less to inflation than demand and technology shocks when inflation is receding (even if it is well above its steady state). Hence, cost-push shocks generate conditional heteroscedasticity in inflation and inflation risk in our nonlinear model. Since these shocks are key drivers of price inflation in our estimated model and commonly believed to have played an important role during the Post-Covid period, we argue that our model can account better for inflation dynamics during this period than a standard linearized macroeconomic model. Regression analysis supports the view that cost-push-type shocks have a larger impact on inflation if inflation is high to begin with, see e.g. Gelos and Ustyugova (2017), Forbes, Gagnon and Collins (2021a) and Forbes, Gagnon and Collins (2021b).

We establish our main results using the nonlinear formulation of the benchmark Smets and

[1] See e.g. Federal Reserve Chair J. Powell a the 2021 Jackson Hole conference as well as the debate between L. Summers and P. Krugman that took place since early 2021.

Wouters (2007, SW henceforth) model, which builds on the seminal model of Christiano, Eichenbaum and Evans (2005). The SW model has been shown to have empiricial properties that are on par with standard Bayesian vector autoregressions prior to the Great Recession, see. e.g. del Negro et al. (2007). The only difference relative to the original SW estimated model is that we follow Harding, Lindé and Trabandt (2022) and use the nonlinear formulation of the SW model allow for a more prominent role for Kimball (1995) quasi-kinked demand in goods markets. The more prominent role for quasi-kinked demand increases the marginal data density provided that the average markup aligns with micro- and macroeconomic empirical evidence.

An important policy implication from our analysis is that the central bank faces a more severe trade-off between inflation and output stabilization when inflation is high. This finding is driven by the fact that cost-push shocks propagate more strongly to inflation than monetary policy shocks when inflation is surging above the central banks inflation target. We use the nonlinear model to filter data up to 2021Q4 and then examine the propagation of positive cost-push and monetary policy shocks given the filtered state. Because inflation is elevated at this state, the nonlinear model implies a twice as large jump in inflation compared to the standard linearized model in response to the cost-push shock. The larger transmission to inflation implies that the central bank endogenously, through its policy rule, tightens the policy rate by twice as much. As a result of the tighter policy stance and the elevated level of inflation, the output gap falls nearly twice as much in the nonlinear model compared to the linearized model. If the central bank chooses to fully offset the pass-through to inflation, the required policy tightening triggers large output costs although policy is more effective than in normal times in the nonlinear model.

Our results rest on two key model elements. First, we introduce real rigidities in price-setting. To do this, we follow Dotsey and King (2005) and Smets and Wouters (2007) and use the Kimball (1995) aggregator instead of the standard Dixit-Stiglitz (1977) aggregator. The Kimball aggregator introduces additional strategic complementarities in firms price-setting behavior, which lowers the sensitivity of prices to marginal cost for a given degree of price-stickiness. As such, the Kimball aggregator is commonly used in New Keynesian models, see e.g. Smets and Wouters (2007), as it allows to simultaneously account for the macroeconomic evidence of a low Phillips curve slope and the microeconomic evidence of frequent price changes.

Second, we argue that the standard procedure of linearizing all equilibrium equations around the steady state, except for the zero lower bound (ZLB) constraint on the nominal interest rate, introduces large approximation errors when large shocks hit the economy as was the case during the

Great Recession and the Post-Covid period. Implicit in the linearization procedure is the assumption that the linearized solution is accurate even far away from the steady state. Our analysis shows that the linearized solution is very inaccurate far away from the steady state. In particular, we show that cost-push shocks can propagate more than 4 times stronger in the nonlinear model relative to the linearized model when inflation is high. This implies that the nonlinear model can account for the large run-up in inflation with notably smaller cost-push shocks during the Post-Covid period. We show that the nonlinearity implied by the Kimball aggregator is a key model feature that accounts for the differences between the linearized and nonlinear model solutions. The Kimball aggregator implies that the demand elasticity for intermediate goods is state-dependent, i.e. firms' demand elasticity is an increasing function of its relative price and the demand curve is quasi-kinked.[2] Due to the rising demand elasticity firms' marginal revenues are a concave function of their prices. Consequently, the optimal price-setting becomes asymmetric since firms equate marginal revenue to marginal cost. If the latter rise or fall the resulting optimal price setting becomes asymmetric in the sense that firms find it optimal to increase prices more than to decrease prices. While the fully nonlinear model takes the state-dependence of the quasi-kinked demand curve explicitly into account, a linear approximation replaces this key nonlinearity by a linear function. When the economy is exposed to large shocks the state-dependence of the quasi-kinked demand curve becomes quantitatively important and the linear approximation ceases to provide accurate results.

All told, our key contribution is to provide a structural general equilibrium model which can jointly account for the small drop in inflation during the Great Recession and the large surge in inflation during the Post-Covid period.

The remainder of the paper is organized as follows. Section 2 presents a stylized static model of optimal price-setting with kinked demand that conveys the key mechanism driving our results. Section 3 presents the workhorse macroeconomic model with real rigidities in a dynamic stochastic general equilibrium framework with nominal price and wage stickiness. Section 4 discusses our results. Finally, section 5 provides concluding remarks.

2 Optimal Price-Setting with Quasi-kinked Demand

In this section we provide intution about firms' optimal price-setting behavior when the demand curve is quasi-kinked. We consider a stylized static model to convey the intution why firms increase their prices by more when marginal costs go up compared to the case when marginal costs fall.

[2] See Dupraz (2018) for a microfoundation of a kinked demand curve theory.

Firms maximize profits, π:

$$\pi = p * y - mc * y$$

subject the following quasi-kinked demand curve:

$$y = a - p^b$$

where p denotes the price, y denotes quantity, mc denotes marginal costs, and $a, b > 0$ are parameters of the demand function. The functional form of the demand function follows e.g. Kimball (1995) as has been used by Harding, Lindé and Trabandt (2022), among many others. The elasticity of demand is given by:

$$\varepsilon \equiv -\frac{dy}{dp}\frac{p}{y} = \frac{b}{\frac{a}{p^b} - 1}$$

which is increasing in the price, p.

Optimal price setting results in the following first order condition:

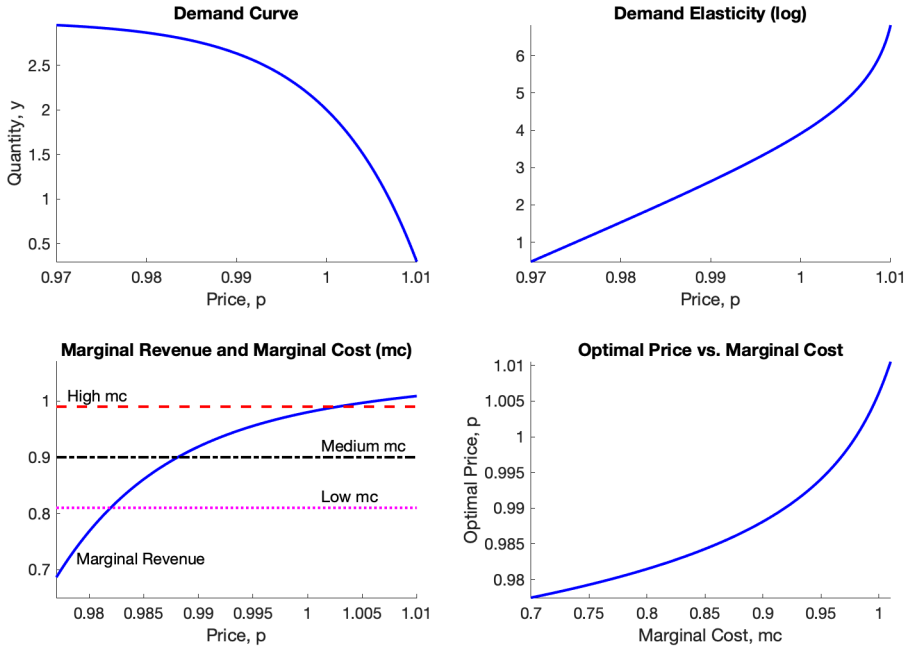
$$\underbrace{p\left[1 - \frac{1}{b}\left(\frac{a}{p^b} - 1\right)\right]}_{\text{marginal revenue}} = mc$$

Note that marginal revenue is a concave function of the price which is due to the increasing elasticity of demand. As an example, consider the following parameter values which are broadly in line with the parameterization of the workhorse model in Section 3: $a = 3, b = 100$ and $mc = 0.9$.

Figure 1 shows the demand curve and the elasticity of demand in the top panels. In the bottom left panel shows marginal revenue as a function of the price for low, medium and high marginal costs. Note that firms increase their prices by more than they cut their prices for a given percentage change in marginal costs. The reason for this is that firms intend to stabilize their markup. When marginal cost are low, the markup is high and firms have a small incentive to cut their prices, especially when demand is quasi-kinked, i.e. firms cannot crowd in a lot of extra demand by cutting their prices. When marginal costs are high, the markup is low and firms have a large incentive to increase their prices – even when it entails a substantial drop in demand. Note that the concavity of marginal revenue (which results from the rising elasticity of demand as a function of the price) is key for the asymmetric pricing behavior. Finally, the bottom right panel shows the optimal price as a function of marginal cost, showcasing a convex banana-type optimal pricing schedule as a function of marginal cost. This nonlinear relationship between marginal costs and optimal price-setting is the key force that will generate our nonlinear Phillips curve in the dynamic

general equilibrium model that we consider in the next section.

Figure 1: Optimal Price-Setting With Quasi-Kinked Demand



3 The Workhorse Macroeconomic Model

In this section we present the workhorse macroeconomic model that we argue is useful to understand how nonlinearities in real rigidities in price- and wage-setting affect inflation dynamics in a quantitatively realistic model environment. Specifically, we use the workhorse Smets and Wouters (2007) model of the US economy which is a multi-shock version of the seminal model of Christiano, Eichenbaum and Evans (2005) with endogenous capital accumulation.

In the following, we first give a brief overview of the model with a focus on the pricing bloc of the model and how we compute the linearized and nonlinear solutions of the model. Appendix A provides a detailed description of the model environment, as well as the resulting linearized and nonlinear equations.

3.1 Model Overview

The workhorse model of Smets and Wouters (2007) features monopolistic competition in the goods and labour markets and nominal frictions in the form of sticky prices and wages. However, it allows

non-optimizing firms (households) to index prices (wages) to a composite of steady-state and lagged inflation. Households can also save in physical capital, with a one-period time to build before new investments turns into productive capital. In addition to the EHL model, the Smets and Wouters model also features several real rigidities in the form of habit formation in consumption, investment adjustment costs, variable capital utilization, and fixed costs in production. The model dynamics are driven by seven structural shocks. Monetary policy shocks follow an AR(1) process and two additional inefficient cost-push shocks in wage and price setting follow an ARMA(1,1) process. Four efficient shocks (total factor productivity, risk premium, investment-specific technology, and government spending shocks) follow AR(1) processes. The exact specification of the nonlinear model and its linearized representation is described in detail in Appendix A, here we just present the linearized and nonlinear pricing equations that are central for our paper.

