Firm Size and Complementarity between Geography and Products

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Abstract

I develop a model of heterogeneous multi-product firms operating in multiple markets that comprise consumers with heterogeneous tastes. In the model, firms choose two extensive margins: product scope (how many products to sell) and geographic scope (how many markets to sell products to). Their decisions on product and geographic scopes interact in two ways: (1) the more products firms sell and the larger profit they expect, the more likely they are to enter a market, and (2) the more markets they enter, the greater variety of products they sell to cater to heterogeneous local tastes. I estimate the model to match moments in the Japanese barcode-level transaction data using the simulated method of moments. I run counterfactual exercises and find that eliminating either product or geographic scope underestimates firm size variations by 86% to 94% due to the interaction between the two margins. Furthermore, I analyze how missing one of the margins affects the welfare implication of size-dependent policies, taking an actual SME subsidy in Japan as an example. The results indicate that omitting the product scope margin underestimates the welfare costs of the subsidy by more than half.

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1. Introduction

In many countries, markets consist of a few large firms and many small firms. The same is true for the Japanese manufacturing industry, where small and medium enterprises (SMEs) account for 98.5% of the total number of firms, and the remaining 1.5% of large firms command over half of the market share.[1] What is the economic mechanism that generates this enormous firm size heterogeneity?[2] Answering this question is essential for quantifying models of heterogeneous firms and for understanding the welfare implication of size-dependent policies.

Early research on firm size heterogeneity focused on productivity differences across firms; higher-productivity firms could charge lower prices and occupy larger market shares.[3,4] Recently, the availability of firm- or product-level sales data allows researchers to point to two extensive margins that amplify productivity differences: product scope (the number of products firms sell) and geographic scope (the number of markets in which firms sell their products to consumers).[5] The mechanism of how each scope amplifies

[1]The numbers are from the 2016 Economic Census for Business Activity. In the manufacturing industry, small and medium enterprises are defined as firms with fewer than 300 employees.

[2]Throughout the paper, my focus is on manufacturing firms, not retailers. I use the term "firm size" to mean the total sales of the manufacturing firm.

[3]For example, see Jovanovic (1982), Hopenhayn (1992), Melitz (2003), Luttmer (2007), and Bartelsman et al. (2013).

[4]More broadly speaking, there are models in which firm productivity differences emerge as the equilibrium outcome of ex ante homogeneous firms. For example, Bagwell and Ramey (1994) feature a model where homogeneous firms choose the degree of advertisement as a mixed strategy. Ex post productivity of firms depends on the degree of advertising. Such equilibria can be purified if firms have small cost differences about which they are privately informed, leading to a framework close to the typical models of heterogeneous productivity firms. For example, see Bagwell and Wolinsky (2002).

[5]The role of product scope in firm size heterogeneity is documented in Arkolakis et al. (2010), Bernard et al. (2010), Eckel and Neary (2010), Bernard et al. (2011), Mayer et al. (2014), and Hottman et al. (2016). The role of geographic scope in determining firm size heterogeneity is documented in recent trade literature such as Eaton et al. (2011). More recently, Bernard et al. (2019) finds the number of connections to buyers carries explanatory power for firm size, which has a similar role for geographic scope.

firm size heterogeneity is intuitive. As higher-productivity firms sell more products—or sell their products in more markets—they become larger.

Despite the extent to which previous literature has considered the role of the extensive margins, it has typically examined only *one* extensive margin at a time.[6] These models tend to leave one-third to two-thirds of the firm size variation unexplained.[7] Furthermore, these models do not account for the ways in which firms' product and geographic scopes interact with each other.

In this paper, I develop a structural model that incorporates both extensive margins. This model highlights the quantitative importance of having *both* geographic and product scopes to explain firm size heterogeneity and understand the welfare implications of size-dependent policies. Counterfactual exercises show that missing either of the extensive margins in the model results in a dramatic underestimation of firm size heterogeneity due to the absence of complementarity between the two margins. Furthermore, omitting either of the extensive margins distorts the welfare implications of size-dependent policies.

I begin by documenting four empirical patterns that motivate the structure of the model. I use barcode-level retail scanner data from the Japanese consumer packaged goods industry. Unlike the household scanner or export data, which most existing literature uses, the retail scanner data allows me

[6] An exception is Bernard et al. (2011) which models multi-product firms exporting to multiple markets. While their focus is on explaining firms' exporting behavior using U.S. firm-level export data, my focus is to quantify the interaction between product and geographic scope in generating firm size heterogeneity in a domestic economy context using Japanese product-level transaction data. In particular, this paper departs from Bernard et al. (2011) in two ways: First, I utilize product-level transaction data to show empirical patterns of product and geographic scopes. Second, I utilize the data to structurally estimate model parameters, which I use to quantify how eliminating either of the extensive margins affects firm size heterogeneity and the welfare implications of size-dependent policies.

[7] For example, Eaton et al. (2011) incorporate geographic scope into the model by considering single-product firms exporting to multiple countries. Although the model fits important geographical patterns of firm exporting behaviors, "it leaves the vastly different performance of the same firm in different markets as a residual."

to observe barcode-level sales and the sales locations for each transaction at once. The visibility of both barcode-level sales and sales location enables me to explore the roles of geographic and product scope in firm size heterogeneity.

The first two patterns suggest that firms' decisions regarding market entry and the number of products to sell in a market are associated with the market size; namely, larger firms enter both large and small markets, whereas smaller firms enter only large markets. Moreover, firms sell more products in larger markets.

The last two patterns imply that firms' decisions about product and geographic scopes complement one another. Larger firms sell more products in a given market, suggesting that as firms expect to sell more products and generate larger profits, they are more likely to enter the market. Moreover, firms sell (partially) different sets of products across markets, which is consistent with the fact that consumer tastes are heterogeneous across markets. As firms enter more markets, they may increase their product scope by offering different ranges of products to cater to various local consumer tastes.

To determine which factors are likely to account for these empirical patterns, I conduct a survey of Japanese packaged food manufacturing firms. Previous studies proposed various model elements that generate the product or geographic scope variation across firms. These model elements usually take the form of barriers for the firms, preventing them from increasing their product or geographic scope. Rather than making an arbitrary choice based on the approach used in existing literature, I make inquires to the surveyed firms regarding what barriers exist to expanding their product and geographic scopes. The survey results indicate that there are two fixed costs of practical importance: (1) the fixed costs of entering a market, and (2) the fixed costs of selling a product.

Motivated by the empirical patterns and surveys, I develop a model of

heterogeneous firms. My model is a multi-product firm version of the one developed by Melitz (2003), as augmented by Bernard et al. (2011), featuring heterogeneity in consumer tastes across markets. Firms make static decisions on which products to sell in which markets. When making such decisions, firms incur fixed costs from both entering a market and selling a product in each market.

The model offers insight about how geographic and product scopes jointly amplify productivity differences. Higher-productivity firms sell more products in a given market because they generate larger variable profits to cover the fixed cost of selling a product. Higher-productivity firms enter more markets because they sell more products and generate larger profits in each market to cover the fixed cost of entering a market. Additionally, as higher-productivity firms enter more markets, they adjust the variety of their products, increasing product scope to align with the heterogeneous consumer tastes across the markets. In total, higher-productivity firms sell more products in more markets, gaining a larger market share in the economy.

I quantify the model and evaluate how well it fits the data. I structurally estimate the main model parameters using the simulated method of moments. The quantified model fits the four empirical patterns and successfully replicates the top 10% vs. the median firms' statistics in firm size, product scope, and geographic scope distributions found in the Japanese data.

Using the model, I run two counterfactual exercises to see how the absence of either extensive margin affects firm size heterogeneity. I first eliminate the geographic scope margin by imposing a single market restriction on the model; this reduces product scope variation by 45% due to the lack of complementarity between product and geographic scope through consumer taste heterogeneity. In total, the absence of geographic scope margin and the reduction in product scope variation results in reducing firm size variation by 86%. Next, instead of eliminating geographic scope, I eliminate

product scope margin by imposing single-product firm restriction on the model. This reduces geographic scope variation by 75% because the model lacks complementarity such that producing more products allows firms to enter more markets. In total, eliminating the product scope margin results in reducing firm size variation by 94%. These exercises show that the complementarity between the two extensive margins is quantitatively large and important for generating firm size heterogeneity.

I conclude by using the parameterized model to examinehow the absence of either extensive margin determines the welfare implications of size-dependent policies. I solve the social planner's problem and find that the market equilibrium is efficient (i.e., any policy interventions are sub-optimal).[8] Despite this finding, SME subsidies are very common in Japan, perhaps for political reasons.[9] What is the welfare cost of such sub-optimal subsidies? I use an actual Japanese SME subsidy—given to expand their geographic scope—as an example and estimate the welfare cost of such subsidies under the baseline and single-product firm models. When the product scope margin is absent, the welfare cost is underestimated by more than half: the Japanese SME subsidy covering two-thirds of the fixed costs of entering a market reduces the real consumption index by 1.25% in the baseline model, and by only 0.58% in the single-product firm model.

[8]As I will describe in Section 7, the discussion of the efficiency is related to Dhingra and Morrow (2019), where they analyze allocational efficiency of heterogeneous firm models with various market structure and demand specifications.

[9]One of the reasons why the Japanese government protects SMEs is their voting power: Japanese SMEs employ 34 million individuals, or approximately 70.1% of the private sector labor force, and thus wield a large number of votes in elections (please see https://www.oecd-ilibrary.org/sites/5989eb3a-en/index.html?itemId=/content/component/5989eb3a-en for more information).

2. Four empirical facts in Japanese barcode-level transaction data

I use Japanese barcode-level transaction data to show the importance and empirical patterns of product and geographic scopes. The accounting decomposition shows that product and geographic scopes combined account for approximately 75% of sales variations, while the intensive margin explains the other 25%. Thereafter, four empirical facts about the product and geographic scopes are documented to motivate my model structure.

Data

The data source is Nikkei Point of Sales (POS) data, which enables me to observe the product-level prices and sales, location of sales, and manufacturing firms' information for millions of products with marked a barcode. Nikkei collects the sales, quantity, date, and time of each transaction made at a registered retail store, as well as the retail store information (name and address). I use the barcode's first seven (or nine) digits to identify the manufacturing firm that produces the product. GS1 provides the concordance table between barcodes and firm identifiers.[10] The GS1 concordance table is only available for Japanese firms, and thus the barcodes produced by foreign manufacturers comprising only 1% of overall sales are eliminated.[11] Combining the Nikkei POS data and the GS1 concordance table allows me to observe who produced which products and when and where those products

[10] GS1 is a non-profit organization that develops global standards for business communication, including barcodes. For more detail, see https://www.gs1.org/.

[11] I identify foreign and domestic firms by the first two digits of barcodes; Barcodes produced by Japanese firms start with 45 or 49 (e.g., 45100000001), and those produced by foreign manufacturers start with different numbers. It was not possible to obtain the firm identifiers for under 2% of the barcodes of the products produced in Japan, so these products were dropped from the sample.

are sold to consumers.

Using retail-scanned barcode-level transaction data has many advantages over the household scanner or firm-level export data, which most existing literature relies on. First, compared to the household scanner data, I can observe the sales location of each transaction as an address of the retail store that the barcode got scanned, which enables me to analyze firms' geographic scope. Furthermore, compared to the export data in which products are defined only at the industry category level, products are defined at the barcode level in my data. Barcode-level sales data allows me to analyze the role of product scope more accurately.

I use the Nikkei POS data from 2014 to 2018. Table 1 presents the number of retail stores, retail chains, prefectures, manufacturing firms, and barcodes in the sample. The data covers on average 350 retail stores a year. These stores represent a geographically balanced sample of 43 out of 47 prefectures located in Japan. The sales volume in each prefecture in the data is correlated with the actual market size of the prefecture.[12] For example, the three largest prefectures in terms of their populations (Tokyo, Osaka, and Kanagawa) are the three largest prefectures in terms of the number of stores and sales in the sample.

I observe the wide range of goods purchased by consumers at retail stores in the data. The database covers approximately 420 billion Japanese yen (≈ 4 billion USD) worth of transactions and $400,000$ barcodes spanning across $16,000$ manufacturing firms per year. Because Nikkei mainly samples supermarkets and convenience stores, the products in the data comprise mostly consumer packaged goods, such as processed foods, beverages, and household goods. Overall, the data covers around 20% of all expenditure on goods

[12]The market size of each prefecture can be measured by prefecture-level employment (https://www.stat.go.jp/data/roudou/pref/index.html) or GDP (https://www.esri.cao.go.jp/jp/sna/data/data_list/kenmin/files/files_kenmin.html). Both measures show a positive correlation with the prefecture sale volume in the Nikkei POS from 2014 to 2018.

in the Japanese official consumer price index.[13]

Nikkei selects retail stores from various chains that cover multiple prefectures. The data samples retail stores from approximately 40 chains a year.[14] Each retail chain covers, on average, four prefectures a year. The largest chain in my sample is a supermarket chain called AEON and covers 22 prefectures a year. All retail stores in the database are relatively large, with average annual sales of 1.3 billion Japanese yen ($\approx$ 11 million USD). Brick-and-mortar grocery or liquor stores are not included in the sample.

Table 1: Sample statistics

Year	Retail stores	Prefectures	Retail chains	Firms	Barcodes
2014	336	43	39	15,590	393,313
2015	348	43	40	16,289	407,497
2016	351	44	40	16,718	411,694
2017	348	43	39	16,628	407,497
2018	345	43	39	15,393	326,407

Note: The fifth and sixth columns are the numbers of *domestic* manufacturing firms and the barcodes produced by the domestic firms. The total number of prefectures in Japan throughout the sample period is 47.

Product category information is available for each barcode, which allows me to analyze the role of product and geographic scopes in firm size heterogeneity within narrowly defined categories. Nikkei organizes barcodes into 6- and 3-digit product categories; the 6-digit product categories are more detailed, and approximately 2000 categories. Conversely, the 3-digit product categories are more aggregated, and there are 217 categories. The five largest of the 217 product categories in terms of total sales in my sample are

[13]The Nikkei Point of Sales data is used to construct a consumer price index provided by the University of Tokyo called UTokyo Daily Price Index (https://www.cmdlab.co.jp/price_u-tokyo/media_e/).

