Fraud-proof non-market allocation mechanisms

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The allocation problem

Problem: allocate $m \leq 1$ mass **homogeneous** objects to unit mass of **heterogeneous** agents **without transfers**

Examples: vaccines, ICU beds, public housing, food stamps

Agents: characterized by a type vector (i, v, γ, s)

- Group – $i \in I$, $|I| < \infty$, unfalsifiable characteristic(s), public (age, gender, DOB, race)
- Valuation – $v \in (0, \overline{v}]$, willingness to pay, private; group sizes μ_i.
- Gaming ability – $\gamma \in (0, \overline{\gamma}]$, falsification cost scaler, private.
- Score – $s \in S = [\underline{s}, \overline{s}]$, fundamental characteristic, falsifiable at a cost.

Planner:

- Social value – w, social (or planner's) value unknown to both
- Conditional social value: $w_i(s) = \mathbb{E}(w|i, s)$ is increasing in s: positive correlation between score and social value

Fraud and payoffs

Planner's payoff: from assigning an object to group i agent with score s:

$$w_i(s)$$

Agent's payoff:

$$\Pr(\text{object})v - \frac{1}{\gamma}c_i(\underbrace{t}_{\text{falsified score}} \mid \underbrace{s}_{\text{true score}})$$

Assumptions:

- $c_i(t|s)$ is measurable and strictly increasing in $|t-s|$.
- $c_i(s|s) = 0$
- Regularity (REG): $c_i(t|s)$ is continuously differentiable in t on $[s, \bar{s}]$ and in s on $[\underline{s}, t]$, and there exists $K > 0$ such that, for every $t > s$,

$$c(t|s) \leq K(t-s).$$

Distributional assumptions

- agents draw their types independently.
- Conditional value and gaming ability distribution: conditional on i, the distribution of (γ, v, s) has full support on $(0, \bar{\gamma}_i] \times (0, \bar{v}_i] \times [\underline{s}_i, \bar{s}_i]$
- Conditional score distribution: $s \sim F_i(s)$ conditional on i with support $S_i = [\underline{s}_i, \bar{s}_i]$.

Non-market mechanisms

Non-market mechanisms: planner cannot use transfers; only objects are allocated; Ben-Porath, Dekel and Lipman (2014), Mylovanov and Zapechelnyuk (2017), Kattwinkel (2020), Condorelli (2013)

A direct non-market mechanism, denoted by α, consists of an allocation rule $\alpha = (\alpha_i)_{i \in I}$ where

$$\alpha_i : [0, \bar{v}] \times (0, \bar{\gamma}] \times [\underline{s}, \bar{s}] \to [0, 1]$$

Incentive compatible non-market mechanisms. An allocation mechanism α is *incentive compatible* if and only if:

$$\alpha_i(s, v, \gamma)v \geq \alpha_i(t, v', \gamma')v - \frac{1}{\gamma} c_i(t|s) \forall v, v' \in V; \gamma, \gamma' \in \Gamma; s, t \in S.$$

Constraints divide into 2 groups:

1. truth-telling constraints involving soft information
2. falsification-proofness constraints involving costly falsification

Note: This is analogous to the division of incentive compatibility constraints in Myerson (1982) into adverse selection and into moral hazard constraints.

Non-market allocation mechanisms

- in the absence of transfers, impossible to implement any allocation that depends on (v, γ)
  - all agents of a given s have incentive to report the values that lead to the highest assignment probability. If $\alpha(s, v', \gamma')$ varies with v', γ' then, all agents that find optimal to report s will claim a value v^*, γ^* associated with the maximal assignment probability.

- Hence the allocation rule cannot depend on (v, γ): any variation would generate false reports that cancel the variation in equilibrium.

- Therefore, a non-market mechanism is an allocation rule as a function of reported and observed characteristics $\alpha = (\alpha_i)_{i \in I}$:

$$\alpha_i : S \to [0, 1]$$

- Akbarpour et al. (2020) study related allocation problem with transfers; in their setting only v can be elicited (through transfers) but no other dimension can be elicited

Constraints

- Resource Constraint:

$$\sum_i \mu_i \int \alpha_i(s) dF_i(s) \leq m \qquad \text{(RC)}$$

- Falsification-proofness constraints:

$$\forall i, s, t \quad \alpha_i(t) - \alpha_i(s) \leq \frac{1}{\bar{\gamma}_i \bar{v}_i} c_i(t|s) \qquad \text{(FPIC)}$$

- Probability constraints:

$$\forall i, s, \quad 0 \leq \alpha_i(s) \leq 1 \qquad \text{(PROB)}$$

- Quotas: for a given quota system $\phi_i \in [0, 1]$, with $\sum_i \phi_i \leq 1$,

$$\forall i, \quad \mu_i \int \alpha_i(s) dF_i(s) \geq \phi_i \sum_j \mu_j \int \alpha_j(s) dF_j(s) \qquad \text{(QC)}$$

