The Welfare Economics of Reference Dependence *

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Abstract

Empirical evidence suggests individuals often evaluate options relative to a reference point, especially seeking to avoid losses. We analyze welfare under reference dependence. We describe how welfare effects of policies depend on normative judgments about whether reference dependence reflects a bias or normative preference. Lowering reference points generally improves welfare, absent countervailing externalities or biases. Conversely, welfare effects of price changes depend strongly on normative judgments. We apply our theory to reference dependence exhibited in German workers' retirement decisions. Our results suggest positive welfare effects of increasing the Normal Retirement Age but ambiguous effects of financial incentives to postpone retirement.

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"Blessed is he who expects nothing, for he shall never be disappointed." - Alexander Pope

1 Introduction

Reference-dependent preferences are a cornerstone of behavioral economics.[1] In a vast array of settings, decision-makers apparently evaluate options relative to a reference point, and they evaluate losses relative to the reference point more strongly than equivalent gains - *loss aversion*. Early evidence of such behavior came from now-classic laboratory experiments by Tversky and Kahneman (1974). Since then, the experiments have been replicated and extended in myriad ways, in parallel with a rich theoretical literature seeking to model reference dependence (see O'Donoghue and Sprenger (2018) for a review). Researchers have found evidence of reference dependence in experiments around the world (Ruggeri et al., 2020), and in studies of observational data from diverse contexts including the hourly labor supply of cab drivers (Camerer et al., 1997; Crawford and Meng, 2011; Thakral and Tô, 2021) and bicycle messengers (Fehr and Goette, 2007), job search (DellaVigna et al., 2017), behavioral responses to taxation (Homonoff, 2018; Rees-Jones, 2018), and the timing of retirement (Seibold, 2021).

As policymakers take notice of mounting evidence of the fundamental importance of reference dependence, difficult questions loom large. How should we evaluate welfare in the presence of reference dependence? What are the policy implications of all the evidence that reference dependence matters? As in other behavioral settings, the influence of reference points on behavior creates ambiguity over welfare. One possibility is that individuals simply have strange normative preferences, so that reference dependence and loss aversion should be viewed as a part of revealed preferences (Samuelson, 1938). Alternatively, one might suppose that these factors distort behavior relative to what is welfare-maximizing. Mainly because of this difficulty, the vast literature on reference dependence has virtually entirely avoided welfare analysis.[2]

This paper undertakes the first study of the welfare economics of reference dependence. We tackle the central challenge by explicitly analyzing the normative ambiguity inherent in the theory. We view the resolution of the ambiguity over welfare as a normative judgment that must be made by an observer or social planner, and we map such judgments to welfarist quantities. Specifically, we characterize the welfare impact of changes in the reference point, and of changes in prices (or, very similarly, taxes), under varying judgments about welfare. Goldin and Reck (2021) applied a similar approach to optimal default policy. We illustrate the findings in the retirement setting of Seibold (2021), where statutory retirement ages set by public policy influence individuals' reference points and implicit prices are given by financial retirement incentives.

Beyond normative ambiguity, a second challenge in analyzing welfare in the presence of reference dependence is the multitude of proposed model varieties. Our basic approach of embracing normative ambiguity and mapping judgments to welfare quantities can be used in any of the popular models, but more complicated models introduce more nuanced normative ambiguities and they become more difficult to map to empirical data. Following much prior literature (see e.g. the discussion in O'Donoghue and Sprenger (2018)), our approach is to start with the simplest model capable of delivering some insight, and then de-

[1] For instance, (DellaVigna, 2018, p. 699) describes models of reference-dependent preferences as "perhaps the most influential model in behavioral economics."

[2] In their review, O'Donoghue and Sprenger (2018) state that in existing literature "there is relatively little discussion of the welfare implications of reference-dependent preferences." In discussing why this is the case, they write, "When one takes a normative approach to reference-dependent preferences, a number of issues arise. Perhaps first and foremost is the question of whether gain-loss utility should be given normative weight – i.e., whether we should assume that the same preferences that rationalize behavior should also be used for welfare analysis."

velop extensions. Specifically, we start with a simple model based on Tversky and Kahneman (1991), where the only deviation from classical preferences comes from loss aversion over the consumption of a single good, with an exogenous reference point.[3] We consider this model in a static, non-stochastic setting, ruling out aspects of the theory like "diminishing sensitivity" (Kahneman and Tversky, 1979). This simple model of reference dependence is often used in applied work, as it is sufficient to rationalize observed behavior in many contexts. Adopting it allows us to analyze welfare and develop our intuition in a simple and tractable way. We then relax our key simplifying assumptions and extend the model in various ways later on.

Within our simple model, we begin by analyzing the effect of a policy that influences the reference point, holding all else fixed. We show that a change in the reference point has two potential first-order welfare effects; which of these matters depends on the planner's normative judgment. If the planner judges that reference dependence is normative, the sole first-order welfare effect is the *direct effect* of a change in the reference point on gain-loss utility, holding behavior fixed. The effect of a change in the reference point on behavior in this case has no bearing on welfare due to the envelope theorem. In contrast, if the planner judges that reference dependence is not normative, then the sole first-order welfare effect comes from the *behavioral effect* of a change in the reference point, and the direct effect is immaterial for welfare.

Characterizing these direct and behavioral effects, we find that in the simple model, *decreasing the reference point is a robust Pareto improvement*: regardless of the view of welfare one takes, lowering the reference point is a Pareto improvement. When reference dependence is normative, the direct effect implies that loss averse agents are better off when the reference point decreases because they incur smaller losses. When reference dependence is not normative, individuals consuming at the reference point or in the loss domain are over-consuming the good to reduce their losses - i.e. there is a *negative internality* (Mullainathan et al., 2012). Decreasing the reference point decreases consumption, mitigating over-consumption.

Empirical evidence suggests that reference dependence also affects the behavioral response to price instruments, like taxes (e.g. Homonoff 2018; Rees-Jones 2018). Motivated by such evidence, we analyze the welfare effect of a price change in our model. A price change also has first-order direct and behavioral effects. Just as in standard models, a price increase has a direct, first-order, negative welfare effect, holding behavior fixed. And as before, when reference dependence is judged to be normative, the change in behavior has no first-order welfare implications. When loss aversion is causing over-consumption in the loss domain, however, the decrease in consumption caused by a price increase has a positive first-order behavioral welfare effect. This logic implies that when reference dependence is a bias, the optimal corrective tax would equal the amount of loss aversion in the loss domain and zero in the gain domain.[4]

We illustrate these theoretical results with an empirical application to retirement policy in Germany, building on Seibold (2021). The setting has two important advantages for our purposes. First, commonly used policies in the retirement context correspond closely to the types of interventions we analyze theoretically. On the one hand, pension systems typically feature statutory retirement ages such as a Normal Retirement Age. These age thresholds are framed as a "normal" time to retire and are perceived as reference points for individuals' retirement decisions. On the other hand, pension systems provide financial retirement incentives, influencing the marginal return to working longer or the implicit price of leisure. The second advantage of the empirical setting is that the relevant parameters governing individual behavior can be transparently estimated. In particular, we use high-quality administrative data on German retirees and

[3] The determinants of reference points are a subject of much discussion in the literature. We discuss this issue in detail in Section 2.2.1.

[4] We note that this conclusion relies on our assumption that choices in the gain domain are normative, i.e. that individuals are over-valuing the mitigation of losses and not under-valuing the intensification of gains. See also the discussion of this issue in Section 2.1. Caveats involving heterogeneity are also discussed below.

exploit the bunching strategy of Seibold (2021) in order to estimate the responsiveness of these individuals to financial incentives and to the Normal Retirement Age as a reference point.

We simulate the effects of two types of pension reforms often discussed as policy options to induce workers to postpone retirement. The first reform is an increase in the Normal Retirement Age by one year. This reform increases the reference age of retirement, or equivalently lowers the reference point in terms of leisure, the corresponding good. We show that in addition to the positive fiscal effects it entails, such a reform increases overall welfare in a robust fashion, i.e. regardless of the planner's normative judgment of reference dependence. If reference dependence is judged as a bias, a lower reference point in terms of lifetime leisure counteracts some of the initial sub-optimal early retirement behavior, bringing individuals closer to their optimal retirement age. If reference dependence is judged to be normative, a lower reference point yields direct welfare gains instead, as individuals compare their lifetime leisure more favorably to the higher Normal Retirement Age.

The second reform we consider is an increase in the Delayed Retirement Credit, that is higher actuarial pension adjustment for working beyond the Normal Retirement Age. Again, this type of reform is the subject of real-world policy debates around incentivizing individuals to work longer, but it also relates directly to our theoretical results. In particular, a higher Delayed Retirement Credit increases the marginal return to working, implying a higher implicit price of leisure in the loss domain above the Normal Retirement Age. We find that the welfare effects of such a subsidy for later retirement depend on strongly on normative judgments. On the one hand, a higher credit can improve welfare when reference dependence is judged as a bias, because incentivizing workers to retire later mitigates sub-optimal early retirement decisions. In fact, it is possible to calculate an optimal level of corrective subsidies for later retirement in this case. Due to the relatively strong estimated reference dependence in our setting, such a subsidy would have to be very large in order to sufficiently correct behavior. If reference dependence is judged normative, on the other hand, the welfare effects of the Delayed Retirement Credit are much more muted. Moderate actuarial adjustment can help correct fiscal externalities in the pension system, while an overly large Delayed Retirement Credit would distort retirement behavior, reduce the fiscal balance of the pension system and ultimately lower welfare.

Finally, we consider a number of extensions, building on the simple model. Our first extension concerns the dimensionality of reference dependence, which arguably has the most relevance for our empirical application. For example, one could suppose that there is reference dependence over labor supply/leisure or over consumption or over both, as prior work on reference dependence and labor supply points out (Crawford and Meng, 2011; Behaghel and Blau, 2012). Because individuals retiring earlier consume less, loss aversion over consumption might work against our finding that increasing the Normal Retirement Age improves welfare. We theoretically derive the main social welfare effects of interest in a two-dimensional model of reference dependence. As before, there are direct and behavioral welfare effects of a change in price or the reference point, and normative judgments similarly shape which of these matter for welfare. However, the sign of these effects can indeed differ from the simple model in some cases.

We then turn to the implications of two-dimensional reference dependence for our empirical application. To begin with, we examine more closely the observed bunching patterns around the Normal Retirement Age. We argue that the shape of the empirical retirement age distribution is well in line with reference dependence over labor supply/leisure, but less well in line with reference dependence over consumption. This observations motivates the assumptions in our main simulations. In addition, we investigate the robustness of our simulation results to allowing for two-dimensional reference dependence. First, we show that the

signs of all main welfare effects of both types of pension reforms persist under our preferred estimate of the strength of consumption reference dependence. Second, we calculate welfare effects as a function of the strength of consumption reference dependence. This analysis reveals a caveat: if reference dependence is judged to be normative and consumption reference dependence is sufficiently strong, the welfare effects of increasing the Normal Retirement Age can turn negative, since workers retiring early experience additional loss disutility relative to a higher consumption reference point. Yet, the empirical retirement age distribution we observe suggests that consumption reference dependence is probably not strong enough for this caveat to bite in our context.

Our second extension considers an implication of some common formulations of reference dependence (Tversky and Kahneman, 1991; Kőszegi and Rabin, 2006), namely that decisions over gains may be affected by giving the individual an extra payoff from comparing outcomes to the reference point in the gain domain. Although reference dependence is often formulated in this way in the literature, including such gain utility is not necessary to explain the empirical patterns typically attributed to reference dependence.[5] However, we show that including reference dependence over gains in the model does affect welfare calculations, depending on whether this is judged to be normative. The main changes are that (1) behavioral welfare effects now include potential negative internalities in the gain domain, and (2) increasing the reference point can have a direct negative welfare effect on individuals in the gain domain if gain-domain reference dependence is normative. Crucially, decreasing the reference point remains a robust Pareto improvement - in fact the welfare gains from lower reference points would become larger if we adopt this formulation. All this implies that incorporating gain utility into our empirical application would strengthen our findings on the desirability of increasing the Normal Retirement Age, regardless of normative judgments.

Our third extension considers the deliberate setting of reference points by individuals. Our findings up to this point suggest that individuals generally prefer low reference points, but in some contexts individuals may set their own, high reference points out of some other concern, like a separate behavioral bias they wish to overcome. We study this in a model of goal-setting like Koch and Nafziger (2011). Unsurprisingly, when individuals optimally set their own reference points, inducing them to set a higher or lower reference point entails no first-order welfare effects due to the envelope theorem, and a second-order welfare loss. Whether individuals do optimally set their own reference point turns on the question of whether they are sophisticated in their knowledge of their own biases (DellaVigna and Malmendier, 2006). This type of reasoning does not seem applicable to our empirical application, because we observe exogenous policy changes influencing reference points in a fashion inconsistent with individuals' deliberately and optimally setting their own reference points. Nevertheless, in other settings where e.g. individuals use reference points to exert self control, this type of model may be applicable.

We finally discuss a number of considerations for future work based on further possible extensions. These include adding "diminishing sensitivity" to the model, studying reference dependence under risk and uncertainty, and the question of "narrow bracketing" (Tversky and Kahneman, 1981). A common theme of these is that they are theoretically feasible, in that one can write down the model and characterize welfare, but they are also difficult to implement empirically, especially outside of highly stylized experiments.

The remainder of this paper proceeds as follows. In Section 2, we introduce the basic model and derive our main theoretical results, Section 3 presents the empirical application to retirement behavior, Section 4 discusses extensions, and Section 5 concludes.

[5] We formalize and prove this claim in Appendix C. See also Barseghyan et al. (2013).

2 A Simple Model of Welfare Under Reference Dependence

In this section, we lay out a simple model of reference dependence and characterize behavior and welfare within this model. As described above, the contemporary theory of reference dependence is in reality a family of theories. We will focus here on the simplest model that incorporates the key predictions of the theory borne out in many empirical contexts. We will then enrich the theory in several ways in Section 4.

2.1 Setup

Behavior. A population of individuals of measure one, indexed by i, chooses a good $x \in \mathbb{R}$ and background good $y \in \mathbb{R}$ subject to a standard budget constraint with income z_i. The exogenous price of x is p, and the price of y is normalized to 1. Each individual chooses according to a utility function $U(x, y)$. We assume $U(x, y)$ consists of a quasi-linear utility function over x and y plus a reference dependence term for consumption over x, with a reference point $r \in \mathbb{R}$.

$$\begin{aligned} &\max_{x,y} u_i(x) + y + v_i(x|r) \\ &\text{subject to } px + y = z_i. \end{aligned} \tag{1}$$

As it generates behavior, Kahneman et al. (1997) would call $U(x, y)$ *decision utility*, to distinguish it from *experienced utility*, or welfare. We assume an interior solution, and that $u_i' > 0$ and $u_i'' < 0$ always.

Empirical patterns in observed choices documented in the literature suggests that reference dependence generates a kink in marginal utility over x at the reference point r, so that losses incur a penalty on the margin relative to gains.[6] In our starting model, the only deviation from classical models we adopt is the modification to decision utility necessary to rationalize such an empirical observation. We therefore specify the deviation from the classical model in the $v_i(x|r)$ term as follows:

$$v_i(x|r) = \begin{cases} 0 & x > r \\ \Lambda_i(x - r) & x \leq r. \end{cases} \tag{2}$$

The parameter $\Lambda_i > 0$ governs the extent of *loss aversion*, the size of the penalty that losses incur relative to gains on the margin. The domain of x where $x > r$ is called the *gain domain*, and the domain where $x < r$ is called the *loss domain*.

Simplifying Assumptions. The model described by Equations (1) and (2) embeds three key simplifying assumptions, which we relax in Section 4.

First, the formulation of reference dependent payoffs in equation (2) is slightly different from that proposed canonically by Tversky and Kahneman (1991).[7] The simplification we use here is common in the literature and immaterial for explaining behavior, but it may matter for welfare. The Tversky and Kahne-

[6] There have been some suggestions in the literature of a notch in utility rather than a kink, see e.g. Allen et al. (2017). The kink formulation is more commonly adopted, so we adopt it here, and defer consideration of alternative forms of reference dependence to future work. Moreover in our empirical application below, a kink fits the observed bunching better than a notch, which one would expect to generate noticeable "missing mass."

[7] In Tversky and Kahneman (1991), gain-loss utility is posited as the sole component of preferences, while later applications incorporated non-reference-dependent concerns as we do in the first part of equation (1) (see e.g. Kőszegi and Rabin (2006); O'Donoghue and Sprenger (2018)). We also disregard *diminishing sensitivity*, which would require that $v_i'' > 0$ in the loss domain and $v_i'' < 0$ in the gain domain. We discuss this further in Section 4.4.

man (1991) formulation is

$$v_i(x,r) = \begin{cases} \eta_i(x-r) & x > r \\ \eta_i\lambda_i(x-r) & x \leq r, \end{cases} \tag{3}$$

where η_i governs the relative importance of reference dependence overall, and λ_i governs loss aversion. With reference dependence over just one good x, this model is behaviorally indistinguishable from the model in Equation (2). We show this formally in Appendix C; Barseghyan et al. (2013) demonstrate a similar equivalence with a focus on the stochastic case. Relatedly, the presence and size of the η_i parameter in the Tversky and Kahneman (1991) formulation is seldom if ever analyzed empirically. As such, following much prior literature, our simple model focuses on the case where the main friction in the model is loss aversion, with no reference-dependence effects on demand in the gain domain. We consider the question of whether reference dependence in the gain domain might justify an additional deviation from revealed preference in Section 4.2.

By ruling out gain domain payoffs, we also rule out another alternative to the specification in equation (2), which would be to keep loss-domain payoffs at zero in $v(.)$ and include a term in the gain domain with a negative payoff proportional to $(x-r)$. Such a specification would be equivalent to equations (1) and (2) for predicting behavior but different for welfare. Nevertheless we argue that our specification is the appropriate one for welfare analysis because a sizable literature in psychology and neuroeconomics suggests that loss aversion is driven by a negative emotional response to the incursion of perceived losses. In other words, the psychology literature provides good reasons to assume "loss aversion" rather than "gain penalization." We review this literature in detail below. For one particularly compelling piece of evidence on this question, suggesting that reference dependence affects loss payoffs and not gain payoffs, we refer readers to the analysis of the neurological responses to the incursion of perceived gains and losses in Sokol-Hessner et al. (2013), especially Figure 4.

In revealed preference terms, a key implication of our first simplifying assumption is that choices in the gain domain are deemed "welfare relevant," while choices in the loss domain are more suspect (Bernheim and Rangel, 2009) - see Appendix D for further explanation.[8] In addition to our ruling out gain domain payoffs, we make two further simplifying assumptions, each of which we discuss in more detail when we relax them in Section 4. Namely, we assume that reference dependence affects payoffs over a single dimension, ruling out additional reference dependent payoffs over good y. And we assume that the reference point is exogenous.

The combined effect of all these assumptions is to focus the model on a specific behavioral phenomenon related to reference dependence: loss aversion over a single good. This is the main behavioral phenomenon of interest in most of the settings where we have seen evidence that reference dependence matters for the behavioral response to policy changes, including for instance tax sheltering behavior in Rees-Jones (2018), job search behavior in DellaVigna et al. (2017), responses to corrective plastic bag incentives in Homonoff (2018), and retirement decisions in Seibold (2021). So we argue that this simple model is the best place to start.

Behavior. We now characterize demand $x_i(p,r)$ in the simple model. Following our assumption of quasi-linear preferences, we suppress z_i as an input to demand and other functions. We first characterize two potentially optimal choices as follows:

$$u'(x_i^G(p)) = p, \tag{4}$$

[8] Appendix D also contains a thorough accounting of how we can fit our analysis into the general behavioral revealed preference framework of Bernheim and Rangel (2009).

$$u'(x_i^L(p)) + \Lambda_i = p. \tag{5}$$

Because $u_i'' < 0$ and $\Lambda_i > 0$, we know that for any i and any p, $x_i^G(p) < x_i^L(p)$. Demand for a given individual will therefore be

$$x_i(p,r) = \begin{cases} x_i^G(p), \text{ if } x_i^G(p) > r & (G) \\ x_i^L(p), \text{ if } x_i^L(p) < r & (L) \\ r, \qquad \text{otherwise.} & (R) \end{cases} \tag{6}$$

At any given price and reference point in this model, there are three groups of individuals, group G in the gain domain, group L in the loss domain, and group R at the reference point. Furthermore, we know that for individuals in group R, $x_i^G(p) \leq r \leq x_i^L(p)$.

Individual Welfare. The planner must judge whether reference-dependent decision utility should be given normative weight, i.e. whether to respect loss aversion or regard it as a bias. We parametrize this decision by $\pi \in \{0, 1\}$, where $\pi = 1$ if the planner respects loss aversion. We focus on the cases where $\pi = 0$ and $\pi = 1$ for clarity, but the analytic expressions we derive below could be evaluated for intermediate values of $\pi \in [0, 1]$ as well, as in Goldin and Reck (2021). We express normative preferences according to

$$U_i^*(x, y) = u_i(x) + y + \pi v_i(x|r), \tag{7}$$

We denote indirect utility, or welfare at a given price, income and reference point, by

$$w_i(p, r) \equiv U_i^*(x_i(p, r), z_i - px_i(p, r)). \tag{8}$$

Given the judgment encoded by π, U_i^* is a money-metric measure of welfare.

Mechanisms and the Correct Value of π. The psychological mechanisms behind loss aversion has some bearing on the question of what judgments planners should make about π. Broadly speaking, what evidence we have suggests that loss aversion has emotional origins (see Rick (2011) for a review). The findings of an influential study by Kermer et al. (2006) suggested that loss aversion derives from an *affective forecasting error*: people wrongly project that they will experience emotional pain if they incur a loss, so they try to avoid losses. In this case, it is arguably appropriate to set $\pi = 0$. However, more recent evidence suggests that the emotional pain of incurring losses is real rather than a forecasting error and that emotional regulation strategies mitigate loss aversion (Sokol-Hessner et al., 2009). This notion has been further borne out by neurological evidence associating activity in the amygdala with loss aversion and the incursion of perceived losses in a number of ways (De Martino et al., 2010; Sokol-Hessner et al., 2013; Sokol-Hessner and Rutledge, 2019). If we accept this premise, the question of π becomes a deeper question about whether individuals should let negative emotions like fear or regret influence their choices, or whether individuals should make decisions dispassionately – see Loewenstein and O'Donoghue (2006) for a thoughtful discussion of this question. Understanding the mechanisms at play here can lead to an interesting philosophical discussion, but it does not resolve the normative ambiguity over π. So henceforth we remain totally agnostic about the value of π.

Social Welfare. Some of our results can be derived from individual welfare alone, but other policy changes will create winners and losers. We therefore require a notion of social welfare to evaluate such

policies. We will adopt a simple utilitarian social welfare function here for simplicity:

$$W(p,r) = \int_i w_i(p,r)di. \tag{9}$$

Due to our assumption of quasi-linear preferences, maximizing this social welfare function is equivalent to maximizing the sum of compensating or equivalent variation, relative to any arbitrary benchmark. One could relax the assumption of strict utilitarian preferences and quasi-linearity with a straightforward application of Saez and Stantcheva (2016), or with a more sophisticated approach. Likewise, we do not address distributional concerns here, but defer them to future work.

2.2 Results

This section lays out the main theoretical results of the paper for the simple model. We begin with an intuition-building characterization of welfare, and then derive the effects of references points and prices on welfare. Finally, to help us understand the fiscal externality in our empirical application, we discuss additional social welfare effects in the presence of an externality.

The Marginal Internality. A key statistic for welfare analysis is the *marginal internality* (Mullainathan et al., 2012; Allcott and Taubinsky, 2015; Allcott et al., 2019). In our context, this is the welfare effect of a marginal change in x along the budget constraint, evaluated at observed demand. Using the first-order conditions in Equations (4) and (5) and the behavioral characterization in (6), it is straightforward to derive the following:

Lemma 1. ***The Marginal Internality.*** *Let* $m_i(x,r) = \frac{dU_i^*(x,z_i-px)}{dx}\Big|_{x=x_i(p,r)}$.

L1.1. *If* $x_i(p,r) > r$, $m_i(p,r) = 0$.

L1.2. *If* $x_i(p,r) < r$, $m_i(p,r) = -(1-\pi)\Lambda_i \equiv m_i^L$

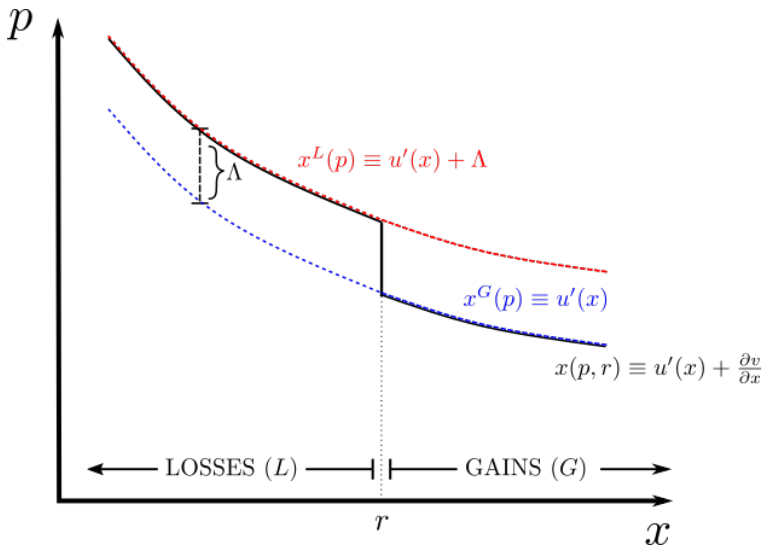
L1.3. *If* $x_i(p,r) = r$,

- $m_i(p,r)$ *is undefined when* $\pi = 1$.
- $m_i(p,r) = u_i'(r) - p$ *when* $\pi = 0$, *with* $-\Lambda_i \leq m_i \leq 0$

We interpret the marginal internality as the marginal welfare effect of paternalistically inducing the consumer to choose a little bit more of good x, starting from observed demand. When the planner judges that observed demand is welfare-maximizing, $\pi = 1$ and there are no marginal internalities as a consequence of the envelope theorem. The marginal internality is undefined when $x = r$ in this case because of the kink in utility at $x = r$, but it remains the case that no induced change in behavior would improve welfare. When $\pi = 0$, in contrast, individuals consuming $x_i \leq r$ are over-consuming good x out of loss aversion. Forcing individuals to consume more x would harm them, so the marginal internality is negative.

Building on Lemma 1 and our characterization of behavior, Figure 1 plots individual demand for good x, which coincides with welfare-maximizing demand when $\pi = 1$, along with $u_i'(x)$, which coincides with welfare-maximizing demand when $\pi = 0$. We also plot demand according to $x^G(p)$ and $x^L(p)$, as defined in Equations (4) and (5), for illustration. Given the judgment by the planner about what is welfare-maximizing, the marginal internality is the vertical difference between observed demand and welfare-maximizing demand in Figure 1.

FIGURE 1: OBSERVED DEMAND, WELFARE-MAXIMIZING DEMAND, AND MARGINAL INTERNALITIES



Note: This figure depicts observed demand $x(p, r)$ at a given reference point as prices vary, in black. We also plot demand in the gain and loss domain, $x^G(p)$ (in blue) and $x^L(p)$ (in red). The vertical distance between these in the loss domain is the loss aversion parameter Λ. When $\pi = 1$, observed demand is welfare maximizing. When $\pi = 0$, $x^G(p)$ is welfare maximizing, and, by Lemma 1, the marginal internality is $-\Lambda$ in the loss domain.

2.2.1 Reference Point Effects

We next characterize the welfare effect of a discrete or marginal change in the reference point.

Can We Change Reference Points? In our model the reference point is exogenous and known to the observer or social planner, but the origin of reference points is the subject of much discussion in the literature. In the design of their early experiments, Kahneman and Tversky apparently viewed reference points as exogenous features of the environment, which that could be manipulated by changing the environment.[9] The psychological literature suggests that reference points may be influenced by several factors, including salient options or features of options (Rosch, 1975) and goals set by the self or others (Heath et al., 1999). Recent economic theory rather attempts to systematically endogenize reference points in terms of beliefs or expectations, though much of this work focuses on the uncertainty case (see e.g. Kőszegi and Rabin, 2006; 2007).

We argue that characterizing the welfare effect of a change in the reference point is important regardless of the precise origins of the reference point in a given context. For any policy P that affects the reference point, we may wish to know the effect of this policy on welfare, which expressing social welfare as a function of P and a reference point $r(P)$ would be $\frac{dW}{dP} = \frac{\partial W}{\partial P} + \frac{\partial W}{\partial r}\frac{\partial r}{\partial P}$. The first term captures any direct effect of the policy on welfare, while the second effect captures the welfare effect that this policy has because it changes the reference point. Our analysis here will characterize the $\frac{\partial W}{\partial r}$ term, abstracting away from other effects policy changes might have.

In some practical settings, it appears to be the case that policymakers can in fact shift reference points, consistent with psychological theories positing that salient features of the environment can induce reference

[9] For example, in the famous "Asian disease experiment" from Tversky and Kahneman (1981), in one experimental condition the potential options are described in terms of gains (lives saved) and in the other condition the same options are described in terms losses (lives lost). The predominant expressed preference – a greater appetite for risk under the loss framing – implies that the change in framing shifted the reference point against which the options were compared.

dependence. For example, Seibold (2021) found significant reference dependence in relation to statutory retirement ages that can be changed by pension policy. Given compelling evidence that the behavioral response to changes in these statutory ages exhibits the signature characteristics of a change in the reference point, we can therefore ask what would happen to welfare if the government changed the Normal Retirement Age, for instance, which is our empirical application below. Some other examples we cited above also suggest that policy can shift reference points. Homonoff (2018) found that framing a plastic bag tax as a tax penalty rather than a discount for re-usable bag use had a large positive effect on the use of re-usable bags. We can interpret this effect as coming from a change in the reference point whereby consumers evaluate the discount in the gain domain and the tax in the loss domain. Likewise, income tax withholding rates can be manipulated by policy and they appear to create a reference point at zero tax due at the time of tax filing (Rees-Jones, 2018).

Consistent with expectations-based origins of reference points, there is experimental evidence that changing expectations can change reference points (Abeler et al., 2011; Ericson and Fuster, 2011). [10] The literature has not yet settled on the best way to model expectations-based reference points. Doing so is a challenging theoretical problem (Masatlioglu and Raymond, 2016), and experimental evidence suggests that simple models of expectations-based reference points have limited explanatory power (Gneezy et al., 2017; Goette et al., 2021). Relatedly, DellaVigna et al. (2017) and Thakral and Tô (2021) use theory and evidence to suggest that past experiences can influence reference points, but whether this is driven by belief updating or some other cognitive process is not entirely clear. In any case, a policy that changes expectations or some similar determinant of the reference point might have effects on welfare that are not driven by reference dependence itself, as above, but in order to characterize the full welfare effect we would still need to consider the effect on welfare through the reference dependence channel, which we do here.