[14]The retail chains are identified by the first word of the store names (e.g., Safeway Menlo park).

Bento (packaged meal in a box for take-out), rice, frozen food, yogurt, and pastry. The 6- or 3-digit product categories are used to generate the below empirical facts depending on the appropriate level of aggregation.

Firm size, product and geographic scopes

Table 2 presents the market share, product scope, and geographic scope by ranked decile in the sample. I calculate the decile values for each quarter and 3-digit product category and weigh the values by the product category-quarter sales. Three striking features emerge. First, the markets are skewed toward a few large firms — the largest 10% of firms occupy approximately 90% of the market share. Second, larger firms sell more products than smaller firms — the largest 10% of firms sell 77 barcodes, while the median firms sell four barcodes. Finally, larger firms sell their products in more locations — the largest 10% of firms sell their products in 28 out of 43 prefectures, while the median firms sell in six.[15]

How important are product and geographic scopes, compared with intensive margin, to account for the observed firm size distribution? Figure 1 presents an accounting decomposition of the firm sales into the number of products firms sell (product scope), the number of prefectures in which firms sell their products (geographic scope), and the sales per product per prefecture (intensive margin) for the 50 firms with the largest average market share in each product category. Each point represents an average contribution of a particular margin. For example, the column of points at the right of the figure indicates that the largest firm in a product group is, on average, almost eight log units larger than an average firm. Of this 8 log unit

[15]A common concern regarding product scope is that larger firms might be connected one large retail chain and sell their products in many stores and prefectures within the chain. As shown in Section A.1, this is not the case: larger firms sell in more retail chains, and these retail chains are not systematically larger than the retail chains that smaller firms sell their products in.

Table 2: Firm size, number of barcodes and prefectures by decile

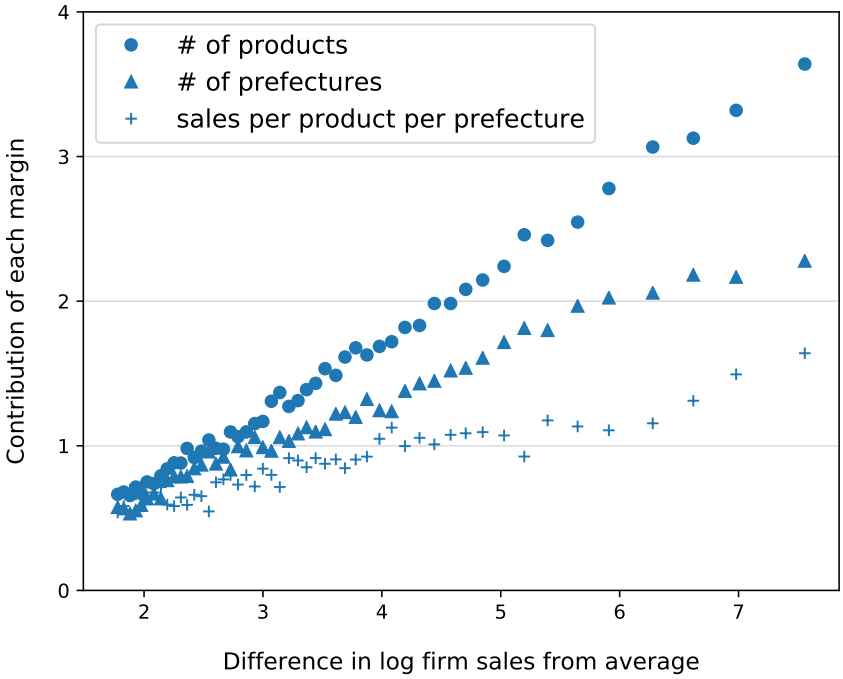
Ranked decile	Decile market share (%)	Avg. no. barcodes	Median no. barcodes	Avg. no. prefectures	Median no. prefectures
1	89.95	73.38	51.76	28.08	29.56
2	6.93	19.74	15.56	17.30	16.54
3	2.47	10.24	8.08	12.46	11.16
4	1.09	6.86	5.30	9.50	7.98
5	0.55	4.82	3.78	7.40	6.02
6	0.29	3.58	2.78	5.50	4.40
7	0.14	2.76	2.14	4.04	3.06
8	0.07	2.18	1.60	2.86	2.10
9	0.03	1.72	1.24	2.10	1.46
10	0.01	1.30	1.02	1.42	1.08

Note: The largest decile is ranked first. The values are weighed by 3-digit product category sales per quarter. Number of prefectures is calculated as the number of prefectures in which each manufacturing firm sells at least one product.

difference, the dotted point indicates that, on average, approximately 3.5 log units (43%) are attributed to the product scope differences. The triangles reveal that approximately 2.3 log units (31%) are, on average, attributable to the geographic scope differences, whereas the intensive margin is responsible for the rest (25%). Thus, approximately 75% of sales variations across the top 50 firms are attributed to product and geographic scope variations.

The sales decomposition confirms that product and geographic scopes are two quantitatively important margins for firm size heterogeneity. Next, I present four empirical facts that motivate my model structure. Facts 1 and 2 show that firms' decisions on whether to enter a market and how many products to sell in the market depend on the market size. Facts 3 and 4 reveal that firms' decisions on geographic and product scopes complement each other.

Figure 1: Accounting decomposition of the firm sales by rank

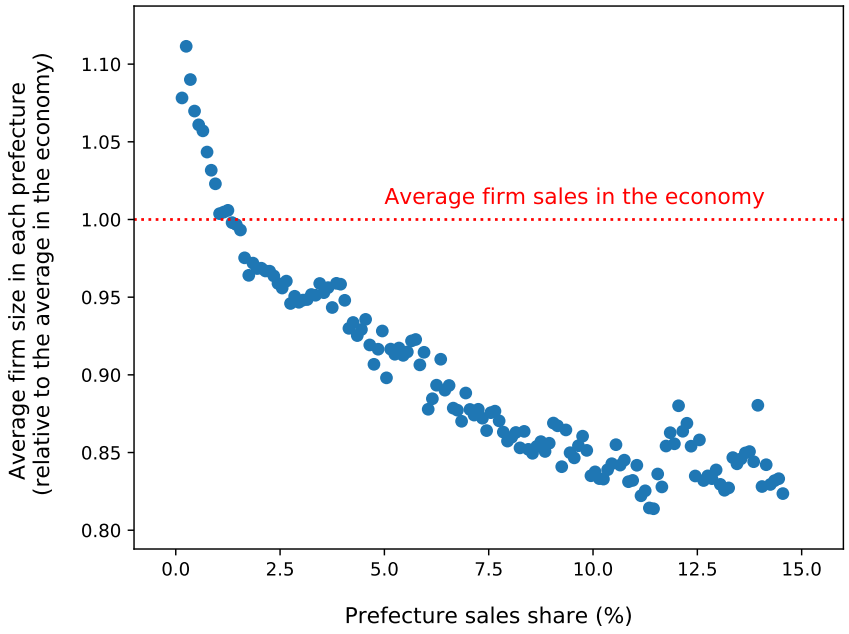


Fact 1: Larger firms penetrate smaller markets

Larger firms enter more prefectures (from Table 2), but is there any pattern on which prefectures larger firms enter? Figure 2 shows that larger firms enter both small and large markets, while smaller firms enter only large markets. The figure plots average firm sales in a particular prefecture relative to the average firm sales in the whole country on the y-axis and the prefecture market size (as a share of total sales in all the prefectures) on the x-axis. I calculated the average firm size in each prefecture (y-axis) and prefecture sales share (x-axis) in each 3-digit product category and then created the binned scatter plot where each bin is defined as 0.1% of prefecture sales share. Because the y-axis is normalized by the average firm size in the country, if the value is above one, it means that the firms in the prefecture are larger than

the average firms in the country.[16] The negative slope in the figure indicates that larger firms are more likely to penetrate smaller prefectures.

Figure 2: Average firm size in each prefecture



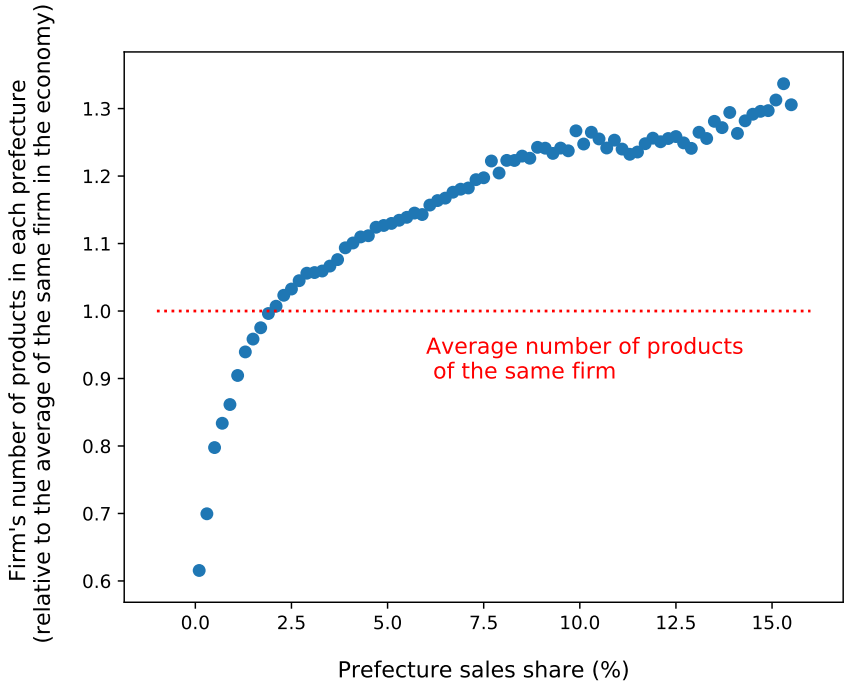
Fact 2: Firms sell more products in larger markets

How do firms' decisions on how many products to sell in a market depend on the market size? Figure 3 shows that the number of products firms sell increases with the market size. The figure plots firms' number of products in each prefecture (relative to the average number of products of the same firm across prefectures) on the y-axis and prefecture market size (as a share of total sales in all the prefectures) on the x-axis. I calculated the average number of products of firms in each prefecture (y-axis) and prefecture sales

[16] For example, the point on the left end shows that the average firm size in the prefectures that have 0.0 to 0.1% of prefecture sales shares is 1.12 times larger than the average firm size in the economy.

share (x-axis) in each 3-digit product category and created the binned scatter plot, where each bin is defined as 0.1% of prefecture sales share.[17] The positive slope in the figure suggests that the same firms sell more products in larger markets.

Figure 3: Firm's number of products in each prefecture



Fact 3: Larger firms sell more products in a given market

According to Figure 4, larger firms not only sell more products in the economy (see Table 2), but they also sell more products in each particular market. The figure plots firms' number of products in each prefecture on the y-axis and the firm sales share on the x-axis. The y-axis is normalized by the average number of products per firm in the same prefecture. I calculated

[17] For example, the figure suggests that if firms sell their products in prefectures that have 2.5% and 5% of sales shares, then the same firms sell 10% more products in the prefecture with 5% of sales share than in the prefecture with 2.5% of sales share.

firms' number of products in each prefecture relative to the average number of products per firm in the same prefecture (y-axis) and firm sales share (x-axis) in each 3-digit product category and created the binned scatter plot where each bin is defined as 0.1% of firm sales share.[18] The positive slope indicates that the firms with a larger sales share in the economy sell more products in each market.

Fact 3 implies that firms' choices on product and geographic scopes are interconnected. As firms expect to sell more products and generate larger profits in a market, they are more likely to enter the market.[19] Therefore, a larger product scope generates a wider geographic scope.

Fact 4: Firms adjust varieties to heterogeneous local taste

Fact 4 suggests another complementarity between geographic and product scopes. First, I show that firms sell (partially) different sets of products across markets. Second, I present the evidence of consumer taste heterogeneity across markets. The evidence is consistent with the story that firms offer different varieties across markets to cater to heterogeneous consumer tastes.

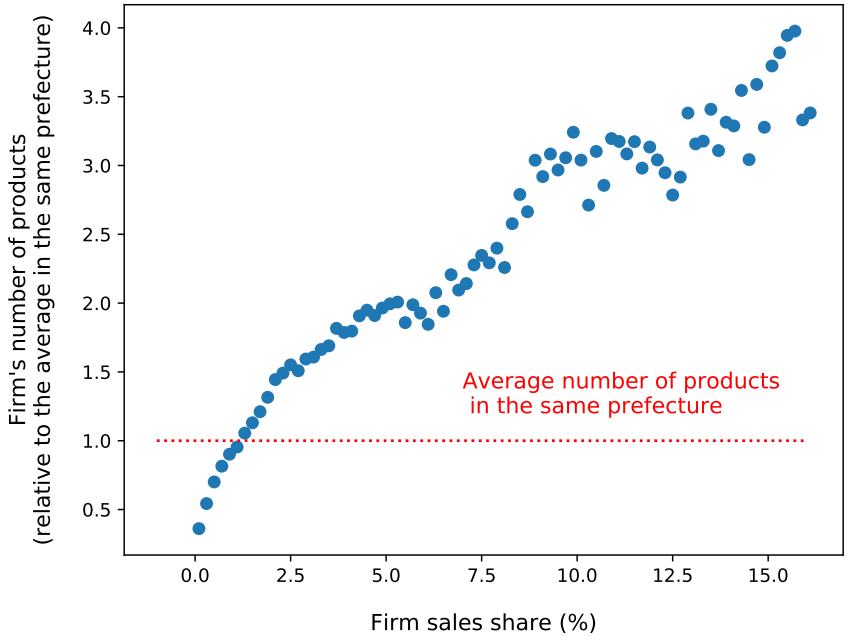
Fact 4-1: Firms sell different sets of products across markets

Do firms sell the same sets of products in all the markets they operate in? Or do they sell different sets of products across markets? To answer the question, I measure the similarity of firms' product sets across prefectures. For

[18] For example, the figure suggests that firms with 2.5% of sales share in the economy sells about 50% more products than the average firms in the same prefecture in the prefectures that the firm sells their products in.

