Monetary-Based Asset Pricing: A Mixed-Frequency Structural Approach

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Abstract

We integrate a high-frequency monetary event study into a mixed-frequency macro-finance model and structural estimation. The model and estimation allow for jumps at Fed announcements in investor beliefs, providing granular detail on why markets react to central bank communications. We find that the reasons involve a mix of revisions in investor beliefs about the economic state and/or future regime change in the conduct of monetary policy, and subjective reassessments of financial market risk. However, the structural estimation also finds that much of the causal impact of monetary policy on markets occurs outside of tight windows around policy announcements.

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1 Introduction

It is practically a truism that the stock market is highly attuned to monetary news. Academic studies are generally consistent with this maxim, finding that the real values of long-term financial assets fluctuate sharply in response to the actions and announcements of central banks. Why?

A growing academic literature has offered a myriad of competing explanations. A classic view is that surprise central bank announcements proxy for shocks to a nominal interest rate rule of the type emphasized by Taylor (1993), which have short-run affects on the real economy in a manner consistent with canonical New Keynesian models (e.g., Christiano, Eichenbaum, and Evans (2005)). But other explanations abound, including the effects such announcements have on financial market risk premia, the information they impart about the state of the economy (the "Fed information effect"), or the role they play in revising the public's understanding of the central bank's reaction function and objectives. For the most part, the empirical facts of these asset market fluctuations have been established from high-frequency event studies in tight windows around Federal Reserve (Fed) communications, combined with estimations of reduced-form empirical specifications. By contrast, the interpretations of these facts largely follow from carefully calibrated theoretical models designed to show that one of the competing explanations fits with certain aspects of the reduced-form evidence.

Yet, as the mushrooming debate over how to interpret this evidence indicates, many questions about the interplay between markets and monetary policy remain unanswered. In this paper we consider three of them. First, theories focused on a single channel of monetary transmission are useful for elucidating its marginal effects, but may reveal only part of the overall picture. To what extent are several competing explanations or others entirely playing a role simultaneously? Second, monetary policy communications cover a range of topics, from interest rate policy, to forward guidance, to quantitative interventions, to the macroeconomic outlook. How do these varied communications affect market participants' perceptions of the primitive economic sources of risk hitting the economy? Third, by design, high frequency events studies only capture the causal effects of the surprise component of a monetary policy announcement. At best, this represents a lower bound on the overall causal impact of monetary policy on markets; at worst, it represents a gross underestimate. How much of the causal influence of shifting monetary policy occurs outside of tight windows around Fed communications, effects that are by construction impossible to observe from high-frequency event studies alone?

Our contribution to these questions is to integrate a high-frequency event study into a mixed-frequency structural model and estimation. We examine Fed communications alongside both high- and lower-frequency data through the lens of a structural equilibrium asset pricing model with New Keynesian style macroeconomic dynamics. The model and estimation allow

for jumps in investor beliefs about the latent state of the economy, the perceived sources of economic risk, and the future conduct of monetary policy. The novelty of this approach allows us to investigate a variety of possible explanations for why markets respond strongly and swiftly to central bank actions and announcements, not merely by delineating which expectations are revised, but also by providing granular detail on *why* they are revised, with a decomposition of market responses into the primitive economic sources of risk responsible for observed forecast revisions. The mixed-frequency structural estimation further permits us to quantify the causal effects of shifts in monetary policy that may occur outside of tight windows surrounding Fed communications. Structural asset pricing models are especially valuable in this context because they place cross-equation restrictions on the type of news capable of moving the real values of extreme long-duration assets like the stock market, where expected payout accrues not just over the next business cycle or even the next decade, but indefinitely. The general approach can be applied in a wide variety of other settings, when a granular understanding of financial market fluctuations in response to almost any news source is desired.

The model economy is comprised of two different representative agents, "investors" and "households," in addition to a central bank. Risky asset wealth is concentrated in the hands of a small number of identical investors who take macroeconomic dynamics as given and earn all of their income from investments in two assets: the aggregate stock market and a one-period nominal bond. Macroeconomic dynamics are specified by a set of equations similar to those commonly featured in New Keynesian models, driven by a representative household that has access to the nominal bond but holds no stock market wealth.

An important feature of our model is that the conduct of monetary policy is not static over time, but is instead subject to infrequent nonrecurrent regime shifts, or "structural breaks." These take the form of shifts in the parameters of a nominal interest rate rule that include both the inflation target and the activism coefficients governing how strongly the monetary authority responds to inflation-target deviations and to economic growth. Revisions in expectations about the Fed's future reaction function and objectives are one reason that investors in the model react to what the central bank does and says. Such changes in what we refer to as the *conduct* of monetary policy give rise to movements in the nominal interest rate that are conceptually distinct from those generated by the monetary policy *shock*, an innovation in the nominal rate that is uncorrelated with inflation, economic growth, and shifts in the policy rule parameters.

We explicitly model investor beliefs about changes in the monetary policy rule. Investors in the model can estimate the current rule, but face uncertainty over how long the current rule will remain in place and what will follow, once the current regime ends. Central bank communications are closely monitored for information that would cause agents to revise the perceived likelihood of transitioning from the current policy regime to a perceived "Alternative regime" that they believe will come next. Investors are aware that they may change their minds subsequently in response to new information, and take that into account when forming

expectations.

Importantly, however, investor reactions to central bank communications are not restricted to be only about the path of short rates driven by shifts in the policy rule. A Fed announcement in our model is an actual news event in which agents may revise their "nowcasts" of the current economic state, their beliefs about the future conduct of monetary policy, their subjective expectations about future economic aggregates, and the perceived risk in the stock market. To ensure that model expectations evolve in a manner that closely aligns with observed expectations, we map the theoretical implications for these beliefs into data on numerous forward-looking variables, including household and professional forecast surveys and financial market indicators from spot and futures markets.

The full structural framework is solved and estimated using Bayesian methods. The equilibrium solution illustrates the rich endogenous interactions between beliefs about central bank policy and the rest of the economy. Specifically, beliefs about the future conduct of monetary policy not only have direct effects on the real economy, they also amplify and propagate economic shocks that are entirely non-monetary in nature and cause the perceived quantity of stock market risk to vary with the expected future conduct of monetary policy.

Before undertaking the full structural model estimation, we establish model-free evidence of regime change in the conduct of monetary policy over our sample. To do so, we document the existence of decades-long deviations of the real federal funds rate from a widely used measure of the neutral rate of interest, as in Bianchi, Lettau, and Ludvigson (2022). We refer to the difference between the two as the *monetary policy spread*, or *mps* for brevity. We estimate infrequent nonrecurrent regime shifts, i.e., "structural breaks," in the mean of *mps* that divide the sample from 1961:Q1 to 2020:Q1 into three distinct subperiods: a "Great Inflation" regime (1961:Q1-1978:Q3), a "Great Moderation" regime (1978:Q4-2001:Q3), and a "Post Millennial" regime (2001:Q4-2020:Q1). The estimates imply the existence of quantitatively important structural change in the mean of *mps* across these regime subperiods, with the Great Inflation and Post Millennial subperiods representing extended accommodative episodes in monetary history, and the Great Moderation subperiod an extended restrictive episode. We take these infrequent breaks to be model-free estimates on the timing of realized regime changes in the conduct of monetary policy over the course of our sample, and use the structural model to assess whether estimated policy rules actually shifted across these subperiods. We find that they did, with the Great Moderation regime exhibiting a much more hawkish policy rule compared to the dovish Great Inflation and Post Millennial regimes.

Our main empirical results from the structural model estimation may be summarized as follows. First, from the high frequency component of our structural estimation we find that investors seldom learn only about conventional monetary policy shocks from central bank announcements. Instead, our estimates imply that jumps in financial market variables are typically the result of a mix of factors, including revisions in investor beliefs about the economic state

and/or future regime change in the conduct of monetary policy, which endogenously cause subjective reassessments of financial market risk. Indeed, the most quantitatively important FOMC announcements in our sample are associated with large high frequency revisions in professional forecasts of inflation and GDP growth, as well as jumps in federal funds futures and the stock market. These fluctuations are themselves associated with announcement-driven revisions in the composition of primitive shocks that investors perceive are hitting the economy. For example, the FOMC announcement of January 22, 2008–at the height of the Great Financial Crisis–is associated with a 2.6% decline in the stock market but a 0.2% increase in professional forecasts of one-year-ahead inflation. Our estimates imply that the market's nosedive in the minutes surrounding this announcement was driven almost entirely by large downward revisions in investors' nowcasts for aggregate demand and for the S&P 500 earnings share of output, the combination of which outweighed other announcement-related factors that contributed to a rise in the market, such as the perception of an accommodative monetary policy shock and faster trend economic growth. By contrast, the jump upward in the inflation outlook was driven almost entirely by an upward revision in the nowcast for markups, which outweighed the negative contribution from lower perceived demand.

Second, a key aspect of the model for explaining stock market behavior is the evolution of investor beliefs about infrequent shifts in the monetary policy rule. We find that fluctuating beliefs about the future conduct of monetary policy can cause significant market volatility even if the current policy rule and target interest rate remain unchanged. However, the estimates also show that investor beliefs about the probability of a regime change in monetary policy continuously evolve *out*side of tight windows around policy announcements and that most of the variation in these beliefs occurs at times that are not close to an FOMC announcement. One explanation for this result is that most Fed announcements are not immediately associated with a change in the policy rule, but instead provide "forward guidance" in the form of a data-dependent recipe for what could trigger a change in the conduct of policy down the road. This finding underscores the challenges with relying solely on high-frequency event studies for quantifying the channels of monetary transmission to markets and the real economy.

Third, a large fraction of the variation in the stock market and in the short-term real interest rate across time is explained by the combination of realized regime changes in the conduct of monetary policy, and fluctuating real-time beliefs about the possibility of future regime change. Investor beliefs about a future regime change are especially important for the stock market because of their role in shaping perceptions of equity market risk. We find that the S&P 500 (relative to lagged aggregate output) would have been 50% higher than it was in February of 2020, had investors counterfactually believed that the Fed was very likely to shift to a policy rule in the next month that featured greater activism in stabilizing the real economy.

The research in this paper touches on several different strands of literature that connect monetary policy to movements in asset values. On the empirical side, the high frequency com-

ponent of our structural estimation connects with a body of evidence that finds the values of long-term financial assets and expected return premia respond sharply to the announcements of central banks (Cochrane and Piazzesi (2002), Piazzesi (2005), Bernanke and Kuttner (2005), Hanson and Stein (2015), Gertler and Karadi (2015), Gilchrist, López-Salido, and Zakrajšek (2015), Boyarchenko, Haddad, and Plosser (2016), Kekre and Lenel (2021), Pflueger and Rinaldi (2020)).[1] Brooks, Katz, and Lustig (2018) document evidence of persistent post-FOMC announcement drift in longer term Treasury yields, implying that monetary policy has a long-lasting influence on markets outside of tight windows around FOMC announcements, a finding shared by our study. We further complement this literature by providing evidence that expected return premia vary in part because the perceived quantity of stock market risk fluctuates with beliefs about the future conduct of monetary policy, with a higher perceived likelihood of transitioning to a more active policy rule where economic fluctuations are more aggressively stabilized associated with a lower perceived quantity of risk.

A classic assumption in the extant literature is that high-frequency financial market reactions to Fed announcements proxy for conventional monetary policy "shocks," i.e., innovations in a Taylor (1993)-type nominal interest rate rule (e.g., Cochrane and Piazzesi (2002), Piazzesi (2005), Hanson and Stein (2015), Kekre and Lenel (2021); Pflueger and Rinaldi (2020)). By contrast, Jarocinski and Karadi (2020) and Cieslak and Schrimpf (2019) argue that some of the fluctuations are likely driven by the revelation of private information by the Fed, a "Fed information effect" channel emphasized in earlier work by Romer and Romer (2000), Campbell, Evans, Fisher, Justiniano, Calomiris, and Woodford (2012), Melosi (2017), and Nakamura and Steinsson (2018). Hillenbrand (2021) argues that bond markets learn about the secular trend in interest rates from Fed announcements. Bauer and Swanson (2021) also find important announcement effects of Fed communications, but emphasize a "response to news" channel whereby markets are surprised by the response of the Fed to recent economic events. This explanation suggests that surprise announcements cause financial markets to revise their understanding of the central bank's policy rule rather than their understanding of the economic state.

Other empirical studies of Fed announcements have focused specifically on crisis episodes (Krishnamurthy and Vissing-Jorgensen (2011), Cox, Greenwald, and Ludvigson (2020), Haddad, Moreira, and Muir (2020)). These studies find large effects of Fed announcements in markets for government bonds during the financial crisis of 2008, and in the stock market and corporate bond market during the Covid crisis of 2020.

All of the papers cited above form their conclusions from reduced-form empirical event

[1] These studies follow on earlier work finding a link between monetary policy surprises and short-term assets in high-frequency data (Cook and Hahn (1989), Gürkaynak, Sack, and Swanson (2005)). A separate literature studies the timing of when premia in the aggregate stock market are earned in weeks related to Federal Open Market Committee- (FOMC)-cycle time (Lucca and Moench (2015), Cieslak, Morse, and Vissing-Jorgensen (2019)).

studies, a natural starting point. Yet the absence of a rich structural interpretation of these events makes it impossible to provide granular detail on why markets react so strongly or to investigate whether multiple channels may be playing a role simultaneously, gaps our mixed-frequency structural approach is designed to fill.

Beyond event studies, contemporaneous work by Bauer, Pflueger, and Sundaram (2022) uses monthly survey data to estimate perceived policy rules. They find that perceptions about policy rules are subject to substantial time-variation, a result similar to our structural estimation finding that there is significant time-variation in beliefs about the future conduct of monetary policy. Their study differs from ours in that they do not investigate the joint determination of beliefs, the macroeconomy, financial markets, and the policy rule in a macro-finance model, or integrate a high-frequency event study into a structural framework, as is the focus of this paper.

On the theoretical side, Piazzesi (2005) studies the effect of monetary policy shocks in a continuous time, arbitrage-free yield curve model and finds that accounting for monetary policy significantly improves the performance of traditional yield curve models with three latent factors. Kekre and Lenel (2021) and Pflueger and Rinaldi (2020) develop carefully calibrated theoretical models designed to explain why stock market return premia vary in response to a monetary policy shock. Kekre and Lenel (2021) argue that accommodative monetary policy shocks shift the composition of wealth toward investors with greater risk-bearing capacity, while Pflueger and Rinaldi (2020) argue that they cause consumption to deviate strongly and persistently from an external habit, resulting in large changes in the representative investor's risk aversion. These models are silent on the possible role of Fed information effects or changing policy rules in market fluctuations.

The structural model and estimation of this paper builds on Bianchi, Lettau, and Ludvigson (2022) (BLL hereafter), who find that infrequent changes in the conduct of monetary policy generate large and persistent fluctuations in the real interest rate, in asset valuations, and in the equity premium. This study differs substantively from BLL in a number of ways. First, in contrast to the present paper, the estimation of BLL used only a handful of observable series available at lower frequencies with the aim of explaining low frequency asset price variation. BLL is thus silent on why markets react at high frequency to Fed announcements, a question at the heart of the investigation in this paper. Second, in comparison to BLL, this study uses a much larger dataset on forward-looking information, culls relevant information at different frequencies, and explicitly models high-frequency revisions in beliefs in the minutes surrounding Fed announcements, as well as at lower frequencies. The mixed-frequency structural approach of this paper offers a significant methodological advance over BLL and, to the best of our knowledge, the extant literature. Third, unlike BLL, we model regime changes in the conduct of monetary policy as nonrecurrent regimes, i.e., structural breaks, rather than recurrent regime-switching. We argue below that this is both a more flexible and a more plausible specification,

since new policy regimes never exactly repeat old ones. We use forward looking variables such as survey expectations and asset prices to extract beliefs about the nature of the Alternative policy rule and the probability of a policy change. Thus, the model of this paper also innovates with respect to the literature on regime changes in general equilibrium models that typically only considers recurrent regime-switching.

The rest of this paper is organized as follows. The next section presents preliminary evidence consistent with structural change in the conduct of monetary policy over the full sample. We use the evidence in this section to pin down the timing of monetary regime changes in our sample, and avoid having to establish evidence on break dates that would be contingent on the details of the structural model. Section 3 describes the mixed-frequency structural macro-finance model and equilibrium solution. Section 4 describes the structural estimation, while Section 5 presents our empirical findings from the structural estimation. Section 6 concludes. A large amount of additional material on the model, estimation, and data has been placed in an Online Appendix.

2 Preliminary Evidence

Figure 1 plots the behavior over time of a key instrument of monetary policy, namely the federal funds rate, measured for the purposes of this plot in real terms as the nominal rate minus a four quarter moving average of inflation. The left panel plots this series along with an estimate of r^* from Laubach and Williams (2003).[2] The data are quarterly and span the sample 1961:Q1-2020:Q1.[3] The left panel shows that there are large, lower frequency fluctuations in the real federal funds rate over the full sample, but little long-term trend. By contrast, the neutral rate of interest exhibits a clear downward trend over the full sample.

The right panel plots the spread between the real funds rate and this measure of the neutral rate of interest, a variable we refer to as the *monetary policy spread*, and denote its time t value as mps_t.[4] Since the Federal Reserve targets the federal funds rate but in theory has no control over the neutral rate, a non-zero value for mps_t may be considered a measure of the stance of monetary policy, i.e., whether monetary policy is accommodative or restrictive, with spreads above zero indicative of restrictive monetary policy and those below zero indicative of accommodative monetary policy. According to this measure of the mps, monetary policy was accommodative over the sample up until about 1980, then sharply restrictive from about

[2] In Laubach and Williams (2003) the neutral or natural rate is a purely empirical measure that amounts to estimates of the level of the real federal funds rate that consistent with no change in inflation.

[3] The 1961 start date is dictated by the availability of the natural rate of interest measure.

[4] mps_t is computed as

$$FFR_t - (\text{Expected Inflation})_t - r_t^*,$$

where FFR is the nominal federal funds rate and where expected inflation is a four quarter moving average of inflation. r_t^* is the neutral rate of interest from Laubach and Williams. The quarterly nominal funds rate is the average of monthly values of the effective federal funds rate.

1980 to about 2000, and subsequently mostly accommodative. While there is no secular trend downward in real interest rates over the full sample, there is a noticeable downward trend in both the real interest rate and *mps* since about 1980.

We assume that there may be regime changes in the mean of the *mps*:

$$mps_t = r_{\xi_t^P} + \epsilon_t^r, \tag{1}$$

where $\epsilon_t^r \sim N(0, \sigma_r^2)$, and the coefficient $r_{\xi_t^P}$ is an intercept governed by a discrete valued latent state variable, ξ_t^P, that is presumed to follow a N_P-state nonrecurrent regime-switching Markov process discussed below, with transition matrix $\mathbf{H}$. Let the vector $\boldsymbol{\theta}_r = \left(r_{\xi_t^P}, \sigma_r^2, vec(\mathbf{H})'\right)'$ denote the set of parameters to be estimated. Since high values for mps_t are indicative of restrictive monetary policy while low values are indicative of accommodative policy, we refer to regimes with values for $r_{\xi_t^P} > 0$ as *hawkish* and denote them with an H, and to those with $r_{\xi_t^P} < 0$ as *dovish,* and denote them with a D. We can think of mps_t as a model-free indicator of the stance of monetary policy.

We assume that the true data generating process for ξ_t^P leads to infrequent regime changes in r_{ξ_t} that are *nonrecurrent*. That is, when the stance of monetary policy shifts, there is no expectation that it must move to a regime that is exactly the same as one in the past (mathematically a probability zero event), though it could be quite similar. BLL estimate a similar specification using recurrent regime-switching with two latent states. Here, the estimation is free to choose $r_{\xi_t^P}$ across regimes that are arbitrarily close to those that have occurred in the past, without being identically equal. We view the specification of this paper as both more flexible and more general than a recurrent regime-switching model where parameters can only shift to one of a finite number of values that would necessarily have to recur in a long enough sample. The Online Appendix explains how the structural breaks can be modeled as nonrecurrent regime-switching with transition matrix $\mathbf{H}$ and N_P nonrecurrent regimes ($N_P - 1$ structural breaks).

We use Bayesian methods with flat priors to estimate the model parameters in (1) over the period 1961:Q1-2020:Q1. Let T be the sample size used in the estimation and let the vector of observations as of time t be denoted $z_{r,t}$, here $z_{r,t} = mps_t$. The sequence $\xi_t^P = \{\xi_1^P, ..., \xi_T^P\}$ of regimes in place at each point is unobservable and needs to be inferred jointly with the other parameters of the model. We use the Hamilton filter (Hamilton (1994)) to estimate the smoothed regime probabilities $P\left(\xi_t^P = i | z_{r,T}; \boldsymbol{\theta}_r\right)$, where $i = 1, ..., N_P$. We then use these regime probabilities to estimate the most likely historical regime sequence ξ_t^P over our sample. This procedure is described in the Online Appendix.

Figure 2 reports the results for the case of two structural breaks ($N_P = 3$). Table 1 reports the dates corresponding to the three regime subperiods as identified by the most likely regime sequence. We identify a first subperiod of dovish monetary policy from 1961:Q1 to 1978:Q3, when mps_t is persistently negative and its mean $r_{\xi_t^P} = -2.67\%$ at the posterior mode. This

period coincides with the run up in inflation that began in the mid-1960s and with two oil shocks in the 1970s that were arguably exacerbated by a Fed that failed to react sufficiently proactively ((Clarida, Gali, and Gertler (2000); Lubik and Schorfheide (2004); Sims and Zha (2006); Bianchi (2013))). We refer to this first regime the "Great Inflation" regime. In 1978:Q4, a structural break in the series drove a jump in the mps_t upward, leaving its mean $r_{\xi_t^P} = 1.38\%$ at the posterior mode. This period of hawkish monetary policy lasted until 2001:Q1 and covers the Volcker disinflation and moderation in economic volatility that followed. We label this second subperiod the "Great Moderation" regime. The third "Post Millennial" regime starts in 2001:Q2 and represents a new prolonged period of dovish monetary policy, where $r_{\xi_t^P} = -1.27\%$ at the posterior mode. The beginning of this regime is labeled the "Greenspan Put," since it follows shortly after the inception of public narratives on the perceived attempt of Chair Greenspan to prop up securities markets in the wake of the IT bust, a recession, and the aftermath of 9/11, by lowering interest rates. The low mps subperiod at the end of the sample overlaps with the explicit forward guidance "low-for-long" policies under Chair Bernanke that promised in 2011 to keep interest rates at ultra low levels for an extended period of time, possibly longer than that warranted by a 2% inflation objective. Below we refer to the Great Inflation, the Great Moderation and the Post Millennial regimes in abbreviated terms as the GI, GM, and PM regimes.

Figure 2 shows that the low frequency deviations of the mps_t from zero are quantitatively large and persistent across the three estimated regime subperiods, providing model-free empirical evidence of structural change in the conduct of monetary policy. An objective of our study is to use the structural macro-finance model described in the next section to assess whether estimated monetary policy rules actually shifted across these subperiods. To accomplish this, we set the break dates for regime changes in the policy rule in the structural estimation to coincide with the regime sequence ξ_t^P estimated using mps_t. We use Bayesian model comparison of different estimated structural models to decide on the appropriate number N_P of policy regimes, and find $N_P = 3$ works well. With this, our structural estimation spans three different policy regimes across the Great Inflation, the Great Moderation, and the Post Millennial subperiods shown in Figure 2.

The preliminary evidence in this section allows us to build a structural model to fit these model-free empirical facts, rather than establishing evidence about the sequence of regimes that would be contingent on the details of the structural model. It should be emphasized, however, that this approach only sets the timing of policy regime changes in the structural model. In particular, all regime-dependent parameters of the policy rule are freely estimated under flat priors, and in principle could show no change across the regime subperiods for ξ_t^P. We describe the structural model next.

3 A Mixed-Frequency Macro-Finance Model

This section proposes a dynamic macro-finance model of monetary policy transmission with two "blocks," an asset pricing block and a macro block. These blocks describe the behavior of two different representative agents, "investors" and "households," along with a central bank. The two agent structure follows BLL, although the details of the models differ substantively as described below. In both blocks we work with a loglinear approximation to the model that can be solved analytically in which all random variables are conditionally lognormally distributed. Before getting into the details of this model, we provide a descriptive overview.

3.1 Overview

The asset pricing block of the model describes the behavior of a representative investor who may be thought of as a relatively sophisticated agent such as a wealthy individual or large institution. Although she owns the overwhelming majority of highly concentrated financial wealth in the U.S., she is small enough relative to the overall population that she takes macroeconomic dynamics as given. Households own no stocks but their expectations are key drives of macro dynamics, which are driven by a set of reduced-form equations similar to those in New Keynesian models. Following BLL, macroeconomic dynamics here are influenced by two distinctive features not found in standard New Keynesian models: sticky household inflation expectations of the type documented in Malmendier and Nagel (2016) (MN), and regime changes in the conduct of monetary policy, though in contrast to BLL regimes are nonrecurrent. Taken together, these departures imply that the model can endogenously generate persistent (though not permanent) departures from monetary neutrality.[5] As a consequence, investors in the model have a strong incentive to play close attention to central bank communications that may offer clues about when and under what conditions the conduct of monetary policy might change.

To understand the impetus for modeling two types of agents, note that the motivating evidence of the previous section suggests that the target interest rate directly controlled by the Fed exhibits prolonged deviations from the neutral rate of interest, or more generally from a low-frequency trend in interest rates consistent with no increase in expected inflation. Such persistent real effects are inconsistent with canonical New Keynesian models because agents in those models are presumed to have full information and rational expectations, and thus quickly adapt to changes in the conduct of monetary policy. The observed long-lasting deviations of mps_t from zero suggests that macro expectations are subject to more inertia than what full information, rational expectations would imply. On the other hand, it is evident that financial markets react swiftly to central bank communications and actions. This suggests that the

[5] As in Bianchi, Lettau, and Ludvigson (2022), persistent monetary non-neutrality is itself an endogenous outcome of the inertia in household inflation expectations evident from household surveys, as discussed further below.

expectations of financial market participants are subject to little inertia. The framework below reconciles these seemingly contradictory observations by considering two types of agents with different beliefs.

In the model, both types of agents are specified to have a monthly decision interval; hence t denotes a month. However, investors are attentive to central bank communications, so their beliefs may exhibit jumps in response to those communications within a month consistent with the notation that investors in the real world spend significant resources on "Fed watching" and swiftly react to surprise announcements. Investors in the model have enough information to estimate the current policy rule, and thus they can observe when actual shifts in the policy rule occur. However, investors are uncertain about how long the current policy regime will last, and what will come after the current policy regime ends. This assumption can be motivated by noting that, in practice, Fed communications in the last 20 years have clearly promulgated the central bank's intention to change the stance of monetary policy, but have been comparatively vague about how long those changes will last.

Finally, the central bank in the model is presumed to follow a nominal interest rule with time-varying parameters subject to structural breaks. We treat the policy rule parameters across nonrecurrent regimes as latent parameters to be estimated, an approach that side-steps the need to take a stand on why the Fed changes its policy rule. This is advantageous, since the reasons for such changes are likely to be difficult to accurately parameterize as a function of historical data, due to the degree of discretion the Fed has in interpreting its dual mandate and the possibility that distinct policy regimes are partly the result of a slow learning mechanism operating in tandem with the bespoke beliefs of different central bank leaders across time.

We now describe the two blocks of the model.

3.2 Model Description

Asset Pricing Block The model allows for a continuum of identical investors indexed by i who derive utility from consumption at time t. Investors trade in two assets: a one-period nominal risk-free bond and a stock market. We assume that investors derive income only from asset holdings and that the nominal bond is in zero net supply among them. In equilibrium, assets are priced by a representative investor who consumes per-capita aggregate equity payout, D_t. We therefore drop the i index from here on and denote the consumption of the representative investor D_t.

The representative investor's intertemporal marginal rate of substitution in consumption is the stochastic discount factor (SDF) and takes the form:

$$M_{t+1} = \beta_{p,t} \left(D_{t+1} / D_t \right)^{-\sigma_p}, \tag{2}$$

where $\beta_{p,t} \equiv \beta_p \exp\left(\vartheta_{pt}\right)$ is a time-varying subjective time discount factor. The time discount factor is subject to an externality in the form of a patience shifter ϑ_{pt} that individual investors

take as given, driven by the market as a whole. The preference shifter is taken as exogenous processes akin to an external habit that is the same for each shareholder. A time-varying specification for the subjective time-discount factor is essential for ensuring that, in equilibrium, investors are willing to hold the nominal bond at the interest rate set by the central bank's policy rule, specified below.

Let lowercase variables denote log variables, e.g., $\ln(D_t) = d_t$. We assume that aggregate payout is derived from a time-varying "capital share" K_t of real output Y_t, implying $D_t = K_t Y_t$. The log payout to output ratio is $d_t - \ln(Y_t) = k_t$. Differencing this relation implies

$$\Delta d_t = \Delta k_t + \Delta \ln(Y_t). \tag{3}$$

Variation in the payout share, k_t, is modeled as exogenous and latent following a specification listed below.

The first-order-condition for optimal holdings of the one-period nominal risk-free bond with a face value equal to one nominal unit is

$$LP_t^{-1} Q_t = \mathbb{E}_t^b \left[M_{t+1} \Pi_{t+1}^{-1} \right], \tag{4}$$

where Q_t is the nominal bond price, $\mathbb{E}_t^b$ denotes the subjective expectations of the investor, and $\Pi_{t+1} = P_{t+1}/P_t$ is the gross rate of general price inflation. Investors' subjective beliefs, indicated with a "b" superscript on the expectation operator, play a central role in asset pricing and are discussed in detail below. We further assume that investors have a time-varying preference for nominal risk-free assets over equity, accounted for in a reduced-form way with the term $LP_t > 1$ in (4), implying that the bond price Q_t is higher than it would be absent these benefits, i.e., when $LP_t = 1$. We discuss the role of LP_t further below.

Taking logs of (4) and using the properties of conditional lognormality delivers an expression for the real interest rate as perceived by the investor:

$$i_t - \mathbb{E}_t^b [\pi_{t+1}] = -\mathbb{E}_t^b [m_{t+1}] - .5 \mathbb{V}_t^b [m_{t+1} - \pi_{t+1}] - lp_t \tag{5}$$

where the nominal interest rate $i_t = -\ln(Q_t)$, $\pi_{t+1} \equiv \ln(\Pi_{t+1})$ is net inflation, $\mathbb{V}_t^b[\cdot]$ is the conditional variance under the subjective beliefs of the investor, and $lp_t \equiv \ln(LP_t) > 0$.

Let P_t^D denote total value of market equity, i.e., price per share times shares outstanding. Then the first-order-condition for optimal shareholder consumption implies the following Euler equation:

$$\begin{aligned} P_t^D &= \mathbb{E}_t^b \left[M_{t+1} \left(P_{t+1}^D + D_{t+1} \right) \right] \\ \frac{P_t^D}{D_t} &= \mathbb{E}_t^b \left[M_{t+1} \frac{D_{t+1}}{D_t} \frac{P_{t+1}^D + D_{t+1}}{D_{t+1}} \right]. \end{aligned}$$

Taking logs on both sides of the above and using the properties of conditional lognormality, we obtain an expression for the log price-payout ratio $pd_t \equiv \ln\left(P_t^D / D_t\right)$:

$$\begin{aligned} pd_t &= \kappa_{pd,0} + \mathbb{E}_t^b \left[m_{t+1} + \Delta d_{t+1} + \kappa_{pd,1} pd_{t+1} \right] + \\ &\quad + .5 \mathbb{V}_t^b \left[m_{t+1} + \Delta d_{t+1} + \kappa_{pd,1} pd_{t+1} \right]. \end{aligned}$$

The log equity return $r^D_{t+1} \equiv \ln\left(P^D_{t+1} + D_{t+1}\right) - \ln\left(P^D_t\right)$ obeys the following approximate identity (Campbell and Shiller (1989)):

$$r^D_{t+1} = \kappa_{pd,0} + \kappa_{pd,1} pd_{t+1} - pd_t + \Delta d_{t+1},$$

where $\kappa_{pd,1} = \exp(\overline{pd})/(1+\exp(\overline{pd}))$, and $\kappa_{pd,0} = \log\left(\exp(\overline{pd})+1\right) - \kappa_{pd,1}\overline{pd}$. Combining all of the above, the log equity premium as perceived by the investor is:

$$\underbrace{\mathbb{E}^b_t\left[r^D_{t+1}\right] - \left(i_t - \mathbb{E}^b_t\left[\pi_{t+1}\right]\right)}_{\text{equity premium}} = \underbrace{\left[\begin{array}{c} -.5\mathbb{V}^b_t\left[r^D_{t+1}\right] - \mathbb{COV}^b_t\left[m_{t+1}, r^D_{t+1}\right] \\ +.5\mathbb{V}^b_t\left[\pi_{t+1}\right] - \mathbb{COV}^b_t\left[m_{t+1}, \pi_{t+1}\right] \end{array}\right]}_{\text{subj. risk premium}} + \underbrace{lp_t}_{\text{liquidity Premium}}, \tag{6}$$

where $\mathbb{COV}^p_t[\cdot]$ is the conditional covariance under the subjective beliefs of the agent. The equity premium has two components. The component labeled "subj. risk premium" is the part attributable to the agent's subjective perception of risk. In the model, these fluctuate only with fluctuations in investor beliefs about future regime change in the conduct of monetary policy, as explained below. The term labeled "liquidity premium," which we model as exogenous and latent, represents a time-varying preference for risk-free nominal debt over equity and captures all sources of time-variation in the equity premium other than those attributable to subjective beliefs about the monetary policy rule. These include variation in the liquidity and safety attributes of nominal risk-free assets (e.g., Krishnamurthy and Vissing-Jorgensen (2012)), variation in risk aversion, flights to quality, or jumps in sentiment. We refer to this catchall component of the equity premium simply as the *liquidity premium* hereafter.

