Debt Moratorium: Theory and Evidence[1]

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———— PRELIMINARY AND INCOMPLETE ————

Abstract

We provide a theory on the effects of debt moratorium policy, which stipulates the suspension of payments during stress episodes. The theory predicts an increase in loan rates while loan amounts depend on demand and supply elasticities. To empirically corroborate these findings and using Colombia as case study at the onset of the 2020 pandemic, we conduct a regression discontinuity design and compare the lending behavior of banks to firms that barely met and missed the criteria of receiving the treatment and investigate how real variables such as investment and labor are affected. We finally shed some theoretical insights on the effects of short and long-run gains and losses of debt moratorium policies with a quantitative default model.

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1 Introduction

Debt moratorium policies, which stipulate payment suspensions or "grace periods" for both households and firms, have been widely discussed in the literature to alleviate debt crisis. However, the enactment of these policies only materialized at the onset of the 2020 pandemic crisis. Particularly, in response to a mix of demand and supply shocks, governments around the globe reached out to the public with numerous credit alleviation measures to troubled borrowers. One common policy among nations, including advanced and emerging economies, is on forbearance measures (see Appendix A for a summary of such enactments among nations).

Our paper first theoretically predicts the effect of debt moratorium policies in a three period closed-economy model. We prove that the policy increases the interest rate on the loan while the effect on the amount is indeterminate. Intuition relies on banks' unwillingness to supply loans when resources are needed the most while firms demand more loans.

On the empirical side, the key challenge for a causal inference is that the policy is a response to the pandemic, which is an unprecedented macroeconomic shock to the economy. Therefore, in our empirical analysis we have to disentangle the causal effects of the policy from the pandemic shock. Our identification strategy relies on the rich borrower heterogeneity in the data. In particular, we will exploit a discontinuity in the following qualification criteria to be eligible for debt suspensions policy: the borrower must have had (*i*) no payment default as of 1 February 2020 or (*ii*), if a payment default existed as of 29 February 2020, the date of the policy enactment, it should not exceed 60 days. Thus, we will compare firms who had a payment default right before January 1^{st} with the ones who had a payment default between January 1^{st} and February 29^{th} employing highly granular Colombian data. With this, we can identify firms who barely missed the criteria of the debt suspension policy with the ones who barely made it. Therefore, any change in our outcome variables can be attributed to the policy.

Lastly, we provide an endogenous quantitative model to gain insights on the effects of suspension policies that cannot be otherwise analyzed in the empirical and theoretical section and measure the short and long-run gains and losses of such policies. We, also, further investigate an optimal haircut on the loan in terms of a reduction in the existing interest rate that the loan bears.

2 A three-period model without default

In this three period model, we consider one-good, closed economy with competitive lenders and firms. Firms and lenders differ in their initial endowment as well as in their preferences so that intertemporal trade can exist. In particular, firms have zero endowment in the first period, that is, $y_1 = 0$ and they discount the future at rate $\beta_f < 1$ while banks discount rate is taken to be unity for simplicity.[1] In period 2 and 3, the firm is endowed with a unit of good. In period 2, the firm faces a shock where it incurs a loss l portion of its endowment with a probability π. The utility function for both the bank and the firm is assumed to take the form $u(c) = Ac + \phi c^2$.

The only option for the government to finance its consumption for the first period is to borrow. A long-term debt b with a price q issued in the first period promises δ units of good in the second period and the remaining $(1-\delta)$ unit of good is paid in period 3.

The maximization problem of the firm without the debt moratorium policy can be written as

$$\max_{b} \; u\left(qb\right) + \beta\left[(1-\pi)u\left(1-\delta b\right) + \pi u\left(1-\delta b - l\right)\right] + \beta^2 u\left(1-(1-\delta)\,b\right) \tag{1}$$
$$\text{subject to } c \geq 0.$$

The FOC with respect to b with $\delta = 1/2$ yields:

$$b(q): \quad \frac{4qA - 2(A-\phi)\beta(1+\beta) - 2\beta\pi\phi l}{4\phi q^2 + \phi\beta(1+\beta)}. \tag{2}$$

And in an economy with the debt moratorium policy, the firm's repayments are deferred to the next period, Thus, instead of paying δb this period, the firm is going to pay all of its debt b in the next period.[2] The maximization problem of the firm becomes

$$\max_{b^p} \; u(qb) + \beta\Big[(1-\pi)u(1-\delta b^p) + \pi u(1-l)\Big] + \tag{3}$$
$$\beta^2\Big[(1-\pi)u(1-(1-\delta)b^p) + \pi u\left(1-b^p\right)\Big]$$
$$\text{subject to } c \geq 0.$$

We can now write the lenders' maximization problem as follows.

[1] What matters is that banks' discount rate should be bigger than firms' in order to have an intertemporal trade.

[2] For simplicity, we assume that the deferred payments do not accrue interest rate.

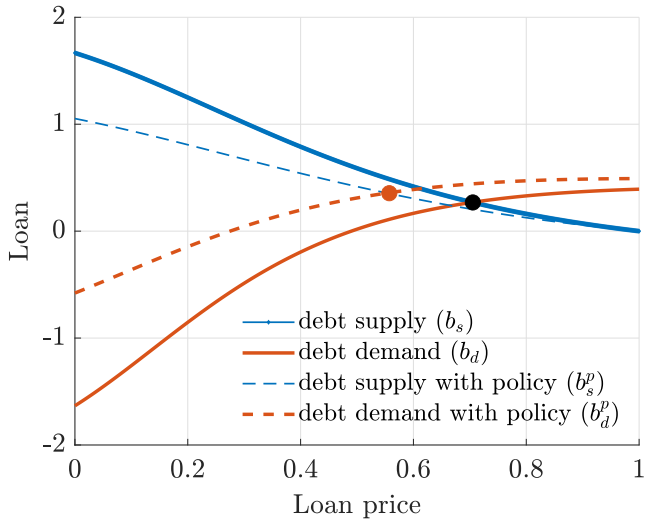


Figure 1: Demand and supply of loans with and without the policy.

Theorem 1 *Let $b_s^p(q)$ and $b_s(q)$ denote debt supplied to firms by lenders with and without the policy, respectively. Given that both firms and lenders have quasi-linear utility such that $u(c) = Ac + \phi c^2$, the amount of debt lenders are willing to lend to firms is lower for the same price q under the debt moratorium policy. That is, $b_s^p < b_s$. And let $b_d^p(q)$ and $b_d(q)$ denote debt demanded by firms with and without the policy, respectively. Firms demand more debt from lenders for the same price q under the policy. That is, $b_d^p > b_d$.*

Details of the proof is relegated to Appendix. Intuitively, lenders are not willing to lend to firms when they need the resources the most. Figure 1 visualizes this intuition. For each level of price (which is inversely related with the interest rate) lenders are willing to save more and asks for a higher interest rate to meet the exact demand of the firms without the policy. Firms, on the other hand, are willing to borrow more for each level of interest rate.