[19] One caveat of this argument that firms with more products might sell less per product and thus, these firms do not expect larger profits. However, as confirmed in Figure 1, firms' number of product and sales per product are positively correlated. Therefore, when firms expect to sell more products in a market, they will expect to generate larger profits in the market.

Figure 4: Firms' number of products in each prefecture



each firm f and prefecture-pair A and B, I calculate *product overlap* statistic, defined in Quan and Williams (2018), as the ratio of sales of products that are sold in both prefectures, i.e.,

$$\text{Product overlap}_f^{A,B} = \frac{\sum_{i=A,B} \text{sales}(A \cap B)_f^i}{\sum_{i=A,B} \text{sales}^i}.$$

This measure is bounded between zero and one. If firm f sells the same set of products in both prefectures, the statistic equals one. I calculate the overlap statistic for firms that sell their products in multiple prefectures in a particular quarter in a particular product category and weigh the result by category-quarter sales. The average overlap statistic is 0.65, and the median overlap is 0.75. The average statistic value of less than one means that the firms sell partially different sets of products across markets.

Fact 4-2: Consumer tastes are heterogeneous across prefectures

The most likely reason for the incomplete product overlap is that firms offer different sets of products across prefectures to cater to heterogeneous local taste.[20,21] To see whether consumer taste is indeed heterogeneous across prefectures in Japan, I calculated the variations of market share of the same products across different prefectures.

The data suggest that the taste across the prefectures is indeed heterogeneous. The first row of Table 3 presents the standard deviation of the log of market share of the top 20 products ranked by national sales in each 9-digit product category. If consumer tastes were homogeneous across prefectures, the share of sales accruing to these products would be expected to be the same in any prefectures; therefore the standard deviations are close to zero. The mean and median standard deviations of the log of market share in all samples are 0.91 and 0.81, respectively. Positive standard deviations indicate the existence of across-prefecture taste heterogeneity in Japan.

The second to fifth rows of Table 3 show the standard deviations of the log of market share for refrigerated, room temperature and frozen foods, and household goods, respectively. Food categories have larger market share variations across prefectures than household goods, reflecting that each pre-

[20]There is extensive literature that contains empirical evidence on across-market taste heterogeneity in the industrial organization field (for example, Waldfogel (2003), Waldfogel et al. (2004), Bronnenberg et al. (2009), Choi and Bell (2011), and Bronnenberg et al. (2012)) Furthermore, Waldfogel (2008) and Quan and Williams (2018) document the evidence of supply-side response to the across-market consumer taste heterogeneity.

[21]I acknowledge that there may be some supply-side factors that affect product overlap between prefectures, such as shipping costs. However, the products covered in the data are consumer packaged goods, and shipping costs of these products are usually small. Frozen foods involve relatively higher shipping costs within consumer packaged goods, but they occupy only less than 3% of overall sales in my sample.

fecture in Japan has a distinctive food culture.[22,23]

Table 3: Across-prefecture market share variations

Std. dev. of log (market share)	Mean	Median	Std. dev.	Min	Max
All sample	0.91	0.81	0.47	0.00	7.29
Refrigerated food/beverage	1.05	0.95	0.56	0.00	7.29
Room temperature food/beverage	0.92	0.84	0.46	0.00	7.11
Frozen food	1.04	0.91	0.61	0.00	5.62
Household goods	0.76	0.69	0.34	0.00	4.26

Note: The table shows the standard deviation of the log of market shares in each 6-digit product category each quarter. In measuring market share variations, the set of products in each prefecture is narrowed down to the top 20 products in each product category based on national sales.

Facts 4-1 and 4-2 combined suggest the following complementarity between geographic and product scopes; as firms enter more markets, they adjust their varieties to deal with different consumer tastes across markets, thereby offering more products. Therefore, Fact 4 implies that a wider geographic scope generates a larger product scope.

[22] I do not go into detail about how the consumer taste heterogeneity is formed in Japan, but as described in Atkin (2013), habit formation is a likely explanation of the across-prefecture taste heterogeneity. For example, in the food category, consumer taste evolves to favor foods consumed as a child, and the taste is inherited to the next generation in the prefecture.

[23] Alternative explanation for demand heterogeneity is income-specific tastes. Handbury (2019) uses retail scanner data in the U.S. to show that high-income households purchase different, more expensive, products. In my data, I still find positive standard deviations of the log of market share even if we compare the prefectures with similar market sizes (and thus, similar income levels), which makes me believe that cultural differences are the main driver of the market share variations.

3. Survey of Japanese food manufacturing firms

Next, I conduct a survey of Japanese food manufacturing firms to help me determine a modeling structure. The literature proposes various model structures that generate the product or geographic variations across firms. The survey results suggest that, among these candidate model structures, two types of fixed costs are of practical importance: the fixed cost of entering a market and the fixed cost of selling a product.

3.1 Survey background and goal

Two candidate structures generate product scope variations across firms: the fixed cost of selling a product and the cost of investment into production capital. The fixed cost of selling a product is usually interpreted as the cost of marketing research or product development/customization (Hottman et al., 2016). The cost of investment into production capital manifests as the so-called flexible manufacturing allowing a firm to add another variety to the existing product line but with an increase in the marginal cost for the new products (Eckel and Neary, 2010). The difference between the two costs is that investment cost increases as the firm adds more products, whereas the fixed cost of selling a product does not depend on the firm's number of products.

Similarly, the literature proposes two candidate model elements for geographic scope variations across firms: the fixed cost of entering a market and the marketing cost. The fixed cost of entering a market is frequently used in many international trade papers and takes the form of the fixed cost of exporting (Melitz, 2003). Meanwhile, the marketing cost generates geographic scope variations in asymmetric information settings. When consumers do not know what products are available, firms incur marketing costs to in-

crease their consumer base (Arkolakis, 2010). The difference between the two costs is due to the fact that the marketing cost to reach additional consumer decreases with the market size, while the fixed cost of entering a market does not depend on the market size.

Instead of making an arbitrary choice from candidate model structures, I surveyed Japanese packaged food manufacturing firms to find out which model structures are of the most practical relevance. The survey took place in one of the biggest food supplier-buyer matching events in Japan, called Food Selection 2020 (the survey and matching event are described in detail in Appendix A). The event gathers packaged food manufacturing firms (suppliers) from all over Japan. The suppliers show their latest products and try to make sales contracts with buyers, while the buyers, ranging from wholesale and retail to restaurants, try to find new products to purchase in the event.

Before conducting the survey, I observed 50 supplier-buyer negotiations in the matching event to find counterparts of these four model elements (fixed cost of entering a market and selling a product, marketing cost, and investment cost) in the business world. The marketing cost and the cost of investment into production capital have direct counterparts in the business world: the advertisement cost to increase consumer awareness and the cost of investment into factories. The fixed cost of entering a market corresponds to the cost of overcoming contract issues with buyers. In the supplier-buyer negotiations, buyers frequently ask suppliers to use the same logistic company as the buyer, open a new bank account specific to the contract, and go through other administrative processes related to the contract.[24] Lastly, the fixed cost of selling a product in a market corresponds to the cost of consumer market research and customizing each product to cater to con-

[24]The quantitative importance of the fixed cost of overcoming contract friction on welfare gains from trade in the international trade context is documented by Startz (2016).

sumers' tastes. In the negotiations, buyers often ask suppliers to customize their products to match the consumers' tastes in the buyers' areas.

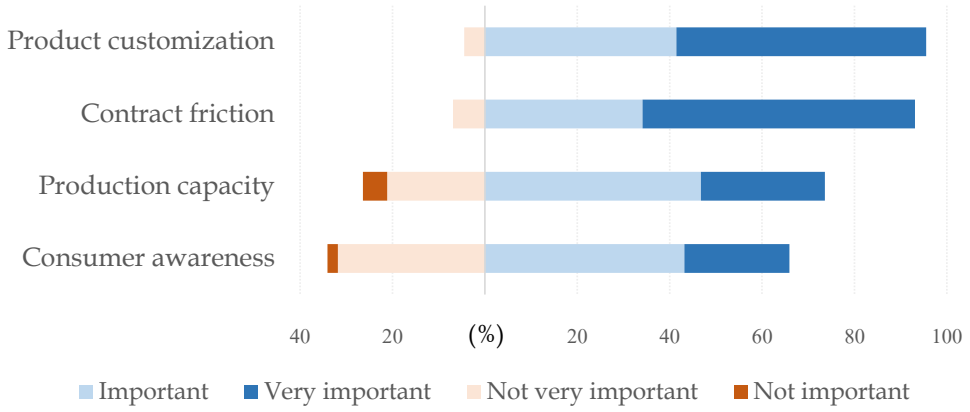
3.2 Survey results

Based on the observations in 50 supplier-buyer negotiations, I asked the suppliers about the relative importance of the four cost counterparts in their businesses. Specifically, I asked: How important are the following elements in expanding into new geographic markets and increasing product range? (1) Overcoming contracting issues with the buyers, (2) understanding consumers' tastes and customizing products to cater to the tastes, (3) having sufficient production capacity for the new market, and (4) advertising products to increase consumer awareness.

Figure 5 shows the survey results. The surveyed firms think that product customization and overcoming contracting issues with buyers are of equal and practical importance. Having sufficient production capacity and increasing consumer awareness turn out to be less important. The cost of contracting with a buyer corresponds to the fixed cost of entering a market, and the cost of understanding consumer tastes and customizing products corresponds to the fixed costs of selling a product in each market. Therefore, the results imply that the two fixed costs have more practical importance than the other two candidate structures.

It should be noted that the survey results are specific to the firms in the Japanese packaged food manufacturing industry, and different contexts would generate different insights for the model structures. For example, increasing consumer awareness can be more important in other sectors such as technology, service, and healthcare (see Hulcr et al., 2019).

Figure 5: Survey: How important are the following elements?



Note: The graph contains the survey result of 246 (out of 802) packaged food manufacturing firms that participated in Food Selection 2020. Product customization = understanding consumer taste and products to the taste. Contract friction = overcoming contract issues with buyers. Production capacity = having sufficient production capacity for the new market. Consumer awareness = advertising products to increase consumer awareness.

4. A heterogeneous-firm model

Motivated by the empirical patterns and survey results, I develop a heterogeneous firm model, where firms choose how many products to sell in which markets. The model is based on Melitz (2003) with an addition of multi-product firms and multiple markets with heterogeneous market sizes. Firms are heterogeneous in their productivity and face two fixed costs: the fixed costs when entering a market and when selling a product in each market.

4.1 Preference

There are N number of consumption markets in the economy. Each consumption market is indexed by r and endowed with L^r amount of labor that are supplied inelastically with no disutility in the same market. A representative consumer in consumption market r chooses consumption amount $(C^r_{k_f})$

of each product k_f sold by firm f to maximize the following constant elasticity of substitution (CES) utility function subject to the budget constraint:

$$U^r = \left[\int_{f \in \Omega^r} \int_{k_f \in \Omega_f^r} \left(\phi_{k_f}^r C_{k_f}^r \right)^{\frac{\sigma-1}{\sigma}} \, \mathrm{d}k_f \, \mathrm{d}f \right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where Ω^r is the endogenous set of firms selling in consumption market r, Ω_f^r is the endogenous set of products that firm f sells in consumption market r, and $p_{k_f}^r$ and $\phi_{k_f}^r$ are the price and taste shifter of product k_f in consumption market r, respectively. The parameter $\sigma > 1$ is the elasticity of substitution across the products.

The taste shifter $\phi_{k_f}^r$ captures consumer taste heterogeneity across markets. The taste shifter is a product of a common component, ϕ_{k_f}, and an i.i.d. regional shock to the common component, $\epsilon_{k_f}^r$, i.e., $\phi_{k_f}^r = \phi_{k_f} \times \epsilon_{k_f}^r$. The common component captures a taste for a product that is common across all markets and is drawn from a log-normal distribution with mean μ and variance σ^2: $\log \phi_{k_f} \sim N(\mu, \sigma^2)$. The regional shock captures an idiosyncratic taste difference across markets and is drawn for each product and market from a log-normal distribution with mean 0 and variance σ_ϵ^2: $\log \epsilon_{k_f}^r \sim N(0, \sigma_\epsilon^2)$.[25] Because a product of two log-normally distributed random variables follows a log-normal distribution, the taste shifter also follows log-normal distribution with mean μ and variance $\sigma^2 + \sigma_\epsilon^2$: $\log \phi_{k_f}^r \sim N(\mu, \sigma^2 + \sigma_\epsilon^2)$.[26]

[25] This taste heterogeneity can also be thought as taste differences coming from wealthiness of the prefectures, i.e., households in wealthy prefectures prefer different products from households in poor prefectures. However, this model does not capture the possibility that households in wealthier cities prefer more expensive products.

[26] Using a log-normal distribution for consumer tastes has an advantage in fitting the data. With the consumer tastes following log-normal and productivity following Pareto distributions, sales per product per region increases with productivity, which fits the empirical finding in the Japanese data (see Figure 1). This is in contrast with assuming Pareto distribution for consumer tastes and productivity, where sales per product per region becomes independent of productivity. See Fernandes et al. (2019) for more discussion.

