Interdependence of water quality and quantity policies: the case of nutrient runoff in the Mississippi River Basin

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Abstract

Agricultural production is a significant determinant for water resource sustainability in terms of both quality and quantity. The interdependence between quality and quantity outcomes calls for studies that account for joint policy impacts. We investigate the effects of water quality policy in the Gulf of Mexico on the water use in the Mississippi River Basin (MRB) and the effects of reduced irrigation in the MRB on water quality outcomes in the Gulf of Mexico. Results show that a 45% nitrogen (N) runoff reduction from the targeted Hypoxia Task Force (HTF) regions in the MRB to the Gulf of Mexico increases (decreases) irrigation from 0.2 (4.6) to 1.4 (4.5) million acre-feet in the HTF (non-HTF) regions. Although irrigation declines in most non-HTF areas, increased use of scarce water for agricultural production is observed in some critical regions, including the counties that rely on the Ogallala Aquifer. The results also show that a 20% decrease in irrigation throughout the MRB increases N use within the MRB by 100 thousand metric tons and N runoff to the Gulf of Mexico from 22 metric tons.

1. Introduction

Agriculture is an important factor for water resource sustainability in quantity and quality. Policy objectives for improving downstream water quality and reducing water use in agriculture are often interdependent. Yet, there is a lack of scientific peer-reviewed literature on the interdependencies between scarcity and quality priorities. We address this gap in the literature by examining agricultural nutrient runoff to the Gulf of Mexico and irrigation within the Mississippi River Basin (MRB). We use a novel integrated hydro-economic agricultural land use (IHEAL) model, which combines a spatially explicit price endogenous partial equilibrium formulation with the process-based ecohydrological Soil and Water Assessment Tool (SWAT).

Agriculture is the largest sector in the U.S. in terms of consumptive water use. In 2015, agriculture withdrew 42 percent of the Nation's total freshwater (USDA ERS,

2021). According to the 2017 Census of Agriculture, irrigated land represented 18 percent of harvested cropland, and more than half of crop sales in terms of dollar value is attributed to irrigated production (USDA NASS, 2019). Droughts and irrigated agriculture have contributed to declining groundwater levels and a growing number of disputes over the allocation of scarce water resources in the western U.S. (Boehlert and Jaeger, 2010; Elbakidze et al., 2012; Elbakidze et al., 2017). The sustainability of agricultural production using water from the Ogallala Aquifer has been a long-standing water policy priority (Torell et al., 1990). Forecasted climate change is expected to intensify droughts and shortages of surface and groundwater (McDonald and Girvetz, 2013; Taylor et al., 2013)

Nutrient runoff from agricultural production to coastal waters has become a serious problem in many parts of the world, including the Gulf of Mexico. The use of nitrogen (N) fertilizers for crop production in the MRB is a major contributor to eutrophication in the Gulf, with detrimental consequences for the aquatic ecosystem (US EPA, 2019). Hypoxia in the Gulf has been a public concern for decades, and in 2001 the EPA formed the Gulf of Mexico Hypoxia Task Force (HTF) to reduce the size of the Hypoxic zone 000 km^2 by 2035 (US EPA, 2014). In 2021, the hypoxic zone in the Gulf reached 16,405 km^2, significantly exceeding the goal (US EPA, 2021a).

Numerous studies have examined spatial characteristics of agricultural best management practices and identified regions of high priority for reducing N loading (Rabotyagov et al., 2014; Ribaudo et al., 2001; Marshall et al., 2018; Xu et al., 2022). The HTF identified critical regions in twelve states [1] for the adoption of best management practices in the MRB (US EPA, 2021b). We rely on this guidance as a reference point for our analyses of N runoff reduction from the targeted regions.

Nonpoint source pollution problems are difficult to solve using performance-based strategies. Instead, practice-based strategies that focus on the regulation of input use are

[1] HTF states include Arkansas, Indiana, Illinois, Iowa, Kentucky, Louisiana, Minnesota, Mississippi, Missouri, Ohio, Tennessee, and Wisconsin (US EPA, 2021b). The determination of targeted regions is based on the relative magnitude of N delivery to the Gulf.

employed. For example, Iowa Department of Agriculture and Land Stewardship, Iowa Department of Natural Resources, and Iowa State University call for reducing the N use rate to 149 kg/ha for corn-soybean rotation and 213kg/ha for continuous corn (ISU, 2017). Similarly, in Illinois, recommendations include reduction of N use to the Maximum Return to N Rate (MRTN)[2] and conversion of corn/soybean tile-drained land to perennials (IEPA, IDOA, and University of Illinois Extension, 2015). Iowa and Illinois are two of the states identified as a high priority for N runoff reduction to the Gulf.

Reducing the per-ha N application rate and switching corn/soybean acres to other land-uses can have the desired effect of lowering the N runoff to waterways. However, these practices can reduce N-intensive crop production at both intensive and extensive margins. These changes can lead to N intensive crop production relocation across targeted and non-targeted regions with implications for both N use and irrigation.

Previous literature documents N intensive crop production relocation potential in response to constrained N use in targeted regions (Ribaudo et al., 2001; Xu et al., 2022). Ribaudo et al. (2001) examined the cost-effectiveness of N fertilizer reduction and wetland restoration for improving water quality in the Gulf of Mexico using a partial equilibrium model linked with the Environmental Policy Integrated Climate (EPIC) tool. Their results showed that reducing N runoff in the MRB could lead to the relocation of crop production to regions outside the MRB. Similarly, Xu et al. (2022) developed the IHEAL model to estimate the HTF objective's costs through N fertilizer use reduction and land-use change. Their results show that targeting production regions in Illinois, Indiana and Iowa is the most cost-effective strategy for mitigating agricultural N runoff to the Gulf of Mexico. However, N intensive crop production moves to the western part of the MRB, including Kansas, Nebraska, and South Dakota. Some of these regions heavily rely on irrigation from scarce water sources, including the Ogallala Aquifer (Ding and Peterson, 2012; Mrad et al., 2020).

The Ogallala Aquifer underlies 45 million ha of land in the U.S. states of Colorado,

[2] MRTN is the N rate that maximizes the dollar return to the use of N (Nafziger, 2018).

Kansas, Nebraska, New Mexico, Oklahoma, South Dakota, Texas, and Wyoming and is the largest freshwater aquifer in the world. The Ogallala Aquifer region accounts for approximately 30% of all U.S. irrigation. More than 30% of U.S. crops and livestock are produced in this area (Steiner et al., 2021). The development of irrigation technology enabled access to Ogallala Aquifer after World War II, and extraction grew rapidly. The depletion of the aquifer in the High Plains has been a public concern since the 1970s (Peterson et al., 2003). Famiglietti and Ferguson (2021) reported that the Southern Ogallala Aquifer was among the most rapidly depleting aquifers in the past two decades.

This study examines agricultural N runoff to the Gulf of Mexico and irrigation within the MRB. In particular, the primary objectives of this study are to (i) evaluate the effects of N runoff reduction in the HTF targeted regions on water use in the MRB, including HTF and non-HTF areas; (ii) evaluate the effect of water use reduction in the MRB on spatial N use in the MRB and N runoff to the Gulf of Mexico, and (iii) quantify the combined effect of N runoff and irrigation water use reduction on agricultural production, prices, and welfare estimates.