Falsification proofness

- With costly falsification, no transfers, more than two states (scores) we know that the revelation principle fails truth-telling is with loss– Perez-Richet and Skreta (forthcoming)
- Optimal mechanisms may have to induce fraud.
- So imposing falsification-proofness is costly for the planner.
- Why the constraint?
  1. Mechanisms (especially non-market) that induce illegal fraud might come under scrutiny and be difficult to defend (political / stability argument).
  2. Fraud might have a negative social externality: spreading dishonest and corrupt behavior (see e.g. Galbiati and Zanella (2012); Ajzenman (2018); Alm et al. (2017); Rincke and Traxler (2011)).
  3. Fraud is costly to agents, and may generate lack of fairness (due to heterogeneous gaming ability).
  4. Theoretical justification: measure the cost of falsification-proofness.

planner's problem

$$\max_{\alpha} \quad \sum_i \mu_i \int \alpha_i(s) w_i(s) dF_i(s)$$

$$\text{s.t.} \quad \alpha_i(t) - \alpha_i(s) \leq \frac{1}{\bar{\gamma}_i \bar{v}_i} c_i(t|s) \quad \forall i, s, t \qquad \text{(FPIC)}$$

$$0 \leq \alpha_i(s) \leq 1 \quad \forall i, s \qquad \text{(PROB)}$$

$$\sum_i \mu_i \int \alpha_i(s) dF_i(s) \leq m \qquad \text{(RC)}$$

$$\mu_i \int \alpha_i(s) dF_i(s) \geq \phi_i \sum_j \mu_j \int \alpha_j(s) dF_j(s) \quad \forall i \qquad \text{(QC)}$$

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$$0 \leq \alpha_i(s) \leq 1 \quad \forall i, s \qquad \text{(PROB)}$$

$$[\lambda_R] \quad \sum_i \mu_i \int \alpha_i(s) dF_i(s) \leq m \qquad \text{(RC)}$$

$$[\lambda_i] \quad \mu_i \int \alpha_i(s) dF_i(s) \geq \phi_i \sum_j \mu_j \int \alpha_j(s) dF_j(s) \quad \forall i \qquad \text{(QC)}$$

Quota and resources as desired allocation thresholds

- Lagrangian:

$$\sum_i \mu_i \int \left\{ w_i(s) - \hat{w}_i(\lambda, \phi) \right\} \alpha_i(s) dF_i(s) + K(\lambda),$$

where

$$\hat{w}_i(\lambda, \phi) = \lambda_R - \lambda_i + \sum_j \lambda_j \phi_j$$

Remark

- *vector of Lagrange multipliers and quota system define a vector of desired allocation thresholds $\hat{w}$ that can be adjusted to satisfy the resource and quota constraints*
- *resulting program is separable across groups*
- *vector of group thresholds $\hat{w}$: endogenous prioritization of groups*
- *without quotas, $\hat{w}$ is homogeneous across groups*
- *$\hat{w}$ governs across group allocation, focus first on within group allocation*

The within group problem

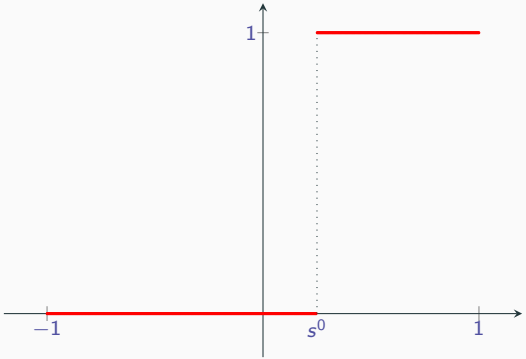
$$\max_{\alpha} \quad \int \alpha(s)\{w(s) - \hat{w}\}dF(s)$$

$$\text{s.t.} \quad \alpha(t) - \alpha(s) \leq \frac{1}{\bar{\gamma}\bar{v}} c(t|s) \quad \forall s, t \qquad \text{(FPIC)}$$

$$0 \leq \alpha(s) \leq 1 \quad \forall s \qquad \text{(PROB)}$$

Some intuition

- The desired score threshold is s^0 such that $w(s^0) = \hat{w}$.

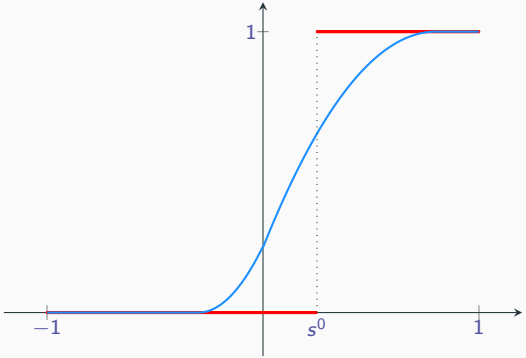


$F = U([-1, 1]),\ w(s) = s,\ \hat{w} = 1/4$

Note: With transfers this is typical shape of optimal allocations. Sudden increase in payment makes this allocation incentive compatible. Not possible without transfers!

