Fiscal Policy and the Monetary Transmission Mechanism*

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Abstract

The economy's response to monetary policy depends on its fiscal backing. We present a novel decomposition of the equilibrium that links the wealth effect, i.e. the revaluation of households' financial and human wealth, to the fiscal response to monetary policy. When monetary policy has fiscal consequences, monetary variables affect the timing of aggregate output while fiscal variables determine its present value and the wealth effect. Consequently, a contractionary monetary policy reduces inflation only if followed by contractionary fiscal policy. The slope of the Phillips curve determines the importance of monetary-fiscal coordination for the effectiveness of monetary policy.

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1 Introduction

The monetary transmission mechanism is often described as the effect that changes in a policy instrument, usually the stock of money or the short-term interest rate, have on aggregate variables such as inflation, output, consumption, and investment.[1] This description limits the scope of the monetary transmission mechanism to *monetary policy*, i.e. actions generally undertaken by a central bank. However, this characterization depicts an incomplete account of all the policy actions involved, as monetary policy usually has fiscal consequences: it affects the value of government debt, debt servicing costs, and primary surpluses (through changes in revenues and other automatic stabilizers). This paper revisits the monetary transmission mechanism with a focus on monetary and fiscal interactions. The analysis isolates the role that the different policy instruments play in determining the economy's equilibrium, with a focus on the wealth effect, i.e. the revaluation of households' financial and human wealth.

The fiscal response to monetary policy is almost entirely overlooked in textbook formulations of the monetary transmission mechanism.[2] This approach usually acknowledges the importance of an appropriate fiscal backing in supporting the equilibrium, but its role is relegated to an adjustment in the background. This is the so-called *Taylor equilibrium*, as it is characterized by an interest rate rule that satisfies the principles of Taylor (1993). Alternative formulations put fiscal policy at the forefront and emphasize the role of government debt and primary surpluses in determining the equilibrium. Much of this literature's focus has been on the determination of the price level, which is why it is generally known as the *Fiscal Theory of the Price Level* (FTPL).[3] In this paper, we present a unifying framework that identifies the channels through which the different policy instruments (monetary and fiscal) affect the main macroeconomic variables. The analysis highlights the role played by fiscal policy and uncovers its quantitative importance. Crucially, the approach is agnostic about the policy rules that gave rise to the *equilibrium paths* of the policy variables, and it is sufficiently general to accommodate any framework in which monetary policy has fiscal consequences. In particular, it nests the Taylor equilibrium and the FTPL as special cases.

We study the dynamic response of the economy to a monetary shock, which results in a deviation of the path of the nominal interest rate from its steady-state level and a simultaneous response of the fiscal authority. A novel finding emerges: when monetary policy has fiscal consequences,

[1] See, e.g., the definition in The New Palgrave Dictionary of Economics (Ireland, 2008).
[2] See Galí (2015). Woodford (2003) presents a limited analysis in Section 4 of Chapter 4.
[3] See Leeper and Leith (2016) for a review and Cochrane (2021) for a detailed analysis.

monetary variables affect the *timing* of aggregate output, while it is fiscal variables that determine its *present value*, a counterpart of the households' wealth. This result implies that contractionary monetary policy reduces inflation *only if* followed by contractionary fiscal policy.[4] Moreover, a stronger initial response of inflation to monetary policy requires a larger fiscal contraction. In particular, in the absence of a change in the fiscal policy stance, an increase in nominal interest rates has no effect on inflation on impact, and it actually raises inflation in future dates. Therefore, the ability of the central bank to control inflation in the New Keynesian model implicitly relies on the response of fiscal policy.

Underlying the analysis, there is a novel decomposition of the dynamic response of aggregate consumption into two terms that represent distinct economic forces. One term is uniquely determined by the equilibrium path of the nominal interest rate and captures the change in the timing of aggregate demand due to the monetary shock without affecting its present value. This term has an interpretation in terms of the substitution effect from microeconomic theory: it corresponds to the households' *Hicksian demand* extended to a general equilibrium setting. We call this term the *intertemporal substitution effect* (ISE).[5] The second term depends on the *wealth effect*. The wealth effect corresponds to the revaluation of the households' financial and human wealth net of the change in the cost of households' initial consumption bundle. Under this definition, a positive wealth effect implies that households can afford higher consumption paths than at the steady-state equilibrium (and vice-versa). Even though monetary shocks generate only transitory changes in income and households conform to the permanent income hypothesis in the RANK model, the general equilibrium dynamics of inflation can significantly amplify the impact of the wealth effect on initial output. We present a numerical exercise in which, for a standard calibration, the wealth effect is amplified by a factor of 30, and it explains more than half of the initial response of consumption to a monetary shock in the Taylor equilibrium.

A significant feature of the decomposition is that the ISE is uniquely determined by the equilibrium path of the nominal interest rate, while the wealth effect is indeterminate under an interest rate peg. Indeed, it is possible to index *all* the bounded equilibria of the New Keynesian model by the wealth revaluation they generate. Moreover, as long as monetary policy has fiscal conse-

[4]Contractionary fiscal policy can take the form of an increase in lump-sum taxes or a reduction in the value of government bonds that is not followed by a reduction in taxes (or a combination of both).

[5]King (1991) and Leeper and Yun (2006) provide an analogous decomposition to study the effects of government spending and tax changes in DSGE models. An important distinction is that, in this paper, the inflation rate used to compute the substitution effect is consistent with the New Keynesian Phillips curve evaluated at the Hicksian demand. This is the sense in which it corresponds to a general equilibrium extension of the standard substitution effect.

quences, we show that the wealth effect can be expressed in terms of fiscal variables according to a *Fiscal Keynesian Cross* in the spirit of the old-Keynesian analysis. The autonomous portion is comprised of government lump-sum transfers and a wealth effect generated by government bonds. The multiplier depends on the proportional tax rate and the inflation tax on nominal bond holdings. This characterization underscores the main result of the paper: in the New Keynesian model, the magnitude of the wealth effect depends on the fiscal response to monetary policy rather than on the change in the path of the nominal interest rate *per se*. This result *does not* require that fiscal policy is set independently of monetary policy. Even in a monetary-active regime (see Leeper, 1991), fiscal policy needs to adjust to guarantee that the government's budget constraint is satisfied in equilibrium. The result states that it is this adjustment that determines the wealth effect. In this sense, the Taylor equilibrium could be interpreted as acting through two separate channels: i) changing the path of the nominal interest rate, which affects the timing of output (i.e. the ISE); and ii) triggering a fiscal response that changes the present value of output (i.e. the wealth effect). Thus, combining the fiscal determination of the wealth effect and the general equilibrium amplification described before, we conclude that the fiscal response to monetary policy is not just an adjustment that happens in the background but a significant determinant of the monetary transmission mechanism.[6]

The importance of the wealth effect and the fiscal response associated with monetary policy becomes even more apparent when considering the dynamics of inflation. We find that the initial response of inflation is entirely determined by the wealth effect rather than by the contemporaneous response of consumption. Moreover, absent a wealth effect, the inflation dynamics after the initial period has a *Neo-Fisherian* property: it moves in the same direction as the change in the nominal interest rate. These results shed new light on the channels through which the central bank controls inflation in these models. A contractionary monetary shock reduces initial inflation, not because of a reduction in the *contemporaneous* level of consumption, but because households are overall *poorer* after the shock. Put differently, initial inflation decreases after a contractionary monetary shock if and only if there is a simultaneous *contractionary fiscal response*.

Naturally, the numerical results depend on the calibration. In a sensitivity analysis, we show that the degree of price stickiness is a crucial parameter determining the relevance of fiscal backing

[6] A noteworthy exception is when monetary policy does not have fiscal consequences. This is the only case that renders the households' budget constraint irrelevant: since output is demand determined, any level of the households' demand can be consistent with equilibrium. The monetary-active equilibrium selection solves this indeterminacy by making only one equilibrium to be bounded. However, this case is non-generic, in the sense that even a small fiscal effect leads to the fiscal characterization, and it is also at odds with reality.

in the monetary transmission mechanism. The general equilibrium amplification of the wealth effect relies on an inflation channel: a reduction in households' wealth reduces aggregate demand, which puts downward pressure on inflation and, for a given path of the nominal interest rate, increases the real rate, generating a second-round reduction in aggregate demand. Thus, this amplification mechanism *increases* with the degree of price flexibility. Since the wealth effect depends on the fiscal response to monetary policy, it follows that fiscal policy has a stronger effect in economies with a high degree of price flexibility. This implies that for low degrees of price flexibility, the Taylor equilibrium and the FTPL generate virtually identical aggregate dynamics for the same given path of the nominal interest rate. This finding can prove relevant to assess the effectiveness of monetary policy in economies with different institutional arrangements, as the degree of monetary-fiscal coordination appears to be more important in economies with a steeper Phillips curve. Moreover, even if fiscal policy might not be crucial for macroeconomic stabilization in an economy with relatively rigid prices, an uncoordinated policy may eventually trigger a regime change.[7]

As a final exercise, we show that all the intuitions built in the simple RANK model extend to richer settings: a liquidity trap scenario,a two-agent New Keynesian (TANK) model, and an analytical RANK model with capital. On top of highlighting the importance of fiscal policy in the monetary transmission mechanism, each model provides some setting-specific new insights.

Literature. There is a long tradition that studies the role of monetary and fiscal policies as macroeconomic stabilizers (see Keynes, 1936; Friedman, 1948). Perhaps one of the most famous quotes related to the origins of inflation is Friedman's "Inflation is always and everywhere a monetary phenomenon," (Friedman, 1963). This view is reflected in much of the modern analysis of the monetary transmission mechanism. However, careful inspection of the government's budget constraint highlights the tight connection between monetary and fiscal policy (see Sargent and Wallace, 1981, for an early formalization). We contribute to this literature by providing a novel characterization of the role that monetary and fiscal policy play in the monetary transmission mechanism.

The paper shares several features emphasized by the Fiscal Theory of the Price Level (FTPL) (see Leeper, 1991; Sims, 1994; Woodford, 1995, 2001, for early developments).[8] We make four im-

[7] Alvarez et al. (2019) estimate the firms' price-setting behavior in Argentina for different inflation rates. They find that the frequency of price changes is relatively constant for low inflation levels but increases for higher rates, suggesting that the degree of price flexibility depends on the policy regime.

[8] A related literature emphasizes the importance of fiscal policy to understand several historical events such as the

portant contributions. First, we formalize the interpretation of the monetary transmission mechanism in terms of substitution and wealth effects and show that the wealth effect is linked to fiscal policy. The connection between the wealth effect and fiscal policy is a recurrent narrative in this literature, but, to the best of our knowledge, the formalization was missing.[9] Second, the paper expands on a recent approach that characterizes equilibria in terms of equilibrium paths for policy variables rather than on policy rules (see Werning, 2012; Cochrane, 2017, 2018a). A significant difference with these papers is that we explicitly consider the joint determination of monetary and fiscal variables. We characterize the restrictions that the choice of *policy rules* impose on the joint dynamics of policy variables and equilibrium allocations, clarifying the extent of observational equivalence emphasized by previous work.[10] Third, we identify the importance of the slope of the Phillips curve in the results, noting that monetary-fiscal coordination is more relevant in economies with relatively flexible prices. Finally, we extend the analysis to three settings of independent interest: a liquidity trap scenario, a TANK model, and a model with endogenous capital accumulation.

The HANK literature has also recognized the importance of fiscal policy in heterogenous agents models in which Ricardian equivalence does not hold (see Kaplan et al., 2018). Our paper makes two contributions. First, it emphasizes that fiscal policy matters even when Ricardian equivalence holds. Second, it shows the extent to which fiscal redistribution can affect the channels of transmission, uncovering the importance of a dynamic component.

Finally, Caramp and Silva (2020) extend the decomposition in this paper to a setting with aggregate risk and private debt. They show how to use fiscal data to discipline the magnitude of the wealth effect and find that the fiscal backing implied by the standard Taylor equilibrium is significantly larger than the one obtained in the data.

The rest of the paper is organized as follows. Section 2 describes the model and presents the equilibrium decomposition. Section 3 shows that the wealth effect can be expressed in terms of the fiscal response to monetary policy. Section 4 extends the analysis to a liquidity trap scenario and a TANK model. Finally, Section 5 concludes. All proofs are presented in the Appendix A.

recovery from the Great Depression (Jacobson et al., 2019), the run-up and end of the Great Inflation (Bianchi and Ilut, 2017) and the missing inflation during the Great Recession (Bianchi and Melosi, 2017).

[9] Kaplan et al. (2018) present an alternative decomposition in terms of the *direct* and *indirect* effects of monetary policy. We compare the two approaches in Appendix C.3.

[10] This is related to the literature that tries to identify the policy regime in the data. See, e.g., Canzoneri et al. (2001).

2 The Model

We study a standard RANK model in discrete time augmented to incorporate fiscal variables and explicitly account for the households' budget constraint. We present a characterization of the equilibrium paths of consumption and inflation that identify two major economic forces: an intertemporal substitution effect and a wealth effect.

We study the dynamic response of an economy hit by a monetary shock, resulting in a deviation of the path of the nominal interest rate from its steady-state level and a simultaneous response of the fiscal authority. We analyze the reaction of the economy to the resulting *equilibrium paths* of the monetary and fiscal variables. By focusing on the equilibrium paths of policy variables, we obtain results that are robust to any monetary-fiscal regime that generated those paths. In particular, we show that the Taylor equilibrium and the FTPL are special cases of the general approach.

2.1 Environment

Time is discrete and denoted by $t \in \{0, 1, 2, \dots\}$. The economy is populated by a large number of identical, infinitely-lived households and a government. There is also a continuum of firms that produce final and intermediate goods. Final-goods producers operate competitively and combine intermediate goods using a CES aggregator with elasticity $\epsilon > 1$. Intermediate-goods producers use labor as the only factor of production to produce a differentiated good that is traded in a monopolistically competitive market. As is standard, we assume that intermediate-goods firms face a pricing friction à la Calvo, so that only a fraction $1 - \theta$ of firms can set a new price each period. Finally, the government chooses monetary policy, which entails a path for the nominal interest rate, and fiscal policy, comprised of proportional sales taxes, nominal debt, and lump-sum transfers.[11] We assume that government debt consists of perpetuities that pay coupons that decay exponentially at a rate $\rho \in [0, \beta^{-1})$. The case with $\rho = 0$ corresponds to one-period bonds, while $\rho = 1$ corresponds to consols. More generally, the duration of these bonds in a steady-state equilibrium is given by $\frac{1}{1-\beta\rho}$. This assumption allows us to study the effects of long-term debt with a minimal departure from the standard model (see Woodford, 2001). As is standard in the literature, we log-linearize the model around its zero inflation steady-state equilibrium and consider the first-order approximation of the response of the economy to exogenous shocks.[12]

[11] Given the focus on monetary policy shocks, we follow the literature and abstract from government spending.
[12] For the detailed derivation of the model, see Appendix B.

Given a path of interest rates $\{i_t\}_{t=0}^{\infty}$ and transfers $\{T_t\}_{t=0}^{\infty}$, the log-linearized solution to the model can be characterized by four equations: the households' intertemporal Euler equation

$$c_t = c_{t+1} - \sigma^{-1}(i_t - \pi_{t+1} - r_n), \tag{1}$$

the New Keynesian Phillips curve

$$\pi_t = \beta\pi_{t+1} + \kappa c_t, \tag{2}$$

the households' intertemporal budget constraint

$$\sum_{t=0}^{\infty} \beta^t c_t = \sum_{t=0}^{\infty} \beta^t \left[(1-\tau)\, y_t + (i_t - \pi_{t+1} - r_n)\, Qb + T_t\right] - \left[\sum_{t=0}^{\infty} (\beta\rho)^t \,(i_t - r_n)\,\rho + \frac{1}{\beta}\pi_0\right] Qb, \tag{3}$$

and the resource constraint

$$c_t = y_t, \tag{4}$$

where c_t and y_t denote, respectively, the percentage difference between actual consumption and output and their corresponding levels in the steady state; π_t denotes the inflation rate; i_t denotes the short-term, risk-free nominal interest rate; σ denotes the inverse of the intertemporal elasticity of substitution; β denotes the households' subjective discount factor, and $r_n \equiv -\log\beta$; κ is the slope of the Phillips curve; τ is the steady-state rate of the proportional sales tax; Qb denotes the steady-state *value* of government debt as a fraction of output, where Q is the steady-state price of a unit of the nominal bond; and T_t denotes the lump-sum taxes as a fraction of output.

Since the analysis emphasizes the role of the households' budget constraint in the dynamic behavior of consumption, it is helpful to briefly describe its components. The left-hand side of equation (3) is the present value of consumption, discounted at the steady-state real interest rate. The right-hand side is the present value of the households' incomes and the revaluation of initial assets. This includes changes in the after-tax profits and wages, which combined equal $(1-\tau)y_t$, the interest income from government bond holdings, lump-sum transfers, and the revaluation of initial bond holdings. The right-hand side can be interpreted as the *wealth effect* associated with the policy change, that is, the revaluation of real and financial assets net of the change in the cost of the steady-state consumption bundle. This definition takes into account that monetary policy affects households directly through changes in wealth and indirectly through changes in the cost of future consumption. Therefore, the wealth effect is positive if and only if the household can

afford to consume more than the steady-state consumption at every period.

Note that there are three channels through which fiscal variables affect the households' budget constraint. First, they affect non-interest income through τ and T_t.[13] Second, the level of government debt determines the households' exposure to changes in the real interest rate. While changes in the real interest rate affect the present discounted value of both consumption and after-tax income, in a closed economy the net impact depends only on the steady-state level of government debt.[14] Finally, the change in the path of the nominal interest rate, $\{i_t\}_{t=0}^{\infty}$, and initial inflation, π_0, affect the real return of initial nominal bond holdings. On the one hand, the change in nominal interest rates generates a revaluation of long-term bonds, given by $-\sum_{t=0}^{\infty}(\beta\rho)^t(i_t - r_n)\rho Qb$. This effect is absent when bonds are one-period, i.e. when $\rho = 0$. On the other hand, initial inflation affects the realized return on initial nominal bond holdings, summarized by $-\frac{1}{\beta}\pi_0 Qb$.

Policy rules. The exercise focuses on the paths of policy variables and studies the channels through which these paths affect equilibrium dynamics. This exercise differs from the standard approach, which typically assumes monetary and fiscal rules and then determines the equilibrium path of policy variables endogenously. While the choice in this paper might seem more restrictive, it is, in fact, more general and can accommodate *any* monetary-fiscal interactions that generate a particular path for monetary and fiscal variables.

A popular approach is to assume that monetary policy follows an interest rate rule of the form

$$i_t = r_n + \phi_\pi \pi_t + \phi_y y_t + \varepsilon_t, \tag{5}$$

where $\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$ and ε_t represents an innovation of the rule relative to its systematic response to inflation and output. Fiscal policy is assumed to be *passive* or *Ricardian*, and the exogenous monetary shock is represented by a path for $\{\varepsilon_t\}_{t=0}^{\infty}$ rather than a path for the nominal interest rate.[15] Under these assumptions, equation (3) is often dropped when finding an equilibrium of the economy because transfers $\{T_t\}_{t=0}^{\infty}$ are assumed to automatically adjust so that

[13] To keep the analysis as close as possible to the standard approach, we have assumed that the proportional tax τ is fixed at its steady-state level, and only lump-sum transfers adjust to a monetary shock.

[14] Formally, the impact of changes in the interest rate on the present discounted value of consumption is $-\frac{\beta}{1-\beta}c\sum_{t=0}^{\infty}\beta^t(i_t - \pi_t - r_n)$, and the corresponding impact on after-tax income is $-\frac{\beta}{1-\beta}[(1-\tau)y + T]\sum_0^{\infty}\beta^t(i_t - \pi_{t+1} - r_n)$, where T denotes the steady-state level of transfers. Combining the two and using that $c = y = \frac{1-\beta}{\beta}Qb + (1-\tau)y + T$, we obtain $\sum_0^{\infty}\beta^t(i_t - \pi_{t+1} - r_n)Qb$.

[15] Note that Ricardian equivalence holds in the model regardless of the monetary-fiscal regime, so only the present value of transfers, $\sum_{t=0}^{\infty}\beta^t T_t$, rather than the whole path, $\{T_t\}_{t=0}^{\infty}$, matters for the equilibrium.

the government's budget constraint is satisfied for any path of the endogenous and exogenous variables.[16] Since lump-sum transfers do not affect any of the other equations characterizing the equilibrium, they represent a free variable that adjusts to guarantee that any solution to the system given by (1), (2), (4) and (5) is an equilibrium of the economy. We call this case the *Taylor equilibrium*.

An alternative approach follows the *Fiscal Theory of the Price Level* (FTPL), which in its simplest specification assumes an exogenous path for the primary surplus, given by $s_t \equiv \tau y_t - T_t$, and an interest rate rule (5) with $\kappa(\phi_\pi - 1) + (1-\beta)\phi_y < 0$ and $\phi_\pi, \phi_y \geq 0$. Then, an equilibrium of the economy is a solution to the system (1)-(5) given $\{s_t\}_{t=0}^{\infty}$.

Despite the stark differences between the two approaches, our formulation is consistent with both. The determination of the paths of policy variables, $\{i_t\}_{t=0}^{\infty}$ and $\{T_t\}_{t=0}^{\infty}$ (which implicitly determine $\{s_t\}_{t=0}^{\infty}$), depends on the specific monetary-fiscal regime in place. However, by analyzing the impact of the policy variables *directly* on consumption and inflation, we are able to bypass the debate on the monetary-fiscal policy regime and obtain results about the monetary transmission mechanism that are robust to any regime. Given equilibrium paths, we can always find rules that lead to these paths, although only certain paths will be consistent with specific rules.

The generality of this approach is particularly useful to assess the role of fiscal variables in a monetary-active regime. Because a monetary shock generates simultaneous movements in $\{i_t\}_{t=0}^{\infty}$ and $\{T_t\}_{t=0}^{\infty}$, it is unclear how much of the overall effect depends on the response of nominal interest rates and fiscal variables. By focusing on how output and inflation react to policy variables directly, we are able to isolate the role of fiscal variables for a given path of nominal interest rates. Proposition 2 and the analysis in Section 3 below show how to interpret the standard Taylor equilibrium as a particular solution of this general approach.

2.2 Special Case: Rigid Prices and no Government Debt

Suppose prices are fully rigid and there is no government debt, that is $\kappa = b = 0$. Naturally, this case does not allow us to study the determination of the price level and inflation. However, this benchmark is useful for isolating the main channels of transmission on consumption in a model with the minimum set of necessary ingredients.

[16] See Woodford (2003) for a discussion.

Consider the Euler equation (1). Iterating it backward, we get

$$c_t = c_0 + \sigma^{-1} \sum_{s=0}^{t-1} (i_s - r_n). \tag{6}$$

Multiplying both sides by β^t and summing across time, we get

$$\sum_{t=0}^{\infty} \beta^t c_t = \frac{1}{1-\beta} c_0 + \sigma^{-1} \frac{\beta}{1-\beta} \sum_{s=0}^{\infty} \beta^s (i_s - r_n). \tag{7}$$

Note that the left-hand side of this expression is the left-hand side of the households' intertemporal budget constraint (3). Let

$$\Omega_0 \equiv (1-\beta) \sum_{t=0}^{\infty} \beta^t \left[(1-\tau) y_t + T_t \right] = (1-\beta) \sum_{t=0}^{\infty} \beta^t c_t.$$

We call Ω_0 the *wealth effect*. Replacing into (7), we get

$$c_0 = \Omega_0 - \sigma^{-1} \beta \sum_{s=0}^{\infty} \beta^s (i_s - r_n).$$

And introducing this expression into (6), we get

$$c_t = \sigma^{-1} \sum_{s=0}^{t-1} (i_s - r_n) - \beta \sigma^{-1} \sum_{s=0}^{\infty} \beta^s (i_s - r_n) + \Omega_0. \tag{8}$$

Given a path for the nominal interest rate, $\{i_t\}_{t=0}^{\infty}$, *all* solutions of the New Keynesian model with rigid prices can be written as the sum of two terms. The first term is *uniquely* pinned down by the path of the nominal rate, and its present value discounted by β^t is equal to zero. We will associate this term with an *intertemporal substitution effect*, as it changes the *timing* of consumption but not its present value. The second term is the *wealth effect* associated with the policy change. This term is not uniquely pinned down by the path of the nominal rate, so one can index the solutions of the model by the change in wealth they generate.

Furthermore, a simple manipulation of the households' intertemporal budget constraint allows us to express Ω_0 in terms of policy variables. Noting that $\frac{\Omega_0}{1-\beta} = \sum_{t=0}^{\infty} \beta^t c_t = \sum_{t=0}^{\infty} \beta^t y_t$, we can write the households' intertemporal budget constraint as follows:

$$\Omega_0 = (1-\tau)\, \Omega_0 + (1-\beta) \sum_{t=0}^{\infty} \beta^t T_t. \tag{9}$$

If $\tau > 0$, we get

$$\Omega_0 = \frac{1-\beta}{\tau} \sum_{t=0}^{\infty} \beta^t T_t. \tag{10}$$

This expression presents the main insight of the paper and is the building block of the analysis that follows. It establishes that we can express the change in the households' wealth (i.e. the present value of the households' human and financial wealth) as a function of *fiscal variables only*: the lump-sum transfers and the sales tax. Therefore, a contractionary monetary policy leads to a drop in *intertemporal aggregate demand*, measured by the present value of consumption, only if followed by contractionary fiscal policy. In the absence of a fiscal response, so $\sum_{t=0}^{\infty} \beta^t T_t = 0$, monetary policy can at best shift demand over time. This implies that fiscal policy plays a role in the monetary transmission mechanism even when Ricardian equivalence holds.

Equation (9) can be interpreted as a *Fiscal Keynesian Cross*, in the spirit of the old-Keynesian logic found in many introductory textbooks. Suppose that $\tau > 0$ and the present value of government transfers increases by $\frac{1}{1-\beta}$. From the households' intertemporal budget constraint, this represents an increase in the households' permanent income of 1, so households increase their demand by 1 every period. This increased demand translates into an increase in the households' income. However, for a given path of transfers, the households do not receive the entire change in income but only a fraction $1-\tau$, as τ is taxed away by the government. The households spend this additional income and a second round of spending implies an increase in income of $(1-\tau)^2$. Continuing this process, it is immediate that the change in the households' wealth is: $1 + (1-\tau) + (1-\tau)^2 + \cdots = \frac{1}{\tau}$. Moreover, since the households' budget constraint is satisfied, the government's budget constraint is also satisfied. The government spends $\frac{1}{1-\beta}$ in present value and obtains $\tau + \tau(1-\tau) + \tau(1-\tau)^2 + \cdots = 1$ every period, hence $\frac{1}{1-\beta}$. A simple manipulation leads to equation (10).

Finally, we show how to connect our approach, which looks at equation (10), with the standard Taylor equilibrium selection. Consider the interest rate rule (5) with $\phi_y > 0$ and, for simplicity, assume that there exists $T > 0$ such that $\varepsilon_t = 0$ for all $t \geq T$ (we study the more general case below). Then, the Taylor equilibrium implies that $c_t = 0 \; \forall t \geq T$. For $t < T$, we have

$$c_t = -\sigma^{-1} \sum_{s=t}^{T-1} (i_s - r_n).$$

Combining with equation (8), we can back out the wealth effect in the Taylor equilibrium:

$$\Omega_0 = -\sigma^{-1} \sum_{s=0}^{T-1} \left(1 - \beta^{s+1}\right) (i_s - r_n) .$$

And using equation (10), we can obtain the fiscal backing necessary to sustain this equilibrium:

$$\sum_{t=0}^{\infty} \beta^t T_t = -\frac{\sigma^{-1}\tau}{1-\beta} \sum_{s=0}^{T-1} \left(1 - \beta^{s+1}\right) (i_s - r_n) . \tag{11}$$

Thus, one can recover the Taylor equilibrium by assuming *equilibrium* paths $\{i_t, T_t\}_{t=0}^{\infty}$ that satisfy equations (5) and (11). But by looking directly at equilibrium paths, our analysis extends beyond this equilibrium selection and uncovers new insights about the monetary transmission mechanism.

Note that the previous results require that $\tau > 0$. A natural question is how the results change if, instead, $\tau = 0$. If $\tau = 0$, the only prediction of the model is that $\sum_{t=0}^{\infty} \beta^t T_t = 0$. Other than that, the households' intertemporal budget constraint imposes no restrictions on the present value of consumption. In particular, consumption and the wealth effect have a self-fulfilling nature: if households expect to receive a higher income, they increase consumption accordingly, and since output is demand determined, output increases to satisfy that demand. But since households' income equals the value of output, the increase in consumption becomes self-fulfilling. This logic resembles the case in which the MPC is equal to one in the old-Keynesian analysis. In the standard equilibrium selection, the Taylor rule pins down Ω_0 by imposing that only a specific path of output is consistent with a bounded equilibrium. In contrast, the wealth effect can be determined by the equilibrium path of policy variables when $\tau > 0$.

The derivation above illustrates three important lessons. First, given the equilibrium paths of policy variables $\{i_t, T_t\}_{t=0}^{\infty}$, (generically) the equilibrium is uniquely determined. The equilibrium multiplicity associated with interest rate pegs can then be indexed by the fiscal backing to monetary shocks (the present value of T_t). Second, the path of nominal interest rates affects the timing of consumption, while the fiscal backing pins down its present value. Therefore, contractionary monetary policy reduces the present value of consumption only if it is accompanied by contractionary fiscal policy in the form of a negative fiscal backing. In the Taylor equilibrium, the fiscal adjustment happens *by assumption*, and it is often overlooked. In this paper, we want to understand its role in shaping the equilibrium dynamics. Third, policy rules affect the economy by

selecting the equilibrium path of policy variables. Different policy rules, particularly different policy regimes, generate different consumption behavior only if they lead to different fiscal backing for a given path of nominal interest rates.

The rest of this section extends the model in two dimensions. First, we bring back sticky prices. We show that this introduces a powerful general equilibrium amplification mechanism. Second, we incorporate government debt, which is an important component of the wealth effect generated by monetary policy. We show how the three lessons derived above extend to a more general setting and use the model to study several dimensions of monetary-fiscal interactions.

2.3 The General Case: Sticky Prices and Long-Term Debt

Assume that $\kappa > 0$ and $b > 0$. The eigenvalues of the system of difference equations (1)-(2) are given by

$$\overline{\lambda} = \frac{1+\beta+\sigma^{-1}\kappa+\sqrt{(1+\beta+\sigma^{-1}\kappa)^2-4\beta}}{2\beta} > 1,$$
$$\underline{\lambda} = \frac{1+\beta+\sigma^{-1}\kappa-\sqrt{(1+\beta+\sigma^{-1}\kappa)^2-4\beta}}{2\beta} \in (0,1).$$

Note that the system has one eigenvalue outside the unit circle and one inside the unit circle. Focusing on bounded solutions, we need one additional condition to determine the equilibrium. A standard approach is to index all solutions of the system by the response of consumption or inflation in period 0, that is, index the equilibria by the value of c_0 or π_0 (see Cochrane, 2017). More generally, one can use the value of consumption or inflation at any point in time, or a combination of different periods, as the extra terminal condition of the system. Following the analysis in Section 2.2, we choose to index the solutions by the change in the households' wealth, or the *wealth effect*, defined as

$$\Omega_0 \equiv (1-\beta)\left[\sum_{t=0}^{\infty}\beta^t\left[(1-\tau)\,y_t+(i_t-\pi_{t+1}-r_n)\,Qb+T_t\right]-\left[\sum_{t=0}^{\infty}(\beta\rho)^t\,(i_t-r_n)\,\rho+\frac{1}{\beta}\pi_0\right]Qb\right]. \quad (12)$$

In Appendix B, we show that Ω_0 is the first-order approximation of the change in households' wealth to a monetary shock net of the change in the cost of consumption (see footnote 14). Then, the households' intertemporal budget constraint implies $(1-\beta)\sum_{t=0}^{\infty}\beta^t c_t = \Omega_0$. Thus, Ω_0 equals the change in the households' *average* consumption (and, from market-clearing, output).

The following proposition provides a characterization of the equilibrium path of consumption in *any* solution to the New Keynesian model for a given path of the nominal interest rate, $\{i_t\}_{t=0}^{\infty}$.

It shows that consumption can be decomposed into the sum of a term that is *uniquely* determined by the path of the nominal interest rate and a term that depends on the households' wealth effect.