The corresponding price index for the consumption market r is given by

$$P^r \equiv \left[\int_{f \in \Omega^r} \int_{k_f \in \Omega_f^r} \left(\frac{p_{k_f}^r}{\phi_{k_f}^r} \right)^{1-\sigma} \mathrm{d}k_f \, \mathrm{d}f \right]^{\frac{1}{1-\sigma}}. \tag{2}$$

4.2 Technology and market structure

There is an unbounded measure of potential firms who are identical prior to their entry into the economy. To enter, firms must incur a fixed cost of entering the economy, F_e in units of labor. Once the fixed cost is incurred, each firm f draws productivity φ_f from a Pareto distribution with a shape parameter θ and a scale parameter $\bar{\varphi}$: $\varphi_f \sim \text{Pareto}(\theta, \bar{\varphi})$. Similarly, once the fixed cost of entering the economy is incurred, each firm observes a common and a regional shock component of taste shifter for each potential product k_f in consumption market r.[27] Each firm is endowed with mass K of potential products. The mass of potential products is constant across firms, but the sets of potential products are differentiated across firms. Among the sets of potential products, each firm chooses which products to sell in a particular market.

Once the fixed cost of entering the economy has been incurred, and firm's productivity and taste shifter have been observed, the firm decides whether to enter each market and which products to sell in each market. Upon choosing which products to sell in which markets, firms face two types of fixed costs. When a firm enters a consumption market (i.e., sells at least one product in a consumption market), the firm pays the fixed cost of entering a market, F. In addition, firms incur the fixed cost of selling per product in a given

[27] To exploit the low of large numbers' feature, it is assumed that the productivity, common component, and regional shock components are independent of one another and independently distributed across firms. Similarly, it is assumed that the common component is independently distributed across the products, while the regional shock component is independently distributed across products and consumption markets.

market, F_p.[28] Two types of fixed costs are paid in units of labor.

In addition to the fixed costs, there is also a marginal cost of producing each product. The marginal cost of selling a product is the same across products and markets within a firm due to the following assumptions. First, the production function is the same across products and firms and is linear in labor, the sole factor of production. The total amount of labor needed to produce one unit of output is $\frac{1}{\varphi_f}$. Second, I assume the free mobility of firms in the economy. The representative consumers supply L^r unit of labor in consumption market r, and if there are any wage differences across markets, firms keep moving until the wages are equalized. Therefore, there is only a single wage in the economy.[29] A single wage in the economy is defined as w and normalized as one. Third, I assume there are no variable costs of trade across markets. In total, firm f's marginal cost of selling a product is given by $w\varphi_f^{-1} = \varphi_f^{-1}$, where the equality comes from the normalization of $w = 1$. This leads to an observation that the marginal cost is common across products and markets within a firm.

Each consumption market is monopolistically competitive. With the CES utility function specified in (1), firms' pricing decision is given by constant markup times the marginal cost:

$$p_{k_f}^r = \frac{\sigma}{\sigma - 1}\varphi_f^{-1}, \tag{3}$$

where the pricing is common across products and markets within a firm.

[28] For example, when a firm enters one market and sells two products there, it pays $F+2F_p$ amount of fixed costs.

[29] A free mobility of firms across markets can be interpreted as a free mobility of workers, which also equates wages across markets and generates a single wage in the economy.

4.3 Firms' decisions on which products to sell where

Firms' profit maximization problem is separable across consumption markets. In each consumption market r, firms decide which products to sell in the market and whether to enter the market with the following two steps: First, given entering a market, firms choose an optimal set of products to sell in the market. Next, given the optimal set of products, firms decide whether to enter the market. The first step generates a cutoff taste shifter condition, and the second step generates a cutoff productivity condition.

The cutoff taste shifter condition. Using the pricing rule defined by expression (3), the equilibrium revenue received by a firm f in consumption market r from selling a product k_f is:

$$r(\varphi_f, \phi^r_{k_f}) = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} (\phi^r_{k_f} \varphi_f P^r)^{\sigma-1} L^r. \tag{4}$$

The corresponding equilibrium profits from selling the product to that market are:

$$\pi(\varphi_f, \phi^r_{k_f}) = \frac{r(\varphi_f, \phi^r_{k_f})}{\sigma} - F_p. \tag{5}$$

Firm f sells product k_f if and only if the profit from selling the product is positive, i.e., $\pi(\varphi_f, \phi^r_{k_f}) > 0$. Because the demand for a product is monotonically increasing in the taste shifter, and $\frac{r(\varphi_f,\infty}{\sigma} > F_p$ and $\frac{r(\varphi_f,0)}{\sigma} < F_p$ hold for any given productivity level φ_f, a *cutoff taste shifter* for each firm f in market r, ϕ^{r*}_f, is defined as

$$\pi(\varphi_f, \phi^{r*}_f) = 0. \tag{6}$$

Condition (6) is called a cutoff taste shifter condition. Given entering a market, firm f sells products with a taste shifter higher than the cutoff value, ϕ^{r*}_f.

The cutoff productivity condition. Given the optimal set of products defined as a set of products with a taste shifter above the cutoff taste shifter, the firm chooses whether to enter the consumption market. Firm f's profit in consumption market r is given by

$$\Pi^r(\varphi_f) = \int_{k_f \in \Omega_f^r} \pi(\varphi_f, \phi_f^{r*}) \, \mathrm{d}k_f - F. \tag{7}$$

The first term in (7) is the variable profit from selling the optimal set of products Ω_f^r (chosen by the cutoff taste shifter condition (6)), and the last term reflects the fixed cost of entering the market.

Each firm f enters consumption market r if and only if entering a market generates a positive profit, i.e., $\Pi^r(\varphi_f) > 0$. Because the demand for products is monotonically increasing with productivity of the firm, and $\Pi^r(\infty) > 0$ and $\Pi^r(0) < 0$ hold for any market r, the *cutoff productivity* in each market r, φ^{r*}, is defined as

$$\Pi^r(\varphi^{r*}) = 0. \tag{8}$$

This condition is called the cutoff productivity condition. Firms with productivity higher than the cutoff productivity enter the market.

4.4 Free-entry

The mass of firms in the economy, M, is determined by a free-entry condition. The free-entry condition ensures that firms keep entering the economy until the expected profit of entering the economy becomes equal to zero, i.e.,

$$\frac{\Sigma_r \Pi^r}{M} = F_e,$$

where $\Pi^r \equiv \int_{f \in \Omega^r} \Pi^r(\varphi_f) \, \mathrm{d}f$ is the sum of profits of all surviving firms in the consumption market r.

Among the firms entering the economy, firms with productivity above the cutoff productivity φ^{r*} enter the consumption market r. Define $G(\varphi_f)$ as the cumulative distribution function of the Pareto distribution for productivity. Then, the mass of firms entering each market r, M^r, is given by

$$M^r = M(1 - G(\varphi^{r*})).$$

4.5 Aggregation

Using the cutoff taste shifter condition for each firm (6), the cutoff productivity condition (8), and the pricing strategy for each firm (3), the price index in the consumption market r (2) can be rewritten as

$$P^r = \left[M \int_{\varphi^{r*}}^{\infty} p(\varphi_f)^{1-\sigma} K \int_{\phi_f^{r*}}^{\infty} (\phi_{k_f}^r)^{\sigma-1} \mathrm{d}F(\phi_{k_f}^r) \mathrm{d}G(\varphi_f) \right]^{\frac{1}{1-\sigma}}, \tag{9}$$

where $F(\phi_{k_f}^r)$ is a cumulative distribution function of taste shifter.

For a given number of firms in the market, a decrease in the cutoff taste shifter for each firm affects the price index in two ways. First, a lower cutoff taste shifter means each firm sells more products, which decreases the price index because of the love-of-variety effect. Second, it lowers the average taste shifter for each firm as more marginal products are introduced to the market, and a lower average taste shifter, in turn, increases the price index.

Similarly, for a given number of products for each firm, a decrease in the cutoff productivity in each market has two effects on the price index. On the one hand, it decreases the price index due to the love of variety effect. On the other hand, it increases the price index because the average productivity decreases as more marginal firms enter the market. These insights will be dealt with further when discussing the efficiency of the market equilibrium

in section 7.

5. Estimation and model evaluation

I structurally estimate model parameters by the simulated method of moments using the Nikkei POS data. With the quantified model, I evaluate model performance by checking whether the model fits (1) four empirical patterns found in Section 2, and (2) the firm size, product scope, and geographic scope distributions observed in the data.

5.1 Structural estimation

First, I explain the identification issues in estimating productivity (φ_f) and the constant elasticity of substitution (σ). Second, I explain how I simulate a set of firms given a particular set of parameter values and how I use the simulated method of moments (SMM) to overcome the identification issue. Finally, I describe the simulation algorithm.

The model parameters are divided into four categories. The first category is the parameters that can be directly determined by the data. The number of markets (R) and the labor endowment of each market (L^r) are included in this category. The second category comprises the parameters that are estimated given from the data. The parameters for taste shifter distributions $\{\log(\phi_k) \sim N(\mu, \sigma^2), \log(\epsilon_k^r) \sim N(0, \sigma_\epsilon^2)\}$ are included in this category. The third category is the parameters that are estimated to match some moments in the data. The mass of potential products for each firm (K) is set so that the lowest-productivity firm produces at least one product. There are three types of fixed costs $\{F, F_p, F_e\}$ set to match the top 10% vs. median firms' stats of firm size, product, and geographic scopes. Specifically, compared to the median firms, the top 10% firms generates 188 times more sales, pro-

duce 18.8 times more products, and enter 4.8 times more markets, and the three fixed costs are set to match the three targeted values. The final category is the parameters that are estimated using SMM to deal with identification issues. The elasticity of substitution (σ) and parameters for productivity distribution $\{\varphi_f \sim \text{Pareto}(\theta, \bar{\varphi})\}$ are in this category. I define $\Theta_1 = \{F, F_p, F_e\}$ as a set of parameters that are estimated using GMM, and $\Theta_2 = \{\sigma, (\theta, \bar{\varphi})\}$ as a set of parameters that are estimated using SMM.

5.1.1 Identification issues

First, it is necessary to explain how it is possible to potentially estimate the values of the elasticity of substitution (σ) from the firm-level expenditure shares and prices. Define firm f's expenditure share in the consumption market r as S_f^r and the number of products firm f produces in market r as N_f^r. Then, firm f's expenditure share per product in market r is given by

$$\log \frac{S_f^r}{N_f^r} = (1-\sigma)\log p_f^r + (\sigma-1)\log P^r + \frac{1}{N_f^r}\int_{k_f \in \Omega_f^{r*}} (\phi_{k,f}^r)^{\sigma-1} dk_f, \tag{10}$$

It is possible to estimate σ from using equation (10), i.e., by regressing the firm sales share per product on firm price and market fixed effect.[30] However, such an estimation suffers from selection bias. The assumption required for the identification is firm prices and the average taste shifter are orthogonal, i.e., $p_f^r \perp \frac{1}{N_f^r}\int_{k_f \in \Omega_f^{r*}} (\phi_{k,f}^r)^{\sigma-1} dk_f$, which is likely to be violated. Firms with a higher productivity (and thus a lower price) can sell products with a lower taste shifter, which leads to a positive correlation between firms' prices and the average taste shifter, underestimating σ.

The estimation of productivity (φ_f) suffers from a similar selection bias. Productivity can be estimated from the data on the product-level expendi-

[30] Upon estimating σ, I compute firm-level price as the average of product-level prices in each market weighted by expenditure share of each product within firm.

ture shares and prices, as well as the known value of the constant elasticity of substitution (σ). Define $S^r_{k_f}$ as the expenditure share of product k sold by firm f in consumption market r. Taking the log of the expenditure share gives the following expression:

$$\log S^r_{k_f} = \log(\frac{\sigma}{\sigma-1})^{1-\sigma} + (\sigma-1)(\log(\phi_{k_f}) + \log(\varphi_f) + \log(P^r)) + (\sigma-1)\log(\epsilon^r_{k_f}). \tag{11}$$

Given the known value of the elasticity of substitution (σ), I can compute productivity (φ_f) for each firm by (1) regressing the log of product sales in each consumption market onto product, firm, and consumption market fixed effects, and (2) dividing the coefficients of the firm fixed effects by $(\sigma - 1)$ and take an exponential of it. The identification assumption in expression (11) is $\epsilon^r_{k_f} \perp \varphi_f$ (i.e., the regional shock to the taste shifter is orthogonal to the firm's productivity), which is likely to be violated. If firms have higher productivity, then the products with a negative (or smaller) regional shock to the taste shifter (and thus lower taste shifter) are still likely to be produced. The selection bias underestimates the coefficient of firm fixed effects and thus the productivity of higher-productivity firms.

I use the simulated method of moments (SMM) to help find unbiased estimates of these parameter values. The key purpose of using SMM is to find true parameter values $\{\sigma, (\theta, \bar{\varphi})\}$ so that the biased estimates of these parameters (estimated using expressions (10) and (11)) in the actual data match the biased estimates in the model-generated data.

5.1.2 Estimation algorithm

The estimation of the model parameters involves the following 6 steps. Throughout the steps, the same data as in Section 2 is used , the Nikkei POS data from 2014 to 2018.

Preparation. Pin down the values of the number of markets (R) and the size of each market r (L^r) from the data. The number of market is set to match the median number of prefectures in Nikkei POS data from 2014-2018. The size of each market is set to match the average total sales volumes in each prefecture in the Nikkei POS data from 2014 to 2018.

Step 1. Assume true values of elasticity (σ) and productivity distribution $\varphi_f \sim$ Pareto$(\theta, \bar{\varphi})$ in the model.

Step 2. Given the value of σ, compute the values of taste shifter ($\phi^r_{k,f}$). Then, compute the parameter values of the common component distribution ($\log(\phi_k) \sim N(\mu, \sigma^2)$) and the regional shock distribution ($\log(\epsilon^r_k) \sim N(0, \sigma^2_\epsilon)$). To obtain unique values for the taste shifter for each product k for firm f in market r ($\phi^r_{k,f}$) , I use the variation of sales across products within each firm. The relative expenditure shares between two products within a firm in the same consumption market is given by

$$\frac{p_f C^r_{k_f}}{p_f C^r_{k'_f}} = \left(\frac{\phi^r_{k_f}}{\phi^r_{k'_f}} \right)^{\sigma-1}. \tag{12}$$

For each firm f, I choose a product that is sold in the largest number of prefectures prefectures in the firm as a base product such that the taste shifter of the product is normalized as one in all consumption markets, i.e., $\phi^r_{k_f} = 1$ for all r. I then estimate the relative taste shifter of other products using expression (12).[31] Finally, I obtain the common components of taste shifter (ϕ_{k_f}) as the average value of taste shifter of the product across all consumption markets and regional shocks ($\epsilon^r_{k_f}$) from the expression $\phi^r_{k_f} = \phi_{k_f} \times \epsilon^r_{k_f}$.

