Market opacity and fragility*

Giovanni Cespa† and Xavier Vives‡

July 2022

Abstract

We show that, consistent with empirical evidence, access to order flow information allows traders to supply liquidity via marketable orders. An informational friction resulting from lack of market transparency can, however, make liquidity demand upward sloping, inducing strategic complementarities: traders demand more liquidity when the market becomes less liquid, fostering market illiquidity. This can generate instability with an initial dearth of liquidity degenerating into a liquidity rout (as in a flash crash), an event that is more likely to occur when market opacity hampers liquidity supply via marketable orders. Our theory also predicts that, when the market is fragile, traders faced with the largest price impact are those consuming more liquidity at equilibrium.

Keywords: Liquidity fragility, flash crash, market information.
JEL Classification Numbers: G10, G12, G14

*This is a fully revised version of a previous working paper of ours under the title "Market transparency and fragility".

†Bayes Business School, City, University of London, and CEPR. 106, Bunhill Row, London EC1Y 8TZ, UK. e-mail: giovanni.cespa@gmail.com

‡IESE Business School, Avinguda Pearson, 21 08034 Barcelona, Spain. e-mail: xvives@iese.edu

Introduction

Concern for crashes has recently revived, in the wake of the sizeable number of "flash events" that have affected different markets.[1] However, episodes of extreme market turbulence, where liquidity seems to inexplicably disappear have also occurred in the past. As the stock market crash of October 19, 1987 makes clear, (apparently) fundamentals-unrelated crashes have been a worrying, regular feature of financial markets.[2]

A unifying characteristic of these episodes seems to be the jamming of the "rationing" function of market illiquidity. In normal market conditions, traders perceive a lack of liquidity as a cost, while arbitrageurs and liquidity suppliers regard it as an opportunity. This, in turn, leads the former to limit their demand for immediacy, and the latter to increase their supply of liquidity (i.e., the demand for and supply of liquidity, are respectively decreasing and increasing in the illiquidity of the market). In normal conditions, then, an illiquidity hike leads traders to contain their demand, producing a stabilizing effect on the market. However, on occasions, a bout of illiquidity, which can hardly be construed as fundamentals-driven, has a destabilizing impact, and fosters a disorderly "run for the exit" that is conducive to a rout. In these cases, traders attempt to place orders *despite* the liquidity shortage, and arbitrageurs flee the market, foregoing profitable (but risky) opportunities. In such conditions, liquidity is fragile. What can account for such a dualistic feature of market illiquidity?

In this paper, we argue that lack of transparency about relevant market conditions is an important ingredient in the answer to this question. In current markets, trading automation arguably creates informational frictions by hampering some traders' access to reliable and timely market information (Ding et al. (2014)).[3] In less automated markets, impaired access to market information arose because of different reasons. For example, in the 80s, access to the NYSE trading floor was crucial to have a good view of market conditions, but obviously constrained by physical limitations. Importantly, such frictions seem to have a bearing on episodes of liquidity crashes. Several accounts of the August 24, 2015 "flash-crash," point to the fact that uncertainty over the price of ETF constituents contributed to a huge investors' sellout, and sidelined the actions of arbitrageurs, exacerbating the liquidity dry-up in some ETFs.[4] In a

[1] Starting with the May 6, 2010 U.S. "flash-crash" where the Dow Jones Industrial Average dropped by 9% in the middle of the trading day, and partially recovered by the end of trading; moving to the October 15, 2014 Treasury Bond crash, where the yield on the benchmark 10-year U.S. government bond, dipped 33 basis points to 1.86% and reversed to 2.13% by the end of the trading day; to end with the August 25, 2015 ETF market freeze, during which more than a fifth of all U.S.-listed exchange traded funds and products were forced to stop trading. More evidence of flash events is provided by NANEX and Bank of International Settlements (2017).

[2] See https://en.wikipedia.org/wiki/List_of_stock_market_crashes_and_bear_markets and also Ian Domowitz's "Will the real market failure please stand up?" for an account of a 1962 flash-crash forerunner.

[3] Ding et al. (2014) argue that in the U.S. "[n]ot all market participants have equal access to trade and quote information. Both physical proximity to the exchange and the technology of the trading system contribute to the latency." In the EU the situation is possibly even worse, as testified by the lack of a consolidated tape in a market environment displaying an even higher degree of market fragmentation than in the US (see, e.g.European Commission progress update on action 14 of the capital markets union 2020 action plan. Action 14: Consolidated tape.).

[4] In the morning of August 24, 2015, the Dow dropped roughly 1,100 points in the first five minutes of

somewhat similar fashion, Amihud et al. (1990), in their analysis of the 1987 "Black Monday," argue that a number of operational issues affected the opening trade session on the day of the event "[O]rders could not be executed, and information on market conditions, and on order execution was delayed." This impaired the ability of traders outside of the market to provide liquidity, restricting total liquidity supply.

We use a stylized model of liquidity provision to show that the absence of reliable market information about *liquidity traders' demand* can seriously dent the risk bearing capacity of a market, to the extent that, in extreme conditions, it can cause a market crash. While, consistent with empirical evidence,[5] access to order flow information allows traders to supply liquidity via marketable orders, an informational friction due to market opacity can block this mechanism and make the market fragile.

More in detail, we analyse a two-period (trading rounds) model of a market in which a risky security is traded by liquidity providers and traders who hedge an endowment shock. Liquidity demand comes from two cohorts of risk-averse liquidity traders who submit market orders. The first cohort observes its endowment shock exposure to a non-tradable good (whose value is perfectly correlated with that of the risky security) prior to the first period and trades at both rounds. The second cohort enters the market at the second period, observes its endowment shock exposure, which is independent from the first period traders' one, possibly a signal about the first period order imbalance (which reflects that period endowment shock), and trades. Liquidity supply is offered by a continuum of risk-averse dealers who who post limit orders at both rounds and are thus able to efficiently rebalance their risk exposure.[6]

Being in the market at both rounds, first period traders split their hedging (liquidity demand) across periods: when they receive a positive (negative) endowment shock, they sell (buy) the risky security in both periods. With full transparency, second period traders perfectly observe the first period imbalance (endowment shock), and take a contrarian position against first period liquidity traders' second period order–in this way de-facto providing liquidity to them. In this case we show that traders' demand for liquidity is a decreasing function of the price impact it induces–that is, *higher illiquidity discourages liquidity demand* and illiquidity works as a *rationing* device. Additionally, a unique equilibrium obtains. Along this equilibrium, we show that at the first round, dealers supply liquidity and also speculate on the anticipated im-

trading, and trading in several stocks was halted due to unusual market turbulence. The ensuing lack of reliable price information allowed profitable, but risky, arbitrage opportunities to go unexploited, leading to a widening of spreads and a thinning of market depth. For example, during the event, the spread between the SPDR S&P500 (SPY) and the Guggenheim S&P 500 Equal Weight ETF (RSP), two very similar ETFs whose prices are normally in sync, at one point reached $21 (see What The E-T-F Happened On August 24?). In a similar vein, in their account of the May 10, 2010 "Flash Crash" Easley et al. (2011) state: "This generalized severe mismatch in liquidity was exacerbated by the withdrawal of liquidity by some electronic market makers and by uncertainty about, or delays in, market data affecting the actions of market participants."

[5]Several authors find that liquidity is provided by (contrarian) market orders both at high trading frequencies (Brogaard et al. (2014) and Biais et al. (2017)) and at lower frequencies (Biais et al. (2017), Anand et al. (2021), Anand et al. (2013)).

[6]In a related paper (Cespa and Vives (2019)), we study the case in which first period liquidity traders have a short-term trading horizon, obtaining qualitatively similar results.

pact of first period traders' order at the second round. Indeed, due to their ability to be in the market in both periods, dealers also demand liquidity by trading in the same direction as first period liquidity traders, thus exploiting their predictability. At the second round, dealers absorb the orders of both cohorts of liquidity traders. First period liquidity traders' split liquidity demand is also responsible for the positive return autocovariance that obtains at equilibrium. That is, in our model, returns are positively autocovariant in the *absence* of any fundamentals information.

A deterioration of second period traders' information (about the first period order imbalance) impairs these traders' ability to supply liquidity via contrarian orders. This reduces the risk-bearing capacity of the market and can increase market fragility. Specifically, we find that, for some plausible parameterizations, the model displays multiple equilibria with different levels of market depth. In this case, a larger price impact leads traders to demand more liquidity and *higher illiquidity incentivizes liquidity demand*.

The intuition is as follows. At the second round, the two cohorts of liquidity traders compete for the liquidity offered by dealers, inducing a *different* impact on the equilibrium price. This is because endowment shocks are independent across traders' cohorts, which makes the liquidity demands of each cohort and their associated effects on the price also independent. This, in turn and provided information on endowment shocks is *opaque*, may generate a self-sustaining demand loop. To see this, assume that the market impact of cohort 2 investors' demand increases. This mitigates these traders' expected return (as a larger impact moves more strongly the price against them when they demand liquidity) and increases the execution risk faced by investors in cohort 1 (as cohort 2 investors' demand affects the equilibrium price more strongly).[7] As a consequence, cohort 1 investors scale down their hedging activity, which reduces the price impact of their endowment shock (because liquidity providers absorb a smaller share of cohort 1's risk exposure). If the investors in cohort 2 *cannot* observe cohort 1's endowment shock, their execution risk depends positively on the cohort 1's price impact.[8] As such, a lower price impact associated with cohort 1's investors' demands *lowers* cohort 2 investors' execution risk. This potentially leads the latter to scale up their liquidity demand, boosting the price impact of their endowment shock (because liquidity providers absorb a larger share of cohort 2's risk exposure), which reinforces the initial spike. In sum, when the market is opaque, an increase in the price impact of cohort 2 liquidity traders' orders hikes the execution risk faced by the traders belonging to cohort 1. This lowers (increases) the liquidity demand and consumption of the latter (former). Thus, the initial second period illiquidity spike leads second period traders to demand more (rather than less) liquidity.

We show that when dealers' risk bearing capacity is small, liquidity traders need to trade

[7]The price impact of the endowment shock hitting investors in a given cohort, does not affect the uncertainty faced by the investors' cohort who hedges the shock, but that of the investors competing to consume liquidity, the more, the higher it is.

[8]To be sure: inability to observe the endowment shock forces traders to estimate the effect this has on the price, which adds to traders' 2 execution risk.

the most (because the dispersion of their endowment shock is large and they have a low risk tolerance) and the security's payoff volatility is large, if the market is fully opaque (second period traders have no information on the first period imbalance), the above described loop generates three equilibria, which can be ranked in terms of market liquidity. Indeed, in these conditions dealers cannot count on the additional risk sharing provided by liquidity traders' contrarian orders. When traders' demand for liquidity spikes, this widens the gap between liquidity demand and supply, making the market fragile. We also prove that only the extreme equilibria are stable and that trading costs for traders at the second round are heterogeneous.[9] At the two stable solutions of the model, first and second period traders' price impact (of endowment shocks) *and* their liquidity consumption are negatively correlated–liquidity provision and consumption work as a zero-sum game. Thus, a spike in liquidity consumption by second (first) period traders crowds out first (second) period traders' liquidity consumption.

Importantly, in this situation, illiquidity stops working as a rationing device of liquidity consumption. That is, at equilibrium the trader cohort facing the highest price for liquidity is also the one consuming more of it (hedging a larger proportion of the endowment shock). We show that, as long as the market is fully opaque, a change in the conditions that generate liquidity fragility (e.g., an increase in the risk-tolerance of liquidity traders or a reduction in the dispersion of their endowment shock), weakens strategic complementarities, leading to a unique equilibrium. However, in such equilibrium liquidity demand is still positively related to illiquidity. Thus, even when strategic complementarities are not strong enough to generate multiple equilibria, order flow opaqueness jams the rationing role of illiquidity.

In the last part of the paper, we consider the effect of two extensions to the baseline model. We first allow second period traders to observe a noisy signal about the first period endowment shock. In this context we show, by way of numerical simulations, that a low precision of such signal delivers equilibrium multiplicity, generalizing the results we obtain in the "fully opaque" case. We also show that an increase in the precision of such signal leads to a unique equilibrium in which second period traders' liquidity demand and price impact are higher than the ones of their first period peers. Thus, an increase in order flow transparency makes the market less fragile, but allows second period traders' liquidity demand to crowd out that of first period traders. In the second extension, we consider the case in which liquidity is also supplied by a mass of dealers who are in the market only at the first round, which we refer to as "standard" dealers.

Our paper is related to–and has implications for–four streams of the finance literature. First, it is related to the literature on liquidity fragility (see, e.g., Brunnermeier and Pedersen (2009)). Most of the contributions in this framework focus on the possibility that liquidity may evaporate because of self-sustaining loops that limit the ability of *liquidity providers* to meet customers' demand, be it because of funding problems (Brunnermeier and Pedersen (2009) and Gromb and Vayanos (2002)), lack of price information (Cespa and Foucault (2014)), or the

[9]Stability is with respect to the best response stability criterion.

effect of retrospective learning about the security's payoff (Cespa and Vives (2015)). In light of such effects, scholars have argued that regulation impairing access to capital for financial institutions may have a negative impact on the risk sharing capacity of the liquidity provision sector, precisely when this is needed the most (see, e.g. Bao et al. (2018)). However, accounts of market crashes often attribute the inception of these events to "aggressive" or "unusually large" liquidity demand realizations which are not met by a sufficiently responsive increase in liquidity supply.[10] In this paper we thus propose a theory in which liquidity fragility arises because of a self-sustaining loop affecting *liquidity demanders*, which exhausts liquidity suppliers' limited risk-bearing capacity. Indeed, in our model poor market information impairs second period traders' ability to speculate against the aggregate order imbalance, creating the loop which impairs risk sharing.[11] In view of the documented decline in quoted depth that has occurred over the past twenty years, this should reinforce regulatory concerns over the paucity of *public, affordable* order flow information in current markets.

Second, the paper is also related to the literature documenting liquidity provision via (contrarian) market orders. Several authors find this phenomenon at high trading frequencies (Brogaard et al. (2014) and Biais et al. (2017)). There is, however, evidence that it also occurs at lower frequencies (Biais et al. (2017)). Anand et al. (2021) provide evidence that far from contributing to market fragility, some corporate bond mutual funds actively supply liquidity during periods of market stress. A similar behavior is also found in equity mutual funds during the recent financial crisis (see Anand et al. (2013)). In this respect, our paper argues that informational impediments to liquidity provision via market orders can negatively affect risk sharing and make liquidity fragile.

Third, the paper is related to the early literature on price crashes. Gennotte and Leland (1990) provide a model tracing the 1987 stock market crash to traders not taking into account the possibility of portfolio insurance affecting the security demand. Jacklin et al. (1992) also analyse the crash-inducing effect of mis-estimating the actual magnitude of portfolio insurance in a dynamic model à la Glosten and Milgrom (1985). Madrigal and Scheinkman (1997) study a model in which traders have private fundamental information and together with noise traders post orders who are accommodated by market makers who act strategically to control the information flow implied by the security price. The authors show that, under some conditions, the need to control the information flow conveyed by prices leads to crashes. All of the above papers rely on some form of irrationality either due to the presence of noise trading, or to the fact that some rational traders are unaware of one component of the aggregate demand for the stock, to generate price discontinuities. In our model, as explained above, all traders are rational expected utility maximizers, and the crash occurs because of the self-sustaining loop triggered by traders' liquidity demand.

[10] For example, the CFTC-SEC report on the flash-crash attributes the inception of the crash to an aggressive E-mini S&P500 futures sell order initiated by a large mutual fund identified as Waddell & Reed (see CFTC and SEC (2010) and Aldrich et al. (2017)).

[11] For evidence of demand driven "commonality" in liquidity, see e.g. Karolyi et al. (2012).

Finally, the paper is related to the literature highlighting the impact of multi-dimensional fundamentals for price discovery and the equilibrium properties of the market (see, e.g., Subrahmanyam and Titman (1999), Cespa and Foucault (2014), Goldstein and Yang (2015), and Goldstein et al. (2021)). Differently from this literature, in this paper we assume that prices are driven by multiple, independent, non-fundamentals-driven shocks (i.e., the hedging demands of different liquidity traders' cohorts) and show that, when liquidity demand reacts to prices, this can have important consequences for market stability.