We approximate our nonlinear SDF (2) as

$$m_{t+1} \simeq \ln\left(\beta_p\right) + \vartheta_{pt} - \sigma_p\left(\Delta d_{t+1}\right). \tag{7}$$

Combining (5) and (7), we see that $\vartheta_{p,t}$ is implicitly defined as

$$\vartheta^p_t = -\left[i_t - \mathbb{E}^b_t\left[\pi_{t+1}\right]\right] + \mathbb{E}^b_t\left[\sigma_p \Delta d_{t+1}\right] - .5\mathbb{V}^b_t\left[-\sigma_p \Delta d_{p,t+1} - \pi_{t+1}\right] - lp_t - \ln\left(\beta_p\right). \tag{8}$$

Summarizing, the model implies the following asset pricing relations:

1. Log SDF:
$$m_{t+1} = \log\left(\beta_p\right) + \vartheta_{pt} - \sigma_p\left(\Delta d_{t+1}\right) \tag{9}$$

2. Log price-payout ratio:
$$\begin{aligned} pd_t &= \kappa_{pd,0} + \mu + \mathbb{E}^b_t\left[m_{t+1} + \Delta d_{t+1} + \kappa_{pd,1} pd_{t+1}\right] + \\ &\quad +.5\mathbb{V}^b_t\left[m_{t+1} + \Delta d_{t+1} + \kappa_1 pd_{t+1}\right] \end{aligned} \tag{10}$$

3. Log Euler equation for bonds:
$$i_t - \mathbb{E}^b_t\left[\pi_{t+1}\right] = -\mathbb{E}^b_t\left[m_{t+1}\right] - .5\mathbb{V}^b_t\left[m_{t+1} + i_t - \pi_{t+1}\right] - lp_t \tag{11}$$

4. Log excess stock market return:

$$er_{t+1}^{D} = r_{t+1}^{D} - (i_t - \pi_{t+1}) = \kappa_{pd,0} + \kappa_{pd,1} pd_{t+1} - pd_t + \Delta d_{t+1} + \mu - (i_t - \pi_{t+1}) \tag{12}$$

5. Laws of motion for exogenous processes:

$$k_t - \overline{k} = (1-\rho_k)\left(\lambda_{k,y}\widetilde{y}_t + \lambda_{k,\Delta y}\Delta y_t\right) + \rho_k\left(k_{t-1} - \overline{k}\right) + \sigma_k \varepsilon_{k,t} \tag{13}$$

$$lp_t - \overline{lp} = \rho_{lp}\left(lp_{t-1} - \overline{lp}\right) + \sigma_{lp}\varepsilon_{lp,t}. \tag{14}$$

In the above, $\widetilde{y}_t$ is the output gap, defined below. Equation (13) allows the payout share k_t to vary with the output gap and economic growth, as well as an independent i.i.d. shock $\varepsilon_{kt} \sim N(0,1)$. Equation (14) is specified to follow a first-order autoregressive (AR(1)) process subject to an i.i.d. shock $\varepsilon_{lp,t} \sim N(0,1)$.

Macro Dynamics Macroeconomic dynamics are driven by a set of equations similar to those commonly featured in New Keynesian models, with two distinctive features: sticky expectations about inflation and output, and regime changes in the conduct of monetary policy.[6] These distinctions are discussed below.

Let $\ln(A_t/A_{t-1}) \equiv g_t$ represents stochastic trend growth of the economy, which follows an AR(1) process $g_t = g + \rho_g(g_{t-1} - g) + \sigma_g \varepsilon_{g,t}$, $\varepsilon_{g,t} \sim N(0,1)$. Log of detrended output in the model is defined as $\ln(Y_t/A_t)$. As above, log variables are denoted in lower case, while log-detrended variables are denoted with a tilde, e.g., $\widetilde{y}_t = \ln(Y_t/A_t)$. This implies that $\widetilde{y}_t$ is positive when y_t is above potential output, and negative when it is below. Thus $\widetilde{y}_t > 0$ represents a positive output gap, and vice versa for $\widetilde{y}_t < 0$. In keeping with New Keynesian models, write most equations in the macro block in terms of detrended real variables.

As in prototypical New Keynesian models, macroeconomic dynamics satisfy a loglinear Euler equation. In our setting this Euler equation is driven by the behavior of a representative household referred to as the "macro agent" that consumes a labor share $(1-K_t)$ of Y_t. This agent can be considered typical of a household in the general population who holds small amounts of wealth in the form of nominal bonds and no equity. The linearized Euler equation takes the form

$$\widetilde{y}_t = \mathbb{E}_t^m(\widetilde{y}_{t+1}) - \sigma\left[i_t - \mathbb{E}_t^m(\pi_{t+1}) - \overline{r}\right] + f_t \tag{15}$$

where i_t is the short-term nominal interest rate, $\mathbb{E}_t^m(\pi_{t+1})$ is the subjective expected inflation of the macro agent, $\overline{r}$ is the steady state real interest rate, and f_t is a demand shock and also absorbs any variation in the macro agent's consumption attributable to movements in the labor share, $\ln(1-K_t)$. This shock follows an AR(1) process $f_t = \rho_f f_{t-1} + \sigma_f \varepsilon_f$, $\varepsilon_f \sim N(0,1)$. The

[6] Outside of these two distinctive features, macroeconomic dynamics are essentially the same as those that arise from the prototypical New Keynesian model of Galí (2015), Chapter 3.

coefficient σ in (15) is a positive parameter. We discuss the way macro expectations are formed below.

We introduce two equations for inflation and the nominal interest rate rule. Inflation dynamics are described by the following equation, which takes the form of a New Keynesian Phillips curve:

$$\begin{aligned}\pi_t - \overline{\pi}_t &= \beta\left(1-\lambda_{\pi,1}-\lambda_{\pi,2}\right)\mathbb{E}_t^m\left[\pi_{t+1}-\overline{\pi}_t\right]+\beta\lambda_{\pi,1}\left[\pi_{t-1}-\overline{\pi}_t\right]+\beta\lambda_{\pi,2}\left[\pi_{t-2}-\overline{\pi}_t\right] \\ &\quad +\kappa_0\widetilde{y}_t+\kappa_1\widetilde{y}_{t-1}+\sigma_\mu\varepsilon_{\mu,t}\end{aligned} \tag{16}$$

where $\overline{\pi}_t$ denotes the perceived long term value of inflation that depends on the macro agent's subjective expectations, and $\varepsilon_{\mu,t} \sim N(0,1)$ is a markup shock. The specification in (16) implies that deviations of inflation from macro agent's perception of trend inflation are a function of the expected future value and lagged value of such deviations, as well as the current and lagged output gap. Lags beyond the current values of these variables are used to capture persistent inflation dynamics. The coefficients β, λ_{π_1}, λ_{π_2}, κ_0, and κ_1 are positive parameters.

The central bank obeys the following nominal interest rate rule subject to nonrecurrent regime changes in the policy rule parameters:

$$\begin{aligned}i_t-\left(\overline{r}+\pi_{\xi_t^p}^T\right) &= \left(1-\rho_{i,\xi_t^p}-\rho_{i_2,\xi_t^p}\right)\left[\psi_{\pi,\xi_t^p}\left(\pi_{t,t-3}-3\pi_{\xi_t^p}^T\right)+\psi_{\Delta y,\xi_t^p}\left(\Delta y_{t,t-3}\right)\right] \\ &\quad +\rho_{i_1,\xi_t^p}\left[i_{t-1}-\left(\overline{r}+\pi_{\xi_t^p}^T\right)\right]+\rho_{i_2,\xi_t^p}\left[i_{t-2}-\left(\overline{r}+\pi_{\xi_t^p}^T\right)\right]+\sigma_i\varepsilon_i,\end{aligned} \tag{17}$$

where $\pi_{t,t-3} \equiv \pi_t+\pi_{t-1}+\pi_{t-2}$ is quarterly inflation, $y_{t,t-3} \equiv y_t - y_{t-3} = \widetilde{y}_t - \widetilde{y}_{t-3} + 3g + \widehat{g}_t + \widehat{g}_{t-1} + \widehat{g}_{t-2}$ is quarterly output growth, $\varepsilon_{i,t} \sim N(0,1)$ is a monetary policy shock, and where the parameters of the rule depend on the discrete-valued latent random variable ξ_t^p. In the above policy rule, the central bank reacts to quarterly data even though the baseline decision interval of agents is monthly. Lags of the left-hand-side variable appear in the rule to capture the observed smoothness in adjustments to the central bank's target interest rate. Note the interest rate rule is written in deviations from the steady state nominal rate conditional on being in a particular regime dictated by ξ_t^P. This means that, once inflation reaches the desired target, the economy stabilizes around it, absent shocks.

An important feature of this interest rate policy rule, and a departure from the prototypical model, is that it allows for nonrecurrent regime changes in the conduct of monetary policy driven by ξ_t^p. These manifest as regime shifts in the inflation target $\pi_{\xi_t^P}^T$ and in the activism coefficients ψ_{π,ξ_t^P}, and $\psi_{\Delta y,\xi_t^P}$ that govern how strongly the central bank responds to deviation from the target and to economic growth. The rule also allows for potential regime shifts in the autocorrelation coefficient ρ_{i,ξ_t^P}. As discussed, these coefficients are modeled as varying with the same discrete-valued random variable ξ_t^P that drives the Markov process for $r_{\xi_t^P}$, previously referred to as following accommodative and restrictive regimes. It is important to emphasize, however, that these labels do not imply that we impose any constraints on the estimated

values of policy rule parameters across the previously estimated regimes. Instead, the previous estimation merely identifies the timing of any structural breaks. Since we freely estimate the policy rule parameters under flat priors, they could in principle show no shift across regimes, or shifts that go in the "wrong" direction with respect to the previously estimated *mps* regimes.

We interpret equations (15) through (17) as equilibrium dynamics and not a micro-founded structural model. We consider an equilibrium in which bonds are in zero-net-supply in both the macro and asset pricing blocks and thus there is no trade between the asset pricing agent and macro agent.

The macro agent's expectations about inflation are formed using an adaptive algorithm, following survey evidence in Malmendier and Nagel (2016) (MN). Specifically, macro agent expectations about inflation are formed using an autoregressive process, $\pi_t = \alpha + \phi\pi_{t-1} + \eta_t$, where the agent must learn about the parameter α.[7] Each period, agents form a belief about α, denoted α_t^m, that is updated over time. Updating affects beliefs about next period inflation as well as beliefs about long-term trend inflation. Define *perceived trend inflation* to be the $\lim_{h\to\infty} \mathbb{E}_t^m [\pi_{t+h}]$ and denote it by $\overline{\pi}_t$. Given the presumed autoregressive process, it can be shown that $\overline{\pi}_t = (1-\phi)^{-1} \alpha_t^m$. This implies that expectations of one step ahead inflation are a weighted average of perceived trend inflation and current inflation:

$$\mathbb{E}_t^m [\pi_{t+1}] = \alpha_t^m + \phi\pi_t = (1-\phi)\,\overline{\pi}_t + \phi\pi_t. \tag{18}$$

We allow the evolution of beliefs about α_t^m and $\overline{\pi}_t$ to potentially reflect both an adaptive learning component as well as a signal about the central bank's inflation target. For the adaptive learning component, we follow evidence in MN that the University of Michigan Survey of Consumers (SOC) mean inflation forecast is well described by a constant gain learning algorithm. For the signal component, we assume that beliefs could be partly shaped by additional information the agent receives about the current inflation target. This signal could reflect the opinion of experts (as in MN) or a credible central bank announcement. Combining these two yields updating rules for α_t^m and $\overline{\pi}_t$ that are a weighted averages of two terms:

$$\alpha_t^m = \left(1-\gamma^T\right) \underbrace{\left[\alpha_{t-1}^m + \gamma\left(\pi_t - \phi\pi_{t-1} - \alpha_{t-1}^m\right)\right]}_{\alpha_t^{mCG}} + \gamma^T \left[(1-\phi)\,\pi_{\xi_t}^T\right] \tag{19}$$

$$\overline{\pi}_t = \left(1-\gamma^T\right) \underbrace{\left[\overline{\pi}_{t-1} + \gamma\,(1-\phi)^{-1}\left(\pi_t - \phi\pi_{t-1} - (1-\phi)\,\overline{\pi}_{t-1}\right)\right]}_{\overline{\pi}_t^{CG}} + \gamma^T \pi_{\xi_t}^T \tag{20}$$

The first terms in square brackets, α_t^{mCG} and $\overline{\pi}_t^{CG}$, are the recursive updating rules implied by constant gain learning, where γ is the constant gain parameter that governs how much

[7] In principle one could introduce learning about ϕ as well. We forgo doing this in order to keep the estimation tractable, since the most important learning aspects in the model involve those parameters such as α that bear most closely on trend inflation.

last period's beliefs α_{t-1}^m and $\overline{\pi}_{t-1}$ are updated given new information, π_t. The second term in square brackets captures the effect of the signal about the current inflation target $\pi_{\xi_t}^T$. If $\gamma^T = 1$, the signal is completely informative and the agent's belief about trend inflation is the same as the perceived inflation target. If $\gamma^T = 0$, the signal is completely uninformative and the agent's belief about trend inflation depends only on the adaptive learning algorithm. Overall perceived trend inflation is a weighted average of the trend implied by the constant gain learning rule and the perceived central bank inflation target. A weight of less than one on the target could arise either because the target is imperfectly observed, or because central bank announcements about the target are not viewed as fully informative or credible. Note that the parameter γ^T is closely related to the speed with which the macro agent learns about a new inflation target. The magnitude of the parameter is also related to the credibility of Fed announcements about changes in the implicit inflation target. Small values for γ^T are indicative of low credibility, since in that case the macro agent continues to base inflation expectations mostly on a backward looking rule even when there has been a shift in the inflation target. Since γ^T is freely estimated, we can empirically assess the magnitude of this learning speed/credibility and its role in macroeconomic fluctuations.

The macro agent forms expectations about detrended output using a simple backward looking rule:

$$\mathbb{E}_t^m\left(\widetilde{y}_{t+1}\right) = \varrho_1\widetilde{y}_{t-1} + \varrho_2\widetilde{y}_{t-2} + \varrho_3\widetilde{y}_{t-3}. \tag{21}$$

Unlike inflation, agents do not perceive a moving mean for detrended output. This assumption is consistent with the equilibrium of the model, which implies that the central bank cannot have a permanent effect on real activity.

Using equations (18), (20), and (21), we substitute out $\mathbb{E}_t^m\left[\pi_{t+1}\right]$, $\overline{\pi}_t$, and $\mathbb{E}_t^m\left(\widetilde{y}_{t+1}\right)$ in the model equations (15), (16), and (17) to obtain the following system of equations that must hold in equilibrium:

1. Real activity

$$\widetilde{y}_t = \varrho_1\widetilde{y}_{t-1} + \varrho_2\widetilde{y}_{t-2} + \varrho_3\widetilde{y}_{t-3} - \sigma\left[i_t - \phi\pi_t - (1-\phi)\,\overline{\pi}_t - r_{ss}\right] + f_t. \tag{22}$$

2. Phillips curve:

$$\pi_t - \overline{\pi}_t = \widetilde{\phi}\beta\lambda_{\pi,1}\left[\pi_{t-1} - \overline{\pi}_t\right] + \widetilde{\phi}\beta\lambda_{\pi,2}\left[\pi_{t-2} - \overline{\pi}_t\right] + \widetilde{\phi}\kappa_0\widetilde{y}_t + \widetilde{\phi}\kappa_1\widetilde{y}_{t-1} + \widetilde{\phi}\sigma_\mu\varepsilon_{\mu,t}. \tag{23}$$

where $\widetilde{\phi} = \left[1 - \beta\left(1 - \lambda_{\pi,1} - \lambda_{\pi,2}\right)\phi\right]^{-1}$.

3. Monetary policy rule:

$$\begin{aligned} i_t - \left(\overline{r} + \pi_{\xi_t^p}^T\right) &= \left(1 - \rho_{i,\xi_t^p} - \rho_{i_2,\xi_t^p}\right)\left[\psi_{\pi,\xi_t^p}\left(\pi_{t,t-3} - 3\pi_{\xi_t^p}^T\right) + \psi_{\Delta y,\xi_t^p}\left(\Delta y_{t,t-3}\right)\right] \\ &\quad + \rho_{i_1,\xi_t^p}\left[i_{t-1} - \left(\overline{r} + \pi_{\xi_t^p}^T\right)\right] + \rho_{i_2,\xi_t^p}\left[i_{t-2} - \left(\overline{r} + \pi_{\xi_t^p}^T\right)\right] + \sigma_i\varepsilon_i, \end{aligned} \tag{24}$$

4. Law of motion for f_t:

$$f_t = \rho_f f_{t-1} + \sigma_f \varepsilon_{ft},\ \varepsilon_{ft} \sim N(0,1). \tag{25}$$

5. Law of motion for $g_t \equiv \ln(A_t/A_{t-1})$:

$$g_t = g + \rho_g (g_{t-1} - g) + \sigma_g \varepsilon_g,\ \varepsilon_{gt} \sim N(0,1). \tag{26}$$

6. Perceived trend inflation:

$$\overline{\pi}_t = \left[1 - \gamma^T\right] \left[\overline{\pi}_{t-1} + \gamma (1-\phi)^{-1} \left(\pi_t - \phi \pi_{t-1} - (1-\phi)\overline{\pi}_{t-1}\right)\right] + \gamma^T \pi^T_{\xi_t}. \tag{27}$$

Investors understand the macro block, can observe equations (22)-(27), and take those dynamics as given. But they form beliefs as described next about the persistence of the current policy regime and the alternative regime that will come afterwards.

3.3 Investor Beliefs

We now describe how investor beliefs about monetary policy regime changes evolve over time.

Investors understand that the true data generating process for the monetary policy rule is subject to infrequent, nonrecurrent regime changes. We further assume that investors closely follow central bank communications and are therefore capable of estimating the current policy rule indexed by ξ^P_t. What they are uncertain about is how long the current regime will last, and what will come after the current regime ends. These considerations require a model of investor expectation formation in the face of infrequent structural breaks. Investors must contemplate a future with a central bank that could operate differently from the one today or any that has come before.

To model these ideas, we assume that, for each time t policy rule regime indexed by ξ^P_t, investors hold in their minds an "Alternative policy rule" indexed by ξ^A_t that they believe will come next, whenever the current policy regime ends. The Alternative policy rule is isomorphic to the current policy rule, except that it has different parameters, i.e.,

$$\begin{aligned} i_t - \left(\overline{r} + \pi^T_{\xi^A_t}\right) &= \left(1 - \rho_{i,\xi^A_t} - \rho_{i_2,\xi^A_t}\right) \left[\psi_{\pi,\xi^A_t} \left(\pi_{t,t-3} - 3\pi^T_{\xi^A_t}\right) + \psi_{\Delta y,\xi^A_t} (\Delta y_{t,t-3})\right] \\ &\quad + \rho_{i_1,\xi^A_t} \left[i_{t-1} - \left(\overline{r} + \pi^T_{\xi^A_t}\right)\right] + \rho_{i_2,\xi^A_t} \left[i_{t-2} - \left(\overline{r} + \pi^T_{\xi^A_t}\right)\right] + \sigma_i \varepsilon_i, \end{aligned} \tag{28}$$

Investors form beliefs about the probability of staying in the current regime ξ^P_t versus switching to the Alternative regime ξ^A_t. For each ξ^P_t, investors hold in their minds a "grid" of B beliefs about the probability of remaining in ξ^P_t versus changing to the Alternative ξ^A_t, and do not consider anything after that. This can be considered a form of bounded rationality.[8]

[8] In theory it is straightforward to consider multiple alternative policy rules, and multiple alternatives to the alternatives. In practice the number of parameters can quickly proliferate creating an intractable estimation problem. We consider a single alternative at each t in order to keep the solution and estimation tractable.

In the nonrecurrent regime setup of the model, this implies that the pondered Alternative is treated as an absorbing state as of time t, since the probability of returning precisely to any previous policy rule must be zero by definition. When the current policy regime ends, the new policy regime that replaces it will never be exactly as previously imagined by the investor. Nevertheless, at that time investors update their understanding of the current policy rule and along with it their perceived Alternative for what comes next.

These ideas can be formalized by introducing the notion of a *belief regime* sequence governed by a discrete-valued variable $\xi_t^b \in \{1, 2, ...B, B+1\}$ with $B+1$ states. The overall policy regime process includes the regime in place, and investors beliefs about transitioning out of that regime and moving to the Alternative. Specifically, each *overall policy regime* $\xi_t = \left\{\xi_t^P, \xi_t^b\right\}$ is characterized by knowledge of the current policy regime ξ_t^P and a belief about the probability of staying in the current policy rule ξ_t^P versus moving to ξ_t^A. To keep notation simple, we exclude ξ_t^A in the set of arguments of ξ_t. It should be kept in mind, however, that each policy rule regime ξ_t^P has associated with it a single perceived Alternative policy rule ξ_t^A. Thus if there are a total of N_p true policy regimes over the course of the sample, there are also N_p perceived alternative policy regimes associated over the same time span.

The regimes $\xi_t^b = 1, 2, ...B$ represent a grid of beliefs taking the form of perceived probabilities that the current policy rule will still be in place next period, given that it is in place this period. The regime $\xi_t^b = B+1$ is a belief regime capturing the perceived probability of staying in the Alternative regime once it is reached. We order these so that belief regime $\xi_t^b = 1$ is the lowest perceived probability that the current policy rule will remain in place and belief regime $\xi_t^b = B$ is the highest.

These perceived regimes are modeled with a perceived transition matrix taking the form:

$$\mathbf{H}^b = \begin{bmatrix} p_{b1}p_s & p_{b2}\left(1-p_s\right)/(B-1) & \cdots & p_{bB}\left(1-p_s\right)/(B-1) & 0 \\ p_{b1}\left(1-p_s\right)/(B-1) & p_{b2}p_s & & p_{bB}\left(1-p_s\right)/(B-1) & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ p_{b1}\left(1-p_s\right)/(B-1) & & & p_{bB}p_s & 0 \\ 1-p_{b1} & 1-p_{b2} & \cdots & 1-p_{bB} & p_{B+1,B+1}=1 \end{bmatrix}, \tag{29}$$

where $\mathbf{H}_{ij}^b \equiv p\left(\xi_t^b = i | \xi_{t-1}^b = j\right)$. In the above, p_{b1} is the subjective probability of remaining in the current policy rule under belief 1. For example, belief 1 could mean that investors believe with $p_{b1} = 0.05$ that the current policy rule will still be in place next period; belief 2 could mean that investors believe with $p_{b2} = 0.10$ that the current rule will still be in place, and so on. The non-zero off diagonal elements in the upper left $B \times B$ submatrix allow for the possibility that investors might receive subsequent information that could change their beliefs, and take that into account when forming expectations. Thus, $\left(1-p_s\right)/(B-1)$ is the probability investors assign to possibility that they will change their beliefs tomorrow as the result of new information or sentiment, while p_s is the probability investors assign to not changing their

minds, i.e., to having the same beliefs tomorrow as today. To avoid overparameterization, we assume that investors assign an equal probability of having any of the other of the $(B-1)$ beliefs tomorrow. With this parameterization, $p_{b1}p_s$ measures the subjective probability of being in belief regime 1 tomorrow, conditional on having belief 1 today, while $p_{b1}\left(1-p_s\right)/(B-1)$ is the subjective probability of being in any of belief regimes 2,3,...,or B tomorrow, conditional on having belief 1 today. Finally, $1-p_{bi}$ is the probability of having belief i today but exiting to the Alternative regime tomorrow. Note that $p_{B+1,B+1}$ is the perceived probability of remaining in the Alternative regime conditional on having moved there. With perceived nonrecurrent regimes, this probability is unity by definition.

Equilibrium An equilibrium is defined as a set of prices (bond prices, stock prices), macro quantities (inflation, output growth, inflation expectations), laws of motion, and investor beliefs such that equations (9)-(14) in the asset pricing block are satisfied, equations (22)-(27) in the macro block are satisfied, and investors beliefs about the persistence of policy regimes are characterized by the perceived Alternative policy rule (28) and the perceived belief regime sequence described above with transition matrix (29).

3.4 Model Solution

The asset pricing block of equations involves conditional subjective variance terms that are affected by Markov-switching random variables in the model. The subsection "Risk Adjustment with Lognormal Approximation," in the Online Appendix explains the approximation used to preserve lognormality of the entire system. This part uses the approach in Bianchi, Kung, and Tirskikh (2018) who in turn build on Bansal and Zhou (2002). We then use the algorithm of Farmer, Waggoner, and Zha (2011) to solve the system of model equations that must hold in equilibrium, where agents form expectations taking into account the probability of regime change in the future. The solution of the model takes the form of a Markov-switching vector autoregression (MS-VAR) in the continuous-valued latent state vector

$$S_t = \left[S_t^M, m_t, pd_t, k_t, lp_t, \mathbb{E}_t^b\left(m_{t+1}\right), \mathbb{E}_t^b\left(pd_{t+1}\right)\right],$$

where S_t^M is a vector of latent macro block state variables given by $S_t^M \equiv [\widetilde{y}_t, g_t, \pi_t, i_t, \overline{\pi}_t, f_t]'$. Once the solution is in hand, investor expectations multiple steps ahead maybe be computed for any variable. The Online Appendix explains how these are computed in the presence of nonrecurrent regime switching with the perceived Alternative policy rule.

The MS-VAR solution consists of a system of equations taking the form

$$S_t = \underbrace{C\left(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b\right)}_{\text{level}} + \underbrace{T(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b)}_{\text{propagation}} S_{t-1} + \underbrace{R(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b)}_{\text{amplification}} Q\varepsilon_t, \tag{30}$$

where $\varepsilon_t = (\varepsilon_{f,t}, \varepsilon_{i,t}, \varepsilon_{g,t}, \varepsilon_{k,t}, \varepsilon_{lp,t}, \varepsilon_{\mu,t})$ is the vector of primitive Gaussian shocks. The solution depends on the realized policy rule in place (ξ_t^P), but also on the investor's subjective beliefs about staying in the current policy regime next period, which depend on ξ_t^b and $\mathbf{H}^b$. Notice that the parameter vector $\theta_{\xi_t^P}$ includes the parameters of the Alternative policy rule ξ_t^A, since there is a single such Alternative for each realized policy rule indexed by ξ_t^P.

Equation (30) shows that the realized policy regime ξ_t^P and investor beliefs ξ_t^b about future changes in the policy rule amplify and propagate shocks in three ways. First, they have "level" effects captured by the coefficients $C\left(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b\right)$ that affect the economy absent shocks, driven by changes in the central bank's objectives such as the inflation target, as well as by the perceived risk of the stock market given by the risk-premium terms in (6). These factors affect the economic state even if the current and past state is held fixed. Second, changes in actual policy and beliefs about future policy have "propagation" effects captured by the matrix coefficient $T(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b)$ that determine how today's economic state is related to tomorrow's. Third, both the current policy regime and investors' subjective beliefs about the future conduct of policy have "amplification" effects governed by the matrix coefficient $R(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b)$ that generate endogenous heteroskedasticity of the primitive Gaussian shocks.

An implication of this endogenous heteroskedasticity for the equilibrium behavior of the stock market is that *perceived quantity of risk* varies with the expected conduct of future monetary policy. Indeed, it is only through $R(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b)$ that the subjective risk premium in (6) varies, which in turn varies only with (i) realized regime changes ξ_t^P in the conduct of monetary policy, and (ii) time-varying beliefs ξ_t^b regarding future policy. The only other source of variation in the equity premium (6) are the liquidity shocks. Below we decompose variation in the equity risk premium into that driven by variation in the subjective risk premium and that driven by liquidity shocks.

3.5 Investor Decision Interval and Updating

We now discuss how investors update their expectations over time.

Agents have a monthly decision interval. The time t information set $\mathbb{I}_t$ of investors includes their current belief, ξ_t^b, the current policy regime ξ_t^P and their perceived Alternative regime ξ_t^A, and additional data observable as of time t. These agents can observe the economic state S_t at the end of each month. Within month t, however, S_t is still latent and investors filter information $\mathbb{I}_t$ to produce *nowcasts* of S_t.

Investors use their information in two ways. First, given a baseline monthly decision interval, they update their previous nowcasts of S_t and their subjective expectations on the basis of new information at the end of every month. Second, investors allocate additional attention to updating their nowcasts of S_t and beliefs ξ_t^b about the future conduct of monetary policy at specific times *within* a month, corresponding to moments when the central bank releases information. This higher-frequency attentiveness to Fed news echoes real-world "Fed watching"

and is the mechanism through which the model accommodates swift market reactions to surprise central bank announcements. These updates in the immediate aftermath of a Fed announcement lead to endogenous jumps in subjective expectations, financial market returns, and in investor perceptions of stock market risk, driven by $\mathbb{COV}_t^b\left[m_{t+1}, r_{t+1}^D\right]$.

The updating of nowcasts and investor beliefs at different sampling intervals requires a mixed-frequency filtering algorithm. A precise description of such an algorithm is given in the Online Appendix. In econometric application, we use the algorithm to filter and update our estimates of investor ξ_t^b and S_t at mixed frequencies, based on an observation vector X_t described below.

4 Structural Estimation

The solution to the model may be written in state-space form by combining the system of state equations (30) with an observation equation taking the form

$$\begin{aligned} X_t &= D_{\xi_t,t} + Z_{\xi_t,t}\left[S_t', \widetilde{y}_{t-1}\right]' + U_t v_t \\ v_t &\sim N\left(0, I\right), \end{aligned} \tag{31}$$

where X_t denotes our vector of data, v_t is a vector of observation errors, U_t is a diagonal matrix with the standard deviations of the observation errors on the main diagonal, and $D_{\xi_t,t}$, and $Z_{\xi_t,t}$ are parameters mapping the model counterparts of X_t into the latent discrete- and continuous-valued state variables ξ_t and S_t, respectively, in the model. We note that the matrices $Z_{\xi_t,t}$, U_t, and the vector $D_{\xi_t,t}$ depend on t because some of our observable series are not available at all frequencies and/or over the full sample. As a result, the state-space estimation uses different measurement equations to include these series when the relevant data are available, and exclude them when they are missing. In particular, in the mixed-frequency estimation, intramonth data available in tight windows around FOMC announcements are used to provide "advanced" estimates of the state space and belief regimes, while "final" estimates are obtained using a more complete set of macro and financial data market data available at the end of each month.

We estimate the state-space representation using Bayesian methods, with the parameters of the monetary policy rule estimated under flat priors. The estimation approach uses Kim's (Kim (1994)) basic filter and approximation to the likelihood for Markov-switching state space models, and a random-walk metropolis Hastings MCMC algorithm to characterize uncertainty. Details on the estimation are provided in the Online Appendix.

4.1 Data and Measurement

This section describes the data X_t used in our structural estimation, which spans January 1961 through February 2020.

Since the evolution of investor expectations about the economy are key variables for the model's implications on asset prices, and to ensure that model expectations evolve in a manner that closely aligns with observed expectations, we map the model implications for these concepts into data on numerous forward looking variables, including household and professional surveys, and financial market indicators from equities and futures markets. The complete estimation relies on data at different frequencies. All of our data are available at monthly sampling intervals, while a subset of these are also available at higher frequencies. Lower frequency (monthly) macro data inform estimates of the policy rule and structural equations driving macroeconomic and stock market dynamics over the full sample. High frequency (daily and minutely) data provide information on market reactions in response to FOMC announcements. We now summarize the observations we use in X_t. Every series listed below is used at monthly sampling intervals, while a subset available at higher frequency are used intramonth in the minutes or days surrounding FOMC press releases. An explicit description of the mapping between our observables and their model counterparts, as well as a complete description of each data series and our sources is given in the Online Appendix.

Among the observations available at monthly sampling intervals but not at higher frequency, we use a monthly 12-month GDP growth estimate (see the Online Appendix), 12-month CPI inflation, the University of Michigan Survey of Consumers (SOC) 12- and 60-month ahead mean inflation forecast, the Bluechip (BC) survey, Survey of Professional Forecasters (SPF) and Livingston (LIV) surveys' mean 12-month- and 120-month-ahead CPI inflation forecasts, the SPF mean 12-month ahead GDP deflator inflation forecast, the means of the BC and SPF 12-month ahead GDP growth forecasts, and the ratio of S&P 500 (SP500) earnings relative to last month's GDP observation, a variable we refer to as the earnings-lagged GDP ratio. We also use the Moody's Baa 20-year bond return minus the 20-year U.S. Treasury bond (referred to hereafter as the "Baa spread").

Among data available at daily sampling intervals, we use the mean of the Bloomberg (BBG) consensus 12-month ahead inflation and GDP growth forecasts and the effective federal funds rate (FFR). Although FFR is available daily, we use observations on this variable only at the end of each month, instead relying on the current contract fed funds futures rate to measure jumps in the funds rate following an FOMC announcement, since these are available on a minutely basis. At the end of the month, FFR the current contract fed funds futures rate coincide.

Among data available at minutely sampling intervals, we use the ratio of SP500 market capitalization relative to lagged GDP, which we refer to as the SP500-lagged GDP ratio. At minutely frequency we also use the current contract and the 6, 10, 20, and 35 month contracts of the federal funds futures (FFF) prices.