Some intuition

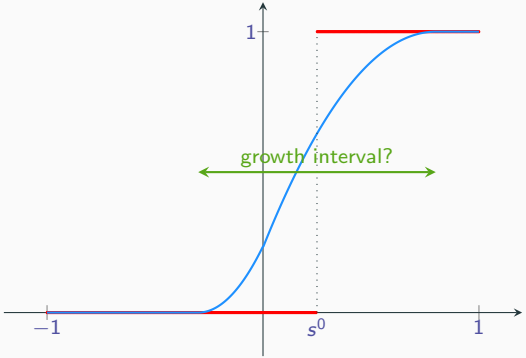
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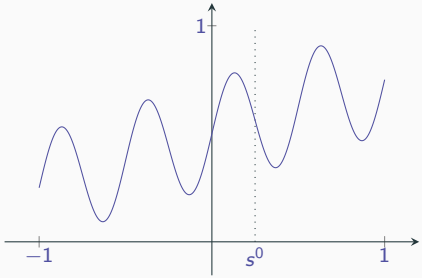
Smooth and monotonic allocation rules

Lemma

Optimal allocation can be without loss taken to be monotonic and Lipschitz continuous. So we can write:

$$\alpha(s) = \underline{\alpha} + \int_{\underline{s}}^{s} \alpha'(z)dz = \overline{\alpha} - \int_{s}^{\overline{s}} \alpha'(z)dz$$

- Lipschitz continuity is implied by (FPIC) and (REG).
- Monotonicity $\rightarrow$



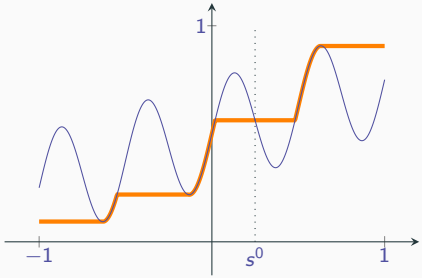
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Optimal within group-allocation: high versus low priority groups

a group has high priority if

$$\overline{w} = \int w(s)dF(s) \geq \hat{w}$$

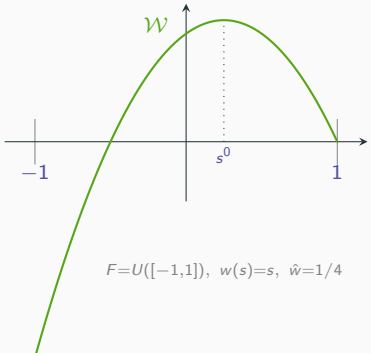
otherwise; it has low priority

Cumulative surplus and growth intervals

Low priority group: $\overline{w} < \hat{w}$

The *upward cumulative surplus* is the marginal gain of uniformly increasing the allocation probability of all scores above some cutoff s:

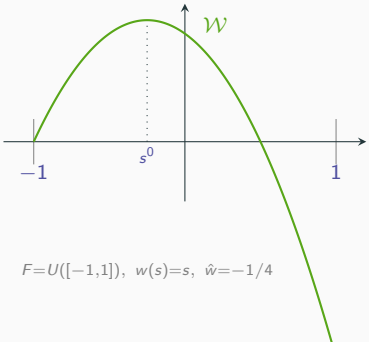
$$\mathcal{W}^{+}(s, \hat{w}) = \int_{s}^{\overline{s}} \{w(x) - \hat{w}\} dF(x)$$



$F=U([-1,1])$, $w(s)=s$, $\hat{w}=1/4$

High priority group: $\overline{w} \geq \hat{w}$ The *downward cumulative surplus* is the marginal gain of uniformly decreasing the allocation probability of all scores below some cutoff s:

$$\mathcal{W}^{-}(s, \hat{w}) = -\int_{\underline{s}}^{s} \{w(x) - \hat{w}\} dF(x)$$



$F=U([-1,1])$, $w(s)=s$, $\hat{w}=-1/4$

The differential program

Low priority group: $\overline{w} < \hat{w}$

$$\max_{\alpha} \ \underline{\alpha}(\overline{w} - \hat{w}) + \int \alpha'(s)\mathcal{W}^{+}(s, \hat{w})ds$$

$$\text{s.t.} \quad 0 \leq \alpha'(s) \quad \forall s \qquad \text{(MON)}$$

$$0 \leq \underline{\alpha} \qquad \text{(PROB)}$$

$$\underline{\alpha} + \int_{\underline{s}}^{s} \alpha'(z)dz \leq 1 \quad \forall s \qquad \text{(PROB)}$$

$$\int_{s}^{t} \alpha'(z)dz \leq \frac{1}{\overline{\gamma}\,\overline{v}} c(t|s) \quad \forall s < t \qquad \text{(FPIC)}$$

High priority group: $\overline{w} \geq \hat{w}$

$$\max_{\alpha} \ \overline{\alpha}(\overline{w} - \hat{w}) + \int \alpha'(s)\mathcal{W}^{-}(s, \hat{w}))ds$$

$$\text{s.t.} \quad 0 \leq \alpha'(s) \quad \forall s \qquad \text{(MON)}$$

$$\overline{\alpha} \leq 1 \qquad \text{(PROB)}$$

$$0 \leq \overline{\alpha} - \int_{s}^{\overline{s}} \alpha'(z)dz \quad \forall s \qquad \text{(PROB)}$$

$$\int_{s}^{t} \alpha'(z)dz \leq \frac{1}{\overline{\gamma}\,\overline{v}} c(t|s) \quad \forall s < t \qquad \text{(FPIC)}$$