The rest of the paper is organized as follows. In the next section we present the model. In Section 2 we study the fully transparent benchmark, in which we assume that second period traders perfectly observe the endowment shock affecting their first period peers. In the following section we assume that such information is not available (the fully opaque case) and prove that this can generate multiple equilibria. In Section 5, we consider the general case in which liquidity is also supplied by a class of dealers that can only trade at the first round and second period traders observe an informative signal about the first period endowment shock. The final section contains concluding remarks. Most of the proofs are relegated to the Appendix.

1 The model

A single risky asset with liquidation value $v \sim N(0, \tau_v^{-1})$, and a risk-less asset with unit return are exchanged in a market during two periods (we interchangeably also use the expression "trading rounds"). Two classes of traders are in the market. First, a continuum of competitive, risk-averse dealers in the interval $[0, 1]$, active in both periods. Second, a unit mass of liquidity traders who enter the market at the first round and post their orders at round 1 and 2. In the second period, a new cohort of liquidity traders (of unit mass) enters the market and trades. The asset is liquidated in period 3. We now illustrate the preferences and orders of the different players.[12]

1.1 Liquidity providers

A dealer has CARA preferences with risk-tolerance γ, and submits price-contingent orders x_t^D, $t = 1, 2$, to maximize the expected utility of his final wealth: $W^D = (v - p_2)x_2^D + (p_2 - p_1)x_1^D$.[13]

1.2 Liquidity traders

The liquidity demand side of the model is represented by a unit mass of risk-averse traders who, prior to entering the market at time t, learn about the value of an endowment shock u_t

[12] In Section 5, we show that our results are qualitatively robust to a generalization of the model which includes a class of "Standard Dealers" who can only trade at the first round. A different version of the trading model with "standard" and "full" dealers is expounded in Cespa and Vives (2021).

[13] We assume, without loss of generality with CARA preferences, that the non-random endowment of dealers is zero. Also, as equilibrium strategies will be symmetric, we drop the subindex i.

in a non-tradable security that they will receive at the liquidation date. We assume that the non-tradable security's value is perfectly correlated with that of the risky security traded in the market. This assumption, which is common in the literature (see, e.g. Wang (1994), Vayanos and Wang (2012), and Llorente et al. (2002)), induces a hedging demand for the risky security.

More in detail, in the first period, a unit mass of CARA traders with risk-tolerance γ_L is in the market. Traders learn the value of the endowment shock u_1 and post a market order x_{t1}, at round $t \in \{1, 2\}$ to maximize the expected utility of their wealth $\pi_1 = u_1 v + (v - p_2)x_{21} + (p_2 - p_1)x_{11}$:

$$E\left[-\exp\{-\pi_1/\gamma_L\}|\Omega_1\right],$$

where $\Omega_1 \equiv \{u_1\}$ denotes their information set. In period 2, a new (unit) mass of CARA traders (with the same risk tolerance γ_L) enters the market, learns the realization of the non-tradable endowment shock u_2 that they will receive at $t = 3$, and observes a noisy signal of the previous period endowment shock $s_{u_1} = u_1 + \eta$. Second period traders submit a market order to maximize the expected utility of their wealth $\pi_2 = u_2 v + (v - p_2)x_2$:

$$E\left[-\exp\{-\pi_2/\gamma_L\}|\Omega_2\right],$$

where $\Omega_2 \equiv \{u_2, s_{u_1}\}$ denotes their information set.We assume $u_t \sim N(0, \tau_u^{-1})$, $\eta \sim N(0, \tau_\eta^{-1})$ and $\text{Cov}[u_t, v] = \text{Cov}[u_t, \eta] = \text{Cov}[u_1, u_2] = 0$, $t = 1, 2$.

For examples of the "non-tradable" security, one can think of a portfolio of assets that traders are unwilling to liquidate (or that are intrinsically illiquid). In view of the assumed correlation structure, protection against changes in the non-tradable value is then obtained by taking an offsetting position in the risky security. For instance, traders could be long in a portfolio of stocks that tracks the market, say a fund, and hedge by shorting a market-tracking ETF; alternatively, they could be long on a S&P500 ETF, like the SPY, and setup a hedge by trading the Emini (while the former trades from 6am to 8pm, including extended trading hours, the latter trades 24/7, thus allowing overnight hedging).[14]

To simplify notation, in the following we denote by $E_t^D[Y]$, and $\text{Var}_t^D[Y]$, the conditional expectation and variance that a dealer forms about the random variable Y, in period $t = 1, 2$. Similarly, $E_t[Y]$, and $\text{Var}_t[Y]$ denote the conditional expectation and variance that a period-t liquidity trader forms about Y.

1.3 Market clearing

We will restrict attention to equilibria in which prices are linear (affine) functions of the endowment shocks and the error term affecting second period traders' signal. With insight, they

[14]For an example involving SPY, see https://money.stackexchange.com/questions/54373/why-dont-spy-spx-and-the-e-mini-sp-500-track-perfectly-with-each-other, and http://tastytradenetwork.squarespace.com/tt/blog/equating-futures-to-etfs, and for other ETF related examples, see https://investorplace.com/2017/10/portfolio-hedge-fund-consider-etfs/.

will be of the form:

$$p_1 = -\Lambda_1 u_1 \tag{1a}$$

$$p_2 = -\Lambda_2 u_2 - \Lambda_{21} u_1 - \Lambda_{22} \eta, \tag{1b}$$

where Λ_1, Λ_2, Λ_{21}, Λ_{22} are coefficients which will be pinned down at equilibrium. The intuition for (1a) and (1b) is as follows. At equilibrium, dealers absorb the orders of first period traders:

$$x_1^D + x_{11} = 0. \tag{2}$$

Traders know u_1, while, at equilibrium, dealers infer it from the price, which justifies (1a).

Consider now the second period equilibrium condition. First period liquidity traders split their hedging needs by posting an order x_{21} together with second period traders. Additionally, dealers rebalance their position at the second round. Formally, from the second period market clearing equation we have

$$(x_2^D - x_1^D) + (x_{21} - x_{11}) + x_2 = 0 \iff x_2^D + x_{21} + x_2 = 0, \tag{3}$$

where the expression on the right hand side in (3) follows from using the first period market clearing condition (2). At equilibrium, dealers' and traders' strategies are a function of their information sets–$\{p_1, p_2\}$ for FDs and $\Omega_2 = \{u_2, s_{u_1}\}$ for second period traders. As a consequence, the price will load on $\{u_1, u_2, \eta\}$, justifying (1b). Figure 1 displays the timeline of the model.

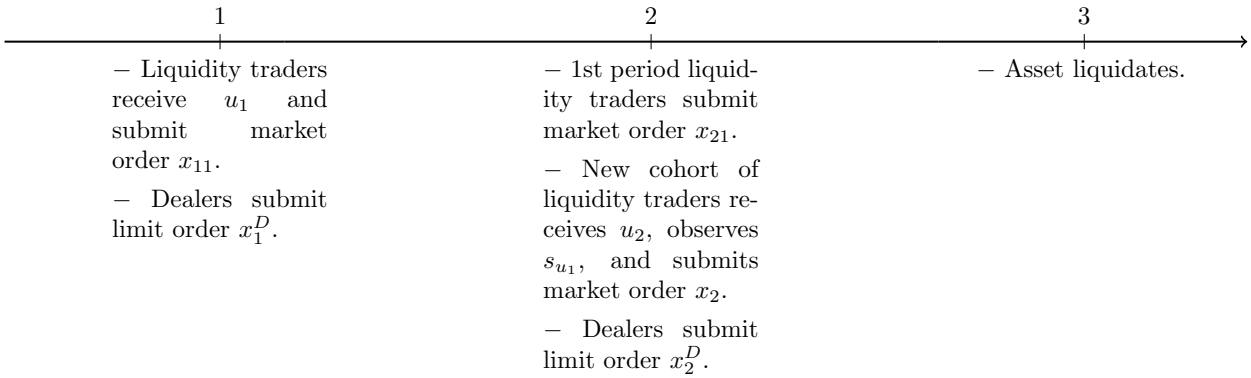


Figure 1: The timeline.

2 Fully transparent benchmark

We start the analysis by assuming that second period traders observe a perfectly informative signal of u_1 (i.e., $\tau_\eta \to \infty$). This assumption implies that the market is fully transparent and

has a direct impact on the second period equilibrium condition, since with a perfect signal, the information set of second period traders is given by $\Omega_2 = \{u_2, u_1\}$. Therefore, the second period price only reflects endowment shocks:

$$p_2 = -\Lambda_2 u_2 - \Lambda_{21} u_1, \tag{4}$$

while the first period price is as in (1a).

Due to the linearity assumption for prices, equilibrium strategies will also be linear. Specifically, we have $x_{11} = a_1 u_1$, $x_{21} = a_{21} u_1$, $x_2 = a_2 u_2 + b u_1$, where the coefficients a_1, a_{21} and a_2 denote the hedging or "liquidity" demand (per unit of endowment shock) of liquidity traders and the corresponding absolute values of such coefficients denote their hedging aggressiveness. The coefficient b denotes second period traders "speculative" aggressiveness. In the Appendix, we show that in this case the equilibrium is identified by the unique solution to a system of simultaneous equations in $\Lambda_1, \Lambda_{21}, \Lambda_2$. We thus obtain the following:

Proposition 1. *When the market is fully transparent, there exists a unique equilibrium in linear strategies $x_{11} = a_1 u_1$, $x_{21} = a_{21} u_1$, and $x_2 = a_2 u_2 + b u_1$. The equilibrium prices' coefficients are given by:*

$$\Lambda_2 = -\frac{1}{\gamma \tau_v} a_2 \tag{5a}$$

$$\Lambda_1 = -\frac{1}{\gamma \tau_v} \frac{\gamma + \gamma_L}{\gamma_L} a_1 \tag{5b}$$

$$\Lambda_{21} = -\frac{1}{\gamma \tau_v} \big(b + a_{21}\big). \tag{5c}$$

The coefficients of traders' strategies are as follows:

$$a_1 = -\gamma_L \frac{\Lambda_{21} - \Lambda_1}{\mathrm{Var}_1[p_2]}, \;\; a_{21} = \frac{\gamma_L \tau_v \Lambda_{21} - 1}{\tau_v \mathrm{Var}_1[v - p_2]}, \tag{6a}$$

$$a_2 = \frac{\gamma_L \tau_v \Lambda_2 - 1}{\tau_v \mathrm{Var}_2[v - p_2]} \in (-1, 0), \;\; b = \gamma_L \tau_v \Lambda_{21} > 0, \tag{6b}$$

where $-1 < a_{21} < a_1 < 0$, $\mathrm{Var}_1[p_2] = \Lambda_2^2 \tau_u^{-1}$, $\mathrm{Var}_1[v - p_2] = \Lambda_2^2 \tau_u^{-1} + \tau_v^{-1}$ and $\mathrm{Var}_2[v - p_2] = \tau_v^{-1}$. Furthermore, $0 < \Lambda_1 < \Lambda_{21} < \Lambda_2$ (explicit expressions for the price coefficients are in (A.4a), (A.6a) and (A.6b)).

According to (6a), first period liquidity traders demand liquidity by hedging part of their risk exposure at both trading rounds. Comparing their strategy coefficients yields: $a_{21} - a_1 < 0$. Hence, if $u_1 > 0$, they hedge their exposure shorting at the first round, and increasing their short position at the second round (that is, their second period *trade* is a sell). This is because, as the price impact of their order worsens across trading rounds ($\Lambda_{21} - \Lambda_1 > 0$), they anticipate a deterioration of the terms of trades at the second round (if $u_1 > 0$, a price decline), and start hedging to take advantage of the better market conditions at the first round.

The liquidity that accommodates such demand is offered by dealers. In the Appendix (see (A.45)), we show that a dealer's strategy is given by:

$$x_1^D = -\gamma \frac{\Lambda_{21} - \Lambda_1}{\text{Var}_1[p_2]} u_1 - \gamma \tau_v p_1. \tag{7}$$

According to the above expression, a dealer's strategy reflects two trading motives: liquidity supply (captured by the price dependent component in (7), $-\gamma\tau_v p_1$), and short-term return speculation (captured by the "market order" component $-u_1\gamma(\Lambda_{21} - \Lambda_1)/\text{Var}_1[p_2]$). To see this, note that at the second round FD in aggregate hold (see (A.30))

$$\begin{aligned} x_2^D &= -\gamma\tau_v p_2, \\ &= \gamma\tau_v \Lambda_{21} u_1 + \gamma\tau_v \Lambda_2 u_2, \end{aligned} \tag{8}$$

where the expression at the second line in (8), originates from substituting (1b) in dealers' second period aggregate position. Expression (8) implies that at the second round dealers hold $\gamma\tau_v\Lambda_{21}$ of the first period endowment shock. Based on (7), at the first round their position is given by

$$x_1^D = \gamma \left(\tau_u \frac{\Lambda_1 - \Lambda_{21}}{\Lambda_2^2} + \tau_v \Lambda_1 \right) u_1.$$

Hence, keeping the assumption $u_1 > 0$, at the first round dealers provide liquidity by absorbing part of first period traders' endowment shock ($\Lambda_1 > 0$). Additionally, they *consume* liquidity by taking a short position in the risky security ($\Lambda_1 - \Lambda_{21} < 0$).

At the second round, based on what said above, they provide liquidity to the additional sell order of first period traders: their trade with respect to the latter is given by

$$\gamma\tau_v\Lambda_{21}u_1 - x_1^D = \gamma \frac{\tau_u + \Lambda_2^2\tau_v}{\Lambda_2^2}(\Lambda_{21} - \Lambda_1)u_1,$$

i.e., a buy order. Thus, because of their ability to anticipate returns, dealers gain from short term speculation at the first round (selling at a higher price at the first round and buying back at a lower price at the second round).[15] At the second round, their activity is instead limited to liquidity provision (see (8)).

At the second round, based on (6b), liquidity traders hedge their risk exposure ($a_2 \in (-1, 0)$). Additionally, because of their ability to perfectly infer the direction of the demand pressure due to first period traders' second round trade, they also post a contrarian market order ($b > 0$), which provides additional risk-sharing.

Corollary 1. *When the market is transparent, second period liquidity traders supply liquidity*

[15] This is akin to "order anticipation" which, according to SEC (2010), occurs when "... a proprietary firm seeks to ascertain the existence of one or more large buyers (sellers) in the market and to buy (sell) ahead of the large orders with the goal of capturing a price movement in the direction of the large trading interest (a price rise for buyers and a price decline for sellers)."

by posting a contrarian market order with aggressiveness $b > 0$ (see (6b)).

In our setup, trading occurs because liquidity traders are exposed to a non-tradable endowment shock which induces a hedging demand. Due to risk aversion, dealers have a limited capacity to bear risk. This implies the following

Corollary 2. *The price coefficients in (5a)–(5c) capture the risk-tolerance weighted risk compensation liquidity providers require to absorb the aggregate liquidity demand.*

To see this note that at the first round a_1 reflects the marginal position of liquidity traders, that is their "liquidity demand":

$$a_1 = \frac{\partial x_{11}}{\partial u_1} = -\gamma_L \frac{\Lambda_{21} - \Lambda_1}{\mathrm{Var}_1[p_2]}. \tag{9}$$

As observed above, dealers also demand liquidity, since they speculate on the price impact of u_1 and their aggregate liquidity demand is given by

$$-\gamma \frac{\Lambda_{21} - \Lambda_1}{\mathrm{Var}_1[p_2]} = \gamma \frac{a_1}{\gamma_L}.$$

Aggregating across liquidity traders and dealers demands yields the aggregate liquidity demand at the first round:

$$a_1 + \gamma \frac{a_1}{\gamma_L} = \frac{\gamma + \gamma_L}{\gamma_L} a_1.$$

At equilibrium, replacing dealers and liquidity traders' equilibrium strategies (respectively, (7) and the first in (6a)) in the first period equilibrium condition (2), we have:

$$\begin{aligned} x_1^D + x_{11} = 0 &\iff \gamma \frac{a_1}{\gamma_L} u_1 - \gamma \tau_v p_1 + a_1 u_1 = 0 \\ &\iff \frac{\gamma + \gamma_L}{\gamma_L} a_1 u_1 = \gamma \tau_v p_1 \end{aligned} \tag{10}$$

At a linear equilibrium the price is proportional to the aggregate endowment shock u_1: $p_1 = -\Lambda_1 u_1$. Identifying $-\Lambda_1$ in the latter expression yields:

$$\underbrace{\frac{1}{\gamma \tau_v} \frac{\gamma + \gamma_L}{\gamma_L} a_1}_{-\Lambda_1} u_1 = p_1. \tag{11}$$

Thus, at the first round we have the following expression for illiquidity:

$$\Lambda_1 = -\frac{1}{\gamma \tau_v} \frac{\gamma + \gamma_L}{\gamma_L} a_1, \tag{12}$$

which captures the risk-weighted compensation that liquidity suppliers demand to absorb the aggregate marginal position of liquidity traders and dealers (the aggregate "liquidity demand").