3 The Colombian Case

3.1 Matching Firm- and Bank-level data

In our empirical exercises, we use highly granular data, comprising the entire Colombian credit registry (at the loan level) from 2019 to 2020. This database, from the Financial Superintendency (*Superintendencia Financiera de Colombia, Formato 341*), contains over 4.4 million observations, with information on all loans extended to corporates, such as interest rate, loan amount, maturity, issuance date, expiration date, delinquency rate, and ex-ante

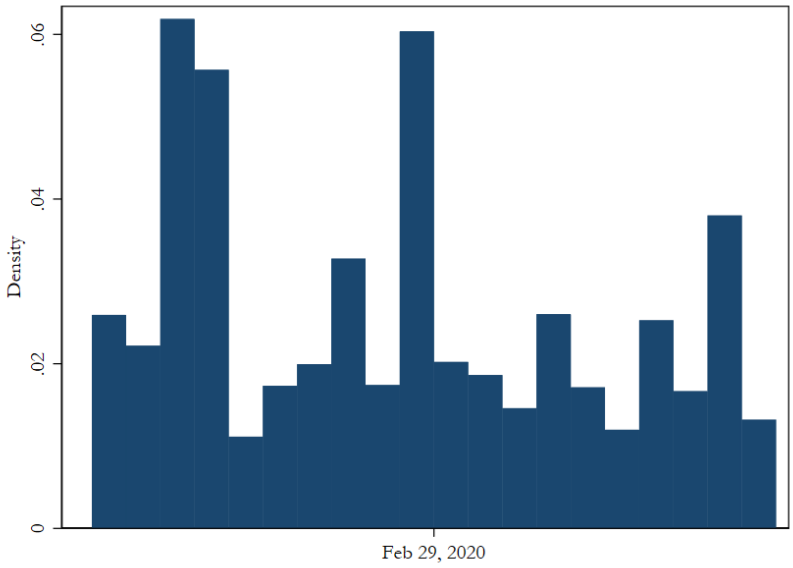
credit rating. We merge these data with debt moratorium data to have information on the loans that were benefit by the government program and with quarterly firm-level balance sheet information from the Corporate Superintendency (*Superintendencia de Sociedades*) in order to include firm-specific variables such as asset size, liabilities, profits, wages, investment, and equity. We obtain data on employment from the Colombian Department of Labor, although only for the second half of the sample, as per data availability. After merging these sources, we match 563.000 loans, a total of 37 private banks and 9,000 firms.

Given that our unit of measurement consists of new loans disbursed from bank j to firm i, we observe 420,000 new loans. Also, as a fundamental part of the study, we use private banks' total bond holdings of sovereign debt (*Títulos de deuda pública — TES*).

To give some initial context, an average Colombian bank has 3,400 new loans with different firms per month. However, large banks, with assets in the top 75th percentile of the banking system, account for 44% of the bank-corporate relationship. Additionally, a large bank has, on average, $2.4 trillion COP (0.24% of GDP) in government bonds, whereas the banking average reports an amount of $1.78 trillion.

3.2 Contextual characteristics

Figure 2: Histogram days past due



3.3 Identification

On the empirical side, the key challenge for causal inference is that the policy measure directly responded to the pandemic, which amounted to an unprecedented global shock. Therefore, the pandemic acts as a confounding factor that also affects loan and corporate variables. Hence, our identification strategy relies on the rich borrower heterogeneity in our data, which allow us to compare similar firm-bank relationships with and without treatment. In particular, we exploit a discontinuity in the eligibility criterion according to how the Colombian regulation was enacted: eligible borrowers could net exceed 60 past due days on their credit as of the 29^{th} of February 2020. Given that the alleviation measures started in the second quarter of 2020, the assignment variable is free from a simultaneity bias. Thus, we compare credit conditions of firms who barely met and missed the criterion in subsequent quarters.

Employing highly granular data and using Colombia as case study, we empirically evaluate the effects of these policies within a localized approach. Specifically, we exploit a rule-based mechanism embedded in the credit relief criteria, in which treatment eligibility was based on a number of past-due (non-performing) days below a given threshold. For identification, we argue that borrowers just below and above this threshold are similar (and comparable) ex-ante across various types of financial variables, and mainly differ in receiving treatment. Hence, borrowers barely missing the cutoff serve as counterfactuals to those who barely crossed the cutoff, had they not received the loan moratorium. Distribution of firms that are barely eligible and ineligible are plotted in Figure 2.

Table 1: Rdd Sharp all loans

	Loan amount	Provision	Credit Rating	Days past due	Interest rate	Maturity
Bias-corrected	-0.304***	-0.179***	0.453***	-8.066**	7.219***	0.644***
	(0.0832)	(0.0404)	(0.0494)	(3.229)	(0.419)	(0.189)
Obs	477,845	477,620	477,620	471,726	454,564	477,620

Table 2: Rdd Sharp all loans - only grace periods

	Loan amount	Provision	Credit Rating	Days past due	Interest rate	Maturity
Bias-corrected	-0.306***	-0.185***	0.467***	-7.675**	7.463***	0.647***
	(0.0848)	(0.0414)	(0.0515)	(3.146)	(0.422)	(0.189)
Obs	464,897	464,691	464,691	458,867	441,829	464,691

Table 3: Rdd Sharp all loans - only treated who are eligible

	Loan amount	Provision	Credit Rating	Days past due	Interest rate	Maturity
Bias-corrected	-0.197**	-0.0872***	0.557***	-18.07**	5.092***	0.308
	(0.0993)	(0.0328)	(0.126)	(7.796)	(0.695)	(0.432)
Obs	120,781	120,556	120,556	118,064	115,933	120,556

Table 4: Rdd Sharp existing loans

	Loan amount	Provision	Credit Rating	Days past due	Interest rate	Maturity
Bias-corrected	-0.222***	-0.164***	0.448***	-9.816***	7.408***	0.647***
	(0.058)	(0.035)	(0.055)	(2.934)	(0.415)	(0.192)
Obs	423,391	423,391	423,391	423,391	402,055	423,391

Table 5: Rdd Sharp existing loans - only grace periods

	Loan amount	Provision	Credit Rating	Days past due	Interest rate	Maturity
Bias-corrected	-0.226***	-0.169***	0.447***	-9.469***	7.398***	0.669***
	(0.0591)	(0.0376)	(0.0558)	(2.986)	(0.415)	(0.194)
Obs	417,399	417,399	417,399	417,399	396,108	417,399

Table 6: Rdd Sharp existing loans - only treated who are eligible

	Loan amount	Provision	Credit Rating	Days past due	Interest rate	Maturity
Bias-corrected	-0.0861	-0.0363	0.640***	-20.18*	4.294***	0.200
	(0.0577)	(0.0299)	(0.188)	(10.96)	(0.892)	(0.448)
Obs	87,287	87,287	87,287	87,287	83,416	87,287

Table 7: Rdd Sharp new loans

	Loan amount	Provision	Credit Rating	Days past due	Interest rate	Maturity
Bias-corrected	0.0601	-0.0391*	0.789**	14.91	1.930	1.705**
	(0.0640)	(0.0234)	(0.326)	(20.22)	(2.368)	(0.848)
Obs	24,119	23,894	23,894	23,313	23,290	23,894

Table 8: Rdd Sharp new loans - only grace periods

	Loan amount	Provision	Credit Rating	Days past due	Interest rate	Maturity
Bias-corrected	-0.0292	-0.0297***	1.057***	10.66	2.667*	1.533*
	(0.0468)	(0.00885)	(0.264)	(10.96)	(1.562)	(0.858)
Obs	22,456	22,250	22,250	21,685	21,662	22,250

Table 9: Rdd Sharp new loans - only treated who are eligible

	Loan amount	Provision	Credit Rating	Days past due	Interest rate	Maturity
Bias-corrected	0.200	-0.0230	1.325**	-15.77	-3.375	0.935
	(0.147)	(0.0212)	(0.530)	(16.99)	(2.954)	(0.988)
Obs	9,685	9,460	9,460	9,128	9,375	9,460

Table 10: RDD fuzzy all loans - only treated who are eligible

	Loan amount	Provision	Credit Rating	Days past due	Interest rate	Maturity
Bias-corrected	-0.210**	-0.0913***	0.607***	-19.25**	5.574***	0.332
	(0.106)	(0.0343)	(0.137)	(8.274)	(0.761)	(0.461)
Obs	120,781	120,556	120,556	118,064	115,933	120,556

Table 11: RDD fuzzy existing loans - only treated who are eligible

	Loan amount	Provision	Credit Rating	Days past due	Interest rate	Maturity
Bias-corrected	-0.0886	-0.0374	0.691***	-21.44*	4.572***	0.211
	(0.0592)	(0.0307)	(0.203)	(11.62)	(0.951)	(0.472)
Obs	87,287	87,287	87,287	87,287	83,416	87,287

Table 12: RDD fuzzy new loans - only treated who are eligible

	Loan amount	Provision	Credit Rating	Days past due	Interest rate	Maturity
Bias-corrected	0.200	-0.0230	1.325**	-15.77	-3.375	0.935
	(0.147)	(0.0212)	(0.530)	(16.99)	(2.954)	(0.988)
Obs	9,685	9,460	9,460	9,128	9,375	9,460

4 A Quantitative Model of Debt, Real Sector, and Financial Sector with Payment Suspensions

A salient ingredient of our model is that firms require working capital financing to pay for imports of a subset of foreign investment as well as for a fraction of wage bills before production takes place.

