Average Inflation Targeting: Time Inconsistency and Intentional Ambiguity*

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Current draft: September 9, 2021

Abstract

We study the implications of the Fed's new policy framework of average inflation targeting (AIT) and its ambiguous communication. We show that AIT improves the trade-off between inflation and real activity by tilting the Phillips curve in a favorable way. To fully utilize this feature and maximize social welfare, the central bank has the incentive to deviate from AIT and implement inflation targeting ex post. Next, we rationalize the central bank's ambiguous communication about the horizon over which it averages inflation. Ambiguous communication, together with uncertainty about economic fundamentals, helps the central bank to gain credibility and improve welfare in the long run, in spite of the time-inconsistent nature of AIT.

Keywords: average inflation targeting, time inconsistency, ambiguous communication.

*We are grateful to Oli Coibion, Todd Clark, Drew Creal, Yuriy Gorodnichenko, Kurt Lunsford, Eric Sims, and Michael Woodford, as well as seminar and conference participants at the Federal Reserve Bank of Cleveland. The views stated herein are those of the authors and are not necessarily those of the Federal Reserve Bank of Cleveland or the Board of Governors of the Federal Reserve System. Correspondence: Chengcheng.Jia@clev.frb.org, cynthia.wu@nd.edu.

1 Introduction

At the 2020 Jackson Hole Economic Policy Symposium, Federal Reserve Chair Jerome Powell announced a revision to the Fed's long-run monetary policy framework, replacing inflation targeting (IT) with average inflation targeting (AIT) to achieve its dual mandate; see Powell (2020). However, what the Fed's communication does not make clear is the horizon over which it targets average inflation at 2 percent. Our paper rationalizes such an ambiguous communication and investigates its implications.

We focus on two key issues of AIT: time inconsistency and ambiguous communication. First, we show that AIT is not time consistent, which is the case for any path-dependent policy; see, e.g., Eggertsson and Woodford (2003). By convincing the private sector about its intention to implement AIT, the central bank can improve the trade-off between inflation and real activity, which is captured by the Phillips curve. Ex post, the central bank has the incentive to deviate from its communication and implement IT instead to improve social welfare. This strategy is welfare-improving provided that the central bank is able to convince private agents with its communication, which is different from its actions. But can it? We show that with uncertainty about economic fundamentals, the central bank can. More interestingly, announcing AIT without clearly specifying its horizon helps the central bank further gain credibility and improve welfare compared to the case with clear communication. Therefore, ambiguous communication could be intentional.

We introduce AIT into an otherwise textbook three-equation New Keynesian model by modifying the central bank's objective function. For a standard model, the central bank minimizes a quadratic loss function in inflation and the output gap, both of which are valued at the current period. AIT replaces current inflation in the objective function with average inflation over L periods between $t - L + 1$ and t.[1] In the perfect communication benchmark, the central bank minimizes its loss subject to a forward-looking Phillips curve,

[1] We define AIT as flexible average inflation targeting, which also puts weight on the output gap. This definition is similar to flexible inflation targeting.

which is standard in the literature.

The central bank faces a trade-off between inflation and real activity when a cost-push shock is present. AIT tilts the reduced-form Phillips curve in a favorable way: Compared to IT, AIT is associated with a Phillips curve with a smaller intercept and slope. When inflation is above its target today, the expected inflation next period will be below the target. A lower expected inflation lowers inflation today through the expectation term in the forward-looking Phillips curve.

Although AIT presents the central bank with a better inflation-output trade-off by tilting the Phillips curve in a favorable way, it does not necessarily yield higher welfare. This is because AIT's objective function is different from social welfare. However, after the central bank convinces the private sector about its intention to implement AIT, it has an incentive to deviate from its communicated objective and optimize social welfare instead. This strategy improves social welfare but is time inconsistent.

Is the central bank's welfare-enhancing strategy sustainable? Or can agents learn the truth in the long run? We answer these questions through social learning, where agents meet and update their beliefs based on their performance. We allow two layers of heterogeneity. First, agents have different beliefs about the horizon L over which the central bank averages inflation. Second, agents with the same belief about L observe different private signals about economic fundamentals, and we use this as a device to capture uncertainty. We assume that the central bank has perfect information on both economic fundamentals and beliefs of the private sector.

We find that when agents observe economic fundamentals without uncertainty, they will eventually learn the truth and the central bank will not benefit from its time-inconsistent strategy in the long run. However, uncertainty allows the central bank to keep a fraction of agents believing in AIT in the long run, even if the central bank deviates from it every period. More interestingly, ambiguous communication helps the central bank further gain credibility and improve social welfare in the long run. This result does not depend on whether the

central bank is credible or not initially.

The rest of the paper proceeds as follows: Section 1.1 draws a connection to the existing literature. Section 2 sets up the perfect communication benchmark. Section 3 discusses the two key issues: time inconsistency and the incentive for ambiguous communication. Via social learning, Section 4 assesses whether the central bank will eventually lose credibility because of the time-inconsistent nature of AIT. Finally, Section 5 concludes.

1.1 Literature

Only a handful papers in the literature study the Federal Reserve's new policy framework of AIT. For example, Amano et al. (2020) examine the optimal degree of history dependence under AIT and find it to be relatively short. Mertens and Williams (2019) show that AIT can mitigate the effects of the zero lower bound by raising inflation expectations when inflation is low. Hebden et al. (2020) discuss the robustness of AIT to alternative assumptions about the slope of the Phillips curve and the uncertainty of economic slack. Different from all the above-mentioned papers, we introduce AIT into the central bank's optimization problem, whereas the literature focuses on an interest rate targeting rule. In that sense, Nessén and Vestin (2005) is the closest to our paper. But their paper only studies the case of perfect communication, whereas our paper discusses both perfect and imperfect communication but focuses on the latter, which is more relevant to current policy discussions.

Our paper is also related to the literature on agent-based modeling. We focus on its applications in economics, which refers to it as social learning or social dynamics. Burnside, Eichenbaum, and Rebelo (2016) use social learning to study boom and bust in housing markets. Our paper is related to Hachem and Wu (2017) in the sense that both papers study inflation. It is also related to Arifovic, Bullard, and Kostyshyna (2013) in the sense that both papers use a New Keynesian model. The difference is that we focus on the Fed's new framework of AIT.

2 The Perfect Communication Benchmark

In this section, we lay out the model that characterizes the new policy framework of average inflation targeting under perfect communication.

2.1 A Model of Average Inflation Targeting

As in a canonical New Keynesian model (see, e.g., Woodford (2003), Clarida, Galí, and Gertler (1999), and Galí (2015)), the economy features an intertemporal IS equation

$$\hat{y}_t = \mathbb{E}_t \, \hat{y}_{t+1} - \frac{1}{\gamma} \left(i_t - \mathbb{E}_t \, \pi_{t+1} \right), \tag{2.1}$$

and a forward-looking Phillips curve

$$\pi_t = \beta \, \mathbb{E}_t \, \pi_{t+1} + \kappa \hat{y}_t + u_t, \tag{2.2}$$

where $\hat{y}_t$ is the log deviation of real output from its natural level, π_t is the rate of inflation, and i_t is the central bank's policy instrument, the short-term interest rate. σ is the inverse of the intertemporal elasticity of substitution, β is the discount factor, and κ depends on nominal rigidity. We assume that the cost-push shock, u_t, is the only shock in the economy, which introduces a trade-off between inflation and the output gap.