At the second round, liquidity demand comes from first and second period traders:

$$a_{21} = \frac{\gamma_L \tau_v \Lambda_{21} - 1}{\tau_v \text{Var}_1[v - p_2]}, a_2 = \frac{\gamma_L \tau_v \Lambda_2 - 1}{\tau_v \text{Var}_2[v - p_2]}, \tag{13}$$

We can interpret the expressions for a_{21} and a_2 in the following way. A liquidity trader hedges a larger fraction of his shock (demands more liquidity), the lower is the impact the endowment shock has on p_2 (as a larger price impact reduces a trader's expected return from hedging), and the lower is the return uncertainty he faces (as a higher return variance dents his utility since he is risk averse). Consider now the second period market clearing condition:

$$\begin{aligned}
(x_2^D - x_1^D) + x_{21} - x_{11} + x_2 = 0 &\iff x_2^D + x_{21} + x_2 = 0 \\
&\iff -\gamma \tau_v p_2 + a_2 u_2 + (a_{21} + b) u_1 = 0 \\
&\iff p_2 = \underbrace{\frac{a_2}{\gamma \tau_v}}_{= -\Lambda_2} u_2 + \underbrace{\frac{a_{21} + b}{\gamma \tau_v}}_{= -\Lambda_{21}} u_1.
\end{aligned} \tag{14}$$

At the second line of the above expression we make use of the first period market clearing equation: $x_1^D + x_{11} = 0$. We then replace strategies with their equilibrium expressions and finally solve for p_2, identifying the price coefficients.

Similarly to Λ_1, the coefficients Λ_2 and Λ_{21} reflect the risk-weighted compensation that liquidity suppliers demand to absorb first and second period liquidity traders' aggregate demand. To understand the numerator of Λ_{21}, note that first period liquidity traders' demand at the second round (i.e., the marginal position a_{21}), is not absorbed by dealers in its entirety. Indeed, at the second round part of first period liquidity traders' endowment shock exposure is absorbed by second period traders' speculation (b).

2.1 Liquidity demand and supply in a transparent market

In this section we focus on the behavior of liquidity demand and supply in the fully transparent benchmark. Using the expressions in Proposition 1, liquidity demand and supply functions are as follows:

$$a_2 = \gamma_L \tau_v \Lambda_2 - 1, \ \Lambda_2 = -\frac{a_2}{\gamma \tau_v} \tag{15a}$$

$$a_{21} = \frac{(\gamma + \gamma_L)^2 (\gamma_L \tau_v \Lambda_{21} - 1) \tau_u \tau_v}{1 + (\gamma + \gamma_L)^2 \tau_u \tau_v}, \ \Lambda_{21} = -\frac{a_{21}}{(\gamma + \gamma_L) \tau_v} \tag{15b}$$

$$a_1 = -\gamma_L \tau_u (\gamma + \gamma_L)^2 \tau_u \tau_v^2 (\Lambda_{21} - \Lambda_1), \ \Lambda_1 = -\frac{\gamma + \gamma_L}{\gamma \gamma_L \tau_v} a_1, \tag{15c}$$

where the expressions for Λ_1 and Λ_{21} can be found in the Appendix (see, respectively (A.6a), and (A.6b)). Inspection of the above expressions shows that:

Corollary 3. *When the market is transparent, liquidity demand is decreasing in the price impact it induces and liquidity supply increases in traders' liquidity demand.*

Therefore, in a transparent market, price impact works as a rationing device: the pricier liquidity becomes, the less traders choose to hedge. Conversely, an increase in traders' liquidity demand prompts liquidity providers to make the market less liquid (i.e., make liquidity pricier). In Figure 2 we plot the liquidity supply and demand functions (respectively, in blue and green) for the two traders' cohorts at the second round. The unique equilibrium corresponds to the crossing point between the two curves.

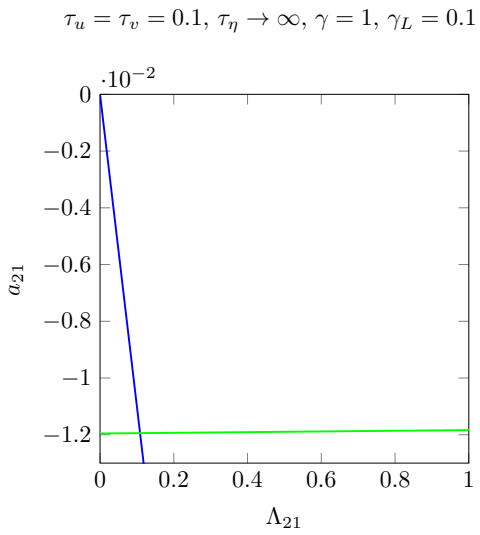


Figure 2: Liquidity demand and supply at the second round with fully transparent market.

Summarizing, when the market is transparent, liquidity demand decreases in price impact coefficients and price impact coefficients increase in liquidity demand, and on the risk-tolerance weighted payoff risk borne by liquidity providers. In these conditions, a unique equilibrium obtains. In this equilibrium dealers speculate on short-term returns and second period liquidity traders hedge their risk exposure and provide liquidity via contrarian market orders, sharing with dealers the risk exposure of first period traders.

3 Liquidity trading and noise trading with persistence

In our model, due to their trading horizon, first period liquidity traders hedge the endowment shock at both trading rounds. This has two implications. First, the equilibrium that obtains can be interpreted as one that arises when noise trading is modeled as an AR(1) process (see, e.g. He and Wang (1995), and Cespa and Vives (2015)). Indeed, we can write the equilibrium prices as follows

$$p_1 = -\Lambda_1 \theta_1, \; p_2 = -\Lambda_2 \theta_2,$$

where

$$\theta_1 \equiv u_1,\ \theta_2 \equiv u_2 + \beta u_1,\ \text{and}\ \beta \equiv \frac{\Lambda_{21}}{\Lambda_2}. \tag{16}$$

The above expression ties liquidity trading persistence to the relative price impact that the endowment shocks of different liquidity traders' cohorts have on p_2. Based on Proposition 1 and the interpretation of the price impact coefficients in Corollary 2, we can also say that with transparency, FD absorb a smaller portion of the first period endowment shock (compared to the second period one), and the noise process is stable: $\beta < 1$.

Second, the first and second period returns are *positively* serially correlated. That is, the model displays momentum, even in the absence of any fundamentals information:

$$\begin{aligned} \text{Cov}[p_2 - p_1, p_1] &= \text{Cov}[-(\Lambda_2 u_2 + (\Lambda_{21} - \Lambda_1)u_1), -\Lambda_1 u_1] \\ &= (\Lambda_{21} - \Lambda_1)\Lambda_1 \tau_u^{-1} > 0, \end{aligned} \tag{17}$$

due to Proposition 1. We collect these results in the following

Corollary 4. *When the market is transparent: (1) liquidity trading behaves as a stable AR(1) process; (2) first and second period returns are positively serially correlated.*

4 The opaque market

Suppose now that second period traders observe a noisy signal of the first period order imbalance ($\tau_\eta \in (0, \infty)$). In this case, $\Omega_2 = \{u_2, s_{u_1}\}$ which implies that second period traders cannot perfectly anticipate p_2. As a consequence, their strategy is affected by their return uncertainty (see (A.26)):

$$x_2 = \underbrace{\frac{\gamma_L \tau_v \Lambda_2 - 1}{\text{Var}_2[v - p_2]}}_{a_2} u_2 + \underbrace{\gamma_L \frac{\Lambda_{21}\tau_\eta + \Lambda_{22}\tau_u}{(\tau_u + \tau_\eta)\text{Var}_2[v - p_2]}}_{b} s_{u_1}, \tag{18}$$

with $\text{Var}_2[v - p_2] = \tau_v^{-1} + (\Lambda_{21} - \Lambda_{22})^2(\tau_u + \tau_\eta)^{-1}$, and the second period price is as in (1b). Intuitively, traders' inability to exactly infer u_1 impacts their return uncertainty, introducing execution risk in their strategy. This, in turn, affects both their hedging and speculative aggressiveness ($|a_2|$ and b) and the price impact of their order. Given the risk-sharing enhancing role of traders' speculation, this impacts market stability. To see this, it is useful to start from the extreme case in which $\tau_\eta \to 0$.

4.1 The fully opaque market

Suppose second period traders' signal becomes unboundedly noisy (i.e., $\tau_\eta \to 0$). In this case, we obtain the following result:

Proposition 2. *When the market is fully opaque, the expressions for the equilibrium price coefficients* Λ_2 *and* Λ_1 *are as in* (5a) *and* (5b)*, while*

$$\Lambda_{21} = -\frac{a_{21}}{\gamma\tau_v}, \tag{19}$$

The coefficients of traders' strategies are as follows:

$$a_1 = -\gamma_L \frac{\Lambda_{21} - \Lambda_1}{\mathrm{Var}_1[p_2]} < 0,\ a_{21} = \frac{\gamma_L\tau_v\Lambda_{21} - 1}{\mathrm{Var}_1[v - p_2]} \in (-1, 0) \tag{20a}$$

$$a_2 = \frac{\gamma_L\tau_v\Lambda_2 - 1}{\mathrm{Var}_2[v - p_2]} \in (-1, 0),\ b = 0, \tag{20b}$$

where $\mathrm{Var}_1[p_2] = \Lambda_2^2\tau_u^{-1}$, $\mathrm{Var}_1[v - p_2] = \tau_v^{-1} + \Lambda_2^2\tau_u^{-1}$ *and* $\mathrm{Var}_2[v - p_2] = \tau_v^{-1} + \Lambda_{21}^2\tau_u^{-1}$. *Furthermore, at equilibrium* $\Lambda_{21} > \Lambda_1 > 0$ *and* $\Lambda_2 > 0$.

According to (19) and the second expression in (20b), when the market is fully opaque, second period traders do not speculate. This is because their signal on u_1 is infinitely noisy, which makes it impossible for them to predict the direction of the first period imbalance. As a consequence, $\Lambda_{22} = 0$ and we have:

Corollary 5. *When the market is fully opaque, second period liquidity traders do not supply liquidity via contrarian market orders and the second period price only reflects traders' endowment shocks.*

According to (20a) and (20b), liquidity traders' second period hedging aggressiveness, $|a_{21}|, |a_2|$ depend on two forces: the expected profit from hedging the endowment shock, and the variance of the second period return $v - p_2$ (respectively captured by the terms at the numerator and denominator of the expressions in (20a) and (20b)).[16] For given return variance, a higher price impact of the t-period traders' endowment shock, reduces the expected hedging profit of the t-period traders, lowering their hedging aggressiveness. Conversely, for given expected hedging profit, a higher price impact of the t-period traders' endowment shock, increases $s \neq t$-period traders' execution risk, lowering the latter hedging aggressiveness. Therefore, the liquidity demands of different investors' cohorts (exposed to different endowment shocks) induce *different* impacts on the equilibrium price and this effect can be responsible for self-sustaining demand loops.

To see this, assume that the market impact of the second period traders' endowment shock (Λ_2) increases. This mitigates these traders' expected return and increases the cohort 1 traders' execution risk, leading them to scale down their liquidity demand ($|a_{21}|$ decreases). All else equal, this reduces the price impact of cohort 1's endowment shock (Λ_{21} decreases), because liquidity providers need to absorb a smaller share of cohort 1's endowment shock. This in turn

[16] In the Appendix we show that $E_2[v - p_2] = \Lambda_2 u_2$, $\mathrm{Var}_2[v - p_2] = \tau_v^{-1} + \Lambda_{21}^2\tau_u^{-1}$, $E_1[v - p_2] = \Lambda_{21}u_1$, $\mathrm{Var}_1[v - p_2] = \tau_v^{-1} + \Lambda_{21}^2\tau_u^{-1}$, $\mathrm{Cov}_1[v, v - p_2] = \mathrm{Cov}_2[v, v - p_2] = \tau_v^{-1}$.

lowers the execution risk faced by cohort 2 traders, potentially leading them to scale up their liquidity demand ($|a_2|$ increases), and further boosting Λ_2, because liquidity providers need to absorb a larger share of cohort 2's endowment shock, which reinforces the initial spike.

The loop described above can lead to multiple equilibria, as we illustrate in Figure 3, where we plot the aggregate best response function that we obtain in the Appendix (see (A.11)):

$$\Lambda_{21} = \Phi(\Lambda_{21}) \equiv \frac{A^2}{\gamma\tau_u + (\gamma + \gamma_L)A^2\tau_v}, \tag{21}$$

where $A \equiv (\gamma + \gamma_L)\tau_u + \gamma\Lambda_{21}^2\tau_v$. As the figure shows, for the chosen parameter values, we obtain three equilibria where $\Lambda_{21} \in \{0.12, 1.98, 8.96\}$, and the corresponding values for the other price coefficients are $\Lambda_2 \in \{8.96, 1.98, 0.12\}$, $\Lambda_1 \in \{0.1 \times 10^{-2}, 0.43, 8.84\}$. Our simulations suggest that equilibrium multiplicity is more likely to obtain when payoff and endowment shock dispersion are larger and liquidity traders are more risk averse.

$\tau_u = \tau_v = 0.1$, $\tau_\eta = 0$, $\gamma = 1$, $\gamma_L = 0.1$

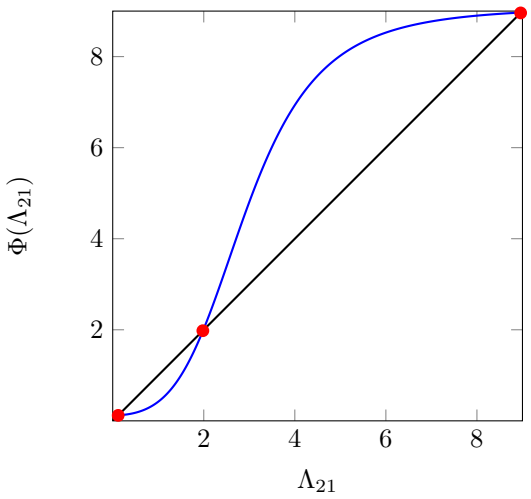


Figure 3: Market opacity and multiple equilibria.

In fact, for $\tau_\eta \to 0$, the system of equations which pins down the price impacts becomes:

$$\Lambda_2 = \frac{\tau_u}{((\gamma + \gamma_L)\tau_u + \gamma\tau_v\Lambda_{21}^2)\tau_v} \tag{22a}$$

$$\Lambda_{21} = \frac{\tau_u}{((\gamma + \gamma_L)\tau_u + \gamma\tau_v\Lambda_2^2)\tau_v} \tag{22b}$$

$$\Lambda_1 = \frac{(\gamma + \gamma_L)\tau_u\Lambda_{21}}{(\gamma + \gamma_L)\tau_u + \gamma\tau_v\Lambda_2^2}. \tag{22c}$$

Manipulating (22a) and (22b) in the Appendix we show that in this case the equilibrium obtains

as a solution to the following quadratic equation:

$$(\gamma + \gamma_L)\gamma\tau_v\Lambda_2^2 - \gamma\Lambda_2 + (\gamma + \gamma_L)^2\tau_u = 0, \tag{23}$$

which, thus, has a closed form solution. Note that in this case, the price impact of the first period endowment shock (Λ_1) does not affect the second period price coefficients (Λ_2, Λ_{21}) but is determined by their equilibrium values. Formally, we obtain the following corollary of the previous result:

Corollary 6. *When the market is fully opaque, at equilibrium*

$$\Lambda_1 = (\gamma + \gamma_L)\tau_v\Lambda_{21}^2. \tag{24}$$

If

$$0 < \tau_u\tau_v < \gamma/(4(\gamma + \gamma_L)^3), \tag{25}$$

three equilibria arise, where

$$\Lambda_2 = \frac{\gamma \pm \sqrt{(\gamma - 4(\gamma + \gamma_L)^3\tau_u\tau_v)\gamma}}{2(\gamma + \gamma_L)\gamma\tau_v},\ \Lambda_{21} = \frac{\gamma \mp \sqrt{(\gamma - 4(\gamma + \gamma_L)^3\tau_u\tau_v)\gamma}}{2(\gamma + \gamma_L)\gamma\tau_v}, \tag{26a}$$

and $\Lambda_2 = \Lambda_{21}$ *obtaining as the unique root of the following cubic*

$$((\gamma + \gamma_L)\tau_u + \gamma\tau_v\Lambda_2^2)\Lambda_2\tau_v = \tau_u. \tag{26b}$$

If $\tau_u\tau_v \geq \gamma/(4(\gamma + \gamma_L)^3)$*, then there is a unique equilibrium where* $\Lambda_2 = \Lambda_{21}$ *is the unique root of the cubic* (26b).