2. Theoretical Framework

This section presents a theoretical framework and simplified analytical results illustrating the impact of N use restriction on irrigation. A parsimonious welfare maximization model assumes two production regions and a representative commodity market. Both regions use irrigation and N in agricultural production. Region 2 is targeted in N use reduction programs because the majority of N runoff to the coast is attributable to production in region 2.

$$\max_{\mathrm{x},x_1,x_2,n_1,n_2,w_1} \pi = \int_0^x \mathrm{p}(t)\,dt - C_n(n_1+n_2) - C_w(w_1+w_2), \quad (1)$$

$$\text{subject to}$$

$$x_1 + x_2 \geq x \quad (2)$$

$$f(n_1, w_1) \geq x_1 \quad (3)$$

$$g(n_2, w_2) \geq x_2 \quad (4)$$

$$\tau \geq n_2 \quad (5)$$

where x is commodity consumption; x_1 and x_2 are commodity production in regions 1 and 2, respectively; suppressing subscript for regions, n and w denote the N fertilizer and irrigation inputs in crop production, respectively. $\mathrm{p}(t)$ is the inverse commodity demand function. C_n and C_w are unit costs for inputs n and w, respectively. $f(n,w)$ is the production function in region 1, with f_n and $f_w > 0$, and $f_{nn}\ and\ f_{ww} < 0$. The cross partial derivatives $f_{nw} = f_{wn}$ can be either negative or positive depending on whether N and water are substitutes or complements. Similarly, g is the production function in region 2, with properties analogous to the production function in region 1.

Equation (2) requires that total crop consumption cannot exceed the sum of production in regions 1 and 2. Equations (3) and (4) are technology constraints for production in regions 1 and 2, respectively. Equation (5) sets the exogenous restriction on N use (τ) in region 2.

The appendix provides the Lagrangian and the first order conditions, which are used to form the Hessian matrix. Comparative statics for changes in variables of interest with respect to the change in τ are obtained using the Cramer's rule.

The signs of comparative static expressions depend on the signs of cross partial derivative. In particular, the effects of nitrogen use restriction in region 2 on irrigation in regions 1 and 2 mainly depend on the signs of $f_{n_1w_1}$ and $g_{n_2w_2}$ and relative magnitudes of $f_{n_1w_1}$ versus $f_{n_1n_1}\ and\ f_{w_1w_1}$.

$$\frac{\partial w_1}{\partial \tau} = \frac{-p_x\left(f_{n_1}f_{n_1w_1} - f_{w_1}f_{n_1n_1}\right)\left(-g_{n_2w_2}g_{w_2} + \left(f_{w_1} + g_{n_2}\right)g_{w_2w_2}\right)}{g_{w_2w_2}\left(p_x f_{n_1}f_{n_1w_1}f_{w_1} - p_x {f_{n_1}}^2 f_{w_1w_1} + \rho\left({f_{n_1w_1}}^2 - f_{n_1n_1}f_{w_1w_1}\right)\right)} \quad (6)$$

$$\frac{\partial w_2}{\partial \tau} = \frac{\begin{pmatrix} -f_{n_1} f_{n_1 w_1} f_{w_1} g_{n_2 w_2} p_x \mu + f_{n_1 w_1}{}^2 f_{w_1 w_1} g_{n_2 w_2} p_x \mu \\ -f_{n_1 w_1}{}^2 \rho \left(f_{w_1} g_{w_2} p_x + g_{n_2} g_{w_2} p_x + g_{n_2 w_2} \mu \right) \\ +\rho f_{n_1 n_1} f_{w_1 w_1} \left(f_{w_1} g_{n_2} p_x + g_{n_2} g_{w_2} p_x + g_{n_2 w_2} \mu \right) \end{pmatrix}}{\mu g_{w_2 w_2} \left(p_x f_{n_1} f_{n_1 w_1} f_{w_1} - p_x f_{n_1}{}^2 f_{w_1 w_1} + \rho \left(f_{n_1 w_1}{}^2 - f_{n_1 n_1} f_{w_1 w_1} \right) \right)} \quad (7)$$

Hence, the effect on irrigation partly depends on whether N and irrigation are substitutes or complements. If N and water are complements, then decreased N use in region 2 can decrease irrigation in region 2. Stricter regulation of N use in region 2 can lead to a relocation of N intensive crop production to region 1, and complementary relationship between N and irrigation can increase water use for N intensive crop production in region 1. If N and irrigation are substitutes, then reduced N use in region 2 can result in greater irrigation in region 2. On the other hand, region 1 may decrease irrigation if N use increases.

The simplified analytical model provides theoretical insights for the effect of N use restriction on irrigation in targeted and non-targeted regions depending on the assumptions regarding the complementary or substitute relationship between N and irrigation and the relative magnitudes of the second order production function derivatives. However, in practice, these assumptions are difficult to assess due to spatial heterogeneity within and across the HTF and non-HTF regions. We address this heterogeneity empirically.

3. Methods and data

We use the IHEAL model (Xu et al., 2022) to empirically assess the effects of the water quality goal set in the Gulf of Mexico on water use in MRB. IHEAL is an integrated hydro-economic agricultural land use model, which combines a national price endogenous partial equilibrium commodity market formulation and a process-based SWAT. The model includes county-scale crop planting, fertilizer use, and irrigation. County production activities generate national commodity supply estimates

that are combined with corresponding commodity demand functions to produce equilibrium prices, quantities, and producer and consumer surplus estimates. The model endogenously determines annual county crop planting acreage, N use, and irrigation based on constrained consumer and producer welfare maximization in the select crop markets.

The IHEAL model maximizes consumer and produces welfare in select commodity markets in the U.S., subject to the supply-demand balance including exports and imports, production technology constraints, water use constraints, N runoff constraints, and the land allocation constraints that represents a convex combination of historical and synthetic acreages. Synthetic acreages are obtained using own and cross-price elasticities and own and cross acreage price elasticities following Chen and Onal (2012). Required elasticity estimated are obtained using fixed effect Arellano-Bond estimator and county production and price data from 2005 to 2019.

SWAT is used to estimate crop yields and N runoff to the Gulf. County production activities, including crop planting acreage, fertilizer application, and irrigation, are combined with the watershed SWAT delivery ratios to estimate annual N runoff from crop production to the Gulf of Mexico.

The IHEAL model includes crop production activities in 2,788 counties in the contiguous U.S. where at least one of the crops included in this model was planted in at least one year from 2005 to 2019. These counties include 1,620 that are located within the MRB and 1,168 outside. Per acre crop yields in the counties located within the MRB are expressed as functions of N use and irrigation levels using SWAT parameters. Per acre crop yields in the counties located outside of MRB are fixed based on the USDA data. County planted acreages within and outside of MRB are endogenously estimated.

Corn, soybean, wheat and sorghum are included in the model because these crops are major agricultural commodities and are the most fertilizer-intensive crops planted in the US (USDA NASS, 2020; Marshall et al., 2015; Steiner et al., 2021).

The parametric model data include crop demand elasticities, market prices, county-specific historical crop acreage, historical maximum irrigated acreage, and input costs,

including energy, fertilizer, water and other production costs. The crop demand elasticities are obtained from previous literature (Westcott and Hoffman, 1999; Piggott and Wohlgenant, 2002; Ishida and Jaime, 2015). The crop market prices and historical crop acreage are collected from USDA NASS (USDA NASS, 2020). The county historical maximum irrigated acreages are obtained from U.S. Geological Survey data (Dieter et al., 2018; USGS, 2018). Energy input, fertilizer, water and other production costs are obtained from USDA ERS (USDA ERS, 2019).