The linearized Phillips curve in the SW model is given by

$$\begin{aligned} \widehat{\pi}_t - \iota_p \widehat{\pi}_{t-1} &= \beta \left(\mathrm{E}_t \widehat{\pi}_{t+1} - \iota_p \widehat{\pi}_t \right) + \kappa \widehat{mc}_t + \hat{\varepsilon}_{p,t}, \\ \kappa &= \frac{(1-\xi_p\beta)(1-\xi_p)}{\xi_p(1+(\phi_p-1)\epsilon_p)} \end{aligned} \tag{1}$$

where $1-\xi_p$ is the probability of each firm being able to re-optimize the price each period, ϵ_p is the curvature of the SW aggregator function, ϕ_p is the steady state gross price markup, and the cost-push, or markup, shock $\hat{\varepsilon}_{p,t}$ has been re-scaled with $1/\kappa$ to enter the Phillips curve with a unit coefficient. The corresponding nonlinear recursive pricing equations are given by

$$\frac{1+\phi_p\epsilon_p}{1+\epsilon_p} p_t^* \gamma_{1,t}^p = \phi_p \gamma_{2,t}^p + \frac{\epsilon_p \left(\phi_p - 1\right)}{1+\epsilon_p} \left(p_t^*\right)^{\frac{\phi_p(1+\epsilon_p)}{\phi_p-1}+1} \gamma_{3,t}^p, \tag{2}$$

$$\gamma_{1,t}^p = \left(\delta_t^p\right)^{\frac{\phi_p(1+\epsilon_p)}{\phi_p-1}} y_t + \left(\beta\gamma^{1-\sigma}\right) \xi_p E_t \frac{\xi_{t+1}}{\xi_t} \left(\frac{\pi^{1-\iota_p} \pi_t^{\iota_p}}{\pi_{t+1}} \right)^{-\frac{1+\phi_p\epsilon_p}{\phi_p-1}} \gamma_{1,t+1}^p, \tag{3}$$

$$\gamma_{2,t}^p = \left(\delta_t^p\right)^{\frac{\phi_p(1+\epsilon_p)}{\phi_p-1}} mc_t \varepsilon_{p,t} y_t + \left(\beta\gamma^{1-\sigma}\right) \xi_p E_t \frac{\xi_{t+1}}{\xi_t} \left(\frac{\pi^{1-\iota_p} \pi_t^{\iota_p}}{\pi_{t+1}} \right)^{-\frac{\phi_p(1+\epsilon_p)}{\phi_p-1}} \gamma_{2,t+1}^p, \tag{4}$$

$$\gamma_{3,t}^p = y_t + \left(\beta\gamma^{1-\sigma}\right) \xi_p E_t \frac{\xi_{t+1}}{\xi_t} \left(\frac{\pi^{1-\iota_p} \pi_t^{\iota_p}}{\pi_{t+1}} \right) \gamma_{3,t+1}^p, \tag{5}$$

where the different endogenous variables are defined in the appendix. Importantly, you can see from eq. (A.46) that the markup shock $\varepsilon_{p,t}$ (which is assumed to have unit mean) in the nonlinear pricing system multiplies marginal costs mc_t. So in this sense, it represents an exogenous shock to the desired markup. We will in the following refer to them as markup or cost-push shocks.

3.2 Calibration and Solution

The model is estimated on seven key macroeconomic quarterly U.S. time series as observable variables: the log differences of real per capita GDP, consumption and investment; log-differences of compensation per hour and the GDP deflator; log deviations of hours per capita from its average as well as the Federal Funds rate. Further details about the data and the measurement equations linking the model variables to their data counterparts are provided in the Appendix.

SW use full information Bayesian techniques to estimate the model. Bayesian inference starts out from a prior distribution that describes the available information prior to observing the data used in the estimation. The observed data is subsequently used to update the prior, via Bayes' theorem, to a posterior distribution of the model's parameters which can be summarized in the usual measures of location (e.g. mode or mean) and spread (e.g. standard deviation and probability intervals).[3] We adapt the parameters estimated by Smets and Wouters (2007) on pre-Global Financial Crisis and Covid periods data, with the exception of the pricing parameters ϕ_p, ξ_p, and ϵ_p. In particular, we reestimate the linearized model for the sample $1965Q1 - 2007Q4$ imposing an alternative prior $\phi_p \sim N(1.2, 0.05)$ to obtain a lower gross markup than the 61 percent estimated by SW. Harding, Lindé and Trabandt shows a large body of calibrated or estimated New Keynesian DSGE models suggests a markup that is notably lower than 61 percent. Moreover, we calibrate $\xi_p = 0.667$ in line with micro evidence (which is close to the SW posterior mode of .65) and instead estimate the Kimball curvature parameter using the prior $\epsilon_p \sim N(75, 25)$. This results in a posterior mode where $\epsilon_p = 64.5$ and $\phi_p = 1.34$. In addition, κ in eq. (1) equals .008 which is somewhat smaller than SW's estimate of .026.[4]

To compute the linearized and nonlinear solutions, we use the Fair and Taylor (1983) solution algorithm. This solution algorithm is also known as a two-point boundary value solution algorithm or time-stacking algorithm. The Fair-Taylor solution algorithm imposes certainty equivalence on the nonlinear model, just as the linearized model solution does by definition. In other words, the Fair-Taylor solution algorithm allows us to trace out the implications of not linearizing the equilibrium equations which is exactly our objective. All relevant information for solving the nonlinear and linearized model is captured by the current state of the economy, including the contemporaneous realization of the exogenous shocks.

[3] We refer the reader to Smets and Wouters (2003, 2007) for a more detailed description of the estimation procedure.

[4] Our lower value for κ partly reflects the different prior and partly the extended sample period (recalling that SW sample ends 2004:4). For our sample period ending 2007:4, we obtain $\kappa = .018$ with the SW priors. Note that the re-estimated model is associated with an improvement of the marginal likelihood compared to the original SW parameterization by roughly 5 log points.

An alternative approach to solve the model would have been to compute solutions where uncertainty about future shock realizations matters for the dynamics of the economy following e.g. Aruoba, Cuba-Borda, and Frank Schorfheide (2018), Adam and Billi (2006, 2007), Fernández-Villaverde et al. (2015), Gust, Herbst, López-Salido and Smith (2017) and Nakata (2017). These authors have shown that allowing for future shock uncertainty can have potentially important implications for equilibrium dynamics. Importantly, none of these authors have considered a model with Kimball aggregation. Lindé and Trabandt (2018) solve a simplified version of our model with sticky prices and Kimball aggregation under shock uncertainty using global methods, and show that the effects of future shock uncertainty on the global solution of the nonlinear model are quantitatively negligible lending support for using the Fair-Taylor solution method for our baseline results.[5]

As a practical matter, we feed the equilibrium equations of the nonlinear and linearized model into Dynare. Dynare is a pre-processor and a collection of MATLAB routines which can solve nonlinear and linearized dynamic models with forward looking variables. The details about the implementation of the algorithm used can be found in Juillard (1996). We use the perfect foresight/deterministic simulation algorithm implemented in Dynare using the 'simul' command.[6] The algorithm can also easily handle the ZLB constraint: one just writes the Taylor rule including the max operator in the model equations, and the solution algorithm reliably calculates the model solution in fractions of a second.

4 Results

In this section, we report our main results for the linearized and nonlinear solution of the model outlined in the previous section. We begin in Section 4.1 by considering the effects of positive cost-push shocks in long simulations of the model as function of the output gap when the business cycle is driven by demand shocks. Next, we proceed in Section 4.2 to characterize how the transmission of markup shocks vary with the initial inflation level for all the shocks in the estimated SW model. Finally, in Section 4.3 we compare the transmission of cost-push and monetary policy shocks in the linearized and nonlinear model in 2021Q4 by filtering the data with the inversion filter on the 7 observable time series $1965Q1 - 2021Q4$ used by Smets and Wouters (2007) when estimating the

[5] The introduction of wage stickiness and Kimball aggregation in the labor market in the present paper (in addition to price stickiness and Kimball aggregation in the goods market as in Lindé and Trabandt, 2018) should temper the effect of shock uncertainty in the nonlinear model even further. To the extent that allowing for shock uncertainty impacts notably the linearized solution, the differences between the linearized and nonlinear solutions we report in this paper are conservative: they would be even larger if we had allowed for shock uncertainty.

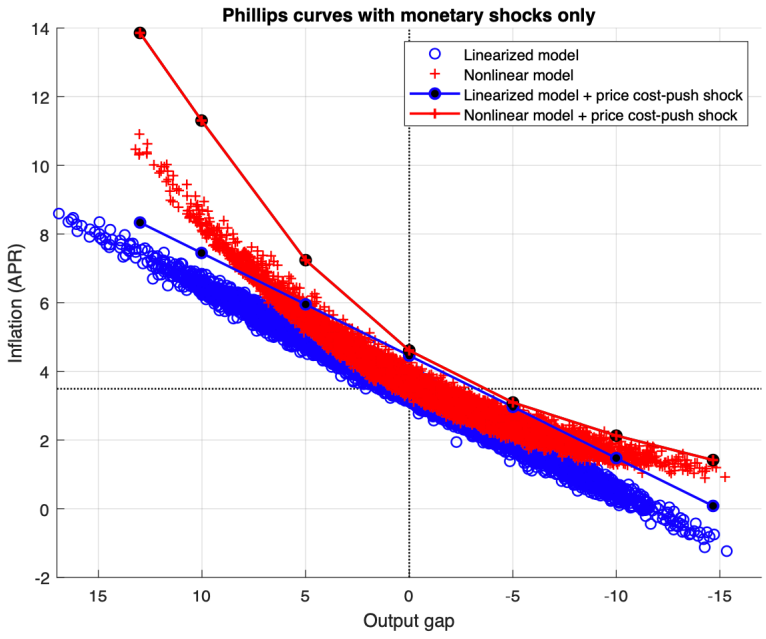
[6] The solution algorithm implemented in Dynare's simul command is the method developed in Fair and Taylor (1983).

model. This allows us to characterize monetary policy trade-offs at the current juncture.

4.1 The Phillips Curve

To understand the differences in the dynamics implied by the linearized and nonlinear solutions, we undertake stochastic simulations of the model for monetary policy shocks ($\varepsilon_{r,t}$) only. We size the standard deviation of monetary policy shocks (σ_r) to imply variations in the model consistent output gap between roughly minus 15 and plus 15 percent in the linearized solution. We solve and simulate the linearized and nonlinear model solutions for a long sample of $10,000$ periods contingent on exactly the same sequence of shocks $\{\varepsilon_{r,t}\}_{t=1}^{10,000}$, disregarding any binding constraints on policy rates to begin with.

Figure 2: Linear and Nonlinear Phillips Curve with Cost-Push Shocks



The blue circles in Figure 2 are the simulated inflation and output gap observations in the linearized model with the output gap on the x-axis (inverted scale) and inflation on the y-axis. The red crosses shows the corresponding observations for the nonlinear model solution. The black dotted horizontal and vertical lines indicate the deterministic steady states for inflation and output gap, respectively. The figure shows a noticeable difference between the linearized and nonlinear model for the relationship between inflation and the output gap. As expected, the linearized model

implies a constant downward-sloped Phillips curve whereas the nonlinear solution is associated with a "banana-shaped" Phillips curve, as in the seminal paper by Phillips (1958). This reflects that the Kimball aggregator implies that firms are reluctant to change prices much when relative demand is low. On the other hand, in periods when relative demand is high, firms are more willing to change their prices. As a result, the nonlinear model produces episodes with more elevated price inflation than the linearized solution in which households and firms are equally sensitive to changes in desired price markups in recessions and booms.

The blue and red solid lines show how the Phillips curves for the linearized and nonlinear solutions shift for positive same-sized markup shocks. These lines have been generated by selecting all economic states with an output gap close to $-15, -10, -5, ...15$. For all those selected states, we simulate the contemporaneous effects when adding same-sized one σ_p positive markup shock to the state and then average the resulting inflation levels in the blue and red dots. This shows how the Phillips curve shifts out in the linearized and nonlinear model solutions for positive cost-push shocks.

