The Costs and Benefits of Rules of Origin in Modern Free Trade Agreements

Emanuel Ornelas and John L. Turner†

January 19, 2022

Abstract

We study the welfare impact of rules of origin in free trade agreements where final-good producers source specialized inputs from suppliers within the trading bloc. We employ a property-rights framework where terms of trade for intermediate inputs are determined via bargaining, creating hold-up problems. Supplier underinvestment is more severe for higher productivity firms. A rule of origin offers preferred market access for final goods if a sufficiently high fraction of inputs used in the production process is sourced within the trading bloc. Such a rule alters behavior for only a subset of suppliers, as some (very-high-productivity) suppliers always comply with the rule in an unconstrained way and some (very-low-productivity) suppliers choose not to comply. For those suppliers it does affect, the rule increases investment, but it also induces excessive sourcing within the trading bloc. From a social standpoint, it is best to have a rule that affects high-productivity suppliers, since they are more affected by the original underinvestment problem. This calls for a relatively *strict* rule, which requires a high share of within-bloc specialized inputs. Furthermore, provided that the rule binds, we show that the marginal net welfare gain from tightening the rule increases with productivity. Therefore, when industry productivity is higher, a stricter rule of origin is more socially desirable.

JEL Classification: F13; F15; L22; D23.
Keywords: Hold-up problem; Sourcing; Incomplete contracts; Regionalism.

†Ornelas: Sao Paulo School of Economics-FGV, CEPR, CESifo and CEP-LSE; eaornelas@gmail.com. Turner: Department of Economics, Terry College of Business, University of Georgia, Athens, GA 30602; jlturner@uga.edu. We thank Bruno Daré for competent research assistance and brownbag participants at the University of Georgia for helpful comments.

1 Introduction

Rules of origin (ROOs) are a pervasive feature of existing free trade agreements (FTAs). Under these rules, final goods whose inputs come substantially from within-bloc countries are traded freely, but otherwise face tariffs. Their formal goal is to prevent the trans-shipment of imported goods from low-tariff to high-tariff countries within the same trading bloc, but the nature and the stringency of the rules vary substantially across agreements and across products within agreements.[1] Generally, economists take a dim view of ROOs, as they tend to lower welfare in competitive models (Grossman, 1981; Krishna, 2006). The typical view is that ROOs are distortionary because they "prevent final good producers from choosing the most efficient input suppliers around the world" (Conconi et al., 2018, p. 2336). Accordingly, the empirical literature customarily interprets findings that ROOs have a negative effect on imports from third countries as evidence of welfare-reducing trade diversion.

While this is true in some contexts, it may not be when within-FTA sourcing is inefficiently low absent ROOs. That is the case when firms need to make relationship-specific, unrecoverable investments that cannot be fully contracted upon, a situation that typically arises when inputs are customized. And in the modern era of global sourcing, customization is indeed prevalent in many industries. This is particularly common in the context of FTAs, which both promote and are promoted by global/regional value chains (Baldwin, 2011; Ruta, 2017). In such cases, terms of trade are determined via bargaining and input suppliers may face hold-up problems. But then, if a ROO induces changes in input mixes, there is not necessarily a welfare loss, and there may be welfare gains. We capture and study these effects in this paper, where we provide the first analysis of ROOs using a property-rights model.

Building on Ornelas, Turner and Bickwit (2021) – henceforth OTB – we consider an environment where specialized input suppliers with heterogeneous productivity form vertical chains with producers of final goods. Both firms belong to countries in the same FTA

[1]For example, mixing of agricultural products (e.g., blended beverages) typically confers origin when a threshold amount of domestic goods are included (Inama, 2009, pp. 80-81). In the US, coating of fabrics is origin-conferring if the weight of rubber or plastics of the finished good is more than 50% of the weight of the product (Inama, 2009, p. 85). Value-added rules are also common in the assembly of machines (Inama, 2009, pp. 93, 100). Moreover, countries often disagree over what is origin-conferring, and these disagreements tend to reflect the positions of domestic producers. For example, Colombia is a major producer of coffee and wishes to retain origin, so it does not consider roasting or decaffeination as conferring origin.

and investments are non-contractible. Within each vertical chain, the supplier invests in technology (marginal cost curve reduction) prior to bargaining with the producer over inputs. The producer also has the option to purchase generic inputs from a competitive world market, and the model is set up so that the producer always sources a mix of specialized and generic inputs in equilibrium. Under free trade, investments are inefficient due to the possibility of hold up. Crucially, because investment affects more units of inputs in more productive firms, the underinvestment problem is more severe for higher productivity firms.

To this model we add a rule such that the final goods produced by a vertical chain enjoy preferred market access as long as the fraction of inputs exceeds a specified level. We find that this rule may increase or decrease welfare, but that welfare is more likely to rise when it is *stricter*. The reason is twofold. First, any rule of origin will affect choices made by just a subset of suppliers, and a stricter rule targets higher productivity firms. Second, for such high-productivity suppliers, a relatively strict rule will be desirable because the marginal net welfare gain from tightening the rule increases with productivity.

The heterogeneous incidence echoes prior and recent work on ROOs (Grossman 1981; Head et al., 2021), where the rule matters only for particular levels of the supply curve and there are three cases to consider. In our model, we can classify the cases according to supplier productivity. The highest-productivity suppliers have the lowest marginal cost curves and produce very high levels of intermediate inputs. Their vertical chains comply with the ROO without altering investments, so the rule has no effect on their behavior. At the opposite extreme, the lowest-productivity suppliers have the highest marginal cost curves and produce very low levels of intermediate inputs. Because compliance would be too costly for them, their vertical chains are unwilling to comply with the ROO, so the rule does not affect their sourcing behavior either. Finally, vertical chains with suppliers within an intermediate productivity range find it optimal to source extra specialized inputs within the FTA to gain preferential access for final goods.

For these suppliers, we say the rule is *effective*. The extra inputs serve as a commitment device for the supplier to invest more. This additional investment tends to improve efficiency when the supplier is initially underinvesting. Because the underinvestment problem is more severe for high-productivity suppliers, a rule that is effective only for relatively

high-productivity suppliers tends to improve welfare. This obtains for a relatively strict rule, which low-productivity suppliers do not comply with.

This result is mediated by the level of the final-good tariff in the FTA partner, which determines the extent of the gains from compliance with the ROO. Most obviously, if that tariff is very close to zero, any ROO will be ineffective because vertical chains have nothing to gain by altering their sourcing choices. But for a sufficiently high final-good tariff, there will be a socially optimal level for the rule, which trades off the benefits from mitigating the hold-up problem against the costs due to excessive within-FTA sourcing. Because the marginal gain from tightening the rule is higher, and the corresponding marginal cost is lower, for high-productivity firms, a hypothetical optimal rule set at the firm level would be increasing in productivity. Thus, a relatively strict rule is both effective for more productive suppliers and is particularly beneficial (from a social standpoint) when it affects their behavior.

Considering the whole distribution of suppliers, analogous results obtain. With a continuous density of productivity, a stricter ROO is more likely to generate a positive welfare effect for all affected suppliers. The intuition is similar: a stricter ROO is effective for more productive suppliers, and the potential welfare gains are higher for higher-productivity suppliers. Now, whether aggregate welfare rises or falls with the stringency of a ROO does depend on the distribution of supplier productivity. Notably, however, welfare strictly increases with the stringency of a ROO for the benchmark case of a uniform distribution.

An external tariff on intermediate inputs alters the welfare consequences of a ROO. Without an external tariff on inputs, we show that a sufficiently strict ROO will raise welfare for *any* distribution of suppliers. But a positive input tariff increases investment by all suppliers, leading some to overinvest. An even tighter ROO is then needed to boost welfare, as it is necessary to tailor the ROO to affect higher-productivity suppliers, who are still underinvesting. However, when the external tariff on inputs is sufficiently high, all suppliers overinvest and any ROO would only worsen welfare, because it is ill-suited to address overinvestment problems.

If we consider limiting cases of our environment, then it is possible to find other situations where a ROO always worsens welfare. First, if there is no investment decision by the supplier,

then there is no underinvestment problem and the ROO leads just to trade diversion. This is the case usually considered in the literature, which justifies the interpretation of many empirical analyses. Second, if suppliers have full bargaining ability, then investments are efficient without external tariffs and ROOs, and any rule will lead to trade diversion and excessive investments. Third, if suppliers have no bargaining ability, then ROOs do not affect investments and yield only trade diversion. Put together, these cases make clear that ROOs can be beneficial only if (1) hold-up problems are present and (2) investment responds to policy.

The paper is organized as follows. After the discussion of the related literature in the next subsection, we set up the basic model in section 2. In section 3, we study the choice of input mix for given level of investment. We then move to understand firms' choice of investment in section 4. This allows us to analyze the welfare impacts of ROOs in section 5. In section 6, we extend the analysis to the case where there is a strictly positive external tariff on inputs, and in section 7 we discuss modeling alternatives and extensions of our model. We conclude in section 8.

1.1 Related Literature

The theoretical literature on ROOs goes back to Grossman (1981), who studies the consequences of local content requirements rules, including the case where access to preferential treatment for exports requires a minimum level of domestic value added. For that situation, in a competitive setting, he identifies the three cases of non-compliance, unconstrained compliance and constrained compliance, as we also find in our environment. Krishna (2006) offers a similar perspective. Also in a competitive model, Falvey and Reed (1998) show how ROOs can distort allocative efficiency and underline the many ways in which "origin" can be defined. In a recent paper, Chung and Perroni (2021) depart from the competitive benchmark to study the effects of ROOs when markets are oligopolistic and show that stricter ROOs tend to lead to higher prices for intermediate goods.

Recently, Head et al. (2021) evaluate the effect of NAFTA's ROOs for auto parts. They use a structural model that yields a "ROO Laffer curve" with respect to the share of intra-bloc inputs used. The curve reflects the tripartite separation of firms regarding compliance

depending on their cost structures. Intuitively, at lenient levels tightening the rule induces more within-bloc sourcing, but eventually the effect reverses, as more firms choose to not comply under a strict rule. A similar pattern emerges in our model as well.

Naturally, the motives behind ROOs are diverse, and raising aggregate welfare may be far from the main goal in some circumstances. In particular, as Krueger (1999) forcefully notes, ROOs can be imposed for protectionist reasons and constitute a source of economic inefficiency in FTAs. We do not study how or why ROOs are actually chosen, but we show that a stricter rule can in many cases be welfare improving, regardless of its motivations.[2]

Empirically, several papers evaluate the impact of ROOs on trade flows from a reduced-form perspective (see, for example, the studies in Cadot et al., 2006). In contrast, Cherkashin et al. (2015) structurally estimate an heterogeneous-firm model with firm-market specific demand shocks to study the impact of changing the costs of meeting ROOs faced by Bangladeshi exports in apparel. They find that fewer requirements for meeting ROOs are associated with greater exports and more entry in the long run. Conconi et al. (2018) provide an insightful product-level analysis of NAFTA, finding that ROOs induce a relocation of sourcing from outside to inside the bloc. That type of result is often taken as evidence that ROOs are distortionary. Yet here we show that greater within-FTA sourcing need not be distortionary and can be welfare enhancing. Thus, an implication of our analysis is that market structure and firm organization must be considered in the interpretation of all empirical results about the effects of ROOs.

Other empirical papers document the incomplete use of preferences in FTAs, and often associate that to poorly designed ROOs. Recently, Crivelli et al. (2021) reveal that preferences in the European Union FTAs, although widely used, are still far from being used by all potential beneficiaries. They provide a detailed account of product-specific ROOs that are too strict to be useful by some firms. Our analysis makes clear that partial utilization is generally inevitable, as firms with different levels of productivity will have different incentives to comply. In particular, the vertical chains with *low-productivity* suppliers in the FTA will

[2] [24], building on the theoretical analysis of FTA formation by Saggi and Yildiz (2010), shows that ROOs could be useful to facilitate the achievement of global free trade. Although in a very distinct context, our analysis connects with Tsirekidze's in that he also uncovers a potentially beneficial role for ROOs, in contrast with the largely negative view from most of the literature.

choose not to comply. From a social welfare perspective, this tends to be beneficial, because they are the least affected by hold-up problems. Related to this point, Krishna et al. (2021) show how documentation costs to satisfy ROOs prevent the full use of preferences in FTAs. Interestingly, they find that firms learn how to satisfy ROOs as they accumulate experience exporting a product to an FTA partner, implying that the fixed cost of documentation falls over time at the firm level. We do not consider fixed costs to use ROOs in our main analysis, but we discuss how they could be incorporated in section 7.

Finally, as indicated above, methodologically this paper is closest to OTB. The main difference is the focus. In OTB we study the welfare effects of an FTA due to within-bloc reduction of input tariffs.[3] Considering the institutional design of FTAs around the world, here we take the next logical step. Given free trade in inputs within the FTA, we analyze how free trade in final goods within the bloc, when mediated by ROOs – as in all existing FTAs – affect the desirability of the trading bloc. Thus, this paper helps toward a fuller understanding of the welfare implications of free trade agreements. Our general approach is also related to that of Antràs and Staiger (2012). Like here, in their model buyers and producers must negotiate their terms of trade through bargaining. Their goal is, however, quite distinct from ours – to study the optimal design of nondiscriminatory trade agreements, where ROOs are not an issue.[4]

2 The Model

We build on OTB's model, with the key departure being the introduction of a ROO requirement to make the trade of final goods tariff-free within an FTA. As we will see, the ROO alters sourcing decisions which, in turn, alter investment decisions. This has implications for the incidence of hold-up problems and for the welfare consequences of FTAs.

There is a continuum of differentiated final consumption goods in the world economy. Consumption of those goods increases consumers' utility at a decreasing rate, and there are

[3] In Ornelas and Turner (2008, 2012), in turn, we seek to rationalize the worldwide growth of trade flows relative to GDP until 2007 and to study optimal unilateral tariffs when firms' organizational mode is endogenous, respectively.

[4] For a recent review of models of international trade featuring incomplete contracts, bargaining, specialized components, as well as search and matching, in the context of global value chains, see Antràs (2020).

no cross-product effects. There is also an homogeneous numéraire good y that enters consumers' utility function linearly. Thus, if they purchase any y, extra income will be directed to the consumption of that good. We assume relative prices are such that consumers always purchase some y. Production of one unit of y requires one unit of labor, the market for y is perfectly competitive and y is traded freely. This sets the wage rate in the economy to unity. Under this structure, there are no general equilibrium effects across sectors. Thus, without further loss of generality, we develop the analysis as if there were a single differentiated sector, which produces final good x. Entirely analogous analyses could be carried out for other differentiated sectors.

Production of the differentiated final good x is carried out by final-good producers (F) – or simply *producers*, for brevity – located in the *Home* country. Those firms transform intermediate inputs, all produced only with labor, into good x, using production function $V(Q)$, where Q denotes the number of inputs purchased. The world price of good x is p_w^x. We consider that $Home$ is small in world markets, and therefore its producers take world prices as given.

Home is in a free trade agreement with *Foreign*. This means that trade between them is free, provided that rules of origin, when present, are respected. We consider that, under free trade, *Home* is an exporter of x while *Foreign* is an importer of x. The FTA's ROO requires that fraction $r \in (0, 1)$ of the inputs used to produce x come from within the FTA. If final-good producers fail to comply with the ROO, they must pay *Foreign*'s MFN specific tariff $\tau > 0$ on final goods sold to *Foreign*. Hence, compliance yields savings of τ times $Home$'s exports of x to $Foreign$.

When sourcing, each producer may purchase generic inputs z available in the world market (ROW) and/or specialized inputs q from a *Home* supplier (S).[5] As in Grossman (1981), we assume that intermediates goods are perfect substitutes. Generic inputs are produced competitively, require p_w^z units of labor, and are thus priced in the world market at p_w^z. Again, decisions in *Home* do not affect p_w^z. In our baseline model, there are no tariffs assessed on z. We relax this assumption in section 6.

[5]It would make no difference for the analysis if specialized suppliers were located in *Home*'s FTA partner, *Foreign*, provided that they could sell to producers in *Home* without incurring tariffs.

We define units so that one unit of generic input and one of specialized input have the same revenue-generating value for a producer. Under this normalization, F's revenue depends on just the total number of intermediate inputs he purchases, Q, and not its composition. Note that the ROO requires $q \geq rQ$.