Generic changes in the reference point Let r_1 and r_0 denote two arbitrary reference points, where $r_1 > r_0$ without loss of generality. Based on (6), we know that there are three cases to consider under a given reference point. It is straightforward to show that $x(p, r_1) \geq x(p, r_0)$, so we have 6 potential cases for behavior under these two reference points. Let group GR be the set of individuals for whom $x_i(p, r_0) = x_i^G$ and $x_i(p, r_1) = r_1$, and define the other groups analogously. The change in individual welfare from a change in the reference point is:

$$w_i(p, r_1) - w_i(p, r_0) = \begin{cases} 0, \ i \in GG \\ u_i(r_1) - u_i(x_i^G) - p(r_1 - x_i^G), \ i \in GR \\ u_i(x_i^L) - u_i(x_i^G) - p(x_i^L - x_i^G) + \pi\Lambda_i(x^L - r_1), \ i \in GL \\ u_i(r_1) - u_i(r_0) - p(r_1 - r_0), \ i \in RR \\ u_i(x^L) - u_i(r_0) - p(x^L - r_0) + \pi\Lambda(x_i^L - r_0), \ i \in RL \\ -\pi\Lambda_i(r_1 - r_0), \ i \in LL \end{cases} \tag{10}$$

The effect of the change in the reference point on social welfare will be the summation of the individual change in welfare across all six of these groups. We denote the probability of an individual being in a given group by, e.g., $P[i \in GG]$. Signing the welfare effects in all six cases yields the following result.

Proposition 1. ***The Desirability of Low Reference Points.*** *Consider a change from r_0 to $r_1 > r_0$. Denote*

[10] These possibilities are not entirely mutually exclusive. Points made salient by policy could influence expectations when individuals have weak priors.

the effect on individual and social welfare given the planners' judgments by $\Delta w_i(\pi) \equiv w_i(p, r_1) - w_i(p, r_0)$ *and* $\Delta W(\pi) \equiv W(p, r_1) - W(p, r_0)$.

P1.1. *For any* i *and any* $\pi \in \{0, 1\}$, $\Delta w_i(\pi) \leq 0$.

P1.2. *If* $P[i \in GG] < 1$, *then* $\Delta W(1) < 0$, *and* r_0 *Pareto dominates* r_1 *for* $\pi = 1$.

P1.3. *If* $P[i \in GG] + P[i \in LL] < 1$, *then* $\Delta W(0) < 0$ *and* r_0 *Pareto dominates* r_1 *for* $\pi = 0$.

Proposition 1 implies that in the model we have laid out so far, notably including no externalities, a benevolent social planner would always prefer to decrease the reference point, holding all else fixed. So long as the reference point has any non-trivial effect on behavior, decreases in the reference point are robust Pareto improvements, meaning that they are Pareto improvements for any value of π. In Appendix D, we show that this notion of a robust improvement in welfare is closely related to the welfare criterion proposed by Bernheim and Rangel (2009); in other words, we find that their welfare criterion also suggests that lower reference points are welfare-improving. A full proof of Proposition 1 is provided in the Appendix.

The basic intuition of Proposition 1 is as follows. A change in the reference point has two effects: a direct effect and a behavioral effect. The direct effect comes from the fact that in the loss domain, $v_i(x|r)$ is decreasing in r. This implies that when $\pi = 1$, increasing the reference point decreases welfare holding behavior fixed. The behavioral effect comes from the impact of changing the reference point. As increasing the reference point tends to increase x and, as discussed above, individuals tend to over-consume in this model, the behavioral effect tends to decrease welfare, particularly when $\pi = 0$ so that over-consumption occurs.

To shed further light on these two effects, we next consider a first-order approximation of Equation (10), that is, the effect of a marginal change in r.

Proposition 2. ***First-Order Individual Welfare Effect of a Change in the Reference Point.*** *Consider a change in the reference point that is small enough that the* GL *group is negligible. Let* $\Delta r = r_1 - r_0$, *and let* $\Delta x_i = x_i(p, r_1) - x_i(p, r_0)$. *The first-order individual welfare effect of this change in the reference point is approximately:*

$$\Delta w_i \approx \begin{cases} 0, \ i \in GG, GR \\ (u_i'(r_0) - p)\Delta r, \ i \in RR \\ -(1-\pi)\Lambda_i \Delta x_i - \pi \Lambda_i \Delta r, \ i \in RL, \\ -\pi \Lambda_i \Delta r, \ i \in LL. \end{cases} \tag{11}$$

To understand how these decompose into the behavioral and direct effects, consider that in every case the change in welfare is approximately

$$\Delta w_i \approx m_i(\pi)\Delta x + \pi \left. \frac{\partial v_i}{\partial r} \right|_{x = x_i(p,r)} \Delta r \tag{12}$$

The first term represents the behavioral effect and the second the direct effect. In the GG and LL cases, $\Delta x = 0$, so the behavioral effect vanishes. The direct effect is zero in the GG case because $\left. \frac{\partial v_i}{\partial r} \right|_{x = x_i(p,r)} = 0$, but it equals $-\pi \Lambda_i \Delta r$ in the LL case. In the GR case, the direct effect similarly vanishes and $m_i = 0$ causes the behavioral effect to vanish. In the RL group we can explicitly see both effects in equation (11). The RR group somewhat more nuanced. We observe in equation (11) that the magnitude of the welfare effect of a change in the reference point on this group does not depend on π, but it turns out that this welfare effect

represents the direct welfare effect when $\pi = 1$ and the behavioral welfare effect when $\pi = 0$ – note that $\Delta x = \Delta r$ in this case. We illustrate this last point more concretely in our empirical application.

Figure 2 depicts the welfare effect through the behavioral response for the many different cases, building on Figure 1. In every case, we integrate the marginal internality over the the change in consumption to obtain the effect of the change in behavior on welfare. The direct effect is visualized (in the cases where $\pi = 1$ and it is present) by noting that the distance between $x(p, r)$ and r is the size of the gain or loss, and the vertical distance between $u_i'(x)$ and observed demand equals $\frac{\partial v_i}{\partial x} = -\frac{\partial v_i}{\partial r}$.

We now turn from individual to social welfare for a change in the reference point. Because welfare and behavior are continuous in this model, the change in social welfare from the GR and RL groups is second-order - a marginal change in welfare for a marginal group. Building on Proposition 2, we obtain the following result.

Proposition 3. ***The First-Order Social Welfare Effect of a Change in the Reference Point.*** *Starting from any given price and an initial reference point, define groups G, L and R. The effect of a small change in the reference point of Δr on social welfare is approximately*

$$\begin{aligned} \Delta W \approx &- \Delta r \pi E[\Lambda_i \mid i \in L] P[i \in L] \\ &- \Delta r E[p - u_i'(r) \mid i \in R] P[i \in R]. \end{aligned} \tag{13}$$

Proposition 3 underscores the results in Proposition 1, that social welfare is robustly decreasing in the reference point but the magnitude of the welfare effect of a decrease in the reference point depends on normative judgments. We knew from Proposition 1 that policy should always seek to decrease reference points in this model, but if we incorporate other concerns, such as an externality, Proposition 3 helps us understand how the planner should balance externality concerns with these private welfare concerns, and how that trade-off depends on normative judgments.

2.2.2 Price Effects

We next consider the effect of a change in price on welfare. For simplicity, we will formally derive the first-order welfare effects, but we illustrate the exact welfare effects in our figures.

Observed demand in Figure 1 is negatively inclined, and it is straightforward to show formally that for a price change from p_0 to p_1 with $p_1 > p_0$, $x_i(p_1, r) \leq x_i(p_0, r)$. As with a change in the reference point, there are six potential cases in general, which we denote using similar notation to before as GG, GR, GL, RR, RL and LL. The characterization of these groups is obviously different here because we are comparing different values of p rather than r.

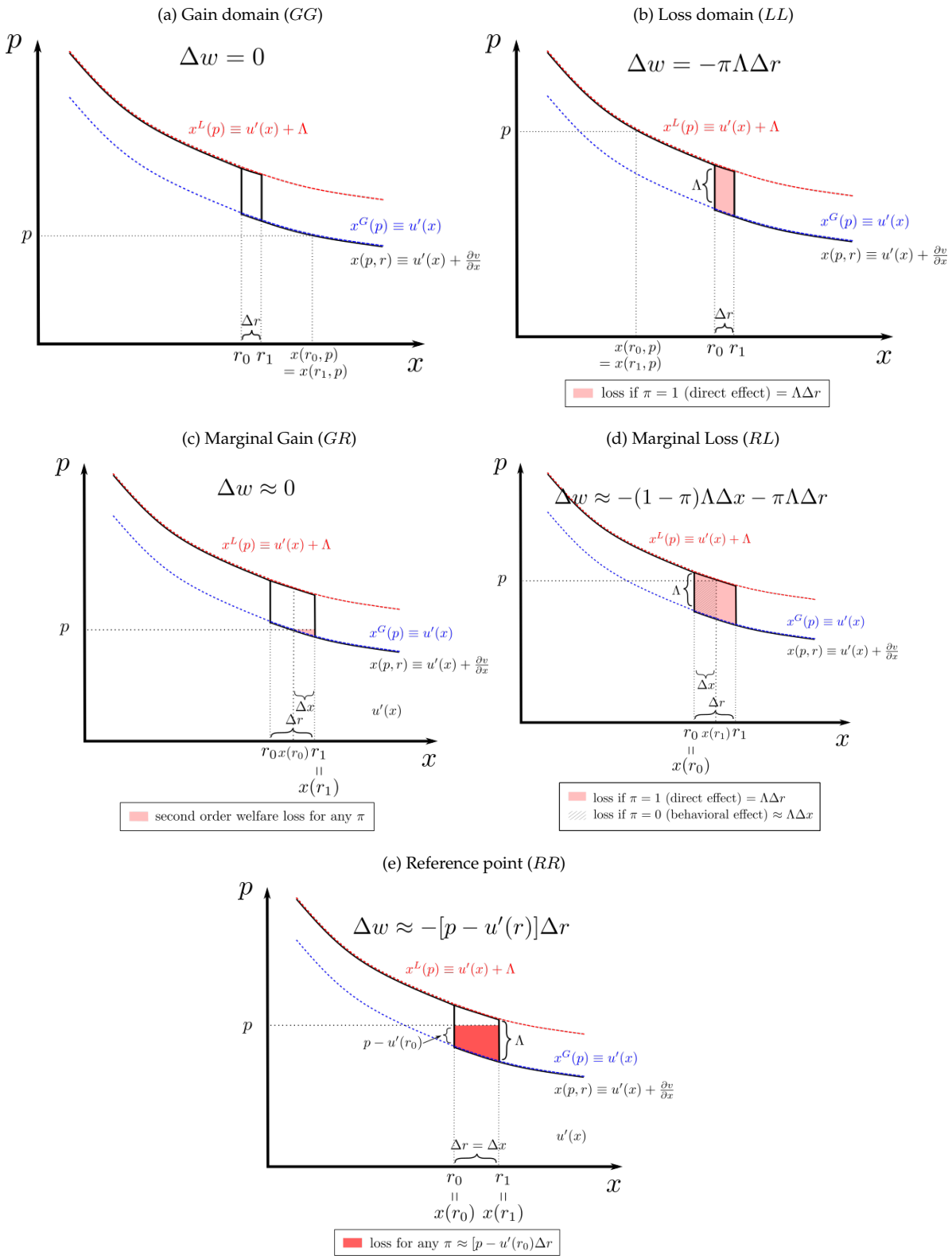
Proposition 4. ***First-Order Welfare Effect of a Change in Price.*** *Consider a change in price that is small enough that the GL group is negligible. Let $\Delta p = p_1 - p_0$, and let $\Delta x_i = x_i(p_1, r) - x_i(p_0, r)$. The first-order welfare effect of a change in price is approximately:*

$$w_i(p_1, r) - w_i(p_0, r) \approx m_i(\hat{p}, r)\Delta x - x_i(\hat{p}, r)\Delta p, \tag{14}$$

where $m_i(\hat{p}, r)$ is defined as in Lemma 1, $\hat{p} = p_0$ if $i \in GG, GR$, and $\hat{p} = p_1$ if $i \in LL, RL$. For $i \in RR$, $\hat{p}$ can be either price for this first-order approximation, and the first term in (14) is zero even when m_i is undefined - in this group $\Delta x = 0$.

A price change has two first-order effects in this model, which are illustrated in Figure 3. First, there is a

FIGURE 2: WELFARE AND CHANGES IN THE REFERENCE POINT



Note: This figure plots the welfare effect of changing the reference point in each domain. We denote observed demand in black and gain and loss domain demand in blue and red, respectively, as in Figure 1. All welfare changes here are losses, shaded in red, reflecting the main result that increasing the reference point unambiguously decreases welfare. Welfare losses from the direct effect are depicted in light red shaded regions, while losses due to the behavioral effect are shaded with diagonal hatching. In panel (e), we find that the change in welfare in the RR case is the same regardless of π, but whether the depicted welfare loss actually represents a behavioral welfare effect or a direct welfare effect does depend on π, so we use dark red shading.

conventional direct, mechanical effect, analogous to the first-order decline in consumer surplus that would occur in a classical model. To value this we simply multiply the change in price by observed demand. Second, there is a change in the internality, which we value by multiplying the relevant marginal internality from Lemma 1 by the change in demand. In the case where demand does not change because the individual sticks at the reference point before and after the price change, this term is absent. When $\pi = 1$, the marginal internality is always 0 and the envelope theorem once again implies the change in x has no first-order welfare effect. In this case, increases in price strictly decrease welfare. When $\pi = 0$, however, the individual over-consumes in the loss domain, and inducing the individual to consume less via a higher price can improve welfare by counteracting the internality. In this case, whether the price change improves or harms individual welfare is ambiguous.

As before, the marginal groups are second order when we turn to the first-order social welfare effect of a change in price. We obtain the following characterization of the first-order welfare effect of a change in price:

Proposition 5. ***The First-Order Social Welfare Effect of a Price Change.*** *Starting from any given reference point r_0 and an initial price p_0, define groups G, L and R. The effect of a small price change Δp on social welfare is approximately*

$$\Delta W \approx \left\{ E[-(1-\pi)\Lambda_i \frac{\partial x_i^L}{\partial p} \Delta p \mid i \in L] \right\} P[i \in L] - E[x_i(p_0, r_0)]\Delta p, \tag{15}$$

where $\frac{\partial x_i^L}{\partial p}$ is evaluated at (p_0, r_0).

Corollary 5.1. ***Corrective Taxes for Reference Dependence.*** *The efficient non-linear, person-specific tax on x in the model given a reference point, $t_i(x, r)$, is*

$$t_i(x, r) = \begin{cases} -(1-\pi)\Lambda_i(x - r), & x < r \\ 0 & x \geq r \end{cases} \tag{16}$$

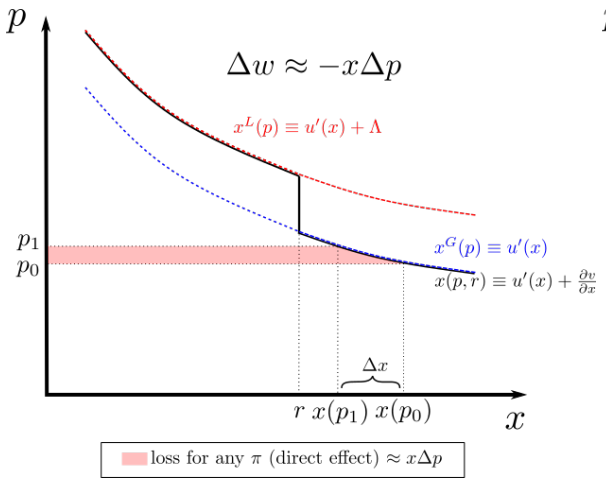
The first term in equation (15) comes from the negative internality in L group from Lemma 1. The final term is the conventional direct first-order effect of a price change, i.e. the mechanical effect on total expenditure. As this final term is equal to the effect on tax revenues from the introduction of tax, Proposition 3 has straightforward implications for optimal corrective taxes in the presence of reference dependence, which we express in Corollary 5.1.

As in other contexts, the optimal corrective tax tends to equal the marginal internality. In this case, the marginal internality is non-linear and only nonzero in the loss domain, so the optimal corrective tax has these properties as well. And naturally, when $\pi = 1$, there are no internalities to correct and the optimal corrective tax is zero. For simplicity, we presume the planner knows Λ_i for each individual. This assumption is obviously unrealistic if Λ_i is highly heterogeneous. In this case, one would need to account for the covariance between the demand elasticity and the internality embodied by Λ_i to set an optimal corrective tax (see e.g. Allcott and Taubinsky, 2015; Allcott et al., 2019). Likewise, we presume the government knows r, which precludes cases where some individuals use different reference points than others in a given situation, and the planner does not know each individual's reference point. We defer further consideration of these issues to future work.

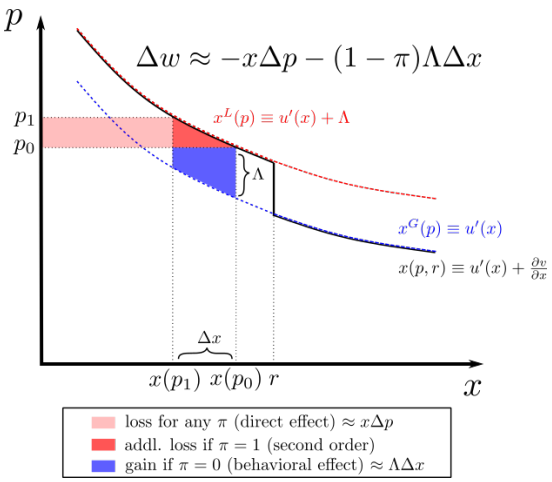
Externalities. In many settings, including our empirical setting, we may wish to incorporate a fiscal or other externality into our welfare framework. For a simple linear atmospheric externality equal to $\alpha E[x_i]$,

FIGURE 3: WELFARE AND PRICE CHANGES

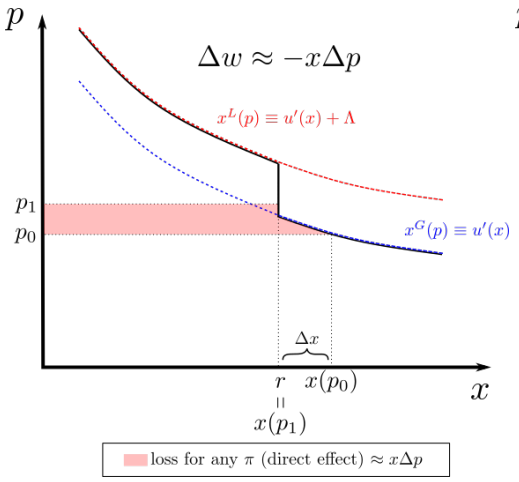
(a) Gain domain (GG)



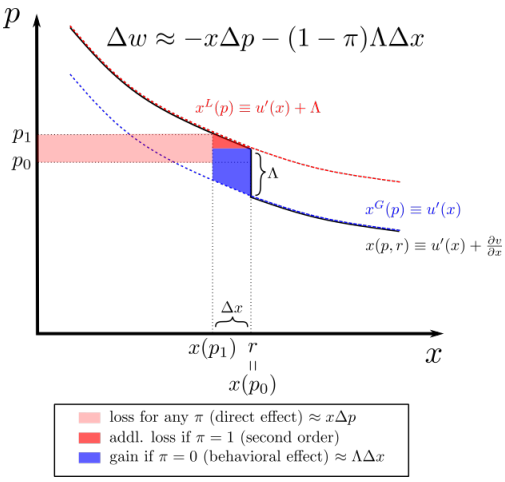
(b) Loss domain (LL)



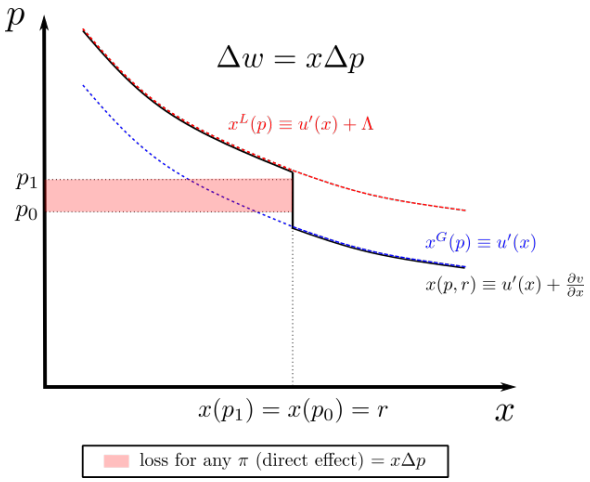
(c) Marginal Gain domain (GR)



(d) Marginal Loss domain (RL)



(e) Reference point (RR)



Note: This figure plots the welfare effect of a price change in each domain. We denote observed demand in black and gain and loss domain demand in blue and red, respectively, as in Figure 1. The direct, mechanical negative effect of a price change is depicted with red shaded regions. The positive behavioral welfare effect attributable to internalities, the normative significance of which depends on π, is depicted with blue shaded regions.

the social welfare effect of either a change in price or a change in the reference point will be the social welfare effect described above plus $\alpha E[\Delta x_i]$, i.e. the marginal externality times the change in aggregate demand for good x.

Consider, for example, the plastic bag incentives studied by Homonoff (2018). Framing the incentive as a tax loss rather than a bonus discount effectively raises the reference point, so that the cost of using a plastic bag is evaluated in the loss domain. Without an externality, our results above suggest that this intervention would decrease welfare, and the decrease would be larger when $\pi = 1$. However, with a large enough negative externality for plastic bag use, which of course was the motivation for introducing the incentive to begin with, the policy implication could go in the opposite direction, in favor of the loss framing. We can further infer from equation (13) that the size of the externality needed to justify the loss framing is larger when $\pi = 1$ than when $\pi = 0$, which reflects that in the $\pi = 1$ case changing the reference point has a direct welfare effect on the individuals using plastic bags who do not change their behavior.

3 Application: Reference Dependence in Retirement Behavior

In this section, we present an empirical application of our theoretical results. Retirement behavior is arguably one of the most important empirical contexts in which reference-dependent preferences have been documented in recent literature (Behaghel and Blau, 2012; Seibold, 2021). Our empirical setting is that of Seibold (2021), who documents large bunching in the retirement distribution around *statutory retirement ages* in Germany and argues that this phenomenon can be explained by workers perceiving those ages as reference points in their retirement decision. In this context, our goal is to characterize the welfare effects of changes to the Normal Retirement Age, and of financial incentives for delayed retirement. These policy reforms are closely related to the welfare effects analyzed in previous sections, and to the types of pension reforms often debated in practice.

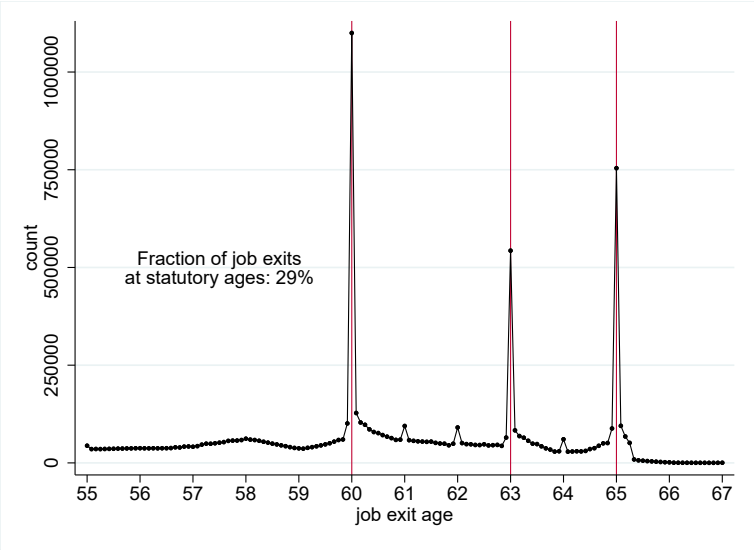
3.1 Institutional Setting and Data

Germany has a pay-as-you-go pension system sharing many of its key characteristics with public pension systems in other developed countries. The vast majority of German workers are covered by public pensions, as enrollment is mandatory for all private-sector employees. Pension contributions are levied as a payroll tax on gross earnings. Benefits are defined according to a pension formula based on a worker's lifetime contribution history. Pension benefits are roughly proportional to lifetime income and there is relatively little redistribution. The average net replacement rate is just over 50% (OECD, 2019), and public pensions are the main source of income for most recipients.

The first key policy dimension for the purpose of this paper are statutory retirement ages, i.e. saliently presented age thresholds used as reference points in the framing of retirement and benefit rules. Most importantly, the *Normal Retirement Age (NRA)* is presented to workers as a "normal" age or time to retire in information material, pension statement letters, and other official government communication. This framing translates into a general perception of the NRA as the reference age of retirement: for instance, a pension reform that will increase the NRA to 67 is commonly known as "retirement at 67" in Germany.

The NRA is the most salient and latest statutory retirement age, but there are others in the system. In addition to the NRA, there is a Full Retirement Age (FRA) from which a "full" pension is available. For most workers in the birth cohorts considered in our analysis, the Normal and Full Retirement Ages coincide, but

FIGURE 4: RETIREMENT AGE DISTRIBUTION IN GERMANY (COHORTS 1933 TO 1949)



Note: The figure shows the retirement (job exit) age distribution among German workers born between 1933 and 1949. The vertical red lines indicate the main locations of statutory retirement ages faced by these workers. Source: Seibold (2021).

they can differ for some. Thirdly, the pension system has an Early Retirement Age (ERA), the earliest age from which a pension can be claimed, which we do not analyze directly. Overall, statutory retirement ages induce strong retirement responses. Figure 4 shows the retirement age distribution among German workers born between 1933 and 1949, among which 29% retire exactly in the month when they reach a statutory retirement age.[11]

The second key policy dimension is financial retirement incentives. Similarly to other pension systems, there is actuarial adjustment of pension benefits as a function of an individual's retirement age. Hence, when a worker chooses to retire later, there is an explicit upward adjustment of pension benefits in addition to the increase in their baseline pension due to additional contributions. In Germany, actuarial adjustment is relatively low, however. Pensions increase by 3.6% per year of later retirement below the FRA, and there is no explicit adjustment between the FRA and the NRA, should they differ for a worker. The largest actuarial adjustment occurs above the NRA, where a *Delayed Retirement Credit* of 6% per year applies.

Two important implications of these pension adjustment rules are worth noting here. First, benefit adjustment is generally less than actuarially fair. For instance, Börsch-Supan and Wilke (2004) calculate that pension adjustment around age 65 would have to be between 7% and 8% per year in order to be actuarially fair. This implies that workers create a fiscal externality when changing their retirement decision, whereby later retirement entails a fiscal benefit to the pension system not internalized by workers. Second, the German pension adjustment schedule creates a *non-convex kink* - an increase in the marginal return to work - at the NRA. This situation is similar to U.S. Social Security, which features approximately actuarially fair benefit adjustment below the NRA, but higher marginal pension adjustment via the Delayed Retirement Credit above the NRA.

[11] Most individuals in Figure 4 face a NRA of 65, a FRA of 63 or 65, and an ERA of 60 or 63. However, the precise location of all three statutory retirement ages differs across workers based on birth cohort and other characteristics such as gender and contribution histories. For this reason, our simulations focus on birth cohort 1946, where the NRA is 65 and coincides with the FRA for most workers. See Seibold (2021) for more details of the institutional setting.

In the empirical analysis, we use the data set of Seibold (2021), which is based on administrative data covering the universe of German retirees who claim a public pension between 1992 and 2014 provided by the German State Pension Fund (Forschungsdatenzentrum der Rentenversicherung (FDZ-RV), 2015).[12] We apply the same sample restrictions as Seibold (2021) and additionally restrict the sample to birth cohort 1946. The main reason to focus on one birth cohort is to simplify the analysis, as different cohorts face different statutory retirement ages and benefit schedules due to various cohort-based pension reforms.

3.2 Model and Parameter Estimation

From our theoretical results, the most important factors for welfare analysis are the strength of reference dependence, the number of individuals in the G, R, L groups, and the behavioral response to price changes. We capture these three components parsimoniously in a static model of retirement behavior with reference dependence, as in Seibold (2021). Preferences of a reference-dependent agent are[13]

$$U_i(C,R) = C - \frac{n_i}{1+\frac{1}{\varepsilon}}\left(\frac{R}{n_i}\right)^{1+\frac{1}{\varepsilon}} - \begin{cases} 0 & R < \hat{R} \\ \widetilde{\Lambda}(R-\hat{R}) & R \geq \hat{R} \end{cases} \tag{17}$$

where C is lifetime consumption and R is the worker's retirement age relative to a career starting age normalized to 0. The parameter $\widetilde{\Lambda}$ captures the strength of reference dependence. The heterogeneous parameter n_i reflects earnings ability at old age, where low ability increases disutility from postponing retirement; for our purposes the distribution of n_i will determine whether an individual is in the $G, R,$ or L group in any given situation. The parameter ε is the elasticity of the retirement age with respect to the implicit net-of-tax rate, which is the relevant elasticity to price changes for our context. Loosely speaking, $\widetilde{\Lambda}$ can be identified by the behavioral response to statutory retirement ages holding financial incentives fixed, ε can be identified by the behavioral response to financial incentives, and the distribution of n_i can be specified to fit the observed variation in retirement ages.

Importantly, equation (17) assumes *loss aversion in lifetime leisure*. The last term in the equation captures reference dependence as in the simple model presented in Section 2. Intuitively, our formulation implies that marginal disutility from increasing labor supply beyond the retirement reference point $\hat{R}$ is greater than marginal disutility from approaching $\hat{R}$ from the left, and the parameter $\widetilde{\Lambda}$ determines the size of this kink in the utility function. Such reference dependence in terms of the retirement age can be interpreted as loss aversion in lifetime leisure, where workers perceive postponing retirement as a loss relative to a normal time to retire.

Workers face a lifetime budget constraint that expresses consumption C as a function of R:

$$C(R) = \sum_{t=0}^{R-1} \delta^t w_t (1-\tilde{\tau}_t) + \sum_{t=R}^{T} \delta^t B(R) \tag{18}$$

where w is the gross wage per period, $\tilde{\tau}$ is the payroll tax/pension contribution rate, T is the time of death, and δ is the discount factor.[14] The slope of the budget constraint, that is the marginal gain in lifetime consumption possibilities C from delaying retirement by one period, defines the implicit net wage $w^{net} =$

[12] Due to the closure of the research data center since the start of the pandemic, the results shown in this version of the paper had to be obtained from a random 1% sample. Final quantitative results based on the full data may thus differ slightly.

[13] Assuming quasi-linear utility in consumption and iso-elastic in lifetime labor supply is convenient for the bunching strategy described below, and, though not strictly necessary, it matches our theory above.

[14] For simplicity, we abstract from the fact that pension benefits can only be claimed from the Early Retirement Age (ERA) onwards if the worker retires before the ERA.

dC/dR. Expressing the consumption gain as a fraction of the gross wage, the *implicit net-of-tax rate* is $1-\tau = w^{net}/w$.