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$$\underline{\alpha} + \int_{\underline{s}}^{s} \alpha'(z)dz \leq 1 \quad \forall s \qquad \text{(PROB)}$$

$$\int_{s}^{t} \alpha'(z)dz \leq \frac{1}{\overline{\gamma}\overline{v}} c(t|s) \quad \forall s < t \qquad \text{(FPIC)}$$

$$\Rightarrow \underline{\alpha} = 0$$

Lowest score does not get it

High priority group: $\overline{w} \geq \hat{w}$

$$\max_{\alpha} \ \overline{\alpha}(\overline{w} - \hat{w}) + \int \alpha'(s)\mathcal{W}^{-}(s, \hat{w}))ds$$

$$\text{s.t.} \quad 0 \leq \alpha'(s) \quad \forall s \qquad \text{(MON)}$$

$$\overline{\alpha} \leq 1 \qquad \text{(PROB)}$$

$$0 \leq \overline{\alpha} - \int_{s}^{\overline{s}} \alpha'(z)dz \quad \forall s \qquad \text{(PROB)}$$

$$\int_{s}^{t} \alpha'(z)dz \leq \frac{1}{\overline{\gamma}\overline{v}} c(t|s) \quad \forall s < t \qquad \text{(FPIC)}$$

$$\Rightarrow \overline{\alpha} = 1$$

Highest score gets it for sure

The differential program

$$\max_{\alpha'} \int \alpha'(s)\mathcal{W}(s,\hat{w})ds$$

$$\text{s.t. } 0 \leq \alpha'(s), \quad \forall s \tag{MON}$$

$$\int_s^t \alpha'(z)dz \leq \frac{1}{\bar{\gamma}\bar{v}}c(t|s), \quad \forall s < t \tag{FPIC}$$

$$\int \alpha'(z)dz \leq 1 \tag{PROB}$$

The differential program

$$\max_{\alpha'} \int \alpha'(s)\mathcal{W}(s,\hat{w})ds$$

$$\text{s.t. } 0 \leq \alpha'(s), \quad \forall s \tag{MON}$$

$$\int_s^t \alpha'(z)dz \leq \frac{1}{\bar{\gamma}\bar{v}}c(t|s), \quad \forall s < t \tag{FPIC}$$

$$[\nu] \quad \int \alpha'(z)dz \leq 1 \tag{PROB}$$

The differential program

$$\max_{\alpha'} \int \alpha'(s)\{\mathcal{W}(s,\hat{w}) - \nu\}ds + K(\nu)$$

$$\text{s.t. } 0 \leq \alpha'(s), \quad \forall s \tag{MON}$$

$$\int_s^t \alpha'(z)dz \leq \frac{1}{\bar{\gamma}\bar{v}} c(t|s), \quad \forall s < t \tag{FPIC}$$

$\Rightarrow$ So $\alpha'(s) = 0$ on $\{s : \mathcal{W}(s,\hat{w}) < \nu\}$.

Note: In Myerson (1981) the objective is assn. prob times marginal revenue (virtual valuation); here opposite in some sense

Matching function

- Matching function $m : [s_{\hat{w}}, s^o] \to [s^o, \bar{s}]$ defined by:

$$\mathcal{W}(s, \hat{w}) = \mathcal{W}(m(s), \hat{w})$$

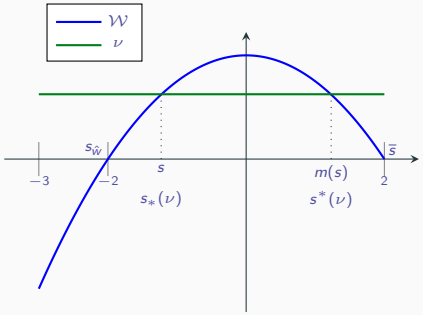
$$\Updownarrow$$

$$\int_s^{m(s)} \{w(s) - \hat{w}\} dF(s) = 0$$

so $\mathcal{W}(s_*(\nu), \hat{w}) = \mathcal{W}(s^*(\nu), \hat{w}) = \nu$

$$\Updownarrow$$

$$\int_{s_*(\nu)}^{s^*(\nu)} \{w(s) - \hat{w}\} dF(s) = 0$$

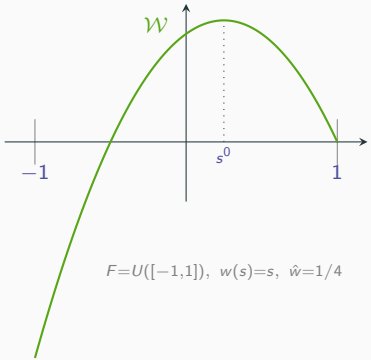


$F_i = Uniform([-3,2])$, $w(s) = s$, $\hat{w} = 0$

Cumulative surplus and growth intervals

Low priority group: $\overline{w} < \hat{w}$

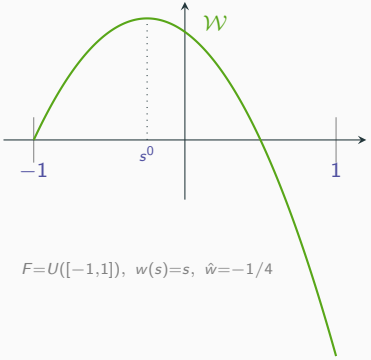
$$\mathcal{W}^{+}(s,\hat{w})(s) = \int_{s}^{\overline{s}} \{w(x) - \hat{w}\} dF(x)$$