A detailed description of the IHEAL model and data is provided in Xu et al. (2022). In this study, HTF-targeted counties are identified following EPA HTF recommendations and solutions obtained in Xu et al. (2022), where county scale changes in land-use and reductions in N use are estimated to achieve a 45% reduction in N runoff to the Gulf. Unlike Xu et al. (2022), we limit the N runoff reduction to counties located in the HTF targeted states, while counties in non-HTF states are not subject to N use reduction. Model solutions provide optimal N use reduction and land-use change in HTF states to achieve the 45% reduction.

4. Results and discussion

We first present the validation and baseline results. Next, we discuss the results obtained from the scenarios with N runoff restriction in the HTF targeted counties and irrigation restrictions throughout MRB.

4.1 Validation and baseline results

The purpose of this section is twofold. One is to validate the model solutions in terms of replication of observed data. The other is to obtain baseline estimates of N runoff in the Gulf and county scale irrigation, which are used as benchmarks for the subsequent scenario analyses.

For model validation purposes, the model is solved using observed county historical crop mix data. We present some of the key baseline model solutions as part of model validation, including crop production, crop prices, the amount of N delivered

to the Gulf of Mexico, the irrigated crop acreages, and the amount of irrigation water used within the MRB, compared with the observed values in the baseline year 2018 (Table 1). The model overestimates cumulative crop acreage for corn, soybean and sorghum by 10.3%, 7.5%, and 4.4%, respectively, relative to the acreages observed in 2018. On the other hand, wheat acreage is 2.6% lower than that observed in 2018. All estimated crop prices are close to the observed values in 2018, with all deviations less than 2%.

Water use, N use and N delivered to the Gulf of Mexico from N fertilizer use are presented in Table 1. Within the MRB, there are 932 and 688 counties located within and outside the HTF targeted states, respectively. We refer to these counties as HTF and non-HTF counties hereafter. The total estimated irrigated acreage for select crops within the MRB is 3.9 million ha, which represents 17.3% of total irrigated acreages in the U.S. The annual water use within the MRB is 5.2 million acre-feet, which accounts for 6.2% of the total observed irrigation water use in the U.S. Water use in non-HTF counties accounts for 92.3% of the total water use within the MRB. N applied within the MRB for corn, soybean, wheat and sorghum is 6,984 thousand metric tons, which is 55.4% of the total observed N use in the U.S. N use in Non-HTF and HTF counties accounts for 43.7% and 56.3%, respectively. The corresponding N delivered to the Gulf of Mexico from fertilizer use in corn, soybean, wheat, and sorghum fields is 368,660 metric tons, accounting for 89.9% of total N delivered to the Gulf of Mexico from the agricultural sector in the MRB (White et al., 2014). N deliveries from non-HTF and HTF counties to the Gulf of Mexico are 5.6% and 94.4% of the estimated N loading, respectively. The dominance of N delivery from HTF counties relative to non-HTF counties is consistent with the targeting scheme proposed by the EPA (US EPA, 2021b). These solutions provide a firm footing and benchmark for the subsequent analysis of N runoff scenarios.

We use the model with historical and synthetic crop mix data to generate baseline model results to serve as a benchmark for comparison (column 3, Table 1). Synthetic crop acreages allow for greater model flexibility relative to the model that uses only

historical crop mix. The added flexibility is advantageous for the scenarios with constraints that fall outside of historically observed settings. The scenario restrictions, including significant N use and irrigation reductions, fall outside of the conditions observed during the 2005 - 2019 period. We use these baseline results as benchmarks for comparing key estimates, rather than the results in column 1, for greater consistency between long-run equilibrium results of scenarios with and without added restrictions. The baseline N runoff to the Gulf of Mexico is 365,350 metric tons with 345,428 metric tons N delivered from HTF states.

4.2 N use reduction in HTF counties

This section presents the results for the scenario corresponding to the EPA's objective to reduce N delivered to the Gulf of Mexico from the HTF counties. We constrain the model to reduce N runoff to the Gulf of Mexico by 45% relative to its baseline value by reducing N use in the HTF counties[3].

Table 2 shows detailed baseline and N runoff curtailment scenario MRB county results for crop acreage and production, irrigated acreage, water use, N fertilizer use and corresponding runoff to the Gulf of Mexico. The results are also decomposed into HTF and non-HTF counties within the MRB.

The combined consumer and producer surplus value decreases from $212 to $206 billion dollars as N runoff from HTF states to the Gulf is restricted. Within the MRB, corn, soybean and sorghum acreage decrease by 8.7%, 12.2% and 6.3%, respectively; wheat acreage slightly increases by 0.9%. Crop production within the MRB decreases due to the reduction in N fertilizer use in HTF counties. N use in HTF counties decreases by 36.9%. The large reduction in N use leads to a drastic increase in water use in HTF counties, increasing from 0.2 to 1.4 million acre-feet. However, irrigated acreage decreases from 938,364 to 706,968 ha; non-irrigated acreage declines from

[3] Baseline N runoff to the Gulf of Mexico from crop production in the MRB is 345,428 metric tons (table 1). Hence, the upper bound on N runoff to the Gulf in the constrained N runoff scenario is 189,986 metric tons.

95.8 to 93.1 million ha. In general, irrigation can be a substitute for N use to some degree in HTF counties. N runoff restriction in HTF counties reduces N use and N-intensive crop acreage but intensifies production in terms of the water use to partially compensate crop production loss due to reduced N use.

In non-HTF counties, crop acreage slightly increases from 29 to 30.3 million ha due to the relocation of N intensive crop production from HTF counties. Corn, soybean and wheat production increase by 6.2%, 1.8% and 1.8%, respectively. On the other hand, sorghum production decreases by 10.7%. This is a reasonable result because sorghum is not as N intensive as corn is, and the production of corn becomes more profitable in non-HTF counties when HTF counties reduce N use. The reallocation of crop production leads to a 6.6% increase in N use in this region, which partially substitutes the water use. The irrigated acreage and water use in non-HTF counties overall decline by 1% and 2.2%, respectively.

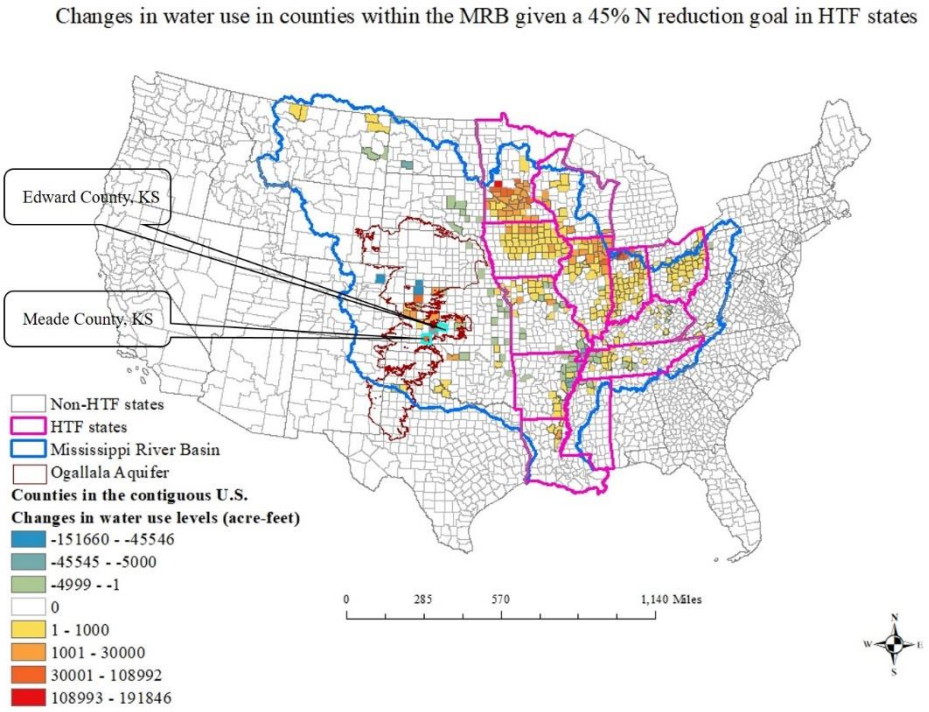


Figure 1. Irrigation change in MRB counties in response to a 45% N runoff from HTF states

