Liability for Non-Disclosure in Equity Financing*

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Abstract

The paper analyzes the effects of holding firms liable for non-disclosure of material information when raising capital. We develop a model in which a privately-informed entrepreneur can choose to withhold information from prospective investors when issuing and selling stock and the investors can bring suit against the firm ex post for (alleged) non-disclosure. The damage payment received by the investors is partially offset by the reduced value of their equity stake. The optimal degree of liability may be zero, compensatory, or punitive; and depends on (1) the level of the entrepreneur's personal investment, (2) the frequency with which the entrepreneur is privately informed (the degree of adverse selection), and (3) the capital required from outside investors. Court errors decrease social welfare by both weakening deterrence and potentially chilling socially valuable financing. Litigation costs may increase social welfare by deterring the inefficient types or decrease social welfare through wasteful litigation spending. The effects of liability waivers and holding entrepreneurs personally liable for non-disclosure are also explored.

1 Introduction

On May 17, 2012, Facebook went public by selling more than 421 million shares of common stock at $38 per share to public investors on the Nasdaq and raised about $16

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billion from the investors. Unlike many other initial public offerings that experience an initial price surge, Facebook's stock price declined shortly after the initial public offering (IPO), hitting a low of $18. It took more than a year for the stock price to rebound to the IPO price of $38. Many public investors, who bought Facebook's shares at the IPO or shortly after, were quite unhappy and brought class action suits against the company under the US Securities Act, claiming that Facebook failed to disclose the fact that more users were using their mobile phones to access Facebook's websites instead of their computers, and the company's advertising revenues were lower than as described in the IPO documents.[1] After more than five years of pre-trial procedures, immediately before the case was to go to trial, the litigants agreed to settle the case for $35 million in 2018.[2]

As the Facebook story demonstrates, when a company offers raises capital by selling securities to outside investors, the US securities laws require the company to disclose all material information it possesses to the prospective investors. In case the company fails to do so, the investors can bring suit against the company to recover compensatory damages. Presumably, such a liability regime ensures that the outside investors will receive all material information from the company and make an informed decision as to whether to purchase the offered security. At the same time, though, critics have argued that the private enforcement regime, especially the class action system, is too costly and encourages indiscriminate lawsuits against even innocent companies.[3] To what extent do such a liability regime induce the company to disclose all material information to the investors? Is such a private liability regime necessary in the first place? If so, in what form? Should the investors be allowed to bring class actions or be required to bring suit on an individual basis, as some advocates have argued? What is the role of the plaintiff class action lawyers? The objective of this paper is to answer some of these questions with the help of game theoretic modeling.

The paper presents a model in which an entrepreneur sells stock to the outside investors while deciding whether to disclose all material information she possesses to the prospective investors. The investors can make rational inference based on the entrepreneur's decision to disclose and, in case it is revealed that the entrepreneur hid material information, the investors can bring suit against the company to recover damages. The model demonstrates, foremost, that when the entrepreneur needs

[1] See Atkins (2018) and Graf (2018).

[2] Id. According to Graf (2018), out of $35 million settlement plaintiff's attorneys are getting almost $14 million as fees and costs.

[3] See Scott (2017 and 2019). Scott has argued that most of the securities class actions are without merit and the companies should be allowed to bar securities class actions through a mandatory non-class arbitration provision in their charters or bylaws.

to commit enough of her own resources for investment, even without any liability, the entrepreneur will have sufficient incentive to disclose all material information to the outside investors. In case the entrepreneur does not need to expend sufficient resources, however, holding the company liable is necessary to fully deter inefficient non-disclosure. The socially optimal level of liability may be zero, compensatory, or punitive.

After presenting the baseline model, the paper also examines various extensions. In the first extension, the entrepreneur is held personally liable for withholding information from investors. Since the damage award is paid by the entrepreneur rather than the firm, the firm's equity value is unaffected by the lawsuit. We show that the level of liability required to deter non-disclosure is smaller than in the baseline model. The second extension allows for the liability system to falsely find uninformed (and non-disclosing) entrepreneurs liable. With false convictions, it becomes more difficult to deter the informed entrepreneur from not disclosing her information. When the liability against uninformed, non-disclosing entrepreneur is sufficiently high, it can also chill efficient financing. In such circumstances, it may be social welfare increasing to eliminate firm liability altogether. In the third extension, the analysis allows for positive litigation costs. With costly litigation, in certain parameter space of litigation costs, we actually can get better deterrence against inefficient non-disclosure. Positive litigation costs can, however, also reduce overall social welfare, especially when the liability system does not deter inefficient non-disclosure.

This paper extends the literature on the disclosure of information prior to the sale of an asset. Grossman (1981) and Milgrom (1981) introduced the famous unravelling result when sellers are privately informed about asset quality. Sellers of high quality assets have a clear incentive to disclose this information (to obtain a better selling price) and, as a consequence, sophisticated buyers draw adverse inferences when sellers *do not* disclose. Grossman and Hart (1981) explored the implications of full unravelling for disclosure laws in corporate takeovers.[4] Dye (1985), Farrell (1986), and Shavell (1994) show that complete unraveling does not occur when buyers are uncertain whether the sellers actually have private information. In their models, sellers with low quality assets have an incentive to withhold this information from the market and pool with the uninformed types.[5] These papers all assume that disclo-

[4]Grossman and Hart conclude that "the commonly held view that firms withhold information (which is free to release) in order to mislead traders into giving them better terms is false." (p. 333)

[5]Shavell (1994) focuses on the incentive of sellers to acquire information prior to a sale. Although mandatory disclosure may be socially desirable conditional on the seller acquiring the information, mandatory disclosure may chill the collection of socially valuable information. Polinsky and Shavell (2012) show that when sellers are strictly liable for consumer harms stemming from defective prod-

sure, if mandated, is perfectly enforced and that the seller does not retain an equity stake in the asset.

Dye (2017) explores a model where mandatory disclosure is imperfectly enforced. As in our model, if the seller fails to disclose material information then the court awards damages that are proportional to the over-payment by the buyers (relative to what they would have paid had the seller disclosed the information). Our analysis differs from Dye's in several important respects. First, in Dye (2017), the sale of the asset, even the lowest-value asset, is assumed to be socially efficient.[6] In our model, the sale of the lowest-value asset is socially inefficient; disclosure is socially valuable in our setting because it prevents the mis-allocation of capital.[7] Second, in Dye (2017), *the seller is personally liable* for the damage payment. In our analysis, *the firm itself* is liable for the damage payment. The seller's accountability for non-disclosure is limited to their (endogenous) financial stake in the firm. In our model, the damage award received by the buyers is paid, in part, by the buyers themselves: The buyers are in effect taking money out of one pocket and putting it into the other.

[Review of law literature to be written.]

The paper is organized as follows. In section 2, we present the baseline model. We start with the case of no asymmetric information (the first best benchmark) and the case of no liability against non-disclosing firm/entrepreneur. We then present the case with liability against non-disclosure, examining, for instance, the degree of liability necessary to deter inefficient non-disclosure. Section 3 examines various extensions, including (i) placing liability on the entrepreneur (rather than on the firm); (ii) court errors where an uninformed, non-disclosing entrepreneur/firm may be falsely held liable too; and (iii) positive litigation costs. The last section concludes with thoughts for future research.

2 The Model

Suppose that an entrepreneur E owns a firm that needs capital of $c > 0$. When E raises c and incurs a personal cost of $e > 0$, $c + e$ is invested and the cash-flow

ucts, and can take precautions to reduce product risks, then mandatory and voluntary disclosure are equivalent.

[6] Dye's primary analysis is descriptive, characterizing the effect of liability on disclosure strategies.

[7] There are other differences. In Dye (2017), social welfare falls when the seller retains a larger fraction of the asset (see Dye section 7). In our model, increasing the seller's stake is socially efficient insofar as it deters sellers with low-value assets from participating.

stream of $x > 0$ is realized, where $x \in \{x_h, x_l\}$ and $prob(x = x_l) = q \in (0,1)$.[8] Let $k \equiv c + e$, where k stands for the total investment necessary for the project, and $\overline{x} \equiv q \cdot x_l + (1-q) \cdot x_h$. We assume that $max\{e, c\} < x_l < k < x_h$ and $\overline{x} > k$ so that financing is efficient either when $x = x_h$ or when E is uninformed, but not when $x = x_l$.[9] Note that this implies that $q \in (0, \overline{q})$ where:

$$\overline{q} \equiv \frac{x_h - k}{x_h - x_l} \in (0,1). \tag{1}$$

We assume that E raises capital from a competitive capital market by having the firm sell fraction $\alpha \in [0,1]$ of the firm's equity to outside investors, whose reservation value is normalized to zero. For instance, with complete information and when $x = x_h$, with competitive capital markets, the outside investors would pay c for a fraction $\alpha = c/x_h$ of the equity of the firm. The outside investors break even in this complete information scenario, since $\alpha \cdot x_h - c = (c/x_h) \cdot x_h - c = 0$.

There are five periods in the game with no discounting, $t \in \{0,1,2,3,4\}$, and the timing of the game is as follows. At $t = 0$, Nature chooses $x = x_l$ with probability q and $x = x_h$ with probability $1-q$. E learns the realized x with probability $\pi \in (0,1)$. E who learns x is "informed" while E who does not learn x is "uninformed." Among the informed E, we denote E (and the firm) who knows that $x = x_h$ as the "h-type" and that $x = x_l$, the "l-type." We denote the uninformed E as the "u-type." Hence, there are three possible types of E (or the firm): u-type, h-type, and l-type.

At $t = 1$, E decides whether to participate and raise capital. If E chooses not to participate then the game ends and E gets a return of zero. If E chooses participate then the game continues.[10]

At $t = 2$, the informed E chooses whether to disclose x or not disclose ("withhold") x. Following the literature, disclosures are accurate, i.e., E cannot present false evidence, and that the uninformed E cannot pretend to know x. The outside investors observe the entrepreneur's disclosure decision and the disclosed information, if any, and update their belief about x.

At $t = 3$, the firm attempts to raise capital c by issuing equity stake α to outside investors. Outside investors are rational and forward looking and the capital market

[8] We can think of E's personal cost of e as either E's costly effort or the amount of personal financial capital E has to pledge to get financing.

[9] The assumption that $e < x_l$ rules out the uninteresting case where the l-type would never participate, even if $\alpha = 0$. Also, the assumption that $c < x_l$ is made for largely technical convenience.