We model average inflation targeting as follows: the central bank minimizes a weighted sum of squared average inflation over L periods and the squared output gap:

$$\mathbb{L}_t^{cb}(L) = \frac{1}{2} \left(\left(\frac{\pi_t + \pi_{t-1} + ... \pi_{t-L+1}}{L} \right)^2 + \lambda^{cb} \hat{y}_t^2 \right) + \beta \, \mathbb{E}_t \, \mathbb{L}_{t+1}^{cb}(L), \tag{2.3}$$

where λ^{cb} is the relative weight between the output gap and average inflation. The central bank's objective function coincides with the social welfare function when $L = 1$ and $\lambda^{cb} = \lambda$, where λ is derived from the second-order approximation of the household's utility function

(Rotemberg and Woodford, 1999).

The equilibrium is

$$\pi_t = a_{\pi,1}^{(L)}\pi_{t-1} + ... + a_{\pi,L-1}^{(L)}\pi_{t-L+1} + b_{\pi}^{(L)}u_t \tag{2.4}$$

$$\hat{y}_t = a_{y,1}^{(L)}\pi_{t-1} + ... + a_{y,L-1}^{(L)}\pi_{t-L+1} + b_{y}^{(L)}u_t. \tag{2.5}$$

For analytic tractability and intuition, we first focus on the case where $L = 2$ and temporary cost-push shocks. We will extend the result to $L > 2$ in Section 3.2 and allow for serially correlated shocks in Section 4.

For 2-period AIT, the coefficients are given by the following set of fixed-point equations:

$$a_\pi \equiv a_{\pi,1}^{(2)} = -\vartheta\frac{\kappa}{4(1-\beta a_\pi)} \tag{2.6}$$

$$b_\pi \equiv b_{\pi}^{(2)} = \vartheta\frac{\lambda}{\kappa} \tag{2.7}$$

$$a_y \equiv a_{y,1}^{(2)} = \frac{1}{\kappa}\left(1-\beta a_\pi\right)a_\pi \tag{2.8}$$

$$b_y \equiv b_{y}^{(2)} = \frac{1}{\kappa}\left(1-\beta a_\pi\right)b_\pi - \frac{1}{\kappa} \tag{2.9}$$

where

$$\vartheta = \left\{\frac{\kappa}{1-\beta a_\pi}\left[\frac{1}{4} + \frac{1}{4}\left(1 + \frac{2}{a_\pi} + \frac{1}{a_\pi^2}\right)\frac{\beta a_\pi^2}{1-\beta a_\pi^2} + \lambda\frac{\beta a_y^2}{1-\beta a_y^2}\right] + \frac{\lambda}{\kappa}\left(1-\beta a_\pi\right)\right\}^{-1}$$

See Appendix A for a detailed derivation.

We characterize the equilibrium with the following proposition:

Proposition 1 *With AIT, higher inflation leads to lower inflation and a lower output gap in the next period; i.e., $a_\pi < 0$, and $a_y < 0$.*

Proof: See Appendix B.

This result, especially a negative $a_\pi = \frac{\partial \pi_t}{\partial \pi_{t-1}}$, forms the basis for our main results discussed below.

2.2 Comparison to Inflation Targeting

We define inflation targeting (IT) as the textbook version of optimal discretionary policy (see Woodford (2003) and Galí (2015), for examples). Its objective function is a special case of equation (2.3) when $L = 1$ and $\lambda^{cb} = \lambda$. Under a discretionary policy, the central bank cannot influence future equilibrium. Therefore, its objective function can be reduced to the period loss:

$$\mathcal{L}_t = \frac{1}{2}\left(\pi_t^2 + \lambda \hat{y}_t^2\right). \tag{2.10}$$

Note that is not the case for AIT, where the central bank considers how its current policy decision affects future equilibrium through expectations as in Proposition 1.

The equilibrium for IT is given by

$$\pi_t = b_\pi^{(1)} u_t \tag{2.11}$$

$$\hat{y}_t = b_y^{(1)} u_t. \tag{2.12}$$

Comparing the equilibrium under IT in (2.11) - (2.12) with that under AIT in (2.4) - (2.5), the latter is path-dependent; i.e. the lagged inflation becomes a state variable and expected future values depend on current inflation.

We further illustrate the difference between IT and AIT by impulse response functions. Figure 1 plots the impulse responses of inflation and the output gap to a temporary cost-push shock of 1 percent. Parameters are calibrated in line with the traditional New Keynesian literature. For details, see Appendix C. On the left side, we plot the impulse responses under IT. In response to a temporary cost-push shock, inflation increases and the output gap decreases on impact. After the first period, the economy is back to the steady state. On the right-hand side, with 2-period AIT, the economy is not restored to the steady state in the second period. But rather, inflation oscillates around zero. When the central bank

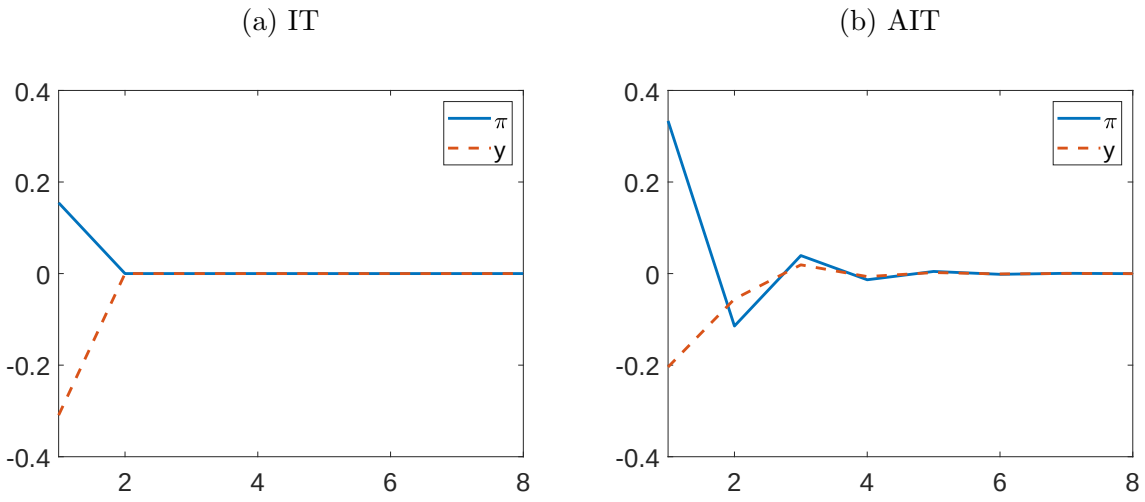


Figure 1: Impulse responses to a cost-push shock

Blue solid lines: inflation; red dashed lines: output gap. Left panel: IT; right panel: AIT. $\lambda^{cb} = \lambda$, $u_t = 0.1$.

implements AIT, it tightens monetary policy when the lagged inflation rate is positive, leading to a negative inflation rate and a negative output gap in the current period.

