Is the Volatility of the Market Price of Risk due to Intermittent Portfolio Re-balancing? *

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Abstract

Our paper examines whether intermittent portfolio re-balancing on the part of some stock market investors can help to explain the counter-cyclical volatility of aggregate risk compensation in financial markets. To answer this question, we set up an incomplete markets model in which CRRA-utility investors are subject to aggregate and idiosyncratic shocks and have heterogeneous trading technologies. In our model, a large mass of passive investors do not re-balance their portfolio shares in response to aggregate shocks, while a smaller mass of active investors adjust their portfolio each period to respond to changes in the investment opportunity set. We find that intermittent re-balancers amplify the effect of aggregate shocks on the time variation in risk premia by a factor of four in a calibrated version of our model.

Keywords: Asset Pricing, Household Finance, Risk Sharing, Limited Participation (JEL code G12)

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1 Introduction

One of the largest challenges for standard dynamic asset pricing models is to explain the enormous countercylical variation in the risk-return trade-off in equity markets. Our paper establishes that the participation of passive investors in equity markets who do not respond to changes in the investment opportunity set and who fail to continuously rebalance should be considered as a plausible alternative explanation.[1]

There is a large group of households that invest in equities but only change their portfolio shares infrequently, even after large common shocks to asset returns. Ameriks and Zeldes (2004) find that over a period of 10 years 44% of households in a TIAA-CREF panel made no changes to either flow or asset allocations, while 17 % of households made only a single change. Calvet, Campbell, and Sodini (2009), in a comprehensive dataset of Swedish households, found a weak response of portfolio shares to common variation in returns: between 1999 and 2002, the equal-weighted share of household financial wealth invested in risky assets drops from 57% to 45% in 2002, which is indicative of very weak re-balancing by the average Swedish household during the bear market. Finally, Brunnermeier and Nagel (2008) conclude that inertia is the main driver of asset allocation in US household portfolios, while changes in wealth only play a minor role.

Without a specific model in mind, it is hard to know what effect, if any, infrequent re-balancing would have on equilibrium asset prices. In an equilibrium where all households are equally exposed to aggregate shocks, there is no need for any single household to re-balance his or her portfolio in response to an aggregate shock. This is clearest in a representative agent economy. However, in an environment in which households have heterogeneous exposures to aggregate shocks, the frequency of re-balancing may have important aggregate effects.

We conjecture that infrequent re-balancing on the part of passive investors may contribute to countercylical volatility in risk prices because intermittent rebalancers mimick the portfolio behavior of households with countercyclical risk aversion. To check the validity of this conjecture, we set up a standard incomplete markets model in which investors are subject to idiosyncratic and aggregate risk. The investors have heterogeneous trading technologies; a large mass of households are passive investors, who do not change their portfolio in response to changes in the investment opportunity set, but a smaller mass of active investors do. We consider two types of passive investors: those that re-balance their portfolio each period to keep their portfolio shares constant, and those that re-balance intermittently. In the economy with passive investors, who rebalance infrequently, we find that the volatility of the price of aggregate risk is four times higher than in the economy with continuously re-balancing passive investors. While the individual welfare loss associated with intermittent rebalancing is small, and hence small costs would suffice to explain this

[1] The standard explanations rely on countercyclical risk aversion and heteroscedasticity in aggregate consumption growth, but they fall short quantitatively.

behavior, the aggregate effects of non-rebalancing are large. When the economy is affected by an adverse aggregate shock and the price of equity declines as a result, passive investors who re-balance end up buying equities to keep their portfolio shares constant, while intermittent rebalancers do not. This means that in the case of intermittent rebalancers more aggregate risk is concentrated among the smaller pool of active investors whenever the economy is affected by a negative aggregate shock. Hence, in a way, these intermittent rebalancers act like households with countercyclical risk aversion.

In our approach, the intermittent rebalancers choose an intertemporal consumption path to satisfy the Euler equation in each period, but, in between rebalancing times, their savings decisions can only affect their holdings of the risk-free assets. Only in rebalancing periods can they actually change their equity holdings. Following Duffie and Sun (1990), we assume that dividends are re-invested in equity and interest earnings are re-invested in the risk-free asset during non-adjustment periods. Hence, our approach is different from the one adopted by Lynch (1996) and Gabaix and Laibson (2002). They consider households who do not adjust consumption in each period; these investors are off their Euler equation in non-adjustment periods.[2]Gabaix and Laibson (2002) assume the portfolio is continuously rebalanced. This approach potentially introduces serial correlation and predictability in aggregate consumption growth (Piazzesi (2002)). In contrast, we fix the properties of aggregate consumption growth; we rely on a small mass of active investors to clear the goods and asset markets in each period.

From the perspective of existing Dynamic Asset Pricing Models (DAPM's), there is a puzzling amount of variation in the risk-return trade-off in financial markets. In standard asset pricing models, the price of aggregate risk is constant (see, e.g., the Capital Asset Pricing Model of Sharpe (1964) and Lintner (1965)) or approximately constant (see, e.g., Mehra and Prescott (1985)'s calibration of the Consumption-CAPM). In the data, there is some variation in the conditional volatility of aggregate consumption growth that can deliver time-varying risk prices in a standard Consumption-CAPM, but probably not enough –and not of the right type– to explain the variation in the data. Recently, Campbell and Cochrane (1999) and Barberis, Huang, and Santos (2001), among others, have shown that standard representative agent models with different, non-standard preferences can rationalize counter-cyclical variation in Sharpe ratios.

Habit formation preferences can help to match the counter-cyclicality of risk premia in the data (Campbell and Cochrane, 1999; Constantinides, 1990), as well as other feautures of the joint distribution of asset returns and macro-economic outcomes over the business cycle (see Boldrin, Christiano, and Fisher, 2001; Jermann, 1998). However, Lettau and Ludvigson (2001) measure the time-variation in the Sharpe ratio on equities in the data. This time variation is driven by variation in the conditional mean of returns (i.e. the predictability of returns) as well

[2] Reis (2006) adopts a rational inattention approach to rationalize this type of behavior.

the variation in the conditional volatility of stock returns. In the data, these two objects are negatively correlated, according to Lettau and Ludvigson (2001), and this gives rise to a considerable amount of variation in the conditional Sharpe ratio: the annual standard deviation of the estimated Sharpe ratio is on the order of 50% per annum. Lettau and Ludvigson (2001) compare their estimate of the conditional Sharpe ratio to that implied by the Campbell and Cochrane (1999) external habit model, and they find that their model dramatically understates the volatility of the conditional Sharpe ratio.[3] Moreover, it is not clear whether households actually have preferences defined over the difference between a habit and actual consumption. In fact, a key prediction of these preferences is that the household's risk aversion, and hence their allocation to risky assets, varies with wealth. According to Brunnermeier and Nagel (2008), there is little evidence of this in the data.

Other channels for time-variation in risk premia that have been explored in the literature include differences in risk aversion (Chan and Kogan, 2002; Gomes and Michaelides, 2008), in exposure to nontradeable risk (Garleanu and Panageas, 2007), participation constraints (Basak and Cuoco, 1998; Guvenen, 2009; Saito, 1996), differences in beliefs (Detemple and Murthy, 1997) and differences in information (Schneider, Hatchondo, and Krusell, 2005). Our paper imposes temporary participation constraints on the intermittent rebalancers instead of permanent ones, and it explores heterogeneity in trading technologies instead of heterogeneity in preferences.

Related Literature There is a large literature on infrequent consumption adjustment starting with Grossman and Laroque (1990)'s analysis of durable consumption in a representative agent setting. Lynch (1996) specifically focuses on the aggregate effects of infrequent consumption adjustment by heterogeneous consumers to explain the equity premium puzzle. Gabaix and Laibson (2002) extend this analysis to a continuous-time setup that allows for closed-form solutions. Our paper is more narrowly focused on the aggregate effects of infrequent portfolio adjustment, but the households in our model face aggregate as well as idiosyncratic risk. This feature is critical to generate reasonable consumption implications. In our model, sophisticated investors load up on aggregate consumption risk. This seems consistent with the data. The consumption of the 10 % wealthiest households is five times more exposed to aggregate consumption growth than that of the average US household (Parker and Vissing-Jorgensen (2009)). In contrast, less sophisticated investors are more exposed to idiosyncratic risk. This is broadly in line with the data. Malloy, Moskowitz, and Vissing-Jorgensen (2009) find that the average consumption growth rate for stock-holders is between 1.4 and two times as volatile as that of non-stock holders. They also find that aggregate stockholder consumption growth for the wealthiest segment (upper third) is up to 3 times as sensitive to aggregate consumption growth shocks as that of non-stock holders.

[3] In addition, they find that the heteroscedasticity in U.S. aggregate consumption growth does not help to explain the variation in the Sharpe ratio either.

To solve for the equilibrium allocations and prices, we develop an extension of the multiplier method developed by Chien, Cole, and Lustig (2007) to handle intermittent rebalancers. In continuous-time finance, Cuoco and He (2001) and Basak and Cuoco (1998) used stochastic weighting schemes to characterize allocations and prices. Our approach differs because it provides a tractable and computationally efficient algorithm for computing equilibria in environments with a large number of agents subject to idiosyncratic risk as well as aggregate risk, and heterogeneity in trading opportunities. The use of cumulative multipliers in solving macro-economic equilibrium models was pioneered by Kehoe and Perri (2002), building on earlier work by Marcet and Marimon (1999). Our use of measurability constraints to capture portfolio restrictions is similar to that in Aiyagari, Marcet, Sargent, and Seppala (2002) and Lustig, Sleet, and Yeltekin (2007), who consider an optimal taxation problem, while the aggregation result extends that in Chien and Lustig (2009) to an incomplete markets environment.

Abel, Eberly, and Panageas (2006) consider a portfolio problem in which the investor pays a cost to observe her portfolio, and they show that even small costs can rationalize fairly large intervals in which the household does not check its portfolio, and finances its consumption out of the riskless account. We do not endogenize the decision to observe the value of the portfolio, but, instead, we focus on the aggregate equilibrium implications of what Abel et al. (2006) call 'stock market inattention'. However, we assume that our investor knows the value of his holdings when making consumption decisions, even in non-rebalancing periods. Hence, we are implicitly assuming that it is the cost of reallocating his portfolio that is prevent continuous adjustment rather than the cost of finding out about the value of his portfolio.

2 Counter-cyclical and volatile Sharpe ratios

Lettau and Ludvigson (2001) measure the conditional Sharpe ratio on U.S. equities by forecasting stock market returns and realized volatility (of stock returns) using different predictors, and they obtain highly countercyclical and volatile Sharpe ratios. To get a clear sense of the link with business cycles, we consider a simple exercise. In expansions (recessions), the investor buys the stock market index in the n-th quarter after the NBER through (peak) and sells after 4 quarters. The NBER defines recessions as periods that stretch from the peak to the through. Strictly speaking, this is not an implementable investment strategy, because NBER peaks and troughs are only announced with a delay.[4] Nonetheless, the average returns on this investment strategy provide a clear indication of the cyclical behavior of the expected returns conditional on the aggregate state being expansion (recessions).

[4] However, there is recent evidence that agents realize a recession has started about 1 quarter after the peak (see Doms and Morin, 2004).

Figure 1 plots the Sharpe ratio on this investment strategy in the U.S. stock market, conditioning on the quarter of the NBER recession/expansion. We plot the (sample) Sharpe ratios obtained in both subsamples. This Sharpe ratio, which conditions only on the stage in the NBER business cycle, clearly increases in recessions (after the peak) and decreases in expansions (after the through). The smoothed version of the conditional Sharpe ratio peaks three quarters into the recession at about 0.60, and it reaches its low three quarters after the trough at about 0.1. The details of the computation are in section C of the appendix.

[Figure 1 about here.]

3 Model

We consider an endowment economy in which households sequentially trade assets and consume. All households are ex ante identical, except for the restrictions they face on the menu of assets that they can trade. These restrictions are imposed exogenously. We refer to the set of restrictions that a household faces as a household trading technology. The goal of these restrictions is to capture the observed portfolio behavior of most households.

We will refer to households as being *passive traders* if they take their portfolio composition as given and simply choose how much to save or dissave in each period. We will also allow for other households to optimally change their portfolio in response to changes in the investment opportunity set. We refer to these traders as *active traders* since they actively manage the composition of their portfolio each period. To solve for the equilibrium allocations and prices, we extend the method developed by Chien et al. (2007) (hereafter CCL) to allow for passive traders who only intermittently adjust their portfolio. In this section we describe the environment, and we describe the household problem for each of different asset trading technologies. We also define an equilibrium for this economy.

3.1 Environment

We consider an endowment economy with a unit measure of households who are subject to both aggregate and idiosyncratic income shocks. Households are ex ante identical, except for the trading technology they are endowed with. Ex post, these households differ in terms of their idiosyncratic income shock realizations. All of the households face the same stochastic process for idiosyncratic income shocks, and all households start with the same present value of tradeable wealth.

In the model time is discrete, infinite, and indexed by $t = 0, 1, 2, ...$ The first period, $t = 0$, is a planning period in which financial contracting takes place. We use $z_t \in Z$ to denote the aggregate shock in period t and $\eta_t \in N$ to denote the idiosyncratic shock in period t. z^t denotes the history

of aggregate shocks, and, similarly, η^t, denotes the history of idiosyncratic shocks for a household. The idiosyncratic events η are i.i.d. across households. We use $\pi(z^t, \eta^t)$ to denote the unconditional probability of state (z^t, η^t) being realized. The events are first-order Markov, and we assume that

$$\pi(z^{t+1}, \eta^{t+1}|z^t, \eta^t) = \pi(z_{t+1}|z_t)\pi(\eta_{t+1}|z_{t+1}, \eta_t).$$

Since we can appeal to a law of large number, $\pi(z^t, \eta^t)/\pi(z^t)$ also denotes the fraction of agents in state z^t that have drawn a history η^t. We use $\pi(\eta^t|z^t)$ to denote that fraction. We introduce some additional notation: $z^{t+1} \succ z^t$ or $y^{t+1} \succ y^t$ means that the left hand side node is a successor node to the right hand side node. We denote by $\{z^\tau \succ z^t\}$ the set of successor aggregate histories for z^t including those many periods in the future; ditto for $\{\eta^\tau \succ \eta^t\}$. When we use $\succeq$, we include the current nodes z^t or η^t in the summation.