[10] We are imagining that E's choice to participate or not participate is a commitment, and cannot change her mind later. This will get rid of the equilibria where investors make positive profits and simplifies the equilibrium characterization.

is competitive. The equity stake α allows investors to break even given their (endogenous) beliefs about the value of the firm x and their returns from future litigation. If the entrepreneur fails to raise capital then the game ends.

At $t = 4$, investments c and e are sunk and all returns are realized. Investors (now, shareholders of the firm) learn (1) value $x \in \{x_h, x_l\}$ and (2) whether E withheld information at $t = 2$, and can decide to bring a suit against the firm.[11] If E withheld information, the court awards damages $d \in [0, x_l]$.[12] Note that we are assuming that the limited liability principle applies and the firm cannot be responsible for more than its cash-flow (x_l). Also, while it is natural to assume that the damages are compensatory and equal to the overcharge paid by investors we also allow for no damages ($d = 0$) and punitive damages.

Several observations will simplify the analysis of this game. First, in any equilibrium, the h-type will participate in the market, disclose its information (of x_h), and succeed in raising capital. To see why, note that the capital market's equilibrium belief $\widehat{x}$ when there is no disclosure must be weakly lower than x_h. If the capital market believed that the firm that seeks financing is the h-type, $\widehat{x} = x_h$, investors would demand $\widehat{\alpha} = c/x_h < c/\overline{x}$. This cannot happen in equilibrium since the uninformed u-type firms would want to participate as well. Since the capital market's belief with no disclosure is $\widehat{x} < x_h$, the h-type firm has an incentive to disclose x_h to secure a better deal with investors.

Second, if the l-type chooses to participate, it will not disclose x_l to the market. To see why, suppose that the l-type did disclose x_l to the market. Investors would demand equity stake $\alpha_l = c/x_l$ in return for investing c in the business venture. The l-type's net return if it participates and discloses x_l is $(1 - \alpha_l)x_l - e = x_l - c - e < 0$: the l-type is losing money. So, if the l-type participates at $t = 1$, it will not disclose x_l to the capital market. (Depending on the parameter values, the l-type may or may not participate in equilibrium.)

Third, in any equilibrium where the l-type participates and raises capital with a positive probability, the u-type participates, too. To see why, let α^* be the equilibrium

[11] We are assuming here than the entrepreneur is not directly liable for non-disclosure. Technically, the security is being sold by and the representations are being made by the firm. Hence, imposing liability on the firm would seem natural. If we assume that the entrepreneur does not have sufficient assets to pay for monetary damages, such an assumption may also be realistic. Nevertheless, we later analyze the case of holding the entrepreneur directly liable for non-disclosure. See section 3.1.

[12] For now, we are assuming that there are no "false positives," the u-type cannot be found liable for non-disclosure after x_l has been realized. We also assume that there are no costs of litigation. Latter sections will relax these assumptions.

equity stake demanded by investors when there is no disclosure. The l-type's return, $(1-\alpha^*)x_l - e$ (minus any damages owed to outside investors), is strictly smaller than the u-type's return, $(1-\alpha^*)\overline{x} - e$. So, if the l-type's financial return is non-negative, then the u-type's return is strictly positive. These findings are summarized in the following Lemma.

Lemma 1. *In any PBNE, the h-type participates, discloses x_h, and issues equity stake $\alpha_h = c/x_h$; the u-type participates and issues equity stake $\alpha^* \geq c/\overline{x} > \alpha_h$; the l-type participates with probability $\beta^* \in [0,1]$, does not disclose x_l, and pools with the u-type. The capital market's point estimate of x conditional on non-disclosure is $x^* \in (x_l, \overline{x}]$.*

In the analysis that follows, we will construct the equilibrium values (α^*, β^*) where α^* is the associated equity stake demanded by investors and β^* is the l-type's participation probability. The equilibrium concept is perfect Bayesian equilibrium (PBNE).

2.1 Full-Information Benchmark

Since $x_l < k < \overline{x} < x_h$, it is socially efficient for the entrepreneur to raise capital and pursue the venture *unless* the project is known to have low value (x_l). If a social planner possessed the same information as the entrepreneur, the social planner would fund the project if the value was known to be high ($x = x_h$) or if the project had unknown value, but not if $x = x_l$. As the following proposition demonstrates, this outcome would be obtained in a competitive market if the investors have the same information as the entrepreneur.

Proposition 1. *Suppose that the capital market has the same information as the entrepreneur.*

1. *If the entrepreneur and capital market learn that $x = x_h$, then the investors pay c for equity stake $\alpha_h = c/x_h$. E's return is $x_h - c - e > 0$.*
2. *If the entrepreneur and capital market learn that $x = x_l$, then no capital is raised. E's return is zero.*
3. *If the entrepreneur and capital market do not learn x, then investors pay c for equity stake $\overline{\alpha} = c/\overline{x}$. E's return is $\overline{x} - c - e > 0$.*

The entrepreneur raises capital and pursues the business venture *unless* the project is commonly known to be of low value (i.e., unless the market knows $x = x_l$). Finally, note that when x is not observed then the equity stake demanded by investors reflects

the average value ($\overline{x}$). In terms of simple comparative statics, as $q \to 0$, so that the probability of facing the l-type is very low, $\overline{x} \to x_h$ and $\overline{\alpha} \to \alpha_h$. As $q \to \overline{q}$ (defined in (1) above), $\overline{x} \to c + e$ and $\overline{\alpha} \to \frac{c}{c+e}$: E's equity stake $1 - \overline{\alpha}$ is just large enough to allow E to break even on average.

2.2 Equilibrium Characterization

We now characterize the PBNE where the outside investors demand an equity stake $\alpha^* \in [0, 1]$ and the l-types participate and raise capital with probability β^*. Let's begin with the l-type's decision to participate and raise capital. If the l-type participates, the outside investors will bring suit against the firm for damages d. After the payment of damages to the investors, the residual firm value of $x_l - d \geq 0$ is divided between the outside investors and the entrepreneur in proportions α^* and $(1 - \alpha^*)$. The l-type raises capital if the gross return for the entrepreneur, $(1 - \alpha^*)(x_l - d)$, exceeds the personal investment e. We have the following characterization of β^*:

$$\begin{cases} \beta^* = 0 & \text{if} \quad (1 - \alpha^*)(x_l - d) - e < 0 \\ \beta^* \in [0, 1] & \text{if} \quad (1 - \alpha^*)(x_l - d) - e = 0 \\ \beta^* = 1 & \text{if} \quad (1 - \alpha^*)(x_l - d) - e > 0 \end{cases} \tag{2}$$

Depending on the level of damages d, the equilibrium may involve full deterrence of the l-type ($\beta^* = 0$), partial deterrence ($\beta^* \in (0, 1)$), or no deterrence ($\beta^* = 1$).

Next, consider the equity stake demanded by outside investors. If the entrepreneur is the uninformed u-type, by assumption, there is no lawsuit and the investors get, in expectation, a net return of $\alpha^* \overline{x} - c$. If the entrepreneur is the informed l-type, the outside investors bring suit and collect damages of d from the firm. Because the damages are paid by the firm, the value of the outside investors' equity stake falls to $(1 - \alpha^*)(x_l - d)$.[13] Thus, the damages awarded to the investors are paid, in part, by the investors themselves. The investors' break-even condition (from the *ex ante*

[13] We are assuming here that the investors have not sold their shares when they bring suit. We can relax this assumption later. When the parameters of the lawsuit are common knowledge and the financial market is sufficiently forward-looking, when the investors sell the stock, the stock price would reflect the returns from prospective lawsuits.

perspective) is:[14]

$$(1-\pi)(\alpha^*\overline{x}-c)+\pi q\beta^*(\alpha^* x_l - c + (1-\alpha^*)d) = 0. \tag{3}$$

As the investors' break-even condition shows, the equilibrium α^* will depend on the equilibrium probability of participation by the l-type ($\beta^* \in [0,1]$).

If the damage award d is above a threshold, l-type entrepreneur will be fully deterred from raising capital: $\beta^* = 0$. We now characterize this upper threshold, $\overline{d}$. First, consider the investors' willingness to supply capital. Setting $\beta^* = 0$ in the investors' break-even condition (3), we have $\alpha^*\overline{x} - c = 0$. Thus, if the outside investors believe that the l-type entrepreneur is fully deterred, they will demand equity stake of

$$\alpha^* = \overline{\alpha} = \frac{c}{\overline{x}} < 1. \tag{4}$$

The equity stake when no information is disclosed reflects the average value of the uninformed u-type only. Now, consider the l-type's decision to participate and raise capital. From (2), the l-type is fully deterred when $(1-\overline{\alpha})(x_l - d) - e < 0$. Since the left-hand side is a decreasing function of d, the threshold damage award $\overline{d}$ satisfies this expression with equality. The following Lemma characterizes this threshold.

Lemma 2. *(Full Deterrence.) The l-type is fully deterred, $\beta^*(d,\pi) = 0$, if $d > \overline{d}$ where*

$$\overline{d} = x_l - \frac{e}{1 - c/\overline{x}}. \tag{5}$$

If $e < (1-c/\overline{x})x_l$, then $\overline{d} > 0$ and if $e > (1-c/\overline{x})x_l$ then $\overline{d} < 0$.[15]

The full-deterrence threshold $\overline{d}$ in (5) has several notable properties. First, $\overline{d}$ is independent of π. With the complete deterrence of the l-type, the outside investors need not worry about the "degree" of adverse selection, represented by π. Second, $\overline{d}$ is a strictly decreasing function of e. Deterrence is easier to achieve when the entrepreneur has more at stake in the venture. Third, as e approaches 0, $\overline{d}$ approaches x_l. When the entrepreneur personally invests very little in the venture, full-deterrence requires damages that effectively liquidate the firm's assets with nothing remaining

[14] Conditional on non-disclosure by the entrepreneur, the investors' break-even constraint is:

$$\frac{1-\pi}{(1-\pi)+\pi q}(\alpha^*\overline{x}-c) + \frac{\pi q}{(1-\pi)+\pi q}\beta^*(\alpha^* x_l - c + (1-\alpha^*)d) = 0.$$

This is equivalent to (3). The investors' equilibrium ex ante return from the h-type is $\alpha_h x_h - c = 0$.