Our motivation for these choices is as follows.

First, our use of high-frequency pre- and post-FOMC observations on survey expectations of inflation, GDP growth, several federal funds rate futures contracts, stock market valuation ratios, and the Baa spread is important for two reasons. First, it allows us to measure the causal effect of Fed announcements on the stock market and other variables, which is of interest in its own right. Second, the use of these high frequency data allow us to control for news about inflation, GDP growth, interest rates, the stock market, and the corporate bond market that may have arrived between the end of the month immediately preceding an FOMC announcement-month and the announcement itself. This is important because the arrival of news within this particular window can lead to revisions in monthly survey forecast data (e.g., the monthly BC survey) around FOMC announcements that appear to support a Fed information effect, when in reality markets may have been surprised by the reaction of the Fed to known economic news that pre-dated the FOMC announcement but arrived after last month's BC survey was taken (Bauer and Swanson (2021)). By conditioning on close-range, pre-announcement observations for inflation and GDP growth expectations (the day before), and pre-announcement observations on interest rate futures, and the stock market, we explicitly control for any economic news reflected in these forward looking variables that came out in the weeks between the last monthly BC forecast and the FOMC announcement. It follows that jumps in the post-announcement observations on these forward-looking variables cannot be readily attributed to stale economic news that came out earlier in the announcement month.

A second motivation for these data series is the ability to use multiple observables on a single variable of interest, especially on expectations. Investor expectations, for example, are unlikely to be perfectly well represented by any single professional survey. We therefore measure investor expectations of inflation and GDP growth using four different professional surveys (BBG, BC, LIV, SPF) and regard each of these as a noisy signal on the true underlying expectations process. In the filtering algorithm these different series explicitly provide four signals on the same latent variable; thus we allow measurement error on all of these variables.

Third, a number of series are used because they have obvious model counterparts. Data for GDP growth and inflation are mapped into the model implications for output growth and inflation, while data on SOC inflation forecasts are mapped into the model's implications for household inflation forecasts. Likewise, data on the current effective federal funds rate are mapped into the model's implications for the current nominal interest rate, while data on the FFF market are mapped into the model's implications for investor expectations of the future federal funds rate, as is the mean of the BC survey measure of the federal funds rate 12 months-ahead. In principle, fed funds futures market rates may contain a risk premium that varies over time. If such variation exists, it is absorbed in the estimation by the measurement error for these equations. In practice, risk premia variation in fed funds futures is known to be small when that variation is measured over the short 30-minute windows surrounding FOMC announcements

that we analyze (Piazzesi and Swanson (2008)). For longer-horizon fed funds rate expectations, we compliment the futures market data with the BC survey measure of expectations that is not subject this issue.

Fourth, we discipline observations on payout D_t and the shareholder payout share of output K_t with data on the SP500 earnings and the earnings share of GDP. Recall that output Y_t in the model is divided between shareholder payout $D_t = K_t Y_t$ and labor income $(1 - K_t) Y_t$. This abstracts from any non-labor charges against corporate earnings, such as net new investment and taxes, which create a wedge between corporate earnings and payout. Thus, in the model, earnings and payout are synonymous. To account for the fact that earnings in the data differs from the payout shareholders actually receive, D_t and K_t are mapped into their respective data series using a linear specifications with a freely estimated constant and slope coefficients, while also allowing for measurement error in both observation equations.

Finally, data on the Baa spread are mapped into the model's implications for the liquidity premium, lp_t. This premium is a catchall for factors outside of the model that could effect the equity premium, such as changes in the liquidity and safety attributes of Treasuries and other forms of equity return premia variation that are likely to correlate with the risky corporate bond spread, such as default risk, flights to quality, and sentiment. We use the Baa spread as an observable likely to be correlated with many of these factors, but our measurement equation allows for both a constant and a slope coefficient on the Baa spread along with measurement error, to soak up variation in this latent component of the equity premium that may not move identically with the spread.

With these data, and our full sample of FOMC events, we estimate the model using Bayesian techniques. Our full sample of FOMC announcements consists of 220 FOMC press releases spanning February 4th, 1994 to February 28th, 2020.

4.2 Estimating Beliefs

Beliefs are modeled with $B + 1$ belief regimes governed by the perceived transition matrix $\mathbf{H}^b$ given in (29). In the applied estimation, we set $B = 11$ and take the parameters p_{bi} from a discretized estimated beta distribution, where the mean and variance of the beta distribution are estimated along with the rest of the model parameters. In our model simulations we use the modal parameter estimates of these parameters.

Estimates of $\mathbf{H}^b$ may be used to estimate investors' perceived probabilities of a change in the policy rule multiple steps ahead. Let T be the sample size used in the estimation and let the vector of observations as of time t be denoted X_t. Let $P\left(\xi_t^b = i | X_T; \boldsymbol{\theta}\right) \equiv \pi_{t|T}^i$ denote the probability that $\xi_t^b = i$, for $i = 1, 2, ..., B + 1$, based on information that can be extracted from the whole sample and knowledge of the parameters $\boldsymbol{\theta}$, while $\pi_{t|T}$ is a $(B + 1) \times 1$ vector containing the elements $\left\{\pi_{t|T}^i\right\}_{i=1}^{B+1}$. We refer to these as the smoothed regime probabilities.

The time t perceived probability of exiting the current policy rule, i.e., of transitioning in the next period to the alternative policy regime ξ_t^A, is given by $\overline{P}_t^{bE} \equiv \sum_{i=1}^{B} \pi_{t|T}^i (1 - p_{bi})$. The time t perceived probability of exiting the current policy rule and transitioning in h periods to ξ_t^A is $\overline{P}_{t+h,t}^{bE} = \mathbf{1}'_{B+1} \left(\mathbf{H}^b\right)^h \pi_{t|T}$, where $\mathbf{1}'_{B+1}$ is an indicator vector with 1 in the $(B+1)$th position and zeros elsewhere. We use these estimated regime probabilities to compute the most likely belief regime at each point in time and track how it changes around Fed announcements and the whole sample.

As mentioned, since we are interested in understanding the connection between the previously estimated dovish/hawkish regimes for mps_t and the interest rate rule in the theoretical model, we force the regime sequence ξ_t^P for the policy rule parameters to correspond to the estimated sequence for the mean of mps_t. This sets the timing of the structural breaks in the policy rule. Importantly, however, the parameters characterizing the policy rule regimes are freely estimated under flat priors.[9] Thus, there is no implication from this procedure that the parameters of the policy rule must necessarily show evidence of structural change. Moreover, since we freely estimate the parameters of the policy regime under flat priors, there is nothing in the model estimation that restricts the low- (high-) mps subperiods to coincide with parameters of the interest rate rule that would imply relatively accommodative (restrictive) monetary policy.

A crucial role for the structural estimation is played by measures of expectations and asset prices. For a given policy rule ξ_t^P in place, the model implies that forward-looking variables depend both on the alternative policy rule indexed by ξ_t^A, and on the probability assigned to visiting that alternative. The Online Appendix provides a description of how expectations are computed in this setting with structural breaks and a perceived alternative policy rule.

5 Structural Estimation Results

This section presents results from the structural estimation. The first subsection discusses the parameter and latent state estimates. The next three subsections discuss the model implications for investor anticipation of realized policy rule regime changes, high frequency analysis around FOMC announcements, and the connection between markets and monetary policy changes both inside and outside of tight windows around FOMC announcements.

Before getting into these results, its worth pointing out that the estimated model-implied series track their empirical counterparts quite well, as shown in Figure 3.[10] In the estimation,

[9] We use the regime sequence $\hat{\xi}^{PT} = \{\hat{\xi}_1^P, ..., \hat{\xi}_T^P\}$ that is most likely to have occurred, given our estimated posterior mode parameter values. See the Online Appendix for details.

[10] The model-implied counterparts are based on smoothed estimates $S_{t|T}$ of S_t, using observations through then end of the sample at date T, which exploit the mapping to observables in (31) using the modal parameter estimates. The difference between the model-implied series and the observed counterpart is attributable to observation errors.

we allow for observation errors on all variables except for inflation, GDP growth, the FFR, and the SP500-lagged GDP ratio. For professional forecasters, we have multiple measures of expectations, each noisy signals on the latent "market's" expectation. It is therefore unsurprising that the fit appears to be better for some of these measures than others.[11]

5.1 Parameter and Latent State Estimates

We begin with parameter estimates for the monetary policy rule. Table 2 reports the posterior distributions for the policy rule parameters $\pi^T_{\xi^P_t}$, ψ_{π,ξ^P_t}, $\psi_{\Delta y,\xi^P_t}$ and ρ_{i,ξ^P_t}, where we use flat priors. A key finding is that the previously estimated regime subperiods (given in Table 1) are associated with quantitatively large changes in the estimated policy rule, as well as in the associated Alternative policy rules that we estimate investors perceived would come after the current rule of each regime subperiod ended. We report the values of the activism coefficients ψ_{π,ξ^P_t} and $\psi_{\Delta y,\xi^P_t}$ separately, as well as the ratio $\psi_{\pi,\xi^P_t}/\psi_{\Delta y,\xi^P_t}$. When output fluctuations are dominated by demand shocks (as in our sample according to parameter estimates below), the ratio $\psi_{\pi,\xi^P_t}/\psi_{\Delta y,\xi^P_t}$ is also relevant for the central bank's commitment to stabilizing the real economy around potential, since below target inflation tends to coincide with output below potential, and vice versa for above target inflation. Moreover, under these circumstances, a central bank cannot stabilize real activity without stabilizing inflation. For this reason, we label a central bank policy rule as more "active" with regard to stabilizing output fluctuations if either the ratio $\psi_{\pi,\xi^P_t}/\psi_{\Delta y,\xi^P_t}$ is higher or both ψ_{π,ξ^P_t} and $\psi_{\Delta y,\xi^P_t}$ are higher.

Table 2 shows that the Great Inflation (GI) regime (1961:Q1-1978:Q3) is characterized by a high estimated inflation target and a modest level of inflation activism (ψ_{π,ξ^P_t}) relative to output activism ($\psi_{\Delta y,\xi^P_t}$). The perceived Alternative policy rule for this subperiod also has a high inflation target, but differs in terms of the focus on inflation and output growth, with inflation stabilization perceived as the main objective. The anticipation of a heightened focus on inflation stabilization is in fact a defining feature of the realized policy rule during the Great Moderation (GM) regime that began in 1978:Q4, when the activism coefficient ψ_{π,ξ^P_t} on inflation is estimated to be 3, far greater than what is required for inflation stabilization according to the Taylor principle (Taylor (1993)), and much larger than the coefficient on output growth. The greater activism against inflation under Volcker in the GM regime is consistent with an older empirical literature (e.g., Clarida, Gali, and Gertler (2000)). The GM regime also features a much lower estimated inflation target than the GI regime, indicative of the much more restrictive monetary policy that characterizes the regime. This aspect of the realized GM regime was not well anticipated by investors during the GI regime according to the estimates of

[11] For household inflation surveys, we see larger observation errors in some episodes. These are mostly at higher frequencies, which we do not view as a source of concern since we are interested in these surveys for the purpose of pinning down the evolution of household's perceived trend inflation. Unlike investors, households are modeled as relatively inattentive to the central bank, thus any revisions at high frequency following Fed announcements are irrelevant for the model estimates.

the Alternative rule in the GI subperiod. Moving to the Post-Millennial (PM) regime, we find that policy rule parameters then shifted back to accommodative values with far less activism: the PM rule has both a higher inflation target compared to the GM regime, virtually no activism on inflation ($\psi_{\pi,\xi_t^P} = 0$) and very little activism on output ($\psi_{\Delta y,\xi_t^P} = 0.08$). The PM regime is also characterized a large increase in the persistence of the federal funds rate, consistent with the forward guidance policies implemented at the zero lower bound (ZLB) that promised to keep interest rates low for a prolonged period of time.

Investors' perceived Alternative policy rules also show marked differences across the three regime subperiods. In the GM regime, the perceived Alternative rule indicates that they expected the next rule to have an inflation target that was lower than what was actively in place during that period, along with greater activism in stabilizing both inflation and economic growth. Thus, the perceived Alternative rule in the GM period was both more hawkish and more active. Likewise, the estimated perceived Alternative policy rule of the PM regime has an inflation target that is even lower still, suggesting that, while policies implemented at the ZLB may have succeeded in increasing inflation, financial markets may not believed them to be fully time-consistent. Investors' perceived Alternative rule in the PM period also implies that investors expected a more active Federal Reserve compared to lackluster activism of the realized rule for the PM subperiod. Thus both the GM and PM periods are characterized by expectations that the next policy rule would be more active than the realized rule for that period. Since, as discussed above, a more active rule is associated with more aggressive action to stabilize the real economy, these features of the rule are related to perceived risk in the stock market that fluctuate with investor beliefs about the probability of exiting the current rule, as discussed below.

A comment is in order about the estimated magnitudes for $\pi^T_{\xi_t^P}$ shown in Table 2. Although this parameter plays the role of an "inflation target" in the interest rate rule, unlike traditional New Keynesian models, $\pi^T_{\xi_t}$ is not a value to which true inflation and inflation expectations in the model necessarily tend in the long-run. This happens because the model here differs in two ways from the traditional New Keynesian models: macro expectations are strongly backward looking, and the policy rule parameters are not constant but instead vary over time. In this setting, $\pi^T_{\xi_t^P}$ is more appropriately thought of as an implicit target rather than an explicit objective. In the PM period for example, the estimates imply that to achieve observed average inflation over this period, the implicit target needed to be higher than what became the explicitly stated 2% explicit objective in 2012 (2.5% according to the estimates), due to the string of negative demand shocks that occurred over two recessions. Such higher implicit objectives are important when the central bank operates at or close to the ZLB, as it did over much of this period. Forward guidance and quantitative easing, two tools that were employed at the ZLB, are channels that manifest in the model as a higher values for the implicit inflation target $\pi^T_{\xi_t^P}$, since this mechanism generates higher expected inflation even as nominal interest rates remain

unchanged at the ZLB.

Table 3 presents estimation results for key model parameters other than those of the policy rule.[12] It is worth emphasizing that the estimates imply a very high level of inertia in household inflation expectations. The constant gain parameter γ, controlling the speed with which beliefs about long-term inflation are updated with new information on inflation, is estimated to be quite low ($\gamma = 0.0001$). Furthermore, the parameter γ^T, controlling the extent to which perceived trend inflation is influenced by shifts in the central bank inflation target, is estimated to be small ($\gamma^T = 0.005$). Taken together, these findings imply that households revise their beliefs about long term inflation only very slowly over time and mostly based on past realizations of inflation rather than on changes in the inflation target. The low value for γ^T implies that, when changes in the implicit inflation target $\pi^T_{\xi^P_t}$ have occurred, the long-term inflation expectations of households react very little initially and subsequently converge only very slowly over time to the new value for $\pi^T_{\xi^P_t}$. This estimate therefore implies that Fed actions and announcements designed to actively change the inflation target over the sample had limited credibility to quickly alter longer term household inflation expectations.

We estimate a moderate level of risk aversion for the investor ($\sigma_P = 6.0$). In terms of the magnitude of the primitive economic shocks, monthly demand shocks are estimated to be the most quantitatively important by a large margin ($\sigma_f = 17$), compared to "supply side" shocks such as the shock to trend growth ($\sigma_g = 1.9$) or the markup shock ($\sigma_\mu = 0.13$). These shocks directly effect real output and are thus stabilized to some degree by the central bank. The estimated shock to the earnings share–not a stabilization target–has a healthy standard deviation of $\sigma_k = 6.13$, while that for the liquidity premium is $\sigma_{lp} = 0.62$.Finally, the parameter p_s is of interest from a behavioral perspective, since it gives the probability that investors assign to having the same beliefs tomorrow as they have today. The estimated value for this parameter is 0.9875, indicating that investors maintain very firmly held beliefs, rarely contemplating the possibility that they may change their minds in the future on the basis of new information.

Before leaving this section we report the model implications for basic asset pricing moments. Table 4 shows the annualized mean and standard deviation of the log excess return on equity, as measured by the log difference in the S&P 500 stock market value, the real interest rate, as measured by the difference between the annualized FFR and the average of the one-year-ahead forecast of inflation averaged across the SPF, BC, SOC, and Livingston surveys,[13] and the log difference in real, per capita S&P 500 earnings growth. The model based moments for these series are based on the modal parameter and latent state estimates and match their data counterparts closely.

[12] The model has a large number of additional auxiliary parameters that are used to map observables into their model counterparts. To conserve space, these additional parameters are reported in the Online Appendix.

[13] We interpolate the biannual Livingston survey observations to obtain monthly values, and only average in the observations for the quarterly SPF and SOC with the monthly BC and interpolated Livingston surveys when observations on the SPF and SOC are not missing.

5.2 Investor Beliefs About Monetary Policy Over the Sample

This section presents results on the evolution of investor beliefs about future changes in the monetary policy regime over the sample.

Figure 4 plots the estimated perceived probability that investors assign to being in a new policy rule regime in one year's time. Specifically, the figure reports the end-of-the-month value for $\overline{P}^{bE}_{t+12,t} \equiv \pi^{B+1}_{t+h,t|T} = \mathbf{1}'_{B+1}\left(\mathbf{H}^b\right)^{12}\pi_{t|T}$, where $\mathbf{1}'_{B+1}$ is an indicator vector with 1 in the $(B+1)$th position and zeros elsewhere and $\pi_{t|T}$ is the smoothed estimate of the time t belief regime probabilities. The vertical lines mark the timing of the two realized policy regime changes over our sample.

Figure 4 shows that the perceived probability of a policy rule regime change fluctuates strongly over the sample and typically increases before a realized policy change, suggesting that financial markets have some ability to anticipate the realized shifts in the conduct of policy. This occurs despite the fact that investors do not perfectly predict what the new policy rule will look like. The perceived probability of a policy rule change occasionally shoots up at times during which no actual change subsequently occurs over the next year, though these movements in beliefs are typically short-lasting. Particularly noticeable in this regard is the sharp increase in the estimated perceived probability of a policy rule change during the 2008/9 financial crisis. The GM regime is associated with sharp increases in the perceived probability of a regime change at both the beginning and the end of that subperiod. These fluctuations in investor beliefs drive expectations about future central bank conduct and thus movements in asset prices in the model, as we discuss below.

An important feature of the findings displayed in Figure 4 is that investor beliefs about the probability of a regime change in the Fed's policy rule continuously evolve *out*side of tight windows surrounding policy announcements. Indeed, most of the variation in investor beliefs about the future conduct of monetary policy occurs at times over the sample that are not close temporally to an FOMC announcement. This indicates that the causal effect of central bank policy on investor beliefs and therefore on markets is substantially more far reaching than what can be observed from market reactions in tight windows surrounding Fed announcements. An obvious explanation for this result is that many if not most Fed announcements provide forward guidance that often includes data-dependent criteria for what is likely to trigger a change in the policy stance down the road, even as it communicates no shift in policy at the time of the announcement. As new data is revealed in between Fed communications, investor beliefs about monetary policy–shaped by what was previously communicated–evolve and have consequences for markets. Because high frequency event studies surrounding Fed communications only capture the causal effects of the surprise component of any announcement, they are by construction incapable of accommodating these additional channels of influence outside of tight windows around events. The estimates portrayed in Figure 4 suggest that event studies alone provide a substantial underestimate of the overall causal impact, underscoring the

challenges with relying solely on such analyses for quantifying the effects of monetary policy on markets.

Figure 5 presents similar information, this time for the *change* in the estimated perceived probability of a monetary policy regime change within the next year in tight windows around every FOMC announcement in our sample. For this figure we focus on the post 1994 period, when we have data for FOMC announcements. We see that most FOMC announcements result in little if any change in the perceived probability of a regime change in monetary policy, again implying that financial markets do not learn about the possibility of policy rule changes only from policy announcements. Naturally, many FOMC announcements carry little news of any kind, consistent with the majority of points lining up along the horizontal line at zero and the idea that significant changes in the policy rule are infrequent. However, some announcements are associated with sizable changes in the perceived probability of exiting the current policy regime. The largest declines occur in the aftermath of the financial crisis, namely on June 24th, 2009, and October 29th, 2008, where in each case the perceived probability of a regime change in the next year declined by more than 1% in the 30 minutes surrounding the FOMC press release. The largest increase in the perceived probability of a policy regime change occurs on April 18th, 2001, with the probability increasing more than 1%. For the first two, a likely relevant aspect of these specific announcements is that they repeated the statement that the FOMC committee "anticipates that economic conditions are likely to warrant exceptionally low levels of the federal funds rate for an extended period." The FOMC press release for April 18, 2001 announced the decision to lower its target for the federal funds rate by 50 basis points.

5.3 High-Frequency Analysis

In this subsection, we present estimation results relevant for the question of why markets respond strongly to central bank announcements. In the model, central bank announcements affect investor expectations through two broad channels. First, they may surprise investors by changing investor beliefs about the likelihood of a regime shift in the conduct of monetary policy. Second, they may convey surprise information about the state of the economy. The subjective equity premium (equation (6)) is endogenously affected by both channels because investor beliefs about changes in the conduct of monetary policy affect the subjective risk premium component, and because the information conveyed about the economic state could cause a revision in the investor's nowcast for lp_t.

As explained above, in our estimation we include a number of variables available at high frequency, including daily measures of professional forecasts of inflation and GDP growth from Bloomberg, and minutely observations on the federal funds futures market observations and observations on the SP500-lagged GDP ratio, among other variables.

Figure 6 displays, for each FOMC announcement in our sample, the log change in pre-/post-announcement values of these variables, where the pre-FOMC value is either 10 minutes before

or the day before the FOMC press release time, depending on data availability (daily versus minutely), and the post-FOMC value is either 20 minutes after or the day after the release. The figure shows that some FOMC announcements have large effects on these forward looking variables, with jumps that are especially pronounced around the 2000/01 recession and tech bust in the stock market, and the 2008/9 financial crisis. Some of these announcements are associated with declines within 30 minutes surrounding the FOMC press release in the stock market that exceed 2% in absolute terms or increases above 4%, as in the announcement of January 3, 2001 in which the FOMC met off-cycle to lower its target for the federal funds rate by 50 basis points.

The mixed-frequency structural approach developed in this paper allows us to investigate a variety of possible explanations for why markets responded strongly to central bank actions and announcements, and to provide granular detail on why investor expectations are revised. This is accomplished by decomposing jumps in variables around FOMC announcements into the primitive economic sources of risk responsible for observed forecast revisions. Before presenting those findings, we briefly discuss how we use the filtering algorithm to obtain these results. The complete description and all technical details on the algorithm are relegated to the Online Appendix.

Consider an FOMC announcement in month t. Let d_i denote the number of time units that have passed within a month when we have reached a particular point in time, and let nd denote the total number of time units in the month. Then $0 \leq d_i/nd \leq 1$, and the end of month t is denoted $t-1+d_i/nd$ with $d_i = nd$. Take as given the state of the economy S_{t-1} and the belief regime ξ^b_{t-1} in place at time $t-1$, i.e., at the end of the previous month. Let $S^j_{t|t-1+d_i/nd}$ denote a filtered estimate of the economic state in time t from data up to time $t-1+d_i/nd$, conditional on $\xi^b_t = j$. We use our high-frequency, forward-looking data on investor expectations and financial markets and Kim's (Kim (1994)) basic filter for Markov-switching state space models to obtain estimates of the pre- and post-FOMC announcement values of $S^j_{t|t-1+d_i/nd}$, and the associated regime probabilities for the belief regimes $\pi^j_{t|t-1+d_i/nd}$, where d_i here assumes distinct values d_{pre} and d_{post} that denote the time right before and right after the FOMC meeting. We compute announcement-related revisions in S and in the belief regime probabilities π^j by taking the difference between the estimated values for these variables pre- and post-announcement. These differences represent our estimates of the market's revised nowcasts for S and beliefs about the future conduct of monetary policy that are attributable to the FOMC announcement.

Recall that the continuous-valued latent state vector $S_t = \left[S^M_t, m_t, pd_t, k_t, lp_t, \mathbb{E}^b_t\left(m_{t+1}\right), \mathbb{E}^b_t\left(pd_{t+1}\right)\right]$ where, S^M_t is a vector of latent macro state variables with $S^M_t \equiv [\widetilde{y}_t, g_t, \pi_t, i_t, \overline{\pi}_t, f_t]'$. Figure 7 displays the percent changes in pre-/post- announcement values of different elements of S_t for every FOMC announcement in our sample, providing an estimate of how investor perceptions about the current state of the economy shifted in the minutes surrounding a Fed announcement. These represent our estimates of how investors' nowcasts shifted in the wake of each announce-

ment. The figure shows that FOMC meetings during the financial crisis led to frequent and large changes in investor perceptions about trend growth g_t, detrended output, $\widetilde{y}_t$, inflation, current demand f_t, the earnings share k_t, and the liquidity premium lp_t. This evidence implies that FOMC announcements occasionally convey substantive information that causes investors to significantly revise their beliefs about the state of the economy and its core driving forces.

To make further progress of our understanding of what markets learn from FOMC announcements, we select the most relevant FOMC announcements for various series and decompose movements in expected inflation, expected GDP growth, the 6-month FFF rates, and the stock market into revisions in beliefs about the primitive shocks affecting the economy. This decomposition is computed as follows. The high-frequency version of our observation equation $X_{t-1+d_i/nd} = D_{\xi_t^b,t-1+d_i/nd} + Z_{\xi_t^b,t-1+d_i/nd}\left[S'_{t|t-1+d_i/nd}, \widetilde{y}_{t-1}\right]$ filters data in $X_{t-1+d_i/nd}$ around announcements to obtain estimates of $S^j_{t|t-1+d_i/nd}$ and the belief regimes $\pi^j_{t-1+d_i/nd|t-1}$ in the minutes and days surrounding an FOMC meeting, we observe different estimates for the shocks that investors *perceive* hit the economy using the state equation:

$$S^j_{t|t-1+d_i/nd} = C\left(\theta_{\xi_t^P}, \xi_t^b = j, \mathbf{H}^b\right) + T(\theta_{\xi_t^P}, \xi_t^b = j, \mathbf{H}^b)S_{t-1} + R(\theta_{\xi_t^P}, \xi_t^b = j, \mathbf{H}^b)Q\varepsilon^j_{t|t-1+d_i/nd},$$

where $\varepsilon^j_{t|t-1+d_i/nd}$ denotes the estimated Gaussian shocks based on the data available at time $t-1+d_i/nd$, conditional on being in belief regime $\xi_t^b = j$. For each FOMC announcement, we compute the contribution of one particular shock in the perceived shock vector $\varepsilon^j_{t|t-1+d_i/nd}$ by setting all other shocks to zero and integrating out the belief regimes. Thus, the contribution of shock k is measured by:

$$S^{\cdot,k}_{t|t-1+d_i/nd} = \textstyle\sum_{j=1}^{B} \pi^j_{t-1+d_i/nd|t-1} R(\theta_{\xi_t^P}, \xi_t^b = j, \mathbf{H}^b)Q\varepsilon^{j,k}_{t|t-1+d_i/nd} \tag{32}$$

The contribution of the belief regime is the remaining part:

$$S^{\cdot,b}_{t|t-1+d_i/nd} = \textstyle\sum_{j=1}^{B} \pi^j_{t-1+d_i/nd|t-1} \left[C\left(\theta_{\xi_t^P}, \xi_t^b = j, \mathbf{H}^b\right) + T(\theta_{\xi_t^P}, \xi_t^b = j, \mathbf{H}^b)S_{t-1}\right]. \tag{33}$$

We can then compute the contribution of revisions in investors perceptions of the shocks and/or about regime shifts in the policy rule to jumps in observed variables by taking the difference between the pre- and post-announcement values of $S^{\cdot,k}_{t|t-1+d_i/nd}$ and $S^{\cdot,b}_{t|t-1+d_i/nd}$.

Although (32) measures the contribution of shock k to revisions in the perceived state vector, it should be kept in mind that jumps in belief regime probabilities still matter for this component because they amplify (or dampen) the affects of Gaussian shocks through the $R(\theta_{\xi_t^P}, \xi_t^b = j, \mathbf{H}^b)$ coefficient. The endogenous heteroskedasticity created by fluctuating beliefs implies that (32) likely understates their role in generating jumps in financial market variables around FOMC announcements, since it is unlikely that those beliefs would remain fixed at the very moment that investors undertake large revisions in their understanding of the composition of shocks hitting the economy.

The next several figures display the decomposition above for four different high-frequency observable variables in X_t: the BBG consensus forecasts of inflation and GDP growth 12-months ahead, the 6-month FFF contract rate, and the SP500-lagged GDP ratio. Note that jumps in the SP500-lagged GDP ratio at FOMC announcements are entirely attributable to jumps in the stock market, since GDP is lagged one month. To keep the figures manageable, we report the decomposition for the 10 most quantitatively important announcements according to the absolute value of the pre-/post-announcement change in a particular variable. For the four variables of interest, the figures report black dots to indicate the observed change in the series, and red triangles to indicate the model implied change. For the stock market variable the black dot and red triangles coincide as we do not allow for observation error in that series. The plots show the decomposition of jumps in these variables into components driven by different elements of the perceived vector of Gaussian shocks $\varepsilon_t = (\varepsilon_{f,t}, \varepsilon_{it}, \varepsilon_{gt}, \varepsilon_{kt}, \varepsilon_{zt}, \varepsilon_{\mu t})$ and by investor beliefs π^j about the probability of a regime shift in the conduct of monetary policy.

Figure 8 reports the decomposition for a selection of FOMC announcements based on 10 most important absolute changes in the 6-month FFF rate. For all such events the model is able to match the direction of the jump in the observed series and in most cases the magnitude is also in line with the data. Many of these jumps are associated with times of important economic change, the largest of which occurs during the financial crisis on January 22, 2008 when the FOMC announced the lowering of the target for the FFR by an unusually large 75 basis point increment. From panel (c) we observe that most of the selected FOMC announcements are associated with a downward revision in the 6-month FFF rate, implying that markets were surprised by monetary policy that was more accommodative than anticipated, consistent with evidence in Cieslak (2018) and Schmeling, Schrimpf, and Steffensen (2020) who argue that markets systematically underestimated the Fed's response to large adverse economic shocks, and more generally with the arguments of Bauer and Swanson (2021), who argue that markets are often surprised by the Fed's response to economic events. Importantly, however, these surprise movements in the FFF rate are rarely estimated to be solely the attributable to a perceived monetary policy shock. Indeed, most announcements convey information about non-monetary shocks as well. For example, the January 22, 2008 announcement is estimated to have caused a downward revision in investor nowcasts of the earnings share and the component of output that investors' perceived was attributable to demand shocks, and a large upward revision in their perception of the component attributable to mark-up and trend growth shocks. The liquidity premium also jumped up in the wake of this announcement. The higher perceived markup shock results in a jump upward in the BBG expected inflation measure. This event is also associated with a jump upward in the BBG forecast of GDP growth over the next year, driven mostly by the revision upward in perceived trend growth but also by the revision downward in perceived demand, which causes survey respondents to expect faster economic growth from a lower current nowcast (the base effect). The stock market declined by more

than 1.9% in the 30 minutes surrounding the January 22, 2008 announcement, dragged down by a lower nowcast for the earnings share, a higher liquidity premium and lower perceived demand.[14] Investor perceptions of a surprisingly accommodative monetary policy shock on this date helped to support the stock market as did the upwardly revised nowcast for trend growth, but the role of these factors was outweighed by revisions in beliefs about the state of the economy that were overwhelmingly in a pessimistic direction. This finding speaks to the importance of "information effects" of these announcements as emphasized by Romer and Romer (2000), Campbell, Evans, Fisher, Justiniano, Calomiris, and Woodford (2012), and Nakamura and Steinsson (2018). We complement and add to the results from these studies by providing more granular detail on *why* expectations about aggregate variables are sometimes revised in the aftermath of Fed announcements, with a decomposition of market responses into the perceived economic sources of risk responsible for jumps in observed forecast revisions and financial markets.

Figures 9-12 report the same decomposition for a selection of FOMC announcements based on 10 largest absolute changes in the BBG consensus inflation forecast, the stock market, the BBG consensus GDP growth forecast, and in the perceived probability of exiting the current policy rule in the next year, respectively. We again find that the January 22, 2008 FOMC announcement shows up as an important surprise event for BBG inflation forecasts. When we sort on the 10 most important events for the BBG GDP growth forecast (Figure 11), we find that the FOMC press release of March 11, 2008–when the Fed announced an expansion of its securities lending programs in coordination with similar efforts at other central banks–is associated with both a negative FFF rate surprise and a negative revision in GDP growth forecasts, a positive comovement that Nakamura and Steinsson (2018) argue is consistent with a strong Fed information effect. Monetary policy shocks played essentially no role in the downward revision in the GDP growth forecast for this event, which was instead driven by a revision downward in the nowcast for trend growth.