Proposition 1 (Consumption Decomposition in General Equilibrium)**.** *Given an equilibrium path for the nominal interest rate,* $\{i_t\}_{t=0}^{\infty}$*, all bounded solutions of the system* (1)-(2) *generate a path of consumption that is given by*

$$c_t = \underbrace{c_t^S}_{ISE} + \underbrace{\frac{1-\beta\underline{\lambda}}{1-\beta}\underline{\lambda}^t}_{GE\ multiplier} \times \underbrace{\Omega_0}_{WE},$$

where $\{c_t^S\}_{t=0}^{\infty}$ *is uniquely determined by the path of the nominal interest rate,* $\{i_t\}_{t=0}^{\infty}$*, and satisfies* $\sum_{t=0}^{\infty}\beta^t c_t^S = 0$*, and* Ω_0 *is given by* (12)*.*

Proposition 1 shows that the equilibrium response of consumption to a monetary shock can be decomposed into two terms.[17] The first term corresponds to an *intertemporal substitution effect* (ISE). The ISE can be interpreted as the path of consumption when the change in households' wealth exactly compensates for the change in the cost of the steady-state consumption bundle, that is, when $\Omega_0 = 0$.[18] A change in the nominal interest rate represents a change in the relative price of present versus future consumption. The households' response to this change in relative prices corresponds to a substitution effect: they change the *timing* of consumption while keeping the total cost of the bundle constant.[19] Moreover, given a path for the nominal interest rate, the ISE is unique. This result is in sharp contrast to the multiplicity of equilibria present in the New Keynesian model for an interest rate peg, and it will help shed some light on the sources of this multiplicity.

The second term has two components: the *wealth effect* (WE) and a *general equilibrium (GE) multiplier*. The wealth effect captures the revaluation of the households' after-tax financial and human wealth after a change in the path of the nominal interest rate. As is common in representative-agent models, the permanent income hypothesis implies that households try to smooth any changes in their transitory income, which generates small changes in each period's consumption for standard-sized shocks. This term was present in the rigid-prices case of Section 2.2. However, Proposition 1

[17] In Appendix C.3, we compare the decomposition in Proposition 1 with the one found in Kaplan et al. (2018).

[18] Of course, Ω_0 is determined endogenously, so it is not yet clear that an equilibrium with $\Omega_0 = 0$, in fact, exists. We postpone the determination of Ω_0 until Section 3, but it should be clear from the analysis in Section 2.2 and the characterization in Proposition 1 that Ω_0 is a free variable of the system. Corollary 1, below, makes this claim formally.

[19] Strictly speaking, the households' choice of the timing of consumption depends on the real rather than the nominal interest rate. Appendix C shows that c_t^S corresponds to the Hicksian demand from microeconomic theory evaluated at the inflation rate *consistent* with the consumption plan $\{c_t^S\}_{t=0}^{\infty}$ according to the New Keynesian Phillips curve (2). This is the sense in which the ISE can be interpreted as an intertemporal substitution channel.

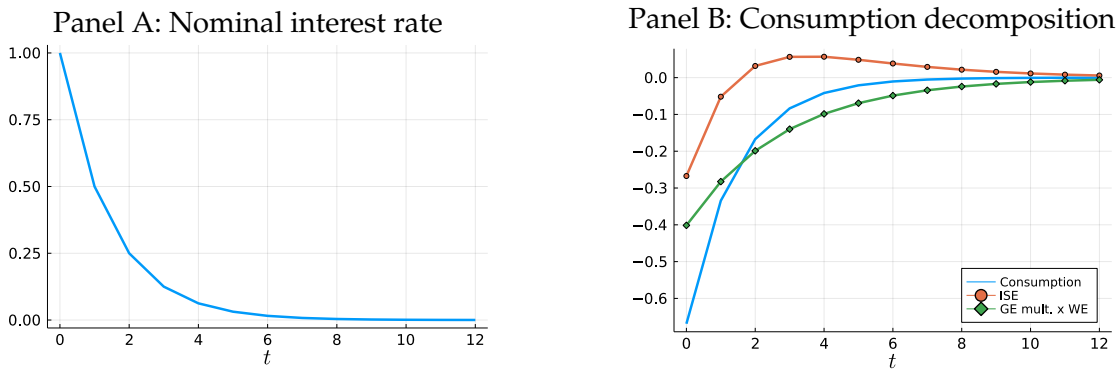


Figure 1: Decomposition of the consumption response to a nominal interest rate change

Calibration: quarterly time period, $\beta = 0.99$, $\sigma = 1$, $\kappa = 0.1275$. The nominal interest rate follows $i_t = r_n + \rho_r^t(i_0 - r_n)$, with $\rho_r = 0.5$ (which implies a half-life of the monetary shock of three months). We set $i_0 - r_n$ to 25bps (100bps annualized). The solution corresponds to the unique purely forward-looking equilibrium.

shows that when prices are sticky, the wealth effect can be amplified in general equilibrium by the GE multiplier. The intuition is as follows. When their wealth decreases, households reduce their consumption, which puts downward pressure on inflation. For a given equilibrium path of the nominal interest rate, the reduction in inflation increases the real interest rate, further contracting the economy. Since $\underline{\lambda} < 1$, the GE multiplier at $t = 0$ is greater than one. In fact, as we show below, the GE multiplier can be very large.

In order to get a sense of the quantitative importance of each component, we present a numerical example in Figure 1. In this section, we use a standard calibration found, for example, in Galí (2015). A crucial parameter is κ, the slope of the Phillips curve. We study the sensitivity of the results to alternative calibrations in Section 3. The solid lines represent the equilibrium paths of the nominal interest rate (Panel A) and the households' consumption (Panel B). The interest rate follows an AR(1) process with an autoregressive coefficient of 0.5, implying a half-life of the monetary shock of 3 months. We depicted a standard equilibrium in which an increase in the nominal interest rate reduces the consumption path.[20] Panel B also decomposes the equilibrium response of consumption into the components defined in Proposition 1. Both components of consumption are negative on impact. Regarding their contribution to the total response, the ISE accounts for 40% of the initial response, while the GE amplified wealth effect (i.e. the GE multiplier times the wealth effect) accounts for 60%. That is, even in the RANK model, more than half of the response of the economy to a monetary shock is explained by a term that depends on the wealth effect

[20] In Figure 1, we focus on the unique purely forward-looking solution to the system (1)-(2), which coincides with the standard Taylor equilibrium, as shown in Proposition 2 below.

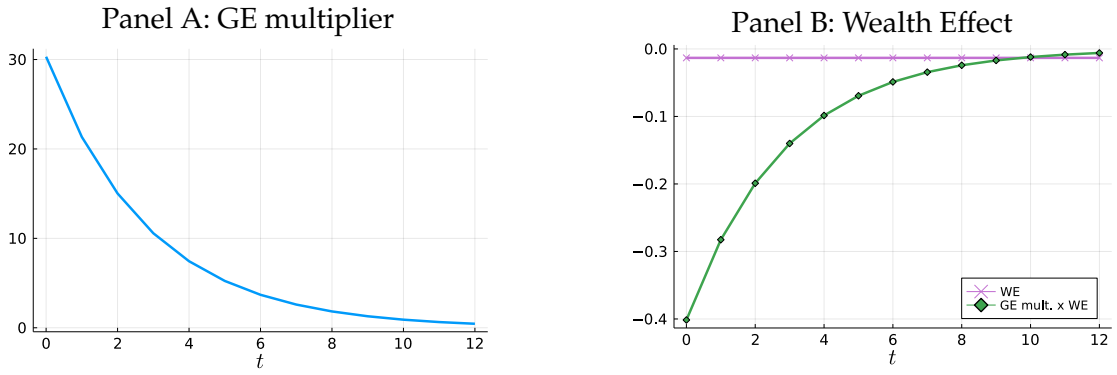


Figure 2: GE multiplier and the wealth effect

Calibration: quarterly time period, $\beta = 0.99$, $\sigma = 1$, $\kappa = 0.1275$. The nominal interest rate follows $i_t = r_n + \rho_r^t(i_0 - r_n)$, with $\rho_r = 0.5$ (which implies a half-life of the monetary shock of three months). We set $i_0 - r_n$ to 25bps (100bps annualized). The solution corresponds to the unique purely forward-looking equilibrium.

rather than the ISE.

Let us consider the GE amplified wealth effect in more detail. Figure 2 plots the dynamics of the GE multiplier (Panel A) and the wealth effect together with the GE amplified wealth effect (Panel B). The wealth effect alone only explains 2% of the consumption response in period 0. The small impact of the wealth effect on equilibrium consumption is consistent with the fact that households in the model conform to the permanent income hypothesis and that the shock is transitory. However, the GE multiplier magnifies the wealth effect to the point that the GE amplified wealth effect accounts for more than half of the total initial response. Notably, the baseline calibration generates a GE multiplier in period 0 equal to 30. These results show that the wealth effect can play a substantial role in the RANK model, though indirectly, through powerful endogenous amplification mechanisms. In Section 3, we show that this observation has important implications for the role of fiscal policy in the monetary transmission mechanism.

Moreover, the decomposition in Proposition 1 provides new insights about the source of multiplicity in the New Keynesian model.

Corollary 1 (Multiplicity in the New Keynesian model)**.** *Given a path for the nominal interest rate,* $\{i_t\}_{t=0}^{\infty}$*, all bounded solutions of the system* (1)-(2) *generate the same ISE and GE multiplier. All bounded solutions to the New Keynesian model for a given path of the nominal interest rate can be indexed by* Ω_0*.*

The decomposition in Proposition 1 characterizes *all* the bounded solutions of the system (1)-(2) for a given path of the nominal interest rate. Corollary 1 establishes that all these solutions generate the same ISE and GE multiplier. This result provides a new perspective on the multi-

plicity of equilibria of the New Keynesian model under an interest rate peg. The solutions of the model can be indexed by the level of wealth effect they generate, i.e. the extent of revaluation of households' financial assets and human wealth. In this sense, the standard Taylor rule equilibrium and the FTPL are ways of selecting a particular level of the wealth effect.[21] Next, we consider the Taylor rule equilibrium in detail.

The Taylor equilibrium. Consider the interest rate rule (5) with $\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$ and $\phi_y > \frac{\sqrt{4\beta\sigma^{-1}\left(\kappa(\phi_\pi-1)+(1-\beta)\phi_y\right)}-\left(1-\beta+\sigma^{-1}\kappa\right)}{\sigma^{-1}\beta}$.[22] We say that a sequence of monetary shocks $\{\varepsilon_t\}_{t=0}^{\infty}$ *decays sufficiently fast* if $\varepsilon_t = \mathcal{O}(\psi^t)$, where $\psi < \underline{\lambda}$.[23] Under these assumptions, the Taylor equilibrium is the unique purely forward-looking solution to the system (1)-(2).

Proposition 2 (Taylor equilibrium)**.** *Suppose that the equilibrium path of the nominal interest rate, $\{i_t\}_{t=0}^{\infty}$, was generated by an interest rate rule (5) with $\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$ and $\phi_y > \frac{\sqrt{4\beta\sigma^{-1}\left(\kappa(\phi_\pi-1)+(1-\beta)\phi_y\right)}-\left(1-\beta+\sigma^{-1}\kappa\right)}{\sigma^{-1}\beta}$, given a sequence of shocks $\{\varepsilon_t\}_{t=0}^{\infty}$ that decays sufficiently fast. Then, the equilibrium path of consumption is the unique purely forward-looking solution to the system (1)-(2), that is,*

$$c_t^{Taylor} = -\frac{\sigma^{-1}}{\overline{\lambda}-\underline{\lambda}}\sum_{s=t}^{\infty}\left(\frac{\overline{\lambda}-1}{\underline{\lambda}^{s-t}} + \frac{1-\underline{\lambda}}{\overline{\lambda}^{s-t}}\right)\left(i_s^{Taylor} - r_n\right).$$

The corresponding wealth effect is

$$\Omega_0^{Taylor} = -(1-\beta)\frac{\sigma^{-1}}{\overline{\lambda}-\underline{\lambda}}\sum_{s=0}^{\infty}\left(\frac{\overline{\lambda}}{\underline{\lambda}^{s}} - \frac{\underline{\lambda}}{\overline{\lambda}^{s}}\right)\left(i_s^{Taylor} - r_n\right). \tag{13}$$

If $\varepsilon_t = \psi\varepsilon_{t-1}$ with $\psi \in (0, \underline{\lambda})$ and ε_0 given, the nominal interest rate satisfies

$$i_t^{Taylor} = r_n + \psi^t \nu \varepsilon_0,$$

with $\nu > 0$.

Proposition 2 shows how consumption responds to changes in the nominal interest rate in the Taylor equilibrium. Two features of the solution are particularly relevant. First, the Taylor

[21] This interpretation is valid conditional on these regimes generating the same equilibrium path for the nominal interest rate. In more general exercises, the two regimes could potentially have different implications for the equilibrium path of the interest rate and, therefore, for the decomposition.

[22] The second restriction implies real-valued eigenvalues.

[23] If the shock follows an AR(1) process, the condition $\psi < \underline{\lambda}$ implies that a positive monetary shock leads to an increase in the nominal interest rate, as in standard calibrations of the New Keynesian model.

equilibrium corresponds to the unique purely forward-looking solution to the system (1)-(2) given an equilibrium path for the nominal interest rate. The substitution effect on date t depends on both past and future interest rates, while the wealth effect can depend, in principle, on the entire path of nominal interest rates. Therefore, the solution to the system (1)-(2) has, in general, both backward- and forward-looking components. Proposition 2 says that there is a unique value of Ω_0 such that the effect of past interest rates on the substitution effect and the GE amplified wealth effect cancel out exactly, and this corresponds to the Taylor equilibrium. Thus, the Taylor equilibrium is the *unique* solution in which only current and future values of the interest rate matter for the determination of aggregate variables.[24] Second, an increase in nominal interest rates leads to a decline in consumption at all dates. Thus, the wealth effect has to be sufficiently negative to offset the increase in consumption embedded in the ISE.

Consider again Figure 1. The equilibrium plotted can be reinterpreted as the response of consumption in the Taylor equilibrium with $\varepsilon_t = \psi\varepsilon_{t-1}$, $\psi = \rho_r$ and $\varepsilon_0 = \frac{i_0 - r_n}{\nu}$. Note that the equilibrium interest rate inherits the persistence and sign of the monetary shock. Moreover, the quantitative importance of the wealth effect in the Taylor equilibrium can be seen by comparing the equilibrium path of consumption with the corresponding ISE. Recall that in the example of Figure 1 the ISE is only 40% of the initial consumption response. The remaining 60% is due to the GE amplified wealth effect. The role of the wealth effect becomes even starker when considering the dynamics of inflation, to which we turn next.

Inflation. Proposition 1 presented a decomposition of consumption. There is a similar decomposition of inflation.

Proposition 3 (Inflation Decomposition)**.** *In the bounded solutions of the system* (1)-(2)*, inflation is given by*

$$\pi_t = \pi_t^S + \frac{\kappa}{1-\beta}\underline{\lambda}^t\Omega_0,$$

where $\{\pi_t^S\}_{t=0}^{\infty}$ *is uniquely determined by the path of the nominal interest rate,* $\{i_t\}_{t=0}^{\infty}$*, and* Ω_0 *is given by* (12)*. In* $t = 0$*,*

$$\pi_0 = \frac{\kappa}{1-\beta}\Omega_0.$$

The decomposition in Proposition 3 uncovers a novel result: inflation in period 0 is entirely

[24] This result depends on assuming that the sequence of monetary shocks decays sufficiently fast, which is the standard assumption in the literature. For more general processes, observational equivalence of monetary-active and fiscally-active regimes obtains (Cochrane, 1998, 2018a). See Appendix D for a discussion.

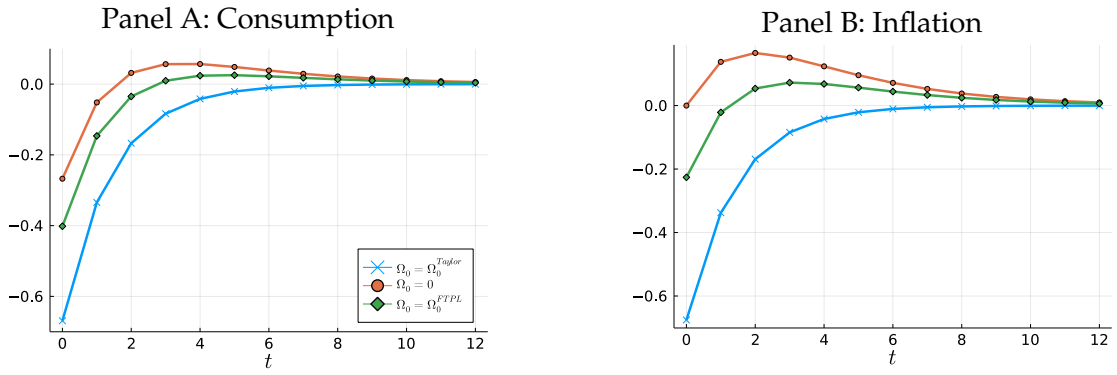


Figure 3: Consumption and inflation response to a nominal interest rate change

Calibration: quarterly time period, $\beta = 0.99$, $\sigma = 1$, $\kappa = 0.1275$. The nominal interest rate follows $i_t = r_n + \rho_r(i_0 - r_n)$, with $\rho_r = 0.5$ (which implies a half-life of the monetary shock of three months). We set $i_0 - r_n$ to 25bps (100bps annualized). The duration of government debt is set to 62 months (20.67 quarters), $\tau = 0.3$, and the debt-to-GDP (annual) is 1.

determined by the wealth effect. In particular, initial inflation does not depend on the change in *initial* consumption, but on whether the households' *lifetime* consumption is on average higher or lower after the shock. That is, initial inflation depends on whether households are richer or poorer rather than on the specific timing of the consumption path. To understand this result, it is helpful to note the forward-looking nature of the New Keynesian Phillips curve, which depends only on the present value of *future* consumption. Since the present value of the ISE is zero, initial inflation is determined solely by the wealth effect. In particular, the old-Keynesian idea that lowering consumption in a period is sufficient to lower inflation contemporaneously does not apply to this New Keynesian environment. Hence, in the absence of wealth effects, the monetary authority is unable to control initial inflation.

Moreover, inflation has *Neo-Fisherian* features under the ISE, as an increase in nominal interest rates actually *raises* future inflation, $\frac{\partial \pi_t^S}{\partial i_s} > 0$, for $t > 0$ and any $s \geq 0$.[25] Therefore, the inverse relation between the nominal interest rate and inflation under the Taylor equilibrium is driven entirely by a negative wealth effect. In the absence of such wealth effects, not only does the monetary authority lose control of initial inflation, but the effect on future inflation has the opposite sign than in the standard result.

Figure 3 shows the paths of consumption (Panel A) and inflation (Panel B) for different values of Ω_0. When $\Omega_0 = 0$, the path is simply given by $\{c_t^S, \pi_t^S\}_{t=0}^{\infty}$. Note that inflation starts at zero and then becomes strictly positive, converging back to zero from above in the limit. When

[25] See the proof of Proposition 3 for a formal derivation of this result.

$\Omega_0 = \Omega_0^{Taylor}$, as calculated in Proposition 2, the negative wealth effect more than compensates the positive effect of the ISE, and both the paths of consumption and inflation are negative and converge back to zero from below. Finally, the figure shows the case in which Ω_0 corresponds to a fiscal policy in which the government's primary surpluses do not change after the monetary shock, which we label as the FTPL case. We study the role of fiscal policy in the determination of Ω_0 in detail in Section 3, but it is worth noting that, in this case, the response of consumption and inflation are attenuated relative to the Taylor case, though inflation is negative on impact, implying that $\Omega_0^{FTPL} < 0$ in the baseline calibration.

Taking stock. The previous analysis shows that the wealth effect, represented by Ω_0, plays a crucial role in shaping the response of consumption and inflation to a monetary shock. Until now, the analysis has mostly taken Ω_0 as given. In Section 2.2 we showed that Ω_0 can be expressed in terms of fiscal variables when prices are rigid. The next section extends this result to the sticky price setting and performs several comparative statics.

3 The Fiscal Determination of the Wealth Effect

In this section, we study the determination of the wealth effect. Recall that the wealth effect is given by

$$\Omega_0 = (1-\beta)\left[\sum_{t=0}^{\infty}\beta^t\left[(1-\tau)\,y_t + (i_t - \pi_{t+1} - r_n)\,Qb + T_t\right] - \left[\sum_{t=0}^{\infty}(\beta\rho)^t\,(i_t - r_n)\,\rho + \frac{1}{\beta}\pi_0\right]Qb\right]. \tag{14}$$

Two forces will help characterize it: the *spending-income spiral* and the *spending-inflation spiral*, given, respectively, by

$$(1-\beta)\sum_{t=0}^{\infty}\beta^t c_t = \Omega_0 = (1-\beta)\sum_{t=0}^{\infty}\beta^t y_t, \qquad \text{and} \qquad \pi_t = \pi_t^S + \frac{\kappa}{1-\beta}\underline{\lambda}^t\Omega_0,$$

where recall that $\{\pi_t^S\}_{t=0}^{\infty}$ is uniquely determined by the path of the nominal interest rate (see Proposition 3). The spending-income spiral states that the wealth effect equals average consumption and, by the resource constraint, average income, and that higher income leads to higher consumption. The spending-inflation spiral states that, given a path for the nominal interest rate, the inflation rate increases with aggregate consumption and, therefore, with the wealth effect. Plug-

ging these two relations into (14), we get[26]

$$\Omega_0 = \left[1 - \left(\tau + \left(\frac{1}{\beta} + \frac{\underline{\lambda}}{1-\beta\underline{\lambda}}\right)\kappa Qb\right)\right]\Omega_0 + (1-\beta)\left[\sum_{t=0}^{\infty}\beta^t\left[\left(i_t - \pi_{t+1}^S - r_n\right)Qb - \rho^t\left(i_t - r_n\right)\rho Qb + T_t\right]\right]. \quad (15)$$

In the same spirit as equation (9), equation (15) states that the wealth effect is determined according to a *Fiscal Keynesian Cross*. To determine the equilibrium value of Ω_0, thus, we need to consider two separate cases: *i*) monetary policy has no fiscal consequences, that is, $\tau = b = 0$; and *ii*) monetary policy has fiscal consequences, that is, either $\tau > 0$ or $b > 0$ (or both). The equilibrium implications of the model are very different in these two cases.

Consider first the case $\tau = b = 0$. This is a knife-edge case and not the empirically relevant one, but it is still important to study as it is commonly assumed in the literature. Evaluating equation (15) at $\tau = b = 0$, we get

$$\Omega_0 = \Omega_0 + (1-\beta)\sum_{t=0}^{\infty}\beta^t T_t \implies \sum_{t=0}^{\infty}\beta^t T_t = 0.$$

As we found in Section 2.2, the only restriction we get from the households' intertemporal budget constraint is that the present value of transfers must be zero. But beyond that, the households' budget constraint imposes no restrictions on the present value of consumption. In the standard equilibrium selection, the Taylor rule pins down Ω_0 by imposing that only a specific path of inflation and output is consistent with a bounded equilibrium.

In contrast, the indeterminacy of the wealth effect disappears when monetary policy has fiscal consequences. As we move away from $\tau = b = 0$, the wealth effect can be characterized by the observed paths of policy variables.

Proposition 4 (Fiscal Keynesian Cross)**.** *Suppose $\tau > 0$ or $b > 0$ (or both). The wealth effect, Ω_0, is given by*

$$\Omega_0 = \frac{1-\beta}{\tau + \left(\frac{1}{\beta} + \frac{\underline{\lambda}}{1-\beta\underline{\lambda}}\right)\kappa Qb}\left[\sum_{t=0}^{\infty}\beta^t\left(\underline{\lambda}^{t+1} - \rho^{t+1}\right)\left(i_t - r_n\right)Qb + \sum_{t=0}^{\infty}\beta^t T_t\right]. \quad (16)$$

Proposition 4 states that, given a path for the nominal interest rate and government transfers,

[26] Recall that $\{\pi_t^S\}_{t=0}^{\infty}$ is uniquely determined by the path of the nominal interest rate.

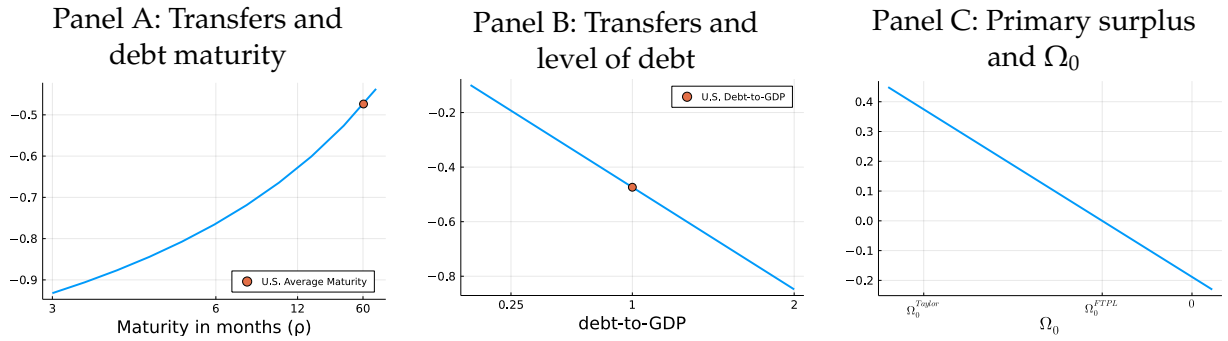


Figure 4: Fiscal policy in the New Keynesian model

Calibration: quarterly time period, $\beta = 0.99$, $\sigma = 1$, $\kappa = 0.1275$. The nominal interest rate follows $i_t = r_n + \rho_r(i_0 - r_n)$, with $\rho_r = 0.5$ (which implies a half-life of the monetary shock of three months). We set $i_0 - r_n$ to 25bps (100bps annualized). The duration of government debt is set to 62 months (20.67 quarters), $\tau = 0.3$ and debt-to-GDP (annual) is 1. Transfers and primary surplus are in percentage of annual steady-state level of output. Panels A and B correspond to the Taylor equilibrium.

the wealth effect is uniquely determined. This is an important result that provides a novel link between the economy's equilibrium and the fiscal response to a monetary shock, which is often overlooked in standard analysis.[27] Crucially, the characterization in Proposition 4 is relevant independently of whether fiscal policy is active or passive, and it helps understand the role of fiscal policy in the monetary transmission mechanism.

It may sound surprising that the wealth effect can be expressed in terms of fiscal variables in the Taylor equilibrium. After all, it is well-known that, in monetary-active regimes, fiscal policy is irrelevant for determining the economy's response to monetary policy as long as it is guaranteed that the government's intertemporal budget constraint is satisfied. The analysis here, however, does not contradict conventional wisdom. Proposition 2 showed that, in the Taylor equilibrium, there exists a unique value of the wealth effect that is consistent with a bounded equilibrium, and this value is independent of fiscal variables. However, the *Fiscal Keynesian Cross* allows us to recover the fiscal backing necessary to *sustain* such equilibrium. Rewriting (16), we obtain an expression for the fiscal transfers that are necessary to sustain a particular level of the wealth effect:

$$\sum_{t=0}^{\infty} \beta^t T_t = \frac{\tau + \left(\frac{1}{\beta} + \frac{\lambda}{1-\beta\lambda}\right)\kappa Q b}{1-\beta}\Omega_0 - \sum_{t=0}^{\infty} \beta^t \left(\underline{\lambda}^{t+1} - \rho^{t+1}\right)(i_t - r_n)\, Q b. \tag{17}$$

For example, the transfers in the Taylor equilibrium can be recovered by evaluating this expression at $\Omega_0 = \Omega_0^{Taylor}$, given by equation (13).

[27] Note that while the nominal interest rate enters in the expression for Ω_0, it does so as the cost of servicing debt. Since this channel operates through the government's budget constraint, we label it as part of fiscal policy.

To get a quantitative sense of the importance of fiscal policy in the Taylor equilibrium, Figure 4 Panel A plots $\sum_{t=0}^{\infty} \beta^t T_t$ as a function of ρ, which controls the maturity of government debt, while Panel B plots $\sum_{t=0}^{\infty} \beta^t T_t$ as a function of Qb. For our calibration of the duration of government bonds, the fiscal backing in the Taylor equilibrium is 0.47% of steady-state annual output, a considerable adjustment.[28] Panel A shows that transfers in the Taylor equilibrium decrease in absolute value with the duration of government bonds. In particular, if government debt had a duration of one quarter, the transfers would need to be almost 97% larger to sustain the Taylor equilibrium. Finally, Panel B shows that the fiscal backing also depends on the level of government debt. If government debt were 25% of GDP (like at the beginning of the Volcker era), the fiscal backing necessary to sustain the Taylor equilibrium would be cut by almost 60%, to 0.19% of annual output. In contrast, it would increase by 79%, to 0.85% of annual output, if the debt-to-GDP ratio increased to 2, as projected by the CBO for 2051 (see Congressional Budget Office, 2021). These observations can prove helpful for the design of debt maturity management, and to understand potential tensions between the monetary and fiscal authorities as the debt-to-GDP ratio increases.

An alternative measure of the fiscal backing is the change in the present discounted value of primary surpluses, which is given by

$$\sum_{t=0}^{\infty} \beta^t s_t = -\frac{\left(\frac{1}{\beta} + \frac{\lambda}{1-\beta\lambda}\right) \kappa Qb}{1-\beta} \Omega_0 + \sum_{t=0}^{\infty} \beta^t \left(\underline{\lambda}^{t+1} - \rho^{t+1}\right) (i_t - r_n)\, Qb.$$

Figure 4 Panel C plots this measure of fiscal backing for different values of Ω_0. The three equilibria highlighted in the figure entail substantial differences in the required fiscal response. The Taylor equilibrium requires a substantial increase in the primary surpluses, translating into a strong neg-

[28] The duration of government debt is set to the average maturity of the U.S. debt, which is 62 months. The value of ρ is relatively insensitive to changes in duration in the neighborhood of 62 months.

ative wealth effect in equilibrium.[29] In contrast, in the equilibrium with $\Omega_0 = 0$, we have

$$\begin{aligned}\sum_{t=0}^{\infty} \beta^t s_t &= \sum_{t=0}^{\infty} \beta^t \left[\tau y_t - T_t\right] \\ &= \underbrace{\sum_{t=0}^{\infty} \beta^t \tau c_t^S}_{=0} + \sum_{t=0}^{\infty} \beta^t T_t \\ &= \sum_{t=0}^{\infty} \beta^t \left(\underline{\lambda}^{t+1} - \rho^{t+1}\right) (i_t - r_n)\, Qb,\end{aligned}$$

where the second equality uses that $y_t = c_t$ and that when $\Omega_0 = 0$, $c_t = c_t^S$, and the third equality uses that $\sum_{t=0}^{\infty} \beta^t c_t^S = 0$. That is, the fiscal surpluses *exactly offset* any wealth effect arising from the holdings of government bonds. With government bonds of sufficiently long duration (in particular, $\rho > \underline{\lambda}$), the $\Omega_0 = 0$ equilibrium requires a *reduction* in the present value of primary surpluses (i.e. expansionary fiscal policy) after a contractionary monetary shock.

Next, we consider the final policy response of independent interest: the FTPL case.

Wealth effects in the FTPL. In the spirit of the canonical formulation of the FTPL, we consider the case in which the change in the path of the nominal interest rate does not affect the present value of the government's primary surpluses, i.e. $\sum_{t=0}^{\infty} \beta^t s_t = 0$. In this case, we get

$$\Omega_0^{FTPL} = \frac{1-\beta}{\left(\frac{1}{\beta} + \frac{\underline{\lambda}}{1-\beta\underline{\lambda}}\right)\kappa} \sum_{t=0}^{\infty} \beta^t \left(\underline{\lambda}^{t+1} - \rho^{t+1}\right) (i_t - r_n)\,.$$

Note that relative to equation (16), this expression does not feature τ or $\{T_t\}_{t=0}^{\infty}$, as they were set to exactly cancel out. Thus, only government bonds generate wealth effects in this economy. Interestingly, the determination of Ω_0^{FTPL} features two opposing forces. An increase in the nominal interest rate leads to an increase in real rates, so households can reinvest their savings at higher rates after the monetary shock, generating a positive wealth effect. However, an increase in nominal interest rates also reduces the value of long-term government bonds, negatively affecting households' wealth. Which effect dominates depends on the duration of public debt. In particular, for a sufficiently long duration, the second effect prevails, and an increase in interest

[29] Note that the adjustment in the primary surplus in the Taylor equilibrium is smaller than the adjustment in transfers. The reason is that counter-cyclical fiscal automatic stabilizers (such as the proportional tax in the model) generate an *expansionary* fiscal response after a contractionary monetary shock which, in the Taylor equilibrium, has to be neutralized by the transfers.