Bunching methods can be used to transparently identify key parameters of the model.[15] As Seibold (2021) shows, the model predicts bunching at the Normal Retirement Age when it is perceived as a reference point by workers. One can identify a marginal bunching individual, whose indifference curve would be tangent to the budget line at some retirement age R^* without reference dependence, and who is tangent exactly at $\hat{R}$ with reference dependence. All workers initially located between $\hat{R}$ and R^* bunch at the reference point, while all individuals initially to the right of R^* retire earlier but stay above the reference point. Individuals initially to the left of the reference point leave their retirement age unchanged. Hence, the bunching mass B at a retirement age reference point is given by

$$B = \int_{\hat{R}}^{R^*} h_0(R)dR \approx h_0(\hat{R})(R^* - \hat{R})$$

where $h_0(\hat{R})$ is the height of the counterfactual retirement density at $\hat{R}$. Based on the tangency conditions of the marginal bunching individual, the excess mass $b = B/h_0(\hat{R})$ at a statutory retirement age can be expressed as

$$\frac{b}{\hat{R}} = \left(\frac{1-\tau}{1-\tau-\Delta\tau-\Lambda}\right)^{\varepsilon} - 1, \tag{19}$$

where $\Lambda = \widetilde{\Lambda}/w$ is the reference dependence parameter normalized by the wage per period and $\Delta\tau$ is the size of the budget constraint kink that may be present at the threshold.[16]

We use the identification strategy of Seibold (2021) in order to estimate Λ and ε. In particular, we leverage the fact that bunching is observed at the Normal Retirement Age, but also at some standard, "pure" financial incentive discontinuities, i.e. budget constraint kinks or notches without the presence of a statutory age. Indexing these various thresholds by i, bunching can be written as

$$\frac{b_i}{\hat{R}_i} = \left(\frac{1-\tau_i}{1-\tau_i-\Delta\tau_i-\Lambda\cdot D_i}\right)^{\varepsilon} - 1 + \xi_i \tag{20}$$

where D_i is an indicator for the Normal Retirement Age and ξ_i is an error term.[17]

Figure 5 shows the empirical retirement age distribution around the Normal Retirement Age among birth cohort 1946. There is sharp, large bunching at age 65, the location of the NRA. The presence of bunching is in line with the NRA serving as a reference point for retirement. While sizable bunching at the NRA has been documented across a number of countries, it is particularly striking in the German case because there is a non-convex kink of size -0.28 at the NRA, providing a negative incentive to retire exactly at this age. The figure also shows a counterfactual density fitted as a polynomial to the empirical distribution, excluding the bunching region. Expressing the bunching mass relative to the counterfactual, the overall excess mass at the NRA is around 31, implying that workers are roughly thirty times more likely to retire exactly in the month of the NRA than we would expect from the smooth counterfactual distribution.

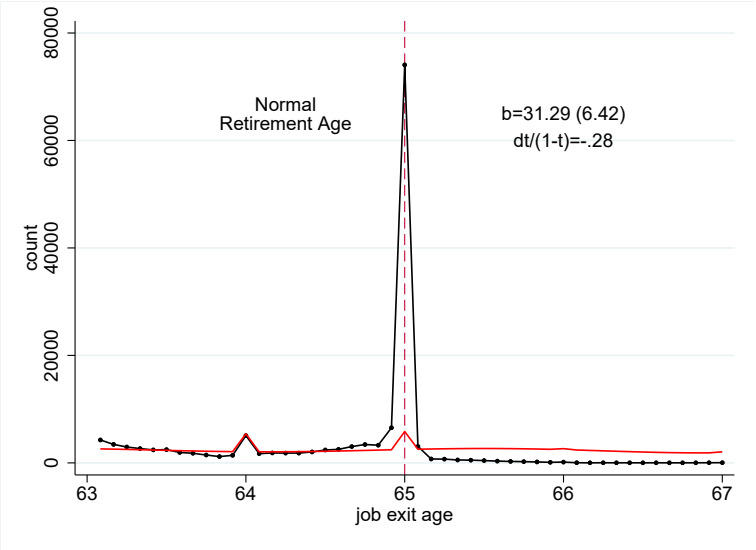
Appendix Table A1 shows these bunching estimates and resulting parameter estimates. The bunching estimates for birth cohort 1946 are a subset of the estimates from Seibold (2021), where bunching is estimated for groups of workers defined by birth cohorts and retirement pathways, the level at which statutory ages and financial incentives vary. In Panel A of the table, the average excess mass at the NRA is 31.3, although

[15] See Kleven (2016) for a general overview of bunching methods.

[16] We assume this transformed parameter Λ is homogeneous for simplicity. Heterogeneity in Λ is difficult to identify empirically given the design used here, but we acknowledge the limitation.

[17] The empirical specification also controls for whether the Normal Retirement Age coincides with the Full Retirement Age.

FIGURE 5: BUNCHING AT THE NORMAL RETIREMENT AGE



Note: The figure shows the pooled distribution of retirement (job exit) ages around the Normal Retirement Age among workers born in 1946. The black connected dots show the actual distribution, while the red line shows the counterfactual density estimated as a seventh-order polynomial excluding the bunching region. The counterfactual density also allows for round-number bunching and features an upward correction to the right of the NRA, where a shift of the retirement age distribution is predicted. The parameter b denotes the average excess mass at the NRA, corresponding to the estimate from Appendix Table A1.

there is a negative local financial incentive to retire corresponding to a kink size of –0.28. At other, pure financial incentive discontinuities faced by the same workers, the average excess mass of 6.73 is smaller, although these entail sizable financial incentives to retire with an average kink size of 0.47. The bunching observations can be used to estimate equation (20), yielding the estimates of $\Lambda = 0.46$ and $\varepsilon = 0.06$ shown in Panel B of the table. These parameter estimates for birth cohort 1946 are similar to the estimates reported in Seibold (2021) for a broader range of cohorts.

3.3 Simulated Welfare Effects of Pension Reforms

3.3.1 Conceptualizing Pension Reforms

In the light of demographic change and resulting fiscal challenges for pension systems, two types of pension reforms are often considered in order to induce workers to postpone retirement. A first common policy is an increase in the Normal Retirement Age (or similar statutory retirement ages). For example, the NRA will be increased to age 67 in the U.S. by 2027, to 67 in Germany by 2031, and to 68 in the U.K. by 2046. This type of reform entails large effects on retirement behavior (Mastrobuoni, 2009; Staubli and Zweimüller, 2013; Manoli and Weber, 2016; Cribb et al., 2016), which is largely driven by shifting individuals' reference points to a higher retirement age (Behaghel and Blau, 2012; Seibold, 2021).

Two important aspects are worth noting about NRA reforms. First, while an increase in the NRA sets the reference point at a higher retirement age, such a reform corresponds to decreasing the reference point in terms of lifetime leisure in the model from Section 3.2. Thus, we should conceptually think of a reform that increases the NRA as one that *lowers individuals' reference points* in the sense of Proposition 1. Second, while our theory considered changes to reference points holding all else fixed, changes to the NRA typically entail

some change in individuals' lifetime budget constraints, because pension benefit schedules are linked to the NRA. In the German context, the Delayed Retirement Credit is only available from the NRA onward. If this feature is maintained, increasing the NRA would also move the non-convex kink in the budget constraint to the new NRA. Moreover, if the NRA coincides with the FRA, the age from which the "full" pension is available may move upwards with the NRA, such that increasing the NRA effectively implies a benefit cut across the board.

The second type of policy often considered for pension reforms are changes to financial incentives. In particular, a natural way to incentivize workers to retire later is to offer higher marginal pension benefit increases for later retirement. This is typically done by increasing the Delayed Retirement Credit, providing higher actuarial adjustment to workers retiring beyond the NRA. For instance, the U.S. Delayed Retirement Credit has been gradually increased from 3% to 8% per year over the last decades (Duggan et al., 2021). Conceptually, a higher Delayed Retirement Credit corresponds to a higher marginal return to work, or a *higher price of lifetime leisure in the loss domain* above the NRA. Whether intentionally or not, the Delayed Retirement Credit can thus be interpreted as an implicit corrective "tax" in the sense of Corollary 5.1, which can incentivize individuals to move away from the reference point of the NRA by increasing their retirement age.

3.3.2 Simulations

We simulate the welfare effects of two pension reforms of the types discussed above, building on Seibold (2021), who calculates effects of similar reforms on behavior and fiscal balances. The first reform is an increase in the NRA from 65 to 66. The reform shifts individuals' retirement reference points and entails a (relatively small) change in the budget constraint. In order to maintain the feature of a budget constraint kink at the NRA, the Delayed Retirement Credit only applies above the new NRA in the simulation. However, the counterfactual scenario does not feature a benefit cut across the board below the NRA in order to avoid confounding the effects of influencing reference points with large mechanical fiscal and consumption effects. The second reform is an increase in the Delayed Retirement Credit. In order to anchor the second reform, we increase the credit from the current level of 6% to 10.4% per year, which yields the same effect on the average retirement age as the first reform.

The policy simulations proceed in the following steps. First, we require a counterfactual distribution of retirement ages – a distribution of retirement ages in the absence of reference dependence. We obtain this counterfactual distribution by fitting a polynomial to the observed distribution, excluding the bunching region around the NRA. In the absence of reference dependence, individuals bunching at the NRA would be distributed across retirement ages above the NRA, and we simulate this un-bunching by distributing the bunching mass across the age range 65 to 68.[18] We then assign counterfactual retirement ages to individuals in the data based on ranks of actually observed retirement ages.

Second, we simulate optimal retirement ages for each individual under the baseline policy environment where the NRA is 65 and the Delayed Retirement Credit is 6% per year. Third, we simulate optimal retirement ages under the two counterfactual policy scenarios. For this, we simulate individual lifetime budget constraints from equation (18) as in Seibold (2021), based on observed individual earnings and contribution

[18]The empirical retirement age distribution offers little information about the counterfactual shape of this upper tail, as few individuals actually retire above the NRA in the data (see Figure 5). In the baseline simulations, we distribute the bunching mass following a fitted Pareto distribution above age 65, corresponding to a moderately decreasing shape above the NRA. Appendix Figure A1 shows the counterfactual density under alternative assumptions about the tail of the distribution, including a uniform and a lognormal distribution above the NRA. Reassuringly, these alternative distributional assumptions have little impact on our simulation results, as Appendix Table A2 shows.

histories, and choose the retirement age that maximizes utility from equation (17) given the level of $C(R)$ pinned down by the budget constraint and the location of the NRA $\hat{R}$.

Fourth, we compute the difference between each counterfactual scenario and the baseline scenario for each of the following outcomes: contributions to the pension system, benefits paid to workers, workers' lifetime consumption, all in terms of net present value at age 65. Moreover, we calculate the effects on disutility from working and reference dependence payoffs given the preferences in equation (17). Based on these, we can calculate the effects of each reform on the fiscal balance of the pension system, on the welfare of workers, and on total welfare – the sum of fiscal effects and individual welfare effects.

3.3.3 Main Results

Table 1 summarizes the effects of the two simulated pension reforms. Appendix Figure A2 provides further illustration of how the different components sum up to welfare effects.

Increasing the NRA. Column (1) shows the effects of the NRA increase. Shifting the NRA by one year increases average actual retirement ages by 7.5 months. As shown in Seibold (2021), such a reform improves the fiscal balance of the pension system. The positive fiscal effect arises due to a combination of a higher net present value of contributions collected and a lower value of benefit payments, both of which arise when individuals work longer and postpone retirement. The magnitude of the net fiscal effect is around +€10.8k per worker. Next, the reform affects individual welfare. Lifetime consumption increases by around +€7.2k along with later retirement. Disutility from work becomes larger because increasing the NRA to 66 induces workers to work up to one year longer, which enters negatively into individual welfare. However the increase in consumption outweighs the disutility from work. This reflects the behavioral welfare effect of a change in the reference point from Propositions 2 and 3, under $\pi = 0$. In words, the individual is consuming too much leisure when $\pi = 0$, so decreasing the reference point over leisure by increasing the NRA has a corrective effect on behavior that improves individual welfare. Thus, we find that worker welfare improves when $\pi = 0$ in the table. The effect on total welfare is given by the sum of the individual welfare effect and the net fiscal effect. Under $\pi = 0$, we find that total welfare increases by around +€13.0k per worker.

In addition, if the planner places normative weight on reference dependence ($\pi = 1$), we should also account for changes in reference dependence payoffs due to the lower reference point in terms of lifetime leisure. We can conceive of the overall change in reference dependence loss disutility as the sum of two components: a negative component of about -€11.7k from additional disutility from working longer, and a positive component +€13.5k from the increase in the reference point $\hat{R}$ itself.[19] When $\pi = 1$, the first of these modifies the behavioral effect relative to the case when $\pi = 0$. The total behavioral welfare effect when $\pi = 1$ is the sum of worker consumption (+€7.2k), disutility from work (-€5.1k), and reference dependent disutility from work (-€11.7k), totalling -€9.5k. We observe that behavioral welfare effect and the net fiscal effect (+€10.8k) approximately offset one another. This cancellation is a consequence of the envelope theorem, reflecting the theoretical idea that the change in behavior induced by a change in the reference point has no first-order consequences for welfare when $\pi = 1$. Panel (b) of Figure A2 provides a visual illustration of this offsetting.[20]

[19] See Appendix E.1 for details of this decomposition of reference dependence payoffs.

[20] If existing pension incentives were completely actuarially fair, so that pension incentives did not distort behavior at all, these two effects would even more completely offset one another. Because the distortions are relatively small under the status quo, we find almost complete offsetting.

With the behavioral effect largely eliminated under $\pi = 1$, the direct welfare effect, i.e. the effect on reference dependence payoffs from the change in reference point itself, becomes the primary determinant of the total welfare effect. We find a total welfare gain under $\pi = 1$ of around +€14.8k, even larger than under $\pi = 0$. Building on the logic of the previous paragraph, almost all of this welfare effect (+€13.5k) is attributable to the direct effect of the change in the reference point. Note that a larger effect under $\pi = 1$ is consistent with Proposition 3.[21]

Appendix Table A3 shows average welfare effects of the NRA increase by groups, where each group is defined by their retirement age relative to the NRA before and after the reform analogously to Section 2.2.1. For instance, the LR group consists of workers who retire in the loss domain above the old NRA before the reform, but retire at the new NRA after the reform. Most importantly, Table A3 shows that the key intuition about behavioral and direct effects we get from the theory and Table 1 carries through to each sub-group. When $\pi = 0$, workers retire sub-optimally early, so the groups whose behavior is affected by the NRA change – all but the LL group – experience a positive total welfare effect (net of fiscal effects). But when $\pi = 1$, the additional behavioral effect via reference dependent disutility from work eliminates this effect. The main determinant of welfare when $\pi = 1$ is the direct welfare effect, via reference dependent utility from the reference point itself; as in the theory, this effect is not present in the gain domain.[22]

TABLE 1: WELFARE EFFECTS OF PENSION REFORMS

	(1) Policy 1: Normal Retirement Age to 66	(2) Policy 2: Delayed Retirement Credit to 10.4%
Contributions collected	+4,042	+3,706
Benefits paid	+6,790	–6,557
Net fiscal effect	+10,821	–2,822
Worker consumption	+7,231	+19,756
Disutility from work	–5,064	–3,427
Worker welfare ($\pi = 0$)	+2,166	+16,329
Ref. dep. disutility from work	–11,712	–14,215
Ref. dep. utility from ref. point	+13,532	0
Worker welfare ($\pi = 1$)	+3,986	+2,113
Total welfare ($\pi = 0$)	+12,987	+13,507
Total welfare ($\pi = 1$)	+14,808	–708

Note: The table shows results from simulations of two pension reforms, an increase in the NRA from 65 to 66 and an increase in the Delayed Retirement Credit to 10.4%. Both reforms yield the same effect on the average actual retirement age (+7.5 months). Simulations are conducted for birth cohort 1946. All effects are calculated among workers retiring at age 65 and above, and are in Euros per worker, in terms of net present value at age 65. The signs the effects correspond to influence on welfare. Total welfare is the sum of net fiscal effect and change in worker welfare.

[21] This occurs because the R group in Proposition 3 experience the same welfare effect regardless of π, as discussed in Section 2.2.1, while the L group experiences a positive direct welfare effect of a lower reference point for leisure when $\pi = 1$, and no welfare effect when $\pi = 0$. See also footnote 24.

[22] Relatedly, we observe that for the RR and RG groups, the total welfare effect of a change in the NRA does not depend on π, because in this case the direct and behavioral welfare effects have equal magnitude.

Increasing the Delayed Retirement Credit. Column (2) of the table shows the effects of the increase in the Delayed Retirement Credit to 10.4%. By construction, this policy achieves a sizable increase in the average retirement age like the NRA increase. However, a first important difference to the NRA reform is the fiscal effect. The net fiscal effect is negative at –€2.8k per worker. Workers also contribute for longer in this scenario, but the positive effect on contributions is more than offset by the large increase in benefit payments.[23] Due to the higher pension benefits and the additional income from working another year, worker consumption increases strongly, as workers retiring later receive large pension benefit increases. Disutility from work becomes larger, but less so than under the NRA reform because workers account for their own marginal disutility of work in deciding just how much later to retire under the higher credit. Thus, there is a large positive effect of +€16.3k on worker welfare under $\pi = 0$.

However, the sizable behavioral response leads to a large increase in reference dependent disutility from work, reducing individual welfare by –€14.2k when this concern carries normative weight under $\pi = 1$. This sizable negative effect arises because workers increase their retirement ages relative to an unchanged reference point, pushing them further into the loss domain over leisure. Taking this additional welfare effect into account, individual welfare increases only by +€2.1k under $\pi = 1$. Finally, the total welfare effect is positive at +€13.5k under $\pi = 0$, as the large gain in individual welfare strongly dominates the negative fiscal effects. However, the total welfare effect turns slightly negative under $\pi = 1$, when workers experience large disutility from being "pulled away" from the reference point.

The sizable difference in worker welfare between the $\pi = 0$ and the $\pi = 1$ cases is directly related to the theoretical results in Propositions 4 and 5. When $\pi = 0$, there is an internality from workers consuming too much leisure out of loss aversion. Increasing the Delayed Retirement Credit acts as a corrective tax on leisure under $\pi = 0$, so this reform has a large positive welfare effect by (partially) correcting the internality from over-consuming leisure. In contrast, when $\pi = 1$, the change in worker welfare is much smaller because this internality is not present. Moreover, in this case, the basic intuition of the envelope theorem implies that first-order effects on worker welfare will be virtually entirely offset by the net fiscal effect. Thus, the Delayed Retirement Credit acts as a distortionary tax on leisure under $\pi = 1$. The initial 6% credit is relatively close to actuarial fairness, so the distortions are second-order in the main simulation. However, distortions can become large when considering larger changes to the credit, which we explore further in the extended simulations below.

3.3.4 Extended Simulations

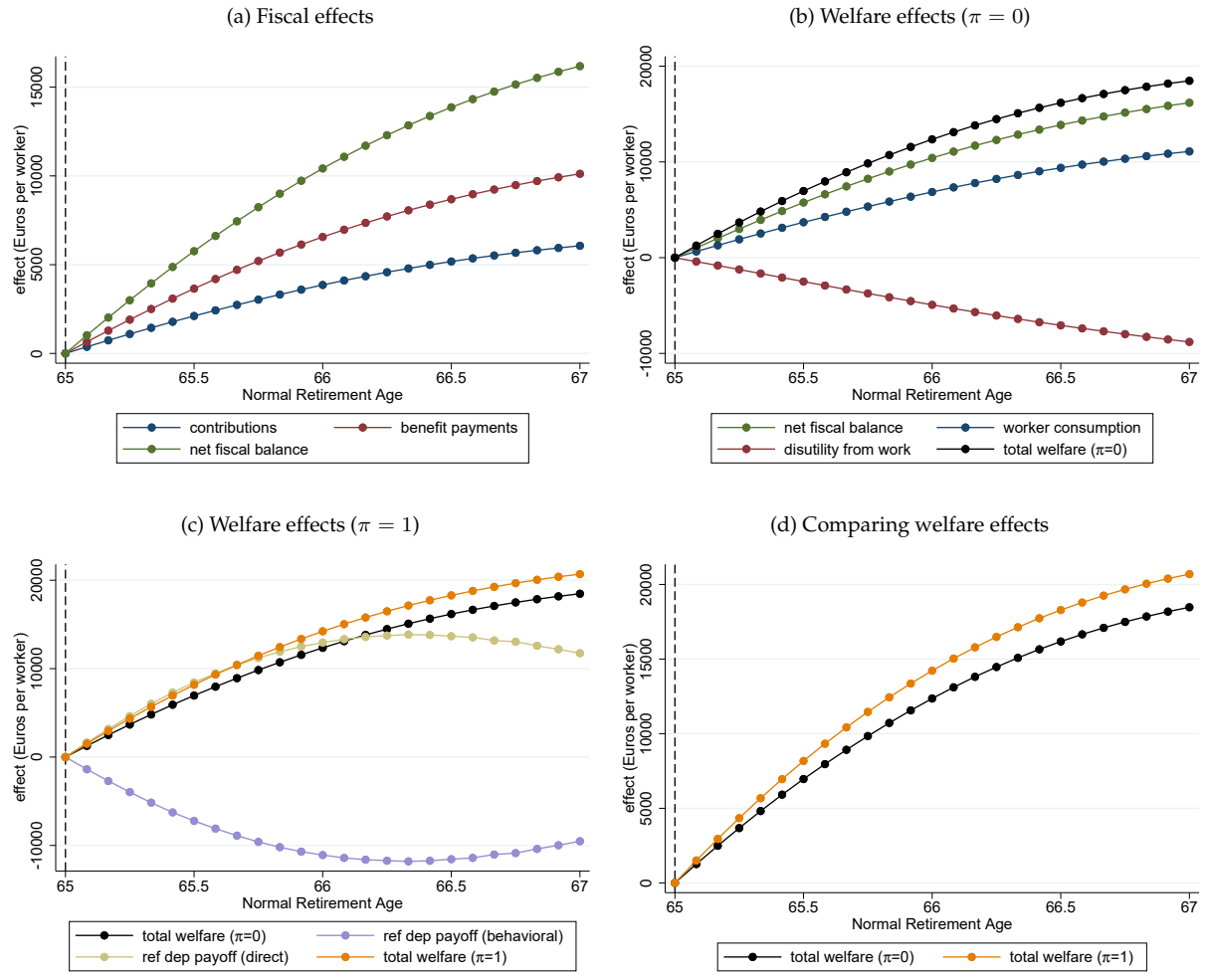
We next extend the simulations to a wider range of policy reforms. This provides further insights into the relationship between the policy simulations and our theoretical results, albeit some additional caution may be warranted in interpreting the findings as we are extrapolating further from observed data than above.

While Table 1 considers a specific change to the Normal Retirement Age, Figure 6 shows results for a range of simulated counterfactual NRAs. The figure is based on simulations for NRAs between 65 and 67 in monthly increments. Overall, the figure confirms the robust positive welfare effects of increasing the NRA. To begin with, Panel (a) shows that the fiscal balance of the pension system increases with the NRA, as more contributions are collected and pension benefits are paid for shorter periods. In Panel (b), individual consumption increases with the NRA, but workers also experience higher disutility from working longer. Adding up those two components of standard preferences and the net fiscal effect, total welfare under $\pi = 0$

[23] That increasing the Delayed Retirement Credit is less fiscally desirable reflects an idea from Loewenstein and O'Donoghue (2006). Policies like increasing the NRA, which they might call a "psychic subsidy" for working, are less fiscally costly than an actual subsidy for working.

increases with the NRA. In Panel (c), we add reference dependence payoffs in order to obtain welfare effects under $\pi = 1$. Again, workers experience additional disutility from work due to reference dependence, but the lower reference point in terms of lifetime leisure exerts a positive direct effect on welfare. As in Table 1, incorporating reference dependent disutility from work eliminates most of the behavioral welfare effect when $\pi = 1$, but the simultaneous introduction of a positive, direct welfare effect of increasing the reference point leads to an overall increase in welfare when $\pi = 1$. As Panel (d) shows, total welfare increases monotonically with the NRA both under $\pi = 0$ and $\pi = 1$, where the welfare increase is somewhat stronger under $\pi = 1$.[24]

FIGURE 6: INCREASING THE NORMAL RETIREMENT AGE



Note: The figure shows results from simulations of pension reforms that increase the NRA to ages between 65 and 67 in monthly increments. Simulations are conducted for birth cohort 1946. All effects are calculated among workers retiring at age 65 and above, and are in Euros per worker, in terms of net present value at age 65. The signs the effects correspond to influence on welfare. Total welfare is the sum of net fiscal effect and change in worker welfare.

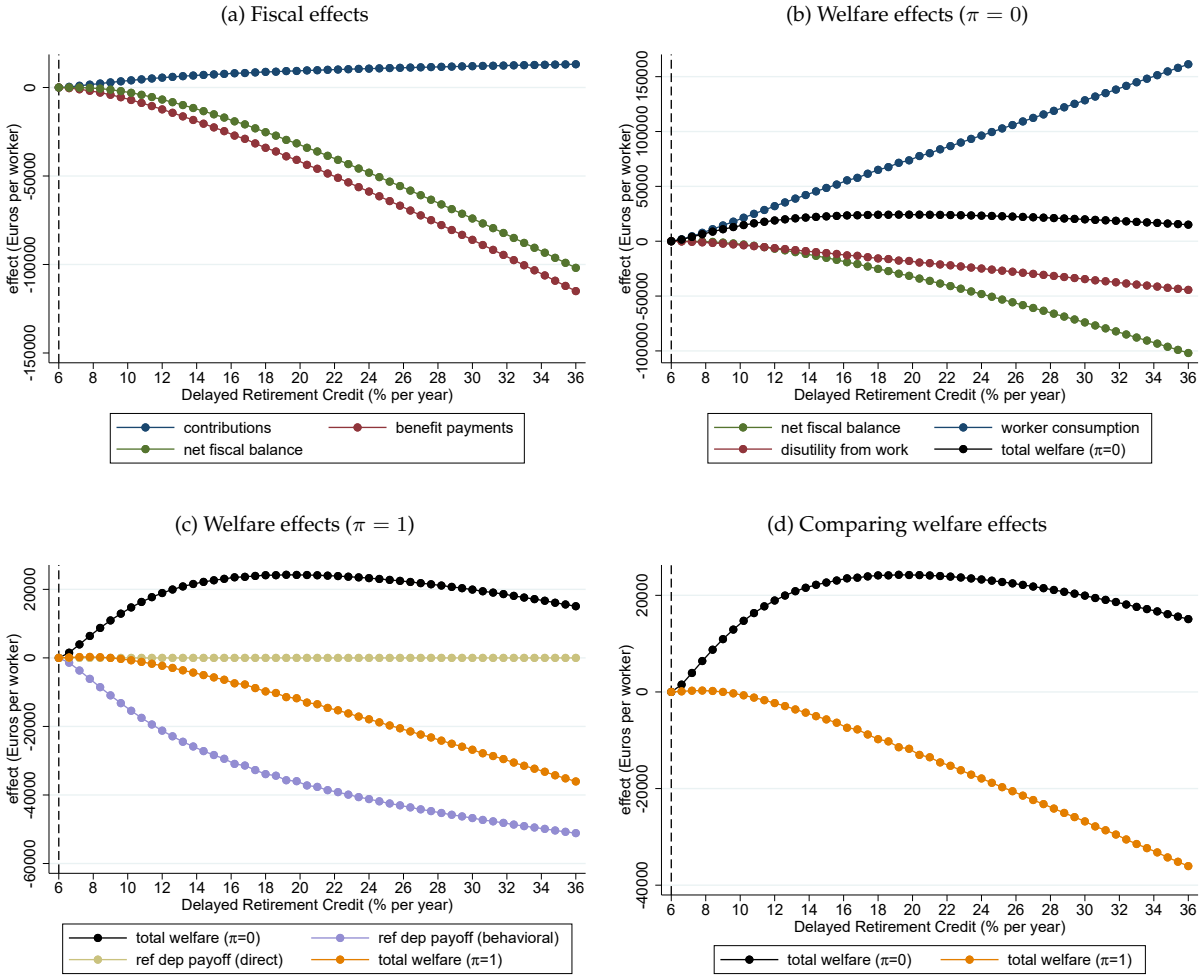
[24] The quantitative similarity between total welfare under $\pi = 0$ and $\pi = 1$ is not a generic feature of the theory. Rather, it occurs in this context because the number of individuals retiring exactly at the NRA (the R group in the theory) is relatively large throughout the range of NRAs we consider. This feature of the environment implies that the behavioral welfare effect of a change in the reference point, which mainly matters when $\pi = 0$, and the direct effect, which mainly matters when $\pi = 1$, will be quantitatively similar. Recall that for the RR group in Proposition 2, the magnitude of the change in welfare from a change in the reference point does not depend on π, but whether we should interpret this change as a direct or behavioral welfare effect does depend on π. With a larger L group, the direct effect, and thus the change in welfare for $\pi = 1$, could be significantly larger.

Similarly, Figure 7 shows results for a range of simulated values of the Delayed Retirement Credit. We simulate credits between 3% and 36% per year in half-percentage point increments. In Panel (a), the fiscal effects of increasing the Delayed Retirement Credit tend to be large and negative, since the large increases in pension benefit payments dominate increases in contributions received by the pension system. There is, however, a small range just above the current value of 6% over which the net fiscal effect of increasing the credit is positive, as the pension system moves closer to actuarial fairness. Panel (b) shows that under $\pi = 0$ the total welfare effect of increasing the credit is positive throughout the large range we consider, since consumption increases by more than disutility from work and the negative fiscal effects, reflecting workers' initial over-consumption of leisure. Under $\pi = 1$, however, the corrective benefits of a higher credit are wiped out by reference dependent disutility from later retirement, so that total welfare decreases for all but small increases in the credit.

A key difference between increasing the NRA and changing the Delayed Retirement Credit is that the total welfare effects of the latter reforms are not monotonic. While increasing the NRA always increases total welfare, both under $\pi = 0$ and $\pi = 1$, Panels (b) to (d) of Figure 7 show that it is possible to find an optimal level of the Delayed Retirement Credit. Importantly, the welfare-maximizing credit depends strongly on whether the planner places normative weight on reference dependence. Under $\pi = 0$, total welfare is maximized at a very large Delayed Retirement Credit of 20.6% p.a., more than three times its current level. This results speaks to a possible role for the Delayed Retirement Credit to correct inefficiently early retirement under $\pi = 0$, as in Corollary 5.1. Such a large marginal financial return to working longer, or implicit price of leisure, can induce workers to retire later and move towards their optimal retirement age. In Panels (c) and (d), the optimal level of the Delayed Retirement Credit is much lower under $\pi = 1$. Intuitively, there is no reason for the planner to incentivize workers to move away from the NRA and retire later when reference dependence is not judged as a bias. The only rationale to increase the Delayed Retirement Credit slightly above its current level is to correct the inefficiency that arises from the fiscal externality, due to less than actuarially fair pension adjustment. Indeed, the optimal Delayed Retirement Credit of 7.7% p.a. that we find appears to be close to previous calculations of actuarially fair adjustment in the German context (Börsch-Supan and Wilke, 2004).

Overall, these simulations illustrate essentially all of the main ideas from our theoretical results. Increasing the NRA, which corresponds to lowering reference points in terms of lifetime leisure, yields robust increases in total welfare. Increasing the Delayed Retirement Credit, which corresponds to an implicit tax on leisure in the loss domain, increases total welfare if reference dependence is judged as a bias. However, increasing the credit beyond its actuarially fair level decreases welfare if reference dependence carries normative weight. Two further aspects of these types of reforms are worth considering. First, if policymakers are mainly concerned about the fiscal sustainability of pension systems, increasing the NRA may be attractive in its own right as this yields sizable positive fiscal effects. Higher late retirement subsidies tend to yield negative fiscal effects, on the other hand. Second, if policymakers are uncertain about the appropriate welfare judgment of reference dependence, NRA increases may appear even more attractive, as their positive welfare effect is robust to the choice of π. The sign and magnitude of the welfare effects of the Delayed Retirement Credit, on the other hand, fully depend on this normative judgment.