[31] Upon estimating the taste shifter, I restrict the sample to the firms that sell multiple products in more than five markets(prefectures).

Step 3. Compute the equilibrium in the model given the values of σ and productivity distribution set in Step 1 and the taste shifter distribution estimated in Step 2.

Step 4. Find a set of parameter values of three fixed costs $\{F, F_p, F_e\}$ that matches top 10% vs. median firms' statistics in the data. Specifically, I match between top 10% vs. median firms' sales, product scope (number of products), and geographic scope (number of markets) in the equilibrium and data. Define $m(\Theta_1)$ as a vector of the top 10% vs. median moments in the model and $m(1)$ as the corresponding moments in the data. Then, I seek a set of parameter values $\hat{\Theta}_1$ to minimize the distance between the moments in the data and model using the criterion function:

$$\hat{\Theta}_1 = \text{argmin } (m(1) - m(\Theta_1))W_1(m(1) - m(\Theta_1))', \tag{13}$$

where W_1 is the variance-covariance matrix of the error function, $m(1) - m(\Theta_1)$.

Step 5. Simulate 10,000 artificial firms and compute product- and firm-level expenditure shares in each market, firm-level prices, and the number of products for each firm (see Appendix A.3.1 for a detailed simulation algorithm). Using the model-generated data, estimate (biased) σ by expression (10) and productivity φ_f by expression (8). Then, compute the best-fitted parameter values $(\theta, \bar{\varphi})$ for the productivity distribution. By using expressions (10) and (8), the estimations suffer from the same selection bias as they do in the actual data.

I iterate these steps until the biased estimates of $(\sigma, (\theta, \bar{\varphi}))$ in the model-generated data converges to the biased estimates of these parameter values estimated in the actual data. The SMM selects a vector of parameter values

$\hat{\Theta}_2$ that minimizes the distance between the moments in the actual data and the model-generated data, using the criterion function:

$$\hat{\Theta}_2 = \text{argmin } (m(2) - m(\Theta_2))W_2(m(2) - m(\Theta_2))', \tag{14}$$

where $m(\Theta_2)$ is a vector of moments, $\{\hat{\sigma}, (\hat{\theta}, \hat{\bar{\varphi}})\}$, estimated in the model-generated data, $m(2)$ is a vector of corresponding moments in the actual data, and W_2 is the variance-covariance matrix of the error function, $m(2) - m(\Theta_2)$.

5.2 Estimation results

Table 4 presents the parameter values. Our estimate of σ confirms the upward bias from the selection bias. The biased estimate of σ in the actual data using expression (10) is 1.32, and the estimated value of σ that generates the same value of biased estimate in the model generated data is 2.89, which confirms the point that selection bias underestimates elasticity σ. The estimated value ($\sigma = 2.89$) is consistent with the literature in trade and macro; for example, the estimates in Ossa (2014) range from 2 to 4 for most categories including non-food consumer goods.

The biased estimates of shape and scale parameters for productivity distribution in the actual data were $(\theta, \bar{\varphi}) = (3.52, 0.69)$ and the parameter values that produces the same value of biased estimates in the model-generated data are $(\theta, \bar{\varphi}) = (2.80, 0.78)$. The biased estimate of scale parameter is larger than the true value, which confirms that the selection bias underestimates high-productivity firms' productivity. The results are consistent with the literature (for example, see Eaton et al., 2011) and satisfy the technical restriction for the distribution of firm revenue to have a finite mean, that is $\theta > \sigma - 1$.

The interpretation of the magnitude of the fixed costs are as follows. The model features a free-entry condition, which ensures that firms' variable profits finance three types of fixed costs exactly. Given that the estimated σ is 2.89 and firms' profit margin is given by $\frac{1}{\sigma} = 0.34$, it means that, on average, 34% of firms' sales are used to pay the three fixed costs. In this economy, firms have a large market power and use large variable profits to pay the fixed costs.

Table 4: Parameter values

Parameter	Definition	Value	Target
K	Mass of potential products for each firm	100	≥ 1 product per firm
N	Number of markets	43	Number of prefectures
L^r	Labor endowment for each region		Sales volume of prefectures
$\log(\phi_k) \sim N(\mu, \sigma^2)$	Distribution of common component	$\mu = 0.47, \sigma = 0.92$	Estimated from the data
$\log(\epsilon_k^r) \sim N(0, \sigma_\epsilon^2)$	Distribution of regional component	$\sigma_\epsilon = 0.49$	Estimated from the data
$\varphi_f \sim$ Pareto$(\theta, \bar{\varphi})$	Distribution of productivity	$\theta = 2.80, \bar{\varphi} = 0.78$	Estimated from the data
σ	Elasticity of substitution in CES	2.89	Estimated from the data
F	Fixed cost of entering a region	0.0009	Parameters to fit the data
F_p	Fixed cost of selling per product in each region	0.05	
F_e	Fixed cost of entering the production economy	0.501	

5.3 Qualitative fit: Four empirical patterns

I start by showing important equilibrium characteristics of the model: the price index, cutoff productivity, and cutoff taste shifter (of each firm) decrease with the market size. These equilibrium characteristics are then connected to the observed empirical patterns in Facts 1-4.

Figure 6 plots the price index, cutoff productivity, and cutoff taste shifter of a firm in each consumption market on the y-axis, and prefecture market

size as a share of the aggregate market size in the economy on the x-axis.[32] All the values in the figure are normalized by the value in the smallest market.

Panel A of Figure 6 shows that the cutoff productivity decreases with the market size. A larger market size has two offsetting effects on the cutoff productivity. Given the price index, a larger market size means demand for each good, allowing the lower-productivity firms to profit sufficiently to cover the fixed cost of entering the market. A larger market size also induces more competition across entering firms, which decreases the firms' profits. Overall, the first effect dominates, and the cutoff productivity is lower in larger markets, i.e., more firms enter larger markets. For the same reason, the cutoff taste shifter of a given firm decreases with the market size (see Panel B of Figure 6), which means that each firm produces more products in larger markets.

Panel C of Figure 6 shows that the price index also decreases with the market size. As described in Section 4.3, when more lower-productivity firms enter larger markets, there are two offsetting effects on the price index: the love-of-variety effect decreases the price index, while lower average productivity increases the price index. In total, the love-of-variety effect exceeds the effect of lower average productivity; thus the price index falls with the market size. It also means that the welfare in a market increases with the market size; since the wage is equalized across markets, the real wage increases as the price index decreases with the market size. Consumers in larger markets enjoy higher welfare generated by more varieties because more firms enter and sell more products in larger markets.[33] These three cross-sectional re-

[32]The cutoff taste shifter in a given market differs across firms. Panel C of Figure 6 plots the cutoff taste shifter of the firm with productivity at 99th percentile as an example. The decreasing pattern of the cutoff taste shifter does not depend on the level of productivity.

[33]Literature in urban economics points to welfare differences across geographic locations due to different varieties available in each location. For example, Handbury (2019) shows that wealthier consumers enjoy a lower price index in wealthier cities because more varieties that are catered to wealthier households are available in these cities.

sults are used to explain the observed patterns in Facts 1-4.

Fact 1: Larger firms penetrate smaller markets. Because the cutoff productivity decreases with market size, only high-productivity firms enter small markets. This follows the empirical pattern, according to which the average firm size decreases with the market size. The model also replicates the convex decreasing shape of an average firm size in the data.

Fact 2: Firms sell more products in larger markets. Panel C of Figure 6 provides direct evidence for Fact 2. It shows that the same firms have a higher cutoff taste shifter in smaller markets, i.e., the same firms sell more products in larger markets. The larger market size has two opposite impacts on the cutoff taste shifter. First, a larger market size decreases the cutoff taste shifter because there is a larger pie of consumer demand. Second, a larger market induces more competition, which means a lower price index. All in all, the first effect dominates, and the cutoff taste shifter decreases with the market size, i.e., firms sell more products in larger markets.

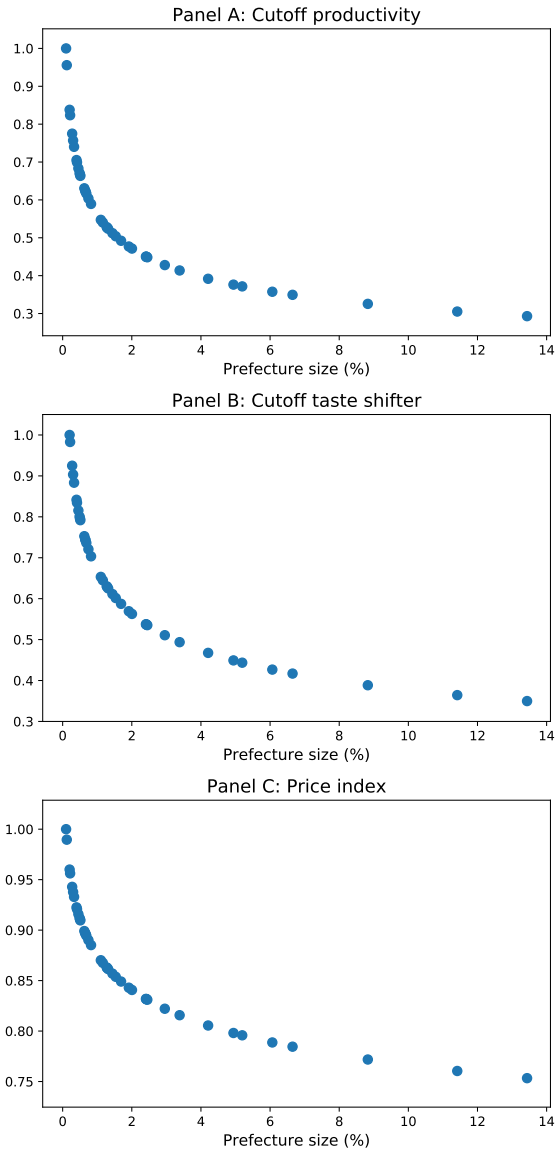
Fact 3: Larger firms sell more products in a given market. Fact 3 is implied by the cutoff taste shifter condition (6). The cutoff taste shifter condition can be rewritten as

$$\phi_f^{r*} = \left(\frac{F_p(\sigma-1)^{\frac{\sigma}{\sigma-1}}}{L^r}\right)^{\frac{1}{\sigma-1}} \frac{1}{\varphi_f P^r}. \tag{15}$$

Given price index P^r and market size L^r, higher productivity lowers the cutoff taste shifter. Therefore, a higher-productivity firms sell more products upon entering a market.

Fact 4: Firms sell different sets of products across markets. The model fits the observed pattern of the imperfect product overlap within a firm across markets. The model implies two mechanisms of the imperfect product overlap. First, the average number of products of a firm increases with the market size. If a firm enters two markets with different market sizes, the firm

Figure 6: Cross-sectional result of the model



Note: All the values are normalized by its value of the smallest market. Panel C plots cutoff taste shifter for the firm with the productivity at the 99th percentile as an example.

sells a smaller set of products in the smaller market. Second, because of the across-market taste heterogeneity, even if a firm enters two markets that have identical market sizes, it produces a different set of products in each of the markets.

5.4 Quantitative fit: firm size, product, and geographic scope distributions

Next, I compare firm size, product scope, and geographic scope distributions in the model vs. the data (Nikkei POS). When comparing the model and data, I plot and visually compare the firm size, product scope, and geographic scope distributions in the model and data.

As stated in Section 5.1, I estimate three fixed costs $\{F, F_p, F_e\}$ to match the largest 10% vs. the median firm statistics in firm size, product scope, and geographic scope. The first row of Table 5 confirms that the model fits these three moments in the data.

Next, I show that not only these targeted moments in the data, but the model also replicates the observed distributions of firm size, product scope, and geographic scope. Panel A of Figure 7 plots the log of average decile sales normalized by top decile sales. The x-axis of the figure shows the decile from the largest to the smallest, from right to left. The model performs well in replicating the sales distribution. Panel B and C of Figure 7 plot the number of products and prefectures in each decile normalized by the top decile value, respectively. The model replicates entire distributions of firms' product scope and geographic scope.

