# Socially Responsible Divestment*

Alex Edmans
LBS, CEPR, and ECGI

Doron Levit
University of Washington and ECGI

Jan Schneemeier
Indiana University

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**Abstract**

Blanket exclusion of "brown" stocks is seen as the best way to reduce their negative externalities by starving them of capital. We show that a more effective strategy may be tilting – holding a brown stock if the firm has taken a corrective action. While such holdings allow the firm to expand, they also encourage the action. We derive conditions under which tilting dominates exclusion for externality reduction. If the action is not publicly observable, the investor might not tilt even if she has perfect information – doing so would lead her to hold a company that has reformed but the market thinks it has not, leading to accusations of greenwashing. The presence of an arbitrageur who buys underpriced stocks increases the relative effectiveness of tilting. A responsible investor who is partially profit-motivated may be more likely to tilt than one whose sole objective is minimizing externalities.

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*aedmans@london.edu, dlevit@uw.edu, jschnee@iu.edu. We thank Tom Gosling, Evgeny Lyandres and seminar/conference participants at UNC and the ECGI conference on Corporate Purpose, Stakeholderism and ESG for valued input.

Responsible investing – the practice of incorporating environmental, social, and governance ("ESG") factors into investment decisions – is becoming increasingly mainstream. In 2006, the United Nations established the Principles for Responsible Investment ("UN PRI"), which was signed by 63 investors managing a total of $6.5 trillion. By the end of 2021, this had grown to 4,375 investors, representing $121 trillion.

One goal of responsible investing is to improve risk-adjusted returns, by incorporating ESG factors that are not fully priced by the market. However, critics argue that this is simply investing, not responsible investing (e.g. Mackintosh, 2022). The more distinctive goal is to improve companies' ESG performance through two channels – engagement and divestment. The latter involves selling "brown" companies that exert negative externalities and buying "green" companies – indeed, the first of the UN PRI's principles is "We will incorporate ESG issues into investment analysis and decision-making processes". Doing so increases the cost of capital of brown companies, hindering their expansion, while helping green firms to grow.

Under this channel, the most powerful investment strategy is blanket exclusion of externality-producing industries. Nobel Peace Prize winner Desmond Tutu has called for outright divestment from the fossil fuel industry, similar to the anti-apartheid divestment campaign from South Africa in the 1980s. 1,500 institutions, collectively managing $40 trillion have publicly committed to divest from fossil fuels. Practitioners and the general public hold investors accountable for their holdings of brown firms. In 2020, Extinction Rebellion protesters dug up a lawn outside Trinity College, Cambridge in protest of its investment in fossil fuel companies, and many asset owners evaluate asset managers according to whether they manage a "net zero" portfolio. Beyond climate, Morningstar's "globe" ratings of funds are based on the Sustainalytics ESG scores of the stocks they hold and are thus boosted by divesting from brown stocks; Hartzmark and Sussman (2019) find that fund flows are significantly influenced by these ratings.[1] Academic studies of greenwashing by asset managers similarly analyze their portfolio holdings (e.g. Gibson et al. (2022), Kim and Yoon (2021), and Liang, Sun, and Teo (2021)). Gibson et al. (2022) define the goal of responsible investing as "to direct capital towards companies that make the world more sustainable"; under this goal, the average ESG rating of portfolio companies is indeed the relevant measure.

However, this argument considers only one channel through which divestment can affect a

[1] While some rating providers industry-adjust their ratings, others do not, so the average rating of a "brown" industry is lower than of a "green" industry.

company's real actions – the primary markets channel, whereby divestment affects new capital raising. As the survey of Bond, Edmans, and Goldstein (2012) points out, investment decisions can also have real effects through a secondary markets channel. Specifically, trading leads to the stock price reflecting a manager's real actions, thus rewarding or punishing him for taking them. Even if a firm is in an irremediably brown sector, where externalities are unavoidably negative, the manager may be able to take corrective actions to mitigate these externalities. Blanket exclusion fails to reward such actions because the firm is divested no matter what. Thus, it may be optimal for a responsible investor to pursue a "tilting" strategy, where she tilts away from a brown industry but is willing to hold firms that take corrective actions.

We build a model in which responsible investment affects firm behavior through both above channels. There is a single brown firm that emits negative externalities. The firm's manager can take a non-contractible corrective action, such as investing in clean energy, that reduces both externalities and also firm value. The firm also raises capital which it uses to fund an expansion, increasing both firm value and externalities. The firm's manager is concerned with both fundamental value and the stock price; the latter may arise through takeover threat, termination threat, or reputational concerns.

The firm is owned by a continuum of risk-averse, profit-motivated, atomistic investors ("households") and a risk-neutral responsible investor. The responsible investor is able to take large positions and have price impact, and so we refer to her as a blockholder. Her objective is to minimize the externalities produced by the firm. To do so, she announces an investment strategy that depends on whether the firm takes a corrective action. Under exclusion, the blockholder never holds the firm; under tilting, she invests if and only if it takes the action. In the core model, we assume that the blockholder can commit to her investment strategy. For example, some funds advertise themselves as boycotting certain industries; deviation will lead to client withdrawals and potentially regulatory action. Other funds state that they have a tilting strategy which involves investing in leaders in controversial industries. Deviating and excluding entire industries may increase tracking error, reduce risk-adjusted returns, or lead to investors withdrawing to cheaper passive funds that pursue exclusion.

We show that the optimal divestment strategy balances two forces. On the one hand, since the brown firm continues to produce negative externalities even under the corrective action, the blockholder wishes to minimize its size. She does so through blanket exclusion – by holding none of the brown firm's shares, they have to be held entirely by risk-averse households, who

require a risk premium for doing so. This minimizes the stock price, as in Heinkel, Kraus, and Zechner (2001), and thus the new funds the firm can raise. On the other hand, the investor wishes to incentivize the action. Exclusion provides no such incentives, since the firm is always divested. Tilting rewards the manager for taking the action – by buying shares, the blockholder reduces the number that must be held by households, thus increasing the stock price.

Intuitively, the blockholder's strategy is analogous to an incentive contract. Exclusion corresponds to paying the manager a flat salary, which minimizes the cost to the firm but provides no incentives. Tilting incentivizes the action, but is costly – in a contracting setting, the cost is the monetary value of the incentive; in a responsible investment setting, the cost is financing the expansion of a brown firm. This analogy highlights how exclusion may be suboptimal, despite being widely advocated – it is tantamount to giving zero incentives.

We show that the optimal investment strategy involves tilting if the action is effective at reducing the externality, because incentivizing it is particularly important compared to stifling capital raising. Tilting is also optimal if the action is less costly and if the manager's stock price concerns are high, as then the blockholder does not need to offer large share purchases to incentivize the action; thus, the additional expansion and externalities created are low. These results suggest that exclusion may be optimal for industries such as controversial weapons , where it is difficult to reduce the harm produced. In contrast, tilting may be preferred for fossil fuels, where managers can take corrective actions such as developing clean energy[2], and the net cost of these actions may not be high – while developing clean energy requires substantial investment, it also generates significant future cash flows.

One might also think that exclusion is optimal if the firm is raising large amounts of new capital, because it is particularly important to stifle capital raising. This turns out to be not always the case due to an opposing force. The effect of the action is plausibly multiplicative in firm size – reducing the per-unit amount of pollution has a greater impact in a firm that produces more. If the firm is raising large amounts of new capital, it becomes even more important to induce the corrective action. Overall, the amount of capital raised has an ambiguous effect on the optimal investment strategy; for similar reasons, the profitability of the investment opportunity has an ambiguous effect.

We extend the model to the case in which the corrective action is unobservable, so the

[2]Cohen, Gurun, and Nguyen (2021) show that the fossil fuel industry produces more green patents than nearly any other sector, suggesting that companies within this industry can take corrective actions.

investor is unable to condition her investment on it; instead, she can condition it on a noisy public signal of the action. The noisier the signal, the greater the reward the investor needs to offer to induce the action, and the more likely she is to choose exclusion. This result highlights a new benefit of ESG disclosure – it allows investors to induce corrective actions without having to promise large amounts of capital.

Importantly, even if the blockholder can gather perfect information about the manager's action at an arbitrarily small cost, she may not do so. It may seem that such information will allow her to induce the action at lower cost, i.e. promise a lower investment – since the blockholder will always make the investment if the manager has taken the action, he will do so even if the investment is small. However, the blockholder may end up buying a company that has taken the action even though the public signal suggests that it has not. Doing so may lead to the blockholder being accused of greenwashing – buying a brown firm even though, in the eyes of the market, it has not taken mitigating steps. If the blockholder suffers a sufficiently large reputational cost from doing so, she will not base her purchases on her private information. This reduces her incentives to gather it in the first place, and may deter her from inducing the corrective action.

If the manager is able to increase the precision of the public signal through disclosure, he will do so if his stock price concerns are sufficiently high, as then he benefits from the blockholder's purchases if he has taken the action. It might seem that he will disclose a perfect signal, so that he will be given full credit for his action. However, he actually discloses a noisy signal, so that the blockholder has to promise a large investment to induce the action.

A common criticism of divestment is that arbitrageurs can buy divested stocks, attenuating the price impact. We introduce an arbitrageur who has the sole objective of maximizing trading profits. He buys half the shares that are not purchased by the blockholder, thus lessening the impact of her trading decisions. On the one hand, this makes tilting less effective – since the arbitrageur partially offsets the blockholder's trades, he needs to promise an even larger purchase to induce the corrective action, making tilting more costly to implement. On the other hand, the arbitrageur makes exclusion less effective, since she buys up underpriced stock and reduces the impact of exclusion on the cost of capital. Since the arbitrageur buys half of the free float, his impact is greater on exclusion (where the blockholder's trade is zero and the free float is the total shares outstanding) than on tilting. Therefore, tilting is more likely to be the optimal divestment strategy in the presence of arbitrageurs.

In our final extension, the blockholder's objective function includes trading profits as well as externalities. Our core result, that the blockholder tilts if the corrective action is sufficiently effective, continues to hold. Moreover, and surprisingly, there are conditions under which the blockholder will induce the action if she is partially profit-motivated but not if she is only concerned with externalities. In particular, if tilting leads to more externalities, a responsible investor will choose exclusion, but a profit-motivated blockholder may tilt as this involves buying shares from risk-averse households and thus earning a premium for risk-bearing. We also consider the case in which the blockholder is unable to commit to a trading strategy. In the absence of commitment, a blockholder concerned only with externalities cannot induce the action. Once the action has been taken, she cannot change it and thus has no incentive to buy shares – doing so will help the firm expand. Thus, any promise to buy shares upon the corrective action is non-credible. However, a sufficiently profit-motivated investor will buy shares to earn trading profits. She is more willing to do so if the action has been taken, as the action minimizes the additional externalities created by her share purchases. Therefore, a profit motivation makes it credible for the blockholder to tilt (i.e. buy more shares if the action has been taken than if it has not), thus allowing her to induce the action.

This paper is related to the theoretical literature on responsible investing. Heinkel, Kraus, and Zechner (2001) show that divestment reduces the stock price by increasing the shares held by risk-averse investors. Davies and Van Wesep (2018) demonstrate that the resulting lower price raises the number of shares granted to the manager if his equity-based pay is fixed in dollar terms, paradoxically rewarding him. Oehmke and Opp (2020) show that responsible investing is only effective if responsible investors are affected by externalities regardless of whether they own the emitting companies, and if they can co-ordinate. Pedersen, Fitzgibbons, and Pomorski (2021) focus on the asset pricing implications of responsible investing and solve for the ESG-efficient frontier. Goldstein et al. (2022) show that responsible investors can increase the cost of capital, because their trades reflect ESG rather than financial performance, thus making the stock price less informative about financials. The above papers do not involve new financing and investment, so the lower stock price from divestment has no real effects. Pastor, Stambaugh, and Taylor (2021) model how greater taste for green companies increases their valuation and reduces equilibrium expected returns. While firms make investment decisions, they are financed by internal cash flow and so there is no primary markets channel through which the stock price

affects investment.[3] The above papers do not model externalities or study different strategies pursued by responsible investors; instead, investors' demands are automatic given their tastes. Landier and Lovo (2020) find that the more money investors put into ESG funds, the more important it is for an industry to reduce its externalities to obtain financing. The only existing argument against divestment of which we are aware is that it hinders an investor's ability to engage. However, many investors rarely engage – their expertise may be stock selection rather than engagement, or they lack the substantial financial resources needed. For example, Engine No. 1 spent $30 million electing three climate-friendly directors onto Exxon's board compared to its stake of $50 billion.

Relative to the above literature, a unique feature of our model is the incorporation of a secondary market channel through which responsible investing affects externalities. This is related to models on "governance through exit", such as Admati and Pfleiderer (2009), Edmans (2009), and Edmans and Manso (2011), where investor trading causes a manager's actions to be reflected in the stock price. Those papers do not feature primary markets channels through which trading may have real effects; in addition, they are not models of responsible investing as investors' objective is to maximize trading profits and firms produce no externalities.

Some empirical studies question the effectiveness of exclusion as a responsible investing strategy. Teoh, Welch, and Wazzan (1999) show that the South Africa divestment campaign had a negligible effect on company valuations. Berk and van Binsbergen (2021) calculate that ESG-motivated exclusion has little effect on the cost of capital because arbitrageurs can buy the underpriced stocks; our model incorporates arbitrageurs and show that they make tilting relatively more effective than exclusion. Gantchev, Giannetti, and Li (2022) show that the threat of exit following negative environmental and social ("E&S") incidents disciplines managers to improve E&S performance. Exit is only possible if the investor is willing to hold brown firms in the first place.

[3]Instead, the stock price affects investment because investors dislike holding brown stocks, and so demand a higher return for holding them, which reduces the stock price. However, the stock price does not affect the fundamental value created by the investment, unlike in our model where it affects the amount of new capital raised.

# 1 The Model

## 1.1 Players and Timing

We consider a single firm with a risk-neutral manager ("$M$"). The firm is in a "brown" industry and thus emits externalities, to be specified later. The initial number of shares is normalized to one. The financial market consists of a continuum of risk-averse, profit-motivated, atomistic investors, indexed by $i \in [0,1]$, and a risk-neutral responsible investor that aims to minimize the externalities produced by the firm. The responsible investor has the ability to take large positions and thus have price impact, and so we refer to it as a blockholder ("$B$").

There are four dates $t \in \{0,1,2,3\}$. At $t=0$, $B$ announces an investment strategy $x(a)$ that depends on a publicly-observable action $a \in \{0,1\}$ taken by the firm. We will sometimes refer to action $a=1$ as the "corrective action", or simply the "action", such as a fossil fuel company investing in clean energy. The strategy $x(0) = x(1) = 0$ represents "exclusion", where $B$ never holds the firm regardless of its action; the strategy $\{x(0) = 0,\ x(1) > 0\}$ represents "tilting", where $B$ tilts away from the stock – she does not hold it if $a = 0$, but is willing to own a strictly positive amount if $a = 1$. As we show below, in equilibrium, $B$'s strategy will be either exclusion or tilting.