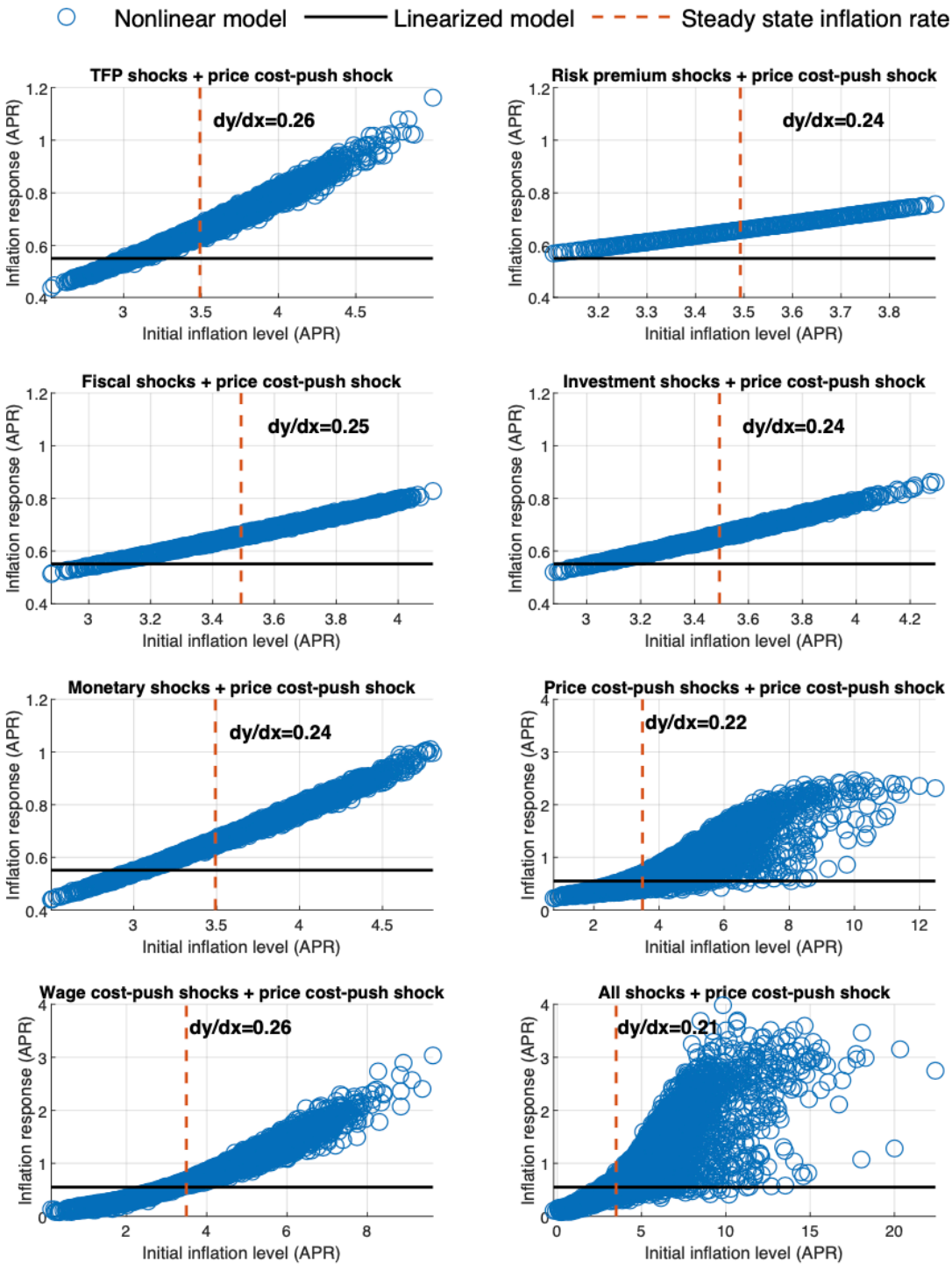
As expected, the transmission of a given-sized markup shock in the linearized model is independent of the output gap level, and the Phillips curve simply shifts out in parallel. In the nonlinear model, the picture is very different. The nonlinear model implies that a markup shock shifts out inflation very little when inflation is subdued. When inflation is high, on the other hand, the same-sized markup shock shifts out the Phillips curve to substantially more than in the linearized model. In other words, the transmission of markup shocks are state dependent and depends importantly on the economic state, lending support for inflation scares in booms (see e.g. Goodfriend, 1993).

4.2 Propagation of Cost-Push Shocks

So far, we have established that cost-push shocks propagate notably stronger in a boom than in a slump, provided that demand shocks is the source of business cycle fluctuations. But what about the case when cost-push shocks rather than demand shocks are the key driver of inflation dynamics? After all, the estimated linearized Smets and Wouters model implies that wage and price markup shocks account for the bulk of inflation volatility. Hence, we now use the estimated model to discuss how the propagation of price cost-push shocks differs for alternative sources of business cycle fluctuations in the estimated Smets and Wouters model. The analysis will show that price cost-push shocks do not propagate more strongly in the nonlinear model only when the economy is experiencing an economic boom with strong demand conditions. Rather, a more reli-

able condition for strong propagation of price cost-push shocks is that inflation is high to begin with.

Figure 3: State Dependent Effects of Cost-Push Shocks on Inflation

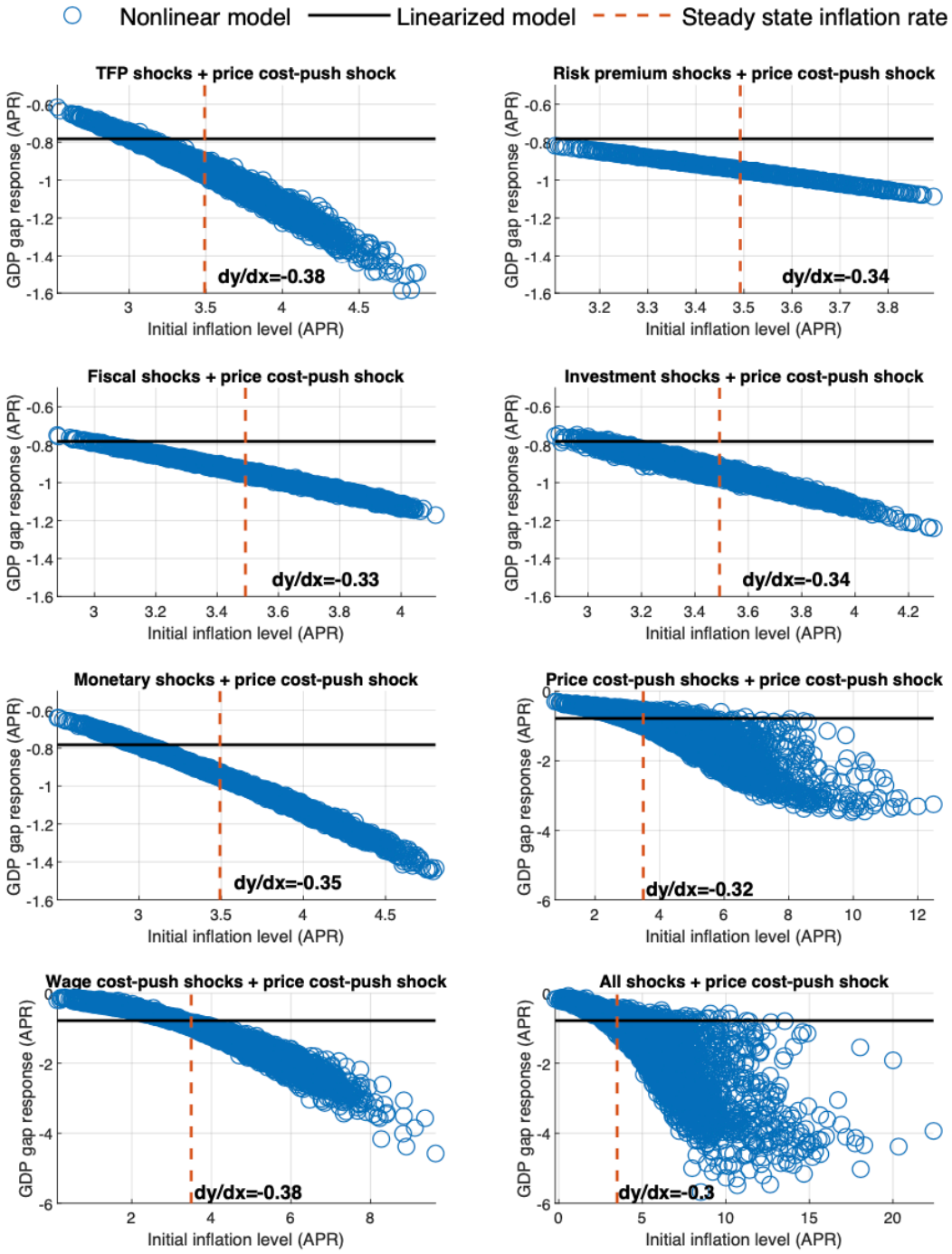


To do this analysis, we begin by simulating the linearized and nonlinear solutions for all 7 shocks

with only one shock active one at a time in the estimated SW model, using the estimated standard deviations for each shock, for $T = 10,000$ periods. This will provide us with simulated paths of all endogenous variables in the nonlinear $\{y_t^{nonlin}\}_{t=1}^{T}$ and linearized $\{y_t^{lin}\}_{t=1}^{T}$ models. Next, we go backwards in time and for each state $t = 1, ..., 10,000$ add one σ_p positive price cost-push shock and compute its impact on inflation and the output gap. Figures 3 and 4 report the results of this exercise for inflation and output, respectively. Each panel in the figure shows the results when inflation dynamics are driven by each of the seven shocks, whereas the lower right panel shows results when we assume all shocks are active. The x-axis show the initial inflation level when the price cost-push shock hit, and the y-axis the impact this shock has on one-year average inflation.

The upper left panel in Figure 3 shows the effects of a positive price cost-push shock on one-year ahead inflation when stationary technology shocks are the sole driver of fluctuations in inflation (apart from the cost-push shocks we add in each point in time). The vertical dashed line shows the deterministic steady state inflation level, while the solid horizontal line shows the impact of the cost-push shocks on one-year ahead inflation in the linearized model, which is independent on the initial inflation level and the same for all seven shocks. However, in the nonlinear model solution, the impact of a same-size cost-push shock is an upward sloping function of the initial inflation level. The intution for the larger inflation response in the nonlinear model is provided in Section 2 and Figure 1 where we discuss the heightened sensitivity of firms' optimal prices in response to increases in marginal costs. The next four panels in Figure 3 for risk premium, fiscal, investment-specific technology, and monetary shocks feature the same upward sloping line, with dy/dx showing the slope of a regression of the inflation impulse on the initial inflation level in the nonlinear model. The slope is essentially the same for all shocks. The only difference is that the shocks induce unequal fluctuations in initial inflation levels. The panels for wage- and price-markup shocks show that these two shocks are most critical for inflation fluctuations in the model, as they generate notably more variation in initial inflation levels. For price markup shocks, we notice that while the regression line has a similar slope as for the other shocks, there is now considerably larger variation in the transmission of additional price cost-push shocks. For instance, with an initial inflation rate of 8 percent, the one-year ahead inflation impulse of the same-sized positive price cost-push shock can be roughly 0.5 or above 2 percent.

Figure 4: State Dependent Effects of Cost-Push Shocks on Output Gap

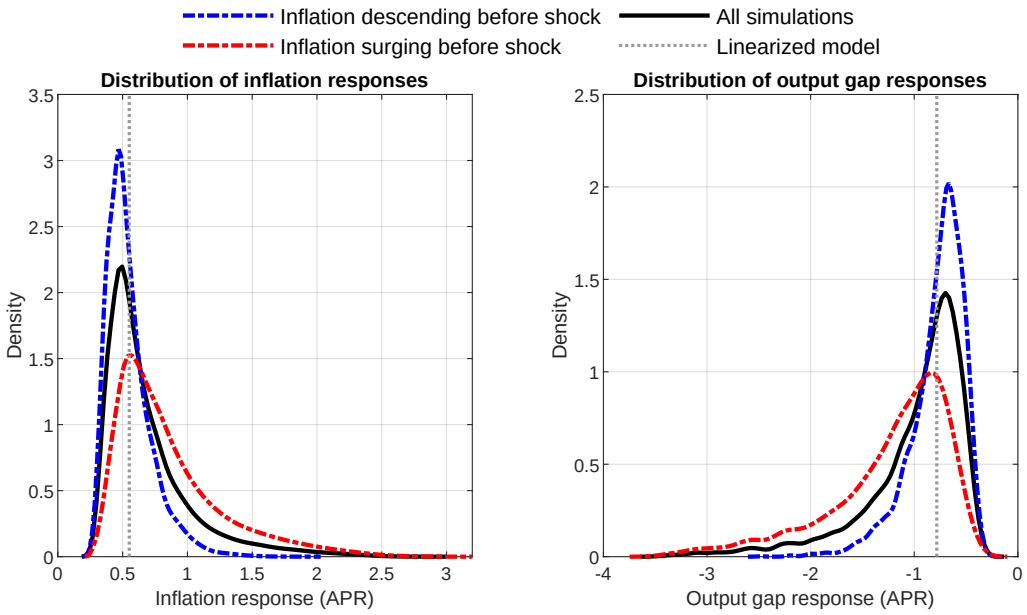


The bottom right panel shows the results when simulating all shocks at the same time, which naturally gives rise to even greater variation in initial inflation levels compared to when simulating each of the seven shocks separately. With all shocks combined, the dispersion increases even further, and for a 10-percent initial inflation level the impact of a positive price markup shock on one-year

ahead inflation can vary from about 0 to over 3 percent. These results imply that the conditional volatility of inflation is increasing in the inflation level. Hence, there is higher inflation risk when inflation is high, whereas inflation risk is constant in the linearized model solution.