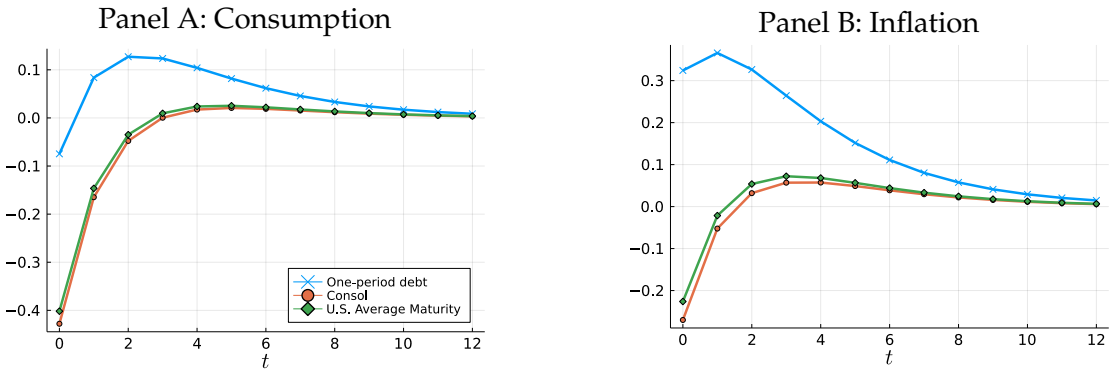


Figure 5: Consumption and inflation response to a monetary shock for various debt duration in the FTPL equilibrium.

Calibration: quarterly time period, $\beta = 0.99$, $\sigma = 1$, $\kappa = 0.1275$. The nominal interest rate follows $i_t = r_n + \rho_r(i_0 - r_n)$, with $\rho_r = 0.5$ (which implies a half-life of the monetary shock of three months). We set $i_0 - r_n$ to 25bps (100bps annualized). The duration of government debt is set to 62 months (20.67 quarters), $\tau = 0.3$ and debt-to-GDP (annual) is 1.

rates generates a *negative* wealth effect.[30]

Proposition 5 (FTPL and Long-Term Bonds)**.** *Suppose* $b > 0$ *and* $\sum_{t=0}^{\infty} \beta^t s_t = 0$*. Then,*

$$\frac{\partial \Omega_0}{\partial i_t} < 0 \iff \rho > \underline{\lambda}.$$

To understand the relevance of Proposition 5, Figure 5 plots the response of consumption and inflation in the FTPL equilibrium for different durations of government bonds. Consider first one-period bonds. The wealth effect in response to an increase in the interest rate is positive. This positive wealth effect explains why consumption decreases only in the first quarter and increases afterward (Panel A). The result is even starker for inflation. A contractionary monetary policy shock uniformly *increases* inflation. Recall that, absent wealth effects, inflation has a strong Neo-Fisherian component. A positive wealth effect exacerbates this force to the extreme that a contractionary monetary shock that increases the nominal interest rate by 100 bps in $t = 0$ generates an increase in inflation of 32 bps.

The results change with long-term bonds. An increase in nominal interest rates reduces the value of government bonds, generating a reduction in households' wealth. If this effect is sufficiently strong, an increase in interest rates generates a *negative* wealth effect. This happens when the duration of government debt satisfies $\rho > \underline{\lambda}$, which in the baseline calibration corresponds to

[30] Notably, the *quantity* of government debt does not matter in this case. This result holds only when $\sum_{t=0}^{\infty} \beta^t s_t$ scales with Qb (including when $\sum_{t=0}^{\infty} \beta^t s_t = 0$).

a duration longer than 10 months (recall that U.S. debt average maturity is 62 months). Figure 5 shows that consumption and inflation drop on impact in the calibrated duration of government debt. However, the negative wealth effect generated by government bonds does not overturn the Neo-Fisherian predictions after the first quarter, and inflation becomes positive until it converges back to zero in the limit. Note that combining the results in Proposition 3 and Proposition 4 it is immediate to see that, in the FTPL, $\pi_0 < 0$ if and only if the duration of government bonds is sufficiently long. That is, in the context of the standard New Keynesian model and absent any change in the present value of primary surpluses, only the maturity of debt can generate a negative co-movement between nominal rates and the inflation rate in period 0. This insight remains approximately true in a model with capital (see Section 4.3). Finally, Figure 5 shows that setting $\rho = 1$ (i.e. a consol) has only a marginal effect relative to the baseline calibration.

The idea that government liabilities are the relevant assets for the assessment of wealth effects is not new. This observation was central to Pigou's argument in his response to Keynesian economics. For instance, Patinkin describes Pigou's argument as follows:[31]

> (...) the private sector considered in isolation is, on balance, neither debtor nor creditor, when in its relationship to the government, it *must be* a net "creditor." (...) If we assume that government activity is not affected by the movements of the price level, then the net effect of a price decline must always be stimulatory.

Two aspects of this quote are important. First, the idea that private assets cancel out in the aggregate, but households are on net creditors of the government. Second, the fact that it is assumed that "government activity is not affected" by the shock. The Pigou effect, as described here, is remarkably similar to the modern formulation of the FTPL.

Moreover, the result in Proposition 5 echoes some of the findings in Cochrane (2018a).[32] The paper highlights the difficulties of the standard RANK model in generating a negative co-movement between initial inflation and the nominal interest rate when primary surpluses do not react to the monetary shock. The paper concludes that long-term bonds help overcome this difficulty. Our findings sharpen these results in two dimensions. First, we show that absent a change in primary surpluses, *only a sufficiently long* bond maturity can generate a negative wealth effect, where the threshold is determined by the lowest eigenvalue of the system. Second, our results show that when monetary policy has fiscal consequences, a necessary condition for a drop of initial inflation

[31] See Patinkin (1948).

[32] The paper builds on results in Sims (2011) and Cochrane (2018b)

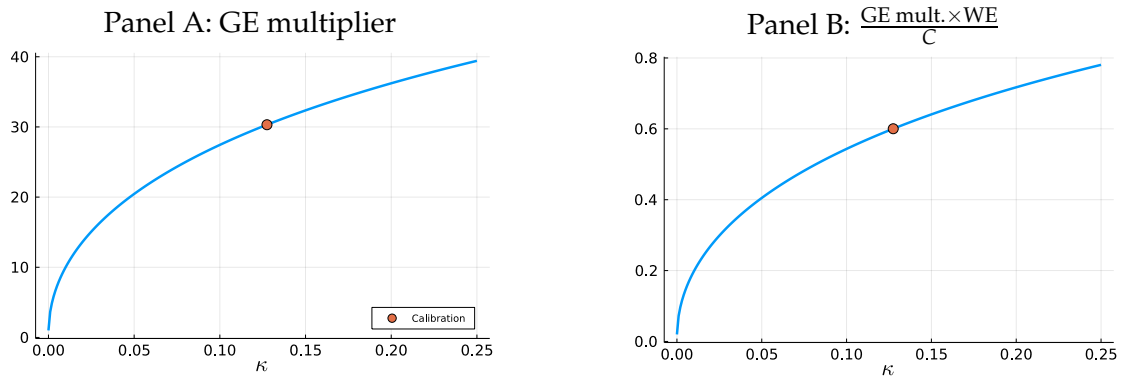


Figure 6: GE multiplier and the degree of price flexibility.

Calibration: quarterly time period, $\beta = 0.99$, $\sigma = 1$. The nominal interest rate follows $i_t = r_n + \rho_r(i_0 - r_n)$, with $\rho_r = 0.5$ (which implies a half-life of the monetary shock of three months). We set $i_0 - r_n$ to 25bps (100bps annualized). The solution in Panel B corresponds to the Taylor equilibrium.

to a contractionary monetary shock is either the presence of (sufficiently long) government debt or contractionary fiscal policy as summarized by the present value of primary surpluses. In particular, the RANK model has no alternative channels to generate a drop in inflation. Section 4 studies several extensions that show the robustness of this result.

Wealth Effect, the GE multiplier, and price stickiness. The previous analysis highlights two novel features of the New Keynesian model. First, it shows that while the direct impact of the wealth effect is small in RANK (consistent with the permanent income hypothesis), its general equilibrium effects can be significantly amplified, as reflected by a potentially large GE multiplier. Second, the wealth effect can be characterized in terms of the fiscal response to the monetary shock. Thus, put together, these results imply that fiscal policy can play an essential role in the monetary transmission mechanism.

Here, we consider in more detail the properties of the GE multiplier. Recall that the GE multiplier captures the effect on consumption (and output) of changes in households' wealth that is mediated through inflation. When their wealth decreases, households reduce consumption putting downward pressure on prices and increasing the real interest rate, further reducing consumption. The baseline calibration of Section 2 finds a strong effect arising from this channel. We now show that this result is highly sensitive to the degree of price flexibility in the economy.

Figure 6 Panel A plots the GE multiplier as a function of κ, indicating the calibrated value from Section 2 as a reference. The GE multiplier is strictly increasing in the degree of price flexibility, achieving a minimum of 1 when prices are perfectly rigid. Panel B shows how the value of κ

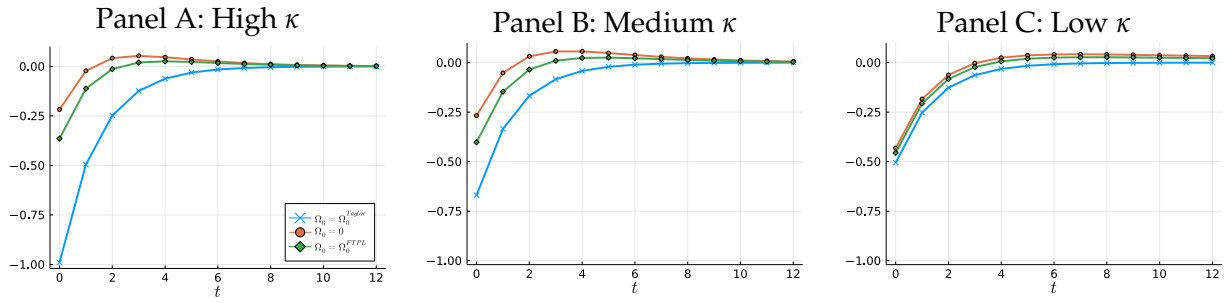


Figure 7: Consumption response to a monetary shock for various values of κ and Ω_0

Calibration: quarterly time period, $\beta = 0.99$, $\sigma = 1$, $\kappa \in \{0.25, 0.1275, 0.005\}$. The nominal interest rate follows $i_t = r_n + \rho_r(i_0 - r_n)$, with $\rho_r = 0.5$ (which implies a half-life of the monetary shock of three months). We set $i_0 - r_n$ to 25bps (100bps annualized). The duration of government debt is set to 62 months (20.67 quarters), $\tau = 0.3$ and debt-to-GDP (annual) to 1.

affects the fraction of the consumption response due to the GE amplified wealth effect in the Taylor equilibrium. For the calibration in Section 2, 60% of the total response of output is explained by the wealth effect. In contrast, with rigid prices, the GE amplified wealth effect explains only 2% of the consumption response, consistent with only the permanent income hypothesis being operative. More generally, the fraction increases with κ.

These results suggest that the importance of fiscal backing in the RANK model depends significantly on the degree of price flexibility. Figure 7 plots the path of consumption after a monetary shock and different values of κ and Ω_0.[33] Panel A plots the path of consumption for different values of Ω_0 when κ is set to 0.25, which is approximately double the value in the calibration of Section 2. Panel B sets κ to the baseline calibration, and it coincides with the plot in Panel A of Figure 3 (with the axis modified to help the comparison across cases). Finally, Panel C presents the response of consumption in a fairly rigid-price environment.[34]

A striking result emerges. While the wealth effect has a substantial impact on the consumption path when prices are relatively flexible, the effect is marginal for lower degrees of price flexibility. In Panel A ($\kappa = 0.25$), the fiscal backing represents 78% of the consumption response in period 0 (taking into account the GE amplification). In contrast, in Panel C ($\kappa = 0.005$), the fiscal backing represents less than 15% of the response. Thus, in the RANK model, monetary-fiscal interactions

[33] To interpret the results for the Taylor equilibrium, note that the path of the nominal interest rate is set the same for all values of κ. From the characterization of Proposition 2, we know that we can achieve this result by adjusting the size of the initial shock, ε_0. Thus, while the Taylor equilibrium features a consumption response to a *monetary shock*, $\{\varepsilon_t\}_{t=0}^{\infty}$, that is decreasing in κ, it features a response to a *path of the nominal interest rate*, $\{i_t\}_{t=0}^{\infty}$, that is increasing in κ.

[34] We set $\kappa = 0.005$. Lower values of κ drastically change the properties of the FTPL equilibrium, as a monetary shock generates a positive wealth effect even with long-term debt (recall that we obtain $\frac{\partial \Omega_0^{FTPL}}{\partial i_t} < 0$ if and only if $\rho > \underline{\lambda}$, and $\underline{\lambda}$ is strictly increasing in κ, with $\underline{\lambda} = 1$ when $\kappa = 0$).

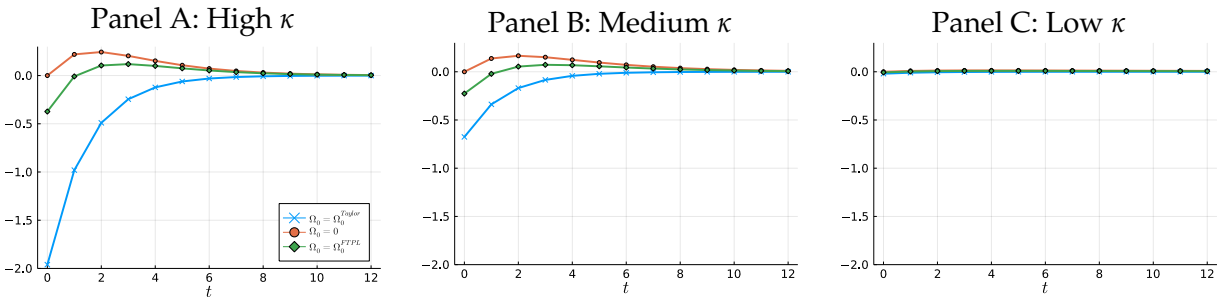


Figure 8: Inflation response to a monetary shock for various values of κ and Ω_0

Calibration: quarterly time period, $\beta = 0.99$, $\sigma = 1$, $\kappa \in \{0.005, 0.1275, 0.25\}$. The nominal interest rate follows $i_t = r_n + \rho_r(i_0 - r_n)$, with $\rho_r = 0.5$ (which implies a half-life of the monetary shock of three months). We set $i_0 - r_n$ to 25bps (100bps annualized). The duration of government debt is set to 62 months (20.67 quarters), $\tau = 0.3$ and debt-to-GDP (annual) to 1.

are particularly relevant in economies with a relatively high degree of price flexibility, while coordination is less relevant when prices are more rigid. Figure 8 shows analogous plots for inflation.

This finding may have important implications for the design of policies in economies that differ in their degree of price flexibility. In economies with a high degree of price flexibility, monetary-fiscal coordination might be a crucial element of an effective stabilization policy, and the monetary authority by itself might have limited power to affect the equilibrium. In contrast, in economies with relatively fixed prices, monetary-fiscal coordination might be secondary, and the monetary authority might be very effective in affecting aggregate variables. Of course, the degree of price flexibility is likely endogenous to the monetary-fiscal institutions. Still, this result suggests that when prices are more rigid, the fiscal authority does not need to fully accommodate monetary policy to approximate the outcomes in the Taylor equilibrium.

4 Extensions

Sections 2 and 3 studied in detail the equilibria in a standard RANK model. While useful, this model presents several limitations relative to the richer model used currently for policy analysis. This section extends the analysis in three dimensions. First, we study the properties of the New Keynesian model in a liquidity trap scenario. Consistent with the previous analysis, we show that when prices are fairly flexible, fiscal policy plays a crucial role in the dynamics of the economy. Then, we solve a simple TANK model in the spirit of Bilbiie (2008). This model allows for a representative household representation, so all the results go through, though redistribution might have subtle effects. Moreover, we argue that the main features of the analysis should extend to

richer HANK models, since all models feature an "aggregate budget constraint" of the household sector in opposition to the government's budget constraint. Finally, we solve analytically a RANK model with capital and investment. While adding a state variable complicates the analysis, the results of the previous sections remain valid (sometimes approximately) in this setting as well. Notably, the wealth effect can still be expressed in terms of fiscal variables. That is, the presence of a real asset in positive net supply does not change the channels of transmission.

4.1 Fiscal Policy in a Liquidity Trap

We follow Werning (2012) and assume a natural interest rate that follows

$$r_{nt} = \begin{cases} -r_n & t \leq T, \\ r_n & t > T, \end{cases}$$

for some known $r_n > 0$ and $T > 0$. We consider monetary responses of the type:

$$i_t = \begin{cases} 0 & t \leq T^*, \\ r_n & t > T^*, \end{cases}$$

for some known $T^* \geq 0$.

To find the equilibrium of the economy, the liquidity trap literature assumes that the economy converges back to its steady state at $T^* + 1$, that is $c_t = \pi_t = 0 \quad \forall t \geq T^* + 1$. We call this the *Standard Liquidity Trap Equilibrium* (SLTE). In the SLTE, a shock that reduces the natural interest rate has a contractionary effect, reducing initial consumption and inflation. Moreover, if $T^* > T$, consumption and inflation eventually become positive, and they revert back to zero at $T^* + 1$.

Figure 9 shows the dynamics of consumption and inflation in the SLTE for different monetary policy responses when the trap lasts until period $T = 4$. The *discretionary monetary policy* corresponds to a policy that sets $T^* = T$. In this case, consumption drops by almost 8% and inflation slightly more than 10%. The *forward guidance* case corresponds to a monetary policy that keeps the nominal rate at zero for an extra period after the natural rate reverts to its long-run level, i.e. $T^* = T + 1$. This policy significantly dampens the recessionary effects of the trap, reducing the drop in initial consumption by 45% and the drop in initial inflation by 57%. Finally, we also consider a *fixed interest rate* stance, which keeps the nominal rate at r_n the whole time, i.e. $T^* = -1$.

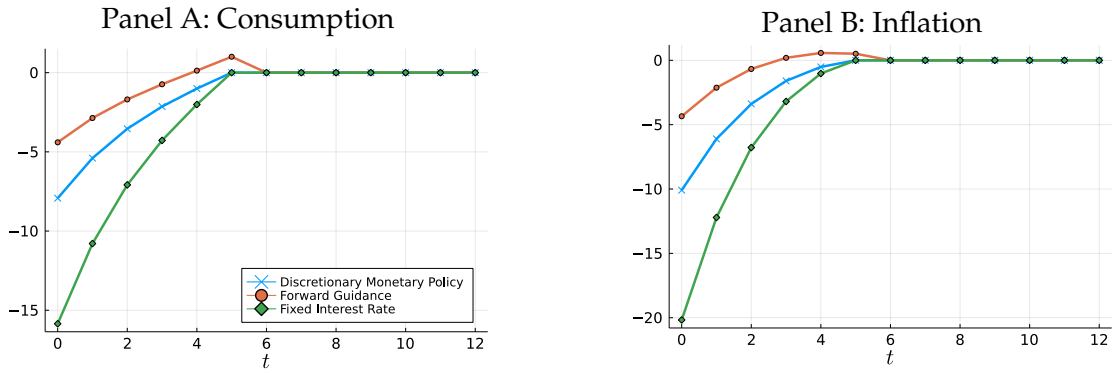


Figure 9: Consumption and inflation response in a liquidity trap in the SLTE.

Calibration: quarterly time period, $\beta = 0.99$, $\sigma = 1$, $\kappa = 0.1275$. The natural rate of interest is set to $-r_n$ until $T = 4$. *Discretionary Monetary Policy* sets $T^* = T$; *Forward Guidance* sets $T^* = T + 1$; *Fixed Interest Rate* sets $T^* = -1$. The solution corresponds to the SLTE.

The drop in consumption and inflation doubles on impact relative to the discretionary monetary policy.

Consider, alternatively, the dynamics of consumption and inflation for the same paths of the nominal interest rate but a fiscal response consistent with the FTPL, that is, a fiscal policy that keeps the present value of primary surpluses unchanged, depicted in Figure 10. For comparison, we also plot the SLTE under a discretionary monetary policy. The results are revealing. The response of consumption under a discretionary monetary policy is 54% smaller in the FTPL equilibrium. Moreover, forward guidance has a relatively small effect on consumption, reducing the magnitude of the initial response by around 0.55 p.p. Finally, we also compare the response of the economy to a fixed interest rate stance. Recall that in the SLTE with a fixed interest rate, consumption decreases by double the magnitude in the discretionary equilibrium. In contrast, in the FTPL equilibrium, the effect becomes 7.5% *smaller* than in the SLTE with discretionary monetary policy. These results highlight that the fiscal policy response to the natural rate shock is a crucial component of the dynamics of the economy in a liquidity trap and the effectiveness of forward guidance. To put it in perspective, while the FTPL features, by construction, no change in the present value of primary surpluses, the SLTE under discretionary monetary policy requires an increase (i.e. contractionary fiscal policy) of 6% of steady-state annual output (and almost 7.5% for transfers). We perform additional exercises in Appendix E.1.

These results provide an alternative interpretation to the findings in Cochrane (2017). Cochrane shows that the selection of equilibrium in the New Keynesian model is crucial in determining the equilibrium dynamics of aggregate variables. He argues that there exists a multiplicity of equilib-

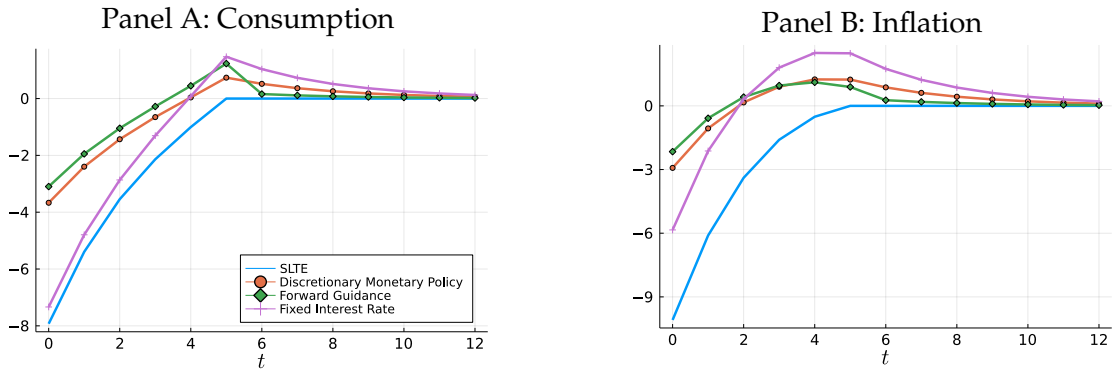


Figure 10: Consumption and inflation response in a liquidity trap in the FTPL equilibrium.

Calibration: quarterly time period, $\beta = 0.99$, $\sigma = 1$, $\kappa = 0.1275$. The natural rate of interest is set to $-r_n$ until $T = 4$. *Discretionary Monetary Policy* sets $T^* = T$; *Forward Guidance* sets $T^* = T + 1$; *Fixed Interest Rate* sets $T^* = -1$. The duration of government debt is set to 62 months (20.67 quarters), $\tau = 0.3$ and debt-to-GDP (annual) is 1. The solution corresponds to the FTPL equilibrium except for the line labeled SLTE, which corresponds to the *Discretionary Monetary Policy* in the SLTE.

ria that feature a substantially milder response of the economy to the liquidity trap and concludes that a sharp recession is not a necessary outcome of the New Keynesian model. Here, we reinforce this argument by connecting the economy's response in a liquidity trap to the fiscal policy implemented. Instead of indexing the solutions in terms of the properties of the system of equations characterizing equilibrium, like in Cochrane (2017), we propose a characterization in terms of the wealth effect and fiscal policy. The conclusion is that absent a strong contractionary fiscal response to the natural rate shock, the economy's response in a liquidity trap is significantly mitigated.[35]

Finally, in Appendix E.3 we show that the decomposition from Section 2 uncovers new insights on the determinants of the so-called *Monetary Paradoxes*, namely the *Fowrward Guidance Puzzle* and the *Paradox of Flexibility*. In particular, we find that the ISE does not feature the puzzles, hence the results are given by the wealth effect and, therefore, by the fiscal response to monetary policy. Moreover, we show that solutions that rely on a *discounted Euler equation* do not solve the puzzles except for extreme calibrations.

4.2 Household Heterogeneity: A TANK Model

We now extend the RANK model from Section 2 to incorporate household heterogeneity, in the spirit of Bilbiie (2008, 2019). The economy is populated by a continuum of measure one of households. A measure $1 - \omega$ of households are *savers*: they are forward-looking and can trade in asset

[35] In Appendix E.2 we provide a mapping between the equilibria studied in Cochrane (2017) and the characterization through wealth effects in this paper.

markets. The complementary fraction ω corresponds to households that are *hand-to-mouth* (HtM): they have no access to financial markets and consume their labor income each period. We log-linearize the model around a symmetric zero-inflation steady state. We provide the details of the model in Appendix B.

Let c_t denote the aggregate consumption, T_t the aggregate government transfers to households, and $T_{h,t}$ the transfers to HtM households. It turns out that the equilibrium of this model can be characterized by the same equations as the RANK model of Section 2, except that the Euler equation is replaced by the following *generalized* version:

$$c_{t+1} = c_t + \tilde{\sigma}^{-1}(i_t - \pi_t - r_n) + v_t,$$

where $\tilde{\sigma}^{-1} \equiv \frac{1-\omega}{1-\omega\chi_y}\sigma^{-1}$ and $v_t \equiv \frac{\omega\chi_T}{1-\omega\chi_y}(T_{h,t+1} - T_{h,t})$, with $\chi_y, \chi_T > 0$. This equation differs from the standard Euler equation in two dimensions. First, the macro-EIS $\tilde{\sigma}^{-1}$ can differ from the micro-EIS σ^{-1}. The difference between the two is determined by χ_y, which denotes the cyclicality of HtM households' income. In particular, the macro-EIS is larger than the micro-EIS if and only if $\chi_y > 1$, echoing the result in Bilbiie (2019). Second, the Euler equation includes an additional term, v_t, which depends on the transfers to the HtM households. Notably, v_t does not depend on the contemporaneous level of the transfer but on future changes. This feature will be important when we describe the channels of transmission below.

Notably, the equilibrium can be characterized by an *aggregate* intertemporal budget constraint, which is given by the sum of all the households' budget constraints and coincides with that of the representative household in the RANK model.

The following proposition extends the decomposition of Proposition 1 to this TANK model.

Proposition 6 (Consumption Decomposition in TANK)**.** *Given an equilibrium path for the nominal interest rate, $\{i_t\}_{t=0}^{\infty}$, all bounded solutions of the TANK model generate a path of consumption that is given by*

$$c_t = \underbrace{c_t^S}_{ISE} + \underbrace{c_t^T}_{\substack{Fiscal \\ redistribution}} + \underbrace{\frac{1-\beta\underline{\lambda}}{1-\beta}\underline{\lambda}^t}_{GE\ multiplier} \times \underbrace{\Omega_0}_{WE}$$

where $\{c_t^S\}_{t=0}^{\infty}$ is uniquely determined by the path of the nominal interest rate, $\{i_t\}_{t=0}^{\infty}$, $\{c_t^T\}_{t=0}^{\infty}$ is uniquely determined by the path of transfers to HtM households, $\{T_{h,t}\}_{t=0}^{\infty}$, with $\sum_{t=0}^{\infty}\beta^t c_t^S = \sum_{t=0}^{\infty}\beta^t c_t^T = 0$, and Ω_0 is given by (12).

Proposition 6 presents a channel of transmission absent in the RANK model: a fiscal redistribution channel. The expression for the fiscal redistribution is

$$c_t^T \propto \underline{\lambda}^t \left[\sum_{s=0}^{t-1} \left(\frac{\overline{\lambda}}{\underline{\lambda}^s} - \frac{\underline{\lambda}}{\overline{\lambda}^s} \right) (T_{h,s+1} - T_{h,s}) + \sum_{s=t}^{\infty} \left(\frac{1-\beta\overline{\lambda}}{1-\beta\underline{\lambda}} \left(\frac{\overline{\lambda}}{\underline{\lambda}} \right)^t - 1 \right) \frac{\underline{\lambda}}{\overline{\lambda}^s} (T_{h,s+1} - T_{h,s}) \right]. \tag{18}$$

The formula clarifies how the redistribution channel operates. First, it shows that fiscal redistribution does not affect the present value of aggregate consumption but only its timing. Second, it shows that only the *growth rate* of the transfers to HtM households affects aggregate demand, not their levels. The reason for these results is that both types of agents have an MPC of 1 to changes in their *permanent* income. Thus, any redistribution that is perfectly smooth over time will only affect the distribution of consumption but not the aggregate level.

Moreover, Proposition 6 also shows the robustness of the results in Sections 2 and 3. Absent any fiscal redistribution effect (i.e. $c_t^T = 0$ for all t), the TANK model can be represented as a RANK model with a different EIS. Put differently, in the TANK model, the EIS and the cyclicality of HtM income contribute to the same *macro* channel of transmission. This also implies that while heterogeneous agents models have the ability to amplify the response of the economy to a monetary shock, the tight connection between the wealth effect and fiscal policy is not affected. Even if at the microeconomic level the channels of transmission are different than in RANK, at the macroeconomic level, the economic forces are similar. The difference is that the TANK model can rely less on a counterfactually large calibration of the micro-EIS and more on household heterogeneity to generate a meaningful output response to monetary policy. We conjecture that this result is robust to richer sources of heterogeneity, as in quantitative HANK models. What matters is the aggregate intertemporal budget constraint, which takes the private sector as a whole in relation to the government.[36]

An important remaining question is whether the results survive if the households are allowed to hold real assets. Next, we solve a RANK model with capital.

4.3 Real Assets: A RANK model with Capital

This section solves a RANK model with capital accumulation analytically. The analysis is based on Li (2002), who studies the determinacy properties of the model. Here, we are interested in understanding whether the channels highlighted in the previous sections extend to a model with

[36] Caramp and Silva (2020) extend these results to a setting with private debt and aggregate risk and show that a version of the Fiscal Keynesian Cross holds.

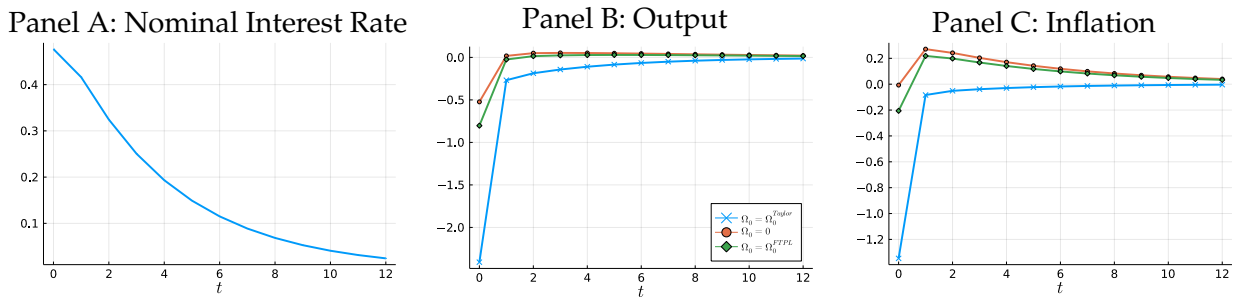


Figure 11: Output and inflation in RANK with capital

Calibration: quarterly time period, $\beta = 0.99$, $\sigma = 1$, $\delta = 0.15$, $\rho_r = 0.8341$, $\phi_\pi = 1.2312$. The duration of government debt is set to 62 months (20.67 quarters), $\tau = 0.3$ and debt-to-GDP (annual) is 1. The path of the nominal interest rate corresponds to the outcome after a monetary shock in the Taylor equilibrium. The other equilibria are computed using the same path for the nominal interest rate.

capital and investment.[37] We show that most of the results go through (e.g., the wealth effect can be stated in terms of the fiscal response to monetary policy), and when they do not, they still hold approximately for standard calibrations (e.g., the determination of initial inflation). We leave the details of the derivation to Appendix F and here focus on the determination of the wealth effect.

Let Ω_0 be defined as in Section 2 (see equation (12)). The next proposition establishes that the wealth effect can be expressed in terms of fiscal variables even in this model with capital.

Proposition 7. *Suppose $\tau > 0$ or $b > 0$ (or both). Then*

$$\Omega_0 = \frac{1-\beta}{\tau + \Psi_\Omega Qb}\left[\sum_{t=0}^{\infty}\beta^t\left[\left(i_t - \pi_t^S - r_n\right)Qb + T_t\right] - \left[\sum_{t=0}^{\infty}\left(\beta\rho\right)^t\left(i_t - r_n\right)\rho + \Psi_\Gamma\Gamma_0\right]Qb\right],$$

where Ψ_Ω and Ψ_Γ are constants independent of monetary and fiscal variables, and $\{\pi_{t+1}^S\}_{t=0}^{\infty}$ and Γ_0 are uniquely determined by the path of the nominal interest rate, $\{i_t\}_{t=0}^{\infty}$.