Now, to acquire specialized inputs, a producer must first form a *vertical chain* with a supplier. Upon forming a vertical chain, F and S specialize their technologies toward each other. This implies that a producer purchases specialized inputs from only one supplier. All producers are identical, whereas each supplier is identified by ω, an heterogeneity parameter that indexes (the inverse of) her productivity. The distribution of matched suppliers follows a continuous and strictly increasing distribution $G(\omega)$, with an associated density $g(\omega)$, where ω lies on $[0, p_w^z]$.[6]

Once F and S are specialized toward each other, S makes a non-contractible relationship-specific investment that lowers her marginal cost prior to trade with F. The investment is observed by both F and S, but is not verifiable in court. Nothing essential would change if the producer also made an analogous ex-ante investment.

After investment is sunk, the firms decide how much to trade and at what price. The specialized inputs are not traded on an open market, and have no scrap value. Furthermore, the parties cannot use contracts to affect their trading decisions.[7] Instead, they need to bargain over price and quantity of specialized inputs. If bargaining breaks down, S produces the numéraire good and earns zero (ex post) profit, while F purchases only generic inputs. If bargaining is successful, F imports generic inputs from ROW and purchases specialized inputs from S.

The timing of events within each vertical chain is therefore as follows. First, (i) S makes

[6]Here we take the FTA, and hence the structure of matching, as given, so that we can focus on the first-order welfare effects of ROOs. Our specification is consistent with the "large natural trading partner" case from OTB, where all specialized suppliers are in the same country and changes to FTA characteristics do not affect matches. Note that OTB also fully characterizes how an FTA alters the matching equilibrium for cases where suppliers are located in multiple countries. See Grossman and Helpman (2021) for a detailed analysis of how discriminatory tariffs affect the structure of global value chains in a setting where matching and bargaining take central stage.

[7]This is the same approach used by Antràs and Staiger (2012), among others analyzing related environments. It can be formally justified if, for example, quality were not verifiable in a court and the supplier could produce either high-quality or low-quality specialized inputs, with low-quality inputs entailing a negligible production cost for the supplier but being useless to the producer.

an irreversible relationship-specific investment. Once the investment is sunk, (*ii*) F and S bargain over price and quantity of q. If bargaining is successful, trade of q takes place; otherwise, $q = 0$ and S produces the numéraire good. Subsequently, (*iii*) F purchases z. Then, (*iv*) production occurs and final good x is sold, with payments dependent on whether the ROO is satisfied. We solve the game by backward induction, from the perspective of a single vertical chain. As in OTB, we use the term *Y-chain* when referring to the entire supply chain, as distinct from the F-S vertical chain.[8]

2.1 Functional Forms

To generate clear-cut analytical solutions, we adopt some specific functional forms. Conditional on investment i, we specify the cost function for specialized units q as

$$C(q, i, \omega) = (\omega - bi)q + \frac{c}{2}q^2.$$

Parameter ω is the intercept of the marginal cost curve; the lower is ω, the more efficient is the supplier. The slope of the marginal cost curve is c, while the effectiveness of investment in reducing production costs is b. In turn, the investment cost is

$$I(i) = i^2.$$

We assume that $2c > b^2$. This ensures that the choice of investment is always finite.

This linear-quadratic specification displays properties that are standard and provide a good representation of the key elements of our environment: investment and productivity reduce both total cost and marginal cost ($C_i < 0, C_{qi} < 0, C_\omega > 0, C_{q\omega} > 0$); the marginal cost curve is positively sloped ($C_{qq} > 0$) but its slope can vary (c is a parameter), and the cost of investment is increasing and convex ($I' > 0, I'' > 0$). This specification has the advantage of permitting full analytical solutions at the level of a single producer-supplier pair and a precise welfare analysis.

We focus on the case where F engages in dual sourcing, purchasing both generic and specialized inputs in equilibrium. This will always happen provided that the marginal cost

[8] We use *Y-chain* because the supply chain includes two sources of upstream supply and one downstream final-goods producer. Visually, this resembles a Y as in Figure 1 in OTB (p. 6).

of the most productive supplier ($\omega = 0$) is high enough so that F wants to purchase some generic inputs even when matched with that best supplier. In addition to being realistic,[9] this simplifies the analysis. More generally, the important requisite is that the producer must have the option of buying generic inputs when negotiating with his specialized supplier, because that establishes the threat point in the bargaining process.

2.2 Welfare

The welfare generated by a single Y-chain has potentially several components: (1) F's profit; (2) S's profit; (3) consumer surplus (CS) from the final good in $Home$; (4) CS from the final good in $Foreign$; (5) tariff revenue (TR) from imports of x in $Foreign$; (6) TR from imports of z in *Home* (in section 6). Thus, there is welfare due to actions both in the inputs and the final-good markets.

The literature on ROOs has concentrated on how they affect the inputs market. As the discussion in the introduction highlights, ROOs are often associated with welfare-reducing trade diversion in the inputs market, because they can induce excessive sourcing within the bloc. We follow the literature by concentrating the analysis on the inputs market.[10] To do so, we shut down welfare effects in the final-goods market.

Specifically, we consider a variation of Grossman and Helpman's (1995) "enhanced protection" case. In that case, final demand in *Foreign* is large enough relative to supply in *Home*, so that *Home* can sell all it produces in *Foreign* without affecting prices there, which is given by the marginal imported unit, which comes from *ROW*. Thus, in *Foreign* the only welfare change is lost tariff revenue on everything it imports from *Home* under the agreement. Each producer in *Home* will sell all of his production in *Foreign*. $Home$ will import everything it consumes, but since the price does not change (it is given by the world price p_w^x before and after the FTA), consumers are indifferent. The implication is that the welfare components (3) and (4) above are constant, unaffected by the FTA and its ROOs.

In Grossman and Helpman's (1995) model, where the goal is to study the final-goods

[9]For example, Boehm and Oberfield (2020) document that mixing customized and standardized inputs is a common practice for Indian manufacturing plants.

[10]We note that this is not without loss of generality. A given ROO could generate trade diversion in inputs but induce trade creation in final goods, for example.

market, under enhanced protection the net welfare effect is negative because the gain for *Home* producers is less than the loss of tariff revenue in *Foreign*. This happens because *Home* producers will work up in their marginal cost curves. The difference, negative, is the cost of trade diversion for the countries. Here, to isolate the welfare generated in the inputs market, we adopt the following specification for the production function of x:

$$x = V(Q) = \begin{cases} Q & \text{if } Q \leq \bar{Q} \\ \bar{Q} & \text{if } Q > \bar{Q}. \end{cases} \tag{1}$$

That is, F can transform Q into x under constant returns to scale until capacity $\bar{Q}$ is reached; beyond that, additional units of Q are no longer useful. We further assume that $p_w^x - p_w^z$ is sufficiently large to ensure that $Q = \bar{Q}$ is always optimal.[11]

Under this specification, in the final-good market we have trade creation = trade diversion = 0. That is, there is no net gain or loss from trade in the final good x with the FTA, only a transfer between the two countries. Specifically, *Home* benefits from the free internal trade in final goods because each of its final-good producers earns $\tau\bar{Q}$ with the higher price earned by exporting to *Foreign*'s market. Meanwhile, *Foreign* loses exactly $\tau\bar{Q}$ in tariff revenue for each variety of final good x imported from *Home* under the FTA. The net effect for the bloc is therefore nil with respect to the trade of the final good.

An implication is that for each Y-chain, the welfare component (5) above is fully offset by the gain in the final-good market of *Home*'s producers, which is part of the welfare component (1). That is, from the bloc's viewpoint, *the FTA is welfare-neutral with respect to the trade of good x.* Thus, in our baseline model the welfare changes due to an FTA between *Home* and *Foreign* stem *only* from changes in S's profit and in F's profit due to expenditures on inputs.

This setup is, obviously, artificial. We know, since Grossman (1981), that ROOs can have either a positive or a negative impact on final good production and exports. We choose to ignore that possibility to focus on a pedagogically more useful setup, which allows us to concentrate on the first-order effects in the market for inputs. Nevertheless, we indicate in section 7 how our analysis and results would change once one allows final-good production

[11] See section 7 and Appendix B for discussion of the case where firms might choose the total inputs $Q < \bar{Q}$.

to vary as well.

3 The Choice of Inputs Conditional on Investment

After S chooses her investment, F and S bargain over the number and price of the specialized intermediate inputs. We assume the outcome follows Generalized Nash Bargaining and specify the supplier as having bargaining ability $\alpha \in (0,1)$. The two firms jointly choose the number of specialized inputs q and their price p^s according to

$$\max_{\{q,p^s\}} \left[U_S^T - U_S^0\right]^{\alpha} \left[U_F^T - U_F^0\right]^{1-\alpha},$$

where U_j^m is the gross profit (i.e., profit absent transfers) that firm j (either F or S) would receive under scenario m. The two possible scenarios are either bargaining and trading ($m = T$) or not reaching a bargain and thus not trading ($m = 0$).

Conditional on inverse productivity ω, investment i, specialized inputs q and total inputs $\bar{Q}$, producer utilities are:

$$\begin{aligned} U_F^T &= \begin{cases} (p_w^x + \tau)\bar{Q} - p_w^z z - p^s q & \text{if } q \geq q_r \\ p_w^x \bar{Q} - p_w^z z - p^s q & \text{else;} \end{cases} \\ U_F^0 &= \begin{cases} (p_w^x + \tau - p_w^z)\bar{Q} & \text{if } q \geq q_r \\ (p_w^x - p_w^z)\bar{Q} & \text{else,} \end{cases} \end{aligned}$$

where $q_r \equiv r\bar{Q}$. Note that the producer's utility depends on whether the ROO is satisfied. If it is, then he obtains an extra surplus of $\tau\bar{Q}$. If the bargain fails, $q = 0$, $z = \bar{Q}$, and the $q \geq q_r$ constraint is satisfied only if $r = 0$.

For supplier utilities, we have

$$\begin{aligned} U_S^T &= p^s q - C(q, i, \omega); \\ U_S^0 &= 0. \end{aligned}$$

Observe that, while the supplier's utility under a bargain U_S^T does not depend directly on whether the ROO is satisfied, the supplier will share part of the additional producer utility through the input price p^s.

For $r > 0$, the bargaining surplus $\Sigma \equiv \left[U_F^T - U_F^0\right] + \left[U_S^T - U_S^0\right]$ is

$$\Sigma = \begin{cases} \Sigma^{RC} & \equiv & \tau\bar{Q} + p_w^z q - C(q,i,\omega) & \text{if } q \geq q_r \\ \Sigma^{NC} & \equiv & p_w^z q - C(q,i,\omega) & \text{else.} \end{cases}$$

The RC superscript denotes rule-of-origin compliance, while the NC superscript denotes non-compliance.[12]

The vertical chain's efficient bargain selects the level of q that maximizes $\Sigma(q,i,\omega)$. It is simplest to first solve the relaxed problem where the $q \geq q_r$ constraint does not bind. The first-order condition is

$$p_w^z = C_q(q,i,\omega).$$

We can solve this under our functional forms to find

$$q_0 = \frac{p_w^z - \omega + bi}{c}. \tag{2}$$

Notice that q_0 is increasing in the world price of the generic alternative. A higher world price favors within-FTA production of specialized inputs, so the supplier produces more. Similarly, q_0 is decreasing in ω and increasing in the investment, which (respectively) increase and decrease S's marginal cost.

This yields the solution for the optimal q as long as $q_0 \geq q_r$. This unconstrained case holds for a sufficiently high investment,

$$i \geq i_{UC}(\omega) \equiv \frac{\omega - p_w^z + cq_r}{b}. \tag{3}$$

Note that, if r is sufficiently close to zero, then $i_{UC} \leq 0$ and q_0 is optimal for any $i \geq 0$.

Now, if $i < i_{UC}(\omega)$, then $q_0 < q_r$ and the vertical chain has an additional consideration. It may choose to comply with the ROO by choosing at least q_r inputs; or it may choose not to comply with the ROO. A vertical chain that fails to comply has bargaining surplus

$$\Sigma^{NC}(q,i,\omega) = p_w^z q - C(q,i,\omega),$$

[12]Note that in the special case of $r = 0$, the bargaining surplus always equals Σ^{NC} even though compliance is assured.

which is lower than $\Sigma^{RC}(q,i,\omega)$ by the constant term $\tau\bar{Q}$. Again, q_0 maximizes $\Sigma^{NC}(q,i,\omega)$.

By choosing $q \geq q_r$, the vertical chain can earn the extra surplus $\tau\bar{Q}$ while sacrificing some efficiency by producing inputs at a marginal cost higher than p_w^z. Indeed, Σ^{NC} is strictly decreasing in q as q rises above q_0, so the best choice satisfying the constraint is $q = q_r$. The resulting bargaining surplus is $\Sigma^{RC}(q_r,i,\omega)$, and it may be lower or higher than $\Sigma^{NC}(q_0,i,\omega)$. For sufficiently low investment

$$i < i_{NC}(\omega,\tau) \equiv \frac{\omega - p_w^z + cq_r - \sqrt{2c\tau\bar{Q}}}{b}, \tag{4}$$

$\Sigma^{RC}(q_r,i,\omega) < \Sigma^{NC}(q_0,i,\omega)$, so input choice q_0 is optimal and fails to satisfy the ROO. In that case, investment is so low that producing q_r would require pushing marginal cost to an excessively high level. For $i \in [i_{NC}(\omega,\tau), i_{UC}(\omega)]$, that distortion is worth incurring, so we have $\Sigma^{RC}(q_r,i,\omega) > \Sigma^{NC}(q_0,i,\omega)$ and constrained compliance with $q = q_r$ is optimal.

The following proposition summarizes optimal sourcing, for given investment levels.

Proposition 1 *Conditional on ω and i, there exist thresholds $i_{NC}(\omega,\tau)$ and $i_{UC}(\omega)$, with $i_{NC}(\omega,\tau) \leq i_{UC}(\omega)$, such that equilibrium level of inputs q_i satisfies the following:*

$$q_i = \begin{cases} q_0 = \frac{p_w^z - \omega + bi}{c} & \text{if } i < i_{NC}(\omega,\tau) \\ q_r = r\bar{Q} & \text{if } i \in [i_{NC}(\omega,\tau), i_{UC}(\omega)] \\ q_0 = \frac{p_w^z - \omega + bi}{c} & \text{if } i > i_{UC}(\omega). \end{cases}$$

In the subsequent analysis, the following definitions are useful.

Definition 1 *A rule of origin that yields output choice $q_r > q_0$ is* **effective***, while a rule of origin that does not yield a change in output choice is* **ineffective**.

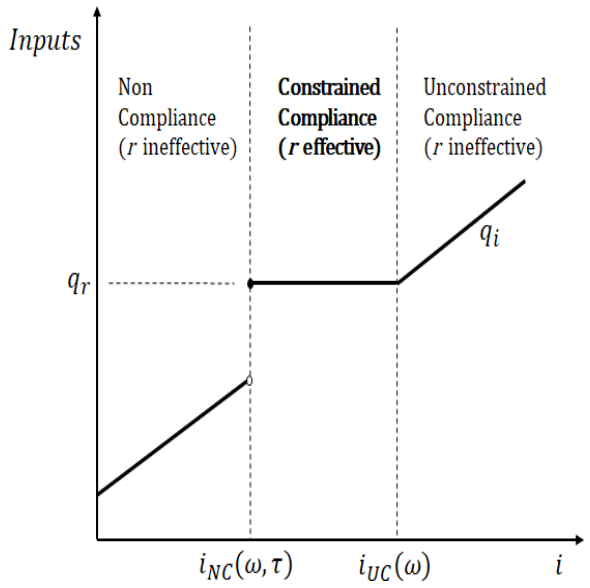
According to Definition 1, a ROO is effective for $i \in [i_{NC}(\omega,\tau), i_{UC}(\omega))$.[13] Figure 1 summarizes, for a given ω, the relationship between investment and the vertical chain's choice

[13]Naturally, "effectiveness" is defined here with respect to the inputs market only. For $i < i_{NC}(\omega,\tau)$, input choices do not change with the ROO, but the vertical chain stops saving $\tau\bar{Q}$ in its exports of x, so the rule does have effects. However, under our specification, that effect is simply a transfer, with no aggregate welfare consequences.

of inputs. The ROO is effective conditional on an investment in the middle range, and is ineffective otherwise. Note from equations (3) and (4) that $i_{NC}(\omega, \tau)$ is a function of τ while $i_{UC}(\omega)$ is not. Clearly, if there were no final-goods tariff ($\tau = 0$), then $i_{NC}(\omega, 0) = i_{UC}(\omega)$ and the ROO would be ineffective, since there would be no reason to alter the input choice away from q_0 to achieve compliance.