Figure 10 shows that the most quantitatively important FOMC announcement in our sample for the stock market was the one on January 3, 2001 discussed above, when the market increased 4.2% in the 30 minutes surrounding the news, driven by a lower now cast for the liquidity premium component of the subjective equity premium, higher nowcasts for aggregate demand and the earnings share, and an accommodative monetary policy shock. The second and third most important FOMC events for the stock market were those on April 18, 2001 and October 29, 2008, respectively, when the market increased 2.5% and declined 2%, respectively, in the 30 minutes surrounding those press releases. For these events, investor beliefs about the probability of regime change in the conduct of monetary policy played large quantitative roles. Indeed, if we

[14]The negative contribution made by the revision in the perceived demand is not necessarily logically inconsistent with the upward revision in GDP growth expectations. A higher growth rate over the next year–from a lower current level of output–can coincide with persistently sluggish growth beyond the next 12 months.

sort events according to their importance for revisions in investor beliefs about the probability of regime change, the events of the April 18, 2001 and October 29, 2008 are the second and third most important according to that cut (Figure 12). The most important FOMC announcement for these belief revisions is the announcement on June 24, 2009, in which the Fed promulgated the continued expansion of its balance sheet, the maintenance of the target range for the federal funds rate at 0 to 0.25%, and the statement that it "anticipates that economic conditions are likely to warrant exceptionally low levels of the federal funds rate for an extended period." This announcement resulted in a large decline in the perceived probability of exiting the current policy rule in the next year that exceeded 2.9% in absolute terms, visible as the lowest dot in Figure 5. Panel (d) of Figure 12 shows that this jump in beliefs made a large negative contribution to the stock market's roughly zero overall response to this announcement, fully offsetting the positive contributions from upward revisions in the nowcasts for the demand component of output, trend growth, and the earnings share. This event also shows that changing beliefs about the policy rule around Fed announcements do not occur in a vacuum and therefore often coincide with changing perceptions about the economic state that have offsetting effects on the market, underscoring the empirical relevance of multiple channels of monetary transmission operating simultaneously in response to Fed communications.

Two other events bear noting. First, the event of October 29, 2008 is the event associated with second largest decline in the probability of exiting the policy rule, visible as the second lowest dot in Figure 5. This shift in investor beliefs about monetary policy regime change is the largest contributor to the market's 2% decline in the 20 minutes following that announcement. Second, the event of April 18, 2001 is the event associated with largest *increase* in the perceived probability of exiting the policy rule, visible as the highest dot in Figure 5. This shift in beliefs was the largest contributor to the market's 2.5% *increase* in the 20 minutes following that announcement.

Why does a decline in the perceived probability of exiting the current policy rule have a negative impact on the market, while an increase in the perceived probability has a positive impact? Since exiting the current policy regime is synonymous with entering the perceived Alternative regime, shifts in this perceived probability directly change perceptions of where the central bank is headed and thus expectations for the future. In the PM regime, the perceived Alternative monetary policy rule is one that features greater activism in stabilizing both inflation and output fluctuations (Table 2). This has quantitatively important implications for the subjective risk premium. For the event of April 18, 2001, the increased expectation of a central bank more actively engaged in stabilizing the real economy lowers uncertainty and thus the perceived quantity of risk, raising stock market valuations. Conversely, the events of June 24, 2009 and October 28, 2008–which lowered the perceived probability that central bank would soon shift to a policy rule dedicated to more actively stabilizing the economy–resulted in higher uncertainty and thus a higher perceived quantity of risk, lowering stock market valuations.

5.4 Markets and Monetary Policy Over the Sample

This section contains results on the role of monetary policy in driving financial market fluctuations over our entire sample.

Figure 13 shows the results of a simulation in which the observables and estimated state vector are taken as they were at the beginning of our sample with all Gaussian shocks shut down. Thus, the only source of variation in the variables plotted in the figure shown in red (dashed) lines arises from realized changes in the policy rule parameters and from changes in investor beliefs about the probability of exiting the current policy rule. These movements are juxtaposed with the observed data for these series, shown in blue (dotted) lines.

The figure shows that realized policy rule regime changes and beliefs about such changes cause large fluctuations in the stock market relative to lagged GDP. This can be observed in panel (e) where the red line tracks the observed series closely over much of the sample. The periods during the sample when the red line deviates substantially from the observed series are mostly within realized regime subperiods and are therefore driven by jumps in investor beliefs about exiting the existing policy rule. For example, the spike upward in the red line in Panel (e) of Figure 13 for the SP500-lagged GDP ratio during the PM regime coincides with the spike upward in the in the perceived probability of exiting the current policy regime that occurred at the end of December 2008 shown in Figure 4. As explained above, in the PM regime an increase in the perceived probability of exiting the current rule is associated with a higher perceived probability of moving to a policy regime with a central bank more actively engaged in stabilizing output fluctuations, which lowers the perceived quantity of risk and thus the subjective risk premium, raising stock market valuations.

The remaining panels of Figure 13 show variation in non-equity market variables. With Gaussian shocks turned off, only realized policy rule regime shifts affect the red (dashed) line for these variables, since investor beliefs about the probability of a future change in the rule play no role in macro dynamics. The red (dashed) lines for these variables evolve dynamically according to the state equation (30) with shocks set to zero. We see that the low frequency swings in the federal funds rate, the real federal funds rate, inflation, and in five-year-ahead SOC expected inflation are all tightly linked to regime changes in the monetary policy rule over the sample. Household inflation expectations adjust only gradually in the wake of policy rule realized regime changes due to the substantial degree of inertia in household inflation expectations (Panel (a)). As a consequence, a large fraction of the secular decline in the real interest rate since about 1980 shown in panel (f) is attributable to regime changes in the conduct of monetary policy, consistent with BLL.

To explore further why stock market valuations move over our sample according to the structural estimation, we decompose the stock price-lagged output ratio into components driven by the representative investor's subjective beliefs about future earnings, future return premia,

and future real interest rates. The price-lagged output ratio is

$$\frac{P_t}{Y_{t-1}} = \frac{P_t}{D_t}\frac{D_t}{Y_t}\frac{Y_t}{Y_{t-1}}$$

or in logs

$$pgdp_t = pd_t + k_t + \Delta y_t,$$

where $pgdp_t \equiv \ln(P_t/Y_{t-1})$ and $pd_t \equiv \ln(P_t/D_t)$. Let r^{ex} denote the log return on the stock market in excess of the log real interest rate, and let rir denote the log real interest rate. We decompose pd_t as in Campbell and Shiller (1989) into the sum of three forward-looking sources of variation:

$$pd_t = \frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + pdv_t(\Delta d) - pdv_t(r^{ex}) - pdv_t(rir) \tag{34}$$

where the first term is a constant, $pdv_t(x) \equiv \sum_{h=0}^{\infty} \beta_p^h \mathbb{E}_t^b[x_{t+1+h}]$, and $rir_{t+1} \equiv \left(i_{t+1} - \mathbb{E}_t^b[\pi_{t+1}]\right)$ is the expected real interest rate from the perspective of the investor.[15] Observe that the subjective expectations of the investor $\mathbb{E}_t^b[\cdot]$ are computed based on the structural estimates and depend on the beliefs about the future conduct of monetary policy as well as the expected paths of Gaussian variables. Subjective equity market return premia embedded in $pdv_t(r^{ex})$ are driven in the model by just three factors: (i), realized regime change in monetary policy ξ_t^P, (ii) changing investor beliefs about the probability of a regime change in monetary policy ξ_t^b, and (iii) the liquidity premium lp_t. Subjective expectations of future real interest rates embedded in $pdv_t(rir)$ depend these factors, as well as expectations about inflation and output growth that enter the monetary policy rule.

With (34), we decompose $pgdp_t$ into the sum of four components:

$$pgdp_t = \underbrace{ey_t}_{\text{earning share}} + \underbrace{pdv_t(\Delta d)}_{\text{earnings}} - \underbrace{pdv_t(r^{ex})}_{\text{premia}} - \underbrace{pdv_t(rir)}_{\text{real int rate}}, \tag{35}$$

where $ey_t \equiv \frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + k_t + \Delta y_t$ is the earnings to lagged output ratio. We refer to ey_t as the "earnings share" for simplicity, though the reader is reminded that this variable depends on both k_t and on output growth Δy_t, and is shifted up by a constant.

Figure 14 reports the empirical decomposition of $pgdp_t$ into estimated components of (35). The solid (blue) line in each panel plots the data for $pgdp_t$ (the SP500-lagged GDP ratio) over our sample. The red lines in panels (a)-(d) successively cumulate the right hand side components in (35) so that they add to the observed $pgdp_t$ as we move from panel (a) to panel (d).

Figure 14, panel (a), shows the data for $pgdp_t$ (in blue) plotted along with the ey_t component alone (in red).[16] This panel shows that the earnings share plays little role in fluctuations in

[15] The derivation of this decomposition is given in the Online Appendix.

[16] Note that the ey_t term includes the constant $\frac{\kappa_{pd,0}}{1-\kappa_{pd,1}}$ so it can be greater than $pgdp_t$.

$pgdp_t$ up to about the year 2000. The ey_t component then declines sharply during the financial crisis of 2008/09 contributing to the sharp drop in the stock market (blue line) during the crisis. Subsequently, the earnings share recovers and increases sharply, helping to boost the market in the years after the financial crisis. These findings echo results in Greenwald, Lettau, and Ludvigson (2019).

Moving to panel (b) of Figure 14, the red line adds $-pdv_t\left(r^{ex}\right)$ to ey_t, showing that subjective return premia play a large role in stock market fluctuations, especially in the PM period. A comparison of panels (a) and (b) shows that adding $-pdv_t\left(r^{ex}\right)$ to ey_t brings the red (dashed) line much closer to the observed $pgdp_t$ data series (blue line) over this subperiod. Panel (b) also plots a counterfactual for the component $-pdv_t\left(r^{ex}\right)+ey_t$ (green line) in which we turn off the liquidity premium shocks lp_t, implying that the only factor causing fluctuations in $pdv_t\left(r^{ex}\right)$ for that counterfactual case are (i) realized policy rule regime changes and (ii) changing investor beliefs about the probability of a regime change. The green, counterfactual, line lies almost on top of the baseline estimate (red line) over most of the sample. This shows that most of the variation in subjective equity premia are driven by fluctuating monetary policy rules and beliefs about future policy rule shifts, rather than by fluctuations in the liquidity premium. Stock market valuations rise sharply at both the beginning and the end of the GM regime primarily because these were times when the perceived probability of a regime change in the conduct of monetary policy rose sharply (Figure 4). Given the estimates of the GM perceived Alternative policy rule, these were then times when investors perceived a greater likelihood that the central bank would move to a regime more actively engaged in stabilizing the economy, especially on the real side (Table 2). The result is a lower perceived quantity of risk in the stock market during these times, and higher valuations.

Panel (c) of Figure 14 adds $-pdv_t\left(rir\right)$ to the components $ey_t-pdv_t\left(r^{ex}\right)$ plotted in panel (b), so that the differences between panels (b) and (c) isolate the role of subjective expectations about future real interest rates in stock market fluctuations. Expectations about future short rates play an important role in equity market valuations early in the sample, from 1961 to about 1990. Expectations of persistently low future real rates helped support the stock market in the Great Inflation regime from 1961:Q1-1978:Q3. By contrast, expectations of higher future real rates pulled down the market in the early part of the Great Moderation regime, when the shift to a hawkish policy rule during the Volcker disinflation took hold. A comparison of panels (b) and (c) shows that, between 1978:Q3 to about 1990, expectations of persistently higher future real interest rates largely explain the low stock market valuations of that time, which panel (b) shows would have been much higher without the changing short rate expectations. This suggests that, while the Volcker disinflation and the Great Moderation that followed set the stage for the high valuations in 1990s by reducing volatility and lowering subjective return premia, initially it dragged the market down through the shift to a more hawkish policy rule with persistently high real interest rates.

Panel (d) of Figure 14 adds $pdv_t(\Delta d)$ to $ey_t - pdv_t(r^{ex}) - pdv_t(rir)$. A comparison of panel (d) with panel (c) shows that expected future cash flow growth plays a small role in these stock market fluctuations. The sum of all four components matches the observed fluctuations in $pgdp_t$ without error, since we do not allow observation error in our state space representation for that series. Taken together, the findings in Figure 14 underscore the importance of investor expectations about future real interest rates and return premia in driving the stock market over the full sample.

To zero in more specifically on the role of investor beliefs about future regime change in monetary policy in driving stock market fluctuations over the sample, Figure 14 exhibits the results of a counterfactual analysis for the PM regime subperiod. For this, we again report a decomposition of $pgdp_t$ into different components, but this time adding only one of the $pdv(\cdot)$ terms in (35) at a time to ey_t. Denote these components as

$$\begin{aligned} pgdp_{r^{ex},t} &\equiv ey_t - pdv_t(r^{ex}) \\ pgdp_{rir,t} &\equiv ey_t - pdv_t(rir) \\ pgdp_{\Delta d,t} &\equiv ey_t + pdv_t(\Delta d). \end{aligned}$$

The solid (blue) line in each panel of Figure 14 plots our baseline estimate of the component series named in the subpanel. For panel (a), which plots $pgdp_t$, our baseline model estimate and the data series coincide by construction. Panels (b)-(c) plot the components $pgdp_{r^{ex},t}$, $pgdp_{rir,t}$, and $pgdp_{\Delta d,t}$, respectively. The red (dashed) line in each panel plots a counterfactual in which investors believe throughout the PM subperiod that the probability of exiting the policy rule was the highest value that they would ever entertain given our estimates on the grid. The purple (dashed-dotted) line in each panel plots a counterfactual in which investors believe that the probability of exiting the policy rule was the lowest value they would entertain.[17]

Figure 14 conveys two main findings. First, it shows that investor beliefs about the conduct of future monetary policy play an outsized role in stock market fluctuations. This can be observed from the quantitatively large gap between the red and purple lines in panel (a). The red (dashed) line shows that, had investors counterfactually maintained the belief that the central bank was very likely to exit the PM policy rule, the stock market would have been much higher than it actually was over most of this period, and substantially higher than if they had counterfactually held the opposite belief shown in the purple (dashed-dotted) line, namely that regime change was very *un*likely. Second, panels (b)-(d) show that the reason for this large

[17] Recall that $P\left(\xi_t^b = i | X_T; \boldsymbol{\theta}\right) \equiv \pi_{t|T}^i$ is the estimated probability that $\xi_t^b = i$, for $i = 1, 2, ..., B+1$, while $\pi_{t|T}$ is a $(B+1)\times 1$ vector containing the elements $\left\{\pi_{t|T}^i\right\}_{i=1}^{B+1}$. The regime $\xi_t^b = 1$ is the belief regime corresponding to the lowest perceived probability that the central bank will stay with the current policy rule, i.e., the highest perceived probability of exiting. The first counterfactual replaces the estimated belief regime probabilities $\pi_{t|T}$ with a vector that has unity as the first element and zeros elsewhere. The second counterfactual replaces $\pi_{t|T}$ with a vector that has unity as the Bth element and zeros elsewhere.

discrepancy has to do with the affect of beliefs on investors' subjective expectations for future equity return premia, rather than with their effect on subjective expectations of future real rates or future payout growth. This can be observed by noting that the red/blue line discrepancy is largest for $pgdp_{r^{ex},t}$ in panel (b), small for $pgdp_{rir,t}$ in panel (c), and non-existent for $pgdp_{\Delta d,t}$ in panel (d). In short, had investors counterfactually believed throughout the PM period that monetary policy regime change was highly likely, the market would have been higher because subjective equity risk premia would have been lower.

Figure 16 examines these forces at high frequency around FOMC announcements. The figure decomposes the fluctuations in the pd_t ratio from the model into fluctuations driven by the $pdv_t(\cdot)$ components on the right-hand-side of (34). Specifically, the figure reports this decomposition for the 30 minute windows around the 10 most relevant FOMC announcements sorted on the basis of jumps in the estimated perceived probability of a regime change in the conduct of monetary policy over the next year. Panel (a) shows the change in the perceived probability of a regime change for each of these 10 events, while panel (b) shows the decomposition of the jump in pd_t into its $pdv_t(\cdot)$ components.

Figure 16, panel (a), shows that the FOMC announcement of June 24, 2009 is associated with a large downward revision in the perceived probability of a regime change in monetary policy. Panel (b) shows that this same event is associated with a large jump upward in subjective expected return premia, as measured by $pdv_t(r^{ex})$, and modest jumps downward in subjective expected real short rates, as measure by $pdv_t(rir)$. Subjective perceptions of risk rise because of the sharp decline in the perceived probability of transitioning to an Alternative policy regime, which in the PM period means a central bank less likely to become actively engaged in stabilizing the real economy. At the same time, the dovish tone of this announcement generated expectations of lower future real interest rates, which helped to support the market. Expected future payout growth $pdv_t(\Delta d)$ plays a small role.

To summarize, why do central banks impact the stock market? The results in this section suggest that they do so primarily because they affect beliefs about how monetary policy will be conducted in the future that turn affect investor perceptions of stock market risk, and because shifts in the policy rule have a persistent influence on short rates. By contrast, Figure 14 suggests that changes in the conduct of monetary policy and uncertainty about that conduct play a small role in driving expected future cash-flow growth. Taken together with the high-frequency results of the last section, this suggests that changes in the conduct of monetary policy plays a large role in stock market fluctuations, and occasionally–typically during times of important economic change–the Fed affects the stock market by providing information about the latent economic state, as it did with its FOMC announcement of January 3, 2001 when the market surged 4.2% in the 30 minutes surrounding the news as a result of sharp upward revisions in the nowcasts for demand and the earnings share of output, and a decrease in the liquidity premium (Figure 10).

6 Conclusion

We integrate a high-frequency monetary event study into a mixed-frequency macro-finance model and structural estimation. The approach allows for jumps at Fed announcements in investor beliefs, providing detailed answers on why markets react strongly to central bank announcements. The methodology can be used in a variety of other settings to provide a granular understanding of the role of news events of almost any category in driving financial market volatility.

Why do financial markets react strongly to central bank communications? In this study we find that the reasons involve a mix of factors, including revisions in investor beliefs about the latent state of the economy ("Fed information effects"), uncertainty over the future conduct of monetary policy, and subjective reassessments of risk in the stock market. Our results imply that investors seldom learn only about conventional monetary policy shocks from central bank announcements. Instead, Fed communications are associated with announcement-driven revisions in the perceived nowcasts of the shocks hitting the economy, including those attributable to demand versus supply factors, markups, and earnings shares.

The mixed-frequency structural approach proposed here also permits us to estimate the effects of monetary policy over an extended sample, not merely in tight windows around Fed announcements. The results suggest that central banks impact stock market valuation ratios at both high and low frequencies primarily because beliefs about the future conduct of monetary policy affect subjective perceptions of stock market risk and because shifts in the policy rule have a persistent influence on short rates, with only a small role played by expected future cash-flow growth. Although there are instances typically associated with unusually large adverse shocks in which the Fed affects the stock market by providing information about the latent economic state, our findings indicate that pure event studies are likely to substantially understate the impact of monetary policy on financial markets, much of which occurs outside of tight windows around central bank communications.

References

Andersen, T. G., T. Bollerslev, and N. Meddahi (2005): "Correcting the errors: Volatility forecast evaluation using high-frequency data and realized volatilities," *Econometrica*, 73(1), 279–296.

Bansal, R., and H. Zhou (2002): "Term structure of interest rates with regime shifts," *The Journal of Finance*, 57(5), 1997–2043.

Barndorff-Nielsen, O. E., P. R. Hansen, A. Lunde, and N. Shephard (2008): "Designing realized kernels to measure the ex post variation of equity prices in the presence of noise," *Econometrica*, 76(6), 1481–1536.

Bauer, M. D., C. Pflueger, and A. Sundaram (2022): "Perceptions About Monetary Policy," Unpublished manuscript, University of Chicago.

Bauer, M. D., and E. T. Swanson (2021): "An Alternative Explanation for the "Fed Information Effect"," *SSRN Electronic Journal*.

Bernanke, B. S., and K. N. Kuttner (2005): "What explains the stock market's reaction to rederal reserve policy?," *Journal of Finance*, 60, 1221–1257.

Bianchi, F. (2013): "Regime switches, agents' beliefs, and post-World war II U.S. macroeconomic dynamics," *Review of Economic Studies*, 80(2), 463–490.

——— (2016): "Methods for measuring expectations and uncertainty in Markov-switching models," *Journal of Econometrics*, 190(1), 79–99.

Bianchi, F., T. Kind, and H. Kung (2019): "Threats to central bank independence: High-frequency identification with Twitter," Discussion paper, National Bureau of Economic Research.

Bianchi, F., H. Kung, and M. Tirskikh (2018): "The origins and effects of macroeconomic uncertainty," Discussion paper, National Bureau of Economic Research.

Bianchi, F., M. Lettau, and S. C. Ludvigson (2022): "Monetary Policy and Asset Valuaton," *The Journal of Finance*, 77(2), 967–1017.

Boyarchenko, N., V. Haddad, and M. C. Plosser (2016): "The Federal Reserve and market confidence," Staff Reports 773, Federal Reserve Bank of New York.

Brooks, J., M. Katz, and H. Lustig (2018): "Post-FOMC announcement drift in US bond markets," Discussion paper, National Bureau of Economic Research.

Brownlees, C. T., and G. M. Gallo (2006): "Financial econometric analysis at ultra-high frequency: Data handling concerns," *Computational statistics & data analysis*, 51(4), 2232–2245.

Campbell, J. R., C. L. Evans, J. D. Fisher, A. Justiniano, C. W. Calomiris, and M. Woodford (2012): "Macroeconomic effects of federal reserve forward guidance [with comments and discussion]," *Brookings papers on economic activity*, pp. 1–80.

Campbell, J. Y., and R. J. Shiller (1989): "The dividend-price ratio and expectations of future dividends and discount factors," *Review of Financial Studies*, 1(3), 195–228.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005): "Nominal rigidities and the dynamic effects of a shock to monetary policy," *Journal of Political Economy*, 113(1), 1–45.

Cieslak, A. (2018): "Short-rate expectations and unexpected returns in treasury bonds," *The Review of Financial Studies*, 31(9), 3265–3306.

Cieslak, A., A. Morse, and A. Vissing-Jorgensen (2019): "Stock returns over the FOMC cycle," *Journal of Finance*, 74, 2201–2248.

Cieslak, A., and A. Schrimpf (2019): "Non-monetary news in central bank communication," *Journal of International Economics*, 118, 293–315.

Clarida, R., J. Gali, and M. Gertler (2000): "Monetary policy rules and macroeconomic stability: evidence and some theory," *Quarterly Journal of Economics*, 115(1), 147–180.

Cochrane, J. H., and M. Piazzesi (2002): "The fed and interest rates-a high-frequency identification," *AEA Papers and Proceedings*, 92(2), 90–95.

Cook, T., and T. Hahn (1989): "The effect of changes in the federal funds rate target on market interest rates in the 1970s," *Journal of Monetary Economics*, 24(3), 331–351.

Cox, J., D. Greenwald, and S. C. Ludvigson (2020): "What Explains the COVID-19 Stock Market?," National Bureau of Economic Research Working Paper No. 27784.

Farmer, R. E., D. F. Waggoner, and T. Zha (2011): "Minimal state variable solutions to Markov-switching rational expectations models," *Journal of Economic Dynamics and Control*, 35(12), 2150–2166.

Galí, J. (2015): *Monetary Policy, Inflation, and the Business Cycle: an Introduction to the New Keynesian Framework and its Applications*. Princeton University Press, Princeton, NJ.

Gertler, M., and P. Karadi (2015): "Monetary policy surprises, credit costs, and economic activity," *American Economic Journal: Macroeconomics*, 7(1), 44–76.

Gilchrist, S., D. López-Salido, and E. Zakrajšek (2015): "Monetary policy and real borrowing costs at the zero lower bound," *American Economic Journal: Macroeconomics*, 7(1), 77–109.

Greenwald, D., M. Lettau, and S. C. Ludvigson (2019): "How the wealth was won: factor shares as market fundamentals," NBER Working paper No. 25769.

Guraynak, R. s., B. Sack, and E. Swanson (2005): "Do actions speak louder than words? The response of asset prices to monetary policy action and statements," *International Journal of Central Banking*, 1, 55–93.

Gürkaynak, R. S., B. Sack, and E. Swanson (2005): "The sensitivity of long-term interest rates to economic news: Evidence and implications for macroeconomic models," *American Economic Review*, 95(1), 425–436.

Haddad, V., A. Moreira, and T. Muir (2020): "When selling becomes viral: Disruptions in debt markets in the covid-19 crisis and the fed's response," Discussion paper, National Bureau of Economic Research.

Hamilton, J. D. (1994): *Time Series Analysis*. Princeton University Press, Princeton, NY.

Hanson, S. G., and J. C. Stein (2015): "Monetary policy and long-term real rates," *Journal of Financial Economics*, 115(3), 429–448.

Hillenbrand, S. (2021): "The Fed and the Secular Decline in Interest Rates," *Available at SSRN 3550593*.

Jarocinski, M., and P. Karadi (2020): "Deconstructing monetary policy surprises: the role of information shocks," *American Economic Journal: Macroeconomics*, 12, 1–43.

Kekre, R., and M. Lenel (2021): "Monetary Policy, Redistribution, and Risk Premia," NBER Working paper No. 28869.

Kim, C.-J. (1994): "Dynamic Linear Models with Markov-Switching," *Journal of Econometrics*, 60, 1–22.

Krishnamurthy, A., and A. Vissing-Jorgensen (2011): "The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy," *Brookings Papers on Economic Activity*, 2011(2), 215–287.

——— (2012): "The aggregate demand for Treasury debt," *Journal of Political Economy*, 120(2), 233–267.

Kuttner, K. N. (2001): "Monetary Policy Surprises and Interest Rates: Evidence from the Federal Funds Futures Market," *Journal of Monetary Economics*, 47, 523–544.

Laubach, T., and J. C. Williams (2003): "Measuring the natural rate of interest," *Review of Economics and Statistics*, 85(4), 1063–1070.

Lubik, T. A., and F. Schorfheide (2004): "Testing for indeterminacy: An application to US monetary policy," *American Economic Review*, 94, 190–217.

Lucca, D. O., and E. Moench (2015): "The pre-FOMC announcement drift," *Journal of Finance*, 70(1), 329–371.

Malmendier, U., and S. Nagel (2016): "Learning from inflation experiences," *Quarterly Journal of Economics*, 131(1), 53–87.

Melosi, L. (2017): "Signalling effects of monetary policy," *The Review of Economic Studies*, 84(2), 853–884.

Nakamura, E., and J. Steinsson (2018): "High-frequency identification of monetary non-neutrality: The information effect," *Quarterly Journal of Economics*, 133(3), 1283–1330.

Pflueger, C., and G. Rinaldi (2020): "Why Does the Fed Move Markets so Much? A Model of Monetary Policy and Time-Varying Risk Aversion," Discussion paper, National Bureau of Economic Research.

Piazzesi, M. (2005): "Bond yields and the Federal Reserve," *Journal of Political Economy*, 113(2), 311–344.

Piazzesi, M., and E. Swanson (2008): "Futures Prices as Risk-Adjusted Forecasts of Monetary Policy," *Journal of Monetary Economics*, 55, 677–691.

Romer, C. D., and D. H. Romer (2000): "Federal Reserve information and the behavior of interest rates," *American economic review*, 90(3), 429–457.

Schmeling, M., A. Schrimpf, and S. Steffensen (2020): "Monetary Policy Expectation Errors," *Available at SSRN 3553496*.

Sims, C. A., and T. Zha (2006): "Were there regime switches in US monetary policy?," *American Economic Review*, 91(1), 54–81.

Stock, J. H., and M. W. Watson (2010): "Research Memorandum," `https://www.princeton.edu/~mwatson/mgdp_gdi/Monthly_GDP_GDI_Sept20.pdf`.

Taylor, J. B. (1993): "Discretion versus policy rules in practice," in *Carnegie-Rochester conference series on public policy*, vol. 39, pp. 195–214. Elsevier.

Table 1: **Regime Subperiods and Parameter Estimates**

	1961:Q1-1978:Q3	1978:Q4-2001:Q1	2001:Q2-2020:Q1
Regime	Great Inflation (D)	Great Moderation (H)	Post-Millennial (D)
$r_{\xi_t^P}$	-2.67%	1.38%	-1.27%

Notes: Table reports the most likely regime sequence based on the posterior mode estimates. (D) refers to the low *mps* regime and (H) refers to the high *mps* regime. The second row reports the model estimate of the mean of *mps* $r_{\xi_t^P}$ at the posterior mode. The estimation sample spans 1961:Q1-2020:Q1.

Table 2: **Taylor Rule Parameters**

	Great Inflation Regime		Great Moderation Regime		Post-Millennial Regime	
	Realized	Alternative	Realized	Alternative	Realized	Alternative
π_ξ^T	12.5288	11.8454	1.9142	0.8244	2.4878	0.0597
ψ_π	1.4831	2.0735	3.0071	3.6075	0.0000	0.6783
ψ_y	1.1985	0.0349	0.0005	0.6763	0.0751	0.5270
ψ_π/ψ_y	1.2375	59.4126	6014.2	5.3342	0.0000	1.2871
$x = \rho_{i,1} + \rho_{i,2}$	0.9960	0.8189	0.9905	0.9852	0.9972	0.9444

Notes: For each realized policy regime, the table reports the posterior mode values of the parameters for the current and alternative policy rules. The estimation sample spans 1961:Q1-2020:Q1.

Table 3: **Other Key Parameters**

Parameter	Mode	Parameter	Mode	Parameter	Mode	Parameter	Mode
σ	0.0522	γ^T	0.0050	σ_f	17.2460	σ_{lp}	0.6211
β	0.7529	σ_p	6.0097	σ_i	0.0344	σ_g	1.9079
ϕ	0.7424	β_p	0.9919	σ_μ	0.1348		
γ	0.0001	p_s	0.9875	σ_k	6.1267		

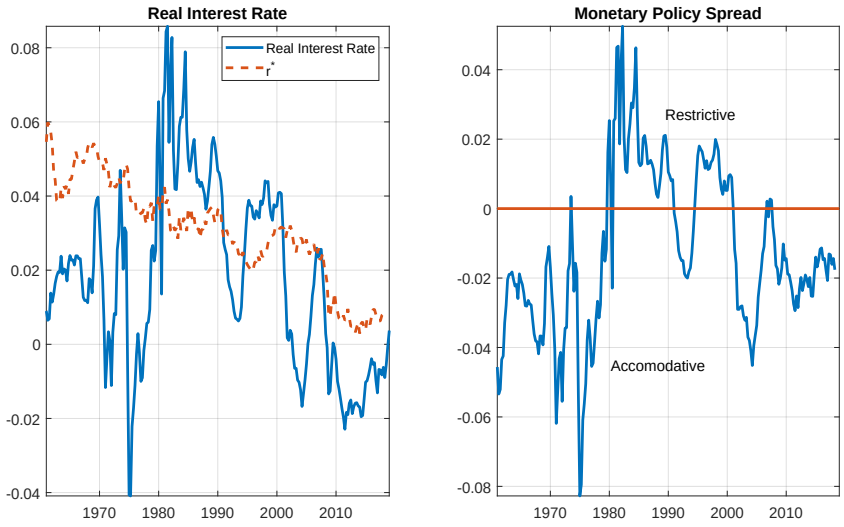
Notes: The table reports the posterior mode values of the parameters named in the row. The estimation sample spans 1961:Q1-2020:Q1.

Table 4: **Asset Pricing Moments**

Moments	Model		Data	
	Mean	StD	Mean	StD
Log Excess Return	7.71	14.92	7.42	14.85
Real Interest Rate	1.63	2.58	1.72	2.53
Log Real Earning Growth	1.97	16.60	1.96	17.24

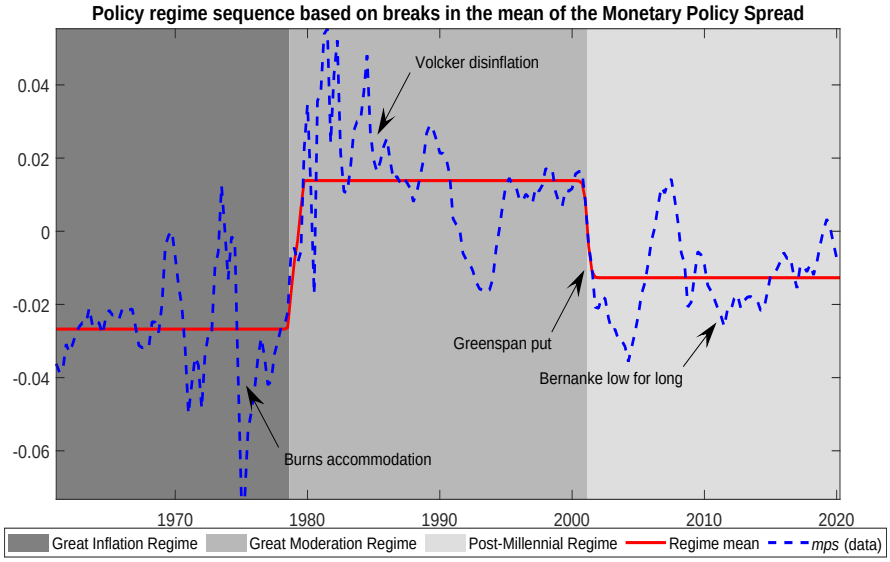
Notes: All reported statistics are annualized monthly statistics (means are multiplied by 12 and standard deviations by $\sqrt{12}$) and reported in units of percent. Excess returns are computed as the log difference in SP500 market capitalization minus FFR. The real interest rate is computed as the difference between FFR and average of the one-year ahead forecast of inflation across different surveys including BC, SPF, SOC, and Livingston. SP500 Earnings is deflated using GDP deflator and divided by population. The sample is 1961:M1 - 2020:M2.