Figure 4 repeats the experiment but instead report effects on the output gap.[7] As was the case for inflation, we see that a same-sized markup shock has a more adverse impact on the output gap when the initial inflation level is high, partly because larger transmission of price markup shocks on inflation (which effectively works as tax by lowering the real wage), but partly because the elevated transmission precipitates a more vigorous endogenous response of the federal funds rate in our model. The Taylor rule implies that the central bank faced with this adverse trade-off shock endogenously raises the policy rate more when initial inflation is high because it puts more weight on the increased inflationary pressure despite the fact that the adverse supply shock causes a larger deterioration in economic activity when initial inflation is high.

Figure 5: Understanding Inflation Risk:
Pass-through of Cost-Push Shocks in Inflation Surge and Descend Episodes



The results in Figures 3 and 4 raise the important question on what accounts for the increased inflation risk in the model. Why is the transmission of a positive price cost-push shock so different when the initial inflation level is elevated? To shed light on this issue, Figure 5 reports the effects of identical price cost-push shocks in inflation surges and when inflation is descending for inflation

[7] Notice that price cost-push shocks do not affect potential output in the SW model, so the response of the output gap coincides with the response of output.

(left panel) and the output gap (right panel). The black line shows the distribution for all shocks in Figure 3, while the red- and blue-dash dotted lines shows the effects when inflation is increasing and decreasing, respectively. We separate the impulses in Figure 3 by simply looking at the 1-period change in inflation (from $t-1$ to t) before a markup shock hits in period t, and then separating the responses to a markup shock across the two states: when the change in inflation is positive (inflation surging) and negative (inflation descending). We then compute the distribution of the responses in each state. The left panel in Figure 5 shows inflation risk is substantially higher when inflation is increasing before the shock hits. The right panel shows that output is at greater risk when inflation is on the rise, explaining the trumpet-like shape for the output gap responses shown in Figure 4.

4.3 Monetary Policy Trade-offs During the Post-Covid Period

We now turn to make an assessment of the transmission of price cost-push shocks during the Post-Covid period and trade-offs for monetary policy to stabilize them. To do so, we filter the shocks in the linearized and nonlinear solutions with the inversion filter described in Fair and Taylor (1983) from 1965Q1 to 2021Q4. Since we have the same number of observables as shocks, the inversion filter provides us with a unique sequence of filtered innovations $\left\{\varepsilon_{t|t}\right\}_{t=1}^{T}$ that maximizes the likelihood for a given parameterization of the linearized and nonlinear model.[8]

Conditional on period t information, let $X_{t|t}$ denote all variables in the model, except for the innovations to the shock processes $\varepsilon_{t|t}$. The solution of the model is a nonlinear function f such that

$$X_{t|t} = f(X_{t-1|t-1}, \varepsilon_{t|t}), \tag{6}$$

and let y_t be a vector of observables and S a selection matrix that matches the observed variables to their model counterparts. Then, in each period t we obtain the filtered structural innovations $\varepsilon_{t|t}$, given the data y_t and state of the model $X_{t-1|t-1}$, as the solution to the nonlinear system

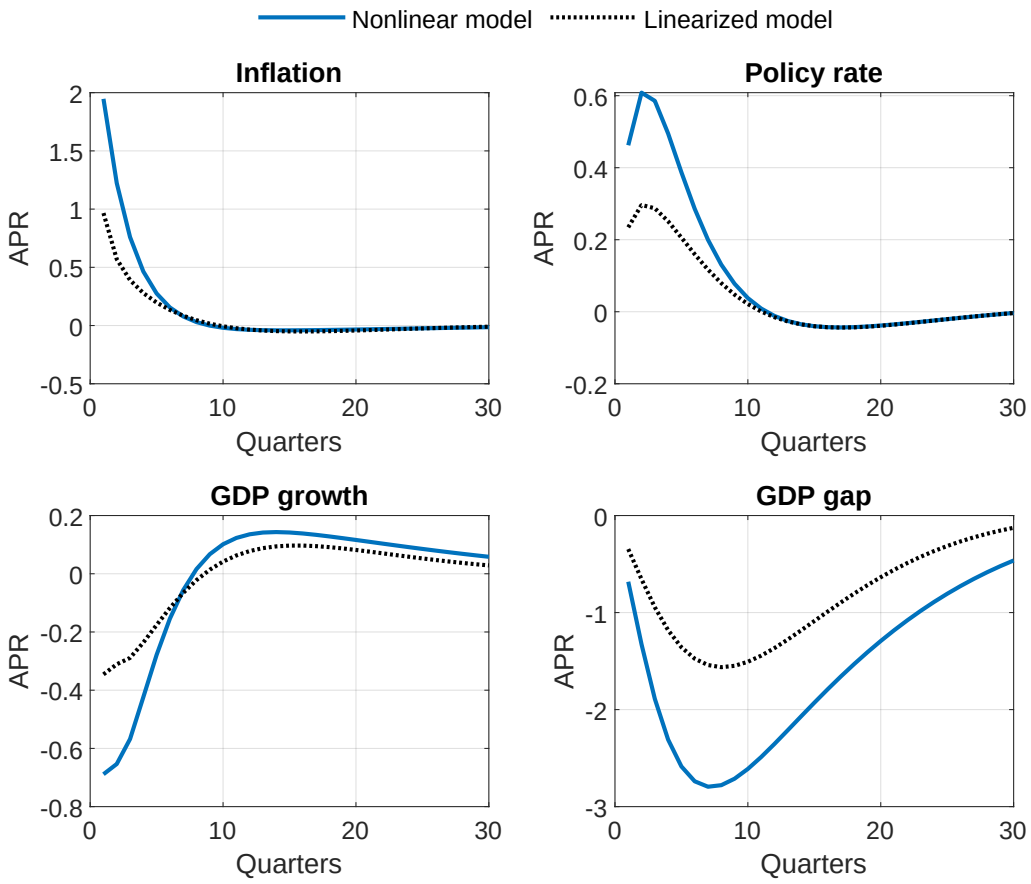
$$y_t = Sf(X_{t-1|t-1}, \varepsilon_{t|t}). \tag{7}$$

We initialize the filter by running the Kalman smoother on the linearized solution and using the implied Kalman smoothed estimates as initial values for the endogenous variables in the nonlinear

[8] A potential drawback of using the inversion filter is that it does not allow to compute the two-sided innovation (i.e. $\varepsilon_{t|T}$), and hence relies on the one-sided filtered shocks $\varepsilon_{t|t}$ in the analysis. That said, in the linearized solution, the one- and two-sided shocks (the latter being based on a standard Kalman filter) are very similar, suggesting that using the one-sided shocks has a limited impact on the findings.

filter. However, the nonlinear model contains some additional endogenous state variables (i.e. price and wage dispersion terms), and we initialize these endogenous variables together with the shock innovations $\varepsilon_{t|t}$ so that the nonlinear model matches the observed variables in y_t.

Figure 6: IRFs to a 1σ Cost-Push Shock in Linearized and Nonlinear Model in 2021Q4

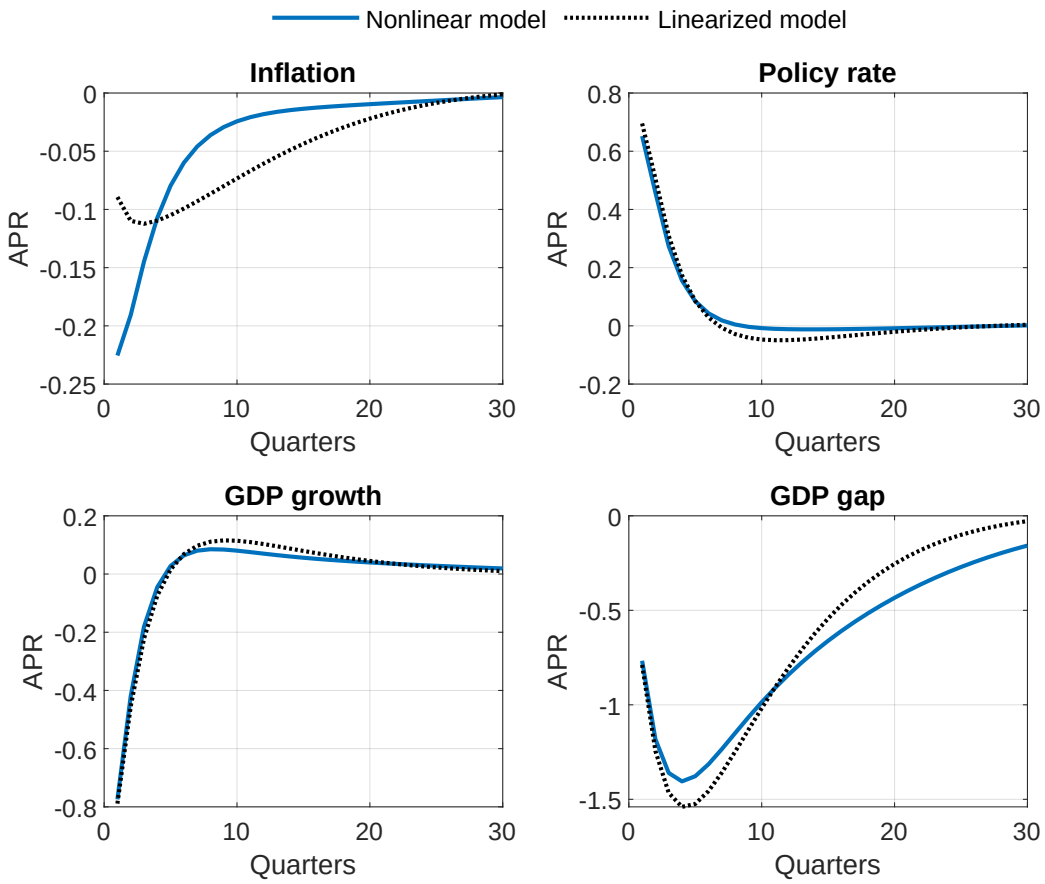


Importantly, we impose the ELB for the federal funds rate when filtering the shocks in the nonlinear model. In the linearized model, we do not impose an effective lower bound through a nonlinear policy rule, so the linearized solution will only prevent the current policy rate to fall below the ELB through unanticipated policy innovations. Since the quarterly average for the periods 2009Q1-2014Q2 and 2020Q2-2021Q4 is not exactly zero and fluctuates a couple of basis points, we adjust the federal funds rate to exactly 10 (annualized) basis points for those quarters. Accordingly, the monetary policy rule in the model is constrained by an Effective Lower Bound (ELB) of 10 (annualized) basis points. One important caveat of using this filtering approach is that, since monetary policy shocks become inconsequential in the model when the ELB is binding, it is not possible to filter these shocks during the ELB period. Following Guerrieri and Iacoviello

(2017) we assume these shocks to be zero when the ELB is binding, except when the model-implied interest rate is predicted to be above the ELB and a monetary policy shock is necessary to obtain an exact match between the observed data and the model counterparts.

Given the filtered states 2021Q4 in the linearized and nonlinear models, we then add a one σ_p price cost-push and σ_r monetary policy shocks to the linearized and nonlinear models, respectively. Figure 6 shows the resulting impulse response functions for the price markup shocks in the linearized and nonlinear model solutions as of 2021Q4. Of course, the results in the linearized model are invariant to the filtered state and the average of the first four quarters in the figure equals 0.55 (black horizontal line in Figure 2). For the nonlinear model, we see that the shock's effect is about twice as large over the first year on inflation and the output gap, despite a twofold increase in the policy rate relative to the linearized model.