$F{=}U([-1,1])$, $w(s){=}s$, $\hat{w}{=}1/4$

High priority group: $\overline{w} \geq \hat{w}$

$$\mathcal{W}^{-}(s,\hat{w})(s) = -\int_{\underline{s}}^{s} \{w(x) - \hat{w}\} dF(x)$$

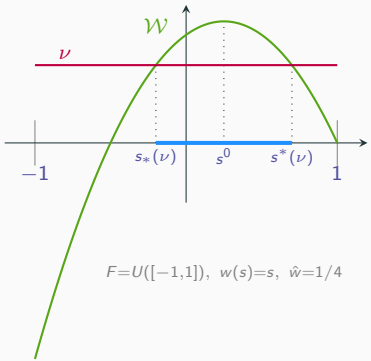


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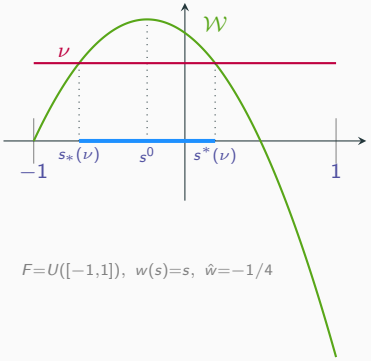
$$\mathcal{W}^+(s,\hat{w})(s) = \int_s^{\bar{s}} \{w(x) - \hat{w}\} dF(x)$$



$F{=}U([-1,1])$, $w(s){=}s$, $\hat{w}{=}1/4$

High priority group: $\overline{w} \geq \hat{w}$

$$\mathcal{W}^-(s,\hat{w})(s) = -\int_{\underline{s}}^{s} \{w(x) - \hat{w}\} dF(x)$$

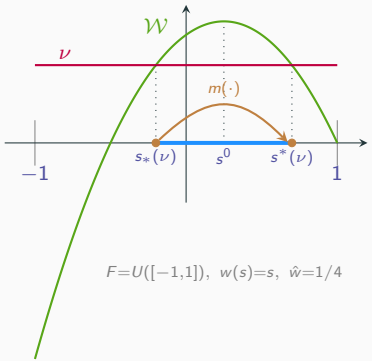


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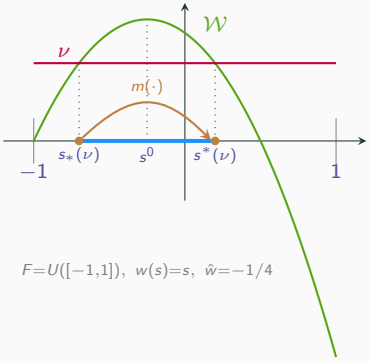
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$F{=}U([-1,1])$, $w(s){=}s$, $\hat{w}{=}-1/4$

Reduced program

$$\max_{\alpha} \int_{s_*}^{s^*} \alpha(s)\{w(s) - \hat{w}\}dF(s)$$

$$\text{s.t. } \alpha(t) - \alpha(s) \leq \frac{1}{\bar{\gamma}\bar{v}} c(t|s) \quad \forall\ s_* \leq s < t \leq s^* \qquad \text{(FPIC)}$$

or

$$\max_{\alpha} \int_{s_*}^{s^*} \alpha'(s)\mathcal{W}(s, \hat{w})(s)dF(s)$$

$$\text{s.t. } \alpha(t) - \alpha(s) \leq \frac{1}{\bar{\gamma}\bar{v}} c(t|s) \quad \forall\ s_* \leq s < t \leq s^* \qquad \text{(FPIC)}$$

where $s_* \leq s^0 \leq s^*$ and $\mathcal{W}(s_*) = \mathcal{W}(s^*, \hat{w}) \geq 0$.

Solving the reduced program

To construct an optimal allocation rule from the program reduced to an interval $[s_*, s^*]$ proceed as follows:

- **Step 1.** Find a solution α of the above reduced program (either in its original or differential form). Note that such a solution is always determined up to addition of a constant.
- **Step 2.** Choose the pair s_*, s^* so that either (i) $\alpha(s^*) - \alpha(s_*) = 1$, or (ii) $s_* = s_*(0)$.
- **Step 3.** Set the additive constant so that $\alpha(s_*) = 0$ in a low priority group, or $\alpha(s^*) = 1$ in a high priority group.
- **Step 4.** Complete the allocation rule by setting $\alpha(s) = \alpha(s_*)$ for all $s < s_*$, and $\alpha(s) = \alpha(s^*)$ for all $s > s^*$.

Cost assumptions

Upward Increasing Differences

$$\forall s < s' \leq t < t', \quad c(t'|s) - c(t|s) \leq c(t'|s') - c(t|s') \tag{UID}$$

Example: $c(t|s) = \Gamma(|t - s|)$, Γ concave: increasing returns to scale of fraud.

Upward Decreasing Differences

$$\forall s < s' \leq t < t', \quad c(t'|s) - c(t|s) \geq c(t'|s') - c(t|s') \tag{UDD}$$

Example: $c(t|s) = \Gamma(|t - s|)$, Γ convex: decreasing returns to scale of fraud.

Cost assumptions

Upward Increasing Differences

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Example: $c(t|s) = \Gamma(|t-s|)$, Γ concave: increasing returns to scale of fraud.