There is a single non-durable good available for consumption in each period, and its aggregate supply is given by $Y_t(z^t)$, which evolves according to

$$Y_t(z^t) = \exp\{z_t\}Y(z^{t-1}), \tag{1}$$

with $Y(z^1) = \exp\{z_1\}$. This endowment good comes in two forms. The first part is non-tradeable income which is subject to idiosyncratic risk and is given by $\gamma Y(z^t)\eta_t$; hence γ is the share of income that is non-tradeable. The second part is diversifiable income, which is not subject to the idiosyncratic shock, and is given by $(1-\gamma)Y_t(z^t)$.

All households are infinitely lived and rank stochastic consumption streams $\{c(z^t, \eta^t)\}$ according to the following criterion

$$U(\{c\}) = \sum_{t\geq 1,(z^t,\eta^t)}^{\infty} \beta^t \pi(z^t, \eta^t) \frac{c_t(z^t, \eta^t)^{1-\alpha}}{1-\alpha}, \tag{2}$$

where $\alpha > 0$ denotes the coefficient of relative risk aversion, and $c_t(z^t, \eta^t)$ denotes the household's consumption in state (z^t, η^t).

3.2 Assets Traded

Households trade assets in securities markets that re-open in every period. These assets are claims on diversifiable income, and the set of traded assets, depending on the trading technology, can include one-period Arrow securities as well as debt and equity claims. Households cannot directly trade their claim to aggregate non-diversifiable income (labor income).

Debt and Equity We follow Abel (1999) in defining equity as a leveraged claim to aggregate diversifiable income (capital income $(1-\gamma)Y_t(z^t)$). We use $V_t[\{X\}](z^t)$ to denote the no-arbitrage price in z^t units of consumption of a claim to a payoff stream $\{X\}$, and we use $R_{t+k,t}[\{X\}](z^{t+k})$ to denote the gross return between t and $t+k$. To construct the debt and the equity claim, we will assume that aggregate diversifiable income in each period is split into a debt component (aggregate interest payments net of new issuance) and an equity component (aggregate dividend payments denoted $D_t(z^t)$). For simplicity, the bonds are taken to be one-period risk-free bonds. Since we assume a constant leverage ratio ψ, the supply of one-period non-contingent bonds $B_t^s(z^t)$ in each period needs to adjust such that:

$$B_t^s(z^t) = \psi\left[(1-\gamma)V_t[\{Y\}](z^t) - B_t^s(z^t)\right],$$

where $V[\{Y\}](z^t)$ denotes the value of a claim to aggregate income in node z^t. The payouts to bond holders are given by $R_{t,t-1}[1](z^{t-1})B_{t-1}^s(z^{t-1}) - B_t^s(z^t)$, and the payments to shareholders, $D_t(z^t)$, are then determined residually as:

$$D_t(z^t) = (1-\gamma)Y_t(z^t) - R_{t,t-1}(z^{t-1})[1]B_{t-1}^s(z^{t-1}) + B_t^s(z^t).$$

A trader who invests a fraction $\psi/(1+\psi)$ in bonds and the rest in debt is holding the market portfolio. We can denote the value of the dividend claim as $V_t[\{D\}](z^t)$. $R_{t-1,t}[\{D\}](z^t)$ denotes the gross return on the dividend claim between $t-1$ and t.

We denote the price of a unit claim to the final good in aggregate state z^{t+1} acquired in aggregate state z^t by $Q_t(z_{t+1}, z^t)$. If there is a group of agents who trade claims with payoffs that are contingent on their idiosyncratic shocks, the absence of arbitrage would imply that the price $Q_t(\eta_{t+1}, z_{t+1}; \eta^t, z^t)$ of a claim to output in state (z^{t+1}, η^{t+1}) acquired in state (z^t, η^t) would be equal to $\pi(\eta^{t+1}|z^{t+1}, \eta^t)Q_t(z_{t+1}, z^t)$.

We consider a household entering the period with net financial wealth $\hat{a}_t(z^t, \eta^t)$. This household buys securities in financial markets (state contingent bonds $a_t(z^{t+1}, \eta^{t+1})$, non-contingent bonds $b_t(z^t, \eta^t)$, and equity shares $s_t^D(z^t, \eta^t)$) and consumption $c_t(z^t, \eta^t)$ in the good markets subject to this one-period budget constraint:

$$\begin{aligned}&\sum_{z^{t+1}\succ z^t, \eta^{t+1}\succ\eta^t} Q_t(\eta_{t+1}, z_{t+1}; \eta^t, z^t)a_t(z^{t+1}, \eta^{t+1}) + s_t^D(z^t, \eta^t)V_t[\{D\}](z^t)\\ &+b_t(z^t, \eta^t) + c_t(z^t, \eta^t) \leq \hat{a}_t(z^t, \eta^t) + \gamma Y_t(z^t)\eta_t, \text{ for all } z^t, \eta^t, \end{aligned} \tag{3}$$

where $\hat{a}_t(z^t, \eta^t)$, the agent's net financial wealth in state (z^t, η^t), and is given by his state-contingent bond payoffs from bonds acquired last period, the payoffs from his equity position and the non-

contingent bond payoffs:

$$\hat{a}_t(z^t, \eta^t) = a_{t-1}(z^t, \eta^t) + s_t^D(z^{t-1}, \eta^{t-1}) \left[D_t(z^t) + V_t[\{D\}](z^t) \right] + R_{t,t-1}[1](z^{t-1}) b_{t-1}(z^{t-1}). \quad (4)$$

3.3 Trading Technology

A trading technology is a restriction on the menu of assets that the agent can trade in any given period. This includes restrictions on the frequency of trading as well; some trades are not allowed in each period for all households. The set of asset trading technologies that we consider can be divided into two main classes: active trading technologies and passive trading technologies.

Agents with an active trading technology optimally choose their portfolio composition given the set of assets that they are allowed to trade in each period and given the state of the investment opportunity set. Passive traders do not. We consider two types of active traders. For *complete traders* this set consists of all state-contingent securities, with payoffs contingent on aggregate and idiosyncratic shocks –including, of course, non-contingent debt and equity. For *z-complete traders* this set consists of only aggregate state-contingent securities –including non-contingent debt and equity.

Finally, for all passive trading technologies, this menu of traded assets only consists of debt and equity claims. A passive trading technology specifies an exogenously assigned target $\varpi^\star$ for the equity share. A *continuous-rebalancer* adjust his equity position continuously (.i.e., in each period) to the target $\varpi^\star$ in each period.[5]. An *intermittent-rebalancer* adjust his equity position to the target only every n periods; in non-rebalancing periods, all (dis-)savings occur through adjusting the holdings of the investor's risk-free asset .

We allow for the possibility that there could be multiple types of both active and passive traders.

All households are initially endowed with a claim to their per capita share of both diversifiable and non-diversifiable income. Finally, each agent's period 1 financial wealth is constrained by the value of their claim to tradeable wealth in the period 0 planning period, which is given by

$$(1-\gamma) V_0[\{Y\}](z^0) \geq \sum_{z_1} Q(z_1, z^0) \hat{a}_0(z^1, \eta^0), \quad (5)$$

where both z^0 and η^0 simply indicate the degenerate starting values for the stochastic income process.

In the quantitative analysis we only look at the ergodic equilibrium of the economy; hence, the assumptions about initial wealth are largely irrelevant. We assume that, during the initial trading period, households with portfolio restriction sell their claim to diversifiable income in exchange for

[5] One could think of this household delegating the management of its portfolio to a fund manager (see Abel et al., 2006)

their type appropriate fixed weighted portfolio of bonds and equities.

The households face exogenous limits on their net asset positions, or solvency constraints,

$$\hat{a}_t(z^t, \eta^t) \geq \underline{M}_t(z^t, \eta^t). \tag{6}$$

In determining the solvency constraint, we assume that the value of the household's net assets must always be greater than $-\xi$ times the value of their non-diversifiable income, where $\xi \in (0, 1)$. We allow households to trade away or borrow up to 100% of the value of their claims to diversifiable capital. We also allow for the possibility that this borrowing constraint may itself be a function of the aggregate history of shocks.

3.4 Measurability Restrictions

To capture the portfolio restrictions, we use measurability constraints.

Active Trader Since idiosyncratic shocks are not spanned for the z-complete trader, his net wealth needs to satisfy:

$$\hat{a}_t\left(z^t, \left[\eta_t, \eta^{t-1}\right]\right) = \hat{a}_t\left(z^t, \left[\tilde{\eta}_t, \eta^{t-1}\right]\right), \tag{7}$$

for all t and η_t, $\tilde{\eta}_t \in N$.

Continuous-Rebalancing (*crb*) Passive Trader Passive traders who re-balance their portfolio in each period to a fixed fraction $\varpi^\star$ in levered equity and $1 - \varpi^{star}$ in non-contingent bonds earn a return:

$$R_t^{crb}(\varpi^\star, z^t) = \varpi^\star R_{t,t-1}[\{D\}](z^t) + (1 - \varpi^\star) R_{t,t-1}[1](z^{t-1})$$

Hence, their net financial wealth will satisfy the measurability restriction:

$$\frac{\hat{a}_t\left([z^{t-1}, z_t], [\eta_t, \eta^{t-1}]\right)}{R_t^{crb}(\varpi^\star, [z^{t-1}, z_t])} = \frac{\hat{a}_t\left([z^{t-1}, \tilde{z}_t], [\tilde{\eta}_t, \eta^{t-1}]\right)}{R_t^{crb}(\varpi^\star, [z^{t-1}, \tilde{z}_t])}, \tag{8}$$

for all t, z_t, $\tilde{z}_t \in Z$, and η_t, $\tilde{\eta}_t \in N$. If $\varpi^\star = \psi/(1+\psi)$, then this trader holds the market in each period and earns the return on a claim to all tradeable income, or $R_{t,t-1}[\{Y\}](z^t)$. We will refer to this type of passive trader as a *diversified trader*. Without loss of generality, we can think of non-participants as *crb* traders with $\varpi^{star} = 0$.

Intermittent-Rebalancing Passive (*irb*) Trader Next we characterize the constraints on a passive trader's type is specified by his portfolio target (denoted $\varpi^\star$) and the periods in which he rebalances (denoted $\mathcal{T}$). We assume that his rebalancing takes place at fixed intervals. For example if he rebalances every other period, then $\mathcal{T} = \{1, 3, 5, ...\}$ or $\mathcal{T} = \{2, 4, 6, ...\}$.

After non-rebalancing periods, traders who do not re-balance their portfolio, with an equity share ϖ_{t-1}, earn a rate of return:

$$R_t^{irb}(\varpi_{t-1}, z^t) = \varpi_{t-1}(z^{t-1})R_{t,t-1}[\{D\}](z^t) + (1 - \varpi_{t-1}(z^{t-1}))R_{t,t-1}[1](z^{t-1})$$

and they face the following measurability restriction on their net wealth:

$$\frac{\hat{a}_t\left([z^{t-1}, z_t], [\eta_t, \eta^{t-1}]\right)}{R_t^{irb}(\varpi_{t-1}, [z^{t-1}, z_t])} = \frac{\hat{a}_t\left([z^{t-1}, \tilde{z}_t], [\tilde{\eta}_t, \eta^{t-1}]\right)}{R_t^{irb}(\varpi_{t-1}, [z^{t-1}, \tilde{z}_t])}, \tag{9}$$

for all t, z_t, $\tilde{z}_t \in Z$, and η_t,$\tilde{\eta}_t \in N$, with $\varpi_t = \varpi^\star$ in rebalancing periods.

We define the trader's equity holdings as $e_t(z^t, \eta^t) = s_t(z^t, \eta^t)V_t[\{D\}](z^t)$. In re-balancing periods, this trader's equity holdings satisfy:

$$\frac{e_t(z^t, \eta^t)}{e_t(z^t, \eta^t) + b_t(z^t, \eta^t)} = \varpi^\star.$$

However, in non-rebalancing periods, the implied equity share is given by $\varpi_t = e_t/(e_t + b_t)$ where e_t, the trader's equity holdings, evolves according to the following law of motion:

$$e_t(z^t, \eta^t) = e_{t-1}(z^{t-1}, \eta^{t-1})R_{t,t-1}[\{D\}](z^t)$$

for each $t \notin \mathcal{T}$. The *irb* trader automatically re-invests the dividends in equity in non-rebalancing periods.

Since this agent cannot hold any type of state-contingent bond, his flow budget constraint in non-rebalancing periods reduces to:

$$\gamma Y_t(z^t)\eta_t + b_{t-1}(z^{t-1}, \eta^{t-1})R_{t,t-1}[1](z^{t-1}) \geq c_t(z^t, \eta^t) + b_t(z^t, \eta^t)\ \forall (z^t, \eta^t).$$

for each $t \notin \mathcal{T}$. Since setting $\mathcal{T} = \{1, 2, 3, ...\}$ generates the continuous-rebalancer's measurability constraint, the continuous-rebalancer can simply be thought of as a degenerate case of the intermittent-rebalancer. Hence, we can state without loss of generality that a passive trading technology is completely characterized by $(\varpi^\star, \mathcal{T})$.

4 Solving the Trader's Optimization Problem

Active Traders For our active traders, we distinguish between two types. The z-complete trader's problem is to choose $\{c_t(z^t, \eta^t), a_t(z^{t+1}, \eta^{t+1}), e_t(z^t, \eta^t), b_t(z^t, \eta^t)\}$, so as to maximize his total expected utility (eq. (2)) subject the flow budget constraint (eq. (3)), the solvency constraint

(eq. (6)), and the appropriate measurability constraint (eq. (7)). The complete trader solves the same optimization problem without the measurability constraint (eq. (7)).

Passive Traders For our passive traders, we distinguish between two types. The *crb* trader's problem is to choose $\{c_t(z^t,\eta^t), a_t(z^{t+1},\eta^{t+1}), e_t(z^t,\eta^t), b_t(z^t,\eta^t)\}$ in each period, so as to maximize his total expected utility (eq. (2)) subject to the flow budget constraint (eq. (3)) in each period, the solvency constraint (eq. (6)), and the appropriate *crb* measurability constraint (eq. (8)). The *irb* solves the same optimization problem with the *irb* measurability constraint (eq. (9)).