Condition (25) defines the parameter restriction for the region where equilibrium multiplicity occurs. According to such condition, multiplicity obtains when liquidity demand is likely to be stronger, the volatility of the security's payoff is larger and traders are more risk averse, i.e. when the gap between liquidity demand and liquidity provision is likely to be *wider*. Indeed, in these conditions traders need to hedge the most (due to the higher unpredictability of their endowment shock and their higher risk aversion), while liquidity providers are less willing to supply liquidity (due to the higher volatility of the security's payoff). Interestingly, an increase in liquidity providers' risk-bearing capacity has a non-monotonic impact on the magnitude of this region. This is because for given hedging aggressiveness ($|a_{21}|$ and $|a_{22}|$), an increase in γ lowers the price impact of trades (see (5a) and (19)) which, for low levels of risk tolerance, induces more liquidity consumption on traders' side (see (20a) and (20b)). However, as γ grows large this effect becomes second order, and an increase in liquidity providers' risk tolerance reduces the magnitude of the multiplicity region.

The expressions in (26a) show that with equilibrium multiplicity, the second period price sensitivities to the endowment shock correspond to the two roots of the quadratic (23). This

implies that at the second round the trading costs faced by traders in different cohorts are heterogeneous: the price impact of first and second period liquidity traders' endowment shocks are *negatively correlated*.

The next result characterizes the stability properties of the equilibrium and the liquidity consumption patterns arising with multiple equilibria. For ease of exposition we denote by Λ_2^* and Λ_2^{***} the low and high root in the first of (26a), and with Λ_2^{**} the unique real root of the cubic (26b). Correspondingly, Λ_{21}^{***}, Λ_{21}^{*}, and Λ_{21}^{**}, denote the low and high root in the second of (26a), and the unique real root of the cubic (26b) (recall that in this case $\Lambda_2 = \Lambda_{21}$). Finally, Λ_1^{***}, Λ_1^{*} and Λ_1^{**} denote the first period price impact coefficient obtained via (24). Accordingly, we rank traders' liquidity demand in a similar way: a_2^* denotes second period traders' demand when $\Lambda_2 = \Lambda_2^*$ (and $\Lambda_{21} = \Lambda_{21}^*$), and so on.

Corollary 7. *When the market is fully opaque, with uniqueness, the equilibrium is stable. When multiple equilibria arise,*

1. *The two extreme equilibria are stable, while the intermediate equilibrium is unstable.*

2. *Equilibria can be ranked in terms of the price sensitivity to first and second period endowment shocks:*

$$\Lambda_2^* < \Lambda_2^{**} < \Lambda_2^{***},\ \Lambda_{21}^{***} < \Lambda_{21}^{**} < \Lambda_{21}^{*},\ \Lambda_1^{***} < \Lambda_1^{**} < \Lambda_1^{*}. \tag{27}$$

   Thus, at a stable equilibrium we have either that p_2 reacts more to u_2 than to u_1, or the opposite. Correspondingly, in the former (latter) case the first period market is more (less) liquid. Comparing liquidity across trading rounds, we have

$$\Lambda_1 < \Lambda_{21}^{***} < \Lambda_2^{***},\ \textit{or}\ \Lambda_1 < \Lambda_2^{*} < \Lambda_{21}^{*}.$$

3. *Liquidity consumption is increasing in the price impact it induces: $-1 < a_2^{***} < a_2^{**} < a_2^{*} < 0$, $-1 < a_{21}^{*} < a_{21}^{**} < a_{21}^{***} < 0$, and $-1 < a_1^{*} < a_1^{**} < a_1^{***} < 0$.*

Therefore, only the extreme equilibria are stable. Additionally, at equilibrium the traders belonging to the cohort that faces the *highest market impact demand more liquidity*. In other words, with multiple equilibria, illiquidity no longer operates as a rationing device. This is because an increase in the price impact induced by the endowment shock (affecting traders in cohort) t, has a proportionally stronger effect on the execution risk faced by cohort $s \neq t$ traders than on the expected return obtained by traders in cohort t.

4.2 Liquidity demand and supply in a fully opaque market

The discussion following the latest result, suggests that when the market is opaque liquidity demand should be an increasing function of the price impact it faces, that is, its *slope* should

change compared to the case where the market is fully transparent. This is exactly what we display in Figure 4, where we substitute (22b) into the first of (20b):

$$a_2 = \frac{\gamma_L \tau_v \Lambda_2 - 1}{\tau_v^{-1} + \Lambda_{21}^2 \tau_u^{-1}},$$

to plot second period traders' liquidity demand (in green) as a function of the price impact it generates and the liquidity supply function (in blue), using the same parameter values of Figure 3. The crossing points between the two curves occur at the three equilibrium points. As shown by the figure, and differently from what shown in Figure 2 with a fully transparent market, a higher Λ_2 leads second period traders to demand more liquidity ($|a_2|$ increases), which leads to the positive association between liquidity consumption and illiquidity at equilibrium.

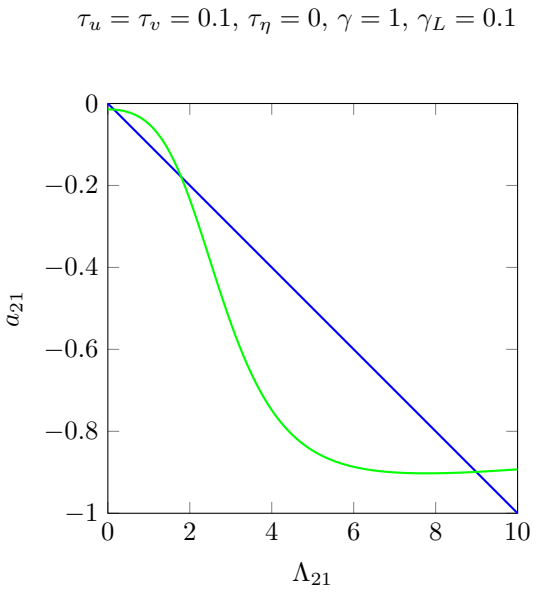


Figure 4: Liquidity demand and supply at the second round with a fully opaque market.

Importantly, the positive relationship between liquidity consumption and illiquidity is preserved even when (25) is not satisfied, and a unique equilibrium arises. In that situation, since $\Lambda_{21} = \Lambda_2$, we have

$$a_{21} = a_2 = \frac{(\gamma_L \tau_v \Lambda_2 - 1)\tau_u}{\tau_u + \Lambda_2^2 \tau_v}. \tag{28}$$

Rearranging (26b) to isolate $\Lambda_2^2 \tau_v$ yields:

$$\Lambda_2^2 \tau_v = \frac{(1 - (\gamma + \gamma_L)\Lambda_2 \tau_v)\tau_u}{\gamma \Lambda_2 \tau_v},$$

which can be substituted at the denominator of (28) to obtain

$$a_2 = -\gamma \tau_v \Lambda_2.$$

This implies that

Corollary 8. *When the market is fully opaque and a unique equilibrium obtains, at the second round (1) both traders' cohorts hedge the same fraction of their endowment shock, facing the same illiquidity: $a_{21} = a_2$ and $\Lambda_{21} = \Lambda_2$, (2) liquidity demand is an increasing function of the equilibrium illiquidity:*

$$a_2 = -\gamma\tau_v\Lambda_2, \tag{29}$$

where Λ_2 is the unique real solution to (26b).

The following corollary derives the implications of multiple equilibria for the time series properties of noise trades and returns autocovariance:

Corollary 9. *With multiple equilibria, (1) $\beta < 1$ ($\beta > 1$) when $\Lambda_2 = \Lambda_2^{***}$ ($\Lambda_2 = \Lambda_2^{*}$); (2) the autocovariance of first and second period returns increases in Λ_{21} and also increases compared to the case with full transparency at both equilibria.*

5 Extensions

In this section, we generalize the model to allow for the possibility (1) that the market can be "partially" opaque (i.e., $\tau_\eta \in (0, \infty)$) and (2) that liquidity is also supplied by "Standard Dealers" (of mass $1-\mu$) who can only trade at the first round–we denote them by RD and use FD (standing for "Full Dealers", of mass $0 < \mu < 1$) to denote the dealers we introduced in Section 1.1. We start by considering the effect of an informative signal, keeping $\mu = 1$.

5.1 An informative signal

When $\tau_\eta \in (0, \infty)$, prices are as in (1a) and (1b), and we have the following result:

Proposition 3. *The equilibrium obtains as a solution to the system of non-linear, simultaneous equations (A.37a)–(A.37c) and (A.46). The expressions for the equilibrium prices' coefficients Λ_2, Λ_1, Λ_{21} and Λ_{22} are as in (A.46), and (A.47a)-(A.47c). The coefficients of traders' strategies are as in Proposition 2, with $\mathrm{Var}_1[p_2] = \Lambda_2^2\tau_u^{-1} + \Lambda_{22}^2\tau_\eta^{-1}$, $\mathrm{Var}_1[v - p_2] = \tau_v^{-1} + \Lambda_2^2\tau_u^{-1} + \Lambda_{22}^2\tau_\eta^{-1}$ and $\mathrm{Var}_2[v - p_2] = \tau_v^{-1} + (\Lambda_{21} - \Lambda_{22})^2(\tau_u + \tau_\eta)^{-1}$, except for b, which is given by:*

$$b = \gamma_L \frac{\Lambda_{21}\tau_\eta + \Lambda_{22}\tau_u}{(\tau_\eta + \tau_u)\mathrm{Var}_2[v - p_2]}. \tag{30}$$

At equilibrium, $\Lambda_2 > 0, \Lambda_{21} > \Lambda_1 > 0$, and $\Lambda_{22} < 0$.

In this case we cannot analytically study the equilibrium and we resort to numerical simulations to investigate the properties of the model.

According to the above result, an informative signal about u_1 ($\tau_\eta \in (0, \infty)$) leads second period traders to speculate against the price pressure created by first period traders' liquidity

demand, taking a contrarian position (in our simulations, $b > 0$), thus enhancing the risk-bearing capacity of the market. This dampens the strategic complementarities responsible for multiple equilibria (see Figure 5) and for τ_η large enough, leads to a unique equilibrium (see Figure 6).

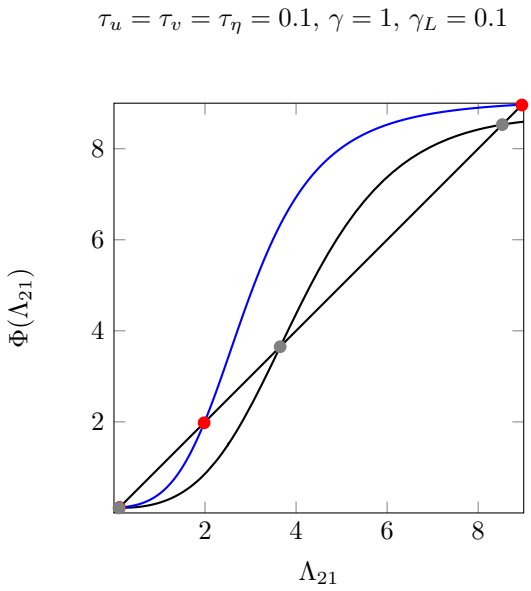


Figure 5: Market transparency and multiple equilibria. In the figure, we plot in black the function $\Phi(\Lambda_{21})$ when $\tau_u = \tau_v = \tau_\eta = 0.1$, $\gamma_L = 0.1$, and $\gamma = 1$. The blue curve shows the case with full opaqueness ($\tau_\eta = 0$) shown in Figure 3.

In Figure 6, we plot the price and strategy coefficients for one of our simulations. As shown in the figure, for τ_η small, three equilibria arise. We plot them using the color green, blue and red to indicate the equilibrium that corresponds to the two "extreme," stable price impacts (respectively in green and red) and the unstable one (in blue) when $\mu = 1$ and $\tau_\eta = 0$. Importantly, when multiple equilibria obtain, order flow partial transparency does not modify an important conclusion we reached in Section 4.2: liquidity demand and illiquidity are positively correlated at equilibrium (see panels (c), (d), (e) and (f) in Figure 6). However, at the unique equilibrium, we have $|a_2| > |a_{21}|$ and $\Lambda_2 > \Lambda_{21}$: a sufficiently high degree of order flow transparency leads second period traders' to demand more liquidity compared to their first period peers (crowding them out), paying a higher price for immediacy.

5.2 Full and Standard dealers

In this section, we assume that liquidity is provided by a mass $\mu \in (0, 1]$ of Full Dealers (FD) who post limit orders at both rounds (as in our base model) and a complementary mass $1 - \mu$ of restricted dealers (RD) that post them only in the first period. An RD has CARA preferences with the same risk-tolerance γ as an FD. However, as he is in the market only in

the first period, he submits a price-contingent order x^{RD} to maximize the expected utility of his wealth $W^{RD} = (v - p_1)x^{RD}$ which, as we show in the Appendix (see (A.34)), has the following expression: $x^{RD} = -\gamma\tau_v p_1$. The inability of RD to trade in the second period captures some liquidity suppliers' limited market participation. This friction could be due to technological reasons as in the case of dealers with impaired access to a technology that allows trading at high frequencies. Alternatively, it could arise from limited access to the trading venue, as in the case of those liquidity suppliers who in the 80s could not access the NYSE trading floor.

Importantly, due to the heterogeneity of liquidity suppliers' types, market clearing conditions change compared to (2)-(3):

$$\mu x_1^{FD} + (1-\mu)x_1^{RD} + x_{11} = 0 \tag{31a}$$

$$(x_2^{FD} - x_1^{FD})\mu + (x_{21} - x_{11}) + x_2 = 0 \iff \mu x_2^{FD} + (1-\mu)x^{RD} + x_{21} + x_2 = 0, \tag{31b}$$

where in the latter we make use of the first period market clearing condition to obtain the expression at the right hand side of (31b). Figure 7 displays the timeline of the model.

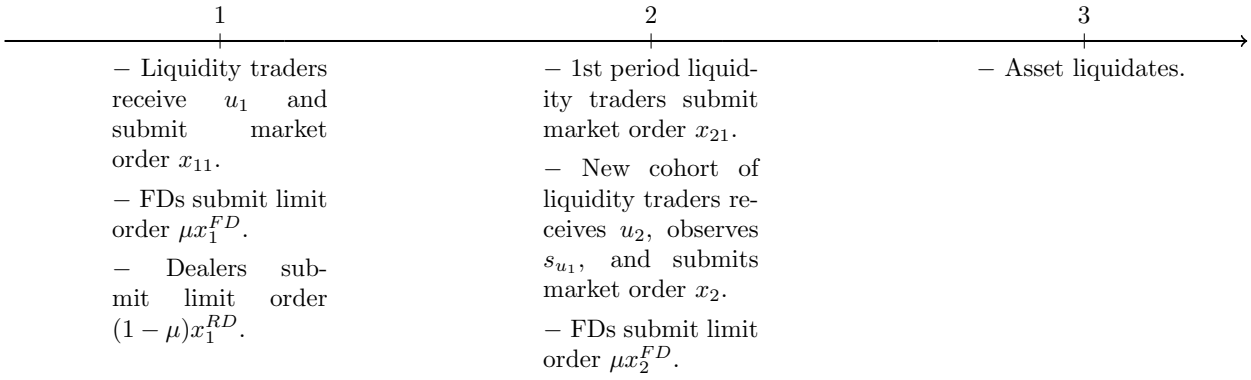


Figure 7: The timeline with heterogeneous liquidity supply.