[15] Since $k = c + e$, we can rewrite $(1-c/\overline{x})x_l = \frac{\overline{x}-k}{\overline{x}-x_l}x_l \equiv \widehat{e}$. Then, $\overline{d} < (>)0$ when $e > (<)\widehat{e}$.

for the entrepreneur. Interestingly, full deterrence may require supra-compensatory damages in the sense that the investors collect more in damages than the overcharge for the assets.[16] Fourth, when e is sufficiently large, the full-deterrence threshold $\overline{d}$ is negative. In particular, if $e \geq (1-\overline{\alpha})x_l$ then the l-type is fully deterred for all $d \geq 0$, so liability is unnecessary for deterrence.

At the opposite end of the spectrum, if the damage award d is below a threshold, the l-type entrepreneur will be completely undeterred, $\beta^* = 1$. Unlike the full deterrence case, with full participation by the l-type, now the degree of adverse selection (π) matters. The next Lemma characterizes the lower threshold, $\underline{d}(\pi)$. As the Lemma (along with its proof) demonstrates, full participation by the l-type (no deterrence) becomes more likely as d, π, or both get smaller.

Lemma 3. *(No Deterrence.) The l-type is undeterred, $\beta^*(d,\pi) = 1$, if $d < \underline{d}(\pi)$ where*

$$\underline{d}(\pi) = x_l - \frac{e}{1 - c/\overline{x} - r(\pi)} \tag{6}$$

and

$$r(\pi) = \frac{\pi q}{1-\pi} \cdot \frac{k - x_l}{\overline{x}}. \tag{7}$$

The function $\underline{d}(\pi)$ is a strictly decreasing function of π with $\underline{d}(0) = \overline{d}$. There exists a unique $\widehat{\pi}_0 \in (0,1)$ where $r(\widehat{\pi}_0) = 1 - c/\overline{x} - e/x_l$ and $\underline{d}(\widehat{\pi}_0) = 0$.

While the formal details are in the proof of Lemma 3, the result may be understood intuitively. The function $r(\pi)$ defined in (7) is the incremental equity stake demanded by the outside investors when the l-type is (just) undeterred, $\beta^* = 1$. To just break even in expectation, the outside investors require equity stake $c/\overline{x} + r(\pi)$. Intuitively, if the l-types are undeterred, the outside investors will demand a "premium" to compensate for the increased risk that they face. The risk premium demanded by outside investors, $r(\pi)$ in equation (7), is higher when the fraction of informed types, π, is larger.

Notice also that the threshold $\underline{d}(\pi)$ in Lemma 3 is a decreasing function of π. When π rises, full participation of the l-types is unsustainable in equilibrium. It is not hard to see why this is true. When π rises and the outside investors demand a larger equity stake compensate them for the increased risk, the entrepreneur's cost of capital becomes higher, too. The higher cost of capital discourages the l-types from

[16] This is true given that $\overline{d}$ is decreasing in e and $\overline{d} = x_l > c - \alpha x_l \ \forall \alpha \in (0,1]$ when $e = 0$.

participating in the market. When π is above a threshold $\widehat{\pi}_0$, therefore, the l-type is partially deterred ($\beta^* < 1$) even when there is no liability ($d = 0$).

When the damage award is in an intermediate range ($\underline{d} < d < \overline{d}$) or when $d < \overline{d}$ and π is sufficiently high ($\pi > \widehat{\pi}_0$), the l-type randomizes between participating and not: $\beta^* \in (0, 1)$. From (2), the l-type entrepreneur is indifferent between participating and not if:

$$(1 - \alpha^*)(x_l - d) - e = 0. \tag{8}$$

Notice that if e or d rise, the outside ownership stake α^* must fall. When the entrepreneur must make a larger personal investment in the venture, or faces greater liability for non-disclosure, it is necessary for the entrepreneur's ownership stake $(1-\alpha^*)$ must be larger (to encourage participation).

Using (8) and allows us to rewrite the investors' break-even condition as:[17]

$$(1 - \pi)(\alpha^* \overline{x} - c) - \pi q \beta^* (c + e - x_l) = 0. \tag{9}$$

The expression, $\alpha^* \overline{x} - c > 0$, is the investors' net return from the uninformed u-type, and the expression, $c + e - x_l = k - x_l > 0$, is the investors' loss associated with the l-type. Notice that the expression $k - x_l > 0$ represents the social loss from the l-type's participation. Since the l-type is indifferent between participating and not, i.e., the l-type's expected return is equal to zero, the outside investors bear the entire social loss in expectation. Solving equations (8) and (9) gives unique closed-form solutions for α^* and β^*. The following Proposition formalizes the results.

Proposition 2. *Suppose that the firm is held liable for the l-type entrepreneur's non-disclosure.*

1. *If $d \geq \overline{d}$, the investors demand equity stake $\alpha^*(d, \pi) = \overline{\alpha} = c/\overline{x}$ and the l-type is fully deterred: $\beta^*(d, \pi) = 0$.*

2. *If $\underline{d}(\pi) < d < \overline{d}$ or $d < \overline{d}$ and $\pi > \widehat{\pi}_0$, the investors demand equity stake*

$$\alpha^*(d, \pi) = 1 - \frac{e}{x_l - d}, \tag{10}$$

where $\alpha^*(d, \pi)$ is decreasing in d and does not depend on π. The l-type is partially deterred,

$$\beta^*(d, \pi) = \frac{1 - \pi}{\pi q} \cdot \frac{\left(1 - \frac{e}{x_l - d}\right)\overline{x} - c}{c + e - x_l}, \tag{11}$$

[17] Expression (8) implies $\alpha^*(x_l - d)x_l - d - e$. Substituting this into equation (2) gives the result.

where $\beta^*(d,\pi)$ *is decreasing in* d *and* π *with* $\lim_{d\to\overline{d}}\beta^*(d,\pi) = 0$ *and* $\lim_{d\to\underline{d}(\pi)}\beta^*(d,\pi) = 1$.

3. *If* $d \leq \underline{d}(\pi)$, *the investors demand equity stake*

$$\alpha^*(d,\pi) = \frac{(1-\pi)c + \pi q(c-d)}{(1-\pi)\overline{x} + \pi q(x_l - d)}, \tag{12}$$

where $\alpha^*(d,\pi)$ *is decreasing in* d *and increasing in* π. *The* l*-type is undeterred,* $\beta^*(d,\pi) = 1$.

The full deterrence and no deterrence equilibria were explained in the earlier Lemmas. Consider the second case of partial deterrence: where $\underline{d}(\pi) < d \leq \overline{d}$ or $d < \overline{d}$ and $\pi > \widehat{\pi}_0$. The equilibrium value $\beta^*(d,\pi)$ characterized (11) has some notable and intuitive properties. First, the equilibrium l-type participation rate β^* is smaller when the damage award d is larger. In other words, the l-type is deterred by legal liability. This is intuitive. When the liability system is stronger then it becomes more costly for the l-type to not disclose the information, and therefore easier to partially deter the l-type from participating in the market. Second, the l-type's participation rate β^* is decreasing in π, the fraction of informed entrepreneurs.[18] When π is larger, the adverse selection problem is worse. To maintain investor indifference, fewer l-types participate. The equilibrium is illustrated in Figure 1.

2.3 Welfare Implications

It is straightforward to evaluate the implications for social welfare. It is inefficient for the l-type entrepreneurs to participate in the market because the gross return from the venture, x_l, is smaller than the cost of the venture, $c + e$. So, from an ex ante perspective, the expected welfare loss $L(d,\pi)$ is

$$L(d,\pi) = \pi q \beta^*(d,\pi)(c + e - x_l) \tag{13}$$

where πq is the probability that the entrepreneur is the l-type and β^* is the probability of l-type non-disclosure and participation. The next result follows immediately from Proposition 2.

Corollary 1. *The expected welfare loss* $L(d,\pi)$ *satisfies the following.*

1. *If* $d > \overline{d}$, *there is full deterrence and* $L(d,\pi) = 0$.

[18] Recall that $\overline{x} = qx_l + (1-q)x_h$ and does not depend on π.

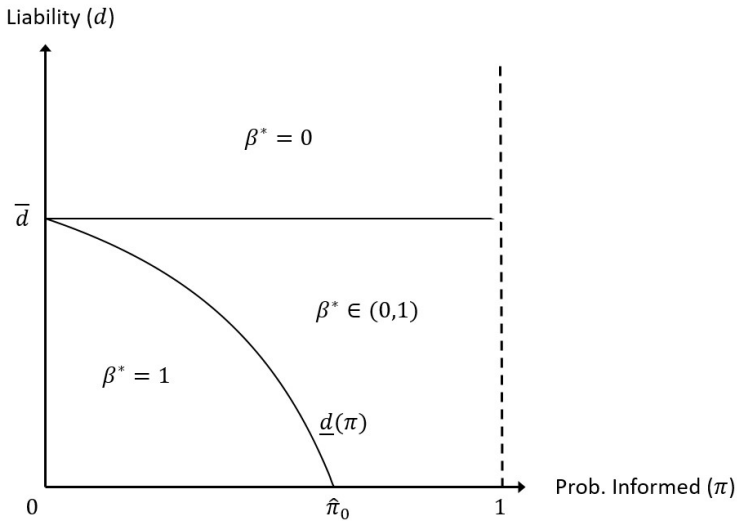


Figure 1: Equilibrium Characterization

2. *If* $\underline{d}(\pi) < d \leq \overline{d}$ *or* $d < \overline{d}$ *and* $\pi > \hat{\pi}_0$*, there is partial deterrence and* $L(d,\pi) = (1-\pi)\left[\left(1 - \frac{e}{x_l - d}\right)\overline{x} - c\right]$*. The function* $L(d,\pi)$ *is strictly decreasing in* π *and* d*.*

3. *If* $d \leq \underline{d}(\pi)$*, there is no deterrence and* $L(d,\pi) = \pi q(k - x_l)$*. The function* $L(d,\pi)$ *is strictly increasing in* π *and is independent of* d*.*

The corollary shows that firm liability plays an important role in deterring entrepreneurs from raising capital for socially wasteful ventures. When firm liability is low, $d \leq \underline{d}(\pi)$, the l-type is undeterred (case 3) and the social loss depends on the ex ante probability of the investors' facing the l-type (πq). When firm liability is in the middle range, however, the l-type is partially deterred and the social welfare depends, in part, on the l-type's equilibrium participation rate (β^*). When firm liability is in the highest range, $d > \overline{d}$ the l-type is fully deterred and there is no social welfare loss (case 1).