Initially, we assume that $B$ can commit to the investment strategy. For example, an asset manager can launch a fund with a stated investment strategy to exclude brown firms, such as the Vanguard ESG Developed World All Cap Equity Index Fund. Deviating will lead to client withdrawals and may prompt regulatory action.[4] Alternatively, an asset manager can launch a fund with a tilting strategy, which generally avoids brown firms but is willing to hold them if they are sustainability leaders in their industry, such as Royal London Asset Management's range of sustainable funds.[5] Such funds claim to add value through active management and analyzing individual companies within a sector. Failing to hold any firms in a controversial industry may also lead to client withdrawals as it would be cheaper to hold a passive fund that pursues an exclusionary strategy. In addition, avoiding entire industries will increase tracking error and may reduce risk-adjusted returns; Hong and Kacperczyk (2009) find that

[4] For example, the UK's Financial Conduct Authority has forced funds to remove "sustainability" labels from their name due to not investing in accordance with their stated strategy

[5] Some passive funds engage in tilting, such as the Legal & General Future World Climate Change Equity Factors Index Fund, which tracks the FTSE Russell All-World ex CW Climate Balanced Factor Index.

alcohol, tobacco, and gaming (often viewed as brown industries) significantly outperform their peers, and Bolton and Kacperzcyk (2021) document higher returns for stocks that emit more carbon dioxide. Practitioners sometimes refer to tilting as a "best-in-class" strategy. We build a parsimonious model with only one firm so the best-in-class concept does not apply literally, but if the model were extended to multiple firms, buying only those that take corrective actions involves investing in those that are best-in-class. In Section 5.1 we consider the case in which $B$ is unable to commit to her investment strategy. Atomistic investors such as households cannot commit to a strategy as they are not accountable to either a regulator or clients; in any case commitment is irrelevant because they are atomistic.

At $t = 1$, $M$ takes action $a \in \{0, 1\}$. Choosing the corrective action ($a = 1$) reduces the firm's externality and decreases firm value by $c$, net of any benefit. We have $c > 0$: the action reduces firm value, otherwise it would automatically be taken without the need for responsible investment. After taking the action, the firm issues $q \in (0, 1)$ additional shares to finance an investment project. At $t = 2$, investors trade claims to the firm's terminal value. At $t = 3$, the firm generates both a terminal cash flow and negative externalities.

## 1.2 Firm Value and Externalities

The firm's terminal value is specified as:

$$V = \widetilde{A} + rI - ca, \tag{1}$$

where $\widetilde{A} \sim N(\mu, \sigma)$ represents the random return generated by the firm's assets in place. The cost of the corrective action is captured by $ca$ and the gross return from the new investment is given by $rI$ with $r > 1$. The firm finances the investment solely by issuing new shares so that $I = pq$. To focus on the main economic mechanism – the blockholder's trade-off between providing incentives for the action and less capital for brown investments – we take the firm's issuance decision $q$ as given. For example, $q$ may be limited by the amount of equity that can be raised without the agency costs of outside equity becoming too severe. With fixed $q$, lower demand for the firm's stock by the blockholder reduces the stock price $p$ and thus the level of investment $I = pq$.

In our setting, investment has constant returns to scale – the firm can invest any amount

$I$, with a constant gross return of $r$. Thus, a divestment-induced decrease in the stock price $p$ has a linear effect on investment $I = pq$. The opposite assumption would be that the firm has a single lumpy investment project which requires $\overline{I}$ dollars of investment. In this case, $p$ (and thus divestment) cannot change the level of investment along the intensive margin. It could only have an effect on the extensive margin, which in turn would require $q$ to be endogenous so that the firm might choose not to raise capital and invest at all. This setting would be less appealing for two reasons. First, it would lead to "bang-bang" solutions as the firm invests either 0 or $\overline{I}$. If the firm is already choosing not to raise capital and invest, a further decrease in $p$ would have no effect on investment; conversely, if the firm is already investing, a further increase in $p$ is ineffective. Second, and more importantly, there would be a substantial loss of tractability. The per-share value of the firm is given by

$$v \equiv \frac{V}{1+q}, \tag{2}$$

and so $q$ affects both the numerator (through affecting investment and thus aggregate value) and denominator. Thus, if $q$ were endogenous, it would not be solvable in closed form. An intermediate assumption would be diminishing returns to scale, in which case a lower $p$ will always reduce investment but the effect is less than one-for-one. We choose the constant returns to scale assumption because it is more tractable, and also because it is the setting in which divestment is most effective; despite this, we will show that divestment is not always optimal.

The firm's operations generate a negative externality $f$ to society:

$$f(\widetilde{A}, rI, a) = \lambda(\widetilde{A} + rI)(1 - \xi a). \tag{3}$$

The externality depends on the firm's assets in place $\tilde{A}$, the payoff from investment $rI$, and the action $a$.[6] The parameter $\lambda > 0$ scales the externality and $0 < \xi < 1$ determines the efficacy of the corrective action. The functional form for $f$ implies that the action reduces the externality in a multiplicative way. For example, if the corrective action involves developing a less polluting technology, this is implemented firm-wide and thus has a larger effect on larger firms.[7] The functional form also means that greater investment increases the externality by

[6] We assume that the externality does not depend on the $ca$ term. We implicitly assume that the firm has cash on hand to pay this cost and does not need to disinvest to do so.

[7] If the action has an additive effect on externalities, it remains the case that tilting is more effective than

increasing the size of the firm, i.e., $\frac{\partial f(\widetilde{A},rI,a)}{\partial I} > 0$. As a result, there are two ways in which the blockholder's investment strategy can reduce externalities. The first is by increasing the cost of capital and thus constraining the externality-producing investment that the manager undertakes. This is typically the stated rationale for divestment strategies. The second is by directly rewarding the manager for taking the corrective action.

## 1.3 Manager's Problem

The manager's utility function depends on the equilibrium stock price $p$ and the per-share firm value $v$:

$$U_m = \omega p + (1 - \omega)v, \tag{4}$$

with $\omega \in [0, 1]$. The concern for the short-term stock price $\omega$ is standard in the literature and can arise from a number of sources introduced by prior research, such as takeover threat (Stein, 1988), termination threat (Edmans, 2011), or reputational concerns (Narayanan, 1985; Scharfstein and Stein, 1990). Another common justification is that the firm intends to raise equity at $t = 2$ (Stein, 1996). We do not include this as a source of $\omega > 0$ because we explicitly model equity issuance. Indeed, we show that even if the manager is fully aligned with the firm's long-term value ($\omega = 0$), he will still care about the stock price $p$ as it will affect the terms at which he will raise equity. While prior papers typically group equity issuance together with other justifications for $\omega > 0$, separating out equity issuance is important in our model as it has different implications for the channels through which the blockholder can reduce externalities. As we will show, exclusion is more effective if the firm is raising more equity; tilting is more effective if $\omega$ is higher.

At $t = 1$, the manager solves:

$$\max_{a \in \{0,1\}} \mathbb{E}[U_m], \tag{5}$$

where the expectation is taken over $\widetilde{A}$. Importantly, the manager takes the blockholder's investment policy $x(a)$ as given when choosing $a$.

exclusion if $\xi$ is sufficiently high. Only some of the comparative statics change.

## 1.4 Financial Market

The blockholder commits to a demand schedule $x(a)$. Households maximize a standard mean-variance objective with constant absolute risk aversion parameter $\gamma > 0$. When submitting their demands, households condition on the action $a$ and the stock price $p$:

$$\max_{x_i} \mathbb{E}[x_i(v-p)|a,p] - \frac{\gamma}{2} Var(x_i(v-p)|a,p). \tag{6}$$

Their demand function is thus given by:

$$x_i = \frac{\mathbb{E}[v|a,p] - p}{\gamma Var(v|a,p)}. \tag{7}$$

Market clearing requires that total demand equals supply:

$$x(a) + \int_0^1 x_i di = 1 + q. \tag{8}$$

Solving for $p$ yields:

$$p = \mathbb{E}[v|a,p] - \gamma Var(v|a,p)\,(1 + q - x(a)) \tag{9}$$

with $\mathbb{E}[v|a,p] = \frac{\mu + rI - ca}{1+q}$, $Var(v|a,p) = \frac{\sigma^2}{(1+q)^2}$, and $I = qp$.

The stock price $p$ is the certainty equivalent per-share value of the firm. The second term represents the risk discount, which is increasing in risk $Var(\cdot)$, household risk aversion $\gamma$, and the number of shares held by households $1+q-x(a)$. An increase in the blockholder's demand raises the stock price by reducing the number of shares that risk-averse investors need to hold.

## 1.5 Blockholder's Problem

The blockholder chooses the investment strategy $x(a)$ to minimize the expected externality:

$$\min_{x(a)} \mathbb{E}[f(\widetilde{A}, rI, a)]. \tag{10}$$

We assume that $0 \leq x(a) \leq 1 + q$. The assumption $x(a) \geq 0$ results from short-sale constraints, which are standard in the blockholder exit literature (e.g. Admati and Pfleiderer, 2009; Edmans, 2009); without short-sales constraints, blockholders have no special role as any

investor can exit, regardless of her initial stake. However, this assumption is not necessary for our results. If short-sales are possible, all the results continue to apply except that $B$ need not be a blockholder – she can be any large investor that can commit to an investment strategy.[8] Similarly, $x(a) \leq 1+q$ means that the blockholder cannot buy more than the entire firm, i.e. households cannot short sell. If this assumption is relaxed, our results become stronger as the blockholder has a greater ability to reward the corrective action.

# 2 Optimal Investment Strategies

We solve the model by backwards induction. We first re-write the equilibrium stock price as a function of $B$'s strategy and the corrective action. We take the stock price in equation (9), plug in $I = pq$ and solve for $p$:

$$p(a) = \frac{\mu - ca - \left(1 - \frac{x(a)}{1+q}\right)\gamma\sigma^2}{1+q-rq}. \tag{11}$$

The intuition is as follows. In the absence of an investment decision, the stock price is the certainty equivalent firm value divided by the number of shares $(1+q)$. One may think that investment should add an additional term to firm value in the numerator. However, since the value of the investment is $rqp(a)$, it effectively reduces the number of shares by $rq$ in the denominator.[9]

To ensure that $p(a)$ is positive, we assume that $\mu > \gamma\sigma^2 + c$ so that expected firm value is not outweighed by the risk premium and the cost of the action, and that $1+q-rq > 0$ so the effective number of shares does not turn negative. The second condition can be rewritten $r < \frac{1+q}{q}$. Intuitively, if $r$ is sufficiently large, then households demand more shares when the price is higher, since their funds will be invested in a very profitable investment opportunity,

[8] We would only need a limit on the maximum possible short-sales to prevent the stock price in equation (9) from turning negative.

[9] To see this, we have:

$$\begin{aligned} \text{Market value of firm} &= \text{Certainty equivalent fundamental value of firm} \\ p(1+q) &= \text{Certainty equivalent assets in place} + rqp \\ p(1+q-rq) &= \text{Certainty equivalent of assets in place} \end{aligned}$$

leading to an upward-sloping demand curve.

We next solve for $M$'s optimal choice of $a$. He takes the action if $\mathbb{E}[U_m|a=1] \geq \mathbb{E}[U_m|a=0]$. Plugging in the earlier expressions for $p(a)$ and $\mathbb{E}[v]$ shows that this inequality is satisfied if and only if:

$$x(1) - x(0) \geq \frac{c(1+q)}{\gamma\sigma^2\left[\omega + (1-\omega)z\right]} \equiv \overline{\Delta}_x \tag{12}$$

where

$$z \equiv \frac{rq}{1+q} \in (0,1). \tag{13}$$

The action $a=1$ has two effects on $M$'s objective function. First, it incurs a cost $c$ which reduces fundamental value and thus the stock price; the latter in turn lowers investment and further reduces fundamental value. Second, it increases the blockholder's demand from $x(0)$ to $x(1)$, raising the stock price $p(a)$ and thus investment and firm value. Thus, $M$ takes the action if the second force is sufficiently strong, i.e. $B$ pursues a tilting strategy where $x(1) - x(0)$ is sufficiently high.

The last step is to solve for $B$'s optimal policy $x(a)$. The previous assumptions $\mathbb{E}[f(\widetilde{A}, rI, 1)] > \mathbb{E}[f(\widetilde{A}, rI, 0)]$ and $\frac{\partial f\left(\widetilde{A}, rI, a\right)}{\partial I} > 0$, and the fact that $p(0)$ increases with $x(0)$, imply that the blockholder optimally sets $x(0) = 0$. It immediately follows from equation (12) that the blockholder can implement $a=1$ by setting $x(1) \geq \overline{\Delta}_x$ and $a=0$ by setting $x(1) \in [0, \overline{\Delta}_x)$. We assume that

$$c \leq \gamma\sigma^2\left[\omega + (1-\omega)z\right] \tag{14}$$

so that the constraint $x(1) \leq 1+q$ does not bind in the main model. (In the extensions, this condition will differ, and we will state the new condition required). Proposition 1 states the blockholder's optimal strategy to minimize externalities; all proofs are given in the Appendix.

**Proposition 1** *(Blockholder's strategy): The blockholder's optimal strategy is given as follows:*

(*i*) *If $\xi \geq \overline{\xi}$, the optimal strategy is tilting, i.e. $x(1) = \overline{\Delta}_x$ and $x(0) = 0$, and the manager chooses $a = 1$;*

(*ii*) *If $\xi < \overline{\xi}$, the optimal strategy is exclusion, i.e. $x(1) = x(0) = 0$, and the manager chooses $a = 0$.*

*The threshold $\overline{\xi} \equiv \frac{(1-\omega)c}{(1-\omega)c + (\frac{\mu}{z} - \gamma\sigma^2)(\omega + \frac{z}{1-z})}$ is increasing in $(c, \gamma, \sigma)$ and decreasing in $(\omega, \mu)$.*

*If $\omega = 0$, then $\overline{\xi}$ is decreasing in $(r, q)$. If $\omega \in (0, 1)$, then $\overline{\xi}$ is hump-shaped in $(r, q)$. The threshold $\overline{\Delta}_x$ is defined in equation (12).*

The intuition is as follows. The blockholder's investment strategy $x(a)$ is analogous to an incentive contract provided to a manager, except that incentives are not provided by cash, but through purchasing shares which raises the stock price.[10] A higher stock price increases the manager's objective function directly as the manager places weight $\omega$ on the stock price, and indirectly by increasing the amount of investment and thus fundamental value (on which the manager places weight $1 - \omega$). As in a compensation model, it is optimal to give the lowest possible reward upon $a = 0$. In a contracting setting with limited liability, this involves zero pay; in an investment setting with short-sales constraints, this involves zero demand. Whether to reward $a = 1$ depends on whether the benefits of the action exceed the costs. In a contracting setting, the cost is the financial cost of pay. In our investment setting, the cost is that positive demand increases the stock price, raising investment and thus the externality. This analogy highlights the drawback of exclusion strategies, despite them being practiced by many investors – they are tantamount to giving the manager zero reward for desirable actions.

The blockholder chooses tilting if the effectiveness of the action $\xi$ is sufficiently high. Then, the most effective way to reduce externalities is to incentivize the manager to take the action through tilting, rather than to starve the firm of funds through exclusion. The threshold $\overline{\xi}$ is lower (i.e. tilting is more likely to be optimal) if the manager has greater stock price concerns ($\omega$ is high[11]) and the action is less costly ($c$ is low). This reduces the number of shares $\overline{\Delta}_x$ that $B$ needs to purchase to induce the corrective action, meaning that doing so is possible without raising investment by much.

Tilting is also preferred if the firm is large (high $\mu$) and risk $\sigma$ and risk aversion $\gamma$ are small, because this increases the stock price and thus the amount of investment. In addition, high $\mu$ increases assets in place. Both factors lead to greater firm value and thus higher externalities. Since the action $a$ has a multiplicative effect on the externality, the greater the firm value, the

[10] A second difference is that an incentive contract is contingent upon output, whereas the blockholder's strategy is contingent upon the action $a$. We only require the action $a$ to be publicly observable, but not contractible. Section 3 studies the case in which $a$ is not publicly observable.

[11] One might think that there is a force in the opposite direction – higher $\omega$ means a lower weight $(1 - \omega)$ on fundamental value, and so the manager has less incentive to boost the stock price to increase the amount raised by the new investment. However, the cost of the investment $c$ also affects fundamental value, so a lower weight on fundamental value makes the manager more willing to pay the cost.

greater the benefit from the action.