This version of the model is also analytically challenging, and we once again resort to numerical simulations to investigate its properties.[17] We first show that multiple equilibria also arise when second period traders observe an informative signal about u_1 and $\mu \in (0, 1]$. In Figure 8 we partition the space $\mu \in (0, 1], \tau_\eta > 0$ in two regions: points above (below) the blue curve correspond to values of μ and τ_η for which our numerical simulations yield a unique equilibrium (three equilibria). According to the figure, uniqueness obtains when second period traders' signal is of a sufficiently good quality, confirming our previous results. The effect of an increase in μ is less obvious. As panel (a) illustrates, we find that when τ_η is low, an increase in μ leads the market to switch from a unique equilibrium, to multiple equilibria and, eventually, back to a unique equilibrium. The intuition is as follows: an increase in μ increases the mass of

[17] The analytical characterization of the equilibrium is very similar to the one illustrated in Proposition 3.

liquidity providers who (1) provide liquidity at the second round and (2) benefit from second period traders' risk-sharing enhancing speculation. For small values of μ, the mass of FD is small and the need for additional risk-sharing is reduced, which explains uniqueness. However, as μ increases this is no longer the case. Finally, as μ grows even larger, the risk-bearing capacity of FD is sufficiently large to require less second period traders' risk-sharing contribution, which brings the market back to the unique equilibrium region.

Consistently with what we have found in Corollary 6, an increase in γ_L or τ_v tends to reduce the chances of liquidity fragility (compare the areas below the blue curve in panel (a) and panels (c) and (d)). The effect of an increase in τ_u is more complicated. Comparing panels (a) and (b) in the figure indicates that when τ_η is low, for extreme values of μ an increase in τ_u increases the chances of liquidity fragility, while the opposite occurs for intermediate values of μ. The intuition is as follows. When $\tau_\eta \in (0, \infty)$, second period traders use their signal, s_{u_1}, and p_2 to learn u_1. Other things equal, an increase in τ_u reduces the effect of first period liquidity traders demand on p_2, reducing second period traders' reliance on p_2 to learn u_1, which works to reduce their speculative activity on the propagated imbalance. For extreme values of μ, the ensuing reduction in risk-sharing produces a more dramatic effect on fragility as either the risk bearing capacity of liquidity providers is small (μ close to 0) or FD bear most of the risk exposure (μ high).

In Figure 9, we plot the price and strategy coefficients for one such simulation, using the same coloring of Figure 6. As in Figure 6, when τ_η increases, only the equilibrium where Λ_1, Λ_{21} are small and Λ_2 is large survive.[18] This confirms the intuition gained via the benchmark (and Figure 8) that an increase in order flow transparency attenuates the demand loop responsible for equilibrium multiplicity.

Next, we explore the impact of order flow transparency on price impact and liquidity consumption, along the stable equilibrium where Λ_{21} is small. In Figure 10 we plot the price impact coefficient of u_1 on p_2 and the strategy coefficient a_{21} as a function of τ_η (respectively Panels (a) and (b)), and then a_{21} as a function of Λ_{21}. The plots confirm that opacity can be responsible for traders consuming more liquidity as the price impact they produce increases (Panel (c)). This confirms the finding of Corollary 8: in the general case too, even when the demand loop is not sufficiently strong to generate multiple equilibria, it can nonetheless impede the rationing function of illiquidity. Additionally, the figure shows that Λ_{21} declines with τ_η. Together with the positive relationship between b and τ_η shown in Figure 9 (Panel (h)), this offers an explanation for one of the findings in Anand et al. (2013). These authors study the trading behavior of equity mutual funds during the crisis, offering evidence that some of them actively participate in the market by providing liquidity as "contrarian" traders and showing that resiliency is enhanced by a larger market participation of such funds. Through the lenses

[18] In all our simulations, the switch from multiple equilibria to a unique equilibrium always occurs with the market reverting to the case with a low Λ_{21} and high Λ_2.

of our model, this is exactly what is shown in the figure: as τ_η increase, second period traders speculate more aggressively against the order imbalance due to first period traders' endowment shock. This improves risk sharing by lowering the risk exposure of FD, which produces a decline in Λ_{21}.

6 Concluding remarks

We analyse a two-period market in which a risky security is traded by liquidity providers and traders who hedge an endowment shock. We show that the properties of the market equilibrium crucially depend on the information environment. With full transparency, second period traders perfectly observe the first period endowment shock. This allows them to take a contrarian position against first period liquidity traders' second period order–in this way de-facto providing liquidity to them. In this case we show that traders' demand for liquidity is a decreasing function of the price impact it induces–that is, illiquidity works as a *rationing* device. Additionally, a unique equilibrium obtains. A deterioration of second period traders' information impairs these traders' ability to supply liquidity via contrarian orders. This reduces the risk-bearing capacity of the market and can increase market fragility. Specifically, we find that with market opacity, an increase in the price impact induced by the endowment shock (affecting traders in cohort) 2 (1), has a proportionally stronger effect of the execution risk faced by cohort 1 (2) traders than on the expected return obtained by traders in cohort 2 (1). In this case, the model displays multiple equilibria with different levels of market depth. Additionally, a larger price impact leads traders to demand and consume more liquidity. Thus, our model predicts that market opacity can make markets fragile and jam the rationing function of illiquidity.

Our model offers a plausible explanation for a number of recent events in which market liquidity "crashes" in the absence of any observable change in fundamentals. In these events, it looks as if traders chased liquidity while liquidity providers withdrew it from the market. We argue that opacity of the trading process can be the responsible for this type of effect, as it can severely impair the participation of "non-standard" liquidity providers. This triggers a self-sustaining loop in which different traders compete to consume a limited amount of liquidity, eventually destabilizing the market. Our model also predicts that when the market is fragile, trading costs are heterogeneous across different cohorts of investors. Specifically, the investors paying most for liquidity are those that consume more of it.

Our model also provides an additional argument in support of the introduction of a "consolidated tape" in the EU. Indeed, the level of stock market fragmentation in the EU is higher than in the US. However, differently from their US peers, traders in the EU cannot rely on a common signal displaying the best quotes available across trading venues. To obtain such

a "consolidated" market view, they need to piece together the more expensive feeds offered by each exchange, which creates a suboptimal two-tiered market (Cespa and Foucault (2013); Brogaard et al. (2021)). In an attempt to level the playing field, the European Commission is seeking to introduce the supply of a consolidated tape, at a reasonable price. However, this effort is facing a fierce resistance from exchanges. Such resistance is likely to lower the transparency of the trading process which, through the lenses of our model, can have undesirable side effects on market stability.

References

Aldrich, E., J. Grundfest, and G. Laughlin (2017). The flash crash: a new deconstruction. *Working Paper*.

Amihud, Y., H. Mendelson, and R. A. Wood (1990). Liquidity and the 1987 stock market crash. *Journal of Portfolio Management 16*(3), 65–69.

Anand, A., P. Irvine, A. Puckett, and K. Venkataraman (2013). Institutional trading and stock resiliency: Evidence from the 2007-2009 financial crisis. *Journal of Financial Economics 108*(3), 773–797.

Anand, A., C. Jotikasthira, and K. Venkataraman (2021). Mutual fund trading style and bond market fragility. *Review of Financial Studies 34*(6), 2993–3044.

Bank of International Settlements (2017). The Sterling 'flash event' of 7 October 2016. *BIS Working Paper*.

Bao, J., M. O'Hara, and X. (Alex) Zhou (2018). The volcker rule and corporate bond market making in times of stress. *Journal of Financial Economics 130*(1), 95–113.

Biais, B., F. Declerck, and S. Moinas (2017). Who supplies liquidity, how and when? *Working Paper, Toulouse school of Economics*.

Brogaard, J., J. Brugler, and D. Rösch (2021). Competition and exchange data fees. *Working Paper*.

Brogaard, J., T. Hendershott, and R. Riordan (2014). High-frequency trading and price discovery. *Review of Financial Studies 27*(8), 2267–2306.

Brunnermeier, M. K. and L. H. Pedersen (2009). Market liquidity and funding liquidity. *Review of Financial Studies 22*(6), 2201–2238.

Cespa, G. and T. Foucault (2013). Sale of price information by exchanges: Does it promote price discovery? *Management Science 60*(1), 148–165.

Cespa, G. and T. Foucault (2014). Illiquidity contagion and liquidity crashes. *Review of Financial Studies 27*(6), 1615–1660.

Cespa, G. and X. Vives (2015). The Beauty Contest and short-term trading. *Journal of Finance 70*(5), 2099–2154.

Cespa, G. and X. Vives (2019). Market transparency and fragility. *Working Paper*.

Cespa, G. and X. Vives (2021). Exchange competition, entry, and welfare. *The Review of Financial Studies 35*(5), 2570–2624.

CFTC and SEC (2010). Findings regarding the market events of may 6, 2010. *Staff Report*.

Ding, S., J. Hanna, and T. Hendershott (2014). How slow is the NBBO? a comparison with direct exchange feeds. *Financial Review* (49), 313–332.

Easley, D., M. O'Hara, and M. López de Prado (2011). The microstructure of the Flash-Crash: flow toxicity, liquidity crashes, and the probability of informed trading. *Journal of Portfolio Management 37*, 118–128.

Gennotte, G. and H. Leland (1990). Market liquidity, hedging, and crashes. *American Economic Review 80*(5), 999–1021.

Glosten, L. and P. Milgrom (1985). Bid, ask and transaction prices in a specialist market with heterogeneously informed traders. *Journal of Financial Economics 14*, 71–100.

Goldstein, I., A. Kopytov, L. Shen, and H. Xiang (2021). On ESG investing: Heterogeneous preferences, information, and asset prices. *Working Paper*.

Goldstein, I. and L. Yang (2015). Information diversity and complementarities in trading and information acquisition. *Journal of Finance 70*, 1723–1765.

Gromb, D. and D. Vayanos (2002). Equilibrium and welfare in markets with financially constrained arbitrageurs. *Journal of Financial Economics 66*(2-3), 361–407.

He, H. and J. Wang (1995). Differential information and dynamic behavior of stock trading volume. *8*(4), 919–972.

Jacklin, C. J., A. W. Kleidon, and P. Pfleiderer (1992). Underestimation of portfolio insurance and the crash of october 1987. *Review of Financial Studies 5*(1), 35–63.

Karolyi, G., K.-H. Lee, and M. van Dijk (2012). Understanding commonality in liquidity around the world. *Journal of Financial Economics 105*(1), 82–112.

Llorente, G., R. Michaely, G. Saar, and J. Wang (2002). Dynamic volume-return relation of individual stocks. *Review of Financial Studies 15*(4), 1005–1047.

Madrigal, V. and J. A. Scheinkman (1997). Price crashes, information aggregation, and market-making. *Journal of Economic Theory 75*(1), 16–63.

SEC (2010). Concept release on equity market structure. *Federal Register*.

Subrahmanyam, A. and S. Titman (1999). The going-public decision and the development of financial markets. *The Journal of Finance 54*(3), 1045–1082.

Vayanos, D. and J. Wang (2012). Liquidity and asset returns under asymmetric information and imperfect competition. *Review of Financial Studies 25*(5), 1339–1365.

Vives, X. (2008). *Information and learning in markets: the impact of market microstructure.* Princeton University Press.

Wang, J. (1994). A model of competitive stock trading volume. *Journal of Political Economy 102*(1), 2099–2154.

A Appendix

The following is a standard results (see, e.g. Vives (2008), Technical Appendix, pp. 382–383) that allows us to compute the unconditional expected utility of market participants.

Lemma 1. *Let the n-dimensional random vector $z \sim N(0, \Sigma)$, and $w = c + b'z + z'Az$, where $c \in \mathbb{R}$, $b \in \mathbb{R}^n$, and A is a $n \times n$ matrix. If the matrix $\Sigma^{-1} + 2\rho A$ is positive definite, and $\rho > 0$, then*

$$E[-\exp\{-\rho w\}] = -|I + 2\rho\Sigma A|^{-1/2} \exp\{-\rho(c - \rho b'(\Sigma + 2\rho A)^{-1} b)\}.$$

Proof of Proposition 1

We prove here that that when second period traders observe a perfectly informative signal of u_1 (i.e., $\tau_\eta \to \infty$), the equilibrium obtained in Proposition 3, is unique. Note that this assumption has a direct impact on the second period equilibrium condition, since with a perfect signal, the information set of second period traders' is given by $\Omega_2 = \{u_2, u_1\}$. Therefore, the second period price only reflects endowment shocks:

$$p_2 = -\Lambda_2 u_2 - \Lambda_{21} u_1,$$

and, using (A.24), second period traders' position reads as follows:

$$\begin{aligned} x_2 &= \gamma_L \frac{E_2[v - p_2]}{\mathrm{Var}_2[v - p_2]} - \frac{\mathrm{Cov}_2[v, v - p_2]}{\mathrm{Var}_2[v - p_2]} u_2 \\ &= \underbrace{(\gamma_L \tau_v \Lambda_2 - 1)}_{= a_2} u_2 + \underbrace{\gamma_L \tau_v \Lambda_{21}}_{= b} u_1, \end{aligned} \tag{A.1}$$

where we note that since second period traders perfectly observe u_1, $\mathrm{Var}_2[v - p_2] = \tau_v^{-1}$. First period traders, trading at the second round, can only anticipate the impact of u_1 on p_2. Thus, using (A.35), we obtain:

$$\begin{aligned} x_{21} &= \gamma_L \frac{E_1[v - p_2]}{\mathrm{Var}_1[v - p_2]} - \frac{\mathrm{Cov}_1[v, v - p_2]}{\mathrm{Var}_1[v - p_2]} u_1 \\ &= \underbrace{\frac{(\gamma_L \tau_v \Lambda_{21} - 1)\tau_u}{\tau_u + \Lambda_2^2 \tau_v}}_{= a_{21}} u_1. \end{aligned} \tag{A.2}$$

The strategies for FD and RD are as in (A.30) and (A.34), so that plugging them in the second period market clearing condition yields:

$$\mu x_2^{FD} + x_2 + (1 - \mu) x^{RD} + x_{21} = 0 \iff p_2 = \underbrace{\frac{a_2}{\mu\gamma\tau_v}}_{= -\Lambda_2} u_2 + \underbrace{\frac{b + (1 - \mu)\gamma\tau_v\Lambda_1 + a_{21}}{\mu\gamma\tau_v}}_{= -\Lambda_{21}} u_1. \tag{A.3}$$

Based on the above, we can immediately identify the second period price impact coefficients:

$$\Lambda_2 = \frac{1}{(\mu\gamma + \gamma_L)\tau_v} \tag{A.4a}$$

$$\Lambda_{21} = \frac{\tau_u + \Lambda_2^2\tau_v}{((\mu\gamma + \gamma_L)(\tau_u + \Lambda_2^2\tau_v) + \gamma_L\tau_u)\tau_v}\left(\frac{\tau_u}{\tau_u + \Lambda_2^2\tau_v} - (1-\mu)\gamma\tau_v\Lambda_1\right). \tag{A.4b}$$

Finally, turning to the first period market, we have the following expression for the market clearing equation:

$$\mu x_1^{FD} + (1-\mu)x^{RD} + x_1 = 0.$$

Replacing the expressions for traders and liquidity providers' strategies (see, respectively (A.44), (A.45), and (A.34)), taking the limit for $\tau_\eta \to \infty$ and identifying the endowment shock price coefficient yields

$$p_1 = \underbrace{\frac{(\mu\gamma + \gamma_L)\tau_u\Lambda_{21}}{(\mu\gamma + \gamma_L)\tau_u + \gamma\tau_v\Lambda_2^2}}_{=-\Lambda_1} u_1. \tag{A.5}$$

The equilibrium is uniquely pinned down by the solution to the linear system given by the expressions for the price coefficients of u_1 at the two trading rounds:

$$\Lambda_1 = \frac{\tau_u^2\tau_v(\gamma\mu + \gamma_L)^4}{\tau_u\tau_v\left(2\gamma^2\mu + \gamma\gamma_L(\mu+3) + \gamma_L^2\right)(\gamma\mu + \gamma_L) + \tau_u^2\tau_v^2(\gamma + 2\gamma_L)(\gamma\mu + \gamma_L)^4 + \gamma} \tag{A.6a}$$

$$\Lambda_{21} = \frac{\tau_u(\gamma\mu + \gamma_L)\left(\tau_u\tau_v(\gamma\mu + \gamma_L)^3 + \gamma\right)}{\tau_u\tau_v\left(2\gamma^2\mu + \gamma\gamma_L(\mu+3) + \gamma_L^2\right)(\gamma\mu + \gamma_L) + \tau_u^2\tau_v^2(\gamma + 2\gamma_L)(\gamma\mu + \gamma_L)^4 + \gamma}, \tag{A.6b}$$

which possesses the unique solution illustrated in the text of the proposition. The ranking across the price impact coefficients follows immediately from their comparison.