The corollary also delivers some nuanced comparative statics on π, the fraction of informed entrepreneurs. When firm liability is low, $d \leq \underline{d}(\pi)$, then the l-type is undeterred and the social loss rises with the fraction of l-type in the population, πq. When the level of liability is in middle range, $\underline{d}(\pi) < d \leq \overline{d}$, the social loss falls when π rises. Although there is a higher probability of facing the l-type who could potentially raise capital, the outside investors recognize the risk and discount the investment accordingly. In equilibrium, the participation rate of the l-types falls and social welfare rises.

3 Extensions

The previous section presented the baseline model. While the model was able to produce interesting results, the model also relied on some simplifying assumptions, such as no litigation cost and no false positives. In this section, we relax some of the assumptions and consider various extensions. The extensions include: (1) the possibility of holding the entrepreneur personally liable; (2) the possibility that the court can find u-type entrepreneur liable (false positive); (3) positive litigation costs for both the defendant-firm and the plaintiff-shareholders; and (4) the possibility of a liability waiver. We consider these extensions in turn.

3.1 Entrepreneur Liability

Now, suppose that an informed E may be held *personally* liable (and the firm is not held liable) for withholding evidence about x. The outside investors, after observing the realized x, can bring suit against the entrepreneur at $t = 4$ to recover damages d. Since the damage award is paid by the entrepreneur instead of the firm, the value of the firm's equity is unaffected by the lawsuit.

Not surprisingly, holding the entrepreneur personally liable creates stronger deterrence. To see why, consider the case of partial deterrence where the l-type is just indifferent between participating and not and the outside investors break even in expectation. The l-type is indifferent if

$$(1 - \alpha^*)x_l - e - d = 0. \tag{14}$$

Comparing this condition to (8) reveals an important difference. When the entrepreneur is personally liable he bears one hundred percent of the damage award d rather than bearing only their proportional share via the reduction in firm value, $(1 - \alpha^*)d$. The outside investors' break-even condition is the same as in (9) above.

As in the baseline model, if d is above a threshold, the l-type entrepreneur is fully deterred: $\beta^* = 0$. The investors' break-even condition in (9) implies that the outside investors demand equity stake $\alpha^* = c/\overline{x}$ as in (4). Equation (14) implies that the l-type is just indifferent between participating and not when $d > \overline{d}_{EL}(e)$ where

$$\overline{d}_{EL} = (1 - c/\overline{x})x_l - e. \tag{15}$$

Comparing this to equation (5), we see that $\overline{d}_{EL} < \overline{d}$, the threshold in the baseline model with firm liability. That is, the l-type is fully deterred for a broader range of parameter values than before.

Similarly, if the damages d are at the threshold where the l-type is (just) undeterred, $\beta^* = 1$, then the outside investors demand equity stake $c/x + r(\pi)$ defined in (7). Using the l-type's indifference condition (14)

$$\underline{d}_{EL}(\pi) = (1 - c/\overline{x} - r(\pi))x_l - e. \tag{16}$$

Comparing this to equation (6) establishes $\underline{d}_{EL}(\pi) < \underline{d}(\pi)$, so the l-type is undeterred for a smaller range of parameter values than before. When $\pi = 0$ then the risk premium $r(\pi) = 0$ and $\underline{d}_{EL}(0) = \overline{d}_{EL}$, and when $\pi = \widehat{\pi}_0$ then $r(\widehat{\pi}_0) = 1 - c/\overline{x} - e/x_l$ and $\underline{d}_{EL}(\widehat{\pi}_0) = 0$. The following Proposition presents the main results of the entrepreneur's personal liability case.

Proposition 3. *(Entrepreneur Liability.) Suppose that the entrepreneur is held personally liable for non-disclosure.*

1. *If $d \geq \overline{d}_{EL}$, then investors demand equity stake $\alpha^*_{EL}(d, \pi) = c/\overline{x}$ and the l-type is fully deterred: $\beta^*_{EL}(d, \pi) = 0$.*

2. *If $\underline{d}_{EL}(\pi) < d < \overline{d}_{EL}$ or $d < \overline{d}_{EL}$ and $\pi > \widehat{\pi}_0$, then investors demand equity stake*

$$\alpha^*_{EL}(d, \pi) = 1 - \frac{d + e}{x_l} \tag{17}$$

where $\alpha^*_{EL}(d, \pi) > c/\overline{x}$. The l-type is partially deterred,

$$\beta^*_{EL}(d, \pi) = \frac{1 - \pi}{\pi q} \cdot \frac{\left(1 - \frac{d+e}{x_l}\right)\overline{x} - c}{c + e - x_l} \in (0, 1). \tag{18}$$

3. *If $d \leq \underline{d}_{EL}(\pi)$ then investors demand equity stake*

$$\alpha^*_{EL}(d, \pi) = \frac{(1 - \pi)c + \pi q(c - d)}{(1 - \pi)\overline{x} + \pi q x_l}. \tag{19}$$

The l-type is undeterred, $\beta^*_{EL}(d, \pi) = 1$.

These results are illustrated in Figure 2. Notice that since $\overline{d}_{EL} < \overline{d}$, the l-type entrepreneur is fully deterred for a larger range of parameter values than before. Furthermore, since $\underline{d}_{EL}(\pi) < \underline{d}(\pi)$, the range where there is no deterrence is smaller, too. When $d = 0$, since there is no liability against either the firm or the entrepreneur, we get the same cut-off point of $\widehat{\pi}_0$. For the middle range with partial deterrence, one can easily verify that $\beta^*_{EL}(d, \pi)$ in equation (18) is smaller than $\beta^*(d, \pi)$ in equation (11). So deterrence is stronger when the entrepreneur is held personally liable for non-disclosure to outside investors.

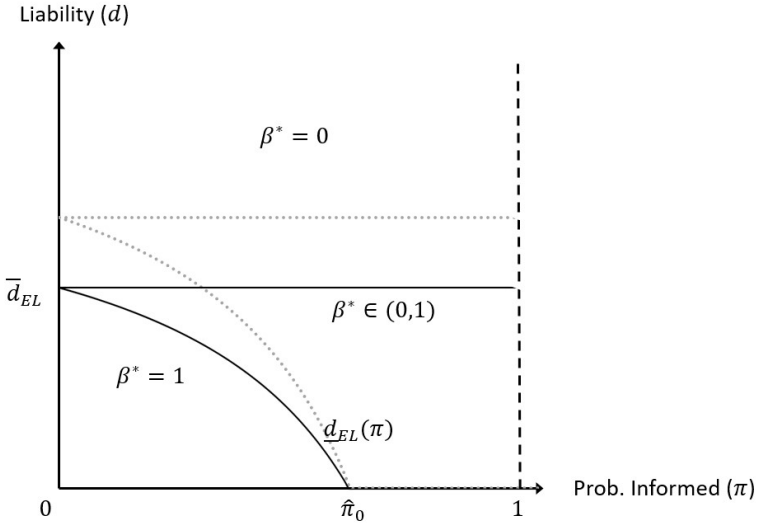


Figure 2: Entrepreneur Liability

Corollary 2. *The social loss is (weakly) lower, and social welfare (weakly) higher, when the entrepreneur is held personally liable.*

3.2 False Positives

In the baseline model, we assumed that only the l-type could be found liable when the investors observe x_l. This is tantamount to assuming that when x_l is observed, the investors (and the court) can distinguish whether it is the l- or that u-type that has generated that cash-flow or, equivalently, the investors (and the court) to know whether the entrepreneur had earlier concealed the information on future cash-flow. Such informational assumption, of course, may be quite strong especially when the investors (and the court) need to verify whether the entrepreneur, in fact, concealed information at the time of financing. In this section, we relax that assumption and allow for the possibility of the court holding also the u-type (falsely) liable.

Suppose, after the investors have financed the project conditional on non-disclosure and x_l has been observed, the investors do not know whether they are facing the l-type or the u-type, and can bring lawsuit against either one. As before, if the lawsuit is brought against the l-type, the l-type will be liable (in expectation) for d. On the other hand, we assume that the liability for the u-type is given by λd where $\lambda \in [0, 1]$. The court is (weakly) competent at distinguishing the types. If $\lambda = 0$, for instance, we're back to the baseline model. To focus on the impact of allowing false positives, we'll continue to assume that litigation is costless.

It is not difficult to imagine that once the court (and the investors) can falsely hold the u-type liable (after x_l is realized), it becomes more difficult to deter the l-type from participating. For instance, the region where the l-type participates in equilibrium will be larger and in the region where the l-type is partially deterred, the l-type will participate with a higher probability. As will be demonstrated more formally in the following proposition and the proof, the l-type is fully deterred, $\beta^* = 0$, when the damages exceed the threshold

$$\overline{d}_{FP} = x_l - \frac{e}{1 - \overline{\alpha}_{FP}} \tag{20}$$

where $\overline{\alpha}_{FP} = \frac{c - q\lambda d}{\overline{x} - q\lambda d}$. Since the investors expect a "rebate" λd from the u-type, they demand a smaller fraction of the firm, $\overline{\alpha}_{FP} < \overline{\alpha} = c/\overline{x}$. Comparing the formula for $\overline{d}_{FP}$ in (20) to the formula for $\overline{d}$ in (5) we see that $\overline{d}_{FP} > \overline{d}$ whenever $\lambda > 0$. The l-type is totally undeterred, $\beta^* = 1$, when the damages are below the threshold

$$\underline{d}_{FP}(\pi) = x_l - \frac{e}{1 - \overline{\alpha}_{FP} - r_{FP}(\pi)} \tag{21}$$

where $r_{FP}(\pi)$ is the risk premium required by the investors when $d = r_{FP}(\pi)$,

$$r_{FP}(\pi) = \frac{\pi q}{1 - \pi} \cdot \frac{k - x_l}{\overline{x} - q\lambda d}. \tag{22}$$

Comparing these expressions with (6) and (7) verifies that $\underline{d}_{FP}(\pi) > \underline{d}(\pi)$. The following proposition, along with the proof, demonstrates this more formally.