FIGURE 7: INCREASING THE DELAYED RETIREMENT CREDIT



Note: The figure shows results from simulations of pension reforms that increase the Delayed Retirement Credit to values between 6% and 36% per year in half-percentage point increments. Simulations are conducted for birth cohort 1946. All effects are calculated among workers retiring at age 65 and above, and are in Euros per worker, in terms of net present value at age 65. The signs the effects correspond to influence on welfare. Total welfare is the sum of net fiscal effect and change in worker welfare.

4 Extensions

4.1 Multi-Dimensional Reference Dependence

Thus far, we have considered reference dependence along a single dimension of the menu space. In some contexts, individuals apparently exhibit reference dependence in more than one dimension. In this section, we consider welfare in a two-dimensional model of reference dependence.[25] For instance, in our empirical application, we could interpret the two dimensions as representing reference dependence over leisure and consumption, similarly to Crawford and Meng (2011), who argue that a two-dimensional model explains well the daily labor supply behavior of cab drivers. Here, we present a framework for normative analysis in a two-dimensional model, and present an extension of our empirical application considering two-dimensional reference dependence.

4.1.1 A Two Dimensional Model

Behavior. We continue to assume that the individual has quasi-linear preferences over goods x and y as before. We introduce a reference point s for good y and model behavior as follows:

$$\begin{aligned} &\max_{x,y} u_i(x) + y + v_i(x|r) + w_i(y|s) \\ &\text{subject to } px + y = z_i \end{aligned} \tag{21}$$

The reference-dependent term in the x dimension is given by equation (2) and the new reference dependence in the y dimension, $w_i(y|s)$, is given by:

$$w_i(y|s) = \begin{cases} 0, & y > s \\ \Gamma_i(y - s) & y \leq s. \end{cases} \tag{22}$$

We continue not to include potential distortions in the gain domain, like η_i from equation (3) and the next section, when we specify reference dependence over x and y.[26] Also for simplicity, we restrict our attention to the situation where the two-dimensional reference point (r, s) is on the budget constraint: $pr + s = z_i$.[27] With this restriction $x > r \iff y < s$. As in equations (4) and (5), the first order conditions for $x > r$ and $x < r$ are now given by

$$\frac{u_i'(x_i^G(p))}{1 + \Gamma_i} = p, \tag{23}$$

$$u_i'(x_i^L(p)) + \Lambda_i = p \tag{24}$$

Behavior is given by equation (6) with the new potential demand curves $x^L(p)$ and $x^G(p)$ implied by equations (23) and (24).

[25] Extending this analysis to arbitrary dimensionality of the goods space is straightforward, though as will be apparent from our application below, it can be difficult to identify a many-dimensional model empirically.

[26] Including an η_i-like term for both dimensions is a straightforward extension of this model, but such a model is very difficult to identify empirically without some methodological progress. Identifying η_i for a single dimension is difficult due to the issues we discuss; separately identifying such a parameter for two different dimensions would be even more challenging.

[27] Implicitly this entails that holding r fixed for everyone, s is heterogeneous across individuals with different z_i; our welfare effects below account for this.

Welfare. As in equation (7), we specify welfare given a normative judgment $\pi \in \{0, 1\}$ as follows:[28]

$$U_i^*(x, y) = u_i(x) + y + \pi[v_i(x|r) + w_i(y|s)]. \tag{25}$$

Lemma 2. ***The Marginal Internality in the 2-D Model.*** *Let m_i be the derivative of $U_i^*(x, z_i - px)$ from equation (25) with respect to x, evaluated at $x_i(p, r)$.*[29]

L2.1. *If $x_i(p, r) > r$, $m_i(p, r) = (1 - \pi)\Gamma_i p$.*

L2.2. *If $x_i(p, r) < r$, $m_i(p, r) = -(1 - \pi)\Lambda_i \equiv m_i^L$*

L2.3. *If $x_i(p, r) = r$,*

- *$m_i(p, r)$ is undefined when $\pi = 1$.*
- *$m_i(p, r) = u_i'(r) - p$ when $\pi = 0$, with $-\Lambda_i \leq m_i \leq \Gamma_i p$*

Note that when $\pi = 0$, the marginal internality is positive in the gain domain, unlike before, while it continues to be negative in the loss domain. The individual under-consumes x to reduce losses in the y domain when $x > r$, and over-consumes x to reduce losses in the x domain when $x < r$. Figure 8 illustrates demand in this model. We plot demand in the gain and loss domain, x^L and x^G according to equations (23) and (24). The main difference in the two-dimensional model is that demand with no reference dependence, pinned down by $p = u'(x)$, now falls *between* observed demand in the gain and loss domains. As $p = u'(x)$ describes welfare-maximizing demand when $\pi = 0$, the vertical distance between this demand curve and observed demand equals the marginal internality when $\pi = 0$, which we know from Lemma 2 is now positive in the gain domain and negative in the loss domain. When $\pi = 1$, observed demand is welfare-maximizing and there is no marginal internality.

4.1.2 Main Social Welfare Effects with Two Dimensions

Proposition 6. ***First-Order Social Welfare Effects in the 2-D Model*** *Starting from any given price and an initial reference point, define groups G, L and R based on $x_i(p, r)$.*

P6.1. *The effect of a small change in the reference point of Δr on social welfare in this model is approximately*

$$\begin{aligned} \Delta W \approx & \Delta r \pi \left\{ E\left[\Gamma_i p \mid i \in G\right] P\left[i \in G\right] - E\left[\Lambda_i \mid i \in L\right] P\left[i \in L\right] \right\} \\ & - \Delta r E\left[p - u_i'(r) \mid i \in R\right] P\left[i \in R\right]. \end{aligned} \tag{26}$$

P6.2. *The effect of a small change in price, Δp, on social welfare in this model is approximately*[30]

$$\begin{aligned} \Delta W \approx & \Delta p \left\{ E\left[(1-\pi)\Gamma_i p \frac{\partial x_i^G}{\partial p} \;\middle|\; i \in G \right] P[i \in G] - E\left[(1-\pi)\Lambda_i \frac{\partial x_i^L}{\partial p} \;\middle|\; i \in L \right] P[i \in L] \right\} \\ & - E[x_i(p_0, r_0)]\Delta p, \end{aligned} \tag{27}$$

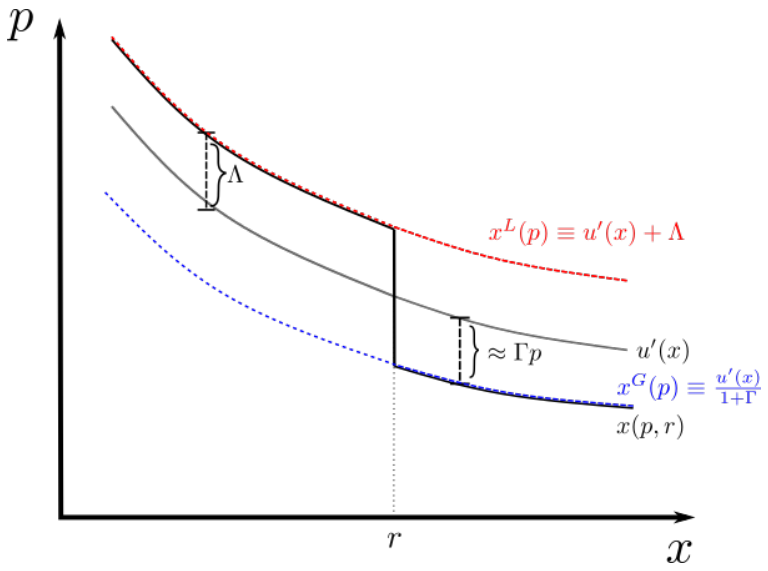
where $\frac{\partial x_i^L}{\partial p}$ and $\frac{\partial x_i^G}{\partial p}$ are evaluated at (p_0, r_0).

[28] We use the same π in both dimensions. Relaxing this assumption is straightforward, but we have difficulty imagining why one might judge that loss aversion is normative in one dimension and not in another.

[29] In this model, the marginal welfare effect of a change in the endowment z_i is no longer unity. In particular when $x < r$, $\partial w / \partial z_i = 1 + \pi\Gamma_i$. As such the marginal internalities derived here might not be money metric. It turns out, however that this does not matter for $\pi \in \{0, 1\}$, because the marginal internality is zero when $\pi = 1$, so that it does not matter if we scale it by $1 + \Gamma$ when $x < r$.

[30] In this approximation we do not allow the reference point over y, s, to change when the price changes. Doing so would introduce a direct welfare effect of a change in that reference point.

FIGURE 8: OBSERVED DEMAND, WELFARE-MAXIMIZING DEMAND, AND MARGINAL INTERNALITIES UNDER 2-DIMENSIONAL REFERENCE DEPENDENCE



Note: This figure depicts observed demand $x(p, r)$, in black, at a given reference point as prices vary. We also plot $u'(x)$, in grey, which coincides with welfare-maximizing demand when $\pi = 1$. The marginal internality when $\pi = 0$ in this model is the vertical distance between observed demand and $u'(x)$, which is depicted in both the gain and loss domains. In the loss domain, observed demand coincides with $x^L(p)$ (red). In the gain domain, observed demand coincides with $x^G(p)$ (blue). In this model, when $\pi = 0$, the marginal internality is therefore positive in the gain domain and negative in the loss domain.

Proposition 6 characterizes the main social welfare effects in the two-dimensional model. The Proposition nests Propositions 3 and 5 above when $\Gamma_i = 0$ for all i, i.e. when there is no loss aversion over y.

Proposition 6.1 considers changes in the reference point. We also depict the welfare effects of changes in the reference point visually in Figure A6. When the reference point is a point on the budget constraint $s = z - pr$, decreasing r must increase s, which is the policy change we consider in equation (26). The main change from previous results is therefore that we observe an additional, positive direct effect of increasing the reference point for individuals consuming in the loss domain over y or the gain domain for x ($i \in G$). Additionally, we should now expect that there are some individuals in the R group for whom $p > u_i'(r)$ and some for whom $p < u_i'(r)$, so the third term in equation (26) is ambiguously signed.

In the two dimensional model, decreasing the reference point along a single dimension – i.e. decreasing r or decreasing s in isolation – would be a robust Pareto improvement, but decreasing the reference point r *along the budget constraint* is not generally a robust social welfare improvement. We can infer from equation (26) that such a decrease in the reference point r would be welfare-improving only if 1) Λ_i is sufficiently large compared to $p\Gamma_i$ on average, 2) there are more individuals in the loss domain compared to the gain domain, and 3) individuals consuming at the reference point tend to be over-consuming rather than under-consuming relative to marginal utility u'.[31]

Proposition 6.2 and Figure A7 consider price changes in the two-dimensional model. For a price change, we continue to observe a behavioral welfare effect valued by the marginal internality m_i, and a direct effect. The main difference in the two-dimensional model is that the marginal internality is positive for $x > r$, which implies that decreasing consumption in response to a change in price decreases welfare for $i \in G$.

We take a few key lessons away from our extension of the simple model from Section 2 to allow for two-

[31] Technically, when $\pi = 0$ only condition 3) matters for welfare. Nevertheless under typical regularity conditions on the distribution of primitives, conditions 1) and 2) will tend to be satisfied when 3) is also satisfied.

dimensional reference dependence. Most importantly, we can see from our results that the application a model like this one to concrete settings will require empirically separating the effect of reference dependence along different dimensions, or further restrictions on the relative strength of reference dependence in a given dimension. One intuitive and influential restriction was proposed by Kőszegi and Rabin (2006), but whether and when this restriction is empirically justified is less clear to us and adopting it would impose substantial structure on welfare. We next consider how one might discipline two-dimensional reference dependence in our empirical application, where we argue that the empirical retirement age distribution can be informative in this regard.

We can also imagine number of further extensions building based on the two-dimensional model. One would be to include the gain domain payoffs we examine in Section 4.2. Another would be to consider more than two dimensions, for instance to study multi-attribute reference dependence and brand choice (Hardie et al., 1993). Finally, we assumed that the reference point must be on the budget constraint because we found this restriction simple and intuitive, but it may not be appropriate in all contexts. In short, a large number of extensions are theoretically feasible, but whether the resulting models will be empirically useful is far less clear.

4.1.3 Disciplining Dimensionality Empirically

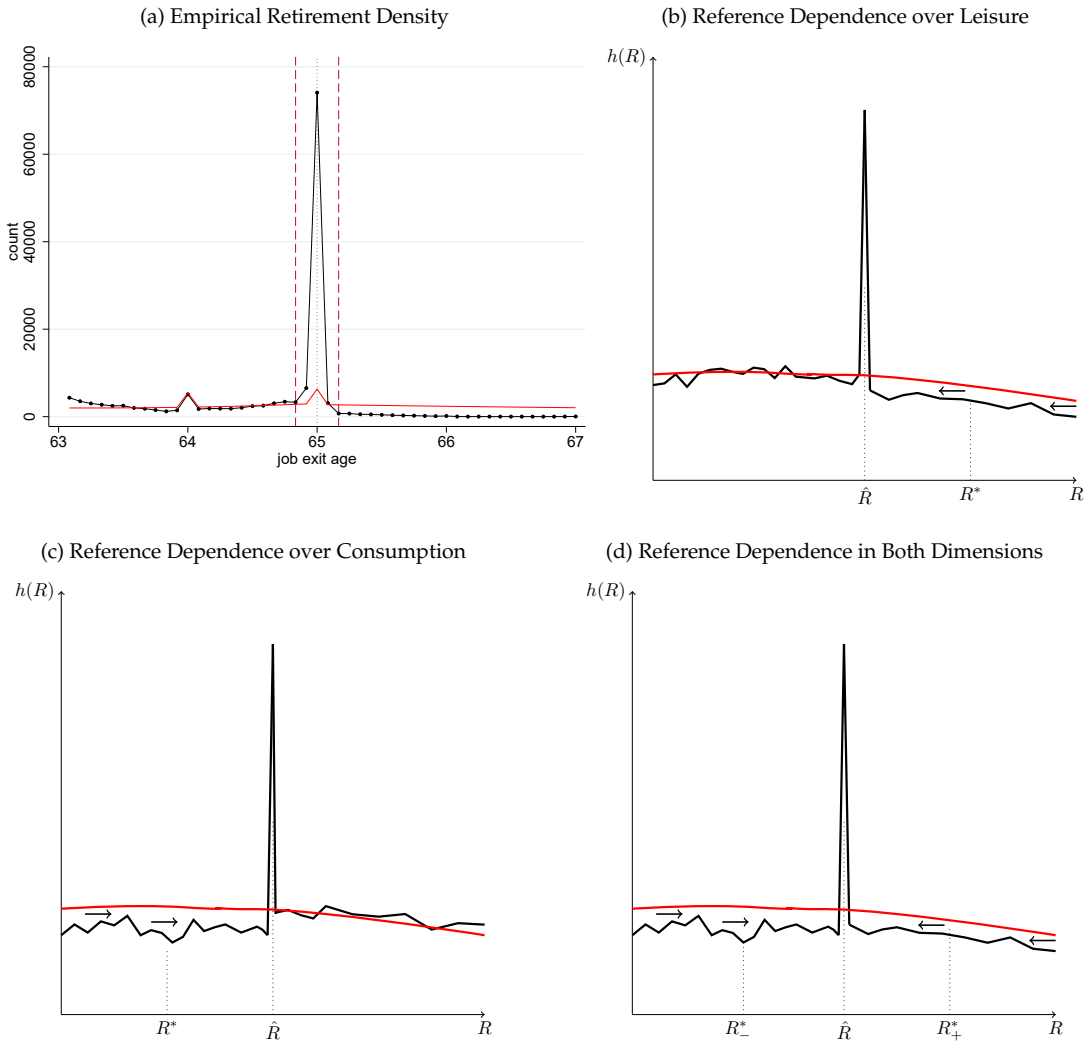
We can infer from the similarities between Figures 1 and 8 that separately identifying reference dependence over different goods will be challenging using within-individual demand curves. However, it turns out that the character of the bunching around the reference point, which comes from between-individual comparisons of demand, can be informative of the relative importance of reference dependence in a given dimension. Specifically, holding p fixed, we can closely examine the distribution of individual choices around the reference point, $x_i(p, r)$.

In our empirical application, besides reference dependence in leisure, there could also be reference dependence in the consumption dimension, for instance because "full" pension benefits become available at the NRA and individuals perceive the associated consumption level as a reference point (Behaghel and Blau 2012). We specify a model with both reference dependence over leisure and over consumption in Appendix E.2. Preferences in the model are identical to the initial specification in equation (17) except that (1) we add a component of utility to capture reference dependent payoffs over consumption (good y as in equation (22)), and (2) we denote the loss aversion parameter in the leisure dimension by Λ_l and in the consumption dimension by Λ_c.

Figure 9 compares the empirical retirement age distribution around the NRA (Panel a) to stylized predicted distributions in three cases: when reference dependence is present over leisure and not consumption (Panel b), over consumption and not leisure (Panel c), and over both consumption and leisure (Panel d). Under reference dependence over leisure, the model from Section 3.2 predicts a density shift towards the NRA from above, as individuals retire earlier due to reference dependence. Under consumption reference dependence, on the other hand, a density shift towards the NRA from below is predicted, since workers postpone retirement in order to increase consumption towards the reference point (see Appendix E.2 for details). Thus, a downward shift of the density should occur above the NRA under reference dependence over leisure, whereas there should be such missing density below the NRA under consumption reference dependence. If reference dependence is present in both dimensions, there may be no visible density shift around the NRA, as it occurs simultaneously on both sides. In Panel (a) of Figure 9, the empirical density exhibits a clearly visible density shift above the NRA, suggesting that bunching at the reference point appears

to be driven by individuals who would be located in the loss domain over leisure otherwise. This pattern suggests that reference dependence in leisure dominates potential reference dependence in consumption (corresponding to the numeraire good in the model), which was our primary motivation for assuming this type of reference dependence in the baseline simulations.

FIGURE 9: BUNCHING AND THE DIMENSIONS OF REFERENCE DEPENDENCE



Note: The figure compares the empirical retirement age distribution around the Normal Retirement Age to the predicted distribution under different models of reference dependence. Panel (a) shows the empirical retirement age distribution among German workers born in 1946 as in Figure 5, with the retirement age normalized to zero at the NRA. Panels (b) to (d) show stylized density graphs, illustrating the predicted shape of the density around statutory ages under different reference dependence models, adapted to the shape of the empirical density. Panel (b) corresponds to reference dependence over leisure as described in Section 3.2, Panel (c) corresponds to reference dependence over consumption as described in Appendix E.2, and Panel (d) corresponds to reference dependence in both dimensions.

However, even though reference dependence over leisure appears to dominate empirically, this does not necessarily exclude *any* degree of reference dependence over consumption. The main empirical challenge is that reference dependence parameters in both dimensions cannot be separately identified based on observed bunching at the NRA alone. A given amount of bunching could be rationalized by reference dependence over leisure, reference dependence over consumption or a combination of the two. We propose two approaches to make further progress on two-dimensional reference dependence in our empirical ap-

plication. First, we can calculate a range of combinations of Λ_l and Λ_c consistent with observed bunching. We obtain these combinations by gradually moving the assumed share of bunching from the left between 0 and 50%. Panel (a) of Appendix Figure A3 shows estimated parameter combinations consistent with the observed amount of excess mass. The negative slope of the relationship illustrates the intuition that the two types of reference dependence are substitutes in terms of rationalizing observed excess mass. The higher the assumed share of bunching from the left, the larger the implied Λ_c, but the smaller the implied Λ_l. The labeled dots mark parameter combinations corresponding to selected left bunching shares.

The possible range of Λ_c shown in Panel (a) of the figure is still wide, and this will create ambiguity in welfare. Thus, a second approach is to narrow down Λ_c to obtain a preferred estimate, using the information contained in the observed retirement age distribution around the NRA. Appendix E.2.2 provides more details of this procedure. Intuitively, the counterfactual density – which would prevail in the absence of any reference dependence – is assumed to be continuous around the threshold, and the relative number of bunchers from the left and from the right are inferred from the vertical difference between the counterfactual and the actually observed density on both sides of the threshold. In general, this approach requires a stronger assumption about the true relative density shifts being reasonably well approximated by locally observed relative shifts. Panel (b) of Appendix Figure A3 illustrates the procedure and shows that the implied density shift is much more substantial above than below the NRA, as we should expect given Panel (a) of Figure 9, implying a point estimate of the left bunching share of 13.3%. This magnitude of relative bunching implies a consumption reference dependence parameter of $\Lambda_c \approx 0.67$ and a leisure reference dependence parameter of $\Lambda_l \approx 0.46$.

4.1.4 Policy Simulations with Two-Dimensional Reference Dependence

In line with the two approaches laid out above, we present two sets of results on the welfare effects of pension reforms under two-dimensional reference dependence. First, Table 2 shows simulated welfare effects of the same policies considered in Table 1 under our preferred two-dimensional reference dependence parameter estimates. Note that the NRA reform can now be interpreted as decreasing the reference point over leisure, while simultaneously increasing the reference point over consumption. The DRC reform still corresponds to a price change in the loss domain over leisure.

Fiscal effects of the two reforms remain similar to the baseline simulations: The NRA increase has strong positive fiscal effects, whereas the DRC increase worsens the fiscal balance. More generally, the effects of the DRC increase are similar to the baseline simulations. This occurs because the effects of the DRC on retirement behavior are concentrated among workers at or above the NRA, where consumption reference does not affect utility or behavior. The estimated welfare effects of increasing the DRC only change because the value of Λ_l changed slightly from Table 1 to Table 2.

The effects of the NRA reform differ substantially from the baseline simulation. As before, behavioral and fiscal effects are the main determinants of welfare when reference dependence is a bias ($\pi = 0$) and direct effects are the main determinant when reference dependence is judged to be rational ($\pi = 1$).[32] In the $\pi = 0$ case, some of the behavioral effect now comes from workers who are retiring *too late* out of loss aversion over consumpion, and increasing the NRA exacerbates this internality (even as it continues to

[32]When $\pi = 1$, all behavioral effects net out against the fiscal effects due to the envelope theorem once again, and the direct effects of changes in both reference points over consumption and leisure are the main determinants of welfare. This cancellation works slightly differently in the two-dimensional model. Workers now receive some additional marginal utility from consumption when retiring later via reference dependent payoffs. This particular effect is relatively small, since retirement behavior changes little below the NRA under our preferred parameter estimates.

mitigate the internality from those retiring after the NRA who retire too early). As a result, the behavioral effect under $\pi = 0$, i.e. the net effect of increased worker consumption and larger disutility form work, is smaller than in Table 1 and becomes slightly negative. The total welfare effect under $\pi = 0$ is thereby somewhat reduced.

When $\pi = 1$ the main difference between Table 2 and our baseline simulations comes from workers retiring before the NRA, i.e. in the gain domain for leisure (the G group in the theory). Note that equation (26) implies that we should have a new, direct welfare effect for this group in the two-dimensional model, which is opposite in sign to the direct effects in the one-dimensional model. Intuitively, workers retiring after the NRA face a lower reference point for lifetime leisure, increasing their utility in a similar fashion to Table 1, but workers retiring before the NRA face a higher reference point for lifetime consumption, which reduces their welfare. Since many individuals retire before the NRA, this direct effect substantially reduces the total welfare effect under $\pi = 1$. Yet, the welfare effects of increasing the NRA remain positive both under $\pi = 0$ and $\pi = 1$ under our preferred parameter estimates.

TABLE 2: WELFARE EFFECTS OF PENSION REFORMS UNDER TWO-DIMENSIONAL REFERENCE DEPENDENCE

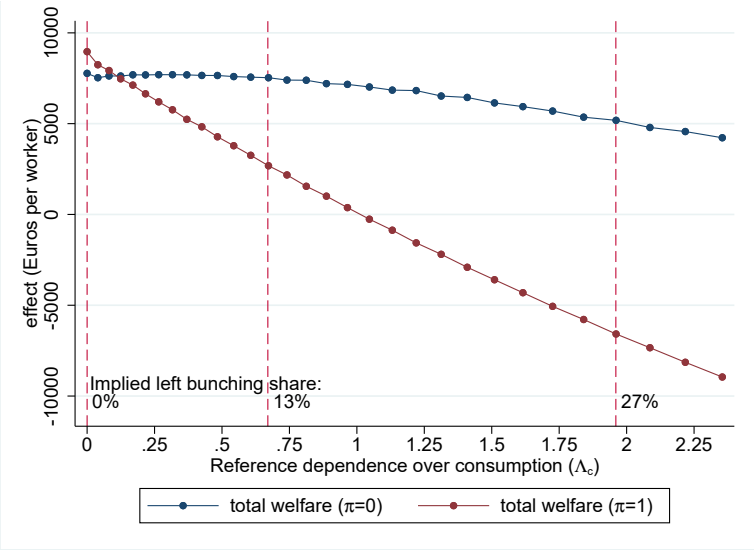
	(1) Policy 1: Normal Retirement Age to 66	(2) Policy 2: Delayed Retirement Credit to 10.4%
Contributions collected	+2,879	+2,253
Benefits paid	+4,807	-4,062
Net fiscal effect	+7,686	-1,809
Worker consumption	+5,298	+12,017
Disutility from work	-5,441	-2,220
Worker welfare ($\pi = 0$)	-143	+9,796
Ref dep disutility from work	-8,997	-8,504
Utility from retirement ref point	+10,207	0
Ref dep utility from consumption	+718	0
Disutility from consumption ref point	-6,784	0
Worker welfare ($\pi = 1$)	-5,000	+1,293
Total welfare ($\pi = 0$)	+7,543	+7,987
Total welfare ($\pi = 1$)	+2,686	-517

Note: The table shows results from simulations of two pension reforms under two-dimensional reference dependence. Both reforms yield the same effect on the average actual retirement age (+5.6 months). Simulations are conducted for birth cohort 1946. All effects are calculated among workers retiring at age 63 and above, and are in Euros per worker, in terms of net present value at age 65. The signs the effects correspond to influence on welfare. Total welfare is the sum of net fiscal effect and change in worker welfare.

We also calculate welfare effects of the NRA reform for a wider range of values of Λ_c in order to account for some uncertainty which remains in estimating the left-bunching share in Figure A3. Figure 10 shows welfare effects under $\pi = 0$ and $\pi = 1$ for Λ_c between zero and around 2. The dashed vertical lines mark the cases where the left bunching share is zero (corresponding to the baseline simulation), 13% (our preferred estimate) and 27% (double our preferred estimate). The effects of the NRA reform decrease with Λ_c under

both $\pi = 0$ and $\pi = 1$. Under $\pi = 0$, the welfare effect remains positive over the range in the graph (at even more extreme values we would find a negative effect), but under $\pi = 1$, the effect turns negative around $\Lambda_c = 1$, corresponding to a left bunching share of around 18%. The faster decline of the welfare effect with Λ_c under $\pi = 1$ is due to the negative direct effects of increasing the NRA on pre-NRA retirees.[33]

FIGURE 10: WELFARE EFFECTS OF INCREASING THE NRA BY STRENGTH OF CONSUMPTION REFERENCE DEPENDENCE



Note: The figure shows the total welfare effects of increasing the NRA from 65 to 66 by the strength of consumption reference dependence Λ_c. Simulations are conducted for birth cohort 1946. The effects are calculated among workers retiring at age 63 and above, and are in Euros per worker, in terms of net present value at age 65. The dashed vertical lines denote selected values of Λ_c, corresponding to implied left bunching shares of 0 (no consumption reference dependence), 13% (our preferred estimate, on which the results in Table 2 are based), and 27% (twice our preferred estimate).

Overall, these results highlight an important caveat to our findings from Section 3 that increasing the NRA would robustly increase welfare. Namely, the welfare effects of a change in the NRA under $\pi = 1$ can turn negative under strong consumption reference dependence. However, we note that the welfare effects of increasing the NRA remain positive under our preferred estimates of two-dimensional reference dependence parameters regardless of the value of π, and the positive welfare effects under $\pi = 0$ are robust to a wide range of reference dependence parameters.

4.2 Alternate Formulation of Reference Dependence

In this section, we return to one-dimensional reference dependence and consider the alternate formulation of reference dependence for $v(x|r)$, from equation (3), which takes the formulation of Tversky and Kahneman (1991) for reference dependence over riskless choice at face value. As discussed above, the main difference between this formulation and the one we use above is the presence of the η_i parameter in the formulation we consider here.[34]

[33] We suppose that all workers retiring at 63 or later use the NRA as a reference point in the two-dimensional simulations. One potential caveat is that some pre-NRA retirees might not actually use the NRA as a reference point for consumption and leisure, but some other reference point like the Early Retirement Age. In this case the behavior and welfare of pre-NRA retirees would not be as much affected by a change in the NRA as we found in Table 2, and the overall welfare effects would more closely resemble those from Table 1. Conversely, the difference in welfare effects would be exacerbated if many workers retiring far below the NRA use it as a reference point.

[34] A similar friction is sometimes studied with or without reliance on loss aversion in the literature on the relative income hypothesis, see e.g. Clark et al. (2017). Eliminating loss aversion (setting Λ equal to 0) is a simple extension to our work in this Section.

Setup. The η_i parameter makes the individual consume more x by virtue of comparing x to the reference point, in both the gain and the loss domain. We think of η_i as governing the importance of reference dependence itself, while λ_i governs the strength of loss aversion. We first note that we can re-formulate the reference dependence in equation (3) as follows, to make it slightly more comparable to our earlier model:

$$\tilde{U}_i(x,y) = \tilde{u}_i(x) + y + \tilde{v}_i(x|r), \tag{28}$$

$$\tilde{v}_i(x|r) = \begin{cases} \eta_i(x-r), & x > r \\ [\eta_i + \Lambda_i](x-r), & x < r, \end{cases} \tag{29}$$

The formulation used here (and in equation 3) is behaviorally equivalent to the model from section 2, with $\tilde{u}_i(x) = u_i(x) - \eta_i x$ and $\Lambda_i = \eta_i(\lambda_i - 1)$ (see Appendix C for a full proof). We can therefore compare the question of how, holding observed behavior fixed, adopting this formulation for welfare instead of the one used in Section 2 affects our normative results.

We consider the question of whether each friction, reference dependence or loss aversion, separately, reflects a behavioral bias or a normative preference, using the parameters $\pi^{RD} \in \{0,1\}$ and $\pi^{LA} \in \{0,1\}$. We therefore use the following specification for welfare:

$$\tilde{U}_i^*(x,y) = \tilde{u}_i(x) + y + \tilde{v}_i^*(x|r), \tag{30}$$

$$\tilde{v}_i^*(x|r) = \begin{cases} \pi^{RD}\eta_i(x-r), & x > r \\ [\pi^{RD}\eta_i + \pi^{LA}\Lambda_i](x-r), & x < r. \end{cases} \tag{31}$$

We do not consider the case where $\pi^{RD} = 0$ and $\pi^{LA} = 1$, because this judgment would imply that reference dependence over gains and losses is a bias, but loss aversion is normative, which does not seem sensible.