Figure1 presents the county-scale change in irrigation in response to the 45% reduction in the Gulf N runoff from the HTF counties. HTF counties in Minnesota, Iowa, Illinois and Indiana, increase irrigation to offset production losses due to reduced N use. However, some of the HTF counties, like those located in Arkansas, decrease irrigation as acreage is removed from production.

Although cumulative irrigation slightly decreases in the non-HTF counties, some spatial heterogeneity is observed. Irrigation increases in some of the non-HTF counties and decreases in others. Some of the increase in irrigation takes place in regions with a critical scarcity of irrigation water. For example, some of the counties where irrigation heavily relies on the Ogallala aquifer increase irrigation.

Table 3 shows the cumulative results for crop acreage, irrigated acreage, irrigation and N use in the Ogallala counties. Crop acreage in these counties cumulatively increases by 7%. Corn and soybean production increase by 6.7% and 3.2%, respectively. However, wheat and sorghum production decrease by 2% and 8.3%, respectively. Again, the production of relatively N-intensive crops increases at the expense of less N-intensive crops. Similar to other non-HTF counties overall, N use increases in Ogallala counties by 4.7%. On the other hand, irrigated acreage and total water use decline by 1.3% and 0.2% relative to the baseline scenario, respectively.

Although the results show that cumulative irrigation in the Ogallala counties does not change significantly, the disaggregated county-scale results reveal that changes are spatially heterogeneous. Irrigation changes in 20 Ogallala counties with decrease (increase) documented in 10 counties (10 counties). The largest reduction in water use occurs in Edwards County, KS, where irrigated acreage for corn, soybean and sorghum decreases from 12,474 to 9,315 ha, resulting in the total water use reduction from 18,476 to 13,797 acre-feet. The greatest irrigation increase is observed in Meade County, KS, where irrigated acreage for selected crops does not change significantly relative to the baseline as the upper bound on acreage constraint is binding. However, irrigation at the intensive margin increases from 270,036 to 302,763 acre-feet.

4.3 Irrigation decrease in the MRB regions

Table 4 presents results for the water scarcity scenario. For the sake of illustration, in this scenario county-scale water use obtained in the baseline scenario is restricted by up to 20% with increments of 5%. The results show that cumulative N use within the MRB increases by roughly 5 thousand metric tons. Almost all of this increase in N use takes place in non-HTF counties. Non-HTF regions have low N delivery ratios, which is why these regions are not targeted by the N runoff reduction efforts to begin with. Therefore, no significant change is observed in terms of N runoff to the Gulf of Mexico. The N use increase in non-HTF regions is reasonable because irrigation water is a more significant input in non-HTF regions than in the HTF regions. Hence, decreased water use is compensated with an increase in N use.

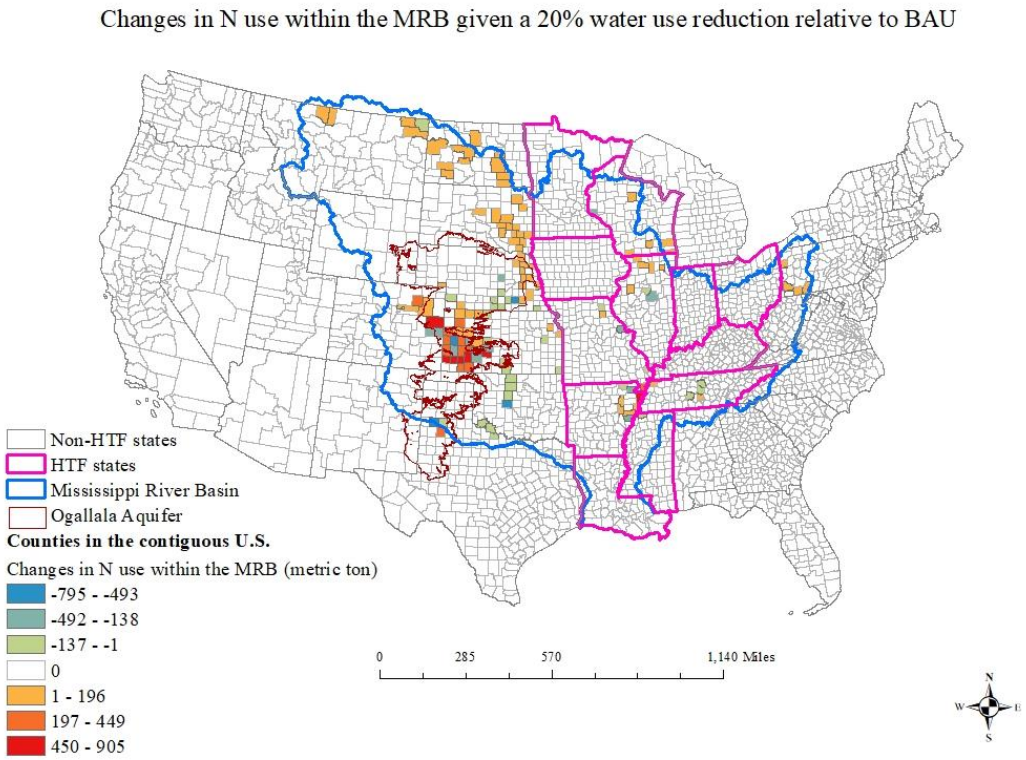


Figure 2. Spatial distribution of changes in N use in response to a 20% reduction in irrigation

Cumulatively, N use increases in the non-HTF region with a slight increase in N

runoff to the Gulf. However, county-scale change in N use is spatially heterogeneous. Figure 2 presents the change in county scale N use for a 20% irrigation reduction relative to the baseline scenario. Most of the change in N use takes place in the non-HTF counties, with most non-HTF counties experiencing an increase in N use. Most significant increases in N use are observed in Ogallala counties, with Stanton County, KS, showing the largest increase from 15,313 to 16,219 metric tons. N use in Stanton County, KS increases as a) corn production moves from other counties where corn acreage decreases due to water scarcity, and b) N is used to compensate for losses due to less irrigation.

These results illustrate that reduced irrigation throughout the MRB is not likely to significantly affect N runoff to the Gulf of Mexico because N use does not increase in HTF areas with high N delivery ratios. In the HTF region, irrigation is not a significant factor. Therefore, a significant change in production is not observed. However, there is a significant increase in N use in some non-HTF counties, which can lead to localized negative environmental impacts.