2.3 Phillips Curve

This section investigates how AIT affects the Phillips curve, which captures the central bank's available trade-off between inflation and the output gap. While IT and AIT share the same structural form of the Phillips curve as in (2.2), the reduced-form Phillips curves are different. Under IT, after a temporary cost-push shock, the economy goes back to the steady state the next period. Therefore, the expected future inflation is at its zero steady state, and (2.2) implies the following reduced-form Phillips curve:

$$\pi_t = \kappa \hat{y}_t + u_t. \tag{2.13}$$

In contrast, under AIT, the private sector forms expectations of the next period's inflation conditional on current inflation (i.e., $\mathbb{E}_t \, \pi_{t+1} = a_\pi \pi_t$). We solve for the reduced-form Phillips curve as follows:

(a) Phillips curve with $u_t = 0.1$ (b) Phillips curve with $\pi_{t-1} = 0.1$

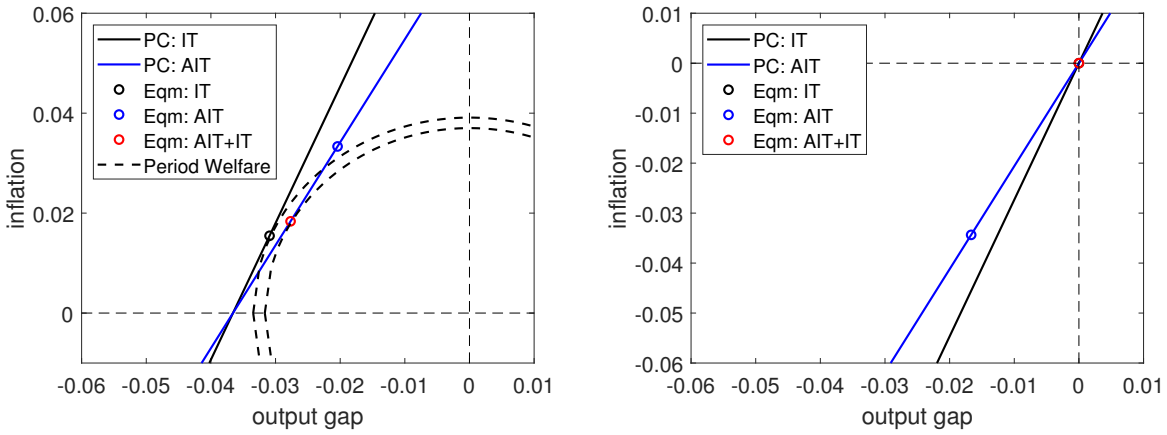


Figure 2: The incentive of imperfect communication

$$\pi_t = \frac{\kappa}{1 - \beta a_\pi} \hat{y}_t + \frac{1}{1 - \beta a_\pi} u_t \tag{2.14}$$

To gain some intuition, we single out one state variable at a time. First, let's focus on the case with a cost-push shock. Comparing the reduced-form Phillips curves under IT in (2.13) and AIT (2.14) leads to the following proposition:

Proposition 2 *The Phillips curve under AIT has*

- *the same x-intercept as,*
- *a smaller absolute value of the y-intercept than,*
- *and a smaller slope than*

that under IT after a cost-push shock.

Proof: See Appendix B.

The y-intercept of the Phillips curve captures the equilibrium when there is a cost-push shock and the policy rate does not respond. We use a positive cost-push shock for illustration. In the absence of a policy response, a positive cost-push shock leads to a positive inflation

rate and a zero output gap. When the central bank implements AIT, positive inflation in the current period leads to negative expected inflation in the next period, which in turn lowers current inflation. Therefore, the intercept of the Phillips curve after a positive shock is smaller under AIT compared to IT after a positive shock. The x-intercept of the Phillips curve represents the equilibrium when inflation is completely stabilized after a cost-push shock. Zero inflation this period implies zero expected inflation and a zero output gap next period for both policies, eliminating any feedback to the current gap. Therefore, the two policies share the same x-intercept. See the two lines in the left panel of Figure 2 for an illustration.

Combining the results on both intercepts, AIT has a smaller slope than IT. What drives the Phillips curve to be flatter? After an expansionary policy that increases the output gap by 1 percent, the direct effect increases the current inflation rate by κ percent. With an AIT policy, higher inflation today lowers expected inflation in the next period, which feeds back to a lower inflation today. In contrast, IT does not have an indirect effect through expectations. Therefore, the response of inflation is smaller under AIT than that under IT, which yields a smaller slope of the Phillips curve for AIT than IT.

Next, we turn to the case with non-zero lagged inflation. IT is not path dependent, and therefore, lagged inflation does not introduce a trade-off between inflation and the output gap. Although lagged inflation is a state variable for AIT, it does not introduce an additional trade-off between inflation and the output gap for the 2-period case (see the lines in the right panel of Figure 2).[2] We summarize this result in the following proposition:

Proposition 3 *The Phillips curves under 2-period AIT and IT both cross the origin after non-zero lagged inflation.*

What drives this result? The private sector forms its expectations next period conditional on current inflation only. Therefore, lagged inflation does not introduce an indirect effect.

The following lemma is a direct result of Propositions 2 and 3.

[2]We will show later that this is not the case for $L > 2$.

Lemma 1 *AIT yields a weakly better available trade-off for the central bank between inflation and the output gap.*

This is one key result of the paper. AIT tilts the Phillips curve in a (weakly) favorable way – closer to the origin in the relevant space. This result will serve as the basis for many discussions that follow.

2.4 Welfare

This section compares the welfare implications of the equilibria under AIT and IT, which combine the Phillips curve and the central bank's objective function.

As discussed in Lemma 1, AIT tilts the Phillips curve in a favorable way, resulting in a (weakly) better available trade-off today between inflation and the output gap. Graphically, in Figure 2, AIT moves the Phillips from the black line to the blue line.

However, this change does not guarantee an improvement in welfare because the central bank's objective function is different from the social welfare. The first difference lies in the current period loss, where the weight on current inflation is potentially different. This means that the equilibrium under AIT potentially does not maximize period welfare, where the maximum is on the tangent point between the Phillips curve and the period loss (the ellipses in Figure 2). When a cost-push shock hits, AIT could yield a higher or lower period loss than IT; the left panel of Figure 2 shows an example that AIT (blue circle) reduces period welfare relative to IT (black circle). After non-zero lagged inflation, IT is always better than AIT; see the right panel. The second difference resides in expected future losses. With non-zero lagged inflation, in the IT case, inflation and the output gap are restored to their steady states. However, AIT results in persistent deviations (see Figure 1), which increases expected future losses.

Although AIT might not be welfare-enhancing in an environment with perfect communication, it could potentially improve welfare when the central bank implements a policy that's different from its announced policy. We will discuss this strategy further in Section 3.

3 Time Inconsistency and Incentive for Ambiguity

This section demonstrates the time inconsistency of AIT and argues that a strategy whereby the central bank announces AIT but implements IT ex post dominates both IT and AIT with commitment. In this section, we assume that the central bank can successfully convince the private sector of its intention to implement AIT, which allows a more favorable Phillips curve. In Section 4, we will assess this assumption and investigate whether the central bank can fool the public consistently. Section 3.1 follows Section 2 and focuses on 2-period AIT, and Section 3.2 extends the argument to multiple periods.

3.1 Two-Period AIT

Once the central bank convinces the private sector of its intended policy rule, it can improve welfare ex post by minimizing the period loss in (2.10) subject to the Phillips curve in (2.14). The equilibrium after this one-time deviation is given by

$$\pi_t = \left(\frac{\kappa}{1-\beta a} + \lambda\frac{1-\beta a}{\kappa}\right)^{-1}\frac{\lambda}{\kappa}u_t \tag{3.1}$$

$$\hat{y}_t = \left[\left(\frac{\kappa}{1-\beta a} + \lambda\frac{1-\beta a}{\kappa}\right)^{-1}\frac{\lambda}{\kappa}\frac{1-\beta a}{\kappa} - \frac{1}{\kappa}\right]u_t \tag{3.2}$$

We can show the following proposition under 2-period AIT:

Proposition 4 *The strategy whereby the central bank announces AIT but implements IT ex post dominates both pure IT and AIT.*

We will explain the intuition one state variable at a time. First, after non-zero lagged inflation, equations (3.1)-(3.2) suggest that the central bank deviates from AIT to the equilibrium of dual stabilization $\pi_t = x_t = 0$ (the red circle in the right panel of Figure 2). This new equilibrium improves welfare from the AIT equilibrium (blue circle), which involves a positive loss, and has the same zero loss as the IT equilibrium (black circle).