4.1 Time Zero Trading

We find it useful to write agent's problems in terms of their equivalent time-zero trading problem in which they select the optimal policy sequence given a complete set of Arrow-Debreu securities, subject to a sequence of measurability and debt constraints (see Chien et al., 2007). This section reformulates the household's problem in terms of a present-value budget constraint, and sequences of measurability constraints and solvency constraints. These measurability constraints capture the restrictions imposed by the different trading technologies of households.

From the aggregate contingent claim prices, we can back out the present-value state prices recursively as follows:

$$\pi(z^t,\eta^t)P(z^t,\eta^t) = Q(z_t,z^{t-1})Q(z_{t-1},z^{t-2})\cdots Q(z_1,z^0)Q(z_0).$$

We use $\widetilde{P}_t(z^t,\eta^t)$ to denote the state prices $P_t(z^t)\pi(z^t,\eta^t)$. Let $M_{t+1,t}(z^{t+1}|z^t) = P(z^{t+1})/P(z^t)$ denote the stochastic discount factor that prices any random payoffs. Using these state prices, we can compute the no-arbitrage price of a claim to random payoffs $\{X\}$ as:

$$V_t[\{X\}](z^t) = \sum_{\tau\geq t, z^\tau \succ z^t} \frac{\widetilde{P}_\tau(z^\tau,\eta^\tau)}{\widetilde{P}_t(z^t,\eta^t)} X_\tau(z^\tau,\eta^\tau).$$

Given this, we can also state the solvency constraint as:

$$\underline{M}_t(z^t,\eta^t) = -\xi V\left[\{\gamma\eta Y - c\}\right](z^t,\eta^t)$$

Active Traders The complete trader chooses a consumption plan $\{c_t(z^t,\eta^t)\}$ to maximize her expected utility $U(\{c\})$ (in eq. (2)) subject to a single time zero budget constraint:

$$V_t\left[\{\gamma\eta Y - c\}\right](z^0) + (1-\gamma)V_0[\{Y\}](z^0) \geq 0. \tag{10}$$

and the solvency constraint in each node (z^t, η^t):

$$V_t\left[\{\gamma\eta Y - c\}\right](z^t, \eta^t) \leq -\underline{M}_t(z^t, \eta^t). \tag{11}$$

This is a standard Arrow-Debreu household optimization problem.

The z-complete trader's problem is the same as the complete-trader's problem except that we need to enforce his measurability constraint (eq. (7)) in each node (z^t, η^t):

$$V\left[\{\gamma\eta Y - c\}\right](z^t, \eta^t) \textit{ is measurable w.r.t. } (z^t, \eta^{t-1}).$$

Hence, we can think of the the z-complete trader choosing a consumption plan $\{c_t(z^t, \eta^t)\}$ and a net wealth plan $\{\widehat{a}_t(z^t, \eta^{t-1})\}$ to maximize her expected utility $U(\{c\})$ subject to the time zero budget constraint (eq. (10)), the solvency constraints (eq. (11)) in each node (z^t, η^t), and the measurability constraint in each node (z^t, η^t):

$$V_t\left[\{\gamma\eta Y - c\}\right](z^t, \eta^t) = \widehat{a}_t(z^t, \eta^{t-1}). \tag{12}$$

The appendix contains a detailed description of the corresponding saddle point problem in section A. Since the complete-trader's problem is merely a simplification of the z-complete's, we focus on the z-complete trader in our discussion.

Let χ denote the multiplier on the time zero budget constraint in eq. (10), let $\varphi_t(z^t, \eta^t)$ denote the multiplier on the debt constraint in node (z^t, η^t) (eq. (11)), and, finally, let $\nu_t(z^t, \eta^t)$ denote the multiplier on the measurability constraint (eq. (12)) in node (z^t, η^t), . We will show how to use the multipliers on these constraints to fully characterize equilibrium allocations and prices.

Following Chien et al. (2007), we can construct new weights for this Lagrangian as follows. First, we define the initial cumulative multiplier to be equal to the multiplier on the budget constraint: $\zeta_0 = \chi$. Second, the multiplier evolves over time as follows for all $t \geq 1$:

$$\zeta_t(z^t, \eta^t) = \zeta_t(z^{t-1}, \eta^{t-1}) + \nu_t\left(z^t, \eta^t\right) - \varphi_t(z^t, \eta^t). \tag{13}$$

The first order condition for consumption leads to a consumption sharing rule that does not depend on the trading technology. Using the law of motion for cumulative multipliers in eq. (13) to restate the first order condition for consumption from the saddle point problem, in terms of our cumulative multiplier, we obtain the following condition:

$$\frac{\beta^t u'(c(z^t, \eta^t))}{P(z^t)} = \zeta_t(z^t, \eta^t). \tag{14}$$

This condition is common to all of our traders irrespective of their trading technology because

differences in their trading technology does not effect the way in which $c_t(z^t, \eta^t)$ enters the objective function or the constraint. This implies that the marginal utility of households is proportional to their cumulative multiplier, regardless of their trading technology. As a result, we can derive a consumption sharing rule. The household consumption share, for all traders is given by

$$\frac{c(z^t, \eta^t)}{C(z^t)} = \frac{\zeta(z^t, \eta^t)^{\frac{-1}{\alpha}}}{h(z^t)}, \text{ where } h(z^t) = \sum_{\eta^t} \zeta(z^t, \eta^t)^{\frac{-1}{\alpha}} \pi(\eta^t | z^t). \tag{15}$$

Moreover, the SDF is given by the Breeden-Lucas SDF and a multiplicative adjustment:

$$M_{t,t+1}(z^{t+1}|z^t) \equiv \beta \left(\frac{C(z^{t+1})}{C(z^t)} \right)^{-\alpha} \left(\frac{h(z^{t+1})}{h(z^t)} \right)^{\alpha}. \tag{16}$$

The first order condition for net financial wealth leads to a martingale condition for the cumulative multipliers which does depend on the trading technology. The first order condition with respect to net wealth $\widehat{a}_t(z^{t+1}, \eta^t)$ is given by:

$$\sum_{\eta^{t+1} \succ \eta^t} \nu \left(z^{t+1}, \eta^{t+1} \right) \pi(z^{t+1}, \eta^{t+1}) P(z^{t+1}) = 0. \tag{17}$$

This condition, which determines the dynamics of the multipliers, is specific to the trading technology. For the z-complete trader, it implies that the average measurability multiplier across idiosyncratic states η^{t+1} is zero since $P(z^{t+1})$ is independent of η^{t+1}. In each aggregate node z^{t+1}, the household's marginal utility innovations not driven by the solvency constraints ν_{t+1} have to be white noise. The trader has high marginal utility growth in low η states and low marginal utility growth in high η states, but these innovations to marginal utility growth average out to zero in each node (z^t, z_{t+1}).

Combining eq. (17) with eq. (13), we obtain the following supermartingale result:

$$E\left[\zeta_{t+1} | z^{t+1}\right] \leq \zeta_t,$$

which holds with equality if the solvency constraint do not bind in z^{t+1}. For the unconstrained z-complete market trader, the martingale condition $E_{t+1}\left[\zeta_{t+1} | z^{t+1}\right] = \zeta_t$ and the consumption sharing rule imply that his IMRS equals the SDF on average in each aggregate node z^{t+1}, averaged over idiosyncratic all states:

$$M_{t,t+1} \geq E_{t+1} \left[\beta \left(\frac{c_{t+1}}{c_t} \right)^{-\alpha} | z^{t+1} \right],$$

with equality if the solvency constraints do not bind in z^{t+1}.

For the complete trader, the first-order condition for to net wealth $\widehat{a}_t(z^{t+1}, \eta^{t+1})$ is given by:

$$\nu\left(z^{t+1}, \eta^{t+1}\right) \pi(z^{t+1}, \eta^{t+1}) P(z^{t+1}) = 0, \tag{18}$$

and this implies that if the solvency constraints do not bind, the cumulative multipliers are constant. For the complete market trader, the martingale condition $\zeta_{t+1} = \zeta_t$ and the consumption sharing rule imply that his IMRS is less than or equal to the SDF, state-by-state:

$$M_{t,t+1} \geq \beta \left(\frac{c_{t+1}}{c_t}\right)^{-\alpha},$$

with equality if the solvency constraint does not bind in (z^{t+1}, η^{t+1}).

As in Chien et al. (2007), we can characterize equilibrium prices and allocations using the household's multipliers and the aggregate multipliers. Consumption is allocated on the basis of a consumption-sharing rule which is independent of

Passive Traders Since the *crb* passive trader is a special case of the *irb* passive trader, we start with the *irb*. The passive trader faces an additional restriction on the dynamics of his equity position. The passive traders' equity position evolves according to:

$$e_t(z^t, \eta^t) = \begin{cases} \frac{\varpi^\star}{1-\varpi^\star} b_t(z^t, \eta^t) \text{ if } t \in \mathcal{T} \\ \\ R_{t,t-1}[\{D\}](z^t) e_{t-1}(z^{t-1}, \eta^{t-1}) \text{ everywhere else} \end{cases}. \tag{19}$$

The passive trader's equity position is being determined in rebalancing periods by his current debt position b_t, and in nonrebalancing periods by his past equity position e_{t-1}. Thus, it is completely determined by the bond position he took in rebalancing periods and the returns on equity.

The *irb* passive trader trader chooses a consumption plan $\{c_t(z^t, \eta^t)\}$ and a net wealth plan $\{\widehat{a}_t(z^t, \eta^{t-1})\}$ to maximize her expected utility $U(\{c\})$ subject to the time zero budget constraint (eq. (10)), the solvency constraints (eq. (11)), the measurability constraint in each node (z^t, η^t):

$$V_t\left[\{\gamma \eta Y - c\}\right](z^t, \eta^t) = \widehat{a}_t(z^t, \eta^{t-1}), \tag{20}$$

where net financial wealth in node z^t, η^t is given by the non-contingent bond holdings and equity holdings:

$$\widehat{a}_t(z^t, \eta^{t-1}) = b_{t-1}(z^{t-1}, \eta^{t-1}) R_{t,t-1}[1](z^{t-1}) + e_{t-1}(z^{t-1}, \eta^{t-1}) R_{t,t-1}[\{D\}](z^t),$$

and, finally, subject to the equity transition restriction in eq. (19).

As before, let χ denote the multiplier on the time-zero budget constraint in (10), let $\varphi(z^t, \eta^t)$ denote the multiplier on the solvency constraint in (11), let $\kappa(z^t, \eta^t)$ denote the multiplier on the equity transition condition in (19), and let $\nu(z^t, \eta^t)$ denote the multiplier on the measurability constraint in node (z^t, η^t) in (20).

The saddle point problem of a passive trader with trading technology $(\phi^*, \mathcal{T})$ is stated in section A of the appendix. As before, we define the cumulative multipliers as in eq. (13).

To keep the notation tractable, we define the continuous-rebalancing one-period portfolio return as:

$$R^{crb}_{t+1,t}(\varpi^\star, z^{t+1}) = \varpi^\star R_{t+1,t}[1](z^t) + (1-\varpi^\star) R_{t+1,t}[\{D\}](z^{t+1}),$$

and we define the intermittent-rebalancing two-period portfolio return as:

$$R^{irb}_{t+2,t}(\varpi^\star, z^{t+2}) = \varpi^\star R_{t+2,t}[1](z^t) + (1-\varpi^\star) R_{t+2,t}[\{D\}](z^{t+2}).$$

To develop some intuition, consider the simplest case in which the rebalancing takes place every other period. The intermittent-rebalancer's first-order condition for net financial wealth can be stated as follows:

1. in the rebalancing periods $t \in \mathcal{T}$:

$$\begin{aligned} 0 \;=\; & \sum_{(z^{t+1},\eta^{t+1})} \nu_{t+1}(z^{t+1},\eta^{t+1}) \widetilde{P}(z^{t+1},\eta^{t+1}) R^{crb}_{t+1,t}(z^{t+1}) \\ & + \sum_{(z^{t+2},\eta^{t+2})} \nu(z^{t+2},\eta^{t+2}) \widetilde{P}(z^{t+2},\eta^{t+2}) R^{irb}_{t+2,t}(z^{t+2}). \end{aligned} \tag{21}$$

2. in the nonrebalancing periods $t \notin \mathcal{T}$:

$$\varpi^\star \sum_{(z^{t+1},\eta^{t+1})} \nu(z^{t+1},\eta^{t+1}) \widetilde{P}(z^{t+1},\eta^{t+1}) R_{t+1,t}[1](z^t) = 0. \tag{22}$$

In the non-rebalancing periods, the passive trader faces the same first order condition as the non-participant in eq. (22), but in re-balancing periods, the standard martingale condition is augmented with a *forward looking* component, because the passive trader anticipates that the next period is not a rebalancing period. Combining eq. (21) with the law of motion for the cumulative multiplier in eq. (13) leads to a martingale condition under a different measure that looks two periods ahead:

$$E_t\left[\left(M_{t,t+2} R^{irb}_{t+2,t}\right)\zeta_{t+2} | z^t, \eta^t\right] \leq \zeta_t,$$

with equality if the passive trader's solvency constraints do not bind in period $t+1$. This martingale condition, combined with the consumption sharing rule, leads to the following Euler equation for

an unconstrained passive trader, who is re-balancing at t, who is not re-balancing at t+1:

$$E_t\left[\beta\left(\frac{c_{t+2}}{c_t}\right)^{-\alpha} R^{irb}_{t+2,t}|z^t,\eta^t\right] \leq 1,\ t\in\mathcal{T}, t+1\notin\mathcal{T}, t+2\in\mathcal{T}$$

4.2 Equilibrium

We allow for the possibility that there may be a positive measure of multiple types of active and passive traders. We assume there is always a non-zero measure of either complete or z-complete traders to guarantee the uniqueness of the stochastic discount factor. For our active traders, let μ_c denote the measure of complete traders and μ_z denote the measure of z-complete traders. For our passive traders, we will assume for simplicity that there are only two types participating passive traders: irb (crb) traders with measure μ_{irb} (μ_{crb}) and portfolio target $\varpi^\star$, and nonparticipants with measure μ_{np} and portfolio target equal to zero. The non-state-contingent bond market clearing condition is given by

$$\sum_{\eta^t}\begin{bmatrix}\mu_c b^c_t(z^t,\eta^t)+\mu_z b^z_t(z^t,\eta^t)\\ +\mu_{irb}b^{icb}_t(z^t,\eta^t)+\mu_{np}b^{np}_t(z^t,\eta^t)\end{bmatrix}\pi(\eta^t|z^t)=V[\{(1-\gamma)Y-D\}](z^t)), \tag{23}$$

and the equity market clearing condition is given by

$$\sum_{\eta^t}\begin{bmatrix}\mu_c e^c_t(z^t,\eta^t)+\mu_z e^z_t(z^t,\eta^t)\\ +\mu_{irb}e^{icb}_t(z^t,\eta^t)\end{bmatrix}\pi(\eta^t|z^t)=V[\{D\}](z^t), \tag{24}$$

where we index the holdings by $\{c,z,irb,np\}$ of the complete-markets, z-complete, intermittent rebalancers, and non-participants respectively. For the sake of clarity, we use (e.g.) $\eta^{t-1}(\eta^t)$ to denote the history from zero to $t-1$ contained in η^t. We use the same convention for the aggregate histories. Using this notation, the market clearing condition in the state-contingent bond market is given by:

$$\sum_{\eta^t}\left[\mu_c a^c_{t-1}(z^t,\eta^t)+\mu_z a^z_{t-1}(z^t,\eta^{t-1}(\eta^t))\right]\pi(\eta^t|z^t)=0. \tag{25}$$

An equilibrium for this economy is defined in the standard way. It consists of a list of bond and dividend claim holdings, a consumption allocation and a list of bond and tradeable output claim prices such that: (i) given these prices, a trader's asset and consumption choices maximizer her expected utility subject to the budget constraints, the solvency constraints and the measurability constraints, and (ii) the asset markets clear (eqs. (23), (24),(25)).