□

Proof of Proposition 2

We obtain the equilibrium in the case with full opacity by taking the limit for $\tau_\eta \to 0$ of the equilibrium price coefficients obtained in the proof of Proposition 3.

Starting from Λ_{22}:

$$\Lambda_{22} = \lim_{\tau_\eta \to 0} -\frac{\gamma_L\Lambda_2\Lambda_{21}\tau_v\tau_\eta}{\tau_u + (1 - \gamma_2\tau_v\Lambda_2)} = 0. \tag{A.7a}$$

Based on (A.7a) we then have

$$\Lambda_2 = \lim_{\tau_\eta \to 0} \frac{1}{(\gamma_L + \mu\gamma\tau_v\mathrm{Var}_2[v - p_2])\tau_v} = \frac{\tau_u}{((\mu\gamma + \gamma_L)\tau_u + \mu\gamma\tau_v(\Lambda_{21} - \Lambda_{22})^2)\tau_v} = \frac{\tau_u}{((\mu\gamma + \gamma_L)\tau_u + \mu\gamma\tau_v\Lambda_{21}^2)\tau_v} \tag{A.7b}$$

and

$$\begin{aligned}
&\Lambda_{21} \\
&= \lim_{\tau_\eta \to 0} -\frac{(\tau_v \mathrm{Var}_1[v-p_2])^{-1}(\gamma_L \tau_v \Lambda_{21} - 1) + \gamma_L((\tau_u+\tau_\eta)\mathrm{Var}_2[v-p_2])^{-1}(\Lambda_{21}\tau_\eta + \Lambda_{22}\tau_u)}{\mu\gamma\tau_v} - \frac{1-\mu}{\mu}\Lambda_1 \\
&= -\frac{(\gamma_L\tau_v\Lambda_{21}-1)\tau_u}{(\tau_u+\Lambda_2^2\tau_v)\mu\gamma\tau_v} - \frac{1-\mu}{\mu}\Lambda_1. \qquad \text{(A.7c)}
\end{aligned}$$

Also,

$$\begin{aligned}
\lim_{\tau_\eta \to 0} \frac{\Lambda_{22}^2}{\tau_\eta} &= \lim_{\tau_\eta \to 0} \left(\frac{\gamma_L \Lambda_2 \Lambda_{21} \tau_v}{(\tau_u/\tau_\eta^{1/2}) + (1-\gamma_L\tau_v\Lambda_2)\tau_\eta^{1/2}} \right)^2 \\
&= 0,
\end{aligned}$$

which implies that

$$\begin{aligned}
\Lambda_1 &= \lim_{\tau_\eta \to 0} \frac{(\gamma_L + \mu\gamma)\tau_u \Lambda_{21}}{\gamma_L\tau_u + \gamma(((\Lambda_{22}^2/\tau_\eta)\tau_u + \Lambda_2^2)\tau_v + \mu\tau_u)} \qquad \text{(A.7d)} \\
&= \frac{(\gamma_L+\mu\gamma)\tau_u\Lambda_{21}}{\gamma_L\tau_u + \gamma(\Lambda_2^2\tau_v + \mu\tau_u)}.
\end{aligned}$$

Based on the limits (A.7a)-(A.7d), the coefficients of traders' strategies are given by

$$a_1 = -\gamma_L\tau_u \frac{\Lambda_{21}-\Lambda_1}{\Lambda_2^2} < 0 \qquad \text{(A.8a)}$$

$$a_{21} = \tau_u \frac{\gamma_L\tau_v\Lambda_{21}-1}{\tau_u+\Lambda_2^2\tau_v} \in (-1,0) \qquad \text{(A.8b)}$$

$$a_2 = \tau_u \frac{\gamma_L\tau_v\Lambda_2-1}{\tau_u+\Lambda_{21}^2\tau_v} \in (-1,0) \qquad \text{(A.8c)}$$

$$b = 0. \qquad \text{(A.8d)}$$

Additionally, an equilibrium is pinned down by solving the following system of simultaneous equations:

$$\Lambda_2 = \Phi_1(\Lambda_2) \equiv \frac{\tau_u}{((\mu\gamma+\gamma_L)\tau_u + \mu\gamma\tau_v\Lambda_{21}^2)\tau_v} \qquad \text{(A.9a)}$$

$$\Lambda_{21} = \Phi_2(\Lambda_{21}) \equiv \frac{\tau_u}{((\mu\gamma+\gamma_L)\tau_u + \mu\gamma\tau_v\Lambda_2^2)\tau_v} - \frac{(1-\mu)(\tau_u+\Lambda_2^2\tau_v)\gamma}{(\mu\gamma+\gamma_L)\tau_u + \mu\gamma\tau_v\Lambda_2^2}\Lambda_1 \qquad \text{(A.9b)}$$

$$\Lambda_1 = \frac{(\mu\gamma+\gamma_L)\tau_u\Lambda_{21}}{(\mu\gamma+\gamma_L)\tau_u + \gamma\tau_v\Lambda_2^2}. \qquad \text{(A.9c)}$$

An equilibrium obtains via the solution of the system (A.9a)-(A.9c). To see that such a solution

exists, replace (A.9c) in (A.9b). Rearranging, yields

$$\Lambda_{21} = \frac{((\mu\gamma + \gamma_L)\tau_u + \gamma\Lambda_2^2\tau_v)\tau_u}{(((\gamma + \gamma_L)\tau_u + \gamma\Lambda_2^2\tau_v)(\mu\gamma + \gamma_L)\tau_u + ((\mu\gamma + \gamma_L)\tau_u + \mu\gamma\Lambda_2^2\tau_v)\gamma\Lambda_2^2\tau_v)\tau_v}. \tag{A.10}$$

Finally, replace (A.9a) into the above to obtain:

$$\Lambda_{21} = \Phi_2(\Lambda_{21}) \equiv \frac{(\gamma\tau_u + (\mu\gamma + \gamma_L)A^2\tau_v)A^2}{(\gamma + \gamma_L)(\mu\gamma + \gamma_L)A^4\tau_v^2 + 2(\mu\gamma + \gamma_L)\gamma A^2\tau_u\tau_v + \mu\gamma^2\tau_u^2}, \tag{A.11}$$

where $A \equiv (\mu\gamma + \gamma_L)\tau_u + \mu\gamma\Lambda_{21}^2\tau_v$. Inspection of (A.11) reveals (i) that $\Phi_2(\Lambda_{21}) > 0$, (ii) that $\Phi_2(0) > 0$, and (iii) that $\Lambda_{21} - \Phi_2(\Lambda_{21})$ is proportional to a 9-the degree polynomial in Λ_{21}, which thus always possesses at least one positive root Λ_{21}^*. Recursive substitution of such root first in (A.9a) and then in (A.9c) allows to pin down the set of equilibrium coefficients for p_1 and p_2.

Comparison of (A.9c) and (A.9b) shows that $\Lambda_{21}, \Lambda_1 > 0$ and $\Lambda_1 < \Lambda_{21}$. To see this, suppose $\Lambda_{21} \leq 0$. Then, because of (A.9c), $\Lambda_1 \leq 0$. However, because of (A.9b) this implies that $\Lambda_{21} > 0$, contradicting the initial assumption. Next, using (A.9c)

$$\begin{aligned}\Lambda_{21} - \Lambda_1 &= \Lambda_{21} - \frac{(\mu\gamma + \gamma_L)\tau_u\Lambda_{21}}{(\mu\gamma + \gamma_L)\tau_u + \gamma\tau_v\Lambda_2^2} \\ &= \frac{\gamma\tau_v\Lambda_2^2\Lambda_{21}}{(\mu\gamma + \gamma_L)\tau_u + \gamma\tau_v\Lambda_2^2},\end{aligned}$$

which is positive. □

Proof of Corollary 6

Divide (22a) by (22b) to obtain

$$\frac{\Lambda_2}{\Lambda_{21}} = \frac{(\gamma + \gamma_L)\tau_u + \gamma\tau_v\Lambda_2^2}{(\gamma + \gamma_L)\tau_u + \gamma\tau_v\Lambda_{21}^2}.$$

Rearranging the above, yields the following equation

$$(\Lambda_2 - \Lambda_{21})((\gamma + \gamma_L)\tau_u - \gamma\tau_v\Lambda_{21}\Lambda_2) = 0. \tag{A.12}$$

One solution to the above equation is $\Lambda_2 = \Lambda_{21}$ which, substituted into (22a) after rearranging yields the following cubic in Λ_2:

$$\varphi(\Lambda_2) \equiv ((\gamma + \gamma_L)\tau_u + \gamma\tau_v\Lambda_2^2)\Lambda_2\tau_v - \tau_u, \tag{A.13}$$

which, since $\varphi(0) < 0$ and $\varphi'(\Lambda_2) > 0$, is easily seen to posses a unique, positive root. Suppose

instead that $\Lambda_{21} \neq \Lambda_2$. In this case, for (A.12) to be satisfied, we need

$$\Lambda_{21}\Lambda_2 = \frac{(\gamma + \gamma_L)\tau_u}{\gamma\tau_v}. \tag{A.14}$$

Solving the above for Λ_{21} and substituting the result into (22a), yields the following quadratic in Λ_2:

$$(\gamma + \gamma_L)\gamma\tau_v\Lambda_2^2 - \gamma\Lambda_2 + (\gamma + \gamma_L)^2\tau_u = 0. \tag{A.15}$$

The roots of the equation are given by

$$\Lambda_2^{*,**} = \frac{\gamma \pm \sqrt{(\gamma - 4(\gamma + \gamma_L)^3\tau_u\tau_v)\gamma}}{2(\gamma + \gamma_L)\gamma\tau_v}.$$

Both roots are positive, which implies that, provided

$$0 < \tau_u\tau_v < \frac{\gamma}{4(\gamma + \gamma_L)^3},$$

there are two additional equilibria of the model and the corresponding value of Λ_{21} obtains by substituting either root into (A.14). Finally, note that when

$$\frac{\gamma}{4(\gamma + \gamma_L)^3} \leq \tau_u\tau_v,$$

the quadratic (A.15) has either two identical solutions $\Lambda_2^* = \Lambda_2^{**} = \Lambda_2 = 1/(2(\gamma + \gamma_L)\tau_v)$, or does not have a real solution, and only the equilibrium with $\Lambda_{21} = \Lambda_2$ obtains. □

Proof of Corollary 7

To analyze the stability properties of the equilibrium in this case, we use the aggregate best response function (A.11) which for $\mu = 1$ has the following expression:

$$\Phi_2(\Lambda_{21}) = \frac{((\gamma + \gamma_L)\tau_u + \Lambda_{21}^2\gamma\tau_v)^2}{\gamma\tau_u + ((\gamma + \gamma_L)\tau_u + \Lambda_{21}^2\gamma\tau_v)^2(\gamma + \gamma_L)\tau_v}. \tag{A.16}$$

1. First, based on the above expression, $\Phi_2(0) > 0$ and differentiating (A.16) with respect to Λ_2 yields:

$$\Phi_2'(\Lambda_{21}) = \frac{4((\gamma + \gamma_L)\tau_u + \Lambda_{21}^2\gamma\tau_v)\gamma^2\Lambda_{21}\tau_u\tau_v}{(\gamma\tau_u + ((\gamma + \gamma_L)\tau_u + \Lambda_{21}^2\gamma\tau_v)^2(\gamma + \gamma_L)\tau_v)^2} > 0, \tag{A.17}$$

   implying that the best response is always upward sloping. Thus, with uniqueness $\Phi_2(\Lambda_{21})$ cuts the 45-degree line from "above" implying that the equilibrium is stable. When multiple equilibria arise, it instead crosses the 45-degree line at three points, with a slope smaller (larger) than one at the two extreme (intermediate) crossings, which correspond to the three equilibria of the market. Hence, with multiplicity, the two extreme equilibria

are stable, while the intermediate one is unstable.

2. Second, evaluating the cubic (A.13) at the low and high roots of the quadratic (A.15) yields

$$\varphi\left(\frac{\gamma-\sqrt{(\gamma-4(\gamma+\gamma_L)^3\tau_u\tau_v)\gamma}}{2(\gamma+\gamma_L)\gamma\tau_v}\right)=\frac{\gamma-4\tau_u\tau_v(\gamma+\gamma_L)^3-\sqrt{\gamma\left(\gamma-4\tau_u\tau_v(\gamma+\gamma_L)^3\right)}}{2\tau_v(\gamma+\gamma_L)^3}<0 \tag{A.18a}$$

$$\varphi\left(\frac{\gamma+\sqrt{(\gamma-4(\gamma+\gamma_L)^3\tau_u\tau_v)\gamma}}{2(\gamma+\gamma_L)\gamma\tau_v}\right)=\frac{\gamma-4\tau_u\tau_v(\gamma+\gamma_L)^3+\sqrt{\gamma\left(\gamma-4\tau_u\tau_v(\gamma+\gamma_L)^3\right)}}{2\tau_v(\gamma+\gamma_L)^3}>0, \tag{A.18b}$$

for $0<\tau_u\tau_v<\gamma/(4(\gamma+\gamma_L)^3)$. Hence, when multiple equilibria arise, the roots of the quadratic equation (A.15) "straddle" the root of the cubic (A.13).

3. Third, taking the product of the two extreme equilibrium values for Λ_2 yields:

$$\frac{\gamma+\sqrt{(\gamma-4(\gamma+\gamma_L)^3\tau_u\tau_v)\gamma}}{2(\gamma+\gamma_L)\gamma\tau_v}\times\frac{\gamma-\sqrt{(\gamma-4(\gamma+\gamma_L)^3\tau_u\tau_v)\gamma}}{2(\gamma+\gamma_L)\gamma\tau_v}=\frac{(\gamma+\gamma_L)\tau_u}{\gamma\tau_v}.$$

Thus, in view of the second expression in (A.29b), at a stable equilibrium we have either that the price reacts more to u_2 than to u_1, or the opposite. Additionally, because of (24), when p_2 reacts more to u_1 than to u_2, the market is also less liquid at the first round.

4. Fourth, evaluating a_2 at the two extreme equilibria, we obtain:

$$a_2|_{\Lambda_2=\Lambda_2^{***}}=-\frac{\gamma+\sqrt{(\gamma-4(\gamma+\gamma_L)^3\tau_u\tau_v)\gamma}}{2(\gamma+\gamma_L)}>a_2|_{\Lambda_2=\Lambda_2^{*}}=\frac{-\gamma+\sqrt{(\gamma-4(\gamma+\gamma_L)^3\tau_u\tau_v)\gamma}}{2(\gamma+\gamma_L)},$$

which always holds within the parameter restriction needed for multiple equilibria to obtain. Given the symmetry of the equilibrium solution, this result also implies that when second period traders consume more liquidity, first period traders consume less of it. Finally, replacing (24) and (A.14) in the expression for a_1 yields:

$$\begin{aligned}a_1&=-\gamma_L\frac{\Lambda_{21}-\Lambda_1}{\Lambda_2^2\tau_u^{-1}}\\&=-\gamma_L\frac{(1-(\gamma+\gamma_L)\tau_v\Lambda_{21})\gamma^2\tau_v^2\Lambda_{21}^3}{(\gamma+\gamma_L)^2\tau_u},\end{aligned} \tag{A.19}$$

implying that also at the first round, liquidity consumption increases in the price impact it induces.

□

Proof of Corollary 9

1. We can interpret the model as one that endogenously yields persistence in noise trading shocks. To see this, note that the equilibrium prices can be written as follows:

$$p_2 = -\Lambda_2\theta_2,\ p_1 = -\Lambda_1\theta_1,$$

where $\theta_1 = u_1$, and

$$\theta_2 = u_2 + \underbrace{\frac{\Lambda_{21}}{\Lambda_2}}_{\beta} u_1.$$

The properties of the noise process are related to the equilibrium that obtains. That is, if

$$\tau_u\tau_v \in (0, \gamma/(4(\gamma+\gamma_L)^3),$$

then $\beta \lessgtr 1$ depending on which equilibrium obtains. If $\tau_u\tau_v \geq \gamma/(4(\gamma+\gamma_L)^3$, $\beta = 1$.