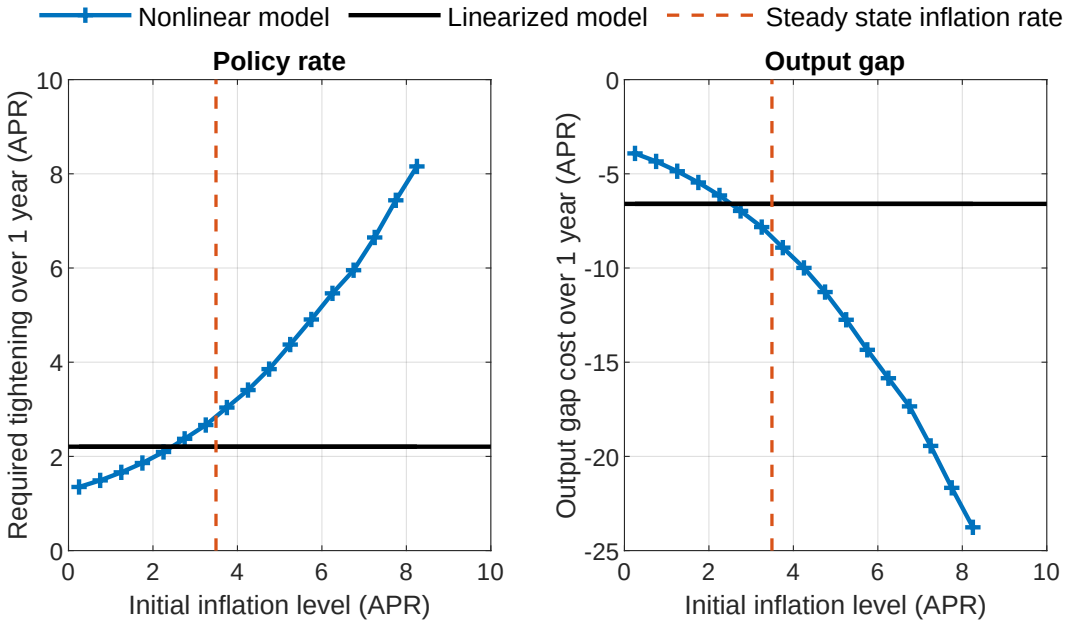
Figure 7: IRFs to a 1σ Monetary Shock in Linearized and Nonlinear Model in 2021Q4



In Figure 7, we show the corresponding effects of a monetary policy shock. The figure shows that the effects of a monetary policy shock has the same effect on output gap and output growth,

but the impact on inflation is about twice as high initially relative to the linearized model. Even so, the model implies a rather unfavorable monetary policy inflation-output gap trade-off. To reduce inflation with 0.1 percentage points the policy maker needs to accept a decline in the output gap of more than 1% over one year. Figure 6 implies this trade-off is further exacerbated in the nonlinear model in the current situation of high inflation risk, as it takes even tighter monetary policy than shown in the figure to maintain a pass-through of price cost-push shocks to inflation at normal levels according to our estimated model.

Figure 8: Tradeoffs for Interest Rate Policy:
Tightening and Output Cost to Provide Full Inflation Stabilization



To demonstrate the more unfavorable monetary policy trade-off when inflation is elevated, Figure 8 reports the required policy tightening, in addition to that prescribed by the endogenous reaction according to the estimated rule, during the first year to fully stabilize any impulses to one-year ahead inflation from price cost-push shocks as function of the initial inflation level. For the linearized model the tightening needed is invariant to the initial inflation level and is a little more than a two percent on average during the first year to stabilize inflation during the first year following a price markup shock. This magnitude can be derived by combining the impulses in the linearized model for inflation and the policy rate in Figures 6 and 7 (i.e. how big policy rate movement you need according to Figure 7 to remove the 0.55 average increase in inflation shown in Figure 6 in the linearized model). The output cost of such an inflation nutter policy is fairly large,

a little above 6 percent in the linearized model. The blue line with crosses plot the corresponding interest rate tightening and output cost in the nonlinear model. As in Figure 2, we compute this trade-off curve as averages for states clustered around certain initial inflation levels. As is evident from the model, the nonlinear formulation of the model implies that increasing adverse trade-off to stabilize inflation, even though the nonlinear model implies that monetary policy has a stronger effect on inflation when inflation is high. This adverse trade-off is driven by the fact that even though both monetary policy and price markup shocks become equally more potent as function of the initial inflation rate on average, the price markup shocks have much larger absolute effects and hence increasingly more policy tightening is needed to keep inflation in check as the initial inflation level increases.

5 Conclusions

We propose a macroeconomic model with a nonlinear Phillips curve, which is flat when inflationary pressure is subdued and steep when inflationary pressure is elevated. The nonlinear Phillips curve in our model arises due to a quasi-kinked demand schedule for goods produced by firms. Our model can jointly account for the modest decline in inflation during the Great Recession and the surge in inflation during the Post-Covid period. Because our model implies a stronger transmission of shocks when inflation is high, it generates conditional heteroscedasticity in inflation and inflation risk. Hence, our model can generate more sizeable inflation surges due to cost-push and demand shocks than a standard linearized model. Finally, our model implies that the central bank faces a more severe trade-off between inflation and output stabilization when inflation is high.

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Appendix A The Smets and Wouters (2007) Model

This appendix contains the linearized and nonlinear equilibrium model equations of the Smets and Wouters (2007) model. We describe the firms' and households' problem in the model, and state the market clearing conditions.[A.1]

A.1 Firms and Price Setting

Final Goods Production The single final output good Y_t is produced using a continuum of differentiated intermediate goods $Y_t(f)$. Following Kimball (1995), the technology for transforming these intermediate goods into the final output good is

$$\int_0^1 G_Y\left(\frac{Y_t\left(f\right)}{Y_t}\right) df = 1. \tag{A.1}$$

[A.1] For a description of the model which derives the log-linearized equations, we refer the reader to the appendix of the Smets and Wouters paper, which is available online at http://www.aeaweb.org/aer/data/june07/20041254_app.pdf.

Following Dotsey and King (2005) and Levin, López-Salido and Yun (2007) we assume that $G_Y(.)$ is given by a strictly concave and increasing function; its particular parameterization follows SW:

$$G_Y\left(\frac{Y_t(f)}{Y_t}\right) = \left(\frac{\phi_p}{1-(\phi_p-1)\epsilon_p}\left[\left(\frac{\phi_p+(1-\phi_p)\epsilon_p}{\phi_p}\right)\frac{Y_t(f)}{Y_t} + \frac{(\phi_p-1)\epsilon_p}{\phi_p}\right]^{\frac{1-(\phi_p-1)\epsilon_p}{\phi_p-(\phi_p-1)\epsilon_p}} + \left[1 - \frac{\phi_p}{1-(\phi_p-1)\epsilon_p}\right]\right), \quad \text{(A.2)}$$

where $\phi_p \geq 1$ denotes the gross markup of the intermediate firms. The parameter ϵ_p governs the degree of curvature of the intermediate firm's demand curve. When $\epsilon_p = 0$, the demand curve exhibits constant elasticity as with the standard Dixit-Stiglitz aggregator. When ϵ_p is positive—as in SW—this introduces more strategic complementarity in price setting which causes intermediate firms to adjust prices less to a given change in marginal cost.

Firms that produce the final output good Y_t are perfectly competitive in both the product and factor markets, and take as given the price $P_t(f)$ of each intermediate good $Y_t(f)$. They sell units of the final output good at a price P_t, and hence solve the following problem:

$$\max_{\{Y_t, Y_t(f)\}} P_t Y_t - \int_0^1 P_t(f) Y_t(f)\, df, \quad \text{(A.3)}$$

subject to the constraint (A.1).

Intermediate Goods Production A continuum of intermediate goods $Y_t(f)$ for $f \in [0,1]$ is produced by monopolistically competitive firms, which utilize capital services $K_t(f)$ and a labour index $L_t(f)$ (defined below) to produce its respective output good. The form of the production function is Cobb-Douglas:

$$Y_t(f) = \varepsilon_t^a K_t(f)^\alpha \left[\gamma^t L_t(f)\right]^{1-\alpha} - \gamma^t \Phi, \quad \text{(A.4)}$$

where γ^t represents the labour-augmenting deterministic growth rate in the economy, Φ denotes the fixed cost (which is related to the gross markup ϕ_p so that profits are zero in the steady state), and ε_t^a is total factor productivity which follows the process

$$\ln \varepsilon_t^a = (1-\rho_a)\ln \varepsilon^a + \rho_a \ln \varepsilon_{t-1}^a + \eta_t^a, \eta_t^a \sim N(0, \sigma_a). \quad \text{(A.5)}$$

Firms face perfectly competitive factor markets for renting capital at price R_{Kt} and hiring labour at a price given by the aggregate wage index W_t (defined below). As firms can costlessly adjust either factor of production, the standard static first-order conditions for cost minimization imply that all firms have identical marginal cost per unit of output.

The prices of the intermediate goods are determined by Calvo-Yun (1996) style staggered nominal contracts. The probability $1-\xi_p$ that any firm f receives a signal to re-optimize its price

$P_t(f)$ is assumed to be independent of the time that it last reset its price. If a firm is not allowed to optimize its price, it adjusts its price by a weighted combination of the lagged and steady-state rate of inflation, i.e., $P_t(f) = (1+\pi_{t-1})^{\iota_p}(1+\pi)^{1-\iota_p} P_{t-1}(f)$ where $0 \leq \iota_p \leq 1$ and π_{t-1} denotes net inflation in period $t-1$, and π the steady-state net inflation rate. A positive value of ι_p introduces structural inertia into the inflation process. All told, this leads to the following optimization problem for the intermediate firms

$$\max_{\tilde{P}_t(f)} \mathrm{E}_t \sum_{j=0}^{\infty} (\beta\xi_p)^j \frac{\Xi_{t+j} P_t}{\Xi_t P_{t+j}} \left[\tilde{P}_t(f) \left(\Pi_{s=1}^{j} (1+\pi_{t+s-1})^{\iota_p} (1+\pi)^{1-\iota_p}\right) - MC_{t+j}\right] Y_{t+j}(f), \quad \text{(A.6)}$$

where $\tilde{P}_t(f)$ is the newly set price. Notice that with our assumptions all firms that re-optimize their prices actually set the same price.

It would be ideal if the markup in $(A.2)$ can be made stochastic and the model can be written in a recursive form. However, such an expression is not available, and we instead directly introduce a shock ε_t^p in the first-order condition to the problem in (A.6). And following SW, we assume the shock is given by an exogenous ARMA(1,1) process:

$$\ln \varepsilon_t^p = (1-\rho_p)\ln \varepsilon^p + \rho_p \ln \varepsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p, \eta_t^p \sim N(0, \sigma_p). \quad \text{(A.7)}$$

When this shock is introduced in the non-linear model, we put a scaling factor on it so that it enters exactly the same way in a log-linearized representation of the model as the price markup shock does in the SW model.

A.2 Households and Wage Setting

We assume a continuum of monopolistically competitive households (indexed on the unit interval), each of which supplies a differentiated labour service to the production sector; that is, goods-producing firms regard each household's labour services $L_t(h)$, $h \in [0,1]$, as imperfect substitutes for the labour services of other households. It is convenient to assume that a representative labour aggregator combines households' labour hours in the same proportions as firms would choose. Thus, the aggregator's demand for each household's labour is equal to the sum of firms' demands. The aggregated labour index L_t has the Kimball (1995) form:

$$L_t = \int_0^1 G_L\left(\frac{L_t(h)}{L_t}\right) dh = 1, \quad \text{(A.8)}$$

where the function $G_L(.)$ has the same functional form as $(A.2)$, but is characterized by the corresponding parameters ϵ_w (governing convexity of labour demand by the aggregator) and ϕ_w

(gross wage markup). The aggregator minimizes the cost of producing a given amount of the aggregate labour index L_t, taking each household's wage rate $W_t(h)$ as given, and then sells units of the labour index to the intermediate goods sector at unit cost W_t, which can naturally be interpreted as the aggregate wage rate.