The finding that the ROO is effective for some but not all producers resembles early findings from the literature (Grossman, 1981; Krishna, 2006). However, as we will see, our model yields quite novel implications for welfare. We now turn to the investment decision.

Figure 1: *Optimal Specialized Inputs Conditional on Investment*



Note: Conditional on a given ω, this diagram illustrates the relationship between investment i and the choice of specialized inputs q_i that obtains under Generalized Nash Bargaining.

4 The Choice of Investment

Now consider the supplier's choice of investment. She solves

$$max_i \quad U_S(i) \equiv \alpha\Sigma(\omega, i) - I(i), \tag{5}$$

where

$$\Sigma(i,\omega) = \begin{cases} \Sigma^{NC}(q_0, i, \omega) & \text{if} \quad i < i_{NC}(\omega) \\ \Sigma^{RC}(q_r, i, \omega) & \text{if} \quad i \in [i_{NC}(\omega,\tau), i_{UC}(\omega)] \\ \Sigma^{RC}(q_0, i, \omega) & \text{if} \quad i > i_{UC}(\omega) \end{cases}$$

incorporates ROO-effectiveness.

Since q_i is a discontinuous function of investment i, it is not obvious that the supplier's objective function is continuous in investment. However, the following lemma shows that $U_S(i)$ is indeed continuous.

Lemma 1 *The function $U_S(i)$ is continuous on $\left[0, \frac{bp_w^z}{2c-b^2}\right]$.*

Lemma 1 implies that $U_S(i)$ achieves a maximum on the set $\left[0, \frac{bp_w^z}{2c-b^2}\right]$.[14] However, the equilibrium investment that solves (5) is not continuous. Just as the choice of inputs conditional on investment is tripartite, the investment decision as a function of inverse productivity ω is also tripartite in the interior case, which holds under the following assumption.[15]

Assumption 1 $r \in \left(2\sqrt{\frac{\tau}{\bar{Q}(2c-\alpha b^2)}}, \frac{2p_w^z}{\bar{Q}(2c-\alpha b^2)}\right)$.

We now characterize equilibrium investment.

Proposition 2 *Conditional on (p_w^z, b, c), under Assumption 1 there exist thresholds*

$$\begin{aligned} \omega_{UC} \equiv \quad & p_w^z - \tfrac{r\bar{Q}(2c-\alpha b^2)}{2} \text{ and} \\ \omega_{NC}(\tau) \equiv \quad & p_w^z - \tfrac{r\bar{Q}(2c-\alpha b^2)}{2} + \sqrt{(2c-\alpha b^2)\tau\bar{Q}}, \end{aligned}$$

such that equilibrium investment i^* satisfies the following:

$$i^* = \begin{cases} i_0^* = \left(\frac{\alpha b}{2c-\alpha b^2}\right)(p_w^z - \omega) & \text{if } \omega < \omega_{UC} & \textit{High Productivity;} \\ i_r^* = \frac{\alpha b q_r}{2} & \text{if } \omega \in [\omega_{UC}, \omega_{NC}(\tau)] & \textit{Medium Productivity;} \\ i_0^* = \left(\frac{\alpha b}{2c-\alpha b^2}\right)(p_w^z - \omega) & \text{if } \omega > \omega_{NC}(\tau) & \textit{Low Productivity.} \end{cases}$$

[14] The right boundary of Lemma 1 is the equilibrium investment from the highest-productivity supplier ($\omega = 0$) with complete bargaining ability ($\alpha = 1$).

[15] To guarantee that this assumption holds, note that we implicitly assume that $\tau < \frac{p_w^z}{\sqrt{\bar{Q}(2c-\alpha b^2)}}$.

The solution to the investment problem has a highly intuitive pattern. For a given r, suppliers can be grouped into high, low and medium productivity categories, partitioned by the cutoffs ω_{UC} and $\omega_{NC}(\tau)$. For high-productivity suppliers, the ROO is ineffective because they comply in an unconstrained way. For low-productivity suppliers, the ROO is also ineffective, but because they fail to comply. The ROO is effective only for medium-productivity suppliers. In complying, they produce more inputs than they otherwise would, so they also invest more than they otherwise would.

Note that, regardless of whether the ROO is effective, the envelope theorem implies that investment satisfies

$$I'(i^*) = \alpha C_i(q_{i^*}, i^*, \omega). \tag{6}$$

This shows that the supplier will always choose investment by equalizing its marginal cost with a fraction $\alpha < 1$ of its marginal benefit. That defines the nature of the fundamental hold-up problem in our setting, which leads to underinvestment in the absence of policies. With our functional forms, the choice of investment satisfies

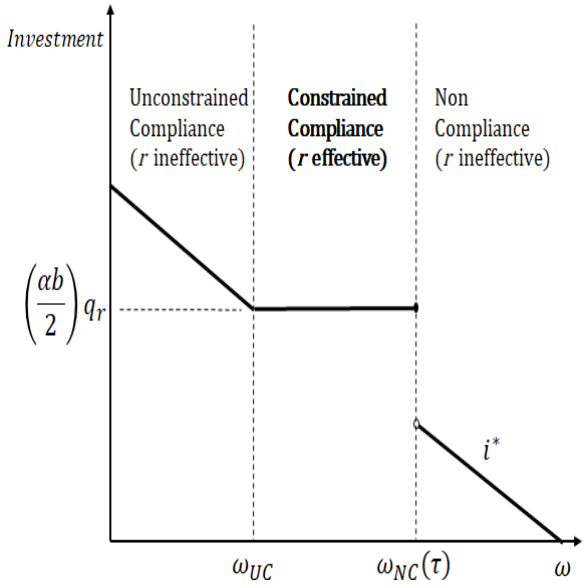
$$i^* = \frac{\alpha b}{2} q_{i^*}. \tag{7}$$

Substituting, we can then write equilibrium inputs as

$$q_{i^*} = \begin{cases} q_0^* = \frac{2(p_w^z - \omega)}{2c - \alpha b^2} & \text{if } \omega < \omega_{UC} \\ q_r^* = r\bar{Q} & \text{if } i \in [\omega_{UC}, \omega_{NC}(\tau)] \\ q_0^* = \frac{2(p_w^z - \omega)}{2c - \alpha b^2} & \text{if } \omega > \omega_{NC}(\tau). \end{cases}$$

Figure 2 summarizes the relationship between inverse productivity and S's investment. High-productivity suppliers have strong incentives to invest, since the resulting reduction in marginal cost will affect a large number of units. As a result, the vertical chain's choices of i and q, under no ROO, satisfy r. The opposite happens in vertical chains with low-productivity suppliers: with weak incentives to invest, they would need to alter their decisions by too much to satisfy r, so the chain forgoes the benefit from compliance. Only vertical chains with suppliers with intermediate levels of productivity find it worthwhile to increase investment (and the production of specialized inputs) beyond their choices under no ROO to satisfy r.

Figure 2: *Equilibrium Investment Conditional on* ω



Note: The relationship between inverse productivity ω and equilibrium investment i^*.

We denote

$$\rho(\tau, b) \equiv \omega_{NC}(\tau) - \omega_{UC} = \sqrt{(2c - \alpha b^2)\tau\bar{Q}}$$

as the *range effect* of the ROO. Interestingly, if ω_{UC} and $\omega_{NC}(\tau)$ are interior, as in Figure 2, then the range of supplier productivity affected by the ROO is independent of the stringency of the rule, r.[16]

Unsurprisingly, the range effect is increasing in $\tau\bar{Q}$, the bargaining-surplus bonus from compliance. Moreover, it is decreasing in both supplier bargaining ability, α, and in the effectiveness of investment, b. When investment is very effective to reduce marginal cost (b is high) and the investing party is very responsive (α is high), the range effect gets small. Intuitively, in that case investment is a "key decision" for suppliers, and therefore most of

[16]This characteristic depends on the supplier's marginal cost being linear. One can rearrange the condition equating profits from compliance and non-compliance, $U_S^{RC}(i_r^*) = U_S^{NC}(i_0^*)$, and represent it as

$$\tau\bar{Q} = \left[\frac{q_r - q_0}{2(2c - \alpha b^2)}\right]^2.$$

The left-hand side is the benefit from compliance. The right-hand side is the cost. Whatever r happens to be, the q_0 that satisfies this condition is always the same distance from q_r.

them are unwilling to distort it to reap the gain from compliance. Conversely, for low α and low b, the role of investment is diminished for suppliers, and therefore most of them become willing to distort it to earn $\tau\bar{Q}$.

Observe, however, that ω_{UC} and $\omega_{NC}(\tau)$ will not be interior if Assumption 1 is not satisfied. For example, when r is very high, $\omega_{UC} < 0$ and even vertical chains with the highest-productivity suppliers comply in a constrained way. When instead τ is very high, $\omega_{NC}(\tau) > p_w^z$ and even the lowest-productivity suppliers choose to alter their investment choices to comply with the ROO. Thus, we could have situations where r and τ are high enough so that all suppliers comply in a constrained way because the gain from compliance is sufficiently high (τ is very high) and compliance requires a great share of within-FTA inputs (r is very high). We can think of that as the limiting case when the tripartite equilibrium described in Proposition 2 collapses to one where the ROO is effective for every vertical chain.

5 Welfare Effects

We now consider the welfare effects of a rule of origin r. Following the analysis of section 2.2, joint welfare Ψ due to a single Y-chain and conditional on ω, input levels q and z, and investment level i, can be written as the sum of producer utility (net of the private gain from ROO compliance) and supplier utility, if we ignore terms that do not change with the FTA. Specifically:[17]

$$\Psi(q, i, \omega) = [p_w^x \bar{Q} - p_w^z(\bar{Q} - q) - p^s q] + [p^s q - C(q, i, \omega) - I(i)].$$

Subtracting the constant $(p_w^x - p_w^z)\bar{Q}$ and rearranging, we obtain (with a slight abuse of notation)

$$\Psi(q, i, \omega) = p_w^z q - C(q, i, \omega) - I(i). \tag{8}$$

Thus, ignoring constant terms, the contribution of a single Y-chain to aggregate welfare corresponds to the savings due to the production of q units of inputs by the specialized

[17]Recall that, as discussed in section 2.2, in our setting welfare changes generated by the FTA affect only the inputs market.

supplier, rather than importing those units of generic inputs, net of investment costs. Note that Ψ does not depend directly on the rule of origin r. However, equilibrium welfare does depend on r through its effects on equilibrium investment and input choice.

Recall that, if $r = 0$, then $i^* = i_0^*$ and $q_{i^*} = q_0^*$ for all ω. We denote $\Delta\Psi(r,\omega) \equiv \Psi\left(q_{i^*}, i^*, \omega\right) - \Psi\left(q_0^*, i_0^*, \omega\right)$ as the welfare effect of moving from no rule of origin to rule of origin r for a single chain involving a supplier with parameter ω – the "Y-chain-level welfare effect." The aggregate welfare effect integrates this term over all levels of productivity:

$$\Delta W(r) = \int_0^{p_w^z} \Delta\Psi(r,\omega) g(\omega) d\omega. \tag{9}$$

In subsection 5.1, we focus on the Y-chain-level welfare effect for a given ω. This analysis identifies complete welfare effects for the case of a degenerate $g(\omega)$ distribution – that is, absent firm heterogeneity. Comparative statics analysis also yields insights that are useful for the aggregate welfare analysis. We turn to aggregate welfare effects with non-degenerate distributions of suppliers in subsection 5.2. Finally, to highlight the role of the hold-up problem in our results, in subsection 5.3 we consider the special cases where investment is useless ($b = 0$) and where the hold-up problem is either unsolvable ($\alpha = 0$) or nonexistent ($\alpha = 1$).

5.1 Homogeneous Suppliers

Fix $\omega \in [0, p_w^z)$ and focus on the investment choice. Conditional on q_i satisfying the optimal bargaining condition, $p_w^z = C_q(q_i, i, \omega)$, the *first-best* investment that maximizes (8) is

$$i^{fb} = \frac{b(p_w^z - \omega)}{2c - b^2}.$$

However, first-best welfare is not achievable generally due to the hold-up problem. This is illustrated in the simplest way for the case of no ROO. Equilibrium investment is

$$i^* = i_0^* = \frac{\alpha b(p_w^z - \omega)}{2c - \alpha b^2} < i^{fb},$$

inefficiently low because the supplier captures only share α of the returns from the investment but pays all investment costs.

Following OTB, we define the extent of the hold-up problem as

$$HUP_0 \equiv i^{fb} - i_0^*.$$

Noting that HUP_0 decreases in ω, we establish a result that holds here and is likely to apply more generally:

Remark 1 *The fundamental hold-up problem is more severe for higher-productivity suppliers.*

The inefficiency in the supplier's investment choice is proportional to the share $(1-\alpha)$ of the returns to investment.[18] In turn, those returns are increasing in supplier productivity, because higher-productivity suppliers produce more, and therefore investment lowers the cost of more units when productivity is higher.

Now, because a ROO can affect investment decisions, it can also affect the severity of the hold-up problem. This is a potential source of gain. However, there are three potential difficulties. First, the ROO may not be effective. Second, when the ROO is effective, it will distort the choice of inputs. Third, the resulting investment may exceed the first best. For future use, it is useful to define the (potential) excess of investment under an effective ROO as

$$EXC_r \equiv i_r^* - i^{fb}.$$

Considering all dimensions, the following lemma shows that the welfare impact of a rule of origin is tripartite in nature.

Lemma 2 *For any $\omega \in [0, p_w^z)$, there exist values*

$$\underline{r}(\omega) \equiv \tfrac{2(p_w^z-\omega)}{\bar{Q}(2c-\alpha b^2)} \text{ and}$$

$$\bar{r}(\omega,\tau) \equiv \tfrac{2(p_w^z-\omega)}{\bar{Q}(2c-\alpha b^2)} + \tfrac{2\sqrt{(2c-\alpha b^2)\tau\bar{Q}}}{\bar{Q}(2c-\alpha b^2)}$$

[18] Specifically, $HUP_0 = \frac{2(1-\alpha)bc(p_w^z-\omega)}{(2c-b^2)(2c-\alpha b^2)}$. If $\alpha = 1$, then $i_0^* = i^{fb}$ and $HUP_0 = 0$.

such that the rule of origin is effective if and only if $r \in (\underline{r}(\omega), \bar{r}(\omega,\tau)]$. *The Y-chain-level welfare effect of the rule of origin is*

$$\Delta\Psi(r,\omega) = \begin{cases} 0 & \text{if } r \leq \underline{r}(\omega) \\ (p_w^z - \omega)(q_r^* - q_0^*) - \frac{\left(2c-2\alpha b^2+\alpha^2 b^2\right)\left(q_r^{*2}-q_0^{*2}\right)}{4} & \text{if } r \in (\underline{r}(\omega), \bar{r}(\omega,\tau)] \\ 0 & \text{if } r > \bar{r}(\omega,\tau). \end{cases}$$

Lemma 2 identifies the levels in which the rule of origin is effective and shows its welfare effect for a given ω, which is non-zero only if the ROO is effective. The lower cutoff, $\underline{r}(\omega)$, comes from setting $\omega = \omega_{UC}$ and solving for r. Any $r < \underline{r}(\omega)$ requires a level of within-FTA sourcing lower than what a vertical chain with parameter ω chooses to do regardless of the ROO, which is therefore redundant. The higher cutoff, $\bar{r}(\omega,\tau)$, stems from setting $\omega = \omega_{NC}(\tau)$ and solving for r. Any $r > \bar{r}(\omega,\tau)$ requires a level of within-FTA sourcing higher than what a vertical chain with parameter ω is willing to meet to avoid paying tariff τ on its final-good exports.

Clearly, when τ is small, the difference between $\underline{r}(\omega)$ and $\bar{r}(\omega,\tau)$ is also small: if the private gain from compliance is modest, most levels of r will be unable to affect investment and sourcing decisions. In the limit when $\tau = 0$, $\underline{r}(\omega) = \bar{r}(\omega, 0)$ and there is no effective rule of origin. Observe also that both $\underline{r}(\omega)$ and $\bar{r}(\omega,\tau)$ decrease with ω. This happens because higher-productivity suppliers always invest more and sell more inputs; thus, it takes a higher r to match the fraction of inputs they produce absent a ROO.

Relying on Lemma 2, the following proposition characterizes the welfare effects of r for the case of homogeneous suppliers.

