A Model of Polarization

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Abstract

The paper models an economy in which output is produced by teams of size two, and in which the match-partners' qualities are complements. Agents' histories are private information and firms use contracts that screen for quality. The initial situation is a random assignment, caused by a technological change that alters agents' productive skills. Agents must relearn their types by observing the output that they and their match partners produce. The economy starts from a garbled situation and adjusts towards its long-run positive assignment at a geometric rate. Labor mobility and firm entry also decline geometrically. Contracting problems may slow down the adjustment whereas certain types of mergers can speed it up.

1 Introduction

This paper analyzes an economy's adjustment to a major technological shock such as the arrival of a general-purpose technology. When the shock hits, agents' abilities are displaced or "garbled," and agents must relearn their types – as suggested by

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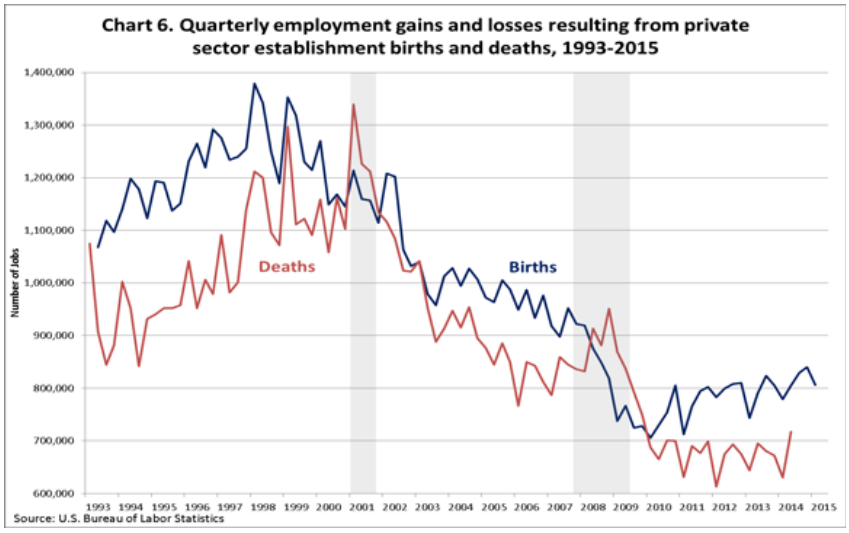


Figure 1: DECLINE OF ENTRY: Chart 6 of BLS's "Entrepreneurship in the U.S. Economy." 2016 https://www.bls.gov/bdm/entrepreneurship/entrepreneurship.htm

Andolfatto and Smith (2001) who refer to it as a "paradigm shift." Agents then gradually learn their abilities by observing what they produce in teams with other workers.

The model seeks to explain several phenomena that have been observed in the last two or three decades: Middle income jobs have been disappearing, earnings inequality has been rising, and firm entry has declined. For U.S. facts see Decker, Haltiwanger, Jarmin and Miranda (2016) and Pries and Rogerson (2022), and for Denmark see Traiberman (2019). Fig. 1 reproduces Chart 6 of Bureau of Labor Statistics (2016) showing the slowdown in the birth and death of plants, and Pries and Rogerson (2022) document a similar decline in labor market turnover, especially at short-tenure durations of one and two quarters, with a declining hires rate and a declining separations hazard each accounting for about half of the decline.

In the model production occurs in pairs and, as in Anderson and Smith (2010), only the output of the match is observed and, not the individual members' contributions. Agents' qualities are complements in production and an agent's output history

is private information.

An agent's type is their belief about their ability. If the contracting mechanism screens perfectly for the types, and the noncooperative solution converges geometrically to the perfect positive assignment and half of adjustment is achieved in just one period. But if such contracts do not work, the half life would take several years, and sometimes the adjustment halts in midstream. That is, if convergence does take place, it is arithmetic. Adverse selection slows things down and the process sometime stops before the economy reaches the positive assignment – and sometimes does not occur at all, being impeded by adverse selection. When it does occur, convergence is to the first-best configuration of a perfect positive assignment, and the half life of adjustment is between 2 and 5 periods.

The main contribution of the paper is the analysis of the dynamics of adjustment of a market involving firms and workers that learn their types gradually. Ljungqvist & Sargent (1998) model a rise in economic turbulence that manifests itself as a stochastic incidence of an individual worker's skill loss at a layoff. Guerrieri, Shimer and Wright (2010) and Guerrieri and Shimer (2014) consider directed search with adverse selection, the former in a static and the latter in a dynamic model. By contrast to Doppelt (2015) where reputations evolve publicly, here histories are private. Anderson and Smith (2010) have individuals learning of types but do not study aggregate dynamics and Moreno and Wooders (2016) study aggregate dynamics without individual learning. Greenwald (1986), Inderst (2005) and Kugler and Saint Paul (2004) discuss adverse selection on one side of the market and without agents learning their types. Moen and Rosen (2005) analyze the role of performance pay and Alder (2016) analyzes mismatch distortions but not aggregate dynamics. Burdett and Coles (1997) and Shimer and Smith (2000) study search where qualities are observed – inspection search. Kremer and Maskin (1996) had noted that growth in wage inequality has been accompanied by greater segregation of high- and low-skill workers into separate firms, but their explanation was rising complementarity between workers skills.

Private information slows down the adjustment. The same conclusion emerges in a different context in two papers. Kyle (1985) studies adjustment as the market learns an insider seller's private information about the payoff of an asset that he owns, and Golosov, Lorenzoni and Tsyvinski (2014) assume agents have private information about an aggregate shock to the payoff of a risky asset's common value, and have private information in an over the counter model of asset trade.[1]

If contracts can separate types, however, as in Guerrieri, Shimer and Wright (2010), then adjustment is faster and takes place at a geometric rate. And if all individual histories were public as in Doppelt (2016), adjustment would be even faster, mergers and takeovers could decentralize first best outcome. Two teams that produced the same first-period output and that look identical, reshuffle the production partners and raise the teams' combined output. Rhodes-Kropf and Robinson (2008) document mergers among equals in terms of market-to-book values, Jovanovic and Braguinsky (2004) analyze the role of mergers in delivering a positive assignment.

The next section introduces the model and the separating contracts that achieve geometric convergence to the positive sorting limit. Section 3 assumes contracts do not work and that agents simply divide their output equally and derives the arithmetically converging sequences that sometimes never reach the positive sorting limit. Section 4 summarizes the implications and briefly relates them to the evidence on polarization. Section 5 discusses some of the model's limitations and the generalizability of its implications, and Section 6 concludes. Several appendixes contain derivations and proofs.

[1] A recent model dealing with aggregate dynamics in situations where sorting matters is Jovanovic (2014), but that was an overlapping generations model where people from different generations meet only once. The private information version, however, is included only in the NBER version of the paper.

2 Model

A continuum of risk neutral firms and workers exists; all are risk neutral and infinitely lived with discount factor β. A firm has just two workers, and its output per worker, y, depends on the qualities x_1 and x_2 of its workers as follows:

$$y = x_1 x_2. \tag{1}$$

The total output of the firm is $2y$. Time is discrete and output is produced at the end of the period.

There are two types of workers with qualities $x \in \{1, r\}$ with $r > 1$ and with ϕ being the fraction of high-ability types. Then three levels of output are possible, $y \in \{1, r, r^2\}$. In period zero matches have already formed randomly and at the outset no one knows their ability. While y is observed without error, it also is the only signal about x_1 and x_2. Outputs $y \in \{1, r^2\}$ perfectly reveal x_1 and x_2, an output $y = r$ does not. Thus at the end of each period there are three types of workers indexed by the belief $\pi \in \left\{0, \frac{1}{2}, 1\right\}$ of being a high type. The parties to the match may remain together or separate. Matching is instantaneous, and workers' output histories are private.

The model is a special case of Anderson and Smith (2020, henceforth "AS") who also have a random component in the output equation.

Contingent contracts

We assume firms post vacancies with associated contracts at zero-expected profit each period. Two of the applicants are chosen and their payments are contingent on the output that their match produced. A firm will design its contracts that attracts only those types that it wants. Denote the wage by w_y for $y \in \{1, r, r^2\}$. The firm is committed to delivering these payments. The match then remains in place forever if $y \in \{1, r^2\}$ and workers are paid y thereafter. But if $y = r$, the workers are paid w_r and are then fired and will seek a new match.

Equilibrium contracts will screen out workers that are in high and in low matches and that have beliefs $\pi \in \{0, 1\}$. All applications will then come from workers that have worked in $y = r$ firms since the beginning, and that as a result have belief $\pi = 1/2$. [2]In that equilibrium, taking a job will, for any type π, mean that one's coworker is equally likely to be of a high or a low type. The equilibrium will have positive sorting on π.

An equilibrium contract must satisfy four conditions. If only the $y_{-1} = r$ workers apply (where y_{-1} is the previous period output of an individual), then in each group (firms or applicants), half are high types and half low types. A firm that posts a vacancy will offer the contingent contract w_y that separates the two types in that it

1. *Induces all $y_{-1} = r$ firm members whose beliefs are $\pi = 1/2$ to abandon their matches and apply.*—Given that one's coworker also has $\pi = 1/2$, the outputs $y \in \{1, r, r^2\}$ have probabilities $\frac{1}{4}, \frac{1}{2}$, and $\frac{1}{4}$ respectively. An output $y = r$ leads to a separation, whereas $y = 1$ and $y = r^2$ lead to a lifetime value of $\beta/(1-\beta)$ and $\beta r^2/(1-\beta)$ respectively. Let W be a $\pi = 1/2$ type's lifetime value associated with applying. Then W must satisfy the Bellman equation and inequality

$$W = \frac{w_1 + \frac{\beta}{1-\beta}}{4} + \frac{w_{r^2} + \frac{\beta r^2}{1-\beta}}{4} + \frac{w_r + \beta W}{2} \geq \frac{r}{1-\beta}. \tag{2}$$

2. *Deters the low ($\pi = 0$) types from applying.*—Knowing that he is a low type, someone coming from a $y_{-1} = 1$ firm expects this new match to be with a $\pi = 1/2$ type and therefore expects output to be either $y = 1$ or $y = r$, each with probability $1/2$. Both outcomes lead to a separation. Therefore the low type will not apply if his one-period expected income is less than the output in his current match, i.e., less

[2]For those in $y = r$ firms, even the initial belief immediately becomes $1/2$ because

$$\pi = \frac{(1-\phi)\,\phi}{\phi\,(1-\phi) + (1-\phi)\,\phi} = \frac{1}{2}$$

than unity:

$$\frac{1}{2}w_1 + \frac{1}{2}w_r \leq 1. \tag{3}$$

3. *Deters the high ($\pi = 1$) types from applying.*—Knowing that he is a high type, someone coming from a $y_{-1} = r^2$ firm expects two possible outputs: $y = r$ or $y = r^2$, each with probability 1/2. This type's initial expected wage must be less than r^2 which is what he earns in his current high-output match:

$$\frac{w_r + w_{r^2}}{2} \leq r^2. \tag{4}$$

4. *Is preferred to matching $\pi = 1/2$ types with $\pi = 0$ (i.e., $x = 1$) types or with $\pi = 1$ (i.e., $x = r$) types for informational reasons.*—This would be the AS type of match that sacrifices output in return for an informational gain. This requires that the joint expected discounted returns to such a match, denoted here by $J^{0,1/2}$, should be smaller than the discounted returns to matching two agents with types $\pi = 1/2$, denoted here by $J^{1/2,1/2}$. If this requirement holds, then the rents to matching $\pi = 1/2$ types with $\pi = 1$ (i.e., $x = r$) types would also be unprofitable. Then no firm would try to form such a heterogeneous-π match because if it succeeded in drawing agents away from a 1/2&1/2 match, its expected profits would be negative.