Figure 11 shows the equilibrium path of output and inflation for $\Omega_0 \in \left\{0, \Omega_0^{Taylor}, \Omega_0^{FTPL}\right\}$, where Ω_0^{Taylor} corresponds to the wealth effect in the Taylor equilibrium and Ω_0^{FTPL} is the wealth effect when the present value of primary surpluses does not change. The path of the nominal interest rate is the corresponding path in the Taylor equilibrium after a monetary shock. The other equilibria are computed using the same equilibrium path of the nominal rate. It is clear from the figure that the general patterns obtained in Section 3 extend to the model with capital. The Taylor equilibrium generates the strongest responses of output and inflation, while the zero-wealth effect equilibrium features similar Neo-Fisherian features as in the model without capital. However, the

[37] Rupert and Šustek (2019) also study the monetary transmission mechanisms in a RANK model with capital. Their focus is on the real interest rate channel.

response of initial inflation is not zero when $\Omega_0 = 0$, but a small drop of 0.7 bps. As a reference, the drop in initial inflation in the Taylor equilibrium is 135 bps while in the FTPL is 22 bps. Thus, we can conclude that the results from Sections 2 and 3 extend approximately to the model with capital.

5 Conclusion

Despite being often overlooked, the fiscal response to monetary policy is central to how the economy responds to monetary shocks. In this paper, we provided novel analytical tools to understand the role of fiscal policy and wealth effects in the monetary transmission mechanism. We presented a decomposition of the equilibrium response of consumption into an intertemporal substitution effect and a wealth effect. General equilibrium forces resulting from inflation dynamics can significantly amplify the impact of the wealth effect on households' consumption, even in a RANK model. Moreover, initial inflation is entirely determined by the wealth effect and not by the initial response of consumption. Crucially, when monetary policy has fiscal consequences, the wealth effect is uniquely determined by fiscal variables. This implies that contractionary monetary policy reduces inflation only if followed by contractionary fiscal policy. Thus, these results highlight the importance of fiscal policy in the monetary transmission mechanism.

The analysis in the paper provides a comprehensive analysis of the role of fiscal policy in the monetary transmission mechanism. Future work should focus on applying these insights to identify and test the channels empirically. This task will require building models that incorporate realistic features absent in the models studied here. Caramp and Silva (2020) take a step in this direction by extending the analysis to a setting with aggregate risk and richer household heterogeneity.

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A Proofs

Proofs of Propositions 1 and 3. The system of equations characterizing equilibrium is given by

$$\begin{bmatrix} c_{t+1} \\ \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1+\sigma^{-1}\frac{\kappa}{\beta} & -\frac{\sigma^{-1}}{\beta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{bmatrix} \begin{bmatrix} c_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \sigma^{-1}(i_t - r_n) \\ 0 \end{bmatrix}.$$

The eigenvalues of the system are

$$\overline{\lambda} = \frac{1+\beta+\sigma^{-1}\kappa + \sqrt{(1+\beta+\sigma^{-1}\kappa)^2 - 4\beta}}{2\beta}, \quad \underline{\lambda} = \frac{1+\beta+\sigma^{-1}\kappa - \sqrt{(1+\beta+\sigma^{-1}\kappa)^2 - 4\beta}}{2\beta}.$$

It is immediate that $\overline{\lambda} > 1$ and $\underline{\lambda} \in (0,1)$. The eigenvectors are given by

$$\overline{v} = \left(\frac{1-\beta\overline{\lambda}}{\kappa}, 1\right), \quad \underline{v} = \left(\frac{1-\beta\underline{\lambda}}{\kappa}, 1\right).$$

Let

$$P \equiv \begin{bmatrix} \frac{1-\beta\overline{\lambda}}{\kappa} & \frac{1-\beta\underline{\lambda}}{\kappa} \\ 1 & 1 \end{bmatrix}.$$

Then, we can write the system as

$$\begin{bmatrix} Z_{1,t+1} \\ Z_{2,t+1} \end{bmatrix} = \begin{bmatrix} \overline{\lambda} & 0 \\ 0 & \underline{\lambda} \end{bmatrix} \begin{bmatrix} Z_{1,t} \\ Z_{2,t} \end{bmatrix} + \frac{\sigma^{-1}\kappa}{\beta(\overline{\lambda}-\underline{\lambda})} \begin{bmatrix} -(i_t - r_n) \\ (i_t - r_n) \end{bmatrix},$$

where $Z_t \equiv P^{-1}\begin{bmatrix} c_t \\ \pi_t \end{bmatrix}$. Since $\overline{\lambda} > 1$, we can solve the first equation forward

$$Z_{1,t} = \frac{\sigma^{-1}\kappa}{\overline{\lambda}-\underline{\lambda}}\overline{\lambda}^t \sum_{s=t}^{\infty} \frac{\underline{\lambda}}{\overline{\lambda}^s}(i_s - r_n).$$

Moreover, since $\underline{\lambda} \in (0,1)$, we can solve the second equation backward

$$Z_{2.t} = \underline{\lambda}^t Z_{2,0} + \frac{\sigma^{-1}\kappa}{\overline{\lambda}-\underline{\lambda}}\underline{\lambda}^t \sum_{s=0}^{t-1} \frac{\overline{\lambda}}{\underline{\lambda}^s}(i_s - r_n).$$

Recall that

$$Z_t = -\frac{\kappa}{\beta(\overline{\lambda}-\underline{\lambda})} \begin{bmatrix} 1 & -\frac{1-\beta\underline{\lambda}}{\kappa} \\ -1 & \frac{1-\beta\overline{\lambda}}{\kappa} \end{bmatrix} \begin{bmatrix} c_t \\ \pi_t \end{bmatrix}.$$

Hence

$$Z_{1,t} = -\frac{\kappa}{\beta(\overline{\lambda}-\underline{\lambda})}\left(c_t - \frac{1-\beta\underline{\lambda}}{\kappa}\pi_t\right),$$
$$Z_{2,t} = -\frac{\kappa}{\beta(\overline{\lambda}-\underline{\lambda})}\left(-c_t + \frac{1-\beta\overline{\lambda}}{\kappa}\pi_t\right).$$

And therefore

$$c_t = \frac{1-\beta\underline{\lambda}}{\kappa}\pi_t - \sigma^{-1}\overline{\lambda}^t \sum_{s=t}^{\infty} \frac{1}{\overline{\lambda}^{s+1}} (i_s - r_n), \tag{19}$$

$$\pi_t = \frac{\kappa}{1-\beta\overline{\lambda}} c_t - \frac{\beta(\overline{\lambda}-\underline{\lambda})}{1-\beta\overline{\lambda}} \underline{\lambda}^t Z_{2,0} - \frac{\sigma^{-1}\kappa}{1-\beta\overline{\lambda}} \underline{\lambda}^t \sum_{s=0}^{t-1} \frac{1}{\underline{\lambda}^{s+1}} (i_s - r_n). \tag{20}$$

Introducing (33) into (34), we get

$$\pi_t = \underline{\lambda}^t Z_{2,0} + \frac{\sigma^{-1}\kappa}{\overline{\lambda}-\underline{\lambda}} \underline{\lambda}^t \sum_{s=0}^{t-1} \frac{\overline{\lambda}}{\underline{\lambda}^s} (i_t - r_n) + \frac{\sigma^{-1}\kappa}{\overline{\lambda}-\underline{\lambda}} \overline{\lambda}^t \sum_{s=t}^{\infty} \frac{\underline{\lambda}}{\overline{\lambda}^s} (i_t - r_n). \tag{21}$$

Introducing (35) into (33), we get

$$c_t = \frac{1-\beta\underline{\lambda}}{\kappa} \underline{\lambda}^t Z_{2,0} + \sigma^{-1} \frac{1-\beta\underline{\lambda}}{\overline{\lambda}-\underline{\lambda}} \underline{\lambda}^t \sum_{s=0}^{t-1} \frac{\overline{\lambda}}{\underline{\lambda}^s} (i_s - r_n) + \sigma^{-1} \frac{1-\beta\overline{\lambda}}{\overline{\lambda}-\underline{\lambda}} \overline{\lambda}^t \sum_{s=t}^{\infty} \frac{\underline{\lambda}}{\overline{\lambda}^s} (i_s - r_n). \tag{22}$$

Multiplying (36) by β^t and summing across time, we get

$$\frac{\Omega_0}{1-\beta} = \sum_{t=0}^{\infty} \beta^t c_t = \frac{1}{\kappa} Z_{2,0} + \frac{\sigma^{-1}}{\overline{\lambda}-\underline{\lambda}} \sum_{s=0}^{\infty} \frac{\underline{\lambda}}{\overline{\lambda}^s} (i_s - r_n).$$

Hence

$$Z_{2,0} = \frac{\kappa}{1-\beta}\Omega_0 - \frac{\sigma^{-1}\kappa}{\overline{\lambda}-\underline{\lambda}} \sum_{s=0}^{\infty} \frac{\underline{\lambda}}{\overline{\lambda}^s} (i_s - r_n). \tag{23}$$

Introducing (37) in (36), we get

$$c_t = c_t^S + \frac{1-\beta\underline{\lambda}}{1-\beta} \underline{\lambda}^t \Omega_0,$$

where

$$c_t^S \equiv \sigma^{-1} \frac{1-\beta\underline{\lambda}}{\overline{\lambda}-\underline{\lambda}} \underline{\lambda}^t \left[\sum_{s=0}^{t-1} \left(\frac{\overline{\lambda}}{\underline{\lambda}^s} - \frac{\underline{\lambda}}{\overline{\lambda}^s} \right) (i_s - r_n) + \sum_{s=t}^{\infty} \left(\frac{1-\beta\overline{\lambda}}{1-\beta\underline{\lambda}} \left(\frac{\overline{\lambda}}{\underline{\lambda}} \right)^t - 1 \right) \frac{\underline{\lambda}}{\overline{\lambda}^s} (i_s - r_n) \right].$$

Similarly

$$\pi_t = \pi_t^S + \frac{\kappa}{1-\beta} \underline{\lambda}^t \Omega_0,$$

where

$$\pi_t^S \equiv \frac{\sigma^{-1}\kappa}{\overline{\lambda}-\underline{\lambda}} \underline{\lambda}^t \left[\sum_{s=0}^{t-1} \left(\frac{\overline{\lambda}}{\underline{\lambda}^s} - \frac{\underline{\lambda}}{\overline{\lambda}^s} \right) (i_s - r_n) + \sum_{s=t}^{\infty} \left(\left(\frac{\overline{\lambda}}{\underline{\lambda}} \right)^t - 1 \right) \frac{\underline{\lambda}}{\overline{\lambda}^s} (i_s - r_n) \right].$$

Note that for $t > 0$,

$$\frac{\partial \pi_t^S}{\partial i_s} = \begin{cases} \frac{\sigma^{-1}\kappa}{\overline{\lambda}-\underline{\lambda}} \underline{\lambda}^t \left(\overline{\lambda}^{s+1} - \underline{\lambda}^{s+1} \right) \beta^s > 0 & \text{if } s < t \\ \frac{\sigma^{-1}\kappa}{\overline{\lambda}-\underline{\lambda}} \underline{\lambda}^t \left(\left(\frac{\overline{\lambda}}{\underline{\lambda}} \right)^t - 1 \right) \frac{\underline{\lambda}}{\overline{\lambda}^s} > 0 & \text{if } s \geq t \end{cases}$$

□

Proof of Corollary 1. Immediate from the fact that, given $\{i_t\}_{t=0}^{\infty}$, $\{c_t^S, \pi_t^S\}_{t=0}^{\infty}$ is unique and $\frac{1-\beta\underline{\lambda}}{1-\beta} \underline{\lambda}^t$ depends

only on the parameters of the model.

□

Proof of Proposition 2. Consider first the Taylor equilibrium. The economy is characterized by the following system of equations:

$$\begin{aligned} c_{t+1} &= c_t + \sigma^{-1}\left(i_t - \pi_{t+1} - r_n\right), \\ \pi_t &= \beta\pi_{t+1} + \kappa c_t, \\ i_t &= r_n + \phi_\pi \pi_t + \phi_y c_t + \varepsilon_t, \end{aligned}$$

where $\kappa\left(\phi_\pi - 1\right) + \left(1-\beta\right)\phi_y > 0$ and $\phi_y > \frac{\sqrt{4\beta\sigma^{-1}\left(\kappa(\phi_\pi-1)+(1-\beta)\phi_y\right)}-\left(1-\beta+\sigma^{-1}\kappa\right)}{\sigma^{-1}\beta}$. The solution to this system is given by

$$c_t^* = -\frac{\sigma^{-1}}{\beta\left(\overline{\delta}-\underline{\delta}\right)}\sum_{s=t}^{\infty}\left(\frac{\beta\overline{\delta}-1}{\overline{\delta}^{s+1-t}} - \frac{\beta\underline{\delta}-1}{\underline{\delta}^{s+1-t}}\right)\varepsilon_s, \qquad \pi_t^* = -\frac{\kappa\sigma^{-1}}{\beta\left(\overline{\delta}-\underline{\delta}\right)}\sum_{s=t}^{\infty}\left(\frac{1}{\underline{\delta}^{s+1-t}} - \frac{1}{\overline{\delta}^{s+1-t}}\right)\varepsilon_s,$$

and

$$i_t^* = r_n - \phi_\pi\frac{\kappa\sigma^{-1}}{\beta\left(\overline{\delta}-\underline{\delta}\right)}\sum_{s=t}^{\infty}\left(\frac{1}{\underline{\delta}^{s+1-t}} - \frac{1}{\overline{\delta}^{s+1-t}}\right)\varepsilon_s - \phi_y\frac{\sigma^{-1}}{\beta\left(\overline{\delta}-\underline{\delta}\right)}\sum_{s=t}^{\infty}\left(\frac{\beta\overline{\delta}-1}{\overline{\delta}^{s+1-t}} - \frac{\beta\underline{\delta}-1}{\underline{\delta}^{s+1-t}}\right)\varepsilon_s + \varepsilon_t,$$

where

$$\begin{aligned} \overline{\delta} &= \frac{\left(1+\beta+\sigma^{-1}\kappa+\sigma^{-1}\beta\phi_y\right) + \sqrt{\left(1+\beta+\sigma^{-1}\kappa+\sigma^{-1}\beta\phi_y\right)^2 - 4\beta\left(1+\sigma^{-1}\kappa\phi_\pi+\sigma^{-1}\phi_y\right)}}{2\beta} > 1, \\ \underline{\delta} &= \frac{\left(1+\beta+\sigma^{-1}\kappa+\sigma^{-1}\beta\phi_y\right) - \sqrt{\left(1+\beta+\sigma^{-1}\kappa+\sigma^{-1}\beta\phi_y\right)^2 - 4\beta\left(1+\sigma^{-1}\kappa\phi_\pi+\sigma^{-1}\phi_y\right)}}{2\beta} > 1. \end{aligned}$$

Note that if the sequence of shocks decays sufficiently fast, then the sequence of nominal interest rates $\{i_t^*\}_{t=0}^{\infty}$ also decays sufficiently fast.

Next, we compute the unique purely forward-looking solution to the system (1)-(2). Recall that the solution to the system can be written as

$$c_t = c_t^S + \frac{1-\beta\underline{\lambda}}{1-\beta}\underline{\lambda}^t\Omega_0, \tag{24}$$

with

$$c_t^S = \sigma^{-1}\frac{1-\beta\underline{\lambda}}{\overline{\lambda}-\underline{\lambda}}\underline{\lambda}^t\left[\sum_{s=0}^{t-1}\left(\frac{\overline{\lambda}}{\underline{\lambda}^s} - \frac{\underline{\lambda}}{\overline{\lambda}^s}\right)\left(i_s - r_n\right) + \sum_{s=t}^{\infty}\left(\frac{1-\beta\overline{\lambda}}{1-\beta\underline{\lambda}}\left(\frac{\overline{\lambda}}{\underline{\lambda}}\right)^t - 1\right)\frac{\underline{\lambda}}{\overline{\lambda}^s}\left(i_s - r_n\right)\right].$$

Since Ω_0 is independent of t and equal to zero when $i_t = r_n$ for all t, it has to take the following form:

$$\Omega_0 = \sum_{s=0}^{\infty}\omega_s\left(i_s - r_n\right),$$

for some $\{\omega_s\}_{s=0}^{\infty}$. Plugging c_t^S and Ω_0 into (24) and combining terms, we get

$$c_t = \frac{1-\beta\underline{\lambda}}{1-\beta}\underline{\lambda}^t \sum_{s=0}^{t-1}\left[\sigma^{-1}\frac{1-\beta}{\overline{\lambda}-\underline{\lambda}}\left(\frac{\overline{\lambda}}{\underline{\lambda}^s}-\frac{\underline{\lambda}}{\overline{\lambda}^s}\right)+\omega_s\right](i_s-r_n)+$$
$$\frac{1-\beta\underline{\lambda}}{1-\beta}\underline{\lambda}^t \sum_{s=t}^{\infty}\left[\sigma^{-1}\frac{1-\beta}{\overline{\lambda}-\underline{\lambda}}\left(\frac{1-\beta\overline{\lambda}}{1-\beta\underline{\lambda}}\left(\frac{\overline{\lambda}}{\underline{\lambda}}\right)^t-1\right)\frac{\underline{\lambda}}{\overline{\lambda}^s}+\omega_s\right](i_s-r_n), \quad (25)$$

where we divided the summation in Ω_0 into a backward-looking and a forward-looking term. This expression is purely forward-looking if and only if $\sigma^{-1}\frac{1-\beta}{\overline{\lambda}-\underline{\lambda}}\left(\frac{\overline{\lambda}}{\underline{\lambda}^s}-\frac{\underline{\lambda}}{\overline{\lambda}^s}\right)+\omega_s=0$ for all s, or $\omega_s=-\sigma^{-1}\frac{1-\beta}{\overline{\lambda}-\underline{\lambda}}\left(\frac{\overline{\lambda}}{\underline{\lambda}^s}-\frac{\underline{\lambda}}{\overline{\lambda}^s}\right)$. Plugging this expression into (25), we get

$$c_t = -\frac{\sigma^{-1}}{\overline{\lambda}-\underline{\lambda}}\sum_{s=t}^{\infty}\left(\frac{\overline{\lambda}-1}{\underline{\lambda}^{s-t}}+\frac{1-\underline{\lambda}}{\overline{\lambda}^{s-t}}\right)(i_s-r_n),$$

where

$$\Omega_0 = -(1-\beta)\frac{\sigma^{-1}}{\overline{\lambda}-\underline{\lambda}}\sum_{s=0}^{\infty}\left(\frac{\overline{\lambda}}{\underline{\lambda}^s}-\frac{\underline{\lambda}}{\overline{\lambda}^s}\right)(i_s-r_n).$$

Note that $\lim_{t\to\infty} c_t$ and Ω_0 are finite if and only if $i_t=\mathcal{O}\left(\xi^t\right)$ for $\xi<\underline{\lambda}$. This condition holds for sequences of nominal interest rates generated by a Taylor rule and shocks that decay sufficiently fast.

We want to show that the Taylor solution coincides with the purely forward-looking solution to the system (1)-(2) evaluated at $\{i_t^*\}_{t=0}^{\infty}$, that is

$$-\frac{\sigma^{-1}}{\overline{\lambda}-\underline{\lambda}}\sum_{s=t}^{\infty}\left(\frac{\overline{\lambda}-1}{\underline{\lambda}^{s-t}}+\frac{1-\underline{\lambda}}{\overline{\lambda}^{s-t}}\right)(i_s^*-r_n)=-\frac{\sigma^{-1}}{\beta\left(\overline{\delta}-\underline{\delta}\right)}\sum_{s=t}^{\infty}\left(\frac{\beta\overline{\delta}-1}{\overline{\delta}^{s+1-t}}-\frac{\beta\underline{\delta}-1}{\underline{\delta}^{s+1-t}}\right)\varepsilon_s.$$

We will work with the LHS of this equality. Plugging in the solution for $\{i_t^*\}_{t=0}^{\infty}$, we get

$$-\frac{\sigma^{-1}}{\overline{\lambda}-\underline{\lambda}}\sum_{s=t}^{\infty}\left(\frac{\overline{\lambda}-1}{\underline{\lambda}^{s-t}}+\frac{1-\underline{\lambda}}{\overline{\lambda}^{s-t}}\right)(i_s^*-r_n)=-\frac{\sigma^{-1}}{\overline{\lambda}-\underline{\lambda}}\sum_{s=t}^{\infty}\left(\frac{\overline{\lambda}-1}{\underline{\lambda}^{s-t}}+\frac{1-\underline{\lambda}}{\overline{\lambda}^{s-t}}\right)\varepsilon_s+$$
$$\frac{\sigma^{-1}}{\overline{\lambda}-\underline{\lambda}}\frac{\sigma^{-1}}{\beta\left(\overline{\delta}-\underline{\delta}\right)}\sum_{s=t}^{\infty}\sum_{z=s}^{\infty}\left(\frac{\overline{\lambda}-1}{\underline{\lambda}^{s-t}}+\frac{1-\underline{\lambda}}{\overline{\lambda}^{s-t}}\right)\left[\frac{\phi_\pi\kappa-\phi_y\left(\beta\underline{\delta}-1\right)}{\underline{\delta}^{z+1-s}}-\frac{\phi_\pi\kappa-\phi_y\left(\beta\overline{\delta}-1\right)}{\overline{\delta}^{z+1-s}}\right]\varepsilon_z. \quad (26)$$

Consider the last term

$$\sum_{s=t}^{\infty}\sum_{z=s}^{\infty}\left(\frac{\overline{\lambda}-1}{\underline{\lambda}^{s-t}}+\frac{1-\underline{\lambda}}{\overline{\lambda}^{s-t}}\right)\left[\frac{\phi_\pi\kappa-\phi_y\left(\beta\underline{\delta}-1\right)}{\underline{\delta}^{z+1-s}}-\frac{\phi_\pi\kappa-\phi_y\left(\beta\overline{\delta}-1\right)}{\overline{\delta}^{z+1-s}}\right]\varepsilon_z=$$
$$\sum_{z=t}^{\infty}\left[-\left(\phi_\pi\kappa-\phi_y\left(\beta\overline{\delta}-1\right)\right)\frac{\left(\overline{\lambda}-\underline{\lambda}\right)\left(\frac{1}{\beta}-\overline{\delta}\right)}{\left(\overline{\lambda}-\overline{\delta}\right)\left(\underline{\lambda}-\overline{\delta}\right)}\overline{\delta}^{t-(z+1)}+\left(\phi_\pi\kappa-\phi_y\left(\beta\underline{\delta}-1\right)\right)\frac{\left(\overline{\lambda}-\underline{\lambda}\right)\left(\frac{1}{\beta}-\underline{\delta}\right)}{\left(\overline{\lambda}-\underline{\delta}\right)\left(\underline{\lambda}-\underline{\delta}\right)}\underline{\delta}^{t-(z+1)}-\right.$$
$$\left.\left(\phi_\pi\kappa-\phi_y\left(\beta\overline{\lambda}-1\right)\right)\frac{\left(\overline{\delta}-\underline{\delta}\right)\left(\frac{1}{\beta}-\overline{\lambda}\right)}{\left(\overline{\lambda}-\overline{\delta}\right)\left(\overline{\lambda}-\underline{\delta}\right)}\overline{\lambda}^{t-(z+1)}+\left(\phi_\pi\kappa-\phi_y\left(\beta\underline{\lambda}-1\right)\right)\frac{\left(\overline{\delta}-\underline{\delta}\right)\left(\frac{1}{\beta}-\underline{\lambda}\right)}{\left(\underline{\lambda}-\overline{\delta}\right)\left(\underline{\lambda}-\underline{\delta}\right)}\underline{\lambda}^{t-(z+1)}\right]\varepsilon_z.$$

Note that

$$\left(\overline{\lambda}-\overline{\delta}\right)\left(\underline{\lambda}-\overline{\delta}\right)=-\sigma^{-1}\frac{\phi_\pi\kappa-\phi_y\left(\beta\overline{\delta}-1\right)}{\beta},\qquad \left(\overline{\lambda}-\underline{\delta}\right)\left(\underline{\lambda}-\underline{\delta}\right)=-\sigma^{-1}\frac{\phi_\pi\kappa-\phi_y\left(\beta\underline{\delta}-1\right)}{\beta},$$
$$\left(\overline{\lambda}-\overline{\delta}\right)\left(\overline{\lambda}-\underline{\delta}\right)=\sigma^{-1}\frac{\phi_\pi\kappa-\phi_y\left(\beta\overline{\lambda}-1\right)}{\beta},\qquad \left(\underline{\lambda}-\overline{\delta}\right)\left(\underline{\lambda}-\underline{\delta}\right)=\sigma^{-1}\frac{\phi_\pi\kappa-\phi_y\left(\beta\underline{\lambda}-1\right)}{\beta}.$$

Hence

$$\sum_{s=t}^{\infty}\sum_{z=s}^{\infty}\left(\frac{\overline{\lambda}-1}{\underline{\lambda}^{s-t}}+\frac{1-\underline{\lambda}}{\overline{\lambda}^{s-t}}\right)\left[\frac{\phi_\pi\kappa-\phi_y\left(\beta\underline{\delta}-1\right)}{\underline{\delta}^{z+1-s}}-\frac{\phi_\pi\kappa-\phi_y\left(\beta\overline{\delta}-1\right)}{\overline{\delta}^{z+1-s}}\right]\varepsilon_z=$$
$$\sum_{z=t}^{\infty}\left[\sigma\left(\overline{\lambda}-\underline{\lambda}\right)\left(1-\beta\overline{\delta}\right)\overline{\delta}^{t-(z+1)}-\sigma\left(\overline{\lambda}-\underline{\lambda}\right)\left(1-\beta\underline{\delta}\right)\underline{\delta}^{t-(z+1)}-\sigma\left(\overline{\delta}-\underline{\delta}\right)\left(1-\beta\overline{\lambda}\right)\overline{\lambda}^{t-(z+1)}+\right.$$
$$\left.\sigma\left(\overline{\delta}-\underline{\delta}\right)\left(1-\beta\underline{\lambda}\right)\underline{\lambda}^{t-(z+1)}\right]\varepsilon_z.$$

Plugging back into (26), we get

$$c_t=-\frac{\sigma^{-1}}{\beta\left(\overline{\delta}-\underline{\delta}\right)}\sum_{s=t}^{\infty}\left(\frac{\beta\overline{\delta}-1}{\overline{\delta}^{s+1-t}}-\frac{\beta\underline{\delta}-1}{\underline{\delta}^{s+1-t}}\right)\varepsilon_s.$$

Finally, suppose $\varepsilon_t=\psi^t\varepsilon_0$ with $\psi\in(0,\underline{\lambda})$ and for some $\varepsilon_0>0$. Then

$$i_t^*=r_n-\phi_\pi\frac{\kappa\sigma^{-1}}{\beta\left(\overline{\delta}-\underline{\delta}\right)}\sum_{s=t}^{\infty}\left(\frac{1}{\underline{\delta}^{s+1-t}}-\frac{1}{\overline{\delta}^{s+1-t}}\right)\psi^s\varepsilon_0-\phi_y\frac{\sigma^{-1}}{\beta\left(\overline{\delta}-\underline{\delta}\right)}\sum_{s=t}^{\infty}\left(\frac{\beta\overline{\delta}-1}{\overline{\delta}^{s+1-t}}-\frac{\beta\underline{\delta}-1}{\underline{\delta}^{s+1-t}}\right)\psi^s\varepsilon_0+\varepsilon_t.$$

After some algebra, we get

$$i_t^*=r_n+\psi^t\nu\varepsilon_0,$$

where

$$\nu\equiv1-\phi_\pi\frac{\kappa\sigma^{-1}}{\beta\left(\overline{\delta}-\psi\right)\left(\underline{\delta}-\psi\right)}-\phi_y\frac{\left(1-\beta\psi\right)\sigma^{-1}}{\beta\left(\overline{\delta}-\psi\right)\left(\underline{\delta}-\psi\right)}.$$

Note that $\frac{\partial i_0^*}{\partial\varepsilon_0}>0$ if and only if $\nu>0$, or

$$\beta\psi^2-\left(1+\beta+\sigma^{-1}\kappa\right)\psi+1>0,$$

where we used that $\overline{\delta}\underline{\delta}=\frac{1+\sigma^{-1}\kappa\phi_\pi+\sigma^{-1}\phi_y}{\beta}$ and $\overline{\delta}+\underline{\delta}=\frac{1+\beta+\sigma^{-1}\kappa+\sigma^{-1}\beta\phi_y}{\beta}$. Since $\psi\in(0,1)$, this condition holds if and only if $\psi<\underline{\lambda}$.