Proposition 4. *(False Positives.) Suppose that the firm is erroneously held liable for λd when the entrepreneur is uninformed and x_l is realized, where $\lambda \in [0, 1]$.*

1. *If $d \geq \overline{d}_{FP}$, then investors demand equity stake $\alpha^*_{FP}(d, \pi) = \frac{c - q\lambda d}{\overline{x} - q\lambda d}$ and the l-type is fully deterred: $\beta^*_{FP}(d, \pi) = 0$.*

2. *If $\underline{d}_{FP}(\pi) < d < \overline{d}_{FP}$ then investors demand equity stake $\alpha^*_{FP}(d, \pi) = 1 - \frac{e}{x_l - d}$, and the l-type is partially deterred:*

$$\beta^*_{FP}(d, \pi) = \frac{1 - \pi}{\pi q} \cdot \frac{\left(1 - \frac{e}{x_l - d}\right)(\overline{x} - q\lambda d) - (c - q\lambda d)}{c + e - x_l} \in (0, 1). \tag{23}$$

3. *If $d \leq \underline{d}_{FP}(\pi)$ then the l-type is undeterred: $\beta^*_{FP}(d, \pi) = 1$. Investors demand equity stake $\alpha^*_{FP}(d, \pi) = \frac{(1-\pi)(c - q\lambda d) + \pi q(c - d)}{(1-\pi)(\overline{x} - q\lambda d) + \pi q(x_l - d)}$.*

While the general attributes of the equilibrium is similar to that in the baseline case, when the court can falsely find the u-type liable ($\lambda > 0$), there are three notable

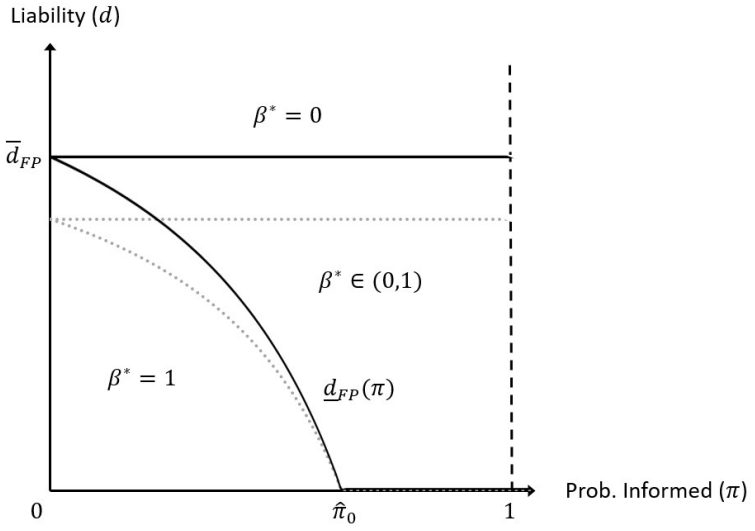


Figure 3: False Positives

differences. First, compared to the baseline case, now it becomes more difficult to achieve full deterrence: $\overline{d}_{FP}(e) > \overline{d}(e)\ \forall \pi \in [0,1]$ whenever $\lambda > 0$. Because the investors collect some damages (λd) from the u-type when x_l is observed, ex ante, they demand a lower fraction of the firm (α^* is smaller) and this provides a stronger incentive for the l-type to not disclose and participate. Second, and for similar reasons, the region where there is no deterrence ($\beta^* = 1$) is larger: $\underline{d}_{FP}(e,\pi) > \underline{d}(e,\pi)\ \forall \pi \in [0, \widehat{\pi}_0(e))$ whenever $\lambda > 0$. Third, when the l-type is partially deterred ($\beta^* \in (0,1)$), even with the same level of liability (the same d), with false positives, the l-type participates with a higher probability. Conditional on d, because the investors collect damages from the u-type, their expected return will be higher with false positives. In order to satisfy their break-even condition, therefore, the l-type has to participate with a higher probability. When compared to the baseline case, all three results will tend to lower social welfare.

Corollary 3. *The social loss is (weakly) higher, and social welfare (weakly) lower, when there are false positives so the court may hold the u-type liable.*

3.3 Litigation Costs

In the baseline model, we assumed that litigation was costless, so the investors brought suit against the firm whenever the l-type participated (and x_l is observed). This section relaxes this assumption, and assumes that the investors and the firm must pay litigation costs of $\phi_p \geq 0$ and $\phi_d \geq 0$, respectively. We let $\phi = \phi_p + \phi_d$.

When litigation is costly, the outside investors may or may not have a financial interest to bring suit against the firm even after discovering the l-type's non-disclosure. The investors' net return from the litigation is positive if $d - \phi_p > 0$. However, the market value of the equity stake held by the investors falls. After the payment of damages d and defense costs ϕ_d, the value of the firm's equity is $x_l - d - \phi_d$. Since the outside investors own fraction α of the equity, their stake falls by $\alpha(d + \phi_d)$. The outside investors will bring suit if and only if the net return from litigation exceeds the the lost equity value:

$$\alpha \leq \frac{d - \phi_p}{d + \phi_d}. \tag{24}$$

The outside investors' incentive to sue the firm is stronger when the damages d are higher, their equity stake α is smaller, or the litigation costs ϕ_p and ϕ_d smaller. Notice that the credibility constraint does not depend on π, c, or e.[19]

Suppose that (24) is satisfied so the investor's incentive to bring the lawsuit is credible. As in the baseline model, there exists a threshold $\overline{d}_{LC}$ above which the l-type entrepreneur is fully deterred: $\beta^* = 0$. In an equilibrium with full deterrence, there is no litigation and the outside investors demand equity stake $\overline{\alpha} = c/\overline{x}$, just as in the baseline model. The l-type is just indifferent between participating and not when $(1 - c/\overline{x})(x_l - d - \phi_d) - e = 0$, so the l-type does not participate when

$$\overline{d}_{LC} = x_l - \phi_d - \frac{e}{1 - c/\overline{x}}. \tag{25}$$

Comparing this to equation (5) establishes $\overline{d}_{LC} < \overline{d}$ since $\phi_d > 0$. In other words, when litigation is costly for the firm, the l-type is fully deterred for a broader range of parameter values than before. The boundary for the partial deterrence region is

$$\underline{d}_{LC} = x_l - \phi_d - \frac{e}{1 - c/\overline{x} - r_{LC}(\pi)} \tag{26}$$

where $r_{LC}(\pi)$ is the risk premium demanded by investors when $d = \underline{d}_{LC}$,

$$r_{LC}(\pi) = \frac{\pi q}{1 - \pi} \cdot \frac{k + \phi - x_l}{\overline{x}}. \tag{27}$$

Comparing these expressions with (6) and (7) verifies that $\underline{d}_{LC}(\pi) < \underline{d}(\pi)$.

If (24) is not satisfied then investor's incentive to bring the lawsuit is not credible.

[19] Note that in the conventional litigation model (with no fee-shifting), plaintiffs bring suit (lawsuit is credible) whenever $d - \phi_p > 0$. Since $\alpha^* \in (0, 1)$ and $d + \phi_d > d - \phi_p$, the credibility constraint is more likely to bind in our setup.

Intuitively, this situation is similar to our baseline model without litigation costs when $d = 0$. Following the logic surrounding Lemma 3, when $\pi < \widehat{\pi}_0$ then $\underline{d}(\pi) > 0$ and so the l-type is undeterred, $\beta^* = 1$. When π is small, the investors face little risk of financing an l-type and so the risk premium that they demand is small. This implies that the (rare) l-types have access to cheap capital, and participate fully in raising capital. When $\pi > \widehat{\pi}_0$ then $\underline{d}(\pi) < 0$ and so the l-type is partially deterred. Intuitively, when π is large, the investors face considerable risk when supplying capital. Since capital is more expensive, the l-types are partially deterred.

Proposition 5. *Suppose that the investors and the firm bear litigation costs ϕ_p and ϕ_d, respectively. There exists a continuous, weakly increasing function function $d_0(\pi)$ ("litigation credibility threshold"), along with $\overline{d}_{LC}$ and $\underline{d}_{LC}(\pi)$, such that we have the following as an equilibrium.*

1. *If $d \geq \overline{d}_{LC}$, then investors demand equity stake $\alpha^*_{LC}(d,\pi) = \overline{\alpha} = c/\overline{x}$ and the l-type is fully deterred: $\beta^*_{LC}(d,\pi) = 0$.*
2. *If $d < d_0(\pi)$, then investors do not have a credible threat to sue.*
   (a) *If $\pi \leq \widehat{\pi}_0$, then then the investors demand equity stake $\alpha^*_{LC}(d,\pi) = \frac{(1-\pi)c+\pi qc}{(1-\pi)\overline{x}+\pi q x_l}$ and the l-type is undeterred: $\beta^*_{LC}(d,\pi) = 1$.*
   (b) *If $\pi > \widehat{\pi}_0$, then the investors demand equity stake $\alpha^*_{LC}(d,\pi) = \frac{x_l - e}{x_l}$ and the l-type is partially deterred:*

$$\beta^*_{LC}(d,\pi) = \frac{1-\pi}{\pi q} \cdot \frac{\left(1 - \frac{e}{x_l}\right)\overline{x} - c}{c + e - x_l} \in (0,1). \tag{28}$$

3. *If $d \geq d_0(\pi)$, then the investors have a credible threat to bring suit.*
   (a) *If $d \leq \underline{d}_{LC}(\pi)$, then the l-type is undeterred: $\beta^*_{LC}(d,\pi) = 1$. The investors demand equity stake $\alpha^*_{LC}(d,\pi) = \frac{(1-\pi)c+\pi q(c+\phi_p-d)}{(1-\pi)\overline{x}+\pi q(x_l-d-\phi_d)}$.*
   (b) *If $\underline{d}_{LC}(\pi) < d < \overline{d}_{LC}$ or $d_0(\pi) < d < \overline{d}_{LC}$, then investors demand equity stake $\alpha^*_{LC}(d,\pi) = 1 - \frac{e}{x_l - d - \phi_d}$. The l-type is partially deterred:*

$$\beta^*_{LC}(d,\pi) = \frac{1-\pi}{\pi q} \cdot \frac{\left(1 - \frac{e}{x_l - d - \phi_d}\right)\overline{x} - c}{c + e + \phi_p + \phi_d - x_l} \in (0,1). \tag{29}$$

When the firm and the investors must incur cost in litigation ($\phi_i > 0$), litigation credibility constraint will affect the equilibrium. The equilibrium is a bit involved

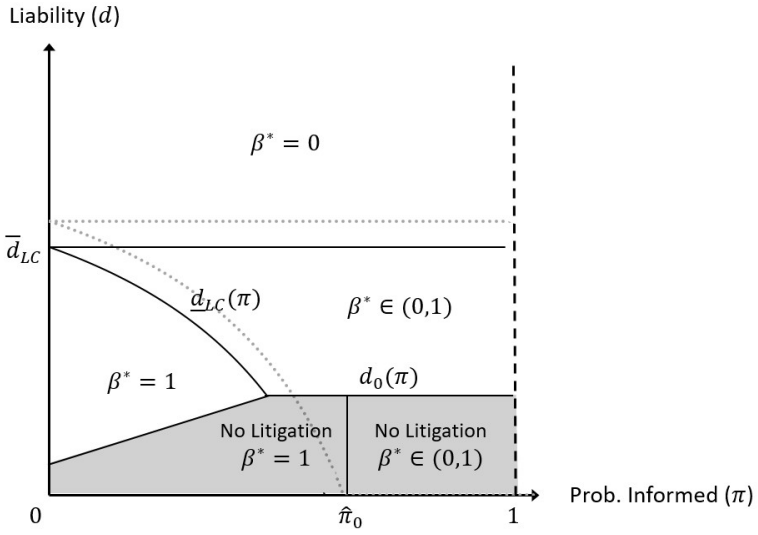


Figure 4: Litigation Costs

due to the fact that the litigation credibility constraint depends, in part, on the equilibrium fraction of the firm sold to the investors, $\alpha \leq \frac{d-\phi_p}{d+\phi_d}$, and the fact that α^* depends on various parameters, including d and π. For instance, as damages decrease, the investors will demand a larger fraction of the firm and this will, in turn, make the litigation constraint more likely to bind. From the Proposition, we see that we have a litigation credibility threshold on damages, given by the function $d_0(\pi)$. In addition, as shown in the earlier equilibria, we also have the other two thresholds, $\underline{d}_{LC}(\pi)$ and $\overline{d}_{LC}$, that represent the no-deterrence and full-deterrence thresholds, respectively.