4.3 The Importance of Rebalancing

We define the aggregate multiplier for each trading segment:

$$h^j(z^t) = \sum_{\eta^t} \zeta^j(z^t, \eta^t)^{\frac{-1}{\alpha}} \pi(\eta^t | z^t).$$

By aggregating household wealth across all households in a trading segment j, we can define the aggregate net wealth for each group of traders $j \in \{c, z, crb\}$

$$\widehat{A}_t^j(z^t) = V_t \left[\{ (\frac{h^j}{h} - \gamma \mu^j) Y \} \right] (z^t),$$

where we use the linearity of the pricing functional. Finally, total aggregate wealth equals the market portfolio:

$$\sum_{j \in \{c,z,irb,np\}} \widehat{A}_t^j(z^t) = (1-\gamma) V_t[\{Y\}](z^t).$$

Now, in an i.i.d. version of our economy, in which aggregate consumption growth is not predictable, there is an equilibrium with passive traders in the market in which the ratio of A^j to total financial wealth is constant, but only if the passive traders rebalance continuously, not if they rebalance intermittently.

IID Example To grasp the importance of rebalancing for aggregate risk sharing, we consider a stylized example in which the aggregate consumption growth shocks are i.i.d. :

$$\phi(z'|z) = \phi(z'), \tag{26}$$

and the distribution of idiosyncratic shocks is independent of aggregate shocks:

$$\pi(\eta', z'|\eta, z)/\phi(z') = \varphi(\eta'|\eta). \tag{27}$$

Suppose that the passive trader belongs to the class of continuous-rebalancers (crb), and holds the market portfolios: $\varpi^\star = \psi$. Also, suppose that there are no non-participants.

One household consumption path that is feasible for the crb trader is proportional to aggregate output:

$$c_t(z^t, \eta^t) = \widehat{c}_t(\eta^t) Y_t(z^t). \tag{28}$$

Krueger and Lustig (2009) show that we can analyze an equivalent stationary economy without aggregate consumption growth (with a unit aggregate endowment) and an adjusted transition probability matrix to solve for the equilibrium allocations and prices. To do so, we transform our

growing model into a stationary model with a stochastic time discount rate and a growth-adjusted probability matrix, following Alvarez and Jermann (2001). First, we define growth deflated consumption allocations (or consumption shares) as in eq. (28). Next, we define *growth-adjusted* probabilities and the growth-adjusted discount factor as:

$$\hat{\phi}(z_{t+1}|z_t) = \frac{\phi(z_{t+1}|z_t)\exp(z_{t+1})^{1-\gamma}}{\sum_{z_{t+1}}\phi(z_{t+1}|z_t)\exp(z_{t+1})^{1-\gamma}} \text{ and } \hat{\beta} = \beta\sum_{z_{t+1}}\phi(z_{t+1}|z_t)\exp(z_{t+1})^{1-\gamma}.$$

Note that $\hat{\pi}$ is a well-defined Markov matrix in that $\sum_{z_{t+1}}\hat{\phi}(z_{t+1}|z_t) = 1$ for all z_t. Finally, we let $\hat{U}(\hat{c})(s^t)$ denote the lifetime expected continuation utility in node s^t, under the new transition probabilities and discount factor, defined over consumption shares $\{\hat{c}_t(s^t)\}$[6]

$$\hat{U}(\hat{c})(s^t) = u(\hat{c}_t(s^t)) + \hat{\beta}\sum_{s_{t+1}}\hat{\pi}(s_{t+1}|s_t)\hat{U}(\hat{c})(s^t, s_{t+1}). \tag{29}$$

Finally, we use $\widehat{V}_t[\{\widehat{X}\}$ to denote the no-arbitrage price of a claim to $\widehat{X}_t$ in the stationary economy, where the payoffs $\widehat{X}_t$ only depend on η^t. In this stationary economy, the measurability constraints of the passive continuous-rebalancers can be stated as:

$$\widehat{V}_t\left[\{\gamma\eta - \widehat{c}\}\right](\eta^t) \textit{ is measurable w.r.t. } \eta^{t-1}. \tag{30}$$

Hence, these measurability constraints in the stationary economy do not depend on the aggregate history. As a result, the active z-complete traders face the exact same measurability constraint in the stationary economy as the passive *crb* traders. Hence, given the assumptions we have imposed on the nature of aggregate and idiosyncratic shocks, the distinction between active and passive trader becomes moot. We can solve for $\{\widehat{c}_t\}$ in the stationary economy, scale it by Y_t, to obtain equilibrium household consumption $\{\widehat{c}_t Y_t\}$. In this equilibrium, the relative wealth of the passive *crb* , traders $\widehat{A}_t^{crb}(z^t)/\sum_{j\in\{z,crb\}}\widehat{A}_t^j(z^t)$ is invariant to aggregate shocks.

Now, this particular consumption path in eq. (28) is feasible for the passive trader simply by trading a claim to aggregate consumption (the market), .i.e., maintaining a portfolio with $\varpi^\star = 1/(1+\psi)$ invested in equity. However, for the *irb* trader, this consumption path is not feasible, because holding the market requires re-balancing every period. Instead, consider what happens to an *irb* trader who starts out by holding the aggregate consumption claim. After a negative aggregate consumption growth shocks z_t, the equity share of his portfolio drops below ψ, and the passive trader no longer holds the market. After a series of negative aggregate consumption growth shocks, the equity share ϖ_{t-1} would be much lower than what is prescribed to hold the

[6]It is easy to show that this transformation does not alter the agents' ranking of different consumption streams. Households rank consumption share allocations in the de-trended model in exactly the same way as they rank the corresponding consumption allocations in the original model with growth.

market, and $R_t^{irb}(\varpi_{t-1}, \tilde{z}_t)$ is increasingly less exposed to aggregate consumption risk. In this equilibrium, the relative wealth of the passive *crb* , traders $\widehat{A}_t^{irb}(z^t)/\sum_{j\in\{z,irb\}}\widehat{A}_t^{j}(z^t)$ cannot be invariant w.r.t aggregate shocks. Hence, these intermittent rebalancers act like households with counter-cyclical risk aversion, because of the nature of the trading technology: adverse aggregate shocks endogenously concentrate aggregate risk among the active traders. In contrast, the *crb* trader would be buying equity after each negative aggregate shock, to re-balance his portfolio. Even in the case of i.i.d. aggregate shocks, without non-participants, the irrelevance result in Krueger and Lustig (2009) no longer holds if some of the passive traders do not continuously rebalance.

In the calibrated version of the model, we introduce non-participants as well. These non-participants create residual aggregate risk that needs to be transferred to the other market participants.

5 Quantitative Results

This section evaluates a calibrated version of the model to examine the extent to which the our model can account for the empirical moments of asset prices, and in particular the counter-cyclical volatility. The first subsection discusses the calibration of the parameters and the endowment processes. We follow the algorithm described Chien et al. (2007) for computing the equilibrium of this economy. To forecast the growth rate of the aggregate multiplier, we use a truncated version σ of the aggregate history z^t; we keep track of the last 6 aggregate shocks. The details are in section D of the appendix.

We then examine the response of the moments of equilibrium asset prices, consumption growth, portfolio returns and the distribution of financial wealth respond to changes in the frequency of rebalancing by passive equity holders, the level of their equity target, and the composition of the active trader traders between z-complete and complete traders.

5.1 Calibration

Preferences and Endowments The model is calibrated to annual data. We choose a coefficient of relative risk aversion α of five and a time discount factor β of .95. These preference parameters allow us to match the collaterizable wealth to income ratio in the data when the tradeable or collateralizable income share $1-\gamma$ is 10%, as discussed below. Non-diversifiable income includes both labor income and entrepreneurial income, among other forms.

Our model is calibrated to match the aggregate consumption growth moments from Alvarez and Jermann (2001). The average consumption growth rate is 1.8%. The standard deviation is 3.15%. Recessions are less frequent than expansions: 27% of realizations are low aggregate consumption growth

states. The first-order autocorrelation coefficient of aggregate consumption growth (ρ_z) is -.14.

We calibrate the labor income process as in Storesletten, Telmer, and Yaron (2004, 2007), who find evidence that the cross-sectional variance of labor income risk is counter-cyclical (henceforth CCV). The Markov process for $\log \eta(y, z)$ has a standard deviation of .60, and the autocorrelation is 0.89. We use a 4-state discretization for both aggregate and idiosyncratic risk. The elements of the process for $\log \eta$ are $\{0.38, 1.61\}$.

In addition, we also report the results for a second calibration of the Markov process for $\log \eta(y, z)$ in which we follow Storesletten et al. (2007)'s calibration, except that we eliminate the CCV and we eliminate the autocorrelation in aggregate consumption growth (see section 5.3).

The average ratio of household wealth to aggregate income in the US is 4.30 between 1950 and 2005. The wealth measure is total net wealth of households and non-profit organizations (Flow of Funds Tables). We choose a collateralizable income ratio α of 10%. The implied ratio of wealth to consumption is 4.88 in the model's benchmark calibration.[7]

Equity in our model is simply a leveraged claim to diversifiable income. In the Flow of Funds, the ratio of corporate debt-to-net worth is around 0.65, suggesting a leverage parameter ψ of 2. However, Cecchetti, Lam, and Mark (1990) report that standard deviation of the growth rate of dividends is at least 3.6 times that of aggregate consumption, suggesting that the appropriate leverage level is over 3. Following Abel (1999) and Bansal and Yaron (2004), we choose to set the leverage parameter ψ to 3.

Composition of Trader Pools In the most recent Survey of Consumer Finance, 51.1 % reported owning stocks directly or indirectly. We set the share of passive traders who hold equities equal to 45%, and the overall share of active traders to 5%. We consider two types of passive equity holders: (1) those who rebalance every period and those who rebalance every 3 periods. These shares can be interpreted as shares of human wealth. We also consider four different rebalancing targets for our passive equity holders: 30%, 33%, 35% and 40%. We will assume that our traders cannot borrow against their labor income, $\xi = 0$.

5.2 Results in Benchmark Economy

Table 1 reports moments of asset prices generated by simulating data from a model with 3,000 agents for 10,000 periods. The panel on the left reports result for the case of active traders that are unable to insure against idiosyncratic shocks (z-complete traders). The panel on the right reports for the case in which the active traders are able to (partly) insure against idiosyncratic shocks.

[7] As is standard in this literature, we compare the ratio of total outside wealth to aggregate non-durable consumption in our endowment economy to the ratio of total tradeable wealth to aggregate income in the data. Aggregate income exceeds aggregate non-durable consumption because of durable consumption and investment.

The top panel reports result for the case when $\varpi^\star$, the target equity share of the passive trader is 30%, the second panel consider 33%, the third panel looks at the case of 35 %, and, finally, the bottom panel looks at the case of 40 %. The target equity share of 33% for the passive investors turns out to be the optimal equity share for the *irb* traders in the case with z-complete traders, as we will show below. Hence, this case is a natural benchmark.

We report the maximum unconditional Sharpe ratio ($\frac{\sigma(m)}{E(m)}$), the standard deviation of the maximum SR ($Std(\frac{\sigma_t(m)}{E_t(m)})$), the equity risk premium $E\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)$, the standard deviation of excess returns $\sigma\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)$, the Sharpe ratio on equity, the mean risk-free rate and the standard deviation of the risk-free rate.

The last line in each panel reports the standard deviation of the allocation error that results from our approximation in percentage points. The standard deviation of the percentage forecast error is between 0.90 and 0.10 % in the benchmark case of a 33 % equity target. This means our approximation is relatively accurate compared to other results reported in the literature for models with heterogeneous agents and incomplete markets. The implied R^2 in a linear regression of realized aggregate multiplier growth rates on the truncated aggregate histories exceed 0.998 in all cases.

[Table 1 about here.]

The participation of passive traders tends to increase the volatility in risk premia. In our model, this force operates in two ways: (i) as we increase the target share of equity in the passive trader's portfolio and (ii) as we shift passive traders from the *crb* type to the *irb* type. We discuss both of these effects.

5.2.1 Passive Trader Pool

Asset Prices First, we focus on the importance of the composition of the passive trader pool. We start with the case of a 30 % equity target (top panel). In the benchmark economy with only *crb* passive traders, the maximum SR is .43 and its standard deviation is only 3 %. The equity premium is 8.9 %. The mean risk-free rate is 1.58 %, and the volatility of the risk-free rate is 3%. When we change the *crb* to *irb* passive traders, the volatility of the market price of risk increases threefold to 11.5 %. That is the main effect of this change in the composition of the passive trader segment. The equity premium drops by 51 basis points, the maximum SR drops by two percentage points. Most importantly, the volatility of the risk-free rate drops to 2.7%.