4.1 Households

Households choose how much to consume and supply labor to maximize expected discounted utility streams, $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, L_t)$, where $0 < \beta < 1$ is the subjective discount factor, and c_t and L_t denote consumption and labor, respectively. The period utility function $u(.)$ is continuous, strictly increasing in consumption, strictly decreasing in labor, and strictly concave in both arguments. The expectation operator E_t is conditional on the information set available at period t.

Households receive a real wage per labor hour w_t, profits paid by the final goods producers π_t^f, and net proceeds obtained from banking activities π_t^b and transfers from the government (T_t). Formally, the household solves the following problem:

$$\max_{c_t, L_t} E_t \left[\sum_{t=0}^{\infty} \beta^t u(c_t, L_t) \right] \tag{4}$$

subject to the period budget constraint

$$c_t = w_t L_t + \pi_t^f + \pi_t^b + T_t. \tag{5}$$

Household preferences are governed by a utility function of the Greenwood et al. (1988) type, which ensures no wealth effect on labor supply.[3] In particular, we use the utility function of the form:

$$u(c_t, L_t) = \frac{\left(c_t - \frac{L_t^{1+\omega}}{1+\omega} \right)^{1-\sigma} - 1}{1-\sigma}, \tag{6}$$

[3]This formulation for preferences removes the wealth effect on labor supply and helps to explain key business cycle facts for small open economies. See, for example, Mendoza and Yue (2012).

where $\sigma > 0$ is the constant relative risk aversion, and $\omega > 0$ governs the (inverse) Frisch elasticity of labor supply. The optimal labor supply is given by

$$L_t^{\omega} = w_t. \tag{7}$$

4.2 Final Goods Producers

We formulate the final goods producers' problem following Aguiar et al. (2009). In particular, these firms maximize profits, π_t, net of depreciation and discounted at the country's borrowing rate:

$$E_0 \left[\sum_t \left(\frac{1}{1 + r_t^*} \right) \left(\pi_t^f - p_t^I \left(K_t - (1 - \delta) K_{t-1} \right) \right) \right], \tag{8}$$

where p_t^I is the price of the investment good, $I_t \equiv (K_t - (1 - \delta)K_{t-1})$ denotes investment, and π_t^f is the output net of labor and working capital costs. As in Neumeyer and Perri (2005) and Uribe and Yue (2006), costs include both wage and working capital costs, which means that the firms need to pay a θ fraction of the wage bills at the beginning of the period. Since the production takes place at the end of the period, firms borrow from the banks at an interest rate of r_t^*.[4] The working capital constraint thus determines producers' demand for loans:

$$\ell_t = \theta w_t L_t. \tag{9}$$

Note that the timing of production is an important ingredient of the model that enables the production economy to be linked with the banking sector. This assumption raises the need for banks as firms need to finance production inputs in advance. Taken together, the profits of the final goods producers are $\pi_t^f = A_t K_t^{\alpha^K} L_t^{\alpha^L} - w_t L_t - \theta(1 + r_t^*) w_t L_t$. The production technology is governed by a Cobb-Douglas function given by

$$Y_t = A_t K_t^{\alpha^K} L_t^{\alpha^L}, \tag{10}$$

where α^K is the output elasticity of capital, α^L is the output elasticity of labor, and A_t is the shock to the level of total factor productivity. They all operate under decreasing returns

[4] This working capital assumption is common in the literature. To name a few examples, see Mendoza and Yue (2012), Perez (2018), Önder (2019) and Mallucci (2015).

to scale, that is, $0 < \alpha^K + \alpha^L < 1$.[5] Producers of final goods take factor prices, w_t and p_t^I, and the country borrowing rate, r_t^*, as given and maximize their profits given by equation (8). Multiplying both sides of equation (7) by θL, we can obtain labor as a function of loan supply ℓ:

$$L_t = \left(\frac{\ell_t}{\theta}\right)^{\frac{1}{\omega}}. \tag{11}$$

If loan demand exceeds loan supply, then the employment adjusts (falls) to clear the loan market. This is one of the key frictions linking the banking sector to the real economy.

We next assume that firms perceive the TFP shock as i.i.d., which then renders an optimal level of capital as a function of the country borrowing rate as in Aguiar et al. (2009). That is, for a given r_t^* and a country's borrowing rule, there is a unique and constant level of optimal K_t. The simplification, without loss of generality, is that final goods producers are myopic, and therefore, their maximization problem satisfies $K \equiv K(r^*)$ and $I(r^*) = \delta K(r^*)$. This assumption is not as restrictive as it might appear, since in the sovereign's dynamic problem that we postulate below, r^* fluctuates endogenously from fluctuations in government external borrowing and in TFP, which then results in endogenously time-varying K_t and I_t.[6] Since we assume myopic firms, our problem can be reduced to a static profit maximization problem as in Mendoza and Yue (2012) and Na et al. (2018). With that, there is no precautionary saving from the firm's perspective.
We can now obtain optimal factor demand schedules with profit maximization:

$$\delta P_t^I = \alpha^K A_t K_t^{\alpha^K - 1} L_t^{\alpha^L} \tag{12}$$

$$w_t = \frac{1}{1 + \theta(1 + r_t^*)} \alpha^L A_t K_t^{\alpha^K} L_t^{\alpha^L - 1}. \tag{13}$$

Notice that the wage is equal to the marginal product of labor multiplied by an additional term that encompasses the financial costs of labor. This equation implies that tight conditions in the financial sector are equivalent to an adverse productivity shock.

[5] Under a decreasing returns to scale production technology, the firm makes positive economic profits (which are then transferred to households). Such a transfer of profits helps to better match the consumption dynamics for default episodes.

[6] Our assumption also boils down to the *non-contractibility of investment*, as laid out by Occhino and Pescatori (2010), that is, the firm cannot commit to an exact investment level when signing the contract, and afterward, it is free to choose investment to maximize its own objective function. Or similar in notion to Eggertsson and Krugman (2012), in the face of a realization of the shock in the next period, firms move quickly to bring investment to the new level, given the new endogenous borrowing constraint due to a given (r_{t+1}, A_{t+1}).

Everything else equal, an increase in the cost of working capital translates into higher wages and lower employment. The borrowing cost r_t^* is always away from zero because the bond price that the government faces (q^D) is higher than zero during exclusion periods with positive recovery. To this end, introducing positive recovery into the model facilitates production during exclusion periods, albeit at a higher cost.