The utility function of a typical member of household h is

$$\mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \left[\frac{1}{1-\sigma_c} \left(C_{t+j}(h) - \varkappa C_{t+j-1} \right) \right]^{1-\sigma_c} \exp\left(\frac{\sigma_c - 1}{1+\sigma_l} L_{t+j}(h)^{1+\sigma_l} \right), \tag{A.9}$$

where the discount factor β satisfies $0 < \beta < 1$. The period utility function depends on household h's current consumption $C_t(h)$, as well as lagged aggregate per capita consumption to allow for external habit persistence through the parameter $0 \leq \varkappa \leq 1$. The period utility function also depends inversely on hours worked $L_t(h)$.

Household h's budget constraint in period t states that its expenditure on goods and net purchases of financial assets must equal its disposable income:

$$\begin{aligned} &P_t C_t(h) + P_t I_t(h) + \frac{B_{t+1}(h)}{\varepsilon_t^b R_t} + \int_s \xi_{t,t+1} B_{D,t+1}(h) - B_{D,t}(h) \\ &= B_t(h) + W_t(h) L_t(h) + R_t^k Z_t(h) K_t^p(h) - a\left(Z_t(h)\right) K_t^p(h) + \Gamma_t(h) - T_t(h). \end{aligned} \tag{A.10}$$

Thus, the household purchases part of the final output good (at a price of P_t), which it chooses either to consume $C_t(h)$ or invest $I_t(h)$ in physical capital. Following Christiano, Eichenbaum, and Evans (2005), investment augments the household's (end-of-period) physical capital stock $K_{t+1}^p(h)$ according to

$$K_{t+1}^p(h) = (1-\delta) K_t^p(h) + \varepsilon_t^i \left[1 - S\left(\frac{I_t(h)}{I_{t-1}(h)} \right) \right] I_t(h). \tag{A.11}$$

The extent to which investment by each household h turns into physical capital is assumed to depend on an exogenous shock ε_t^i and how rapidly the household changes its rate of investment according to the function $S\left(\frac{I_t(h)}{I_{t-1}(h)}\right)$, which we specify as

$$S(x_t) = \tfrac{\varphi}{2} (x_t - \gamma)^2. \tag{A.12}$$

Notice that this function satisfies $S(\gamma) = 0$, $S'(\gamma) = 0$ and $S''(\gamma) = \varphi$. The stationary investment-specific shock ε_t^i follows

$$\ln \varepsilon_t^i = \rho_i \ln \varepsilon_{t-1}^i + \eta_t^i, \eta_t^i \sim N(0, \sigma_i). \tag{A.13}$$

In addition to accumulating physical capital, households may augment their financial assets through increasing their government nominal bond holdings (B_{t+1}), from which they earn an interest rate

of R_t. The return on these bonds is also subject to a risk-shock, ε_t^b, which follows

$$\ln \varepsilon_t^b = \rho_b \ln \varepsilon_{t-1}^b + \eta_t^b, \eta_t^b \sim N(0, \sigma_b). \tag{A.14}$$

Agents can engage in frictionless trading of a complete set of contingent claims to diversify away idiosyncratic risk. The term $\int_s \xi_{t,t+1} B_{D,t+1}(h) - B_{D,t}(h)$ represents net purchases of these state-contingent domestic bonds, with $\xi_{t,t+1}$ denoting the state-dependent price, and $B_{D,t+1}(h)$ the quantity of such claims purchased at time t.

On the income side, each member of household h earns after-tax labour income $W_t(h) L_t(h)$, after-tax capital rental income of $R_t^k Z_t(h) K_t^p(h)$, and pays a utilization cost of the physical capital equal to $a(Z_t(h)) K_t^p(h)$ where $Z_t(h)$ is the capital utilization rate, so that capital services provided by household h, $K_t(h)$, equals $Z_t(h) K_t^p(h)$. The capital utilization adjustment function $a(Z_t(h))$ is assumed to be given by

$$a(Z_t(h)) = \frac{r^k}{\tilde{z}_1} \left[\exp\left(\tilde{z}_1 \left(Z_t(h) - 1\right)\right) - 1\right], \tag{A.15}$$

where r^k is the steady state net real interest rate $(\bar{R}_t^K / \bar{P}_t)$. Notice that the adjustment function satisfies $a(1) = 0$, $a'(1) = r^k$, and $a''(1) \equiv r^k \tilde{z}_1$. Following SW, we want to write $a''(1) = z_1 = \psi/(1-\psi) > 0$, where $\psi \in [0,1)$ and a higher value of ψ implies a higher cost of changing the utilization rate. Our parameterization of the adjustment cost function then implies that we need to set $\tilde{z}_1 \equiv z_1 / r^k$. Finally, each member also receives an aliquot share $\Gamma_t(h)$ of the profits of all firms, and pays a lump-sum tax of $T_t(h)$ (regarded as taxes net of any transfers).

In every period t, each member of household h maximizes the utility function (A.9) with respect to its consumption, investment, (end-of-period) physical capital stock, capital utilization rate, bond holdings, and holdings of contingent claims, subject to its labour demand function, budget constraint (A.10), and transition equation for capital (A.11).

Households also set nominal wages in Calvo-style staggered contracts that are generally similar to the price contracts described previously. Thus, the probability that a household receives a signal to re-optimize its wage contract in a given period is denoted by $1 - \xi_w$. In addition, SW specify the following dynamic indexation scheme for the adjustment of the wages of those households that do not get a signal to re-optimize: $W_t(h) = \gamma (1 + \pi_{t-1})^{\iota_w} (1+\pi)^{1-\iota_w} W_{t-1}(h)$. All told, this leads to the following optimization problem for the households

$$\max_{\tilde{W}_t(h)} \mathrm{E}_t \sum_{j=0}^{\infty} (\beta \xi_w)^j \frac{\Xi_{t+j} P_t}{\Xi_t P_{t+j}} \left[\tilde{W}_t(h) \left(\Pi_{s=1}^{j} \gamma (1 + \pi_{t+s-1})^{\iota_w} (1+\pi)^{1-\iota_w}\right) - W_{t+j}\right] L_{t+j}(h), \tag{A.16}$$

where $\tilde{W}_t(h)$ is the newly set wage; notice that with our assumptions all households that reoptimize their wages will actually set the same wage.

Following the same approach as with the intermediate-goods firms, we introduce a shock ε_t^w in the resulting first-order condition. This shock, following SW, is assumed to be given by an exogenous ARMA(1,1) process

$$\ln \varepsilon_t^w = (1-\rho_w)\ln \varepsilon^w + \rho_w \ln \varepsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w, \eta_t^w \sim N(0,\sigma_w). \tag{A.17}$$

As discussed previously, we use a scaling factor for this shock so that it enters in exactly the same way as the wage markup shock in SW in the log-linearized representation of the model.

A.3 Monetary policy

The monetary authority follows a Taylor rule adjusting the interest rate in response to changes in inflation and the output gap. An important difference with respect to the standard SW model is that we consider a nonlinear policy rule to explicitly take the ZLB into account:

$$R_t = \max\left[1+\bar{b}, R_{t-1}^{\rho_R}\bar{R}^{(1-\rho_R)}\left(\frac{\pi_t}{\bar{\pi}}\right)^{(r_\pi)(1-\rho_R)}\left(\frac{y_t}{y_t^{pot}}\right)^{(r_y)(1-\rho_R)}\left(\frac{y_t/y_t^{pot}}{y_{t-1}/y_{t-1}^{pot}}\right)^{r_{\Delta y}}\varepsilon_{r,t}\right], \tag{A.18}$$

where $\bar{b} > 0$ is a constant reflecting the level of the Effective Lower Bound (ELB). Since the Federal Funds Rate never reached exactly zero but oscillated around 10 (annualized) basis points throughout the ELB period, we set $\bar{b}$ equal to 10 (annualized) basis points. y_t^{pot} is the output prevailing in the flexible price and wage economy in absence of the inefficient monetary policy, price and wage markup shocks.

A.4 Market Clearing Conditions

Government purchases G_t are exogenous, and the process for government spending relative to trend output, i.e. $g_t = G_t/\left(\gamma^t Y\right)$, is given by the following exogenous AR(1) process:

$$\ln g_t = (1-\rho_g)\ln g + \rho_g\left(\ln g_{t-1} - \rho_{ga}\ln \varepsilon_{t-1}^a\right) + \varepsilon_t^g, \varepsilon_t^g \sim N(0,\sigma_g). \tag{A.19}$$

Government purchases have no effect on the marginal utility of private consumption, nor do they serve as an input into goods production. Moreover, the government is assumed to balance its budget through lump-sum taxes (which are irrelevant since Ricardian equivalence holds in the model).

Total output of the final goods sector is used as follows:

$$Y_t = C_t + I_t + G_t + a\left(Z_t\right)\bar{K}_t, \tag{A.20}$$

where $a\left(Z_t\right)\bar{K}_t$ is the capital utilization adjustment cost.

Finally, one can derive an aggregate production constraint, which depends on aggregate technology, capital, labour, fixed costs, as well as the price and wage dispersion terms.[A.2]

Table A.1: Parameter values in Smets and Wouters (2007).

Panel A: Calibrated

Parameter	Description	Value	Parameter	Description	Value
δ	Depreciation rate	0.025	ϵ_p	Kimball Elast. GM	10
ϕ_w	Gross wage markup	1.50	ϵ_w	Kimball Elast. LM	10
g_y	Gov't G/Y ss-ratio	0.18			

Panel B: Estimated

Parameter	Description	Value	Parameter	Description	Value
φ	Investment adj. cost	5.48	ϕ_p	Gross price markup	1.61
σ_c	Inv subs. elast. of cons.	1.39	γ	Steady state gross growth	1.0043
$\varkappa$	Degree of ext. habit	0.71	$\bar{l}$	Steady state hours worked	0.25
ξ_w	Calvo prob. wages	0.73	π	Steady state net infl. rate	0.0081
σ_l	Labour supply elas.	1.92	β	Discount factor	0.9984
ξ_p	Calvo prob. prices	0.65	ρ_R	Taylor rule, int. rate smooth.	0.81
ι_w	Ind. for non-opt. wages	0.59	$r_{\Delta y}$	Taylor rule, coef. Δ out. gap	0.22
ι_p	Ind. for non-opt. prices	0.22	r_y	Taylor rule, coef. out. gap	0.08
α	Capital production share	0.19	r_π	Taylor rule, coef. inflation	2.03
ψ	Capital utilization cost	0.54			

Panel C: Shock Processes

Shock		Persistence	MA(1)		Std. of Innovation (%)	
Neutral Technology	ρ_a	0.95		-	σ_a	0.45
Risk premium	ρ_b	0.18		-	σ_b	0.24
Gov't spending	ρ_g	0.97	ρ_{ga}	0.52	σ_g	0.52
Inv. Specific Tech.	ρ_i	0.71			σ_i	0.45
Price markup	ρ_p	0.90	μ_p	0.74	σ_p	0.14
Wage markup	ρ_w	0.97	μ_w	0.88	σ_w	0.24
Monetary policy	ρ_r	0.2		-	σ_r	0.24

[A.2] We refer the interested reader to Adjemian, Paries and Moyen (2008) for further details.

A.5 Model Parameterization

When solving the model, we consider two alternative parameterizations. First, we adopt the parameter estimates (posterior mode) in Table A.1 of SW. Second, we re-estimate the SW model setting a lower mean and standard deviation for the steady state price markup (ϕ_p) prior and a higher value for the curvature of the intermediate firm's demand curve (ϵ_p). Specifically, we consider a prior $\phi_p \sim N(1.2, 0.05)$, compared to the SW prior of $\phi_p \sim N(1.25, 0.125)$, and we estimate $\epsilon_p = 64.5$ instead of it being set to 10 as in SW. In both cases we use the same values as SW for the calibrated parameters. Table A.1. provides the values of SW's baseline estimation and Table A.2. provides the parameter values with prior $\phi_p \sim N(1.2, 0.05)$ and $\epsilon_p = 64.5$.