To show that $J^{1/2,1/2} > J^{0,1/2}$, we first note that matching a $\pi = 1/2$ agent matched with a $\pi = 0$ agent implies that

a) With probability 1/2 today's output will be $2y = 2$, both types are low and in that event the lifetime joint payoff then is $2/(1-\beta)$.

b) With probability 1/2 output will be $2r$ in the first period and thereafter (provided in the best-case that the newly discovered high type can match with another high type in the following period) it is $1 + r^2$, and the joint expected lifetime payoff then is $2r + \frac{\beta}{1-\beta}(1 + r^2)$.

Thus the joint expected discounted output of agents in the **"0&1/2" contract** is

$$J^{0,1/2} = \frac{1}{2}\left(\frac{2}{1-\beta}\right) + \frac{1}{2}\left(2r + \frac{\beta}{1-\beta}\left(1+r^2\right)\right) \tag{5}$$

whereas for agents in the **"1/2&1/2" contract** is

$$J^{1/2,1/2} = \frac{1}{4}\left(\frac{2}{1-\beta}\right) + \frac{1}{4}\left(\frac{2r^2}{1-\beta}\right) + \frac{1}{2}\left(2r + \beta J^{1/2,1/2}\right) \tag{6}$$

Then the Appendix proves

Lemma 1 *For all $r > 1$ and all $\beta < 1$,*

$$J^{1/2,1/2} > J^{0,1/2}. \tag{7}$$

Intuition for why $J^{0,1/2} < J^{1/2,1/2}$.—Matching $\pi = 1/2$ types with $\pi = 0$ *(i.e.,* $x = 1$) types indeed does give full learning with probability 1 for the $\pi = 1/2$ type, but no learning for the $\pi = 0$ type. On the other hand, the match of two $\pi = 1/2$ types gives each member of the match full learning with probability 1/2. And since $1/2 + 1/2 = 1$ the informational gain benefit is the same in expected terms. But, the expected output of the 0&1/2 match is lower than the expected output of the 1/2&1/2 match. Similarly for the rents to matching π=1/2 types with π=1 (i.e., $x = r$) types.

If we wanted to, we could always find enough low and high firms to experiment with; that number always exceeds the number of middle firms, regardless of ϕ:

$$(1-\phi)^2 + \phi^2 \geq 2\phi\,(1-\phi) \tag{8}$$

the total number of low firms and high firms is always greater than the number of middle firms. We can use both low firms and high firms to match with the members of middle firm and achieve the output path in AS. In either case, however, a 0&1/2 match or a 1/2&1 match, firms would then expect to lose money if they formed such matches, and neither would be a profitable deviation from the assortative matching equilibrium we shall describe next.

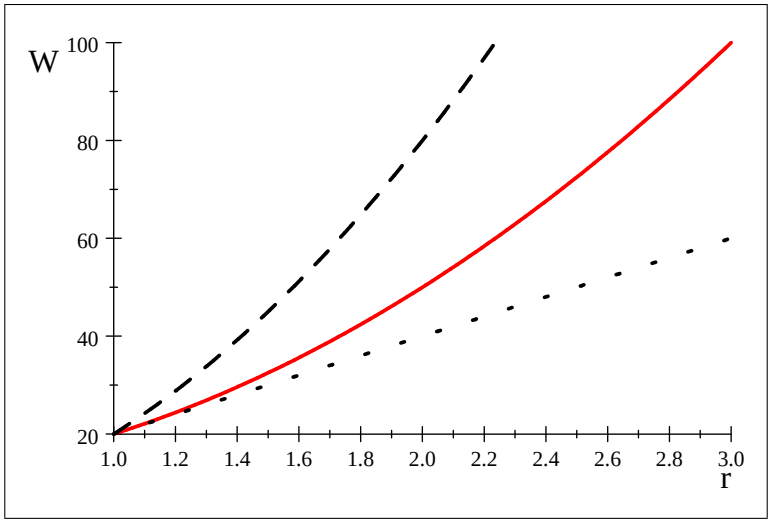


Figure 2: W (solid), $r/(1-\beta)$ (dotted) and $r^2/(1-\beta)$ (dashed)

Proposition 1 *For any $r > 1, \phi \in (0,1)$ and $\beta < 1$, there exist (w_1, w_r, w_{r^2}) satisfying (2) - (4). In general there are many solutions and one in particular is*

$$w_1 = 0, \;\; w_r = 2, \quad \textit{and} \quad w_{r^2} = 2\left(r^2 - 1\right). \tag{9}$$

PROOF: Evidently, (3) and (4) hold as equalities. Solving (2) for W yields

$$\begin{aligned} W &= \frac{1}{2-\beta}\left(w_r + \frac{1}{2}\left(w_{r^2} + \frac{\beta\left(1+r^2\right)}{1-\beta}\right)\right) = \frac{1}{2-\beta}\left(2 + \left(r^2-1\right) + \frac{1}{2}\frac{\beta\left(1+r^2\right)}{1-\beta}\right) \\ &= \frac{1}{2-\beta}\left(\left(1 + \frac{1}{2}\frac{\beta}{1-\beta}\right)\left(1+r^2\right)\right) > \frac{2r}{2-\beta}\left(\left(1 + \frac{1}{2}\frac{\beta}{1-\beta}\right)\right) = \frac{r}{1-\beta}. \end{aligned}$$

because $(1+r^2) > 2r$. And since $\frac{2}{2-\beta} > 1$, this implies that $W > r/(1-\beta)$, i.e., that (2) holds. ■

Two comments: First, ϕ does not matter because the $y_{-1} \in \{1, r^2\}$ types are excluded and the proportions of the two qualities always equal 1/2, and so does the participants' belief. Second, conditions (2)-(4) contain one equality and two inequality restrictions on the three unknowns (w_1, w_r, w_{r^2}). Therefore a continuum of solutions likely exists. Let us use the contract in (9) to plot W in relation to the payoffs agents get on the low match and the high match:

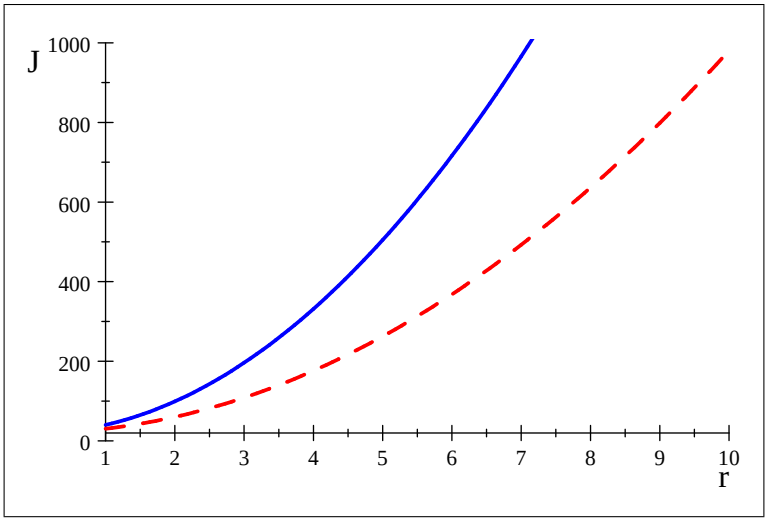


Figure 3: $J^{1/2,1/2}$ (solid) and $J^{0,1/2}$ (dashed)

Next, we plot $J^{1/2,1/2}$ (solid) and $J^{0,1/2}$ (dashed) as a function of r when $\beta = 0.95$:

The Contract solution.—We may sometimes refer to the separating contract mechanism for match formation as yielding the "Becker" solution because it achieves a positive sorting of types by their beliefs π.

First best

If a planner knew who worked with whom in the previous period and how much they produced, he could achieve the first best allocations in two periods as follows: Aggregate output is maximized by a positive assignment, and the firms with $y \in \{1, r^2\}$ are already composed optimally. Note that a set of 4 objects $\{A, B, C, D\}$ can be arranged in pairs in only 3 ways: (i) A,B and C,D, (ii) A,C and B,D, and (iii) A,D and B,C. Keeping the 4-worker set fixed, if one arrangement generated outputs (r, r) in the previous period, at most two more trials are needed to reach the allocation in Eq. (27).

Mergers could achieve first best if histories were public. Merging parties could then identify each other's output y_{-1} and get positive rents by merging.

3 Equilibrium if separating contracts fail

If contracts cannot separate types, there will be adverse selection in meetings. To simplify, we assume that the output of each match is divided equally. Little would change if rents were divided by randomly choosing one of the parties to be the proposer as Golosov, Lorenzoni and Tsyvinski (2013) do.[3] Based only on the history of the outputs y in the firms in which they participated, the workers must decide whether to dissolve the match or not. Matches can recombine costlessly and instantly at the beginning of a period. No one is ever left with no match, i.e., there is no unemployment. Agents recombine in order to find a better partner. Some match dissolutions are involuntary in that one partner wants to dissolve the match while the other wants to keep it.

Adverse selection in meetings.—While meetings continue, they comprise agents who have produced $y \in \{1, r\}$. A team with $y = r^2$ is composed of high types and withdraws from the process. A team that produced 1 contains members who know that they are low types, but they can hope to raise their earnings if they meet a high type, and so they separate voluntarily and enter the matching pool. The only non-trivial decision rests with members of the middle firms.

Beliefs among the middle-firms' team members.—No worker that finds himself in a middle firm could ever have worked for a high-quality firm because such firms do not dissolve. Thus, at the end of period 2 and beyond, after at least two production events, the relevant histories of the two members are of at most two kinds. Either one of the members was previously in $y = 1$ teams and the other always in $y = r$ teams, or both have always been in $y = r$ teams. Thus there are only two beginning-of-period

[3]Differences would arise if, as in Wilson (2001) a mediator makes alternating proposals; as proposal offers get very frequent the solution approaches the Nash bargain outcome with equal weights if the discount factors are the same, and this would allocate more rents to the agent with the higher π because his outside option is higher. We could not do directed search and queuing like Guerrieri and Shimer (2014) do because all our matches occur instantly.

beliefs among the team members in the $y = r$ firms (the latter becoming less and less likely as t grows): either exactly one member knows he is a low type, or neither knows his type.

Separations among the middle firms.—If a $y = r$ team has an agent who knows he is a low type, that agent wishes that the team would stay intact. But the team member who is uncertain of his type may wish to dissolve it. Such an agent has always been in $y = r$ teams.