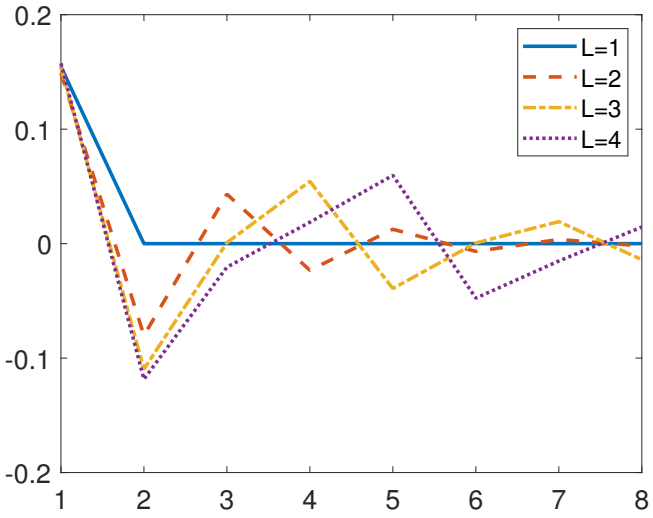


Figure 3: Inflation responses to a cost-push shock for different L

Notes: different lines represent different $L = \{1, 2, 3, 4\}$. $u_t = 0.1$.

Next, in response to a cost-push shock, equations (3.1) - (3.2) suggest that the central bank might pick a different equilibrium than the equilibrium under AIT described in (2.4) - (2.5), or the red circle could be different from the blue circle in the left panel of Figure 2. The equilibrium where the central bank announces AIT and implements IT (the red circle) is on the tangent point between the more favorable Phillips curve (the blue line) and social welfare (the ellipses). Therefore, it (weakly) dominates both IT, which is on the worse (black) Phillips curve, and AIT, where period welfare might not be minimized because the central bank's objective function is different from social welfare.

3.2 Multi-period Model

We have shown that when $L = \{1, 2\}$, the central bank's best strategy is to announce AIT ($L = 2$) and deviate to the discretionary policy ($L = 1$) ex post. What happens when L is allowed to go beyond 2 periods? We focus on such a case in this section. We show that all our results go through and further argue for the case of ambiguity.

First, we extend the result in Proposition 1 and show that $a_{\pi,1}^{(L)} = \frac{\partial \pi_t}{\partial \pi_{t-1}} < 0$ in the multi-

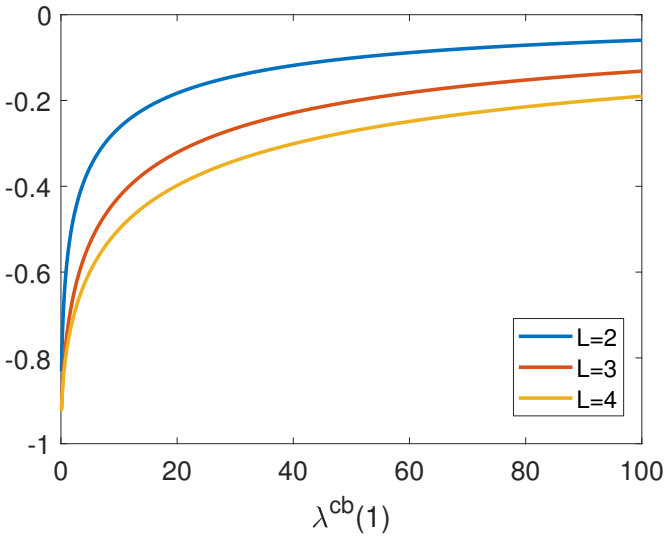


Figure 4: $a_{\pi,1}^{(L)}$

period case. We do so numerically; see Figure 3. To make results of different Ls comparable, we assume $\lambda^{cb}(L)$ is proportional to $1/L^2$, or

$$\lambda^{cb}(L) = \lambda^{cb}(1)/L^2. \tag{3.3}$$

In response to a temporary positive cost-push shock, inflation increases on impact. To stabilize average inflation over L periods, the central bank tightens monetary policy, which lowers inflation subsequently.

Next, the Phillips curve under multi-period AIT is

$$\pi_t = \frac{\kappa}{1-\beta a_{\pi,1}^{(L)}}\hat{y}_t + \sum_{l=1}^{L-2} \frac{\beta a_{\pi,l+1}^{(L)}}{1-\beta a_{\pi,1}^{(L)}}\pi_{t-l} + \frac{1}{1-\beta a_{\pi,1}^{(L)}}u_t \tag{3.4}$$

There are two countervailing forces on the intercept: the cost-push shock and lagged inflation.

We first focus on the cost-push shock. With a negative $a_{\pi,1}^{(L)}$, AIT yields a smaller intercept for the Phillips curve than IT regardless of L, which implies a better available trade-off between inflation and the output gap. This extends the result in Lemma 1.

	t	$t+1$	$t+2$	$t+3$
$L=1$	0.1170	0	0	0
$L=2$	0.0980	0	0	0
$L=3$	0.0920	0.0101	0	0
$L=4$	0.0904	0.0249	0.0545	0

Table 1: Period loss

Notes: The central bank announces L-period average inflation targeting, where each row corresponds to a different L, but implements IT. We calculate the period-by-period loss after a cost-push shock of $u_t = 0.1$.

Next, we compare the intercept across different Ls. For a larger L, the intercept of the Phillips curve on impact of u_t, $\frac{1}{1-\beta a_{\pi,1}^{(L)}} u_t$, is smaller, which makes the trade-off between inflation and the output gap more favorable. We can see this from Figure 4, which compares $a_{\pi,1}^{(L)}$ across different L over different values of λ^{cb} ($\frac{1}{1-\beta a_{\pi,1}^{(L)}}$ is an increasing function of $a_{\pi,1}^{(L)}$). For any $\lambda^{cb}(1) \in (0, \infty)$, $a_{\pi,1}^{(4)} < a_{\pi,1}^{(3)} < a_{\pi,1}^{(2)}$.

Therefore, to maximize period t welfare, the central bank should announce a large L but implement IT. We corroborate this result with Table 1, which computes period loss when the central bank announces L-period AIT but implements IT. We assume $\lambda^{cb}(1) = \lambda$ hereafter. For period t, the largest L ($L = 4$) corresponds to the smallest loss.

However, non-zero inflation has the opposite effect on the intercept. After t, cases with $L > 2$ incur additional loss. This is because unlike the 2-period AIT in equation (2.14), the intercept of the Phillips curve in (3.4) also depends on past inflation and $\frac{\beta a_{\pi,l+1}^{(L)}}{1-\beta a_{\pi,1}^{(L)}} > 0$. Therefore, a larger L corresponds to a larger intercept of the Phillips curve.

The best strategy for the central bank is to announce a large L when the shock hits but change its announcement to $L = 2$ in the subsequent periods to avoid additional loss. However, this strategy does not seem to be attainable under reasonable assumptions. We will argue in Section 4 that agents might believe the central bank when its announcement is different from the actual implementation when uncertainties are present. But agents will not believe the central bank when it changes its announcements all the time. The fact that the central bank would like to change its announcements about the length of AIT over time but cannot argues for ambiguous communication.