Overall, the model replicated the observed firm size, product scope, and geographic scope heterogeneity well. The differences between the actual and the predicted distributions come from the fact that I approximate the actual productivity and taste shifter distributions by the best fitted distribu-

tions (Pareto and log-normal distributions). However, the best fitted distributions still do not match the actual distributions perfectly.[34]

Table 5: Top 10% vs. median firms' statistics in the data and model

$\frac{\text{Top 10\% firms}}{\text{median firms}}$	Firm size		Product scope		Geographic scope	
	Data	Model	Data	Model	Data	Model
Baseline	188.0	178.8	18.3	18.1	4.8	5.2
CF1	188.0	25.2	18.3	10.1	4.8	1
CF2	188.0	9.2	18.3	1	4.8	1.1

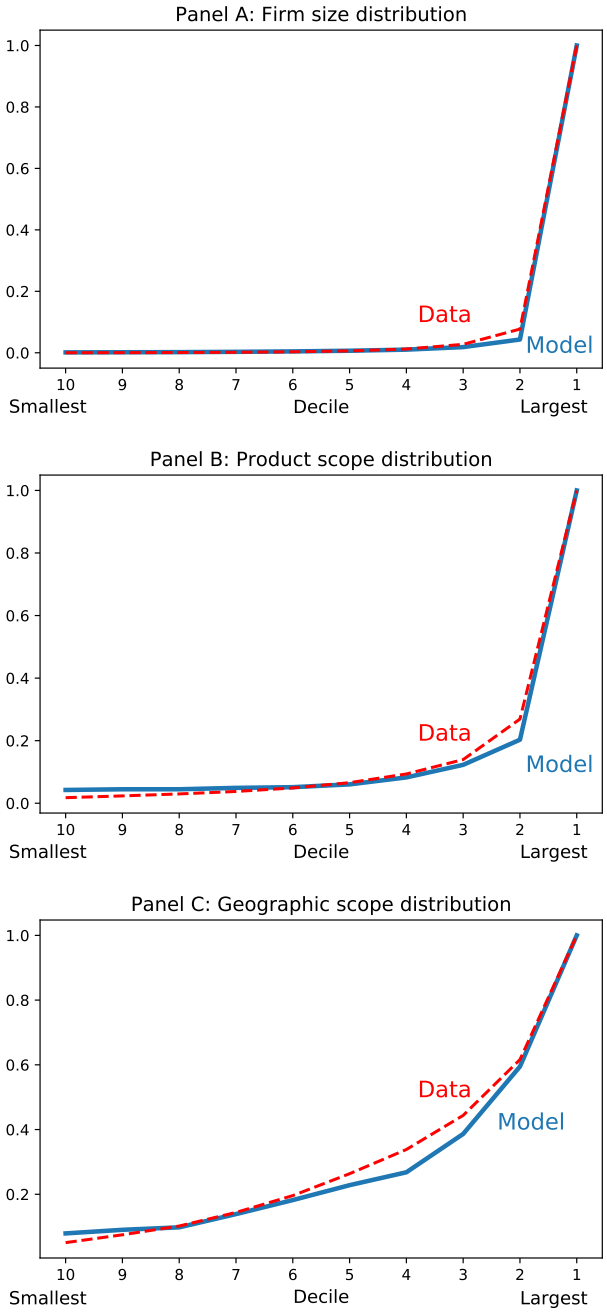
Note: The model contains top 10% firms vs. median firms' statistics for firm size, product and geographic scopes in the Nikkei POS data from 2014 to 2018 and in the model. CF1 corresponds to the counterfactual equilibrium where geographic scope is eliminated by imposing single-market in the model. CF2 correseponds to the counterfactual equilibrium where the product scope is eliminated by imposing single-product firms in the model.

6. Counterfactual Exercises

To see the quantitative importance of geographic and product scopes to account for the firm size variations, I run two counterfactual exercises that eliminate each scope and see how it affects the model fit to the data. The counterfactual exercises show that eliminating either geographic or product scopes reduces not only a significant fraction of firm size variations but also the variations in the other margin.

[34]For example, it is well known that the Pareto distribution overestimates productivity of low-productivity firms. This is reflected as the overestimation of the product and geographic scope in the smallest decile in Figure 7.

Figure 7: Firm size, product and geographic scope distributions in the data and model: Baseline



Note: In all panels, the x-axis is the number of decile, from the smallest 10% (left) to the largest 10% (right). Panel A shows the average firm sales in each decile in the data and the model. The average decile sales are normalized by the top decile sales. Panel B shows the average number of products of the firms in each decile in the data and the model. The average number of products in each decile is normalized by the top decile value. Panel C shows the average number of markets that firms in each decile sell at least one product in. Again, the values are normalized by the top decile value.

CF1: Eliminating geographic scope

To eliminate geographic margin from the model, I solve a single-market version of the model. Specifically, I assume that there is a single market in the economy in which the labor endowment is equal to the average labor endowment in the original economy, i.e., $L = \frac{1}{N}\Sigma_r L^r$.

Moving from the original to the single-market model shuts off geographic scope variations across firms because all the surviving firms enter one market. Therefore, compared with the original equilibrium, where higher-productivity firms enter more markets and acquire larger revenues, the firm size variations are expected to be smaller. In addition, unlike the original equilibrium where firms sell different sets of products across markets due to consumer taste heterogeneity, the product scope variations are expected to be smaller in the single-market model.

The second column of Table 5 reports the result from eliminating geographic scope. First, because I eliminate geographic scope, the top 10% firms vs. median firms' value of the geographic scope is reduced from 4.8 to 1 in terms of the top 10% vs. median firms' values. Second, product scope heterogeneity is also reduced by 45% (from 18.3 to 10.1). This is explained by a lack of complementarity between geographic and product scope through consumer taste heterogeneity across markets. In total, moving from the multiple-market to the single-market models reduces firm size heterogeneity by 86% (from 188 to 25.2).[35]

[35]Another way to eliminate geographic scope is to assume symmetric market size across markets. As an alternative counterfactual exercise, I impose a symmetric market size assumption with labor endowment in each market equal to the average labor endowment in the original economy, $L^r = \frac{1}{N}\Sigma_r L^r$. It reduces product scope heterogeneity by 28% (18.3 to 13.1) and firm size heterogeneity by 53% (188 to 88.4). The difference between the original and alternative counterfactual exercise is the total market size in the economy. Because the alternative exercise has a larger market size of the economy, it involves a larger mass of firms, and thus, competition is harsher.

CF2: Eliminating product scope

To eliminate product scope from the model, I solve a single-product firm version of the model. Specifically, each firm is endowed with one product for which the taste shifter equals one in any consumption market. The demand for the product of firm f in market r equals

$$C_f^r = \left(\frac{p_f}{P^r}\right)^{-\sigma} \frac{L^r}{P^r}.$$

The model is now similar to Melitz (2003) but with asymmetric market size.

Moving from the original to the single-product firm model shuts off product scope variations across firms because all the surviving firms produce only one product. Therefore, compared with the original equilibrium where higher-productivity firms produce more products and acquire larger revenues, the firm size variations are expected to be smaller. In addition, unlike the original equilibrium, where higher-productivity firms produce more products in a given market, when all the firms produce only one product, lower-productivity firms have a higher chance to enter the market. Therefore, geographic scope variations are expected to be smaller as well.

The third column of Table 5 reports the result of eliminating product scope. When product scope heterogeneity is completely eliminated (18.8 to 1), geographic scope heterogeneity is also reduced by 77% (from 4.8 to 1.1) in terms of the top 10% versus median firms' values. This is due to the lack of complementarity between product and geographic scopes described above. In total, moving from the multi-product firm model to the single-product firm model reduces firm size heterogeneity by 95% (from 188.8 to 9.2).

The two counterfactuals show that complementarity between product and geographic scope is quantitatively important. Existing models of firm heterogeneity with only either one of the extensive margins miss firm size heterogeneity due to a lack of variation in missed margin but also the vari-

ation in the other margin coming from complementarity between the two margins. It should be noted that these counterfactual exercises are conducted with all the parameters value fixed at the baseline values presented in Table 4. In Appendix A.3.2, I explain how parameter values change if the model is re-estimated to match the top 10% vs. the median firm statistics in firm size, product scope, and geographic scope in two counterfactual exercises.

7. Efficiency of the market equilibrium and welfare implication of size-dependent policies

I solve the social planner's problem and show that the market equilibrium is efficient. Given the result, I discuss the welfare implication of sub-optimal size-dependent policies, taking actual policy in Japan as an example. I compare the welfare cost between the baseline and single-product firm model and find that the missing product scope margin largely underestimates the welfare cost of such policies.

7.1 Social planner's problem

It is well known that the market equilibrium in my baseline model, Melitz (2003), is efficient.[36]To understand whether this efficiency is carried over when multi-product firms and multiple regions with heterogeneous market sizes are added, I solve the social planner's problem, whose objective is to maximize the real consumption index of the representative consumers in the economy.

[36]For example, Melitz and Redding (2015) show that with CES preferences and monopolistic competition, the open economy equilibrium in the heterogeneous firm model is efficient. Dhingra and Morrow (2019) analyze the efficiency of heterogeneous firm models in a more generalized setting.

The real consumption index for the representative consumer in the economy is defined as

$$Q = \Sigma_r \beta^r \left[M \int_{\varphi_s^r}^{\infty} K \int_{\phi_{s,f}^{r*}}^{\infty} \left(\phi_{k,f}^r C_{k,f}^r \right)^{\frac{\sigma-1}{\sigma}} \mathrm{d}F(\phi_{k_f}^r) \mathrm{d}G(\varphi_f) \right]^{\frac{\sigma}{\sigma-1}}, \tag{16}$$

where β^r is the welfare weight on the real consumption index of each market r. The welfare weight is set as proportional to the labor endowment, i.e., $\beta^r = \frac{L^r}{\sum_r L^r}$. For each consumption market r, the social planner chooses the productivity cutoff φ_s^r, the taste shifter cutoff $\phi_{f,s}^r$ for all producing firms $\varphi_f \geq \varphi_s^r$, and the output level $C_{k,f}^r$ for all produced goods $\phi_{k,f}^r \geq \phi_{f,s}^r$ for all producing firms $\varphi_f \geq \varphi_s^r$. Lastly, the planner chooses the mass of firms M subject to the aggregate labor constraint:

$$\Sigma_r L^r = \Sigma_r M \left\{ \int_{\varphi_s^r}^{\infty} K \int_{\phi_{s,f}^{r*}}^{\infty} \frac{C_{k,f}^r}{\varphi_f} + F_p \mathrm{d}F(\phi_{k_f}^r) \mathrm{d}G(\varphi_f) + \int_{\varphi_s^r}^{\infty} F \mathrm{d}G(\varphi_f) + F_e \right\}, \tag{17}$$

where the social planner faces the same productivity distribution $G(\varphi_f)$, taste shifter distribution $F(\phi_{k,f}^r)$, and fixed costs. The following proposition summarizes the social planner's choice with general distributions of productivity and taste shifter.

Proposition 1 *The social planner's choice of the output level, taste shifter cutoff, productivity cutoff, and mass of firms entering the economy coincides with the market allocation, i.e., the market equilibrium is efficient.*

The proof of this proposition is presented in Appendix A.4.

The intuition behind the efficiency of the market equilibrium is the following. First, the CES utility function and monopolistic competition leads to constant markups across firms, $\frac{\sigma}{\sigma-1}$. Therefore, there is no markup distortion across products or firms; hence the output level in the market equilibrium is optimal. Next, as for the taste shifter cutoff and productivity cutoff,

there are two sources of potential inefficiency. When introducing another product or firm, firms do not internalize the positive effect on consumer surplus (the love-of-variety effect) and the negative effect on the sales of other products and firms (the business-stealing effect). The CES utility function uniquely ensures that these two externalities exactly offset each other. Lastly, as for the mass of firms in the economy, a potential inefficiency comes from the fact that, with heterogeneous productivity, some firms make profits and some others make a loss. A free-entry condition ensures that the profits and losses exactly finance the fixed costs of entering, which means that the mass of firms entering the economy is at an efficient level. Therefore, the market implements the efficient allocation.

7.2 Welfare cost of sub-optimal SME subsidies

The efficiency of the market equilibrium implies that any policy distorting the market allocation is sub-optimal. In this section, I estimate the social cost of the subsidies for Small and Medium Enterprises (SMEs) to expand their geographic scope, a policy tool frequently used in Japan. I consider two cases: the baseline and the single-product firms. Comparison between the welfare costs of the subsidies under the baseline model and the single-product firm model allows us to measure the importance of having both product and geographic scopes for quantifying effects of size-dependent policies.

7.2.1 SME subsidies in Japan

Although my model suggests that the market equilibrium with a vast difference in firm performance is efficient, the Japanese government has con-

ducted various policies to promote SME growth.[37] One of the main policy tools to promote SME growth is subsidy for SMEs to expand their markets.

There are many types of subsidies for SMEs to expand their markets.[38] Coverage and financial sources of the subsidies vary across different types of subsidies, but most of them share the eligibility requirement and the types of costs that are subsidized. The condition for the companies to be eligible for subsidies is to have equity or employment less than a cutoff value.[39] The types of the costs covered by subsidies include the fees of joining supplier-buyer matching events, traveling expenses to meet buyers, and the costs associated with making new contracts with buyers.

Among these SME subsidies, I take the subsidy provided by the Tokyo Metropolitan Small and Medium Enterprise Support Center (a government owned financial agency) in 2020-2021 financial year as an example to analyze the welfare cost of the subsidy. This particular SME subsidy aims to promote growth of the SMEs located in Tokyo prefecture; there are also similar SME subsidies available at government owned financial agencies. The firms eligible for the subsidy are manufacturing firms with less than 300 employees. Whether a particular firm is eligible for the subsidy is determined based on the number of employees listed in the previous year's tax return documents of the enterprise tax. The subsidy covers the costs of participating in supplier-buyer matching events (including the participation fee, traveling costs, etc). The coverage of the subsidy is up to two-thirds of the total costs. The total costs are first paid by the companies, and the agency reviews the

[37] The rationale for the Japanese government to support SMEs growth that is not captured in my model includes the political power of SMEs. Since SMEs represent 70% of all employment in Japan, the voting power of SMEs is larger than their economic presence.

[38] For example, a list of subsidies available as of June 4th, 2021 can be found here https://hojyokin-portal.jp/subsidies/list. Note that the website is available only in Japanese.

[39] For the manufacturing firms, the usual cutoff values are 300 million Japanese yen (approximately 3 million USD) for equity and 300 people for employment. Firms in the service and other industries have lower cutoff values.

receipts and determines whether these costs are eligible for a subsidy. After getting it approved by the agency, two-thirds of the costs are reimbursed back to the companies.[40]

7.2.2 Modeling the SME subsidy

The eligibility, types of the costs covered by the subsidy, and the coverage of the subsidy are modeled in the following ways. In the model, firms' employments have a one-to-one relationship with their productivity, i.e., higher-productivity firms use more labor. Thus, imposing an eligibility threshold for the subsidy by employment (less than 300 employess) is identical to imposing an eligibility threshold on productivity in the model.[41] Define φ_s^* as the threshold productivity allowing the firms with the productivity below the threshold to be considered eligible for a subsidy. Because the firms who have less than 300 employees comprise 98.5% of all firms in the manufacturing industry in Japan, I set φ_s^* such that surviving firms with productivity less than φ_s^* consist of 98.5% of all surviving firms:

$$\frac{G(\varphi_s^*) - G(\varphi^*)}{1 - G(\varphi^*)} = 0.985, \tag{18}$$

where φ^* is the smallest cutoff productivity in the market equilibrium (before the subsidy is imposed).[42]

[40] See more detail about the subsidy in https://www.tokyo-kosha.or.jp/support/josei/jigyo/hankaku.html. Note that the document is available only in Japanese.