Finally, the capital raised by the firm ($q$) and the profitability of the investment ($r$) have non-monotonic effects on $\overline{\xi}$. One might think that these parameters should have an unambiguous effect – the greater the capital raised, the more important the cost of capital channel, and thus the more valuable exclusion is to increase the cost of capital. However, there is a force in the opposite direction – the greater the capital raised, the more important the action is to reduce the externalities from the new investment. If $q$ and $r$ are sufficiently high, such a large amount of capital is raised that this second force dominates and further increases in $q$ and $r$ make tilting more effective.

If $\xi > \overline{\xi}$, i.e. the blockholder chooses tilting, then her holdings of the brown firm upon the action are higher if $\gamma$, $\sigma$, and $\omega$ are low. If risk and risk aversion are small, then changes in demand have small effects on the stock price; thus, a large change is needed to incentivize $a = 1$. Similarly, if the manager has little concern $\omega$ for the stock price, a large increase is necessary to induce the action.

# 3 Unobservable Corrective Action

In this section, we consider the case in which the corrective action is not publicly observable. As a result, $B$ cannot condition her holdings on $a$. Instead, there is a public signal $s \in \{0, 1\}$ which is correlated with the action, such as an ESG rating. The signal precision is given by

$$\tau \equiv \Pr[s = a|a] \in [0.5, 1). \tag{15}$$

This signal is publicly observed at $t = 2$, before trade in the secondary market takes place. The blockholder is able to condition her holdings on this signal, $x(s) \geq 0$. Households have rational expectations and correctly conjecture the manager's equilibrium action.

We proceed in two steps. First, we take the signal precision as given and analyze how $\tau$ affects the optimal investment strategy. Second, we endogenize the signal precision and allow the manager to choose $\tau$ ex ante.

## 3.1 Optimal Investment Strategies

Following the same steps as in the baseline model, $M$ takes the action if and only if:

$$x(1) - x(0) \geq \frac{1}{2\tau - 1} \frac{(1-\omega)c(1+q)(1-z)}{\gamma\sigma^2[\omega + (1-\omega)z]} \equiv \widehat{\Delta}_x(\tau). \tag{16}$$

As in the baseline model, the manager chooses the action if it leads to the blockholder buying a sufficiently large amount. The threshold $\widehat{\Delta}_x$ is decreasing in signal precision $\tau$: $\frac{\partial \widehat{\Delta}_x}{\partial \tau} < 0$. Intuitively, $B$ has to provide $M$ stronger incentives to take the action when the public signal is less precise. If $\tau = 1/2$, the signal is uninformative about the action. Since the blockholder is unable to reward the action, the manager always chooses $a = 0$.

The blockholder optimally chooses tilting if the expected externality with $a = 1$ and $x(1) = \widehat{\Delta}_x$ is lower than that under $a = 0$ and $x(1) = 0$, and $\widehat{\Delta}_x(\tau) \leq 1 + q$. The equivalent of condition (14), to ensure $x(1) \leq 1 + q$, is $c \leq \frac{\gamma\sigma^2[\omega+(1-\omega)z](2\tau-1)}{(1-\omega)(1-z)}$. Under this assumption, $B$'s optimal strategy is given by Proposition 2:

**Proposition 2** *(Unobservable corrective action): The blockholder's optimal strategy is tilting and the manager chooses $a = 1$ if and only if*

$$\xi \geq \overline{\xi}_{unob}(\tau) \equiv 1 - \frac{\frac{\mu}{z} - \gamma\sigma^2}{\frac{\mu}{z} - \gamma\sigma^2 + c(\frac{\tau}{2\tau-1}\frac{1-\omega}{\omega+\frac{z}{1-z}} - 1)}. \tag{17}$$

*Otherwise, it is exclusion and the manager chooses $a = 0$. The tilting strategy involves $x(1) = \widehat{\Delta}_x(\tau)$. $\overline{\xi}_{unob}(\tau)$ is increasing in $(c, \gamma, \sigma)$ and decreasing in $(\omega, \mu, \tau)$.*

As in the baseline model, $B$ chooses tilting if the effectiveness of the action exceeds a threshold. This threshold is decreasing in signal precision $\tau$. A higher $\tau$ means that it is less costly for $B$ to implement the action, and so tilting is preferable to exclusion. Note that $\overline{\xi}_{unob}(1/2) = 1$: if the signal is pure noise, then $B$ always chooses exclusion.

Proposition 2 thus highlights a new benefit of superior ESG disclosure. Common arguments are that ESG disclosure allows investors to allocate capital according to ESG performance, and to hold managers to account. Both of these channels operate here, but there is an additional force – by allowing investors to allocate capital according to ESG performance, they can induce corrective actions without having to commit to a significant investment in a brown firm, thus making them more likely to incentivize the action in the first place.

## 3.2 Blockholder Private Information

We now allow the blockholder to gather private information on the manager's action $a$, after the action has been taken but before she trades. We assume the cost of information acquisition is arbitrarily small, and if $B$ is indifferent between acquiring information and remaining uninformed, she prefers the latter. Other market participants remain uninformed about $a$, although they continue to observe the public signal $s$.

If the blockholder acquires private information on $a$, we assume that she can commit to an investment that conditions on $a$. Since both the manager and blockholder know when $a = 1$, the blockholder can commit to rewarding the manager by purchasing stock if $a = 1$. If the blockholder reneges on this commitment, and the action becomes publicly observable with a lag so it becomes known that she has reneged, then she will be unable to induce the action in any other firms going forwards.[12] If the blockholder does not acquire information, then she can only commit to an investment strategy that is based on the public signal $s$. If $B$ acquires a stake in the firm (i.e., $x > 0$), but the public signal indicates that the action has not been taken by the firm (i.e., $s = 0$), then she incurs a reputational cost of $g > 0$. This cost results from accusations of greenwashing – the blockholder has invested in a brown firm despite there being no public evidence that it has taken a corrective action.[13] The blockholder's objective function is to minimize the sum of the firm's externalities, her cost of information acquisition, and her reputational costs.

**Proposition 3** *(Blockholder private information):*

(*i*) *If* $\tau \geq \frac{1}{2} + \frac{1}{2}\frac{1-\omega}{\frac{1+z}{1-z}+\omega}$, *the blockholder remains uninformed and chooses tilting if and only if* $\xi > \overline{\xi}_{unob}(\tau)$, *as in Proposition 2.*

[12] The same logic means that $B$ is able to commit to acquiring information. If she reneges on this commitment, she will be uninformed about $a$ and thus will not be able to reward the manager by purchasing stock if $a = 1$. Allowing for a divestment strategy that conditions both on $a$ and $s$ would not change the result since the action $a$ is perfectly predictable in equilibrium.

[13] Note that the "public" is different from the atomistic investors who have rational expectations about the manager's action. For example, the public may include the blockholder's current or future clients.

(*ii*) *Suppose* $\frac{1}{2} < \tau < \frac{1}{2} + \frac{1}{2}\frac{1-\omega}{\frac{1+z}{1-z}+\omega}$ *and let*

$$\overline{\xi}_{in}(g) \equiv \frac{(1-\omega)c + \frac{(1-\tau)g}{\lambda}\frac{1-z}{z}\left(\omega + \frac{z}{1-z}\right)}{(1-\omega)c + \left(\frac{\mu}{z} - \gamma\sigma^2\right)\left(\omega + \frac{z}{1-z}\right)}. \tag{18}$$

$$\overline{\xi}_{un}(g) \equiv 1 - \frac{\frac{(1-\tau)g}{\lambda}\frac{1-z}{z}\left(\omega + \frac{z}{1-z}\right)}{c\left[\frac{\tau}{2\tau-1}(1-\omega) - \frac{1}{1-z}\right]}. \tag{19}$$

*Then, there exists* $g^* > 0$ *that satisfies* $\overline{\xi}_{unob}(\tau) = \overline{\xi}_{in}(g^*)$ *such that:*

(*a*) *If* $g \geq g^*$*, the blockholder remains uninformed and chooses tilting if and only if* $\xi > \overline{\xi}_{unob}(\tau)$*.*

(*b*) *If* $g < g^*$*, the blockholder chooses exclusion if* $\xi < \overline{\xi}_{in}(g)$*, informed tilting if* $\overline{\xi}_{in}(g) < \xi < \overline{\xi}_{un}(g)$*, and uninformed tilting if* $\xi > \overline{\xi}_{un}(g)$*. In this case* $\overline{\xi}_{in}(g) < \overline{\xi}_{unob}(\tau) < \overline{\xi}_{un}(g)$*, with* $\lim_{g \nearrow g^*} \overline{\xi}_{in}(g) = \overline{\xi}_{unob}(\tau) = \lim_{g \nearrow g^*} \overline{\xi}_{un}(g)$*.*

(*c*) *The equilibrium expected externality is increasing in* $g$*.*

Overall, Proposition 3 shows that $B$ is less likely to acquire information when the cost from greenwashing accusations $g$ is large. Absent reputational concerns, it is efficient for $B$ to acquire private information as she does not need to promise as large a purchase to induce the action – since $M$ knows that $B$ will have observed that he has taken the action, he will be willing to do so even if the promised purchases are low. However, by committing to condition her strategy on $a$, the blockholder exposes herself to the risk that she ends up purchasing shares if $a = 1$ even if $s = 0$ – investing in a company that the public thinks has taken no corrective action. If the cost of greenwashing accusations is sufficiently large, the blockholder is less likely to acquire private information which, in turn, increases the cost of inducing the action and deters her from doing so in the first place.

In reality, many responsible investors claim to gather private information on firms' social performance. Indeed, one might think that doing so makes them more effective, since they can hold firms more accountable for their social performance. However, contrary to their claims, they have no incentive to gather private information if they are unable to trade on it, due to being evaluated on how their investments vary with publicly observable signals.

## 3.3 Optimal Disclosure

In this section, we return to the case in which $a$ is unobservable to all investors, and allow the manager to choose $\tau$ ex ante by engaging in disclosure. We assume that if the manager is indifferent between different values of $\tau$, he chooses the lowest possible $\tau$ of $\frac{1}{2}$ as this would be strictly optimal if disclosure were costly. We also assume the choice of $\tau$ is made public, so that $B$ can condition her investment strategy on $\tau$.

The equivalent of condition (14), to ensure $x(1) \leq 1+q$, is $c \leq \frac{\gamma\sigma^2[\omega+(1-\omega)z]}{1+\omega+(1-\omega)z}$. Under this assumption, Proposition 4 gives $M$'s optimal disclosure policy and shows how it affects $B$'s optimal investment strategy.

**Proposition 4** *(Optimal disclosure policy). If and only if*

$$\xi \geq 1 - \frac{\frac{\mu}{z} - \gamma\sigma^2}{\frac{\mu}{z} - \gamma\sigma^2 + c\frac{1-\omega}{\omega+\frac{z}{1-z}}} \equiv \overline{\xi}_{disc}, \tag{20}$$

*then the manager chooses* $\tau^* = \max\{\widehat{\tau}(\xi), \tau^{min}\} \in \left(\frac{1}{2}, 1\right)$ *where* $\widehat{\tau}(\xi)$ *satisfies* $\xi = \overline{\xi}_{unob}(\tau)$ *and* $\tau^{min}$ *satisfies* $\widehat{\Delta}_x(\tau^{min}) = 1+q$*, the blockholder chooses tilting, and the manager chooses* $a = 1$*. Otherwise, the manager chooses* $\tau = \frac{1}{2}$*, the blockholder chooses exclusion, and the manager chooses* $a = 0$*. The threshold* $\overline{\xi}_{disc}$ *decreases in* $(\mu, \omega)$*, it increases in* $(c, \gamma, \sigma)$*, and it is hump-shaped in* $(r, q)$*.*

The manager discloses information (i.e. chooses $\tau > \frac{1}{2}$), and the blockholder chooses tilting, if and only if the action is sufficiently effective. The threshold for $\xi$ decreases in the manager's stock price concerns. This is because disclosure increases the stock price if the manager has taken the action. One might think that $M$ should choose full disclosure ($\tau = 1$) so that his action is always reflected in the public signal ($s = 1$). In contrast, the manager deliberately discloses noisy signals, so that the blockholder has to promise a high investment $x(1)$ upon the action in order to induce it. Indeed, $\widehat{\tau}(\xi)$ is the minimum disclosure that persuades the blockholder to implement the action.

The model considers a blockholder who chooses optimally between tilting and exclusion strategies. Stepping outside the model, if there was a probability that the blockholder only implements exclusion strategies (e.g. due to lack of sophistication, or its clients believing that exclusion is the best way to invest responsibly), then the greater this probability, the more likely

it is for the manager to choose minimal disclosure ($\tau = \frac{1}{2}$). Thus, if the economy contains more responsible investors that are open to adopting a tilting strategy, this would encourage firms to disclose more information about their ESG activities, in turn reinforcing investors' incentives to adopt the tilting strategy.

# 4 Presence of Arbitrageur

A common criticism of divestment strategies is they allow arbitrageurs to buy brown firms at depressed prices, attenuating the impact of divestment on prices. This section extends the model to incorporating an arbitrageur, $A$, who is purely profit-motivated like households, and is risk-neutral and can take large stakes and have price impact like the blockholder. We return to the case in which the action $a$ is publicly observable; this simplifies the analysis as it means that firm value (which is net of $c$, if $a = 1$) is publicly observable.

With probability $\eta > 0$, $A$ arrives after $B$ has announced her investment strategy and $M$ has taken action $a$. The presence of the arbitrageur is public information. He trades an amount $y$ at $t = 2$ to maximize $\Pi_A(y) = y(v - p)$. The equivalent of condition (14), to ensure $x(1) \leq 1 + q$, is $c \leq \gamma\sigma^2[\omega + (1-\omega)z]\left(1 - \frac{\eta}{2}\right)$. Under this assumption, the solution is given in Proposition 5:

**Proposition 5** *(Arbitrageur). If the arbitrageur is present, his trading volume and profit are given by*

$$y^*(x) = \arg\max_y \Pi_A(y) = \frac{1+q-x}{2} \tag{21}$$

$$\Pi_A(y^*(x)) = \left(\frac{1}{2}\frac{1+q-x}{1+q}\right)^2 \gamma\sigma^2 \tag{22}$$

*and, conditional on $x$, the stock price is given by*

$$p(x, a, y^*(x)) = \frac{\mu - ca - \left(1 - \frac{\eta}{2}\right)\left(1 - \frac{x}{1+q}\right)\gamma\sigma^2}{1+q-rq}. \tag{23}$$

*The blockholder's optimal strategy is tilting and the manager chooses $a = 1$ if and only if*

$$\xi \geq \overline{\xi}_{arb} \equiv \frac{(1-\omega)\,c}{(1-\omega)\,c + \left(\frac{\mu}{z} - \left(1-\frac{\eta}{2}\right)\gamma\sigma^2\right)\left(\omega + \frac{z}{1-z}\right)}. \tag{24}$$

*Otherwise, it is exclusion and the manager chooses $a = 0$. The tilting strategy involves*

$$x\,(1) = \frac{(1+q)\,c}{\left(1-\frac{\eta}{2}\right)\gamma\sigma^2\left[\omega + (1-\omega)z\right]}. \tag{25}$$

As is standard, $A$ buys half of the free float not acquired by $B$, as shown in equation (21). Comparing (23) with (11), there is an additional $\left(1-\frac{\eta}{2}\right)$ term in the numerator, which multiplies the term containing $x$ and means that the blockholder's trade has a lower effect on the stock price. Intuitively, if $A$ is present, she buys half of the free float, so $B$'s impact is halved. As a consequence, equation (25) contains an additional $\left(1-\frac{\eta}{2}\right)$ term in the denominator – since the blockholder has smaller price impact, she must promise a higher purchase to induce the action, which makes tilting more expensive to implement. Exclusion also becomes less effective because the arbitrageur partially reverses the impact of exclusion on the stock price and the cost of capital. Since the arbitrageur buys half of the free float, his impact is decreasing in the blockholder's trade. Thus, while the arbitrageur makes both exclusion and tilting less effective, the impact is greater on exclusion as the blockholder's trade is zero. As a result, the threshold in (24) is decreasing in $\eta$ – the greater the probability of the arbitrageur appearing, the more likely the blockholder is to tilt.