5. Conclusion

This paper examines the effect of water quality policy in the Gulf of Mexico on the reallocation of water use within the MRB. We obtain two main findings. First, for HTF counties (targeted region for N runoff reduction within the MRB), the water use increases to compensate the yield and production losses due to the N use restriction and land-use change, especially in the Corn Belt. Second, some N-intensive crop production moves from HTF to non-HTF counties (region not targeted by water quality policy within the MRB). Cumulatively, we observe a slight drop in water use and an increase in total N use in non-HTF counties. However, the effects are heterogeneous among non-HTF counties. Irrigation can increase in some of the sensitive counties where depletion of groundwater is a major concern. Such unintended consequences of water quality policies like N runoff reduction to the Gulf of Mexico warrant further evaluation and consideration in policy design.

Our results also show the reduction of N runoff to the Gulf of Mexico could result in an increase of crop production and/or a decrease in irrigation in some of the non-HTF counties. While these co-benefits maybe welcome in water-scarce regions like the Ogallala Aquifer region, one has to keep in mind that irrigation decrease in these non-HTF counties comes with a significant increase in N use. Although N runoff from the non-HTF counties to the Gulf of Mexico is modest, local watersheds and ecosystems may be harmed by the increased use of N fertilizers. These consequences are also important for policy evaluation.

The results also document the impacts of water quantity restrictions on water quality outcomes. We observe that irrigation curtailments in the MRB are not likely to have significant impacts on N runoff to the Gulf of Mexico. The reason for the modest impact is that in our context, most critical regions for N runoff to the Gulf do not rely on irrigation, and N use in the areas that heavily rely on irrigation does not have a significant impact on N runoff to the Gulf. Nevertheless, the increase in N use is observed in the areas that rely on irrigation when irrigation is curtailed. This increase in N use can result in the degradation of local ecosystems. Therefore, evaluation of policies that aim to decrease irrigation use should consider water quality impacts from potential increases in fertilizer use as a substitute for irrigation water.

Two limitations of this study should be mentioned. First, the IHEAL model corresponds to the social planner's problem with perfect information. Crop production, land-use, and input use (N and water) are obtained based on the maximization of social welfare. This framework is consistent with Potential Pareto Optimality criteria but does not explicitly consider implications for strict Pareto Optimality (Griffin, 1995). Second, crop production outside of MRB is not subject to changes imposed on production activities within the MRB. No N use reduction or water use reduction constraints are imposed on the production outside of MRB. The hypoxic zone in the Gulf of Mexico is the largest in the U.S. and examining the N runoff to the Gulf in isolation is reasonable. However, droughts and water scarcity are not unique to irrigated agriculture in the MRB. Many major agricultural regions in the western U.S. also experience shortages of

irrigation water. This is an important caveat for our examination of the impact of reduced irrigation in the MRB on water quality outcomes. An important assumption is that the reduction in water use is limited to the areas within the MRB.

Table 1. Validation and baseline results

	Validation results (historical crop mix)	Observed in 2018[ab]	Baseline results (historical and synthetic crop mix)
	LAND USE (MILLION HECTARES) FOR THE CONTIGUOUS UNITED STATES		
Corn	39.7	36.0	39.2
Soybean	38.8	36.1	38.1
Wheat	18.8	19.3	17.0
Sorghum	2.4	2.3	2.4
	PRICES ($/METRIC TON)		
Corn Price	139.2	142	140.5
Soybean Price	311.1	314	315.2
Wheat Price	190	190	212.8
Sorghum Price	118.5	117	112.9

	Validation results (historical crop mix)	Values from literature	Baseline results (historical and synthetic crop mix)
Total irrigated acreage (million ha)	3.9 (MRB)	22.6 (U.S.)	3.9 (MRB)
Non HTF counties	3.0		3.0
HTF counties	0.9		0.9
Total water use (million acre-feet)	5.2 (MRB)	83.4 (U.S.)[a]	4.8 (MRB)
Non HTF counties	4.8		4.6
HTF counties	0.4		0.2
N applied within the MRB (1000 metric ton)	6,984 (MRB)	12,610 (U.S.)[e]	6,832 (MRB)
Non HTF counties	3,054		2,951

HTF counties	3,930		3,881
N delivered to the Gulf of Mexico from fertilizer application (metric ton)	368,660 (MRB)	410,190 (MRB)[fg]	365,350 (MRB)
Non HTF counties	20,463		19,922
HTF counties	348,197		345,428

[a] Source: USDA NASS, 2019

[b] Baseline model data, including prices and quantities for commodity demands are from 2018. Hence, we compare the baseline results with data observed in 2018.

[c] Source: Rosenberg et al., 1999

[d] There are 237 counties partially or entirely located above the Ogallala Aquifer. 66 of these counties are entirely or partially located in the MRB.

[e] The sum of county-level farm N fertilizer use (Falcone, 2021).

[f] Source: White et al., 2014.

[g] N fertilizer use in crop production accounts for 68% of N delivered to the Gulf of Mexico from agriculture. The rest of N exported to the Gulf from agriculture comes from confined animal operations and legume crops (USGS, 2017).

Table 2. N runoff reduction from HTF states by 45%

	Baseline	Constrained N runoff
Corn acreage within the MRB (million ha)	30.9	28.2
Corn production within the MRB (million metric ton)	309.9	267.4
Soybean acreage within the MRB (million ha)	28.6	25.1
Soybean production within the MRB (million metric ton)	94.9	82.7
Wheat acreage within the MRB (million ha)	10.9	11
Wheat production within the MRB (million metric ton)	27.1	26.1
Sorghum acreage within the MRB (million ha)	1.6	1.5
Soeghum production within the MRB (million metric ton)	6.7	5.6
Irrigated Acreage within the MRB (ha)	3,930,805	3,669,178
Non HTF counties	2,992,441	2,962,210
HTF counties	938,364	706,968
Total water use within the MRB (million acre-feet)	4.8	5.9
Non HTF counties	4.6	4.5
HTF counties	0.2	1.4
N applied within the MRB (1000 metric ton)	6,832	5,595
Non HTF counties	2,951	3,145
HTF counties	3,881	2,450
N delivered to the Gulf of Mexico from fertilizer application (metric ton)	365,350	211,000
Non HTF counties	19,922	21,014
HTF counties	345,428	189,986
social welfare (million $)	212,500	206,100

Table 3. Results for the Ogallala counties under a 45% N runoff reduction goal for the HTF counties

	Baseline results	Constrained N runoff
Crop acreage (million ha)	8.28	8.86
Irrigated acreage (million ha)	2.36	2.33
Total water use (million acre-feet)	4.37	4.36
Total N use (thousand metric ton)	1,114	1,166