[41] One caveat is that, in reality, firms may modify their number of employees to become eligible for a subsidy. However, a firm's eligibility is determined based on the previous year's tax return documents, and the announcement of the subsidy was made after the deadline for the tax document. Therefore, I conclude that firms did not decrease their number of employees to become eligible for the subsidy.

[42] In the model, there is no *location* of firms, so it is not possible to distinguish firms in the Tokyo prefecture from firms in the other prefectures. Since similar types of SME subsidy are available in most prefectures in Japan, I assume that all prefectures in Japan provide the same type of subsidies. Therefore, it is deduced that a subsidy is available for all firms that are below the productivity threshold of the subsidy.

Lastly, the costs covered by the actual subsidy correspond to the fixed cost of entering a market, F, in the model. Define s as a fraction of subsidy for the fixed cost of entering a market. Then, the government provides subsidy for the firms whose productivity is less than the threshold productivity φ_s^*, i.e., the fixed cost of entering a market = $(1-s)F$ for the firms with productivity $\varphi_f \leq \varphi_s^*$. Lastly, to be consistent with the actual coverage of the subsidy, I set $s = \frac{2}{3}$ in the model.

I assume a balanced budget in each consumption market by imposing lump-sum income tax on consumers. The representative consumer's budget constraint becomes

$$\int_{f\in\Omega^r}\int_{k_f\in\Omega_f^r} p_{k_f}^r C_{k_f}^r \,\mathrm{d}k_f \,\mathrm{d}f \leq L^r - T^r. \tag{19}$$

The lump-sum tax T^r is given by

$$T^r = M\int_{\varphi^{r*}}^{\varphi_s^*} sF \,\mathrm{d}G(\varphi_f), \tag{20}$$

where the subsidy is granted for the firms entering market r ($\varphi_f \geq \varphi^{r*}$) but with the productivity less than the threshold ($\varphi_f \leq \varphi_s^*$). In the single-product firm model, the representative consumer's budget constraint becomes

$$\int_{f\in\Omega^r} p_{k_f}^r C_{k_f}^r \,\mathrm{d}f \leq L^r - T^r, \tag{21}$$

where product k_f denotes a single product produced by firm f. The lump-sum income tax amount T^r follows the same expression as in the baseline model shown in (20).

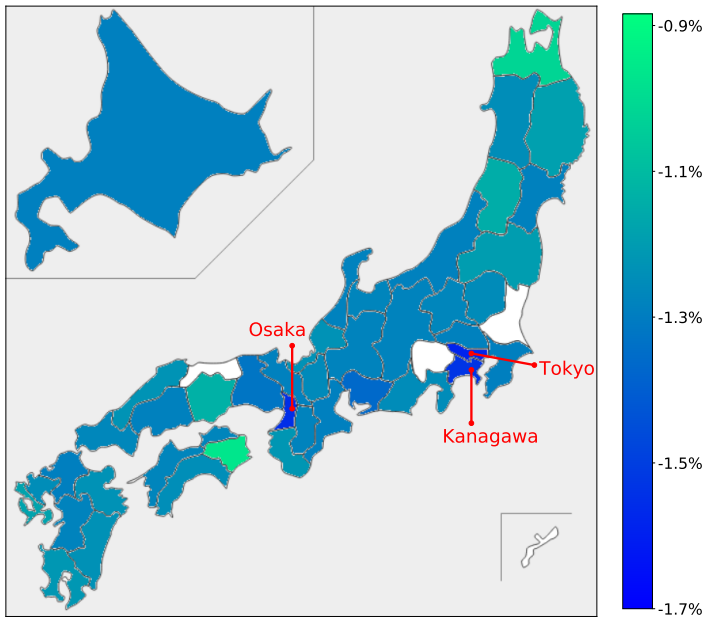
7.2.3 Results: baseline and single-product market model

Moving from the market equilibrium to the equilibrium under the subsidy affects the real consumption index of each market in the following way. The firms whose productivity is less than the threshold productivity receive a subsidy to cover the fixed cost of entering a market; thus they are more likely to enter the market. The subsidy is financed by a lump-sum income tax, which is borne by all the entering firms as a reduction in total consumer demand, including the high-productivity firms that do not receive the subsidy. Furthermore, because of the entry of more firms due to the subsidy, the competition within a market becomes harsher. In total, the subsidy helps the low-productivity firms enter more markets but hurts the high-productivity firms' profits.

In the baseline model, the subsidy results in the loss of the real consumption index in the economy by 1.25%. In the model with single-product firms, the welfare costs of the subsidy are reduced to 0.58% of the real consumption index. There are two reasons why the single-product firm model underestimates the cost of the subsidy. First, the subsidy helps the lower-productivity firms to enter the market, but these firms only sell one product. Thus, the entry of lower-productivity firms induces a minor competition increase. Second, the high-productivity firms that suffer from the reduction in total consumer demand sell only one product with and without subsidy. Therefore, higher-productivity firms' product scope margin is not distorted, and the only effect is reflected on their intensive margins. Taken together, the single-product firm model underestimates the welfare cost of the SME subsidies.

Furthermore, the subsidy affects different markets differently. The subsidy hits larger markets harder because larger markets have lower cutoff firm productivity, and more, lower productivity firms receive a subsidy to en-

Figure 8: Estimated welfare loss in each prefecture: Baseline model



Note: Tokyo, Osaka, Kanagawa are the largest three prefectures in terms of total transaction volumes in the Nikkei POS data from 2014 to 2018. Four prefectures that are painted white are the prefectures that are not covered in the sample.

ter the markets. Therefore, a larger pie of consumer demand is allocated to lower productivity firms, which lowers the profits of higher productivity firms that do not receive the subsidy. Figure 8 shows the estimated welfare cost in each prefecture under baseline model. Three prefectures that experienced the largest loss by the subsidy are Tokyo, Osaka, and Kanagawa, which are the three largest markets in terms of total sales volume in the data. This geographical difference in the welfare effect of the subsidy suggests that size-dependent policies might have a geographical re-allocational effect through changing the firm entry decisions to each market.

8. Conclusion

The recent literature in firm heterogeneity has documented the importance of two extensive margins to account for firm size heterogeneity: product and geographic scopes. Although much attention has been paid to the role of each extensive margin separately, I argue in this paper that the two extensive margins interact in a major way and thus, for the need to have both extensive margins in one model. I develop a heterogeneous-firm model with multi-product firms and multiple markets that comprise consumers with heterogeneous tastes. In the model, firms' decisions on product and geographic scopes interact in two ways. First, as firms expect to sell more products and larger profits, they are more likely to enter a market. Second, as firms enter more markets, they sell greater varieties of products because each market exhibits different consumer tastes. The calibrated model reveals that each complementarity is quantitatively important. Eliminating either geographic or product scope from the model reduces firm size variations by 86 to 94%, failing to incorporate the complementarity between two margins. I further use the model to quantify the welfare implication of the actual size-dependent subsidy in Japan. Notably, the model with only a single extensive margin underestimates the welfare implication of such subsidies by more than half.

There are a couple of additional directions for research suggested by the paper. First, my analysis focuses on explaining cross-sectional firm size variations, and it is left unknown how the firms' performance and product and geographic scopes evolve. Exploring these aspects requires access to panel data that would allow analyzing observations made over some course of time. Second, my framework does not incorporate distance. Including distance in the model lets us explore what role shipping costs and taste heterogeneity, which are correlated with the distance between the markets, play in

generating firm size heterogeneity.

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A. Appendix

A.1 Retail chains and firm sizes

This section answers whether larger firms are connected to a few larger retail chains and sell their products in many stores within the few retail chains. My data shows that this is not the case; larger firms sell in more retail chains, and these retail chains are not systematically larger than the retail chains that smaller firms are connected.

The first column of Table 6 shows the average number of prefectures that firms sell their products in increases with firm size. If larger firms have a wider geographic scope because they sell to a few retail chains with many stores in many prefectures, then I would expect the average number of retail chains to not increase with firm size. The second column of Table 6 shows that the average number of retail chains monotonically increases with firm size. It means that larger firms sell not only in many prefectures but also in many retail chains.

I also check whether the retail chains that larger firms connect have more stores than those that smaller firms connect. The last column of Table 6 shows that it is not the case; the average number of stores per retail chain does not increase monotonically with firm size. The smallest 10% firms sell to retail chains that have, on average, the largest number of stores. The findings in Table 6 suggest that the reason why larger firms sell their products in more prefectures is not that they sell to large retail chains, but they sell to many retail chains and stores.

Table 6: Number and size of retail chains by decile

Ranked decile	Avg. no. prefectures	Avg. no. retail chains	Avg. no. stores per chain
1	28.08	14.99	17.39
2	17.30	10.11	14.24
3	12.46	7.59	14.63
4	9.50	5.68	14.38
5	7.40	4.53	16.90
6	5.50	3.65	14.90
7	4.04	2.94	16.67
8	2.86	2.39	16.74
9	2.10	1.93	17.60
10	1.42	1.43	18.33

Note: The largest decile is ranked first. The values are weighed by 3-digit product category sales per quarter. Number of prefectures is calculated as the number of prefectures in which each manufacturing firm sells at least one product. Number of retail chains is calculated as the total number of retail chains firms sell their products to. Finally, average number of stores per chain in each decile is calculated as the average number of stores of each retail chain that firms in each decile sell their products to.

A.2 Survey at a Japanese supplier-buyer matching event

A.2.1 Matching event summary

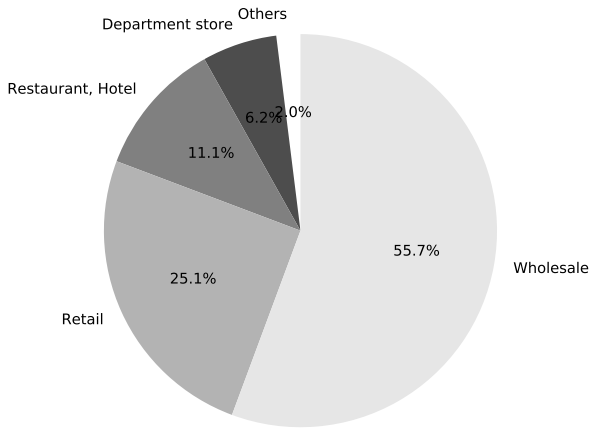
Food Selection 2020 is one of the biggest food supplier-buyer matching events in Japan, enriched with a fifteen-year-old history.[43] There are approximately 800 food manufacturing suppliers and 2500 food buyers participating in the event. The participating food manufacturing suppliers try to make new sales contracts with buyers, while the buyers try to find local products and make

[43] See http://food-selection.com/gaiyo.html for more detail. Please note that the website is only available in Japanese.

new contracts with suppliers.

The participating suppliers come from all over Japan (from all 47 prefectures). Product types vary across suppliers from seaweed to packaged Udon noodles, but all suppliers produce packaged foods. The buyers come mainly from big cities, such as Tokyo, Kanagawa, and Saitama, ranging from wholesale, retail, hotels and restaurants to department stores. The composition of buyers is summarized in Figure 9.

Figure 9: Buyer sector composition at the event



Note: Others include food manufacturing and school meal preparation.

The matching event takes place as follows. First, each supplier registers their product information (picture, weight, height, volume, ingredients, etc.) in a database. Second, buyers browse the products, and if they like a product, they notify the organizing company that they want to start a negotiation about the product. Figure 9a shows an example search page in the database. If the buyer clicks a specific product on the search page, the database will take them to a product information page, as shown in Figure 9b. Third, the supplier and buyer negotiate the contract terms (order quantity, price, pack-

aging, etc.), and if the negotiation goes well, they will sign a contract. Figure 11 show shows the supplier-buyer negotiations in Zoom (due to COVID-19, in 2020, the event was held online). Negotiations often start with the supplier providing a detailed product explanation, following which the buyer and the supplier negotiate potential contract terms.

The event registration started in June 2020, and the database opened (and thus, negotiations started) on October 1st, 2020. The event lasted one year and ended on September 31st, 2021. Each supplier pays $80,000$ Japanese yen (≈ 800 USD) to register and access the database. All buyers can register and browse the products in the database free of charge. Each supplier-buyer negotiation costs $5,000$ Japanese yen (≈ 50 USD) for both the supplier and the buyer. .

As mentioned in Section 3, I joined 50 supplier-buyer negotiations in the event to find out the cost counterparts of the four modeling elements. Table 7 summarizes the counterpart costs of the candidate model elements and the frequency of these costs discussed in the 50 negotiations. Although the survey results indicate that the fixed costs of entering a market and selling a product are practically more important, all four costs are frequently discussed in these negotiations.

Table 7: Frequently discussed issues and corresponding model structures: 50 observations

Discussed issue	Corresponding model structure	Frequency
Understanding consumer taste and product customization	FC of selling a product	50%
Contracting issues between suppliers and buyers	FC of entering a market	48%
Consumer awareness of the product	Marketing cost	48%
Production capacity and lead time	Lumpy investment	34%

Note: The frequency reflects the number of times each topic was discussed during the negotiations. FC stands for fixed costs.

Figure 10: An overview of the database

(a) A search page example.

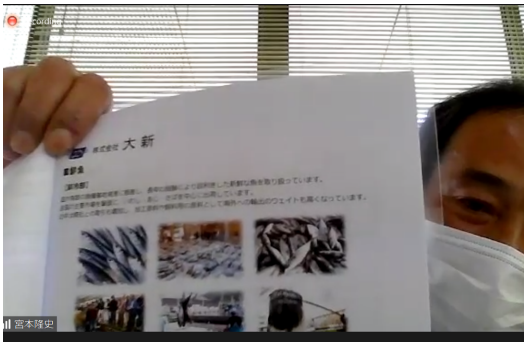


(b) A product information example.