Figure 1: Real Interest Rate



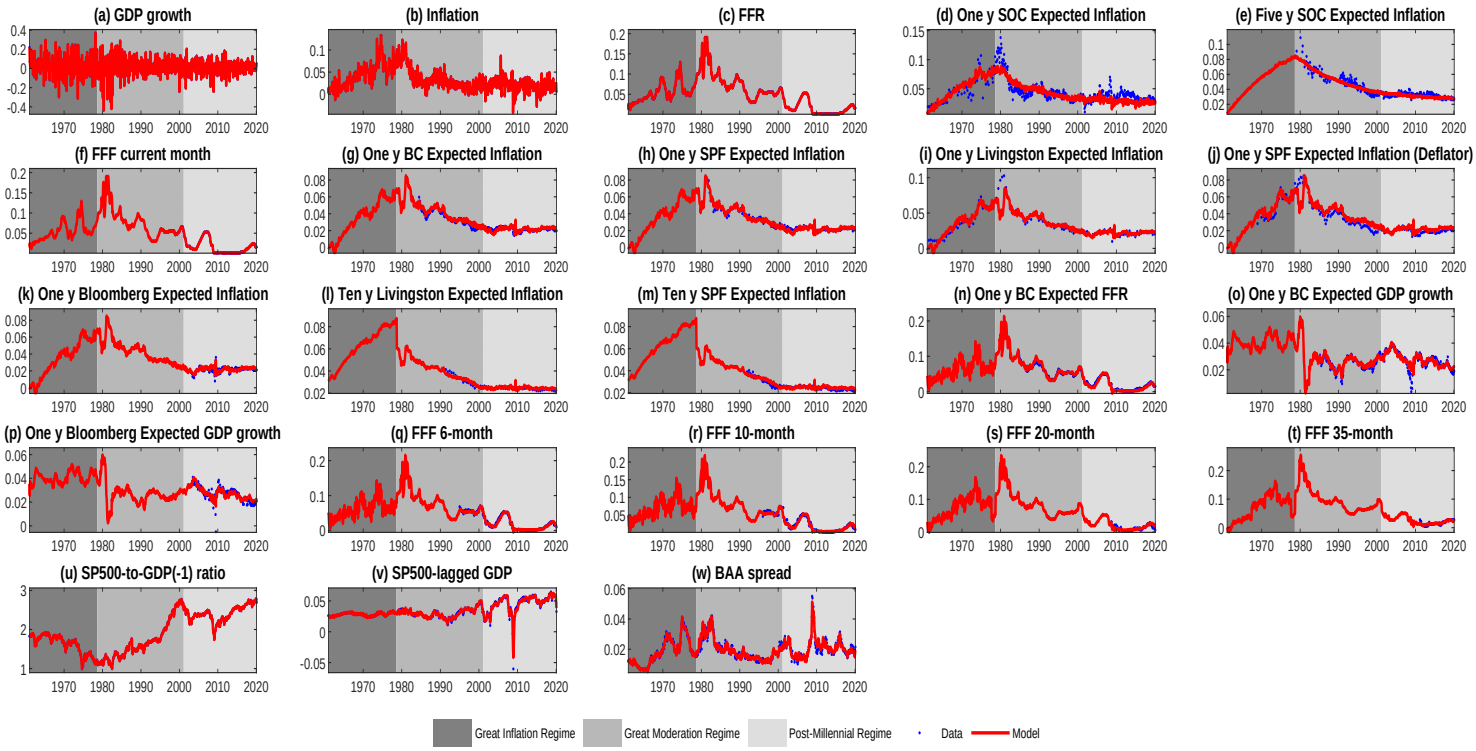
Notes: The real interest rate is measured as the federal funds rate minus a four quarter moving average of inflation. The left panel plots this observed series along with an estimate of r^* from Laubach and Williams (2003). The right panel plots the monetary policy spread, i.e., the spread between the real funds rate and the Laubach and Williams (2003) natural rate of interest. The sample spans 1961:Q1-2020:Q1.

Figure 2: Breaks in Monetary Policy



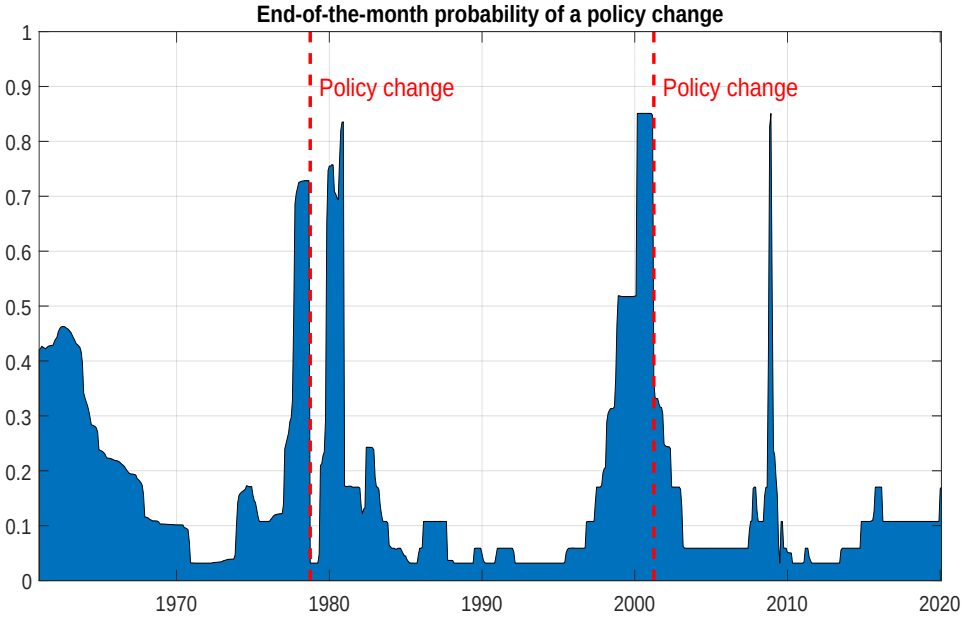
Notes: Monetary policy spread $mps_t \equiv FFR_t - \text{Expected Inflation}_t - r_t^*$. r^* is from Laubach and Williams (2003). The red (dashed) line represents the data. The blue (solid) line is the estimated regime mean. Accomdative regimes have $mps_t < 0$; restrictive regimes have $mps_t > 0$. The sample spans 1961:Q1-2020:Q1.

Figure 3: Smoothed Series



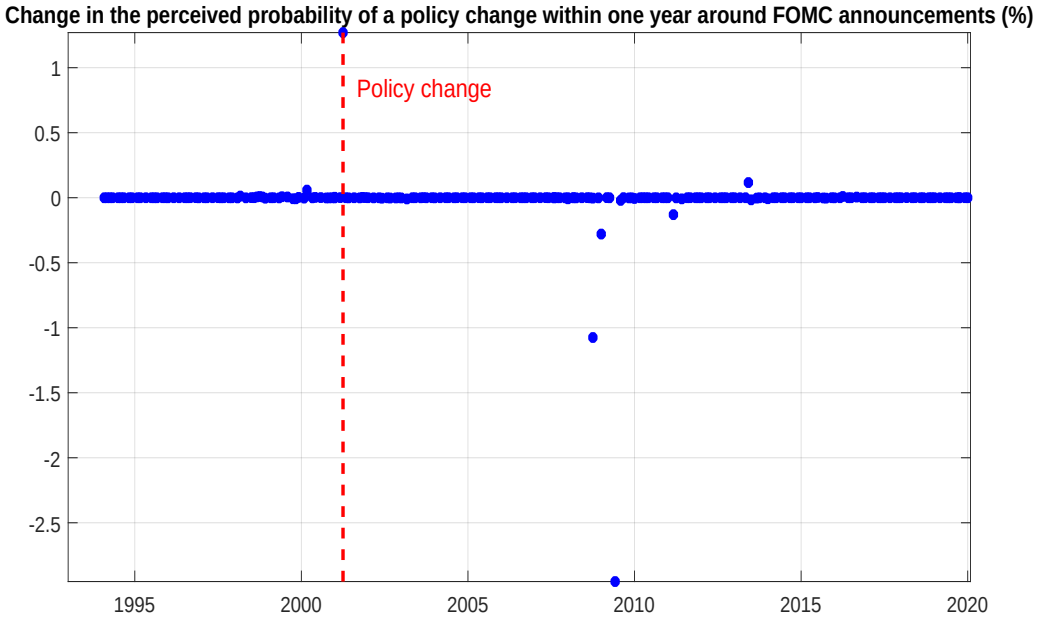
Notes: The figure displays the model-implied series (red, solid line) and the actual series (blue dotted line). The model-implied series are based on smoothed estimates $S_{t|T}$ of S_t, using observations through then end of the sample at date T, and exploit the mapping to observables in (31) using the modal parameter estimates. The difference between the model-implied series and the observed counterpart is attributable to observation error. We allow for observation errors on all variables except for GDP growth, inflation, the FFR, and the SP500 capitalization to GDP ratio. The sample is 1961:M1-2020:M2.

Figure 4: Perceived Probability of Monetary Policy Regime Change



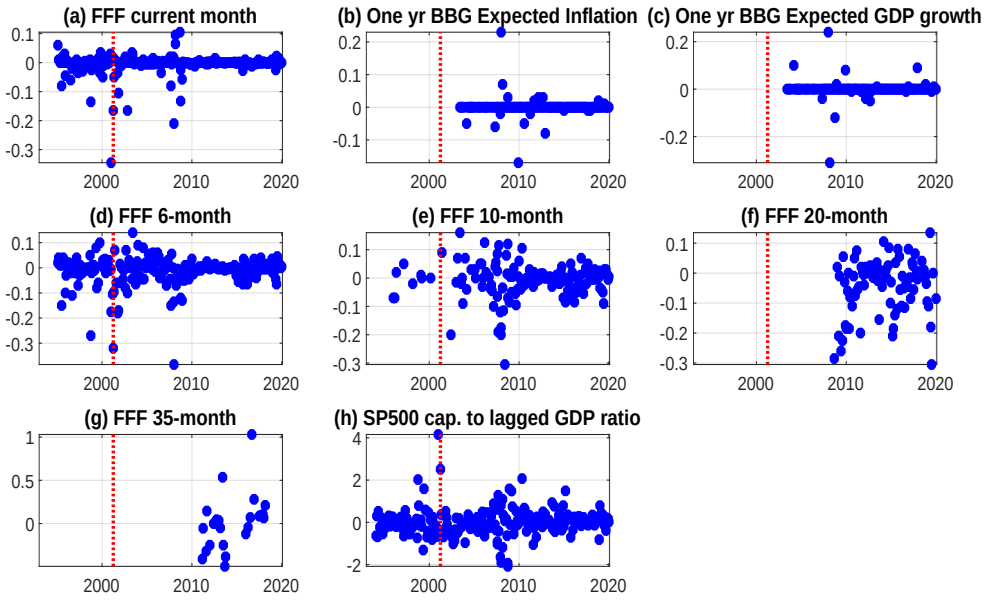
Notes: The figure displays the estimated end-of-month perceived probability investors assign to exiting the current monetary policy rule within one year, computed as the estimated perceived transition probability of being in the Alternative rule at $t + 12$ under each $\xi_t^b = i$, weighted by the smoothed regime probabilities $\Pr(\xi_t^b = i | X_T; \boldsymbol{\theta})$. The sample spans 1961:M1-2020:M2.

Figure 5: Change in the probability of a policy switch around FOMC announcements



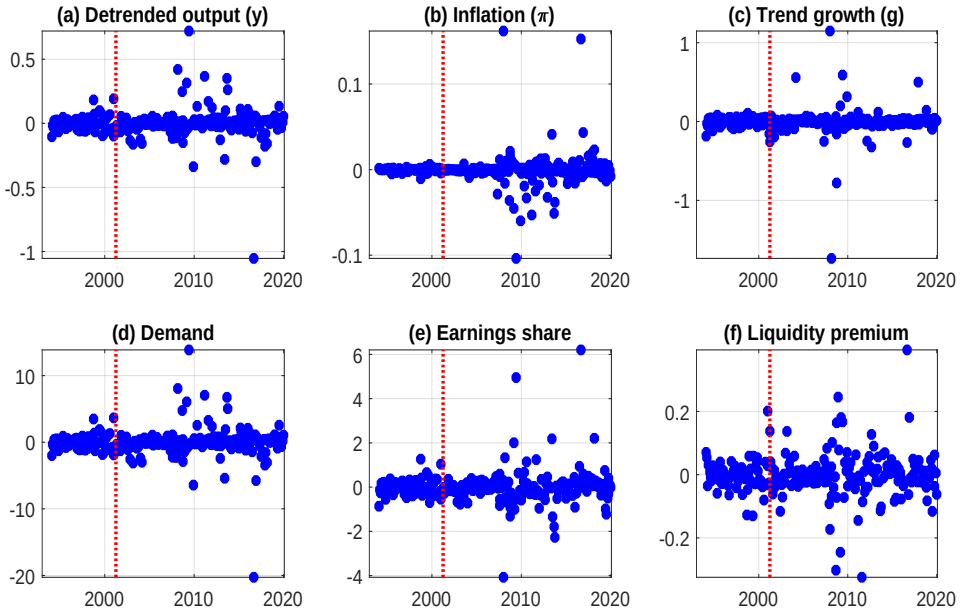
Notes: The figure displays, for each FOMC announcement in our sample, the pre-/post- FOMC announcement log change (10 minutes before/20 minutes after) in the probability that financial markets assign to a switch in the monetary policy rule occurring within one year. The full sample has 220 announcements spanning February 4th, 1994 to February 28th, 2020. The sample reported in the figure is 1993:M1-2020:M2.

Figure 6: HF Changes in Prices and Expectations



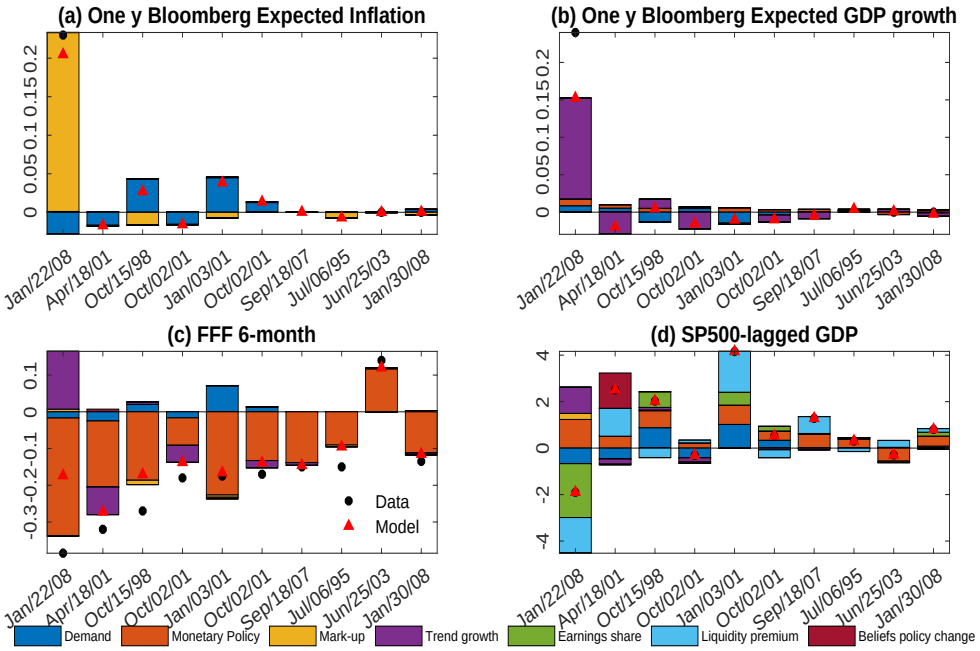
Notes: The figure displays, for each FOMC meeting in our sample, the log change in the observed variables in a short time-window around FOMC meetings. For all but panels (b) and (c), this corresponds to a change measured from 10 minutes before to 20 minutes after an FOMC statement is released. For panels (b) and (c), this corresponds to one day before to one day after the FOMC statement is released. The full sample has 220 FOMC announcements spanning February 4th, 1994 to February 28th, 2020. The sample reported in the figure is 1993:M1-2020:M2.

Figure 7: HF Changes in State Variables



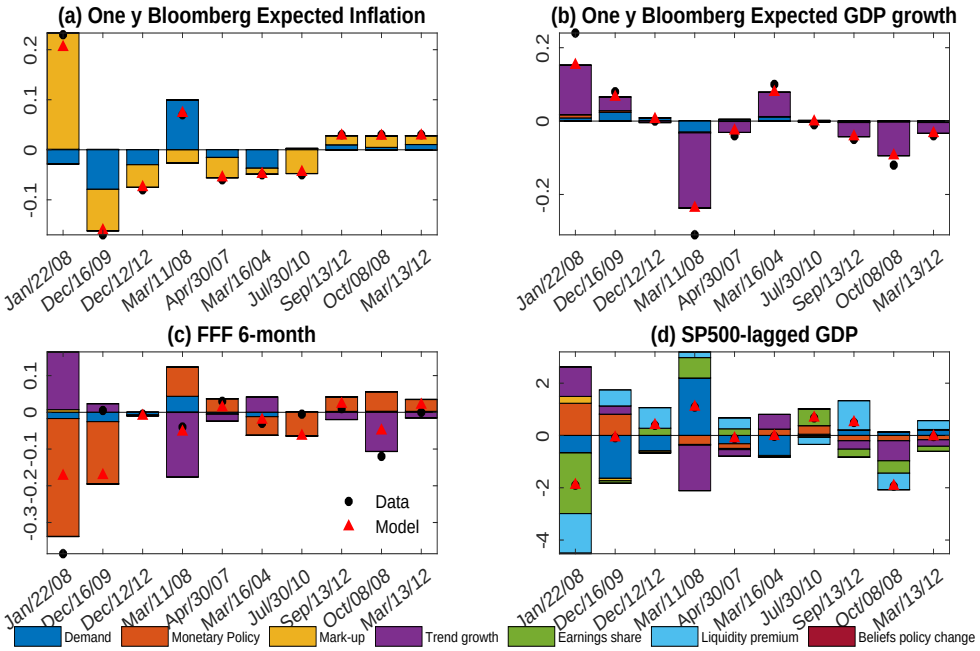
Notes: The figure displays, for each FOMC announcement in our sample, the change in the perceived state of the economy from 10 minutes before to 20 minutes after an FOMC statement is released. The full sample has 220 FOMC announcements spanning February 4th, 1994 to February 28th, 2020. The sample reported in the figure is 1993:M1-2020:M2.

Figure 8: Top Ten FOMC: 6-month FFF rate



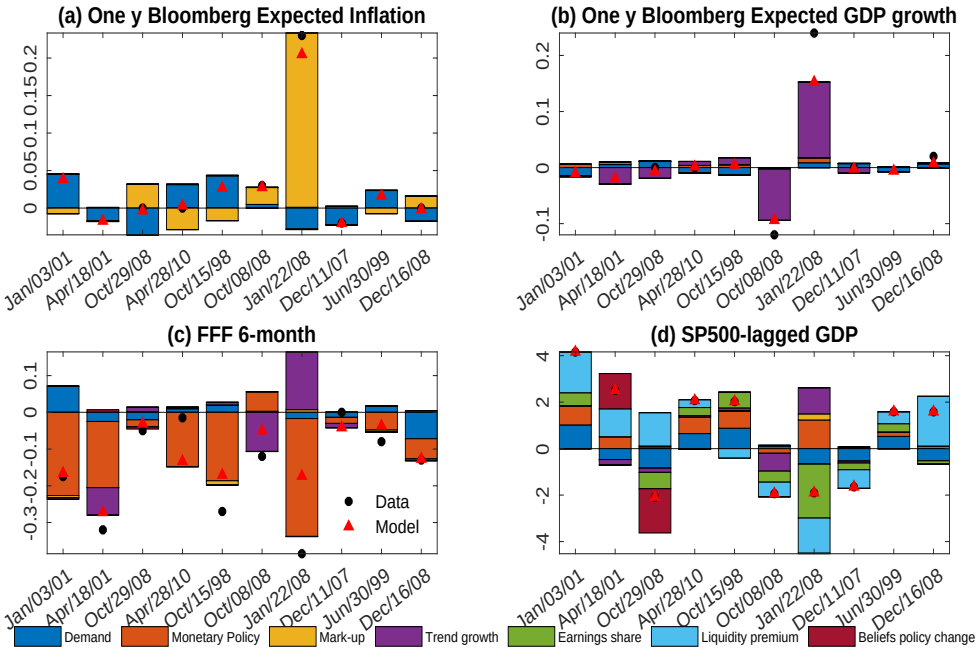
Notes: The figure reports the decomposition of movements in Bloomberg expected inflation, Bloomberg expected GDP growth, the 6-month FFF rates, and the stock market attributable to revisions in the perceived shocks hitting the economy and in the belief regimes for the 10 most relevant FOMC announcements based on changes in the 6-month FFF rate. For panel (d), because we do not have measurement error in the equations for the SP500 to lagged GDP ratio, the black dot (data) and the red triangles (estimation) lie on top of each other, so the black dot is obscured. The sample is 1961:M1-2020:M2.

Figure 9: Top Ten FOMC: Bloomberg Expected Inflation



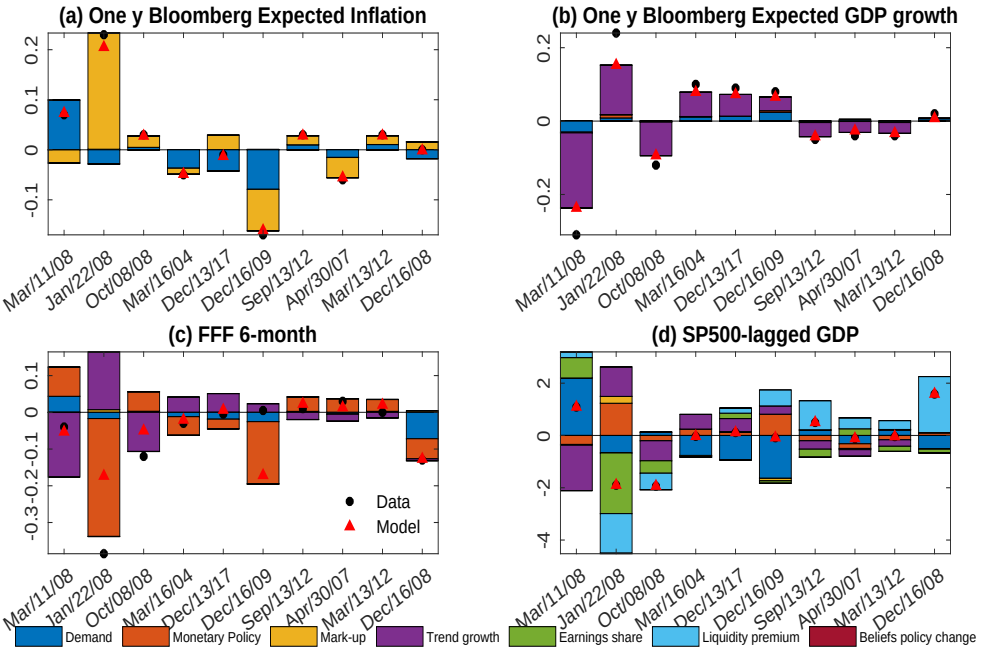
Notes: See Figure 8. The figure reports the decomposition of movements in Bloomberg expected inflation, Bloomberg expected GDP growth, the 6-month FFF rates, and the stock market attributable to revisions in the perceived shocks hitting the economy and in the belief regimes for the 10 most relevant FOMC announcements based on changes in the Bloomberg one-year inflation expectations. The sample is 1961:M1-2020:M2.

Figure 10: Top Ten FOMC: SP-to-GDP ratio



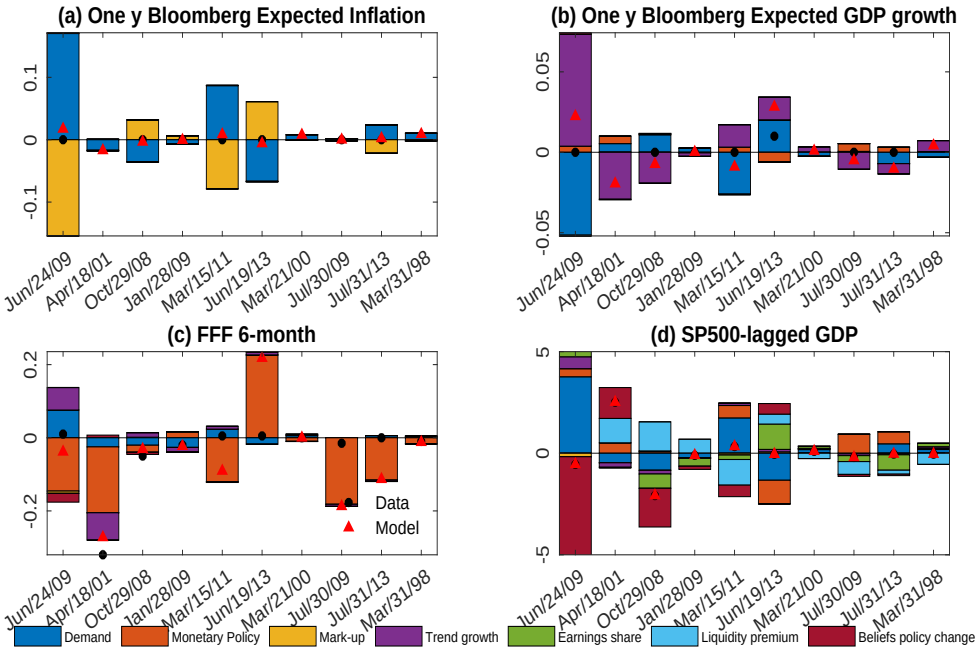
Notes: See Figure 8. The figure reports the decomposition of movements in Bloomberg expected inflation, Bloomberg expected GDP growth, the 6-month FFF rates, and the stock market attributable to revisions in the perceived shocks hitting the economy and in the belief regimes for the 10 most relevant FOMC announcements based on changes in the SP500-lagged GDP ratio. The sample is 1961:M1-2020:M2.

Figure 11: Top Ten FOMC: Bloomberg Expected GDP growth



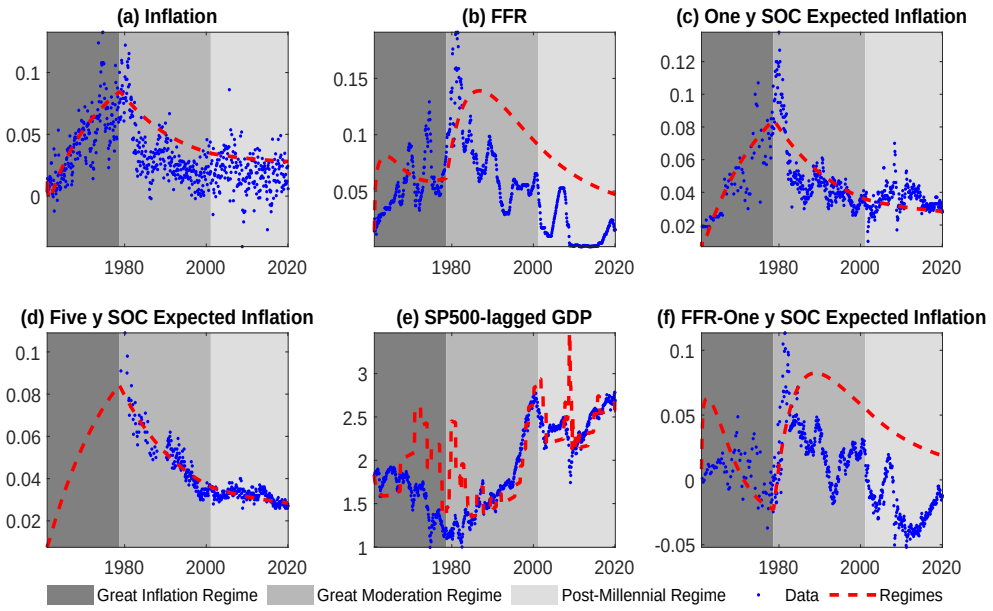
Notes: See Figure 8. The figure reports the decomposition of movements in Bloomberg expected inflation, Bloomberg expected GDP growth, the 6-month FFF rates, and the stock market attributable to revisions in the perceived shocks hitting the economy and in the belief regimes for the 10 most relevant FOMC announcements based on changes in the Bloomberg one-year GDP growth expectations. The sample is 1961:M1-2020:M2.

Figure 12: Top Ten FOMC: Probability of Exiting Policy Rule over the Next Year



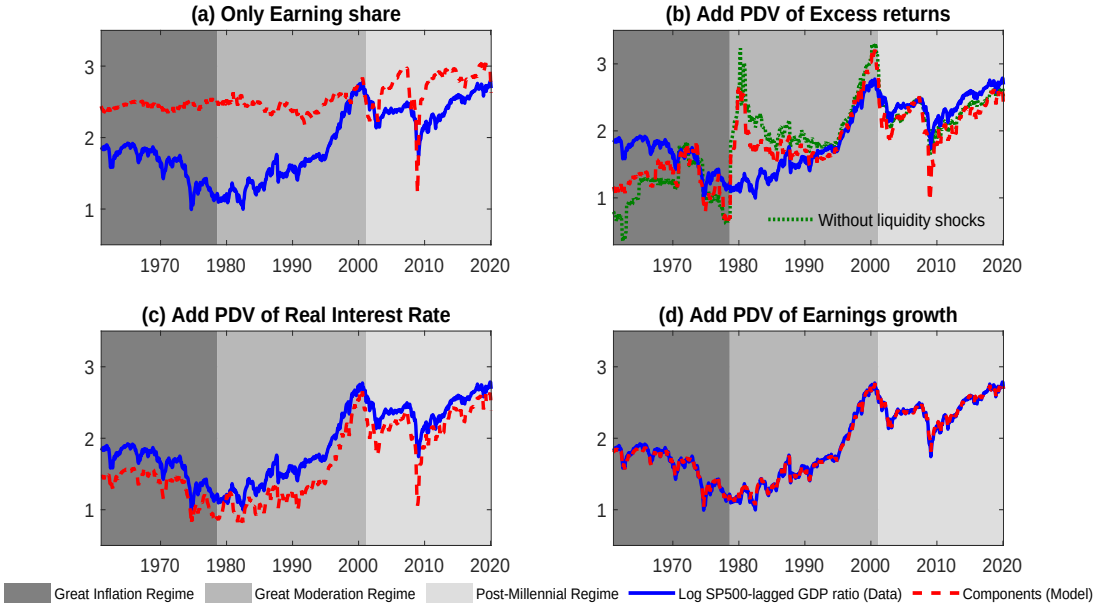
Notes: See Figure 8. The figure reports the decomposition of movements in Bloomberg expected inflation, Bloomberg expected GDP growth, the 6-month FFF rates, and the stock market attributable to revisions in the perceived shocks hitting the economy and in the belief regimes for the 10 most relevant FOMC announcements based on changes in the beliefs about the probability of exiting the policy rule over the next 12 months. The sample is 1961:M1-2020:M2.

Figure 13: Effects of Monetary Policy and Belief Regimes



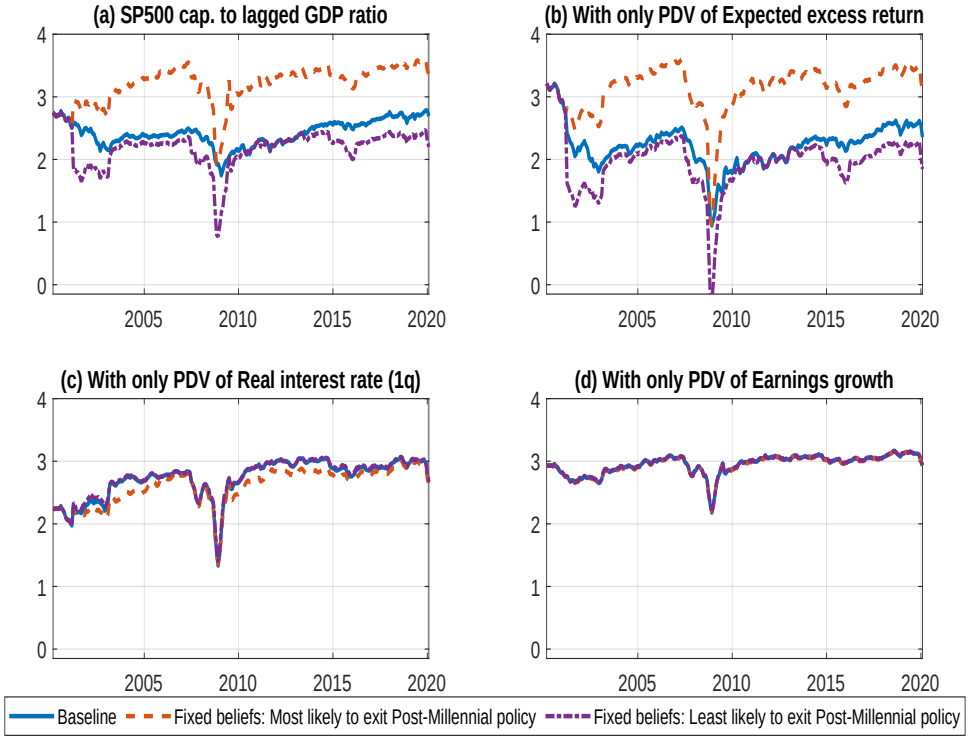
Notes: The figure displays the contribution of changes in policy regimes and belief regimes combined (dashed line). The blue (dotted) lines represent the data on each series. The red (dashed) lines show the component of the series fluctuations attributable solely to realized regime changes in the policy rule and investor beliefs about shifts in the rule. The sample spans 1961:M1 - 2020:M2.