Proposition 3 *Let the distribution of inverse productivity be degenerate and centered on any* $\omega \in [0, p_w^z)$. *There exist*

$$\begin{aligned} \underline{\tau}(\omega) &\equiv \frac{1}{\bar{Q}(2c-\alpha b^2)}\left[\frac{(p_w^z-\omega)\alpha(1-\alpha)b^2}{2c-2\alpha b^2+\alpha^2 b^2}\right]^2 \text{ and} \\ \hat{r}(\omega) &\equiv \frac{2(p_w^z-\omega)}{\bar{Q}(2c-2\alpha b^2+\alpha^2 b^2)} \end{aligned}$$

such that:

(i) If $\tau > \underline{\tau}$, then $\Delta\Psi(r,\omega)$ is an inverted-U function of r on $[\underline{r}(\omega), \bar{r}(\omega,\tau)]$ and is maximized with rule of origin $r^* = \hat{r}(\omega)$;

(ii) If $\tau \leq \underline{\tau}$, then $\Delta\Psi(r,\omega)$ is a strictly increasing function of r on $[\underline{r}(\omega), \bar{r}(\omega,\tau)]$ and is maximized with rule of origin $r^* = \bar{r}(\omega,\tau)$;

(iii) $r^* = \min\{\hat{r}(\omega), \bar{r}(\omega,\tau)\}$.

If the $r \in (\underline{r}(\omega), \bar{r}(\omega,\tau)]$ constraint is relaxed, then $\Delta\Psi(\omega, r)$ is an inverted-U function of r with a unique maximizer $\hat{r}(\omega)$. But because of Lemma 2, $\hat{r}(\omega)$ maximizes welfare only if it is effective for Y-chains with inverse productivity ω. Because of the hold-up problem, $\hat{r}(\omega)$ is always above $\underline{r}(\omega)$. If $\hat{r}(\omega)$ is below $\bar{r}(\omega,\tau)$, then $\hat{r}(\omega)$ is optimal and is independent of τ. But if $\hat{r}(\omega)$ is above $\bar{r}(\omega,\tau)$, then $\bar{r}(\omega,\tau)$ is the highest r that yields constrained compliance and is optimal. Specifically, the threshold $\underline{\tau}$ is the level of τ that implicitly solves $\hat{r}(\omega) = \bar{r}(\omega,\tau)$. For any $\tau < \underline{\tau}$, $\hat{r}(\omega) > \bar{r}(\omega,\tau)$, and hence setting $r = \hat{r}(\omega)$ in that case would induce non-compliance by a vertical chain with parameter ω. The best choice of r is therefore $\bar{r}(\omega,\tau)$, which is the highest r that is effective and induces more investment.

Now, the reason why $\Delta\Psi$ is an inverted-U function of r when r is effective is that the ROO has two opposing effects on welfare. On the one hand, it induces more investment, helping to alleviate the hold-up problem (although it can also induce too much investment). On the other hand, it yields a socially excessive number of specialized inputs because, for any i, the supplier's marginal cost of the final unit exceeds the unit price of generic inputs, $C_q(q_r, i) > C_q(q_0, i) = p_w^z$. Analogously to OTB, we call the former effect *relationship strengthening* and the latter effect *sourcing diversion*.[19]

More precisely, we define relationship strengthening as

$$\Delta\Psi_{RS} = p_w^z\,(q_1 - q_0^*) + [C(q_0^*, i_0^*, \omega) - C(q_1, i_r^*, \omega)] - [I(i_r^*) - I(i_0^*)]\,,$$

where q_1 denotes the level of specialized inputs that equalizes the marginal costs of the two types of inputs when $i = i_r^*$. That is, q_1 solves $C_q(q_1, i_r^*) = p_w^z$. Using our functional forms

[19] OTB define those forces to describe the effects of tariff preferences, without any ROO. Here they are defined to describe the effects of a ROO, without any tariff preference (until section 6, when we consider both). Indeed, those two terms can be used to describe the effects of any policy that affects firm choice of investment and its subsequent impact on production. However, the precise form of the two effects vary with the specifics of the policy in analysis.

and manipulating, this expression can be rewritten as

$$\Delta\Psi_{RS} = \left(\frac{2c - b^2}{2c}\right)(i_r^* - i_0^*)\,(HUP_0 - EXC_r)\,. \tag{10}$$

This effect is positive if and only if the ROO moves the level of investment *closer* to the first-best level.[20] Differentiating equation (10) with respect to r, while noticing that r affects it only via i_r^* and EXC_r (through its effect on i_r^*), one finds that

$$\frac{\partial \Delta\Psi_{RS}}{\partial r} = \left(\frac{2c - b^2}{2c}\right)\alpha b\bar{Q}\left(i^{fb} - i_r^*\right),$$

which is positive if and only if $i_r^* < i^{fb}$. That is, increasing r improves welfare through the relationship-strengthening effect provided that it does not induce investment in excess of i^{fb}. Since $\partial\Delta\Psi_{RS}/\partial r^2 < 0$, it follows that $\Delta\Psi_{RS}$ is concave in r.

In turn, we define sourcing diversion as

$$\Delta\Psi_{SD} = C(q_1, i_r^*, \omega) - C(q_r, i_r^*, \omega) + p_w^z(q_r^* - q_1),$$

With our functional forms, this expression can be rewritten as

$$\begin{aligned} \Delta\Psi_{SD} &= -\tfrac{c}{2}\left(q_r^* - q_1\right)^2 \\ &= -\tfrac{(2c-\alpha b^2)^2}{2\alpha^2 b^2 c}\left(i_r^* - i_0^*\right)^2. \end{aligned} \tag{11}$$

This effect is always negative[21] and increases in magnitude as ω rises. The reason is that, with lower productivity, i_0^* is lower, so ROO compliance generates a bigger increase in investment above i_0^*, which yields a greater sourcing distortion, $q_r^* - q_1^*$. Differentiating equation (11) with respect to r, we have that

$$\begin{aligned} \tfrac{\partial\Delta\Psi_{SD}}{\partial r} &= -\left(\tfrac{2c-b^2}{4c}\right)(q_r^* - q_0^*)\bar{Q} \\ &= \tfrac{-(2c-\alpha b^2)^2\bar{Q}}{2\alpha bc}\left(i_r^* - i_0^*\right) < 0, \end{aligned}$$

[20] It can also be expressed as $\Delta\Psi_{RS} = \left(\frac{\alpha b(2c-b^2)}{4c}\right)(q_r^* - q_0^*)\,(HUP_0 - EXC_r)$.

[21] It can also be expressed as $\Delta\Psi_{SD} = -\frac{(2c-\alpha b^2)^2}{8c}(q_r^* - q_0^*)^2$.

so increasing r makes sourcing diversion monotonically worse. Furthermore, since $\partial^2 \Delta\Psi_{SD}/\partial r^2 < 0$, $\Delta\Psi_{SD}$ is also concave in r, and so is the full $\Delta\Psi$.

Note that, if $r = \underline{r}(\omega)$, then $i_r^* = i_0^*$ and the ROO is ineffective. For slightly higher r, i_r^* rises and $\Delta\Psi_{SD}$ grows in magnitude, but the loss due to sourcing diversion is of second order. By contrast, the relationship-strengthening effect is of first order. However, as r increases further, the loss due to sourcing diversion grows larger, while the relationship-strengthening effect starts to decrease eventually. Hence, the net effect is an inverted-U curve, reaching its maximum at $\hat{r}(\omega)$.

The level of investment that obtains when $r = \hat{r}(\omega)$ is effective is

$$i^{sb} = \frac{\alpha b(p_w^z - \omega)}{2c - 2\alpha b^2 + \alpha^2 b^2}.$$

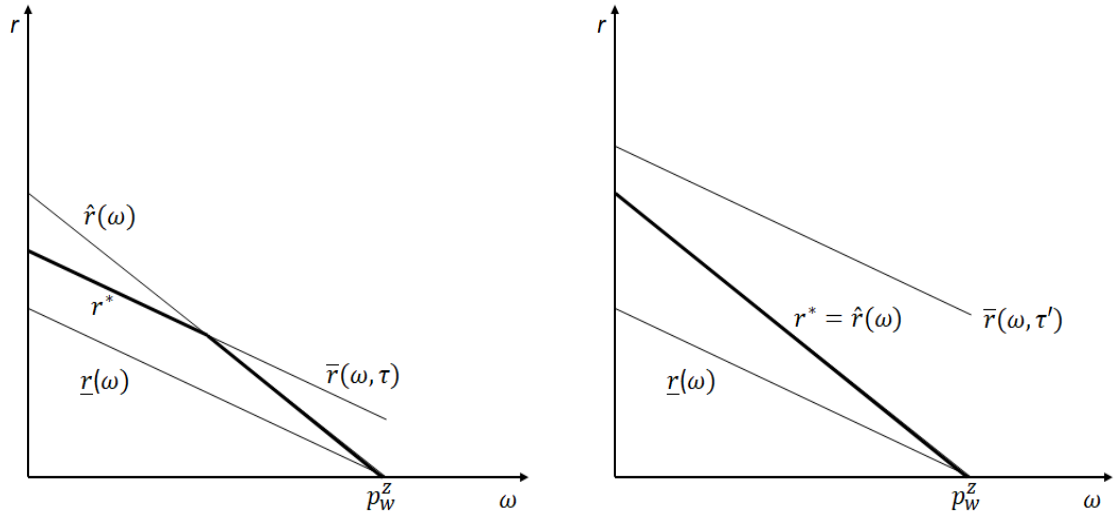
This level of investment is, essentially, a *second-best* level and satisfies the following lemma.

Lemma 3 *$i^{fb} > i^{sb} > i_0^*$. Moreover, the differences $(i^{fb} - i^{sb})$ and $(i^{sb} - i_0^*)$ decrease with ω.*

We have that $i^{sb} > i_0^*$ because i^{sb} is the level of investment that optimally trades off the gains from relationship strengthening against the losses from sourcing diversion. In turn, $i^{fb} > i^{sb}$ because i^{fb} is the level of investment that solves the hold-up problem without inducing any sourcing diversion. Notice that the gap between i^{fb} and i_0^* is wider for lower ω. This reflects the fact that the hold-up problem becomes more severe as productivity increases (Remark 1). The two differences indicated in Lemma 3 decrease with ω because i^{sb} incorporates elements from both i^{fb} and i_0^*.

Figures 3a-b highlight the optimal $r^*(\omega)$ and its complementarity with τ. For the limiting case where all suppliers have the least-possible level of productivity ($\omega = p_w^z$), $r^* = \hat{r} = 0$. Generally, $r^*(\omega)$ is decreasing in ω. In Figure 3a, it equals $\bar{r}(\omega, \tau)$ if ω is low, because $\hat{r}(\omega) > \bar{r}(\omega, \tau)$ is too high to be effective. It does equal $\hat{r}(\omega)$ if ω is high. In Figure 3b, τ is high enough so that $\hat{r}(\omega)$ is always effective, so $r^*(\omega) = \hat{r}(\omega)$ everywhere. Generally, welfare under the optimal r^* increases with τ and strictly increases with τ on $[0, \underline{\tau}]$, i.e., whenever $\hat{r}(\omega) > \bar{r}(\omega, \tau)$. Intuitively, a higher τ provides greater scope of action for a ROO, which allows for higher welfare if r is chosen optimally. Thus, if the FTA-importing country

Figure 3: *Optimal ROO, Homogeneous Suppliers, Varying τ*



(a) *Final Goods Tariff τ*

(b) *Final Goods Tariff $\tau' > \tau$*

Note: These diagrams highlight the optimal ROO for the case where all suppliers have the same level of inverse productivity. The x-axis is the level of inverse productivity ω that all suppliers have, while the bold-face function is the optimal ROO $r^*(\omega)$ for that set of suppliers. In the left panel, τ is relatively small, which constrains $r^*(\omega)$ for low ω. In the right panel, τ is high enough so that $r^*(\omega)$ is unconstrained.

has high tariffs on the final good, the ROO has the potential to be more useful to mitigate hold-up inefficiencies and increase welfare.

As Figures 3a-b illustrate, the function $\hat{r}$ decreases in ω. This reflects strategic substitutability in $\Delta\Psi$ between r and ω (or equivalently, strategic complementarity between r and productivity). To understand that, note first that the marginal social gain (due to higher investment) when r increases is greater for high productivity Y-chains, because the hold-up problem is more severe for them (Remark 1). Given an effective ROO,

$$\frac{\partial^2 \Delta\Psi_{RS}}{\partial r \partial \omega} = \left(\frac{2c - b^2}{2c}\right) \alpha b \bar{Q} \frac{\partial i^{fb}}{\partial \omega} < 0.$$

In addition, the marginal social loss (due to sourcing diversion) when r increases is smaller for high productivity Y-chains, because they already use more within-FTA specialized inputs anyway, so the distortion due to raising q up to q_r is smaller. Formally,

$$\frac{\partial^2 \Delta\Psi_{SD}}{\partial r \partial \omega} = \left(\frac{2c - \alpha b^2}{4c}\right) \bar{Q} \frac{\partial q_0}{\partial \omega} < 0.$$

Thus, considering the Y-chain-level welfare effect, it is best to raise r as productivity increases. Every supplier with $\omega < p_w^z$ underinvests. The smaller ω is (that is, the higher the supplier productivity), the more severe is the hold-up problem and the less severe is sourcing diversion. Therefore, the rule of origin should be tighter, inducing a greater increase in investment, the smaller ω is.

5.2 Heterogeneous Suppliers

Now consider the effect of a ROO on a distribution of heterogeneous suppliers. Incorporating ROO-effectiveness, the aggregate welfare effect is

$$\Delta W(r) = \int_{\omega_{UC}}^{\omega_{NC}(\tau)} \Delta\Psi(r,\omega) g(\omega) d\omega.$$

This integrates out over Y-chains with suppliers $\omega \in [\omega_{UC}, \omega_{NC}(\tau)]$, who respond to the ROO with an investment higher than i_0^*. The following proposition provides an initial characterization of how the level of r affects $\Delta\Psi(r,\omega)$.

Proposition 4 *There exist values*

$$\begin{aligned} r^+ &\equiv \left[\tfrac{2c-\alpha^2b^2}{\alpha(1-\alpha)b^2}\right]\sqrt{\tfrac{\tau}{Q(2c-\alpha b^2)}} \text{ and} \\ \omega^0 &\equiv p_w^z - q_r\left[\tfrac{(2c-\alpha b^2)(2c-2\alpha b^2+\alpha^2 b^2)}{2(2c-\alpha^2 b^2)}\right] \end{aligned}$$

such that the Y-chain-level welfare effect, $\Delta\Psi(r,\omega)$, has the following properties:
(i) If $r \le r^+$, then $\Delta\Psi(r,\omega)$ is an inverted-U function of ω, is positive for $\omega \in [\omega_{UC}, \omega^0]$ and negative for $\omega \in [\omega^0, \omega_{NC}]$.

(ii) If $r \in (r^+, 2r^+)$, then $\Delta\Psi(r,\omega)$ is a positive inverted-U function of ω on $\omega \in [\omega_{UC}, \omega_{NC}]$.

(iii) If $r > 2r^+$, then $\Delta\Psi(r,\omega)$ is positive and strictly increasing in ω for all $\omega \in [\omega_{UC}, \omega_{NC}]$.

The intuition for this result is as follows. Ignoring ROO-effectiveness, $\Delta\Psi(r,\omega)$ is an inverted-U function of ω, with $\Delta\Psi(r,\omega_{UC}) = 0$ and $\Delta\Psi(r,\omega)$ increasing in ω at $\omega = \omega_{UC}$. Thus, if $\Delta\Psi(r,\omega_{NC}) \ge 0$ at $\omega = \omega_{NC}$, then $\Delta\Psi(r,\omega) \ge 0$ for all ω such that the ROO is effective. That condition is equivalent to $r \ge r^+$. Now, if $\Delta\Psi(r,\omega_{NC}) < 0$ at $\omega = \omega_{NC}$, then

$\Delta\Psi(r,\omega)$ will be positive for ω relatively close to ω_{UC} but negative for ω relatively close to ω_{NC}. The cutoff ω^0 determines what "close" represents.

Hence, any rule of origin improves welfare for some Y-chains, and a sufficiently tight rule of origin (weakly) improves welfare for all Y-chains. To see the reasons behind the first claim, note that welfare is unchanged for Y-chain ω_{UC}. But for any rule of origin, no matter how "light," welfare must increase for a Y-chain with ω that is higher but sufficiently close to ω_{UC}. The reason is that the ROO induces a first-order relationship-strengthening effect, which is higher than the second-order loss from the sourcing diversion induced by the ROO.[22] For Y-chains with ω well above ω_{UC}, however, the sourcing-diversion effect may dominate.