□

Proof of Proposition 4. From equation (15), we have

$$\Omega_0=\left[1-\left(\tau+\left(\frac{1}{\beta}+\frac{\underline{\lambda}}{1-\beta\underline{\lambda}}\right)\kappa Qb\right)\right]\Omega_0+$$
$$\left(1-\beta\right)\left[\sum_{t=0}^{\infty}\beta^t\left[\left(i_t-\pi_{t+1}^S-r_n\right)Qb-\rho^t\left(i_t-r_n\right)\rho Qb+T_t\right]\right].$$

Note that

$$\sum_{t=0}^{\infty}\beta^t\pi_{t+1}^S=\sum_{t=0}^{\infty}\beta^t\left(1-\underline{\lambda}^{t+1}\right)(i_t-r_n).$$

Then

$$\Omega_0=\left[1-\left(\tau+\left(\frac{1}{\beta}+\frac{\underline{\lambda}}{1-\beta\underline{\lambda}}\right)\kappa Qb\right)\right]\Omega_0+(1-\beta)\left[\sum_{t=0}^{\infty}\beta^t\left(\underline{\lambda}^{t+1}-\rho^{t+1}\right)(i_t-r_n)Qb+\sum_{t=0}^{\infty}\beta^tT_t\right].$$

If $\tau>0$ or $b>0$ (or both), we get

$$\Omega_0=\frac{1-\beta}{\tau+\left(\frac{1}{\beta}+\frac{\underline{\lambda}}{1-\beta\underline{\lambda}}\right)\kappa Qb}\left[\sum_{t=0}^{\infty}\beta^t\left(\underline{\lambda}^{t+1}-\rho^{t+1}\right)(i_t-r_n)Qb+\sum_{t=0}^{\infty}\beta^tT_t\right].$$

□

Proof of Proposition 5. The wealth effect in the FTLP is given by

$$\Omega_0^{FTPL}=\frac{1-\beta}{\left(\frac{1}{\beta}+\frac{\underline{\lambda}}{1-\beta\underline{\lambda}}\right)\kappa}\sum_{t=0}^{\infty}\beta^t\left(\underline{\lambda}^{t+1}-\rho^{t+1}\right)(i_t-r_n). \tag{27}$$

Thus,

$$\frac{\partial\Omega_0^{FTPL}}{\partial i_t}=\frac{1-\beta}{\left(\frac{1}{\beta}+\frac{\underline{\lambda}}{1-\beta\underline{\lambda}}\right)\kappa}\beta^t\left(\underline{\lambda}^{t+1}-\rho^{t+1}\right)<0\iff\rho>\underline{\lambda}.$$

□

Proof of Proposition 6. The system of equations characterizing equilibrium is given by

$$\begin{bmatrix}c_{t+1}\\ \pi_{t+1}\end{bmatrix}=\begin{bmatrix}1+\tilde{\sigma}^{-1}\frac{\kappa}{\beta} & -\frac{\tilde{\sigma}^{-1}}{\beta}\\ -\frac{\kappa}{\beta} & \frac{1}{\beta}\end{bmatrix}\begin{bmatrix}c_t\\ \pi_t\end{bmatrix}+\begin{bmatrix}\tilde{\sigma}^{-1}(i_t-r_{n,t})\\ 0\end{bmatrix},$$

where

$$r_{n,t}=r_n-\tilde{\sigma}\frac{\omega\chi_T}{1-\omega\chi_y}(T_{h,t+1}-T_{h,t}).$$

The solution is analogous to the one in the RANK model (see proof of Proposition 1), with $\tilde{\sigma}^{-1}$ taking the place of σ^{-1} and $r_{n,t}$ taking the place of r_n. Then, we can separate v_t to get

$$c_t^S=\tilde{\sigma}^{-1}\frac{1-\beta\underline{\lambda}}{\overline{\lambda}-\underline{\lambda}}\underline{\lambda}^t\left[\sum_{s=0}^{t-1}\left(\frac{\overline{\lambda}}{\underline{\lambda}^s}-\frac{\underline{\lambda}}{\overline{\lambda}^s}\right)(i_s-r_n)+\sum_{s=t}^{\infty}\left(\frac{1-\beta\overline{\lambda}}{1-\beta\underline{\lambda}}\left(\frac{\overline{\lambda}}{\underline{\lambda}}\right)^t-1\right)\frac{\underline{\lambda}}{\overline{\lambda}^s}(i_s-r_n)\right],$$

$$c_t^T\equiv\frac{\omega\chi_T}{1-\omega\chi_y}\frac{1-\beta\underline{\lambda}}{\overline{\lambda}-\underline{\lambda}}\underline{\lambda}^t\left[\sum_{s=0}^{t-1}\left(\frac{\overline{\lambda}}{\underline{\lambda}^s}-\frac{\underline{\lambda}}{\overline{\lambda}^s}\right)(\hat{T}_{h,s+1}-\hat{T}_{h,s})+\sum_{s=t}^{\infty}\left(\frac{1-\beta\overline{\lambda}}{1-\beta\underline{\lambda}}\left(\frac{\overline{\lambda}}{\underline{\lambda}}\right)^t-1\right)\frac{\underline{\lambda}}{\overline{\lambda}^s}(\hat{T}_{h,s+1}-\hat{T}_{h,s})\right].$$

□

Proof of Proposition 7. The system of equations characterizing equilibrium is given by

$$\underbrace{\begin{bmatrix} \pi_{t+2} \\ \pi_{t+1} \\ c_{t+1} \\ k_{t+1} \end{bmatrix}}_{=X_{t+1}} = \underbrace{\begin{bmatrix} \xi_{\pi\pi} & 0 & -\xi_{\pi c} & 0 \\ 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -\beta\xi_{k\pi} & \xi_{k\pi} & -\xi_{kc} & \xi_{kk} \end{bmatrix}}_{=A} \underbrace{\begin{bmatrix} \pi_{t+1} \\ \pi_t \\ c_t \\ k_t \end{bmatrix}}_{=X_t} + \underbrace{\begin{bmatrix} -\xi_{\pi i}\left(i_t - r_n\right) \\ 0 \\ \left(i_t - r_n\right) \\ 0 \end{bmatrix}}_{=b_t},$$

where

$$\xi_{\pi\pi} \equiv \frac{1}{\beta}\left(1 + \psi + \frac{\psi\gamma\left(1-\delta\right)}{r}\right), \quad \xi_{\pi i} \equiv \frac{\psi}{\beta}\left(1 + \frac{\gamma\left(1-\delta\right)}{r}\right), \quad \xi_{\pi c} \equiv \frac{\psi}{\beta}\left(1-\gamma\right),$$
$$\xi_{k\pi} \equiv \frac{1}{\psi}\frac{1-\gamma}{\gamma}\frac{\delta}{s_I}, \quad \xi_{kc} \equiv \frac{\delta}{s_I}\left(\frac{1-\gamma}{\gamma} + s_c\right), \quad \xi_{kk} \equiv 1 - \delta + \frac{\delta}{s_I}.$$

Since k_t is the only predetermined variable in the system, the Rational Expectations Equilibrium of this economy is determinate if and only if the matrix A has exactly one eigenvalue inside the unit circle and three eigenvalues outside. It is immediate to see that $\xi_{kk} > 1$ is an eigenvalue of the matrix A. Therefore, the characteristic polynomial of A can be written as

$$p(\lambda) = (\xi_{kk} - \lambda)(-\lambda)\left[(\xi_{\pi\pi} - \lambda)(1-\lambda) - \xi_{\pi c}\right].$$

The roots of the polynomial are thus given by

$$\lambda_1 = 0,$$
$$\lambda_2 = \frac{\xi_{\pi\pi} + 1 - \sqrt{\left(\xi_{\pi\pi}+1\right)^2 - 4\left(\xi_{\pi\pi} - \xi_{\pi c}\right)}}{2},$$
$$\lambda_3 = \frac{\xi_{\pi\pi} + 1 + \sqrt{\left(\xi_{\pi\pi}+1\right)^2 - 4\left(\xi_{\pi\pi} - \xi_{\pi c}\right)}}{2},$$
$$\lambda_4 = \xi_{kk}.$$

Since $\xi_{\pi\pi} > 1$ and $\xi_{\pi\pi} - \xi_{\pi' c} > 0$, $\lambda_2 \in (0,1)$ and $\lambda_3 > 1$. The associated eigenvectors are

$$v_1 = \begin{bmatrix} 0 & \xi_{kk} & 0 & -\xi_{k\pi} \end{bmatrix}',$$
$$v_2 = \begin{bmatrix} 1 & \frac{1}{\lambda_2} & \frac{1}{1-\lambda_2} & \frac{\beta\xi_{k\pi} - \frac{\xi_{k\pi}}{\lambda_2} + \frac{\xi_{kc}}{1-\lambda_2}}{\xi_{kk} - \lambda_2} \end{bmatrix}',$$
$$v_3 = \begin{bmatrix} 1 & \frac{1}{\lambda_3} & \frac{1}{1-\lambda_3} & \frac{\beta\xi_{k\pi} - \frac{\xi_{k\pi}}{\lambda_3} + \frac{\xi_{kc}}{1-\lambda_3}}{\xi_{kk} - \lambda_3} \end{bmatrix}',$$
$$v_4 = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}'.$$

Let

$$\Lambda \equiv \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 \\ 0 & 0 & 0 & \lambda_4 \end{bmatrix},$$

and

$$V \equiv \begin{bmatrix} 0 & 1 & 1 & 0 \\ \xi_{kk} & v_{22} & v_{32} & 0 \\ 0 & v_{23} & v_{33} & 0 \\ -\xi_{k\pi} & v_{24} & v_{34} & 1 \end{bmatrix},$$

where

$$v_{22} \equiv \frac{1}{\lambda_2}, \quad v_{23} \equiv \frac{1}{1-\lambda_2}, \quad v_{24} = \frac{\beta\xi_{k\pi} - \frac{\xi_{k\pi}}{\lambda_2} + \frac{\xi_{kc}}{1-\lambda_2}}{\xi_{kk} - \lambda_2},$$
$$v_{32} \equiv \frac{1}{\lambda_3}, \quad v_{33} \equiv \frac{1}{1-\lambda_3}, \quad v_{34} = \frac{\beta\xi_{k\pi} - \frac{\xi_{k\pi}}{\lambda_3} + \frac{\xi_{kc}}{1-\lambda_3}}{\xi_{kk} - \lambda_3}.$$

After some algebra, it is possible to get that

$$V^{-1} = \begin{bmatrix} \tilde{v}_{11} & \frac{1}{\xi_{kk}} & \tilde{v}_{13} & 0 \\ \tilde{v}_{21} & 0 & \tilde{v}_{23} & 0 \\ \tilde{v}_{31} & 0 & \tilde{v}_{33} & 0 \\ \tilde{v}_{41} & \frac{\xi_{k\pi}}{\xi_{kk}} & \tilde{v}_{43} & 1 \end{bmatrix},$$

where

$$\tilde{v}_{11} = -\frac{1}{\xi_{kk}}\frac{1}{\lambda_2\lambda_3}, \quad \tilde{v}_{21} = \frac{1-\lambda_2}{\lambda_3-\lambda_2}, \quad \tilde{v}_{31} = -\frac{1-\lambda_3}{\lambda_3-\lambda_2},$$
$$\tilde{v}_{41} = \frac{\beta\xi_{k\pi} + \xi_{k\pi} + \xi_{kc} - \beta\xi_{k\pi}\xi_{kk} + \frac{\xi_{kk}-(\lambda_2+\lambda_3)}{\lambda_2\lambda_3}\xi_{k\pi}}{(\xi_{kk}-\lambda_2)(\xi_{kk}-\lambda_3)} - \frac{\xi_{k\pi}}{\xi_{kk}}\frac{1}{\lambda_2\lambda_3},$$
$$\tilde{v}_{13} = \frac{1}{\xi_{kk}}\frac{(1-\lambda_2)(1-\lambda_3)}{\lambda_2\lambda_3}, \quad \tilde{v}_{23} = -\frac{(1-\lambda_2)(1-\lambda_3)}{\lambda_3-\lambda_2}, \quad \tilde{v}_{33} = \frac{(1-\lambda_2)(1-\lambda_3)}{\lambda_3-\lambda_2},$$
$$\tilde{v}_{43} = \frac{(1-\lambda_2)(1-\lambda_3)\left[-\beta\xi_{k\pi} + \frac{(\lambda_2+\lambda_3)-\xi_{kk}}{\lambda_2\lambda_3}\xi_{k\pi} + \frac{(\lambda_2+\lambda_3)-\xi_{kk}-1}{(1-\lambda_2)(1-\lambda_3)}\xi_{kc}\right]}{(\xi_{kk}-\lambda_2)(\xi_{kk}-\lambda_3)} + \frac{\xi_{k\pi}}{\xi_{kk}}\frac{(1-\lambda_2)(1-\lambda_3)}{\lambda_2\lambda_3}.$$

Then, we can rewrite the system as

$$X_{t+1} = V\Lambda V^{-1}X_t + b_t,$$

or

$$Y_{t+1} = \Lambda Y_t + m_t,$$

where $Y_t \equiv V^{-1}X_t$, $m_t \equiv V^{-1}b_t$. Then

$$y_{1,t} = \begin{cases} y_{1,0} & \text{if } t = 0 \\ m_{1,t-1} & \text{if } t \geq 1 \end{cases}$$

$$y_{2,t} = \lambda_2^t y_{2,0} + \lambda_2^t \sum_{k=0}^{t-1} \lambda_2^{-(k+1)} m_{2,k}$$

$$y_{3,t} = -\lambda_3^t \sum_{k=t}^{\infty} \lambda_3^{-(k+1)} m_{3,k}$$

$$y_{4,t} = -\lambda_4^t \sum_{k=t}^{\infty} \lambda_4^{-(k+1)} m_{4,k}$$

Note that

$$m_t = \begin{bmatrix} \tilde{\boldsymbol{v}}_{11} & \frac{1}{\xi_{kk}} & \tilde{\boldsymbol{v}}_{13} & 0 \\ \tilde{\boldsymbol{v}}_{21} & 0 & \tilde{\boldsymbol{v}}_{23} & 0 \\ \tilde{\boldsymbol{v}}_{31} & 0 & \tilde{\boldsymbol{v}}_{33} & 0 \\ \tilde{\boldsymbol{v}}_{41} & \frac{\xi_{k\pi}}{\xi_{kk}} & \tilde{\boldsymbol{v}}_{43} & 1 \end{bmatrix} \begin{bmatrix} -\xi_{\pi i}\left(i_t - r_n\right) \\ 0 \\ \left(i_t - r_n\right) \\ 0 \end{bmatrix} = \begin{bmatrix} \left(\tilde{\boldsymbol{v}}_{13} - \tilde{\boldsymbol{v}}_{11}\xi_{\pi i}\right)\left(i_t - r_n\right) \\ \left(\tilde{\boldsymbol{v}}_{23} - \tilde{\boldsymbol{v}}_{21}\xi_{\pi i}\right)\left(i_t - r_n\right) \\ \left(\tilde{\boldsymbol{v}}_{33} - \tilde{\boldsymbol{v}}_{31}\xi_{\pi i}\right)\left(i_t - r_n\right) \\ \left(\tilde{\boldsymbol{v}}_{43} - \tilde{\boldsymbol{v}}_{41}\xi_{\pi i}\right)\left(i_t - r_n\right) \end{bmatrix}.$$

Hence

$$y_{1,t} = \begin{cases} \underbrace{\tilde{\boldsymbol{v}}_{11}\pi_1 + \frac{1}{\xi_{kk}}\pi_0 + \tilde{\boldsymbol{v}}_{13}c_0}_{y_{1,0}} & \text{if } t = 0 \\ \left(\tilde{\boldsymbol{v}}_{13} - \tilde{\boldsymbol{v}}_{11}\xi_{\pi i}\right)\left(i_{t-1} - r_n\right) & \text{if } t \geq 1 \end{cases}$$

$$y_{2,t} = \lambda_2^t \underbrace{\left[\tilde{\boldsymbol{v}}_{21}\pi_1 + \tilde{\boldsymbol{v}}_{23}c_0\right]}_{y_{2,0}} + \left(\tilde{\boldsymbol{v}}_{23} - \tilde{\boldsymbol{v}}_{21}\xi_{\pi i}\right)\lambda_2^t \sum_{k=0}^{t-1} \lambda_2^{-(k+1)}\left(i_k - r_n\right),$$

$$y_{3,t} = -\left(\tilde{\boldsymbol{v}}_{33} - \tilde{\boldsymbol{v}}_{31}\xi_{\pi i}\right)\lambda_3^t \sum_{k=t}^{\infty} \lambda_3^{-(k+1)}\left(i_k - r_n\right)$$

$$y_{4,t} = -\left(\tilde{\boldsymbol{v}}_{43} - \tilde{\boldsymbol{v}}_{41}\xi_{\pi i}\right)\lambda_4^t \sum_{k=t}^{\infty} \lambda_4^{-(k+1)}\left(i_k - r_n\right)$$

where we used that $Y_0 = V^{-1}X_0$. We can recover the original variables using that $X_t = VY_t$, which implies that

$$\pi_{t+1} = \lambda_2^t\left[\tilde{\boldsymbol{v}}_{21}\pi_1 + \tilde{\boldsymbol{v}}_{23}c_0\right] + \left(\tilde{\boldsymbol{v}}_{23} - \tilde{\boldsymbol{v}}_{21}\xi_{\pi i}\right)\lambda_2^t \sum_{k=0}^{t-1} \lambda_2^{-(k+1)}\left(i_k - r_n\right) - \left(\tilde{\boldsymbol{v}}_{33} - \tilde{\boldsymbol{v}}_{31}\xi_{\pi i}\right)\lambda_3^t \sum_{k=t}^{\infty} \lambda_3^{-(k+1)}\left(i_k - r_n\right)$$

$$\pi_t = \begin{cases} \xi_{kk}\left[\tilde{\boldsymbol{v}}_{11}\pi_1 + \frac{1}{\xi_{kk}}\pi_0 + \tilde{\boldsymbol{v}}_{13}c_0\right] + \boldsymbol{v}_{22}\left[\tilde{\boldsymbol{v}}_{21}\pi_1 + \tilde{\boldsymbol{v}}_{23}c_0\right] - \boldsymbol{v}_{32}\left(\tilde{\boldsymbol{v}}_{33} - \tilde{\boldsymbol{v}}_{31}\xi_{\pi i}\right)\sum_{k=0}^{\infty} \lambda_3^{-(k+1)}\left(i_k - r_n\right) & \text{if } t = 0 \\ \xi_{kk}\left(\tilde{\boldsymbol{v}}_{13} - \tilde{\boldsymbol{v}}_{11}\xi_{\pi i}\right)\left(i_{t-1} - r_n\right) + \boldsymbol{v}_{22}\lambda_2^t\left[\left[\tilde{\boldsymbol{v}}_{21}\pi_1 + \tilde{\boldsymbol{v}}_{23}c_0\right] + \left(\tilde{\boldsymbol{v}}_{23} - \tilde{\boldsymbol{v}}_{21}\xi_{\pi i}\right)\sum_{k=0}^{t-1} \lambda_2^{-(k+1)}\left(i_k - r_n\right)\right] & \\ \qquad - \boldsymbol{v}_{32}\left(\tilde{\boldsymbol{v}}_{33} - \tilde{\boldsymbol{v}}_{31}\xi_{\pi i}\right)\lambda_3^t \sum_{k=t}^{\infty} \lambda_3^{-(k+1)}\left(i_k - r_n\right) & \text{if } t \geq 1 \end{cases}$$

$$c_t = \boldsymbol{v}_{23}\lambda_2^t\left[\left[\tilde{\boldsymbol{v}}_{21}\pi_1 + \tilde{\boldsymbol{v}}_{23}c_0\right] + \left(\tilde{\boldsymbol{v}}_{23} - \tilde{\boldsymbol{v}}_{21}\xi_{\pi i}\right)\sum_{k=0}^{t-1} \lambda_2^{-(k+1)}\left(i_k - r_n\right)\right] - \boldsymbol{v}_{33}\left(\tilde{\boldsymbol{v}}_{33} - \tilde{\boldsymbol{v}}_{31}\xi_{\pi i}\right)\lambda_3^t \sum_{k=t}^{\infty} \lambda_3^{-(k+1)}\left(i_k - r_n\right)$$

$$k_t = \begin{cases} -\xi_{k\pi}\left[\tilde{v}_{11}\pi_1 + \dfrac{1}{\xi_{kk}}\pi_0 + \tilde{v}_{13}c_0\right] + v_{24}\left[\tilde{v}_{21}\pi_1 + \tilde{v}_{23}c_0\right] - v_{34}\left(\tilde{v}_{33} - \tilde{v}_{31}\xi_{\pi i}\right)\sum_{k=0}^{\infty}\lambda_3^{-(k+1)}\left(i_k - r_n\right) - \\ \qquad\qquad \left(\tilde{v}_{43} - \tilde{v}_{41}\xi_{\pi i}\right)\sum_{k=0}^{\infty}\lambda_4^{-(k+1)}\left(i_k - r_n\right) & \text{if } t = 0 \\ -\xi_{k\pi}\left(\tilde{v}_{13} - \tilde{v}_{11}\xi_{\pi i}\right)\left(i_{t-1} - r_n\right) + v_{24}\lambda_2^t\left[\left[\tilde{v}_{21}\pi_1 + \tilde{v}_{23}c_0\right] + \left(\tilde{v}_{23} - \tilde{v}_{21}\xi_{\pi i}\right)\sum_{k=0}^{t-1}\lambda_2^{-(k+1)}\left(i_k - r_n\right)\right] - \\ \qquad v_{34}\left(\tilde{v}_{33} - \tilde{v}_{31}\xi_{\pi i}\right)\lambda_3^t\sum_{k=t}^{\infty}\lambda_3^{-(k+1)}\left(i_k - r_n\right) - \left(\tilde{v}_{43} - \tilde{v}_{41}\xi_{\pi i}\right)\lambda_4^t\sum_{k=t}^{\infty}\lambda_4^{-(k+1)}\left(i_k - r_n\right) & \text{if } t \geq 1 \end{cases}$$

Let $\Omega_0 \equiv (1-\beta)\sum_{t=0}^{\infty}\beta^t y_t$, $\Omega_0^c \equiv (1-\beta)\sum_{t=0}^{\infty}\beta^t c_t$ and $\Omega_0^I \equiv (1-\beta)\sum_{t=0}^{\infty}\beta^t \iota_t$. Then

$$\Omega_0 = s_c\Omega_0^c + s_I\Omega_0^I$$

Using that $\sum_{t=0}^{\infty}\beta^t\iota_t = \frac{\frac{1}{\beta}-(1-\delta)}{\delta}\sum_{t=0}^{\infty}\beta^t k_t$ and letting $\Omega_0^k \equiv (1-\beta)\sum_{t=0}^{\infty}\beta^t k_t$, we get

$$\Omega_0 = s_c\Omega_0^c + \frac{s_I}{\delta}\left[\frac{1}{\beta} + (1-\delta)\right]\Omega_0^k \tag{28}$$

We can use the expressions above to compute the sums. For consumption we have

$$\frac{\Omega_0^c}{1-\beta} = \sum_{t=0}^{\infty}\beta^t c_t = \frac{\boldsymbol{v}_{23}}{1-\beta\lambda_2}\left[\left[\tilde{\boldsymbol{v}}_{21}\pi_1 + \tilde{\boldsymbol{v}}_{23}c_0\right] + \left(\tilde{\boldsymbol{v}}_{23} - \tilde{\boldsymbol{v}}_{21}\xi_{\pi i}\right)\sum_{t=0}^{\infty}\beta^{t+1}\left(i_t - r_n\right)\right] - \\ \frac{\boldsymbol{v}_{33}}{1-\beta\lambda_3}\left(\tilde{\boldsymbol{v}}_{33} - \tilde{\boldsymbol{v}}_{31}\xi_{\pi i}\right)\sum_{t=0}^{\infty}\left(\lambda_3^{-(t+1)} - \beta^{t+1}\right)\left(i_t - r_n\right)$$

hence

$$\tilde{\boldsymbol{v}}_{21}\pi_1 + \tilde{\boldsymbol{v}}_{23}c_0 = \frac{1}{\boldsymbol{v}_{23}}\frac{1-\beta\lambda_2}{1-\beta}\Omega_0^c + \frac{1-\beta\lambda_2}{1-\beta\lambda_3}\frac{\boldsymbol{v}_{33}}{\boldsymbol{v}_{23}}\left(\tilde{\boldsymbol{v}}_{33} - \tilde{\boldsymbol{v}}_{31}\xi_{\pi i}\right)\sum_{t=0}^{\infty}\left(\lambda_3^{-(t+1)} - \beta^{t+1}\right)\left(i_t - r_n\right) - \\ \left(\tilde{\boldsymbol{v}}_{23} - \tilde{\boldsymbol{v}}_{21}\xi_{\pi i}\right)\sum_{t=0}^{\infty}\beta^{t+1}\left(i_t - r_n\right) \tag{29}$$

For capital, we have

$$\frac{\Omega_0^k}{1-\beta} \equiv \sum_{t=1}^{\infty}\beta^t k_t = \frac{v_{24}}{1-\beta\lambda_2}\beta\lambda_2\left[\tilde{v}_{21}\pi_1 + \tilde{v}_{23}c_0\right] + \\ \left[-\xi_{k\pi}\left(\tilde{v}_{13} - \tilde{v}_{11}\xi_{\pi i}\right) + \frac{v_{24}}{1-\beta\lambda_2}\left(\tilde{v}_{23} - \tilde{v}_{21}\xi_{\pi i}\right)\right]\sum_{t=0}^{\infty}\beta^{t+1}\left(i_t - r_n\right) - \\ \frac{v_{34}}{1-\beta\lambda_3}\left(\tilde{v}_{33} - \tilde{v}_{31}\xi_{\pi i}\right)\sum_{t=0}^{\infty}\left(\beta\lambda_3^{-t} - \beta^{t+1}\right)\left(i_t - r_n\right) - \\ \frac{1}{1-\beta\lambda_4}\left(\tilde{v}_{43} - \tilde{v}_{41}\xi_{\pi i}\right)\sum_{t=0}^{\infty}\left(\beta\lambda_4^{-t} - \beta^{t+1}\right)\left(i_t - r_n\right)$$

Combining with (29), we get

$$\Omega_0^k = (1-\beta)\,\Gamma_0 + \frac{\boldsymbol{v}_{24}}{\boldsymbol{v}_{23}}\beta\lambda_2\Omega_0^c \tag{30}$$

where

$$\Gamma_0 \equiv \frac{\beta\lambda_2}{1-\beta\lambda_3}\frac{v_{24}\boldsymbol{v}_{33}}{\boldsymbol{v}_{23}}\left(\tilde{\boldsymbol{v}}_{33}-\tilde{\boldsymbol{v}}_{31}\xi_{\pi i}\right)\sum_{t=0}^{\infty}\left(\lambda_3^{-(t+1)}-\beta^{t+1}\right)\left(i_t-r_n\right)-$$
$$\frac{\beta\lambda_2}{1-\beta\lambda_2}v_{24}\left(\tilde{\boldsymbol{v}}_{23}-\tilde{\boldsymbol{v}}_{21}\xi_{\pi i}\right)\sum_{t=0}^{\infty}\beta^{t+1}\left(i_t-r_n\right)+$$
$$\left[-\xi_{k\pi}\left(\tilde{v}_{13}-\tilde{v}_{11}\xi_{\pi i}\right)+\frac{v_{24}}{1-\beta\lambda_2}\left(\tilde{v}_{23}-\tilde{v}_{21}\xi_{\pi i}\right)\right]\sum_{t=0}^{\infty}\beta^{t+1}\left(i_t-r_n\right)-$$
$$\frac{1}{1-\beta\lambda_3}v_{34}\left(\tilde{v}_{33}-\tilde{v}_{31}\xi_{\pi i}\right)\sum_{t=0}^{\infty}\left(\beta\lambda_3^{-t}-\beta^{t+1}\right)\left(i_t-r_n\right)-$$
$$\frac{1}{1-\beta\lambda_4}\left(\tilde{v}_{43}-\tilde{v}_{41}\xi_{\pi i}\right)\sum_{t=0}^{\infty}\left(\beta\lambda_4^{-t}-\beta^{t+1}\right)\left(i_t-r_n\right)$$

Introducing this expression into (28), we get

$$\Omega_0^c=-\frac{\frac{s_I}{\delta}\left[\frac{1}{\beta}+(1-\delta)\right]}{\frac{s_I}{\delta}\left[\frac{1}{\beta}+(1-\delta)\right]\frac{v_{24}}{v_{23}}\beta\lambda_2+s_c}\left(1-\beta\right)\Gamma_0+\frac{1}{\frac{s_I}{\delta}\left[\frac{1}{\beta}+(1-\delta)\right]\frac{v_{24}}{v_{23}}\beta\lambda_2+s_c}\Omega_0$$

and

$$\Omega_0^k=\frac{s_c}{\frac{s_I}{\delta}\left[\frac{1}{\beta}+(1-\delta)\right]\frac{v_{24}}{v_{23}}\beta\lambda_2+s_c}\left(1-\beta\right)\Gamma_0+\frac{\frac{v_{24}}{v_{23}}\beta\lambda_2}{\frac{s_I}{\delta}\left[\frac{1}{\beta}+(1-\delta)\right]\frac{v_{24}}{v_{23}}\beta\lambda_2+s_c}\Omega_0$$

Thus, we can rewrite consumption as

$$c_t=c_t^s-\frac{\frac{s_I}{\delta}\left[\frac{1}{\beta}+(1-\delta)\right]}{\frac{s_I}{\delta}\left[\frac{1}{\beta}+(1-\delta)\right]\frac{v_{24}}{v_{23}}\beta\lambda_2+s_c}\left(1-\beta\lambda_2\right)\lambda_2^t\Gamma_0+\frac{1}{\frac{s_I}{\delta}\left[\frac{1}{\beta}+(1-\delta)\right]\frac{v_{24}}{v_{23}}\beta\lambda_2+s_c}\frac{1-\beta\lambda_2}{1-\beta}\lambda_2^t\Omega_0$$

where

$$c_t^S\equiv\frac{1-\beta\lambda_2}{1-\beta\lambda_3}\boldsymbol{v}_{33}\left(\tilde{\boldsymbol{v}}_{33}-\tilde{\boldsymbol{v}}_{31}\xi_{\pi i}\right)\lambda_2^t\sum_{t=0}^{\infty}\left(\lambda_3^{-(t+1)}-\beta^{t+1}\right)\left(i_t-r_n\right)-$$
$$\boldsymbol{v}_{23}\left(\tilde{\boldsymbol{v}}_{23}-\tilde{\boldsymbol{v}}_{21}\xi_{\pi i}\right)\lambda_2^t\sum_{t=0}^{\infty}\beta^{t+1}\left(i_t-r_n\right)+\boldsymbol{v}_{23}\left(\tilde{\boldsymbol{v}}_{23}-\tilde{\boldsymbol{v}}_{21}\xi_{\pi i}\right)\lambda_2^t\sum_{k=0}^{t-1}\lambda_2^{-(k+1)}\left(i_k-r_n\right)-$$
$$\boldsymbol{v}_{33}\left(\tilde{\boldsymbol{v}}_{33}-\tilde{\boldsymbol{v}}_{31}\xi_{\pi i}\right)\lambda_3^t\sum_{k=t}^{\infty}\lambda_3^{-(k+1)}\left(i_k-r_n\right)$$

and it satisfies $\sum_{t=0}^{\infty}\beta^t c_t^S=0$. Moreover, we can rewrite capital as

$$k_t=k_t^S+\frac{s_c}{\frac{s_I}{\delta}\left[\frac{1}{\beta}+(1-\delta)\right]\frac{v_{24}}{v_{23}}\beta\lambda_2+s_c}\frac{1-\beta\lambda_2}{\beta\lambda_2}\lambda_2^t\Gamma_0+\frac{\frac{v_{24}}{v_{23}}}{\frac{s_I}{\delta}\left[\frac{1}{\beta}+(1-\delta)\right]\frac{v_{24}}{v_{23}}\beta\lambda_2+s_c}\frac{1-\beta\lambda_2}{1-\beta}\lambda_2^t\Omega_0$$

where

$$k_t^S = -\xi_{k\pi}\left(\tilde{v}_{13} - \tilde{v}_{11}\xi_{\pi i}\right)\left(i_{t-1} - r_n\right) - \frac{1-\beta\lambda_2}{\beta\lambda_2}\left[-\xi_{k\pi}\left(\tilde{v}_{13} - \tilde{v}_{11}\xi_{\pi i}\right) + \frac{v_{24}}{1-\beta\lambda_2}\left(\tilde{v}_{23} - \tilde{v}_{21}\xi_{\pi i}\right)\right]\lambda_2^t \sum_{t=0}^{\infty}\beta^{t+1}\left(i_t - r_n\right) + \frac{1}{\beta\lambda_2}v_{34}\frac{1-\beta\lambda_2}{1-\beta\lambda_3}\left(\tilde{v}_{33} - \tilde{v}_{31}\xi_{\pi i}\right)\lambda_2^t \sum_{t=0}^{\infty}\left(\beta\lambda_3^{-t} - \beta^{t+1}\right)\left(i_t - r_n\right) + \frac{1}{\beta\lambda_2}\frac{1-\beta\lambda_2}{1-\beta\lambda_4}\left(\tilde{v}_{43} - \tilde{v}_{41}\xi_{\pi i}\right)\lambda_2^t \sum_{t=0}^{\infty}\left(\beta\lambda_4^{-t} - \beta^{t+1}\right)\left(i_t - r_n\right) + v_{24}\left(\tilde{v}_{23} - \tilde{v}_{21}\xi_{\pi i}\right)\lambda_2^t \sum_{k=0}^{t-1}\lambda_2^{-(k+1)}\left(i_k - r_n\right) - v_{34}\left(\tilde{v}_{33} - \tilde{v}_{31}\xi_{\pi i}\right)\lambda_3^t \sum_{k=t}^{\infty}\lambda_3^{-(k+1)}\left(i_k - r_n\right) - \left(\tilde{v}_{43} - \tilde{v}_{41}\xi_{\pi i}\right)\lambda_4^t \sum_{k=t}^{\infty}\lambda_4^{-(k+1)}\left(i_k - r_n\right)$$

$$k_t^i \equiv \xi_{\pi i}\left[\xi_{k\pi}\tilde{\boldsymbol{v}}_{\mathbf{11}}\left(i_{t-1} - r_n\right) - \frac{1-\beta\lambda_2}{\beta\lambda_2}\lambda_2^t \sum_{k=0}^{\infty}\left[\xi_{k\pi}\tilde{\boldsymbol{v}}_{\mathbf{11}} - \frac{\boldsymbol{v_{24}}\tilde{\boldsymbol{v}}_{\mathbf{21}}}{1-\beta\lambda_2} - \frac{\boldsymbol{v_{34}}\tilde{\boldsymbol{v}}_{\mathbf{31}}}{1-\beta\lambda_3} - \frac{\tilde{\boldsymbol{v}}_{\mathbf{41}}}{1-\beta\lambda_4}\right]\beta^{k+1}\left(i_k - r_n\right) - \frac{1-\beta\lambda_2}{\lambda_2}\lambda_2^t \sum_{k=0}^{\infty}\left[\frac{\boldsymbol{v_{34}}\tilde{\boldsymbol{v}}_{\mathbf{31}}}{1-\beta\lambda_3}\lambda_3^{-k} + \frac{\tilde{\boldsymbol{v}}_{\mathbf{41}}}{1-\beta\lambda_4}\lambda_4^{-k}\right]\left(i_k - r_n\right) - \boldsymbol{v_{24}}\tilde{\boldsymbol{v}}_{\mathbf{21}}\lambda_2^t \sum_{k=0}^{t-1}\lambda_2^{-(k+1)}\left(i_k - r_n\right) + \boldsymbol{v_{34}}\tilde{\boldsymbol{v}}_{\mathbf{31}}\lambda_3^t \sum_{k=t}^{\infty}\lambda_3^{-(k+1)}\left(i_k - r_n\right) + \lambda_4^t \sum_{k=t}^{\infty}\lambda_4^{-(k+1)}\tilde{\boldsymbol{v}}_{\mathbf{41}}\xi_{\pi i}\left(i_k - r_n\right)\right]$$

and it satisfies $\sum_{t=0}^{\infty}\beta^t k_t^S = 0$.