4 Will Agents Be Fooled Forever?

Section 3 demonstrates AIT's time- inconsistent nature and suggests that the central bank should make ambiguous announcement about the horizon of AIT. In Section 3, we assume that the central bank has full credibility. This section assesses this assumption and focuses on the following questions: Can agents eventually learn the truth about the central bank's behavior? Is the central bank's welfare-enhancing strategy sustainable and beneficial over time in light of learning? Section 4.1 sets up the environment of social learning. Section 4.2 studies the impact of uncertainty about underlying economic conditions on learning, Section 4.3 further introduces ambiguous communication, and Section 4.4 assesses the impact of initial credibility. Parameter values are in Appendix C.

4.1 Social Learning

We model social learning similar to Arifovic, Bullard, and Kostyshyna (2013) and Hachem and Wu (2017). Beliefs are rule-based. There are L_{max} groups of beliefs: $L \in \{1, 2, ...L_{max}\}$, and beliefs can also be heterogeneous within each group. Agents update their beliefs via tournament selection and mutation.

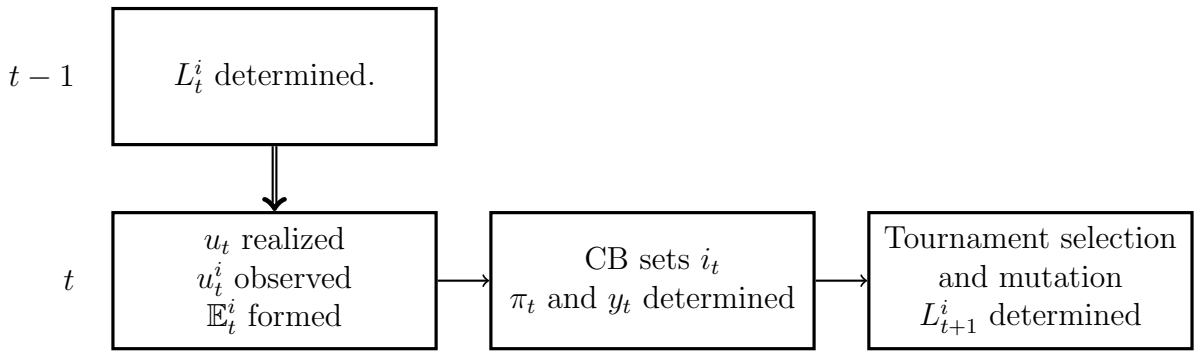


Figure 5: The sequence of events

The sequence of events is drawn in Figure 5. Before entering period t, each agent has a belief about L, and we label it L_t^i. When agents enter period t, a shock u_t is realized,

and agent i observes it with a private signal u_t^i. Within-group heterogeneity is present when $u_t^i \neq u_t$. An agent believes all other agents in the economy have the same belief set as his/hers, and forms his/her expectations about variables at time t using equations (2.4) - (2.5).

$$\mathbb{E}_t^i \pi_t = a_{\pi,1}^{(L_t^i)} \pi_{t-1} + ... + a_{\pi,L_t^i-1}^{(L_t^i)} \pi_{t-L_t^i+1} + b_\pi^{(L_t^i)} u_t^i \tag{4.1}$$

$$\mathbb{E}_t^i \hat{y}_t = a_{y,1}^{(L_t^i)} \pi_{t-1} + ... + a_{y,L_t^i-1}^{(L_t^i)} \pi_{t-L_t^i+1} + b_y^{(L_t^i)} u_t^i, \tag{4.2}$$

Next, agent i forms expectations about inflation in the next period. In this section, we allow the cost-push shock to be serially correlated:

$$u_t = \rho u_{t-1} + \epsilon_t, \qquad \epsilon_t \sim N(0, \sigma_\epsilon^2) \tag{4.3}$$

Then, agent i's expectations about the cost-push shock next period are

$$\mathbb{E}_t^i u_{t+1} = \rho u_t^i. \tag{4.4}$$

Consequently,

$$\mathbb{E}_t^i \pi_{t+1} = a_{\pi,1}^{(L_t^i)} \mathbb{E}_t^i \pi_t + ... + a_{\pi,L_t^i-1}^{(L_t^i)} \pi_{t-L_t^i+2} + b_\pi^{(L_t^i)} \rho u_t^i. \tag{4.5}$$

The average expectation is

$$\bar{\mathbb{E}}_t^i \pi_{t+1} = \frac{1}{N} \sum_i \mathbb{E}_t^i \pi_{t+1} \tag{4.6}$$

where N is the total number of agents.

Therefore, the Phillips curve in equation (2.2) becomes

$$\pi_t = \beta \bar{\mathbb{E}}_t^i \pi_{t+1} + \kappa \hat{y}_t + u_t. \tag{4.7}$$

The central bank now picks an i_t to minimize the period loss $\mathcal{L}_t$ defined in (2.10) subject to the Phillips curve in equation (4.7) and the IS curve in (2.1). The realized variables are π_t and $\hat{y}_t$, whose law of motion is different from equations (2.4) - (2.5).

Tournament selection works as follows: At each time period t, agents are randomly selected to meet in pairs, and we draw N meetings in total. When two agents meet, they update their beliefs by comparing forecast errors, which are defined as

$$\varepsilon_t^i = |\, \mathbb{E}_t^i \, \pi_t - \pi_t| + |\, \mathbb{E}_t^i \, \hat{y}_t - \hat{y}_t|. \tag{4.8}$$

When two agents are from the same group, i.e., $L_t^i = L_t^j$, they stay in this group. When two agents come from two different groups, suppose agent i has a smaller forecast error $\varepsilon_t^i < \varepsilon_t^j$. Agent i stays in his/her current group, whereas agent j switches. At the end of period t, agents can mutate, i.e., randomly switch to another rule. This step further updates the belief to L_{t+1}^i, which will be used in time $t+1$.

4.2 Uncertainty

This section assesses the impact of uncertainty about underlying economic conditions on learning. We introduce uncertainty by allowing agents to observe noisy signals of the underlying shock:

$$u_t^i = u_t + v_t^i, \qquad v_t^i \sim N(0, \sigma_v^2). \tag{4.9}$$

When agents observe the economic fundamental with certainty, i.e., $\sigma_v = 0$, there is no within-group heterogeneity and each agent within the group L behaves the same. $\sigma_v > 0$ captures uncertainty and allows within group heterogeneity.

(a) No uncertainty (b) Low uncertainty (c) High uncertainty

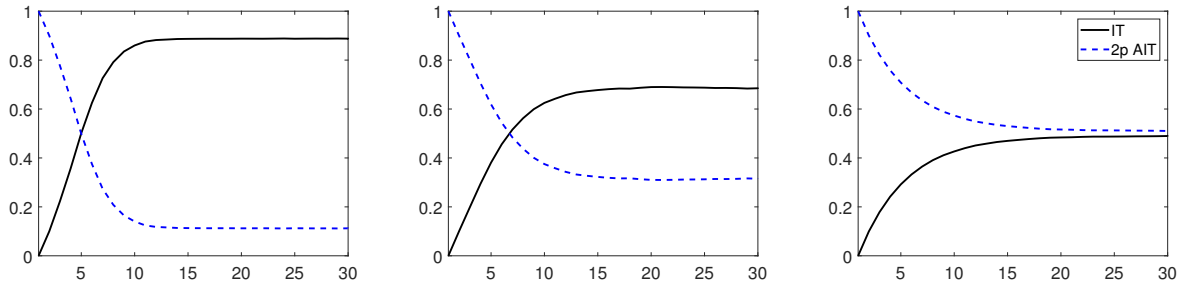


Figure 6: Uncertainty

In this section, the central bank announces 2-period AIT. In this case, we have two groups of beliefs, $L_{max} = 2$. They either believe IT or 2-period AIT. We define the fraction of agents in each group as $p^{(1)}$ and $p^{(2)}$, where $p^{(1)} + p^{(2)} = 1$.