Now, for a sufficiently high r, the welfare effect is positive for all $\omega \in [\omega_{UC}, \omega_{NC}]$. Intuitively, a "tight" rule of origin only affects the behavior of high-productivity suppliers (because low-productivity ones will choose to not comply). Since those suppliers' underinvestment is more severe, the rule has a more beneficial role in mitigating that problem, relative to the sourcing distortion (for given investment) that it also engenders. Importantly, the range effect of the ROO, $\rho(\tau, b)$, does not depend on r. Hence, as r increases, the range of affected suppliers covers more productive suppliers but does not change in size. We have the following corollary.

Corollary 1 *If $r > r^+$, then for any distribution of suppliers, the welfare effect of the rule of origin, $\Delta W(r)$, is positive.*

This result implies that, with a continuous density of productivity, a stricter ROO is more likely to generate a positive welfare effect for all affected suppliers.[23] Figure 4 highlights this

[22] The relationship-strengthening effect can be rewritten as

$$\Delta\Psi_{RS} = \left(\frac{\alpha b(2c - b^2)}{2c(2c - \alpha b^2)}\right)(\omega - \omega_{UC})\,(HUP_0 - EXC_r)\,,$$

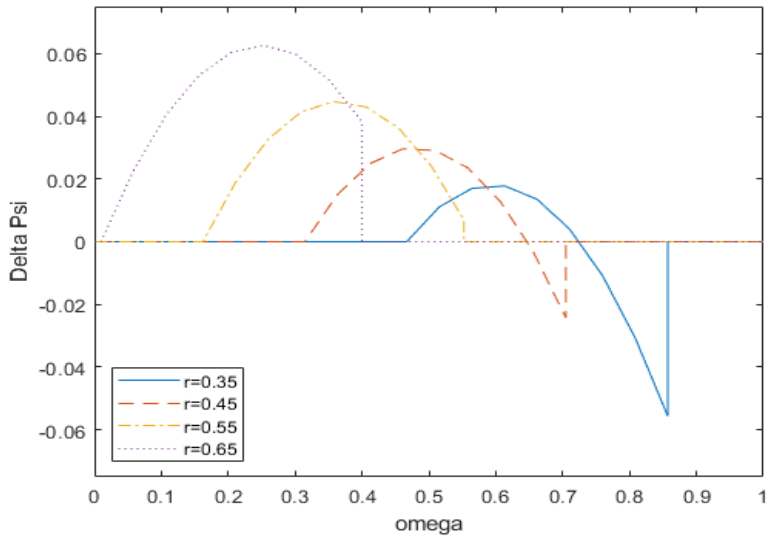
while the sourcing diversion effect can be rewritten as

$$\Delta\Psi_{SD} = -\frac{1}{2c}\,(\omega - \omega_{UC})^2\,.$$

The former is first-order and positive for ω just above ω_{UC}, while the latter is negative but second-order for such ω.

[23] Note that with a discrete distribution, e.g., homogeneous suppliers with inverse productivity ω, this does

Figure 4: *Welfare Effects of a ROO: An Example*



Note: This diagram illustrates the Y-chain-level welfare effect $\Delta\Psi(r,\omega)$ using an example. The ROO $r \in \{0.35, 0.45, 0.55, 0.65\}$. For each r, $\Delta\Psi(r,\omega)$ is shown for all ω. Other parameters are $\alpha = 0.5$, $b = 1.25$, $c = p_w^z = 1$, $\tau = 0.05$, $g(\omega) = 1$ and $\bar{Q} = 2.5$.

using an example where we consider different levels of r. Parameters are such that all ROO values satisfy Assumption 1. Following Proposition 4, $\Delta\Psi(r,\omega)$ is increasing at $\omega = \omega_{UC}$ for all r and is concave on $\omega \in [\omega_{UC}, \omega_{NC}]$. For the particular τ chosen here, $\Delta\Psi(r,\omega)$ is an inverted-U on $\omega \in [\omega_{UC}, \omega_{NC}]$.

Using the results in Proposition 4, we can solve to find $r^+ \approx .53$ under the parametrization used in Figure 4. Hence, in the figure, $\Delta\Psi(r,\omega) > 0$ for all $\omega \in [\omega_{UC}, \omega_{NC}]$ for $r = 0.55$ and $r = 0.65$. By Corollary 1, if r is in this upper region, then the aggregate welfare effect of the ROO is positive for any distribution of ω. We also see in the figure that $\Delta\Psi(r,\omega) < 0$ at $\omega = \omega_{NC}$ for $r = 0.45$ and $r = 0.35$. If r is in this lower region, then the aggregate welfare effect of the ROO may be positive or negative depending upon the distribution of ω.

The ultimate magnitude of $\Delta W(r)$, and how it changes with r, depends on the distribution of ω. If Assumption 1 holds, $[\omega_{UC}, \omega_{NC}]$ shifts to the left as r increases while its width

not hold. If $r > \hat{r}(\omega)$ but the ROO remains effective, then increasing the ROO further reduces welfare and can make it negative.

$\rho(\tau)$ does not change. And as shown in Figure 4, the Y-chain-level welfare effect at the left boundary ($\omega = \omega_{UC}$) is nil for any r, but the effect at the right boundary ($\omega = \omega_{NC}$) strictly increases with r. It can also be shown that, fixing the *distance* between one of the boundaries and some value in the strict interior of $[\omega_{UC}, \omega_{NC}]$ and evaluating the Y-chain-level welfare effect for the Y-chain with that productivity, welfare strictly increases with r.[24] For a uniform density, then, as long as Assumption 1 holds, aggregate welfare strictly increases with r.

Proposition 5 *Let r satisfy Assumption 1 and let $g(\omega)$ be uniform on $[0, p_w^z]$. Then $\Delta W(r)$ strictly increases with r.*

More generally, if the density is concentrated at higher values of ω, then as r increases, there is a better welfare effect for the affected suppliers but fewer of them will be affected. The change in $\Delta W(r)$ with r may then be positive or negative depending on the distribution. Moreover, if Assumption 1 does not hold, then suppliers at one or both boundaries are affected by the ROO. The range effect then depends upon r, which also affects how $\Delta W(r)$ changes with r.

5.3 Special Cases

Several prior papers have studied rules of origin in competitive environments. Generally, their view is that rules of origin reduce welfare. This is consistent with our analysis. Intuitively, when firms act as price takers, there are no hold-up problems. Accordingly, there is no scope for welfare-enhancing relationship strengthening and ROOs produce only distortions in the inputs market.

Indeed, welfare also falls for several special limiting cases in our model. Consider first the possibility that either investment is useless ($b = 0$) or the hold-up problem cannot be solved ($\alpha = 0$). Then equilibrium investments are zero and do not change with a rule of origin

[24] Intuitively, imagine taking the curve for $r = 0.35$ in Figure 4 and sliding it to the left so that it sits on the same $[\omega_{UC}, \omega_{NC}]$ as $r = 0.45$. This would show the curve to be lower everywhere than the curve for $r = 0.45$. More precisely, let $d \leq \rho(\tau)$ be arbitrary. Then, for any r, there is some ω that satisfies $d = \omega - \omega_{UC}$. Noting that this $\omega = \omega_{UC}(r) + d$ is a function of r, it is possible to write the Y-chain-level welfare effect for productivity distance d from $\omega_{UC}(r)$ as a function of r. Written this way, the effect strictly increases in r.

(recall equation (7)). Hence, the relationship-strengthening effect in equation (10) is zero. Meanwhile, the firms still have some incentive to comply with the ROO, inducing sourcing diversion. Welfare cannot improve, and it strictly decreases whenever the ROO is effective.

Welfare also falls for sure when the hold-up problem is non-existent, $\alpha = 1$. In this case, investment equals the first-best with no ROO. Hence, $HUP_0 = 0$ and $EXC_r \geq 0$, so the relationship-strengthening effect cannot be positive and is strictly negative if the ROO is effective. Again, welfare cannot improve, and it strictly decreases whenever the ROO is effective.

Proposition 6 *In the limiting cases of ineffective investment ($b = 0$) or extreme supplier bargaining ability ($\alpha = 0$ or $\alpha = 1$), any effective ROO strictly decreases welfare.*

6 Positive Input Tariff

Now consider the case where the input external tariff is strictly positive, $t > 0$. Absent an effective ROO, equilibrium investment and output become

$$i^* = i_t^* = \frac{\alpha b(p_w^z + t - \omega)}{2c - \alpha b^2},$$
$$q_i^* = q_t^* = \frac{2(p_w^z + t - \omega)}{2c - \alpha b^2}.$$

Note that $i_t^* > i_0^*$ and $q_t^* > q_0^*$. The input tariff increases the privately optimal number of inputs in the F-S bargain; in response, S invests more.[25]

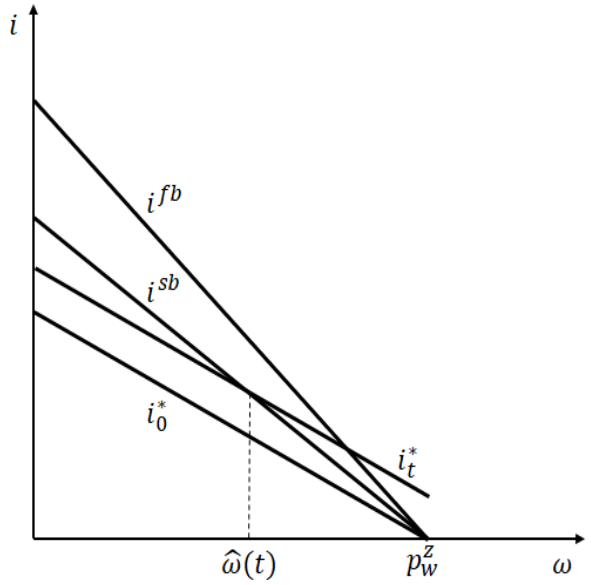
This can have either a positive or negative effect on welfare. Figure 5 shows benchmark investment levels from subsection 5.1, together with i_t^*. Clearly, investment absent a ROO may now be either below or above i^{sb}. Indeed, $i_t^* \geq i^{sb}$ if and only if $\omega \geq \hat{\omega}(t)$ in the diagram.

The input tariff alters both the effectiveness of a ROO and its ability to increase welfare. Regarding effectiveness, we return to the homogeneous supplier case of subsection 5.1 and redefine the Y-chain-level welfare effect as $\Delta\Psi(r, t, \omega)$. The cutoffs $\underline{r}(\omega)$ and $\bar{r}(\omega, \tau)$ become now functions of t :

$$\underline{r}(\omega, t) \equiv \frac{2(p_w^z + t - \omega)}{Q(2c - \alpha b^2)} \text{ and}$$
$$\bar{r}(\omega, \tau, t) \equiv \frac{2(p_w^z + t - \omega)}{Q(2c - \alpha b^2)} + 2\sqrt{\frac{\tau}{Q(2c - \alpha b^2)}}.$$

[25] This captures the equilibrium from section 3.2 of OTB.

Figure 5: *Benchmark Investment Levels*



Note: This diagram illustrates the relationship between inverse productivity ω and first-best, second-best and equilibrium investment, with and without an input external tariff.

Because specialized sourcing is increasing in t, it becomes more likely that vertical chains will meet a given r unconstrained. Thus, to be effective, the ROO needs to be tighter. The upshot is that the input tariff shifts $[\underline{r}(\omega), \bar{r}(\omega, \tau)]$ higher.

The shift in $[\underline{r}(\omega, t), \bar{r}(\omega, \tau, t)]$ may improve welfare, provided that τ is relatively low. In particular, if $\bar{r}(\omega, \tau, 0) < \hat{r}(\omega) < \bar{r}(\omega, \tau, t)$, then input tariff t enables the use of ROO $\hat{r}(\omega)$ to yield investment i^{sb}. On the other hand, if τ is high enough so that $\hat{r}(\omega) < \bar{r}(\omega, \tau, 0)$, then $\hat{r}(\omega)$ is effective with no input tariff but there is no additional welfare improvement to be had. And if t is sufficiently high, then $\hat{r}(\omega) \leq \underline{r}(\omega, t)$, so that the optimal ROO at the Y-chain level is infeasible. In this latter case, $i_t^* \geq i^{sb}$, as in Figure 5 for $\omega \geq \hat{\omega}(t)$.[26] In such cases, it is optimal to choose an ineffective ROO. We have the following proposition.

Proposition 7 *Let the distribution of inverse productivity be degenerate and centered on*

[26]Note that $\hat{\omega}(t)$ is the level of inverse productivity such that the tariff is exactly $t = \hat{t}(\hat{\omega}(t))$, where $\hat{t}$ is derived in Proposition 7.

any $\omega \in (0, p_w^z)$. *There exists*

$$\hat{t}(\omega) = \frac{\alpha(1-\alpha)b^2(p_w^z - \omega)}{(2c - 2\alpha b^2 + \alpha^2 b^2)}$$

such that:

(i) If $t < \hat{t}(\omega) - \sqrt{(2c - \alpha b^2)\tau\bar{Q}}$, *then* $\Delta\Psi(r, t, \omega)$ *is maximized with rule of origin* $r^*(\omega, t) = \bar{r}(\omega, \tau, t)$*;*

(ii) If $t \in [\hat{t}(\omega) - \sqrt{(2c - \alpha b^2)\tau\bar{Q}}, \hat{t}(\omega)]$, *then* $\Delta\Psi(r, t, \omega)$ *is maximized with rule of origin* $r^*(\omega, t) = \hat{r}(\omega)$*;*

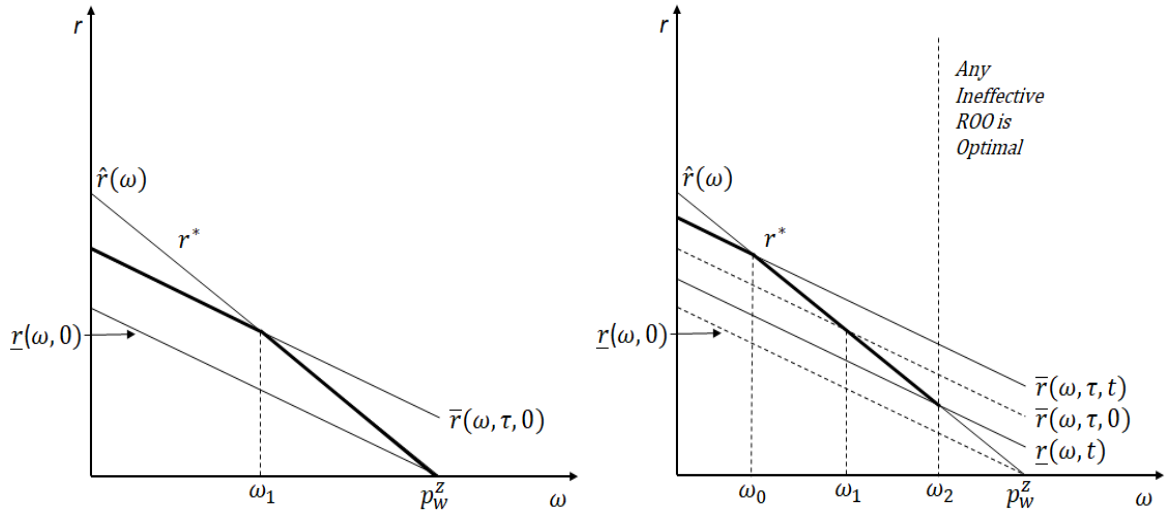
(iii) If $t > \hat{t}(\omega)$, *then welfare cannot be improved with a rule of origin.*

The lower cutoff, $\hat{t}(\omega) - \sqrt{(2c - \alpha b^2)\tau\bar{Q}}$, is found by setting $\hat{r}(\omega) = \bar{r}(\omega, \tau, t)$ and solving for t. The higher cutoff, $\hat{t}(\omega)$, is found by setting $\hat{r}(\omega) = \underline{r}(\omega, t)$ and solving for t. It maximizes the Y-chain-level welfare effect when an input tariff is used but no ROO is used. If t is higher than that, then investment already exceeds i^{sb} with no ROO. Because the ROO can only increase investment, it is best not to have an effective ROO.

Figures 6a-b show how $r^*(\omega)$ changes with ω. Figure 6a is the same as 3a, and therefore has the same optimal unconstrained ROO $\hat{r}(\omega)$. In Figure 6b, the input external tariff shifts the range $[\underline{r}(\omega, t), \bar{r}(\omega, \tau, t)]$ up for all ω. Generally, the input tariff is more likely to be helpful when productivity is higher; it makes $r^*(\omega)$ strictly higher for $\omega \leq \omega_1$. For $\omega \leq \omega_0$, $r^*(\omega) = \bar{r}(\omega, \tau, t)$. For $\omega \in (\omega_0, \omega_1]$, $r^*(\omega) = \hat{r}(\omega)$. Welfare rises in each of these cases. For $\omega \in (\omega_1, \omega_2)$, $r^*(\omega)$ is the same as in Figure 3a. For $\omega > \omega_2$, the input tariff eliminates the usefulness of a ROO. To the right of the vertical dashed line, any effective ROO will worsen welfare. An ineffective ROO is therefore optimal.