2. We now evaluate the expression for returns autocovariance at the equilibrium with full transparency (and $\mu = 1$):

$$\mathrm{Cov}[p_2 - p_1, p_1] = \frac{\gamma\tau_u^2\tau_v(\gamma+\gamma_L)^5}{\left(\tau_u\tau_v\left(2\gamma^2+4\gamma\gamma_L+\gamma_L^2\right)(\gamma+\gamma_L) + \tau_u^2\tau_v^2(\gamma+2\gamma_L)(\gamma+\gamma_L)^4 + \gamma\right)^2} \tag{A.20}$$

and at both the equilibria that obtain under the parameter restriction ensuring multiplicity, when the market is fully opaque, when $\Lambda_{21} = \Lambda_{21}^*$ we have

$$\mathrm{Cov}[p_2 - p_1, p_1] = \frac{\left(\gamma - \sqrt{\gamma\left(\gamma - 4\tau_u\tau_v(\gamma+\gamma_L)^3\right)}\right)^3\left(\sqrt{\gamma\left(\gamma - 4\tau_u\tau_v(\gamma+\gamma_L)^3\right)} + \gamma\right)}{16\gamma^4\tau_u\tau_v^2(\gamma+\gamma_L)^2}, \tag{A.21}$$

and when $\Lambda_{21} = \Lambda_{21}^{***}$ we have instead

$$\mathrm{Cov}[p_2 - p_1, p_1] = \frac{\left(\gamma - \sqrt{\gamma\left(\gamma - 4\tau_u\tau_v(\gamma+\gamma_L)^3\right)}\right)\left(\sqrt{\gamma\left(\gamma - 4\tau_u\tau_v(\gamma+\gamma_L)^3\right)} + \gamma\right)^3}{16\gamma^4\tau_u\tau_v^2(\gamma+\gamma_L)^2}. \tag{A.22}$$

Comparing the two latter expressions shows that return autocovariance is higher when $\Lambda_{21} = \Lambda_{21}^*$. While comparing the latter expression above with (A.20) shows that it increases with respect to the case with full transparency.

□

Proof of Proposition 3

This proof works for the general case in which $\tau_\eta \in (0, \infty)$ and $\mu \in (0, 1]$. To concentrate on the effect of order flow transparency, set $\mu = 1$ in the equilibrium expressions.

Based on the market clearing condition (3), to pin down p_2 we need the strategies of first and second period traders, and liquidity providers. We work by backward induction. In the second period, CARA and normality assumptions imply that the objective function of a liquidity trader is given by

$$E_2[-\exp\{-\pi_2/\gamma_L\}] = -\exp\left\{-\frac{1}{\gamma_L}\left(E_2[\pi_2] - \frac{1}{2\gamma_L}\text{Var}_2[\pi_2]\right)\right\}, \tag{A.23}$$

where $\pi_2 \equiv (v-p_2)x_2 + u_2 v$. Maximizing (A.23) with respect to x_2, and solving for the optimal strategy yields:

$$x_2 = \gamma_L \frac{E_2[v-p_2]}{\text{Var}_2[v-p_2]} - \frac{\text{Cov}_2[v-p_2, v]}{\text{Var}_2[v-p_2]} u_2, \tag{A.24}$$

where,

$$E_2[v-p_2] = \Lambda_2 u_2 + \frac{\Lambda_{21}\tau_\eta + \Lambda_{22}\tau_u}{\tau_\eta + \tau_u} s_{u_1} \tag{A.25a}$$

$$\text{Var}_2[v-p_2] = \frac{1}{\tau_v} + \frac{(\Lambda_{21}-\Lambda_{22})^2}{\tau_\eta + \tau_u} \tag{A.25b}$$

$$\text{Cov}_2[v-p_2, v] = \frac{1}{\tau_v}. \tag{A.25c}$$

Substituting (A.25a) and (A.25c) in (A.24), and rearranging yields:

$$x_2 = \underbrace{\frac{\gamma_L \tau_v \Lambda_2 - 1}{\tau_v \text{Var}_2[v-p_2]}}_{a_2} u_2 + \underbrace{\gamma_L \frac{\Lambda_{21}\tau_\eta + \Lambda_{22}\tau_u}{(\tau_\eta + \tau_u)\text{Var}_2[v-p_2]}}_{b} s_{u_1}. \tag{A.26}$$

A FD maximizes the expected utility of his second period wealth:

$$\begin{aligned} &E_2^{FD}\left[-\exp\left\{-\frac{1}{\gamma}\left((p_2-p_1)x_1^{FD} + (v-p_2)x_2^{FD}\right)\right\}\right] = \\ &= \exp\left\{-\frac{1}{\gamma}(p_2-p_1)x_1^{FD}\right\}\left(-\exp\left\{-\frac{1}{\gamma}\left(E_2^{FD}[v-p_2]x_2^{FD} - \frac{(x_2^{FD})^2}{2\gamma}\text{Var}_2^{FD}[v-p_2]\right)\right\}\right). \end{aligned} \tag{A.27}$$

For given x_1^{FD} the above is a concave function of the second period strategy x_2^{FD}. Solving the first order condition, yields that a second period FD's strategy is given by:

$$x_2^{FD} = \gamma \frac{E_2^{FD}[v-p_2]}{\text{Var}_2^{FD}[v-p_2]}. \tag{A.28}$$

Computing expectation and variance in the above expression:

$$E_2^{FD}[v-p_2] = -p_2 \tag{A.29a}$$

$$\text{Var}_2^{FD}[v-p_2] = \frac{1}{\tau_v}, \tag{A.29b}$$

and substituting these in x_2^{FD} yields:

$$x_2^{FD} = -\gamma \tau_v p_2. \tag{A.30}$$

Similarly, due to CARA and normality, in the first period an RD maximizes

$$E_1^{RD}\left[-\exp\left\{ -\frac{1}{\gamma}(v - p_1)x_1^{RD} \right\}\right] = \tag{A.31}$$
$$-\exp\left\{ -\frac{1}{\gamma}\left(E_1^{RD}[v - p_1]x_1^{RD} - \frac{(x_1^{RD})^2}{2\gamma}\mathrm{Var}_1^{RD}[v - p_1] \right)\right\}.$$

Maximizing the above and solving for x_1^{RD} yields:

$$x_1^{RD} = \gamma \frac{E_1^{RD}[v - p_1]}{\mathrm{Var}_1^{RD}[v - p_1]}. \tag{A.32}$$

Computing the conditional expectation and variance:

$$E_1^{RD}[v - p_1] = -p_1 \tag{A.33a}$$
$$\mathrm{Var}_1^{RD}[v - p_1] = \frac{1}{\tau_v}, \tag{A.33b}$$

so that

$$x^{RD} = -\gamma \tau_v p_1. \tag{A.34}$$

At the second round, first and second period traders face the same utility maximization problem. This is because they both need to hedge the endowment shock, and have only one round to go. As a consequence, a first period trader's strategy reads as follows:

$$\begin{aligned} x_{21} &= \gamma_L \frac{E_1[v - p_2]}{\mathrm{Var}_1[v - p_2]} - \frac{\mathrm{Cov}_1[v, v - p_2]}{\mathrm{Var}_1[v - p_2]} u_1 \\ &= \underbrace{\frac{\gamma_L \Lambda_{21} \tau_v - 1}{\tau_v \mathrm{Var}_1[v - p_2]}}_{a_{21}} u_1, \end{aligned} \tag{A.35}$$

where

$$\mathrm{Var}_1[v - p_2] = \frac{1}{\tau_v} + \frac{\Lambda_2^2}{\tau_u} + \frac{\Lambda_{22}^2}{\tau_\eta}. \tag{A.36}$$

Substituting (A.26), (A.30), (A.34), and (A.35) in (3), solving for p_2 and identifying the equi-

librium price coefficients yields:

$$\Lambda_2 = -\frac{a_2}{\mu\gamma\tau_v} \tag{A.37a}$$

$$\Lambda_{21} = -\frac{b + a_{21} + (1-\mu)\gamma\tau_v\Lambda_1}{\mu\gamma\tau_v} \tag{A.37b}$$

$$\Lambda_{22} = -\frac{b}{\mu\gamma\tau_v} \tag{A.37c}$$

According to (A.37a), at an equilibrium

$$\Lambda_2 = \frac{1}{(\gamma_L + \mu\gamma\tau_v \mathrm{Var}_2[v - p_2])\tau_v},$$

so that at equilibrium $\Lambda_2 > 0$, and $\gamma_L\Lambda_2\tau_v < 1$. Based on the expression for a_2 in (A.26), this implies that

$$a_2 \in (-1, 0). \tag{A.38}$$

To pin down the first period equilibrium price, we use the expression for liquidity providers' strategies in the first period market clearing equation and turn to determine x_{11}. To obtain it, we start from the second period value function of a first period trader, substituting (A.35) into the trader's objective function:

$$E[-\exp\{-((v-p_2)x_{21}+vu_1)/\gamma_L\}|u_1] = -\exp\{-(\mathrm{Var}_1[v-p_2]x_{21}^2 - \mathrm{Var}[v]u_1^2)/2\gamma_L^2\}, \tag{A.39}$$

so that at the first round, the objective function reads as follows:

$$\begin{aligned} &E[-\exp\{-\pi_1/\gamma_L\}|u_1] \\ &\quad = E[-\exp\{-((p_2-p_1)x_1 + (\mathrm{Var}_1[v-p_2]a_{21}^2u_1^2 - \mathrm{Var}[v]u_1^2)/2\gamma_L)/\gamma_L\}|u_1] \\ &\quad = E[-\exp\{-((p_2-p_1)x_1 + \underbrace{((\mathrm{Var}_1[v-p_2]a_{21}^2 - \mathrm{Var}[v])/2\gamma_L)}_{C}u_1^2)/\gamma_L\}|u_1], \end{aligned} \tag{A.40}$$

where $\pi_1 = vu_1 + (v-p_2)x_{21} + (p_2-p_1)x_{11}$. Using the expression for p_2 in (1b), the argument of the exponential in the latter expression of (A.40) can be written as follows:

$$(p_2-p_1)x_{11} + Cu_1^2 = -(\Lambda_{21}-\Lambda_1)u_1x_{11} + Cu_1^2 - (\Lambda_2u_2 + \Lambda_{22}\eta)x_{11}, \tag{A.41}$$

which is a quadratic form of the normal random variable $Z \equiv -(\Lambda_2u_2+\Lambda_{22}\eta)|u_1 \sim N(0, \mathrm{Var}_1[p_2-p_1])$ (the constant multiplying the squared term of Z in the quadratic form is in this case null), where

$$\mathrm{Var}_1[p_2-p_1] = \mathrm{Var}_1[p_2] = \Lambda_2^2\tau_u^{-1} + \Lambda_{22}^2\tau_\eta^{-1}. \tag{A.42}$$

We can then write the objective function of a trader at the first round as follows:

$$E[-\exp\{-\pi_1/\gamma_1\}|u_1] = -\exp\{-(-(\Lambda_{21}-\Lambda_1)u_1x_{11}+Cu_1^2-x_{11}^2\text{Var}_1[p_2-p_1]/2\gamma_L)/\gamma_L\}. \quad \text{(A.43)}$$

Maximizing the above function with respect to x_{11} yields

$$x_{11} = \underbrace{-\gamma_L \frac{\Lambda_{21}-\Lambda_1}{\text{Var}_1[p_2]}}_{a_1} u_1. \quad \text{(A.44)}$$

Recall that at the first trading round an FD's strategy is given by:

$$x_1^{FD} = -\gamma \frac{\Lambda_{21}-\Lambda_1}{\text{Var}_1[p_2]} u_1 - \gamma\tau_v p_1. \quad \text{(A.45)}$$

Comparing (A.45) with (A.44) shows that in this model at the first round FD and liquidity traders submit the same type of market order. That is, we can think of the strategy of a liquidity trader as being similar to the "directional bet" part of the FD strategy (more on this in section 2).

Substituting (A.34), (A.44) and (A.45) into the first period market clearing condition (2) and identifying the equilibrium price coefficient yields:

$$\Lambda_1 = -\frac{\mu\gamma+\gamma_L}{\gamma\gamma_L\tau_v} a_1. \quad \text{(A.46)}$$

We have already signed Λ_2. To sign the remaining price coefficients, we substitute the expressions for the strategy coefficients into (A.37b), (A.37c), and (A.46), obtaining:

$$\Lambda_{21} = \left(1 + \frac{\tau_\eta}{\mu\gamma\tau_v(\tau_u+\tau_\eta)\text{Var}_2[v-p_2]+\gamma_L\tau_u} + \frac{\gamma_L\tau_u}{\mu\gamma\tau_u\tau_v\text{Var}_1[v-p_2]}\right)^{-1} \times \left(\frac{1}{\tau_u\text{Var}_1[v-p_2]} - (1-\mu)\gamma\tau_v\Lambda_1\right)\frac{1}{\mu\gamma\tau_v} \quad \text{(A.47a)}$$

$$\Lambda_{22} = -\frac{\Lambda_{21}\tau_\eta}{\mu\gamma\tau_v(\tau_u+\tau_\eta)\text{Var}_2[v-p_2]+\gamma_L\tau_u} \quad \text{(A.47b)}$$

$$\Lambda_1 = \frac{(\mu\gamma+\gamma_L)\Lambda_{21}\tau_u\tau_\eta}{(\mu\gamma+\gamma_L)\tau_u\tau_\eta + (\Lambda_{22}^2\tau_u+\Lambda_2^2\tau_\eta)\gamma\tau_v}. \quad \text{(A.47c)}$$

Note that from (A.47c), the sign of Λ_{21} coincides with that of Λ_1. Now, suppose that $\Lambda_{21} \leq 0$, then this implies that $\Lambda_1 \leq 0$. However, because of (A.47a), we then have that $\Lambda_{21} > 0$, which is a contradiction. Once we have signed Λ_{21}, because of (A.47b), we know that $\Lambda_{22} < 0$, and by computing $\Lambda_{21} - \Lambda_1$ with (A.47c), we obtain $\Lambda_{21} - \Lambda_1 > 0$. □

A.1 Welfare analysis

Denoting by EU^{FD}, EU^{RD}, and EU_t^L, respectively the unconditional expected utilities of FD, RD and round $t \in \{1,2\}$ liquidity traders, we measure traders' payoffs by computing the certainty equivalents:

$$CE^{FD} = -\gamma \ln(-EU^{FD}),\ CE^{RD} = -\gamma \ln(-EU^{RD}), CE_t^L = -\gamma_L \ln(-EU_t^L).$$

The next result provides the expressions for traders' payoffs.

Proposition 4. *If an equilibrium exists, assuming*

$$\gamma_L^2 \tau_u \tau_v > 1, \tag{A.48}$$

traders' payoffs are given by

$$CE^{FD} = \frac{\gamma}{2}\left(\ln\left(1 + \frac{\text{Var}[E[v-p_1|p_1]]}{\text{Var}[v-p_1|p_1]} + \frac{\text{Var}[E[p_2-p_1|p_1]]}{\text{Var}[p_2-p_1|p_1]}\right) + \ln\left(1 + \frac{\text{Var}[E[v-p_2|p_1,p_2]]}{\text{Var}[v-p_2|p_1,p_2]}\right)\right) \tag{A.49a}$$

$$CE^{SD} = \frac{\gamma}{2} \ln\left(1 + \frac{\text{Var}[E[v-p_1|p_1]]}{\text{Var}[v-p_1|p_1]}\right) \tag{A.49b}$$

$$CE_1^L = \frac{\gamma_L}{2} \ln\left(1 + \frac{(a_1^2 \text{Var}_1[p_2] + a_{21}^2 \text{Var}_1[v-p_2])\tau_v - 1}{\gamma_L^2 \tau_u \tau_v}\right) \tag{A.49c}$$

$$CE_2^L = \frac{\gamma_L}{2} \ln\left(1 + \frac{a_2^2 \text{Var}_2[v-p_2]\tau_v - 1}{\gamma_L^2 \tau_u \tau_v} + \frac{b^2 \text{Var}[s_{u_1}](\gamma_L^2 \tau_u \tau_v - 1)}{\gamma_L^4 \tau_u \tau_v}\right). \tag{A.49d}$$

Proof. The expressions in (A.49a) and (A.49b) come straight from the ones we derived in the platforms' paper.