In the case of a 35 % equity target, the maximum Sharpe ratio is substantially lower (.37), and the average equity premium is 7.44 %. Changing the *crb* traders into *irb* traders has a similar effect on the volatility of risk prices in this case; the standard deviation increases from 3.9 % to 18 %, more than fourfold. The equity premium drops by 71 basis points and the volatility of the riskfree

declines from 3 % to 2.64 %. Finally, in the case of a 40 % equity target, reported in the bottom panel, the volatility of the market price increases from 6.1 % to 19.8%. Hence, as we increase the equity holdings of the passive traders from 30% to 40%, the unconditional market price of risk (and the equity premium) decreases, but the volatility of risk prices increases from 11.5 % to 19.8 %.

Table 2 decomposes the variation in the conditional Sharpe ratio on equity into the variation in the equity risk premium and the variation in the conditional volatility of stock returns. In the benchmark case with a 33 % equity target, the standard deviation of the conditional equity premium increases from 1.09% to 2.76 % as we compare the *crb* case to the *irb* case, the standard deviation of the vol increases from 2.33 % to 3.05 %. The combined effect translates into an increase in the volatility of the conditional Sharpe ratio from 3.59 to 16.47 % per annum. In the case of a 40 % equity target, the volatility of the equity risk premium increases even more, from 1.18 % to 5.36 %, and the volatility of the conditional SR increases from 6.18 % to 24 %. The range of equity targets that we explore here includes the optimal equity share for the *irb* trader: 33%.

Clearly, the shift from *crb* to *irb* traders increases the volatility of risk prices by a factor of four. However, increasing the target share of equity for *crb* traders also increases the volatility substantially from 3.1 % (11% in the *irb* case), in the case of the 30 % target (see top panel of Table 1), to 6 % (24 %) in the case of the 40 % target (see bottom panel of Table 1). The more equity passive traders hold, the higher the volatility of risk prices. A 10 percentage point increase in the target share doubles the volatility of risk prices.

[Table 2 about here.]

The variation created by the *irb* traders is counter-cyclical. Figure 2 plots the conditional risk premium on equity, the conditional standard deviation and the conditional Sharpe ratio on equity against the history of aggregate consumption growth shocks for the benchmark case of a 33 % equity share. The shaded areas denote the low aggregate consumption growth realizations. The dotted line shows 4-period moving average of aggregate consumption growth; the full line shows the conditional Sharpe ratio. Clearly, in the *irb* case, the conditional risk premium on equity increases with each low aggregate consumption growth realization, and decreases with each high aggregate consumption growth realization.

[Figure 2 about here.]

The conditional Sharpe ratio is even more counter-cyclical, because the conditional volatility decreases with each negative aggregate consumption growth realization. Figure 3 shows this in a scatter plot representation of the same 100 simulations, with the weighted average of aggregate

consumption growth shocks on the x-axis and the conditional equity premium (top panel), the conditional standard deviation (middle panel) and the conditional Sharpe ratio (bottom panel). This is in line with the findings of Lettau and Ludvigson (2001) who document that the conditional risk premium and the conditional volatility of stock returns are negatively correlated. On the other hand, in the *crb* case, the conditional risk premium does not vary as much, and it is not clearly counter-cyclical. Moreover, the Sharpe ratio is only weakly counter-cyclical.

[Figure 3 about here.]

As we increase the target equity share to 40 %, the equity risk premium actually turns negative after a series of high aggregate consumption growth shocks. This explains why the volatility of the Sharpe ratio surpasses that of the market price of risk.

Portfolio, Wealth, Consumption and Welfare Costs The top panel in Table 3 reports the moments of household portfolio returns. We fix the equity share at the optimal level for the *irb* traders. The active z-traders realize an excess return of 4.75% and a SR of .42, compared to only 2.42 and .311% respectively for the *irb* trader. The optimal average portfolio share for a passive *crb* trader is only 51 % (compared to 61 % in the crb case), because the equity premium is lower. Interestingly, these numbers change to 27 % (and 34 %) if we increase the target to 35 %, which reflects the sensitivity of the optimal portfolio share to changes in the average risk premium.

However, the cost of being a passive *crb* trader in this case is 3 times higher in the *irb* case than in the *crb* case, simply because the risk premium is much more volatile and hence the cost of not responding to variation in the investment opportunity set is much larger. We also report the cost of being a *crb* trader compared to an *irb* trader. The cost is small (-.14%). This is surprising, but can easily be understood by looking at the optimal equity share (51 %), which is higher than 33 %. The *irb* traders gets closer on average to this target, because expansions are more frequent than recessions, and because their equity share drifts up in expansions. This benefit almost outweighs the cost of intermittent rebelancing.[8] So, the *crb* trader is willing to give up some of his consumption to attain a larger average equity share.

The second panel in Table 3 reports the moments of household consumption growth, and the moments of aggregate consumption growth for each group of traders. In the case of *crb* traders, the volatility of household consumption growth is inversely related to the degree of sophistication of the trader: 3.00 % for active traders, 3.23 % for the *crb* traders, for 3.65 % for non-participants. However, it is important to point out that these traders are exposed to different types of risk. This becomes apparent when we consider the moments for group consumption. The volatility for the active trader segment is 1.87 %, compared to 1.20 % for the passive equity holders, and 0.67 %

[8]However, if we would force the average equity shares to be the same for these traders, the cost would obviously be negative. In any case, this shows that the direct cost of intermittent rebalancing has to be small.

for the non-participants. The relation between consumption volatility and trader sophistication reverses itself at the group level. Now, in the case of the *irb* traders, the volatility of the active trader's consumption growth (at the group level) decreases to 1.73 %, while, at the household level, the volatility of household consumption growth for passive equity holders increases from 3.15 % to 3.61 %. Other than that, the consumption numbers are very similar.

Finally, the bottom panel in Table 3 reports the household wealth statistics. The active (z-complete) trader accumulates 2.16 times as much wealth as the average household in the baseline *crb* case, while the passive trader accumulates 1.14 times as much and the non-participant .76. These fractions are virtually unchanged in the *irb* case. However, the wealth of the passive trader (expressed as a fraction of average wealth) becomes more volatile –it increases from 8.4 to 12 %.

[Table 3 about here.]

5.2.2 Active Trader Pool

While the results reported sofar show that *irb* passive traders amplify the volatility of risk prices, the numbers are still small compared to the 50 % standard deviation of the SR reported by Lettau and Ludvigson (2001). However, the composition of the active trader pool is equally important for the volatility of the market price of risk. The z-complete traders are subject to idiosyncratic risk and hence have a precautionary motive to accumulate wealth. We now look at what happens when we introduce traders who are not subject to idiosyncratic risk or can hedge against it. We think of these traders as a stand-in for highly levered, active market participants like hedge funds. These participants will tend to increase the volatility of risk premia if they are subject to occasionally binding solvency constraints.

As we change the active traders from z-complete traders to complete traders, the volatility of the market price of risk increases from 11.5% in the *irb* case to 21.5 % in the top panel (30 % target); from 18% to 30 % in the case of a 40 % target. As shown in Table 2, the volatility of the conditional Sharpe ratio on equity increases to 18 % in the case of a 30 % target, 30 % in the benchmark case with a 35 % equity target, and finally, 27 % in the case of 40 %. This means we do get much closer to the target in the data if we introduce these complete active traders.

The volatility of risk prices is much higher because these complete traders have no precautionary motive to accumulate wealth, and hence run into more binding solvency constraints more frequently. Moreover, the maximum SR increases as well from .41 to .52 in the 30 % case; from .33 to .46 in the 40 % case. The welfare cost of being a passive trader increases from 4.9 % to 14 % of aggregate consumption, simply because the volatility of risk premia is so much higher.

In addition, these complete traders load up on more aggregate risk, as is apparent from the results in the right panel of Table 3. The complete traders realize average excess returns of up to 11.5 % per annum. At the household level, in the baseline case with *crb* traders, we get the same

relation between trader sophistication and consumption growth volatility: the standard deviation of household consumption growth is 2.30 % for the active traders, compared to 3.15 % for the passive equity holders and 3.64 % for the non-participants. However, the composition is very different: the group volatility is 1.88 for the active traders, compared to 1.20 for the passive equity holders and .68 for the non-participants.

Figure 4 plots the the Sharpe ratio against the history of aggregate consumption growth shocks in the benchmark case with a 33 % equity target. For the same 100 simulations, we plot the conditional risk premium on equity (top panel), the conditional standard deviation (middle panel) and the conditional Sharpe ratio (bottom panel) against the history of aggregate growth shocks in Figure 5. The conditional SR varies between 1.0 and zero, and it declines monotonically as the weighted average of aggregate consumption growth shocks increases.

[Figure 4 about here.]

[Figure 5 about here.]

Overall, what is striking is how similar the unconditional moments are in the case of *crb* and *irb* traders, both in terms of portfolio returns and consumption growth. The main quantitative difference is the increase in the volatility of household consumption growth for the passive equity holders.

5.2.3 Size of Active Trader Pool

The volatility of risk premia depends critically on the size of the active trader pool. We fix the target equity share at 33%. As we grow the size of the active trader pool, the volatility of the market price of risk decreases at a fast rate. Table 4 reports the conditional moments in the case of a 10% active trader pool (up from 5% in the benchmark case).

The first two columns report the case with z-complete traders. The amplification channel is still operative, but the effect is much smaller. In the case with 10% z-complete traders, and 40% *crb* traders, the volatility of the market price of risk is 3.9%, and this number increases to 6.5% when we replace the *crb* traders with *irb* traders. In the benchmark case with only 5% active traders, these numbers were 3.6% and 16.5% respectively, as reported in Table 1. So, the amplification channel has weakened considerably. The standard deviation of the conditional Sharpe ratio on equity increases from 3.89% to 6.53%, compared to an increase from 3.5% to 16.47% in the benchmark case with 5% active traders.

As the mass of z-complete traders increases, the amplification channel weakens because aggregate risk is not concentrated enough. This is not the case if we replace these z-complete traders with complete traders because the latter have no precautionary motive to accumulate wealth, and

as result, their solvency constraints will still bind in equilibrium. These results are reported in the last two columns of Table 4. In this case, the standard deviation of the market price of risk increases from 8.44% to 28.79%, compared to 8.40% and 30.4% respectively in the benchmark case with only 5% active traders. In this case, the amplification channel is not mitigated by an increase in the supply of active capital.

[Table 4 about here.]

5.3 IID Economy

Alvarez and Jermann (2001) match the first-order autocorrelation of aggregate consumption growth shocks reported by Mehra and Prescott (1985) ($\rho_z = . - .14$). In addition, the Storesletten et al. (2007) calibration of idiosyncratic risk turns on the CCV (counter-cyclical cross-sectional variation) mechanism; in low aggregate consumption growth states, the volatility of idiosyncratic risk increases. We check the sensitivity of our results to the negative autocorrelation of aggregate consumption growth shocks and the CCV mechanism by choosing an IID calibration of aggregate consumption growth shocks without the CCV mechanism. This calibration satisfies the assumption we imposed in the IID example (see 4.3). In this version of model, without non-participants, the representative agent risk premium obtains if all passive traders are of *crb* type.

The key moments of the stochastic discount factor are reported in Table 5. In the benchmark case of a 35 , the standard deviation of the market price of risk increases from 5.4% in the *crb* case to 11 % in the *irb* case, a smaller 111 percent increase, compared to a 190 percent increase in the volatility in the benchmark calibration (see results reported in Table 1). The volatility is smaller in the IID economy, but the *irb* traders do amplify the volatility of the market prices of risk.

[Table 5 about here.]

6 Conclusion

Our paper shows that intermittent re-balancing should be considered as a potential explanation for the puzzling volatility of Sharpe ratios in equity markets. This explanation does not rely on non-standard preferences, but instead it simply assumes that some traders fail to continuously re-balance their portfolios. However, the welfare cost calculations suggest that small costs might suffice to deter households from continuously re-balancing. Even though the individual welfare loss from not rebalancing may be small, the aggregate impact on pricing is large. This makes it an appealing friction.

References

Abel, A. B. (1999). Risk premia and term premia in general equilibrium. *Journal of Monetary Economics 43*, 3–33. 8, 21

Abel, A. B., J. C. Eberly, and S. Panageas (2006). Optimal inattention to the stock market. *American Economic Review Papers and Proceedings 97(2)*, 244–248. 5, 9

Aiyagari, S. R., A. Marcet, T. J. Sargent, and J. Seppala (2002). Optimal taxation without state-contingent debt. *Journal of Political Economy 110(6)*, 1220–1254. 5

Alvarez, F. and U. Jermann (2001). Quantitative asset pricing implications of endogenous solvency constraints. *Review of Financial Studies 14*, 1117–1152. 20, 27, 39, 40, 41, 42, 45, 46, 47, 48

Ameriks, J. and S. P. Zeldes (2004, September). How do household portfolio shares vary with age? Working Paper Columbia University. 2

Bansal, R. and A. Yaron (2004, August). Risks for the long run: A potential resolution of asset prizing puzzles. *The Journal of Finance 59*, 1481–1509. 21

Barberis, N., M. Huang, and T. Santos (2001, February). Prospect theory and asset prices. *Quarterly Journal of Economics 116*(1), 1–53. 3

Basak, S. and D. Cuoco (1998). An equilibrium model with restricted stock market participation. *Review of Financial Studies 11*(2), 309–341. 4, 5

Boldrin, M., L. Christiano, and J. Fisher (2001, March). Habit persistence, asset returns, and the business cycle. *American Economic Review 91*(1), 149–66. 3

Brunnermeier, M. K. and S. Nagel (2008). Do wealth fluctuations generate time-varying risk aversion? micro-evidence on individuals' asset allocation. *American Economic Review 98:3*, 713–736. 2, 4

Calvet, L. E., J. Y. Campbell, and P. Sodini (2009). Fight or flight? portfolio rebalancing by individual investors. *The Quarterly Journal of Economics February*, 301–346. 2

Campbell, J. Y. and J. H. Cochrane (1999). By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy 107*(2), 205–251. 3, 4

Cecchetti, S. G., P.-S. Lam, and N. C. Mark (1990). Mean reversion in equilibrium asset prices. *American Economic Review Vol. 80, No. 3*, 398–418. 21