Note that τ may be both a complement and a substitute for t. In Figure 6b, τ and t are complements for some $\omega < \omega_0$. If τ rises, then an increase in t enables higher welfare for additional levels of ω. But if $\tau = \tau'$ is as large as in Figure 3b, then with no input tariff, $r^*(\omega) = \hat{r}(\omega)$ for all ω. With an input tariff, $\underline{r}(\omega, t)$ shifts up as in Figure 6b, and for $\omega > \omega_2$, a ROO now cannot help. But because τ' is so large, there are no ω where the input tariff enables an increase in welfare. The final-goods tariff only substitutes for the input tariff.

Figure 6: *Optimal ROO, Homogeneous Suppliers, Varying t*



(a) *No Input External Tariff* ($t = 0$) (b) *Input External Tariff* ($t > 0$)

Note: These diagrams highlight the optimal ROO for the case where all suppliers have the same level of inverse productivity. The x-axis is the level of inverse productivity ω that all suppliers have, while the bold-face function is the optimal ROO $r^*(\omega)$ for that set of suppliers. The left panel presents the same case as in Figure 3a, while the right panel presents that case with an input tariff $t > 0$.

Now consider the effect of a ROO for the case of heterogeneous suppliers. Incorporating ROO effectiveness, the aggregate welfare effect can now be written as

$$\Delta W(r) = \int_{\omega_{UC}}^{\omega_{NC}(\tau)} \Delta\Psi(r,t,\omega)g(\omega)d\omega.$$

The following result characterizes how the level of r affects $\Delta\Psi(r,t,\omega)$.

Proposition 8 *There exist values*

$$\begin{aligned} r_t^- &\equiv t\left(\tfrac{2}{\bar{Q}\alpha(1-\alpha)b^2}\right), \\ r_t^+ &\equiv t\left(\tfrac{2}{\bar{Q}\alpha(1-\alpha)b^2}\right)\left(\tfrac{(2c-\alpha^2b^2)(4c-3\alpha b^2+\alpha b^2)}{2(2c-\alpha b^2)^2}\right)+\left(\tfrac{(2c-\alpha^2b^2)\sqrt{\tau\bar{Q}}}{\sqrt{2c-\alpha b^2}\alpha(1-\alpha)b^2}\right) \end{aligned}$$

and

$$\omega_t^0 \equiv p_w^z - t\left(\frac{2c-2\alpha b^2+\alpha^2b^2}{2c-\alpha^2b^2}\right) - q_r\left[\frac{(2c-\alpha b^2)(2c-2\alpha b^2+\alpha^2b^2)}{2(2c-\alpha^2b^2)}\right]$$

such that the Y-chain-level welfare effect of the rule of origin, $\Delta\Psi(\omega,r)$*, has the following properties:*

(i) If $r < r_t^-$, *then* $\Delta\Psi(r,t,\omega) < 0$ *for all* $\omega \in (\omega_{UC}, \omega_{NC}]$.
(ii) If $r \in [r_t^-, r_t^+]$, *then* $\Delta\Psi(r,t,\omega) \geq 0$ *for all* $\omega \in [\omega_{UC}, \omega_t^0]$ *and* $\Delta\Psi(r,t,\omega) \leq 0$ *for all* $\omega \in [\omega_t^0, \omega_{NC}]$.
(iii) If $r > r_t^+$, *then* $\Delta\Psi(r,t,\omega) > 0$ *for all* $\omega \in [\omega_{UC}, \omega_{NC}]$.

As with $t = 0$, a ROO is more likely to be beneficial if it is tighter, and for the same reasons: a tight rule affects the behavior of high-productivity suppliers. The key difference when $t > 0$ is that the aggregate welfare effect can be strictly *negative* for any distribution of ω, because a ROO may aggravate overinvestment by *all* affected suppliers.

Corollary 2 *If* $r < r_t^-$, *then the welfare effect of the rule of origin is strictly negative for any continuous distribution of suppliers. If* $r > r_t^+$, *then the welfare effect of the rule of origin is strictly positive for any continuous distribution of suppliers.*

Note that both r_t^- and r_t^+ increase with t. With a higher external tariff on intermediate inputs, more suppliers overinvest and fewer underinvest, so a higher r is needed to affect only those that underinvest. For a sufficiently high t, all suppliers overinvest and welfare improvements are impossible.

Corollary 3 *If* $t > \frac{\alpha(1-\alpha)b^2\bar{Q}}{2}$, *then the introduction of any rule of origin strictly decreases welfare.*

7 Extensions

In this section, we consider some important alternative specifications to our benchmark model and discuss how our results would change under those specifications.

7.1 Location of Inputs

Since a large fraction of global value chains are actually regional, and are often circumscribed to members of FTAs, we conduct our analysis assuming that the key economic relationship between firms takes place within a trading bloc. But, of course, this is not always the case,

and specialized and generic suppliers may operate in geographic regions that are different from our baseline model.

Consider first the situation where both types of suppliers are in ROW. In that case, the ROO would be irrelevant because there would be no within-bloc sources of input supply. The ROO would be equally mute if both types of inputs are fully available within the trading bloc: regardless of the input mix, compliance would be assured.

The more interesting alternative obtains when specialized suppliers are in ROW while generic suppliers are in either $Home$ or $Foreign$. Essentially, this reverses the location considered in our analysis. In that case, constrained compliance with a ROO would induce more sourcing of generic within-bloc inputs, crowding out the supply of specialized inputs. Thus, the ROO would yield inefficient sourcing while also reducing relationship-specific investments, thereby worsening hold-up problems. This is clearly bad for welfare. Extending the model in this direction would therefore reinforce a key insight from our analysis, made clear in section 5.3: if ROOs do not stimulate relationship-specific investments, then they cannot improve welfare.

7.2 Adjustments in Q

In our benchmark model, we impose assumptions that make it optimal for vertical chains to always choose $Q = \bar{Q}$. This is very helpful to highlight the implications of a ROO for investment, the key variable for the welfare analysis, but is of course an artificial assumption. More generally, vertical chains have two margins of adjustment when complying with a ROO: (1) the mix of inputs for given level of production; and (2) the level of production for a given mix of inputs. Our analysis shuts down the second margin, but in general firms will find optimal to alter both margins.

In particular, firms might reduce the total number of inputs, Q, to comply with the ROO. Intuitively, lowering Q also reduces the need for higher q, and could be a less costly way to comply with the ROO in some settings. When this occurs, the welfare benefit of the ROO is affected both because fewer final goods are produced within the bloc and because this reduces the need to increase investments. Importantly, whether lower production of the

final good within the bloc increases or decreases welfare by itself is, in principle, ambiguous. In the case of enhanced protection considered here, it is welfare-improving because it lowers trade diversion, but the opposite would happen when there is trade creation in final goods.

Handling this possibility is relatively easy in our setting, but cases where $Q < \bar{Q}$ obtains in equilibrium require a number of conditions to hold simultaneously.[27] To see the intuition, note that to preclude this outcome, it suffices to assume that $(p_w^x - p_w^z)$ is sufficiently large. By reducing Q, a vertical chain costs itself the margin $(p_w^x - p_w^z)$ on every unit not produced in the equilibrium bargain that would be produced absent that bargain. When this margin is large, the chain prefers not to set $Q < \bar{Q}$, and instead just adjusts the mix of inputs to comply with the ROO, as in our benchmark analysis. A high final-good tariff also pushes the chain in this direction, because reducing Q also sacrifices τ of surplus per unit of foregone final-good production. A lenient ROO and high supplier productivity similarly push the chain to keep Q high, because both changes make it easier to comply by increasing specialized inputs. Thus, the most likely situation where a chain would adjust Q downwards would be if r is both large and effective for relatively unproductive suppliers, but where the final-good tariff is relatively low. Recalling that a high r tends to center the effectiveness on high-productivity suppliers and a low τ decreases the range of effectiveness of the ROO, it follows that the conditions that make $Q < \bar{Q}$ optimal are relatively difficult to satisfy and, when they are satisfied, the ROO is not particularly relevant (i.e., τ is low and the rule has little bite).

In the general case where the production function of the final good is smooth in the number of inputs, Q, constrained compliance will typically imply both lower Q and more specialized inputs, q.[28] Thus, our current analysis of how a ROO induces more sourcing of within-bloc specialized inputs and its novel welfare implications would carry over that general case. The difference is that the conventional – and ambiguous – welfare changes due to the final-goods market would need to be taken into account as well.

[27] See Appendix B for the basic analysis.

[28] No compliance, in turn, will be associated with lower Q but no changes in q, because under dual sourcing $p_w^z + t$ determines q when the ROO does not bind.

7.3 Documentation Cost of Compliance

We assume that, if a vertical chain chooses quantities so that $q \geq rQ$, then it automatically satisfies the ROO. In reality, there are additional costs of compliance related to the administrative costs of proving to the customs authority that, indeed, the firm's choice of inputs satisfies $q \geq rQ$. These include the cost of keeping additional records of transactions, of filling out additional border documents, and the like. Such costs are often considered to be equivalent to an increase in marginal costs (or to a reduction of the preferential tariff), although the magnitudes vary with the study and with the trading bloc in analysis (e.g., in the analyses of Cadot et al., 2006, they vary from 2 to 7 percent). They could also include a fixed cost component. Numerous authors argue that such costs are significant enough to induce non-compliance by many firms.

It is relatively straightforward to incorporate such costs into our setup. Recall that the gain from compliance for a vertical chain is $\tau\bar{Q}$, the total tariff savings when it exports $\bar{Q}$ units of the final good to the FTA partner. Following the literature, the documentation costs of compliance could be represented as $\delta\bar{Q}$, thus reducing the gain from compliance to $(\tau-\delta)\bar{Q}$. Naturally, if the per-unit documentation cost were larger than the final-good tariff, $\delta \geq \tau$, then no vertical chain complies and the ROO is ineffective in affecting the inputs market. Otherwise, the analysis of the private decisions of the vertical chains carries over just as before, but for an "adjusted" final-good tariff of $\tau' \equiv \tau - \delta$.[29]

Now, the welfare analysis does change with the introduction of the fixed documentation/administrative costs, because such costs are a real burden to the economy, and not just a transfer (as in the case of the tariff proceeds, $\tau\bar{Q}$). Therefore, whenever there is rule compliance, we would need to subtract $\delta\bar{Q}$ from the welfare calculation. The upshot is that, the higher the documentation and administrative costs, the less attractive are ROOs for the society, all else equal.

[29] Observe that, in our setting, we can think of $\delta\bar{Q}$ also as a fixed cost.

8 Conclusion

We study the welfare effects of rules of origin in free trade agreements with a property-rights model. Given the nature of modern global value chains and their prevalence within FTAs, this approach seems natural. We design the details of the model so that we can derive clear-cut analytical solutions, but the two key assumptions on which the analysis rests are probably much more general. First, hold-up problems matter. Second, trade policies affect investment incentives. This makes clear when our conclusion do and do not hold. A prominent case when they do not is a competitive market.

Indeed, in addition to being novel, the implications of ROOs from a property-rights perspective are vastly different from those under a competitive setting. We show that welfare may rise or fall with the imposition of such a rule, but that the effects tend to be more positive when the rule requires a higher fraction of within-bloc inputs. Moreover, a sufficiently strict rule can ensure a positive welfare impact, at least if input external tariffs are not exceedingly high.

Some analysts (e.g., Crivelli et al., 2021) suggest that more lenient rules should be used to induce higher levels of preference utilization. This is also a common message of several papers that find that ROOs induce changes in sourcing patterns, which implicitly assume that sourcing is efficient in the absence of ROOs. Our analysis recommends caution in such policy proposals. In our setting, making rules of origin more lenient would induce some low-productivity suppliers to comply and would make the rule redundant for some high-productivity suppliers. While the former effect is unlikely to raise welfare by much, the latter may imply forgoing gains from mitigating hold-up problems precisely when they matter more. More generally, our analysis shows that understanding the organization of the firms affected by the rules is critical for their normative assessment.

Clearly, the design of ROOs has several practical dimensions that we bypass in our analysis. The lack of transparency and clarity in actual ROOs, their multiplicity across products and agreements, and the distinct ways of defining origin are important dimensions from a practical perspective. Attempts at defining "best practices" and at "multilateralizing" the rules under the auspices of the World Trade Organization have so far failed, but develop-

ments in those directions would surely be helpful. Indeed, the insights from our analysis (as well as other theoretical analyses) would be more useful following those developments.

References

Antràs, Pol. "Conceptual Aspects of Global Value Chains." *World Bank Economic Review* 34 (3), 551-574, 2020.

Antràs, Pol and Robert Staiger. "Offshoring and the Role of Trade Agreements," *American Economic Review* 102(7), 3140-3183, 2012.

Baldwin, Richard. "21st century regionalism: Filling the gap between 21st century trade and 20th century trade rules," World Trade Organization, Staff Working Paper ERSD-2011-08, 2011.

Boehm, Johannes and Ezra Oberfield. "Misallocation in the Market for Inputs: Enforcement and the Organization of Production," *Quarterly Journal of Economics* 135(4), 2007–2058, 2020.

Cadot, Olivier, Antoni Estevadeordal, Akiko Suwa-Eisenmann, and Thierry Verdier (2006). The origin of goods: Rules of origin in regional trade agreements. Oxford: Oxford University Press, 2006.

Cherkashin, Ivan, Svetlana Demidova, Hiau L. Kee, and Kala Krishna (2015). Firm heterogeneity and costly trade: A new estimation strategy and policy experiments. Journal of International Economics 96(1), 18–36.

Chung, Wanyu and Carlo Perroni. "Rules of Origin and Market Power," CEPR Discussion Paper DP15897, 2021.

Conconi, Paola; Garcia-Santana, Manuel; Puccio, Laura; Venturini, Roberto. "From Final Goods to Inputs: The Protectionist Effects of Rules of Origin," *American Economic Review* 108, 2335-65, 2018.

Crivelli, Pramila, Stefano Inama and Jonas Kasteng. "Using Utilization Rates to Identify Rules of Origin Reforms: The Case of EU Free Trade Area Agreements," EUI Working Paper RSC 2021/21, 2021.

Falvey, Rod and Geoff Reed. "Economic effects of rules of origin," *Review of World Economics* 134, 209-229, 1998.

Grossman, Gene. "The Theory of Domestic Content Protection and Content Preference," *Quarterly Journal of Economics* 96, 583-603, 1981.

Grossman, Gene and Helpman, Elhanan. "The Politics of Free-Trade Agreements," *American Economic Review* 85, 667-690, 1995.

Grossman, Gene and Helpman, Elhanan. When Tariffs Disrupt Global Supply Chains. Mimeo, 2021.

Head, Keith, Thierry Mayer and Marc Melitz. "Moving parts: When more restrictive content rules backfire," Mimeo, 2021.

Inama, Stefano. Rules of Origin in International Trade, Cambridge University Press: New York, NY. 2009.

Krishna, Kala. "Understanding Rules of Origin." In: Olivier Cadot et al. (eds.), The Origin of Goods: Rules of Origin in Regional Trade Agreements, 191–212. Oxford: Oxford University Press, 2006.

Krishna, Kala, Carlos Salamanca, Yuta Suzuki and Christian Volpe Martincus. "Learning to Use Trade Agreements," NBER Working Paper 29319, 2021.

Krueger, Anne. "Free trade agreements as protectionist devices: rules of origin." In J. Melvin, J. Moore and R. Riezmond (eds.), Trade, Theory and Econometrics: Essays in Honor of John C. Chipman, 91–102. London: Routledge, 1999

Ornelas, Emanuel and John L. Turner. "Trade Liberalization, Outsourcing and the Hold-Up Problem," *Journal of International Economics* 74, 225-41, 2008.

Ornelas, Emanuel and John L. Turner. Protection and international sourcing. *Economic Journal* 122, 26–63, 2012.

Ornelas, Emanuel; Turner, John L.; Bickwit, Grant. "Preferential Trade Agreements and Global Sourcing," *Journal of International Economics* 128, Article 103395, 2021.

Ruta, Michele. "Preferential trade agreements and global value chains: Theory, evidence, and open questions." In: Global Value Chain Development

Report: Measuring and Analyzing the Impact of GVCs on Economic Development, World Bank, Washington DC., 175-185, 2017.