For inflation, recall that we have

$$\pi_t = \beta\pi_{t+1} + \psi mc_t$$

hence

$$\pi_t = \psi\sum_{k=0}^{\infty}\beta^k mc_{t+k}$$

Moreover,

$$mc_t = c_t + \frac{\gamma}{1-\gamma}y_t - \frac{\gamma}{1-\gamma}k_t$$

therefore

$$\pi_t = \psi\sum_{k=0}^{\infty}\beta^k\left[c_{t+k} + \frac{\gamma}{1-\gamma}y_{t+k} - \frac{\gamma}{1-\gamma}k_{t+k}\right]$$

or

$$\pi_t = \pi_t^S + \widetilde{\Psi}_\Gamma\lambda_2^t\Gamma_0 + \frac{\widetilde{\Psi}_\Omega}{1-\beta}\lambda_2^t\Omega_0$$

where

$$\pi_t^S \equiv \psi \sum_{k=0}^{\infty} \beta^k \left[\left(1 + \frac{\gamma}{1-\gamma} s_c \right) c_{t+k}^s + \frac{\gamma}{1-\gamma} \frac{s_I}{\delta} k_{t+1+k}^S - \frac{\gamma}{1-\gamma} \left(1 + \frac{1-\delta}{\delta} s_I \right) k_{t+k}^S \right]$$

$$\widetilde{\Psi}_\Gamma \equiv \psi \frac{-\left(1 + \frac{\gamma}{1-\gamma} s_c\right) \frac{s_I}{\delta} \left[\frac{1}{\beta} + (1-\delta) \right] + \frac{\gamma}{1-\gamma} \frac{s_I}{\delta} s_c \frac{1}{\beta} - \frac{\gamma}{1-\gamma} \left(1 + \frac{1-\delta}{\delta} s_I \right) s_c \frac{1}{\beta \lambda_2}}{\frac{s_I}{\delta} \left[\frac{1}{\beta} + (1-\delta) \right] \frac{v_{24}}{v_{23}} \beta \lambda_2 + s_c}$$

$$\widetilde{\Psi}_\Omega \equiv \psi \frac{1 + \frac{\gamma}{1-\gamma} s_c + \frac{\gamma}{1-\gamma} \frac{s_I}{\delta} \frac{v_{24}}{v_{23}} \lambda_2 - \frac{\gamma}{1-\gamma} \left(1 + \frac{1-\delta}{\delta} s_I \right) \frac{v_{24}}{v_{23}}}{\frac{s_I}{\delta} \left[\frac{1}{\beta} + (1-\delta) \right] \frac{v_{24}}{v_{23}} \beta \lambda_2 + s_c}$$

For $t = 0$

$$\pi_0 = \psi \left[-\frac{\frac{s_I}{\delta} \left[\frac{1}{\beta} + (1-\delta) \right] + \frac{\gamma}{1-\gamma} \frac{s_c}{\beta \lambda_2}}{\frac{s_I}{\delta} \left[\frac{1}{\beta} + (1-\delta) \right] \frac{v_{24}}{v_{23}} \beta \lambda_2 + s_c} \Gamma_0 + \left(\frac{\gamma}{1-\gamma} + \frac{1 - \frac{\gamma}{1-\gamma} \frac{v_{24}}{v_{23}}}{\frac{s_I}{\delta} \left[\frac{1}{\beta} + (1-\delta) \right] \frac{v_{24}}{v_{23}} \beta \lambda_2 + s_c} \right) \frac{\Omega_0}{1-\beta} \right]$$

Hence, in general, $\pi_0 \neq 0$ when $\Omega_0 = 0$.

Finally, recall that

$$\Omega_0 = \sum_{t=0}^{\infty} \beta^t \left[(1-\tau) y_t + (i_t - \pi_{t+1} - r_n) Qb + T_t \right] - \left[\sum_{t=0}^{\infty} (\beta \rho)^t (i_t - r_n) \rho + \frac{1}{\beta} \pi_0 \right] Qb$$

Thus, we can write Ω_0 as

$$\Omega_0 = \frac{1-\beta}{\tau + \Psi_\Omega Qb} \left[\sum_{t=0}^{\infty} \beta^t \left[\left(i_t - \pi_t^S - r_n \right) Qb + T_t \right] - \left[\sum_{t=0}^{\infty} (\beta \rho)^t (i_t - r_n) \rho + \Psi_\Gamma \Gamma_0 \right] Qb \right]$$

where

$$\Psi_\Gamma \equiv \frac{1 - \beta \lambda_2 + \beta}{\beta (1 - \beta \lambda_2)} \widetilde{\Psi}_\Gamma, \qquad \Psi_\Omega \equiv \frac{1 - \beta \lambda_2 + \beta}{\beta (1 - \beta \lambda_2)} \widetilde{\Psi}_\Omega.$$

□

B Derivation of TANK model

This section derives the TANK model. Time is discrete and runs forever. The economy is populated by households, firms and a government. The model nests the standard RANK model as a special case.

Households. The economy is populated by a continuum of measure one of households. A measure $1-\omega$ of households are savers, indexed by s: they are forward-looking and can trade in asset markets. The complementary fraction ω corresponds to households that are hand-to-mouth (HtM), indexed by h: they have no access to financial markets and consume their labor income each period. The RANK model is a particular case in which $\omega = 0$.

Households receive labor income $W_t N_{j,t}$, profits from corporate holdings $\Pi_{j,t}$, and government transfers $P_t \widetilde{T}_{j,t}$, for $j \in \{s, h\}$. We assume that corporations are owned by savers, so $\Pi_{h,t} = 0$ for all $t \geq 0$.

The problem of a saver is given by

$$\max_{\{C_{s,t}, N_{s,t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(C_{s,t}, N_{s,t})$$

subject to the flow budget constraint

$$P_t C_{s,t} + Q_t B_{s,t+1} \leq (1 + \rho Q_t) B_{s,t} + W_t N_{s,t} + \Pi_{s,t} + P_t \widetilde{T}_{s,t},$$

where the price of long-term bonds satisfies $Q_t = \frac{1+\rho Q_{t+1}}{1+i_t}$.

The saver's optimality conditions are given by

$$-\frac{U^n_{s,t}}{U^c_{s,t}} = \frac{W_t}{P_t}$$
$$1 = (1 + i_t)\beta \frac{U^c_{s,t+1}}{U^c_{s,t}} \frac{P_t}{P_{t+1}},$$

where $U^c_{j,t} \equiv \frac{\partial U(C_{j,t}, N_{j,t})}{\partial C_{j,t}}$ and $U^n_{j,t} \equiv \frac{\partial U(C_{j,t}, N_{j,t})}{\partial N_{j,t}}$, and we used that $1 + i_t = \frac{1+\rho Q_{t+1}}{Q_t}$.

The problem of a HtM household is

$$\max_{\{C_{h,t}, N_{h,t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(C_{h,t}, N_{h,t})$$

subject to

$$P_t C_{h,t} \leq W_t N_{h,t} + P_t \widetilde{T}_{h,t}.$$

The HtM household's optimality condition is given by

$$-\frac{U^n_{h,t}}{U^c_{h,t}} = \frac{W_t}{P_t}.$$

In what follows, we assume that $U(C_{j,t}, N_{j,t}) = \frac{C_{j,t}^{1-\sigma}}{1-\sigma} - \frac{N_{j,t}^{1+\varphi}}{1+\varphi}$.

Firms. There are two types of firms in the economy: final-goods producers and intermediate-goods producers. Final-goods producers operate in a perfectly competitive market and combine a unit mass of intermediate goods $Y_t(i)$, for $i \in [0,1]$, using the production function

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}. \tag{31}$$

The problem of the final-good producer is given by

$$\max_{[Y_t(i)]_{i\in[0,1]}} P_t Y_t - \int_0^1 P_t(i) Y_t(i)$$

subject to (31). The solution to this problem gives the standard CES demand

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t, \tag{32}$$

where $P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$.

Intermediate goods are produced using the following technology:

$$Y_t(i) = N_t(i)^{1-\gamma},$$

with $\gamma \in [0,1)$. Firms choose the price for their good, $P_t(i)$, subject to the demand for their good, given by (32), taking the aggregate price level P_t and aggregate output, Y_t, as given. As is standard in New Keynesian models, we assume that firms are subject to a pricing friction à la Calvo: each firm may set a new price with probability $1-\theta$ in each period. Let P_t^* denote the price chosen by a firm that is able to set the price in period t. Then, P_t^* is the solution to the following problem:

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} [(1-\tau) P_t^* Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t})]$$

subject to

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} Y_{t+k},$$

where $Q_{t,t+k} \equiv \beta^k U^c_{s,t+k} / U^c_{s,t} P_t / P_{t+k}$ is the savers' stochastic discount factor for nominal payoffs, $\Psi_t(Y_{t+k|t}) = W_{t+k} Y_{t+k|t}^{\frac{1}{1-\gamma}}$ is the cost function, $Y_{t+k|t}$ denotes output in period $t+k$ for a firm that last set price in period t, and τ is a proportional sales tax. The first-order condition associated with this problem is given by

$$\sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k|t} \left[(1-\tau) P_t^* - \frac{\epsilon}{\epsilon-1} \Psi_t'(Y_{t+k|t}) \right] = 0.$$

Dividing this expression by P_t, we get

$$\sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k|t} \left[(1-\tau) \frac{P_t^*}{P_t} - \frac{\epsilon}{\epsilon-1} MC_{t+k|t} \frac{P_{t+k}}{P_t} \right] = 0,$$

where $MC_{t+k|t} \equiv \Psi_t'(Y_{t+k|t})/P_{t+k}$ is the real marginal cost in period $t+k$ for a firm whose price was last set in period t.

Government. We assume that the monetary authority follows an interest rate rule of the form

$$\log(1+i_t) = r_n + \phi_\pi \pi_t + \phi_y \log\left(\frac{Y_t}{Y}\right) + \varepsilon_{m,t},$$

where $r_n \equiv -\log\beta$, $\pi_t \equiv \log\left(\frac{P_t}{P_{t-1}}\right)$, Y is the zero-inflation steady-state level of output, and $\varepsilon_{m,t}$ denotes a monetary policy shock.

Moreover, the government chooses transfers to savers and HtM households, $\{\widetilde{T}_{s,t}, \widetilde{T}_{h,t}\}_{t=0}^{\infty}$, and the sales tax rate τ, to satisfy the flow budget constraint

$$Q_t B_{t+1} = (1+\rho Q_t)B_t + P_t(\omega \widetilde{T}_{h,t} + (1-\omega)\widetilde{T}_{s,t}) - \tau \int_0^1 P_t(i)Y_t(i)di$$

and the No-Ponzi condition $\lim_{t\to\infty} Q_t B_{t+1} = 0$.

Market clearing. The market clearing conditions for goods, labor and bonds are given by

$$\begin{aligned}
\omega C_{h,t} + (1-\omega)C_{s,t} &= Y_t, \\
\omega N_{h,t} + (1-\omega)N_{s,t} &= N_t, \\
(1-\omega)B_{s,t} &= B_t,
\end{aligned}$$

where $N_t = \int_0^1 N_t(i)di$ denotes the aggregate labor demand in period t.

Because of the Calvo friction, the price level can be written as

$$P_t = \left[(1-\theta)(P_t^*)^{1-\epsilon} + \int_{S(t)} (P_{t-1}(i))^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}},$$

where $S(t) \subset [0,1]$ is the set of firms that do not set a new price in period t. Since a random set of firms is able to change prices every period (independent of any firm characteristic), we have that

$$\int_{S(t)} (P_{t-1}(i))^{1-\epsilon} di = \theta P_{t-1}^{1-\epsilon}.$$

Hence, we can write the price level as

$$P_t = \left[(1-\theta)(P_t^*)^{1-\epsilon} + \theta P_{t-1}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}.$$

Steady state. Let the variables without subscript denote the value of the variables in a zero-inflation steady state.

Consumption of the HtM households is given by

$$C_h = \frac{W}{P} N_h + \widetilde{T}_h.$$

Consumption of savers is given by

$$C_s = \frac{W}{P}N_s + \frac{(1-\tau)Y - \frac{W}{P}N}{1-\omega} + \tilde{T}_s + \frac{1-\beta}{\beta}\frac{QB_s}{P},$$

where $B_s = \frac{B}{1-\omega}$, and $Q = \frac{\beta}{1-\beta\rho}$. Combining these two conditions, we obtain the government's budget constraint

$$\tau Y - \omega\tilde{T}_h - (1-\omega)\tilde{T}_s = \frac{1-\beta}{\beta}\frac{QB}{P}.$$

Optimal labor implies

$$\frac{W}{P} = N_j^{\varphi} C_j^{\sigma}$$

From the optimal pricing equation, we obtain

$$P = \frac{\epsilon}{\epsilon-1}\frac{W}{1-\tau}\frac{Y^{\frac{\gamma}{1-\gamma}}}{1-\gamma},$$

with $Y = N^{1-\gamma}$. Note that

$$\frac{WN}{PY} = (1-\tau)(1-\gamma)\frac{\epsilon-1}{\epsilon}.$$

The distribution of consumption in steady state will depend on fiscal policy. Fix a steady state with a given value for (C_h, C_s) and government debt B. The required value of transfers that implement the given level of consumption are

$$\widetilde{T}_h = C_h - \left(\frac{W}{P}\right)^{\frac{1+\varphi}{\varphi}} C_h^{-\frac{\sigma}{\varphi}},$$

$$\widetilde{T}_s = C_s - \left(\frac{W}{P}\right)^{\frac{1+\varphi}{\varphi}} C_s^{-\frac{\sigma}{\varphi}} - \frac{1+(\epsilon-1)\gamma}{\epsilon}\frac{1-\tau}{1-\omega}Y - \frac{1-\beta}{\beta}\frac{QB}{1-\omega},$$

where $Y = \omega C_h + (1-\omega)C_s$.

Log-linearization. As is standard, we study the dynamics of the economy around a steady-state equilibrium with zero inflation. For a variable X_t, let $x_t \equiv \log\left(\frac{X_t}{X}\right)$, where X denotes the zero-inflation steady-state value. We derive the equilibrium conditions for the general case where C_h may differ from C_s, and then specialize to the $C_h = C_s$ case considered in Section 4.2.

The log-linearized version of the savers' Euler equation is given by

$$c_{s,t+1} = c_{s,t} + \sigma^{-1}\left(i_t - \pi_{t+1} - r_n\right),$$

where we used that $\log 1 + i_t \approx i_t$.

The labor supply condition can be written as

$$w_t - p_t = \varphi n_{j,t} + \sigma c_{j,t}.$$

Log-linearizing the market clearing conditions for consumption and labor, we obtain

$$\omega_c c_{h,t} + (1-\omega_c)c_{s,t} = y_t, \quad \omega_n n_{h,t} + (1-\omega_n)n_{s,t} = n_t,$$

where $\omega_c \equiv \frac{\omega C_h}{Y}$ and $\omega_n \equiv \frac{\omega N_h}{N}$.

From the labor-supply condition, we obtain

$$\begin{aligned} n_{s,t} &= n_{h,t} + \varphi^{-1}\sigma(c_{h,t} - c_{s,t}) \\ &= n_{h,t} + \varphi^{-1}\sigma(1-\omega_c)^{-1}(c_{h,t} - y_t), \end{aligned}$$

using the market-clearing condition for goods to eliminate $c_{s,t}$. Plugging this expression into the market-clearing condition for labor, we obtain

$$n_{h,t} = \left(\frac{1}{1-\gamma} + \varphi^{-1}\sigma\right) y_t - \varphi^{-1}\sigma c_{h,t} + \varphi^{-1}\sigma \frac{\omega_c - \omega_n}{1-\omega_c}(y_t - c_{h,t}),$$

where we used that $n_t = \frac{1}{1-\gamma} y_t$. The real wage is then given by

$$w_t - p_t = \left(\frac{\varphi}{1-\gamma} + \sigma\right) y_t + \sigma \frac{\omega_c - \omega_n}{1-\omega_c}(y_t - c_{h,t}).$$

Linearizing the borrowers' budget constraint, we obtain

$$c_{h,t} = \frac{WN_b}{PC_b}(w_t - p_t + n_{h,t}) + T_{h,t},$$

where $T_{h,t} \equiv \frac{\widetilde{T}_{h,t} - \widetilde{T}_h}{C_h}$. Plugging the expressions for the real wage and labor supply into this expression, we obtain

$$c_{h,t} = \frac{WN_h}{PC_h}\left[\left(1+\varphi^{-1}\right)\left(\frac{\varphi}{1-\gamma} + \sigma\right) y_t - \varphi^{-1}\sigma c_{h,t} + \left(1+\varphi^{-1}\right)\sigma \frac{\omega_c - \omega_n}{1-\omega_c}\left(y_t - c_{h,t}\right)\right] + T_{h,t}.$$

Then,

$$c_{h,t} = \chi_y y_t + \chi_T T_{h,t},$$

where

$$\begin{aligned} \chi_y &\equiv \frac{\frac{WN_h}{PC_h}\left[\left(1+\varphi^{-1}\right)\left(\frac{\varphi}{1-\gamma} + \sigma\right) + \left(1+\varphi^{-1}\right)\sigma\frac{\omega_c-\omega_n}{1-\omega_c}\right]}{1 + \frac{WN_h}{PC_h}\left[\varphi^{-1}\sigma + \left(1+\varphi^{-1}\right)\sigma\frac{\omega_c-\omega_n}{1-\omega_c}\right]} \\ \chi_T &\equiv \frac{1}{1 + \frac{WN_h}{PC_h}\left[\varphi^{-1}\sigma + \left(1+\varphi^{-1}\right)\sigma\frac{\omega_c-\omega_n}{1-\omega_c}\right]} \end{aligned}$$

The symmetric steady state case is obtained by imposing $C_h = C_s = Y$, so $\omega_c = \omega_n = \omega$, and $1 - \alpha \equiv \frac{WN}{PY} = (1-\tau)(1-\gamma)\frac{\epsilon-1}{\epsilon}$.

From the borrower's consumption and market clearing, we obtain

$$c_{s,t} = \frac{1 - \omega_c\chi_y}{1-\omega_c} y_t - \frac{\omega_c \chi_T}{1-\omega_c} T_{h,t}.$$

Introducing this expression into the saver's Euler equation, we get

$$y_{t+1} = y_t + \tilde{\sigma}^{-1}\left(i_t - \pi_{t+1} - r_n\right) + v_t,$$

where

$$\tilde{\sigma}^{-1} \equiv \frac{1-\omega_c}{1-\omega_c\chi_y}\sigma^{-1}, \quad v_t \equiv \frac{\omega_c\chi_T}{1-\omega_c\chi_y}\left(T_{h,t+1}-T_{h,t}\right).$$

The flow budget constraint for savers can be written as

$$c_{s,t}+\frac{QB_s}{PC_s}b_{s,t+1}+\frac{QB_s}{PC_s}\left(1-\rho\right)q_t+\frac{QB_s}{PC_s}\pi_{t+1} \leq \frac{1}{\beta}\frac{QB_s}{PC_s}b_{s,t}+\frac{WN_s}{PC_s}\left(w_t-p_t+n_{s,t}\right)+ \\ \frac{\left(1-\tau\right)y_t-\frac{WN}{PY}\left(w_t-p_t+n_t\right)}{1-\omega_c}+T_{s,t}.$$

Multiplying by β^t, summing over time and using the government's No-Ponzi condition, we get

$$\sum_{t=0}^{\infty}\beta^t c_{s,t} \leq -\frac{QB_s}{PC_s}\frac{1}{\beta}\pi_0+\frac{QB_s}{PC_s}\rho q_0+\sum_{t=0}^{\infty}\beta^t\left[\frac{1-\tau}{1-\omega_c}y_t+\frac{WN_s}{PC_s}\left(w_t-p_t+n_{s,t}\right)- \right. \\ \left. \frac{1}{1-\omega_c}\frac{WN}{PY}\left(w_t-p_t+n_t\right)+\left(i_t-\pi_{t+1}-r_n\right)\frac{QB_s}{PC_s}+T_{s,t}\right],$$

where we used that $\beta\rho q_{t+1}-q_t=i_t-r_n$. Finally, noting that $q_0=-\sum_{t=0}^{\infty}\left(\beta\rho\right)^t\left(i_t-r_n\right)$, we get

$$\sum_{t=0}^{\infty}\beta^t c_{s,t} \leq \sum_{t=0}^{\infty}\beta^t\left[\frac{1-\tau}{1-\omega_c}y_t+\frac{WN_s}{PC_s}\left(w_t-p_t+n_{s,t}\right)-\frac{1}{1-\omega_c}\frac{WN}{PY}\left(w_t-p_t+n_t\right)+ \right. \\ \left. \left(i_t-\pi_{t+1}-r_n\right)\frac{QB_s}{PC_s}+T_{s,t}\right]-\left[\sum_{t=0}^{\infty}\left(\beta\rho\right)^t\left(i_t-r_n\right)\rho+\frac{1}{\beta}\pi_0\right]\frac{QB_s}{PC_s}.$$

Evaluating this expression at $\omega=0$, we obtain equation (3) in Section 2.

Note that summing the flow budget constraint of savers and HtM households, we get

$$c_t+\frac{QB}{PY}b_{t+1}+\frac{QB}{PY}\left(1-\rho\right)q_t+\frac{QB}{PY}\pi_{t+1} \leq \left(1-\tau\right)y_t+\frac{1}{\beta}\frac{QB}{PY}b_t+T_t,$$

where $T_t=\left(1-\omega_c\right)T_{s,t}+\omega_c T_{h,t}$ and $b_t=b_{s,t}$. Thus, we obtain the intertemporal aggregate budget constraint of the households,

$$\sum_{t=0}^{\infty}\beta^t c_t \leq \sum_{t=0}^{\infty}\beta^t\left[\left(1-\tau\right)y_t+\left(i_t-\pi_{t+1}-r_n\right)\frac{QB}{PY}+T_t\right]-\left[\sum_{t=0}^{\infty}\left(\beta\rho\right)^t\left(i_t-r_n\right)\rho+\frac{1}{\beta}\pi_0\right]\frac{QB}{PY}.$$

Now, consider the firms. The log-linear approximation of the intermediate-goods producers' first-order condition around the zero inflation steady state yields

$$p_t^*-p_t=\left(1-\theta\beta\right)\sum_{k=0}^{\infty}\left(\theta\beta\right)^k\left(mc_{t+k|t}+p_{t+k}-p_t\right).$$

Approximating the expression for the marginal cost, we get

$$mc_{t+k|t}=w_{t+k}-p_{t+k}+\frac{\gamma}{1-\gamma}y_{t+k|t},$$

where

$$y_{t+k|t} = -\epsilon \left(p_t^* - p_{t+k}\right) + y_{t+k}.$$

Let mc_{t+k} denote the average marginal cost in the economy, which is given by

$$mc_{t+k} = w_{t+k} - p_{t+k} + \frac{\gamma}{1-\gamma} y_{t+k}.$$

Introducing the labor supply optimality condition, and using that $n_t = \frac{1}{1-\gamma} y_t$, we get

$$mc_{t+k} = \left(\sigma + \frac{\varphi + \gamma}{1-\gamma}\right) y_{t+k} + \sigma \frac{\omega_c - \omega_n}{1-\omega_c} (y_{t+k} - c_{h,t+k}).$$

Moreover, approximating the price level equation we get

$$p_t^* - p_t = \frac{\theta}{1-\theta} \pi_t.$$

Hence, we can write the firm's optimality condition as

$$\pi_t = \beta \pi_{t+1} + \left[\kappa y_t + \kappa_h (y_t - c_{h,t})\right],$$

where $\kappa \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta} \frac{1-\gamma}{1-\gamma+\gamma\epsilon} \left(\sigma + \frac{\varphi+\gamma}{1-\gamma}\right)$ and $\kappa_h \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta} \frac{1-\gamma}{1-\gamma+\gamma\epsilon} \sigma \frac{\omega_c - \omega_n}{1-\omega_c}$. Imposing a symmetric steady state, $\kappa_h = 0$.

Finally, let's calculate the change in aggregate households' wealth. Let

$$\widetilde{\Omega}_0 \equiv (1 + \rho Q_0) \frac{B_0}{P_0} + \sum_{t=0}^{\infty} \prod_{s=0}^{t-1} \left(\frac{P_{s+1}/P_s}{1+i_s}\right) \left[\frac{W_t}{P_t} N_t + \frac{\Pi_t}{P_t} + \widetilde{T}_t\right].$$

Then, the households' intertemporal budget constraint can be written as

$$\sum_{t=0}^{\infty} \prod_{s=0}^{t-1} \left(\frac{P_{t+1}/P_t}{1+i_s}\right) C_t = \widetilde{\Omega}_0.$$

In steady state

$$\sum_{t=0}^{\infty} \beta^t C = \widetilde{\Omega} \implies \frac{\widetilde{\Omega}}{C} = \frac{1}{1-\beta}.$$

Let $c_t \equiv \log \frac{C_t}{C}$, $\hat{\widetilde{\Omega}}_0 \equiv \log \frac{\widetilde{\Omega}_0}{\widetilde{\Omega}}$, and note that, up to first order, $i_t \simeq \log 1 + i_t$. Then, a first order approximation of the households' intertemporal budget constraint around a zero-inflation steady state is given by

$$\sum_{t=0}^{\infty} \beta^t c_t = \frac{1}{1-\beta} \hat{\widetilde{\Omega}}_0 + \frac{\beta}{1-\beta} \sum_{t=0}^{\infty} \beta^t \left(i_t - \pi_{t+1} - r_n\right),$$

where $r_n \equiv -\log \beta$. Let

$$\Omega_0 \equiv \hat{\widetilde{\Omega}}_0 + \beta \sum_{t=0}^{\infty} \beta^t \left(i_t - \pi_{t+1} - r_n\right).$$

Note that

$$\Omega_0 = (1-\beta)\left[\sum_{t=0}^{\infty}\beta^t\left[(1-\tau)\,y_t + T_t\right] - \frac{\beta}{1-\beta}\frac{(1-\tau)\,Y+T}{Y}\sum_{t=0}^{\infty}\beta^t\left(i_t - \pi_{t+1} - r_n\right) - \left[\sum_{t=0}^{\infty}\left(\beta\rho\right)^t\left(i_t - r_n\right)\rho + \frac{1}{\beta}\pi_0\right]\frac{QB}{YP}\right] + \beta\sum_{t=0}^{\infty}\beta^t\left(i_t - \pi_{t+1} - r_n\right),$$

or

$$\Omega_0 = (1-\beta)\left[\sum_{t=0}^{\infty}\beta^t\left[(1-\tau)\,y_t + T_t\right] + \sum_{t=0}^{\infty}\beta^t\left(i_t - \pi_{t+1} - r_n\right)\frac{QB}{PY} - \left[\sum_{t=0}^{\infty}\left(\beta\rho\right)^t\left(i_t - r_n\right)\rho + \frac{1}{\beta}\pi_0\right]\frac{QB}{PY}\right],$$

where we have used that the government's budget constraint in steady state satisfies $\frac{\tau Y - T}{Y} = \frac{1-\beta}{\beta}\frac{QB}{PY}$. Hence

$$\sum_{t=0}^{\infty}\beta^t c_t = \frac{\Omega_0}{1-\beta} = \sum_{t=0}^{\infty}\beta^t\left[(1-\tau)\,y_t + T_t\right] + \sum_{t=0}^{\infty}\beta^t\left(i_t - \pi_{t+1} - r_n\right)\frac{QB}{PY} - \left[\sum_{t=0}^{\infty}\left(\beta\rho\right)^t\left(i_t - r_n\right)\rho + \frac{1}{\beta}\pi_0\right]\frac{QB}{PY}.$$

C Hicksian Demand

This appendix presents an extension of the Slutsky equation of microeconomic theory to a general equilibrium setting. We begin by computing the Hicksian demand, i.e. the solution to the expenditure minimization problem subject to delivering a minimum level of utility. In this setting, the different goods are consumption at different dates, and the price of one unit of consumption at date t is $\prod_{s=0}^{t-1}\left(\frac{P_{s+1}/P_s}{1+i_s}\right)$. After that, we show that $\{c_t^S\}_{t=0}^{\infty}$ in the decomposition of Section 2 (see Proposition 1) can be reinterpreted as the (log-linearized) Hicksian demand evaluated at the inflation rate consistent with the Hicksian demand according to the New Keynesian Phillips curve. Finally, we compare the decomposition in this paper with one that looks at the direct and indirect effects of monetary policy, as in Kaplan, Moll and Violante (2018).

C.1 Derivation of the Hicksian demand

The Hicksian demand of the non-linear model is obtained as the solution to the following problem:

$$\min_{\{C_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty}\prod_{s=0}^{t-1}\left(\frac{P_{s+1}/P_s}{1+i_s}\right)C_t$$

subject to

$$\sum_{t=0}^{\infty}\beta^t\frac{C_t^{1-\sigma}}{1-\sigma}\geq\overline{U},$$

for some $\overline{U}\in\mathbb{R}$. The FOCs of this problem are given by

$$\prod_{s=0}^{t-1}\left(\frac{P_{s+1}/P_s}{1+i_s}\right)=\lambda\beta^t C_t^{-\sigma},$$

where λ is the Lagrange multiplier associated to the constraint. This implies that

$$C_t=\prod_{s=0}^{t-1}\left(\frac{1+i_s}{P_{s+1}/P_s}\right)^{\frac{1}{\sigma}}\lambda^{\frac{1}{\sigma}}\beta^{\frac{1}{\sigma}t}\implies\sum_{t=0}^{\infty}\beta^t\frac{C_t^{1-\sigma}}{1-\sigma}=\sum_{t=0}^{\infty}\beta^{\frac{1}{\sigma}t}\frac{\prod_{s=0}^{t-1}\left(\frac{1+i_s}{P_{s+1}/P_s}\right)^{\frac{1-\sigma}{\sigma}}\lambda^{\frac{1-\sigma}{\sigma}}}{1-\sigma}=\overline{U},$$

and hence

$$\lambda=\left[\frac{(1-\sigma)\overline{U}}{\sum_{t=0}^{\infty}\beta^{\frac{1}{\sigma}t}\prod_{s=0}^{t-1}\left(\frac{1+i_s}{P_{s+1}/P_s}\right)^{\frac{1-\sigma}{\sigma}}}\right]^{\frac{\sigma}{1-\sigma}}.$$

Replacing in the FOC for C_t, we get

$$C_t=\frac{\beta^{\frac{1}{\sigma}t}\prod_{s=0}^{t-1}\left(\frac{1+i_s}{P_{s+1}/P_s}\right)^{\frac{1}{\sigma}}}{\left[\sum_{t=0}^{\infty}\beta^{\frac{1}{\sigma}t}\prod_{s=0}^{t-1}\left(\frac{1+i_s}{P_{s+1}/P_s}\right)^{\frac{1-\sigma}{\sigma}}\right]^{\frac{1}{1-\sigma}}}[(1-\sigma)\overline{U}]^{\frac{1}{1-\sigma}}.$$

Log-linearizing around the zero inflation steady state, we get

$$c_t^H = \frac{1}{\sigma}\sum_{s=0}^{t-1}\left(i_s - \pi_{s+1} - r_n\right) - \frac{1}{\sigma}\sum_{s=0}^{\infty}\beta^{s+1}\left(i_s - \pi_{s+1} - r_n\right),$$

where $r_n \equiv -\log\beta$, and we used that, in steady state, $C = [(1-\beta)(1-\sigma)\overline{U}]^{\frac{1}{1-\sigma}}$. The present value of the Hicksian demand is given by

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t c_t^H &= \sum_{t=0}^{\infty}\beta^t\frac{1}{\sigma}\sum_{s=0}^{t-1}\left(i_s - \pi_{s+1} - \rho\right) - \sum_{t=0}^{\infty}\beta^t\frac{1}{\sigma}\sum_{s=0}^{\infty}\beta^{s+1}\left(i_s - \pi_{s+1} - \rho\right)\\ &= \frac{1}{\sigma}\sum_{s=0}^{\infty}\sum_{t=s+1}^{\infty}\beta^t\left(i_s - \pi_{s+1} - \rho\right) - \frac{1}{1-\beta}\frac{1}{\sigma}\sum_{s=0}^{\infty}\beta^{s+1}\left(i_s - \pi_{s+1} - \rho\right)\\ &= 0.\end{aligned}$$

Moreover, note that

$$c_{t+1}^H = c_t^H + \sigma^{-1}\left(i_t - \pi_{t+1} - r_n\right),$$

that is, the Hicksian demand satisfies the households' Euler equation.

C.2 The Intertemporal Substitution Effect in General Equilibrium

To find the inflation rate consistent with the Hicksian demand we need to solve the following system of difference equations:

$$\begin{aligned}c_{t+1}^H &= c_t^H + \sigma^{-1}\left(i_t - \pi_{t+1}^H - r_n\right)\\ \pi_t^H &= \beta\pi_{t+1}^H + \kappa c_t^H,\end{aligned}$$

with terminal condition

$$\sum_{t=0}^{\infty}\beta^t c_t^H = 0.$$

It should be straightforward that this system is equivalent to the system in Section 2 with the terminal condition $\Omega_0 = 0$. Thus, the solution is

$$c_t^H = c_t^S, \quad \pi_t^H = \pi_t^S.$$

C.3 An alternative consumption decomposition: direct and indirect effects

An alternative decomposition separates the response of equilibrium consumption into a direct effect of the real interest rate, keeping output and fiscal policy fixed, and an indirect effect that incorporates the changes in output and fiscal policy. Let $\tilde{c}_t^H$ denote the Hicksian demand in period t evaluated at the *equilibrium* path of the inflation rate.[38] Recall that Proposition 3 states that the equilibrium inflation rate satisfies

$$\pi_t = \pi_t^S + \frac{\kappa}{1-\beta}\underline{\lambda}^t\Omega_0.$$

[38] In contrast, c_t^S is the Hicksian demand evaluated at the Hicksian inflation rate π_t^S.