Chan, Y. L. and L. Kogan (2002). Catching up with the joneses: Heterogeneous preferences and the dynamics of asset prices. *Journal of Political Economy 110*, 1255–1285. 4

Chien, Y., H. Cole, and H. Lustig (2007). Macro implications of household finance. Working Paper UCLA. 5, 6, 12, 13, 15, 20

Chien, Y.-L. and H. Lustig (2009). The market price of aggregate risk and the wealth distribution. *Forthcoming in Review of Financial Studies*. Available at SSRN: http://ssrn.com/abstract=290917. 5

Constantinides, G. M. (1990). Habit-formation: A resolution of the equity premium puzzle. *Journal of Political Economy 98*, 519–543. 3

Cuoco, D. and H. He (2001). Dynamic equilibrium in infinite dimensional economies with incomplete financial markets. *Annals of Economics and Finance 2*, 265–296. 5

Detemple, J. and S. Murthy (1997). Equilibrium asset prices and no-arbitrage with portfolio constraints. *Review of Financial Studies 10*, 1133–1174. 4

Doms, M. and N. Morin (2004). Consumer sentiment, the economy, and the news media. *FRBSF Working Paper 2004-09*. 5

Duffie, D. and T.-S. Sun (1990). Transactions costs and portfolio choice in a discrete-continuous-time setting. *Journal of Economic Dynamics and Control 14*, 35–51. 3

Gabaix, X. and D. Laibson (2002). The 6d bias and the equity premium puzzle. *NBER Macroeconomics Annual*, 257–312. 3, 4

Garleanu, N. and S. Panageas (2007). Young, old, conservative and bold: The implications of heterogeneity and finite lives for asset pricing. *Working Paper Wharton*. 4

Gomes, F. and A. Michaelides (2008). Asset pricing with limited risk sharing and heterogeneous agents. *Review of Financial Studies 21(1)*, 415–448. 4

Grossman, G. and G. Laroque (1990). Asset pricing and optimal portfolio choice in the presence of illiquid durable consumption goods. *Econometrica 58*(1), 25–51. 4

Guvenen, F. (2009). A parsimonious macroeconomic model for asset pricing. *Econometrica 77(6)*, 1711 – 1750. 4

Jermann, U. (1998). Asset pricing in production economies. *Journal of Monetary Economics*, 257–275. 3

Kehoe, P. J. and F. Perri (2002, May). International business cycles with endogenous incomplete markets. *Econometrica 70*(3), 907–928. 5

Krueger, D. and H. Lustig (2009). When is market incompleteness irrelevant for the price of aggregate risk (and when is it not)? *Journal of Economic Theory 145(1)*, 1–41. 18, 20

Lettau, M. and S. C. Ludvigson (2001). Measuring and modeling variation in the risk-return tradeoff. Unpublished paper, New York University. 3, 4, 5, 24, 25

Lintner, J. (1965). Security prices, risk and maximal gains from diversification. *Journal of Finance 20*, 587–615. 3

Lustig, H., C. Sleet, and S. Yeltekin (2007). Fiscal hedging with nominal assets. *Journal of Monetary Economics 55(4)*, 710–727. 5

Lynch, A. W. (1996). Decision frequency and synchronization across agents: Implications for aggregate consumption and equity return. *The Journal of Finance 51(4)*, 1479–1497. 3, 4

Malloy, C., T. Moskowitz, and A. Vissing-Jorgensen (2009). Long-run stockholder consumption risk and asset returns. *Journal of Finance forthcoming*. 4

Marcet, A. and R. Marimon (1999). Recursive contracts. Working Paper Universitat Pompeu Fabra. 5

Mehra and E. Prescott (1985). The equity premium: A puzzle. *Journal of Monetary Economics 15*(2), 145–161. 3, 27

Parker, J. A. and A. Vissing-Jorgensen (2009). Who bears aggregate fluctuations and how? *NBER Working Paper No. 14665*. 4

Piazzesi, M. (2002). Discussion of the the 6d bias and the equity premium puzzle. *NBER Macroeconomics Annual*, 257–312. 3

Reis, R. (2006). Inattentive consumers. *Journal of Monetary Economics 53*, 1761–1800. 3

Saito, M. (1996). Limited participation and asset prices. *Working Paper University of British Columbia*. 4

Schneider, M., J. C. Hatchondo, and P. Krusell (2005). A quantitative model of competitive asset pricing under private information. Working Paper Stanford University. 4

Sharpe, W. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance 19*, 425–444. 3

Storesletten, K., C. Telmer, and A. Yaron (2004). Cyclical dynamics of idiosyncratic labor market risk. *The Journal of Political Economy*. 21

Storesletten, K., C. Telmer, and A. Yaron (2007). Asset pricing with idiosyncratic risk and overlapping generations. *Review of Economic Dynamics 10(4)*, 519–548. 21, 27, 39, 40, 41, 42, 45, 46, 47, 48

A Saddle Point

A.1 Active

The saddle point problem of an z-complete trader can be stated as:

$$
\begin{aligned}
L \;=\; & \min_{\{\chi,\nu,\varphi\}} \max_{\{c,\hat{a}\}} \sum_{t=1}^{\infty} \beta^t \sum_{(z^t,\eta^t)} u(c(z^t,\eta^t))\pi(z^t,\eta^t) \\
& + \chi \left\{ \sum_{t\geq 1} \sum_{(z^t,\eta^t)} \widetilde{P}(z^t,\eta^t) \left[\gamma Y(z^t)\eta_t - c(z^t,\eta^t)\right] + \varpi(z^0) \right\} \\
& + \sum_{t\geq 1} \sum_{(z^t,\eta^t)} \nu(z^t,\eta^t) \left\{ \sum_{\tau\geq t} \sum_{(z^\tau,\eta^\tau)\succeq(z^t,\eta^t)} \widetilde{P}(z^\tau,\eta^\tau) \left[\gamma Y(z^\tau)\eta_\tau - c(z^\tau,\eta^\tau)\right] + \widetilde{P}(z^t,\eta^t)\hat{a}_{t-1}(z^t,\eta^{t-1}) \right\} \\
& + \sum_{t\geq 1} \sum_{(z^t,\eta^t)} \varphi(z^t,\eta^t) \left\{ -\underline{M}_t(z^t,\eta^t)\widetilde{P}(z^t,\eta^t) - \sum_{\tau\geq t} \sum_{(z^\tau,\eta^\tau)\succeq(z^t,\eta^t)} \widetilde{P}(z^\tau,\eta^\tau) \left[\gamma Y(z^\tau)\eta_\tau - c(z^\tau,\eta^\tau)\right] \right\},
\end{aligned}
$$

where $\widetilde{P}(z^t,\eta^t) = \pi(z^t,\eta^t)P(z^t,\eta^t)$. This is a standard convex programming problem –the constraint set is still convex, even with the measurability conditions and the solvency constraints. The first order conditions are necessary and sufficient. The complete-trader's problem is simply this problem where with net financial wealth allowed to depend on the full idiosyncratic history, or $\hat{a}_{t-1}(z^t,\eta^t)$, and hence this measurability constraint is degenerate.

Let χ denote the multiplier on the present-value budget constraint, let $\nu(z^t,\eta^t)$ denote the multiplier on the measurability constraint in node (z^t,η^t), and, finally, let $\varphi(z^t,\eta^t)$ denote the multiplier on the debt constraint.

The first-order condition for consumption is given by

$$
\beta^t u'(c(z^t,\eta^t))\pi(z^t,\eta^t) = \chi + \sum_{(z^t,\eta^t)\succeq(z^\tau,\eta^\tau)} \left[\nu(z^\tau,\eta^\tau) - \varphi(z^\tau,\eta^\tau)\right] \widetilde{P}(z^t,\eta^t),
$$

A.2 Passive

Here again, we will work with the present-value problem. As before, let χ denote the multiplier on the present-value budget constraint, let $\nu(z^t,\eta^t)$ denote the multiplier on the measurability constraint in node (z^t,η^t), let $\varphi(z^t,\eta^t)$ denote the multiplier on the debt constraint. In addition, let $\kappa(z^t,\eta^t)$ denote the multiplier on the equity transition condition. The saddle point problem of

a passive trader with trading technology (ϕ^*, T) can be stated as:

$$
\begin{aligned}
L &= \min_{\{\chi,\nu,\varphi\}} \max_{\{c,b,e\}} \sum_{t=1}^{\infty} \beta^t \sum_{(z^t,\eta^t)} u(c(z^t,\eta^t))\pi(z^t,\eta^t) \\
&+ \chi \left\{ \sum_{t\geq 1} \sum_{(z^t,\eta^t)} \widetilde{P}(z^t,\eta^t) \left[\gamma Y(z^t)\eta_t - c(z^t,\eta^t)\right] + \varpi(z^0) \right\} \\
&+ \sum_{t\geq 1} \sum_{(z^t,\eta^t)} \nu(z^t,\eta^t) \left\{ \begin{array}{c} \sum_{\tau\geq t} \sum_{(z^\tau,\eta^\tau)\succeq(z^t,\eta^t)} \widetilde{P}(z^\tau,\eta^\tau) \left[\gamma Y(z^\tau)\eta_\tau - c(z^\tau,\eta^\tau)\right] \\ +\widetilde{P}(z^t,\eta^t) \left[b(z^{t-1},\eta^{t-1})R^f(z^{t-1}) + I_{\{t\in T\}} e(z^{t-1},\eta^{t-1})R^e(z^t)\right] \end{array} \right\} \\
&+ \sum_{t\geq 1} \sum_{(z^t,\eta^t)} \varphi(z^t,\eta^t) \left\{ \begin{array}{c} -\underline{M}_t(z^t,\eta^t)\widetilde{P}(z^t,\eta^t) \\ -\sum_{\tau\geq t} \sum_{(z^\tau,\eta^\tau)\succeq(z^t,\eta^t)} \widetilde{P}(z^\tau,\eta^\tau) \left[\gamma Y(z^\tau)\eta_\tau - c(z^\tau,\eta^\tau)\right] \end{array} \right\} \\
&+ \sum_{t\geq 1} \sum_{(z^t,\eta^t)} \kappa(z^t,\eta^t) \left\{ \begin{array}{c} I_{\{t\in T\}} \left[e(z^t,\eta^t) - \frac{\varpi^\star}{1-\varpi^\star} b(z^t,\eta^t)\right] \\ +I_{\{t\notin T\}} \left[e(z^t,\eta^t) - R^e(z^t)e(z^{t-1},\eta^{t-1})\right] \end{array} \right\}.
\end{aligned}
$$

where $\widetilde{P}(z^t,\eta^t) = \pi(z^t,\eta^t)P(z^t,\eta^t)$. This is a standard convex programming problem. We list the first-order conditions for consumption c:

$$
\beta^t u'(c(z^t,\eta^t))\pi(z^t,\eta^t) = \left\{ \chi + \sum_{(z^t,\eta^t)\succeq(z^\tau,\eta^\tau)} \left[\nu(z^\tau,\eta^\tau) - \varphi(z^\tau,\eta^\tau)\right] \right\} \widetilde{P}(z^t,\eta^t),
$$

for bonds b_t

$$
\sum_{(z^{t+1},\eta^{t+1})} \nu(z^{t+1},\eta^{t+1})\widetilde{P}(z^{t+1},\eta^{t+1})R^f(z^t) - I_{\{t\in T\}}\kappa(z^t,\eta^t)\frac{\varpi^\star}{1-\varpi^\star} = 0,
$$

and finally for equity holdings e:

$$
\sum_{(z^{t+1},\eta^{t+1})} \left\{ \begin{array}{c} \nu(z^{t+1},\eta^{t+1})I_{\{t+1\in T\}}\widetilde{P}(z^{t+1},\eta^{t+1})R^e(z^{t+1}) \\ -\kappa(z^{t+1},\eta^{t+1})I_{\{t+1\notin T\}}R^e(z^{t+1}) \end{array} \right\} + \kappa(z^t,\eta^t) = 0.
$$

Taxonomy There are four cases with respect to the last two first-order conditions depending upon whether t and/or $t+1$ is an element of $\mathcal{T}$, the set of rebalancing periods. Here is an overview of these different cases:

1. If $t \in \mathcal{T}$ and $t+1 \in \mathcal{T}$ then the last two conditions reduce to

$$
\sum_{(z^{t+1},\eta^{t+1})} \nu_{t+1}(z^{t+1},\eta^{t+1})\widetilde{P}_{t+1}(z^{t+1},\eta^{t+1}) \left[(1-\varpi^\star)R_{t+1,t}[1](z^t) + \varpi^\star R_{t+1,t}[\{D\}](z^{t+1})\right] = 0,
$$

where $[(1-\varpi^\star)R_{t+1,t}[1](z^t) + \varpi^\star R_{t+1,t}[\{D\}](z^{t+1})]$ is the simply overall return on the agent's portfolio conditional on the transition from z^t to z^{t+1}. This is the martingale condition for the continuous-rebalancing trader.