Saggi, Kamal and Halis Yildiz. "Bilateralism, multilateralism, and the quest for global free trade," *Journal of International Economics* 86, 26-37, 2010.

Tsirekidze, David. "Global supply chains, trade agreements and rules of origin," *The World Economy* 44(11), 3111-3140, 2021.

Appendix A: Proofs

Proof of Proposition 1. Ignoring the ROO constraint, note that both Σ^{RC} and Σ^{NC} are strictly concave in q and are maximized by q_0 as in (2). Then because $\Sigma^{RC}(q_0, i, \omega) > \Sigma^{NC}(q_0, i, \omega)$, if q_0 is ROO-compliant, then q_0 is optimal. The expression for $i_{UC}(\omega) \equiv \frac{\omega-(p_w^z+t)+cq_r}{b}$ is found by setting $q_0 = q_r$ and solving for ω, and it follows that $q_0 > q_r$ precisely when $i > i_{UC}$. This shows that $q_i = q_0$ if $i > i_{UC}$.

To find the expression for i_{NC}, note first that Σ^{RC} and Σ^{NC} are both increasing functions of i, with $\frac{d\Sigma^{RC}(q_r,i,\omega)}{di} = bq_r$ and $\frac{d\Sigma^{NC}(q_0,i,\omega)}{di} = bq_0$. As i falls below i_{UC}, we know that $q_r > q_0$; hence $\Sigma^{RC}(q_r, i, \omega)$ falls faster than $\Sigma^{NC}(q_0, i, \omega)$, and $\Sigma^{RC}(q_r, i, \omega) = \Sigma^{NC}(q_0, i, \omega)$ for a unique value of i. Algebra shows that $i_{NC}(\omega)$ solves this. It follows that for $\Sigma^{RC}(q_r, i, \omega) < \Sigma^{NC}(q_0, i, \omega)$ for $i < i_{NC}$, so that $q_i = q_0$ if $i < i_{NC}$. It also follows that $\Sigma^{RC}(q_r, i, \omega) > \Sigma^{NC}(q_0, i, \omega)$ for $i \in (i_{NC}, i_{UC})$, so that $q_i = q_r$ if $i \in (i_{NC}, i_{UC})$. Note that if $i = i_{UC}$, then $q_r = q_0$ and $\Sigma^{RC}(q_r, i_{UC}, \omega) = \Sigma^{RC}(q_0, i_{UC}, \omega)$. ■

Proof of Lemma 1. Because q_0 and q_r are continuous functions, it is obvious that $\Sigma^{NC}(i, q_0, \omega)$ is continuous on $[0, i_{NC}]$, that $\Sigma^{RC}(q_r, i, \omega)$ is continuous on $[i_{NC}, i_{UC}]$, and that $\Sigma^{RC}(q_0, i, \omega)$ is continuous for $i \in [i_{UC}, \frac{bp_w^z}{2c-b^2}]$, where the upper bound of this interval is i_0^* for the case where $\omega = 0$ and $\alpha = 1$. It exceeds i_{UC} by the assumption $C_q(\bar{Q}, b(p_w^z)\,(2c - b^2)^{-1}, 0) > p_w^z$.

It remains to establish that $U_S(i)$ is continuous at $i = i_{NC}$ and at $i = i_{UC}$. For the first condition, the Proof of Proposition 1 shows that i_{NC} is defined as the unique i such that $\Sigma^{RC}(q_r, i, \omega) = \Sigma^{NC}(q_0, i, \omega)$. Hence $\lim_{i \to i_{NC}} U_S(i) = \Sigma^{RC}(q_r, i_{NC}, \omega) = \Sigma^{NC}(q_0, i_{NC}, \omega)$, and $U_S(i)$ is continuous at $i = i_{NC}(\omega)$.

For the second condition, note that $i_{UC}(\omega)$ is defined so that $q_0 = q_r$. Hence, $\Sigma^{RC}(q_0, i_{UC}, \omega) = \Sigma^{RC}(q_r, i_{UC}, \omega)$ and $U_S(i)$ is continuous at $i = i_{UC}$. ■

Proof of Proposition 2. Optimizing either $\Sigma^{RC}(q_i, i, \omega)$ or $\Sigma^{NC}(q_i, i, \omega)$ yields the effi-

ciency condition $I'(i^*) = \alpha C_i(q_{i^*}, i^*, \omega)$. If $q_{i^*} = q_0$, then with our functional forms we can solve to find

$$\begin{aligned} q_0^* &= \tfrac{2(p_w^z - \omega)}{2c - \alpha b^2} \\ i_0^* &= \tfrac{\alpha b(p_w^z - \omega)}{2c - \alpha b^2}. \end{aligned} \tag{12}$$

If $q_{i^*} = q_r$, then $i^* = i_r^* = \frac{\alpha b q_r}{2}$. From Proposition 1, we know that q_0 maximizes Σ when i is such that $q_0 \geq q_r$. Hence, i_0^* maximizes $U_S(i)$ if q_0^* in (12) exceeds $q_r^* = r\bar{Q}$. The expression for ω_{UC} is found by setting $q_0^* = q_r^*$ and solving for ω. Because q_0^* is decreasing in ω, we have shown that $i^* = i_0^*$ if $\omega < \omega_{UC}$.

To find the expression for ω_{NC}, note that $\Sigma^{RC}(q_0^*, i_0^*, \omega)$ equals $\Sigma^{NC}(q_0^*, i_0^*, \omega)$ plus a constant term that reflects the additional producer surplus coming from ROO compliance. For both functions, the envelope theorem implies that the rate of change of supplier profit is

$$\frac{\Sigma^{RC}(q_0^*, i_0^*, \omega)}{d\omega} = \frac{\Sigma^{NC}(q_0^*, i_0^*, \omega)}{d\omega} = -\alpha C_\omega(q_0^*, i_0^*, \omega) = -\alpha q_0^*.$$

Because q_0^* is itself a decreasing function of ω, these profit functions are decreasing and strictly convex functions of ω. The envelope theorem also applies for the rate of change for profit under constrained compliance:

$$\frac{\Sigma^{RC}(q_r^*, i_r^*, \omega)}{d\omega} = -\alpha C_\omega(q_r^*, i_r^*, \omega) = -\alpha q_r^*.$$

If $\omega > \omega_{UC}$, then $q_r^* > q_0^*$. Hence, $\Sigma^{RC}(q_r,^*, i_r^*, \omega)$ has a steeper slope than $\Sigma^{NC}(q_0^*.i_0^*, \omega)$. This implies that there is a unique value ω_{NC} satisfying $\Sigma^{RC}(q_r^*, i_r^*, \omega_{NC}) = \Sigma^{NC}(q_0^*, i_0^*, \omega_{NC})$. Moreover, it follows that for $\Sigma^{RC}(q_r^*, i_r^*, \omega) < \Sigma^{NC}(q_0^*, i_0^*, \omega)$ for $\omega > \omega_{NC}$, so that $i^* = i_0^*$ if $\omega > \omega_{NC}$. It also follows that $\Sigma^{RC}(q_r^*, i_r^*, \omega) > \Sigma^{NC}(q_0^*, i_0^*, \omega)$ for $i \in (\omega_{UC}, \omega_{NC})$, so that $i^* = i_r^*$ if $\omega \in [\omega_{UC}, \omega_{NC}]$. Note that if $\omega = \omega_{UC}$, then $i_r^* = i_0^*$ and $\Sigma^{RC}(q_r^*, i_r^*, \omega_{UC}) = \Sigma^{RC}(q_0^*, i_0^*, \omega_{UC})$. ■

Proof of Lemma 2. By definition, $\Delta\Psi(\omega, r) = \Psi(\omega, q_{i^*}, i^*) - \Psi(\omega, q_0^*, i_0^*)$. For ω outside $[\omega_{UC}, \omega_{NC}]$, $i^* = i_0^*$ and $q_{i^*} = q_0^*$, so $\Delta\Psi(r, \omega) = 0$.

For $\omega \in [\omega_{UC}, \omega_{NC}]$, $i^* = i_r^*$ and $q_{i^*} = q_r^*$. We can then write

$$\begin{aligned} \Psi(q_{i^*}, i^*, \omega) &= (p_w^x - p_w^z)\bar{Q} + p_w^z q_r^* - C(q_r^*, i_r^*, \omega) - I(i_r^*) \\ &= (p_w^x - p_w^z)\bar{Q} + p_w^z q_r^* - (\omega - b i_r^*) q_r^* - \tfrac{c}{2} q_r^{*2} - i_r^{*2} \\ &= (p_w^x - p_w^z)\bar{Q} + (p_w^z - \omega) q_r^* + b\left(\tfrac{\alpha b q_r^*}{2}\right) q_r - \tfrac{c}{2} q_r^{*2} - \left(\tfrac{\alpha b q_r^*}{2}\right)^2 \\ &= (p_w^x - p_w^z)\bar{Q} + (p_w^z - \omega) q_r^* - \tfrac{(2c - 2\alpha b^2 + \alpha^2 b^2) q_r^{*2}}{4}. \end{aligned}$$

Similarly, we can derive

$$\Psi\left(\omega, q_0, i_0^*\right) = (p_w^x - p_w^z)\bar{Q} + (p_w^z - \omega)q_0^* - \frac{(2c - 2\alpha b^2 + \alpha^2 b^2)q_0^{*2}}{4}.$$

Collecting terms, we then have

$$\Delta\Psi(\omega, r) = \left(p_w^z - \omega\right)\left(q_r^* - q_0^*\right) - \frac{\left(2c - 2\alpha b^2 + \alpha^2 b^2\right)\left(q_r^{*2} - q_0^{*2}\right)}{4},$$

concluding the proof. ■

Proof of Proposition 3. Ignoring the effectiveness constraints,

$$\Delta\Psi(r, \omega) = (p_w^z - \omega)(q_r^* - q_0^*) - \frac{\left(2c - 2\alpha b^2 + \alpha^2 b^2\right)\left(q_r^{*2} - q_0^{*2}\right)}{4}.$$

This is a strictly concave function of r that equals 0 at $r = \underline{r}(\omega)$ (where $q_0^* = q_r^*$). The first and second derivatives are

$$\begin{aligned}
&\tfrac{d\Delta\Psi(\omega,r)}{dr} = (p_w^z - \omega)\bar{Q} - \tfrac{(2c-2\alpha b^2+\alpha^2 b^2)\left(2q_r^*\bar{Q}\right)}{4} \\
&\tfrac{d^2\Delta\Psi(\omega,r)}{dr^2} = -\tfrac{(2c-2\alpha b^2+\alpha^2 b^2)\bar{Q}^2}{2}.
\end{aligned}$$

The second-derivative is negative, so the function is strictly concave. Evaluating the first derivative at $r = \underline{r}(\omega)$ and rearranging, we have

$$\frac{d\Delta\Psi(\omega, r)}{dr}\Big|_{r=\underline{r}(\omega)} = \frac{(p_w^z - \omega)\alpha(1-\alpha)b^2\bar{Q}}{2c - \alpha b^2} > 0.$$

Hence $\Delta\Psi(\omega, r)$ is increasing at $r = \underline{r}(\omega)$. Setting the first derivative equal to zero and solving, we find that $\Delta\Psi(\omega, r)$ is maximized for

$$\hat{r}(\omega) = \frac{2(p_w^z - \omega)}{\bar{Q}(2c - 2\alpha b^2 + \alpha^2 b^2)} > \underline{r}(\omega).$$

Hence, if $\hat{r}(\omega) \leq \bar{r}(\omega, \tau)$, then $\hat{r}(\omega)$ is effective and $r^* = \hat{r}(\omega)$. We find $\underline{\tau}(\omega)$ by setting $\hat{r}(\omega) = \bar{r}(\omega, \tau)$ and solving for τ. Because $\bar{r}(\omega, \tau)$ is increasing in τ, it follows that for any $\tau > \underline{\tau}(\omega)$, the unconstrained optimum $\hat{r}(\omega) < \bar{r}(\omega, \tau)$, so $\Delta\Psi(\omega, r)$ is an inverted-U function of r on $[\underline{r}(\omega), \bar{r}(\omega, \tau)]$ and is maximized at $r^* = \hat{r}(\omega)$ (part (i)). If $\tau \leq \underline{\tau}(\omega)$, then $\hat{r}(\omega) \geq \bar{r}(\omega, \tau)$ and $\Delta\Psi(\omega, r)$ is strictly increasing on $[\underline{r}(\omega), \bar{r}(\omega, \tau)]$ and is maximized at $r^* = \bar{r}(\omega, \tau)$ (part (ii)). If $\tau = \underline{\tau}(\omega)$, then $r^* = \bar{r}(\omega, \tau) = \hat{r}(\omega)$. ■

Proof of Lemma 3. We have $i^{fb} = \frac{b(p_w^z - \omega)}{2c - b^2}$, $i^{sb} = \frac{\alpha b(p_w^z - \omega)}{2c - 2\alpha b^2 + \alpha^2 b^2}$, and $i_0^* = \frac{\alpha b(p_w^z - \omega)}{2c - \alpha b^2}$. $i_0^* \leq i^{sb}$ follows from $2c - 2\alpha b^2 + \alpha^2 b^2 \leq 2c - \alpha b^2$ (which holds because $\alpha \leq 1$). Comparing i^{sb} and i^{fb}, we need to show

$$\frac{\alpha b(p_w^z - \omega)}{2c - 2\alpha b^2 + \alpha^2 b^2} \leq \frac{b(p_w^z - \omega)}{2c - b^2}.$$

Manipulating this expression leads to $2c \geq \alpha b^2$, which holds by assumption. ■

Proof of Proposition 4. Relaxing the ROO constraint, we can write

$$\Delta\Psi(r, \omega) = (q_r^* - q_0^*) \left[(p_w^z - \omega) - \frac{(2c - 2\alpha b^2 + \alpha^2 b^2)(q_r^* + q_0^*)}{4} \right]. \tag{13}$$

At $\omega = \omega_{UC}$, $q_r^* = q_0^*$, so $\Delta\Psi(r, \omega_{UC}) = 0$. Recalling that q_0^* is a function of ω, the first derivative is

$$\begin{aligned} \tfrac{d\Delta\Psi(r,\omega)}{d\omega} &= (q_r^* - q_0^*) \left[-1 - \left(\tfrac{2c - 2\alpha b^2 + \alpha^2 b^2}{4} \right) \left(\tfrac{-2}{2c - \alpha b^2} \right) \right] \\ &+ \left(\tfrac{2}{2c - \alpha b^2} \right) \left[(p_w^z - \omega) - \tfrac{(2c - 2\alpha b^2 + \alpha^2 b^2)(q_r + q_0)}{4} \right], \end{aligned}$$

or equivalently,

$$\frac{d\Delta\Psi(r, \omega)}{d\omega} = -(q_r^* - q_0^*) \frac{(2c - \alpha^2 b^2)}{2(2c - \alpha b^2)} + \left(\frac{2}{2c - \alpha b^2} \right) \left[(p_w^z - \omega) - \frac{(2c - 2\alpha b^2 + \alpha^2 b^2)(q_r + q_0)}{4} \right]. \tag{14}$$

At $\omega = \omega_{UC}$, the terms on the first line vanish. We can then solve to find

$$\frac{d\Delta\Psi(r, \omega)}{d\omega}\Big|_{\omega = \omega_{UC}} = \frac{r\bar{Q}\alpha(1 - \alpha)b^2}{2c - \alpha b^2} > 0.$$

Hence, $\Delta\Psi(r, \omega)$ is increasing in ω at $\omega = \omega_{UC}$. The second derivative is, after some rearranging,

$$\frac{d^2\Delta\Psi(r, \omega)}{d\omega^2} = \frac{-2(2c - \alpha^2 b^2)}{2c - \alpha b^2} < 0.$$

Hence, $\Delta\Psi(r, \omega)$ is concave.

With these characteristics, $\Delta\Psi(r, \omega)$ on $\omega \in (\omega_{UC}, \omega_{NC}]$ can cross zero at most once and only from above. The cutoff r^+ is then found by inserting $\omega = \omega_{NC}$ into (13), setting it to zero, and solving for r. This yields

$$r^+ = \left(\frac{2(2c - \alpha^2 b^2)}{\alpha(1 - \alpha)b^2} \right) \sqrt{\frac{\tau}{\bar{Q}(2c - \alpha b^2)}}.$$

Because ω_{NC} is strictly decreasing in r plus the aforementioned concavity and single-crossing characteristics of $\Delta\Psi(r, \omega)$, it follows that for any $r > r^+$ and for any $\omega \in (\omega_{UC}, \omega_{NC})$, $\Delta\Psi(r, \omega) > \Delta\Psi(r, \omega_{NC}) > 0$. This proves part (ii).