Introducing this expression into the Hicksian demand we get

$$\tilde{c}_t^H = \frac{1}{\sigma}\sum_{s=0}^{t-1}\left(i_s - \pi_{s+1}^S - \frac{\kappa}{1-\beta}\underline{\lambda}^{s+1}\Omega_0 - r_n\right) - \frac{1}{\sigma}\sum_{s=0}^{\infty}\beta^{s+1}\left(i_s - \pi_{s+1}^S - \frac{\kappa}{1-\beta}\underline{\lambda}^{s+1}\Omega_0 - r_n\right),$$

and after some algebra,

$$\tilde{c}_t^H = \underbrace{\frac{1}{\sigma}\sum_{s=0}^{t-1}\left(i_s - \pi_{s+1}^S - r_n\right) - \frac{1}{\sigma}\sum_{s=0}^{\infty}\beta^{s+1}\left(i_s - \pi_{s+1}^S - r_n\right)}_{=c_t^S} + \left(\frac{1-\underline{\lambda}\beta}{1-\beta}\underline{\lambda}^t - 1\right)\Omega_0.$$

Hence,

$$c_t = \tilde{c}_t^H + \Omega_0.$$

Introducing the definition of Ω_0, we get

$$c_t = \underbrace{\tilde{c}_t^H + (1-\beta)\left[\sum_{t=0}^{\infty}\beta^t\left(i_t - \pi_{t+1} - r_n\right) - \sum_{t=0}^{\infty}\left(\beta\rho\right)^t\left(i_t - r_n\right)\rho - \frac{1}{\beta}\pi_0\right]Qb}_{\text{direct effect}} +$$

$$\underbrace{(1-\beta)\sum_{t=0}^{\infty}\beta^t\left[(1-\tau)\,y_t + T_t\right]}_{\text{indirect effect}}.$$

This decomposition appears in, for example, Kaplan, Moll and Violante (2018).

There are two main differences between this decomposition and the one proposed in Proposition 1. On the one hand, the direct effect includes the wealth effect arising from the holdings of government bonds (interest payments, revaluation of long-term bonds and surprise inflation). On the other hand, and more importantly, the direct effect and the ISE can both be interpreted as a Hicksian demand, but evaluated at different paths of the inflation rate. While the ISE is evaluated at the Hicksian-consistent inflation rate, the direct effect is evaluated at the equilibrium rate. This is the main distinction between the two approaches and it reflects the different objectives pursued in both papers. Kaplan, Moll and Violante (2018) are interested in understanding the *micro* channels of transmission, which justifies evaluating the households' Hicksian demand at the equilibrium inflation rate. Our focus is on *macro* channels, so distinguishing between the inflation rate arising from the ISE and the wealth effect is crucial. It is this feature that allows us to identify the importance of the wealth effect in the equilibrium dynamics of the economy and later connect to the fiscal response to monetary policy. Notably, while neither the direct nor the indirect effect are uniquely determined by the path of the nominal interest rate, the ISE is. This feature uncovers new insights about the source of multiplicity in the New Keynesian model. Finally, note that both decompositions coincide when $b = 0$ and prices are fully rigid. In this case, there is no wealth effect arising from government bonds and the general equilibrium multiplier is equal to 1.

D Observational Equivalence Between Monetary-Active and Fiscally-Active Regimes

Proposition 2 shows that if the interest rate rule (5) satisfies the Taylor principle and the sequence of monetary shocks decays sufficiently fast, the Taylor equilibrium is the unique purely forward-looking solution to the New Keynesian model for a given equilibrium path of the nominal interest rate. Since a fiscally-active regime can rationalize any solution to the New Keynesian model, it implies that except for one particular value of Ω_0, monetary-active and fiscally-active regimes are not observationally equivalent. If we restrict the sequence of monetary shocks to follow an AR(1) process, imposing that it decays sufficiently fast is equivalent to imposing that a positive shock generates an increase in the nominal interest rate. This is a standard assumption in the monetary literature.

Here, we study two extensions. First, we show that if the sequence of monetary shocks follows an AR(1) process, the monetary-active and fiscally-active regimes are not observationally equivalent even if the sequence of monetary shocks does not decay sufficiently fast. As a byproduct of this result, we show that in the knife-edged case in which the persistence of the shock is equal to the lower eigenvalue of the system (i.e. $\psi = \underline{\lambda}$ in our notation), it is possible to obtain any value for the equilibrium wealth effect without the nominal interest rate ever moving from its steady-state level. This result is crucial for understanding the more general case.

Second, we show that if we instead restrict the sequence of the *nominal interest rate* to follow an AR(1) process (but no restriction on the sequence of shocks), we can recover observational equivalence. Notably, the restriction on the nominal interest rate implies that the sequence of monetary shocks needs to follow an ARMA(2,1) process with appropriately chosen coefficients. We conclude that two necessary conditions for observational equivalence are that the *monetary shocks* do not decay sufficiently fast and that they do not follow an AR(1) process.

D.1 General AR(1) process for the sequence of monetary shocks

Suppose that the sequence of monetary shocks follows an AR(1) process, so that $\varepsilon_t = \psi^t \varepsilon_0$ for some $\varepsilon_0 \neq 0$ and $\psi \in (0, \underline{\lambda}) \cup (\underline{\lambda}, 1)$. Relative to Proposition 2, here we do not impose $\psi < \underline{\lambda}$. We consider the case with $\psi = \underline{\lambda}$ separately.

Under this assumption, we cannot guarantee that the Taylor equilibrium is the unique purely forward-looking solution to the system. However, it is still true that, for a given equilibrium value of i_0, Ω_0 in the Taylor equilibrium is uniquely determined.

Proposition 8. *Suppose that the equilibrium path of the nominal interest rate, $\{i_t\}_{t=0}^{\infty}$, was generated by an interest rate rule (5) with $\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$ and $\phi_y > \frac{\sqrt{4\beta\sigma^{-1}(\kappa(\phi_\pi-1)+(1-\beta)\phi_y)}-(1-\beta+\sigma^{-1}\kappa)}{\sigma^{-1}\beta}$, given a sequence of shocks $\{\varepsilon_t\}_{t=0}^{\infty}$ that satisfies $\varepsilon_t = \psi^t \varepsilon_0$, with $\psi \in (0, \underline{\lambda}) \cup (\underline{\lambda}, 1)$ and $\varepsilon_0 \neq 0$. Then, the equilibrium path of consumption is given by*

$$c_t^{Taylor} = -\frac{1}{\nu} \frac{(1-\psi\beta)\sigma^{-1}}{\beta\left(\overline{\lambda}-\psi\right)\left(\underline{\lambda}-\psi\right)} \psi^t (i_0^{Taylor} - r_n).$$

where $\nu \neq 0$ is a constant independent of $i_0 - r_n$. The corresponding wealth effect is

$$\Omega_0^{Taylor} = -\frac{1}{\nu} \frac{1-\beta}{\beta} \frac{\sigma^{-1}}{\left(\overline{\lambda}-\psi\right)\left(\underline{\lambda}-\psi\right)} (i_0^{Taylor} - r_n).$$

The nominal interest rate satisfies

$$i_t^{Taylor} = r_n + \psi^t \nu \varepsilon_0.$$

Finally,

$$\frac{\partial i_0^{Taylor}}{\partial \varepsilon_0} > 0 \iff \psi < \underline{\lambda}.$$

Proof. Plugging in $\varepsilon_t = \psi^t \varepsilon_0$ in the formulas of the Taylor equilibrium (see proof of Proposition 2), we get

$$c_t^* = -\frac{1}{\nu} \frac{(1-\beta\psi)\,\sigma^{-1}}{\beta\left(\overline{\delta}-\psi\right)\left(\underline{\delta}-\psi\right)} \psi^t \left(i_0^* - r_n\right),$$
$$\pi_t^* = -\frac{1}{\nu} \frac{\kappa\sigma^{-1}}{\beta\left(\overline{\delta}-\psi\right)\left(\underline{\delta}-\psi\right)} \psi^t \left(i_0^* - r_n\right),$$
$$i_t^* = r_n + \psi^t \nu \varepsilon_0,$$

where $\nu \equiv 1 - \phi_\pi \frac{\kappa\sigma^{-1}}{\beta(\overline{\delta}-\psi)(\underline{\delta}-\psi)} - \phi_y \frac{(1-\psi\beta)\sigma^{-1}}{\beta(\overline{\delta}-\psi)(\underline{\delta}-\psi)}$. Moreover, since $\pi_0 = \frac{\kappa}{1-\beta}\Omega_0$, we have

$$\Omega_0^* = -\frac{1}{\nu} \frac{1-\beta}{\beta} \frac{\sigma^{-1}}{\left(\overline{\delta}-\psi\right)\left(\underline{\delta}-\psi\right)} \left(i_0^* - r_n\right).$$

Hence, given i_0^*, Ω_0^* is uniquely determined. Finally, note that $\frac{\partial i_0^*}{\partial \varepsilon_0} > 0$ if and only if $\nu > 0$, or

$$\beta\psi^2 - \left(1 + \beta + \sigma^{-1}\kappa\right)\psi + 1 > 0,$$

where we used that $\overline{\delta}\underline{\delta} = \frac{1+\sigma^{-1}\kappa\phi_\pi+\sigma^{-1}\phi_y}{\beta}$ and $\overline{\delta} + \underline{\delta} = \frac{1+\beta+\sigma^{-1}\kappa+\sigma^{-1}\beta\phi_y}{\beta}$. Since $\psi \in (0, \underline{\lambda}) \cup (\underline{\lambda}, 1)$, this condition holds if and only if

$$\psi < \underline{\lambda}.$$

□

Proposition 8 considers all the possible parametrizations of the AR(1) process, except for $\psi = \underline{\lambda}$. It turns out that this case is rather pathological: the shock and the response of inflation and output are such that all the terms in the interest rate rule cancel out and the nominal interest rate does not change after a monetary shock. In terms of the decomposition, this implies that the ISE is equal to zero and all the response of consumption is given by the GE amplified wealth effect. Notably, we can parametrize *any* magnitude of the wealth effect by the size of the initial shock.

Proposition 9. *Suppose that the equilibrium path of the nominal interest rate, $\{i_t\}_{t=0}^{\infty}$, was generated by an interest rate rule (5) with $\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$ and $\phi_y > \frac{\sqrt{4\beta\sigma^{-1}(\kappa(\phi_\pi-1)+(1-\beta)\phi_y)} - (1-\beta+\sigma^{-1}\kappa)}{\sigma^{-1}\beta}$, given a sequence of shocks $\{\varepsilon_t\}_{t=0}^{\infty}$ that satisfies $\varepsilon_t = \psi^t\varepsilon_0$, with $\psi = \underline{\lambda})$ and $\varepsilon_0 \neq 0$. Then, the equilibrium path of consumption is given by*

$$c_t^{Taylor} = -\frac{\sigma^{-1}}{\beta} \frac{1-\beta\underline{\lambda}}{\left(\overline{\delta}-\underline{\lambda}\right)\left(\underline{\delta}-\underline{\lambda}\right)} \underline{\lambda}^t \varepsilon_0,$$

where $\overline{\delta}, \underline{\delta} > 1$. *The corresponding wealth effect is*

$$\Omega_0^{Taylor} = -\sigma^{-1} \frac{1-\beta}{\beta} \frac{1}{\left(\overline{\delta} - \underline{\lambda}\right)\left(\underline{\delta} - \underline{\lambda}\right)} \varepsilon_0.$$

The nominal interest rate satisfies

$$i_t^{Taylor} = r_n.$$

Proof. Immediate from evaluating the formulas characterizing the Taylor equilibrium at $\varepsilon_t = \underline{\lambda}^t \varepsilon_0$. □

Proposition 9 identifies a Taylor equilibrium in which the monetary shock affects Ω_0 *independently* of any effect on the path of the nominal interest rate. Thus, a promising path towards observational equivalence is to assume that the sequence of monetary shocks follows a process that combines the properties of $\psi \in (0, \underline{\lambda}) \cup (\underline{\lambda}, 1)$ and $\psi = \underline{\lambda}$. Next, we formalize this idea.

D.2 General AR(1) process for the sequence of the nominal interest rate

Suppose that the equilibrium path of the nominal interest rate satisfies

$$i_t - r_n = \psi^t (i_0 - r_n),$$

for some given initial value $i_0 - r_n$. The question we ask is the following: can we find a sequence of monetary shocks that obtains *any* solution of the New Keynesian system as a Taylor equilibrium? It turns out that the answer is yes, and that the sequence of monetary shocks needs to satisfy

$$\varepsilon_t = \left[\chi \psi^t + (1-\chi)\underline{\lambda}^t\right] \varepsilon_0,$$

with $\psi \in (0, \underline{\lambda}) \cup (\underline{\lambda}, 1)$, and $\chi, \varepsilon_0 \neq 0$. Note that this process is neither an AR(1) nor it generates a sequence of shocks that decays sufficiently fast. In particular, it can be represented by an ARMA(2,1) process with appropriately chosen coefficients.[39] Notably, it combines the properties of the AR(1) processes studied above. The first term determines the equilibrium path of the nominal interest rate, while the second term determines the magnitude of the wealth effect. Thus, by choosing χ appropriately, we can obtain any solution to the New Keynesian system as a Taylor equilibrium of the model.

Proposition 10. *Consider an economy characterized by equations* (1)-(2) *and a path of the nominal interest rate,* $\{i_t\}_{t=0}^{\infty}$, *that satisfies* $i_t - r_n = \psi^t (i_0 - r_n)$, *with* $\psi \in (0, \underline{\lambda}) \cup (\underline{\lambda}, 1)$. *Then, any solution to the system* (1)-(2) *given*

[39] To see this, start with the general representation

$$\varepsilon_t = \chi \sum_{s=0}^{\infty} \psi^s \zeta_{t-s} + (1-\chi) \sum_{s=0}^{\infty} \underline{\lambda}^s \zeta_{t-s}.$$

This expression can be written as

$$\varepsilon_t = \chi \frac{\zeta_t}{1-\psi L} + (1-\chi) \frac{\zeta_t}{1-\underline{\lambda} L},$$

or

$$(1-\psi L)(1-\underline{\lambda} L)\varepsilon_t = \chi (1-\underline{\lambda} L)\zeta_t + (1-\chi)(1-\psi L)\zeta_t.$$

Therefore

$$\varepsilon_t = (\psi + \underline{\lambda})\varepsilon_{t-1} - \psi \underline{\lambda} \varepsilon_{t-2} + \zeta_t - \left[(1-\chi)\psi + \chi \underline{\lambda}\right] \zeta_{t-1}.$$

This process is an ARMA(2,1) that simplifies to an AR(2) if $(1-\chi)\psi + \chi\underline{\lambda} = 0$.

$\{i_t\}_{t=0}^{\infty}$ *can be obtained as a Taylor equilibrium. The sequence of monetary shocks satisfies*

$$\varepsilon_t = \left[\chi\psi^t + (1-\chi)\underline{\lambda}^t\right]\varepsilon_0,$$

where χ *and* ε_0 *are appropriately chosen constants.*

Proof. Recall that any solution to the New Keynesian system can be written as

$$c_t = c_t^S + \frac{1-\beta\underline{\lambda}}{1-\beta}\underline{\lambda}^t\Omega_0$$

and

$$\pi_t = \pi_t^S + \frac{\kappa}{1-\beta}\underline{\lambda}^t\Omega_0$$

for some Ω_0. Plugging in the sequence of interest rates $i_t - r_n = \psi^t(i_0 - r_n)$ and after some algebra, we get

$$c_t = -\frac{\sigma^{-1}(1-\beta\psi)}{\beta(\overline{\lambda}-\psi)(\underline{\lambda}-\psi)}\psi^t(i_0-r_n) + \frac{1-\beta\underline{\lambda}}{1-\beta}\underline{\lambda}^t\left[\frac{\sigma^{-1}(1-\beta)}{\beta(\overline{\lambda}-\psi)(\underline{\lambda}-\psi)}(i_0-r_n)+\Omega_0\right]$$
$$\pi_t = -\frac{\sigma^{-1}\kappa}{\beta(\overline{\lambda}-\psi)(\underline{\lambda}-\psi)}\psi^t(i_0-r_n) + \frac{\kappa}{1-\beta}\underline{\lambda}^t\left[\frac{\sigma^{-1}(1-\beta)}{\beta(\overline{\lambda}-\psi)(\underline{\lambda}-\psi)}(i_0-r_n)+\Omega_0\right]$$

Then, we can recover the monetary shocks that lead to this equilibrium as

$$\varepsilon_t = i_t - r_n - \phi_\pi\pi_t - \phi_y c_t.$$

Plugging in the expressions for c_t and π_t, and after some algebra, we get

$$\varepsilon_t = \left(\frac{1}{1+\chi}\psi^t + \frac{\chi}{1+\chi}\underline{\lambda}^t\right)\varepsilon_0$$

where $\varepsilon_0 \equiv (1-\chi)\,\tilde{v}\,(i_0 - r_n)$, $\tilde{v} \equiv 1 + \phi_\pi\frac{\sigma^{-1}\kappa}{\beta(\overline{\lambda}-\psi)(\underline{\lambda}-\psi)} + \phi_y\frac{\sigma^{-1}(1-\beta\psi)}{\beta(\overline{\lambda}-\psi)(\underline{\lambda}-\psi)}$, and $\chi \equiv -\frac{\phi_\pi\frac{\kappa}{1-\beta}+\phi_y\frac{1-\beta\underline{\lambda}}{1-\beta}}{\tilde{v}}$ $\left[\frac{\sigma^{-1}(1-\beta)}{\beta(\overline{\lambda}-\psi)(\underline{\lambda}-\psi)} + \frac{\Omega_0}{i_0-r_n}\right]$. By choosing $\varepsilon_0 = (1-\chi)\tilde{v}\,(i_0 - r_n)$ we get the full path of the nominal interest rate from the Taylor rule. Then, we can freely vary χ depending on the choice of Ω_0. Since the model is linear, the equilibrium can be obtained as the weighted average of two different equilibria: one with $\varepsilon_t = \psi^t\varepsilon_0$ and the other with $\varepsilon_t = \chi\underline{\lambda}^t\varepsilon_0$, for the same value of ε_0. Then, the result follows from Propositions 8 and 9. □

The proof of the proposition shows how each term of the monetary shock process plays a different role. The first term matches the desired path of the nominal interest rate. The second term matches the desired magnitude of the wealth effect.

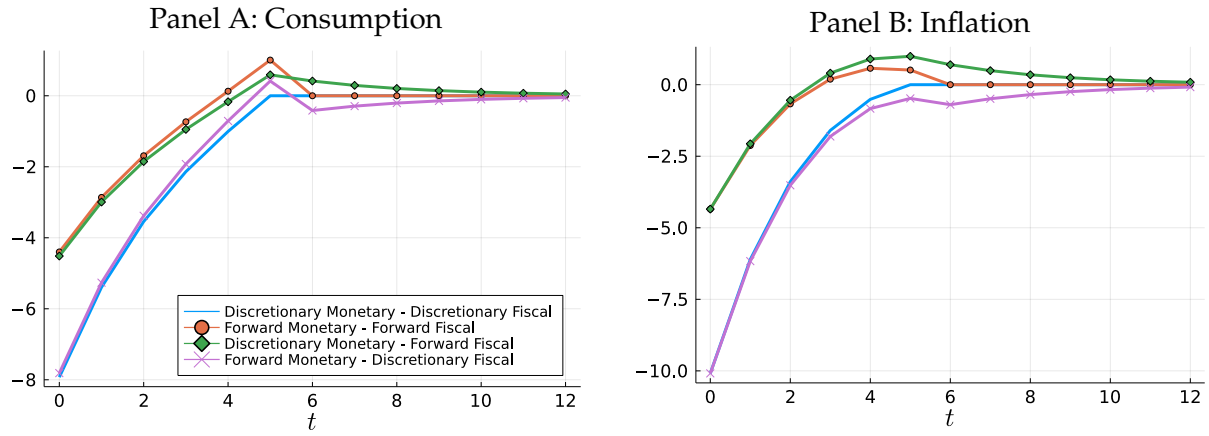


Figure 12: Consumption and inflation in a liquidity trap under different monetary and fiscal responses

Calibration: quarterly time period, $\beta = 0.99$, $\sigma = 1$, $\kappa = 0.1275$. The natural rate of interest is set to $-r_n$ until $T = 4$. *Discretionary Monetary - Discretionary Fiscal* sets $T^* = T$ and the fiscal transfers to the "discretionary" SLTE; *Forward Monetary - Forward Fiscal* sets $T^* = T + 1$ and the fiscal transfers to the "forward" SLTE; *Discretionary Monetary - Forward Fiscal* sets $T^* = T$ and fiscal transfers to the "forward" SLTE; *Forward Monetary - Discretionary Fiscal* sets $T^* = T + 1$ and fiscal transfers to the "discretionary" SLTE.

E Liquidity Trap

This appendix provides additional analysis of the liquidity trap equilibrium.

E.1 Fiscal Policy in the liquidity trap

Figure 12 provides an alternative perspective of the role of fiscal policy in the liquidity trap equilibrium. The figure shows different combinations of monetary and fiscal policy responses. As a benchmark, we plot the dynamics of consumption and inflation under the discretionary monetary policy in the SLTE, which we label the "Discretionary Monetary - Discretionary Fiscal" case. We also plot the forward guidance SLTE: the interest rate is kept at zero for an extra period *and* fiscal policy is at the level that sustains the SLTE equilibrium. The other two lines are the alternative combinations of the monetary and fiscal policies. For example, consider the "Forward Monetary - Discretionary Fiscal." This case corresponds to the nominal interest rate path in the forward guidance SLTE and the fiscal policy of the discretionary SLTE. The figure shows that the path of consumption barely changes relative to the discretionary SLTE. The response in period 0 is virtually the same. The differences arise close to the end of the trap, where the "forward monetary" case generates a small boom and then a small recession before converging back to steady state from below. In contrast, notice what happens when we consider the "Discretionary Monetary - Forward Fiscal." The path of consumption is now virtually equivalent to the forward guidance SLTE. A similar pattern emerges for inflation. Thus, Figures 10 and 12 together highlight the importance of fiscal policy in the liquidity trap dynamics.

E.2 Comparison with Cochrane (2017)

To better compare and contrast with the results in Cochrane (2017), we provide a mapping between the three main equilibria considered in that paper and the indexation through Ω_0 in this one.

The standard equilibrium. The standard equilibrium is selected by imposing $c_t = \pi_t = 0$ for all $t \geq T^*$. Then, for all $t \leq T^*$, consumption and inflation are given by

$$c_t^{SLTE} = -\frac{\sigma^{-1}}{\overline{\lambda} - \underline{\lambda}} \left[\sum_{s=t}^{T} \left(\frac{\overline{\lambda} - 1}{\underline{\lambda}^{s-t}} + \frac{1 - \underline{\lambda}}{\overline{\lambda}^{s-t}} \right) - \sum_{s=T+1}^{T^*} \left(\frac{\overline{\lambda} - 1}{\underline{\lambda}^{s-t}} + \frac{1 - \underline{\lambda}}{\overline{\lambda}^{s-t}} \right) \right] r_n,$$

$$\pi_t^{SLTE} = -\frac{\kappa\sigma^{-1}}{\overline{\lambda} - \underline{\lambda}} \left[\sum_{s=t}^{T} \left(\frac{\overline{\lambda}}{\underline{\lambda}^{s-t}} - \frac{\underline{\lambda}}{\overline{\lambda}^{s-t}} \right) - \sum_{s=T+1}^{T^*} \left(\frac{\overline{\lambda}}{\underline{\lambda}^{s-t}} - \frac{\underline{\lambda}}{\overline{\lambda}^{s-t}} \right) \right] r_n.$$

This equilibrium coincides with the unique purely forward-looking solution to the system (1)-(2) in Section 2. Thus, the solution corresponds to the following level of the wealth effect:

$$\Omega_0 = -(1-\beta)\frac{\sigma^{-1}}{\overline{\lambda} - \underline{\lambda}} \left[\sum_{s=0}^{T} \left(\frac{\overline{\lambda}}{\underline{\lambda}^{s}} - \frac{\underline{\lambda}}{\overline{\lambda}^{s}} \right) - \sum_{s=T+1}^{T^*} \left(\frac{\overline{\lambda}}{\underline{\lambda}^{s}} - \frac{\underline{\lambda}}{\overline{\lambda}^{s}} \right) \right] r_n.$$

The backward-stable equilibrium This equilibrium corresponds to the solution in which, if the shock were anticipated, inflation and consumption would not explode as we move back in time. Consider the path of inflation. We have

$$\lim_{t \to -\infty} \pi_t = \lim_{t \to -\infty} \underline{\lambda}^t \left[-\frac{\sigma^{-1}\kappa}{\overline{\lambda} - \underline{\lambda}} \sum_{s=0}^{\infty} \frac{\underline{\lambda}}{\overline{\lambda}^s} (i_s - r_n) + \frac{\kappa}{1-\beta}\Omega_0 \right].$$

Thus, $\lim_{t \to -\infty} \pi_t = 0$ if and only if

$$\Omega_0 = (1-\beta)\frac{\sigma^{-1}}{\overline{\lambda} - \underline{\lambda}} \sum_{s=0}^{\infty} \frac{\underline{\lambda}}{\overline{\lambda}^s} (i_s - r_n).$$

Note that for the natural rate and discretionary interest rate paths in the liquidity trap equilibrium, $\Omega_0 > 0$ so $\pi_0 > 0$, as found in Cochrane (2017).

The no-inflation-jump equilibrium. This equilibrium is immediate from the analysis in Section 2. We have that $\pi_0 = 0 \iff \Omega_0 = 0$. Thus, the *no-inflation-jump* equilibrium is the unique equilibrium with a zero wealth effect.

E.3 Fiscal Policy and the Monetary Paradoxes

Before ending the section on the liquidity trap, we want to briefly analyze the monetary paradoxes through the lens of our decomposition. In particular, we focus on the Forward Guidance Puzzle and the Paradox of Flexibility.

Definition 1 (Monetary Paradoxes). *Given a path of the nominal interest rate, $\{i_t\}_{t=0}^{\infty}$, an equilibrium features*

the Forward Guidance Puzzle if

$$\frac{\partial^2 c_0}{\partial t \partial i_t} < 0, \quad \frac{\partial^2 \pi_0}{\partial t \partial i_t} < 0.$$

Moreover, the equilibrium features the Paradox of Flexibility if

$$\lim_{\kappa \to \infty} \frac{\partial c_0}{\partial i_t} = -\infty, \quad \lim_{\kappa \to \infty} \frac{\partial \pi_0}{\partial i_t} = -\infty.$$

The Forward Guidance Puzzle refers to the theoretical result that the promise to reduce the interest rates in the future becomes more powerful the further in the future the actual time of intervention is.[40] The Paradox of Flexibility is the result that the effect of monetary policy is unboundedly strong in the limit to flexible prices even though the nominal interest rate has no impact on real variables in a fully flexible price economy. Proposition 11 formally establishes that the Forward Guidance Puzzle and the Paradox of Flexibility are present in the SLTE.

Proposition 11 (Monetary Paradoxes in the SLTE)**.** *The SLTE exhibits the monetary paradoxes.*

Proof. We have

$$c_0^{SLTE} = -\frac{\sigma^{-1}}{\overline{\lambda} - \underline{\lambda}} \left[\sum_{s=0}^{T} \left(\frac{\overline{\lambda} - 1}{\underline{\lambda}^s} + \frac{1 - \underline{\lambda}}{\overline{\lambda}^s} \right) r_n + \sum_{s=T+1}^{T^*} \left(\frac{\overline{\lambda} - 1}{\underline{\lambda}^s} + \frac{1 - \underline{\lambda}}{\overline{\lambda}^s} \right) (i_s - r_n) \right].$$

Hence, for $t > T$, we have

$$\frac{\partial c_0^{SLTE}}{\partial i_t} = -\frac{\sigma^{-1}}{\overline{\lambda} - \underline{\lambda}} \left(\frac{\overline{\lambda} - 1}{\underline{\lambda}^t} + \frac{1 - \underline{\lambda}}{\overline{\lambda}^t} \right) < 0.$$

For the Forward Guidance Puzzle, we have

$$\frac{\partial^2 c_0^{SLTE}}{\partial t \partial i_t} = \frac{\sigma^{-1}}{\overline{\lambda} - \underline{\lambda}} \left(\frac{\overline{\lambda} - 1}{\underline{\lambda}^t} \log \underline{\lambda} + \frac{1 - \underline{\lambda}}{\overline{\lambda}^t} \log \overline{\lambda} \right).$$

Then

$$\frac{\partial^2 c_0^{SLTE}}{\partial t \partial i_t} < 0 \iff \frac{\overline{\lambda} - 1}{\underline{\lambda}^t} \log \underline{\lambda} + \frac{1 - \underline{\lambda}}{\overline{\lambda}^t} \log \overline{\lambda} < 0.$$

A sufficient condition is

$$(\overline{\lambda} - 1) \log \underline{\lambda} + (1 - \underline{\lambda}) \log \overline{\lambda} < 0.$$

Note that

$$\overline{\lambda} - 1 > \log \overline{\lambda} \quad \text{and} \quad \log \underline{\lambda} < \underline{\lambda} - 1.$$

Then

$$(\overline{\lambda} - 1) \log \underline{\lambda} < (\underline{\lambda} - 1) \log \overline{\lambda},$$

or

$$(\overline{\lambda} - 1) \log \underline{\lambda} + (1 - \underline{\lambda}) \log \overline{\lambda} < 0.$$

[40] This term was coined by Del Negro et al. (2015), who find that the estimated response of the US economy to forward guidance shocks are significantly smaller than the ones predicted by the standard New Keynesian model.

Hence

$$\frac{\partial^2 c_0^{SLTE}}{\partial t \partial i_t} < 0.$$

For the Paradox of Flexibility

$$\lim_{\kappa\to\infty} \frac{\partial c_0^{SLTE}}{\partial i_t} = -\lim_{\kappa\to\infty} \frac{\sigma^{-1}}{\overline{\lambda}-\underline{\lambda}} \left(\frac{\overline{\lambda}-1}{\underline{\lambda}^t} + \frac{1-\underline{\lambda}}{\overline{\lambda}^t} \right) = \sigma^{-1} \left(\lim_{\kappa\to\infty} \frac{\overline{\lambda}-1}{\overline{\lambda}-\underline{\lambda}} \lim_{\kappa\to\infty} \frac{1}{\underline{\lambda}^t} + \lim_{\kappa\to\infty} \frac{1-\underline{\lambda}}{\overline{\lambda}-\underline{\lambda}} \lim_{\kappa\to\infty} \frac{1}{\overline{\lambda}^t} \right).$$

Note that

$$\lim_{\kappa\to\infty} \overline{\lambda} = \infty \quad \text{and} \quad \lim_{\kappa\to\infty} \underline{\lambda} = 0,$$

hence

$$\lim_{\kappa\to\infty} \frac{1}{\overline{\lambda}^t} = 0 \quad \text{and} \quad \lim_{\kappa\to\infty} \frac{1}{\underline{\lambda}^t} = \infty \quad \forall t \geq 1.$$

Moreover, using L'Hôpital's rule,

$$\lim_{\kappa\to\infty} \frac{\overline{\lambda}-1}{\overline{\lambda}-\underline{\lambda}} = \lim_{\kappa\to\infty} \frac{\frac{\partial\overline{\lambda}}{\partial\kappa}}{\frac{\partial\overline{\lambda}}{\partial\kappa}-\frac{\partial\underline{\lambda}}{\partial\kappa}} = \lim_{\kappa\to\infty} \left(\frac{1}{2} + \frac{\sqrt{1-\frac{4\beta}{\left(1+\beta+\sigma^{-1}\kappa\right)^2}}}{2} \right) = 1,$$

and

$$\lim_{\kappa\to\infty} \frac{1-\underline{\lambda}}{\overline{\lambda}-\underline{\lambda}} = \lim_{\kappa\to\infty} -\frac{\frac{\partial\underline{\lambda}}{\partial\kappa}}{\frac{\partial\overline{\lambda}}{\partial\kappa}-\frac{\partial\underline{\lambda}}{\partial\kappa}} = \lim_{\kappa\to\infty} \left(\frac{1}{2} - \frac{\sqrt{1-\frac{4\beta}{\left(1+\beta+\sigma^{-1}\kappa\right)^2}}}{2} \right) = 0,$$

where we used that

$$\frac{\partial\overline{\lambda}}{\partial\kappa} = \sigma^{-1} \frac{1+\frac{1+\beta+\sigma^{-1}\kappa}{\sqrt{\left(1+\beta+\sigma^{-1}\kappa\right)^2-4\beta}}}{2\beta} > 0,$$

$$\frac{\partial\underline{\lambda}}{\partial\kappa} = \sigma^{-1} \frac{1-\frac{1+\beta+\sigma^{-1}\kappa}{\sqrt{\left(1+\beta+\sigma^{-1}\kappa\right)^2-4\beta}}}{2\beta} < 0.$$

Hence

$$\lim_{\kappa\to\infty} \frac{\partial c_0^{SLTE}}{\partial i_t} = -\infty.$$

For inflation, we have

$$\pi_0^{SLTE} = -\frac{\kappa\sigma^{-1}}{\overline{\lambda}-\underline{\lambda}} \left[\sum_{s=0}^{T} \left(\frac{\overline{\lambda}}{\underline{\lambda}^s} - \frac{\underline{\lambda}}{\overline{\lambda}^s} \right) r_n + \sum_{s=T+1}^{T^*} \left(\frac{\overline{\lambda}}{\underline{\lambda}^s} - \frac{\underline{\lambda}}{\overline{\lambda}^s} \right) (i_s - r_n) \right].$$

For $t > T$

$$\frac{\partial \pi_0^{SLTE}}{\partial i_t} = -\frac{\kappa\sigma^{-1}}{\overline{\lambda}-\underline{\lambda}} \left(\frac{\overline{\lambda}}{\underline{\lambda}^t} - \frac{\underline{\lambda}}{\overline{\lambda}^t} \right) < 0.$$

For the Forward Guidance Puzzle,

$$\frac{\partial^2 \pi_0^{SLTE}}{\partial t \partial i_t} = -\frac{\kappa\sigma^{-1}}{\overline{\lambda}-\underline{\lambda}}\left(-\frac{\overline{\lambda}}{\underline{\lambda}^t}\log\underline{\lambda} + \frac{\underline{\lambda}}{\overline{\lambda}^t}\log\overline{\lambda}\right) < 0.$$

For the Paradox of Flexibility

$$\lim_{\kappa\to\infty}\frac{\partial \pi_0^{SLTE}}{\partial i_t} = -\lim_{\kappa\to\infty}\kappa\sigma^{-1}\left(\underbrace{\lim_{\kappa\to\infty}\frac{\overline{\lambda}}{\overline{\lambda}-\underline{\lambda}}}_{\to 1}\underbrace{\lim_{\kappa\to\infty}\frac{1}{\underline{\lambda}^t}}_{\to\infty} - \underbrace{\lim_{\kappa\to\infty}\frac{\underline{\lambda}}{\overline{\lambda}-\underline{\lambda}}}_{\to 0}\underbrace{\lim_{\kappa\to\infty}\frac{1}{\overline{\lambda}^t}}_{\to 0}\right) < -\infty.$$

□

Given the decomposition in Section 2, it has to be true that either the ISE or the wealth effect (or both) feature the monetary paradoxes as well. The next proposition shows that the ISE is a force *against* the paradoxes, and the paradoxes are the result of the wealth effect.