2. If $t \in \mathcal{T}$ and $t+1 \notin \mathcal{T}$ then the last two conditions become

$$\frac{1-\varpi^\star}{\varpi^\star} \sum_{(z^{t+1},\eta^{t+1})} \nu_{t+1}(z^{t+1},\eta^{t+1})\widetilde{P}_{t+1}(z^{t+1},\eta^{t+1})R_{t+1,t}[1](z^t) = \kappa(z^t,\eta^t),$$

and

$$\sum_{(z^{t+1},\eta^{t+1})} R_{t+1,t}[\{D\}](z^{t+1}) \left\{\nu_{t+1}(z^{t+1},\eta^{t+1})\widetilde{P}_{t+1}(z^{t+1},\eta^{t+1}) - \kappa_{t+1}(z^{t+1},\eta^{t+1})\right\} = -\kappa_t(z^t,\eta^t).$$

3. If $t \notin \mathcal{T}$ and $t+1 \in \mathcal{T}$ then the last two conditions become

$$\frac{1-\varpi^\star}{\varpi^\star} \sum_{(z^{t+1},\eta^{t+1})} \nu_{t+1}(z^{t+1},\eta^{t+1})\widetilde{P}_{t+1}(z^{t+1},\eta^{t+1})R_{t+1,t}[1](z^t) = 0,$$

and

$$\sum_{(z^{t+1},\eta^{t+1})} \nu_{t+1}(z^{t+1},\eta^{t+1})\widetilde{P}(z^{t+1},\eta^{t+1})R_{t+1,t}[\{D\}](z^{t+1}) = -\kappa_t(z^t,\eta^t).$$

4. If $t \notin \mathcal{T}$ and $t+1 \notin \mathcal{T}$ then the last two conditions become

$$\frac{1-\varpi^\star}{\varpi^\star} \sum_{(z^{t+1},\eta^{t+1})} \nu_{t+1}(z^{t+1},\eta^{t+1})\widetilde{P}_{t+1}(z^{t+1},\eta^{t+1})R_{t+1,t}[1](z^t) = 0,$$

and

$$\sum_{(z^{t+1},\eta^{t+1})} R_{t+1,t}[\{D\}](z^{t+1}) \left\{\nu_{t+1}(z^{t+1},\eta^{t+1})\widetilde{P}(z^{t+1},\eta^{t+1}) - \kappa(z^{t+1},\eta^{t+1})\right\} = -\kappa_t(z^t,\eta^t).$$

In the simple case in which the rebalancing takes place every other period, then these conditions boil down to

$$\begin{aligned} 0 \;=\; & \sum_{(z^{t+1},\eta^{t+1})} \left\{\nu(z^{t+1},\eta^{t+1})\widetilde{P}(z^{t+1},\eta^{t+1})\left[\phi^* R^f(z^t)\right]\right\} \\ & + \sum_{(z^{t+2},\eta^{t+2})} \left\{\nu(z^{t+2},\eta^{t+2})\widetilde{P}(z^{t+2},\eta^{t+2})\left[R_{t+2,t}[\{D\}](z^{t+2})\right]\right\} \end{aligned}$$

in the rebalancing periods, and

$$\phi^* \sum_{(z^{t+1},\eta^{t+1})} \nu(z^{t+1},\eta^{t+1})\widetilde{P}(z^{t+1},\eta^{t+1})R_{t+1,t}[1](z^t) = 0.$$

in the nonrebalancing periods.

B Proofs

Proof of Result in eq. (16):

Proof. The consumption sharing rule follows directly from the ratio of the first order conditions and the market clearing condition. Condition (14) implies that

$$c(z^t,\eta^t) = u'^{-1}\left[\frac{\zeta(z^t,\eta^t)P(z^t)}{\beta^t}\right].$$

In addition, the sum of individual consumptions aggregate up to aggregate consumption:

$$C(z^t) = \sum_{\eta^t} c(z^t,\eta^t)\pi(\eta^t|z^t).$$

This implies that the consumption share of the individual with history (z^t,η^t) is

$$\frac{c(z^t,\eta^t)}{C(z^t)} = \frac{u'^{-1}\left[\frac{\zeta(z^t,\eta^t)P(z^t)}{\beta^t}\right]}{\sum_{\eta^t} u'^{-1}\left[\frac{\zeta(z^t,\eta^t)P(z^t)}{\beta^t}\right]\pi(\eta^t|z^t)}.$$

With CRRA preferences, this implies that the consumption share is given by

$$\frac{c(z^t,\eta^t)}{C(z^t)} = \frac{\zeta(z^t,\eta^t)^{\frac{-1}{\alpha}}}{h(z^t)}, \text{ where } h(z^t) = \sum_{\eta^t}\zeta(z^t,\eta^t)^{\frac{-1}{\alpha}}\pi(\eta^t|z^t).$$

Hence, the $-1/\alpha^{\text{th}}$ moment of the multipliers summarizes risk sharing within this economy. We refer to this moment of the multipliers simply as **the aggregate multiplier**. The equilibrium SDF is the standard Breeden-Lucas SDF times the growth rate of the aggregate multiplier. This aggregate multiplier reflects the aggregate shadow cost of the measurability and the borrowing constraints faced by households. The expression for the SDF can be recovered directly by substituting for the consumption sharing rule in the household's first order condition for consumption (14). □

C Conditioning on NBER Recessions and Expansions

Nonetheless, the average returns on this investment strategy, reported in Table 6, provide a clear indication of the cyclical behavior of the expected returns conditional on the aggregate state being expansion (recessions). We report the returns on buying one quarter through five quarters after the trough (peak). The top panel looks at expansions. The bottom panel looks at recessions. The evidence is striking.

In the bottom panel, we see that expected excess returns increase in a recession. The average returns, conditional on being in a recession, increase as we enter the recession from 6.22% (2.88% in the whole sample) in the 1st quarter after the peak to 12.63% (11.73%) in the third quarter after the peak. After 3 quarters, average returns tend to decrease again. The volatility of stock returns tends to increase increase initially during recessions, but then it declines.

When we compare say the midpoint (3rd quarter) of a recession with the 3rd quarter of an expansion, we see a large difference in volatilities: 8.50% (8.70%) in an expansion and 11.00% (12.07%) in a recession. We also observe a large difference in the risk compensation per unit of risk: the Sharpe ratio is .051 (.20) in the midpoint of an expansion, compared to .54 (.49) in the midpoint of a recession. That is essentially what others have found as well: the compensation per unit of risk is much higher in recessions.

[Table 6 about here.]

D Approximation

We forecast the growth rate of the aggregate multiplier $[h(z^{t+1})/h(z^t)]$ by using a finite partition of the history of aggregate shocks z^t, with each element in the partition being assigned a distinct forecast value.

Algorithm 1. *We construct our partition of aggregate histories Σ by applying the following procedure. σ denotes an element of this partition. We construct a partition based upon the last n aggregate shocks, which we denote by Z^n. The partition simply consists of truncated aggregate histories: $\Sigma = Z^n$. The number of elements in the partition is given by $\#Z^n$, where $\#Z$ is the number of aggregate states.*

The rationale for the first partition with truncated aggregate histories is straightforward. All households start off with the same multiplier at time 0. If we keep track of the history of aggregate shocks z^t through period t, then obviously we know the entire distribution of multipliers at t, and we can compute all of its moments. Hence, the actual growth rate $[h(z^{t+1})/h(z^t)]$ can be determined exactly provided that one knows the entire history of the aggregate shocks z^t. Of course, for large

t, keeping track of the entire aggregate history becomes impractical. However, if there is an ergodic equilibrium, the effect of aggregate shocks has to wear off after some time has passed.

We define $\widehat{g}(\sigma, \sigma')$ as the forecast of the aggregate multiplier growth rate $[h(z^{t+1})/h(z^t)]$, conditional on the the last n elements of z^t equaling σ, and the last n elements of z^{t+1} equaling σ'.

Algorithm 2. *The algorithm we apply is:*

1. *conjecture a function* $\widehat{g}_0(\sigma, \sigma') = 1$.
2. *solve for the equilibrium updating functions* $T_0^j(\sigma', \eta'|\sigma, \eta)(\zeta)$ *for all trader groups* $j \in \{z, bh, np\}$.
3. *By simulating for a panel of* N *households for* T *time periods, we compute a new aggregate weight forecasting function* $\widehat{g}_1(\sigma, \sigma')$.
4. *We continue iterating until* $\widehat{g}_k(\sigma, \sigma')$ *converges.*

In our approximation, we allocate consumption to households with a version of the consumption sharing rule that uses our forecast of the aggregate multiplier $\widehat{g}(\sigma, \sigma')$ in each aggregate node σ, $\zeta^{\frac{-1}{\alpha}}/\widehat{g}(\sigma, \sigma')$. Prices are set using the forecast as well: $m(\sigma', \sigma) \equiv \beta e^{-\alpha z'} \widehat{g}(\sigma, \sigma')^{\alpha}$. Of course, this implies that actually allocated aggregate consumption C^a differs from actual aggregate consumption C:

$$C^a(z^{t+1}) = \frac{g(z^{t+1})}{\widehat{g}(\sigma, \sigma')} Y(z^{t+1}),$$

where $g(z^{t+1})$ is the actual growth rate of the aggregate multiplier in that aggregate node z^{t+1}. This equation simply follows from aggregating our consumption sharing rule across all households. When the forecast $\widehat{g}(\sigma, \sigma')$ deviates from the realized growth rate $g(z^{t+1})$, this causes a gap between total allocated consumption and the aggregate endowment. Hence, the percentage forecast errors ($\log e = \log g - \log \widehat{g}$) are really allocation errors ($\log C^a - \log Y$).[9]

With a slight abuse of notation, we use $z^t \in \sigma$ to denote that the last n aggregate shocks equal σ. The forecasts are simply the conditional sample means of the realized aggregate growth rates in each node (σ, σ'):

$$\log \widehat{g}(\sigma, \sigma') = \frac{1}{N(\sigma, \sigma')} \sum_{(z^{t-n}, z^t, z^{t+1}) \in \sigma \times \sigma'} \log g(z^{t+1}),$$

where $N(\sigma, \sigma')$ denotes the number of observations of this aggregate history in our panel. As one metric of the approximation quality, we report the standard deviation of the forecast errors:

$$std\left[\log e_{t+1}\right] = std\left[\log \widehat{g}(\sigma, \sigma') - \log g(z^{t+1})\right].$$

[9]However, the household's Euler equation holds exactly in each node, given that we have set the prices and allocated consumption in each node on the basis of the forecasted aggregate multiplier, not the realized one.

Equivalently, we can also think of $\log \widehat{g}(\sigma, \sigma')$ as the fitted value in a regression of realized growth rates g_{t+1} on dummy variables $d(\sigma_t, \sigma_{t+1})$, one for each node:

$$\log g_{t+1} = \sum_{(\sigma_t, \sigma_{t+1}) \in \Sigma} \log g(\sigma_t, \sigma_{t+1}) d(\sigma_t, \sigma_{t+1}) + e_{t+1}. \tag{31}$$

As a second metric, we also report the R^2 in the forecasting regression in equation (31).

Table 1: Moments of Asset Prices Moments of annual returns. The *irb* traders re-balance every three periods in a staggered fashion (1/3 each year). Storesletten et al. (2007) of idiosyncratic shocks; Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks. Parameters: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 10%. The results are generated by simulating an economy with $3,000$ agents and $10,000$ periods.

	z-Complete		Complete	
	crb	*irb*	*crb*	*irb*
Active ***z***	5%	5%	0%	0%
Active ***c***	0%	0%	5%	5%
Passive ***crb***	45%	0%	45%	0%
Passive ***irb***	0%	45%	0%	45%
Passive ***np***	50%	50%	50%	50%
	30% equity share passive target ($\varpi^\star$)			
$\frac{\sigma(M)}{E(M)}$	0.439	0.417	0.516	0.527
$Std\left[\frac{\sigma_t(M)}{E_t(M)}\right]$	0.031	0.115	0.065	0.215
$E\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)$	8.919	8.408	10.148	9.546
$\sigma\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)$	20.562	21.756	20.163	21.622
Sharpe Ratio	0.434	0.386	0.503	0.441
$E\left(R_{t+1,t}[1]\right)$	1.582	1.750	1.604	1.758
$\sigma\left(R_{t+1,t}[1]\right)$	3.026	2.727	2.944	2.419
$std[\log(e)](\%)$	0.128	0.109	0.145	0.097
	33% equity share passive target ($\varpi^\star$)			
$\frac{\sigma(M)}{E(M)}$	0.396	0.384	0.489	0.506
$Std\left[\frac{\sigma_t(M)}{E_t(M)}\right]$	0.036	0.165	0.082	0.303
$E\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)$	7.977	7.392	9.223	8.211
$\sigma\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)$	20.433	22.784	19.542	24.310
Sharpe Ratio	0.390	0.324	0.472	0.338
$E\left(R_{t+1,t}[1]\right)$	1.811	1.992	1.747	2.036
$\sigma\left(R_{t+1,t}[1]\right)$	2.997	2.618	2.829	2.230
$std[\log(e)](\%)$	0.100	0.102	0.091	0.110
	35% equity share passive target ($\varpi^\star$)			
$\frac{\sigma(M)}{E(M)}$	0.370	0.360	0.459	0.492
$Std\left[\frac{\sigma_t(M)}{E_t(M)}\right]$	0.039	0.180	0.084	0.304
$E\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)$	7.436	6.722	8.611	7.207
$\sigma\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)$	20.450	23.632	19.516	26.994
Sharpe Ratio	0.364	0.284	0.441	0.267
$E\left(R_{t+1,t}[1]\right)$	1.939	2.145	1.844	2.291
$\sigma\left(R_{t+1,t}[1]\right)$	2.979	2.642	2.802	2.180
$std[\log(e)](\%)$	0.092	0.107	0.074	0.125
	40 % equity share passive target ($\varpi^\star$)			
$\frac{\sigma(M)}{E(M)}$	0.267	0.335	0.314	0.469
$Std\left[\frac{\sigma_t(M)}{E_t(M)}\right]$	0.062	0.198	0.128	0.270
$E\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)$	5.203	4.796	5.671	5.003
$\sigma\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)$	20.532	26.377	21.095	31.396
Sharpe Ratio	0.253	0.182	0.269	0.159
$E\left(R_{t+1,t}[1]\right)$	2.451	2.591	2.440	2.837
$\sigma\left(R_{t+1,t}[1]\right)$	2.891	2.507	2.806	2.099
$std[\log(e)](\%)$	0.090	0.128	0.080	0.162

Table 2: Conditional Moments Moments of annual returns conditional on history of aggregate shocks z^t. The *irb* traders rebalance every three periods in a staggered fashion (1/3 each year). Storesletten et al. (2007) of idiosyncratic shocks; Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks. Parameters: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 10%. The results are generated by simulating an economy with $3,000$ agents and $10,000$ periods.