Table 4. Effects of water scarcity on N use within the MRB

		Decrease in water use relative to baseline			
	Baseline results	**5%**	**10%**	**15%**	**20%**
Corn acreage within the MRB (million ha)	30.91	30.89	30.89	30.89	30.89
Corn production within the MRB (million metric ton)	309.85	309.67	309.65	309.66	309.68
Soybean acreage within the MRB (million ha)	28.63	28.65	28.65	28.65	28.64
Soybean production within the MRB (million metric ton)	94.89	94.94	94.94	94.93	94.93
Wheat acreage within the MRB (million ha)	10.86	10.89	10.89	10.87	10.85
Wheat production within the MRB (million metric ton)	27.12	27.18	27.19	27.15	27.13
Sorghum acreage within the MRB (million ha)	1.61	1.61	1.62	1.62	1.62
Sorghum production within the MRB (million metric ton)	6.74	6.73	6.71	6.69	6.66
Irrigated Acreage within the MRB (ha)	3,930,805	3,915,495	3897712	3879577	3861779
Non HTF counties	2,992,441	2,981,571	2968363	2954351	2942829
HTF counties	938,364	933,924	929349	925226	918950
Total water use within the MRB (million acre-feet)	4.8	4.6	4.4	4.1	3.9
Non HTF counties	4.6	4.41	4.22	3.93	3.74
HTF counties	0.2	0.19	0.18	0.17	0.16
N applied within the MRB (1000 metric ton)	6832.40	6835.22	6836.57	6836.63	6837.42
Non HTF counties	2950.96	2,954.59	2955.74	2955.32	2955.62
HTF counties	3881.44	3,880.63	3880.83	3881.31	3881.8

N delivered to the Gulf of Mexico (metric ton)	365,350	365,347	365,352	365,361	365,372
Non HTF counties	19,922	19,933	19,936	19,936	19,937
HTF counties	345,429	345,414	345,416	345,425	345,435
Social welfare (million $)	212,484	212,431	212,427	212,422	212,418

Reference

Boehlert, B. B., & Jaeger, W. K. 2010. Past and future water conflicts in the Upper Klamath Basin: An economic appraisal. Water Resources Research, 46(10).

Chen, X., and Önal, H. 2012. Modeling agricultural supply response using mathematical programming and crop mixes. Am. J. Agric. Econ. 94(3), 674-686. https://doi.org/10.1093/ajae/aar143

Dieter, C.A., Linsey, K.S., Caldwell, R.R., Harris, M.A., Ivahnenko, T.I., Lovelace, J.K., Maupin, M.A., and Barber, N.L., 2018, Estimated use of water in the United States county-level data for 2015 (ver. 2.0, June 2018): U.S. Geological Survey data release, https://doi.org/10.5066/F7TB15V5.

Ding, Y., Peterson, J. M. 2012. Comparing the cost-effectiveness of water conservation policies in a depleting aquifer: A dynamic analysis of the Kansas High Plains. J. Agric. Appl. Econ. 44(2), 223-234. https://doi.org/10.1017/S1074070800000286.

Elbakidze, L., Shen, X., Taylor, G., & Mooney, S. 2012. Spatiotemporal analysis of prior appropriations water calls. Water Resources Research, 48(6).

Elbakidze, L., Fa'anunu, B., Mamula, A., & Taylor, R. G. 2017. Evaluating economic efficiency of a water buyback program: The Klamath irrigation project. Resource and Energy Economics, 48, 68-82.

Famiglietti, J.S.; Ferguson, G. The hidden crisis beneath our feet: Disappearing groundwater requires widespread action to prevent widespread water scarcity. Science 2021, 372, 344–345.

Griffin, R. C. 1995. On the meaning of economic efficiency in policy analysis. Land Economics, 71: 1-15

HAWQS, 2020, "HAWQS System and Data to model the lower 48 conterminous U.S using the SWAT model", https://doi.org/10.18738/T8/XN3TE0, Texas Data Repository Dataverse, V1

Illinois Environmental Protection Agency (IEPA), Illinois Department of Agriculture (IDOA), and University of Illinois Extension. 2015. Illinois Nutrient Loss Reduction Strategy. Illinois Environmental Protection Agency and Illinois Department of Agriculture; Springfield, Illinois. University of Illinois Extension; Urbana, Illinois. Available at: https://www2.illinois.gov/epa/topics/water-quality/watershed-management/excess-nutrients/Pages/nutrient-loss-reduction-strategy.aspx (Accessed Jan. 2022)

Iowa State University (ISU). 2017. Iowa Nutrient Reduction Strategy. Section 2.1 Executive Summary – Iowa Science Assessment of Nonpoint Source Practices to Reduce Nitrogen and Phosphorus Transport in the Mississippi River Basin. Available at: https://www.nutrientstrategy.iastate.edu/. (Accessed on Dec 2021)

Marshall, K. K., Riche, S. M., Seeley, R. M., Westcott, P. C. 2015. Effects of recent energy price reductions on US agriculture. United States Department of Agriculture, Economic Research Service.

McDonald, R. I., & Girvetz, E. H. 2013. Two challenges for US irrigation due to climate change: increasing irrigated area in wet states and increasing irrigation rates in dry states. PloS one, 8(6), e65589.

Mrad, A., Katul, G. G., Levia, D. F., Guswa, A. J., Boyer, E. W., Bruen, M., Carlyle-

Moses, D. E., Coyte, R., Creed, I. F., van de Giesen, N., Grasso, D., Hannah, D. M., Hudson, J. E., Humphrey, V., Lida, S., Jackson, R. B., Kumagai, T., Llorens, P., Michalzik, B., Nanko, K., Peters, C. A., Selker, J. S., Tetzlaff, D., Zalewski, M., Scanlon, B. R. 2020. Peak grain forecasts for the US High Plains amid withering waters. Proc. Natl. Acad. Sci. U.S.A. 117(42), 26145-26150. https://doi.org/10.1073/pnas.2008383117.

Nafziger, E. (2018). Using the Maximum Return to Nitrogen (MRTN) recommendation system in Illinois. Springfield, IL: Illinois Nutrient Research and Education Council.

Peterson, J. M., Marsh, T. L., & Williams, J. R. (2003). Conserving the Ogallala Aquifer: Efficiency, equity, and moral motives. Choices, 2003(1st qtr.), 15-18.

Ribaudo, M. O., Heimlich, R., Claassen, R., Peters, M. 2001. Least-cost management of nonpoint source pollution: source reduction versus interception strategies for controlling nitrogen loss in the Mississippi Basin. Ecol. Econ. 37(2), 183-197. https://doi.org/10.1016/S0921-8009(00)00273-1.

Rosenberg, N. J., Epstein, D. J., Wang, D., Vail, L., Srinivasan, R., & Arnold, J. G. (1999). Possible impacts of global warming on the hydrology of the Ogallala aquifer region. Climatic change, 42(4), 677-692.

Steiner, J. L., Devlin, D. L., Perkins, S., Aguilar, J. P., Golden, B., Santos, E. A., & Unruh, M. (2021). Policy, Technology, and Management Options for Water Conservation in the Ogallala Aquifer in Kansas, USA. Water, 13(23), 3406.

Taylor, R. G., Scanlon, B., Döll, P., Rodell, M., Van Beek, R., Wada, Y., Longuevergne, L., Leblanc, M., Famiglietti, J. S., Edmunds, M., Konikow, L., Green, T. R., Chen, J., Taniguchi, M., Bierkens, M.F. P., MacDonald, A., Fan, Y., Maxwell, R. M., Yechieli, Y., Gurdak, J. J., Allen, D. M., Shamsudduha, M., Hiscock, K., Yeh, P. J.-F., Holman, I., and Treidel, H. 2013. Ground water and climate change. Nature Climate Change, 3(4), 322-329.

Thomas, A., Groom, B., Koundouri, P., & Nauges, C. (2008). The story of the moment: Risk averse Cypriot farmers respond to drought management. Applied Economics, 40(3), 315-326. https://doi.org/10.1080/00036840600592916.