4.3 Capital Accumulation and Investment Goods Producers

The importance of foreign investment goods in accumulating capital is well recognized (e.g., Eaton and Kortum (2001) and Alfaro and Hammel (2007)). Along with this literature, we assume that the investment good, I_t, is produced via a CES aggregate of domestic and imported foreign investment goods, I_t^d and I_t^f, respectively. This aggregation satisfies:

$$I_t = \left[\lambda (I_t^d)^\epsilon + (1-\lambda)(I_t^f)^\epsilon\right]^{\frac{1}{\epsilon}}, \tag{14}$$

where $\lambda \in [0,1)$ is the weight of I_t^d, and $\epsilon < 1$ governs the (inverse) elasticity of substitution. Moreover, I_t^f is composed of imperfectly substitutable varieties, for reasons that we discuss below. In particular, I_t^f is a CES aggregate of foreign investment goods varieties:

$$I_t^f = \left(\int_{j\in[0,1]} (I_{i,t}^f)^\nu\right)^{\frac{1}{\nu}}, \tag{15}$$

where $I_{i,t}^f$ is the foreign investment good variety i, and the elasticity of substitution across varieties is given by $|\frac{1}{\nu-1}|$.

Producers of investment goods minimize the cost of producing a certain level of investment ($\overline{I_t}$) by solving the problem:

$$\min_{I_t^d, I_t^f} I_t^d + P_t^f I_t^f \tag{16}$$

subject to $\overline{I_t} = \left[\lambda (I_t^d)^\epsilon + (1-\lambda)(I_t^f)^\epsilon\right]^{\frac{1}{\epsilon}}$.
The optimality conditions for the first stage are:

$$I_t^d = \left(\frac{1-\lambda}{\lambda}\frac{1}{P_t^f}\right)^{\frac{1}{\epsilon-1}} I_t^f \tag{17}$$

$$I_t^f = \left(\lambda\left(\frac{1-\lambda}{\lambda}\frac{1}{P_t^f}\right)^{\frac{\epsilon}{\epsilon-1}} + (1-\lambda)\right)^{-\frac{1}{\epsilon}} I_t \tag{18}$$

and given the zero profit condition, $P_t^I I_t = I_t^d + P_t^f I_t^f$, the price of investment goods satisfies

$$P_t^I = \left(\lambda^{\frac{1}{1-\epsilon}} + (P_t^f)^{\frac{\epsilon}{\epsilon-1}}(1-\lambda)^{\frac{1}{1-\epsilon}}\right)^{\frac{\epsilon-1}{\epsilon}}. \tag{19}$$

At the second stage, given their optimal demand for the aggregate foreign investment good, investment good producers choose their demand for each variety i, subject to a working capital constraint. In particular, they need to finance a θ fraction of their purchase of foreign investment good. One can show that the demand for each variety i satisfies:

$$I_{it}^f = \begin{cases} \left(\frac{p_i^f(1+r_t^*)}{P^f(r_t^*)}\right)^{-\frac{1}{1-\nu}} I_t^f, & \text{for } i \in [0,\theta]; \\ \left(\frac{p_i^f}{P^f(r_t^*)}\right)^{-\frac{1}{1-\nu}} I_t^f, & \text{for } i \in [\theta,1]. \end{cases} \tag{20}$$

Note that, unlike Mendoza and Yue (2012), we do not assume that producers of investment goods cannot access working capital loans during exclusion periods. Instead, the interest rate that the producers face depends on the price of bonds during exclusion periods, which is captured up by q^D. Thus, the costs rise but do not tend to infinity. Given the CES aggregator for foreign capital (equation 15), and given that a random θ fraction of varieties needs to be financed by working capital loans, then $P^f(r_t^*)$ satisfies the following:

$$P_t^f(r_t^*) = \left[\int_0^\theta (p_i^f(1+r_t^*))^{\frac{\nu}{\nu-1}} di + \int_\theta^1 (p_i^f)^{\frac{\nu}{\nu-1}} di\right]^{\frac{\nu-1}{\nu}} \tag{21}$$

Without a positive recovery assumption, one would have solved the demand function system with the limit of that system as $r_t^* \to \infty$.

4.4 Primary Dealer Banks

Bankers supply loans to firms and provide credit to the government by holding sovereign securities. For simplicity, we say that the timing of events that bankers undertake is divided into two interim periods, morning and afternoon. In the morning, bankers observe the TFP shock A_t with which the government's default decision d_t becomes public. Primary dealer banks meet the government's debt demand by up to B_{t+1}, and with the remaining resources they meet the demands of firms in the form of loans. Thus, the total amount of loans that can be supplied to the firms in the morning is:

$$\ell_t \leq \epsilon_t - \min\left\{\eta\epsilon_t, B_{t+1}(1-d_t)\right\}, \tag{22}$$

where ℓ stands for loans. Thus, the amount of loans that firms can obtain depends on the government's borrowing rule as well as its repayment decision. Notice that as the government increases its borrowing amount, the amount of available resources to the firms decline. So, this effect introduces a friction into the model: during the government's expansionary episodes, the banks may not be able to meet firms' entire loan demand. Recall that firms use these loans to pay a fraction of labor's wage. With the constrained borrowing, the labor market clears at a lower employment level. Notice that even if the government defaults and is left out of the financial markets, firms can still obtain loans from the banking sector.

Endowment ϵ_t is designed to capture bankers' source of income that is free of the government's borrowing and default decisions such as income from mergers and acquisitions, foreign exchange operations, and so on. The parameter η represents a natural limit on the amount of government debt securities a primary bank can hold. This is to ensure that there is a positive amount of loans available to private firms. Thus, primary dealer banks' portfolio includes at most $\eta\epsilon_t$ amount of government securities.

Note that the government's total new debt issuance is given by $b_{t+1} - (1-\delta_b)b_t$ and primary dealer banks can only supply B_{t+1}. International lenders provide the residual demand, that is, $b_{t+1} - (1-\delta_b)b_t - B_{t+1}$. The terms b_{t+1}, b_t, δ_b, d_t and asset price q_t are defined more extensively under the government's problem in Section 4.5. In our simulations, we confirm that the government almost always borrows more than what primary dealers banks can provide which is denoted by B_{t+1}. This is going to be important because the asset price is now going to be determined by the international lenders and primary dealers take it as given.

In the afternoon, bankers receive an interest rate of r_t^* from their loans to the firms and κB_{t+1} is the total amount of coupon receipts after the government's repayment decision

accrued from the government's borrowing. The interest rate that the bank receives from holding government debt becomes $\frac{1}{q_t(b_{t+1},A_t)}$. Consequently, we can now write the primary dealers' profit as:

$$\pi_t^b = \overbrace{\epsilon_t - B_{t+1}(1-d_t) - \ell_t}^{morning} + \underbrace{(1+r_t^*)\,\ell_t + \frac{\kappa B_{t+1}\,(1-d_t)}{q_t(b_{t+1},A_t)}}_{afternoon}. \tag{23}$$

Modeling primary dealer banks' profit as in equation (23) provides the following advantages: (i) we do not need to introduce B_t as a state variable when solving the competitive equilibrium, (ii) if one writes the morning lending as $q_t B_{t+1}$ and afternoon receivables as $\kappa B_t\,(1-d_t)$, non-monotonicity arises due to the constraint in 22, that is, price q_t declines as debt issuance B_{t+1} increases.

4.5 Government's problem

This section describes the benevolent government's optimization problem. The government lacks a commitment technology, and thus cannot commit to its future default and borrowing decisions, and it decides how much non-state-contingent long-term debt to issue at each period after repayment.[7] The timing of our paper, as in Eaton and Gersovitz (1981), intends to rule out the multiplicity dynamics.[8]

We introduce an endogenous link between sovereign default and private economic activity. This endogenous link relates to the following three ingredients. First, there is a one-to-one mapping between the government's implied one-period borrowing rate and the firm's short-term working capital loan rate. The channel of that link is as follows: a rise in government debt holdings increases the likelihood of government default, and thus amplifies the bond spreads that are priced by risk-neutral international investors. Since local banks take bond prices as given, they pass on the increased cost of government debt holdings to firms by increasing loan rates. The second ingredient is that domestic and foreign goods are imperfectly substitutable. The intuition is that the higher rates would translate into an efficiency loss for the firms for their production because firms will now try to use more local goods with heightened costs of production. Efficiency loss happens since foreign investment goods and their domestic counterparts are assumed to

[7] Allowing for long-term maturity for sovereign debt helps us to obtain more realistic sovereign bond spreads and default frequencies. Note also that, under certain conditions, long-term debt incorporates short-term debt as a special case (Hatchondo and Martinez (2009), Chatterjee and Eyigungor (2012)).