Let T = the first date at which there is no recombination. For $t \in \{0, ..., T-1\}$ all middle firms are dissolving, and *all* the $1-\phi$ low types always recombine, as long as the probability of meeting a high type is positive. Let

n_{t} = # of high types in the meeting pool at the start of period t, and

λ_t = Fraction in the meeting pool that are of the high type r

Then,

$$\lambda_t = \frac{n_{\mathrm{t}}}{n_{\mathrm{t}} + 1 - \phi}, \tag{10}$$

with the initial condition

$$n_0 = \lambda_0 = \phi. \tag{11}$$

When a type r matches with another type r, they produce r^2 per worker, and withdraw from the rematching process. To a type r, withdrawal happens with probability λ_t. Therefore n_t satisfies the difference equation

$$n_{t+1} = (1 - \lambda_t)\, n_t, \tag{12}$$

with initial condition (11). As n_t declines, so does λ_t, making it less attractive.

Solving (10) for $n_t = \frac{\lambda_t(1-\phi)}{1-\lambda_t}$ and substituting into (12) yields the difference equation

$$\lambda_{t+1} = \frac{\lambda_t}{1 + \lambda_t}. \tag{13}$$

The solution for λ_t subject to the initial condition (11) is

$$\lambda_t = \frac{\phi}{1+t\phi}. \tag{14}$$

and substituting this into (10) and simplifying,

$$n_t = \frac{\phi(1-\phi)}{1+(t-1)\,\phi}, \tag{15}$$

and it is valid while meetings continue.

The belief sequence

Let π_t be the beginning of period-t belief of an agent that he is a high type.[4] The likelihood of producing $y = r$ conditional on being a high type is $1 - \lambda_t$, and the likelihood of producing $y = r$ conditional on being a low type is λ_t. Bayes rule then says

$$\pi_{t+1} = \frac{(1-\lambda_t)\,\pi_t}{\lambda_t\,(1-\pi_t) + (1-\lambda_t)\,\pi_t} \quad \text{with } \pi_0 = \phi. \tag{16}$$

Solving backwards for π taking the (λ_t) sequence as given, as Appendix 3 shows that $\pi_1 = 1/2$ and that for $t \geq 2$,

$$\pi_t = \left(1 + \prod_{i=1}^{t-1} \frac{\lambda_i}{1-\lambda_i}\right)^{-1}, \tag{17}$$

which is strictly increasing in t if $\phi \leq 1/2$. As t gets large, we shall show that $\lambda_t \to 0$, and therefore $\pi_t \to 1$.

Lemma 2

$$\pi_{t+1} \gtreqless \pi_t \iff \lambda_t \lesseqgtr \frac{1}{2}$$

Proof. Dividing both sides of (16) by π_t we find that

$$\frac{\pi_{t+1}}{\pi_t} = \frac{1}{\frac{\lambda_t}{1-\lambda_t}\,(1-\pi_t) + \pi_t} \gtreqless 1 \quad \text{as} \quad \lambda_t \lesseqgtr \frac{1}{2}.$$

■

An implication of the Lemma is that

[4] Doppelt (2016), who also has two types, refers to π_t as the worker's résumé.

1. If $\phi < 1/2$, λ_t is always below $1/2$, and therefore π_t is strictly increasing for all t. Conditional on each new match producing $y = r$, the searcher for a new match becomes more and more optimistic about his quality.

2. If $\phi > 1/2$, then an agent initially becomes more pessimistic if his match produces $y = r$. This is because if $\phi > 1/2$, an agent more likely to be matched with a high type. But if he then produces r and not r^2, his belief drops to $\pi = \frac{(1-\phi)\phi}{\phi(1-\phi)+(1-\phi)\phi} = \frac{1}{2}$.

Finitely many recombinations equilibrium

The expected value of recombining.—It depends on two variables: The agent's beginning-of-period belief, π, that he is a high type, and the probability of matching with another high type, λ. The latter is an aggregate state. Suppose there was a last period of search; then the expected output in the current period if the agent recombines is

$$\begin{aligned} v(\pi, \lambda) &= \pi\left(\lambda r^2 + (1-\lambda) r\right) + (1-\pi)(\lambda r + 1 - \lambda) \\ &= (1 + \lambda(r-1))(1 + \pi(r-1)). \end{aligned} \tag{18}$$

In the periods before the last, the expected value of recombining generally exceeds this because it yields a new signal on one's quality. But the last search at $T-1$ yields $v(\pi, \lambda)$ and must be at least as large as the value of staying. This defines an indifference curve in (π, λ) space.

The reservation belief.—The value staying with the match is r. Let $\hat{\pi}$ denote the belief in one's own quality that makes the agents in the firm $y = r$ indifferent between remaining together forever and searching exactly once more and then remaining together forever with whoever the new partner happens to be. Then $\hat{\pi}$ solves

$$r = v(\pi, \lambda)$$

and the solution is a decreasing function of λ denoted by

$$\hat{\pi}(\lambda) = \frac{1 - \lambda_t}{1 + \lambda_t (r-1)}. \tag{19}$$

Recombination continues only as long as $\pi_t \geq \hat{\pi}(\lambda_t)$. As λ converges to zero, this critical belief converges to one.

Maximal finite-duration recombination equilibrium.—We look for incentive compatibility for middle-quality firms to recombine as long as all other middle quality firms are doing so too. Search occurs at the beginning of each period and so the earliest that recombination can end is the beginning of period 1. Now let

$T \geq 0 =$ the last date at which a middle firm member with belief π_t will want to recombine. And since the initial period is date $t = 0$,

$$\text{duration of recombination} = T$$

Then T is the first date t for which the output of remaining together exceeds the expected output on a match with a random searcher so that the following inequality holds,

$$r > v(\pi_t, \lambda_t), \tag{20}$$

No discounting appears on the RHS of (20) because recombination results in a match immediately and production in the new match takes place within the period. Since $\lambda_0 = \pi_0 = \phi$, and since λ_t decreases with t, adverse selection prevents recombination from taking place if ϕ is small enough.

Infinitely many recombinations equilibrium

Here we go the space of sequences. Let V_t be the value of recombination at date t for someone who repeatedly has been in a middle firm.

$$\begin{aligned} V_t &= \pi_t \lambda_t \frac{r^2}{1-\beta} + (1-\pi_t)(1-\lambda_t)\left(1 + \sum_{s=t+1}^{\infty} \beta^{s-t}(\lambda_s r + 1 - \lambda_s)\right) + \\ &\quad + \pi_t (1-\lambda_t)(r + \beta V_{t+1}) + (1-\pi_t)\lambda_t (r + \beta V_{t+1}) \end{aligned}$$

because $1+\sum_{s=t+1}^{\infty}\beta^{s-t}\left(\lambda_s r+1-\lambda_s\right)$ is the value of searching forever for someone who knows he is a low type. This equilibrium exists if

$$V_t \geq \frac{r}{1-\beta} \quad \text{for all } t.$$

Appendix 2 also shows that the equilibrium number of firms of each type at date t is

$$\begin{aligned} N_{1,t}^{\mathrm{No}} &= \left(1-\phi+n_{\mathrm{t}}\right)\left(1-\lambda_t\right)^2 \rightarrow 1-\phi \\ N_{r,t}^{\mathrm{No}} &= \left(1-\phi+n_{\mathrm{t}}\right)\left\{1-\left(1-\lambda_t\right)^2-\lambda_t^2\right\} \rightarrow 0 \\ N_{r^2,t}^{\mathrm{No}} &= \sum_{j=0}^{t}\left(1-\phi+n_{\mathrm{j}}\right)\lambda_{\mathrm{j}}^2 \rightarrow \phi \end{aligned} \tag{21}$$

The convergence statements hold only for parameter values in the $T=\infty$ region in Figs 4 and 5.

The duration of recombination

We have focused on the adjustment process in the first place, rather than just comparing the long-run steady state. One argument for doing so is that the economy gets stuck in the adjustment for a long time. Under some conditions, adverse selection eliminates recombination altogether and $T=0$. Gains from recombination increase in the fraction of high types ϕ and in the productivity of high-quality matches, r^2. With the random allocation in period 0 described in Eq. (26) we now ask whether in period 1 there can be recombination. For there to be recombination we must have $T \geq 1$. Appendix 2 proves the following

Proposition 2 *There are three types of equilibrium in the no-contract case:*

(i) If $r\phi < 1$, *then* $T=0$, *i.e., no recombination*

(ii) If $r\phi \geq 1$ *and* $r < \phi+1$, *then* $T=1$, *i.e., one period of recombination.*

(iii) If $r\phi \geq 1$ *and* $r \geq \phi+1$, *then* $T^{\max}=\infty$, *i.e. recombination forever.*

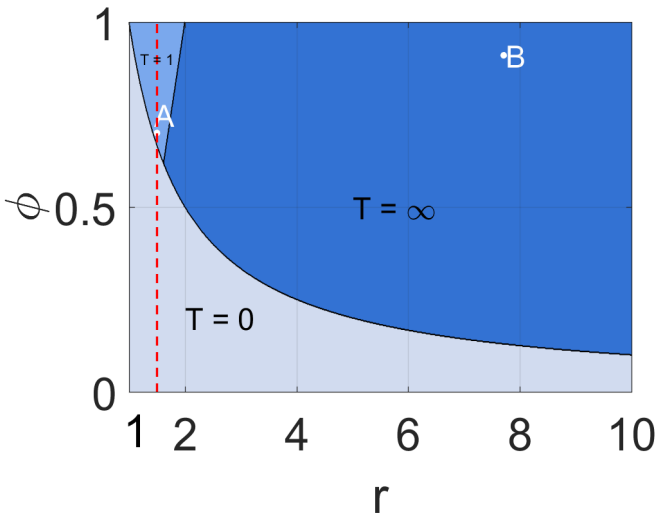


Figure 4: THE THREE REGIONS FOR T WHEN $r \in [1, 10]$.

Table 1 lists two values of the vector (r, ϕ) that will be labeled as points A and B in Figs. 4, 6 and 5.

	r	ϕ	T
case A	1.50	0.70	1
case B	7.71	0.91	∞

Table 1: PARAMETER VALUES USED IN THE PLOTS

Case A.—Case A entails exactly one recombination. Output growth ends after one period at $t = 1$ and, as Appendix 1 shows, for $t \geq 1$, firm types stabilize at the fractions

$$N_{1,1}^{\text{NC}} = \frac{1-\phi}{1+\phi}, \quad N_{r,1}^{\text{NC}} = \frac{2\phi\,(1-\phi)}{1+\phi}, \quad \text{and} \quad N_{r^2,1}^{\text{NC}} = \frac{2\phi^2}{1+\phi}. \tag{22}$$

Then for $t \geq 0$, Eq. (39) yields no change over time in aggregate output: $Y_t^{\text{NC}} = Y_1^{\text{NC}} = \frac{1-\phi}{1+\phi} + \frac{2\phi(1-\phi)}{1+\phi} r + \frac{2\phi^2}{1+\phi} r^2$. Point B was also chosen so that the model fits the earnings of people who originated in $y_{-1} = r$ firms and who want to start new firms each period (while their output $y = r$ keeps repeating) and are labeled as "quitters," relative to earnings of those who originated in $y_{-1} = 1$ firms and who are "laid off". The details are in Appendix 4.