FPIC bind for far apart scores $\rightarrow$ new method based on optimal transport.

Upward Decreasing Differences

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Example: $c(t|s) = \Gamma(|t-s|)$, Γ convex: decreasing returns to scale of fraud.

FPIC bind locally $\rightarrow$ first-order approach.

Optimal transport approach: UID

$$\max_{\alpha} \int_{s_*}^{s^0} \alpha(s)\{w(s) - \hat{w}\}dF(s) + \int_{s^0}^{s^*} \alpha(t)\{w(t) - \hat{w}\}dF(t)$$

$$\text{s.t.} \quad \alpha(t) - \alpha(s) \leq \frac{1}{\bar{\gamma}\bar{\nu}} c(t|s), \quad \forall s_* \leq s < t \leq s^* \qquad \text{(FPIC)}$$

Optimal transport approach: UID

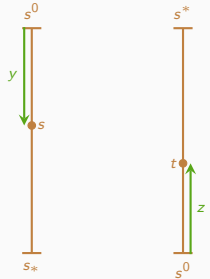
$$\max_{\alpha} \int_{s_*}^{s^0} \alpha(s)\{w(s) - \hat{w}\}dF(s) + \int_{s^0}^{s^*} \alpha(t)\{w(t) - \hat{w}\}dF(t)$$

$$\text{s.t.} \quad \alpha(t) - \alpha(s) \leq \frac{1}{\bar{\gamma}\bar{v}} c(t|s), \quad \forall s_* \leq s \leq s^0 \leq t \leq s^* \qquad \text{(FPIC)}$$

Optimal transport approach: UID

$$\max_{\alpha} \int_{s_*}^{s^0} \alpha(s)\{w(s) - \hat{w}\} dF(s) + \int_{s^0}^{s^*} \alpha(t)\{w(t) - \hat{w}\} dF(t)$$

$$\text{s.t.} \quad \alpha(t) - \alpha(s) \leq \frac{1}{\bar{\gamma}\bar{v}} c(t|s), \quad \forall s_* \leq s \leq s^0 \leq t \leq s^* \qquad \text{(FPIC)}$$



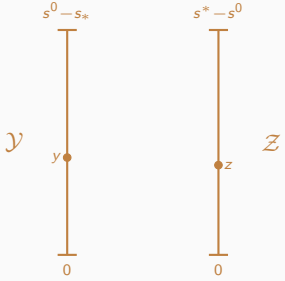
Optimal transport approach: UID

$$\max_{\alpha} \int_{s_*}^{s^0} \alpha(s)\{w(s)-\hat{w}\}dF(s) + \int_{s^0}^{s^*} \alpha(t)\{w(t)-\hat{w}\}dF(t)$$

$$\text{s.t. } \alpha(t)-\alpha(s) \le \frac{1}{\bar{\gamma}\bar{v}}c(t|s), \quad \forall s_* \le s \le s^0 \le t \le s^* \qquad \text{(FPIC)}$$

$P(y)=\frac{\mathcal{W}(\hat{s},\hat{w})-\mathcal{W}(\hat{s}-y,\hat{w})}{\mathcal{W}(\hat{s},\hat{w})-\mathcal{W}(s_*,\hat{w})}$

$\phi(y)=\alpha(s^0-y)$



$Q(z)=\frac{\mathcal{W}(\hat{s},\hat{w})-\mathcal{W}(\hat{s}+z,\hat{w})}{\mathcal{W}(\hat{s},\hat{w})-\mathcal{W}(s^*,\hat{w})}$

$\psi(z)=\alpha(s^0+z)$

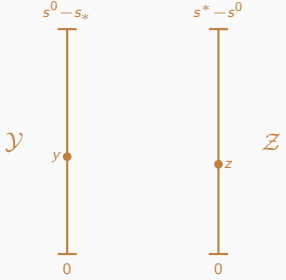
Optimal transport approach: UID

$$\max_{\phi,\psi} \int_{\mathcal{Y}} \phi(y)dP(y) + \int_{\mathcal{Z}} \psi(z)dQ(z)$$

$$\text{s.t. } \psi(z) - \phi(y) \leq \frac{1}{\bar{\gamma}\bar{v}} c(s^0 + z | s^0 - y), \quad \forall y, z \qquad \text{(FPIC)}$$

$P(y) = \frac{\mathcal{W}(\hat{s},\hat{w}) - \mathcal{W}(\hat{s}-y,\hat{w})}{\mathcal{W}(\hat{s},\hat{w}) - \mathcal{W}(s_*,\hat{w})}$

$\phi(y) = \alpha(s^0 - y)$



$Q(z) = \frac{\mathcal{W}(\hat{s},\hat{w}) - \mathcal{W}(\hat{s}+z,\hat{w})}{\mathcal{W}(\hat{s},\hat{w}) - \mathcal{W}(s^*,\hat{w})}$