[8] Önder (2016) documents that the models and extensions of the Eaton-Gersovitz type of sovereign default models are not prone to multiple equilibria.

be imperfectly substitutable. The last ingredient is the positive recovery following default. Without positive recovery, the implied interest rate on the loan would be zero during default episodes. The wages would then tend to infinity, and labor would plummet to zero.

4.5.1 Long-term debt contracts

The government issues long-term-non-state contingent debt in real goods which are traded at a price q_t by infinitely many risk-neutral international lenders. Similar to Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012), a debt issued at time t promises a stream of geometrically decreasing coupons κ, which depreciate at a rate $\delta_b \in (0, 1]$. Thus, a government promises to pay $\kappa(1-\delta_b)^{n-1}$ units of consumption good in period $t+n$ for $n \geq 1$. Coupon payment κ is computed such that in the absence of default risk, the price of non-state-contingent long-term debt is equal to the price of the average one-period debt, denoted as $\frac{r+\delta_b}{1+r}$. This is a common formulation for long-term debt contracts to avoid keeping track of the entire maturity distribution. Hence, the dynamics of the long-term can be represented as follows:

$$b_{t+1} = (1-\delta_b)b_t + m_t, \tag{24}$$

where b_t and b_{t+1} are the total outstanding debt obligations at the beginning of period t and $t+1$ and m_t is the amount of debt that is issued or purchased by the government in period t. Note that one-period debt is a special case of long-term debt where $\delta_b = 1$. The budget constraint of a government, conditional on its repayment decision is:

$$c_t = A_t f(K_t, L_t) - I_t^f P_t^f - \kappa b_t + q(b_{t+1}, A_t)m_t. \tag{25}$$

4.5.2 Defaults

A typical assumption in the literature is that the government is excluded from credit markets for a stochastic period of time following its default decision, and gains access to credit markets the next period with constant probability $\psi \in [0, 1]$. Bond contracts often include acceleration and collective action clauses; thus, all debt obligations become due when a government defaults.

To regain access to credit markets, the government needs to repay an α fraction of its defaulted debt. This is a simple way of introducing recovery into the model. However, in a model with long-term debt, the government issues an infinite amount of debt prior to its default decision, which may distort the moments in the simulations. To account for that,

the assumption is that the government cannot issue bonds with a price lower than $\underline{q}$, as in Hatchondo et al. (2017). In our simulations, we confirm that this constraint never binds. In default, total output is used for consumption and imports of foreign investment goods. Therefore, the budget constraint of an economy in default is denoted as

$$c_t = A_t f(K_t, L_t) - I_t^f P_t^f. \tag{26}$$

4.5.3 Income cost of defaulting

In endogenous quantitative default models, a penalty scheme upon a default decision is required to be able to generate plausible debt-to-income ratios along with spreads. Otherwise, the government would always default, and lenders price it accordingly. Thus, only a limited amount of debt with high spreads can be supported in equilibrium. In our main analysis above, we abstract from assuming an additional income cost and resort to the model's endogenous dynamics. During default episodes, because of the increased cost of firms' working capital financing and the imperfect substitutability of domestic and foreign goods, there is a fall in investment and thus a decline in income. To improve the model's moment-matching success, we additionally assume a quadratic loss function for income during a default episode $\phi(y) = \max\{0, d_0 y + d_1 y^2\}$, as in Chatterjee and Eyigungor (2012).

4.5.4 International lenders

The international risk-neutral lenders invest in non-state-contingent long-term debt, taking decision and borrowing rules as given. Thus, the lenders' supply schedule reflects a no-arbitrage condition with an opportunity cost of funds equal to the world's risk-free rate (r). We denote the pricing schedule of bonds that are repaid as $q_t(b_{t+1}, A_t)$ and for bonds that are in arrears as $q_t^D(b_t, A_t)$. Recall that some portion of the debt is also held by local banks. Thus, the amount of external debt held by international lenders is determined as the residual between the aggregate stock of government debt and the public debt held by banks. In our simulations, we confirm that the entire amount of debt cannot be supplied by the local banks.

4.6 Recursive representation

We now formulate the government's optimization problem recursively. Let V be the value function of the government that has the option of defaulting. The government

chooses to repay if the value of repayment V^R is greater than the value of default V^D. The function V satisfies the following functional equation:

$$V(b,A) = \max\left\{V^R(b,A), V^D(b,A)\right\}, \tag{27}$$

where the government's continuation value for repayment is denoted as

$$\begin{aligned} V^R(b,A) &= \underset{b'\geq 0,\, c\geq 0,\, L\geq 0,\, I^f\geq 0,\, I^d\geq 0}{Max} \left\{u(c,L) + \beta E_A\left[V(b',A')\right]\right\}, \\ \textit{subject to}: & \\ c &= Af(K,L) - I^f P^f(r^*) - b\kappa + q(b',A)\left[b' - (1-\delta_b)b\right], \\ b' &= \hat{b}(b,A), \\ q(b',A) &\geq \underline{q} \text{ if } b' > (1-\delta_b)b, \end{aligned} \tag{28}$$

where $f(K,L) = K_t^{\alpha^K} L_t^{\alpha^L}$. Imported foreign investment goods and the price of these goods are denoted by I^f and $P^f(r^*)$, respectively. The term r^* is the government's implied interest rate on one-period bonds. Note that the benevolent government faces the same allocations of output and factors of production as the agents in the private economy. To be specific, for a given TFP shock, implied short-term borrowing rate r^* and long-term bond price q —which will be obtained below under the price function—, as well as current and next-period government bond holdings, the optimal factor allocations (I^f, I^d, L) chosen by the benevolent government characterize the private sector competitive equilibrium.

The government stays in default at least for one period and regains access to the credit markets with constant probability ψ. The continuation value of default takes into account the positive repayment when the government regains access to the credit markets. We assume that debt in arrears grows at the international risk-free rate r during the sovereign's exclusion from the credit markets. Along these lines, the value of default is

$$\begin{aligned} V^D(b,A) &= \\ & \underset{c\geq 0,\, L\geq 0,\, I^f\geq 0,\, I^d\geq 0}{Max} \left\{u(c,L) + \beta E_A\left[\psi V(\alpha b(1+r),A) + (1-\psi)V^D(b(1+r),A)\right]\right\}, \\ \textit{subject to}: & \\ & c = Af(K,L) - I^f P^f(r^*). \end{aligned} \tag{29}$$

The solution to the government's optimization problem yields decision rules for default $\hat{d}(b,A)$ and borrowing $\hat{b}(b,A)$. A default decision rule is denoted by the binary variable $\hat{d} \in \{0,1\}$; $\hat{d} = 1$ if the government finds the value of defaulting higher than the value

of repayment and 0 otherwise. In equilibrium, defined in Section 4.7, lenders use these decision rules for contract pricing. The price of a bond satisfies:

$$\begin{aligned} q(b',A) &= \frac{1}{1+r}E_A\left[\left(1-\hat{d}\left(b',A'\right)\right)\left(\kappa+(1-\delta_b)\,q\left(b'',A'\right)\right)+\right. \\ &\quad \left.\hat{d}\left(b',A'\right)q^D\left((1+r)\,b',A'\right)\right], \\ b'' &= \hat{b}(b',A'). \end{aligned} \tag{30}$$

Equation (30) indicates that with the no-arbitrage condition, selling a bond and investing it in a risk-free asset today at an international risk-free rate r (the left-hand side of the equation) is equal to the value of keeping the bond (the right-hand side of the equation). If the lender keeps the bond, she will receive one unit of coupon κ and can sell the unamortized portion of its assets $(1-\delta_b)$ at tomorrow's prevailing price $q\left(b'',A'\right)$ conditional on the government's repayment decision ($\hat{d}\left(b',A'\right)=0$). If the government defaults, then lenders can still trade their bonds at the secondary market at a price q^D. The value of a bond in default satisfies:

$$q^D(b',A)=\frac{1}{1+r}E_A\left[\begin{array}{l}(1-\psi)\,(1+r)q^D\left((1+r)b',A'\right)+\\ \alpha\psi\hat{d}(b''^{DD},A')q^D\left(b''^{DD},A'\right)+\\ \alpha\psi\left(1-\hat{d}(\alpha(1+r)b',A')\right)\left[\kappa+(1-\delta_b)q\left(b''^{DR},A'\right)\right]\end{array}\right],$$

where $b''^{DD}=\alpha(1+r)b'$ denotes the next-period end-of-period total debt if the government defaults after exiting the current default and $b''^{DR}=\hat{b}(\alpha b')$ denotes the next-period end-of-period debt obligations chosen by a government that repays after exiting the default.