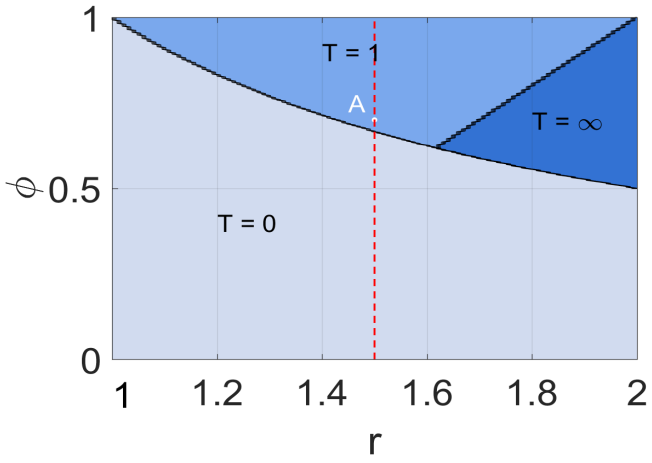


Figure 5: THE THREE REGIONS FOR T WHEN $r \in [1, 2]$.

The Proposition is first illustrated in Figs. 4 and 5. The downward sloping function is $\phi = 1/r$, and it divides the no-recombination region and that where recombination is positive. The upward-sloping line is $\phi = r - 1$, and it divides the $T = 1$ region and the $T = \infty$ region. Appendix 2 also proves

Proposition 3

$$T = \begin{cases} \infty \text{ if } \phi \geq \frac{1}{r+1}, \\ 0 \quad \text{otherwise} \end{cases}$$

Recombination indifference curves.—The two parameters that raise the returns to recombination are r and ϕ. Thus we expect to see negatively sloped indifference curves in (r, ϕ) space, perhaps reaching $T = \infty$. Since recombination starts at the beginning of the period, and since everyone is randomly matched in period 0, having $T = 0$ means that there is no recombination, and T is the number of periods in which some agents recombine.

The line that splits the regions $T = 0$ and $T = 1$ is the line $r\phi = 1$. Parts $(i) - (iii)$ of Proposition 2 imply that the three curves should intersect at the point $\left(\hat{r}, \hat{\phi}\right)$ satisfying the two equalities

$$r = \frac{1}{\phi} = 1 + \phi. \tag{23}$$

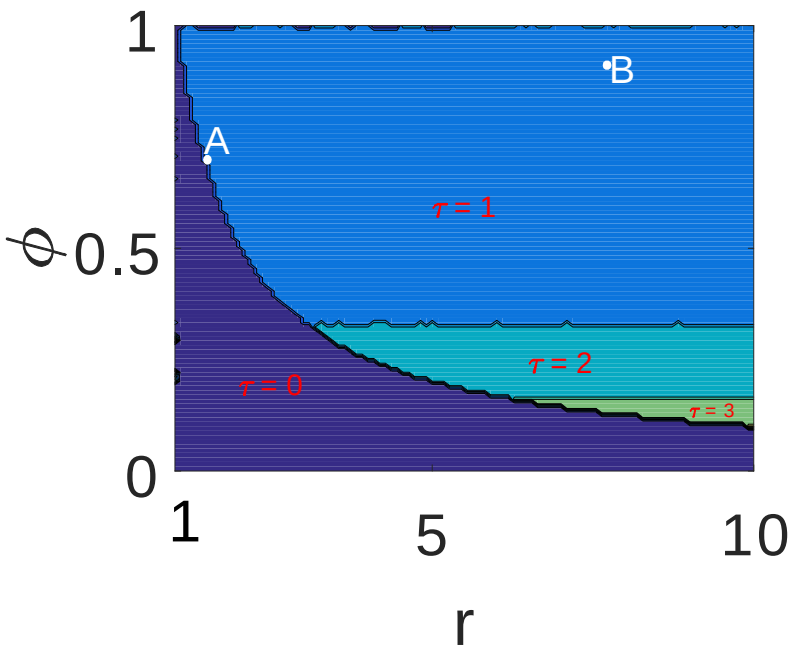


Figure 6: HALF LIVES FOR THE NO CONTRACTS CASE

The solution is $\left(\hat{r},\hat{\phi}\right)=(0.62,1.62)$, which is just below and to the right of point A.

Half lives τ.—To summarize speeds of convergence over the entire parameter space for the private information case we use half lives – the time that it takes to get half way to the limiting output. That is, τ solves (to the nearest integer) for t the equation

$$Y_t^{\mathrm{NC}}=\frac{1}{2}\left(Y_0+Y_\infty^{\mathrm{NC}}\right). \tag{24}$$

In Eq. (24) Y_t^{NC} is given by Eq. (39) with $i=\mathrm{NC}$, $Y_0=(1-\phi)^2+2\phi\,(1-\phi)\,r+\phi^2r^2$, and for Y_∞^{NC} we have

$$Y_\infty^{\mathrm{NC}}=\begin{cases} Y_0 & \text{for } (r,\phi)\in \text{Fig. 4 region } T=0 \\ Y_1^{\mathrm{NC}} & \text{for } (r,\phi)\in \text{region } T=1 \\ Y_\infty^{\mathrm{C}} & \text{for } (r,\phi)\in \text{region } T=\infty \end{cases}. \tag{25}$$

The results are shown in Fig. 6. The half life is largest and adjustment slowest in the South-West region of Fig. 6. The region $\tau=0$ is one where no adjustment at

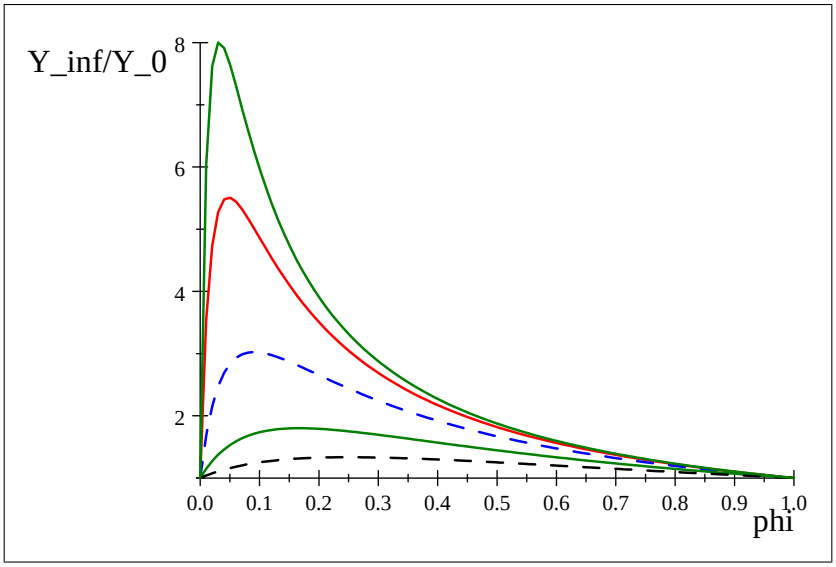


Figure 7: Y_∞/Y_0 when $r = 3, 5, 10,$ 20 and 30

all; the economy remains at its original position – adverse selection does not allow any adjustment. The half life for for $(r, \phi) \in$ region $T = 1$ will be $\tau = 1$. In the other regions the economy converges to Y_∞^{C}, as stated in Eq. (25), but apparently, adverse selection slows things down considerably even when r is high as long as when ϕ is low enough.

Paradoxically, a low ϕ is precisely when the growth of output due to sorting is the largest if r is also large, because

$$\frac{Y_\infty}{Y_0} = \frac{1 - \phi + \phi r^2}{(1-\phi)^2 + 2\phi(1-\phi)r + \phi^2 r^2} \to \frac{1}{\phi} \text{ as } r \to \infty.$$

This is illustrated in Fig. 7, which shows that a technological change that raised the productivity of the top ϕ fraction of agents would have relatively the largest effect when ϕ is small.

4 Polarization

In this section we gather the implications for the path of adjustment and discuss the model's relation to some evidence. We first state the initial and long-run distribution

of firm outputs per worker:

Period 0.—Technological change occurs in existing two-agent matches and the initial period's distribution of output reflects a purely random assignment of agents. No adjustment is possible until the period ends. The initial period distribution of output and earnings would be

$$y_0 = \begin{cases} r^2 & \text{for a fraction } \phi^2 \\ r & \text{for a fraction } 2\phi(1-\phi) \\ 1 & \text{for a fraction } (1-\phi)^2 \end{cases}. \tag{26}$$

Period ∞.—Assuming an eventual positive assignment (which we can guarantee only if contracts work), the distribution of output and wages distribution would then be

$$y_\infty = \begin{cases} r^2 & \text{for a fraction } \phi \\ 1 & \text{for a fraction } 1-\phi \end{cases}. \tag{27}$$

The contract solution converges geometrically.— For $i \in \{1, r, r^2\}$ let $N_{i,t}$ be the number of firms of each type. Each middle firm contains one low type and one high type. After a random pairing, each period, 1/4 of the newly formed matches produce 1, and 1/4 produce r^2. Workers from these firms stop searching, leaving the remaining 1/2 of the firms with agents who will rematch with some other $\pi = 1/2$ type again. The proportions of the worker types in the middle pool are equal to 1/2. The initial number of middle firms is $2\phi(1-\phi)$ and each period the number is halved. By date $t \geq 1$, the cumulative number added to each of the $y \in \{1, r^2\}$ groups is

$$\phi(1-\phi)\sum_{j=1}^{t}\frac{1}{2^j} = \phi(1-\phi)\left(1-\frac{1}{2^t}\right),$$

one half of the new matches still yield r and the other half yield $y \in \{1, r^2\}$. Therefore,

$$\begin{aligned} \mathrm{N}^C_{1,t} &= (1-\phi)\left(1-\frac{\phi}{2^t}\right) \quad \nearrow 1-\phi, \\ \mathrm{N}^C_{r,t} &= 2\phi(1-\phi)\frac{1}{2^t} \quad \searrow 0, \\ \mathrm{N}^C_{r^2,t} &= \phi\left(1-(1-\phi)\frac{1}{2^t}\right) \nearrow \phi. \end{aligned} \tag{28}$$