When the damages are large enough, i.e., $d \geq \overline{d}_{LC}$, as before, we get the full deterrence and the l-type does not participate (case 1). Short of full deterrence, i.e., $d < \overline{d}_{LC}$, we can divide the parameter space into four regions: (1) no deterrence ($\beta^* = 1$) without litigation (case 2(a)); (2) partial deterrence ($\beta^* \in (0,1)$) without litigation (case 2(b)); (3) no deterrence ($\beta^* = 1$) with litigation (case 3(a)); and (4) partial deterrence ($\beta^* \in (0,1)$) with litigation (case 3(b)). Although characterizing the precise thresholds and the equilibrium strategies (α^*_{LC} and β^*_{LC}) is somewhat involved, the division into four separate regions is fairly intuitive. As we showed earlier, even without any lawsuits, when π is sufficiently high ($\pi > \widehat{\pi}_0$), we had partial deterrence against the l-type. This is what's shown in case 2. When litigation is credible (case 3), compared to the case without litigation cost, the equilibrium strategies (α^*_{LC} and β^*_{LC}) depend on the litigation cost (ϕ_i). It is easy to check that as litigation costs go to zero, we get the earlier equilibrium.

4 Conclusion

To be completed.

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Appendix

Proof of Lemma 2. Note that $\beta^* = 0$, $\alpha^* = \overline{\alpha} = c/\overline{x}$ defined in (4), and $d = \overline{d}$ defined in (5) satisfy equilibrium conditions (2) and (3) with equality.

Suppose that $d > \overline{d}$. We will prove that $\beta^* = 0$ by contradiction. Suppose not: $\beta^* > 0$. Condition (2) implies that $(1-\alpha^*)(x_l - d) - e \geq 0$, and so $\alpha^* < \overline{\alpha}$. Condition (2) also implies that $\alpha^*(x_l - d) \geq x_l - d - e$. Substituting this into (3) gives $(1-\pi)(\alpha^*\overline{x} - c) + \pi q \beta^*(x_l - k) \geq 0$. Solving for α^* and using the definition of $\overline{\alpha}$ in (4) gives

$$\alpha^* \geq \overline{\alpha} + \frac{\pi q \beta^*}{1-\pi} \cdot \frac{k - x_l}{\overline{x}}. \tag{30}$$

If $\beta^* > 0$ then $\alpha^* > \overline{\alpha}$, a contradiction. Thus, if $d > \overline{d}$ then $\beta^* = 0$.

We now show $\overline{d}'(e) < 0$. Differentiating the expression for $\overline{d}(e)$ in (5) with respect to e, and using the definition of $\alpha_0(e)$ in (4),

$$\overline{d}'(e) = \frac{-(1-\alpha_0(e)) - e\alpha_0'(e)}{(1-\alpha_0)^2} = \frac{-1 + c/\overline{x} - c/\overline{x}}{(1-\alpha_0)^2} < 0. \tag{31}$$

Threshold $\widehat{e}$ is defined by $\overline{d}(\widehat{e}) = 0$ or equivalently $(1 - \alpha_0(\widehat{e}))x_l - \widehat{e} = 0$. Using (4) this becomes $\left(\frac{\overline{x}-k+\widehat{e}}{\overline{x}}\right)x_l - \widehat{e} = 0$. Solving for $\widehat{e}$ gives the expression in Lemma 2. □

Proof of Lemma 3. Here is an alternative way of writing the result:

$$\underline{d}(\pi) = x_l - \frac{e}{1 - \underline{\alpha}(\pi)} \tag{32}$$

where

$$\underline{\alpha}(\pi) = \frac{c}{\overline{x}} + \frac{\pi q}{1-\pi} \cdot \frac{k - x_l}{\overline{x}}. \tag{33}$$

Note that $\underline{\alpha}(\pi)$ defined in (33), $\underline{d}(\pi)$ defined in (6), and $\beta^* = 1$ satisfy equilibrium conditions (2) and (3) with equality. Next, suppose that $d < \underline{d}(\pi)$. We will prove that $\beta^* = 1$ by contradiction.

If $d < \underline{d}(\pi)$ and $\beta^* < 1$, then condition (2) implies that $(1-\alpha^*)(x_l - d) - e \leq 0$. Since $d < \underline{d}(\pi)$ we must have $\alpha^* > \underline{\alpha}(\pi)$ (defined in (33)). Equation (2) also implies that $\alpha^*(x_l - d) \geq x_l - d - e$. Substituting this into (3) gives $(1-\pi)(\alpha^*\overline{x} - c) + \pi q \beta^*(x_l - k) \leq 0$. Solving for α^* gives

$$\alpha^* \leq \frac{c}{\overline{x}} + \frac{\pi q \beta^*}{1-\pi} \cdot \frac{k - x_l}{\overline{x}}. \tag{34}$$

Since $\beta^* < 1$, we have

$$\alpha^* < \frac{c}{\overline{x}} + \frac{\pi q}{1-\pi} \cdot \frac{k - x_l}{\overline{x}} = \underline{\alpha}(\pi) \tag{35}$$

where $\underline{\alpha}(\pi)$ defined in (33). This is a contradiction. Thus, if $d < \underline{d}(\pi)$, then $\beta^* = 1$.

Since $\underline{\alpha}(\pi)$ in (33) is an increasing function of π it follows that $\underline{d}(\pi)$ in (6) is a decreasing function of π. We now characterize the threshold $\widehat{\pi}_0$ where $\underline{d}(\widehat{\pi}_0) = 0$. Setting $\underline{d}(\widehat{\pi}_0) = 0$ in (6) gives us

$$\underline{\alpha}(\widehat{\pi}_0) = \frac{x_l - e}{x_l} \tag{36}$$

and equation (33) gives us

$$\underline{\alpha}(\widehat{\pi}_0) = \frac{k - e}{\overline{x}} + \frac{\widehat{\pi}_0 q}{1 - \widehat{\pi}_0} \cdot \frac{k - x_l}{\overline{x}}. \tag{37}$$

Setting these equal and rearranging,

$$\frac{1 - \widehat{\pi}_0}{\widehat{\pi}_0 q} = \frac{x_l(k - x_l)}{x_l(\overline{x} - k) - e(\overline{x} - x_l)}. \tag{38}$$

Consider the right-hand side. The numerator is positive. The denominator is positive, too:

$$x_l(\overline{x} - k) - e(\overline{x} - x_l) = x_l(\overline{x} - c - e) - e(\overline{x} - x_l) = x_l(\overline{x} - c) - e\overline{x} \tag{39}$$

$$= \overline{x}\left(\left(1 - \frac{c}{\overline{x}}\right) x_l - e\right) \tag{40}$$

so the denominator is positive if $\overline{d}$ is positive (by Lemma 2). The left-hand side is a decreasing function of $\widehat{\pi}_0$ and a unique interior solution exists. □

Proof of Proposition 2. *Case 1.* The full deterrence result follows from Lemma 2.

Case 2. The partial deterrence result may be found by solving (8) and (9) simultaneously. The comparative statics are immediate.

Case 3. In the no-deterrence range, the equilibrium $\alpha^*(d, \pi)$ in (12) may be found by setting $\beta^* = 1$ in the investors' break-even condition (3). Notice that $\alpha^*(d, 0) = \overline{\alpha} = c/\overline{x} > 0$ and $\alpha^*(0, \pi) > 0$ for all $\pi \in (0, 1)$. We will show that $\alpha^*(d, \pi)$ is decreasing

in d and increasing in π. Letting $z = \pi q/(1-\pi)$, rewrite (12)

$$\alpha^* = \frac{c + z(c-d)}{\overline{x} + z(x_l - d)}. \tag{41}$$

First show that α^* is an increasing function of x. Differentiate with respect to z,

$$\frac{\partial \alpha^*}{\partial z} = \frac{(\overline{x} + z(x_l - d))(c-d) - (c + z(c-d))(x_l - d)}{(\overline{x} + z(x_l - d))^2}. \tag{42}$$

$$= \frac{\overline{x}(c-d) - c(x_l - d)}{(\overline{x} + z(x_l - d))^2} \tag{43}$$

$$= \frac{c(\overline{x} - x_l) - d(\overline{x} - c)}{(\overline{x} + z(x_l - d))^2} \tag{44}$$

The numerator is positive if

$$d < \frac{c(\overline{x} - x_l)}{\overline{x} - c} \tag{45}$$

The right-hand side is larger than $\overline{d}$ defined in (5) above. To prove this,

$$\frac{c(\overline{x} - x_l)}{\overline{x} - c} < \overline{d} = x_l - \frac{e}{1 - c/\overline{x}} = x_l - \frac{e\overline{x}}{\overline{x} - c} = \frac{x_l(\overline{x} - c) - e\overline{x}}{\overline{x} - c} \tag{46}$$

This is true if and only if

$$c(\overline{x} - x_l) < x_l(\overline{x} - c) - e\overline{x} = c\overline{x} - x_l\overline{x} + e\overline{x} = \overline{x}(c + e - x_l) < 0 \tag{47}$$

which is true because $c + e + x_l = k - x_l < 0$ by assumption. So the numerator is positive for all $d < \overline{d}$. Thus α^* is an increasing function of x and by extension π.