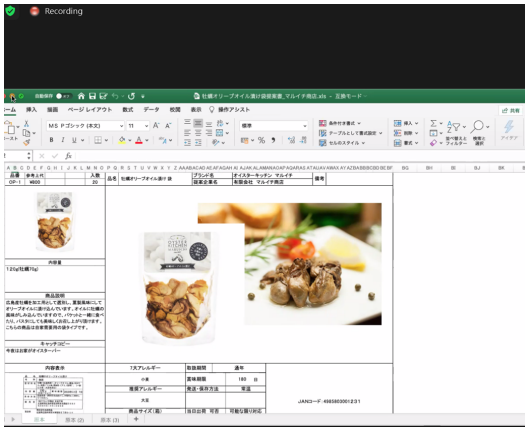


Figure 11: An overview of online negotiations

(a) A supplier explaining the production process.



(b) A supplier explaining the product information.



A.2.2 Survey design and response rates

The survey targeted all the suppliers who participated in the event (802 firms from 47 prefectures in Japan) and was designed to be filled in online through a Google Form. The survey questions were incorporated into the event survey circulated by the organizing company in April 2021 to obtain the participants' feedback about the event.

The objective of the survey for this paper was to determine the relative importance among the four modeling elements generating product or geographic scope variations across firms and choose my model structures that are of the highest practical importance. As it was elaborated in Section 3, the four candidate model elements suggested by the literature are the fixed costs of entering a market and selling a product, the marketing cost, and the cost of investment into production capital.

Based on the observations of 50 supplier-buyer negotiations (see Table 7), I find that the fixed cost of entering a market corresponds to the cost of making a contract with buyers, and the fixed cost of selling a product corresponds to the cost of researching consumer taste and product customization. The marketing cost and the cost of investment into production capital have the straightforward corresponded costs in reality. Therefore, to determine the relative importance between the four model elements, I ask the suppliers the relative importance of the four types of costs each corresponding to one of the model elements. In the online survey, I asked the suppliers the following question. On the scale of 1 to 4 (1: not important to 4: very important), how important are the following elements when expanding into new geographic markets? (1) overcoming contracting issues with buyers, (2) understanding consumer taste and customizing products to cater to the taste, (3) having sufficient production capacity for the new market, and (4) advertising products to increase consumer awareness.

Out of the 802 targeted suppliers, I got responses from 246 suppliers (i.e., 31% response rate). The respondent suppliers come from a wide range of geographic areas from 34 out of 47 prefectures in Japan. Since the survey requires knowledge about the relative importance of business barriers, it is important to make sure that the respondents have enough knowledge about the companies' businesses. Out of 246 respondents, 39% are CEO/presidents, 16% are executive managing directors, 23% are department chief/general

managers, and the rest 22% of the respondents are either sales staff or operators.

A.3 More explanation on structural estimation

Section s:simulation-algorithm explains simulation algorithm used in the SMM, and Section 6 explains the re-estimation of the parameter values in the counterfactual exercises.

A.3.1 Simulation algorithm

The given simulation of the model requires a set of parameters $\Theta_2 = \{\sigma, (\theta, \bar{\varphi})\}$, the data for product- and firm-level sales, (firm-level) prices, and number of products firms sell in each market. It involves the following steps:

Preparation: Draw productivity of 100,000 artificial Japanese firms from the following distribution: $\varphi_f \sim$ Pareto$(\theta, \bar{\varphi})$, given assumed values of θ and $\bar{\varphi}$.

Step 1. Among these artificial Japanese firms, randomly sample 10,000 firms in each simulation. Among the 10,000 artificial firms, use the surviving firms, i.e., the firms with the productivity higher than the lowest cutoff productivity, for the remaining simulation.

Step 2. Calculate the prices of the surviving firms by expression 3. Note that, in my model, the firms' pricing depends only on productivity (φ_f) and the elasticity of substitution (σ).

Step 3. For the surviving firms, draw K common components of the taste shifter for each product from the distribution: $\log(\phi_k) \sim N(\mu, \sigma^2)$. Similarly, for each product and each market, draw the regional shock of the taste shifter from the distribution: $\log(\epsilon_k^r) \sim N(0, \sigma_\epsilon^2)$.

Step 4. Compute the number of products firms produce in each market by the cutoff taste-shifter condition (6).

Step 5. Compute the product-level sales for each firm in each market by the following expression:

$$p_f^r C_{k,f}^r = (\phi_f)^{\sigma-1} \left(\frac{p_f}{P^r}\right)^{1-\sigma} L^r. \tag{22}$$

Compute the firm-level sales in each market by the following expression:

$$\int_{k_f \in \Omega_f^r} p_f C_{k,f}^r \, \mathrm{d}k_f = \int_{k_f \in \Omega_f^r} (\phi_f)^{\sigma-1} \left(\frac{p_f}{P^r}\right)^{1-\sigma} L^r \, \mathrm{d}k_f. \tag{23}$$

Step 6. Estimate σ using expression (10) and φ_f for each surviving firm from expression (11).

Step 7. Compute the best-fitted scale and shape parameters for the estimated productivity distribution: $\varphi_f \sim \text{Pareto}(\theta, \bar{\varphi})$.

These seven steps produce the biased estimates of $\{\hat{\sigma}, (\hat{\theta}, \hat{\bar{\varphi}})\}$. The weighing matrix is calculated as a variance-covariance matrix of the error function using the following bootstrap procedure: (i) re-sample, with replacement, 10,000 firms from our initial data set of the 100,000 firms 1000 times. (ii) For each re-sampling b, calculate moments $m(\Theta_2)$. (iii) Calculate

$$W_2 = \frac{1}{1000} \sum_{b=1}^{1000} (m_2^b - m^b(\Theta_2))(m_2^b - m^b(\Theta_2))'. \tag{24}$$

A.3.2 Re-estimation of the parameter values

In the counterfactual exercises conducted in Section 6, I check how the firm size, the product and geographic scope distributions change when eliminating one of the two extensive margins. In this section, the parameters are re-estimated to match the firm size and the product or geographic scope distributions in the counterfactual economies.

First, I re-estimate parameter values in the first counterfactual scenario,

i.e., the model with a single market. In the counterfactual economy, I re-estimate the fixed costs of selling a product and entering the production economy (F_p and F_e) and the shape parameter of productivity distribution (θ) to match the firm size and product scope distributions. In particular, I find parameter values $\{F_p, F_e, \theta\}$ that match the top 10% vs. median firm statistics in the firm size and product scope distributions. The re-estimated parameter values are $F_p = 0.0023$, $F_e = 0.10$, and $\theta = 2.06$. In order to generate larger firm size heterogeneity without the geographic scope margin, the model requires a larger productivity dispersion across firms, which leads to a lower shape parameter of the Pareto distribution (θ).

I re-estimate the parameter values in the second counterfactual scenario, i.e., the model with single-product firms. In the counterfactual economy, I re-estimate the fixed costs of entering a market and entering the production economy (F and F_e) and the shape parameter of productivity distribution (θ) to match the top 10% vs. median firm statistics in the firm size and geographic scope distributions. The re-estimated parameter values are $F = 0.001$, $F_e = 0.19$, and $\theta = 1.99$.

A.3.3 Welfare cost of the Japanese subsidy with the re-estimated parameter values

In Section 7, I estimate the welfare cost of the sub-optimal SME subsidies in baseline model vs. single-product firm model using the parameter values estimated in the baseline model. This section uses the re-estimated parameter values in the counterfactual single-product firm model to calculate the welfare cost of such subsidies subsidy in the single-product firm model. Re-estimated parameter values are $F = 0.001$, $F_e = 0.19$, and $\theta = 1.99$. The rest of the parameter values are the same as in Table 4.

The welfare cost of the SME subsidy with the re-estimated parameter val-

ues is 1.07%. Compared to the welfare cost with baseline parameter values in the single-product firm model, 0.58%, the loss is larger because the productivity distribution is more disperse with a lower shape parameter (θ). The fatter the right tail of the productivity distribution is, the larger the loss due to the subsidy because these very high-productivity firms' profits decrease from the subsidy. It should be noted that the welfare cost of the subsidy with the re-estimated parameter values in the single-product firm model is still smaller than the cost in the baseline model, 1.25%.

A.4 Social planner's problem

Proof of Proposition 1. First, I derive the planner's choice on intensive margin. The planner chooses the output levels $C^r_{k,f}$ to equate the the marginal rates of transformations (MRT) and the marginal rates of substitution (MRS) for the products of the firms with different productivity and for the products of the same firm in each market. The MRT between the products for any two different firms with productivity φ_1 and φ_2 in market r is:

$$MRS = \left(\frac{\phi^r_{k,1}}{\phi^r_{k,2}}\right)^{\frac{1-\sigma}{\sigma}} \left(\frac{C^r_{k,1}}{C^r_{k,2}}\right)^{\frac{1}{\sigma}}. \tag{25}$$

The MRT for the products for any two different firms with productivity φ_1 and φ_2 in market r is:

$$MRT = \frac{\varphi_1}{\varphi_2}. \tag{26}$$

The social planner chooses an output level to equate the MRS and MRT:

$$MRS = MRT \implies \frac{C^r_{k,1}}{C^r_{k,2}} = \left(\frac{\phi^r_{k,1}}{\phi^r_{k,1}}\right)^{\sigma-1} \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma}, \tag{27}$$

which yields the same relationship between relative quantities, relative productivity, and relative taste shifters as those in the market equilibrium. Similarly, I can show that the planner's choice of the output level for two goods within a same firm also coincides with that in the market equilibrium. Therefore, the output level in the market equilibrium is efficient.

Next, I derive the social planner's choice of the taste shifter cutoff. For a given firm f with productivity φ_f in market r, the social planner sets a taste shifter cutoff $\phi^r_{f,s}$ that satisfies the first order conditions with respect to $\phi^r_{f,s}$ and the $C(\phi^r_{f,s})$, where $C(\phi^r_{f,s})$ is the consumption quantity of the product with the cutoff taste shifter. Combining the two first order conditions, I obtain

$$\left(\frac{1}{\sigma-1}\right)\varphi_f^{-1}C(\phi^r_{f,s}) = F_p. \tag{28}$$

The condition coincides with the cutoff taste shifter condition in the market equilibrium (6). Therefore, the value of the cutoff taste shifter in the market equilibrium is efficient.

Next, I derive the planner's choice of the productivity cutoff. Using the relationship ${}^{C(\phi^r_{k,f})}/_{C(\phi^r_{f,s})} = ({}^{\phi^r_{k,f}}/_{\phi^r_{f,s}})^{\sigma-1}$, I rewrite the real consumption index (16) as

$$Q = \Sigma_r \beta^r (MK)^{\frac{\sigma}{\sigma-1}} \left[\int_{\varphi^r_s}^{\infty} (C(\phi^r_{f,s})\phi^r_{f,s})^{\frac{\sigma-1}{\sigma}} \int_{\phi^r_{s,f}}^{\infty} \left(\phi^r_{k,f}\right)^{\frac{\sigma-1}{\sigma}} \mathrm{d}F(\phi^r_{k_f})\mathrm{d}G(\varphi_f)\right]^{\frac{\sigma}{\sigma-1}}, \tag{29}$$

and the aggregate labor constraint (17) as

$$\Sigma_r L^r = \Sigma_r M \left\{ K \int_{\varphi^r_s}^{\infty} \frac{C(\phi^r_{f,s})\phi^r_{f,s}}{\varphi_f} \int_{\phi^{r*}_{s,f}}^{\infty} \phi^r_{f,s} + F_p \mathrm{d}F(\phi^r_{k_f})\mathrm{d}G(\varphi_f) + \int_{\varphi^r_s}^{\infty} F \mathrm{d}G(\varphi_f) + F_e \right\}. \tag{30}$$

Combining the first order conditions with respect to the productivity cutoff φ^r_s and the consumption quantity of products with the taste shifter cutoff

sold by a firm with the productivity cutoff $C(\phi^r_{f,s}(\varphi^r))$ I obtain

$$\left(\frac{1}{\sigma-1}\right)(\varphi^r_s)^{-1}K\int_{\phi^r_{f,s}}^{\infty}C(\phi^r_{f,s}(\varphi^r))\mathrm{d}F(\phi^r_{k,f})=K\int_{\phi^r_{f,s}}^{\infty}F_p\mathrm{d}F(\phi^r_{k,f})+F, \quad (31)$$

which corresponds to the cutoff productivity condition in the market equilibrium (8). Therefore, the productivity cutoff in the market equilibrium is efficient.

Lastly, I derive the planner's choice of the mass of firms entering the economy. To do so, I take the first order condition with respect to the mass of firms M, combine the condition with the first order condition with respect to $C(\phi^r_{f,s}(\varphi^r))$, and take summation Σ_r:

$$\frac{\frac{1}{\sigma}\Sigma_r L^r-\left\{M\int_{\varphi^r_s}^{\infty}K\int_{\phi^r_{f,s}}^{\infty}F_p\mathrm{d}F(\phi^r_{k,f})+F\mathrm{d}G(\varphi_f)\right\}}{M}=F_e. \quad (32)$$

The term $1/\sigma\Sigma_r L^r$ in the numerator is equal to the total variable profit made in the market equilibrium. The term $\left\{M\int_{\varphi^r_s}^{\infty}K\int_{\phi^r_{f,s}}^{\infty}F_p\mathrm{d}F(\phi^r_{k,f})+F\mathrm{d}G(\varphi_f)\right\}$ in the numerator is equal to the total fixed costs paid by the firms in the market equilibrium. Thus, the numerator on the left-hand side of (32) equals the total profit made by the firms in the market equilibrium. Therefore, the condition (32) coincides with the free-entry condition in the market equilibrium; hence the mass of firms in the market equilibrium is efficient. ■