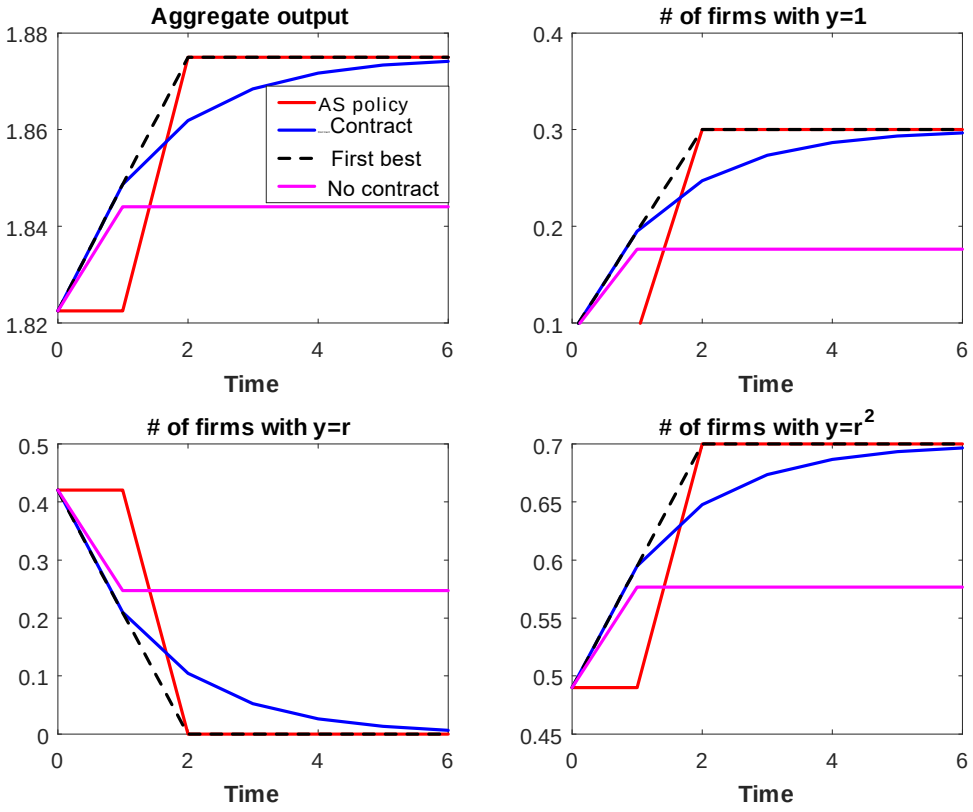


Figure 8: First-best (dashed), contract (blue) AS (red), and No-Contract (violet) when $r = 1.5$, $\phi = 0.7$

Thus convergence to the allocation in Eq. (27) is at a geometric rate, and their half life is just one period. Some derivations are in Appendix 5. If contracts could separate more than three types of history,

The no-contract solution converges arithmetically.—The solutions are in Eqs. (21), in which $n_t = \frac{\phi(1-\phi)}{1+(t-1)\phi}$ is given in (15) and and $\lambda_t = \frac{n_t}{n_t+1-\phi}$ is given in (10),.and both n_t and λ_t converge to zero arithmetically.

Half lives of the adjustment depend on whether contracts work or not. Fig. 8 shows that without separating contracts (the purple line),the no contract economy stops adjusting before half of the effect on aggregate output is reached by first best and by the contract economy. This was for the case where $\phi = 0.7$.

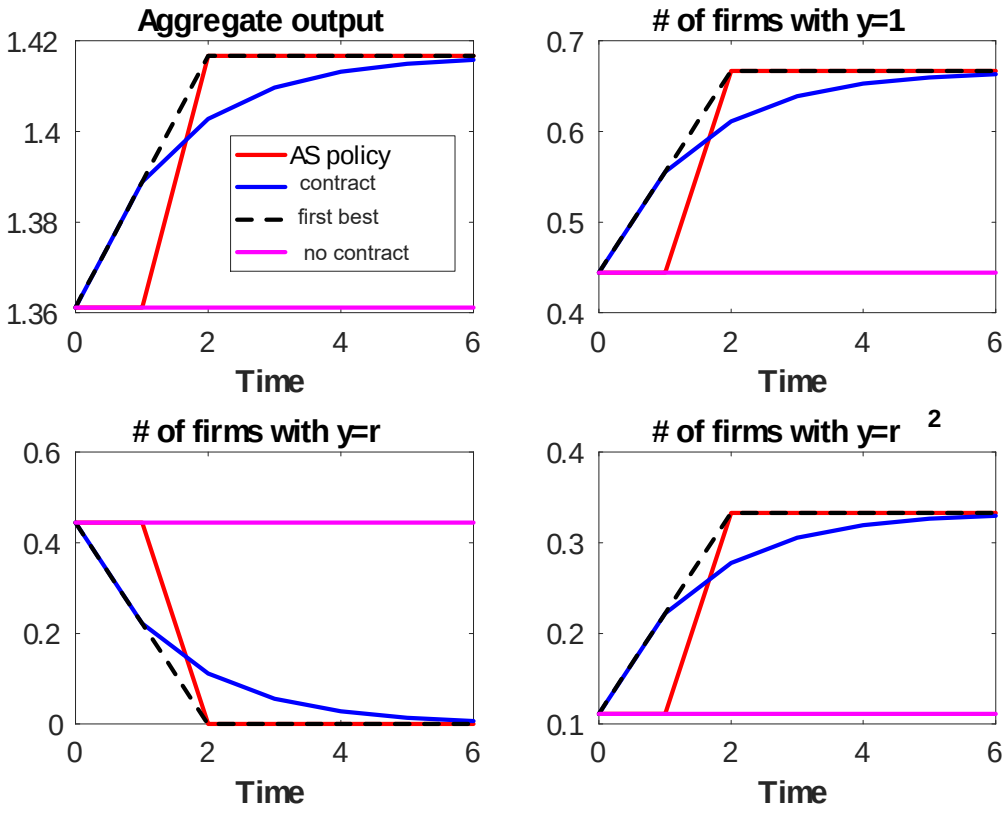


Figure 9: First-best (dashed), Contract (blue) AS (red), and No-Contract (violet) when $r = 1.5, \phi = 1/3$

When we reduce ϕ to 1/3, Fig. 9 shows that the no contract economy does not adjust at all.

Full adjustment of the no-contract economy does take place when $r = 4$ and $\phi = 1/3$, as Fig. 10 shows.

The model relates to several patterns in the data. Each match dissolution results in a new match being form which we can call a startup. So, dissolutions and startups should show the same pattern, and the model predicts both should decline – geometrically if contracts work (the blue lines in Figs. 8 - 10), arithmetically if they do not.(the violet lines in Figs. 8 - 10)

Decline in business dynamism.—Decker *et al.* (2020) and Chart 6 of BLS. (2016)

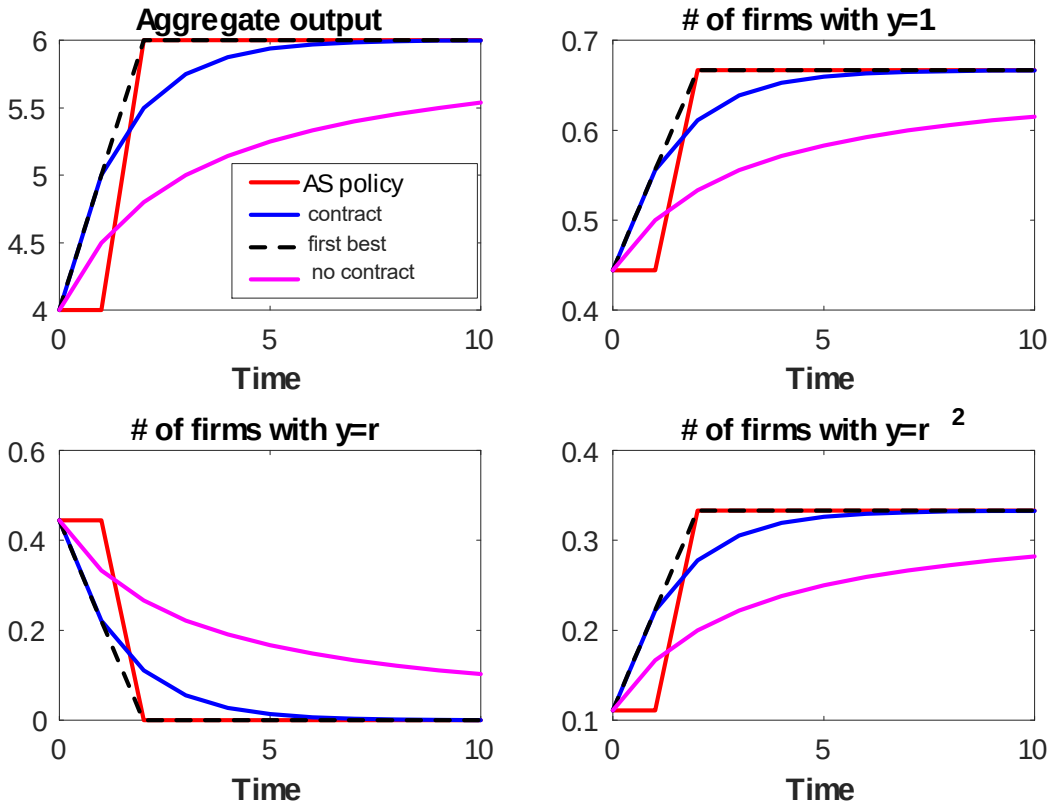


Figure 10: First-best (dashed), Contract (blue) AS (red), and No-Contract (violet) when $r = 4$, $\phi = 1/3$

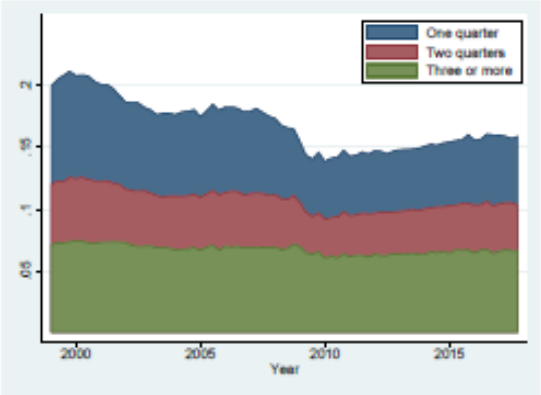


Figure 3: Decomposition of the separations rate s_t into the parts accounted for by employment spells with one quarter, two quarter, and three-plus quarters of tenure.

Figure 11: Separations by Figure 3 from Pries & Rogerson 2021 using Wyatt and Spletzer 2017 data

document a decline in firm births and deaths in the past 2 decades.

Decline in job separations.—The solution in (28) separations and startups implication is consistent with the third and fourth facts that Pries and Rogerson (2022, p. 265) report, namely that over the past two decades or so, namely, that "The decline in the incidence of the shortest employment spells – ones that last less than a quarter – can be broken down into two components: a declining hires rate and a declining hazard rate for first quarter spells. The two components each account for about half of the decline in the incidence of one quarter spells," and that "The hazard rates of employment spells of short tenure have declined by much more than the hazard rates of employment spells with longer tenure."

Figure 11 portrays some of these facts visually. In the model, when contracts work, the only separations that we observe are after one period. No separations occur at higher levels of job tenure, and so the similarity is only qualitative.