Next we show that α^* is a decreasing in d. Differentiating with respect to d,

$$\frac{\partial \alpha^*}{\partial d} = \frac{(\overline{x} + z(x_l - d))(-z) - (c + z(c-d))(-z)}{(\overline{x} + z(x_l - d))^2}. \tag{48}$$

The denominator is positive and the numerator is negative if

$$-\overline{x} - z(x_l - d) + c + z(c-d) < 0 \tag{49}$$

$$-\overline{x} - zx_l + c + zc < 0 \tag{50}$$

$$-z(\overline{x} - c) < \overline{x} - c \tag{51}$$

$$\frac{1}{z}(\overline{x}-c) > -(x_l - c) \tag{52}$$

$$\frac{1-\pi}{\pi q} > \frac{-(x_l-c)}{\overline{x}-c} \tag{53}$$

The right-hand side could be positive or negative. The left hand side is positive, and approaches infinity as $\pi \to 0$. So this is true when π is small. We will now show that it is true when $\pi = \widehat{\pi}_0$. From (38) we have

$$\frac{1-\widehat{\pi}_0}{\widehat{\pi}_0 q} = \frac{x_l(k-x_l)}{x_l(\overline{x}-k) - e(\overline{x}-x_l)} \tag{54}$$

which may be rewritten using $k = c + e$,

$$\frac{1-\widehat{\pi}_0}{\widehat{\pi}_0 q} = \frac{-x_l(x_l-c) + ex_l}{x_l(\overline{x}-c) - e\overline{x}} \tag{55}$$

We will now prove that $\frac{1-\widehat{\pi}_0}{\widehat{\pi}_0 q}$ is greater than $\frac{-(x_l-c)}{\overline{x}-c}$:

$$\frac{-x_l(x_l-c) + ex_l}{x_l(\overline{x}-c) - e\overline{x}} > \frac{-(x_l-c)}{\overline{x}-c} \tag{56}$$

Cross multiplying,

$$(-x_l(x_l-c) + ex_l)(\overline{x}-c) > -(x_l-c)(x_l(\overline{x}-c) - e\overline{x}) \tag{57}$$

$$ex_l(\overline{x}-c) > (x_l-c)e\overline{x} \tag{58}$$

$$-ex_l c > -ce\overline{x} \tag{59}$$

$$ec(\overline{x}-x_l) > 0. \tag{60}$$

We have established that α^* is a decreasing function of d and this completes the proof. □

Proof of Proposition 3. The investors' break-even condition is

$$(1-\pi)(\alpha^*\overline{x} - c) + \pi q \beta^*(\alpha^* x_l - c + d) = 0. \tag{61}$$

When $\beta^* = 1$ then we have

$$\alpha^*[(1-\pi)\overline{x} + \pi q x_l] = (1-\pi)c + \pi q(c-d). \tag{62}$$

Dividing through by $(1-\pi)\overline{x}+\pi q x_l$ gives the result. $\square$

Proof of Proposition 4. Let's first start with the full deterrence case ($\beta^* = 0$). When only the u-type participates in financing, the investors' equilibrium break-even condition is given by:

$$(1-\pi)(\alpha^*\overline{x} - c + q(1-\alpha)\lambda d) = 0.$$

Compared to the case without any false positives ($\lambda = 0$), in equilibrium, the investors receive a litigation return of $(1-\alpha)\lambda d$ from the u-type when x_l is realized, which happens with probability q. From this, we get the equilibrium fraction for the investors:

$$\alpha^* = \frac{c - q\lambda d}{\overline{x} - q\lambda d}$$

Whenever $\lambda > 0$, $\frac{c-q\lambda d}{\overline{x}-q\lambda d} < \frac{c}{\overline{x}}$. Also, $\frac{\partial \alpha}{\partial \lambda} = \frac{-qd(\overline{x}-c)}{(\overline{x}-q\lambda d)^2} < 0$ and $\frac{\partial \alpha}{\partial d} = \frac{-q\lambda(\overline{x}-c)}{(\overline{x}-q\lambda d)^2} < 0$. Compared to the baseline case ($\lambda = 0$), because the investors get additional return from the u-type through litigation, the equilibrium fraction for the investors is lower.

To ensure that the l-type is fully deterred, we need: $(1-\alpha^*)(x_l - d) - e < 0$. Let $\overline{d}$ be given by: $(1-\alpha^*)(x_l - d) - e = 0$. When we substitute in the expression for α^* and simplify, we get:

$$\overline{d}_{FP}(e) = \frac{(\overline{x}-c)x_l - e\overline{x}}{\overline{x} - c - eq\lambda}.$$

When $\lambda = 0$, we get $\overline{d}_{FP}(e) = \frac{(\overline{x}-c)x_l - e\overline{x}}{\overline{x}-c-eq\lambda} = x_l - \frac{e\overline{x}}{\overline{x}-c} = \overline{d}(e)$. Compared to the baseline case, a stronger liability is necessary to fully deter the l-type from participating.

Now, suppose we are in the partial deterrence region ($\beta^* \in (0,1)$). The l-type's indifference condition is given by $(1-\alpha^*)(x_l - d) - e = 0$, from which we get: $\alpha^* = 1 - \frac{e}{x_l - d}$. We can see that when the l-type is indifferent, the u-type strictly prefers to participate: $(1-\alpha^*)(\overline{x} - q\lambda d) - e > 0$. The investors' break-even condition is given by:

$$(1-\pi)(\alpha^*\overline{x} - c + q\lambda d(1-\alpha^*)) + \pi q\beta^*(\alpha^* x_l - c + (1-\alpha^*)d) = 0.$$

When we use the equality $\alpha^*(x_l - d) = x_l - d - e$ and simplify, we get the equilibrium participation rate by the l-type:

$$\beta^* = \frac{1-\pi}{\pi q} \cdot \frac{\alpha^*(\overline{x} - q\lambda d) - c + q\lambda d}{k - x_l}.$$

Recall that, in the baseline case, we had: $\alpha^* = 1 - \frac{e}{x_l - d}$ and $\beta^* = \frac{1-\pi}{\pi q} \cdot \frac{\alpha^* \overline{x} - c}{k - x_l}$. Compared to the baseline case, therefore, conditional on d, whenever $\lambda > 0$, the l-type's participation rate is strictly higher and, therefore, the social welfare lower. Although the equilibrium fraction (α^*) that the investors receive from the l-type (conditional on d) is the same, because the investors get a positive litigation return from the u-type, compared to the baseline case, their return is strictly higher. To satisfy their break-even condition, therefore, the l-type's equilibrium participation rate (β^*) has to be higher. Finally, we see that: $\frac{\partial \alpha^*}{\partial d} = -\frac{e}{(x_l - d)^2} < 0$, $\frac{\partial \alpha^*}{\partial \pi} = 0$, $\frac{\partial \beta^*}{\partial d} = \frac{e}{(x_l - d)^2}(-\overline{x} + q\lambda x_l) < 0$, and $\frac{\partial \beta^*}{\partial \pi} = -\frac{1}{\pi q} \cdot \frac{\alpha^*(\overline{x} - q\lambda d) - c + q\lambda d}{k - x_l} < 0$. As in the baseline case, when the liability gets stronger, social welfare improves.

To establish $\underline{d}_{FP}(e, \pi)$, let's examine the no deterrence case ($\beta^* = 1$). When $\beta^* = 1$, the investors' break-even condition is given by:

$$(1 - \pi)(\alpha^*(\overline{x} - q\lambda d) - c + q\lambda d) + \pi q(\alpha^*(x_l - d) - c + d) = 0,$$

from which, we get $\alpha^* = \frac{(1-\pi)(c - q\lambda d) + \pi q(c - d)}{(1-\pi)(\overline{x} - q\lambda d) + \pi q(x_l - d)}$.

A few preliminary observations regarding the α^*. From the equation $(1 - \pi)(\alpha(\overline{x} - q\lambda d) - c + q\lambda d) + \pi q(\alpha(x_l - d) - c + d) = 0$, we see that the left hand side is strictly increasing with respect to α. We also know that, in equilibrium, $\alpha^* \overline{x} - c + q\lambda d(1 - \alpha^*) > 0$ and $\alpha^* x_l - c + (1 - \alpha^*)d < 0$. Hence, when π increases, investors' return goes down, and to restore equality, α^* must increase. Hence, $\frac{\partial \alpha^*}{\partial \pi} > 0$. Also, $(1-\pi)(\alpha(\overline{x} - q\lambda d) - c + q\lambda d) + \pi q(\alpha(x_l - d) - c + d)$ is strictly increasing with respect to d. When d rises, therefore, to restore the equality, α must decrease. In other words, $\frac{\partial \alpha^*}{\partial d} < 0$. In sum, the equilibrium α^* is increasing with respect to π and decreasing with respect to d.

To establish $\underline{d}_{FP}(e, \pi)$, from the partial deterrence case, we know that:

$$\beta^* = \frac{1 - \pi}{\pi q} \cdot \frac{\left(\frac{x_l - d - e}{x_l - d}\right)(\overline{x} - q\lambda d) - c + q\lambda d}{k - x_l}$$

When we set $\beta^* = 1$ and simplify, we get:

$$\frac{1 - \pi}{\pi q} \cdot \frac{(x_l - \underline{d} - e)(\overline{x} - q\lambda \underline{d}) - (x_l - \underline{d})(c - q\lambda \underline{d})}{(x_l - \underline{d})(k - x_l)} = 1.$$

When we solve for $\underline{d}$, we get:

$$\underline{d}_{FP}(e, \pi) = \frac{(1 - \pi)(x_l(\overline{x} - c) - e\overline{x}) - \pi q x_l(k - x_l)}{(1 - \pi)((\overline{x} - c) - eq\lambda) - \pi q(k - x_l)}.$$

It's straightforward (but somewhat messy) to establish that this expression is decreasing with respect to π. When $\pi = 0$, we get:

$$\underline{d}_{FP}(e, 0) = \frac{x_l(\overline{x} - c) - e\overline{x}}{(\overline{x} - c) - eq\lambda} = \overline{d}.$$

When $\underline{d}_{FP}(e, \pi) = 0$, we need: $(1 - \pi)(x_l(\overline{x} - c) - e\overline{x}) - \pi q x_l(k - x_l) = 0$, from which we get:

$$\frac{1 - \widehat{\pi}_0}{\widehat{\pi}_0 q} \cdot \frac{x_l(\overline{x} - k) - e(\overline{x} - x_l)}{x_l(k - x_l)} = 1.$$

Note that this expression is the same as the baseline case and $\widehat{\pi}_0 < 1$. Therefore, $\underline{d}_{FP}(e, \pi) \in [0, \overline{d}]$ when $\pi \in [0, \widehat{\pi}_0]$. □

Proof of Proposition 5. We start with the construction of the full deterrence threshold, $\overline{d}_{LC}$. In order to have a sufficiently high d so as to fully deter the l-type (with litigation), we need: $(1 - \overline{\alpha})(x_l - d - \phi_d) - e < 0$. When we let $d = \overline{d}_{LC}$ and solve $(1 - \overline{\alpha})(x_l - \overline{d}_{LC} - \phi_d) - e = 0$ for $\overline{d}_{LC}$, we get:

$$\overline{d}_{LC} = x_l - \phi_d - \frac{e\overline{x}}{\overline{x} - c}.$$

Note that $\overline{d}_{LC} < x_l - \frac{e}{1 - c/\overline{x}}$. Because of the positive litigation cost (ϕ_d), compared to the case without litigation cost, a weaker liability is necessary to achieve full deterrence. Hence, when $d > \overline{d}_{LC}$, the investors demand $\alpha^*_{LC} = \frac{c}{\overline{x}}$ and the l-type is fully deterred: $\beta^*_{LC} = 0$.