Torell, L. A., Libbin, J. D., & Miller, M. D. (1990). The market value of water in the Ogallala Aquifer. Land economics, 66(2), 163-175.

U.S. Environmental Protection Agency (US EPA). 2014. Mississippi River Gulf of Mexico Watershed Nutrient Task Force New Goal Framework. https://www.epa.gov/sites/production/files/2015-07/documents/htf-goals-framework-2015.pdf. (Accessed on Dec 2021)

US EPA. 2019. Hypoxia 101. https://www.epa.gov/ms-htf/hypoxia-101. (Accessed on Dec 2021)

US EPA. 2021a. Northern Gulf of Mexico Hypoxic Zone. Available at: https://www.epa.gov/ms-htf/northern-gulf-mexico-hypoxic-zone. (Accessed on Dec 2021)

US EPA. 2021b. Hypoxia Task Force Nutrient Reduction Strategies. Available at: https://www.epa.gov/ms-htf/hypoxia-task-force-nutrient-reduction-strategies. (Accessed on Dec 2021)

USDA ERS (United States Department of Agriculture Economic Research Service). 2019. Fertilizer Use and Price. https://www.ers.usda.gov/data-products/fertilizer-use-and-price. (accessed 30 Oct 2019).

USDA ERS. 2021. Irrigation & Water Use. Available at: https://www.ers.usda.gov/topics/farm-practices-management/irrigation-water-use/. (Accessed on Dec 2021)

U.S. Department of Agriculture National Agricultural Statistics Service (USDA NASS). 2019. 2017 Census of Agriculture. Available at: www.nass.usda.gov/AgCensus. (Accessed on Dec 2021)

USDA NASS. 2020b. U.S. &All States County Data – Crops. Washington, DC. http://www.nass.usda.gov/. (accessed 13 May 2021).

U.S. Geological Survey (USGS). 2018. Water-use data available from USGS. https://water.usgs.gov/watuse/data/index.html. (accessed 1 Jan 2022).

Xu, Y., Elbakidze, L., Yen, H., Arnold, J. G., Gassman, P. W., Hubbart, J., & Strager, M. P. 2022. Integrated assessment of nitrogen runoff to the Gulf of Mexico. Resource and Energy Economics, 67:1-17

Appendix

$$\max_{\mathrm{x},x_1,x_2,n_1,n_2,w_1,w_2} \pi = \int_0^x \mathrm{p}(t)\,dt - C_n(n_1+n_2) - C_w(w_1+w_2) \quad (1)$$

subject to:

$$x_1 + x_2 \geq x \quad (2)$$
$$f(n_1, w_1) \geq x_1 \quad (3)$$
$$g(n_2, w_2) \geq x_2 \quad (4)$$
$$\tau \geq n_2 \quad (5)$$

Lagrangian and corresponding first order conditions are as follows:

$$\mathrm{L} = \int_0^x \mathrm{p}(t)\,dt - C_n(n_1+n_2) - C_w(w_1+w_2) + \lambda(x_1+x_2-x) + \rho[f(n_1,w_1)-x_1] + \mu[g(n_2,w_2)-x_2] + \gamma(\tau-n_2)$$

$[x]$ $\quad \frac{\partial L}{\partial x_1} = \mathrm{p}(x) - \lambda = 0 \quad (6)$

$[x_1]$ $\quad \frac{\partial L}{\partial x_1} = \lambda - \rho = 0$

$[x_2]$ $\quad \frac{\partial L}{\partial x_2} = \lambda - \mu = 0$

$[n_1]$ $\quad \frac{\partial L}{\partial n_1} = -C_n + \rho f_{n_1} = 0$

$[n_2]$ $\quad \frac{\partial L}{\partial n_2} = -C_n + \mu g_{n_2} - \gamma = 0$

$[w_1]$ $\quad \frac{\partial L}{\partial w_1} = -C_w + \rho f_{w_1} = 0$

$[w_2]$ $\quad \frac{\partial L}{\partial w_2} = -C_w + \mu g_{w_2} = 0$

$[\lambda]$ $\quad \frac{\partial L}{\partial \lambda} = x_1 + x_2 - x = 0$

$[\rho]$ $\quad \frac{\partial L}{\partial \rho} = f(n_1, w_1) - x_1 = 0$

$[\mu]$ $\quad \frac{\partial L}{\partial \mu} = g(n_2, w_2) - x_2 = 0$

$[\gamma]$ $\quad \frac{\partial L}{\partial \gamma} = \tau - n_2 = 0$

Total differentiation of the first order conditions with respect to τ gives:

$$[x] \qquad p_x \frac{\partial x}{\partial \tau} - \frac{\partial \lambda}{\partial \tau} = 0 \qquad (7)$$

$$[x_1] \qquad \frac{\partial \lambda}{\partial \tau} - \frac{\partial \rho}{\partial \tau} = 0$$

$$[x_2] \qquad \frac{\partial \lambda}{\partial \tau} - \frac{\partial \mu}{\partial \tau} = 0$$

$$[n_1] \qquad \frac{\partial \rho}{\partial \tau} f_{n_1} + \rho f_{n_1 n_1} \frac{\partial n_1}{\partial \tau} + \rho f_{n_1 w_1} \frac{\partial w_1}{\partial \tau} = 0$$

$$[n_2] \qquad \frac{\partial \mu}{\partial \tau} g_{n_2} + \mu g_{n_2 n_2} \frac{\partial n_2}{\partial \tau} + \mu g_{n_2 w_2} \frac{\partial w_2}{\partial \tau} - \frac{\partial \gamma}{\partial \tau} = 0$$

$$[w_1] \qquad \frac{\partial \rho}{\partial \tau} f_{w_1} + \rho f_{w_1 w_1} \frac{\partial w_1}{\partial \tau} + \rho f_{w_1 n_1} \frac{\partial n_1}{\partial \tau} = 0$$

$$[w_2] \qquad \frac{\partial \mu}{\partial \tau} g_{w_2} + \mu g_{w_2 w_2} \frac{\partial w_2}{\partial \tau} + \mu g_{w_2 n_2} \frac{\partial n_2}{\partial \tau} = 0$$

$$[\lambda] \qquad \frac{\partial x_1}{\partial \tau} + \frac{\partial x_2}{\partial \tau} - \frac{\partial x}{\partial \tau} = 0$$

$$[\rho] \qquad f_{n_1} \frac{\partial n_1}{\partial \tau} + f_{w_1} \frac{\partial w_1}{\partial \tau} - \frac{\partial x_1}{\partial \tau} = 0$$

$$[\mu] \qquad g_{n_2} \frac{\partial n_2}{\partial \tau} + g_{w_2} \frac{\partial w_2}{\partial \tau} - \frac{\partial x_2}{\partial \tau} = 0$$

$$[\gamma] \qquad \frac{\partial n_2}{\partial \tau} = 1$$

The second order conditions can be expressed in terms of the Bordered Hessian representation as $\mathrm{AH} = \mathrm{B}$, where $A = \left[\frac{\partial x}{\partial \tau}, \frac{\partial x_1}{\partial \tau}, \frac{\partial x_2}{\partial \tau}, \frac{\partial n_1}{\partial \tau}, \frac{\partial n_2}{\partial \tau}, \frac{\partial w_1}{\partial \tau}, \frac{\partial w_2}{\partial \tau}, \frac{\partial \lambda}{\partial \tau}, \frac{\partial \rho}{\partial \tau}, \frac{\partial \mu}{\partial \tau}, \frac{\partial \gamma}{\partial \tau}\right]$ is the vector of derivatives of all endogenous variables w.r.t τ. H is the Hessian matrix shown below, and $B = [0,0,0,0,0,0,0,0,0,0,1]$.