Growing right tail in the firm size distribution.—Technological change resulting in a shuffling of productivities reduces inequality and then inequality grows as the optimal matches are implemented. Polarization occurs in the sense that the number

of middle firms converges to zero, with the mass moving to the two tails of the productivity distribution. But the output share of the top x percent of firms and the income share of the top x percent is monotonically increasing only if $\phi < x$. When if $\phi > x$, the fraction of r^2 firms, $\mathrm{N}^C_{r^2 t}$ reaches x at a finite date t. After that date the share of the top x firms falls because their output is fixed, while that of the other agents keeps rising

Mergers.—If histories were public, mergers could achieve first best and there, indeed, was a merger wave at the turn of the 21st century, but brief. In fact, the information involved is not public.[5] Merger negotiations are often protracted and accountants and analysts are heavily involved in evaluating the qualities of the merging parties. Each deal is preceded by an information gathering stage. There is a stage where the target provides information and a deal gets worked out. Then after the deal is 'signed' the buyer continues to do due diligence and can then back away if the facts do not match what they were told before the deal was signed. In fact, in the data some deals that are marked "not completed," in which the deal fell apart in the information gathering stage. Nevertheless, Rhodes-Kropf and Robinson (2008) provide evidence that firms that are alike in terms of their Tobin's Q values are often involved in mergers, contrary to the implications in the models of Jovanovic and Rousseau (2002) and Jovanovic and Braguinsky (2004) where high-Q firms buy low-Q firms. One possible role for "like buys like" is to raise output by reshuffling the team members.

5 Discussion

Here we briefly discuss how the implications of the model may change if some of its key assumptions change, and some questions that come up.

[5] Kahn (2013) finds that incumbent wages track differences in ability distributions more closely than do outside firm wages. Doppelt (2016, note 17) summarizes the evidence on this topic.

More than two types?—With more types and no output noise, if contracts could separate the types the rate of convergence would be geometric but with a smaller base. Let us suppose that there are M agent types $(x_1, ..., x_{\mathrm{M}})$ with population proportions $(\phi_1, ..., \phi_{\mathrm{M}})$. Generically M! distinct match-quality pairs and M! distinct output levels $y_{i,j} = x_i x_j$ would then arise. A random reassignment would have probability $1/N$ of resulting in a match of two agents having the same type. The number of "incorrectly composed" matches, defined here as m_t would equal

$$m_t = \quad 1 - \sum_{i=1}^{M} \phi_i^2 \Bigg) \left(\frac{M-1}{M} \right)^t \tag{29}$$

and $\mathrm{N}_{r,t}^{C} = 2\phi\,(1-\phi)\,\frac{1}{2^t}$ in Eq. (28) being a special case that arises when $M = 2$. So we still would have geometric convergence to the positive assignment, but a slower rate.

This presumes, however, that the situation could be decentralized which, if there was no noise in output, would be impossible for large M. Output losses from experimenting with re-matching adjacent-y team types would be small whereas the informational gain would be proportional to the dispersion of x as in the two type case, and the AS deviation would become profitable. One

Fixed cost of posting a contract?—Such a cost would be similar to the cost of posting a vacancy. If it was large, it would overturn the result of Lemma 1's because with an AS deviation and no output noise, one can instantly learn one's type after one match with a known type. For the results to survive one would need noise in the output equation and more than two types so as to slow down the growth of belief-precisions.

What happens before date zero?—What happens before the polarization episode starts? Andolfatto and Smith (p. 742) write "Technological change is specified as an exogenous labor augmenting process that increases ...productivity... It also disrupts individual production possibilities. Old economic relationships are altered as the economy diverts resources towards adjusting to the new structure of technology. The

disruptive aspects of technological change are modelled here as a random 'shuffling' of abilities; that is, conditional on the arrival of a new technology, an individual draws a new ability parameter at random..."

In other words, the adjustment that the model aims to capture a staggered response to a positive technological shock such as the arrival of a general-purpose technology like the internet. Identifying date zero, however, may be challenging — Kremer and Maskin (1996) cite evidence that the correlation of wages among workers in the same business unit were rising since the mid-1980s.

6 Conclusion

This paper analyzed the role that adverse selection in the labor market may play in slowing down an economy's process of adjustment to a major technological shock that garbles agents' abilities so that agents must relearn their types. We found that this was more likely when there were other frictions preventing contracts and actions to merge that would attain the planner's optimum. The subsequent adjustment process is slowed down by adverse selection. The use of performance pay and of mergers speeds up the adjustment process and can lead to first best.

It is hoped that future empirical work will add this mechanism to the list of other explanations for the phenomena described in the previous section. In particular, the positive role of contingent pay and of mergers suggests that societies with better legal and financial institutions should respond better to the arrival of a general purpose technology or to a major political change such as the drop of the iron curtain.

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Appendix 1: Proof of Lemma 1

Simplifying (5) yields

$$J^{0,1/2} = \frac{1}{1-\beta} + r + \frac{1}{2}\frac{\beta}{1-\beta}\left(1+r^2\right),$$

and simplifying (6) yields

$$\begin{aligned} J^{1/2,1/2} &= \frac{1}{1-\beta/2}\left(\frac{1}{2}\left(\frac{1}{1-\beta}\right)+\frac{1}{2}\left(\frac{r^2}{1-\beta}\right)+r\right) \\ &= \frac{1}{1-\beta/2}\left(\frac{1}{2}\left(\frac{1+r^2}{1-\beta}\right)+r\right) \end{aligned}$$

Then

$$\begin{aligned} J^{1/2,1/2} &\geq J^{0,1/2} \qquad \Longleftrightarrow \\ \frac{1}{1-\beta/2}\left(\frac{1}{2}\left(\frac{1+r^2}{1-\beta}\right)+r\right) &\geq \frac{1}{1-\beta}+r+\frac{1}{2}\frac{\beta}{1-\beta}\left(1+r^2\right) \Longleftrightarrow \\ \frac{1}{1-\beta/2}\left(\frac{1}{2}\left(1+r^2\right)+(1-\beta)\,r\right) &\geq 1+(1-\beta)\,r+\frac{1}{2}\beta\left(1+r^2\right) \Longleftrightarrow \\ \frac{1}{2}\left(1+r^2\right)+(1-\beta)\,r &\geq 1-\frac{\beta}{2}+(1-\beta)\left(1-\frac{\beta}{2}\right)r+\frac{1}{2}\beta\left(1-\frac{\beta}{2}\right)\left(1+r^2\right) \Longleftrightarrow \\ 1+r^2 &\geq 2-\beta-(1-\beta)\,\beta r+\beta\left(1-\frac{\beta}{2}\right)\left(1+r^2\right) \Longleftrightarrow \\ r^2 &\geq 1-\beta-(1-\beta)\,\beta r+\beta\left(1-\frac{\beta}{2}\right)\left(1+r^2\right) \Longleftrightarrow \\ \left(1-\beta+\frac{\beta^2}{2}\right)r^2 &\geq 1-\beta-(1-\beta)\,\beta r+\beta\left(1-\frac{\beta}{2}\right) \end{aligned} \tag{30}$$

Now the LHS of (30) is linear in β whereas the RHS is convex in β. So if the inequality in Eq. (30) holds at $\beta = 0$ and at $\beta = 1$, it will hold everywhere. And that is indeed the case

$$\begin{aligned} \text{At } \beta &= 0 \text{ Eq. (30) reads} \qquad r^2 \geq 1 \\ \text{At } \beta &= 1 \text{ Eq. (30) reads } \frac{1}{2}r^2 \geq \frac{1}{2} \end{aligned}$$

Therefore $J^{1/2,1/2} \geq J^{0,1/2}$ for all $\beta \in [0,1]$ ■

Derivation of Eqs. (21), (22) and the dynamics portrayed in Figs. 8 - 10

—Until T, only firms that produce r^2 stay intact. At time $t < T$, the fraction continuing to search is $1-\lambda_t^2$ where λ_t is defined in (10). At t the total number of dissolutions is

$$S_t = 1-\phi+n_{\rm t} = \prod_{j=0}^{t-1}\left[1-\lambda_t^2\right],$$

where $n_0=\lambda_0=\phi$, and where (λ_t, n_t) evolve according to (10) and (12). Let $N_{i,t}^{\rm NC}$ be the number of firms of type $i \in \{1, r, r^2\}$ at the end of period t. Then, among $1-\phi+n_{\rm t}$ searchers, $(1-\lambda_t)^2$ get a (low, low) match, and produce 1, and λ_t^2 get a (high, high) match, and produce r^2, adding to the previous (high, high) match from time $1, 2.., t-1$. The rest produce a low-high match and produce r. Then the no-contract solution "NC" is

$$\begin{aligned} N_{1,t}^{\rm NC} &= (1-\phi+n_{\rm t})(1-\lambda_t)^2 \\ N_{r,t}^{\rm NC} &= (1-\phi+n_{\rm t})\left\{1-(1-\lambda_t)^2-\lambda_t^2\right\} \\ N_{r^2,t}^{\rm NC} &= \sum_{j=0}^{t}(1-\phi+n_{\rm j})\,\lambda_{\rm j}^2 \end{aligned}$$

This is Eq. (21).

Derivation of Eq. (22).—Using Eqs. (21), and (15)

$$\begin{aligned} N_{1,1}^{\rm NC} &= (1-\phi+n_1)(1-\lambda_1)^2 = (1-\phi+\phi(1-\phi))\left(1-\frac{\phi}{1+\phi}\right)^2 = \frac{1-\phi}{1+\phi}, \\ N_{r,1}^{\rm NC} &= (1-\phi+n_1)\left[1-(1-\lambda_1)^2-\lambda_1^2\right] = \frac{2\phi(1-\phi)}{1+\phi}, \\ N_{r^2,1}^{\rm NC} &= (1-\phi+n_0)\lambda_0^2+(1-\phi+n_1)\lambda_1^2 = \phi^2+\frac{(1-\phi)\phi^2}{1+\phi} = \frac{2\phi^2}{1+\phi}. \end{aligned}$$

Appendix 2: Proof of Propositions 2 and 3

Proof. of Proposition 2. (i) Since $\lambda_0=\phi$, the number of firms that produced r in period 0 is $2\phi(1-\phi)$. Half of the workers in these firms are of high quality. Then

$n_1 = \phi(1-\phi)$. Suppose search proceeds in period 0, then

$$\lambda_1 = \frac{\phi(1-\phi)}{\phi(1-\phi)+1-\phi} = \frac{\phi}{1+\phi} \quad \text{and} \quad \pi_1 = \frac{(1-\phi)\phi}{\phi(1-\phi)+(1-\phi)\phi} = \frac{1}{2},$$

Comparing the RHS of (20) at $t=1$ with r yields

$$\begin{aligned} v(\pi_1,\lambda_1) - r &= \frac{1}{2}\left(\frac{\phi}{1+\phi}r^2 + \frac{1}{1+\phi}r\right) + \frac{1}{2}\left(\frac{\phi}{1+\phi}r + \frac{1}{1+\phi}\right) - r \\ &= \frac{1}{2}\left(\frac{\phi}{1+\phi}r^2 + \frac{1}{1+\phi}\right) - \frac{r}{2} \\ &= \frac{\phi r^2 + 1 - r(1+\phi)}{2(1+\phi)} \\ &= \frac{(r-1)(\phi r - 1)}{2(1+\phi)} \gtrless 0 \quad \text{as} \quad r\phi \gtrless 1 \end{aligned}$$

Therefore, $T=1$ iff $r\phi < 1$.