Figure 14: SP500-to-GDP decomposition



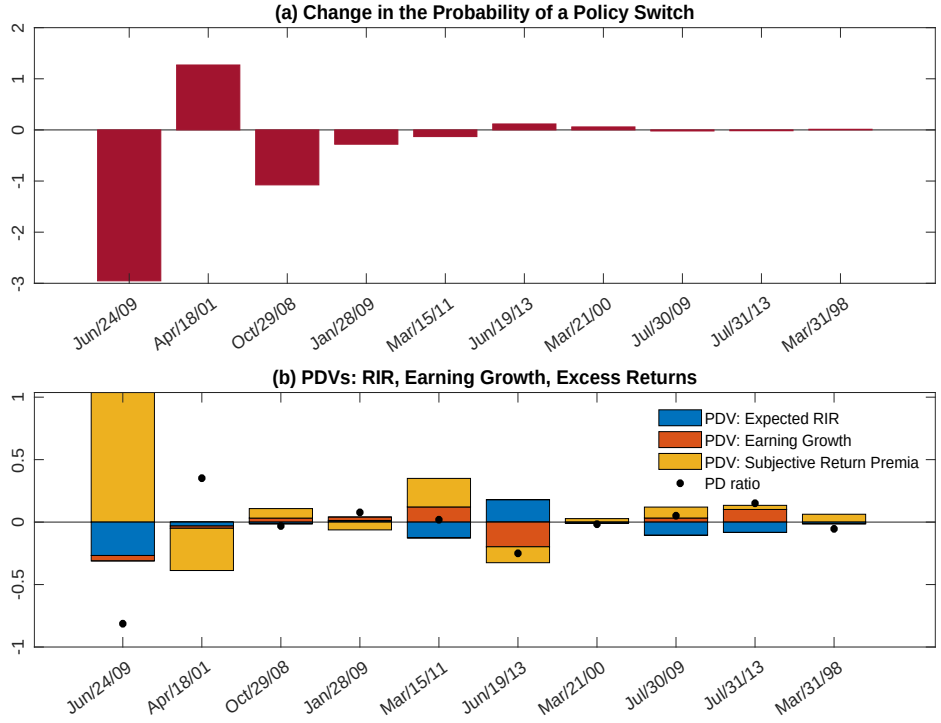
Notes: The figure displays a decomposition of the log SP500-to-lagged GDP ratio. The blue (solid) line represents the data. The dashed (red) lines represent component in the model. The log ratio in the model may be decomposed as $pgdp_t = ey_t + pdv_t(\Delta d) - pdv_t(r^{ex}) - pdv_t(rir)$. Panel (a) plots $pgdp_t$ along with ey_t. Panel (b) plots $pgdp_t$ with $ey_t - pdv_t(r^{ex})$. Panel (c) plots $pgdp_t$ with $ey_t - pdv_t(r^{ex}) - pdv_t(rir)$. Panel (d) plots $pgdp_t$ in the data along with $ey_t + pdv_t(\Delta d) - pdv_t(r^{ex}) - pdv_t(rir)$. The sample spans 1961:M1 - 2020:M2

Figure 15: Counterfactual simulations: The Post-Millennial period



Notes: The figure displays counterfactual simulations for the post-Millennial period. The red (dashed) line corresponds to a counterfactual simulation in which agents' beliefs are set assuming that the $(B+1)$-dimensional belief regime probability vector $\pi_{t|T}$ is replaced by a counterfactual regime probability vector equal to $(1, ..., 0, 0)\prime$ at each t. The purple (dashed-dotted) line corresponds to a counterfactual simulation in which agents' beliefs are set assuming that $\pi_{t|T}$ is replaced by a counterfactual regime probability vector equal to $(0, ..., 1, 0)\prime$ at each t. Panel (a) plots the model implications for the price-lagged output ratio $pgdp_t$. This series perfectly matches our observed series for the SP500-lagged GDP ratio. Panel (b) plots $pgdp_{r^{ex},t}$. Panel (c) plots $pgdp_{rir,t}$. Panel (d) plots $pgdp_{\Delta d,t}$. The sample for the counterfactual spans 2000:M3 to 2020:M2.

Figure 16: Jumps in risk perceptions, short rates, and earnings expectations



Notes: The table reports jumps in subjective expectations of risk, future short rates, and future earnings growth within tight windows around an FOMC announcement. Panel (a) shows the pre-/post-FOMC announcement change (10 minutes before/20 minutes after) in the perceived probability that financial markets assign to a switch in the monetary policy rule occurring within one year, for the 10 most quantitatively important FOMC announcements based on changes in investor beliefs about the future conduct of monetary policy. Panel (b) shows a decomposition of the model's fluctuations in the log price-payout ratio $pd = pdv_t(\Delta d) - pdv_t(r^{ex}) - pdv_t(rir)$ in 30 minute windows around these 10 announcements that are driven by subjective equity risk premium variation, as measured by $pdv_t(r^{ex})$ (yellow bar), subjective expected future real interest rate fluctuations, as measured by $pdv_t(RIR)$ (blue bar), and subjective expected earnings growth, as measured by $pdv_t(\Delta d)$ (red bar). PD ratio is $pdv_t(\Delta d) - pdv_t(r^{ex}) - pdv_t(rir)$. The sample is 1961:M1-2020:M2.

Online Appendix

Table A.1: **Other Parameters**

Parameter	Mode	Parameter	Mode	Parameter	Mode	Parameter	Mode
σ	0.0650	ϕ	0.7436	ρ_k	0.9980	scale Baa	0.3998
ϱ_1	0.5372	$\bar{r}$	0.0000	$\lambda_{k,y}$	26.9629	σ_f	23.4733
β	0.7161	γ^T	0.0051	ρ_{lp}	0.8407	σ_i	0.0331
κ_1	0.0036	$e^{\bar{k}}$	0.0507	ϱ_2	0.2338	σ_μ	0.1379
γ	0.0001	σ_p	5.8542	ϱ_3	0.1887	σ_k	6.2614
ρ_g	0.0914	β_p	0.9936	$\lambda_{k,2}$	10.7499	σ_{lp}	0.5699
κ_0	0.0026	$\overline{lp}$	-0.0130	p_s	0.9876	σ_g	1.7200
β_a	0.3905	$\lambda_{\pi,1}$	0.4244	mean beta be1	0.9286		
γ_π	0.0000	$\lambda_{\pi,2}$	0.3139	std beta be1	0.1090		
ρ_f	0.5010	σ_2	0.0383	int Baa	0.0140		

Note: For each realized policy regime, the table reports the posterior mode values of the parameters for the current and alternative policy rules.

Table A.2: FED Announcements

Date	**Day**	**Hour**	**Event**	**Information**
			Top Ten FOMC: 6-month FFF rate	
1/22/2008	Tuesday	8:30 AM	Conference Call	• The Federal Open Market Committee has decided to lower its target for the federal funds rate 75 basis points to 3-1/2 percent. • The Committee took this action in view of a weakening of the economic outlook and increasing downside risks to growth • The Committee expects inflation to moderate in coming quarters, but it will be necessary to continue to monitor inflation developments carefully. Appreciable downside risks to growth remain.
4/18/2001	Wednesday	10:00 AM	Conference Call	• The Federal Open Market Committee decided today to lower its target for the federal funds rate by 50 basis points to 4-1/2 percent. • Capital investment has continued to soften and the persistent erosion in current and expected profitability, in combination with rising uncertainty about the business outlook, seems poised to dampen capital spending going forward.

Table A.2: FED Announcements (Cont'd)

Date	Day	Hour	Event	Information
10/15/1998	Thursday	3:15 PM	Conference Call	• The Board of Governors approved a reduction in the discount rate by 25 basis points from 5 percent to 4-3/4 percent. • The federal funds rate is expected to fall 25 basis points from around 5-1/4 percent to around 5 percent. • Growing caution by lenders and unsettled conditions in financial markets more generally are likely to be restraining aggregate demand in the future. Against this backdrop, further easing of the stance of monetary policy was judged to be warranted to sustain economic growth in the context of contained inflation.
1/3/2001	Wednesday	1:15 PM	Conference Call	• The Federal Open Market Committee decided today to lower its target for the federal funds rate by 50 basis points to 6 percent. • These actions were taken in light of further weakening of sales and production, and in the context of lower consumer confidence...Moreover, inflation pressures remain contained. Nonetheless, to date there is little evidence to suggest that longer-term advances in technology and associated gains in productivity are abating.
10/2/2001	Tuesday	1:15 PM	FOMC Meeting	• The Federal Open Market Committee decided today to lower its target for the federal funds rate by 50 basis points to 2-1/2 percent • The terrorist attacks have significantly heightened uncertainty in an economy that was already weak...Nonetheless, the long-term prospects for productivity growth and the economy remain favorable and should become evident once the unusual forces restraining demand abate. • The Committee continues to believe that...the risks are weighted mainly toward conditions that may generate economic weakness in the foreseeable future.
9/18/2007	Tuesday	2:15 PM	FOMC Meeting	• The Federal Open Market Committee decided today to lower its target for the federal funds rate 50 basis points to 4-3/4 percent. • Economic growth was moderate during the first half of the year, but the tightening of credit conditions has the potential to intensify the housing correction and to restrain economic growth more generally.
7/6/1995	Thursday	2:15 PM	FOMC Meeting	• Today's action will be reflected in a 25 basis point decline in the federal funds rate from about 6 percent to about 5-3/4 percent. • As a result of the monetary tightening initiated in early 1994, inflationary pressures have receded enough to accommodate a modest adjustment in monetary conditions.

Table A.2: FED Announcements (Cont'd)

Date	Day	Hour	Event	Information
6/25/2003	Wednesday	1:30 PM	FOMC Meeting	• The Federal Open Market Committee decided today to lower its target for the federal funds rate by 25 basis points to 1 percent. • Recent signs point to a firming in spending, markedly improved financial conditions, and labor and product markets that are stabilizing...With inflationary expectations subdued, the Committee judged that a slightly more expansive monetary policy would add further support for an economy which it expects to improve over time. • The Committee perceives that the upside and downside risks to the attainment of sustainable growth for the next few quarters are roughly equal.
1/30/2008	Wednesday	2:15 PM	FOMC Meeting	• The Federal Open Market Committee decided today to lower its target for the federal funds rate 50 basis points to 3 percent. • TodayŠs policy action, combined with those taken earlier, should help to promote moderate growth over time and to mitigate the risks to economic activity. However, downside risks to growth remain. The Committee will continue to assess the effects of financial and other developments on economic prospects and will act in a timely manner as needed to address those risks.

Table A.2: FED Announcements (Cont'd)

Date	Day	Hour	Event	Information
Top Ten FOMC: BBG Expected Inflation				
1/22/2008	Tuesday	8:30 AM	Conference Call	• The Federal Open Market Committee has decided to lower its target for the federal funds rate 75 basis points to 3-1/2 percent. • The Committee took this action in view of a weakening of the economic outlook and increasing downside risks to growth • The Committee expects inflation to moderate in coming quarters, but it will be necessary to continue to monitor inflation developments carefully. Appreciable downside risks to growth remain.
12/16/2009	Wednesday	2:15 PM	FOMC Meeting	• The Committee will maintain the target range for the federal funds rate at 0 to 1/4 percent and continues to anticipate that economic conditions...are likely to warrant exceptionally low levels of the federal funds rate for an extended period. • The Federal Reserve is in the process of purchasing $1.25 trillion of agency mortgage-backed securities and about $175 billion of agency debt...the Committee is gradually slowing the pace of these purchases, and it anticipates that these transactions will be executed by the end of the first quarter of 2010. • In light of ongoing improvements in the functioning of financial markets, the Committee and the Board of Governors anticipate that most of the Federal ReserveŠs special liquidity facilities will expire on February 1, 2010, consistent with the Federal ReserveŠs announcement of June 25, 2009.
12/12/2012	Wednesday	12:30 PM	FOMC Meeting	• The Committee will continue purchasing additional agency mortgage-backed securities at a pace of $40 billion per month. The Committee also will purchase longer-term Treasury securities after its program to extend the average maturity of its holdings of Treasury securities is completed at the end of the year, initially at a pace of $45 billion per month. • To support continued progress toward maximum employment and price stability, the Committee expects that a highly accommodative stance of monetary policy will remain appropriate for a considerable time after the asset purchase program ends and the economic recovery strengthens. In particular, the Committee decided to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that this exceptionally low range for the federal funds rate will be appropriate at least as long as the unemployment rate remains above 6-1/2 percent, inflation between one and two years ahead is projected to be no more than a half percentage point above the CommitteeŠs 2 percent longer-run goal, and longer-term inflation expectations continue to be well anchored.

Table A.2: FED Announcements (Cont'd)

Date	Day	Hour	Event	Information
3/11/2008	Tuesday	8:30 AM	Conference Call	• The Federal Reserve announced today an expansion of its securities lending program. Under this new Term Securities Lending Facility (TSLF), the Federal Reserve will lend up to $200 billion of Treasury securities to primary dealers secured for a term of 28 days. • In addition, the Federal Open Market Committee has authorized increases in its existing temporary reciprocal currency arrangements (swap lines) with the European Central Bank (ECB) and the Swiss National Bank (SNB). The FOMC extended the term of these swap lines through September 30, 2008.
5/9/2007	Wednesday	2:15 PM	FOMC Meeting	• Economic growth slowed in the first part of this year and the adjustment in the housing sector is ongoing. Nevertheless, the economy seems likely to expand at a moderate pace over coming quarters. • Core inflation remains somewhat elevated. Although inflation pressures seem likely to moderate over time, the high level of resource utilization has the potential to sustain those pressures.
3/16/2004	Tuesday	1:15 PM	FOMC Meeting	• The Committee perceives the upside and downside risks to the attainment of sustainable growth for the next few quarters are roughly equal...With inflation quite low and resource use slack, the Committee believes that it can be patient in removing its policy accommodation.
8/10/2010	Tuesday	2:15 PM	FOMC Meeting	• Information received since the Federal Open Market Committee met in June indicates that the pace of recovery in output and employment has slowed in recent months. • The Committee will maintain the target range for the federal funds rate at 0 to 1/4 percent and continues to anticipate that economic conditions...are likely to warrant exceptionally low levels of the federal funds rate for an extended period. • To help support the economic recovery in a context of price stability, the Committee will keep constant the Federal Reserve's holdings of securities at their current level by reinvesting principal payments from agency debt and agency mortgage-backed securities in longer-term Treasury securities. The Committee will continue to roll over the Federal Reserve's holdings of Treasury securities as they mature.

Table A.2: FED Announcements (Cont'd)

Date	Day	Hour	Event	Information
9/13/2012	Thursday	12:30 PM	FOMC Meeting	• The Committee agreed today to increase policy accommodation by purchasing additional agency mortgage-backed securities at a pace of $40 billion per month. The Committee also will continue through the end of the year its program to extend the average maturity of its holdings of securities...and it is maintaining its existing policy of reinvesting principal payments from its holdings of agency debt and agency mortgage-backed securities in agency mortgage-backed securities. • If the outlook for the labor market does not improve substantially, the Committee will continue its purchases of agency mortgage-backed securities, undertake additional asset purchases, and employ its other policy tools as appropriate until such improvement is achieved in a context of price stability. • In particular, the Committee also decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015.
10/8/2008	Wednesday	7:00 AM	Conference Call	• The Federal Open Market Committee has decided to lower its target for the federal funds rate 50 basis points to 1-1/2 percent. The Committee took this action in light of evidence pointing to a weakening of economic activity and a reduction in inflationary pressures. • Inflation expectations are diminishing and remain anchored to price stability. The recent intensification of the financial crisis has augmented the downside risks to growth and thus has diminished further the upside risks to price stability.
3/13/2012	Tuesday	2:15 PM	FOMC Meeting	• The Committee decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that economic conditions...are likely to warrant exceptionally low levels for the federal funds rate at least through late 2014. • The Committee expects moderate economic growth over coming quarters and consequently anticipates that the unemployment rate will decline gradually toward levels that the Committee judges to be consistent with its dual mandate. • The Committee also decided to continue its program to extend the average maturity of its holdings of securities as announced in September. The Committee is maintaining its existing policies of reinvesting principal payments from its holdings of agency debt and agency mortgage-backed securities in agency mortgage-backed securities and of rolling over maturing Treasury securities at auction.

Table A.2: FED Announcements (Cont'd)

Date	Day	Hour	Event	Information
Top Ten FOMC: SP-to-GDP ratio				
1/22/2008	Tuesday	8:30 AM	Conference Call	• The Federal Open Market Committee has decided to lower its target for the federal funds rate 75 basis points to 3-1/2 percent. • The Committee took this action in view of a weakening of the economic outlook and increasing downside risks to growth • The Committee expects inflation to moderate in coming quarters, but it will be necessary to continue to monitor inflation developments carefully. Appreciable downside risks to growth remain.
4/28/2010	Wednesday	2:15 PM	FOMC Meeting	• The Committee will maintain the target range for the federal funds rate at 0 to 1/4 percent and continues to anticipate that economic conditions...are likely to warrant exceptionally low levels of the federal funds rate for an extended period. • The Federal Reserve has closed all but one of the special liquidity facilities...The only remaining such program, the Term Asset-Backed Securities Loan Facility, is scheduled to close on June 30 for loans backed by new-issue commercial mortgage-backed securities; it closed on March 31 for loans backed by all other types of collateral.
3/11/2008	Tuesday	8:30 AM	Conference Call	• The Federal Reserve announced today an expansion of its securities lending program. Under this new Term Securities Lending Facility (TSLF), the Federal Reserve will lend up to $200 billion of Treasury securities to primary dealers secured for a term of 28 days. • In addition, the Federal Open Market Committee has authorized increases in its existing temporary reciprocal currency arrangements (swap lines) with the European Central Bank (ECB) and the Swiss National Bank (SNB). The FOMC extended the term of these swap lines through September 30, 2008.
10/8/2008	Wednesday	7:00 AM	Conference Call	• The Federal Open Market Committee has decided to lower its target for the federal funds rate 50 basis points to 1-1/2 percent. The Committee took this action in light of evidence pointing to a weakening of economic activity and a reduction in inflationary pressures. • Inflation expectations are diminishing and remain anchored to price stability. The recent intensification of the financial crisis has augmented the downside risks to growth and thus has diminished further the upside risks to price stability.

Table A.2: FED Announcements (Cont'd)

Date	Day	Hour	Event	Information
12/16/2008	Tuesday	2:15 PM	FOMC Meeting	• The Federal Open Market Committee decided today to establish a target range for the federal funds rate of 0 to 1/4 percent...The Committee anticipates that weak economic conditions are likely to warrant exceptionally low levels of the federal funds rate for some time. • Since the Committee's last meeting, labor market conditions have deteriorated...Financial markets remain quite strained and credit conditions tight. Overall, the outlook for economic activity has weakened further. • The Committee is also evaluating the potential benefits of purchasing longer-term Treasury securities. Early next year, the Federal Reserve will also implement the Term Asset-Backed Securities Loan Facility to facilitate the extension of credit to households and small businesses.
8/7/2007	Tuesday	2:15 PM	FOMC Meeting	• Economic growth was moderate during the first half of the year...The economy seems likely to continue to expand at a moderate pace over coming quarters, supported by solid growth in employment and incomes and a robust global economy. • Although the downside risks to growth have increased somewhat, the Committee's predominant policy concern remains the risk that inflation will fail to moderate as expected.
10/29/2008	Wednesday	2:15 PM	FOMC Meeting	• The Federal Open Market Committee decided today to lower its target for the federal funds rate 50 basis points to 1 percent. • In light of the declines in the prices of energy and other commodities and the weaker prospects for economic activity, the Committee expects inflation to moderate in coming quarters to levels consistent with price stability.
8/9/2011	Tuesday	2:15 PM	FOMC Meeting	• The Committee now expects a somewhat slower pace of recovery over coming quarters than it did at the time of the previous meeting...Moreover, downside risks to the economic outlook have increased. The Committee also anticipates that inflation will settle, over coming quarters, at levels at or below those consistent with the Committee's dual mandate. • The Committee currently anticipates that economic conditions...are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013. The Committee also will maintain its existing policy of reinvesting principal payments from its securities holdings.

Table A.2: FED Announcements (Cont'd)

Date	Day	Hour	Event	Information
12/19/2018	Wednesday	2:00 PM	FOMC Meeting	• The Committee judges that some further gradual increases in the target range for the federal funds rate will be consistent with sustained expansion of economic activity, strong labor market conditions, and inflation near the Committee's symmetric 2 percent objective over the medium term. • In view of realized and expected labor market conditions and inflation, the Committee decided to raise the target range for the federal funds rate to 2-1/4 to 2-1/2 percent.
2/5/1997	Wednesday	12:00 PM	FOMC Meeting	• The Committee at this meeting established ranges for growth of M2 and M3 of 1 to 5 percent and 2 to 6 percent respectively, measured from the fourth quarter of 1996 to the fourth quarter of 1997. The monitoring range for growth of total domestic nonfinancial debt was set at 3 to 7 percent for the year. • In the context of the Committee's long-run objectives for price stability and sustainable economic growth...somewhat greater reserve restraint would or slightly lesser reserve restraint might be acceptable in the intermeeting period.
Top Ten FOMC: BBG Expected GDP Growth				
3/11/2008	Tuesday	8:30 AM	Conference Call	• The Federal Reserve announced today an expansion of its securities lending program. Under this new Term Securities Lending Facility (TSLF), the Federal Reserve will lend up to $200 billion of Treasury securities to primary dealers secured for a term of 28 days. • In addition, the Federal Open Market Committee has authorized increases in its existing temporary reciprocal currency arrangements (swap lines) with the European Central Bank (ECB) and the Swiss National Bank (SNB). The FOMC extended the term of these swap lines through September 30, 2008.
1/22/2008	Tuesday	8:30 AM	Conference Call	• The Federal Open Market Committee has decided to lower its target for the federal funds rate 75 basis points to 3-1/2 percent. • The Committee took this action in view of a weakening of the economic outlook and increasing downside risks to growth • The Committee expects inflation to moderate in coming quarters, but it will be necessary to continue to monitor inflation developments carefully. Appreciable downside risks to growth remain.
10/8/2008	Wednesday	7:00 AM	Conference Call	• The Federal Open Market Committee has decided to lower its target for the federal funds rate 50 basis points to 1-1/2 percent. The Committee took this action in light of evidence pointing to a weakening of economic activity and a reduction in inflationary pressures. • Inflation expectations are diminishing and remain anchored to price stability. The recent intensification of the financial crisis has augmented the downside risks to growth and thus has diminished further the upside risks to price stability.

Table A.2: FED Announcements (Cont'd)

Date	Day	Hour	Event	Information
3/16/2004	Tuesday	1:15 PM	FOMC Meeting	• The Committee perceives the upside and downside risks to the attainment of sustainable growth for the next few quarters are roughly equal...With inflation quite low and resource use slack, the Committee believes that it can be patient in removing its policy accommodation.
12/13/2017	Wednesday	2:00 PM	FOMC Meeting	• In view of realized and expected labor market conditions and inflation, the Committee decided to raise the target range for the federal funds rate to 1-1/4 to 1-1/2 percent. • The Committee expects that economic conditions will evolve in a manner that will warrant gradual increases in the federal funds rate; the federal funds rate is likely to remain, for some time, below levels that are expected to prevail in the longer run. • Information received since the Federal Open Market Committee met in November indicates that the labor market has continued to strengthen and that economic activity has been rising at a solid rate.
12/16/2009	Wednesday	2:15 PM	FOMC Meeting	• The Committee will maintain the target range for the federal funds rate at 0 to 1/4 percent and continues to anticipate that economic conditions...are likely to warrant exceptionally low levels of the federal funds rate for an extended period. • The Federal Reserve is in the process of purchasing $1.25 trillion of agency mortgage-backed securities and about $175 billion of agency debt...the Committee is gradually slowing the pace of these purchases, and it anticipates that these transactions will be executed by the end of the first quarter of 2010. • In light of ongoing improvements in the functioning of financial markets, the Committee and the Board of Governors anticipate that most of the Federal ReserveŠs special liquidity facilities will expire on February 1, 2010, consistent with the Federal ReserveŠs announcement of June 25, 2009.
9/13/2012	Thursday	12:30 PM	FOMC Meeting	• The Committee agreed today to increase policy accommodation by purchasing additional agency mortgage-backed securities at a pace of $40 billion per month. The Committee also will continue through the end of the year its program to extend the average maturity of its holdings of securities...and it is maintaining its existing policy of reinvesting principal payments from its holdings of agency debt and agency mortgage-backed securities in agency mortgage-backed securities. • If the outlook for the labor market does not improve substantially, the Committee will continue its purchases of agency mortgage-backed securities, undertake additional asset purchases, and employ its other policy tools as appropriate until such improvement is achieved in a context of price stability. • In particular, the Committee also decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015.

Table A.2: FED Announcements (Cont'd)

Date	Day	Hour	Event	Information
5/9/2007	Wednesday	2:15 PM	FOMC Meeting	• Economic growth slowed in the first part of this year and the adjustment in the housing sector is ongoing. Nevertheless, the economy seems likely to expand at a moderate pace over coming quarters. • Core inflation remains somewhat elevated. Although inflation pressures seem likely to moderate over time, the high level of resource utilization has the potential to sustain those pressures.
3/13/2012	Tuesday	2:15 PM	FOMC Meeting	• The Committee decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that economic conditions...are likely to warrant exceptionally low levels for the federal funds rate at least through late 2014. • The Committee expects moderate economic growth over coming quarters and consequently anticipates that the unemployment rate will decline gradually toward levels that the Committee judges to be consistent with its dual mandate. • The Committee also decided to continue its program to extend the average maturity of its holdings of securities as announced in September. The Committee is maintaining its existing policies of reinvesting principal payments from its holdings of agency debt and agency mortgage-backed securities in agency mortgage-backed securities and of rolling over maturing Treasury securities at auction.
12/16/2008	Tuesday	2:15 PM	FOMC Meeting	• The Federal Open Market Committee decided today to establish a target range for the federal funds rate of 0 to 1/4 percent...The Committee anticipates that weak economic conditions are likely to warrant exceptionally low levels of the federal funds rate for some time. • Since the Committee's last meeting, labor market conditions have deteriorated...Financial markets remain quite strained and credit conditions tight. Overall, the outlook for economic activity has weakened further. • The Committee is also evaluating the potential benefits of purchasing longer-term Treasury securities. Early next year, the Federal Reserve will also implement the Term Asset-Backed Securities Loan Facility to facilitate the extension of credit to households and small businesses.

Data

Real GDP

The real Gross Domestic Product is obtained from the US Bureau of Economic Analysis. It is in billions of chained 2012 dollars, quarterly frequency, seasonally adjusted, and at annual rate. The source is from Bureau of Economic Analysis (BEA code: A191RX). The sample spans 1959:Q1 to 2021:Q2. The series was interpolated to monthly frequency using the method in

Stock and Watson (2010). The quarterly series was downloaded on August 20th, 2021.

GDP price deflator

The Gross Domestic Product: implicit price deflator is obtained from the US Bureau of Economic Analysis. Index base is 2012=100, quarterly frequency, and seasonally adjusted. The source is from Bureau of Economic Analysis (BEA code: A191RD). The sample spans 1959:Q1 to 2021:Q2. The series was interpolated to monthly frequency using the method in Stock and Watson (2010). The quarterly series was downloaded on August 20th, 2021.

Capital Share

The capital share is defined as 1 - LS where LS is the nonfarm business sector labor share. Labor share is measured as labor compensation divided by value added. The labor compensation is defined as Compensation of Employees - Government Wages and Salaries- Compensation of Employees of Nonprofit Institutions - Private Compensation (Households) - Farm Compensation of Employees - Housing Compensation of Employees - Imputed Labor Compensation of Self-Employed. The value added is defined as Compensation of Employees + Corporate Profits + Rental Income + Net Interest Income + Proprietors' Income + Indirect Taxes Less Subsidies + Depreciation. The quarterly, seasonally adjusted data spans from 1959:Q1 to 2021:Q2. The source is from Bureau of Labor Statistics. The labor share index is available at http://research.stlouisfed.org/fred2/series/PRS85006173 and the quarterly LS level can be found from the dataset at https://www.bls.gov/lpc/special_requests/msp_dataset.zip. The series was interpolated to monthly frequency using the method in Stock and Watson (2010). The quarterly series was downloaded on September 21th, 2021.

Federal funds rate (FFR)

The Effective Federal Funds Rate is obtained from the Board of Governors of the Federal Reserve System. It is in percentage points, quarterly frequency, and not seasonally adjusted. The sample spans 1960:02 to 2021:06. The series was downloaded on August 20th, 2021

SP500 and SP500 futures

We use tick-by-tick data on SP500 index obtained from tickdata.com. The series was downloaded on September 22th, 2021 from `https://www.tickdata.com/` .We create the minutely data using the close price within each minute. Our sample spans January 2nd 1986 to September 17th, 2021. Within trading hours, we construct SP500 market capitalization by multiplying the SP500 index by the SP500 Divisor. The SP500 Divisor is available at the URL: `https://ycharts.com/indicators/sp\_500\_divisor`. We supplement SP500 index using SP500 futures for events that occur in off-market hours. We use the current-quarter contract futures.

We purchased the SP500 futures from CME group at URL: https://datamine.cmegroup.com/. The SP500 futures data were downloaded on October 6, 2021.

SP500 Earnings and Market Capitalization

We obtained monthly S&P earnings from multpl.com at URL: `https://www.multpl.com/shiller-pe`. For S&P market cap, we obtain the series from Ycharts.com available at `https://ycharts.com/indicators/sp%\_500\_market\_cap`. Both series were downloaded on December 22nd, 2021.

Baa Spread, 20-yr T-bond, Long-term US government securities

We obtained daily Moody's Baa Corporate Bond Yield from FRED (series ID: DBAA) at URL: `https://fred.stlouisfed.org/series/BAA`, US Treasury securities at 20-year constant maturity from FRED (series ID: DGS20) at URL: `https://fred.stlouisfed.org/series/DGS20`, and long-term US government securities from FRED (series ID: LTGOVTBD) at URL: `https://fred.stlouisfed.org/series/LTGOVTBD`. To construct the long term bond yields, we use LTGOVTBD before 2000 and use DGS20 after 2000. The Baa spread is the difference between the Moody's Corporate bond yield and the 20-year US government yield. The excess bond premium is obtained at URL: `https://www.federalreserve.gov/econresdata/notes/feds-notes/2016/recession-risk-and-the-excess-bond-premium-20160408.html`. All series were downloaded on Feb 21, 2022.

Bloomberg Consensus Inflation and GDP forecasts

We obtain the Bloomberg (BBG) US GDP (id: EXGDUS) and inflation (id: ECPIUS) consensus mean forecast from the Bloomberg Terminal available on a daily basis up to a few days before the release of GDP and inflation data. The Bloomberg (BBG) US consensus forecasts are updated daily (except for weekends and holidays) and reports daily quarter-over-quarter real GDP growth and CPI forecasts from 2003:Q1. These forecasts provide more high-frequency information on the professional outlook for economic indicators. Both forecast series were downloaded on October 21, 2021.

Livingston Survey Inflation Forecast

We obtained the Livingston Survey mean 1-year and 10-year CPI inflation forecast from the Federal Reserve Bank of Philadelphia, URL: https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/livingston-historical-data. The forecast series were downloaded on September 20, 2021.

Michigan Survey of Consumers Inflation Forecasts

We construct MS forecasts of annual inflation of respondents answering at time t. Each month, the SOC contains approximately 50 core questions, and a minimum of 500 interviews are conducted by telephone over the course of the entire month, each month. We use two questions from the monthly survey for which the time series begins in January 1978.

1. Annual CPI inflation: To get a point forecast, we combine the information in the survey responses to questions A12 and A12b.

   - Question A12 asks (emphasis in original): *During the next 12 months, do you think that prices in general will go up, or go down, or stay where they are now?*
   - A12b asks (emphasis in original): *By about what percent do you expect prices to go (up/down) on the average, during the next 12 months?*

2. Long-run CPI inflation: To get a point forecast, we combine the information in the survey responses to questions A13 and A13b.

   - Question A13 asks (emphasis in original): *What about the outlook for prices over the next 5 to 10 years? Do you think prices will be higher, about the same, or lower, 5 to 10 years from now?*
   - A13b asks (emphasis in original): *By about what percent per year do you expect prices to go (up/down) on the average, during the next 5 to 10 years?*

All series were downloaded on September 17th, 2021.

Bluechip Inflation and GDP Forecasts

We obtain Blue Chip expectation data from Blue Chip Financial Forecasts. The surveys are conducted each month by sending out surveys to forecasters in around 50 financial firms such as Bank of America, Goldman Sachs & Co., Swiss Re, Loomis, Sayles & Company, and J.P. Morgan Chase. The participants are surveyed around the 25th of each month and the results published a few days later on the 1st of the following month. The forecasters are asked to forecast the average of the level of U.S. interest rates over a particular calendar quarter, e.g. the federal funds rate and the set of H.15 Constant Maturity Treasuries (CMT) of the following maturities: 3-month, 6-month, 1-year, 2-year, 5-year and 10-year, and the quarter over quarter percentage changes in Real GDP, the GDP Price Index and the Consumer Price Index, beginning with the current quarter and extending 4 to 5 quarters into the future.

In this study, we look at a subset of the forecasted variables. Specifically, we use the Blue Chip micro data on individual forecasts of the quarter-over-quarter (Q/Q) percentage change in the Real GDP, the GDP Price Index and the CPI, and convert to quarterly observations as explained below.

1. CPI inflation: We use quarter-over-quarter percentage change in the consumer price index, which is defined as

   "Forecasts for the quarter-over-quarter percentage change in the CPI (consumer prices for all urban consumers). Seasonally adjusted, annual rate."