The cutoff ω^0 is found by setting the term in brackets in (13) equal to zero and solving for ω. It is easily shown that $\omega^0 \in [\omega_{UC}, \omega_{NC}]$ if and only if $r \leq r^+$. Then, assuming $r \leq r^+$ and noting again the concavity and single-crossing characteristics of $\Delta\Psi(r,\omega)$, it follows that $\Delta\Psi(r,\omega) \geq 0$ for all $\omega \in [\omega_{UC}, \omega^0]$ and $\Delta\Psi(r,\omega) < 0$ for all $\omega \in (\omega^0, \omega_{NC}]$. This proves part (i). ■

Proof of Corollary 1. Suppose $r \geq r^+$ and let $g(\omega)$ be arbitrary. From the Proof of Proposition 4, we know that $\Delta\Psi(r,\omega) \geq 0$ for all $\omega \in [\omega_{UC}, \omega_{NC}]$. Further, because $\Delta\Psi(r,\omega)$ is strictly decreasing in ω, we know that $\Delta\Psi(r,\omega) > 0$ for all $\omega \in (\omega_{UC}, \omega_{NC})$. Then $\Delta W(r) = \int_{\omega_{UC}}^{\omega_{NC}(\tau)} \Delta\Psi(r,\omega) g(\omega) d\omega > 0$. ■

Proof of Proposition 5. Noting that ω_{UC} and ω_{NC} are functions of r and ignoring the constant term $(p_w^x - p_w^z)\bar{Q}$ from both $\Psi(r,\omega)$ and $\Psi(0,\omega)$, we can write

$$\Delta W(r) = \int_{\omega_{UC}(r)}^{\omega_{NC}(r)} \Delta\Psi(r,\omega) g(\omega) d\omega.$$

Applying the Leibniz formula, we can write

$$\frac{d\Delta W(r)}{dr} = \int_{\omega_{UC}(r)}^{\omega_{NC}(r)} \left(\frac{d\Delta\Psi(r,\omega)}{dr} \right) g(\omega) d\omega + \omega'_{UC}(r) \Delta\Psi(r,\omega_{UC}) g(\omega_{UC}) - \omega'_{NC}(r) \Delta\Psi(r,\omega_{NC}) g(\omega_{NC}),$$

where " $'$ " denotes a derivative. Because $\Delta\Psi(r,\omega_{UC}) = 0$, the middle term vanishes. We can then easily calculate $\omega'_{NC}(r)$ and insert to obtain the general expression

$$\frac{d\Delta W(r)}{dr} = \int_{\omega_{UC}(r)}^{\omega_{NC}(r)} \left(\frac{d\Delta\Psi(r,\omega)}{dr} \right) g(\omega) d\omega - \left(\frac{\bar{Q}(2c - \alpha b^2)}{2} \right) \Delta\Psi(r,\omega_{NC}) g(\omega_{NC}).$$

Consider the first term, T_1 for short. Inserting from the Proof of Proposition 3 and specifying uniform types, we have

$$T_1 = \frac{1}{p_w^z} \left\{ \int_{\omega_{UC}(r)}^{\omega_{NC}(r)} \left[(p_w^z - \omega)\bar{Q} - \frac{(2c - 2\alpha b^2 + \alpha^2 b^2)\left(2q_r^* \bar{Q}\right)}{4} \right] d\omega \right\}.$$

After some manipulation, this expression can be rewritten as

$$T_1 = \frac{\bar{Q}\sqrt{(2c - \alpha b^2)\tau\bar{Q}}}{2p_w^z} \left\{ r\bar{Q}\alpha(1-\alpha)b^2 - \sqrt{(2c - \alpha b^2)\tau\bar{Q}} \right\}.$$

Now consider the second term, T_2 for short. Inserting from the Proof of Proposition 3 and specifying uniform types, we have

$$T_2 = \left(\frac{\bar{Q}(2c-\alpha b^2)}{2p_w^z}\right)(q_r^* - q_0^*(\omega_{NC}))\left[(p_w^z - \omega_{NC}) - \frac{(2c-2\alpha b^2+\alpha^2 b^2)\,(q_r^* + q_0^*(\omega_{NC}))}{4}\right].$$

After some manipulation, this expression can be rewritten as

$$T_2 = \left(\frac{\bar{Q}\sqrt{(2c-\alpha b^2)\tau\bar{Q}}}{2p_w^z}\right)\left[r\bar{Q}\alpha(1-\alpha)b^2 - \sqrt{(2c-\alpha b^2)\tau\bar{Q}}\left(\frac{2c-\alpha^2 b^2}{2c-\alpha b^2}\right)\right].$$

Collecting terms, we then have

$$\begin{aligned}
T_1 - T_2 &= \left(\tfrac{\bar{Q}\sqrt{(2c-\alpha b^2)\tau\bar{Q}}}{2p_w^z}\right) \\
&\times \left[r\bar{Q}\alpha(1-\alpha)b^2 - \sqrt{(2c-\alpha b^2)\tau\bar{Q}} - \left(r\bar{Q}\alpha(1-\alpha)b^2 - \sqrt{(2c-\alpha b^2)\tau\bar{Q}}\left(\tfrac{2c-\alpha^2 b^2}{2c-\alpha b^2}\right)\right)\right] \\
&= \left(\tfrac{\bar{Q}\sqrt{(2c-\alpha b^2)\tau\bar{Q}}}{2p_w^z}\right)\left[\tfrac{\sqrt{(2c-\alpha b^2)\tau\bar{Q}}(\alpha(1-\alpha)b^2)}{2c-\alpha b^2}\right] \\
&= \left(\tfrac{\bar{Q}^2\tau\alpha(1-\alpha)b^2}{2p_w^z}\right) > 0.
\end{aligned}$$

Hence, when ω is uniformly distributed, because we know that $\Delta W(r=0)$, $\Delta W(r)$ is positive and linear in r. ■

Proof of Proposition 6. Suppose either $b = 0, \alpha = 0$ or $\alpha = 1$. Then from the Proof of Proposition 3, we know that

$$\frac{d\Delta\Psi(r,\omega)}{dr}|_{r=\underline{r}(\omega)} = \frac{(p_w^z - \omega)\alpha(1-\alpha)b^2 r\bar{Q}^2}{2c-\alpha b^2} = 0,$$

and that $\frac{d^2\Delta\Psi(r,\omega)}{dr^2} < 0$. Summarizing, $\Delta\Psi(r,\omega_{UC}) = 0$ and $\Delta\Psi(r,\omega)$ is a decreasing and strictly concave function on $r \in (underliner(\omega), \bar{r}(\omega,\tau)]$. Hence any effective ROO strictly decreases welfare. ■

Proof of Proposition 7. Ignoring the effectiveness constraints,

$$\Delta\Psi(r,t,\omega) = (p_w^z - \omega)(q_r^* - q_t^*) - \frac{(2c-2\alpha b^2+\alpha^2 b^2)\,(q_r^{*2} - q_t^{*2})}{4}.$$

This is a strictly concave function of r that equals 0 at $r = \underline{r}(\omega, t)$ (where $q_t^* = q_r^*$). The first and second derivatives are

$$\begin{aligned}
&\tfrac{d\Delta\Psi(\omega,t,r)}{dr} = (p_w^z - \omega)\bar{Q} - \tfrac{(2c-2\alpha b^2+\alpha^2 b^2)\left(2q_r^*\bar{Q}\right)}{4} \\
&\tfrac{d^2\Delta\Psi(\omega,t,r)}{dr^2} = -\tfrac{(2c-2\alpha b^2+\alpha^2 b^2)\bar{Q}^2}{2}.
\end{aligned}$$

The second-derivative is negative, so the function is strictly concave. Evaluating the first derivative at $r = \underline{r}(\omega, t)$ and rearranging, we have

$$\frac{d\Delta\Psi(\omega,t,r)}{dr}|_{r=\underline{r}(\omega,t)} = \bar{Q}\left[\frac{(p_w^z-\omega)\alpha(1-\alpha)b^2}{2c-\alpha b^2} - \left(\frac{2c-2\alpha b^2+\alpha^2 b^2}{2c-\alpha b^2}\right)t\right].$$

This is strictly negative if $t > \hat{t}(\omega)$. Because $\Delta\Psi(\omega, t, r)$ is strictly concave, if $t > \hat{t}(\omega)$, then $\Delta\Psi(\omega, t, r) < 0$ for $r \in (\underline{r}(\omega, t), \bar{r}(\omega, \tau, t)]$. This proves part (iii).

Setting the first derivative equal to zero and solving, we find that (relaxing the effectiveness constraint) $\Delta\Psi(\omega, t, r)$ is maximized for

$$\hat{r}(\omega) = \frac{2(p_w^z - \omega)}{\bar{Q}(2c - 2\alpha b^2 + \alpha^2 b^2)}.$$

If $\hat{r}(\omega) \in [\underline{r}(\omega, t), \bar{r}(\omega, \tau, t)]$, then $\hat{r}(\omega)$ is effective and $r^* = \hat{r}(\omega)$. If $\hat{r}(\omega) > \bar{r}(\omega, \tau, t)$, then $\hat{r}(\omega)$ is ineffective. Then by the concavity of $\Delta\Psi(\omega, t, r)$, $r^* = \bar{r}(\omega, \tau, t)$. We thus find the upper bound on t in part (ii) of the proof by setting $\hat{r}(\omega) = \underline{r}(\omega, t)$ and solving for t. We find the lower bound on t in part (ii) of the proof and the bound in part (i) by setting $\hat{r}(\omega) = \bar{r}(\omega, \tau, t)$ and solving for t. ■

Proof of Proposition 8. Relaxing the ROO constraint, we can write

$$\Delta\Psi(r, t, \omega) = (q_r^* - q_t^*)\left[(p_w^z - \omega) - \frac{(2c - 2\alpha b^2 + \alpha^2 b^2)(q_r^* + q_t^*)}{4}\right]. \tag{15}$$

At $\omega = \omega_{UC}$, $q_r^* = q_t^*$, so $\Delta\Psi(r, t, \omega_{UC}) = 0$. Recalling that q_t^* is a function of ω, the first derivative is

$$\begin{aligned}
\tfrac{d\Delta\Psi(r,t,\omega)}{d\omega} &= (q_r^* - q_t^*)\left[-1 - \left(\tfrac{2c-2\alpha b^2+\alpha^2 b^2}{4}\right)\left(\tfrac{-2}{2c-\alpha b^2}\right)\right] \\
&+ \left(\tfrac{2}{2c-\alpha b^2}\right)\left[(p_w^z - \omega) - \tfrac{(2c-2\alpha b^2+\alpha^2 b^2)\left(q_r^*+q_t^*\right)}{4}\right],
\end{aligned}$$

At $\omega = \omega_{UC}$, the terms on the first line vanish. We can then solve to find

$$\frac{d\Delta\Psi(r,t,\omega)}{d\omega}|_{\omega=\omega_{UC}} = \left(\frac{2}{2c-\alpha b^2}\right)\left(\frac{r\bar{Q}\alpha(1-\alpha)b^2}{2} - t\right).$$

Hence, $\Delta\Psi(r,t,\omega)$ is increasing in ω at $\omega=\omega_{UC}$ if and only if $r \geq t\left(\frac{2}{\bar{Q}\alpha(1-\alpha)b^2}\right) \equiv r_t^-$. The second derivative is, after some rearranging,

$$\frac{d^2\Delta\Psi(r,t,\omega)}{d\omega^2} = \frac{-2(2c-\alpha^2b^2)}{2c-\alpha b^2} < 0.$$

Hence, $\Delta\Psi(r,t,\omega)$ is concave. Because it is concave and $\Delta\Psi(r,\omega_{UC})=0$, it follows that if $r < r_t^-$, then $\Delta\Psi(r,t,\omega_{UC}) < 0$ for all $\omega \in (\omega_{UC},\omega_{NC}]$. This proves part (i).

If $r > r_t^-$, then $\frac{d\Delta\Psi(r,t,\omega)}{d\omega}|_{\omega=\omega_{UC}} > 0$. Given the concavity of the function, $\Delta\Psi(r,t,\omega)$ on $\omega \in (\omega_{UC},\omega_{NC}]$ can cross zero at most once and only from above. The cutoff r_t^+ is then found by inserting $\omega=\omega_{NC}$ into (15), setting it to zero, and solving for r. Because ω_{NC} is strictly decreasing in r plus the aforementioned concavity and single-crossing characteristics of $\Delta\Psi(r,t,\omega)$, it follows that for any $r > r_t^+$ and for any $\omega \in (\omega_{UC},\omega_{NC})$, $\Delta\Psi(r,t,\omega) > \Delta\Psi(r,t,\omega_{NC}) > 0$. This proves part (iii).

The cutoff ω_t^0 is found by setting the term in brackets in (15) equal to zero and solving for ω. It is easily shown that $\omega_t^0 \in [\omega_{UC},\omega_{NC}]$ if and only if $r \leq r_t^+$. Then, assuming $r \leq r_t^+$ and noting again the concavity and single-crossing characteristics of $\Delta\Psi(r,t,\omega)$, it follows that $\Delta\Psi(r,t,\omega) \geq 0$ for all $\omega \in [\omega_{UC},\omega_t^0]$ and $\Delta\Psi(r,t,\omega) < 0$ for all $\omega \in (\omega_t^0,\omega_{NC}]$. This proves part (ii). ■

Proof of Corollary 2. The welfare effect is

$$\Delta W(r) = \int_{\omega_{UC}}^{\omega_{NC}(\tau)} \Delta\Psi(r,t,\omega)g(\omega)d\omega.$$

If $r < r_t^-$ then by Proposition 8 the integrand is strictly negative for all $\omega > \omega_{UC}$. Hence $\Delta W(r) < 0$. If $r > r_t^-$ then by Proposition 8 the integrand is strictly positive for all $\omega > \omega_{UC}$. Hence $\Delta W(r) > 0$. ■

Proof of Corollary 3. If $t > \frac{\alpha(1-\alpha)b^2\bar{Q}}{2}$, then $r_t^- > 1$. Then any ROO satisfies $r < r_t^-$ so by Proposition 8 welfare is negative for any ω such that the ROO is effective. ■

Appendix B: Endogenous Q

Relax the assumption that $Q=\bar{Q}$. In the default of no bargain and no compliance, production of $\bar{Q}$ units is always optimal. So we can write

$$U_F^T - U_F^0 = [(p_w^x+\tau)Q - p_w^z z - p^s q] - \left[(p_w^x - p_w^z)\,\bar{Q}\right].$$

With this expression the bargaining surplus becomes

$$\Sigma^{RC} \equiv \tau Q + p_w^z q - C(q, i, \omega) - (p_w^x - p_w^z)(\bar{Q} - Q).$$

Obviously, if $Q = \bar{Q}$, then this collapses back to what is in the main analysis. But if $Q < \bar{Q}$, then the bargaining surplus is reduced because there is a margin of $p_w^x - p_w^z$ that is earned on $(\bar{Q} - Q)$ extra units in the default (versus the equilibrium).

So incorporating this, we can then write the maximization problem as

$$\max_{\{q,Q\}} \qquad \Sigma^{RC} \equiv \tau Q + p_w^z q - C(q, i, \omega) - (p_w^x - p_w^z)(\bar{Q} - Q)$$

such that

$$q \geq rQ$$
$$Q \leq \bar{Q}.$$

Imposing the first constraint, relaxing the second and optimizing over Q, we find

$$Q^*(i) = \frac{\tau + (p_w^x - p_w^z) + (p_w^z - (\omega - bi))\, r}{r^2 c}.$$

Substituting for $i^* = \frac{\alpha b \bar{Q}}{2}$, we then get

$$Q^* = \frac{2\left(\tau + (p_w^x - p_w^z) + r(p_w^z - \omega)\right)}{r^2(2c - \alpha b^2)}.$$

It is easy to show that Q^* is decreasing in r and ω. Hence the condition $Q^* \geq \bar{Q}$ holds more easily if r is low or if ω is low. To guarantee that $Q^* \geq \bar{Q}$ for any ω and r, we need it to hold for $\omega = p_w^z$ and $r = 1$. This implies

$$\tau + (p_w^x - p_w^z) \geq \frac{(2c - \alpha b^2)\bar{Q}}{2}.$$

This condition holds more easily if either τ or p_w^x are higher, and can hold for negligible τ as long as p_w^x is sufficiently high.