The short-term interest rate r^* that is used by the investment good producers in the economy is computed by setting $\delta_b=1$. Notice that when $\delta_b=1$, equation (30) boils down to the price of one-period debt, which is determined by tomorrow's default probability and the recovery rate.

4.7 Definition of equilibrium

This paper focuses on a Markov perfect equilibrium. The government cannot commit to any future (repayment and borrowing) decisions. Hence, the government's strategies depend only on the payoff-relevant state variables.

Definition 1 (Markov perfect equilibrium) *A Markov perfect equilibrium is characterized by value functions* $V(b, A), V^D(b, A), V^R(b, A)$*, bond pricing function* $q(b', A)$*, default rule* $\hat{d}$*, and borrowing rule* $\hat{b}$ *such that*

1. *Given the bond pricing function, government policy rules* $\left\{\hat{d}, \hat{b}\right\}$ *solve the utility maximization problem defined in equations (27), (28), and (30).*
2. *Given government policy rules* $\left\{\hat{d}, \hat{b}\right\}$*, the pricing function* q *satisfies condition (30).*

A solution to the Markov perfect equilibrium consists of the solutions to the competitive equilibrium given the government policies. The optimality conditions of the private sector including the factor allocations as well as production with and without access to the credit market, are obtained using Euler equations.

4.8 Numerical Solution

This section briefly sketches the main numerical algorithm, relegating the details of the implementation to the appendix. To solve the model, we take a two-pronged approach. First, the competitive equilibrium, given any government policy rules, is solved using the Euler equations characterized in the text. We obtain the optimal private sector allocations for any productivity shock, the short- and long-term bond price, as well as current and next-period government bond holdings. With these optimal private allocations in hand, we then solve the government's problem with global solution methods. In particular, solving the model relies on iterating the value functions V^R and V^D and price function q as well as an approximation scheme to the private sector's allocation problem. To avoid the potential multiplicity problem outlined in Krusell and Smith (2003), we first solve the equilibrium of the finite-horizon economy. We start with an initial guess for the terminal value and iterate backward until the differences in value and price functions for two subsequent periods are less than 10^{-5}. We then use the obtained values as the equilibrium of the infinite horizon economy.

4.9 Calibration

We calibrate the model economy at a quarterly frequency. For most parameters, we use conventional estimates reported in the literature. For the remaining parameters, we resort to the Colombian data using simulated method of moments (SMM). Table 13 presents the calibrated parameter values.

Table 13: Parameter values

	Parameter	Value	Target
Risk aversion	γ	2	Standard RBC value
Risk-free rate	r	1%	Standard RBC value
Probability of reentry after default	ψ	0.083	Mendoza and Yue (2012), Gelos et al. (2011)
Recovery rate	α	0.55	Cruces and Trebesch (2013)
Bond decay rate	δ_b	0.033	Average duration of 5 years
Lower bound for bond prices	$\underline{q}$	0.5	-
Labor supply curvature parameter	ω	1.455	Frisch wage elasticity (2.2)
Armington weight of domestic investment	λ	0.62	Data
Armington curvature parameter	ϵ	0.62	Mendoza and Yue (2012)
Dixit-Stiglitz curvature parameter	ν	0.59	Mendoza and Yue (2012)
Upper bound to use working capital	θ	0.7	Mendoza and Yue (2012)
Share of capital	α^K	0.45	Zuleta et al. (2009)
Share of labor	α^L	0.45	Bernanke and Gürkaynak (2001); Zuleta et al. (2009)
Depreciation rate of capital	δ	0.035	Data
Limit of govt securities for primary dealer	η	0.15	Colombian Data
Calibrated			
Discount factor	β	0.96	Default frequency (2%)
Income autocorrelation coefficient	ρ	0.97	GDP autocorrelation (0.752)
Standard deviation of innovations	σ_ϵ	2.0%	GDP std. deviation (0.015)
Income cost of defaulting	d_0	-0.99	Spread and Debt
Income cost of defaulting	d_1	1.01	Spread and Debt

The risk aversion parameter is set to 2 and the quarterly world interest rate is set to 1%, the conventional values used in the literature. The labor supply curvature parameter, ω, is set to 1.455, as in Mendoza and Yue (2012). This value yields a Frisch elasticity of labor supply, $\frac{1}{\omega-1}$, equal to 2.2, and is well within the range reported in the literature.[9] The probability of re-entry after default, ψ, is set to 0.083, which implies that the country stays in default about three years before regaining access to international markets (Mendoza and Yue (2012), Gelos et al. (2011)).

The weight of domestic investment goods in the Armington aggregate of investment is based on the OECD Trade-in Value Added (TiVA) database. In particular, about 38% of gross fixed capital formation in Colombia is from imports. We accordingly set λ equal to 0.62. Moreover, we set $1 - \alpha^K - \alpha^L$ equal to 0.1 to have a reasonable profits-to-GDP ratio in the model, consistent with the data. The factors shares α^K and α^L are then set equal to 0.45, in line with Bernanke and Gürkaynak (2001) and Zuleta et al. (2009). The quarterly depreciation rate, δ, is set to 0.035, and is slightly higher than estimates for developed economies. For the remaining parameters for production, ϵ, ν, θ, we follow Mendoza and Yue (2012) and set them equal to 0.62, 0.59, and 0.70, respectively.

[9] See, for example, Rogerson and Wallenius (2009), Christiano et al. (2009), and references therein.

We target the debt-to-GDP ratio of 38.3% using the rich public debt dataset provided by Abbas et al. (2010). The productivity shocks in the model follow an AR(1) process:

$$log(A_t) = \rho log(A_{t-1}) + \varepsilon_t$$

with $|\rho| < 1$ and $\varepsilon_t \sim N\left(0, \sigma_\epsilon^2\right)$. Because of the lack of an actual TFP series, we set ρ and σ_ϵ^2 in the SMM process to target the GDP process of Colombia, using HP-detrended seasonally adjusted quarterly real GDP series for the period 200Q1 to 2017Q4. This is the longest time frame during which the quarterly GDP data are available. The standard deviation coefficient and the autocorrelation coefficient of the cyclical component of estimated GDP is 1.2% and 0.76, respectively. Our SMM procedure spells $\sigma_\epsilon^2 = 0.3\%$ and $\rho = 0.97$.