Proposition 12 (Monetary Paradoxes and the Decomposition)**.** *Suppose $\kappa > 0$ and let $t > T$. The ISE satisfies*

$$\frac{\partial c_0^S}{\partial i_t} < 0, \quad \frac{\partial^2 c_0^S}{\partial t \partial i_t} > 0, \quad \lim_{\kappa\to\infty}\frac{\partial c_0^S}{\partial i_t} = 0.$$

If an equilibrium exhibits the monetary paradoxes, then

$$\frac{\partial^2 \Omega_0}{\partial t \partial i_t} < 0 \quad \textit{and} \quad \lim_{\kappa\to\infty}\frac{\partial \Omega_0}{\partial i_t} = -\infty.$$

Proof. We have

$$c_0^S = \sigma^{-1}\frac{1-\beta\underline{\lambda}}{\overline{\lambda}-\underline{\lambda}}\left[\sum_{s=0}^{T}\left(\frac{1-\beta\overline{\lambda}}{1-\beta\underline{\lambda}}-1\right)\frac{\underline{\lambda}}{\overline{\lambda}^s}r_n + \sum_{s=T+1}^{T^*}\left(\frac{1-\beta\overline{\lambda}}{1-\beta\underline{\lambda}}-1\right)\frac{\underline{\lambda}}{\overline{\lambda}^s}\left(i_s - r_n\right)\right].$$

For $t > T$

$$\frac{\partial c_0^S}{\partial i_t} = -\frac{\sigma^{-1}}{\overline{\lambda}^{t+1}} < 0,$$

and

$$\frac{\partial^2 c_0^S}{\partial t \partial i_t} = \log\overline{\lambda}\frac{\sigma^{-1}}{\overline{\lambda}^{t+1}} > 0.$$

Moreover

$$\frac{\partial^2 c_0^S}{\partial \kappa \partial i_t} = (t+1)\frac{\sigma^{-1}}{\overline{\lambda}^{t+2}}\frac{\partial\overline{\lambda}}{\partial\kappa},$$

where

$$\frac{\partial\overline{\lambda}}{\partial\kappa} = \sigma^{-1}\frac{1+\frac{1+\beta+\sigma^{-1}\kappa}{\sqrt{\left(1+\beta+\sigma^{-1}\kappa\right)^2-4\beta}}}{2\beta} > 0.$$

Hence

$$\frac{\partial^2 c_0^S}{\partial \kappa \partial i_t} > 0.$$

Moreover, since $\lim_{\kappa\to\infty}\overline{\lambda}=\infty$, then

$$\lim_{\kappa\to\infty}\frac{\partial c_0^S}{\partial i_t}=0.$$

Finally, note that if $\frac{\partial^2 c_0}{\partial t\partial i_t}<0$, then

$$\frac{\partial^2 c_0}{\partial t\partial i_t}=\frac{\partial^2 c_0^S}{\partial t\partial i_t}+\frac{1-\beta\underline{\lambda}}{1-\beta}\frac{\partial^2\Omega_0}{\partial t\partial i_t}<0.$$

Since $\frac{\partial^2 c_0^S}{\partial t\partial i_t}>0$ and $\frac{1-\beta\underline{\lambda}}{1-\beta}>0$, then $\frac{\partial^2\Omega_0}{\partial t\partial i_t}<0$. Similarly for the Paradox of Flexibility.

□

Proposition 12 implies that the paradoxes are not the result of the intertemporal substitution of consumption. In the absence of wealth effects, an increase in interest rates shifts consumption from the present to the future. However, the effect becomes weaker as the date of the shock moves further into the future since future rates are discounted by $\frac{1}{\lambda}<1$. Moreover, the substitution effect is continuous in the price flexibility parameter κ, in sharp contrast with the standard liquidity trap equilibrium. Thus, only the wealth effect shares the paradoxical properties of initial consumption. Moreover, note that the wealth effect exhibiting the paradoxes is insufficient to generate the paradoxes on equilibrium consumption. To obtain the paradoxes, the wealth effect must be sufficiently strong to overturn the countervailing force embedded in the ISE. Finally, since fiscal variables determine the wealth effect, we can conclude that the paradoxes are the result of fiscal rather than monetary policy in the standard RANK model.

A popular resolution to the monetary paradoxes has been to build a model that can be characterized by a *discounted* Euler equation of the form:

$$c_t=\delta c_{t+1}-\sigma^{-1}(i_t-\pi_{t+1}-r_n),$$

with $\delta\in(0,1)$.[41] Next, we show for standard calibrations, this specification only mitigates the paradoxes in the SLTE. Only for extreme calibrations (low EIS and high degree of price rigidity), the paradoxes disappear. This is achieved by making both eigenvalues of the system fall outside the unit circle. However, this solution fundamentally changes the properties of the model.

Consider a model characterized by the following system of equations: a discounted Euler equation

$$c_t=\delta c_{t+1}-\sigma^{-1}(i_t-\pi_{t+1}-r_n)$$

with $\delta\in(0,1)$; and a New Keynesian Phillips curve,

$$\pi_t=\beta\pi_{t+1}+\kappa c_t.$$

[41] For example, McKay et al. (2016) propose a heterogeneous agent model with incomplete markets, Angeletos and Lian (2016) relax the assumption of common knowledge, Gabaix (2016) introduces behavioral inattention, and Gertler (2017) adaptive expectations. All these micro-foundations essentially modify the households' Euler equation to reduce the direct impact of future interest rates changes.

This system can be written as

$$\begin{bmatrix} c_{t+1} \\ \pi_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\delta}\left(1+\frac{\sigma^{-1}\kappa}{\beta}\right) & -\frac{\sigma^{-1}}{\delta\beta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{bmatrix} \begin{bmatrix} c_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \sigma^{-1}\left(i_t - r_n\right) \\ 0 \end{bmatrix}.$$

The eigenvalues of the system are

$$\overline{\lambda}_d = \frac{\left(\delta+\beta+\sigma^{-1}\kappa\right) + \sqrt{\left(\delta+\beta+\sigma^{-1}\kappa\right)^2 - 4\beta\delta}}{2\beta\delta},$$
$$\underline{\lambda}_d = \frac{\left(\delta+\beta+\sigma^{-1}\kappa\right) - \sqrt{\left(\delta+\beta+\sigma^{-1}\kappa\right)^2 - 4\beta\delta}}{2\beta\delta}.$$

Note that for $\delta \in [0,1]$, $\overline{\lambda}_d > 1$. Uniqueness of the solution requires that $\underline{\lambda}_d > 1$, which holds if and only if

$$\delta < 1 - \frac{\sigma^{-1}\kappa}{1-\beta}.$$

For standard calibrations, $1 - \frac{\sigma^{-1}\kappa}{1-\beta} < 0$. For example, for $\sigma^{-1} = 1$, $\kappa = 0.1275$ and $\beta = 0.99$, the condition becomes $\delta < -11.75$. Even if we set $\sigma^{-1} = 0.1$, we still have $\delta < -0.2745$. Thus, in what follows we assume that $\delta > 1 - \frac{\sigma^{-1}\kappa}{1-\beta}$, which implies $\underline{\lambda}_d \in (0,1)$.

The eigenvectors are given by

$$\overline{v} = \left(\frac{1-\beta\overline{\lambda}_d}{\kappa}, 1\right),$$
$$\underline{v} = \left(\frac{1-\beta\underline{\lambda}_d}{\kappa}, 1\right).$$

Let

$$P \equiv \begin{bmatrix} \frac{1-\beta\overline{\lambda}_d}{\kappa} & \frac{1-\beta\underline{\lambda}_d}{\kappa} \\ 1 & 1 \end{bmatrix}.$$

Then, we can write the system as

$$\begin{bmatrix} Z_{1,t+1} \\ Z_{2,t+1} \end{bmatrix} = \begin{bmatrix} \overline{\lambda}_d & 0 \\ 0 & \underline{\lambda}_d \end{bmatrix} \begin{bmatrix} Z_{1,t} \\ Z_{2,t} \end{bmatrix} + \frac{\sigma^{-1}\kappa}{\beta(\overline{\lambda}_d - \underline{\lambda}_d)} \begin{bmatrix} -\left(i_t - r_n\right) \\ \left(i_t - r_n\right) \end{bmatrix},$$

where $Z_t \equiv P^{-1}\begin{bmatrix} c_t \\ \pi_t \end{bmatrix}$. Since $\overline{\lambda}_d > 1$, we can solve the first equation forward

$$Z_{1,t} = \delta\frac{\sigma^{-1}\kappa}{\overline{\lambda}_d - \underline{\lambda}_d}\overline{\lambda}_d^t \sum_{s=t}^{\infty} \frac{\underline{\lambda}_d}{\overline{\lambda}_d^s}\left(i_s - r_n\right).$$

Moreover, since $\underline{\lambda}_d \in (0,1)$, we can solve the second equation backward

$$Z_{2,t} = \underline{\lambda}_d^t Z_{2,0} + \delta\frac{\sigma^{-1}\kappa}{\overline{\lambda}_d - \underline{\lambda}_d}\underline{\lambda}_d^t \sum_{s=0}^{t-1} \frac{\overline{\lambda}_d}{\underline{\lambda}_d^s}\left(i_s - r_n\right).$$

Recall that

$$Z_t = -\frac{\kappa}{\beta(\overline{\lambda}_d - \underline{\lambda}_d)} \begin{bmatrix} 1 & -\frac{1-\beta\underline{\lambda}_d}{\kappa} \\ -1 & \frac{1-\beta\overline{\lambda}_d}{\kappa} \end{bmatrix} \begin{bmatrix} c_t \\ \pi_t \end{bmatrix}.$$

Hence

$$Z_{1,t} = -\frac{\kappa}{\beta\left(\overline{\lambda}_d - \underline{\lambda}_d\right)} \left(c_t - \frac{1-\beta\underline{\lambda}_d}{\kappa} \pi_t \right),$$
$$Z_{2,t} = -\frac{\kappa}{\beta\left(\overline{\lambda}_d - \underline{\lambda}_d\right)} \left(-c_t + \frac{1-\beta\overline{\lambda}_d}{\kappa} \pi_t \right).$$

And therefore

$$c_t = \frac{1-\beta\underline{\lambda}_d}{\kappa}\pi_t - \sigma^{-1}\overline{\lambda}_d^t \sum_{s=t}^{\infty} \frac{1}{\overline{\lambda}_d^{s+1}} \left(i_s - r_n\right), \tag{33}$$

$$\pi_t = \frac{\kappa}{1-\beta\overline{\lambda}_d} c_t - \beta\frac{\overline{\lambda}_d - \underline{\lambda}_d}{1-\beta\overline{\lambda}_d}\underline{\lambda}_d^t Z_{2,0} - \frac{\sigma^{-1}\kappa}{1-\beta\overline{\lambda}_d}\underline{\lambda}_d^t \sum_{s=0}^{t-1} \frac{1}{\underline{\lambda}_d^{s+1}} \left(i_s - r_n\right). \tag{34}$$

Introducing (33) into (34), we get

$$\pi_t = \underline{\lambda}_d^t Z_{2,0} + \delta\frac{\sigma^{-1}\kappa}{\overline{\lambda}_d - \underline{\lambda}_d}\underline{\lambda}_d^t \sum_{s=0}^{t-1} \frac{\overline{\lambda}_d}{\underline{\lambda}_d^s} \left(i_s - r_n\right) + \delta\frac{\sigma^{-1}\kappa}{\overline{\lambda}_d - \underline{\lambda}_d}\overline{\lambda}_d^t \sum_{s=t}^{\infty} \frac{\underline{\lambda}_d}{\overline{\lambda}_d^s} \left(i_s - r_n\right). \tag{35}$$

Introducing (35) into (33), we get

$$c_t = \frac{1-\beta\underline{\lambda}_d}{\kappa}\underline{\lambda}_d^t Z_{2,0} + \delta\sigma^{-1}\frac{1-\beta\underline{\lambda}_d}{\overline{\lambda}_d - \underline{\lambda}_d}\underline{\lambda}_d^t \sum_{s=0}^{t-1} \frac{\overline{\lambda}_d}{\underline{\lambda}_d^s} \left(i_s - r_n\right) + \delta\sigma^{-1}\frac{1-\beta\overline{\lambda}_d}{\overline{\lambda}_d - \underline{\lambda}_d}\overline{\lambda}_d^t \sum_{s=t}^{\infty} \frac{\underline{\lambda}_d}{\overline{\lambda}_d^s} \left(i_s - r_n\right). \tag{36}$$

Multiplying (36) by β^t and summing across time, we get

$$\frac{\Omega_0}{1-\beta} = \sum_{t=0}^{\infty} \beta^t c_t = \frac{1}{\kappa} Z_{2,0} + \delta\frac{\sigma^{-1}}{\overline{\lambda}_d - \underline{\lambda}_d} \sum_{s=0}^{\infty} \frac{\underline{\lambda}_d}{\overline{\lambda}_d^s} \left(i_s - r_n\right).$$

Hence

$$Z_{2,0} = \frac{\kappa}{1-\beta}\Omega_0 - \delta\frac{\sigma^{-1}\kappa}{\overline{\lambda}_d - \underline{\lambda}_d} \sum_{s=0}^{\infty} \frac{\underline{\lambda}_d}{\overline{\lambda}_d^s} \left(i_s - r_n\right). \tag{37}$$

Introducing (37) in (36), we get

$$c_t = \delta c_t^S + \frac{1-\beta\underline{\lambda}_d}{1-\beta}\underline{\lambda}_d^t \Omega_0,$$

where

$$c_t^S \equiv \sigma^{-1}\frac{1-\beta\underline{\lambda}_d}{\overline{\lambda}_d - \underline{\lambda}_d}\underline{\lambda}_d^t \left[\sum_{s=0}^{t-1} \left(\frac{\overline{\lambda}_d}{\underline{\lambda}_d^s} - \frac{\underline{\lambda}_d}{\overline{\lambda}_d^s} \right) \left(i_s - r_n\right) + \sum_{s=t}^{\infty} \left(\frac{1-\beta\overline{\lambda}_d}{1-\beta\underline{\lambda}_d} \left(\frac{\overline{\lambda}_d}{\underline{\lambda}_d} \right)^t - 1 \right) \frac{\underline{\lambda}_d}{\overline{\lambda}_d^s} \left(i_s - r_n\right) \right].$$

Recall that the SLTE is the unique purely-forward looking equilibrium. Hence

$$c_t^{SLTE} = \frac{\sigma^{-1}}{\overline{\lambda}_d - \underline{\lambda}_d} \sum_{s=t}^{\infty} \left(\frac{\delta\underline{\lambda}_d - 1}{\overline{\lambda}_d^{s-t}} - \frac{\delta\overline{\lambda}_d - 1}{\underline{\lambda}_d^{s-t}} \right) \left(i_s - r_n\right).$$

Note that

$$\frac{\partial c_0^{SLTE}}{\partial i_t} = -\frac{\sigma^{-1}}{\overline{\lambda}_d - \underline{\lambda}_d}\left(\frac{\delta\overline{\lambda}_d - 1}{\underline{\lambda}_d^t} + \frac{1 - \delta\underline{\lambda}_d}{\overline{\lambda}_d^t}\right).$$

Since $\delta\overline{\lambda}_d > 1$ and $\underline{\lambda}_d < 1$, the model shares the same properties as when $\delta = 1$. Thus, extending the results in Proposition 11 we can establish that the discounted Euler equation equilibrium features the Forward Guidance Puzzle and the Paradox of Flexibility.

F Derivation of the RANK model with capital

This section derives the standard New Keynesian model with capital. Time is discrete and runs forever. The economy is populated by households, firms and a government.

Households. There is a representative infinitely-lived household with preferences over consumption C_t and labor N_t represented by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t).$$

The household faces a per-period budget constraint given by

$$P_t C_t + P_t I_t + Q_t B_{t+1} \leq (1 + \rho Q_t) B_t + W_t N_t + R_t K_t + \Pi_t + P_t \tilde{T}_t,$$

and the law of motion of capital

$$K_{t+1} = (1 - \delta) K_t + I_t,$$

where P_t is the price level, B_{t+1} denotes purchases of long-term risk-free bonds with price Q_t (which satisfies $Q_t = \frac{1+\rho Q_{t+1}}{1+i_t}$, where i_t is the risk-free nominal interest rate), W_t is the nominal wage, Π_t denotes corporate profits, and $\tilde{T}_t$ is a lump-sum government transfer. Moreover, the household faces a solvency condition given by $\lim_{T\to\infty} B_T \geq 0$ for all t.

The household's optimality conditions are given by

$$\begin{aligned} -\frac{U_{n,t}}{U_{c,t}} &= \frac{W_t}{P_t}, \\ 1 &= (1 + i_t)\beta \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}}, \\ 1 &= \left[\frac{R_{t+1}}{P_{t+1}} + 1 - \delta\right] \beta \frac{U_{c,t+1}}{U_{c,t}}. \end{aligned}$$

where $U_{c,t} \equiv \frac{\partial U(C_t, N_t)}{\partial C_t}$ and $U_{n,t} \equiv \frac{\partial U(C_t, N_t)}{\partial N_t}$. In what follows, we assume that $U(C_t, N_t) = \log(C_t) - N_t$.

Firms. There are two types of firms in the economy: final-good producers and intermediate-goods producers. Final-goods producers operate in a perfectly competitive market and combine a unit mass of intermediate goods $Y_t(i)$, for $i \in [0, 1]$, using the production function

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}. \tag{38}$$

The problem of the final-good producer is given by

$$\max_{[Y_t(i)]_{i\in[0,1]}} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$$

subject to (38). The solution to this problem gives the standard CES demand

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t, \tag{39}$$

where $P_t \equiv \left(\int_0^1 P_t\left(i\right)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}$.

Intermediate goods are produced using the following technology:

$$Y_t(i) = K_t\left(i\right)^{\gamma} N_t(i)^{1-\gamma},$$

where $\gamma \in [0,1]$. Given a demand for the intermediate good i, the cost-minimization problem is given by

$$\min_{K_t(i),N_t(i)} R_t K_t\left(i\right) + W_t N_t\left(i\right)$$

subject to

$$Y_t(i) = K_t\left(i\right)^{\gamma} N_t(i)^{1-\gamma}.$$

The optimality conditions imply

$$\frac{K_t\left(i\right)}{N_t\left(i\right)} = \frac{\gamma}{1-\gamma}\frac{W_t}{R_t},$$

or, noting that $N_t(i) = \left(\frac{Y_t(i)}{K_t(i)^{\gamma}}\right)^{\frac{1}{1-\gamma}}$,

$$\frac{K_t\left(i\right)}{Y_t(i)} = \left(\frac{\gamma}{1-\gamma}\frac{W_t}{R_t}\right)^{1-\gamma}.$$

Thus, the firm's cost function is given by

$$\Psi_t\left(Y_t\left(i\right)\right) = \left(\frac{W_t}{1-\gamma}\right)^{1-\gamma}\left(\frac{R_t}{\gamma}\right)^{\gamma} Y_t\left(i\right).$$

Firms choose the price for their good, $P_t(i)$, subject to the demand for their good, given by (39), taking the aggregate price level P_t and aggregeate output, Y_t, as given. As is standard in New Keynesian model, we assume that firms are subject to a pricing friction à la Calvo: each firm may set a new price only with probability $1-\theta$ in any period. Let P_t^* denote the price chosen by a firm that is able to set the price in period t. Then, P_t^* is the solution to the following problem:

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k Q_{t,t+k}\left[(1-\tau)P_t^* Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t})\right]$$

subject to

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k},$$

where $Q_{t,t+k} \equiv \beta^k U_{c,t+k}/U_{c,t}P_t/P_{t+k}$ is the stochastic discount factor for nominal payoffs, $Y_{t+k|t}$ denotes output in period $t+k$ for a firm that last set price in period t, and τ is a proportional sales tax. The first-order condition associated with this problem is given by

$$\sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k|t}\left((1-\tau)P_t^* - \frac{\epsilon}{\epsilon-1}\Psi_t'(Y_{t+k|t})\right) = 0.$$

Dividing this expression by P_t, we get

$$\sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k|t} \left((1-\tau) \frac{P_t^*}{P_t} - \frac{\epsilon}{\epsilon - 1} MC_{t+k|t} \frac{P_{t+k}}{P_t} \right) = 0,$$

where $MC_{t+k|t} \equiv \Psi_t'(Y_{t+k|t}) / P_{t+k}$ is the real marginal cost in period $t+k$ for a firm whose price was last set in period t.

Government. We assume that the monetary authority follows an interest rate rule of the form

$$\log(1+i_t) = \rho_r \log(1+i_{t-1}) + (1-\rho_r)(r_n + \phi_\pi \pi_t) + \varepsilon_{m,t},$$

where $\rho_r \in [0,1)$, $r_n \equiv -\log \beta$, $\pi_t \equiv \log\left(\frac{P_t}{P_{t-1}}\right)$, and $\varepsilon_{m,t}$ denotes a monetary policy shock. The per-period budget constraint of the government is given by

$$Q_t B_{t+1} = (1 + \rho Q_t) B_t - \tau P_t Y_t + P_t \tilde{T}_t,$$

where we used that $\left(\int_0^1 (\tau Y_t(i))^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} = \tau \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} = \tau Y_t$.

Equilibrium. Market clearing in the goods market requires

$$C_t + I_t = Y_t.$$

Market clearing in the factor markets requires

$$N_t = \int_0^1 N_t(i) di, \qquad K_t = \int_0^1 K_t(i)\, di.$$

From the production function, we get

$$\begin{aligned} Y_t(i) &= \left(\frac{N_t(i)}{K_t(i)} \right)^{1-\gamma} K_t(i) \\ &= \left(\frac{1-\gamma}{\gamma} \frac{R_t}{W_t} \right)^{1-\gamma} K_t(i), \end{aligned}$$

where the second equality follows from the firm's cost-minimization problem. Integrating across firms, we get

$$\begin{aligned} K_t &= \left(\frac{\gamma}{1-\gamma} \frac{W_t}{R_t} \right)^{1-\gamma} \int_0^1 Y_t(i)\, di \\ &= \left(\frac{\gamma}{1-\gamma} \frac{W_t}{R_t} \right)^{1-\gamma} Y_t \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} di, \end{aligned}$$

where the second equality follows from the demand for differentiated goods.

Because of the Calvo friction, the price level can be written as

$$P_t = \left[(1-\theta)(P_t^*)^{1-\epsilon} + \int_{S(t)} (P_{t-1}(i))^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}},$$

where $S(t) \subset [0,1]$ is the set of firms that do not set a new price in period t. Since a random set of firms is able to change prices every period (independent of any firm characteristic), we have that

$$\int_{S(t)} (P_{t-1}(i))^{1-\epsilon} di = \theta P_{t-1}^{1-\epsilon}.$$

Hence, we can write the price level as

$$P_t = \left[(1-\theta)(P_t^*)^{1-\epsilon} + \theta P_{t-1}^{1-\varepsilon} \right]^{\frac{1}{1-\epsilon}}.$$

Steady state. Let the variables without subscript denote their value in a zero-inflation steady state. The steady-state equilibrium of this economy is the solution to the following system of equations:

$$\begin{aligned}
Y &= K^{\gamma} N^{1-\gamma} \\
\frac{K}{N} &= \frac{\gamma}{1-\gamma} \frac{W}{R} \\
MC &= \left(\frac{W/P}{1-\gamma} \right)^{1-\gamma} \left(\frac{R/P}{\gamma} \right)^{\gamma} \\
(1-\tau) &= \frac{\epsilon}{\epsilon - 1} MC \\
Y &= C + \delta K \\
\frac{W}{P} &= C \\
i &= \frac{R}{P} - \delta \\
i &= \frac{1-\beta}{\beta} \\
\frac{1-\beta}{\beta} QB &= \tau Y - T.
\end{aligned}$$

Log-linearization. We study the dynamics of the economy around a steady-state equilibrium with zero inflation. For a variable X_t, let $x_t \equiv \log\left(\frac{X_t}{X}\right)$, where X denotes the zero-inflation steady-state value.

The log-linear versions of the household's optimality conditions are given by

$$\begin{aligned}
w_t - p_t &= c_t, \\
c_t &= c_{t+1} - (i_t - \pi_{t+1} - r_n),
\end{aligned}$$

and the rental rate of capital is given by

$$r(r_{t+1} - p_{t+1}) = \beta(i_t - \pi_{t+1} - r_n),$$

where $r = \frac{1-\beta(1-\delta)}{\beta}$.

The log-linear approximation of the firms' first-order condition around the zero inflation steady state yields

$$p_t^* - p_t = (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k\left[mc_{t+k} + p_{t+k} - p_t\right].$$

Moreover, approximating the price level equation we get

$$p_t^* - p_t = \frac{\theta}{1-\theta}\pi_t.$$

Hence, we can write the firm's optimality condition as

$$\pi_t = \beta\pi_{t+1} + \psi mc_t, \tag{40}$$

where $\psi \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}$. Approximating the expression for the marginal cost, we get

$$mc_t = (w_t - p_t) - \gamma\left(w_t - r_t\right). \tag{41}$$

Note that the cost-minimization optimality condition implies

$$k_t - y_t = (1-\gamma)\left(w_t - r_t\right),$$

and the production function implies

$$n_t = \frac{1}{1-\gamma}y_t - \frac{\gamma}{1-\gamma}k_t,$$

where we used that $\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} di \simeq 1$ up to first order. Introducing these two expressions and the labor supply equation into (41), we get

$$mc_t = c_t + \frac{\gamma}{1-\gamma}y_t - \frac{\gamma}{1-\gamma}k_t. \tag{42}$$

Moreover, approximating the resource constraint, we get

$$y_t = s_c c_t + s_I\left(\frac{1}{\delta}k_{t+1} - \frac{1-\delta}{\delta}k_t\right),$$

where $s_c \equiv \frac{C}{Y}$ and $s_I \equiv \frac{I}{Y}$. Introducing into (42),

$$mc_t = \left(1 + \frac{\gamma}{1-\gamma}s_c\right)c_t + \frac{\gamma}{1-\gamma}\frac{s_I}{\delta}k_{t+1} - \frac{\gamma}{1-\gamma}\left(1 + \frac{1-\delta}{\delta}s_I\right)k_t. \tag{43}$$

Introducing (43) into (40) and rearranging, we get

$$k_{t+1} = -\xi_{k\pi}\beta\pi_{t+1} + \xi_{k\pi}\pi_t - \xi_{kc}c_t + \xi_{kk}k_t,$$

where

$$\xi_{k\pi} \equiv \frac{1}{\psi}\frac{1-\gamma}{\gamma}\frac{\delta}{s_I}, \quad \xi_{kc} \equiv \frac{\delta}{s_I}\left(\frac{1-\gamma}{\gamma} + s_c\right), \quad \xi_{kk} \equiv 1 - \delta + \frac{\delta}{s_I}.$$

Consider now the expression for the rental rate of capital

$$r(r_{t+1} - p_{t+1}) = \frac{1}{\beta}(i_t - \pi_{t+1} - r_n). \tag{44}$$

Combining the cost-minimization optimality condition and the labor supply, and iterating forward one period, we get

$$r_{t+1} - p_{t+1} = c_{t+1} + \frac{1}{1-\gamma}\left(y_{t+1} - k_{t+1}\right). \tag{45}$$

Note that introducing (42) into (40) and rearranging, we get

$$y_t - k_t = \frac{1-\gamma}{\gamma}\left[\frac{1}{\psi}\left(\pi_t - \beta\pi_{t+1}\right) - c_t\right].$$

Iterating this expression forward one period and introducing into (45), we get

$$r_{t+1} - p_{t+1} = -\frac{1-\gamma}{\gamma}c_{t+1} + \frac{1}{\gamma}\frac{1}{\psi}\left(\pi_{t+1} - \beta\pi_{t+2}\right).$$

Replacing c_{t+1} by the Euler equation for bonds, we get

$$r_{t+1} - p_{t+1} = -\frac{1-\gamma}{\gamma}c_t - \frac{1-\gamma}{\gamma}\left(i_t - r_n\right) + \left(\frac{1-\gamma}{\gamma} + \frac{1}{\gamma}\frac{1}{\psi}\right)\pi_{t+1} - \frac{1}{\gamma}\frac{1}{\psi}\beta\pi_{t+2}.$$

Introducing this expression into (44) and rearranging, we get

$$\pi_{t+2} = \xi_{\pi\pi}\pi_{t+1} - \xi_{\pi i}\left(i_t - r_n\right) - \xi_{\pi c}c_t,$$

where

$$\xi_{\pi\pi} \equiv \frac{1}{\beta}\left(1 + \psi + \frac{\psi\gamma\left(1-\delta\right)}{r}\right), \quad \xi_{\pi i} \equiv \frac{\psi}{\beta}\left(1 + \frac{\gamma\left(1-\delta\right)}{r}\right), \quad \xi_{\pi c} \equiv \frac{\psi}{\beta}\left(1-\gamma\right).$$

The log-linear approximation of the intertemporal budget constraint is given by

$$\sum_{t=0}^{\infty}\beta^t\left[s_c c_t + s_I \iota_t\right] \leq \sum_{t=0}^{\infty}\beta^t\left[\left(1-\tau\right)y_t + \left(i_t - \pi_{t+1} - r_n\right)Qb + T_t\right] - \left[\sum_{t=0}^{\infty}\left(\beta\rho\right)^t\left(i_t - r_n\right)\rho + \frac{1}{\beta}\pi_0\right]Qb,$$

where $b \equiv \frac{B}{PY}$ and $T_t \approx \frac{\tilde{T}_t - T}{Y}$, and we used that, up to first order, the price of the bond satisfies

$$i_t - r_n = \beta\rho q_{t+1} - q_t.$$

Note that we assume that $B_0 = B$ and $P_{-1} = P$, so that the economy starts in steady state.

As is standard, the approximation of the interest rate rule is given by

$$i_t = \rho_r i_{t-1} + (1-\rho_r)(r_n + \phi_\pi \pi_t) + \varepsilon_{m,t},$$

where, with a slight abuse of notation, we are using that $i_t \approx \log(1+i_t)$.

APPENDIX REFRENCES

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