$$H = \begin{bmatrix} p_x & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & \rho f_{n_1 n_1} & 0 & \rho f_{n_1 w_1} & 0 & 0 & f_{n_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu g_{n_2 n_2} & 0 & \mu g_{n_2 w_2} & 0 & 0 & g_{n_2} & -1 \\ 0 & 0 & 0 & \rho f_{w_1 n_1} & 0 & \rho f_{w_1 w_1} & 0 & 0 & f_{w_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu g_{w_2 n_2} & 0 & \mu g_{w_2 w_2} & 0 & 0 & g_{w_2} & 0 \\ -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & f_{n_1} & f_{w_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & g_{n_2} & 0 & g_{w_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$|H| = \rho\mu g_{w_2w_2}\left(pf_{n_1}f_{n_1w_1}f_{w_1} - p{f_{n_1}}^2 f_{w_1w_1} + \rho\left({f_{n_1w_1}}^2 - f_{n_1n_1}f_{w_1w_1}\right)\right) \quad (8)$$

Using Cramer's rule, we obtain the effect of the N use restriction in region 2 on the crop production and water use in region 1:

$$\frac{\partial x}{\partial \tau} = \frac{|H_x|}{|H|} = \frac{\rho\left(-{f_{n_1w_1}}^2 + f_{n_1n_1}f_{w_1w_1}\right)\left(g_{n_2w_2}g_{w_2} - \left(f_{w_1} + g_{n_2}\right)g_{w_2w_2}\right)}{g_{w_2w_2}\left(p_x f_{n_1}f_{n_1w_1}f_{w_1} - p_x{f_{n_1}}^2 f_{w_1w_1} + \rho\left({f_{n_1w_1}}^2 - f_{n_1n_1}f_{w_1w_1}\right)\right)} \quad (9)$$

$$\frac{\partial x_1}{\partial \tau} = \frac{|H_{x_1}|}{|H|}$$

$$= \frac{-\begin{pmatrix} p{f_{n_1}}^2 f_{w_1w_1}\left(g_{n_2w_2}g_{w_2} - g_{n_2}g_{w_2w_2}\right) + p_x f_{n_1}f_{n_1w_1}f_{w_1}\left(-g_{n_2w_2}g_{w_2} + g_{n_2}g_{w_2w_2}\right) \\ +\rho f_{w_1}g_{w_2w_2}\left(-{f_{n_1w_1}}^2 + f_{n_1n_1}f_{w_1w_1}\right)\end{pmatrix}}{g_{w_2w_2}\left(p_x f_{n_1}f_{n_1w_1}f_{w_1} - p_x{f_{n_1}}^2 f_{w_1w_1} + \rho\left({f_{n_1w_1}}^2 - f_{n_1n_1}f_{w_1w_1}\right)\right)} \quad (10)$$

$$\frac{\partial x_2}{\partial \tau} = \frac{|H_{x_2}|}{|H|}$$

$$= \frac{\left(g_{n_2w_2}g_{w_2} - g_{n_2}g_{w_2w_2}\right)\left(-p_x f_{n_1}f_{n_1w_1}f_{w_1} + p_x{f_{n_1}}^2 f_{w_1w_1} - \rho{f_{n_1w_1}}^2 + \rho f_{n_1n_1}f_{w_1w_1}\right)}{g_{w_2w_2}\left(p_x f_{n_1}f_{n_1w_1}f_{w_1} - p_x{f_{n_1}}^2 f_{w_1w_1} + \rho\left({f_{n_1w_1}}^2 - f_{n_1n_1}f_{w_1w_1}\right)\right)} \quad (11)$$

$$\frac{\partial n_1}{\partial \tau} = \frac{|H_{n_1}|}{|H|}$$

$$= \frac{-p_x\left(f_{w_1}f_{n_1w_1} - f_{n_1}f_{w_1w_1}\right)\left(-g_{n_2w_2}g_{w_2} + \left(f_{w_1} + g_{n_2}\right)g_{w_2w_2}\right)}{g_{w_2w_2}\left(p_x f_{n_1}f_{n_1w_1}f_{w_1} - p_x{f_{n_1}}^2 f_{w_1w_1} + \rho\left({f_{n_1w_1}}^2 - f_{n_1n_1}f_{w_1w_1}\right)\right)} \quad (12)$$

$$\frac{\partial n_2}{\partial \tau} = \frac{|H_{n_2}|}{|H|}$$

$$= \frac{\begin{pmatrix} f_{n_1} f_{n_1 w_1} f_{w_1} g_{n_2 w_2} p_x \mu + f_{n_1 w_1}{}^2 \rho \left(g_{w_2}{}^2 p_x + g_{w_2 w_2} \mu \right) \\ -f_{w_1 w_1} \left(f_{n_1}{}^2 g_{w_2 w_2} \mu p_x + \rho f_{n_1 n_1} \left(g_{w_2}{}^2 p_x + g_{w_2 w_2} \mu \right) \right) \end{pmatrix}}{\mu g_{w_2 w_2} \left(p_x f_{n_1} f_{n_1 w_1} f_{w_1} - p_x f_{n_1}{}^2 f_{w_1 w_1} + \rho \left(f_{n_1 w_1}{}^2 - f_{n_1 n_1} f_{w_1 w_1} \right) \right)} \quad (13)$$

$$\frac{\partial w_1}{\partial \tau} = \frac{|H_{w_1}|}{|H|}$$

$$= \frac{-p_x \left(f_{n_1} f_{n_1 w_1} - f_{w_1} f_{n_1 n_1} \right) \left(-g_{n_2 w_2} g_{w_2} + \left(f_{w_1} + g_{n_2} \right) g_{w_2 w_2} \right)}{g_{w_2 w_2} \left(p_x f_{n_1} f_{n_1 w_1} f_{w_1} - p_x f_{n_1}{}^2 f_{w_1 w_1} + \rho \left(f_{n_1 w_1}{}^2 - f_{n_1 n_1} f_{w_1 w_1} \right) \right)} \quad (14)$$

$$\frac{\partial w_2}{\partial \tau} = \frac{\begin{pmatrix} -f_{n_1} f_{n_1 w_1} f_{w_1} g_{n_2 w_2} p_x \mu + f_{n_1 w_1}{}^2 f_{w_1 w_1} g_{n_2 w_2} p_x \mu \\ -f_{n_1 w_1}{}^2 \rho \left(f_{w_1} g_{w_2} p_x + g_{n_2} g_{w_2} p_x + g_{n_2 w_2} \mu \right) \\ +\rho f_{n_1 n_1} f_{w_1 w_1} \left(f_{w_1} g_{n_2} p_x + g_{n_2} g_{w_2} p_x + g_{n_2 w_2} \mu \right) \end{pmatrix}}{\mu g_{w_2 w_2} \left(p_x f_{n_1} f_{n_1 w_1} f_{w_1} - p_x f_{n_1}{}^2 f_{w_1 w_1} + \rho \left(f_{n_1 w_1}{}^2 - f_{n_1 n_1} f_{w_1 w_1} \right) \right)} \quad (15)$$