We have two main calibration exercises, with and without introducing the income cost of defaulting. A penalty scheme is necessary in this class of models because otherwise the government would always default, and lenders would price it accordingly. So in equilibrium, only a limited amount of debt issuance at very high spreads can be generated. We initially resort to our model's dynamics. The endogenous penalty scheme in our model is the decline in investment that is followed by a fall in output. In this exercise, we target the annual debt-to-income ratio of 38.3 percent with $\beta = 0.92$. The discount factor β is smaller than that in regular business cycle models, but it is still much larger than the existing corresponding studies. For instance, Mendoza and Yue (2012) set β as 0.88 and generate a debt-to-GDP ratio of 23%. Similarly, Sosa Padilla (2018) uses a discount factor of 0.80, which helps to generate a debt-to-GDP ratio of 13%. In the next calibration exercise, we introduce an exogenous income cost of defaulting as in typical quantitative default studies to improve the model's moment-matching success. We calibrate the value of the parameters of the income cost of defaulting (d_0 and d_1) targeting the mean levels of debt and sovereign spreads.

We set δ_b at 3.3%. With that value, the maturity of long-term debt becomes 5 years, which is roughly the average bond duration estimated for emerging economies documented by Cruces et al. (2002)[10]. The definition of duration in Macaulay (1938) is standard in calculating the long-term debt duration. Duration D is the weighted average maturity of future cash flows. A bond issued at time t makes periodic payments κ for the subsequent periods with a geometrically decay rate δ_b.[11] Observe that equation (32) is 1 for $\delta = 1$. The

[10] Cruces et al. (2002) report an average debt duration of 4.8 years with a standard deviation of 1.5 in the year 2000 for emerging economies.

[11] Duration D satisfies,

$$D(\kappa) = \frac{1}{q}\left(\sum_{j=1}^{J} j \frac{\kappa(1-\delta_b)^{j-1}}{(1+i)^j}\right)$$

sovereign spread r_s is defined as the difference between yield i and the risk-free rate r. The annualized spread reported in the tables is computed as

$$1 + r_s = \left(\frac{1+i}{1+r}\right)^4.$$

The debt levels obtained from the simulations are equivalent to the present value of future debt obligations, which are discounted at the risk-free rate and computed as $\frac{b(1+r)}{\delta+r}$.

5 Quantitative Results

This section presents our baseline results. Table 14 compares the data moments from Colombian data with the one obtained from the model. The model features plausible moments and matches both the sovereign debt statistics as well as the business cycle moments reasonably well. We undertake two calibration exercises: (i) one that does not introduce an exogenous income shock during defaults and resorts to the endogenous decline in investment during exclusion episodes, and (ii) one that engineers exogenous income cost of defaulting.

The model without exogenous income cost of defaulting generates debt-to-GDP ratio of 31 percent, on average. This result is mainly a byproduct of an endogenous decline in investment during default episodes which increases the cost of defaulting. Part of the reason why our study does a better job in matching debt-to-GDP ratio and the spreads, lies in the modeling choice. Inclusion of long-term maturity of sovereign debt and partial recovery after default are important ingredients. The role of long-term debt in quantitative

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where i is the periodic yield an investor would earn if the bond is held to maturity without any defaults and it satisfies

$$q = \sum_{j=1}^{\infty} \frac{\kappa(1-\delta)^{j-1}}{(1+i)^j},$$

and the periodic yield i reads as

$$i = \frac{\kappa}{q} - \delta_b$$

with which D becomes

$$D = \frac{1}{q(1-\delta_b)} \lim_{J\to\infty} \sum_{j=1}^{J} j \left(\frac{1-\delta}{1+i}\right)^j \tag{31}$$

$$= \frac{1+i}{i+\delta_b}. \tag{32}$$

sovereign default models are thoroughly discussed in Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012).

As expected and in line with the data, the model generates negative correlation between interest rate spreads and GDP. Quantitative models of sovereign default as well as the business cycles in emerging markets yield countercyclical interest rates. However, in the former one output is an exogenous process and in the latter one the interest rate spreads are exogenous. In contrast, this paper introduces an endogenous income process and endogenously determined interest rate spreads both of which are influenced from each other. We should note that Mendoza and Yue (2012) also generate, even though it is weak, a negative correlation between interest rate spreads and income.

Turning to our second calibration which engineers an exogenous income cost of defaulting, third column of Table 14 shows that the model generates positive correlation between investment-to-GDP ratio and debt-to-GDP ratio. However, as our empirical model shows, the negative feedback between investment and GDP intensifies when the government debt ratio is high. To account for that, we also report the correlation of these two moments for the instances where debt-to-GDP ratio exceeds its 75th percentile. The model now produces significant crowding-out channel. This result hinges on the inter-linkages between private sector and the sovereign risk. There are essentially two forces working in the same direction. First, higher government indebtedness limits the primary dealers' ability to provide loans to firms as they are primarily required to supply credit to the government. Due to this supply crunch, both labor and investment falls. Second, higher government debt induces a rise in default risk resulting in higher bond spreads. Primary dealers then pass on these costs to firms. This chain of events leads a decline in the private sector's investment levels.

Figure 3 displays the dynamics of debt-to-GDP and investment-to-GDP ratios. From the figure it is visible that investment-to-GDP is not responsive for low levels of debt-to-GDP. This follows because the government's implied short-term borrowing rate does not move much so that the banks do not pass their costs to firms. Furthermore, availability of funds from banks to the private sectors is not necessarily hindered. The country risk is almost zero for low enough debt levels (see Figure **??**). Recall that the private sector's investment also depends on the imperfectly substitutable foreign investment goods whose cost depends on the country's short-term risk. As the government's total borrowing increases, the default risk and thus the short-term borrowing rate as well as the price of foreign investment goods rise. This, in turn, reduces per period equilibrium investment and explains the intuition of why crowding-out kicks in for higher levels of sovereign indebtedness. In addition, the model also features a negative correlation between sovereign

Table 14: Long-run Statistical Moments

	(1) Data	(2) Model	(3) Income cost
Long-term debt statistics[a] (%)			
Mean debt-to-GDP	38.3	31	36.3
Mean r_s	2.1	0.2	2.1
Default rate	2	0.5	2.07
Debt duration (years)	5	5	5.8
Mean investment-to-GDP	22.0	26.4	26.6
Inv decline during exclusions(%)	35	25	2.5
Volatilities (%)			
$\sigma(c)/\sigma(GDP)$	0.92	1.1	1
$\sigma(tb/GDP)$	0.86	1.71	0.7
$\sigma(r_s)$	0.69	0.41	0.01
Correlations			
$\rho(c, GDP)$	0.91	0.98	0.7
$\rho(tb/GDP, GDP)$	0.37	0.72	0.35
$\rho(r_s, GDP)$	-0.51	-0.83	-0.08
$\rho(inv, GDP)$	0.86	0.97	0.97
$\rho(r_s, inv)$	-0.54	-0.80	-0.77
$\rho(r_s, tb/GDP)$	-0.11	0.94	0.92
$\rho(inv/GDP, b/GDP)$	0.38	0.63	0.63
$\rho(inv/GDP, b/GDP)^a$	-0.25	-0.60	-0.88

Long-term statistics are annualized. The third column represents the moments obtained with income cost of defaulting. Consumption (c), investment (inv), trade balance (tb) are in real terms. GDP excludes government expenditures (inline with the model). All series are deseasonalized (X13), and HP filtered with a smoothing parameter of 1600. EMBIs (r_s) are quarterly averaged. Public debt (b) is reported annual, so the correlations that involves b are based on annual data. The model moments are calculated using the averages of 250 simulated sample paths of 100 quarters (25 years) without any default and each sample path starts at least five years after a default incident. The business cycle moments are detrended with HP-filter using a smoothing parameter of 1600. If otherwise noted, source of the data is Central Bank of Colombia. $\sigma(h)$ denotes the standard deviation of a variable h. $\rho(h, m)$ denotes the coefficient correlation between variables h and m.

[a] Computes the correlations of instances where b/GDP is higher than its 75th percentile.

spreads and private investment. Intuitively, the banks pass on the costs of government's borrowing to firms by requiring higher loan rates which reduces investment.