Table A.2: Parameter Values with prior $\phi_p \sim N(1.2, 0.05)$ and estimated ϵ_p.

Panel A: Calibrated

Parameter	Description	Value	Parameter	Description	Value
δ	Depreciation rate	0.025	ξ_p	Calvo prob. prices	0.67
ϕ_w	Gross wage markup	1.50	ϵ_w	Kimball Elast. LM	10
g_y	Gov't G/Y ss-ratio	0.18			

Panel B: Estimated

Parameter	Description	Value	Parameter	Description	Value
φ	Investment adj. cost	5.58	ϕ_p	Gross price markup	1.34
σ_c	Inv subs. elast. of cons.	1.41	γ	Steady state gross growth	1.0044
$\varkappa$	Degree of ext. habit	0.68	$\bar{l}$	Steady state hours worked	–
ξ_w	Calvo prob. wages	0.80	π	Steady state net infl. rate	0.0087
σ_l	Labour supply elas.	2.20	β	Discount factor	0.9987
ϵ_p	Kimball elast. GM	64.5	ρ_R	Taylor rule, int. rate smooth.	0.82
ι_w	Ind. for non-opt. wages	0.56	$r_{\Delta y}$	Taylor rule, coef. Δ out. gap	0.25
ι_p	Ind. for non-opt. prices	0.24	r_y	Taylor rule, coef. out. gap	0.097
α	Capital production share	0.18	r_π	Taylor rule, coef. inflation	1.93
ψ	Capital utilization cost	0.49			

Panel C: Shock Processes

Shock		Persistence	MA(1)		Std. of Innovation (%)	
Neutral Technology	ρ_a	0.95		-	σ_a	0.48
Risk premium	ρ_b	0.22		-	σ_b	0.23
Gov't spending	ρ_g	0.97	ρ_{ga}	0.53	σ_g	0.47
Inv. Specific Tech.	ρ_i	0.70			σ_i	0.40
Price markup	ρ_p	0.83	μ_p	0.69	σ_p	0.13
Wage markup	ρ_w	0.97	μ_w	0.93	σ_w	0.28
Monetary policy	ρ_r	0.11		-	σ_r	0.23

Note that we adapt and re-scale the processes of the price and wage markup shocks so that when our model is log-linearized it matches exactly the original SW model.

A.6 Summary of Nonlinear Model Equations

We detrend the variables with a deterministic trend γ and the nominal variables are replaced by their real counterparts. For instance, we use the following definitions:

$$\begin{aligned}
k_t &= \frac{K_t}{\gamma^t}; w_t = \frac{W_t}{P_t\gamma^t}; w_t^* = \frac{W_t^*}{P_t\gamma^t} \\
r_t^k &= \frac{R_t^k}{P_t}; mc_t = \frac{MC_t}{P_t}; p_t^* = \frac{P_t^*}{P_t}; \delta_t^w = \frac{\Lambda_t^w}{P_t\gamma^t}, \delta_t^p = \frac{\Lambda_t^p}{P_t} \\
\xi_t &\equiv \Xi_t\gamma^{\sigma_c t} \\
\gamma_{1,t}^p &= \frac{\Gamma_{1,t}^p}{\gamma^t}, \gamma_{2,t}^p = \frac{\Gamma_{2,t}^p}{\gamma^t}, \gamma_{3,t}^p = \frac{\Gamma_{3t}^p}{\gamma^t}, \\
\gamma_{1,t}^w &= \frac{\Gamma_{1,t}^w}{(\gamma^t)^{\frac{\phi_w(1+\epsilon_w)}{\phi_w-1}}}, \gamma_{2,t}^w = \frac{\Gamma_{2,t}^w}{(\gamma^t)^{1+\frac{\phi_w(1+\epsilon_w)}{\phi_w-1}}},
\end{aligned}$$

Also, we define $g_t \equiv G_t/Y_t$, and re-define the fixed cost $\Phi \equiv (\phi_p - 1)\, y$, where y is the steady-state level of output.

Market clearing

$$c_t + i_t + g_t y + a\left(U_t\right)\frac{k_{t-1}^h}{\gamma} = y_t \tag{A.21}$$

$$y_t\frac{\epsilon_p + s_t^p}{1+\epsilon_p} = \left(\varepsilon_t^a\left(k_t^s\right)^\alpha L_t^{1-\alpha} - (\phi_p - 1)\, y\right) \tag{A.22}$$

$$k_t^s = \frac{1}{\gamma}U_t k_{t-1}^h \tag{A.23}$$

$$k_t^h = \frac{1-\delta}{\gamma}k_{t-1}^h + \varepsilon_t^i\left[1 - S\left(\frac{i_t\gamma}{i_{t-1}}\right)\right]i_t \tag{A.24}$$

$$s_t^p = (1-\xi_p)\left(\frac{p_t^*}{\delta_t^p}\right)^{-\frac{\phi_p(1+\epsilon_p)}{\phi_p-1}} + \xi_p\left(\frac{\pi^{1-\iota_p}\pi_{t-1}^{\iota_p}}{\pi_t}\frac{\delta_{t-1}^p}{\delta_t^p}\right)^{-\frac{\phi_p(1+\epsilon_p)}{\phi_p-1}} s_{t-1}^p \tag{A.25}$$

$$s_t^w = (1-\xi_w)\left(\frac{w_t^*}{\delta_t^w}\right)^{-\frac{\phi_w(1+\epsilon_w)}{\phi_w-1}} + \xi_w\left(\frac{\pi^{1-\iota_w}\pi_{t-1}^{\iota_w}}{\pi_t}\frac{\delta_{t-1}^w}{\delta_t^w}\right)^{-\frac{\phi_w(1+\epsilon_w)}{\phi_w-1}} s_{t-1}^w \tag{A.26}$$

$$1=(1-\xi_p)\left(\frac{p_t^*}{\delta_t^p}\right)^{-\frac{1+\phi_p\epsilon_p}{\phi_p-1}}+\xi_p\left(\frac{\pi^{1-\iota_p}\pi_{t-1}^{\iota_p}}{\pi_t}\frac{\delta_{t-1}^p}{\delta_t^p}\right)^{-\frac{1+\phi_p\epsilon_p}{\phi_p-1}} \tag{A.27}$$

$$1=(1-\xi_w)\left(\frac{w_t^*}{\delta_t^w}\right)^{-\frac{1+\phi_w\epsilon_w}{\phi_w-1}}+\xi_w\left(\frac{\pi^{1-\iota_w}\pi_{t-1}^{\iota_w}}{\pi_t}\frac{\delta_{t-1}^w}{\delta_t^w}\right)^{-\frac{1+\phi_w\epsilon_w}{\phi_w-1}} \tag{A.28}$$

$$(1+\epsilon_w)\,w_t=\left(1+\epsilon_w s_t^{wl}\right)\delta_t^w \tag{A.29}$$

$$(1+\epsilon_p)=\left(1+\epsilon_p s_t^{pl}\right)\delta_t^p \tag{A.30}$$

$$s_t^{pl}=(1-\xi_p)\left(\frac{p_t^*}{\delta_t}\right)+\xi_p\left(\frac{\pi^{1-\iota_p}\pi_{t-1}^{\iota_p}}{\pi_t}\frac{\delta_{t-1}^p}{\delta_t^p}\right)s_{t-1}^{pl} \tag{A.31}$$

$$s_t^{wl}=(1-\xi_w)\left(\frac{w_t^*}{\delta_t^w}\right)+\xi_w\left(\frac{\pi^{1-\iota_w}\pi_{t-1}^{\iota_w}}{\pi_t}\frac{\delta_{t-1}^w}{\delta_t^w}\right)s_{t-1}^{wl} \tag{A.32}$$

Firms

$$\frac{k_t^s}{L_t} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k} \tag{A.33}$$

$$mc_t = \left(\frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k}\right)^{-\alpha}\frac{w_t}{(1-\alpha)\,\varepsilon_t^a} \tag{A.34}$$

Households

$$\xi_t \equiv \Xi_t\gamma^{\sigma_c t}=\varepsilon_t^d\left(c_t-\frac{\varkappa}{\gamma}c_{t-1}\right)^{-\sigma_c}\exp\left((\sigma_c-1)\frac{\left(L_t^h\right)^{1+\sigma_l}}{1+\sigma_l}\right);\;\; L_t^h\equiv L_t\frac{\epsilon_w+s_t^w}{1+\epsilon_w} \tag{A.35}$$

$$1 = Q_t\varepsilon_t^i\left[1-S\left(\frac{i_t\gamma}{i_{t-1}}\right)-S'\left(\frac{i_t\gamma}{i_{t-1}}\right)\frac{i_t\gamma}{i_{t-1}}\right]+\beta\gamma^{-\sigma}E_t\frac{\xi_{t+1}}{\xi_t}Q_{t+1}\varepsilon_{t+1}^i S'\left(\frac{i_{t+1}\gamma}{i_t}\right)\left(\frac{i_{t+1}\gamma}{i_t}\right)^2 \tag{A.36}$$

$$R_t^{-1} = \varepsilon_t^b\beta\gamma^{-\sigma}E_t\frac{\xi_{t+1}}{\xi_t}\Pi_{t+1}^{-1} \tag{A.37}$$

$$Q_t = \beta\gamma^{-\sigma}E_t\frac{\xi_{t+1}}{\xi_t}\left[(1-\delta)\,Q_{t+1}+\left(r_{t+1}^k U_{t+1}-a\left(U_{t+1}\right)\right)\right] \tag{A.38}$$

$$r_t^k \equiv \frac{R_t^k}{P_t}=a'\left(U_t\right) \tag{A.39}$$

Wage Setting

$$\frac{1+\phi_w\epsilon_w}{1+\epsilon_w}w_t^*\gamma_{1,t}^w=\phi_w\gamma_{2,t}^w+\frac{\epsilon_w\left(\phi_w-1\right)}{1+\epsilon_w}\left(w_t^*\right)^{\frac{\phi_w(1+\epsilon_w)}{\phi_w-1}+1}\gamma_{3,t}^w \tag{A.40}$$

$$\gamma_{1,t}^w=\left(\delta_t^w\right)^{\frac{\phi_w(1+\epsilon_w)}{\phi_w-1}}L_t+\left(\beta\gamma^{1-\sigma}\right)\xi_w E_t\frac{\xi_{t+1}}{\xi_t}\left(\frac{\pi^{1-\iota_w}\pi_t^{\iota_w}}{\pi_{t+1}}\right)^{-\frac{1+\phi_w\epsilon_w}{\phi_w-1}}\gamma_{1,t+1}^w \tag{A.41}$$

$$\gamma_{2,t}^w=\left(\delta_t^w\right)^{\frac{\phi_w(1+\epsilon_w)}{\phi_w-1}}w_t^h\varepsilon_t^w L_t+\left(\beta\gamma^{1-\sigma}\right)\xi_w E_t\frac{\xi_{t+1}}{\xi_t}\left(\frac{\pi^{1-\iota_w}\pi_t^{\iota_w}}{\pi_{t+1}}\right)^{-\frac{\phi_w(1+\epsilon_w)}{\phi_w-1}}\gamma_{2,t+1}^w \tag{A.42}$$

$$\gamma_{3,t}^w=L_t+\left(\beta\gamma^{1-\sigma}\right)\xi_w E_t\frac{\xi_{t+1}}{\xi_t}\left(\frac{\pi^{1-\iota_w}\pi_t^{\iota_w}}{\pi_{t+1}}\right)\gamma_{3,t+1}^w \tag{A.43}$$