Figure 6 plots three scenarios: There is no uncertainty in the left panel, or $\sigma_\upsilon = 0$. The middle panel is associated with low uncertainty, and the right panel with high uncertainty. The black lines mark the fraction of IT believers, and the blue lines show the evolution of the fraction of AIT believers. All the fractions are averaged over N simulations.

When agents observe the underlying economic shock with no uncertainty, then agents will eventually learn the truth and all agents converge to believe in IT (subject to mutation), and they learn it relatively quickly. This result is intuitive and expected. When uncertainty increases, more agents will believe AIT in the long run, and it takes a longer time to converge.

Notice that we interpret time in a relative sense instead of an absolute sense. Although we calibrate structural parameters to an annual frequency, the time to converge still very much depends on the parameterization of learning, which is difficult to pin down.

4.3 Ambiguous Communication

In this section, we further introduce ambiguous communication. We extend our analysis to four groups of beliefs: $L = 1$, $L = 2$, $L = 3$, and $L = 4$.

As a benchmark, the left panel of Figure 7 plots the fraction of AIT believers when the

(a) Clear communication (b) Ambiguous communication

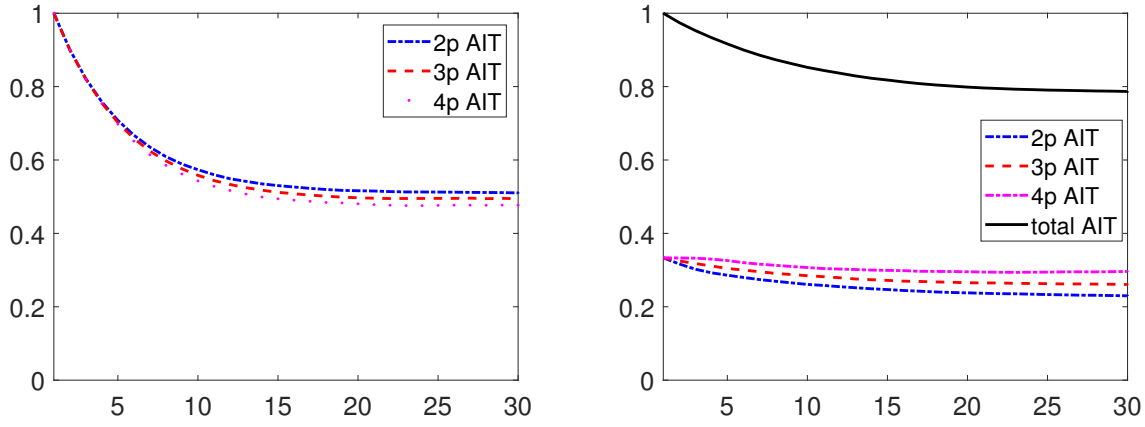


Figure 7: Clear vs. ambiguous communication

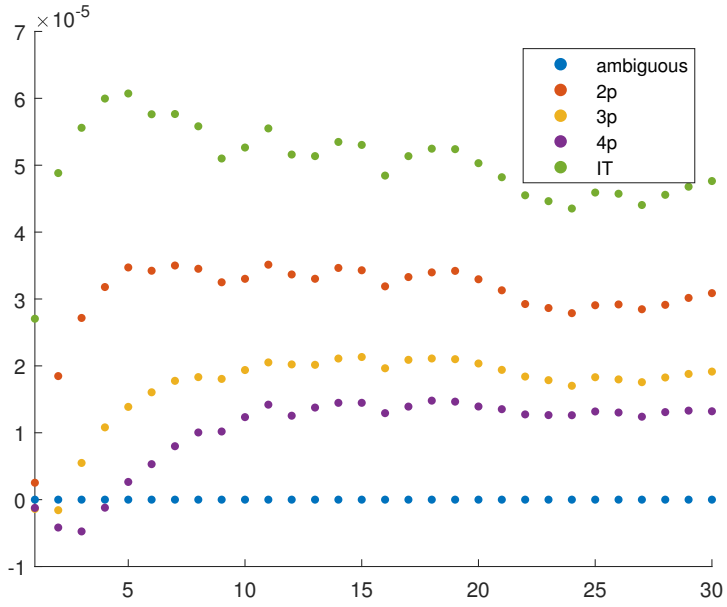
central bank announces AIT with a clear horizon L, for $L = 2, 3$, or 4. In each case, there are only two groups of beliefs: IT, and AIT with the length communicated by the central bank. The blue line corresponds to a central bank that announces 2-period AIT, and replicates the blue line in the right panel of Figure 6. The red and purple lines represent a central bank that announces 3-period or 4-period AIT, respectively.

At each point in time, the fraction of AIT believers is the highest when the central bank announces 2-period AIT, and lowest when the central bank announces 4-period AIT. That is because 2-period AIT is the closest to IT, which is what the central bank actually does.

The right panel of Figure 7 plots the evolution of beliefs under ambiguous communication, where the central bank only announces AIT as its tool to manage inflation but does not specify its length. At the beginning, an equal fraction of agents believe in AIT with different length: $p^{(2)} = p^{(3)} = p^{(4)}$. Then these fractions evolve through social learning. Blue, red, and purple lines capture the fraction of agents in each AIT group, and the black line is the sum of the three, which shows the fraction of total AIT believers.

Comparing the black line in the right panel with each of the lines in the left panel, we find that ambiguous communication allows a larger fraction of agents to believe in the AIT strategy that the central bank announces, even though it implements IT ex post. Therefore,

Figure 8: Welfare loss



ambiguous communication helps the central bank gain more credibility in the long run.

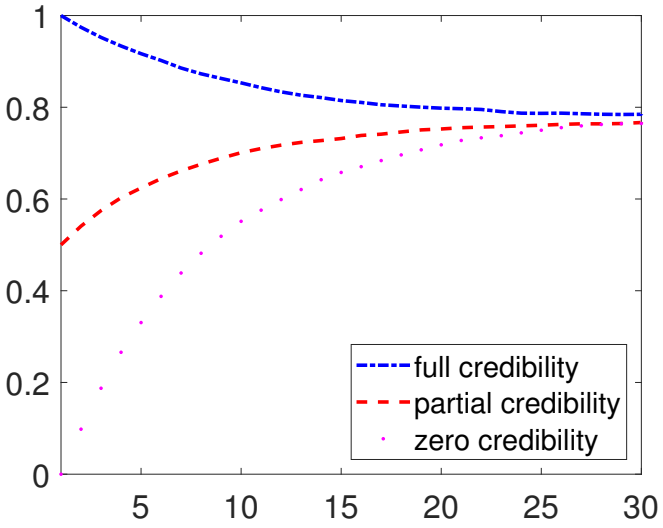
Comparing the three color lines in the right panel, similar to the left panel, more agents believe in 2-period AIT than in 3- or 4-period. Notice that in the left panel, the three colors represent three different types of announcements. For each announcement, only the corresponding color is relevant, whereas, in the right panel, there is one type of announcement - ambiguous announcement - but agents can have different beliefs about L in this scenario.