# 5 Profit-Motivated Responsible Investor

This section extends the blockholder's objective function to comprise trading profits as well as externalities. She now maximizes

$$U_B = \varphi x\,(v-p) - (1-\varphi)\,f(\widetilde{A}, rI, a), \tag{26}$$

where $\varphi \in [0,1]$ parametrizes the blockholder's concern for profits. The baseline model is a special case where $\varphi = 0$. We consider two sub-cases: in the first, $B$ cannot commit to an investment strategy; in the second, she can. We use "profit-motivated" to denote a blockholder

with $\varphi > 0$ and "responsible" for $\varphi = 0$.

## 5.1 No Commitment

Suppose that $B$ cannot commit to an investment strategy, i.e. she chooses her optimal trade at $t = 2$ freely after the action $a$ has become public at $t = 1$. In the core model where $\varphi = 0$ (no profit motives), the blockholder will always choose $x = 0$. Her trading decision has no influence on the action since it has already been taken; her only objective is to minimize the externality which is achieved through $x = 0$. Thus, in equilibrium, the manager will choose $a = 0$ and the expected externality is $\lambda \frac{\mu - z\gamma\sigma^2}{1-z}$.

If $\varphi > 0$, the blockholder's objective function includes profit. Given action $a$, she maximizes her expected utility by choosing:

$$x^*(a) = \frac{1+q}{2} \max\left\{1 - \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda(1-a\xi), 0\right\}. \tag{27}$$

The profit-motivated blockholder buys shares, since there are gains from trade between the risk-neutral blockholder and the risk-averse households. Moreover, $x^*(1) \geq x^*(0)$: the blockholder buys more shares when the manager takes the action, even though it reduces firm value. The intuition is as follows. The action's impact on firm value does not affect trading profits, because the action is public and thus fully reflected in the stock price. However, the action means that buying shares, and thus helping the company expand, has a less positive impact on externalities. Thus, buying shares has the same benefit (trading profits are unchanged) and a lower cost (externalities are smaller) and so the blockholder buys more shares if $a = 1$.

The manager has rational expectations about the blockholder trade $x^*(a)$. As in the baseline model, he takes the action if and only if

$$x^*(1) - x^*(0) \geq \overline{\Delta}_x \tag{28}$$

where $\overline{\Delta}_x$ is given by (12). The equilibrium is given by Proposition 6.

**Proposition 6** *(Profit-motivated blockholder, no commitment): Suppose the blockholder cannot commit to an investment strategy. In equilibrium:*

(*i*) *If* $\xi \leq \frac{2c}{\gamma\sigma^2[\omega+(1-\omega)z]}$, *the blockholder buys* $x^*(0)$ *shares and* $a^* = 0$.

(*ii*) *If* $\xi > \frac{2c}{\gamma\sigma^2[\omega+(1-\omega)z]}$, *there exists* $0 < \underline{\varphi}(\xi) < \overline{\varphi}(\xi) < 1$ *such that:*

(*a*) *If* $\varphi \in \left[\underline{\varphi}(\xi), \overline{\varphi}(\xi)\right]$, *the blockholder buys* $x^*(1)$ *shares and* $a^* = 1$.

(*b*) *If* $\varphi \notin \left[\underline{\varphi}(\xi), \overline{\varphi}(\xi)\right]$, *the blockholder buys* $x^*(0)$ *shares and* $a^* = 0$.

(*iii*) *Let* $f^*(\varphi)$ *be the externalities where the blockholder's weight on profits is* $\varphi$. *Then, there exists* $\xi_{NC} \in (0,1)$ *and* $\varphi_{NC}(\xi) \leq \overline{\varphi}(\xi)$ *such that:*

(*a*) *If* $\xi \leq \max\left\{\frac{2c}{\gamma\sigma^2[\omega+(1-\omega)z]}, \xi_{NC}\right\}$ *then* $f^*(\varphi) = f^*(0)$ *if* $\varphi < \underline{\varphi}(\xi)$, *and* $f^*(\varphi) > f^*(0)$ *if* $\varphi \geq \underline{\varphi}(\xi)$.

(*b*) *If* $\xi > \max\left\{\frac{2c}{\gamma\sigma^2[\omega+(1-\omega)z]}, \xi_{NC}\right\}$ *then* $f^*(\varphi) = f^*(0)$ *if* $\varphi < \underline{\varphi}(\xi)$, $f^*(\varphi) < f^*(0)$ *if* $\varphi \in \left[\underline{\varphi}(\xi), \varphi_{NC}(\xi)\right]$, *and* $f^*(\varphi) > f^*(0)$ *if* $\varphi \geq \varphi_{NC}(\xi)$.

Part (i) states that, as in the core model, the blockholder cannot induce the action if it is sufficiently ineffective. Part (ii) demonstrates that, if the action is sufficiently effective, the blockholder can induce it if profit motives fall within an intermediate range. On the one hand, they need to be sufficiently high to induce the blockholder to buy shares: note that $\varphi$ needs to be sufficiently greater than 0, rather than just strictly positive, since profit motives need to outweigh the increased externalities from buying shares. On the other hand, the blockholder's concern for externalities also has to be sufficiently high that she buys significantly more shares when $a = 1$ than when $a = 0$.

Even if profit motives allow the blockholder to induce the action, this does not automatically mean that externalities fall, since profit motives also lead the blockholder to buy shares which helps the firm expand. Part (iii) shows that, for an intermediate range of profit motives and a sufficiently effective corrective action, the former effect outweighs the latter – i.e. the inducement of the action outweighs the financing of expansion and so externalities fall.

The intuition is as follows. In the core model, the blockholder is able to commit to buying more shares when the action is taken. If the blockholder is unable to commit, and is only concerned with externalities, she will not buy shares, regardless of the manager's action, as doing so increases externalities. Profit motives substitute for the ability to commit – they make it individually rational for the blockholder to buy more shares when $a = 1$, effectively committing her to reward the corrective action.

## 5.2 Commitment

This sub-section allows the blockholder to commit to a trading strategy, as in the core model. The equilibrium is given by Proposition 7.

**Proposition 7** *(Profit-motivated blockholder, commitment):*

(*i*) *The firm's externalities in equilibrium increase with* $\varphi$.

(*ii*) *There exists* $\overline{\xi}_C > 0$ *such that the profit-motivated blockholder induces the corrective action if and only if* $\xi \geq \overline{\xi}_C$. *Moreover,*

(*a*) *If* $\overline{\Delta}_x \leq \frac{1+q}{2}$ *then* $\overline{\xi}_C < 1$, *and if* $\overline{\Delta}_x > \frac{1+q}{2}$ *then there is* $\varphi^* > \frac{z\lambda}{1-z+z\lambda}$ *such that* $\overline{\xi}_C < 1$ *if and only if* $\varphi < \varphi^*$.

(*b*) *If* $\frac{1+q}{2} < \overline{\Delta}_x$ *and* $\varphi < \frac{z\lambda}{1-z+z\lambda}$, *then* $\overline{\xi}_C < \overline{\xi}$.

Part (i) shows that, under commitment, a profit-motivated investor will always generate (weakly) more externalities than a responsible investor, who by definition minimizes externalities. The greater the weight the investor puts on trading profits, the larger are the expected externalities in equilibrium. Part (ii) shows that, as in baseline model, a profit-motivated investor tilts if the action is sufficiently effective. Intuitively, large $\xi$ makes tilting attractive since it reduces externalities (as in the core model), but also because it enables the profit-motivated investor to earn the gains from trade at a lower social cost. Part (iia) confirms that this condition is satisfied for a non-empty range of parameter values, i.e. profit-motivated investors continue to tilt in some cases. Part (iib) gives sufficient conditions for the threshold for $\xi$ to be lower for a profit-motivated investor than a responsible one. If these conditions are satisfied, then when $\xi \in \left(\overline{\xi}_C, \overline{\xi}\right)$, a responsible investor excludes but a profit-motivated investor tilts. Intuitively, even though tilting leads to more externalities, it also generates trading profits which the profit-motivated investor is concerned about but the responsible investor is not.

# 6 Conclusion

This paper has analyzed the optimal investment strategy of a responsible investor who aims to minimize the externalities emitted by a brown firm. While exclusion – never investing in

the firm – minimizes the stock price and thus the amount of externality-enhancing investment the firm can undertake, it provides no incentives for the firm to undertake a corrective action. Tilting provides incentives to take the action, at the cost of providing capital to a brown firm and allowing it to expand. The optimal strategy is for the investor to tilt if the action is effective at reducing externalities and comes at little cost to firm value, and also if the manager's stock price concerns are high, as then the blockholder does not need to promise a large investment to persuade the manager to take the action.

We extend the model to the case in which the corrective action is not observable, but a noisy signal is, and the investor can condition her holdings only on the signal. The noisier the signal, the greater the reward the investor needs to offer to induce the action, and the more likely she is to tilt. If the manager can choose signal precision, he will choose to disclose some information if his stock price concerns are sufficiently high, as the blockholder will buy if he has taken the corrective action, increasing the stock price. However, he will only disclose a noisy signal, so that the investor has to promise high investment upon the corrective action in order to induce it. Even if the blockholder has the option to acquire private information about the manager's action at an arbitrarily small cost, she may refrain from doing so if she suffers a sufficiently large reputational loss from investing in a company that has taken a corrective action but the public is unaware of this fact. If there is an arbitrageur who buys underpriced stock, exclusion becomes relatively less effective compared to tilting as the arbitrageur offsets the negative effect of exclusion on the stock price.

Finally, if the blockholder is partially profit-motivated, this gives her greater incentives to tilt, since doing so involves buying shares from risk-averse households and thus earning trading profits for risk-bearing. In particular, if the blockholder is unable to commit to an investment strategy, one whose sole objective is to minimize externalities will never tilt, since she has no incentives to buy shares after the action has been taken, and thus cannot implement the action. A profit motivation effectively commits the blockholder to buy more shares if the corrective action is taken – i.e., to tilt – thus allowing her to induce the action.

Our paper has a number of potential implications for policymakers. Most obviously, it highlights that regulators should not automatically punish responsible investing funds that hold brown stocks, or use the average holding of green stocks as a measure of an investor's sustainability. Under the EU's Sustainable Finance Disclosure Regulation, Article 9 funds are viewed as the most sustainable, and funds that classify themselves as Article 9 will likely attract

greatest investment from socially-minded clients. However, Article 9 funds are prevented from holding any stocks that are viewed as unsustainable – even if they are best-in-class. Similarly, the EU's Non-Financial Reporting Directive requires institutional investors, such as banks and asset managers, to report the percentage of its portfolio that is environmentally sustainable, as defined by their alignment with the EU Taxonomy. This also encourages investors to exclude rather than tilt. In contrast, regulators should potentially police the opposite behavior – sustainable funds that claim to be actively managed and conduct their own research, but blanketly exclude certain sectors even though the fund policy does not specify them as being excluded. Just as regulators are scrutinizing actively-managed mainstream funds that act like closet indexers, they could also scrutinize actively-managed sustainable funds that engage in blanket exclusion. Doing so would help investors commit to tilting strategies.

Various standard-setting bodies (e.g. the Value Reporting Framework and the World Economic Forum) are developing common metrics for ESG company performance. A frequently-stated advantage of such standards is that they allow policymakers and savers to evaluate which funds are greenwashing according to the average metrics of their portfolio companies. However, such behavior will deter investors from gathering their own private information and using it to tilt in a less costly way, in turn deterring tilting in the first place. A quite separate implication is that ensuring that funds fulfill their fiduciary duty to generate financial returns for their clients is not inconsistent with achieving social returns, and may actually support it by giving funds incentives to tilt if commitment is not possible.

The model also has implications for future research. One potential extension is to multiple firms. In addition to demonstrating that the optimal divestment strategy might involve buying firms that are literally "best-in-class", featuring these firms as competing with each other in the same industry would have interesting implications for how the investor's divestment strategy affects product market interactions. A second implication is for the design of optimal incentive contracts in the presence of responsible investors. Most research suggests that long-term contracts (i.e. minimizing $\omega$) are preferred to encourage long-term behavior, and this might seem to be especially the case in the presence of social objectives. However, short-term contracts allow investors to reward corrective actions by reflecting them in the short-term stock price. A third extension is to multiple responsible investors. They may increase the power of tilting if multiple investors are able to reward the manager for a corrective action. In contrast, if they compete for client flows, and some clients are unsophisticated and view funds that

blanketly exclude as being more sustainable, such competition may discourage tilting.

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# A Proofs

**Proof of Proposition 1.** $B$'s objective function is given by $\mathbb{E}[f(\widetilde{A}, rI, a)]$ with $I = qp(a)$. The equilibrium stock price, as a function of $a$, is given by:

$$p(a) = \begin{cases} \frac{\mu - \gamma\sigma^2 + \frac{x(1)}{1+q}\gamma\sigma^2 - c}{1+q-rq} & \text{if } a = 1 \\ \frac{\mu - \gamma\sigma^2}{1+q-rq} & \text{if } a = 0. \end{cases} \tag{29}$$

If $a = 1$, then the realized, and thus the expected, externality increases in $x(1)$ through its impact on $p(1)$. As a result, $B$'s objective given $a = 1$ is minimized at the smallest possible value that implements $a = 1$, $x(1) = \overline{\Delta}_x$. It follows that $B$ implements $a = 1$ by choosing $x(1) = \overline{\Delta}_x$ if and only if:

$$x(1) = \overline{\Delta}_x \Leftrightarrow \mathbb{E}[f(\widetilde{A}, rqp(0), 0)] \geq \mathbb{E}[f(\widetilde{A}, +rqp(1; x(1) = \overline{\Delta}_x), 1)]. \tag{30}$$

Otherwise, $B$ is better off implementing $a = 0$ and sets $x(1) = x(0) = 0$. Evaluating $\mathbb{E}[f]$ at $a \in \{0, 1\}$ leads to the following condition for tilting:

$$x(1) = \overline{\Delta}_x \Leftrightarrow \xi \geq \frac{rq\left(p\left(1\right) - p\left(0\right)\right)}{\mu + rqp\left(1\right)}. \tag{31}$$

Evaluating $p(a)$ at $a \in \{0, 1\}$ and using $x(1) = \overline{\Delta}_x$ leads to:

$$\overline{\xi} = \frac{\frac{\overline{\Delta}_x}{1+q}\gamma\sigma^2 - c}{\frac{\mu}{z} - \gamma\sigma^2 + \frac{\overline{\Delta}_x}{1+q}\gamma\sigma^2 - c} = \frac{c\left(1-\omega\right)}{\left(\frac{\mu}{z} - \gamma\sigma^2\right)\left(\omega + \frac{z}{1-z}\right) + c\left(1-\omega\right)} \tag{32}$$

where we have used $\overline{\Delta}_x = \frac{c(1+q)}{\gamma\sigma^2[\omega+(1-\omega)z]}$.

It immediately follows from the expression for $\overline{\xi}$ that $\frac{\partial\overline{\xi}}{\partial\mu} < 0$, $\frac{\partial\overline{\xi}}{\partial\gamma} > 0$, and $\frac{\partial\overline{\xi}}{\partial\sigma} > 0$. For the effect of $c$, we can divide the expression above by $c$ to see that $\frac{\partial\overline{\xi}}{\partial c} > 0$. For the comparative statics with respect to $\omega$, we re-write the expression as:

$$\overline{\xi} = \frac{c}{\left(\frac{\mu}{z} - \gamma\sigma^2\right) g_1(\omega) + c} \tag{33}$$

with $g_1(\omega) \equiv \left(\omega + \frac{z}{1-z}\right)\frac{1}{1-\omega}$ and $\frac{\partial g_1'}{\partial\omega} = \frac{1}{(1-\omega)^2(1-z)} > 0$ because $z \in (0, 1)$. It follows that $\frac{\partial\overline{\xi}}{\partial\omega} < 0$

if $\omega \in [0,1)$. If $\omega = 1$, then $\overline{\xi} = 0$.