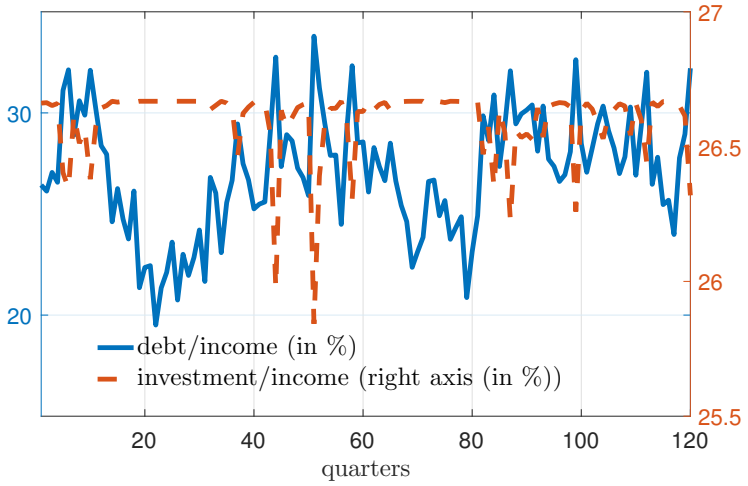


Figure 3: Dynamics of debt-to-GDP and investment-to-GDP in the model without the samples of default. Blue solid line represents debt-to-income ratio and dashed line represents invest-to-income ratio on the right-y axis.

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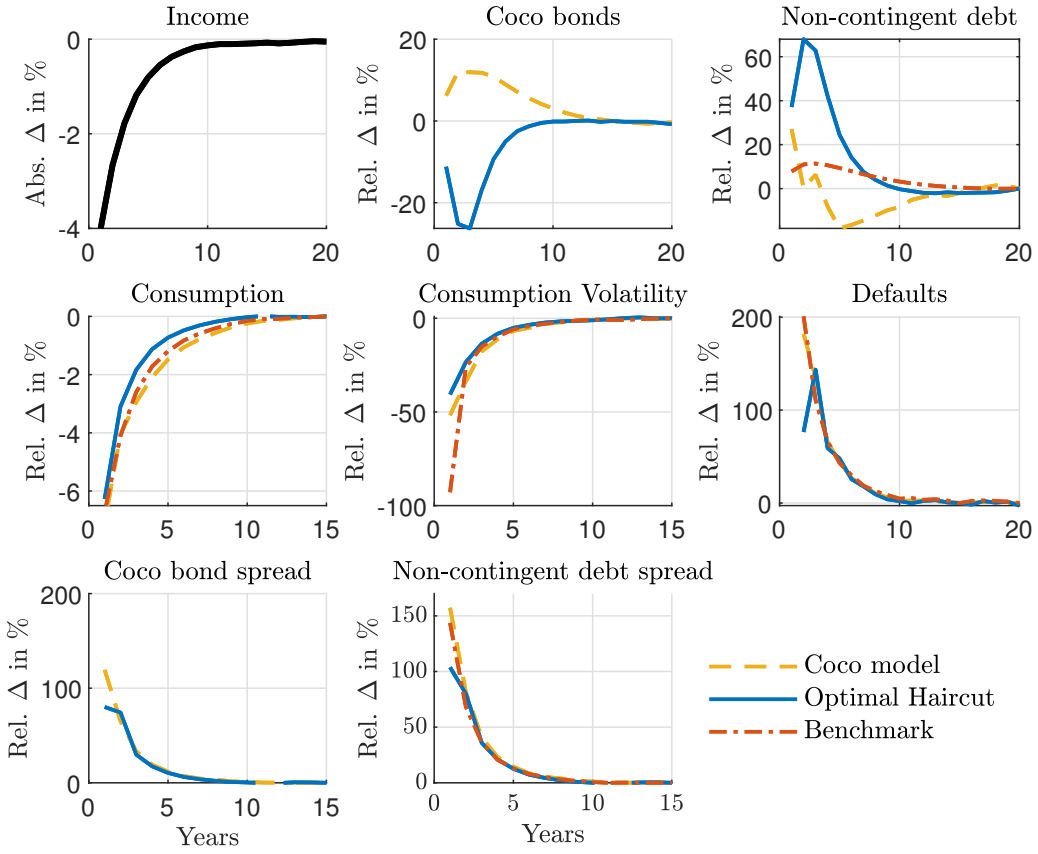


Figure 4: IRFs. Each economy starts with their respective ergodic asset levels when the initial income shock is 4 percent lower than its mean.

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Appendix A Similar measures in other countries

Ecuador

Period: 22/08/2020 - 20/05/2021

According to the measures taken by the National Assembly of Ecuador (2020) at the Organic Law of Humanitarian Support (*Ley Orgánica de Apoyo Humanitario*), by mutual agreement, debtors may subscribe with their creditors pre-bankruptcy agreements of an exceptional nature through which they can establish conditions, terms and the reduction, capitalization or restructuring of the obligations earrings of any nature.

Chile

Period: 23/03/2020 - currently in force

According to the measures taken by the Council of the Commission for the Financial Market (2020) (*Consejo de la Comisión para el Mercado Financiero, CMF*), individuals could apply for regulatory exceptions to facilitate the possibility of postponing up to three installments in the payment of mortgage loans. This special treatment is aimed at debtors who were up to date in their obligations when the state of emergency was decreed by the authorities. Likewise, to help individuals and SMEs, the Commission also made changes to the regulation so that banks can increase the term of consumer loans by up to six months, without this being considered a renegotiation for provisions purposes.

Peru

Period: 27/03/2020 - currently in force

The Superintendency of Banking (2020) (*Superintendencia de Banca, Seguros y AFP, SBS*) empowered financial entities and cooperatives to reschedule the credits of their clients and cooperative partners (companies and / or individuals), who, being up to date with their payments, due to the health emergency, have difficulties in continuing to pay their credits on time. This rescheduling will not affect the risk rating of these clients in the SBS Risk Center.

Argentina

Period: 19/03/2020 - currently in force

In accordance with the provisions of the Central Bank of the Argentine Republic (2020), all financial entities in the system may offer a special line of credit to micro, small and medium-sized enterprises at a maximum annual interest rate of 24%. On the other hand, the BCRA ordered the temporary flexibility of the parameters with which bank debtors are classified. The current classification system takes into account, among other variables, the number of days in arrears for each debtor. From today until September 2020, 60 days will be added to the classification of each debtor for each category, allowing to contemplate the difficulties caused by the crisis in various branches of economic activity.

Brazil

Period: 17/03/2020 - 29/12/2021

The Ministry of Economy of Brasil (2020) made the conditions for the renegotiation of the debt available to all taxpayers, with the exception of debts with the Severance Compensation Fund (FGTS) for criminal fines. This modality allows the initial payment, referred to 1% of the total amount of the debts, in up to three months. The remaining installments will be deferred 90 days. Another benefit is the longer term for the installment payment. For legal persons, the payment of the balance can be divided into up to 81 months. In the case of natural persons, micro or small companies, educational institutions, the deadline to be part of the measure was extended until December 29, 2021.

Uruguay

Period: 19/03/2020 - currently in force

The Superintendency of Financial Services (2020) (*La Superintendencia de Servicios Financieros, SSF*) authorizes financial intermediation Institutions, Financial Services Companies and Credit Administrators with higher assets, to extend the maturity terms of fixed-term or amortizable loans corresponding to the loan portfolio to the Non-Financial Sector, for up to 180 days both of the payment of capital and of interest, without modifications in the accounting classification of the operations, nor in the classification into risk categories of the debtors. These term extensions will be made prior agreement with the clients except when they do not generate interest, in which case the communication to the debtor will suffice so that he can express his refusal. The interest rate to be applied during the extension of the term may not be higher than that agreed in the original loan.

Belgium

Period: 29/02/2020 - 30/06/2020