We have established that ambiguous communication can further help the central bank gain a following of its communicated AIT. More importantly, does it improve welfare in the long run? We illustrate this in Figure 8.

In the figure, we plot the period loss, which is averaged over N simulations, for 5 different announcements. We take the ambiguous communication in the blue dots as the benchmark, and plot loss in other cases relative to this case. Therefore, the blue dots are at zero by construction.

Initially, the purple dots which represent 4-period AIT, show the smallest loss. This result

Figure 9: Initial credibility



is consistent with what we find in Section 3.2: The central bank wants to announce a large L when the shock hits on impact. But the advantage of announcing a large L disappears quickly. That is because 4-period AIT loses its following quickly. In the long run, ambiguous communication is associated with the smallest loss.

Comparing IT in the green dots with the rest of the colors, announcing IT and sticking to it, which is the operating framework before introducing AIT, is universally the worst in terms of welfare. This result is consistent with our argument in Section 3.

4.4 Initial Credibility

We have shown that uncertainty and ambiguous communication can increase the fraction of agents who believe AIT although it is time inconsistent. Our analysis so far in this section starts with a central bank that is endowed with full credibility, which is a stark assumption. For example, Coibion et al. (2020) use survey data to show that the private sector does not understand the impact of AIT soon after the introduction of AIT. In this section, we assess what happens when the central bank is partial credible or not credible initially.

In Figure 9, we plot the evolution of the fraction of AIT believers with different levels of

initial credibility. The blue line starts with 100 percent believers, which replicates the black line in the right panel of Figure 7. The red line starts with partial credibility of 50 percent AIT believers, and the purple line starts with no credibility. In the long run, all three lines converge to the same point where approximately 80 percent of agents believe AIT. Therefore, the initial credibility does not matter in the long run.

5 Conclusion

Our paper studies the implications of AIT. We focus on two key issues: time inconsistency and ambiguous communication. AIT can improve the available trade-off between inflation and real activity that the central bank faces, but this does not automatically improve social welfare. To do that, the central bank has the incentive to deviate from its communicated objective and implement IT ex post. This strategy is welfare improving, assuming that the central bank can convince the private sector of its intention to implement AIT. We assess this assumption using social learning. We show that without uncertainty about economic fundamentals, agents will eventually learn the truth, which is an intuitive result. However, with higher uncertainty, the central bank can convince a higher fraction of agents believe in AIT in the long run. Moreover, ambiguous communication further helps the central bank to gain credibility and improve welfare in the long run despite AIT being time inconsistent. These results do not rely on the central bank having credibility initially.

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A Equilibrium for 2-Period AIT

We solve the equilibrium by the undetermined coefficient method. From the central bank's objective function given in equation (2.10), we know that the equilibrium output gap will be a function of π_{t-1}, and since the loss function is quadratic, we guess the following linear functions for π_t and $\hat{y}_t$:

$$\pi_t = a\pi_{t-1} + bu_t \tag{A.1}$$
$$\hat{y}_t = c\pi_{t-1} + du_t \tag{A.2}$$

which yields

$$E_t\pi_{t+1} = a\pi_t + b\rho u_t \tag{A.3}$$
$$E_t\hat{y}_{t+1} = c\pi_t + d\rho u_t \tag{A.4}$$

Substitute into the Phillips curve and get

$$\pi_t = \beta\left(a\pi_t + b\rho u_t\right) + \kappa\hat{y}_t + u_t = \frac{\kappa}{1-\beta a}\hat{y}_t + \frac{\beta b\rho + 1}{1-\beta a}u_t \tag{A.5}$$

Re-arrange to get

$$\hat{y}_t = \frac{1}{\kappa}\left\{\pi_t - \beta(a\pi_t + b\rho u_t) - u_t\right\} = \frac{1}{\kappa}\left(1-\beta a\right)\pi_t - \frac{1}{\kappa}\left(\beta b\rho + 1\right)u_t \tag{A.6}$$

Therefore, the slope of the Phillips curve becomes

$$\frac{\partial\pi_t}{\partial\hat{y}_t} = \frac{\kappa}{1-\beta a} \tag{A.7}$$

From the central bank's objective function, the first condition on $\hat{y}_t$ is

$$\begin{aligned} 0 = {} & \frac{1}{4}\left(\pi_t + \pi_{t-1}\right)\frac{\partial\pi_t}{\partial\hat{y}_t} + \lambda\hat{y}_t + \frac{1}{4}\beta\left(E_t\pi_{t+1} + \pi_t\right)\left(\frac{\partial E_t\pi_{t+1}}{\partial\pi_t} + \frac{\partial\pi_t}{\partial\pi_t}\right)\frac{\partial\pi_t}{\partial\hat{y}_t} \\ & + \beta\lambda E_t\hat{y}_{t+1}\frac{\partial E_t\hat{y}_{t+1}}{\partial\pi_t}\frac{\partial\pi_t}{\partial\hat{y}_t} + \frac{1}{4}\beta^2\left(E_t\pi_{t+2} + E_t\pi_{t+1}\right)\left(\frac{\partial E_t\pi_{t+2}}{\partial\pi_t} + \frac{\partial E_t\pi_{t+1}}{\partial\pi_t}\right)\frac{\partial\pi_t}{\partial\hat{y}_t} \\ & + \beta^2\lambda E_t\hat{y}_{t+2}\frac{\partial E_t\hat{y}_{t+2}}{\partial\pi_t}\frac{\partial\pi_t}{\partial\hat{y}_t} + ... \end{aligned} \tag{A.8}$$

where

$$E_t\pi_{t+j} = a^j\pi_t + \Sigma_{k=0}^{j-1}a^k b\rho^{j-k}u_t \tag{A.9}$$

and

$$E_t\hat{y}_{t+j} = c^j\pi_t + \Sigma_{k=0}^{j-1}c^k d\rho^{j-k}u_t \tag{A.10}$$

which makes

$$\frac{\partial E_t \pi_{t+j}}{\partial \hat{y}_t} = a^j, \tag{A.11}$$

$$\frac{\partial E_t \hat{y}_{t+j}}{\partial \hat{y}_t} = c^j \tag{A.12}$$

Substitute $E_t\pi_{t+j}$, $E_t\hat{y}_{t+j}$, $\frac{\partial E_t\pi_{t+j}}{\partial \hat{y}_t}$, $\frac{\partial E_t\hat{y}_{t+j}}{\partial \hat{y}_t}$ $\frac{\partial \pi_t}{\partial \hat{y}_t}$ and $\hat{y}_t$ in the first-order condition and re-arrange the equation. We get

$$\frac{\kappa}{1-\beta a}\left\{\frac{1}{4}\left(\pi_t+\pi_{t-1}\right)+\frac{1}{4}\Sigma\beta^j\left(a^j\pi_t+\Sigma_{k=0}^{j-1}a^k b\rho^{j-k}u_t\right)\left(a^j+a^{j-1}\right)+\lambda\Sigma\beta^j\left(c^j\pi_t+\Sigma_{k=0}^{j-1}c^k d\rho^{j-k}u_t\right)c^j\right\}$$
$$=-\lambda\left\{\frac{1}{\kappa}\left(1-\beta a\right)\pi_t-\frac{1}{\kappa}\left(\beta b\rho+1\right)u_t\right\} \tag{A.13}$$