For the comparative statics with respect to $z$, and thus $(r,q)$, we re-write the expression above as:

$$\overline{\xi} = \frac{(1-\omega)\,c}{g_2(z) + (1-\omega)\,c} \tag{34}$$

with $g_2(z) = \left(\frac{\mu}{z} - \gamma\sigma^2\right)\left(\omega + \frac{z}{1-z}\right)$. If $\omega = 1$, then $\overline{\xi}$ does not depend on $z$. If $\omega < 1$, then the sign of $\frac{\partial \overline{\xi}}{\partial z}$ is the opposite of $g_2'(z)$, which is equal to:

$$g_2'(z) = \frac{\mu - \gamma\sigma^2}{(1-z)^2} - \frac{\omega\mu}{z^2}. \tag{35}$$

Also note that $g_2''(z) > 0$, $\lim_{z\to 0} g'(z) = -\infty$ if $\omega > 0$ and $\lim_{z\to 0} g'(z) > 0$ if $\omega = 0$, and that $\lim_{z\to 1} g'(z) = \infty$. It follows that $g_2(z)$ is U-shaped in $z$ if $\omega > 0$ and that it is increasing in $z$ if $\omega = 0$. As a result, $\overline{\xi}$ is hump-shaped in $(r,q)$ if $\omega > 0$ and decreasing in $(r,q)$ if $\omega = 0$. ■

**Proof of Equation (16).** The equilibrium stock price given public signal $s$ is given by:

$$p(\widehat{a}, s) = \frac{\mu - c\widehat{a} - \left(1 - \frac{x(s)}{1+q}\right)\gamma\sigma^2}{1 + q - rq}, \tag{36}$$

where $\widehat{a}$ denotes the action conjectured by households.

If $M$ chooses $a = 1$, his expected utility is given by:

$$\mathbb{E}[U_m | a = 1] = \omega\left[\tau p(\widehat{a},1) + (1-\tau)p(\widehat{a},0)\right] + (1-\omega)\frac{\mu + rq\left[\tau p(\widehat{a},1) + (1-\tau)p(\widehat{a},0)\right] - c}{1+q}.$$

If he chooses $a = 0$, his expected utility is given by:

$$\mathbb{E}[U_m | a = 0] = \omega\left[\tau p(\widehat{a},0) + (1-\tau)p(\widehat{a},1)\right] + (1-\omega)\frac{\mu + rq\left[\tau p(\widehat{a},0) + (1-\tau)p(\widehat{a},1)\right]}{1+q}.$$

Conditional on tilting, $M$ chooses $a = 1$ if and only if $\mathbb{E}[U_m|a=1] \geq \mathbb{E}[U_m|a=0]$, which is equivalent to the condition in equation (16). ■

**Proof of Proposition 2.** For $\tau \in (1/2, 1)$, $B$ chooses tilting, $(x(1) = \widehat{\Delta}_x, x(0) = 0)$ if (i) the expected externality with $a = 1$ and $x(1) = \widehat{\Delta}_x$ is lower than that under $a = 0$ and $x(1) = 0$, and (ii) $x(1) \leq 1 + q$. It follows from the expression for $\widehat{\Delta}_x(\tau)$ that condition (ii) is equivalent

to $c \leq \frac{\gamma\sigma^2[\omega+(1-\omega)z](2\tau-1)}{(1-\omega)(1-z)}$. Otherwise, she chooses exclusion and sets $x(1) = x(0) = 0$. Suppose $c \leq \frac{\gamma\sigma^2[\omega+(1-\omega)z](2\tau-1)}{(1-\omega)(1-z)}$, then $B$ chooses tilting if:

$$\begin{aligned}
\left[\mu + rq\left(\tau p(1,1) + (1-\tau)p(1,0)\right)\right](1-\xi) \leq \mu + rqp(0,0) \Leftrightarrow \\
1 - \xi \leq \frac{\mu + rqp(0,0)}{\mu + rq\left(\tau p(1,1) + (1-\tau)p(1,0)\right)} \Leftrightarrow \\
1 - \xi \leq \frac{\mu + \frac{z}{1-z}\left(\mu - \gamma\sigma^2\right)}{\mu + \frac{z}{1-z}\left(\mu - c - \left(1 - \frac{\tau\hat{\Delta}_x}{1+q}\right)\gamma\sigma^2\right)} \Leftrightarrow \\
\xi \geq 1 - \frac{\frac{\mu}{z} - \gamma\sigma^2}{\frac{\mu}{z} - \gamma\sigma^2 - c + \gamma\sigma^2\frac{\tau\hat{\Delta}_x}{1+q}} \Leftrightarrow \\
\xi \geq 1 - \frac{\frac{\mu}{z} - \gamma\sigma^2}{\frac{\mu}{z} - \gamma\sigma^2 + c\left(\frac{\tau}{2\tau-1}\frac{1-\omega}{\omega+\frac{z}{1-z}} - 1\right)} \equiv \overline{\xi}_{\text{unob}}(\tau).
\end{aligned}$$

It immediately follows that $\overline{\xi}_{\text{unob}}(\tau)$ increases in $(c, \gamma, \sigma)$ and decreases in $(\mu, \tau)$. Moreover, it decreases in $\omega$ because $\frac{1-\omega}{\omega+\frac{z}{1-z}}$ decreases in $\omega$. For $\tau = 1/2$, $B$ always chooses exclusion. ■

**Proof of Proposition 3.** We start by calculating $B$'s payoff in different scenarios, assuming that $c \leq \frac{\gamma\sigma^2[\omega+(1-\omega)z](2\tau-1)}{(1-\omega)(1-z)}$ so that she can implement tilting, which is shown in Proposition 2. If $c > \frac{\gamma\sigma^2[\omega+(1-\omega)z](2\tau-1)}{(1-\omega)(1-z)}$, then $B$ always chooses exclusion. First, if $B$ chooses exclusion, then $M$ chooses $a = 0$, and $B$'s payoff is independent of her private information and given by

$$\Pi_{exclusion} = -\lambda\left[\mu + rqp(0)\right].$$

In particular, $B$ never acquires information if she intends to use exclusion.

Second, if $B$ is uninformed about $a$ and chooses tilting, she must be conditioning her trade on the public signal $s$. Therefore, she never suffers reputational costs and her payoff from tilting is

$$\Pi^{un}_{tilting} = -\lambda\left[\mu + rq\left(\tau p(1,1) + (1-\tau)p(1,0)\right)\right](1-\xi).$$

Third, if $B$ is informed about $a$ and chooses tilting, she has two options. First, if she chooses to condition her trade on $a$, her expected payoff is

$$\Pi^{in}_{tilting} = -\lambda\left(\mu + rqp(1)\right)(1-\xi) - (1-\tau)\,g.$$

Second, if despite being informed she conditions her trade on $s$, her expected payoff is $\Pi^{un}_{tilting}$. Therefore, $B$ has no incentives to acquire information if it is not being used.

Overall, if $B$ prefers uninformed tilting over exclusion if and only if $\Pi^{un}_{tilting} > \Pi_{exclusion} \Leftrightarrow \xi > \overline{\xi}(\tau)$. She prefers informed tilting over exclusion if and only if $\Pi^{in}_{tilting} > \Pi_{exclusion} \Leftrightarrow$

$$\begin{aligned}
-\lambda(\mu + rqp(1))(1-\xi) - (1-\tau)g \geq -\lambda[\mu + rqp(0)] \Leftrightarrow \\
\xi \geq \frac{rq[p(1)-p(0)] + \frac{(1-\tau)g}{\lambda}}{\mu + rqp(1)} \Leftrightarrow \\
\xi \geq \frac{\frac{z}{1-z}\left[-c + \frac{x(1)}{1+q}\gamma\sigma^2\right] + \frac{(1-\tau)g}{\lambda}}{\mu + \frac{z}{1-z}\left[\mu - c - \left(1 - \frac{x(1)}{1+q}\right)\gamma\sigma^2\right]} \Leftrightarrow \\
\xi \geq \frac{-c + \frac{x(1)}{1+q}\gamma\sigma^2 + \frac{(1-\tau)g}{\lambda}\frac{1-z}{z}}{\frac{\mu}{z} - \gamma\sigma^2 - c + \frac{x(1)}{1+q}\gamma\sigma^2} \Leftrightarrow \\
\xi \geq \overline{\xi}_{\text{in}}(g) \equiv \frac{(1-\omega)c + \frac{(1-\tau)g}{\lambda}\frac{1-z}{z}\left(\omega + \frac{z}{1-z}\right)}{(1-\omega)c + \left(\frac{\mu}{z} - \gamma\sigma^2\right)\left(\omega + \frac{z}{1-z}\right)}.
\end{aligned}$$

Notice that $\overline{\xi}_{\text{unob}}(\tau) > \overline{\xi}_{\text{in}}(g) \Leftrightarrow$

$$\begin{aligned}
\overline{\xi}_{\text{unob}}(\tau) > \overline{\xi}_{\text{in}}(g) \Leftrightarrow \\
1 - \frac{\frac{\mu}{z} - \gamma\sigma^2}{\frac{\mu}{z} - \gamma\sigma^2 + c\left(\frac{\tau}{2\tau-1}\frac{1-\omega}{\omega + \frac{z}{1-z}} - 1\right)} > \frac{(1-\omega)c + \frac{(1-\tau)g}{\lambda}\frac{1-z}{z}\left(\omega + \frac{z}{1-z}\right)}{(1-\omega)c + \left(\frac{\mu}{z} - \gamma\sigma^2\right)\left(\omega + \frac{z}{1-z}\right)} \Leftrightarrow \\
\frac{\frac{\mu}{z} - \gamma\sigma^2}{\frac{\mu}{z} - \gamma\sigma^2 + c\left(\frac{\tau}{2\tau-1}\frac{1-\omega}{\omega + \frac{z}{1-z}} - 1\right)} c\left[\frac{\tau}{2\tau-1}(1-\omega) - \frac{1}{1-z}\right] > \frac{(1-\tau)g}{\lambda}\frac{1-z}{z}\left(\omega + \frac{z}{1-z}\right) \Leftrightarrow \\
\left(1 - \overline{\xi}(\tau)\right) c\left[\frac{\tau}{2\tau-1}(1-\omega) - \frac{1}{1-z}\right] > \frac{(1-\tau)g}{\lambda}\frac{1-z}{z}\left(\omega + \frac{z}{1-z}\right)
\end{aligned}$$

Notice $\frac{\tau}{2\tau-1}(1-\omega) - \frac{1}{1-z} > 0 \Leftrightarrow \tau < \frac{1}{2-(1-z)(1-\omega)}$. Thus,

$$\begin{aligned}
\overline{\xi}_{\text{unob}}(\tau) > \overline{\xi}_{\text{in}}(g) \Leftrightarrow \\
\tau < \frac{1}{2-(1-z)(1-\omega)} \text{ and } \overline{\xi}_{\text{unob}}(\tau) < \overline{\xi}_{\text{un}}(g)
\end{aligned}$$

where

$$\overline{\xi}_{\text{un}}(g) \equiv 1 - \frac{\frac{(1-\tau)g}{\lambda}\frac{1-z}{z}\left(\omega + \frac{z}{1-z}\right)}{c\left[\frac{\tau}{2\tau-1}(1-\omega) - \frac{1}{1-z}\right]}$$

$B$ prefers informed tilting over uninformed tilting if and only if $\Pi^{in}_{tilting} > \Pi^{un}_{tilting} \Leftrightarrow$

$$\begin{gathered}
-\lambda(\mu + rqp(1))(1-\xi) - (1-\tau)g > -\lambda[\mu + rq(\tau p(1,1) + (1-\tau)p(1,0))](1-\xi) \Leftrightarrow \\
p(1) + \frac{1}{rq}\frac{(1-\tau)g}{\lambda}\frac{1}{1-\xi} < \tau p(1,1) + (1-\tau)p(1,0) \Leftrightarrow \\
\frac{\mu - c - \left(1 - \frac{x(a=1)}{1+q}\right)\gamma\sigma^2}{1+q-rq} + \frac{1}{rq}\frac{(1-\tau)g}{\lambda(1-\xi)} < \tau\frac{\mu - c - \left(1 - \frac{x(s=1)}{1+q}\right)\gamma\sigma^2}{1+q-rq} + (1-\tau)\frac{\mu - c - \gamma\sigma^2}{1+q-rq} \Leftrightarrow \\
\frac{x(a=1)}{1+q}\gamma\sigma^2 + \frac{1+q-rq}{rq}\frac{(1-\tau)g}{\lambda(1-\xi)} < \tau\frac{x(s=1)}{1+q}\gamma\sigma^2 \Leftrightarrow \\
\frac{\frac{c(1+q)}{\gamma\sigma^2[\omega+(1-\omega)z]}}{1+q}\gamma\sigma^2 + \frac{1-z}{z}\frac{(1-\tau)g}{\lambda(1-\xi)} < \tau\frac{\frac{1}{2\tau-1}\frac{(1-\omega)c(1+q)(1-z)}{\gamma\sigma^2[\omega+(1-\omega)z]}}{1+q}\gamma\sigma^2 \Leftrightarrow \\
\frac{c}{\omega+(1-\omega)z} + \frac{1-z}{z}\frac{(1-\tau)g}{\lambda(1-\xi)} < \frac{\tau}{2\tau-1}(1-\omega)(1-z)\frac{c}{\omega+(1-\omega)z}
\end{gathered}$$

That is, $\Pi^{in}_{tilting} > \Pi^{un}_{tilting} \Leftrightarrow$

$$\tau < \frac{1}{2-(1-z)(1-\omega)} \text{ and } \xi < \overline{\xi}_{\text{un}}(g).$$

We consider two cases:

1. Suppose $\overline{\xi}_{\text{unob}}(\tau) < \overline{\xi}_{\text{in}}(g)$. There are three sub cases:

   (a) If $\xi < \overline{\xi}_{\text{unob}}(\tau)$ then $B$ prefers exclusion over both informed and uninformed tilting and hence she never becomes informed and always chooses exclusion.

   (b) If $\overline{\xi}_{\text{unob}}(\tau) < \xi < \overline{\xi}_{\text{in}}(g)$ then $B$ prefers uninformed tilting over exclusion, and exclusion over informed tilting. Therefore, $B$ never becomes informed and she choose tilting.

   (c) If $\overline{\xi}_{\text{in}}(g) < \xi$, then exclusion is an inferior strategy. Recall $\overline{\xi}(\tau) < \overline{\xi}_{\text{in}}(g)$ implies either $\tau \geq \frac{1}{2-(1-z)(1-\omega)}$, in which case we have $\Pi^{in}_{tilting} < \Pi^{un}_{tilting}$, or $\overline{\xi}_{\text{un}}(g) < \overline{\xi}_{\text{unob}}(\tau)$, which given $\overline{\xi}_{\text{unob}}(\tau) < \overline{\xi}_{\text{in}}(g) < \xi$, implies $\overline{\xi}_{\text{un}}(g) < \xi$, i.e., $\Pi^{in}_{tilting} < \Pi^{un}_{tilting}$. Either way, $B$ remains uninformed.