$\psi(z) = \alpha(s^0 + z)$

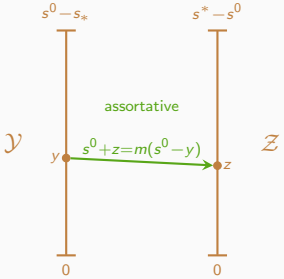
Optimal transport approach: UID

Dual of the Monge-Kantorovich optimal transport problem:

$$\min_{\xi \in \mathcal{M}(P,Q)} \frac{1}{\bar{\gamma}\bar{v}} \int_{\mathcal{Y}\times\mathcal{Z}} \underbrace{c(s^0 + z | s^0 - y)}_{\text{submodular}} \, d\xi(y,z)$$

$$P(y) = \frac{\mathcal{W}(\hat{s},\hat{w}) - \mathcal{W}(\hat{s}-y,\hat{w})}{\mathcal{W}(\hat{s},\hat{w}) - \mathcal{W}(s_*,\hat{w})}$$

$$\phi(y) = \alpha(s^0 - y)$$



$$Q(z) = \frac{\mathcal{W}(\hat{s},\hat{w}) - \mathcal{W}(\hat{s}+z,\hat{w})}{\mathcal{W}(\hat{s},\hat{w}) - \mathcal{W}(s^*,\hat{w})}$$

$$\psi(z) = \alpha(s^0 + z)$$

Optimal allocation rule for UID

$$\alpha^*_{uid} = \begin{cases} 0 & \text{if } s < s_* \\ A\,\mathbb{1}_{\tilde{w}>\hat{w}} - \frac{1}{\bar{\gamma}\bar{v}} \int_{s_*}^{s} c_s\big(m(x)|x\big)\,dx & \text{if } s \in [s_*, s^0] \\ 1 - A\,\mathbb{1}_{\tilde{w}<\hat{w}} - \frac{1}{\bar{\gamma}\bar{v}} \int_{s}^{s^*} c_t\big(x|m^{-1}(x)\big)\,dx & \text{if } s \in [s^0, s^*] \\ 1 & \text{if } s > s^* \end{cases},$$

where $s^* = m(s_*)$,

$$s_* = \min\big\{s \in [s_*(0), s^0] : c\big(m(s)|s\big) \leq \bar{\gamma}\bar{v}\big\}, \tag{1}$$

and

$$A = 1 - \frac{1}{\bar{\gamma}\bar{v}} c(s^*|s_*). \tag{2}$$

Theorem

If the cost function satisfies UID, α^*_{uid} is an optimal allocation rule for the within group problem.

Optimal allocation rule for UID

Low priority group: $\overline{w} < \hat{w}$

- Scores in $[\underline{s}, s_*(0)]$ do not get any object.
- Highest score might not get it for sure if $\bar{\gamma}\bar{v}$ high.
- planner's payoff is decreasing in $\bar{\gamma}\bar{v}$.
- High $\bar{\gamma}\bar{v}$: agent's payoff is decreasing in $\bar{\gamma}\bar{v}$.
- Low $\bar{\gamma}\bar{v}$: decreasing for high scores, increasing for low scores.

High priority group: $\overline{w} \geq \hat{w}$

- Scores in $[s^*(0), \bar{s}]$ get an object for sure.
- Lowest score might get it if $\bar{\gamma}\bar{v}$ high.
- planner's payoff is decreasing in $\bar{\gamma}\bar{v}$.
- High $\bar{\gamma}\bar{v}$: agent's payoff is increasing in $\bar{\gamma}\bar{v}$.
- Low $\bar{\gamma}\bar{v}$: decreasing for high scores, increasing for low scores.

First-order approach: UDD

$$\max_{\alpha} \int_{s_*}^{s^*} \alpha'(s)\mathcal{W}(s,\hat{w})dF(s)$$

$$\text{s.t.} \quad s \in \arg\max_{t} \alpha(t) - c(t|s) \quad \forall s \in [s_*, s^*] \qquad \text{(FPIC)}$$

First-order approach: UDD

$$\max_{\alpha} \int_{s_*}^{s^*} \alpha'(s)\mathcal{W}(s,\hat{w})dF(s)$$

$$\text{s.t. } \alpha'(s) \leq c_{t^+}(s|s) \qquad \text{(FPICFOC)}$$

First-order approach: UDD

$$\max_{\alpha} \int_{s_*}^{s^*} \alpha'(s)\mathcal{W}(s,\hat{w})dF(s)$$

s.t.

$$\Rightarrow \alpha'(s) \leq c_{t^+}(s|s) \text{ on } [s_*, s^*]$$

Optimal allocation rule for UDD

$$\alpha^*_{udd}(s) = \begin{cases} 0 & \text{if } s < s_* \\ B\,\mathbb{1}_{\bar{w}>\hat{w}} + \frac{1}{\bar{\gamma}\bar{v}} \int_{s_*}^{s} c_{t^+}(x|x)dx & \text{if } s \in [s_*, s^*] \\ 1 & \text{if } s > s^* \end{cases},$$

where $s^* = m(s_*)$,

$$s_* = \min\{s \in [s_*(0), s^0] : \int_s^{m(s)} c_{t^+}(x|x)dx \leq \bar{\gamma}\bar{v}\}, \tag{3}$$

and

$$B = 1 - \frac{1}{\bar{\gamma}\bar{v}} \int_{s_*}^{s^*} c_{t^+}(x|x)dx. \tag{4}$$

Theorem

If the cost function satisfies UDD, α^*_{udd} is an optimal allocation rule for the within group problem.