Finally, we denote indirect utility by $\tilde{w}_i(p,r)$, and utilitarian social welfare by $\tilde{W}(p,r)$, defined as above.

Results. We next show how adopting this formulation modifies the marginal internality and the first-order social welfare effects of a change in price and the reference point.

Lemma 3. ***Marginal Internalities Under the Tversky-Kahneman (1991) Form.*** *Let $\tilde{m}_i$ be the derivative of $\tilde{U}_i^*(x, z_i - px)$ with respect to x, evaluated at $x_i(p,r)$.*

L3.1. *If $x_i(p,r) > r$, $\tilde{m}_i = -(1-\pi^{RD})\eta_i \equiv \tilde{m}_i^G$.*

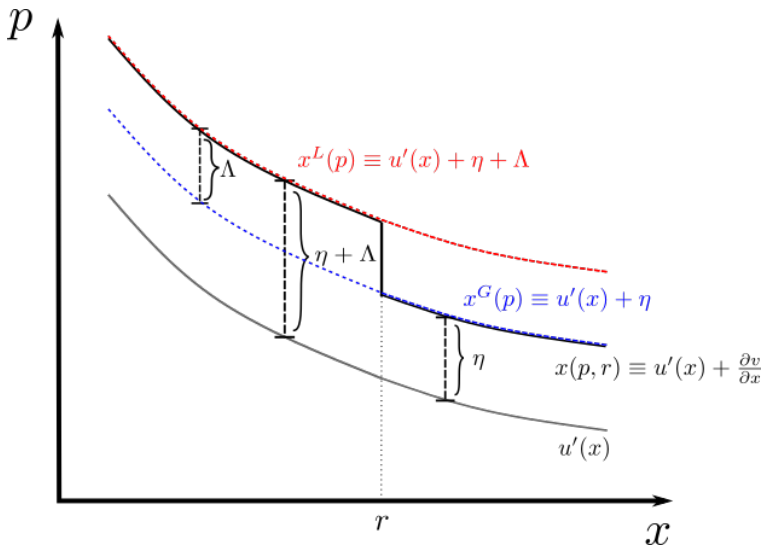
L3.2. *If $x_i(p,r) < r$, $\tilde{m}_i = -(1-\pi^{RD})\eta_i - (1-\pi^{LA})\Lambda_i$.*

L3.3. *If $x_i(p,r) = r$,*

- *$\tilde{m}_i$ is undefined when $\pi^{RD} = \pi^{LA} = 1$.*
- *Otherwise, $\tilde{m}_i = \tilde{u}_i'(r) + \pi^{RD}\eta_i - p$.*
- *Moreover, in the cases where $\tilde{m}_i$ is defined, $\tilde{m}_i^L \leq \tilde{m}_i \leq \tilde{m}_i^G$.*

Comparing Lemma 3 to the analogous Lemma 1 helps us understand how adding the friction embodied by η_i changes the model. When $\pi^{RD} = 1$, these marginal internalities are all exactly the same as in our earlier analysis except that π is now denoted π^{LA} – recall that the models are behaviorally isomorphic if $\tilde{u}_i(x) + \eta_i = u_i(x)$. When $\pi^{RD} = 0$, however, reference dependence is generating additional distortions to the choice of x, leading to more over-consumption than in our earlier formulation. The revised marginal

FIGURE 11: OBSERVED DEMAND, WELFARE-MAXIMIZING DEMAND, AND MARGINAL INTERNALITIES UNDER THE TVERSKY-KAHNEMAN (1991) FORMULATION



Note: This figure depicts observed demand $x(p, r)$, in black, at a given reference point as prices vary. We also plot $u'(x)$, in grey, which coincides with welfare-maximizing demand when $\pi^{RD} = \pi^{LA} = 1$. The marginal internality when $\pi = 0$ in this model is the vertical distance between observed demand and $u'(x)$, which is depicted in both the gain and loss domains. In the loss domain, observed demand coincides with $x^L(p)$ (red). In the gain domain, observed demand coincides with $x^G(p)$ (blue), which is also welfare-maximizing demand when $\pi^{RD} = 1$ and $\pi^{LA} = 0$. In this case there is no marginal internality in the gain domain, and the vertical distance between observed demand and $x^G(p)$ is the marginal internality in the loss domain. When $\pi^{RD} = \pi^{LA} = 1$, observed demand is welfare-maximizing and there are no internalities.

internalities and welfare-maximizing demand are plotted in Figure 11. Welfare maximizing demand corresponds to the line depicting $p = u_i'(x)$, when $\pi^{RD} = 0$. When $\pi^{RD} = 1$, welfare-maximizing demand corresponds to either observed demand or $x_i^G(p)$, depending on whether or not $\pi^{LA} = 1$.

Proposition 7. ***Modified First-Order Welfare Effects with Gain Domain Payoffs*** *Starting from an iniial price and reference point, define G, R, and L groups. Let ΔW denote the change in social welfare from a reform that we obtain adopting the original formulation, as derived in Propositions 3 and 5, with $\pi = \pi^{LA}$.*

P7.1. *The first-order social welfare effect of a change in the reference point is approximated by*

$$\Delta\tilde{W} \approx \Delta W - E[\pi^{RD}\eta_i \Delta r] - E[(1-\pi^{RD})\eta_i \Delta r \mid i \in R]P[i \in R]. \tag{32}$$

P7.2. *The first-order social welfare effect of a change in price is approximated by*

$$\Delta\tilde{W} \approx \Delta W - E[(1-\pi^{RD})\eta_i \Delta x_i], \tag{33}$$

Proposition 7 shows that introducing the potential additional distortion to demand represented by the η_i parameter in this formulation of reference dependence has two potential effects on our welfare calculations. We illustrate welfare effect for change in reference point for the five cases, as in Figure 2, in Appendix Figure A4. Likewise, Appendix Figure A5 shows the modifications to Figure 3.

First, if we judge that this friction is normative, i.e. $\pi^{RD} = 1$, then the direct effect of a change in the reference point becomes larger and is present even in the gain domain. Apart from this extra direct effect, setting $\pi^{RD} = 1$ is equivalent to adopting our original formulation. Second, if we judge that this

new friction is *not* normative, then the internalities in the model become larger negative internalities, which means that any changes in demand caused by a change in prices or reference points will have larger first-order welfare effects. Conditional on these two effects, the role of π^{LA} in essentially identical to the earlier question about π in our original formulation, which makes sense because both of these represent the same underlying normative judgment about loss aversion. Note that both of these channels tend to strengthen the result from Proposition 1 that lowering the reference point leads to an improvement in individual and social welfare regardless of normative judgments.

That this formulation of reference dependent payoffs and the original one we considered are behaviorally indistinguishable but carry differing implications for welfare raises interesting questions for future empirical research. In general, one might obtain specialized choice data, beyond observed demand at one or more reference points, to distinguish between the models. For example, the two models carry different predictions for what would happen if we can observe a situation where the reference dependence were eliminated by some intervention (see e.g. Sokol-Hessner et al., 2009). Alternatively, we might solicit individuals' willingness to pay to change the reference point.[35] The different formulations of reference dependence we consider make detailed predictions about how individuals would respond to either of these interventions. Additionally, with either of these novel designs, one could potentially identify the parameter η.

4.3 Goals

The only potential behavioral friction we considered so far is reference dependence. Part of the literature on reference dependence considers the possibility that reference points may serve as goals, in order to overcome another bias, such as present bias (Koch and Nafziger, 2011, 2016; Loewenstein and O'Donoghue, 2006). Such a model contains two new elements compared to what we have done so far: an additional behavioral bias, and the possibility that individuals set their own reference points with some degree of sophistication about those biases.[36]

To extend our theory along these lines, we first suppose that decision utility governing the choice of x is the same as in our original model, i.e. equations (1) and (2). Because reference dependence induces individuals to consume more of a given good in order to avoid losses, using reference points to overcome biases is useful when biases lead to under-consumption of some good ordinarily.[37] As such, the new component of our theory here is an additional positive internality from consuming good x, which we model in a reduced form fashion as follows:

$$U_i^*(x, y) = u_i(x) + y + b_i(x) + \pi v_i(x|r). \tag{34}$$

The new bias term $b_i(x)$ captures the extent to which the individual under-values x when making a decision. We assume $b_i'(x) > 0$, i.e. a positive marginal internality from $b_i(x)$, and $b_i''(x) \leq 0$, which ensures an interior solution for the welfare-maximizing choice of x. As a benchmark, we assume individuals are *fully sophisticated* about their biases. That is, the individual is perfectly aware of the bias $b(x)$ when they set a reference point. We discuss what happens if we relax this assumption below.

We next make a simplifying assumption, which is that the bias $b_i(x)$ is not so large that it cannot be

[35] The type of meta-willingness-to-pay design we have in mind resembles that of Allcott and Kessler (2019), who studied it for a different intervention.

[36] This notion of sophistication is closely related to "planner-doer" models (Fudenberg and Levine, 2006). Intuitively, the planner sets r with knowledge of what the doer will choose, and then the doer makes choices as in our basic model.

[37] If the individual could set reference points or goals for other goods, then a tendency to over-consume one good (e.g. leisure due to present bias and the up-front costs of work) could potentially be overcome by setting a goal/reference point for the residual good (e.g. a reference point over consumption, or an earnings target). We abstract away from this case here, but the approach we take here can be straightforwardly used in this type of situation too.

overcome by setting a reference point. Formally, let $x_i^* = \arg\max u_i(x) + b_i(x) + z - px$. We assume that $b_i'(x_i^*) \leq \Lambda_i$.[38]

In this setting, it is straightforward to show that a fully sophisticated individual would set an optimal reference point at $r_i = x_i^*$. Because $u'(x^*) + b'(x^*) = p$ and $b_i'(x_i^*) \leq \Lambda_i$, we know that the individual subsequently chooses x at the reference point, i.e. in the R case from above. Appendix Figure A8 illustrates the choice of r in this case. In other words, sophisticated individuals set a goal r to overcome their biases and they meet this goal exactly.

How does this result change our earlier thinking? Unsurprisingly, letting individuals choose r optimally implies that policies that aim to reduce r will no longer improve welfare. More specifically, inducing a marginal increases or decrease in r in this case has no first-order welfare effects due to the envelope theorem, and a second-order loss due to the optimality of r_i. Next, to understand price changes, note that the individual always ends up in the R case, where a marginal change in price has no effect on behavior, only a first-order negative direct effect on welfare. Another interesting implication of this line of reasoning is that if individuals self-regulate their own biases by setting goals, there is no need to correct the bias in $b(x)$ by setting a corrective tax. Note also that all of the above obtains regardless of π: in this situation, the individual never incurs a loss, so whether loss aversion is normative becomes irrelevant.

From the logic we have now laid out, it is straightforward to infer what would happen if we relaxed the assumption of full sophistication about biases. An individual who underestimates their bias, or who neglects it entirely, would set a goal r that is too low. In this case, inducing the individual to set a higher reference point would have a first-order, positive impact on welfare. Similarly, an individual who overestimates their future bias would set an over-optimistic goal r, and could be made better off by setting a reduced reference point, especially when $\pi = 1$ and the losses incurred by failing to meet one's goal generate a negative payoff. Obviously, people sometimes do fail to meet their goals, incurring potentially painful losses, and sometimes they significantly exceed them (Loewenstein and O'Donoghue, 2006). Whether policymakers would have enough information to correct these types of mistakes in the choice of goal reference points is less clear (Glaeser, 2006), but estimates of individuals perceptions of their own biases compared to estimates of their actual biases could inform this question.

This model is just one intuitive model in which individuals deliberately choose their own reference points. Thinking more broadly, the model we laid out in Section 2 implies that if given the ability to manipulate their reference points, individuals would choose low levels of r, low enough to avoid ever incurring losses. Insofar as individuals appear to deliberately set higher reference points, by revealed preference they must have some concern that is not present in the model in Section 2. Once we account for these additional concerns, provided that individuals choose their reference points fully optimally, the envelope theorem bites, implying that changing reference points will not have first-order welfare effects. Moreover, all of this will be true for welfare in other models that endow individuals with other motives to choose high reference points, such as models of anticipatory utility (Sarver, 2012).

[38] Without this assumption, the individual would choose a reference point in order to induce the highest consumption of x possible without incurring a loss. In other words they would set r such that $u_i'(r) + \Lambda_i = p$, so that $r{=}x_i^L(p)$. Another related complication is the question of whether the individual, when setting r, cares about the reference dependent payoff in $v_i(x|r)$. Under the simplifying assumption we make on the size of the bias here, this turns out not to matter for the choice of r: the individual would choose the same r if they only sought to maximize $u_i(x) + b_i(x) + y$ rather than $u_i(x) + b_i(x) + y + v_i(x|r)$. If our simplifying assumption does not obtain, so $b_i'(x_i^*) > \Lambda$, the individual choosing r to maximize $u_i(x) + b_i(x) + y$ would be indifferent between any r such that they made a choice of x in the loss domain.

4.4 Further Theoretical Extensions

In this section, we briefly discuss three further complications that could be added to our model, which may be useful in some applied settings. We defer fully characterizing welfare in these models to future work. Our subjective view is that these complications are best explored in applied contexts where empirical evidence and features of the environment can discipline the structure one imposes on the model.

Diminishing Sensitivity. As discussed above, we have so far ignored diminishing sensitivity, which, compared to equation (3), would require that $v'' < 0$ for $x > r$ and $v'' > 0$ for $x < r$.

Adding diminishing sensitivity is straightforward, but there is limited evidence of diminishing sensitivity for decision-making under certainty. Moreover, with diminishing sensitivity, it becomes difficult to empirically distinguish curvature of intrinsic utility over x, which we denoted $u''(x)$, and curvature over reference dependent utility $v''(x)$ in the gain domain.

Allowing for diminishing sensitivity is unlikely to change much of the qualitative intuition above, about the direct and behavioral effects of a change in the price or the reference point. For example, the presence and sign of the marginal internality will be unaffected, and the result that lower reference points tend to improve welfare will obtain under diminishing sensitivity. However, the presence of v'' would imply that the various demand curves in Figures 1 through 3 are no longer parallel, which implies that the size of various welfare effects will be quantitatively different. Insofar as we only consider choices nearby the reference point, such differences are negligible as in this region the piece-wise linear formulation we use is approximately accurate.

Risk and Uncertainty. We have here considered the case of reference dependence under certainty. Reference dependence under uncertainty is the subject of a rich theoretical and experimental literature.

There are two challenges in adapting the type of welfare analysis we consider here to a model with uncertainty. First, the question of exactly how to specify welfare in such a model with uncertainty can be tricky. Many applications of welfare economics under uncertainty use certainty equivalence welfare metrics (Einav et al., 2010). With reference dependence, at least under $\pi = 1$, state dependence in the model makes certainty equivalence a poor welfare metric, so a generalization of equivalent variation that allows for uncertainty and state dependence would be more appropriate.

Second, as discussed in Section 2.2.1, there is much more of a debate on the origins of reference points for the stochastic case. The mixed empirical evidence on the origins of reference points and the wide variety of models one might use makes it more difficult to choose a formulation of reference-dependent preferences and it may not be clear which policy changes might induce a shift in reference points. Researchers often pose that reference points are based on expectations (e.g. Kőszegi and Rabin (2006)). In this case, changing beliefs would change the reference point, but changing beliefs can also influence welfare and behavior in other ways, which complicates the analysis.

Narrow Bracketing. An important component of prospect theory as laid out by Kahneman and Tversky (1979) is the bracketing of payoffs, which is closely related to the concept of mental accounting (Thaler, 1985). What we have considered above is essentially "broad bracketing." The agents evaluate all purchases of good x, with a reference point over total x consumed. In the labor-leisure model from our application, there is a single reference point for lifetime leisure.

In other contexts, individuals seem to adopt *narrow bracketing*, where what we would ordinarily think of as the same option is partitioned into component parts and evaluated separately, with a reference point for each. For example, narrow bracketing of assets in a financial portfolio, through which individuals receive a jolt of reference-dependent utility each time they sell a specific stock, appears to be an important driver of the disposition to hold stocks that depreciate and sell those that appreciate (Barberis and Xiong, 2012; Imas, 2016). Narrow bracketing implies individuals receive a payoff based on whether a specific stock has gained or lost value (a zero reference point) instead of receiving payoffs based only on the value of their entire portfolio. Modelling welfare in the presence of narrow bracketing would require a normative judgment over not only whether reference-dependent payoffs deserve normative weight, but also the question of whether an individual should be bracketing at all (see e.g. Koch and Nafziger (2016)).

5 Conclusion

In this paper, we provide a first attempt at characterizing the welfare economics of reference dependence. Our most robust finding is that lowering reference points tends to improve welfare, even though different views of welfare provide different answers as to why this is the case. Other welfare effects, such as the welfare effects of prices or taxes, are more inherently ambiguous because evaluating them requires taking a stand on whether some individuals are over-consuming out of loss aversion.

Our empirical application highlights the real-world policy relevance of these results. Reference-dependent behavior has been documented in a number of empirical contexts, raising important questions of how the welfare consequences of different policies are affected. In the context of retirement, we show that increasing the Normal Retirement Age is welfare-improving when it serves as a reference point in the labor supply/leisure dimension. The welfare effects of subsidies for later retirement, on the other hand, are more ambiguous and depend on normative judgments regarding reference dependence.

Taking our results at face value would suggest that lowering reference points to extreme degrees, or in the empirical application, raising the Normal Retirement Age to an extremely high level, may be optimal. This notion should be taken with a grain of salt. In practice, we can imagine several ways in which this result may be disciplined. First, it may be that shifting statutory retirement ages, or other policies influencing reference points, to extreme levels would cause individuals to stop using the policy as a reference point. Relatedly, individuals may be insensitive to extreme reference points due to some form of self-regulation or bounded rationality, and the costs of self-regulation might themselves carry normative importance (Goldin and Reck, 2018). Additionally, in some settings decision-makers may deliberately exercise control over their own reference points due to concerns not present in the simplest model we considered; we showed that this can lead them to prefer higher reference points. Thus, we believe our results are useful in applications considering policies that shift reference points locally, in the absence of countervailing concerns like self-control problems or anticipation. Understanding larger, global changes requires further research, as does analysis of whether and when individuals deliberately control their own reference points. Finally, reference points set by public policy are often linked to other policies in practice, and it may not always be realistic to influence reference points in isolation from other factors. For instance, pension reforms that increase the NRA often feature large pension benefit cuts due to an institutional linkage of benefit levels to the NRA.

More broadly, our results demonstrate that embracing normative ambiguity can provide a way forward for difficult problems at the core of behavioral economics. The question of whether behavioral phenomena arise due to behavioral biases or non-standard normative preferences has complicated applications of

behavioral economics to welfare in many domains. Nevertheless, policy interest in behavioral economics has grown extremely rapidly in recent years, and, as in other policy settings, careful analysis of the welfare effect of policy changes can inform and discipline the policy debate. Embracing normative ambiguity can illuminate the path forward because it lets us separate questions that can be empirically analyzed, such as the influence of a change in reference point or prices on behavior, from normative judgments. This distinction between normative judgments and positive empirical questions has a long tradition in public economics when it comes to questions of equity and efficiency (Mirrlees, 1971; Saez, 2001).

Finally, our work demonstrates that the wide variety of models of reference dependence can pose a significant challenge for welfare analysis. We take a few lessons away from confronting this challenge. First, testing some new hypotheses could help distinguish between alternative formulations of reference dependence. These may involve how reducing or eliminating reference dependence affects behavior or the willingness to pay to change a reference point. Testing these would be would be informative for welfare and interesting from a positive perspective. Second, our work on model extensions leaves room for much future research, especially on reference dependence under uncertainty and the dynamic evolution of reference points.

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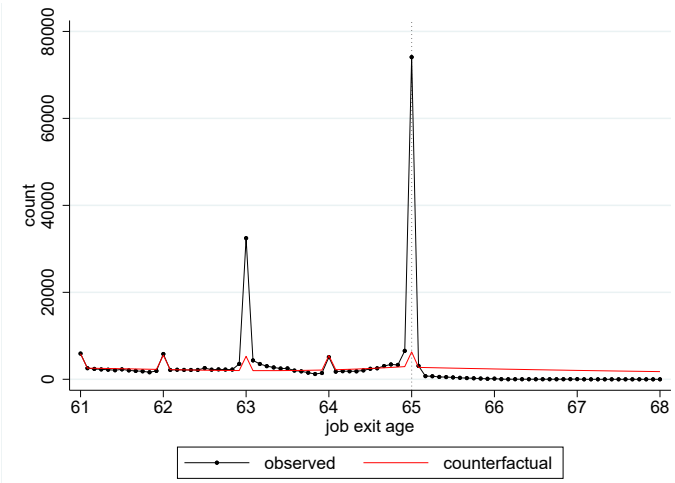
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Appendix (For Online Publication)

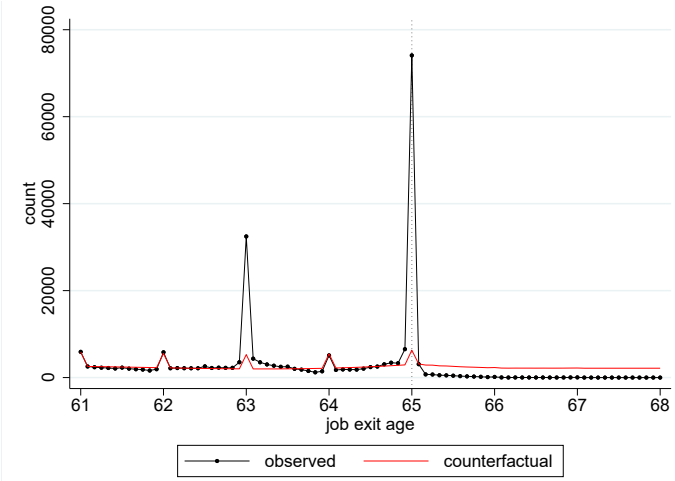
A Additional Figures and Tables

FIGURE A1: COUNTERFACTUAL RETIREMENT AGE DISTRIBUTION

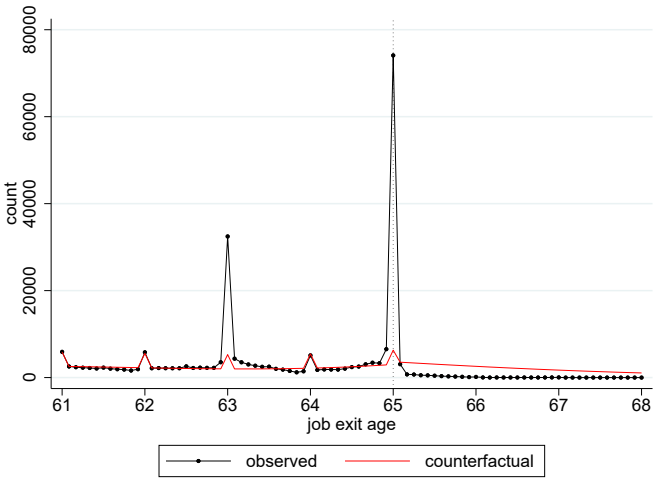
(a) Baseline: Pareto Tail



(b) Alternative 1: Uniform Tail



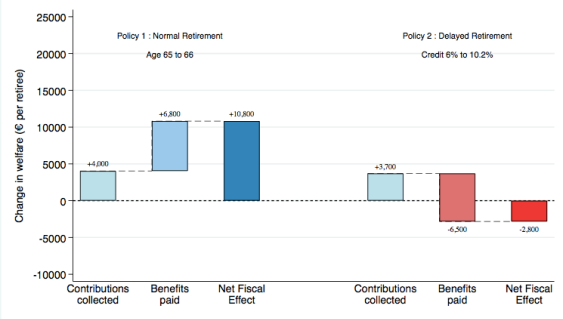
(c) Alternative 2: Lognormal Tail



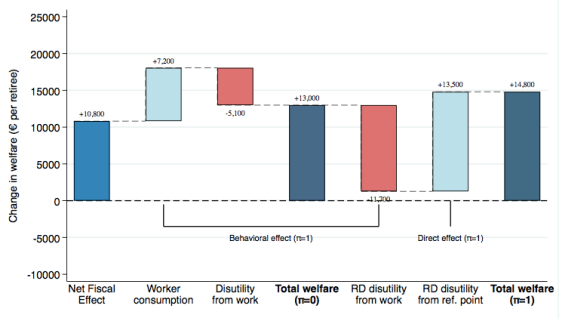
Note: The figures shows counterfactual retirement distributions under different assumptions about the shape of the upper tail of the distribution. In all panels, the counterfactual distribution up until the NRA (age 65) is obtained by fitting a seventh-order polynomial to the observed retirement age distribution, allowing for round-age effects. Panel (a) shows the baseline distribution we use in the simulations, where the upper tail is given by a fitted Pareto distribution. Panels (b) and (c) show alternative counterfactual distributions, where the upper tail is is given by a uniform and lognormal distribution, respectively. Appendix Table A2 shows that our simulation results are robust to the shape of the upper tail of the counterfactual distribution.

FIGURE A2: WELFARE EFFECTS OF PENSION REFORMS

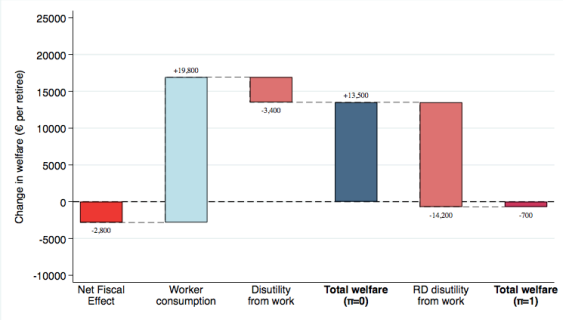
(a) Fiscal Effects



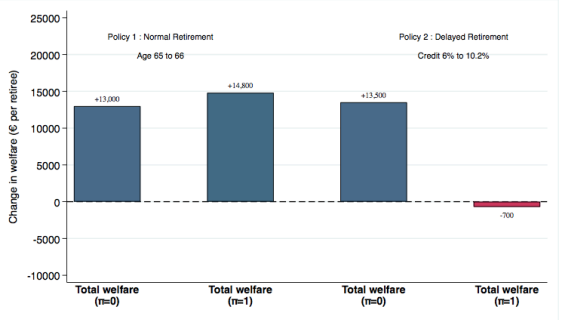
(b) Welfare Effects of NRA Increase



(c) Welfare Effects of DRC Increase



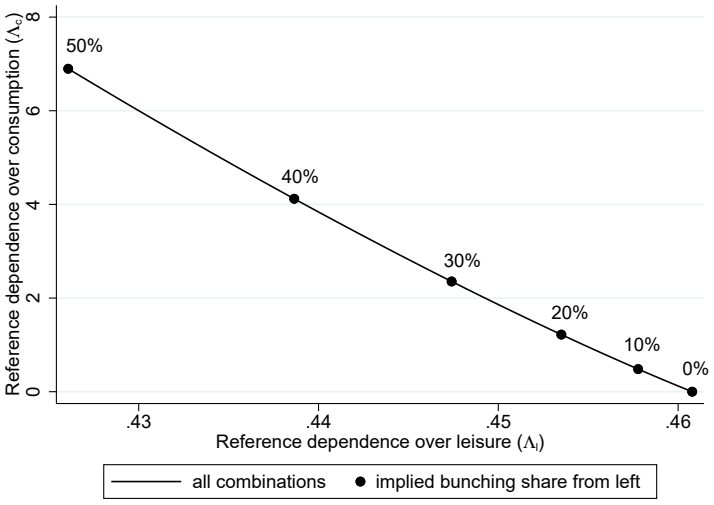
(d) Comparing Welfare Effects



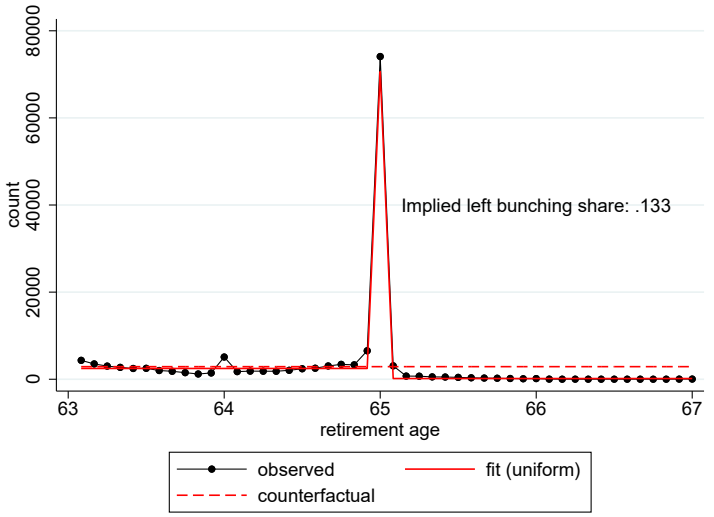
Note: This figure illustrates the aggregation of components of the welfare effects of pension reforms shown in Table 1. All effects are calculated among workers retiring at age 65 and above, and are in Euros per worker, in terms of net present value at age 65.