We conclude, if $\overline{\xi}_{\text{unob}}(\tau) < \overline{\xi}_{\text{in}}(g)$ then $B$ remains uninformed. She chooses exclusion if and only if $\xi < \overline{\xi}_{\text{unob}}(\tau)$. Notice that if $\tau < \frac{1}{2-(1-z)(1-\omega)}$ then $\overline{\xi}_{\text{unob}}(\tau) > \overline{\xi}_{\text{in}}(0)$, and hence, there is $g^* > 0$ that satisfies $\overline{\xi}_{\text{unob}}(\tau) = \overline{\xi}_{\text{in}}(g^*)$, such that $\overline{\xi}_{\text{unob}}(\tau) < \overline{\xi}_{\text{in}}(g) \Leftrightarrow g > g^*$. Notice that if $\overline{\xi}_{\text{unob}}(\tau) = \overline{\xi}_{\text{in}}(g^*)$ and $\tau < \frac{1}{2-(1-z)(1-\omega)}$, then it must be $\overline{\xi}_{\text{unob}}(\tau) = \overline{\xi}_{\text{un}}(g^*)$

2. Suppose $\overline{\xi}_{\text{unob}}(\tau) > \overline{\xi}_{\text{in}}(g)$. There are three sub cases:

   (a) If $\xi < \overline{\xi}_{\text{in}}(g)$ then $B$ prefers exclusion over both informed and uninformed tilting and hence she never becomes informed and always chooses exclusion.

   (b) If $\overline{\xi}_{\text{in}}(g) < \xi < \overline{\xi}_{\text{unob}}(\tau)$ then $B$ prefers informed tilting over exclusion, and exclusion over uninformed tilting. Therefore, $B$ becomes informed and chooses tilting.

   (c) If $\overline{\xi}_{\text{unob}}(\tau) < \xi$, then exclusion is an inferior strategy. Recall $\overline{\xi}_{\text{unob}}(\tau) > \overline{\xi}_{\text{in}}(g)$ implies $\tau < \frac{1}{2-(1-z)(1-\omega)}$ and $\overline{\xi}_{\text{un}}(g) > \overline{\xi}_{\text{unob}}(\tau)$. Therefore, in this case, $\overline{\xi}_{\text{in}}(g) < \overline{\xi}_{\text{unob}}(\tau) < \overline{\xi}_{\text{un}}(g)$. $B$ chooses informed tilting if $\xi < \overline{\xi}_{\text{un}}(g)$, and uninformed tilting if $\xi > \overline{\xi}_{\text{un}}(g)$.

   We conclude that if $\overline{\xi}_{\text{unob}}(\tau) > \overline{\xi}_{\text{in}}(g)$ then $B$ chooses exclusion if $\xi < \overline{\xi}_{\text{in}}(g)$, informed tilting if $\overline{\xi}_{\text{in}}(g) < \xi < \overline{\xi}_{\text{un}}(g)$, and uninformed tilting if $\xi > \overline{\xi}_{\text{un}}(g)$.

Finally, suppose $\xi < \overline{\xi}_{\text{unob}}(\tau)$. Notice that the amount of externalities under exclusion is lower than under informed tilting if and only if $\xi < \overline{\xi}_{\text{in}}(0)$. Therefore, if $\xi < \overline{\xi}_{\text{in}}(0)$ then $g$ has no impact on the externalities in equilibrium. If $\overline{\xi}_{\text{in}}(0) < \xi < \overline{\xi}_{\text{unob}}(\tau)$ then larger $g$ increases the externalities in equilibrium by increasing the likelihood of exclusion in a region where informed tilting generates lower externalities.

Second, suppose $\xi > \overline{\xi}_{\text{unob}}(\tau)$. Notice that the amount of externalities under informed tilting is lower than under uninformed tilting if and only if $\xi < \overline{\xi}_{\text{un}}(0)$. Therefore, if $\xi > \overline{\xi}_{\text{un}}(0)$ then $g$ has no impact on the externalities in equilibrium. If $\overline{\xi}_{\text{unob}}(\tau) < \xi < \overline{\xi}_{\text{un}}(0)$ then larger $g$ increases the externalities in equilibrium by increasing the likelihood of uninformed tilting in a region where informed tilting generates lower externalities. ■

**Proof of Proposition 4.** We have shown before that $\overline{\xi}_{\text{unob}}(\tau)$ is a decreasing function of $\tau$. Moreover, $\lim_{\tau \to 1} \overline{\xi}_{\text{unob}}(\tau) < 1$. If $\lim_{\tau \to 1} \overline{\xi}_{\text{unob}}(\tau) > \xi$ then $B$ chooses exclusion regardless

of $\tau$. In this case, $M$ chooses $\tau = \frac{1}{2}$. Suppose $\lim_{\tau \to 1} \overline{\xi}_{\text{unob}}(\tau) \leq \xi$, there exists $\widehat{\tau}(\xi) \in (\frac{1}{2}, 1)$ such that, $\xi \geq \overline{\xi}_{\text{unob}}(\tau) \Leftrightarrow \tau \geq \widehat{\tau}(\xi)$. Moreover, suppose that $\widehat{\Delta}_x(\tau) \leq 1+q$. We can write the expected payoff and stock price as functions of $\tau$ as follows

$$\mathbb{E}[p(\tau)] = \frac{\mu - \gamma\sigma^2 + \left(\frac{\tau \widehat{\Delta}_x(\tau)}{1+q}\gamma\sigma^2 - c\right)\mathbf{1}_{\tau \geq \widehat{\tau}(\xi)}}{1+q-rq} \tag{37}$$

and

$$\mathbb{E}[v(\tau)] = \frac{\mu + rq\mathbb{E}[p(\tau)] - c\mathbf{1}_{\tau \geq \widehat{\tau}(\xi)}}{1+q}. \tag{38}$$

$M$'s expected utility can be rewritten as:

$$\begin{aligned}
\mathbb{E}[U_m(\tau)] &= \omega\mathbb{E}[p(\tau)] + (1-\omega)\mathbb{E}[v(\tau)] \\
&= [\omega + (1-\omega)z]\,\mathbb{E}[p(\tau)] + (1-\omega)\frac{\mu - c\cdot\mathbf{1}_{\tau \geq \widehat{\tau}(\xi)}}{1+q} \\
&= [\omega + (1-\omega)z]\left(\frac{\mu - \gamma\sigma^2}{1+q-rq} + \frac{\frac{\tau\widehat{\Delta}_x(\tau)}{1+q}\gamma\sigma^2 - c}{1+q-rq}\cdot\mathbf{1}_{\tau \geq \widehat{\tau}(\xi)}\right) + (1-\omega)\frac{\mu - c\cdot\mathbf{1}_{\tau \geq \widehat{\tau}(\xi)}}{1+q} \\
&= [\omega + (1-\omega)z]\,\frac{\mu - \gamma\sigma^2}{1+q-rq} + (1-\omega)\frac{\mu}{1+q} \\
&+ \left([\omega + (1-\omega)z]\,\frac{\frac{\tau\widehat{\Delta}_x(\tau)}{1+q}\gamma\sigma^2 - c}{1+q-rq} - (1-\omega)\frac{c}{1+q}\right)\cdot\mathbf{1}_{\tau \geq \widehat{\tau}(\xi)} \\
&= [\omega + (1-\omega)z]\,\frac{\mu - \gamma\sigma^2}{1+q-rq} + (1-\omega)\frac{\mu}{1+q} \\
&+ \frac{c}{1+q}\left(\frac{\tau}{2\tau-1}(1-\omega) - \frac{1}{1-z}\right)\cdot\mathbf{1}_{\tau \geq \widehat{\tau}(\xi)}
\end{aligned}$$

Notice that $\frac{\tau}{2\tau-1}$ decreases in $\tau$. Thus, $M$ chooses $\tau = \widehat{\tau}(\xi)$ if $\frac{\widehat{\tau}(\xi)}{2\widehat{\tau}(\xi)-1}(1-\omega) - \frac{1}{1-z} > 0$, and $\tau = \frac{1}{2}$ otherwise. Notice that

$$\frac{\tau}{2\tau-1}(1-\omega) - \frac{1}{1-z} > 0 \Leftrightarrow \tau < \frac{1}{1+\omega+(1-\omega)z}$$

Thus, $M$ chooses $\tau = \widehat{\tau}(\xi)$ if $\widehat{\tau}(\xi) < \frac{1}{1+\omega+(1-\omega)z}$, and $\tau = \frac{1}{2}$ otherwise.

Next, we plug in $\tau = \frac{1}{1+\omega+(1-\omega)z}$ into $\widehat{\Delta}_x(\tau)$ to check whether $B$'s position is less than $1+q$. It follows that $\widehat{\Delta}_x\left(\frac{1}{1+\omega+(1-\omega)z}\right) \leq 1+q$ is equivalent to $c \leq \frac{\gamma\sigma^2[\omega+(1-\omega)z]}{1+\omega+(1-\omega)z}$. In this case, $B$

can afford to implement tilting at $\tau = \frac{1}{1+\omega+(1-\omega)z}$. If instead $c > \frac{\gamma\sigma^2[\omega+(1-\omega)z]}{1+\omega+(1-\omega)z}$, then $B$ cannot implement tilting for any $\tau < \frac{1}{1+\omega+(1-\omega)z}$ because $\widehat{\Delta}_x(\tau)$ is decreasing in $\tau$. Hence, $M$ chooses $\tau = \frac{1}{2}$.

Suppose $c \leq \frac{\gamma\sigma^2[\omega+(1-\omega)z]}{1+\omega+(1-\omega)z}$ and recall that $\widehat{\tau}(\xi)$ satisfies $\xi = \overline{\xi}_{\text{unob}}(\tau)$, and since $\overline{\xi}_{\text{unob}}(\tau)$ is a decreasing function,

$$\widehat{\tau}(\xi) < \frac{1}{1+\omega+(1-\omega)z} \Leftrightarrow$$
$$\xi > \overline{\xi}_{\text{unob}}\left(\frac{1}{1+\omega+(1-\omega)z}\right)$$

Next, we use the expression for $\overline{\xi}_{\text{unob}}$ to re-write the condition above as:

$$\xi > 1 - \frac{z^{-1}\mu - \gamma\sigma^2}{z^{-1}\mu - \gamma\sigma^2 + c\frac{(1-z)(1-\omega)}{z+\omega-z\omega}} \equiv \overline{\xi}_{\text{disc}}.$$

The right-hand side of this condition increases in $c, \gamma, \sigma$ and it decreases in $\mu, \omega$. It is hump-shaped in $z$, and thus in $r, q$.

Finally, we solve for the lowest value of $\tau \in \left(\frac{1}{2}, \frac{1}{1+\omega+(1-\omega)z}\right)$ that satisfies $\widehat{\Delta}_x(\tau^{\min}) = 1+q$. This leads to $\tau^{\min} = \frac{1}{2}\left(1 + \frac{c(1-\omega-(1-\omega)z)}{\gamma\sigma^2(\omega+(1-\omega)z)}\right)$. For any $\xi \geq \overline{\xi}(\tau^{\min})$, $M$ sets $\tau^* = \tau^{\min}$ because any $\tau < \tau^{\min}$ would lead to exclusion. ■

**Proof of Proposition 5.** Given $x$, $a$, and $y$, the stock price is given by:

$$p(x,a,y) = \frac{\mu - ca - \left(1 - \frac{x+y}{1+q}\right)\gamma\sigma^2}{1+q-rq}. \tag{39}$$

Thus, $A$'s profit is given by:

$$\begin{aligned}
\Pi_A(y) &= y\left(v(x,a,y) - p(x,a,y)\right) \\
&= y\left(\frac{\mu + rqp(x,a,y) - ac}{1+q} - p(x,a,y)\right) \\
&= y\left(\frac{\mu - (1-q-rq)\,p(x,a,y) - ac}{1+q}\right) \\
&= y\left(\frac{\mu - \left[\mu - ca - \left(1 - \frac{x+y}{1+q}\right)\gamma\sigma^2\right] - ac}{1+q}\right) \\
&= y\frac{\left(1 - \frac{x+y}{1+q}\right)\gamma\sigma^2}{1+q}
\end{aligned}$$

and so his trade is given by:

$$y^*(x) = \arg\max_y \Pi_A(y) = \frac{1+q-x}{2}$$

which yields a profit of

$$\Pi_A\left(y^*(x)\right) = \left(\frac{1}{2}\frac{1+q-x}{1+q}\right)^2\gamma\sigma^2.$$

Thus, $B$ expects the stock price to be

$$\begin{aligned}
p\left(x,a,y^*(x)\right) &= (1-\eta)\frac{\mu - ca - \left(1 - \frac{x}{1+q}\right)\gamma\sigma^2}{1+q-rq} + \eta\frac{\mu - ca - \left(1 - \frac{x+y^*(x)}{1+q}\right)\gamma\sigma^2}{1+q-rq} \\
&= \frac{\mu - ca - \left(1 - \frac{x}{1+q} - \eta\frac{y^*(x)}{1+q}\right)\gamma\sigma^2}{1+q-rq} \\
&= \frac{\mu - ca - \left(1 - \frac{x}{1+q} - \eta\frac{\frac{1+q-x}{2}}{1+q}\right)\gamma\sigma^2}{1+q-rq} \\
&= \frac{\mu - ca - \left(1 - \frac{\eta}{2}\right)\left(1 - \frac{x}{1+q}\right)\gamma\sigma^2}{1+q-rq}
\end{aligned}$$

$M$ chooses $a=1$ if and only if

$$\begin{aligned}
\omega p\left(x\left(1\right),1\right)+\left(1-\omega\right)\frac{\mu+rqp\left(x\left(1\right),1\right)-c}{1+q} &> \omega p\left(x\left(0\right),0\right)+\left(1-\omega\right)\frac{\mu+rqp\left(x\left(0\right),0\right)}{1+q}\\
\left[\omega+\left(1-\omega\right)z\right]\left[p\left(x\left(1\right),1\right)-p\left(x\left(0\right),0\right)\right] &> \left(1-\omega\right)\frac{c}{1+q}\\
x\left(1\right)-x\left(0\right) &> \frac{\left(1+q\right)c}{\left(1-\frac{\eta}{2}\right)\gamma\sigma^{2}\left[\omega+\left(1-\omega\right)z\right]}
\end{aligned}$$

$B$ chooses tilting if and only if

$$\begin{aligned}
\xi &\geq \frac{rq\left(p\left(1\right)-p\left(0\right)\right)}{\mu+rqp\left(1\right)}\\
&= rq\frac{\frac{-c+\left(1-\frac{\eta}{2}\right)\left(\frac{\Delta_{x}}{1+q}\right)\gamma\sigma^{2}}{1+q-rq}}{\mu+rq\frac{\mu-c-\left(1-\frac{\eta}{2}\right)\left(1-\frac{\Delta_{x}}{1+q}\right)\gamma\sigma^{2}}{1+q-rq}}\\
&= \frac{-c+\left(1-\frac{\eta}{2}\right)\left(\frac{\Delta_{x}}{1+q}\right)\gamma\sigma^{2}}{\frac{\mu}{z}-c-\left(1-\frac{\eta}{2}\right)\left(1-\frac{\Delta_{x}}{1+q}\right)\gamma\sigma^{2}}\\
&= \frac{\left(1-\omega\right)c}{\left(1-\omega\right)c+\left(\frac{\mu}{z}-\left(1-\frac{\eta}{2}\right)\gamma\sigma^{2}\right)\left(\omega+\frac{z}{1-z}\right)}.
\end{aligned}$$

The condition $x(1)\leq(1+q)$ is equivalent to $c\leq\gamma\sigma^{2}[\omega+(1-\omega)z]\left(1-\frac{\eta}{2}\right)$. If $c>\gamma\sigma^{2}[\omega+(1-\omega)z]\left(1-\frac{\eta}{2}\right)$, then $B$ cannot implement tilting and chooses $x(1)=x(0)=0$. ■