Second, suppose that we are in the partial deterrence region. Suppose, first, that the litigation is credible. For the l-type's indifference, we need: $(1-\alpha)(x_l-d-\phi_d)-e = 0$. The investors' break-even condition is: $(1-\pi)(\alpha\overline{x}-c)+\pi q\beta(\alpha x_l - c + d - \phi_p - \alpha(d + \phi_d)) = 0$. From the l-type indifference condition, we get the equilibrium α:

$$\alpha^*_{LC}(d, \pi) = \frac{x_l - d - \phi_d - e}{x_l - d - \phi_d}.$$

Note that $\alpha^*_{LC}(d, \pi)$ is independent of π but is decreasing with respect to d. With $\alpha^*_{LC}(d, \pi)$, the investors' break-even condition can be written as: $(1 - \pi)(\alpha\overline{x} - c) +$

$\pi q\beta(x_l - k - \phi) = 0$ and we get the equilibrium participation rate for the l-type:

$$\begin{aligned}\beta^*_{LC}(d,\pi) &= \frac{1-\pi}{\pi q} \cdot \frac{\alpha^*_{LC}(d,\pi)\cdot \overline{x} - c}{k + \phi - x_l} \\ &= \frac{1-\pi}{\pi q} \cdot \frac{(x_l - d - \phi_d - e)\overline{x} - c(x_l - d - \phi_d)}{(x_l - d - \phi_d)(k + \phi - x_l)}.\end{aligned}$$

Given the litigation credibility condition of $\alpha \leq \frac{d-\phi_p}{d+\phi_d}$, this implies that $\exists\ d_0 > \phi_p$ such that the litigation credibility constraint binds: $\frac{x_l - d_0 - \phi_d - e}{x_l - d_0 - \phi_d} = \frac{d_0 - \phi_p}{d_0 + \phi_d}$. When we solve for d_0, we get:

$$d_0^{pd}(\pi) = \frac{x_l\phi - \phi_d(e+\phi)}{e+\phi}.$$

The superscript "pd" stands for the fact that we are defining d_0 in the partial deterrence region. In the partial deterrence region, therefore, litigation is credible only when $d \geq \frac{x_l\phi - \phi_d(e+\phi)}{e+\phi}$. When $d < d_0^{pd}(\pi)$, we get: $\alpha^*_{LC}(d,\pi) = \frac{x_l - e}{x_l}$ and $\beta^*_{LC}(d,\pi) = \frac{1-\pi}{\pi q} \cdot \frac{(x_l - e)\overline{x} - cx_l}{x_l(k - x_l)}$.

Third, suppose we are in the no deterrence region. Suppose, initially, that the litigation is credible. In equilibrium, the l-type participates with probability one ($\beta^*_{LC}(d,\pi) = 1$) and makes, in expectation: $(1-\alpha)(x_l - d - \phi_d) - e > 0$. From the investors' break-even condition of $(1-\pi)(\alpha\overline{x} - c) + \pi q(\alpha x_l - c + d - \phi_p - \alpha(d + \phi_d)) = 0$, we get:

$$\alpha^*_{LC}(d,\pi) = \frac{(1-\pi)c + \pi q(c - d + \phi_p)}{(1-\pi)\overline{x} + \pi q(x_l - d - \phi_d)}.$$

Though a bit tedious, it is straightforward to show that $\frac{\partial \alpha^*_{LC}(d,\pi)}{\partial \pi} > 0$ and $\frac{\partial \alpha^*_{LC}(d,\pi)}{\partial d} < 0$. In the no deterrence region, the litigation credibility condition is given by:

$$\frac{(1-\pi)c + \pi q(c - d + \phi_p)}{(1-\pi)\overline{x} + \pi q(x_l - d - \phi_d)} \leq \frac{d - \phi_p}{d + \phi_d}.$$

When we solve for d, we find the expression for $d_0^{nd}(\pi)$ (superscript "nd" standing for "no deterrence"):

$$d_0^{nd}(\pi) = \frac{((1-\pi)c + \pi qc)\phi_d + ((1-\pi)\overline{x} + \pi q x_l)\phi_p}{(1-\pi)(\overline{x} - c) + \pi q(x_l - c)}.$$

Though bit tedious, it is straightforward to show that $\frac{\partial d_0^{nd}(\pi)}{\partial \pi} > 0$. Note that when $\pi = 0$, we get $d_0^{nd}(0) = \frac{c\phi_d + \overline{x}\phi_p}{\overline{x} - c} > \phi_p$. Hence, to satisfy the litigation credibility constraint, damages will always be higher than ϕ_p. Also, when $d < d_0^{nd}(0)$, the

equilibrium is given by: $\beta^*_{LC}(d,\pi) = 1$ and $\alpha^*_{LC}(d,\pi) = \frac{(1-\pi)c+\pi qc}{(1-\pi)\overline{x}+\pi q x_l}$. Note that, from $\beta^*_{LC}(d,\pi) = \frac{1-\pi}{\pi q} \cdot \frac{(x_l-e)\overline{x}-cx_l}{x_l(k-x_l)}$ in the partial deterrence region, when we let $\beta^*_{LC}(d,\pi) = 1$ and rearrange, we get:

$$\frac{1-\pi}{\pi q} = \frac{x_l(k-x_l)}{(x_l-e)\overline{x}-cx_l}.$$

This is the same expression that defined $\widehat{\pi}_0$. Therefore, conditional on no credible litigation, we get partial deterrence when $\pi > \widehat{\pi}_0$, and no deterrence when $\pi \leq \widehat{\pi}_0$.

So far, we have established the function $d_0(\pi)$ in two separate regions: one in partial deterrence ($d_0^{pd}(\pi)$), and the other in no deterrence ($d_0^{nd}(\pi)$) region. To establish the continuity, we first find the expression for $\underline{d}_{LC}(\pi)$ and then show that $d_0(\pi)$ is continuous. In order to find $\underline{d}_{LC}(\pi)$, from the partial deterrence region, we know that

$$\beta^*_{LC}(d,\pi) = \frac{1-\pi}{\pi q} \cdot \frac{(x_l-d-\phi_d-e)\overline{x}-c(x_l-d-\phi_d)}{(x_l-d-\phi_d)(k+\phi-x_l)}.$$

When $\beta^*_{LC}(d,\pi) \to 1$ and we rearrange, we get:

$$\frac{\widehat{\pi}(d)q}{(1-\widehat{\pi}(d))} = \frac{(x_l-d-\phi_d-e)\overline{x}-c(x_l-d-\phi_d)}{(x_l-d-\phi_d)(k+\phi-x_l)}.$$

This equality implicitly defines $\underline{d}_{LC}(\pi)$. It is straightforward to establish that $\frac{\partial\widehat{\pi}(d)}{\partial d} < 0$ or, equivalently, $\frac{\partial \underline{d}_{LC}(\pi)}{\partial \pi} < 0$. When $\widehat{\pi} = 0$, for instance, we get $\underline{d}_{LC}(0) = x_l - \phi_d - \frac{e\overline{x}}{\overline{x}-c} = \overline{d}_{LC}$.

To show that $d_0(\pi)$ is continuous across the two regions, first, when we can rewrite the expression for $d_0^{nd}(\pi)$ and get:

$$\left(\frac{1-\pi}{\pi q}\right)^{nd} = \frac{c\phi_d + x_l\phi_p - (x_l-c)d_0^{nd}(\pi)}{(\overline{x}-c)d_0^{nd}(\pi) - c\phi_d - \overline{x}\phi_p}.$$

The superscript "nd" on the left hand side is to indicate that the ratio is being evaluated in the no deterrence region. From the partial deterrence region, we know that $d_0^{pd}(\pi) = \frac{x_l\phi-\phi_d(e+\phi)}{e+\phi}$. When we let $d_0^{nd}(\pi) = \frac{x_l\phi-\phi_d(e+\phi)}{e+\phi}$ and rearrange the above expression, we get:

$$\left(\frac{1-\pi}{\pi q}\right)^{nd}_{d=d_0^{pd}(\pi)} = \frac{x_l(e+\phi-x_l+c)}{x_l(\overline{x}-c)-\overline{x}(e+\phi)}.$$

We also know that, from the partial deterrence region, when $\beta^* = 1$, we have

$$\left(\frac{1-\pi}{\pi q}\right)^{pd} = \frac{(x_l - d - \phi_d)(k + \phi - x_l)}{(x_l - d - \phi_d - e)\overline{x} - c(x_l - d - \phi_d)}.$$

When we substitute in the expression $d_0^{pd}(\pi) = \frac{x_l\phi - \phi_d(e+\phi)}{e+\phi}$ for d, after some algebra, we get:

$$\left(\frac{1-\pi}{\pi q}\right)^{pd}_{d=d_0^{pd}(\pi)} = \frac{x_l(e + \phi - x_l + c)}{x_l(\overline{x} - c) - \overline{x}(e + \phi)}.$$

This establishes the continuity of $d_0(\pi)$, which is weakly increasing with respect to π since $d_0(\pi)$ is independent of π in the partial deterrence region but is strictly increasing with respect to π in the no deterrence region. □