FIGURE A3: TWO-DIMENSIONAL REFERENCE DEPENDENCE

(a) Bunching at the NRA Identifies Combinations of Λ_l, Λ_c

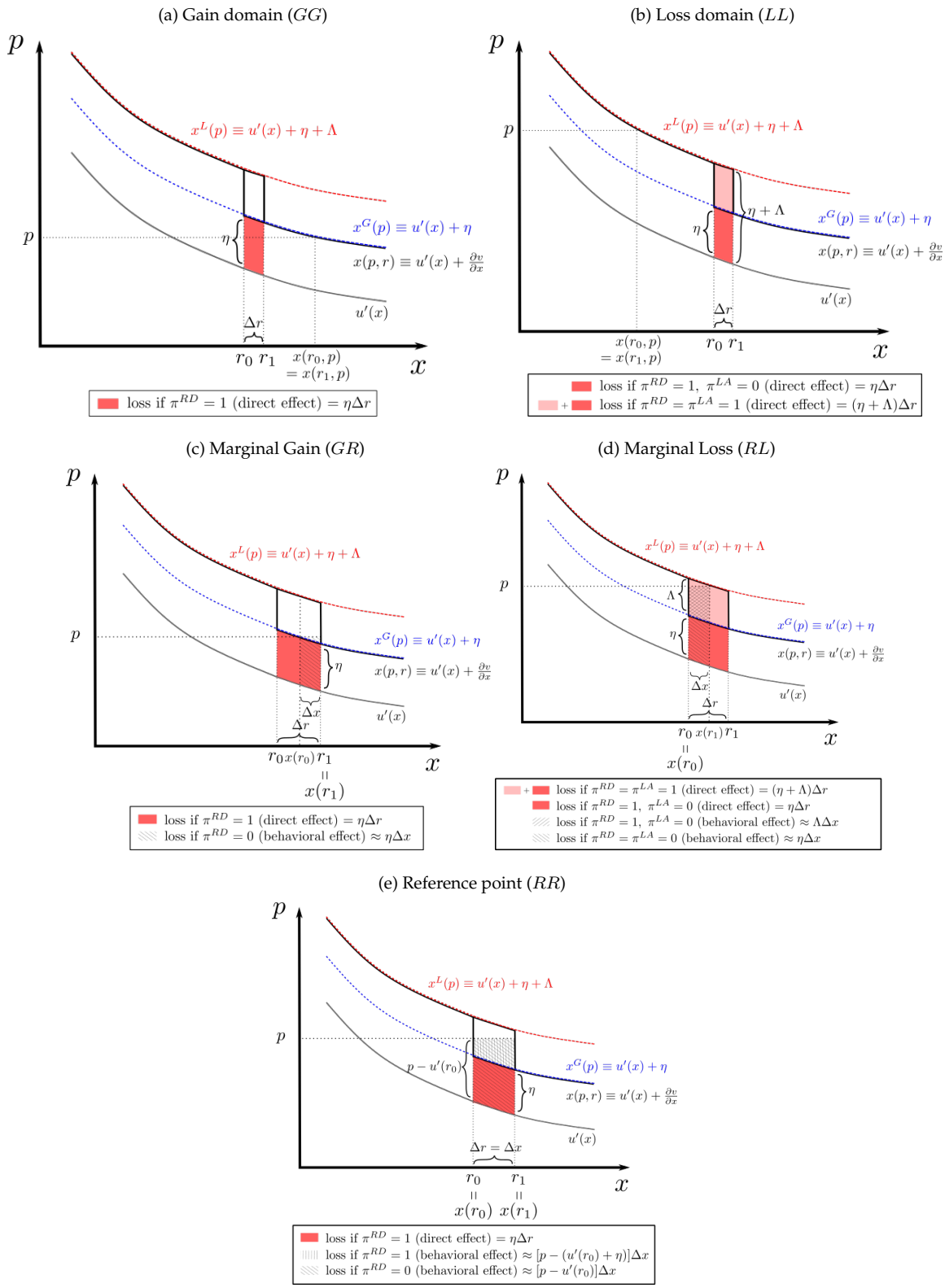


(b) Preferred Bunching Share Estimate



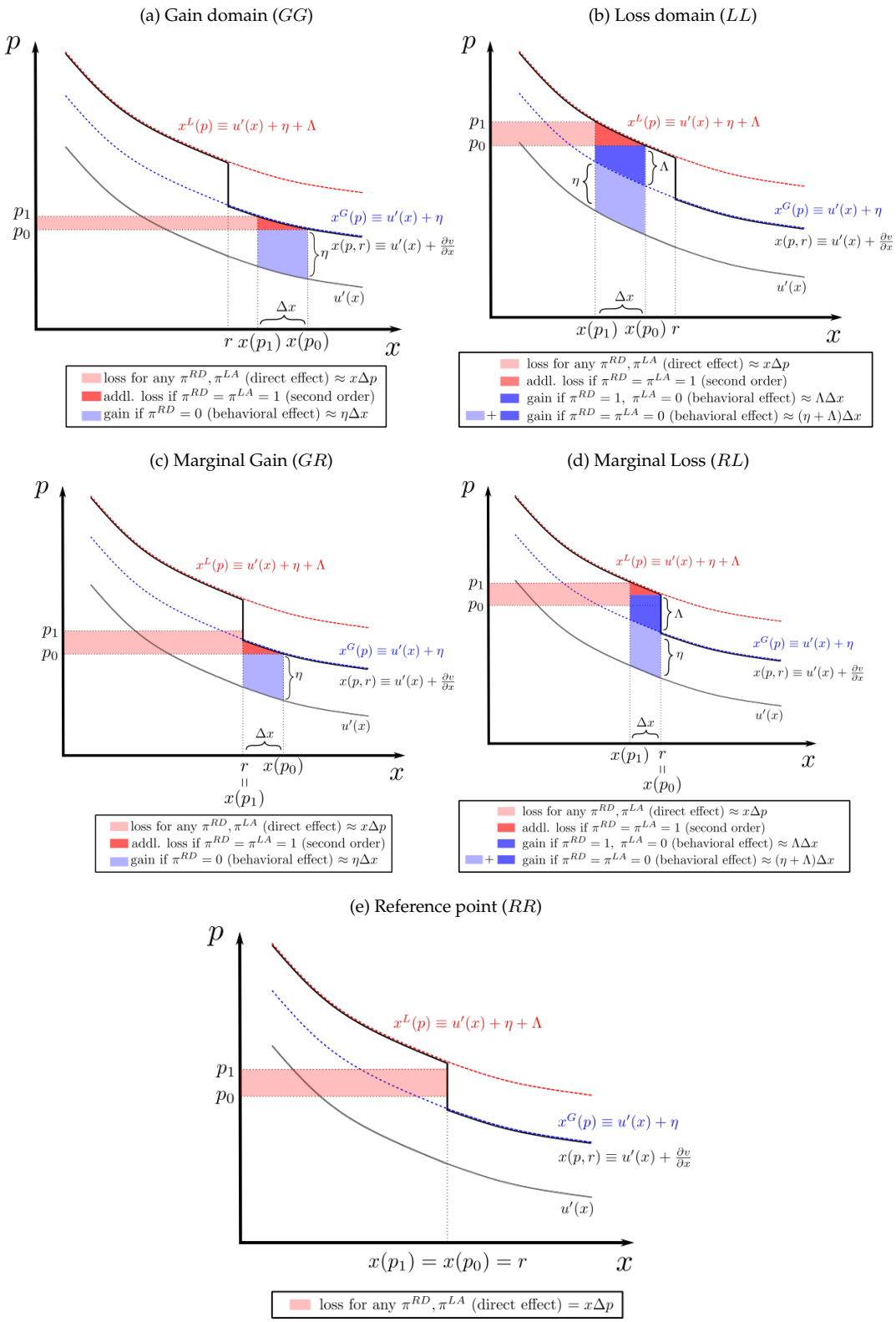
Note: Panel (a) of the figure shows a simulated range of combinations of reference dependence over leisure Λ_l and reference dependence over consumption Λ_c. Parameter combinations are obtained by gradually moving the left bunching share from zero to 50% as described in Appendix E.2. Labeled dots mark parameter combinations implied by selected left bunching shares between 0 and 50%. Panel (b) of the figures illustrates how we obtain our preferred estimate of Λ_c. The black connected dots show the observed retirement age distribution around the NRA among workers born in 1946. The solid red line denotes the average empirical retirement age density on each side of the threshold, and the dashed red line denotes the implied counterfactual density (see Appendix E.2 for details).

FIGURE A4: REFERENCE POINT WELFARE EFFECTS UNDER THE TVERSKY-KAHNEMAN (1991) FORM



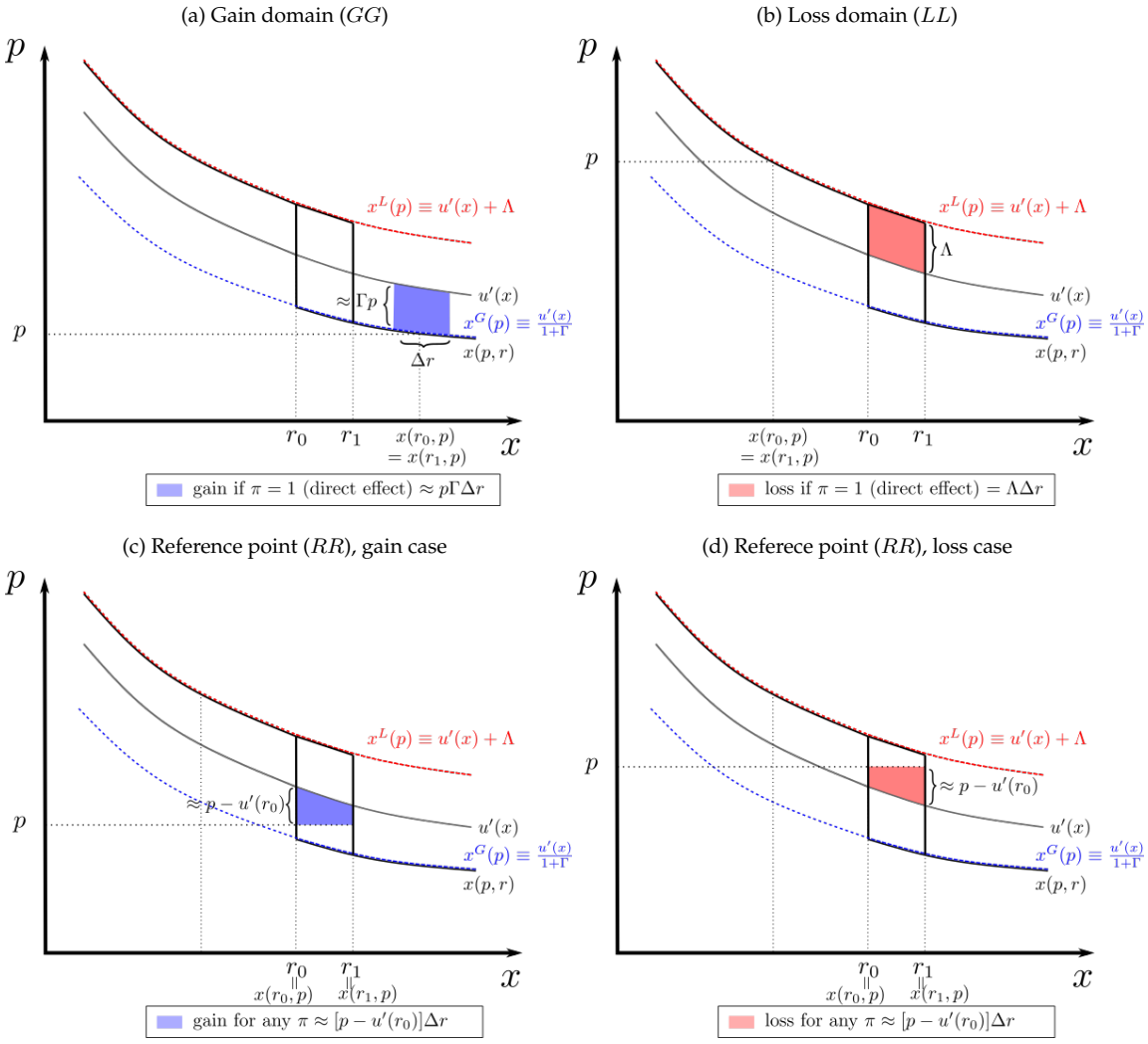
Note: This figure plots the welfare effect of changing the reference point in each domain. Unlike in Figure 2, we adopt the formulation of reference dependence including the η parameter from Tversky and Kahneman (1991) - see Equation: (28). We denote observed demand in black, marginal utility $u'(x)$ in grey, and gain and loss domain demand in blue and red, respectively, as in Figure 11. The direct effect of a change in the reference point is depicted with red shaded regions and the behavioral welfare effects are depicted with hatching. In this case the size of the direct effect can depend on both π^{RD} and π^{LA}. Refer to Section 4.2 for further details.

FIGURE A5: WELFARE EFFECTS OF PRICE CHANGES UNDER THE TVERSKY-KAHNEMAN (1991) FORM



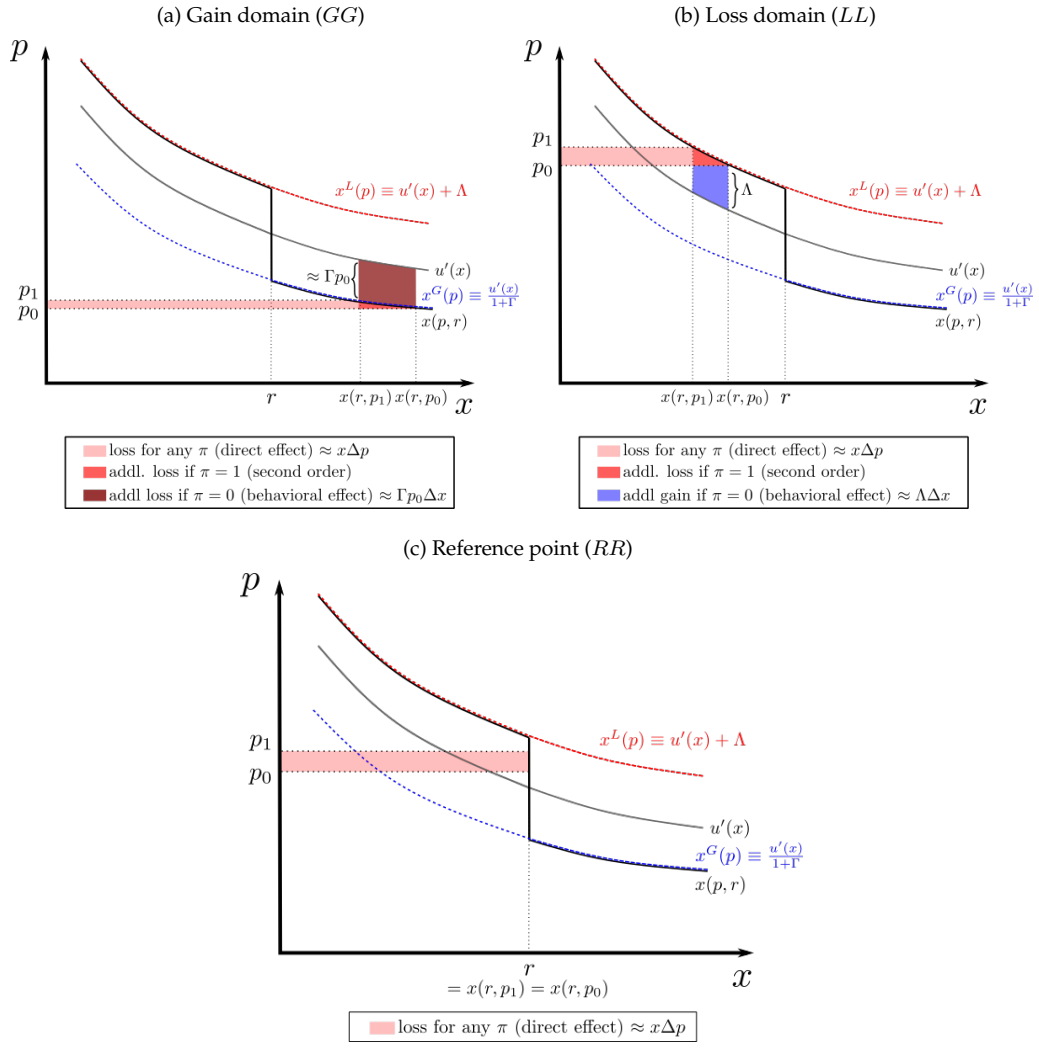
Note: This figure plots the welfare effect of changing the price in each domain. Unlike in Figure 3, we adopt the formulation of reference dependence including the η parameter from Tversky and Kahneman (1991) - see Equation: (28). We denote observed demand in black, marginal utility $u'(x)$ in grey, and gain and loss domain demand in blue and red, respectively, as in Figure 11. The direct (mechanical) negative effect of the price change is depicted with red shaded regions, and the positive behavioral effect is plotted in blue. In this case the size of the behavioral welfare effect depends on both π^{RD} and π^{LA}. Refer to Section 4.2 for further details.

FIGURE A6: REFERENCE POINT WELFARE EFFECTS UNDER 2-DIMENSIONAL REFERENCE DEPENDENCE



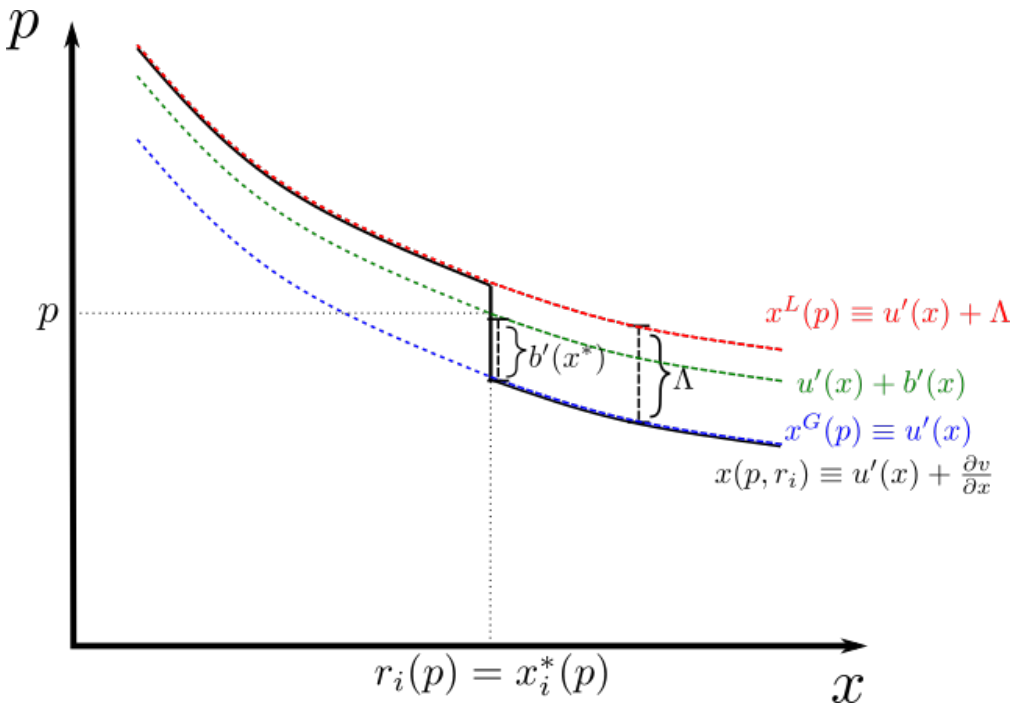
Note: This figure plots the welfare effect of changing the reference point in each domain. Unlike in Figure 2, we assume reference dependence over both x and the background good y. We include only those cases that are relevant for first-order social welfare for simplicity. We also depict the RR case in two situations: those in which the individual experiences a gain or loss. We denote observed demand in black, marginal utility $u'(x)$ in grey, and gain and loss domain demand in blue and red, respectively, as in Figure 8. Gains are depicted in blue shaded regions and losses in red. Because the size of the direct effect on the G group in equation 26 depends on the price, we can no longer illustrate the direct effect of a change in the reference point in the figures like Figure 2 or A4, or Panel (b) of this figure. We depict this quantity slightly differently in Panel (a) for this reason. Refer to Section 4.1for further details.

FIGURE A7: WELFARE EFFECTS OF PRICE CHANGES UNDER 2-DIMENSIONAL REFERENCE DEPENDENCE



Note: This figure plots the welfare effect of changing the price in each domain. Unlike in Figure 2, we assume reference dependence over both x and the background good y. We include only those cases that are relevant for first-order social welfare for simplicity. We denote observed demand in black, marginal utility $u'(x)$ in grey, and gain and loss domain demand in blue and red, respectively, as in Figure 8. Gains are depicted in blue shaded regions and losses in red. The only difference with Figure 3 is the presence of an additional loss from the behavioral effect in the gain domain, depicted in dark red in panel (a). Refer to Section 4.1 for further details.

FIGURE A8: THE OPTIMAL REFERENCE POINT UNDER GOAL-SETTING



Note: This figure illustrates the optimal choice of reference point given an additional bias unrelated to reference dependence, $b(x)$, and reference dependent preferences over x, for the model in Section 4.3. Under the assumption that $b'(x^*) < \Lambda$, which can be seen in the figure, the individual sets a reference point r_i to completely correct the bias and makes a choice in the R domain. Analyzing welfare building on the result illustrated here leads straightforwardly to the results described in the text of section 4.3.

TABLE A1: BUNCHING AND PARAMETER ESTIMATES

Panel A: Bunching Estimates

	(1) Excess mass	(2) Kink size	(3) Number of bunching observations
Normal Retirement Age (NRA)	31.29 (6.42)	-0.28	5
Pure financial incentive discontinuities	6.73 (2.09)	0.47	15

Panel B: Parameter Estimates

Reference dependence w.r.t. NRA Λ	0.461 (0.000)
Retirement age elasticity ε	0.057 (0.014)

Note: Panel A of the table summarizes bunching estimates at the Normal Retirement Age and at pure financial incentive discontinuities. The excess mass figures shown represent the average excess mass estimates at the respective type of threshold among the subset of group-level bunching observations from Seibold (2021) applying to workers in birth cohort 1946, with standard errors in parantheses. The table also shows the average kink size at each type of threshold as well as the number of bunching observations the average estimate is based on. Panel B presents the parameter estimates based on estimating equation (20), using the bunching estimates across thresholds summarized in Panel A. See the main text for more details of the estimation.

TABLE A2: WELFARE EFFECTS OF PENSION REFORMS: ALTERNATIVE COUNTERFACTUAL DISTRIBUTIONS

	(1)	(2)
	Panel A: Uniform Tail	
	Policy 1: Normal Retirement Age to 66	Policy 2: Delayed Retirement Credit to 10.20%
Contributions collected	4,014	3,644
Benefits paid	6,820	-6,222
Net fiscal effect	10,834	-2,578
Worker consumption	7,139	19,136
Disutility from work	-4,933	-3,229
Worker welfare ($\pi = 0$)	2,205	15,906
Ref. dep. utility from ref. point	13,645	0
Ref. dep. disutility from work	-11,727	-13,874
Worker welfare ($\pi = 1$)	4,124	2,032
Total welfare ($\pi = 0$)	13,040	13,328
Total welfare ($\pi = 1$)	14,958	-546
	Panel B: Lognormal Tail	
	Policy 1: Normal Retirement Age to 66	Policy 2: Delayed Retirement Credit to 11.28%
Contributions collected	3,864	3,556
Benefits paid	6,247	-7,109
Net fiscal effect	10,111	-3,553
Worker consumption	7,276	19,738
Disutility from work	-5,495	-3,861
Worker welfare ($\pi = 0$)	1,781	15,877
Ref. dep. utility from ref. point	11,451	0
Ref. dep. disutility from work	-10,189	-13,521
Worker welfare ($\pi = 1$)	3,043	2,356
Total welfare ($\pi = 0$)	11,892	12,324
Total welfare ($\pi = 1$)	13,154	-1,197

Note: The table shows results from pension reform simulations as in Table 1 under alternative assumptions about the upper tail of the retirement age distribution. Simulations are conducted for birth cohort 1946. All effects in Euros per worker, in terms of net present value at age 65. The signs the effects correspond to influence on welfare. Total welfare is the sum of net fiscal effect and change in worker welfare.

TABLE A3: WELFARE EFFECTS OF INCREASING THE NRA BY GROUP

	(1) RR	(2) RG	(3) LR	(4) LG	(5) LL
Contributions collected	+7,287	+2,489	+1,931	+1,383	0
Benefits paid	+10,713	+3,554	+6,630	+2,606	+3,846
Net fiscal effect	+18,000	+6,043	+8,561	+3,989	+3,846
Worker consumption	+13,981	+5,337	+909	+4,672	-3,846
Disutility from work	-6,530	-5,882	-2,170	-4,664	0
Worker welfare ($\pi = 0$)	+7,451	-545	-1,261	+8	-3,846
Ref. dep. disutility from work	-27,543	0	-7,338	+1,298	0
Ref. dep. utility from ref. point	+27,543	0	+14,941	0	+7,819
Worker welfare ($\pi = 1$)	+7,451	-545	+6,342	+1,307	+3,974
Total welfare ($\pi = 0$)	+25,451	+5,498	+7,300	+3,997	0
Total welfare ($\pi = 1$)	+25,451	+5,498	+14,903	+5,295	+7,819

Note: The table shows results from simulations of a pension reform increasing the NRA from 65 to 66. Effects are shown for following groups of workers: those retiring at the old NRA before and at the new NRA after the reform (RR), those retiring at the old NRA before and in the gain domain below the new NRA after (RG), those retiring above the old NRA before and at the new NRA after (LR), those retiring above the old NRA before and below the new NRA after (LG), and those retiring above the old NRA before and above the new NRA after (LL). Effects for the GG group who retire below the old NRA before and below the new NRA after the reform are not shown, as these workers are unaffected by the reform. Simulations are conducted for birth cohort 1946. All effects are calculated among workers retiring at age 65 and above, and are in Euros per worker, in terms of net present value at age 65. The signs the effects correspond to influence on welfare. Total welfare is the sum of net fiscal effect and change in worker welfare.

B Proofs

Lemma 1. The Marginal Internality

L 1.1 $x_i(p,r) > r$,

$$m_i^G = \frac{\partial U_i(x, z_i - px)}{\partial x}|_{x=x^G} = u_i'(x^G) - p = 0 \quad \text{since } u_i'(x^G(p)) = p$$

L 1.2 $x_i(p,r) < r$,

$$m_i^L = \frac{\partial U_i(x, z_i - px)}{\partial x}|_{x=x^L} = u_i'(x^L) - p + \pi\Lambda_i = -\Lambda_i + \pi\Lambda_i \quad \text{since } u_i'(x^L(p)) + \Lambda_i = p$$
$$= -(1-\pi)\Lambda_i$$

L 1.3 $x = r$, $\pi = 1$,
The marginal internality is undefined in this case because of the kink in utility at $x = r$.
$x = r$, $\pi = 0$,

$$m_i = \frac{\partial U_i(x, z_i - px)}{\partial x}|_{x=r} = u_i'(r) - p$$

Proposition 1. The Desirability of Low Reference Points

- **P 1.1.**
  Case 1: $i \in GG$

  $$w_i(p, r_1) - w_i(p, r_0) = 0 \;\Rightarrow\; \Delta w_i(\pi) \leq 0$$

  Case 2: $i \in GR$

  $$w_i(p, r_1) - w_i(p, r_0) = u_i(r_1) - u_i(x_i^G) - p(r_1 - x_i^G)$$

  Holding p fixed, since the individual's consumption switches from the G domain to the R domain as r becomes r_1, we necessarily have $x_i^G < r_1$.
  Since u_i is strictly concave, $u_i(r_1) - u_i(x_i^G) < u_i'(x_i^G)(r_1 - x_i^G)$.
  Following the first-order condition of the individual's welfare-maximizing program, $p = u_i'(x_i^G)$. Then, $u_i(r_1) - u_i(x_i^G) < p(r_1 - x_i^G)$, which implies $\Delta w_i(\pi) < 0$.

  Case 3: $i \in GL$

  $$w_i(p, r_1) - w_i(p, r_0) = u_i(x_i^L) - u_i(x_i^G) - p(x_i^L - x_i^G) + \pi\Lambda_i(x_i^L - r_1)$$

  Holding p fixed, since the individual's consumption switches from the G domain to the L domain as r becomes r_1, we necessarily have $r_1 > x_i^L > x_i^G > r_0$.
  Since u_i is strictly concave, $u_i(x_i^L) - u_i(x_i^G) < u_i'(x_i^G)(x_i^L - x_i^G)$.
  Following the first-order condition of the individual's welfare-maximizing program, $p = u_i'(x_i^G)$. Then,

  $$u_i(x_i^L) - u_i(x_i^G) < p(x_i^L - x_i^G) \;\Rightarrow\; u_i(x_i^L) - u_i(x_i^G) - p(x_i^L - x_i^G) + \pi\Lambda_i(x_i^L - r_1) < \pi\Lambda_i(x_i^L - r_1) \leq 0$$

  Then, for all π, $\Delta w_i(\pi) < 0$
  Case 4: $i \in RR$

$$w_i(p, r_1) - w_i(p, r_0) = u_i(r_1) - u_i(r_0) - p(r_1 - r_0)$$

Holding p fixed, since the individual's consumption remains in the R domain as r becomes r_1, we necessarily have $u_i'(r_0) \leq p \leq u_i'(r_1) + \Lambda$.
Since u_i is strictly concave,

$$u_i(r_1) - u_i(r_0) < u_i'(r_0)(r_1 - r_0) \;\Rightarrow\; u_i'(r_0) > \frac{u_i(r_1) - u_i(r_0)}{r_1 - r_0} \;\Rightarrow\; p > \frac{u_i(r_1) - u_i(r_0)}{r_1 - r_0} \text{ as } p \geq u_i'(r_0)$$

This implies that $u_i(r_1) - u_i(r_0) - p(r_1 - r_0) < 0$, thus $\Delta w_i(\pi) < 0$.
Case 5: $i \in RL$

$$w_i(p, r_1) - w_i(p, r_0) = u_i(x_i^L) - u_i(r_0) - p(x_i^L - r_0) + \pi\Lambda_i(x_i^L - r_1)$$

Holding p fixed, since the individual's consumption switches from the R domain to the L domain as r becomes r_1, we necessarily have $r_1 > x_i^L > r_0$.
Moreover, this setting imposes the two following conditions on p :

$$u_i'(r_0) + \Lambda_i \geq p \geq u_i'(r_1) + \Lambda_i \quad \text{that defines the domain } RL$$

$u_i'(r_0) + \Lambda_i \geq p \geq u_i'(r_0)$ that defines the domain such that the individual is in the R domain at $r = r_0$

Since u_i is strictly concave, $u_i(x_i^L) - u_i(r_0) < u_i'(r_0)(x_i^L - r_0)$.
Then, as $p \geq u_i'(r_0)$ by second condition it must be that

$$u_i(x_i^L) - u_i(r_0) < p(x_i^L - r_0) \;\Rightarrow\; u_i(x_i^L) - u_i(r_0) - p(x_i^L - r_0) + \pi\Lambda_i(x_i^L - r_1) < \pi\Lambda_i(x_i^L - r_1) \leq 0$$

Then, for all π, $\Delta w_i(\pi) < 0$
Case 6: $i \in LL$

$$w_i(p, r_1) - w_i(p, r_0) = -\pi\Lambda_i(r_1 - r_0) \quad \Rightarrow \quad \Delta w_i(\pi) \leq 0$$

- **P 1.2.** $\Delta w_i(1) = 0$ for $i \in GG$ and $\Delta w_i(1) < 0$ for $i \notin GG$. Then if $P[i \in GG] < 1$, there exists at least one individual, the one who does not belong to the GG group, who strictly loses from the increase of r and all others who are left as well off. Consequently, social welfare strictly decreases after the change in r.

$$\Delta W(1) = \int_i \Delta w_i(1)di = \int_{i \in GG} \Delta w_i(1)di + \int_{i \notin GG} \Delta w_i(1)di = \int_{i \notin GG} \Delta w_i(1)di < 0$$

If $P[i \in GG] < 1$, r_0 Pareto dominates r_1 when $\pi = 1$ as all individuals who belong to the GG group are indifferent and all others lose in welfare.

- **P 1.3.** $\Delta w_i(0) = 0$ for $i \in GG \cup LL$ and $\Delta w_i(1) < 0$ otherwise. Then if $P[i \in GG] + P[i \in LL] < 1$, there exists at least one individual, the one who is not GG or LL, who strictly loses from the increase of r and all others who are left as well off. Consequently, social welfare strictly decreases after the change in r.

$$\Delta W(1) = \int_i \Delta w_i(1)di = \int_{i \in GG \cup LL} \Delta w_i(1)di + \int_{i \notin GG \cup LL} \Delta w_i(1)di = \int_{i \notin GG \cup LL} \Delta w_i(1)di < 0$$

If $P[i \in GG] + P[i \in LL] < 1$, r_0 Pareto dominates r_1 when $\pi = 1$ as all individuals GG or LL are indifferent and all others lose in welfare.

Proposition 2. First-Order Individual Welfare Effect of a Change in the Reference Point.