**Proof of Proposition 6.** We define

$$E\left[U\left(a,x\right)\right] \;=\varphi x\left(v\left(x,a\right)-p\left(x,a\right)\right)-\left(1-\varphi\right)\lambda\left(\mu+qrp\left(x,a\right)\right)\left(1-a\xi\right)$$

where $p\left(x,a\right)=\frac{\mu-ca-\left(1-\frac{x}{1+q}\right)\gamma\sigma^{2}}{1+q-rq}$ and $v=\frac{\mu+rqp(x,a)-ca}{1+q}$. Observe that $\varphi x\left(v\left(x,a\right)-p\left(x,a\right)\right)=\varphi\frac{x}{1+q}(1-\frac{x}{1+q})\gamma\sigma^{2}$. Thus

$$x^{*}\left(a\right)=\arg\max_{x\geq0}\;E\left[U\left(a,x\right)\right]=\frac{1+q}{2}\max\left\{1-\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda\left(1-a\xi\right),0\right\}$$

and

$$E\left[U\left(a,x^{*}\left(a\right)\right)\right]=\begin{cases}\varphi\frac{1-\left[\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda(1-a\xi)\right]^{2}}{4}\gamma\sigma^{2}- & \\ \left(1-\varphi\right)\lambda\left(\mu+z\frac{\mu-ca-\frac{1+\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda(1-a\xi)}{2}\gamma\sigma^{2}}{1-z}\right)\left(1-a\xi\right) & \text{if } 1\geq\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda\left(1-a\xi\right)\\ -\left(1-\varphi\right)\lambda\left(\mu+z\frac{\mu-ca-\gamma\sigma^{2}}{1-z}\right)\left(1-a\xi\right) & \text{else.}\end{cases} \tag{40}$$

Consider parts (i) and (ii). Notice that

$$x^{*}(1)-x^{*}(0)=\frac{1+q}{2}\times\begin{cases}\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda\xi & \text{if } \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda\leq1\\ 1-\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda\left(1-\xi\right) & \text{if } 1<\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda<\frac{1}{1-\xi}\\ 0 & \text{if } \frac{1}{1-\xi}\leq\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda\end{cases} \tag{41}$$

Thus, $x^{*}(1)-x^{*}(0)>\overline{\Delta}_{x}$ if and only if $\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda\leq1$ and $\frac{1+q}{2}\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda\xi>\frac{c(1+q)}{\gamma\sigma^{2}[\omega+(1-\omega)z]}$, or

$1<\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda<\frac{1}{1-\xi}$ and $\frac{1+q}{2}\left(1-\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda\left(1-\xi\right)\right)>\frac{c(1+q)}{\gamma\sigma^{2}[\omega+(1-\omega)z]}$. These conditions can be rewritten as

$$\begin{aligned}\frac{1}{\xi}\frac{2c}{\gamma\sigma^{2}\left[\omega+(1-\omega)z\right]}<\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda\leq1 \text{ or}\\ 1<\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda<\frac{1}{1-\xi}\left(1-\frac{2c}{\gamma\sigma^{2}\left[\omega+(1-\omega)z\right]}\right).\end{aligned}$$

Notice

$$\frac{1}{\xi}\frac{2c}{\gamma\sigma^{2}\left[\omega+(1-\omega)z\right]}<\frac{1}{1-\xi}\left(1-\frac{2c}{\gamma\sigma^{2}\left[\omega+(1-\omega)z\right]}\right)\Leftrightarrow\xi>\frac{2c}{\gamma\sigma^{2}\left[\omega+(1-\omega)z\right]}.$$

Thus, if $\xi\leq\frac{2c}{\gamma\sigma^{2}[\omega+(1-\omega)z]}$ then the condition is empty. That is, $x^{*}(1)-x^{*}(0)\leq\overline{\Delta}_{x}$, $a=0$, and $B$ buys $x^{*}\left(0\right)$ shares. If $\xi>\frac{2c}{\gamma\sigma^{2}[\omega+(1-\omega)z]}$ then the condition above is reduced to

$$\frac{1}{\xi}\frac{2c}{\gamma\sigma^{2}\left[\omega+(1-\omega)z\right]}<\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda<\frac{1}{1-\xi}\left(1-\frac{2c}{\gamma\sigma^{2}\left[\omega+(1-\omega)z\right]}\right)\Leftrightarrow\varphi\in\left(\underline{\varphi}\left(\xi\right),\overline{\varphi}\left(\xi\right)\right)$$

where

$$\overline{\varphi}(\xi) \equiv \frac{\frac{z}{1-z}\lambda}{\frac{z}{1-z}\lambda + \frac{1}{\xi}\frac{2c}{\gamma\sigma^2[\omega+(1-\omega)z]}}$$
$$\underline{\varphi}(\xi) \equiv \frac{\frac{z}{1-z}\lambda}{\frac{z}{1-z}\lambda + \frac{1}{1-\xi}\left(1 - \frac{2c}{\gamma\sigma^2[\omega+(1-\omega)z]}\right)}.$$

Thus, if $\varphi \in \left(\underline{\varphi}(\xi), \overline{\varphi}(\xi)\right)$ then $x^*(1) - x^*(0) > \overline{\Delta}_x$, $a = 1$, and $B$ buys $x^*(1)$ shares. If $\varphi \notin \left(\underline{\varphi}(\xi), \overline{\varphi}(\xi)\right)$ then $x^*(1) - x^*(0) \leq \overline{\Delta}_x$, $a = 0$, and $B$ buys $x^*(0)$ shares.

Consider part (iii). Notice that $x^*(a)$ weakly increases in $\varphi$. If $M$ chooses $a^* = 0$ in equilibrium, then the externalities increase with $\varphi$ if $x^*(0) > 0 \Leftrightarrow \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda < 1 \Leftrightarrow \frac{\frac{z}{1-z}\lambda}{1+\frac{z}{1-z}\lambda} < \varphi$, and are invariant to $\varphi$ otherwise. Notice that $\xi > \frac{2c}{\gamma\sigma^2[\omega+(1-\omega)z]}$ implies $\frac{1}{\xi}\frac{2c}{\gamma\sigma^2[\omega+(1-\omega)z]} < 1 < \frac{1}{1-\xi}\left(1 - \frac{2c}{\gamma\sigma^2[\omega+(1-\omega)z]}\right)$ and hence $\frac{\frac{z}{1-z}\lambda}{1+\frac{z}{1-z}\lambda} \in \left(\underline{\varphi}(\xi), \overline{\varphi}(\xi)\right)$. That is, if $\varphi < \underline{\varphi}(\xi)$ then $\varphi < \frac{\frac{z}{1-z}\lambda}{1+\frac{z}{1-z}\lambda}$, and hence, $x^*(0) = 0$.

Suppose $M$ chooses $a^* = 1$ in equilibrium. Then, it must be $\varphi \in \left[\underline{\varphi}(\xi), \overline{\varphi}(\xi)\right]$, and the externalities are given by

$$\lambda\left(\mu + z\frac{\mu - c - \frac{1+\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda(1-\xi)}{2}\gamma\sigma^2}{1-z}\right)(1-\xi),$$

which is increasing in $\varphi$. Notice

$$\lambda\left(\mu + z\frac{\mu - c - \frac{1+\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda(1-\xi)}{2}\gamma\sigma^2}{1-z}\right)(1-\xi) < \lambda\frac{\mu - z\gamma\sigma^2}{1-z} \Leftrightarrow$$
$$\frac{\left(\frac{\mu}{z} - c - \frac{1}{2}\gamma\sigma^2\right)(1-\xi) - \left(\frac{\mu}{z} - \gamma\sigma^2\right)}{\frac{1}{2}(1-\xi)^2\gamma\sigma^2} < \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda \Leftrightarrow$$
$$\varphi < \varphi_{NC}(\xi) \equiv \frac{\frac{z}{1-z}\lambda}{\frac{z}{1-z}\lambda + \frac{\left(\frac{\mu}{z} - c - \frac{1}{2}\gamma\sigma^2\right)(1-\xi) - \left(\frac{\mu}{z} - \gamma\sigma^2\right)}{\frac{1}{2}(1-\xi)^2\gamma\sigma^2}}$$

and

$$\frac{\left(\frac{\mu}{z}-c-\frac{1}{2}\gamma\sigma^2\right)(1-\xi)-\left(\frac{\mu}{z}-\gamma\sigma^2\right)}{\frac{1}{2}(1-\xi)^2\gamma\sigma^2}<\frac{1}{1-\xi}\left(1-\frac{2c}{\gamma\sigma^2\left[\omega+(1-\omega)z\right]}\right)\Leftrightarrow$$

$$\xi>\xi_{NC}\equiv\frac{\frac{4c}{\omega+(1-\omega)z}-c}{\frac{\mu}{z}-\gamma\sigma^2+\frac{4c}{\omega+(1-\omega)z}-c}.$$

Thus, if $\xi\leq\max\left\{\frac{2c}{\gamma\sigma^2[\omega+(1-\omega)z]},\xi_{NC}\right\}$ then $\varphi_{NC}(\xi)\leq\underline{\varphi}(\xi)$ and $f^*(\varphi)\geq f^*(0)$ for all $\varphi>0$, with the inequality being strict if and only if $\varphi>\underline{\varphi}(\xi)$. If $\xi>\max\left\{\frac{2c}{\gamma\sigma^2[\omega+(1-\omega)z]},\xi_{NC}\right\}$ then $\varphi_{NC}(\xi)>\underline{\varphi}(\xi)$. Thus, if $\varphi\in(0,\underline{\varphi}(\xi)]$ then $f^*(\varphi)=f^*(0)$, if $\varphi\in\left(\underline{\varphi}(\xi),\varphi_{NC}(\xi)\right)$ then $f^*(\varphi)<f^*(0)$, and if $\varphi\in(\varphi_{NC}(\xi),1)$ then $f^*(\varphi)>f^*(0)$. ■

**Proof of Proposition 7.** To see part (i), let $x^*(a,\varphi)$ be the optimal trade of $B$ if her type is $\varphi$ and $M$ takes action $a$. We let $f(\varphi)$ and $\pi(\varphi)$ be the externalities and trading profits in equilibrium that are induced by strategies $x^*(a,\varphi)$, respectively. The equilibrium utility of type $\varphi$ is $\varphi\pi(\varphi)-(1-\varphi)f(\varphi)$. Suppose to the contrary there are $\varphi'<\varphi''$ such that $f(\varphi')>f(\varphi'')$, that is, a blockholder with a greater profits motive induces less externalities in equilibrium. This implies either $x^*(0,\varphi')\neq x^*(0,\varphi'')$ or $x^*(1,\varphi')\neq x^*(1,\varphi'')$. Notice $f(\varphi')>f^*(\varphi'')$ implies $\pi(\varphi')>\pi(\varphi'')$. Otherwise,

$$\varphi'\pi(\varphi')-(1-\varphi')f(\varphi')<\varphi'\pi(\varphi'')-(1-\varphi')f(\varphi'')$$

and type $\varphi'$ has a profitable deviation from $x^*(a,\varphi')$ to $x^*(a,\varphi'')$, a contradiction. Suppose $f(\varphi')>f^*(\varphi'')$ and $\pi(\varphi')>\pi(\varphi'')$. By revealed preferences of types $\varphi'$ and $\varphi''$ we have

$$\varphi'\pi(\varphi')-(1-\varphi')f(\varphi')>\varphi'\pi(\varphi'')-(1-\varphi')f(\varphi'')\Leftrightarrow\varphi'>\frac{\frac{f(\varphi')-f(\varphi'')}{\pi(\varphi')-\pi(\varphi'')}}{1+\frac{f(\varphi')-f(\varphi'')}{\pi(\varphi')-\pi(\varphi'')}}$$

$$\varphi''\pi(\varphi'')-(1-\varphi'')f(\varphi'')>\varphi''\pi(\varphi')-(1-\varphi'')f(\varphi')\Leftrightarrow\varphi''<\frac{\frac{f(\varphi')-f(\varphi'')}{\pi(\varphi')-\pi(\varphi'')}}{1+\frac{f(\varphi')-f(\varphi'')}{\pi(\varphi')-\pi(\varphi'')}}$$

But since $\varphi''>\varphi'$, we get a contradiction.

Consider part (ii). Suppose $B$ wants to induce $a=0$ in equilibrium. Given $a=0$, the optimal strategy is $x_C(0)=\frac{1+q}{2}\max\left\{1-\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda,0\right\}$ and $x(1)$ such that $x(1)-x_C(0)\leq\overline{\Delta}_x$

(e.g., $x(1) = x_C(0)$). This will generate $B$ an expected payoff of

$$U_C(0) = \varphi \frac{x_C(0)}{1+q}\left(1 - \frac{x_C(0)}{1+q}\right)\gamma\sigma^2 - (1-\varphi)\lambda\left(\frac{\mu}{1-z} - \frac{z}{1-z}\gamma\sigma^2\left(1 - \frac{x_C(0)}{1+q}\right)\right).$$

Notice that this term does not depend on $\xi$. If $B$ wants to induce $a = 1$ in equilibrium, she will choose $x(0) = 0$ and

$$\begin{aligned} x_C(1) &= \max\left\{\frac{1+q}{2}\max\left\{1 - \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda(1-\xi), 0\right\}, \overline{\Delta}_x\right\} \\ &= \frac{1+q}{2}\max\left\{1 - \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda(1-\xi), \frac{2c}{\gamma\sigma^2[\omega + (1-\omega)z]}\right\}. \end{aligned}$$

This will generate $B$ an expected payoff of

$$U_C(1) \;= \varphi \frac{x_C(1)}{1+q}\left(1 - \frac{x_C(1)}{1+q}\right)\gamma\sigma^2 - (1-\varphi)\lambda\left(\frac{\mu - zc}{1-z} - \frac{z}{1-z}\gamma\sigma^2\left(1 - \frac{x_C(1)}{1+q}\right)\right)(1-\xi).$$

We consider two cases:

1. If $\overline{\Delta}_x > \frac{1+q}{2}\max\left\{1 - \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda(1-\xi), 0\right\}$ then $x_C(1) = \overline{\Delta}_x$ and it does not depend on $\xi$. In this case,

$$\frac{dU_C(1)}{d\xi} = (1-\varphi)\lambda\left(\frac{\mu - zc}{1-z} - \frac{z}{1-z}\gamma\sigma^2\left(1 - \frac{\overline{\Delta}_x}{1+q}\right)\right).$$

Notice

$$\frac{dU_C(1)}{d\xi} > 0 \Leftrightarrow \mu/z - c - \gamma\sigma^2 > -\gamma\sigma^2\frac{\overline{\Delta}_x}{1+q},$$

which always holds given that $\overline{\Delta}_x > 0$, $z \in (0,1)$, and the assumption $\mu - c - \gamma\sigma^2 > 0$.

2. If $\overline{\Delta}_x \leq \frac{1+q}{2}\max\left\{1 - \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda(1-\xi), 0\right\}$ then $x_C(1) = \frac{1+q}{2}\left(1 - \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda(1-\xi)\right) > 0$, which is the optimal trade of $B$. Therefore,

$$\frac{dU_C(1)}{d\xi} = \frac{\partial U_C(1)}{\partial \xi} + \frac{\partial U_C(1)}{\partial x}|_{x=x_C(1)} \times \frac{\partial x_C(1)}{\partial \xi}.$$

By the envelope theorem, $\frac{\partial U_C(1)}{\partial x}|_{x=x_C(1)}=0$. Also,

$$\frac{\partial U_C(1)}{\partial \xi}=(1-\varphi)\lambda\left(\frac{\mu-zc}{1-z}-\frac{z}{1-z}\gamma\sigma^2\frac{1+\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda(1-\xi)}{2}\right)$$

where

$$\frac{\partial U_C(1)}{\partial \xi}>0\Leftrightarrow \mu/z-c-\gamma\sigma^2>-\gamma\sigma^2\frac{1-\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda(1-\xi)}{2}$$

which always holds given that $1-\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda(1-\xi)>0$, $z\in(0,1)$, and the assumption $\mu-c-\gamma\sigma^2>0$.