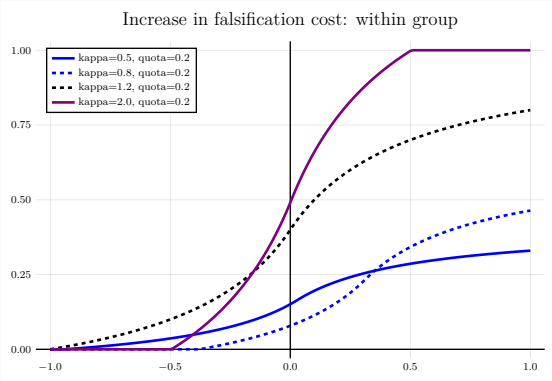


Figure 1: Cost $\kappa c(t|s) = \kappa|t-s|/(1+|t-s|)$ if $t \geq s$, and $F_i = U(-1,1)$, m=0.4

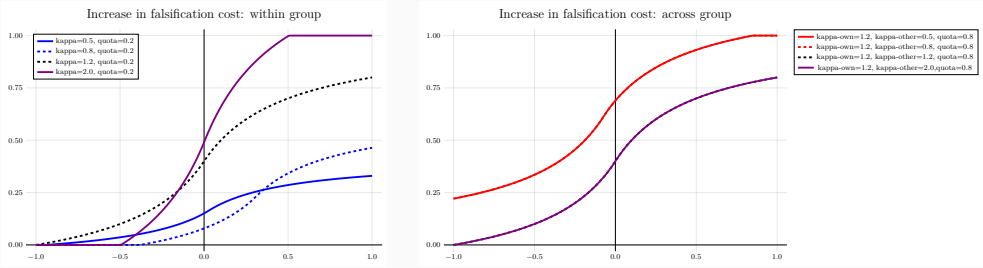


Figure 2: Cost $\kappa c(t|s) = \kappa|t-s|/(1+|t-s|)$ if $t \geq s$, and $F_i = U(-1,1)$, m=0.4

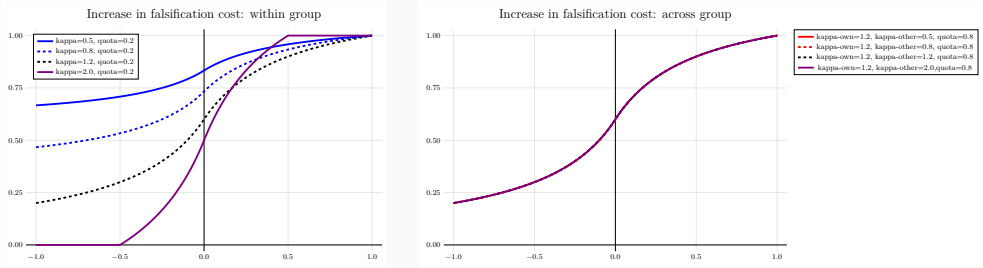
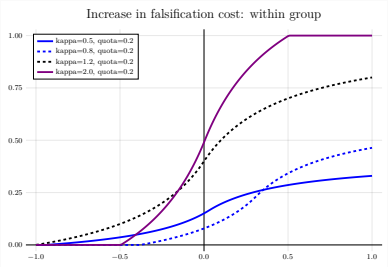


Figure 3: Cost $\kappa c(t|s) = \kappa|t-s|/(1+|t-s|)$ if $t \geq s$, and $F_i = U(-1,1)$, m=1



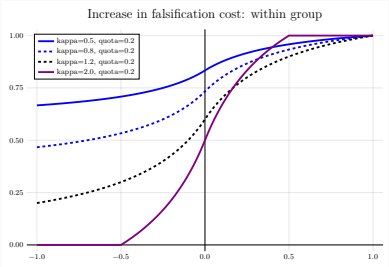


Figure 4: Cost $\kappa c(t|s) = \kappa|t-s|/(1+|t-s|)$ if $t \geq s$, and $F_i = U(-1,1)$, m=0.4 versus m=1

Literature

Mechanism design; design of allocation mechanisms:
Myerson (1981), Akbarpour et al. (2020).

This paper: no transfers, costly state falsification

Mechanism design, allocation mechanisms without transfers:
Ben-Porath, Dekel and Lipman (2014), Mylovanov and Zapechelnyuk (2017), Kattwinkel (2020), Condorelli (2013).

This paper: exploit costly falsification

Literature

Mechanism design with costly misreporting / state falsification:
Deneckere and Severinov (2017), Kephart and Conitzer (2016), Lacker and Weinberg (1989), Landier and Plantin (2016), Severinov and Tam (2019)

This paper: no transfers

Lying Costs; communication under lying costs:
Abeler, Nosenzo and Raymond (2019), Gneezy, Kajackaite and Sobel (2020), Kartik (2009), Kartik, Ottaviani and Squintani (2009), Sobel (2020) This paper: mechanism design without transfers

Thanks

1. References

Akbarpour, Mohammad, Piotr Dworczak, and Scott Duke Kominers (2020): "Redistributive allocation mechanisms," *Available at SSRN 3609182*.

Myerson, R. B. (1982): "Optimal Coordination Mechanisms in Generalized Principal-Agent Problems," *Journal of Mathematical Economics*, 10, 67–81.