Recall that $w_i(p,r) = u(x_i(p,r)) + z_i - px_i(p,r) + \pi v(x_i(p,r)|r)$.
Case 1: $i \in GG : w_i(p,r) = u(x_i(p,r)) + z_i - px_i(p,r)$
By first-order Taylor series approximation :

$$\Delta w_i \equiv w_i(p,r_1) - w_i(p,r_0) \approx \frac{\partial w_i(p,r)}{\partial r}|_{r=r_0}\Delta r$$

And

$$\frac{\partial w_i(p,r)}{\partial r}|_{r=r_0} = \frac{\partial x_i^G(p)}{\partial r}(u_i'(x_i^G(p)) - p)$$

As the demand x^G does not depend on r (according to FOC^G), the derivative $\frac{\partial x_i^G(p)}{\partial r}$ is null. Then, $\Delta w_i = 0$.
Case 2: $i \in GR : w_i(p,r) = u(x_i(p,r)) + z_i - px_i(p,r)$
By first-order Taylor series approximation :

$$\Delta w_i \equiv w_i(p,r_1) - w_i(p,r_0) \approx \frac{\partial w_i(p,r)}{\partial r}|_{r=r_0}\Delta r$$

And

$$\frac{\partial w_i(p,r)}{\partial r}|_{r=r_0} = \frac{\partial x_i(p,r)}{\partial r}(u_i'(x_i^G(p)) - p)$$

According to FOC^G, $u_i'(x_i^G) = p$. Then, $\Delta w_i = 0$.
Case 3: $i \in RR : w_i(p,r) = u(x_i(p,r)) + z_i - px_i(p,r) + \pi\Lambda_i(x_i(p,r) - r)$
The kink at $x = r$ results in an indeterminate form for first-order approximation. However, independently of the change in r, the individual remains with a consumption level equal to his reference point in both situations. Then, there is no direct effect here and we only consider the behavioral effect of the change.
Then, $\Delta w_i = \Delta r(u'(r_0) - p)$.
Case 4: $i \in RL : w_i(p,r) = u(x_i(p,r)) + z_i - px_i(p,r) + \pi\Lambda_i(x_i(p,r) - r)$
By first-order Taylor series approximation :

$$-\Delta w \equiv w_i(p,r_0) - w_i(p,r_1) \approx -\frac{\partial w_i(p,r)}{\partial r}|_{r=r_1}\Delta r$$

$$\Rightarrow \quad \Delta w_i \approx \frac{\partial w_i(p,r)}{\partial r}|_{r=r_1}\Delta r$$

And

$$\begin{aligned}\frac{\partial w_i}{\partial r}|_{r=r_1} &= \frac{\partial x_i}{\partial r}(u_i'(x_i^L(p)) - p) + \frac{\partial x_i}{\partial r}\pi\Lambda_i - \pi\Lambda_i \\ &= -\frac{\partial x_i}{\partial r}\Lambda_i + \frac{\partial x_i}{\partial r}\pi\Lambda_i - \pi\Lambda_i \quad \text{by FOC}^L \\ &= -(1-\pi)\Lambda_i\frac{\partial x_i}{\partial r} - \pi\Lambda_i \\ \Rightarrow \Delta w_i &\approx -(1-\pi)\Lambda_i\Delta x_i - \pi\Lambda_i\Delta r \ \text{ as, by Taylor approximation } \Delta x_i \approx \frac{\partial x_i}{\partial r}\Delta r\end{aligned}$$

Case 5: $i \in LL : w_i(p,r) = u(x_i(p,r)) + z_i - px_i(p,r) + \pi\Lambda_i(x_i(p,r) - r)$
By first-order Taylor series approximation :

$$-\Delta w_i \equiv w_i(p,r_0) - w_i(p,r_1) \approx -\frac{\partial w_i(p,r)}{\partial r}|_{r=r_1}\Delta r$$

$$\Rightarrow \quad \Delta w_i \approx \frac{\partial w_i(p,r)}{\partial r}|_{r=r_1}\Delta r$$

And
$$\frac{\partial w_i(p,r)}{\partial r}|_{r=r_1} = \frac{\partial x_i^L(p)}{\partial r}(u_i'(x_{il(p,r_1)}) - p) + \frac{\partial x_i^L(p)}{\partial r}\pi\Lambda_i - \pi\Lambda_i$$

As the demand x^L does not depend on r (according to FOC^L), the behavioral effect $\frac{\partial x_i^L(p)}{\partial r}$ is null. Then, $\Delta w_i = -\pi\Lambda_i\Delta r$.

Proposition 3. The First-Order Social Welfare Effect of a Change in the Reference Point.
We denote for all i, f_p and f_{Λ_i} the probability density functions respectively of $p|\Lambda_i$ and Λ_i. We also sometimes abbreviate for $k = G, L, R$,
$U_i^k = U_i^*(x_i^k, z_i - px_i^k)$.

$$W = \int_0^{+\infty}\left[\int_{u_i'(r)+\Lambda_i}^{+\infty} U_i^*(x_i^L, z_i - px_i^L)f_p(p)dp + \int_{u_i'(r)}^{u_i'(r)+\Lambda_i} U_i^*(r, z_i - pr)f_p(p)dp \right.$$
$$\left. + \int_0^{u_i'(r)} U_i^*(x_i^G, z_i - px_i^G)f_p(p)dp\right] f_{\Lambda_i}(\Lambda_i)d\Lambda_i$$

Applying Leibniz rule,

$$\frac{\partial W}{\partial r} = \int_0^{+\infty}\left[\int_{u_i'(r)+\Lambda_i}^{+\infty}\frac{\partial U_i^L}{\partial r}f_p(p)dp + \int_{u_i'(r)}^{u_i'(r)+\Lambda_i}\frac{\partial U_i^R}{\partial r}f_p(p)dp + \int_0^{u_i'(r)}\frac{\partial U_i^G}{\partial r}f_p(p)dp\right] f_{\Lambda_i}(\Lambda_i)d\Lambda_i$$
$$+ \int_0^{+\infty}\left[-u_i"(r)U_i^L|_{p=u_i'(r)+\Lambda_i}f_p(u_i'(r)+\Lambda_i) + u_i"(r)U_i^R|_{p=u_i'(r)+\Lambda_i}f_p(u_i'(r)+\Lambda_i)\right.$$
$$\left. - u_i"(r)U_i^R|_{p=u_i'(r)}f_p(u_i'(r)) + u_i"(r)U_i^G|_{p=u_i'(r)}f_p(u_i'(r))\right] f_{\Lambda_i}(\Lambda_i)d\Lambda_i$$

Focus on the second part of the expression. For all i,

$$- u_i"(r)U_i^L|_{p=u_i'(r)+\Lambda_i}f_p(u_i'(r)+\Lambda_i) + u_i"(r)U_i^R|_{p=u_i'(r)+\Lambda_i}f_p(u_i'(r)+\Lambda_i) - u_i"(r)U_i^R|_{p=u_i'(r)}f_p(u_i'(r))$$
$$+ u_i"(r)U_i^G|_{p=u_i'(r)}f_p(u_i'(r))$$
$$= u_i"(r)\left[f_p(u_i'(r)+\Lambda_i)(U_i^R|_{p=u_i'(r)+\Lambda_i} - U_i^L|_{p=u_i'(r)+\Lambda_i}) + f_p(u_i'(r))(U_i^G|_{p=u_i'(r)} - U_i^R|_{p=u_i'(r)})\right]$$

For all i, $p = u_i'(r)$ corresponds to the situation where the individual i is at the threshold between the G and R groups. At this point, the individual's utility is the same regardless of whether $x > r$ or $x \leq r$. Formally,

$$U_i^*(r, z_i - pr)|_{u_i'(r)=p} = u_i(r) + z_i - u_i'(r)r$$
$$U_i^*(x_i^G, z_i - px_i^G)|_{u_i'(r)=p} = u_i(x_i^G) + z_i - u_i'(x_i^G)x_i^G$$

Since $r = x_i^G$ for i such that $p = u_i'(r)$, then $U_i^*(r, z_i - pr)|_{u_i'(r)=p} - U_i^*(x_i^G, z_i - px_i^G)|_{u_i'(r)=p} = 0$.
Similarly for the R and L groups, $U_i^*(x_i^L, z_i - px_i^L)|_{p=u_i'(r)+\Lambda_i} - U_i^*(r, z_i - pr)|_{p=u_i'(r)+\Lambda_i} = 0$.

Consequently,

$$\begin{aligned}
\frac{\partial W}{\partial r} &= \int_0^{+\infty} \left[\int_{u_i'(r)+\Lambda_i}^{+\infty} \frac{\partial U_i^L}{\partial r} f_p(p)dp + \int_{u_i'(r)}^{u_i'(r)+\Lambda_i} \frac{\partial U_i^R}{\partial r} f_p(p)dp + \int_0^{u_i'(r)} \frac{\partial U_i^G}{\partial r} f_p(p)dp \right] f_{\Lambda_i}(\Lambda_i)d\Lambda_i \\
&= \int_0^{+\infty} \left[\int_{u_i'(r)+\Lambda_i}^{+\infty} \frac{\partial U_i^L}{\partial r} f_p(p)dp + \int_{u_i'(r)}^{u_i'(r)+\Lambda_i} \frac{\partial U_i^R}{\partial r} f_p(p)dp \right] f_{\Lambda_i}(\Lambda_i)d\Lambda_i \\
&= \int_0^{+\infty} \left[\int_{u_i'(r)+\Lambda_i}^{+\infty} -\pi\Lambda_i f_p(p)dp + \int_{u_i'(r)}^{u_i'(r)+\Lambda_i} (u_i'(r)-p) f_p(p)dp \right] f_{\Lambda_i}(\Lambda_i)d\Lambda_i \\
&= \int_0^{+\infty} \left\{ E\left[-\pi\Lambda_i | p > u_i'(r)+\Lambda_i, \Lambda_i \right] \right\} P(p > u_i'(r)+\Lambda_i|\Lambda_i)(f_{\Lambda_i}(\Lambda_i)d\Lambda_i \\
&\quad - \int_0^{+\infty} \left\{ E\left[p - u_i'(r) | u_i'(r) > p > u_i'(r)+\Lambda_i, \Lambda_i \right] \right\} P(u_i'(r) > p > u_i'(r)+\Lambda_i|\Lambda_i) f_{\Lambda_i}(\Lambda_i)d\Lambda_i
\end{aligned}$$

By law of iterated expectations,

$$\frac{\partial W}{\partial r} = -\pi E\left[\Lambda_i | i \in L\right] P\left[i \in L\right] - E\left[p - u_i'(r) | i \in R\right] P\left[i \in R\right]$$

Which yields the result by first-order Taylor series approximation.

Proposition 4. First-Order Welfare Effect of a Change in Price.
Case 1: $i \in GG, GR$

$$\Delta w_i \equiv w_i(p_1, r) - w_i(p_0, r) \approx \frac{\partial w_i(p,r)}{\partial p}|_{p=p_0} \Delta p$$

And

$$\begin{aligned}
\frac{\partial w_i(p,r)}{\partial p}|_{p=p_0} &= \frac{\partial x(p,r)}{\partial p}|_{p=p_0} . \frac{\partial U_i(x(p,r), z_i - px(p,r))}{\partial x}|_{p=p_0} - x(p_0, r) \\
&= \frac{\partial x(p,r)}{\partial p}|_{p=p_0} m_i(p_0, r) - x(p_0, r) \\
\Rightarrow \Delta w_i &\approx \frac{\partial x(p,r)}{\partial p}|_{p=p_0} m_i(p_0, r)\Delta p - x(p_0, r)\Delta p
\end{aligned}$$

As

$$\Delta x \equiv x(p_1, r) - x(p_0, r) \approx \frac{\partial x(p,r)}{\partial p}|_{p=p_0} \Delta p$$

We finally obtain for $i \in GG, GR$

$$\Delta w \equiv w_i(p_1, r) - w_i(p_0, r) \approx m_i(p_0, r)\Delta x - x(p_0, r)\Delta p$$

Case 2: $i \in LL, RL$

$$\begin{aligned}
-\Delta w_i \equiv w_i(p_0, r) - w_i(p_1, r) &\approx \frac{\partial w_i(p,r)}{\partial p}|_{p=p_1} (p_0 - p_1) \\
\Rightarrow \Delta w_i &\approx \frac{\partial w_i(p,r)}{\partial p}|_{p=p_1} \Delta p
\end{aligned}$$

And

$$\frac{\partial w_i(p,r)}{\partial p}|_{p=p_1} = \frac{\partial x(p,r)}{\partial p}|_{p=p_1}.\frac{\partial U_i(x(p,r), z-px(p,r))}{\partial x}|_{p=p_1} - x(p_1,r)$$
$$= \frac{\partial x(p,r)}{\partial p}|_{p=p_1} m_i(p_1,r) - x(p_1,r)$$
$$\Rightarrow \Delta w_i \approx \frac{\partial x(p,r)}{\partial p}|_{p=p_1} m_i(p_1,r)\Delta p - x(p_1,r)\Delta p$$

As

$$-\Delta x \equiv x(p_0,r) - x(p_1,r) \approx -\frac{\partial x(p,r)}{\partial p}|_{p=p_1}\Delta p$$
$$\Rightarrow \Delta x \approx \frac{\partial x(p,r)}{\partial p}|_{p=p_1}\Delta p$$

We finally obtain for $i \in LL, RL$

$$\Delta w \equiv w_i(p_1,r) - w_i(p_0,r) \approx m_i(p_1,r)\Delta x - x(p_1,r)\Delta p$$

Case 3: $i \in RR$

The kink at $x = r$ results in an indeterminate form for first-order approximation. However, independently of the change in r, the individual remains with the same consumption level (equal to his reference point) for both prices. Then, there is no behavioral effect here ($\Delta x = 0$) and we only consider the direct effect of the change.

Then, $\Delta w_i = -x(\hat{p}, r)\Delta p$, for any $\hat{p}$.

Proposition 5. The First-Order Social Welfare Effect of a Price Change.

We denote for all i, f_u and f_{Λ_i} the probability density functions respectively of $u_i'(r)|\Lambda_i$ and Λ_i. We also sometimes abbreviate for $k = G, L, R$, $U_i^k = U_i^*(x_i^k, z_i - px_i^k)$.

$$W = \int_0^{+\infty} \left[\int_{-\infty}^{p-\Lambda_i} U_i^*(x_i^L, z_i - px_i^L) f_u(u_i'(r)) du_i'(r) + \int_{p-\Lambda_i}^{p} U_i^*(r, z_i - pr) f_u(u_i'(r)) du_i'(r) \right.$$
$$\left. + \int_p^{+\infty} U_i^*(x_i^G, z_i - px_i^G) f_u(u_i'(r)) du_i'(r) \right] f_{\Lambda_i}(\Lambda_i) d\Lambda_i$$

Applying Leibniz rule,

$$\frac{\partial W}{\partial p} = \int_0^{+\infty} \left[\int_{-\infty}^{p-\Lambda_i} \frac{\partial U_i^L}{\partial p} f_u(u_i'(r)) du_i'(r) + \int_{p-\Lambda_i}^{p} \frac{\partial U_i^R}{\partial p} f_u(u_i'(r)) du_i'(r) + \int_p^{+\infty} \frac{\partial U_i^G}{\partial p} f_u(u_i'(r)) du_i'(r) \right] f_{\Lambda_i}(\Lambda_i) d\Lambda_i$$
$$+ \int_0^{+\infty} \left[U_i^L|_{u_i'(r)=p-\Lambda_i} f_u(p-\Lambda_i) + U_i^R|_{u_i'(r)=p} f_u(p) - U_i^R|_{u_i'(r)=p-\Lambda_i} f_u(p-\Lambda_i) - U_i^G|_{u_i'(r)=p} f_u(p) \right] f_{\Lambda_i}(\Lambda_i) d\Lambda_i$$

Focus on the second part of the expression. For all i,

$$U_i^L|_{u_i'(r)=p-\Lambda_i} f_u(p-\Lambda_i) + U_i^R|_{u_i'(r)=p} f_u(p) - U_i^R|_{u_i'(r)=p-\Lambda_i} f_u(p-\Lambda_i) - U_i^G|_{u_i'(r)=p} f_u(p)$$
$$= f_u(p) \left[U_i^R(r, z_i - pr)|_{u_i'(r)=p} - U_i^G(x_i^G, z_i - px_i^G)|_{u_i'(r)=p} \right] + f_u(p-\Lambda_i) \left[U_i^L(x_i^L, z_i - px_i^L)|_{u_i'(r)=p-\Lambda_i} \right.$$
$$\left. - U_i^*(r, z_i - pr)|_{u_i'(r)=p-\Lambda_i} \right]$$

For all i, $p = u_i'(r)$ corresponds to the situation where the individual i is at the threshold between the G and

R groups. At this point, the individual's utility is the same regardless of whether $x > r$ or $x \leq r$. Formally,

$$U_i^*(r, z_i - pr)|_{u_i'(r)=p} = u_i(r) + z_i - u_i'(r)r$$
$$U_i^*(x_i^G, z_i - px_i^G)|_{u_i'(r)=p} = u_i(x_i^G) + z_i - u_i'(x_i^G)x_i^G$$

Since $r = x_i^G$ for i such that $p = u_i'(r)$, then $U_i^*(r, z_i - pr)|_{u_i'(r)=p} - U_i^*(x_i^G, z_i - px_i^G)|_{u_i'(r)=p} = 0$.
Similarly for the R and L groups, $U_i^*(x_i^L, z_i - px_i^L)|_{u_i'(r)=p-\Lambda_i} - U_i^*(r, z_i - pr)|_{u_i'(r)=p-\Lambda_i} = 0$.
Consequently,

$$\int_0^{+\infty} \left[U_i^L|_{u_i'(r)=p-\Lambda_i} f_u(p-\Lambda_i) + U_i^R|_{u_i'(r)=p} f_u(p) - U_i^R|_{u_i'(r)=p-\Lambda_i} f_u(p-\Lambda_i) - U_i^G|_{u_i'(r)=p} f_u(p)\right] f_{\Lambda_i}(\Lambda_i) d\Lambda_i = 0$$

The social welfare effect now writes as :

$$\begin{aligned}
\frac{\partial W}{\partial p} &= \int_0^{+\infty} \left[\int_{-\infty}^{p-\Lambda_i} \frac{\partial U_i^L}{\partial p} f_u(u_i'(r)) du_i'(r) + \int_{p-\Lambda_i}^{p} \frac{\partial U_i^R}{\partial p} f_u(u_i'(r)) du_i'(r) + \int_p^{+\infty} \frac{\partial U_i^G}{\partial p} f_u(u_i'(r)) du_i'(r)\right] f_{\Lambda_i}(\Lambda_i) d\Lambda_i \\
&= \int_0^{+\infty} \left\{ \int_{-\infty}^{p-\Lambda_i} \left[\frac{\partial x_i^L}{\partial p}(u_i'(x_i^L) - p + \pi\Lambda_i) - x_i^L\right] f_u(u_i'(r)) du_i'(r) + \int_{p-\Lambda_i}^{p} -r f_u(u_i'(r)) du_i'(r) \right. \\
&\qquad \left. + \int_p^{+\infty} \left[\frac{\partial x_i^G}{\partial p}(u_i'(x_i^G) - p) - x_i^G\right] f_u(u_i'(r)) du_i'(r) \right\} f_{\Lambda_i}(\Lambda_i) d\Lambda_i
\end{aligned}$$

By FOC^G, $u_i'(x_i^G) - p = 0$ and by FOC^L, $u_i'(x_i^L) = p - \Lambda_i$

$$\begin{aligned}
&= \int_0^{+\infty} \left\{ \int_{-\infty}^{p-\Lambda_i} \left[-(1-\pi)\Lambda_i \frac{\partial x_i^L}{\partial p} - x_i^L\right] f_u(u_i'(r)) du_i'(r) - \int_{p-\Lambda_i}^{p} r f_u(u_i'(r)) du_i'(r) \right. \\
&\qquad \left. + \int_p^{+\infty} -x_i^G f_u(u_i'(r)) du_i'(r) \right\} f_{\Lambda_i}(\Lambda_i) d\Lambda_i \\
&= \int_0^{+\infty} \left\{ \int_{-\infty}^{p-\Lambda_i} -(1-\pi)\Lambda_i \frac{\partial x_i^L}{\partial p} f_u(u_i'(r)) du_i'(r) - \int_{-\infty}^{+\infty} x_i f_u(u_i'(r)) du_i'(r) \right\} f_{\Lambda_i}(\Lambda_i) d\Lambda_i \\
&= \int_0^{+\infty} \left\{ E\left[-(1-\pi)\Lambda_i \frac{\partial x_i^L}{\partial p} | u_i'(r) < p - \Lambda_i, \Lambda_i\right] P(u_i'(r) < p - \Lambda_i | \Lambda_i) - E\left[x_i | \Lambda_i\right] \right\} f_{\Lambda_i}(\Lambda_i) d\Lambda_i \\
&= \int_0^{+\infty} E\left[-(1-\pi)\Lambda_i \frac{\partial x_i^L}{\partial p} | i \in L, \Lambda_i\right] P(i \in L | \Lambda_i) f_{\Lambda_i}(\Lambda_i) d\Lambda_i - \int_0^{+\infty} E\left[x_i | \Lambda_i\right] f_{\Lambda_i}(\Lambda_i) d\Lambda_i
\end{aligned}$$

By law of iterated expectations

$$\frac{\partial W}{\partial p} = E\left[-(1-\pi)\Lambda_i \frac{\partial x_i^L}{\partial p} | i \in L\right] P\left[i \in L\right] - E\left[x_i\right]$$

Which yields the proposition result, after a first-order Taylor series approximation.

Corollary 5.1. Corrective Taxes for Reference Dependence.

- $x \geq r$: Denoting x_i^G the solution to the individual's welfare-maximizing program, the tax-comprehensive individual welfare objective function writes

$$U_i^*(x_i^G, z_i - px_i^G, t_i(x_i^G, r)) = u_i(x_i^G) + z_i - px_i^G - t_i(x_i^G, r)$$

The efficient individual tax t_i on x solves

$$\frac{\partial U_i^*}{\partial x_i^G} = 0 \Rightarrow u_i'(x_i^G) - p - \frac{\partial t_i}{\partial x_i^G} = 0$$
$$\Rightarrow \frac{\partial t_i}{\partial x_i^G} = 0 \quad \text{by } FOC^G$$

Then, for $x \geq r$, $t_i(x, r) = 0$.

- $x < r$: Denoting x_i^L the solution to the individual's welfare-maximizing program, the tax-comprehensive individual welfare objective function writes

$$U_i^*(x_i^L, z_i - px_i^L, t_i(x_i^L, r)) = u_i(x_i^L) + z_i - px_i^L + \pi\Lambda_i(x_i^L - r) - t_i(x_i^L, r)$$

The efficient individual tax t_i on x solves

$$\frac{\partial U_i^*}{\partial x_i^L} = 0 \Rightarrow u_i'(x_i^L) - p + \pi\Lambda_i - \frac{\partial t_i}{\partial x_i^L} = 0$$
$$\Rightarrow -(1-\pi)\Lambda_i - \frac{\partial t_i}{\partial x_i^L} = 0 \quad \text{by } FOC^L$$
$$\Rightarrow \frac{\partial t_i}{\partial x_i^L} = -(1-\pi)\Lambda_i$$

Then, for $x < r$, $t_i(x, r) = -(1-\pi)\Lambda_i(x - r)$.

Note that the first-order conditions are different in the K-T formulation model.

$$\tilde{u_i}'(\tilde{x}^G) + \eta_i = p \qquad (FOC^G)$$
$$\tilde{u_i}'(\tilde{x}^L) + \eta_i + \Lambda_i = p \qquad (FOC^L)$$

Lemma 3. The Revised Marginal Internality.
L 3.1 $x_i(p, r) > r$

$$\tilde{m_i}^G = \frac{\partial \tilde{U}_i(x, z - px)}{\partial x}|_{x=x^G} = \tilde{u_i}'(x^G) - p + \pi^{RD}\eta_i = -(1-\pi^{RD})\eta_i \quad \text{by } FOC^G$$

L 3.2 $x_i(p, r) < r$

$$\tilde{m_i}^L = \frac{\partial \tilde{U}_i(x, z - px)}{\partial x}|_{x=x^L} = \tilde{u_i}'(x^L) - p + \pi^{RD}\eta_i + \pi^{LA}\Lambda_i = -(1-\pi^{RD})\eta_i - (1-\pi^{LA})\Lambda_i \quad \text{by } FOC^L$$

L 3.3 $x_i(p, r) = r$
Under this model, $\tilde{m_i}$ is undefined for $x = r$ if $\pi^{LA} = 1$. Otherwise,

$$\tilde{m_i} = \tilde{u_i}'(r) - p + \pi^{RD}\eta_i$$

Proposition 7. Modified First-Order Welfare Effects.
P 7.1 $i \in GG$

$$\Delta\tilde{w_i} = \tilde{u_i}(x^G(p)) - \tilde{u_i}(x^G(p)) - p(x^G(p) - x^G(p)) - \pi^{RD}\eta_i(r_1 - r_0) = -\pi^{RD}\eta_i(r_1 - r_0)$$

equal to the desired result as $\Delta x = 0$.

$i \in GR$

$$\begin{aligned}
\Delta\tilde{w}_i &= \tilde{u_i}(r_1) - \tilde{u_i}(x^G(p)) - p(r_1 - x^G(p)) - \pi^{RD}\eta_i(x^G(p) - r_0) \\
&\quad by\ Taylor\ approximation \Rightarrow \tilde{u_i}(r_1) \approx \tilde{u_i}(x^G(p)) + \tilde{u_i}'(x^G(p))(r_1 - x^G(p)) \\
\Rightarrow \Delta\tilde{w}_i &\approx \tilde{u_i}'(x^G(p))(r_1 - x^G(p)) - p(r_1 - x^G(p)) - \pi^{RD}\eta_i(x^G(p) - r_0) \\
&\approx -\eta_i(r_1 - x^G(p)) - \pi^{RD}\eta_i(x^G(p) - r_0) \quad \text{by } FOC^G \\
&\approx -(1 - \pi^{RD})\eta_i(r_1 - x^G(p)) - \pi^{RD}\eta_i(r_1 - r_0) \quad \text{by adding and removing } \pi^{RD}\eta_i r_1. \\
&\approx \Delta w_i - \pi^{RD}\eta_i\Delta r - (1 - \pi^{RD})\eta_i\Delta x
\end{aligned}$$

$i \in RR$

$$\begin{aligned}
\Delta\tilde{w}_i &= \tilde{u_i}(r_1) - \tilde{u_i}(r_0) - p(r_1 - r_0) \\
&\approx (\tilde{u_i}'(r_0) - p)(r_1 - r_0) \quad \text{by first-order Taylor approximation} \\
&\approx (\tilde{u_i}'(r_0) - p - \eta_i)(r_1 - r_0) \quad \text{as } \tilde{u_i}' = \tilde{u_i}' - \eta_i \text{ for } i \in RR
\end{aligned}$$

As $\Delta x = \Delta r$ in this case, we obtain the desired result.

LL and RL cases are analogous to GG and GR.

P 7.2 Analogously and using the same notations as in the Proposition 3 proof,

$$\begin{aligned}
\tilde{W} &= \int_0^{+\infty}\left[\int_{\tilde{u_i}'(r)+\Lambda_i}^{+\infty}\tilde{U_i}^*(x_i^L, z_i - px_i^L)f_p(p)dp + \int_{\tilde{u_i}'(r)}^{\tilde{u_i}'(r)+\Lambda_i}\tilde{U_i}^*(r, z_i - pr)f_p(p)dp\right. \\
&\qquad \left. + \int_0^{\tilde{u_i}'(r)}\tilde{U_i}^*(x_i^G, z_i - px_i^G)f_p(p)dp\right] f_{\Lambda_i}(\Lambda_i)d\Lambda_i \\
\Rightarrow \frac{\partial\tilde{W}}{\partial r} &= \int_0^{+\infty}\left[\int_{\tilde{u_i}'(r)+\Lambda_i}^{+\infty}\frac{\partial\tilde{U_i^L}}{\partial r}f_p(p)dp + \int_{\tilde{u_i}'(r)}^{\tilde{u_i}'(r)+\Lambda_i}\frac{\partial\tilde{U_i^R}}{\partial r}f_p(p)dp + \int_0^{\tilde{u_i}'(r)}\frac{\partial\tilde{U_i^G}}{\partial r}f_p(p)dp\right] f_{\Lambda_i}(\Lambda_i)d\Lambda_i \\
&= \int_0^{+\infty}\left[\int_{\tilde{u_i}'(r)+\Lambda_i}^{+\infty} -(\pi^{RD}\eta_i + \pi^{LA}\Lambda_i)f_p(p)dp + \int_{\tilde{u_i}'(r)}^{\tilde{u_i}'(r)+\Lambda_i}(\tilde{u_i}'(r) - p)f_p(p)dp\right. \\
&\qquad \left. + \int_0^{\tilde{u_i}'(r)} -\pi^{RD}\eta_i f_p(p)dp\right] f_{\Lambda_i}(\Lambda_i)d\Lambda_i \\
&= \int_0^{+\infty}\left\{E\left[-(\pi^{RD}\eta_i + \pi^{LA}\Lambda_i)|p > \tilde{u_i}'(r) + \Lambda_i, \Lambda_i\right]\right\}P(p > \tilde{u_i}'(r) + \Lambda_i|\Lambda_i)(f_{\Lambda_i}(\Lambda_i)d\Lambda_i \\
&\quad - \int_0^{+\infty}\left\{E\left[p - \tilde{u_i}'(r)|\tilde{u_i}'(r) + \Lambda_i > p > \tilde{u_i}'(r), \Lambda_i\right]\right\}P(\tilde{u_i}'(r) + \Lambda_i > p > \tilde{u_i}'(r)|\Lambda_i)f_{\Lambda_i}(\Lambda_i)d\Lambda_i \\
&\quad + \int_0^{+\infty}\left\{E\left[-\pi^{RD}\eta_i|\tilde{u_i}'(r) > p, \Lambda_i\right]\right\}P(\tilde{u_i}'(r) > p|\Lambda_i)f_{\Lambda_i}(\Lambda_i)d\Lambda_i \\
&= \int_0^{+\infty}\left\{E\left[-\pi^{LA}\Lambda_i|i \in L, \Lambda_i\right]P(i \in L|\Lambda_i) - E\left[p - u_i'(r)|i \in R, \Lambda_i\right]P(i \in R|\Lambda_i)\right. \\
&\quad \left. - E\left[\pi^{RD}\eta_i|\Lambda_i\right] - E\left[(1 - \pi^{RD})\eta_i|i \in R, \Lambda_i\right]P(i \in R|\Lambda_i)\right\}f_{\Lambda_i}(\Lambda_i)d\Lambda_i
\end{aligned}$$

Since $\tilde{u_i}'(r) - p = u_i'(r) - p - \eta_i$ from the FOC^R in both models. By law of iterated expectations,

$$\frac{\partial \tilde{W}}{\partial r} = -\pi^{LA}\Lambda_i E\left[\Lambda_i | i \in L\right] P\left[i \in L\right] - E\left[p - \tilde{u_i}'(r) | i \in R\right] P\left[i \in R\right] - E\left[\pi^{RD}\eta_i\right] \\ - E\left[(1-\pi^{RD})\eta_i | i \in R\right] P(i \in R)$$

Then, as π from the first model is now π^{LA},

$$\Rightarrow \frac{\partial \tilde{W}}{\partial r} = \frac{\partial W}{\partial r} - E[\pi^{RD}\eta_i] - E[(1-\pi^{RD})\eta_i | i \in R]P[i \in R]$$

Which yields the result after two first-order Taylor approximations.