Price Setting

$$\frac{1+\phi_p\epsilon_p}{1+\epsilon_p}p_t^*\gamma_{1,t}^p=\phi_p\gamma_{2,t}^p+\frac{\epsilon_p\left(\phi_p-1\right)}{1+\epsilon_p}\left(p_t^*\right)^{\frac{\phi_p(1+\epsilon_p)}{\phi_p-1}+1}\gamma_{3,t}^p, \tag{A.44}$$

$$\gamma_{1,t}^p=\left(\delta_t^p\right)^{\frac{\phi_p(1+\epsilon_p)}{\phi_p-1}}y_t+\left(\beta\gamma^{1-\sigma}\right)\xi_pE_t\frac{\xi_{t+1}}{\xi_t}\left(\frac{\pi^{1-\iota_p}\pi_t^{\iota_p}}{\pi_{t+1}}\right)^{-\frac{1+\phi_p\epsilon_p}{\phi_p-1}}\gamma_{1,t+1}^p, \tag{A.45}$$

$$\gamma_{2,t}^p=\left(\delta_t^p\right)^{\frac{\phi_p(1+\epsilon_p)}{\phi_p-1}}mc_t\varepsilon_{p,t}y_t+\left(\beta\gamma^{1-\sigma}\right)\xi_pE_t\frac{\xi_{t+1}}{\xi_t}\left(\frac{\pi^{1-\iota_p}\pi_t^{\iota_p}}{\pi_{t+1}}\right)^{-\frac{\phi_p(1+\epsilon_p)}{\phi_p-1}}\gamma_{2,t+1}^p, \tag{A.46}$$

$$\gamma_{3,t}^p=y_t+\left(\beta\gamma^{1-\sigma}\right)\xi_pE_t\frac{\xi_{t+1}}{\xi_t}\left(\frac{\pi^{1-\iota_p}\pi_t^{\iota_p}}{\pi_{t+1}}\right)\gamma_{3,t+1}^p, \tag{A.47}$$

Monetary Policy

$$R_t=\max\left[1+\bar{b},R_{t-1}^{\rho_R}\bar{R}^{(1-\rho_R)}\left(\frac{\pi_t}{\pi}\right)^{r_\pi(1-\rho_R)}\left(\frac{y_t}{y_t^{pot}}\right)^{r_y(1-\rho_R)}\left(\frac{y_t/y_t^{pot}}{y_{t-1}/y_{t-1}^{pot}}\right)^{r_{\Delta y}}\varepsilon_{r,t}\right] \tag{A.48}$$

Together, this consitutes an equation system with 28 equations and the following 28 endogenous variables: c_t, y_t, L_t, i_t, k_t^s, k_t^h, U_t, p_t^*, w_t^*, π_t, s_t^p, s_t^w, δ_t^p, δ_t^w, s_t^{pl}, s_t^{wl}, r_t^k, w_t, mc_t, ξ_t, R_t, Q_t, $\gamma_{1,t}^p$, $\gamma_{2,t}^p$, $\gamma_{3,t}^p$, $\gamma_{1,t}^w$, $\gamma_{2,t}^w$, $\gamma_{3,t}^w$.

The flexible price and wage allocations are obtained by setting $\xi_p=\xi_w=0$ which defines y_t^{pot}. Only 4 shocks affect the flex price-wage allocations; $\varepsilon_t^a,\varepsilon_t^b,\varepsilon_t^i$, and g_t.

A.7 Summary of Linearized Model Equations

In this section, we summarize the log-linear equations of the SW-model. The complete model also includes the seven exogenous shocks $\varepsilon_t^a,\varepsilon_t^b,\varepsilon_t^i,\varepsilon_t^p,\varepsilon_t^w,\varepsilon_t^r$, and g_t, but their processes are not stated here as they were presented earlier. Consistent with the notation of the log-linearized *endogenous* variables $\hat{x}_t=dx_t/x$, the exogenous shocks are denoted with a 'hat', i.e. $\hat{\varepsilon}_t=\ln\varepsilon_t$.

First, we have the consumption Euler equation:

$$\widehat{c}_t=\tfrac{1}{(1+\varkappa/\gamma)}\mathrm{E}_t\widehat{c}_{t+1}+\tfrac{\varkappa/\gamma}{(1+\varkappa/\gamma)}\widehat{c}_{t-1}-\tfrac{1-\varkappa/\gamma}{\sigma_c(1+\varkappa/\gamma)}(\hat{R}_t-\mathrm{E}_t\widehat{\pi}_{t+1}+\hat{\varepsilon}_t^b)-\tfrac{(\sigma_c-1)(w_*^hL/c_*)}{\sigma_c(1+\varkappa/\gamma)}(\mathrm{E}_t\widehat{L}_{t+1}-\widehat{L}_t), \tag{A.49}$$

where $\varkappa$ is the external habit parameter, σ_c the reciprocal of the intertemporal substitution elasticity, w_*^hL/c_* the steady state nominal labor earnings to consumption ratio.

Next, we have the investment Euler equation:

$$\widehat{i}_t=\tfrac{1}{(1+\overline{\beta}\gamma)}\left(\widehat{i}_{t-1}+\overline{\beta}\gamma\mathrm{E}_t\widehat{i}_{t+1}+\tfrac{1}{\gamma^2\varphi}\widehat{Q}_t^k\right)+\widehat{\varepsilon}_t^q, \tag{A.50}$$

where $\bar{\beta} = \beta\gamma^{-\sigma_c}$, φ is the investment adjustment cost, and the investment specific technology shock $\widehat{\varepsilon}_t^q$ has been re-scaled so that it enters linearly with a unit coefficient. Additionally $i_1 = 1/(1+\beta)$ and $i_2 = i_1/\psi$, where β is the discount factor and ψ is the elasticity of the capital adjustment cost function.

The price of capital is determined by:

$$\widehat{Q}_t^k = -(\widehat{\bar{R}}_t - \mathrm{E}_t\widehat{\pi}_{t+1} + \widehat{\varepsilon}_t^b) + q_1\mathrm{E}_t r_{t+1}^k + (1-q_1)\mathrm{E}_t Q_{t+1}^k, \tag{A.51}$$

where $q_1 \equiv r_*^k/(r_*^k + (1-\delta))$ in which r_*^k is the steady state rental rate to capital, δ the depreciation rate.

Fourth, we have the optimal condition for the capital utilization rate $\widehat{u}_t$:

$$\widehat{u}_t = (1-\psi)/\psi\widehat{r}_t^k, \tag{A.52}$$

where ψ is the elasticity of the capital utilization cost function and capital services used in production $(\widehat{k}_t)$ is defined as:

$$\widehat{k}_t \;=\; \widehat{u}_t \;+\; \widehat{\bar{k}}_{t-1}, \tag{A.53}$$

where $\widehat{\bar{k}}_{t-1}$ is the physical capital stock which evolves according to the capital accumulation equation:

$$\widehat{\bar{k}}_t = \;\kappa_1\;\widehat{\bar{k}}_{t-1} \;+(1-\kappa_1)\widehat{i}_t + \kappa_2\widehat{\varepsilon}_t^q \tag{A.54}$$

with $\kappa_1 = 1 - (i_*/\overline{k}_*)$ and $\kappa_2 = (i_*/\overline{k}_*)\gamma^2\varphi(1+\bar{\beta}\gamma)$.

The following optimal capital/labor input condition also holds:

$$\widehat{k}_t \;=\; \widehat{w}_t \;-\; \widehat{r}_t^k \;+\; \widehat{L}_t, \tag{A.55}$$

where $\widehat{w}_t$ is the real wage.

The log-linearized production function is given by:

$$\widehat{y}_t = \phi_p\;(\;\alpha\widehat{k}_t \;+\; (1-\alpha)\widehat{L}_t + \widehat{\varepsilon}_t^a), \tag{A.56}$$

in which ϕ_p is the fixed costs of production corresponding to the gross price markup in the steady state, and $\widehat{\varepsilon}_t^a$ is the exogenous TFP process.

Aggregate demand must equal aggregate supply:

$$\widehat{y}_t \;=\; \frac{c_*}{y_*}\widehat{c}_t \;+\; \frac{i_*}{y_*}\widehat{i}_t \;+\; g_t \;+\frac{r_*^k k_*}{y_*}\widehat{u}_t, \tag{A.57}$$

where g_t represents the exogenous demand component.

Next, we have the following log-linearized price-setting equation with dynamic indexation ι_p:

$$\widehat{\pi}_t - \iota_p \widehat{\pi}_{t-1} = \pi_1 \left(\mathrm{E}_t \widehat{\pi}_{t+1} - \iota_p \widehat{\pi}_t \right) - \pi_2 \widehat{\mu}_t^p + \hat{\varepsilon}_t^p, \tag{A.58}$$

where $\pi_1 = \beta$, $\pi_2 = (1-\xi_p\beta)(1-\xi_p)/[\xi_p(1+(\phi_p-1)\epsilon_p)]$, $1-\xi_p$ is the probability of each firm being able to re-optimize the price each period, ϵ_p is the curvature of the aggregator function (eq. (A.1)), and the markup shock $\hat{\varepsilon}_t^p$ has been re-scaled to enter with a unit coefficient. The price markup $\widehat{\mu}_t^p$ equals the inverse of the real marginal cost, $\widehat{\mu}_t^p = -\widehat{mc}_t$, which in turn is given by:

$$\widehat{mc}_t = (1-\alpha)\, \widehat{w}_t^{real} + \alpha\, \widehat{r}_t^k - \widehat{\varepsilon}_t^a. \tag{A.59}$$

We also have the following wage-setting equation allowing for dynamic indexation of wages for non-optimizing households:

$$\begin{aligned} &(1+\overline{\beta}\gamma)\widehat{w}_t^{real} - \widehat{w}_{t-1}^{real} - \overline{\beta}\gamma \mathrm{E}_t \widehat{w}_{t+1}^{real} = \\ &\tfrac{(1-\xi_w\overline{\beta}\gamma)(1-\xi_w)}{[\xi_w(1+(\phi_w-1)\epsilon_w)]} \left(\tfrac{1}{1-\varkappa/\gamma}\widehat{c}_t - \tfrac{\varkappa/\gamma}{1-\varkappa/\gamma}\widehat{c}_{t-1} + \sigma_l \widehat{L}_t - \widehat{w}_t \right) - (1+\overline{\beta}\gamma\iota_w)\widehat{\pi}_t + \iota_w \widehat{\pi}_{t-1} + \overline{\beta}\gamma \mathrm{E}_t \widehat{\pi}_{t+1} + \hat{\varepsilon}_t^w, \end{aligned} \tag{A.60}$$

where ϕ_w the gross wage markup, $1-\xi_p$ is the probability of each household being able to re-optimize its wage each period, ϵ_w is the curvature of the aggregator function (eq, A.8), and σ_l determines the elasticity of labor supply given σ_c (see equation (A.9)). The exogenous wage markup shock $\hat{\varepsilon}_t^w$ has been re-scaled to enter linearly with a unit coefficient.

Finally, we have the following monetary policy rule:

$$\widehat{R}_t = \rho_R \widehat{R}_{t-1} + (1-\rho_R)\left(r_\pi \widehat{\pi}_t + r_y \hat{y}_t^{gap} \right) + r_{\Delta y} \Delta \hat{y}_t^{gap} + \hat{\varepsilon}_t^r, \tag{A.61}$$

where $\hat{y}_t^{gap} = \widehat{y}_t - \widehat{y}_t^{pot}$, or in words: the difference between actual output and the output prevailing in the flexible price and wage economy in absence of the inefficient price and wage markup shocks. We solve for $\widehat{y}_t^{pot}$ by setting $\xi_p = \xi_w = 0$ (or arbitrary close to nil) and removing $\hat{\varepsilon}_t^w$ and $\hat{\varepsilon}_t^p$ from the system of equations given by (A.49) – (A.61). When linearizing the model we do not take the ZLB into account and the policy rate equals the shadow rate implied by the Taylor rule in equation (A.61) at all times.

A.8 Observer Equations and Data

We have the following observer equations in the nonlinear model:

$$\begin{aligned}
\pi_t^{obs} &= 100 \ln \pi_t \\
\Delta w_t^{obs} &= 100 \ln w_t / w_{t-1} + \gamma \\
R_t^{obs} &= 100 \left(R_t - 1 \right) \\
\Delta y_t^{obs} &= 100 \ln y_t / y_{t-1} + \gamma \\
\Delta c_t^{obs} &= 100 \ln c_t / c_{t-1} + \gamma \\
\Delta i_t^{obs} &= 100 \ln i_t / i_{t-1} + \gamma \\
l_t^{obs} &= 100 \ln l_t / l
\end{aligned}$$

We use the same data as Smets and Wouters (2007), except that we update their dataset through 2014Q2.