(ii) From part (i), if $r\phi \geq 1$, then $v(\pi_1,\lambda_1) > r$ and search proceeds in period 0. Suppose search proceeds in period 1, then

$$\lambda_2 = \frac{\phi}{1+2\phi} \quad \text{and} \quad \pi_2 = \frac{(1-\lambda_1)\pi_1}{\lambda_1(1-\pi_1)+(1-\lambda_1)\pi_1} = \frac{1}{1+\phi}$$

because $\lambda_1 = \frac{\phi}{1+\phi}$ and $\pi_1 = \frac{1}{2}$.

Evaluating $v(\pi_t,\lambda_t)$ at $t=2$ gives

$$\begin{aligned} v(\pi_2,\lambda_2) &= \frac{1}{1+\phi}\left(\frac{\phi}{1+2\phi}r^2 + \frac{1+\phi}{1+2\phi}r\right) + \frac{\phi}{1+\phi}\left(\frac{\phi}{1+2\phi}r + \frac{1+\phi}{1+2\phi}\right) \\ &= \frac{r^2\phi + r\phi^2 + r\phi + r + \phi^2 + \phi}{2\phi^2 + 3\phi + 1}. \end{aligned}$$

Then

$$\begin{aligned} v(\pi_2,\lambda_2) - r &= \frac{r^2\phi + r\phi^2 + r\phi + r + \phi^2 + \phi - 2r\phi^2 - 3r\phi - r}{2\phi^2 + 3\phi + 1} \\ &= \frac{r^2\phi + \phi^2 + \phi - r\phi^2 - 2r\phi}{2\phi^2 + 3\phi + 1} \\ &= \frac{\phi(r-1)(r-\phi-1)}{(1+2\phi)(1+\phi)} \gtrless 0 \quad \text{as} \quad r \gtrless 1+\phi. \end{aligned}$$

Therefore, $T=1$ if $r < 1+\phi$ and $r\phi \geq 1$

(iii) Suppose $T > 1$, i.e. $r > 1 + \phi$ and $r\phi \geq 1$. Consider any $t \geq 2$, we have

$$\begin{aligned}\hat{\pi}(\lambda_t) &= \frac{1-\lambda_t}{1+\lambda_t(r-1)} \\ &= \frac{1+(t-1)\phi}{1+t\phi+\phi(r-1)} \\ &= 1-\frac{\phi r}{1+(t-1)\phi+\phi r}.\end{aligned}$$

On the other hand,

$$\begin{aligned}\pi_t &= \frac{(1-\lambda_{t-1})\pi_{t-1}}{\lambda_{t-1}(1-\pi_{t-1})+(1-\lambda_{t-1})\pi_{t-1}} = \frac{(1-\lambda_{t-1})}{\lambda_{t-1}/\pi_{t-1}+(1-2\lambda_{t-1})} \\ &\geq \frac{(1-\lambda_{t-1})}{\lambda_{t-1}/\pi_2+(1-2\lambda_{t-1})} \quad \text{since } \pi_t \text{ is increasing} \\ &= \frac{1+(t-2)\phi}{\phi(\phi+1)+(1+(t-3)\phi)} = 1-\frac{\phi^2}{\phi^2+1+(t-2)\phi}.\end{aligned}$$

Then

$$\begin{aligned}\pi_t-\hat{\pi}(\lambda_t) &\geq \frac{\phi r}{1+(t-1)\phi+\phi r}-\frac{\phi^2}{\phi^2+1+(t-2)\phi} \\ &\geq \frac{\phi(\phi+1)}{1+(t-1)\phi+\phi(\phi+1)}-\frac{\phi^2}{\phi^2+1+(t-2)\phi} \text{ since } r>\phi+1 \\ &= \frac{\phi(t\phi-2\phi-\phi^2+1)}{\left(1+t\phi+\phi^2\right)\left(1+(t-2)\phi+\phi^2\right)} > 0 \text{ as } t \geq 2.\end{aligned}$$

Thus, for any $t \geq 2$, we have $\pi_t > \hat{\pi}(\lambda_t)$, i.e. search will never stop, i.e. $T = \infty$, and that completes the proof of Proposition 2. ■

Proof. of Proposition 3. Suppose there was a last period of search, call it T. In this equilibrium only the incoming young are searching, and then at T in this equilibrium we would replace $v(\pi, \lambda)$ by $v(\pi, \phi)$ in Eq. (18). For search to end at T we then need $v(\pi, \phi) \leq r$, i.e.,

$$(1+\phi(r-1))(1+\pi_{\mathrm{T}}(r-1)) \leq r. \tag{31}$$

We first show that Eq. (31) cannot hold for $T \geq 3$, and that instead we have the following:

$$1+\pi_{\mathrm{T}}(r-1) > \frac{r}{1+\phi(r-1)} \Leftrightarrow \pi_{\mathrm{T}}(r-1) \geq \frac{(r-1)(1-\phi)}{1+\phi(r-1)} \Leftrightarrow \pi_{\mathrm{T}} \geq \frac{1-\phi}{1+\phi(r-1)}. \tag{32}$$

From Eqs. (14) and (17) we have, for $T \geq 3$

$$\pi_{\mathrm{T}} = \quad 1 + \prod_{i=2}^{T-1} \frac{\lambda_i}{1-\lambda_i} \Bigg)^{-1} \quad = \quad 1 + \prod_{i=2}^{T-1} \frac{\phi}{1+(i-2)\,\phi} \Bigg)^{-1}.$$

Substituting this value for π_{T} into (32) yields

$$\begin{aligned} 1 + \prod_{i=2}^{T-1} \frac{\phi}{1+(i-2)\,\phi} \quad &< \quad \frac{1+\phi\,(r-1)}{1-\phi} \prod_{i=2}^{T-1} \frac{\phi}{1+(i-2)\,\phi} < \frac{\phi r}{1-\phi} \Longleftrightarrow \\ \frac{1-\phi}{\phi} \prod_{i=2}^{T-1} \frac{\phi}{1+(i-2)\,\phi} \quad &< \quad r \end{aligned}$$

Since the term $\prod_{i=2}^{T-1} \frac{\phi}{1+(i-2)\phi}$ is decreasing in T, a sufficient condition for the above inequality to hold for all $T \geq 3$ is that the inequality be true for $T = 3$, i.e., that

$$\frac{1-\phi}{\phi} \frac{\phi}{1+(2-2)\,\phi} \leq r \Longleftrightarrow 1-\phi \leq r$$

which is always true. Thus Eq. (32) holds, so that Eq. (31) can not hold for $T \geq 3$. For $T = 2$, however,

$$\pi_2 = \frac{(1-\lambda_1)\,\pi_1}{\lambda_1\,(1-\pi_1) + (1-\lambda_1)\,\pi_1} = \frac{(1-\phi)\,\phi}{\phi\,(1-\phi) + (1-\phi)\,\phi} = \frac{1}{2}$$

and then (31) reads

$$(1+\phi\,(r-1)) \left(1 + \frac{1-\phi^2}{2-\phi}\,(r-1)\right) \leq r \tag{33}$$

and to contradict (31), we have

$$(1+\phi\,(r-1)) \left(1 + \frac{1}{2}\,(r-1)\right) \geq r \Leftrightarrow \frac{1}{2}\,(r-1)\,(\phi + r\phi - 1) \geq 0 \Leftrightarrow \phi \geq \frac{1}{r+1}.$$

That completes the proof of Proposition 3. ■

Appendix 3: Derivation of Eq. (17).

Solving backwards for π taking as given the λ_t sequence: Upon dividing top and bottom by π_t, (16) reads

$$\pi_{t+1} = \frac{1-\lambda_t}{\lambda_t \frac{1-\pi_t}{\pi_t} + 1 - \lambda_t}.$$

Now let $m_t = \frac{1-\pi_t}{\pi_t}$ so that $\pi_t = \frac{1}{1+m_t}$. Then

$$\begin{aligned}\frac{1}{1+m_{t+1}} &= \frac{1-\lambda_t}{\lambda_t m_t + 1 - \lambda_t}, \quad \text{i.e.,} \quad \lambda_t m_t + 1 - \lambda_t = (1-\lambda_t)(1+m_{t+1}) \\ &\Rightarrow \quad \lambda_t m_t = (1-\lambda_t) m_{t+1}.\end{aligned}$$

therefore

$$m_t = \frac{\lambda_{t-1}}{1-\lambda_{t-1}} m_{t-1} = \left(\prod_{i=0}^{t-1} \frac{\lambda_i}{1-\lambda_i} \right) m_0.$$

Finally,

$$\begin{aligned}\pi_t &= \frac{1}{1+m_t} = \left(1 + \prod_{i=0}^{t-1} \frac{\lambda_i}{1-\lambda_i} m_0 \right)^{-1} = \left(1 + \prod_{i=0}^{t-1} \frac{\lambda_i}{1-\lambda_i} \frac{1-\pi_0}{\pi_0} \right)^{-1} \\ &= \left(1 + \prod_{i=1}^{t-1} \frac{\lambda_i}{1-\lambda_i} \right)^{-1} \quad \text{for } t \geq 2\end{aligned}$$

The last equality is the same as (17), and it follows because $\pi_0 = \lambda_0 = \phi$. For $t = 1$,

$$\pi_1 = \left(1 + \frac{\lambda_0}{1-\lambda_0} \frac{1-\pi_0}{\pi_0} \right)^{-1} = \frac{1}{2}$$

Appendix 4. How the parameters for Case B were chosen

Laid-off workers experience larger wage losses than workers who quit or workers who lose jobs because their plant has closed. A match breaks up even if not all workers agree to it, and there is a sense in which a laid-off worker – defined to be one that would prefer to remain in his dissolving match – experiences larger wage losses than one who chooses to dissolve the match or whose firm has shut down, as Gibbons and Katz (1991) find. Using information from Sullivan and To (2014, Table 1), we have

$$\frac{\Delta \log w_{quit}}{\Delta \log w_{layoff}} = \frac{0.081}{-0.071}$$

Quits are "voluntary" and "layoffs" are involuntary. From the model, we have

$$\begin{aligned}w_{quit} &= \pi_t \left(\lambda_t r^2 + (1-\lambda_t) r \right) + (1-\pi_t)(\lambda_t r + 1 - \lambda_t) \\ w_{layoff} &= \lambda_t r + (1-\lambda_t).\end{aligned}$$

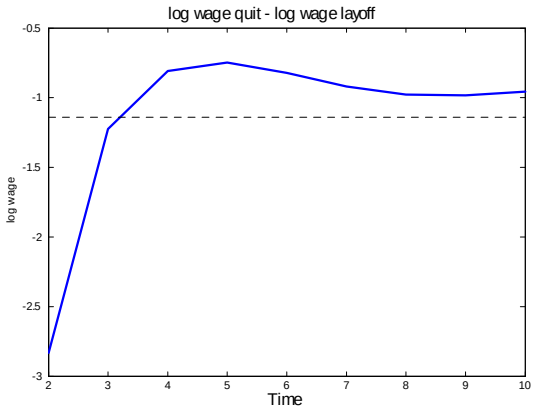


Figure 12: FITTING THE QUITS-TO-LAYOFFS WAGE RATIO

This implies that

$$\frac{\log\left(\pi_t\left(\lambda_t r^2+\left(1-\lambda_t\right)r\right)+\left(1-\pi_t\right)\left(\lambda_t r+1-\lambda_t\right)\right)-\log\left(r\right)}{\log\left(\lambda_t r+1-\lambda_t\right)-\log\left(r\right)}=\frac{0.081}{-0.071}.$$

Using the mean that the model produces during the first ten years we have the parameter values for case B yielding the pattern in Fig. 12. The dotted line is the target ratio $\frac{0.081}{-0.071}$.