P 7.3 The result proven in Proposition 4 holds for the new model:

$$\tilde{w_i}(p_1, r) - \tilde{w_i}(p_0, r) \approx \tilde{m_i}(\hat{p}, r)\Delta x - x(\hat{p}, r)\Delta p$$

Then the difference between the two variations in indirect utility is all due to the combination of the marginal internality and the behavioral effect :

$$\Delta\tilde{w_i} - \Delta w_i \approx (\tilde{m_i}(p, r) - m_i(p, r))\Delta x_i$$

For $i \notin RR$, $\Delta x_i \neq 0$ and the difference in marginal internalities is

$$\tilde{m_i}(p, r) - m_i(p, r) = -(1 - \pi^{RD})\eta_i$$

For $i \in RR$, $\Delta x = 0$, then the result stands independently of the marginal internalities.

P 7.4 Analogously and using similar notations as in Proposition 5 proof,

$$\tilde{W} = \int_0^{+\infty} \left[\int_{-\infty}^{p-\Lambda_i} \tilde{U_i}^*(x_i^L, z_i - px_i^L) f_{\tilde{u}}(\tilde{u_i}'(r)) d\tilde{u_i}'(r) + \int_{p-\Lambda_i}^{p} \tilde{U_i}^*(r, z_i - pr) f_{\tilde{u}}(\tilde{u_i}'(r)) d\tilde{u_i}'(r) \right. \\ \left. + \int_p^{+\infty} \tilde{U_i}^*(x_i^G, z_i - px_i^G) f_{\tilde{u}}(\tilde{u_i}'(r)) d\tilde{u_i}'(r) \right] f_{\Lambda_i}(\Lambda_i) d\Lambda_i$$

$$\frac{\partial W}{\partial p} = \int_0^{+\infty} \left[\int_{-\infty}^{p-\Lambda_i} \frac{\partial \tilde{U_i^L}}{\partial p} f_{\tilde{u}}(\tilde{u_i}'(r)) d\tilde{u_i}'(r) + \int_{p-\Lambda_i}^{p} \frac{\partial \tilde{U_i^R}}{\partial p} f_{\tilde{u}}(\tilde{u_i}'(r)) d\tilde{u_i}'(r) \right. \\ \left. + \int_p^{+\infty} \frac{\partial \tilde{U_i^G}}{\partial p} f_{\tilde{u}}(\tilde{u_i}'(r)) d\tilde{u_i}'(r) \right] f_{\Lambda_i}(\Lambda_i) d\Lambda_i$$

$$= \int_0^{+\infty} \left[\int_{-\infty}^{p-\Lambda_i} \left[\frac{\partial x_i^L}{\partial p}(\tilde{u_i}'(x_i^L) - p + \pi^{RD}\eta_i + \pi^{LA}\Lambda_i) - x_i^L \right] f_{\tilde{u}}(\tilde{u_i}'(r)) d\tilde{u_i}'(r) + \int_{p-\Lambda_i}^{p} -r f_{\tilde{u}}(\tilde{u_i}'(r)) d\tilde{u_i}'(r) \right. \\ \left. + \int_p^{+\infty} \left[\frac{\partial x_i^G}{\partial p}(\tilde{u_i}'(x_i^G) - p + \pi^{RD}\eta_i) - x_i^G \right] f_{\tilde{u}}(\tilde{u_i}'(r)) d\tilde{u_i}'(r) \right] f_{\Lambda_i}(\Lambda_i) d\Lambda_i$$

By FOC^G and FOC^L,

$$
\begin{aligned}
&= \int_0^{+\infty} \left[\int_{-\infty}^{p-\Lambda_i} \left[\frac{\partial x_i^L}{\partial p} - ((1-\pi^{RD})\eta_i + (1-\pi^{LA})\Lambda_i) - x_i^L \right] f_{\tilde{u}}(\tilde{u_i}'(r)) d\tilde{u_i}'(r) + \int_{p-\Lambda_i}^{p} -r f_{\tilde{u}}(\tilde{u_i}'(r)) d\tilde{u_i}'(r) \right. \\
&\quad \left. + \int_p^{+\infty} \left[-\frac{\partial x_i^G}{\partial p}(1-\pi^{RD})\eta_i - x_i^G \right] f_{\tilde{u}}(\tilde{u_i}'(r)) d\tilde{u_i}'(r) \right] f_{\Lambda_i}(\Lambda_i) d\Lambda_i \\
&= \int_0^{+\infty} \left\{ E\left[-((1-\pi^{RD})\eta_i + (1-\pi^{LA})\Lambda_i) \frac{\partial x_i^L}{\partial p} | \tilde{u_i}'(r) < p - \Lambda_i, \Lambda_i \right] P(\tilde{u_i}'(r) < p - \Lambda_i | \Lambda_i) - E\left[x_i | \Lambda_i \right] \right. \\
&\quad \left. - E\left[(1-\pi^{RD})\eta_i \frac{\partial x_i^G}{\partial p} | \tilde{u_i}'(r) > p, \Lambda_i \right] P(\tilde{u_i}'(r) > p | \Lambda_i) \right\} f_{\Lambda_i}(\Lambda_i) d\Lambda_i
\end{aligned}
$$

By law of iterated expectations,

$$
\begin{aligned}
\frac{\partial \tilde{W}}{\partial p} &= E\left[-(1-\pi^{LA})\Lambda_i \frac{\partial x_i^L}{\partial p} | \tilde{u_i}'(r) < p - \Lambda_i \right] P(\tilde{u_i}'(r) < p - \Lambda_i) - E\left[(1-\pi^{RD})\eta_i) \frac{\partial x_i^G}{\partial p} | \tilde{u_i}'(r) > p \right] P(\tilde{u_i}'(r) > p) \\
&\quad - E\left[x_i \right] - E\left[(1-\pi^{RD})\eta_i) \frac{\partial x_i^L}{\partial p} | \tilde{u_i}'(r) < p - \Lambda_i \right] P(\tilde{u_i}'(r) < p - \Lambda_i) \\
&= \frac{\partial W}{\partial p} - E\left[(1-\pi^{RD})\eta_i \frac{\partial x_i}{\partial p} \right] \quad \text{as } \frac{\partial x_i}{\partial p} = 0 \text{ in the } R \text{ group.}
\end{aligned}
$$

Which yields the proposition result after two first-order Taylor approximations.

Proposition 6. First-Order Social Welfare Effects in the 2-D Model.
In this model, welfare is given by:

$$
w_i(p,r) = u_i(x(p,r)) + z_i - p_x + \pi \left[1\{x(p,r) < r\} \Lambda_i (x(p,r) - r) + 11\{x(p,r) > r\} \Gamma_i p (z_i - p x(p,r) - s(r)) \right] \tag{35}
$$

Note that we presume $s(r) = z_i - pr$ but we disregard $\partial s / \partial p$, so that a change in p does not effect the reference point s and cause a direct effect. Taking derivatives of this expression with respect to r or p for the G, R, L, substituting for $u_i'(x)$ using the FOCs for the G and L case from equations (23) and (24), and integrating over the three first-order groups for social welfare, we obtain the desired results.

C The Behavioral Equivalence of Alternative Formulations

In the main text, we considered two formulations of reference dependence. A key fact for our analysis was that these models are behaviorally indistinguishable using observed choices (i.e. demand at given prices and reference points), but that they carry somewhat different implications for welfare. In this Appendix, we formalize the sense in which the models are behaviorally indistinguishable. Crucially, the main reason that these models are indistinguishable is that we assume that choices of x given a reference point (and price and endowment), but we do not observe choices or revealed preferences over chosen options and reference points, i.e. (x, r), jointly. This assumption is consistent with what is observed in typical applications of models of reference-dependent preferences, but it might be relaxed in more stylized experiments. We note that a similar equivalence, implying that the η_i parameter is typically unidentified, is shown for the stochastic case in Barseghyan et al. (2013).

In Section 2 of the main text, we mainly analyzed the following model of behavior, which we will here call *Model 1*:

$$x_i(p,r,z) = \arg\max_x u_i(x) + z - px + 1\{x < r\}\Lambda_i(x-r), \tag{36}$$

for $u_i' > 0, u_i'' < 0$, and $\Lambda_i > 0$.

In Section 4.2 and very briefly earlier on in equation (3), we considered an alternative model in line with the formulation of reference dependence proposed by Tversky and Kahneman (1991), which we here call *Model 2*.

$$x_i(p,r,z) = \arg\max_x \tilde{u}_i(x) + z - px + \begin{cases} \eta_i(x-r) & x > r \\ \eta_i\lambda_i(x-r) & x \leq r, \end{cases} \tag{37}$$

for $\tilde{u}_i' > 0, \tilde{u}'' < 0$, $\eta_i > 0$, and $\lambda_i > 1$.

Consider a demand function $x(p,r,z)$, which describes the choice of x the consumer makes for any (p,r,z). We say $x(p,r,z)$ is *rationalizable* with either model if there are utility functions and parameters such that the optimization problem the model describes generates the observed behavior for any (p,r,z). That is, $x(p,r,z)$ is rationalizable by model 1 if and only if there is a utility function $u(x)$ with $u' > 0, u'' < 0$ and a parameter $\Lambda_i > 0$ such that for any (p,r,z) equation (36) obtains. We say $x(p,r,z)$ is rationalizable by model 2 under similar conditions.

We make one more modest technical assumption for our desired result to obtain, which is that the domain of good x is compact. In Model 1, this ensures that $u'(x)$ has a strictly positive minimum for all values of x, which we denote $\epsilon \equiv \min u'(x)$. The assumption ensures $\epsilon > 0$ exists. Why we need this assumption will become clear in the proof of the result below.

Proposition 8. ***Behavioral equivalence of Model 1 and Model 2.*** *A demand function $x_i(p,r,z)$ is rationalizable by Model 1 if and only if it is rationalizable by Model 2.*

Corollary 8.1. ***A behavioral isomorphism.*** *If $x_i(p,r,z)$ is rationalizable by model 1 with utility $u_i(x)$ and parameter Λ_i and rationalizable by model 2 with utility $\tilde{u}_i(x)$ and parameters η_i, λ_i, then we must have*

$$u_i(x) = \tilde{u}_i(x) + \eta_i x. \tag{38}$$

$$\Lambda_i = \eta_i(\lambda_i - 1). \tag{39}$$

Proof. First suppose that $x_i(p,r,z)$ is rationalizable by model 1 with some utility $u_i(x)$ and parameter Λ_i.

Set any η_i such that $0 < \eta_i < \epsilon$.[39] Specify $\tilde{u}_i$ according to equation (38), i.e. $\tilde{u}_i = u_i(x) - \eta_i x$. Specify λ_i according to equation (39), i.e. $\lambda_i = \frac{\Lambda_i + \eta_i}{\eta_i}$.

Because $u' > \eta_i$ for any x by construction, we know that $\tilde{u}_i' = u_i' - \eta_i >= u_i' - \varepsilon > 0$, and $u'' < 0 \implies \tilde{u}_i'' < 0$. Further, by construction $\eta_i > 0$ and $\lambda_i > 1$. With the necessary restrictions satisfied, we only need to show that with these specifications, the optimization problem in equation (36) is equivalent to the optimization problem in (37). As we have guaranteed equations (38) and (39) hold, we can re-express the optimization problem in model 1 as:

$$x_i(p,r,z) = \arg\max_x \tilde{u}_i(x) + \eta_i x + z - px + 1\{x < r\}\eta_i(\lambda_i - 1)(x-r), \tag{40}$$

Next note that as it has no effect on the optimal x, we may freely subtract $-\eta_i r$ from the maximand. Doing so and re-arranging yields Model 2.

[39] The fact that we can choose such an arbitrary η_i in this step is related to the fact that η_i is typically unidentified from observations of observed demand.

For the converse, suppose that $x_i(p,r,z)$ is rationalizable by model 2 with utility function $\tilde{u}_i(x)$ and parameters $\eta_i > 0$, and $\lambda_i > 1$. Specify $u_i(x)$ using equation (38) and set Λ using (39). Checking the restrictions, we know that $\tilde{u}_i' > 0$ $\eta_>0$, implying that $u_i' = \tilde{u}_i' + \eta > 0$, and $u_i'' = \tilde{u}_i'' < 0$. And we know that $\Lambda_i > 0$ by $\eta_i > 0$ and $\lambda_i > 1$. We can re-express the optimization problem in Model 2 as

$$x_i(p,r,z) = \arg\max_x \tilde{u}_i(x) + \eta_i x + z - px + 1x > r\eta_i(\lambda - 1)(x - r) - \eta r. \tag{41}$$

The last term has no bearing on the optimum so we can eliminate it. Applying our constructed $u_i(x)$ and Λ_i then yields Model 1. □

D Relationship to Bernheim and Rangel (2009)

Bernheim and Rangel (2009) propose a general framework for decision-theoretic behavioral welfare economics. This Appendix describes in detail the relationship between our analysis and this framework. We focus on mapping the model in Section 2 into the Bernheim-Rangel framework; a similar line of reasoning can be applied to the extended models in Section 4.

The first step in applying this framework is to conceive of an observed choice in terms of a menu and an ancillary condition, or *frame* (denoted by f) - see also Bernheim and Taubinsky (2018). In describing this process in Bernheim and Taubinsky (2018), the authors write that frames should be those aspects of the choice situation that "have no direct bearing on well-being, but that instead impact biases."

What are the frames in our context? A naive guess might be that the reference point itself is a frame, but based on the definition above, this seems inappropriate. We showed in the main text that a change in the reference point can have a direct welfare effect - by changing the losses of individuals in the loss domain. Whether this direct effect should carry normative weight is a question of central importance for us, but this question belongs to a later step of the analysis, not the definition of a frame. Similarly, the theory implies that individuals should have a willingness to pay to change the reference point, suggesting that it may have a direct bearing on well-being. As such, we do not conceive of the reference point as a frame. A similar justification is used by Bernheim et al. (2015) in their application of this framework to the welfare economics of default options, to justify the treatment of the default as a component of the menu rather than a frame.

Nevertheless, there is a formal sense in which our results can be interpreted within the Bernheim-Rangel framework, which we now describe. First, we suppose that what we called observed demand in our analysis comes from choices under a single frame, f_1. This frame is analogous to what Bernheim et al. (2015) call a "naturally occurring frame." Under the frame f_1, the individual reveals preferences consistent with the utility function in equation (1), which we re-write here:

$$u(x,y,r,f_1) = u_i(x) + y + v_i(x|r), \tag{42}$$

where v_i takes the simple form described in equation (2).

In order to map our analysis into the Bernheim-Rangel framework, we need to consider a hypothetical choice situation in which reference dependence were eliminated. If we wish to consider the possibility that reference dependence may be a bias, what preferences would be revealed by choices in an unbiased state? We represent choices made in a no-reference-dependence state by encoding a frame f_0. Choices under f_0 maximize

$$u_i(x,y,r,f_0) = u_i(x) + y. \tag{43}$$

Obviously, choices under f_0 are difficult to directly observe in positive empirical analysis, but the applica-

tion of the Bernheim-Rangel framework does not require that all relevant parts of the choice correspondence are empirically observable. Choices under f_0 could potentially be observed by eliminating the effect of the reference point through some experimental intervention, or by inducing individuals to use an arbitrarily low reference point (recall that $v_i = 0$ in the gain domain).

Note that setting $f_1 = 1$ and $f_0 = 0$, we can represent choices in either frame $f \in \{0, 1\}$ by:

$$u_i(x, y, r, f) = u_i(x) + y + f * v_i(x|r), \tag{44}$$

The frame f now obviously plays a very similar role in the model to π, but here we are conceiving of the two different frames purely in terms of choices in different situations.

The second step in applying the framework is to designate a subset of choice situations as the *welfare-relevant domain*, i.e. situations from which we wish to take normative inference. There are three intuitive possibilities for the welfare relevant domain, each of which reflects a normative judgment:

- (J1) include only choices under the naturally occurring frame ($f = 1$),
- (J2) include only choices under the no-reference-dependence frame ($f = 0$), or
- (J3) include choices under both frames.

The third step in the analysis is then to consider what revealed preferences are consistently expressed for choices within the welfare-relevant domain. If a is chosen when b is available for some situation in the welfare-relevant domain, and b is never chosen when a is available for other such situations, then we conclude that a is preferred to b.

If we interpret our results within the Bernheim-Rangel framework, the content of the results is mainly to show how the these alternative judgments about the welfare-relevant domain influence welfare and optimal policy considerations. Under (J1) or (J2), there is a single utility function (either equation (42) or equation (43)) that ranks all options in the menu space (i.e. all combinations of (x, y, r)). Under (J3), however, we obtain only an incomplete ranking. Recall that we used the word "robust" above to describe situations where whether one situation or the other was better for welfare did not depend on π. Our results map into the Bernheim-Rangel framework as follows:

- (J1) Restricting the welfare-relevant domain to choices under $f = 1$ is equivalent to judging $\pi = 1$
- (J2) Restricting the welfare-relevant domain to choices under $f = 0$ is equivalent to judging $\pi = 0$.
- (J3) Including both $f = 0$ and $f = 1$ in the welfare relevant domain is equivalent to only taking welfare inference from robust welfare comparisons, i.e. those under which some option (x_0, y_0, r_0) is preferred to some other option (x_1, y_1, r_1) for any $\pi \in \{0, 1\}$.

As discussed in the main text, we find that decreases in the reference point tend to improve welfare for either value of π. Through the lens of the Bernheim-Rangel framework, this suggests that even if we include choices under f_1 and f_0 in the welfare relevant domain (J3) and use the revealed preference criterion proposed by Bernheim and Rangel, we would conclude that individuals prefer lower reference points.

Note that because $v_i = 0$ in the gain domain, we find that holding the reference point fixed, equations (42) and (43) express the same preferences in the gain domain. Moreover the reference point has no direct on welfare in the gain domain for either value of f or π. If we restrict our attention to individuals making choices in the gain domain, therefore, all welfare comparisons will be robust. The only potential

deviations from revealed preference come from individuals choosing at the reference point or in the loss domain. This finding is the basis for the statement in Section 2 that we respect revealed preference in the gain domain. The potential deviations from revealed preferences in the naturally occurring frame that we consider are material for individuals at the reference point or in the loss domain only. Obviously, the extended models in Section 4 do not necessarily have this property. Importing those extended models requires modifications to the above - for instance we would need to introduce three frames rather than two to import the model in Section 4.2 into the Bernheim-Rangel framework - but the general structure of how we could give those models a behavioral revealed preference interpretation remains the same.

E Empirical Application

E.1 Decomposing Reference Dependence Payoffs

Besides fiscal effects and effects on standard utility components, we calculate the effects of policies on reference dependence payoffs in the simulations. An individual's total reference dependence payoffs are given by

$$v(R|\hat{R}) = -\begin{cases} 0 & R < \hat{R} \\ \widetilde{\Lambda}(R-\hat{R}) & R \geq \hat{R}, \end{cases}$$

where R is the individual's retirement age and $\hat{R}$ is the reference point given by the Normal Retirement Age. We further decompose reference dependence payoffs into additional disutility from work due to reference dependence and direct utility from the reference point. The first component, reference dependence disutility from work, is

$$v_b(R|\hat{R}) = -\begin{cases} \widetilde{\Lambda}\hat{R}_0 & R < \hat{R} \\ \widetilde{\Lambda}R & R \geq \hat{R}, \end{cases}$$

The second component, reference dependence utility from the reference point itself, is

$$v_d(R|\hat{R}) = -\begin{cases} \widetilde{\Lambda}(-\hat{R}_0) & R < \hat{R} \\ \widetilde{\Lambda}(-\hat{R}) & R \geq \hat{R}, \end{cases}$$

Note that we introduce a "base age" $\hat{R}_0$ given by the pre-reform NRA in the case $R < \hat{R}$. This choice is inconsequential for overall welfare effects, since $v_b + v_d = v$ for any base age. However, anchoring v_b and v_d at the initial reference point $\hat{R}_0$ allows to avoid introducing a jump discontinuity in v_b and v_b at $R = \hat{R}$, which would complicate the calculation of direct versus behavioral welfare effects for individuals moving between gain and loss domains relative to $\hat{R}$.

E.2 Two-Dimensional Reference Dependence in the Empirical Application

E.2.1 Two-Dimensional Model

In our empirical application, besides reference dependence over leisure, there could also be reference dependence in the consumption dimension. We can modify the preferences from equation (17) to include consumption reference dependence:

$$U = C - \frac{n}{1+\frac{1}{\epsilon}}\left(\frac{R}{n}\right)^{1+\frac{1}{\epsilon}} - \begin{cases} 0 & R < \hat{R} \\ \widetilde{\Lambda}_l(R-\hat{R}) & R \geq \hat{R}, \end{cases} - \begin{cases} \Lambda_c(\hat{C}-C) & C < \hat{C} \\ 0 & C \geq \hat{C}, \end{cases} \tag{45}$$

where $\hat{C} = C(\hat{R})$ is the consumption reference point, which is assumed to correspond to the consumption level at the NRA $\hat{R}$. The parameter Λ_l captures the strength of reference dependence over leisure and Λ_c captures the strength of reference dependence in the consumption dimension.[40] Such loss aversion in consumption may arise for instance because "full" pension benefits become available at the NRA, and individuals perceive the associated consumption level as a reference point (Behaghel and Blau 2012).[41]

[40] Λ_c implies additional marginal utility from consumption in the loss domain below $\hat{C}$. For instance, $\Lambda_c = 0.5$ corresponds to 50% higher marginal utility from consumption in the loss domain than in the gain domain.

[41] Whether "full" pension benefits become available at the NRA depends on the specifics of the pension system. In the German setting, full benefits become available at the Full Retirement Age, which is in principle distinct from the NRA. However, for most

As in the one-dimensional case, the two-dimensional model predicts bunching at the NRA. However, a crucial difference between the two models lies in the direction of predicted bunching. While reference dependence over leisure induces workers to retire earlier in order to enjoy more leisure, reference dependence over consumption induces individuals to postpone retirement and increase consumption. This occurs because the consumption loss domain is the range of consumption levels and associated retirement ages below the NRA, whereas the loss domain over leisure is above the NRA. Thus, reference dependence over leisure leads to *bunching from above*, but reference dependence over consumption leads to *bunching from below*. Figure 9 illustrates the predicted effect of the two dimensions of reference dependence on the retirement age distribution. Reference dependence over leisure implies a shift in the distribution towards the NRA from above, while reference dependence over consumption leads to a shift in the distribution towards the NRA from below. A combination of the two would imply a shift towards the reference points from both sides. As we argue in Section 4.1.3, the empirically observed retirement age distribution around the NRA suggests that reference dependence over leisure dominates reference dependence over consumption.

The marginal bunching individual from above can be characterized as in Section 3.2. The upper marginal buncher's indifference curve would be tangent to the budget line at some retirement age R_+^* without reference dependence, and another indifference curve is tangent exactly at $\hat{R}$ with reference dependence. All workers initially located between $\hat{R}$ and R_+^* bunch at the reference point from above, while all individuals initially to the right of R_+^* decrease their retirement age but stay above the reference point. The two tangency conditions for the upper marginal buncher imply $R_+^* = n_+^*[w(1-\tau)]^\varepsilon$ and $\hat{R} = n_+^*[w(1-\tau-\Delta\tau-\Lambda_l)]^\varepsilon$, where n_+^* denotes her ability level and $\Lambda_l = \widetilde{\Lambda}_l/w$ is the reference dependence parameter normalized by the wage per period. Hence,

$$\frac{R_+^*}{\hat{R}} = \left(\frac{1-\tau}{1-\tau-\Delta\tau-\Lambda_l}\right)^\varepsilon$$

Similarly, a marginal bunching individual from below can be identified. The lower marginal buncher's indifference curve would be tangent to the budget line at R_-^* without reference dependence, and tangency occurs exactly at $\hat{R}$ with reference dependence. All workers initially located between R_-^* and $\hat{R}$ bunch at the reference point from below, while all individuals initially to the left R_-^* retire later but stay below the reference point. The two tangency conditions of the lower marginal buncher are $R^* = n^*[w(1-\tau)]^\varepsilon$ and $\hat{R} = n^*[(1+\Lambda_c)w(1-\tau)]^\varepsilon$, where n_-^* denotes her ability level. Hence,

$$\frac{R_-^*}{\hat{R}} = \left(\frac{1}{1+\Lambda_c}\right)^\varepsilon$$

The total excess mass $b = B/h_0(\hat{R})$ is

$$\frac{b}{\hat{R}} = \left[\left(\frac{1-\tau}{1-\tau-\Delta\tau-\Lambda_l}\right)^\varepsilon - 1\right] + \left[1 - \left(\frac{1}{1+\Lambda_c}\right)^\varepsilon\right] \tag{46}$$

Hence, bunching has two components. The first term in equation (46) captures bunching from the right (from above) due to the retirement age/leisure reference point in combination with a potential budget set kink present at the threshold. The second term in the equation captures bunching from the left (from below) due to the consumption reference point.

workers among birth cohort 1946 on whom we focus in the simulations, the NRA and FRA coincide and thus full benefits become available at the NRA.

E.2.2 Parameter Estimation and Simulations

Analogously to equation (19), bunching observed at a threshold i, which may be the Normal Retirement Age or a pure financial incentive discontinuity, can be written as

$$\frac{b_i}{\hat{R}_i} = \left[\left(\frac{1-\tau_i}{1-\tau_i-\Delta\tau_i-\Lambda_l\cdot D_i}\right)^{\varepsilon}-1\right]+\left[1-\left(\frac{1}{1+\Lambda_c\cdot D_i}\right)^{\varepsilon}\right]+\xi_i \tag{47}$$

where D_i is an indicator for the Normal Retirement Age and ξ_i is an error term. A key issue with the estimation is that Λ_l and Λ_c cannot be separately identified based solely on equation (47). Intuitively, both retirement age and consumption reference points lead to sharp bunching at the threshold $\hat{R}$ such that a given amount of excess mass could be rationalized by a range of combinations of Λ_l and $Llambda_c$.

In order to make progress, it is useful to write the two components of excess mass separately. Bunching from the right is

$$\frac{b_i^+}{\hat{R}_i} = \left[\left(\frac{1-\tau_i}{1-\tau_i-\Delta\tau_i-\Lambda_l\cdot D_i}\right)^{\varepsilon}-1\right]+\xi_i^+ \tag{48}$$

and bunching from the left is

$$\frac{b_i^-}{\hat{R}_i} = \left[1-\left(\frac{1}{1+\Lambda_c\cdot D_i}\right)^{\varepsilon}\right]+\xi_i^- \tag{49}$$

where $b_i = b_i^+ + b_i^-$. Denoting $\alpha_i = b_i^+/b_i$ the share of excess mass originating from the right, this share ranges between a minimum $\hat{\alpha}_i$ and 1. The minimum right bunching share $\hat{\alpha}_i$ is given by the fraction of bunching that would persist if workers only bunch due to the budget constraint kink.

We follow two approaches in order to obtain joint estimates of Λ_l and Λ_c. First, we can simulate the full range of possible combinations of the two parameters by gradually moving the share of right bunching at the NRA from its minimum to 1 and estimating equations (48) and (49) using the implied values of b_i^+ and b_i^-. Panel (a) of Appendix Figure A3 shows resulting parameter combinations. The negative slope of the relationship illustrates the intuition that the two types of reference dependence are substitutes in terms of rationalizing observed excess mass. The labeled dots in in the figure mark a range of implied left bunching shares between 0 and 50%. These results allow us to simulate the welfare effects of pension reforms as a function of the relative strength of consumption reference dependence, which are shown in Figure 10.

As a second approach, we aim at obtaining a set of preferred "point" estimates of Λ_l and Λ_c. For this, an empirical estimate of α_i is needed. We argue that the empirical retirement age distribution around the NRA is informative of the relative magnitude of bunching from the two sides, and can be used for this purpose under some additional assumptions. In particular, bunching shares from both sides can be computed based on estimates of the corresponding density shifts. Intuitively, we assume the counterfactual density to be continuous around the NRA, and infer the relative number of bunchers from the left and from the right from the vertical difference between the counterfactual density and the actually observed density on both sides of the threshold. This estimation requires a stronger assumption about the true relative density shifts being reasonably well approximated by locally observed relative shifts.

We begin with the observation that bunching at the threshold must equal the total missing density from both sides:

$$B = \int_{R_{min}}^{\hat{R}} \left(h_0(R)-h(R)\right)dR + \int_{\hat{R}}^{R_{max}} \left(h_0(R)-h(R)\right)dR$$

where R_{min} and R_{max} are the minimum and maximum counterfactual retirement ages from which individuals bunch at the NRA.

Measuring the true density shift over the full support is impossible in practice for two reasons. First, the shift $h_0(R)-h(R)$ may vary across R in an unknown way so that $h_0(R)$ cannot be measured for all R

based on the observed density. Second, the full support of the counterfactual density may not be observed. Even if the full support of the actual density could be observed, this does not necessarily correspond to the counterfactual support since some counterfactual density is predicted to "disappear" at the bounds because all individuals shift out a certain range.[42]

One solution to this problem is to approximate the true density shift by a constant shift over a certain range on each side. Denote by h_+ and h_- the observed density immediately to the right and left, respectively, of the threshold $\hat{R}$. Furthermore, denote by h_+^0 and h_-^0 the corresponding counterfactual density in the absence of the threshold. The approximation is

$$B \approx \left(h_-^0 - h_-\right)\left(\hat{R} - R^-\right) + \left(h_+^0 - h_+\right)\left(R^+ - \hat{R}\right)$$

where a constant density shift observed immediately to the left of the threshold over a range $[R^-, \hat{R}]$ approximates for the true shift on the left and a constant shift observed immediately to the right of $\hat{R}$ over $[\hat{R}, R^+]$ approximates for the shift on the right.

Assume also that the counterfactual density is continuous at $\hat{R}$ such that $h_+^0 = h_-^0 = h_0$. Then h_0 can be recovered as

$$h_0 \approx \frac{B + (\hat{R} - R^-)h_- + (R^+ - \hat{R})h_+}{R^+ - R^-}$$

From this, the implied bunching shares from both sides can be computed as $B^- = (h_0 - h_-)(\hat{R} - R^-)$ and $B^+ = (h_0 - h_+)(R^+ - \hat{R})$ since bunching from either side must be equal to the total density shift on that side.

Panel (b) of Appendix Figure A3 illustrates this procedure. The solid red line shows the average empirical retirement density on both sides in a window of +/-2 years around the NRA, h_+ and h_-. The dashed red line shows the implied counterfactual density h_0 calculated as described above. The figure shows that the difference between the observed density and the counterfactual density is much larger on the right, indicating that most "missing density" is on this side, and thus most bunching appears to originate from above. We obtain an estimate of $\alpha_i = 0.867$. Thus, the estimated share of bunching from the right due to reference dependence over leisure is 86.7% and the share of bunching from the left due to reference dependence over consumption is 13.3%. Finally, the parameters Λ_c and Λ_l can be estimated by plugging the bunching shares into equations (48) and (49). We obtain estimates of $\Lambda_c = 0.672$ and $\Lambda_l = 0.457$. The simulations shown in Table 2 are conducted based on these parameter estimates.

[42] Besides, although theory predicts individuals responding to the threshold along the entire density in principle, it is unclear in practice whether those far from the threshold respond in the same way as those closer.