   Quarterly and annual CPI inflation are constructed the same way as for PGDP inflation, except CPI replaces PGDP.

2. For real GDP growth, We use quarter-over-quarter percentage change in the Real GDP, which is defined as

   "Forecasts for the quarter-over-quarter percentage change in the level of chain-weighted real GDP. Seasonally adjusted, annual rate. Prior to 1992, Q/Q % change (SAAR) in real GNP."

The surveys are conducted right before the publication of the newsletter. Each issue is always dated the 1st of the month and the actual survey conducted over a two-day period almost always between 24th and 28th of the month. The major exception is the January issue when the survey is conducted a few days earlier to avoid conflict with the Christmas holiday. Therefore, we assume that the end of the last month (equivalently beginning of current month) is when the forecast is made. For example, for the report in 2008 Feb, we assume that the forecast is made on Feb 1, 2008.

Survey of Professional Forecasters (SPF)

The SPF is conducted each quarter by sending out surveys to professional forecasters, defined as forecasters. The number of surveys sent varies over time, but recent waves sent around 50 surveys each quarter according to officials at the Federal Reserve Bank of Philadelphia. Only forecasters with sufficient academic training and experience as macroeconomic forecasters are eligible to participate. Over the course of our sample, the number of respondents ranges from a minimum of 9, to a maximum of 83, and the mean number of respondents is 37. The surveys are sent out at the end of the first month of each quarter, and they are collected in the second or third week of the middle month of each quarter. Each survey asks respondents to provide nowcasts and quarterly forecasts from one to four quarters ahead for a variety of variables. Specifically, we use the SPF micro data on individual forecasts of the price level, long-run inflation, and real GDP.[1] Below we provide the exact definitions of these variables as well as our method for constructing nowcasts and forecasts of quarterly and annual inflation for each respondent.[2]

The following variables are used on either the right- or left-hand-sides of forecasting models:

[1]Individual forecasts for all variables can be downloaded at https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/historical-data/individual-forecasts.

[2]The SPF documentation file can be found at https://www.philadelphiafed.org/-/media/research-and-data/real-time-center/survey-of-professional-forecasters/spf-documentation.pdf?la=en.

1. Quarterly and annual inflation (1968:Q4 - present): We use survey responses for the level of the GDP price index (PGDP), defined as

   "Forecasts for the quarterly and annual level of the chain-weighted GDP price index. Seasonally adjusted, index, base year varies. 1992-1995, GDP implicit deflator. Prior to 1992, GNP implicit deflator. Annual forecasts are for the annual average of the quarterly levels."

   Since advance BEA estimates of these variables for the current quarter are unavailable at the time SPF respondents turn in their forecasts, four quarter-ahead inflation and GDP growth forecasts are constructed by dividing the forecasted level by the survey respondent-type's nowcast. Let $\mathbb{F}_t^{(i)}[P_{t+h}]$ be forecaster i's prediction of PGDP h quarters ahead and $\mathbb{N}_t^{(i)}[P_t]$ be forecaster i's nowcast of PGDP for the current quarter. Annualized inflation forecasts for forecaster i are

   $$\mathbb{F}_t^{(i)}[\pi_{t+h,t}] = (400/h) \times \ln\left(\frac{\mathbb{F}_t^{(i)}[P_{t+h}]}{\mathbb{N}_t^{(i)}[P_t]}\right),$$

   where $h = 1$ for quarterly inflation and $h = 4$ for annual inflation. Similarly, we construct quarterly and annual nowcasts of inflation as

   $$\mathbb{N}_t^{(i)}[\pi_{t,t-h}] = (400/h) \times \ln\left(\frac{\mathbb{N}_t^{(i)}[P_t]}{P_{t-h}}\right),$$

   where $h = 1$ for quarterly inflation and $h = 4$ for annual inflation, and where P_{t-1} is the BEA's advance estimate of PGDP in the previous quarter observed by the respondent in time t, and P_{t-4} is the BEA's most accurate estimate of PGDP four quarters back. After computing inflation for each survey respondent, we calculate the 5th through the 95th percentiles as well as the average, variance, and skewness of inflation forecasts across respondents.

2. Long-run inflation (1991:Q4 - present): We use survey responses for 10-year-ahead CPI inflation (CPI10), which is defined as

   "Forecasts for the annual average rate of headline CPI inflation over the next 10 years. Seasonally adjusted, annualized percentage points. The "next 10 years" includes the year in which we conducted the survey and the following nine years. Conceptually, the calculation of inflation is one that runs from the fourth quarter of the year before the survey to the fourth quarter of the year that is ten years beyond the survey year, representing a total of 40 quarters or 10 years. The fourth-quarter level is the quarterly average of the underlying monthly levels."

   Only the median response is provided for CPI10, and it is already reported as an inflation rate, so we do not make any adjustments and cannot compute other moments or percentiles.

3. Real GDP growth (1968:Q4 - present): We use the level of real GDP (RGDP), which is defined as

   *"Forecasts for the quarterly and annual level of chain-weighted real GDP. Seasonally adjusted, annual rate, base year varies. 1992-1995, fixed-weighted real GDP. Prior to 1992, fixed-weighted real GNP. Annual forecasts are for the annual average of the quarterly levels. Prior to 1981:Q3, RGDP is computed by using the formula NGDP / PGDP * 100."*

All series were downloaded on September 17th, 2021.

Fed Funds Futures and Eurodollar Futures

We use tick-by-tick data on Fed funds futures (FFF) and Eurodollar futures obtained from the CME Group. Our sample spans January 3, 1995 to June 2, 2020. FFF contracts settle based on the average federal funds rate that prevails over a given calendar month. Fed funds futures are priced at $100 - f_t^{(n)}$, where $f_t^{(n)}$ is the average effective Fed funds rate in month $t+n$ of contract expiry as quoted at the end of month t. Contracts are monthly and expire at month-end, with maturities ranging up to 60 months. For the buyer of the futures contract, the amount of $\left(f_t^{(n)} - r_{t+n}\right) \times \$\text{D}$, where r_{t+n} is the ex post realized value of the federal funds rate for month $t+n$ and \$D is a dollar "deposit", represents the payoff of a zero-cost portfolio.

Eurodollar futures contracts are quarterly, expiring two business days before the third Wednesday in the last month of the quarter. Eurodollar futures are similarly quoted, where $f_t^{(q)}$ is the average 3-month LIBOR in quarter q of contract expiry. Maturities range up to 40 quarters. For both types of contracts, the implied contract rate is recovered by subtracting 100 from the price and multiplying by -1.

Both types of contracts are cleaned following the same procedure following communication with the CME Group. First, trades with zero volume, which indicate a canceled order, are excluded. Floor trades, which do not require a volume on record, are included. Next, trades with a recorded expiry (in YYMM format) of 9900 indicate bad data and are excluded (Only 1390 trades, or less than 0.01% of the raw Fed funds data, have contract delivery dates of 9900). For trades time stamped to the same second, we follow Bianchi, Kind, and Kung (2019) and keep the trade with the lowest sequence number, corresponding to the first trade that second.

Fed funds futures data require additional cleaning. Trade prices were quoted in different units prior to August 2008. To standardize units across our sample, we start by noting that Fed funds futures are priced to the average effective Fed funds rate realized in the contract month. And in our sample, we expect a reasonable effective Fed funds rate to correspond to prices in the 90 to 100 range. As such, we rescale prices to be less than 100 in the pre-August 2008 subsample.[3] After rescaling, a small number of trades still appear to have prices that are far

[3]For trades with prices significantly greater than 100, we repeatedly divide by 10 until prices are in the range of 90 to 100. We exclude all trades otherwise.

away from the effective Fed funds rates at both trade day and contract expiry, along with trades in the immediate transactions. The CME Group could not explain this data issue, so following Bianchi, Kind, and Kung (2019) and others in the high frequency equity literature(Brownlees and Gallo 2006, Barndorff-Nielsen, Hansen, Lunde, and Shephard 2008, Andersen, Bollerslev, and Meddahi 2005), we apply an additional filter to exclude trades with such non-sensible prices. Specifically, for each maturity contract, we only keep trades where

$$|p_t - \overline{p}_t(k, \delta)| < 3\sigma_t(k, \delta) + \gamma,$$

where p_t denotes the trade price (where t corresponds to a second), and $\overline{p}_t(k, \delta)$ and $\sigma_t(k, \delta)$ denote the average price and standard deviation, respectively, centered with $k/2$ observations on each side of t excluding $\delta k/2$ trades with highest price and excluding $\delta k/2$ trades with lowest price. Finally, γ is a positive constant to account for the cases where prices are constant within the window. Our main specification uses $k = 30$, $\delta = 0.05$ and $\gamma = 0.4$, and alternative parameters produce similar results.

High Frequency Changes Around FOMC Meetings

We follow Guraynak, Sack, and Swanson (2005) and Nakamura and Steinsson (2018) among others in constructing high frequency changes around FOMC meetings. Although we do not use these changes directly in the structural model estimation, we constructed these changes as a cross-check on the construction of our high-frequency FFF data around meetings.

First, we compile dates and times of FOMC meetings from 1994 to 2004 from Guraynak, Sack, and Swanson (2005). The dates of the remaining FOMC meetings are collected from the Federal Reserve Board website. The times of statement releases were coalesced in the following priority: the Federal Reserve Board calendar, the Federal Reserve Board minutes, Bloomberg's FOMC page, and the first news article to appear on Bloomberg. We only include scheduled meetings and unscheduled meetings where a statement was released.

Next, we calculate changes in implied futures rates in a tight window around each FOMC statement release. Our main specification uses an inner window of 30 minutes, from 10 minutes before the FOMC announcement to 20 minutes after it, along with an outer window from 12am to noon the next day. Specifically, on the left side of the window, we use the first trade at 10 minutes before the FOMC announcement, or the nearest trade before 10 minutes if there is no trade at 10 minutes exactly, but not before 12am. Similarly, on the right side of the window, we use the first trade at 20 minutes after the announcement, or nearest trade after 20 minutes otherwise, but not after noon the next day. In other words, we use the nearest trades on or outside the inner window, but inside of the outer window.

For example, suppose the FOMC announcement is at 2:15pm. Then the inner window is from 2:05pm to 2:35pm. On the left side, we take the first trade at 2:05 or earlier, but not before 12am. On the right side, we take the first trade at 2:35pm or later, but not after noon the next day. Then we subtract the two implied rates.

As a robustness check, we also consider an inner window of 60 minutes (15 minutes before the FOMC announcement and 45 minutes after), along with outer windows of 12am to 1 hour after the statement release, and 12am to 2 hours after the statement release.

In addition to calculating the change in implied rates from Fed funds futures and Eurodollar futures, we also calculate the surprise component of Fed funds futures. We follow Kuttner (2001) in unwinding the average rate into a surprise measure.

To make notation consistent, for a variable X_t^j let the superscript j index the current or future FOMC meetings ($j = 0$ is the current meeting), and let the subscript t index the "real-time" of when the statement is released ($t - \Delta t$ and $t + \Delta t$ are the inner window before and after the statement, respectively). Let d^j be the day of the jth FOMC meeting and m^j denote the number of calendar days in the month of the FOMC meeting. Let r^j denote the target Fed funds rate prevailing after the jth meeting. And let $f^j_{t-\Delta t}$ and $f^j_{t+\Delta t}$ denote the implied rate from the Fed funds futures contract expiring in the month of the jth meeting, before and after the current meeting. Finally, $\mathbb{E}_{t-\Delta t} \equiv \mathbb{E}[\cdot|\mathcal{I}_{t-\Delta t}]$ is the conditional expectation using information up to an inner window before the FOMC meeting at t.

The implied rate from the Fed funds futures in an inner window around the current FOMC can be written as

$$\begin{aligned} f^0_{t-\Delta t} =& \frac{d^0}{m^0} r^{-1} + \frac{m^0 - d^0}{m^0} \mathbb{E}_{t-\Delta t}(r^0) + \mu^0_{t-\Delta t} \\ f^0_{t+\Delta t} =& \frac{d^0}{m^0} r^{-1} + \frac{m^0 - d^0}{m^0} (r^0) + \mu^0_{t+\Delta t}. \end{aligned}$$

Here we make three assumptions. First, the effective Fed funds rate equals the target rate; if not, then replace r^{-1} by the average effective rate realized so far in the month. Second, r^0 only changes from the FOMC meeting, and is constant for the remainder of the month after the FOMC meeting. In other words, $\mathbb{E}_{t+\Delta t}(r^0) = r^0$. Third, high frequency changes around the term premium μ^0 are negligible. Piazzesi and Swanson (2008) argue the narrow daily window largely "differences out" risk premia that are moving primarily at lower, business cycle frequencies.

With these three assumptions, we can then calculate the current FOMC surprise as a scaled change in the current Fed funds implied rates,

$$e^0_{t+\Delta t} \equiv \frac{m^0}{m^0 - d^0} \left[f^0_{t+\Delta t} - f^0_{t-\Delta t} \right], \tag{A.1}$$

where the scaling is proportional to when in the month the FOMC meeting occurs. And the change in implied rate equals the expected component plus the surprise component.

We also calculate longer horizon surprises around the jth meeting, after the current meeting, as

$$e^j_{t+\Delta t} \equiv \frac{m^j}{m^j - d^j} \left[\left(f^j_{t+\Delta t} - f^j_{t-\Delta t} \right) - \frac{d^j}{m^j} e^{j-1}_t \right]. \tag{A.2}$$

Lastly, the scale factor can get large if the meeting is at the end of the month and Fed funds futures only trade in half a basis point increments. Therefore if a meeting is in the last 3 to 7 days of the month, then we use the current change in next month's Fed funds futures implied rate.

Structural Breaks as Nonrecurrent Regime-Switching

To capture the phenomenon of nonrecurrent regimes, we suppose that ξ_t^P follows a Markov-switching process in which new regimes can arise but do not repeat exactly as before. This is modeled by specifying the transition matrix over nonrecurrent states, or "structural breaks." If the historical sample has N_P nonrecurrent regimes (implying $N_P - 1$ structural breaks), the transition matrix for the Markov process takes the form

$$\mathbf{H} = \begin{bmatrix} p_{11} & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ 1-p_{11} & p_{22} & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & 1-p_{22} & p_{33} & 0 & \cdots & \cdots & \vdots \\ \vdots & 0 & 1-p_{33} & \ddots & & & \\ \vdots & \vdots & 0 & \vdots & \ddots & & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & p_{N_P,N_P} & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & 1-p_{N_P,N_P} & 1 \end{bmatrix}, \tag{A.3}$$

where $\mathbf{H}_{ij} \equiv p\left(\xi_t^P = i | \xi_{t-1}^P = j\right)$. For example, if there were $N_P = 2$ nonrecurrent regimes in the sample, we would have

$$\mathbf{H} = \begin{bmatrix} p_{11} & 0 \\ 1-p_{11} & 1 \end{bmatrix}.$$

The above process implies that, if you are currently in regime 1, you will remain there next period with probability p_{11} or exit to regime 2 with probability $1 - p_{11}$. Upon exiting to regime 2, since there are only two regimes in the sample and the probability p_{12} of returning exactly to the previous regime 1 is zero, $p_{22} = 1$.

Most Likely Regime Sequence

In this section we explain how to compute the most likely regime sequence. This most likely regime sequence is the particular regime sequence $\xi^{P,T} = \{\hat{\xi}_1^P, ..., \hat{\xi}_T^P\}$ that is most likely to have occurred, given our estimated posterior mode parameter values for $\boldsymbol{\theta}$. This sequence is computed as follows. Let $P\left(\xi_t^P = i | z_{t-1}; \boldsymbol{\theta}_r\right) \equiv \pi_{t|t-1}^i$. First, we run Hamilton's filter to get the vector of filtered regime probabilities $\pi_{t|t}$, $t = 1, 2, ..., T$. The Hamilton filter can be expressed iteratively as

$$\begin{aligned} \pi_{t|t} &= \frac{\pi_{t|t-1} \odot \eta_t}{\mathbf{1}'\left(\pi_{t|t-1} \odot \eta_t\right)} \\ \pi_{t+1|t} &= \mathbf{H}\pi_{t|t} \end{aligned}$$

where η_t is a vector whose j-th element contains the conditional density $p(mps_t|\xi_t^P = j; \boldsymbol{\theta}_r)$, the symbol $\odot$ denotes element by element multiplication, and $\mathbf{1}$ is a vector with all elements equal to 1. The final term, $\pi_{T|T}$ is returned with the final step of the filtering algorithm. Then, a recursive algorithm can be implemented to derive the other smoothed probabilities:

$$\pi_{t|T} = \pi_{t|t} \odot \left[\mathbf{H}' \left(\pi_{t+1|T} \left(\div\right) \pi_{t+1|t}\right)\right]$$

where $(\div)$ denotes element by element division. To choose the regime sequence most likely to have occurred given our parameter estimates, consider the recursion in the next to last period $t = T - 1$:

$$\pi_{T-1|T} = \pi_{T-1|T-1} \odot \left[\mathbf{H}' \left(\pi_{T|T} \left(\div\right) \pi_{T|T-1}\right)\right].$$

We first take $\pi_{T|T}$ from the Hamilton filter and choose the regime that is associated with the largest probability, i.e., if $\pi_{T|T} = (.9, .1)$, where the first element corresponds to the probability of regime 1, we select $\hat{\xi}_T^P = 1$, indicating that we are in regime 1 in period T. We now update $\pi_{T|T} = (1, 0)$ and plug into the right-hand-side above along with the estimated filtered probabilities for $\pi_{T-1|T-1}$, $\pi_{T|T-1}$ and estimated transition matrix $\mathbf{H}$ to get $\pi_{T-1|T}$ on the left-hand-side. Now we repeat the same procedure by choosing the regime for $T - 1$ that has the largest probability at $T - 1$, e.g., if $\pi_{T-1|T} = (.2, .8)$ we select $\hat{\xi}_{T-1} = 2$, indicating that we are in regime 2 in period $T - 1$, we then update to $\pi_{T-1|T} = (0, 1)$, which is used again on the right-hand-side now

$$\pi_{T-2|T} = \pi_{T-2|T-2} \odot \left[\mathbf{H}' \left(\pi_{T-1|T} \left(\div\right) \pi_{T-1|T-2}\right)\right].$$

We proceed in this manner until we have a most likely regime sequence $\xi^{P,T}$ for the entire sample $t = 1, 2, ..., T$. Two aspects of this procedure are worth noting. First, it fails if the updated probabilities are exactly $(.5, .5)$. Mathematically this is virtually a zero probability event. Second, note that this procedure allows us to choose the most likely regime sequence by using the recursive formula above to update the filtered probabilities sequentially working backwards from $t = T$ to $t = 1$. This allows us to take into account the time dependence in the regime sequence as dictated by the transition probabilities.

Price-Output Decompositions

Mapping from price to output (measured as GDP_t) is

$$\begin{aligned} \frac{P_t}{GDP_{t-1}} &= \frac{P_t}{D_t}\frac{D_t}{GDP_t}\frac{GDP_t}{GDP_{t-1}} \\ pgdp_t &= pd_t + k_t + \widetilde{y}_t + g_t - \widetilde{y}_{t-1} \end{aligned}$$

Below we decompose pd_t to write:

$$\begin{aligned}
pgdp_t &= \underbrace{\frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + k_t + y_t + g_t - \widetilde{y}_{t-1}}_{\text{earning share component}} + \underbrace{pdv_t\left(\Delta d\right)}_{\text{earnings}} - \underbrace{pdv_t\left(r^{ex}\right)}_{\text{premia}} - \underbrace{pdv_t\left(rir\right)}_{\text{RIR}} \\
pgdp_{r^{ex},t} &= \underbrace{\frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + k_t + \widetilde{y}_t + g_t - \widetilde{y}_{t-1}}_{\text{earning share component}} - \underbrace{pdv_t\left(r^{ex}\right)}_{\text{premia}} \\
pgdp_{rir,t} &= \underbrace{\frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + k_t + \widetilde{y}_t + g_t - \widetilde{y}_{t-1}}_{\text{earning share component}} - \underbrace{pdv_t\left(rir\right)}_{\text{RIR}} \\
pgdp_{\Delta d,t} &= \underbrace{\frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + k_t + \widetilde{y}_t + g_t - \widetilde{y}_{t-1}}_{\text{earning share component}} + \underbrace{pdv_t\left(\Delta d\right)}_{\text{earnings}}
\end{aligned}$$

where

$$\begin{aligned}
pd_t &= \kappa_{pd,0} + \mathbb{E}_t^b\left[m_{t+1} + \Delta d_{t+1} + \kappa_{pd,1} pd_{t+1}\right] + \\
&\quad + .5\mathbb{V}_t^b\left[m_{t+1} + \Delta d_{t+1} + \kappa_{pd,1} pd_{t+1}\right].
\end{aligned}$$

The solution approximates around the balanced growth path with $\frac{D_{t+1}}{D_t} = G$, where G is the gross growth rate of the economy. The Euler equation under the balanced growth path is

$$\begin{aligned}
1 &= \left[M_{t+1}\left(\frac{P_{t+1}/D_{t+1}+1}{P_t/D_t}\right)\frac{D_{t+1}}{D_t}\right] \\
&= \left[\beta_p\left(\frac{D_{t+1}}{D_t}\right)^{-\sigma_p}\left(\frac{P_{t+1}/D_{t+1}+1}{P_t/D_t}\right)\frac{D_{t+1}}{D_t}\right] \\
&= \left[\underbrace{\beta_p G^{1-\sigma_p}}_{\widetilde{\beta}_p}\left(\frac{P/D+1}{P/D}\right)\right] => \\
\frac{1}{\widetilde{\beta}_p} &= \left(\frac{P/D+1}{P/D}\right) => \\
P/D &= \frac{\widetilde{\beta}_p}{1-\widetilde{\beta}_p}.
\end{aligned}$$

Denote the log steady state price-payout ratio as $\ln\left(P/D\right) = \overline{pd}$, thus we have

$$\overline{pd} = \ln\left(\frac{\widetilde{\beta}_p}{1-\widetilde{\beta}_p}\right).$$

$$\begin{aligned}\kappa_{pd,1} &= \exp(\overline{pd})/(1+\exp(\overline{pd})) = \frac{\widetilde{\beta}_p}{1-\widetilde{\beta}_p}\left[1+\frac{\widetilde{\beta}_p}{1-\widetilde{\beta}_p}\right]^{-1} = \widetilde{\beta}_p \\ \kappa_{pd,0} &= \ln(\exp(\overline{pd})+1) - \kappa_{pd,1}pd = \ln\left(\frac{1}{1-\widetilde{\beta}_p}\right) - \widetilde{\beta}_p \ln\frac{\widetilde{\beta}_p}{1-\widetilde{\beta}_p} \\ &= -\widetilde{\beta}_p \ln\widetilde{\beta}_p - \left(1-\widetilde{\beta}_p\right)\ln\left(1-\widetilde{\beta}_p\right)\end{aligned}$$

The log return obeys the following approximate identity (Campbell and Shiller (1989)):

$$r^D_{t+1} = \kappa_{pd,0} + \kappa_{pd,1}pd_{t+1} - pd_t + \Delta d_{t+1},$$

where $\kappa_{pd,1} = \exp(\overline{pd})/(1+\exp(\overline{pd}))$, and $\kappa_{pd,0} = \log\left(\exp(\overline{pd})+1\right) - \kappa_{pd,1}\overline{pd}$. Combining all of the above, the log equity premium is

$$\underbrace{\mathbb{E}^b_t\left[r^D_{t+1}\right] - \left(i_t - \mathbb{E}^b_t\left[\pi_{t+1}\right]\right)}_{\text{Equity Premium}} = \underbrace{\left[\begin{array}{c} -.5\mathbb{V}^b_t\left[r^D_{t+1}\right] - \mathbb{COV}^b_t\left[m_{t+1}, r^D_{t+1}\right] \\ +.5\mathbb{V}^b_t\left[\pi_{t+1}\right] - \mathbb{COV}^b_t\left[m_{t+1}, \pi_{t+1}\right] \end{array}\right]}_{\text{Risk Premium}} + \underbrace{\overline{lp}_t}_{\text{Liquidity Premium}},$$

Then

$$\begin{aligned} pd_t &= \kappa_{pd,0} + \mathbb{E}^b_t\left[\Delta d_{t+1} - r^D_{t+1} + \kappa_{pd,1}pd_{t+1}\right] \\ pd_t &= \kappa_{pd,0} + \mathbb{E}^b_t\left[\Delta d_{t+1} - \left(r^{ex}_{t+1} - rir_{t+1}\right) + \kappa_{pd,1}pd_{t+1}\right]\end{aligned}$$

where $\mathbb{E}^b_t\left[r^{ex}_{t+1}\right] = \mathbb{E}^b_t\left[r^D_{t+1}\right] - rir_{t+1}$, where $rir_{t+1} \equiv \left(i_{t+1} - \mathbb{E}^b_t\left[\pi_{t+1}\right]\right)$.

Solving forward:

$$\begin{aligned} pd_t &= \kappa_{pd,0} + \mathbb{E}^b_t\left[\Delta d_{t+1} - r^{ex}_{t+1} - rir_{t+1}\right] + \\ &\quad + \kappa_{pd,1}\mathbb{E}^b_t\left[\kappa_{pd,0} + \mathbb{E}^b_t\left[\Delta d_{t+2} - r^{ex}_{t+2} - rir_{t+1} + \kappa_{pd,1}pd_{t+2}\right]\right]\end{aligned}$$

Thus:

$$pd_t = \frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + \left(1_{\Delta d} - 1_{\mathbb{E}(r^{ex})} - 1_{rir}\right)\sum_{h=0}^{\infty}\kappa^h_{pd,1}\mathbb{E}^b_t\left[S_{t+1+h}\right]$$

where 1_x is a vector of all zeros except for a 1 in the xth position. This can be written as:

$$pd_t = \frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + pdv_t\left(\Delta d\right) - pdv_t\left(r^{ex}\right) - pdv_t\left(rir\right)$$

Using the solution:

$$pd_t = \frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + \left(1_{\Delta d} - 1_{\mathbb{E}(r^{ex})} - 1_{r^b}\right)\left(\mathbf{I} - \kappa_{pd,1}T_{\xi_t}\right)^{-1}\left[T_{\xi_t}S_t + \left(\mathbf{I}-\kappa_{pd,1}\right)^{-1}C_{\xi_t}\right].$$

Thus, we can decompose movements in the pd_t into those attributable to expected dividends, equity premia, and expected real interest rates:

$$pgdp_t = \underbrace{\frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + k_t + y_t + g_t - y_{t-1}}_{\text{earning share component}} + \underbrace{pdv_t\left(\Delta d\right)}_{\text{earnings}} - \underbrace{pdv_t\left(r^{ex}\right)}_{\text{premia}} - \underbrace{pdv_t\left(rir\right)}_{\text{RIR}}.$$

Solution and Estimation Details

Computing Expectations with Regime Switching and Alternative Policy Rules

In what follows, we explain how to use expectations to infer what alternative regimes agents have in mind. Expectations about inflation, FFR, and GDP growth depend on the regime currently in place, the alternative regime, and the probability of moving to such regime. This note is based on "Methods for measuring expectations and uncertainty in Bianchi (2016). That paper explains how to computed expected values in presence of regime changes. In the models described above, for each policy rule in place, agents would have different beliefs about alternative future policy rules. This would lead to changes in expected values for the endogenous variables of the model.

Consider a MS model:

$$S_t = C_{\xi_t} + T_{\xi_t} S_{t-1} + R_{\xi_t} Q \varepsilon_t \tag{A.4}$$

where $\xi_t = \left\{\xi_t^P, \xi_t^b\right\}$ controls the policy regime ξ_t^P controls the policy rule currently in place and the alternative policy rule, while the belief regime ξ_t^b controls agents' beliefs about the possibility of moving to the alternative policy rule.

Let n be the number of variables in S_t. Let $m = B + 1$ be the number of Markov-switching states and define

$$\xi_t = i \equiv \left\{\xi_t^P, \xi_t^b = i\right\}, \quad i = 1, ..., B+1.$$

Define the $mn \times 1$ column vector q_t as:

$$\underset{mn\times 1}{q_t} = \left[q_t^{1\prime}, ..., q_t^{m\prime}\right]'$$

where the individual $n \times 1$ vectors $q_t^i = \mathbb{E}_0\left(S_t 1_{\xi_t=i}\right) \equiv \mathbb{E}\left(S_t 1_{\xi_t=i} | \mathbb{I}_0\right)$ and $1_{\xi_t=i}$ is an indicator variable that is one when belief regime i is in place and zero otherwise. Note that:

$$q_t^i = \mathbb{E}_0\left(S_t 1_{\xi_t=i}\right) = \mathbb{E}_0\left(S_t | \xi_t = i\right) \pi_t^i$$

where $\pi_t^i = P_0\left(\xi_t = i\right) = P\left(\xi_t = i | \mathbb{I}_0\right)$. Therefore we can express $\mu_t = \mathbb{E}_0\left(S_t\right)$ as:

$$\mu_t = \mathbb{E}_0\left(S_t\right) = \textstyle\sum_{i=1}^m q_t^i = w q_t$$

where the matrix $\underset{n\times mn}{w} = [I_n, ..., I_n]$ is obtained placing side by side m n-dimensional identity matrices. Then the following proposition holds:

PROPOSITION 1: *Consider a Markov-switching model whose law of motion can be described by (A.4) and define* $q_t^i = \mathbb{E}_0\left(S_t 1_{\xi_t=i}\right)$ *for* $i = 1...m$. *Then* $q_t^j = C_j \pi_t^j + \sum_{i=1}^m T_j q_{t-1}^i p_{ji}$.

It is then straightforward to compute expectations conditional on the information available at a particular point in time. Suppose we are interested in $\mu_{t+s|t} \equiv \mathbb{E}_t^b\left(S_{t+s}\right)$, i.e. the expected value for the vector S_{t+s} conditional on the information set available at time t. If we define:

$$q_{t+s|t} = \left[q_{t+s|t}^{1\prime}, ..., q_{t+s|t}^{m\prime}\right]'$$

where $q^i_{t+s|t} = \mathbb{E}^b_t\left(S_{t+s}1_{\xi_t=i}\right) = \mathbb{E}^b_t\left(S_{t+s}|\xi_t = i\right)\pi^i_{t+s|t}$, where $\pi^i_{t+s|t} \equiv P\left(\xi_{t+s} = i|\mathbb{I}_t\right)$, we have

$$\mu_{t+s|t} = \mathbb{E}^b_t\left(S_{t+s}\right) = wq_{t+s|t}, \tag{A.5}$$

where for $s \geq 1$, $q_{t+s|t}$ evolves as:

$$\begin{aligned} q_{t+s|t} &= C\pi_{t+s|t} + \Omega q_{t+s-1|t} && \text{(A.6)} \\ \pi_{t+s|t} &= \mathbf{H}^b\pi_{t+s-1|t} && \text{(A.7)} \end{aligned}$$

with $\pi_{t+s|t} = \left[\pi^1_{t+s|t}, ..., \pi^m_{t+s|t}\right]'$, $\Omega = bdiag\left(T_1, ..., T_m\right)\left(\mathbf{H}^b \otimes I_n\right)$, and $\underset{mn\times m}{C} = bdiag\left(C_1, ..., C_m\right)$, where e.g., C_1 is the $n \times 1$ vector of constants in regime 1, $\otimes$ represents the Kronecker product and *bdiag* is a matrix operator that takes a sequence of matrices and use them to construct a block diagonal matrix.

The formulas above are used to compute expectations conditional on each belief regime ξ^b_t and policy rule regime ξ^P_t. For each composite regime $\xi_t = \left\{\xi^P_t, \xi^b_t\right\}$, we can obtain a forecast for each of the variables of the model. For example, conditional on ξ^P_t and $\xi^b_t = j$ in place we have

$$q_{t,\xi_t=j} = e_j \otimes S_t$$

where e_j is a variable that has elements equal to zero except for the one in position ξ^b_t. For example, with $B = 5$ belief regimes and $\xi^b_t = 3$ we have

$$q_{t,\xi_t=3} = \left[\mathbf{0}', \mathbf{0}', S_t', \mathbf{0}', \mathbf{0}', \mathbf{0}'\right]'.$$

where $\mathbf{0}$ and S_t are column vectors with n rows. We have $B + 1$ subvectors in $q_{t,\xi_t=j}$ to take into account the alternative policy mix. The fact that all subvectors are zero except for the one corresponding to the belief regime $b = 3$ reflects the assumption that agents can observe the current state S_t and, by definition, their own beliefs (while the econometrician cannot observe any of the two and she uses macro data and survey expectations to estimate both S_t and agents' beliefs).