To keep analytic tractability, we consider $\rho = 0$. In this case, the first-order condition reduces to

$$\frac{1}{4}\left(\pi_t+\pi_{t-1}\right)\frac{\partial \pi_t}{\partial \hat{y}_t}+\lambda\hat{y}_t+\frac{1}{4}\frac{\partial \pi_t}{\partial \hat{y}_t}\Sigma_{j=1}^{\infty}\beta^j\left(a^j+a^{j-1}\right)^2\pi_t+\lambda\frac{\partial \pi_t}{\partial \hat{y}_t}\Sigma_{j=1}^{\infty}\beta^j c^{2j}\pi_t=0 \tag{A.14}$$

Since $\Sigma_{j=1}^{\infty}\beta^j\left(a^j+a^{j-1}\right)^2=\left(1+\frac{2}{a}+\frac{1}{a^2}\right)\frac{\beta a^2}{1-\beta a^2}$ and $\Sigma_{j=1}^{\infty}\beta^j c^{2j}=\frac{\beta c^2}{1-\beta c^2}$, we get

$$\left\{\frac{\kappa}{1-\beta a}\left[\frac{1}{4}+\frac{1}{4}\left(1+\frac{2}{a}+\frac{1}{a^2}\right)\frac{\beta a^2}{1-\beta a^2}+\lambda\frac{\beta c^2}{1-\beta c^2}\right]+\frac{\lambda}{\kappa}\left(1-\beta a\right)\right\}\pi_t=-\frac{1}{4}\frac{\kappa}{1-\beta a}\pi_{t-1}+\frac{\lambda}{\kappa}u_t \tag{A.15}$$

Rearranging and comparing with the conjectured form, we get

$$a=-\left\{\frac{\kappa}{1-\beta a}\left[\frac{1}{4}+\frac{1}{4}\left(1+\frac{2}{a}+\frac{1}{a^2}\right)\frac{\beta a^2}{1-\beta a^2}+\lambda\frac{\beta c^2}{1-\beta c^2}\right]+\frac{\lambda}{\kappa}\left(1-\beta a\right)\right\}^{-1}\frac{1}{4}\frac{\kappa}{1-\beta a} \tag{A.16}$$

$$b=\left\{\frac{\kappa}{1-\beta a}\left[\frac{1}{4}+\frac{1}{4}\left(1+\frac{2}{a}+\frac{1}{a^2}\right)\frac{\beta a^2}{1-\beta a^2}+\lambda\frac{\beta c^2}{1-\beta c^2}\right]+\frac{\lambda}{\kappa}\left(1-\beta a\right)\right\}^{-1}\frac{\lambda}{\kappa} \tag{A.17}$$

From equation (A.6), we get

$$\hat{y}_t=\frac{1}{\kappa}\left(1-\beta a\right)a\pi_t+\left(\frac{1}{\kappa}\left(1-\beta a\right)b-\frac{1}{\kappa}\right)u_t \tag{A.18}$$

Comparing with the conjectured form leads to

$$c=\frac{1}{\kappa}\left(1-\beta a\right)a \tag{A.19}$$

$$d=\frac{1}{\kappa}\left(1-\beta a\right)b-\frac{1}{\kappa} \tag{A.20}$$

parameter	description	value
ρ	serial correlation	0.6
σ_ϵ	std. dev. in shocks	0.1
β	discount factor	0.95
θ	price rigidity	0.6
γ	elasticity of intertemporal substitution	1
φ	elasticity of labor supply	1
η	the price elasticity of demand	10

Table C.1: Structural parameters

B Proofs

B.1 Proof of Proposition 1

We first prove that $a < 0$ by contradiction. Suppose on the contrary that $a > 0$. First, it cannot be that $a > 1$; otherwise, for any disturbance in the current inflation rate, the expected long-term inflation will explode, since $E_t\pi_{t+j} = a^j\pi_t \to \infty$ when $j \to \infty$. Second, since $0 < a < 1$, we have $0 < a < \frac{1}{\beta}$. If this is the case, $\frac{\kappa}{1-\beta a} > 0$, $\frac{1}{4}+\frac{1}{4}\left(1+\frac{2}{a}+\frac{1}{a^2}\right)\frac{\beta a^2}{1-\beta a^2} > 0$, and $\frac{\beta c^2}{1-\beta c^2} > 0$, which makes $\frac{\kappa}{1-\beta a}\left[\frac{1}{4}+\frac{1}{4}\left(1+\frac{2}{a}+\frac{1}{a^2}\right)\frac{\beta a^2}{1-\beta a^2}+\lambda\frac{\beta c^2}{1-\beta c^2}\right] > 0$. In addition, $\frac{\lambda}{\kappa}(1-\beta a) > 0$. Therefore, if $0 < a < \frac{1}{\beta}$,

$$a = -\left\{\frac{\kappa}{1-\beta a}\left[\frac{1}{4}+\frac{1}{4}\left(1+\frac{2}{a}+\frac{1}{a^2}\right)\frac{\beta a^2}{1-\beta a^2}+\lambda\frac{\beta c^2}{1-\beta c^2}\right]+\frac{\lambda}{\kappa}(1-\beta a)\right\}^{-1}\frac{1}{4}\frac{\kappa}{1-\beta a} < 0.$$

which is a contradiction.

Therefore, we have proved that $a < 0$. Then, it follows $c = \frac{1}{\kappa}(1-\beta a)\, a < 0$.

B.2 Proof of Proposition 2

This proposition is a result of the comparison between equation (2.14) and equation (2.13). Because $a < 0$, the slope of the Phillips curve under AIT, $\frac{\kappa}{1-\beta a}$, is smaller than the slope under IT, κ. For the same reason, the intercept of the Phillips curve under AIT, which is $\frac{1}{1-\beta a}u_t$, is smaller than the intercept under IT, which is u_t.

C Calibration

C.1 Structural parameters

We calibrate the model at an annual frequency; see parameters in Table C.1. We calibrate the parameters following the New Keynesian literature. We assume a unitary Frisch elasticity of labor supply, $\varphi = 1$, and log utility of consumption, $\sigma = 1$. We set $\beta = 0.95$ to match the annual return on safe financial assets in the U.S. We choose ϵ, the price elasticity of demand

parameter	description	value
N	number of agents	1000
N_{sim}	number of simulations	1000
N_{meet}	number of meetings every period	1000
$p_{mutation}$	probability of mutation	0.1
σ_v	std. dev. in private signals	0 (no)/ 0.03 (low)/ 0.3 (high)

Table C.2: Learning parameters

to be 10. We set the Calvo parameter, which governs price stickiness, to be 0.6. With these underlying parameters, the slope of the Phillips curve, κ, is 2.73, and the weight on output gap stabilization in the welfare function, λ is 0.27.

For the shock process, we assume $\rho = 0$ in Section 2 and Section 3. We set $\rho = 0.6$ when simulating the general version of the model in Section 4. We set $\sigma_\epsilon = 0.1$.

C.2 Learning parameters

We summarize the learning parameters in Table C.2. There are $N = 1000$ agents in the economy, and we draw 1000 meetings at each point in time.

We set the number of agents in the private sector to be 1000. We further assume that the number of random meetings per period is 1000. Our results are reported as the average from 1000 simulations. We follow Arifovic, Bullard, and Kostyshyna (2013) by setting the mutation probability to 10 percent. For the parameters governing the levels of uncertainty, we choose $\sigma_v = 0$ for no uncertainty, 0.03 for low uncertainty, and 0.3 for high uncertainty.