To obtain the expression for first period liquidity traders' payoff in (A.49c), we replace the strategy (A.44) into the objective function (A.43) and rearrange the result, to obtain

$$E[-\exp\{-\pi_1/\gamma_1\}|u_1] = -E\left[\exp\left\{-\frac{u_1^2}{\gamma_L}\left(\frac{\gamma_L(\Lambda_1 - \Lambda_{21})^2}{2\text{Var}_1[p_2]} + \frac{a_{21}^2 \tau_v \text{Var}_1[v-p_2] - 1}{2\gamma_L \tau_v}\right)\right\}\right]. \tag{A.50}$$

The argument in the above expression is a quadratic form of the normal random variable $u_1 \sim N(0, \tau_u^{-1})$. Thus, to compute the unconditional expectation of (A.50), we apply the usual Lemma to obtain

$$E[-\exp\{-\pi_1/\gamma_L\}] = -\left(\frac{\gamma_L^2 \tau_u \tau_v}{\gamma_L^2 \tau_u \tau_v - 1 + (a_1^2 \text{Var}_1[p_2] + a_{21}^2 \text{Var}_1[v-p_2])\tau_v}\right)^{1/2}. \tag{A.51}$$

To obtain the certainty equivalent, we compute

$$\begin{aligned} CE_1^L &= -\gamma_L \ln\left(-E[-\exp\{-\pi_1/\gamma_1\}]\right) \\ &= \frac{\gamma_L}{2}\ln\left(1+\frac{(a_1^2\mathrm{Var}_1[p_2]+a_{21}^2\mathrm{Var}_1[v-p_2])\tau_v-1}{\gamma_L^2\tau_u\tau_v}\right). \end{aligned} \tag{A.52}$$

To obtain the payoff of second period liquidity traders we proceed similarly by replacing their equilibrium strategy into their objective function:

$$E_2[-\exp\{-(1/\gamma_L)((v-p_2)x_2+vu_2)\}] = -\exp\left\{-\frac{1}{\gamma_L}\left(\frac{\mathrm{Var}_2[v-p_2]x_2^2-\mathrm{Var}[v]u_2^2}{2\gamma_L}\right)\right\}. \tag{A.53}$$

The argument of the exponential at the right hand side of the above expression is a quadratic form of the normally distributed random vector

$$\begin{pmatrix} x_2 \\ u_2 \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \underbrace{\begin{pmatrix} \mathrm{Var}[x_2] & a_2/\tau_u \\ a_2/\tau_u & \tau_u^{-1} \end{pmatrix}}_{\Sigma}\right).$$

Indeed, we have

$$\frac{\mathrm{Var}_2[v-p_2]x_2^2-\mathrm{Var}[v]u_2^2}{2\gamma_L} = \frac{1}{2\gamma_L}\begin{pmatrix} x_2 & u_2 \end{pmatrix}\underbrace{\begin{pmatrix} \mathrm{Var}_2[v-p_2] & 0 \\ 0 & -\mathrm{Var}[v] \end{pmatrix}}_{A}\begin{pmatrix} x_2 \\ u_2 \end{pmatrix}.$$

Applying the usual lemma, we then have

$$\begin{aligned} E\left[-\exp\left\{-\frac{1}{\gamma_L}\left(\frac{\mathrm{Var}_2[v-p_2]x_2^2-\mathrm{Var}[v]u_2^2}{2\gamma_L}\right)\right\}\right] &= -\left|I+(2/\gamma_L)\Sigma A\right|^{-1/2} \\ &= -\left(\frac{\gamma_L^4\tau_u^2\tau_v}{a_2^2\mathrm{Var}_2[v-p_2]+(\mathrm{Var}_2[v-p_2]\mathrm{Var}[x_2]+\gamma_L^2)(\gamma_L^2\tau_u\tau_v-1)\tau_u}\right)^{1/2}. \end{aligned} \tag{A.54}$$

Finally, the certainty equivalent obtains by computing

$$\begin{aligned} CE_2^L &= -\gamma_L \ln\left(-E[-\exp\{-\pi_2/\gamma_L\}]\right) \\ &= \frac{\gamma_L}{2}\ln\left(1+\frac{a_2^2\mathrm{Var}_2[v-p_2]\tau_v-1}{\gamma_L^2\tau_u\tau_v}+\frac{b^2\mathrm{Var}[s_{u_1}](\gamma_L^2\tau_u\tau_v-1)}{\gamma_L^4\tau_u\tau_v}\right). \end{aligned} \tag{A.55}$$

□

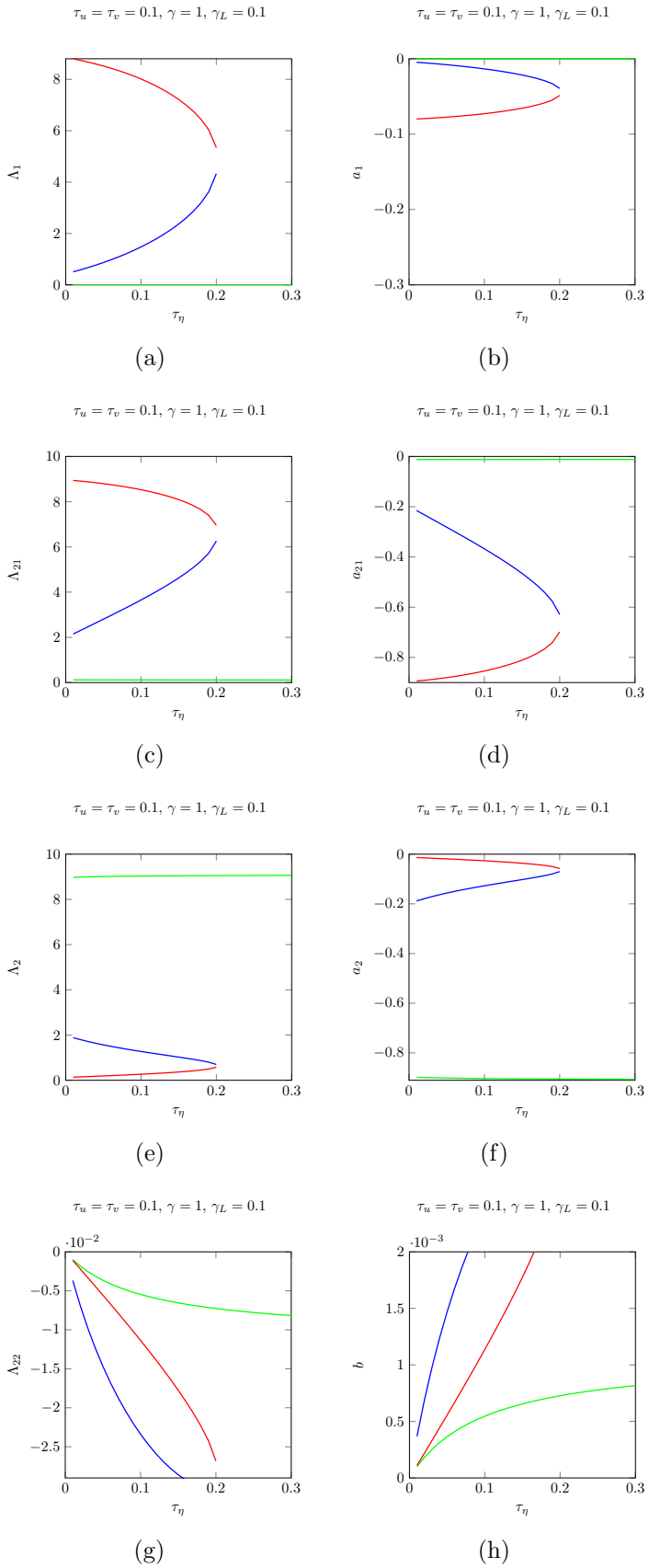


Figure 6: Price impact coefficients (panel (a), (c), (e), (g)) and strategy coefficients in the general case. Parameter values are as in Figure 4 except for $\tau_\eta \in \{0.01, 0.02, \ldots, 1\}$.

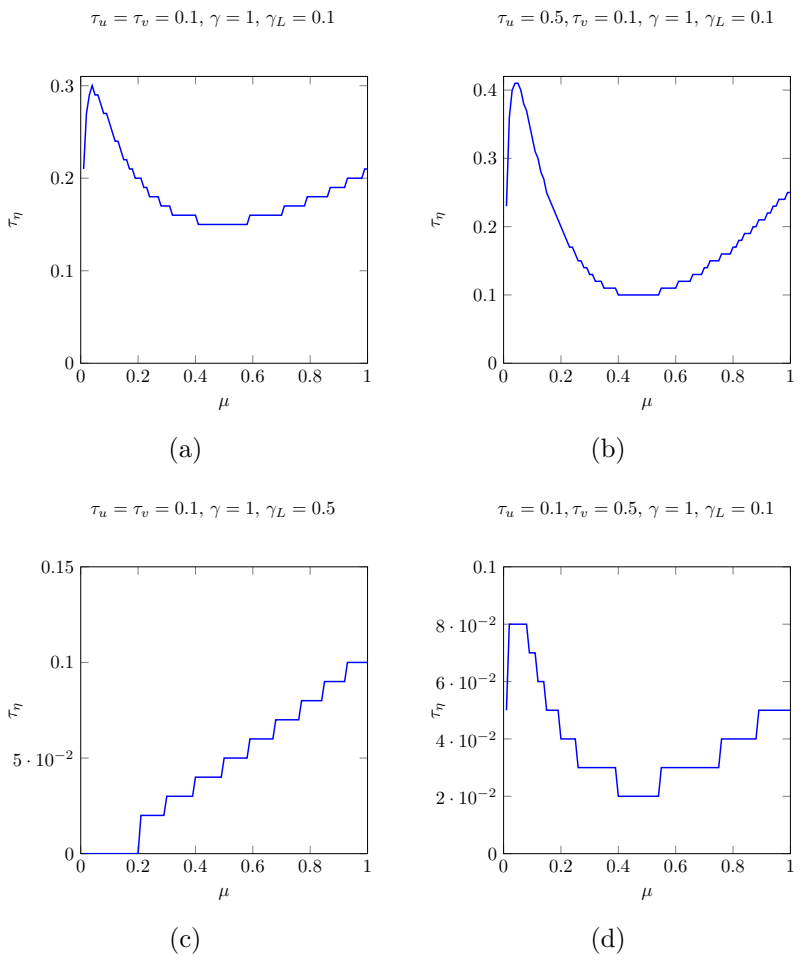


Figure 8: The region above (below) the curve captures values of (μ, τ_η) for which a unique equilibrium (multiple equilibria) obtain.

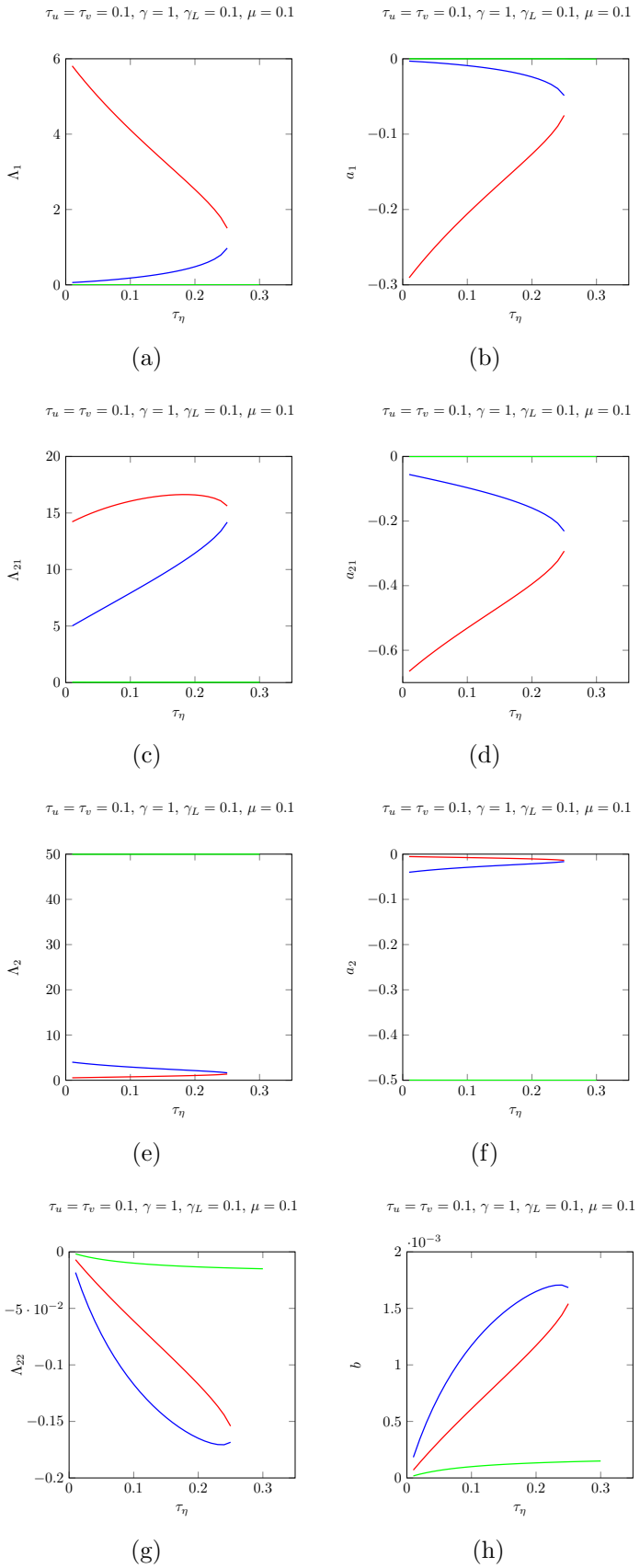


Figure 9: Price impact coefficients (panel (a), (c), (e), (g)) and strategy coefficients in the general case. Parameter values are as in Figure 4 except for $\mu = 0.1$ and $\tau_\eta \in \{0.01, 0.02, \ldots, 1\}$.

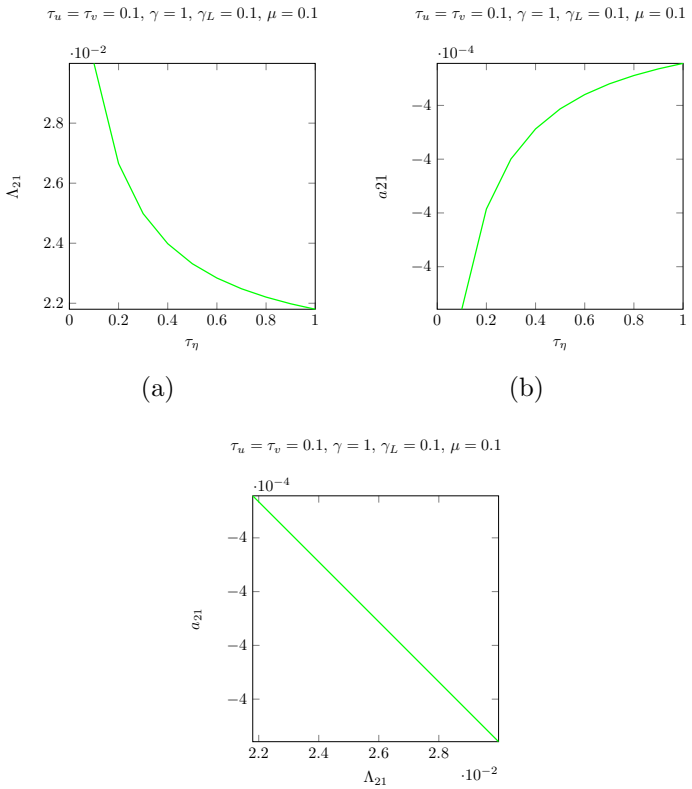


Figure 10: Price impact coefficient of u_1 at the second round and strategy coefficient of first period traders when trading at the second round (respectively, Panel (a) and (b)). In Panel (c) we plot a_{21} against the price impact it generates. Parameter values are as in Figure 9.