Thus, suppose we want to compute the expected value for a variable x over the next year under the assumption that agents' beliefs are $\xi^b_t = j$. With monthly data, we have:

$$\begin{aligned} \mathbb{E}^b_t\left(x_{t,t+s}|\xi_t = j\right) &= \textstyle\sum_{s=1}^{12}\mathbb{E}^b_t\left(x_{t+s}|\xi_t = j\right) \\ &= e_x\textstyle\sum_{s=1}^{12}\mu_{t+s|t,\xi_t=j} \\ &= e_xw\textstyle\sum_{s=1}^{12}q_{t+s|t,\xi_t=j} \end{aligned}$$

where for $s \geq 1$, $q_{t+s|t}$ evolves as:

$$\begin{aligned} q_{t+s|t,\xi_t=j} &= C\pi_{t+s|t} + \Omega q_{t+s-1|t,\xi_t=j} && \text{(A.8)} \\ \pi_{t+s|t,\xi_t=j} &= \mathbf{H}^b\pi_{t+s-1|t,\xi_t=j} && \text{(A.9)} \end{aligned}$$

with $\pi_{t+s|t} = \left[\pi^1_{t+s|t}, ..., \pi^m_{t+s|t}\right]'$, $\Omega = bdiag\left(T_1, ..., T_m\right)\left(\mathbf{H}^b \otimes I_n\right)$, and $\underset{mn\times m}{C} = bdiag\left(C_1, ..., C_m\right)$, where e.g., C_1 is the $n \times 1$ vector of constants in regime 1, $\otimes$ represents the Kronecker product and *bdiag* is a matrix operator that takes a sequence of matrices and use them to construct a block diagonal matrix. The recursive algorithm is initialized with $\pi_{t|t,\xi_t=j} = 1_{\xi_t=j}$ and $q_{t,\xi_t=j} = e_j \otimes S_t$.

The formulas (A.8) and (A.9) can be written in a more compact form. If we define $\widetilde{q}_{t|t} = [q'_{t|t}, \pi'_{t|t}]'$, with $\pi_{t|t}$ a vector with elements $\pi^i_{t|t} \equiv P\left(\xi_t = i|\mathbb{I}_t\right)$ we can compute the conditional expectations in one step:

$$\mu_{t+s|t} = \mathbb{E}^b_t\left(S_{t+s}\right) = \widetilde{w}\widetilde{\Omega}^s\widetilde{q}_{t|t} \tag{A.10}$$

where $\widetilde{w} = [w, 0_{n\times m}]$. The formula above can be used to compute the expected value from the point of view of the agent of the model with beliefs $\xi_t = j$:

$$\mathbb{E}^b_t\left(x_{t+s}|\xi_t = j\right) = e_x\mu_{t+s|t,\xi_t=j} = e_x\widetilde{w}\widetilde{\Omega}^s\widetilde{q}_{t|t,\xi^b_t=j} = \underbrace{e_x w\widetilde{\Omega}^s_{\{1,nm\},\{n(j-1)+1,nj\}}}_{Z_{\xi_t,x_{t+s}}}\underbrace{S_t}_{(n\times 1)} + \underbrace{e_x w\widetilde{\Omega}^s_{\{1,nm\},nm+j}}_{D_{\xi_t,x_{t+s}}} \tag{A.11}$$

where $D_{\xi_t,x_{t+s}}$ is a scalar, $Z_{\xi_t,x_{t+s}}$ is an $(1 \times n)$ vector, $\widetilde{\Omega}^s_{\{1,nm\},\{n(j-1)+1,nj\}}$ is the submatrix obtained taking the first nm rows and the columns from $n(j-1)+1$ to nj of $\widetilde{\Omega}^s$, while $\widetilde{\Omega}^s_{\{1,nm\},nm+j}$ is the submatrix obtained taking the first nm rows and the $nm+j$ column of $\widetilde{\Omega}^s$. Thus, we have that conditional on one belief regime and a policy rule regime, we can map the current state of the economy S_t into the expected value reported in the survey. The matrix algebra in (A.11) returns the same results of the recursion in (A.8) and (A.9).

To see what the formulas above do, consider a simple example with $B = 2$ and we are currently in belief regime $b = 2$:

$$\begin{aligned}
\mathbb{E}^b_t\left(x_{t+s}|\xi_t = 2\right) &= e_x\widetilde{w}\widetilde{\Omega}^s\widetilde{q}_{t|t,\xi_t=2} = e_x\widetilde{w}\widetilde{\Omega}^s\begin{bmatrix}\mathbf{0}_{n\times 1}\\ S_t\\ n\times 1\\ \mathbf{0}_{n\times 1}\\ 0\\ 1\\ 0\end{bmatrix} = e_x\widetilde{w}\begin{bmatrix}\widetilde{\Omega}^s_{11} & \widetilde{\Omega}^s_{12} & \widetilde{\Omega}^s_{13} & \widetilde{\Omega}^s_{14} & \widetilde{\Omega}^s_{15} & \widetilde{\Omega}^s_{16}\\ \widetilde{\Omega}^s_{21} & \widetilde{\Omega}^s_{22} & \widetilde{\Omega}^s_{23} & \widetilde{\Omega}^s_{24} & \widetilde{\Omega}^s_{25} & \widetilde{\Omega}^s_{26}\\ \widetilde{\Omega}^s_{31} & \widetilde{\Omega}^s_{32} & \widetilde{\Omega}^s_{33} & \widetilde{\Omega}^s_{34} & \widetilde{\Omega}^s_{35} & \widetilde{\Omega}^s_{36}\\ & & & \widetilde{\Omega}^s_{44} & \widetilde{\Omega}^s_{45} & \widetilde{\Omega}^s_{46}\\ & & & \widetilde{\Omega}^s_{54} & \widetilde{\Omega}^s_{55} & \widetilde{\Omega}^s_{56}\\ & & & \widetilde{\Omega}^s_{64} & \widetilde{\Omega}^s_{65} & \widetilde{\Omega}^s_{66}\end{bmatrix}\begin{bmatrix}\mathbf{0}_{n\times 1}\\ S_t\\ n\times 1\\ \mathbf{0}_{n\times 1}\\ 0\\ 1\\ 0\end{bmatrix}\\
&= e_x\widetilde{w}\begin{bmatrix}\widetilde{\Omega}^s_{12}S_t + \widetilde{\Omega}^s_{15}\\ \widetilde{\Omega}^s_{22}S_t + \widetilde{\Omega}^s_{25}\\ \widetilde{\Omega}^s_{32}S_t + \widetilde{\Omega}^s_{35}\\ \widetilde{\Omega}^s_{44}\\ \widetilde{\Omega}^s_{54}\\ \widetilde{\Omega}^s_{64}\end{bmatrix} = e_x\left(\widetilde{\Omega}^s_{12} + \widetilde{\Omega}^s_{22} + \widetilde{\Omega}^s_{32}\right)S_t + e_x\left(\widetilde{\Omega}^s_{15} + \widetilde{\Omega}^s_{25} + \widetilde{\Omega}^s_{35}\right)
\end{aligned}$$

Finally, suppose we are interested in the forecast $\mathbb{E}^b_t\left(x_{t,t+s}|\xi^b_t = j, \xi^p_t\right)$:

$$\mathbb{E}^b_t\left(x_{t,t+s}|\xi_t = j\right) = \underbrace{\left[e_x\sum_{s=1}^{12} w\widetilde{\Omega}^s_{\{1,nm\},\{n(j-1)+1,nj\}}\right]}_{Z_{\xi_t,x_{t,t+s}}}\underbrace{S_t}_{(n\times 1)} + \underbrace{e_x\sum_{s=1}^{12} w\widetilde{\Omega}^s_{\{1,nm\},nm+j}}_{D_{\xi_t,x_{t,t+s}}} \tag{A.12}$$

Thus, we can include $Z_{\xi_t,x_{t,t+s}}$ as a row in Z_{ξ_t} and $D_{\xi_t,x_{t,t+s}}$ as a row in D_{ξ_t} in the mapping from the model to the observables described in (A.13). Note that the matrix Z and vector D are now regime dependent.

For GDP growth, we are interested in the average growth over a certain horizon. Our state vector contains $\widetilde{y}_t$. Thus, we can use the following approach:

$$\begin{aligned}\mathbb{E}_t^b\left[\left(gdp_{t+h}-gdp_t\right)h^{-1}|\xi_t=j\right] &= \mathbb{E}_t^b\left[\left(\widetilde{y}_{t+h}-\widetilde{y}_t+h\mu\right)h^{-1}|\xi_t=j\right]\\ &= h^{-1}\mathbb{E}_t^b\left[\widetilde{y}_{t+h}|\xi_t=j\right]-h^{-1}\widetilde{y}_t+\mu\end{aligned}$$

where μ is the average growth rate in the economy and $\widetilde{y}_t$ is GDP in deviations from the trend. With deterministic growth we have $gdp_{t+h}-gdp_t-h\mu\equiv\widetilde{y}_{t+h}-\widetilde{y}_t$. We then have

$$\begin{aligned}\mathbb{E}_t^b\left[\left(gdp_{t+h}-gdp_t\right)h^{-1}|\xi_t=j\right] &= h^{-1}\mathbb{E}_t^b\left[\widetilde{y}_{t+h}|\xi_t=j\right]-h^{-1}\widetilde{y}_t+\mu\\ &= h^{-1}\left[\underbrace{e_{\widetilde{y}}w\widetilde{\Omega}^s_{\{1,nm\},\{n(j-1)+1,nj\}}}_{Z_{\xi_t,\widetilde{y}_{t+s}}}\underbrace{S_t}_{(n\times1)}+\underbrace{e_{\widetilde{y}}w\widetilde{\Omega}^s_{\{1,nm\},nm+j}}_{D_{\xi_t,\widetilde{y}_{t+s}}}-e_{\widetilde{y}}S_t\right]+\mu\\ &= h^{-1}\underbrace{\left[e_{\widetilde{y}}w\widetilde{\Omega}^s_{\{1,nm\},\{n(j-1)+1,nj\}}-e_{\widetilde{y}}\right]}_{Z_{\xi_t,\widetilde{y}_{t+s}-\widetilde{y}_t}}\underbrace{S_t}_{(n\times1)}+h^{-1}\underbrace{e_{\widetilde{y}}w\widetilde{\Omega}^s_{\{1,nm\},nm+j}}_{D_{\xi_t,\widetilde{y}_{t+s}}}+\mu\end{aligned}$$

The expected values for the endogenous variables depend on the perceived transition matrix $\mathbf{H}^b$ and the properties of the alternative regime. The latter can be seen by recalling that the regime $\xi_t=B+1$ applies to the perceived alternative regime. Thus, data on expectations provide information about the perceived probability of moving across belief regimes as well as the parameters of the alternative regime.

Estimation

The solution of the model takes the form of a Makrov-switching vector autoregression (MS-VAR) in the state vector $S_t=\left[S_t^M,m_t,pd_t,k_t,z_t,lp_t,\mathbb{E}_t^b\left(m_{t+1}\right),\mathbb{E}_t^b\left(pd_{t+1}\right)\right]$. Here, S_t^M is a vector of macro block state variables given by $S_t^M\equiv\left[\widetilde{y}_t,g_t,\pi_t,i_t,\overline{\pi}_t,f_t\right]'$. The asset pricing block of equations involves conditional subjective variance terms that are affected by Markov-switching random variables in the model. The subsection "Risk Adjustment with Lognormal Approximation," below, explains the approximation used to preserve lognormality of the entire system.

The model solution in state space form is

$$
\begin{aligned}
X_t &= D_{\xi_t,t} + Z_{\xi_t,t}\left[S_t', \widetilde{y}_{t-1}\right]' + U_t v_t \\
S_t &= C\left(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b\right) + T(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b) S_{t-1} + R(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b) Q \varepsilon_t \\
Q &= diag\left(\sigma_{\varepsilon_1}, ..., \sigma_{\varepsilon_G}\right), \ \varepsilon_t \sim N(0, I) \\
U &= diag\left(\sigma_1, ..., \sigma_X\right), \ v_t \sim N(0, I) \\
\xi_t^P &= 1...N_P, \ \xi_t^b = 1, ...B+1, H_{i,j} = p\left(\xi_t^b = i | \xi_{t-1}^b = j\right).
\end{aligned}
$$

where X_t is a $N_X \times 1$ vector of data, v_t are a vector of observation errors, U_t is a diagonal matrix with the standard deviations of the observation errors on the main diagonal, and $D_{\xi_t,t}$, and $Z_{\xi_t,t}$ are parameters mapping the model counterparts of X_t into the latent discrete- and continuous-valued state variables ξ_t and S_t, respectively, in the model. The vector X_t of observables is explained below. Note that the parameters $D_{\xi_t,t}$, $Z_{\xi_t,t}$, and U_t vary over time in part because not all variables are observed at each data sampling period. In addition, some of the parameters in the system are dependent on the current policy rule and the associated Alternative rule, ξ_t^P, and the unobserved, discrete-valued $(B+1)$-state Markov-switching variable ξ_t^b $\left(\xi_t^b = 1, 2, ..., B+1\right)$ with perceived transition probabilities

$$
\mathbf{H}^b = \begin{bmatrix}
p_{11} & \cdots & p_{1B} & 0 \\
\vdots & \ddots & \vdots & \vdots \\
p_{B1} & \cdots & p_{BB} & 0 \\
1 - \sum_{i=1}^{B} p_{i1} & \cdots & 1 - \sum_{i=1}^{B} p_{iB} & p_{B+1,B+1} = 1
\end{bmatrix},
$$

where $\mathbf{H}_{ij}^b \equiv p\left(\xi_t^b = i | \xi_{t-1}^b = j\right)$. We use the following notation:

$$
\begin{aligned}
C_{\xi_t^P,j} &= C\left(\theta_{\xi_t^P}, \xi_t^b = j\right), \ T_{\xi_t^P,j} = T\left(\theta_{\xi_t^P}, \xi_t^b = j\right), \ R_{\xi_t^P,j} = R\left(\theta_{\xi_t^P}, \xi_t^b = j\right) \\
D_{j,t} &= D_{\xi_t,t|\xi_t^b=j}, \quad Z_{j,t} = Z_{\xi_t,t|\xi_t^b=j}.
\end{aligned}
$$

Kim's Approximation to the Likelihood and Filtering We use Kim's (Kim (1994)) basic filter and approximation to the likelihood.

First note that, from the econometricians viewpoint, only the first B regimes can actually be realized, since the true alternative that arises after one regime ends is never exactly as previously conceived by the investor. Thus, the estimation algorithm involves estimation just the upper $B \times B$ submatrix of $\mathbf{H}^b$, rescaled so that the elements sum to unity. The filtering described here therefore loops over just B states, rather than $B+1$.

The sample is divided into N_P policy regime subperiods indexed by ξ_t^P. Denote the last observation of each regime subperiod of the sample $T_1, ..., T_{N_P}$. The algorithm for the basic filter is described as follows.

Initiate values $\widetilde{S}_{0|0}$, $P_{0|0}$, for the Kalman filter and $\Pr\left(\xi_0^b\right) = \pi_0$ for the Hamilton filter and initialize $\mathcal{L}(\theta) = 0$. Denote $X^{t-1} \equiv \{X_1, ..., X_{t-1}\}$ and $\xi^{PT} = \left\{\xi_1^P, ..., \xi_T^P\right\}$.

In the mixed-frequency estimation, we use intra-month data to provide "early" estimates of the state space, while "final" estimates are obtained using a more complete set of data available at the end of each month. Let t denote a month. Let d_i denote the number of time units that have passed *within* a month when we have reached a particular point in time, and let nd denote the total number of time units in the month. Then $0 \leq d_i/nd \leq 1$, and the end of month t is denoted $t-1+d_i/nd$ with $d_i = nd$. For example, d_{100}/nd could denote the point within the month that is exactly 10 minutes before an FOMC meeting during the month, while d_{130}/nd could denote point in the month 20 minutes after the same meeting. Intra-month observations used just prior to an FOMC meeting will typically include the daily BBG consensus forecasts from the day before the meeting, and the 10-minutes before FFF, ED and stock market data. Intermonth observations for the point of the month right after the FOMC meeting will typically include the daily BBG consensus forecasts from the day after the meeting, and the 20-minutes after FFF, ED and stock market data.

- For $t = 1$ to T_1 and $\theta_{\xi_t^P}$ relevant when $\xi_t^P = 1$:

1. Suppose we have information up through month $t-1$ and new information arrives at $t-1+d_i/nd$. Conditional on $\xi_{t-1}^b = i$ and $\xi_t^b = j$ run the Kalman filter given below for $i, j = 1, 2, ..., B$ to update estimates of the latent state:

$$\begin{aligned}
S_{t|t-1}^{(i,j)} &= C_{\xi_t^P,j} + T_{\xi_t^P,j} S_{t-1|t-1}^i \\
P_{t|t-1}^{(i,j)} &= T_{\xi_t^P,j} P_{t-1|t-1}^i T_{\xi_t^P,j}' + R_{\xi_t^P,j} Q^2 R_{\xi_t^P,j}' \text{ with } Q^2 = QQ' \\
e_{t-1+d_i/nd|t-1}^{(i,j)} &= X_{t-1+d_i/nd} - D_{j,t-1+d_i/nd} - Z_{j,t-1+d_i/nd} \left[\widetilde{S}_{t|t-1}^{(i,j)\prime}, \widetilde{y}_{t-1} \right] \\
f_{t|t-1}^{(i,j)} &= Z_{j,t-1+d_1/nd} P_{t|t-1}^{(i,j)} Z_{j,t-1+d_1/nd}' + U_{t-1+d_1/nd}^2 \\
S_{t|t-1+d_i/nd}^{(i,j)} &= S_{t|t-1}^{(i,j)} + P_{t|t-1}^{(i,j)} Z_{j,t-1+d_i/nd}' \left(f_{t|t-1}^{(i,j)} \right)^{-1} e_{t-1+d_i/nd|t-1}^{(i,j)} \\
P_{t|t-1+d_i/nd}^{(i,j)} &= P_{t|t-1}^{(i,j)} - P_{t|t-1}^{(i,j)} Z_{j,t-1+d_i/nd}' \left(f_{t|t-1}^{(i,j)} \right)^{-1} Z_{j,t-1+d_i/nd} P_{t|t-1}^{(i,j)}
\end{aligned}$$

2. Run the Hamilton filter to calculate new regime probabilities $\Pr\left(\xi_t^b, \xi_{t-1}^b | X_{t-1+d_i/nd}, X^{t-1}\right)$ and $\Pr\left(\xi_t^b | X_{t-1+d_i/nd}, X^{t-1}\right)$, for $i, j = 1, 2, ..., B$

$$\begin{aligned}
\Pr\left(\xi_t^b, \xi_{t-1}^b | X^{t-1}\right) &= \Pr\left(\xi_t^b | \xi_{t-1}^b\right) \Pr\left(\xi_{t-1}^b | X^{t-1}\right) \\
\ell\left(X_{t-1+d_i/nd} | X^{t-1}\right) &= \textstyle\sum_{j=1}^B \sum_{i=1}^B f\left(X_{t-1+d_i/nd} | \xi_{t-1}^b = i, \xi_t^b = j, X^{t-1}\right) \Pr\left[\xi_{t-1}^b = i, \xi_t^b = j | X^{t-1}\right] \\
f\left(X_{t-1+d_i/nd} | \xi_{t-1}^b = i, \xi_t^b = j, X^{t-1}\right) &= (2\pi)^{-N_X/2} |f_{t+1|t}^{(i,j)}|^{-1/2} \exp\left\{ -\frac{1}{2} e_{t-1+d_i/nd|t-1}^{(i,j)\prime} f_{t|t-1}^{(i,j)} e_{t-1+d_i/nd|t-1}^{(i,j)} \right\} \\
\mathcal{L}(\theta) &= \mathcal{L}(\theta) + \ln\left(\ell\left(X_{t-1+d_i/nd} | X^{t-1}\right)\right) \\
\Pr\left(\xi_t^b, \xi_{t-1}^b | X_{t-1+d_i/nd}, X^{t-1}\right) &= \frac{f\left(X_{t-1+d_i/nd} | \xi_t^b, \xi_{t-1}^b, X^{t-1}\right) \Pr\left(\xi_t^b, \xi_{t-1}^b | X^{t-1}\right)}{\ell\left(X_{t-1+d_i/nd} | X^{t-1}\right)} \\
\Pr\left(\xi_t^b | X_{t-1+d_i/nd}, X^{t-1}\right) &= \sum_{i=1}^B \Pr\left(\xi_t^b, \xi_{t-1}^b = i | X_{t-1+d_i/nd}, X^{t-1}\right)
\end{aligned}$$

3. Using $\Pr\left(\xi_t^b, \xi_{t-1}^b | X_{t-1+d_i/nd}, X^{t-1}\right)$ and $\Pr\left(\xi_t^b | X_{t-1+d_i/nd}, X^{t-1}\right)$, collapse the $B \times B$ values of $S_{t|t-1+d_i/nd}^{(i,j)}$ and $P_{t|t-1+d_i/nd}^{(i,j)}$ into B values represented by $S_{t|t-1+d_i/nd}^{j}$ and $P_{t|t-1+d_i/nd}^{j}$:

$$S_{t|t-1+d_i/nd}^{j} = \frac{\sum_{i=1}^{B} \Pr\left[\xi_{t-1}^b = i, \xi_t^b = j | X_{t-1+d_i/nd}, X^{t-1}\right] S_{t|t-1+d_i/nd}^{(i,j)}}{\Pr\left[\xi_t^b = j | X_{t-1+d_i/nd}, X^{t-1}\right]}$$

$$P_{t|t-1+d_i/nd}^{j} = \frac{\sum_{i=1}^{B} \Pr\left[\xi_{t-1}^b = i, \xi_t^b = j | X_{t-1+d_i/nd}, X^{t-1}\right] \left(P_{t|t-1+d_i/nd}^{(i,j)} + \left(S_{t|t-1+d_i/nd}^{j} - S_{t|t-1+d_i/nd}^{(i,j)}\right)\left(S_{t|t-1+d_i/nd}^{j} - S_{t|t-1+d_i/nd}^{(i,j)}\right)'\right)}{\Pr\left[\xi_t^b = j | X_{t-1+d_i/nd}, X^{t-1}\right]}$$

4. If $t - 1 + d_i/nd = t$, move to the next period by setting $t - 1 = t$ and returning to step 1

5. else, store the updated $S_{t|t-1+d_i/nd}^{j}$, $P_{t|t-1+d_i/nd}^{j}$, $\Pr\left(\xi_t^b, \xi_{t-1}^b | X_{t-1+d_i/nd}, X^{t-1}\right)$, and $\Pr\left(\xi_t^b | X_{t-1+d_i/nd}, X^{t-1}\right)$, and repeat steps 1-5 keeping $t - 1$ fixed.

- At $t = T_1 + 1$ use $\theta_{\xi_t^P}$ relevant when $\xi_t^P = 2$, set $t - 1 = t$, and repeat steps 1-5
- At $t = T_2 + 1$ use $\theta_{\xi_t^P}$ relevant when $\xi_t^P = 3$, set $t - 1 = t$, and repeat steps 1-5
- $\vdots$
- At $t = T_{N_P - 1} + 1$ use $\theta_{\xi_t^P}$ relevant when $\xi_t^P = N_P$, set $t - 1 = t$ and repeat steps 1-5
- At $t = T_N = T$ stop. Obtain $\mathcal{L}(\theta) = \sum_{t=1}^{T} \ln\left(\ell\left(X_t | X^{t-1}\right)\right)$.

The algorithm above is described in general terms; in principle the intermonth loop could be repeated at every instant within a month for which we have new data. In application, we repeat steps 1-5 only at certain minutes or days pre- and post-FOMC meeting.

Observation Equation The mapping from the variables of the model to the observables in the data can be written using matrix algebra to obtain the observation equation $X_t = D_{\xi_t,t} + Z_{\xi_t,t}\left[S_t', \widetilde{y}_{t-1}\right]' + U_t v_t$. Denote $\widehat{g}_t = g_t - g$, and $\widehat{lp}_t = lp_t - lp$. Using the definition of stochastically detrended output, we have $\widetilde{y}_t = \ln\left(Y_t/A_t\right)$, $\Delta\ln\left(A_t\right) \equiv g_t = g + \rho_g\left(g_t - g\right) + \sigma_g \varepsilon_{g,t} \Rightarrow \widetilde{y}_t - \widetilde{y}_{t-1} = \Delta\ln\left(Y_t\right) - g_t \Rightarrow \Delta\ln\left(Y_t\right) = \widetilde{y}_t - \widetilde{y}_{t-1} + g_t = \widetilde{y}_t - \widetilde{y}_{t-1} + \widehat{g}_t + g$. Annualizing the monthly growth rates to get annualized GDP growth we have $\Delta\ln\left(GDP_t\right) \equiv 12\Delta\ln\left(Y_t\right) = 12g + 12\left(\widetilde{y}_t + \widehat{g}_t - \widetilde{y}_t\right)$. Thus the vector of observables X_t is defined as:

$$\begin{bmatrix} \Delta \ln(GDP_t) \\ Inflation \\ FFR \\ SOC(Inflation)_{12m} \\ SOC(Inflation)_{60m} \\ f_t^{(0)} \\ BC(Inflation)_{12m} \\ SPF(Inflation)_{12m} \\ Liv(Inflation)_{12m} \\ SPF(GDPDInfl)_{12m} \\ BBG(Inflation)_{12m} \\ Liv(Inflation)_{120m} \\ SPF(Inflation)_{120m} \\ BC(FFR)_{12m} \\ BC(\Delta GDP)_{12m} \\ BBG(\Delta GDP)_{12m} \\ SPF(\Delta GDP)_{12m} \\ f_t^{(n)} \\ Baa_t \\ pgdp_t \\ egdp_t \end{bmatrix} = \begin{bmatrix} 12g \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ D_{\pi_{t,t+12},\xi_t} \\ D_{\pi_{t,t+12},\xi_t} \\ D_{\pi_{t,t+12},\xi_t} \\ D_{\pi_{t,t+12},\xi_t} \\ D_{\pi_{t,t+12},\xi_t} \\ D_{\pi_{t,t+120},\xi_t} \\ D_{\pi_{t,t+120},\xi_t} \\ D_{i_{t,t+12},\xi_t^b} \\ D_{y_{t+s},\xi_t^b} \\ D_{y_{t+s},\xi_t^b} \\ D_{y_{t+s},\xi_t^b} \\ D_{i_{t+X},\xi_t} \\ C_{Baa} \\ \ln(K) + g \\ C_{egdp} \end{bmatrix} + \begin{bmatrix} 12(\widetilde{y}_t + \widehat{g}_t - \widetilde{y}_t) \\ 12\pi_t \\ 12i_t \\ \left[h + (h-1)\phi + (h-2)\phi^2 + ... + \phi^{11}\right](1-\phi)\overline{\pi}_t + \left[\phi + \phi^2 + ... + \phi^{12}\right]\pi_t \\ \left[h + (h-1)\phi + (h-2)\phi^2 + ... + \phi^{59}\right](1-\phi)\overline{\pi}_t + \left[\phi + \phi^2 + ... + \phi^{60}\right]\pi_t \\ 12i_t \\ Z_{\pi_{t,t+12},\xi_t} S_t \\ Z_{\pi_{t,t+12},\xi_t} S_t \\ Z_{\pi_{t,t+12},\xi_t} S_t \\ Z_{\pi_{t,t+12},\xi_t} S_t \\ Z_{\pi_{t,t+12},\xi_t} S_t \\ Z_{\pi_{t,t+120},\xi_t} S_t \\ Z_{\pi_{t,t+120},\xi_t} S_t \\ Z_{i_{t,t+12},\xi_t^b} S_t \\ Z_{\xi_t^b, y_{t+s}-y_t} S_t \\ Z_{\xi_t^b, y_{t+s}-y_t} S_t \\ Z_{\xi_t^b, y_{t+s}-y_t} S_t \\ Z_{i_{t+X},\xi_t^b} S_t \\ B\widetilde{lp}_t \\ \widetilde{k}_t + pd_t + \widehat{g}_t + \widetilde{y}_t - \widetilde{y}_{t-1} \\ K\widetilde{k}_t \end{bmatrix} + U_t v_t \tag{A.13}$$

where in the last row we have used the fact that expectations for the macro agent in the model is:

$$\begin{aligned} \mathbb{E}_t^m[\pi_{t,t+h}] &= \left[h + (h-1)\phi + (h-2)\phi^2 + ... + \phi^{h-1}\right]\alpha_t^m + \left[\phi + \phi^2 + ... + \phi^h\right]\pi_t \\ &= \left[h + (h-1)\phi + (h-2)\phi^2 + ... + \phi^{h-1}\right](1-\phi)\overline{\pi}_t + \left[\phi + \phi^2 + ... + \phi^h\right]\pi_t \end{aligned}$$

The term $Inflation$ in the above stands for CPI inflation; $GDPDInfl$ refers to GDP deflator inflation. The $f_t^{(n)}$ refers to the federal funds futures market return data, we consider, where $f_t^{(n)}$ is the average effective Fed funds rate in month n of contract expiry. Here we use $n = \{6, 10, 20, 35\}$. The variable $pgdp$ is the SP500 capitalization-to-lagged GDP ratio, i.e., P_t/GDP_{t-1}; $egdp_t$ is the SP 500 earnings-to-lagged GDP ratio, i.e., E_t/GDP_{t-1}; Baa_t is the Baa spread described above, where C_{Baa} and B and K are parameters. To allow for the fact that the true convenience yield is only a function of Baa_t, we add a constant C_{Baa} to our model-implied convenience yield lp_t and scale it by the parameter B to be estimated. Unless otherwise indicated, all survey expectations are 12 month-ahead forecasts in annualized units.

The above uses multiple measures of observables on a single variable, e.g., investor expectations of inflation 12 months ahead are measured by four different surveys (BC, SPF, LIV, and BBG). In the filtering algorithm above, these provide four noisy signals on the same latent variable.

Computing the Posterior

The likelihood is computed with the Kim's approximation to the likelihood, as explained above, and then combined with a prior distribution for the parameters to obtain the posterior. A

block algorithm is used to find the posterior mode as a first step. Draws from the posterior are obtained using a standard Metropolis-Hastings algorithm initialized around the posterior mode. Here are the key steps of the Metropolis-Hastings algorithm:

- Step 1: Draw a new set of parameters from the proposal distribution: $\vartheta \sim N\left(\theta_{n-1}, c\overline{\Sigma}\right)$
- Step 2: Compute $\alpha\left(\theta^m;\vartheta\right) = \min\left\{p\left(\vartheta\right)/p\left(\theta^{m-1}\right),1\right\}$ where $p\left(\theta\right)$ is the posterior evaluated at θ.
- Step 3: Accept the new parameter and set $\theta^m = \vartheta$ if $u < \alpha\left(\theta^m;\vartheta\right)$ where $u \sim U\left([0,1]\right)$, otherwise set $\theta^m = \theta^{m-1}$
- Step 4: If $m \geq n^{sim}$, stop. Otherwise, go back to step 1

The matrix $\overline{\Sigma}$ corresponds to the inverse of the Hessian computed at the posterior mode $\overline{\theta}$. The parameter c is set to obtain an acceptance rate of around 30%. We use four chains of $540,000$ draws each (1 of every 200 draws is saved). The four chains combined are used to form an estimate of the posterior distribution from which we make draws. Convergence is checked by using the Brooks-Gelman-Rubin potential reduction scale factor using within and between variances based on the four multiple chains used in the paper.

Risk Adjustment with Lognormal Approximation

The asset pricing block of equations involves conditional subjective variance terms that are affected by Markov-switching random variables in the model. We extend the approach in Bansal and Zhou (2002) of approximating a model with Markov-switching random variables using a risk-adjustment while maintaining conditional log-normality. Consider the forward looking relation for the price-payout ratio:

$$\begin{aligned} P_t^D &= \mathbb{E}_t^b\left[M_{t+1}\left(P_{t+1}^D + D_{t+1}\right)\right] \\ \frac{P_t^D}{D_t} &= \mathbb{E}_t^b\left[M_{t+1}\frac{D_{t+1}}{D_t}\left(\frac{P_{t+1}^D}{D_{t+1}}+1\right)\right]. \end{aligned}$$

Taking logs on both sides, we get:

$$pd_t = \log\left[\mathbb{E}_t^b\left[\exp\left(m_{t+1}+\Delta d_{t+1}+\kappa_{pd,0}+\kappa_{pd,1}pd_{t+1}\right)\right]\right].$$

Applying the approximation implied by conditional log-normality we have:

$$\begin{aligned} pd_t &= \kappa_0 + \mathbb{E}_t^b\left[m_{t+1}+\Delta d_{t+1}+\kappa_{pd,1}pd_{t+1}\right] + \\ &\quad +.5\mathbb{V}_t^b\left[m_{t+1}+\Delta d_{t+1}+\kappa_{pd,1}pd_{t+1}\right]. \end{aligned}$$

To implement the solution, we follow Bansal and Zhou (2002) and approximate the conditional variance as the weighted average of the objective variance across regimes, conditional on ξ_t. Using the simpler notation of the state equation,

$$S_t = C_{\xi_t} + T_{\xi_t}S_{t-1} + R_{\xi_t}Q\varepsilon_t,$$

the approximation takes the form

$$\mathbb{V}_t^b \left[x_{t+1} \right] \approx e_x \mathbb{E}_t^b \left[R_{\xi_{t+1}} Q Q' R'_{\xi_{t+1}} \right] e_x \tag{A.14}$$

where e_x is a vector used to extract the desired linear combination of the variables in S_t. This approximation maintains conditional log-normality of the entire system. In the solution, C_{ξ_t} depends on the risk adjustment term $\mathbb{V}_t^b \left[m_{t+1} + \Delta d_{t+1} + \kappa_{pd,1} pd_{t+1} \right]$ which depends on R_{ξ_t}. To solve this fixed point problem, employ the iterative approach of Bianchi, Kung, and Tirskikh (2018). Specifically, we solve the model and get S_t for a given risk adjustment. Then, given S_t, we compute a new risk adjustment, which gives us a new $R_{\xi_{t+1}}$. If the new $R_{\xi_{t+1}}$ is the same as the $R_{\xi_{t+1}}$ that was implicitly assumed in the given risk adjustment, stop. Otherwise, use it to resolve S_t and repeat the procedure until convergence.