Appendix 5: Time paths of firm and aggregate outputs

The first best "FB", Contract "C", and Anderson-Smith "AS" adjustments of aggregate output plotted in Figs 8 - 10 plot the time series of Y_t

a) first best and Contract have the same period-1 distribution of firms and therefore the same period 1 aggregate output. This is because both first best and Contract re-distribute middle firm members amongst each other at the start of period 1. But first-best then reaches the maximum in period 2 whereas Contract approaches it only gradually.

b) AS gets first best output in period 2 but no output gain in period 1. The Contract policy gives more output early on, but AS overtakes it in period 2. Both converge to the first best limit. Note that

Discounted output

Let us rank C *vs.* AS and FB by comparing for the discounted outputs as of date 0

$$\hat{Y}^i \equiv \sum_{t=0}^{\infty} \beta^t Y_t^i \qquad \text{for } i \in \{\text{FB, C, AS}\} \tag{34}$$

and discuss how β, ϕ and r affect the comparison.

We first calculate the PV of each group of firms separately, and in each case evaluate it as of $t = 0$: Since

$$\mathrm{N}_{1,t}^{\mathrm{C}} = (1-\phi)\left(1 - \frac{\phi}{2^t}\right),\ \mathrm{N}_{r,t}^{\mathrm{C}} = 2\phi(1-\phi)\frac{1}{2^t} \text{ and } \mathrm{N}_{r^2,t}^{\mathrm{C}} = \phi\left(1 - (1-\phi)\frac{1}{2^t}\right),$$

$$\sum_{t=0}^{\infty} \beta^t \mathrm{N}_{1,t}^{\mathrm{C}} = \frac{1-\phi}{1-\beta} - (1-\phi)\phi\sum_{t=0}^{\infty}\left(\frac{\beta}{2}\right)^t = (1-\phi)\left(\frac{1}{1-\beta} - \frac{\phi}{1-\beta/2}\right)$$

$$r\sum_{t=0}^{\infty} \beta^t \mathrm{N}_{r,t}^{\mathrm{C}} = \sum_{t=0}^{\infty} \beta^t 2\phi(1-\phi)\frac{r}{2^t} = \frac{2\phi(1-\phi)}{1-\beta/2}r$$

and

$$r^2\sum_{t=0}^{\infty} \beta^t \mathrm{N}_{r^2,t}^{\mathrm{C}} = r^2\left[\frac{\phi}{1-\beta} - \phi(1-\phi)\sum_{t=0}^{\infty}\left(\frac{\beta}{2}\right)^t\right] = \phi\left(\frac{1}{1-\beta} - \frac{1-\phi}{1-\beta/2}\right)r^2$$

and so the total is

$$\begin{aligned}\hat{Y}^{\mathrm{C}} = &\ (1-\phi)\left(\frac{1}{1-\beta} - \frac{\phi}{1-\beta/2}\right) + \frac{2\phi(1-\phi)}{1-\beta/2}r + \phi\left(\frac{1}{1-\beta} - \frac{1-\phi}{1-\beta/2}\right)r^2 \\ = &\ \frac{1}{1-\beta}\left(1-\phi+\phi r^2\right) - \frac{2(1-\phi)\phi}{2-\beta}(1-r)^2\end{aligned} \tag{35}$$

$\hat{Y}^{\mathrm{FB}} > \hat{Y}^{\mathrm{AS}}$ because FB generates the same information at the same speed as AS but without forcing a negative assignment. We'll compare the discounted profits as of period 0:

$$\begin{aligned}\hat{Y}^{\mathrm{FB}} = &\ (1-\phi)^2 + 2\phi(1-\phi)r + \phi^2 r^2 + \beta\left((1-\phi)^2 + \frac{\phi(1-\phi)}{2} + \phi(1-\phi)r + \left(\phi^2 + \frac{\phi(1-\phi)}{2}\right)r^2\right) \\ &+ \frac{\beta^2}{1-\beta}\left(1-\phi+\phi r^2\right) \\ \hat{Y}^{\mathrm{AS}} = &\ \text{is } (1+\beta)\left((1-\phi)^2 + 2\phi(1-\phi)r + \phi^2 r^2\right) + \frac{\beta^2}{1-\beta}\left(1-\phi+\phi r^2\right)\end{aligned}$$

Since they have the same outputs in period 0 and the second period and beyond, they can be ranked in terms of their period-1 outputs.

$$\begin{aligned}\hat{Y}^{\mathrm{FB}}-\hat{Y}^{\mathrm{AS}} = \quad & \beta\left(\left(1-\phi\right)^{2}+\frac{\phi\left(1-\phi\right)}{2}+\phi\left(1-\phi\right)r+\left(\phi^{2}+\frac{\phi\left(1-\phi\right)}{2}\right)r^{2}\right)\\ & -\beta\left(\left(1-\phi\right)^{2}+2\phi\left(1-\phi\right)r+\phi^{2}r^{2}\right)\\ = \quad & \frac{1}{2}\beta\phi\left(1-\phi\right)\left(r-1\right)^{2}>0\end{aligned}$$

Derivation of (28).—The Contract policy recombines **only** the middle firms. For $i\in\{1,r,r^{2}\}$ let $N_{i,t}$ be the number of firms of each type. Then, each period after a random pairing, 1/4 of the newly formed matches produce 1, and 1/4 produce r^{2}. Workers from these firms stop searching, leaving the remaining 1/2 of the firms with agents who will search again. Let N_{i}^{C} be the number of firms of type $i\in\{1,r,r^{2}\}$. The proportions of the worker types in this pool are equal. Here the table looks like this:

period	# firms 1	# firms r	# firms r^2
0	$(1-\phi)^2$	$2\phi(1-\phi)$	ϕ^2
1	$(1-\phi)^2+\frac{\phi(1-\phi)}{2}$	$\phi(1-\phi)$	$\phi^2+\frac{\phi(1-\phi)}{2}$
..	..	..	..
t	$(1-\phi)^2+\sum_{j=1}^{t}\frac{\phi(1-\phi)}{2^j}$	$2\left(\phi(1-\phi)-\sum_{j=1}^{t}\frac{\phi(1-\phi)}{2^j}\right)$	$\phi^2+\sum_{j=1}^{t}\frac{\phi(1-\phi)}{2^j}$

Now

$$\sum_{j=1}^{t}\left(\frac{1}{2}\right)^{j}=1-\frac{1}{2^{t}}$$

Then

$$\begin{aligned}\mathrm{N}_{1,t}^{\mathrm{C}} &= \left(1-\phi\right)\left(1-\phi+\phi\left(1-\frac{1}{2^{t}}\right)\right)=\left(1-\phi\right)\left(1-\frac{\phi}{2^{t}}\right)\\ \mathrm{N}_{r,t}^{\mathrm{C}} &= 2\phi\left(1-\phi\right)\left(1-1+\frac{1}{2^{t}}\right)=2\phi\left(1-\phi\right)\frac{1}{2^{t}}\\ \mathrm{N}_{r^{2},t}^{\mathrm{C}} &= \phi\left(\phi+\left(1-\phi\right)\left(1-\frac{1}{2^{t}}\right)\right)=\phi\left(1-\left(1-\phi\right)\frac{1}{2^{t}}\right)\end{aligned}$$

I.e., (28)

Next, the difference between **C** and **AS**. The undiscounted output difference between the contract economy labeled **C** and the **AS** policy is zero.—

$$\lim_{\beta \to 1} \left(\hat{Y}^{\mathbf{C}} - Y^{\mathbf{AS}} \right) = 2\phi \left(1-\phi\right) \left(r-1\right)^2 - 2\left(1-\phi\right)\phi \left(1-r\right)^2 = 0. \tag{36}$$

And since $Y^{\mathrm{C}} > Y^{\mathrm{AS}}$ early on, its present value must be higher when $\beta \in (0,1)$. See Eqs. (37) and (38). This is always true in all the cases that the figures portray. Aggregate discounted output is in Eq. (35).

$$Y^{\mathrm{C}} = \frac{1}{1-\beta}\left(1-\phi+\phi r^2\right) - \frac{2\left(1-\phi\right)\phi}{2-\beta}\left(1-r\right)^2$$

and

$$\begin{aligned} Y^{\mathrm{AS}} &= \left(1+\beta\right)\left[\left(1-\phi\right)^2 + 2\phi\left(1-\phi\right)r + \phi^2 r^2\right] + \frac{\beta^2}{1-\beta}\left(1-\phi+\phi r^2\right) \\ &= \frac{1}{1-\beta}\left(1-\phi+\phi r^2\right) - \left(1+\beta\right)\phi\left(1-\phi\right)\left(r-1\right)^2 \end{aligned}$$

So

$$Y^{\mathrm{C}} - Y^{\mathrm{AS}} = \left(1+\beta\right)\phi\left(1-\phi\right)\left(r-1\right)^2 - \frac{2\left(1-\phi\right)\phi}{2-\beta}\left(1-r\right)^2 \tag{37}$$

$\Rightarrow Y^{\mathrm{C}} - Y^{\mathrm{AS}} \geq 0$ if

$$\begin{aligned} 1+\beta &\geq \frac{2}{2-\beta} \Leftrightarrow \left(1+\beta\right)\left(2-\beta\right) \geq 2 \Leftrightarrow \\ 2-\beta+2\beta-\beta^2 &> 2 \Leftrightarrow \beta\left(1-\beta\right) \geq 0. \end{aligned} \tag{38}$$

We now compare the Separating contract-based – Sec. 2.2 and here labeled "C" – with equilibrium evolution– Sec. 4.1.3 and Figs. 4 and 5, and here labeled "NC.". Using the solution for the number of firm types in Eqs. (28) and (21), for $i \in \{\mathrm{C},\ \mathrm{NC}\}$. Aggregate output at date t is

$$Y_t^{\mathrm{i}} = N_{1,t}^{\mathrm{i}} + N_{r,t}^{\mathrm{i}} r + N_{r^2,t}^{\mathrm{i}} r^2 \tag{39}$$

Initial aggregate output is always $Y_0 = (1-\phi)^2 + 2\phi(1-\phi)r + \phi^2 r^2$, and for all parameter values $Y_t^{\mathrm{C}} \to Y_\infty^{\mathrm{C}} = 1 - \phi + \phi r^2$. But the parameter configurations A and B described in Table 1 produce different equilibrium evolutions. We have set $\phi = 1/3$ and $r = 1.5$