Since $U_C(1)$ increases in $\xi$ and $U(0)$ is invariant to $\xi$, there exists $\overline{\xi}_C>0$ such that $U_C(1)>U_C(0)$ if and only if $\xi\geq\overline{\xi}_C$, as required.

Consider part (2.a). We prove $\overline{\xi}_C<1$. Indeed, if $\xi=1$ then inducing $a=1$ implies no externalities and

$$U_C(1)\ =\varphi\frac{x_C(1)}{1+q}\left(1-\frac{x_C(1)}{1+q}\right)\gamma\sigma^2.$$

If $\frac{1}{2}\geq\frac{c}{\gamma\sigma^2[\omega+(1-\omega)z]}$ then $x_C(1)=\frac{1+q}{2}$ which is the quantity that maximizes the unconstrained gains from trade, and therefore it must be $U_C(1)>U_C(0)$. Suppose $\frac{1}{2}<\frac{c}{\gamma\sigma^2[\omega+(1-\omega)z]}$. In this case, $x_C(1)=\frac{(1+q)c}{\gamma\sigma^2[\omega+(1-\omega)z]}$ and

$$U_C(1)=\varphi\frac{c}{\gamma\sigma^2[\omega+(1-\omega)z]}\left(1-\frac{c}{\gamma\sigma^2[\omega+(1-\omega)z]}\right)\gamma\sigma^2$$

Assumption (14), $\frac{c}{\gamma\sigma^2[\omega+(1-\omega)z]}\leq 1$, guarantees $U_C(1)\geq 0$. If $1-\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda\leq 0$ then $x_C(0)=0$ and

$$U_C(0)=-(1-\varphi)\lambda\left(\frac{\mu}{1-z}-\frac{z}{1-z}\gamma\sigma^2\right)<0$$

and thus, $U_C(1)>U_C(0)$, that is, $\overline{\xi}_C<1$. Suppose $1-\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda>0$. Then, $x_C(0)=\frac{1+q}{2}\left(1-\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda\right)$ and

$$\begin{aligned}U_C(0)&=\varphi\frac{x_C(0)}{1+q}\left(1-\frac{x_C(0)}{1+q}\right)\gamma\sigma^2-(1-\varphi)\lambda\left(\frac{\mu}{1-z}-\frac{z}{1-z}\gamma\sigma^2\left(1-\frac{x_C(0)}{1+q}\right)\right)\\&=\varphi\left[\frac{1}{4}\gamma\sigma^2\left(1-\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda\right)^2-\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda\left(\frac{\mu}{z}-\gamma\sigma^2\right)\right].\end{aligned}$$

Notice $U_C(0)$ increases in $\varphi$. Moreover, if $\varphi = 1$ then $U_C(0) > U_C(1)$ and if $\varphi = \frac{\frac{z}{1-z}\lambda}{1+\frac{z}{1-z}\lambda}$ then $U_C(0) < U_C(1)$. Therefore, if $\frac{1}{2} < \frac{c}{\gamma\sigma^2[\omega+(1-\omega)z]}$ then there is $\varphi^* \in (\frac{z\lambda}{1-z+z\lambda}, 1)$ such that if $\varphi < \varphi^*$ then $\overline{\xi}_C < 1$, and otherwise, $\overline{\xi}_C = 1$.

Finally, we prove part (2.b). Recall that in the baseline model, when $\varphi = 0$, then $B$ induces $a = 1$ if and only if $\xi \geq \frac{\gamma\sigma^2\frac{\overline{\Delta}_x}{1+q} - c}{\mu/z - \gamma\sigma^2 + \gamma\sigma^2\frac{\overline{\Delta}_x}{1+q} - c}$. Suppose $\varphi > 0$ but $\frac{1+q}{2}\max\left\{1 - \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda(1-\xi), 0\right\} < \overline{\Delta}_x$ and $1 - \frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda < 0$. Then, $x_C(1) = \overline{\Delta}_x$ and $x_C(0) = 0$. That is, although $\varphi > 0$, $B$'s optimal trade that induces action $a$ is the same as in the baseline model when $\varphi = 0$. Notice that $U_C(1) > U_C(0)$ if and only if

$$\frac{\overline{\Delta}_x}{1+q}\left(1 - \frac{\overline{\Delta}_x}{1+q}\right)\gamma\sigma^2 > \frac{1-\varphi}{\varphi}\lambda\left[\left(\frac{\mu - zc}{1-z} - \frac{z}{1-z}\gamma\sigma^2\left(1 - \frac{\overline{\Delta}_x}{1+q}\right)\right)(1-\xi) - \left(\frac{\mu}{1-z} - \frac{z}{1-z}\gamma\sigma^2\right)\right]$$

$$\xi > \frac{\gamma\sigma^2\frac{\overline{\Delta}_x}{1+q} - c - \frac{\frac{\overline{\Delta}_x}{1+q}\left(1-\frac{\overline{\Delta}_x}{1+q}\right)\gamma\sigma^2}{\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda}}{\mu/z - \gamma\sigma^2 + \gamma\sigma^2\frac{\overline{\Delta}_x}{1+q} - c} = \overline{\xi}_C$$

Thus, if $\frac{\overline{\Delta}_x}{1+q} < 1$ and $\frac{\gamma\sigma^2\frac{\overline{\Delta}_x}{1+q} - c - \frac{\frac{\overline{\Delta}_x}{1+q}\left(1-\frac{\overline{\Delta}_x}{1+q}\right)\gamma\sigma^2}{\frac{1-\varphi}{\varphi}\frac{z}{1-z}\lambda}}{\mu/z - \gamma\sigma^2 + \gamma\sigma^2\frac{\overline{\Delta}_x}{1+q} - c} < \xi < \frac{\gamma\sigma^2\frac{\overline{\Delta}_x}{1+q} - c}{\mu/z - \gamma\sigma^2 + \gamma\sigma^2\frac{\overline{\Delta}_x}{1+q} - c}$ then $B$ chooses $a = 0$ if $\varphi = 0$ but $a = 1$ if $\varphi > 0$. Notice that sufficient conditions that satisfies these conditions are $\frac{1+q}{2} < \overline{\Delta}_x$ and $\varphi < \frac{z\lambda}{1-z+z\lambda}$. ■

# B Implications for Firm Value

Proposition 8 compares expected firm value under tilting or exclusion, to study whether the blockholder's desire to minimize externalities comes at the expense of firm value. The analysis holds irrespective of the functional form for the externalities.

**Proposition 8** *(Firm value comparison): The expected value of the firm under tilting is always lower than under exclusion:*

$$E[V|Tilting] = E[V|Exclusion] - \frac{\omega c}{\omega + (1-\omega)z}. \tag{42}$$

***Proof.*** *Expected firm value under exclusion is given by:*

$$\begin{aligned} E\left[V|Exclusion\right] &= \mu + rqp\left(0\right) \\ &= \mu + rq\left[\frac{\mu - \gamma\sigma^2}{1+q-rq}\right] \\ &= \mu\frac{1}{1-z} - \gamma\sigma^2\frac{z}{1-z} \end{aligned}$$

*Expected firm value under tilting is given by:*

$$\begin{aligned} E\left[V|Tilting\right] &= \mu + rqp\left(1\right) - c \\ &= \mu + rq\left[\frac{\mu - \gamma\sigma^2 + \frac{c}{\omega+(1-\omega)\frac{rq}{1+q}} - c}{1+q-rq}\right] - c \\ &= \mu\frac{1}{1-z} - \gamma\sigma^2\frac{z}{1-z} - c\frac{\omega}{\omega+(1-\omega)z} \end{aligned}$$

*We thus have*

$$E\left[V|Tilting\right] = E\left[V|Exclusion\right] - c\frac{\omega}{\omega+(1-\omega)z}.$$

■

On the one hand, tilting induces the corrective action which reduces firm value by $c$; on the other hand, tilting leads to a higher stock price which allows the firm to invest more in the positive-NPV project. Proposition 8 shows that first force is always greater than the second – firm value is always lower under tilting than under exclusion – for any strictly positive $\omega$. The intuition is as follows. If $\omega > 0$, the manager is concerned about the stock price. Thus, he will take the action partly due to his stock price concerns, rather than because the action increases firm value by allowing the firm to invest more, and so he will take the action even if it reduces firm value. The action increases firm value if and only if $rq\left[p\left(1\right) - p\left(0\right)\right] > c$, but it increases the manager's payoff if and only if $rq\left[p\left(1\right) - p\left(0\right)\right] > \frac{c}{\frac{1+q}{rq}\frac{\omega}{1-\omega}+1}$. Since the manager only places weight $(1-\omega)$ on fundamental value, he does not fully internalize the cost of the action. Since the blockholder chooses $x\left(1\right)$ so that the manager is exactly indifferent between $a = 1$ and $a = 0$, in equilibrium we have $rq\left[p\left(1\right) - p\left(0\right)\right] = \frac{c}{\frac{1+q}{rq}\frac{\omega}{1-\omega}+1} < c$, and so the action always reduces firm value.

Practitioners debate whether there is a trade-off between financial and social value. In our setting, financial value corresponds to firm value $V$, and social value corresponds to the negative of externalities $-f$. From the firm's perspective, there is always a trade-off between financial and social value – actions taken to increase social value are costly to financial value. However, Proposition 8 shows that, from society's perspective, there need not be a trade-off. If exclusion is the optimal strategy, then the presence of a blockholder with purely social objectives is not at the expense of financial value – indeed, the blockholder's investment strategy leads to greater financial value (relative to tilting) even though the blockholder is unconcerned with financial value. However, it would be incorrect to conclude that there is no trade-off from the firm's perspective – forcing the firm to take the action would automatically reduce financial value. Instead, the absence of the trade-off from society's perspective arises because the blockholder can reduce externalities more by starving the firm of capital rather than encouraging it to take the costly action. Overall, whether *investors'* pursuit of social objectives reduces firm value has no bearing on whether *firms* face this trade-off.

Finally, since the blockholder chooses endogenously whether to tilt or exclude a firm, there are conditions under which tilted firms have both higher firm value and lower externalities than excluded firms. This positive correlation between financial and social value may lead to conclusions that there is no trade-off between these objectives, when the positive correlation is driven by selection – the investor is choosing to tilt in companies that are more valuable to begin with. Corollary 1 demonstrates this result.

**Corollary 1** *(No trade-off):*

*(i) Let $i$ denote a firm in which the blockholder optimally tilts, and $j$ denote a firm that she optimally excludes. Let $i$ and $j$ differ only in $(\mu, \xi)$. There exists $\xi^* < 1$ such that if $\xi_i > \xi^*$ and $\mu_i - \mu_j > c\frac{\omega(1-z)}{\omega(1-z)+z}$, then firm $i$ has a higher expected value and lower expected externalities than firm $j$.*

*(ii) The blockholder's trading profits under tilting are greater than under exclusion.*

***Proof.*** *We start with part (i). Under exclusion, expected firm value and externalities are given as follows:*

$$E\left[V_i|Exclusion\right] = \mu_i + r_i q_i p_i\left(0\right) = \frac{\mu_i - \gamma\sigma_i^2 z_i}{1 - z_i}$$
$$E\left[f_i|Exclusion\right] = \lambda_i E\left[V_i|Exclusion\right].$$

*Under tilting, expected firm value and externalities are given by:*

$$E\left[V_i|Tilting\right] = \mu_i + r_i q_i p_i\left(1\right) - c_i = \frac{\mu_i - \gamma\sigma_i^2 z_i}{1 - z_i} - c_i \frac{\omega_i}{\omega_i + \left(1 - \omega_i\right) z_i}$$
$$E\left[f_i|Tilting\right] = \lambda_i(E\left[V_i|Tilting\right] + c_i)(1 - \xi_i).$$

*We thus have* $E\left[V_i|Tilting\right] > E\left[V_j|Exclusion\right]$ *and* $E\left[f_i|Tilting\right] < E\left[f_j|Exclusion\right]$ *if and only if*

$$\frac{\lambda_i}{\lambda_j}(E\left[V_i|Tilting\right] + c_i)(1 - \xi_i) < E\left[V_j|Exclusion\right] < E\left[V_i|Tilting\right]$$

*Suppose that firm* $i$ *and* $j$ *differ only in* $(\mu, \xi)$*, with all other parameters constant. We have* $\lambda_i(E\left[V_i|Tilting\right] + c_i)(1 - \xi_i) < \lambda_j E\left[V_j|Exclusion\right]$ *if and only if:*

$$\frac{\lambda_i}{\lambda_j}(E\left[V_i|Tilting\right] + c_i)(1 - \xi_i) < E\left[V_j|Exclusion\right] \Leftrightarrow$$
$$\frac{\frac{(\mu_i - \mu_j)}{1-z} + c\frac{(1-\omega)}{\omega z^{-1} + (1-\omega)}}{\frac{\mu_i - \gamma\sigma^2 z}{1-z} + c\frac{(1-\omega)}{\omega z^{-1} + (1-\omega)}} < \xi_i.$$

*Note that the left-hand side is strictly smaller than 1 since* $\mu_j > \gamma\sigma^2 + c > z\gamma\sigma^2$.

*The condition* $E\left[V_j|Exclusion\right] < E\left[V_i|Tilting\right]$ *is equivalent to:*

$$\frac{\mu_i - \gamma\sigma^2 z}{1 - z} - c\frac{\omega}{\omega + \left(1 - \omega\right) z} > \frac{\mu_j - \gamma\sigma^2 z}{1 - z} \Leftrightarrow \mu_i - \mu_j > c\frac{\omega(1 - z)}{\omega + (1 - \omega)z}.$$

*We now move to part (ii). Under exclusion, B's trading profits are zero as she owns no shares. Under tilting, her profits are:*

$$E\left[\Pi|Tilting\right] = x\left(1\right)\left(v\left(1\right) - p\left(1\right)\right) = x\left(1\right)\left(\frac{\mu + rqp\left(1\right) - c}{1 + q} - p\left(1\right)\right).$$

*This is positive if:*

$$
\begin{aligned}
x(1)\left(\frac{\mu + rqp(1) - c}{1+q} - p(1)\right) > 0 \Leftrightarrow \\
p(1) < \frac{\mu - c}{1+q-rq} \Leftrightarrow \\
\frac{\mu - c - \left(1 - \frac{x(1)}{1+q}\right)\gamma\sigma^2}{1+q-rq} < \frac{\mu - c}{1+q-rq} \Leftrightarrow \\
x(1) < 1+q \Leftrightarrow \\
c < \gamma\sigma^2\left[\omega + (1-\omega)z\right]
\end{aligned}
$$

*which holds due to inequality (14).* ■

We start with part (i). If $\mu_i$ is sufficiently higher than $\mu_j$, then the value of firm $i$ is higher than firm $j$; if the action is sufficiently powerful, then externalities are also lower. However, this correlation is driven by selection – the blockholder endogenously chooses to tilt in firms in which the action is powerful, and if such firms are also more valuable firms, then it will seem that financial value and and social value coincide. However, there remains a trade-off between both objectives, since if the blockholder chose to exclude firm $i$, its value would be higher.

We now move to part (ii). In the model, the blockholder's objective function is to minimize externalities. A more general objective function would involve a weighted sum of the blockholder's trading profits and (the negative of) externalities – the blockholder does not receive per-share firm value, but firm value minus the price paid. Under exclusion, the blockholder's profit is zero as she owns no shares. Under tilting, the blockholder's profits are positive. Since households are risk-averse, they are only willing to hold a strictly positive amount if the stock price is less than the fundamental value of the firm, and so the blockholder earns trading profits. Intuitively, buying shares upon the corrective action not only rewards the action, but also gives the blockholder a return for bearing risk. Thus, there is no trade-off between social value and the blockholder's trading profits.