	z-Complete		Complete	
	crb	*irb*	*crb*	*irb*
Active ***z***	5%	5%	0%	0%
Active ***c***	0%	0%	5%	5%
Passive ***crb***	45%	0%	45%	0%
Passive ***irb***	0%	45%	0%	45%
Passive ***np***	50%	50%	50%	50%
	30% equity share passive target ($\varpi^\star$)			
$Std\left[E_t\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)\right]$	1.17	1.59	1.07	2.70
$Std\left[\sigma_t\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)\right]$	2.38	2.78	2.40	3.38
$Std\left[SR_t\right]$	3.14	11.51	6.52	21.54
	33% equity share passive target ($\varpi^\star$)			
$Std\left[E_t\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)\right]$	1.09	2.76	0.91	4.93
$Std\left[\sigma_t\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)\right]$	2.35	3.05	2.33	4.52
$Std\left[SR_t\right]$	3.59	16.47	8.17	30.28
	35% equity share passive target ($\varpi^\star$)			
$Std\left[E_t\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)\right]$	1.05	3.18	1.00	6.41
$Std\left[\sigma_t\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)\right]$	2.34	3.66	2.23	6.02
$Std\left[SR_t\right]$	3.89	17.95	8.45	33.84
	40% equity share passive target ($\varpi^\star$)			
$Std\left[E_t\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)\right]$	1.18	5.36	2.23	9.35
$Std\left[\sigma_t\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)\right]$	2.40	4.97	2.95	8.35
$Std\left[SR_t\right]$	6.18	24.47	12.78	37.75

Table 3: Moments of Household Portfolio Returns and Consumption Panel I reports moments of household portfolio returns, Panel II reports moments of household consumption, and Panel III reports moments of household wealth: we report the average excess returns on household portfolios and the Sharpe ratios, we report the standard deviation of household consumption growth (as a multiple of the standard deviation of aggregate consumption growth), and we report the standard deviation of group consumption growth (as a multiple of the standard deviation of aggregate consumption growth); the last panel reports the average household wealth as a share of total wealth, and the standard deviation of household wealth, as a share of total wealth. Results for 33% equity share passive target ($\varpi^\star$).Moments of annual returns and consumption flows. The *irb* traders re-balance every three periods in a staggered fashion (1/3 each year). Storesletten et al. (2007) of idiosyncratic shocks; Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks. Parameters: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 10%. The results are generated by simulating an economy with 3,000 agents and 10,000 periods.

	z-Complete		Complete	
	crb	*irb*	*crb*	*irb*
Active ***z***	5%	5%	0%	0%
Active ***c***	0%	0%	5%	5%
Passive ***crb***	45%	0%	45%	0%
Passive ***irb***	0%	45%	0%	45%
Passive ***np***	50%	50%	50%	50%
	Panel I: Household Portfolio			
	Excess Return			
Active Trader	4.95	4.75	13.12	12.12
Passive Equity Holder	2.62	2.42	3.03	2.62
	Sharpe Ratio			
Active Trader	0.396	0.424	0.119	0.102
Passive Equity Holder	0.391	0.311	0.474	0.320
	Additional Stats			
Optimal Equity Share for irb	0.44	0.33	0.49	0.37
Welfare cost(%) of irb to z at optimal	1.64	5.86	4.31	14.50
Optimal Equity Share for crb	0.62	0.51	0.87	0.57
Welfare cost(%) of crb to z at optimal	1.12	4.16	1.75	11.57
Welfare cost(%) of crb to irb at 33% equity	0.43	−0.14	0.54	−0.52
	Panel II Household Consumption			
	Std. Dev. at Household level			
Active Trader	3.00	3.06	2.35	2.44
Passive Equity Holder	3.23	3.66	3.15	3.61
Passive non-participant	3.65	3.62	3.66	3.61
	Std. Dev. of Group Average			
Active Trader	1.87	1.73	2.09	2.00
Passive Equity Holder	1.20	1.25	1.17	1.24
Passive non-participant	0.67	0.67	0.69	0.68
	Panel III: Household Wealth			
	Average Household Wealth Ratio			
Active Trader	2.163	1.851	0.709	0.644
Passive Equity Holder	1.14	1.16	1.26	1.25
Passive non-participant	0.758	0.772	0.792	0.809
	Stdev. of Household Wealth Ratio			
Active Trader	0.585	0.474	0.248	0.264
Passive Equity Holder	0.069	0.114	0.084	0.123
Passive non-participant	0.114	0.110	0.097	0.108
	Stdev. of Aggregate Equity Share			
Passive Equity Holder	0.069	0.116	0.048	0.100
	Correlation of Aggregate Equity Share			
Passive Equity Holder	0.643	0.792	0.449	0.739

Table 4: Conditional Moments and size of Active Trader Pool Moments of annual returns conditional on history of aggregate shocks z^t. The *irb* traders re-balance every three periods in a staggered fashion (1/3 each year). Storesletten et al. (2007) of idiosyncratic shocks; Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks. Parameters: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 10%. Results for 33% equity share passive target ($\varpi^\star$). The results are generated by simulating an economy with 3,000 agents and 10,000 periods.

	z-Complete		Complete	
	crb	*irb*	*crb*	*irb*
Active ***z***	10%	10%	0%	0%
Active ***c***	0%	0%	10%	10%
Passive ***crb***	40%	0%	40%	0%
Passive ***irb***	0%	40%	0%	40%
Passive ***np***	50%	50%	50%	50%
$\frac{\sigma(m)}{E(m)}$	0.369	0.317	0.459	0.486
$Std\left[\frac{\sigma_t(M)}{E_t(M)}\right]$	0.039	0.065	0.084	0.287
$Std\left[E_t\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)\right]$	1.044	0.947	0.999	4.407
$Std\left[\sigma_t\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)\right]$	2.338	3.072	2.229	5.100
$Std\left[SR_t\right]$	3.899	6.538	8.449	28.795

Table 5: Moments of Asset Prices with IID Calibration Moments of annual returns. The *irb* traders re-balance every three periods in a staggered fashion (1/3 each year). Storesletten et al. (2007) calib of idiosyncratic shocks without CCV; Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks with AR(1) coefficient for aggregate consumption growth $\rho_z = 0$. Parameters: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 10%. The results are generated by simulating an economy with 3,000 agents and 10,000 periods.

	z-Complete		Complete	
	crb	*irb*	*crb*	*irb*
Active ***z***	5%	5%	0%	0%
Active ***c***	0%	0%	5%	5%
Passive ***crb***	45%	0%	45%	0%
Passive ***irb***	0%	45%	0%	45%
Passive ***np***	50%	50%	50%	50%
	30% equity share passive target ($\varpi^\star$)			
$\frac{\sigma(m)}{E(m)}$	0.444	0.430	0.518	0.517
$Std(\frac{\sigma_t(m)}{E_t(m)})$	0.052	0.087	0.075	0.124
	35% equity share passive target ($\varpi^\star$)			
$\frac{\sigma(m)}{E(m)}$	0.400	0.379	0.497	0.490
$Std(\frac{\sigma_t(m)}{E_t(m)})$	0.054	0.110	0.083	0.161
	40% equity share passive target ($\varpi^\star$)			
$\frac{\sigma(m)}{E(m)}$	0.341	0.320	0.433	0.411
$Std(\frac{\sigma_t(m)}{E_t(m)})$	0.063	0.144	0.084	0.246

Table 6: The Conditional Sharpe Ratio in Equity Markets and the Business Cycle

In expansions (recessions), the investor buys the stock market index in the n-th quarter after the NBER hrough (peak) and sells after 4 quarters. Table reports moments of excess returns for thhis investment strategy implemented on the CRSP-VW index of NYSE-AMEX-NASDAQ realized. The riskfree rate is the 90-days T-bill rate (also from CRSP). The entire sample comprises 1925.IV-2009.II. The postwar sample comprises 1945.I-2009.II. We report the average excess return on this investment strategy (annualized) in the first panel, the standard deviation (not annualized) in the second panel and the Sharpe ratio (annualized) in the third panel.

			Expansions		
	Buy in $n-th$ quarter after through				
	1st	2nd	3rd	4th	5th
	Conditional Expected Excess Return				
whole	17.30%	4.29%	3.48%	8.22%	2.11%
postwar	9.85%	1.45%	0.86%	5.51%	5.51%
	Conditional Stdev. of Excess Return				
whole	14.75%	9.17%	8.70%	7.74%	8.57%
postwar	8.62%	8.95%	8.50%	7.77%	6.97%
	Conditional Sharpe Ratio				
whole	0.586	0.234	0.200	0.531	0.123
postwar	0.571	0.081	**0.051**	0.355	0.396
			Recessions		
	Buy in $n-th$ quarter after peak				
	1st	2nd	3rd	4th	5th
	Conditional Expected Excess Return				
whole	2.88%	8.43%	11.73%	10.76%	2.46%
postwar	6.22%	10.70%	12.63%	10.57%	3.82%
	Conditional Stdev. of Excess Return				
whole	12.54%	12.61%	12.07%	10.84%	10.79%
postwar	10.47%	11.20%	11.00%	9.78%	9.87%
	Conditional Sharpe Ratio				
whole	0.115	0.334	0.486	0.496	0.114
postwar	0.297	0.478	**0.574**	0.540	0.193

Figure 1: Counter-Cyclical Time Variation in the Sharpe Ratio on Equities

Conditional Sharpe Ratio on Market (VW-CRSP). Computed by buying equity N quarters after NBER peak/trough and holding for one year. N is on horizontal axis. We plot the results that we obtained on the 1925-2009 sample (dotted line) and the 1945-2009 sample (dashed line). The data is quarterly. The full line is a 4-th degree polynomial approximation.

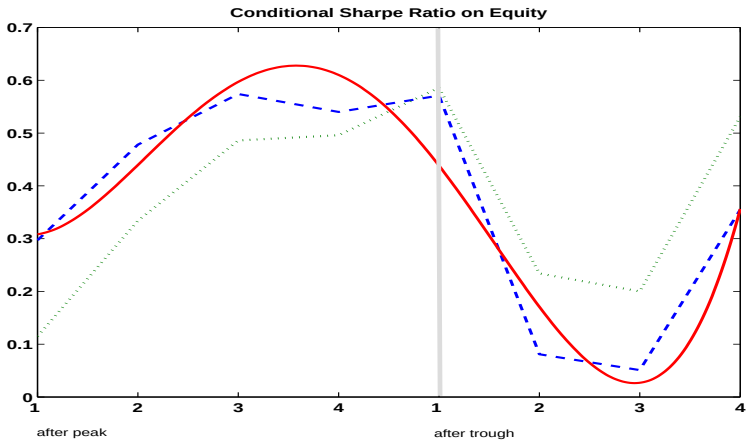


Figure 2: Counter-Cyclical Time Variation: Benchmark case with z-Complete Active Traders

The dashed line is a 4-period moving average of aggregate consumption growth with linearly decreasing weights. This calibration has 50 % non-participants, 5% complete and 45 % either in *crb* or *irb* traders. The target equity share is 33 %. The *irb* traders re-balance every three periods in a staggered fashion (1/3 each year). Storesletten et al. (2007) of idiosyncratic shocks; Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks. Parameters: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 10%. The results are generated by simulating an economy with $3,000$ agents and $10,000$ periods.

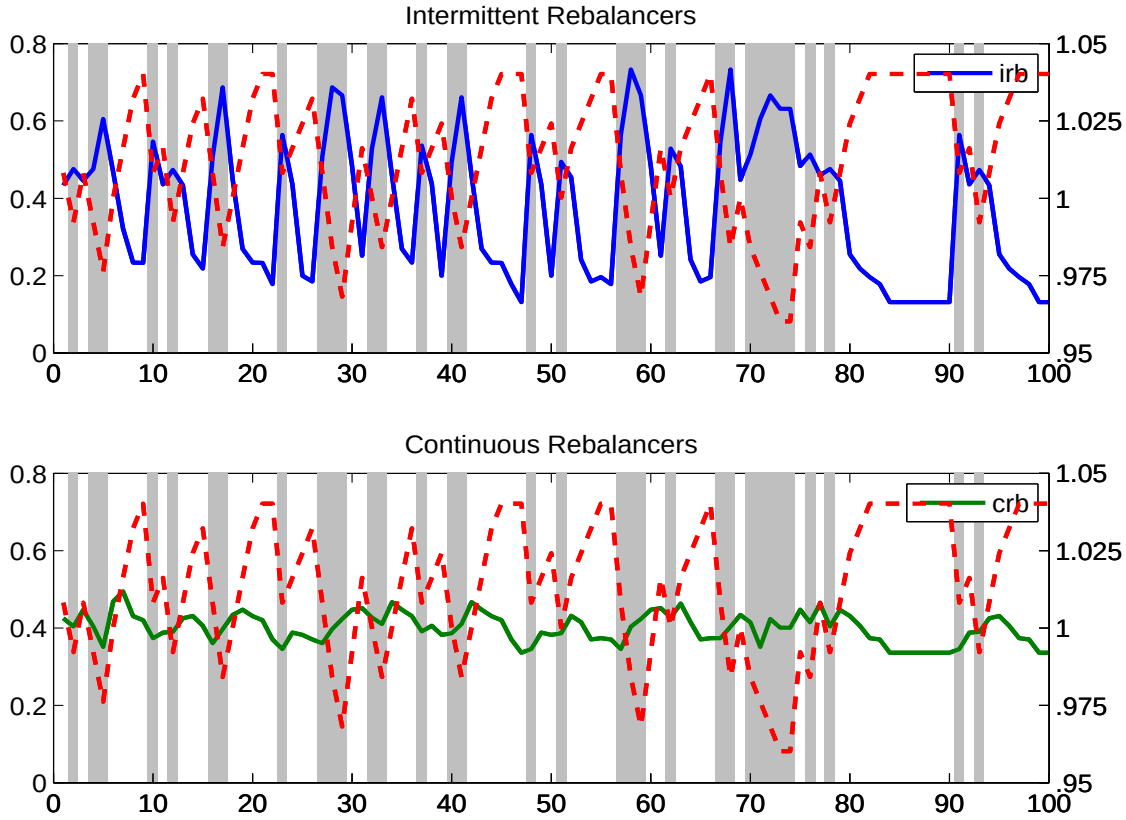


Figure 3: Benchmark case with z-Complete Active Traders and *irb* Passive Traders.

Scatter plot of the 100 data points in figure 2. On the x-axis is a 4-period moving average of aggregate consumption growth with linearly decreasing weights. On the y-axis is the conditional expected excess return on equity (top panel), the conditional standard deviation (middle panel), the conditional SR (bottom panel). This calibration has 50 % non-participants, 5% complete and 45 % *irb* traders. The target equity share is 33 %. The *irb* traders re-balance every three periods in a staggered fashion (1/3 each year). Storesletten et al. (2007) of idiosyncratic shocks; Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks. Parameters: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 10%. The results are generated by simulating an economy with $3,000$ agents and $10,000$ periods.

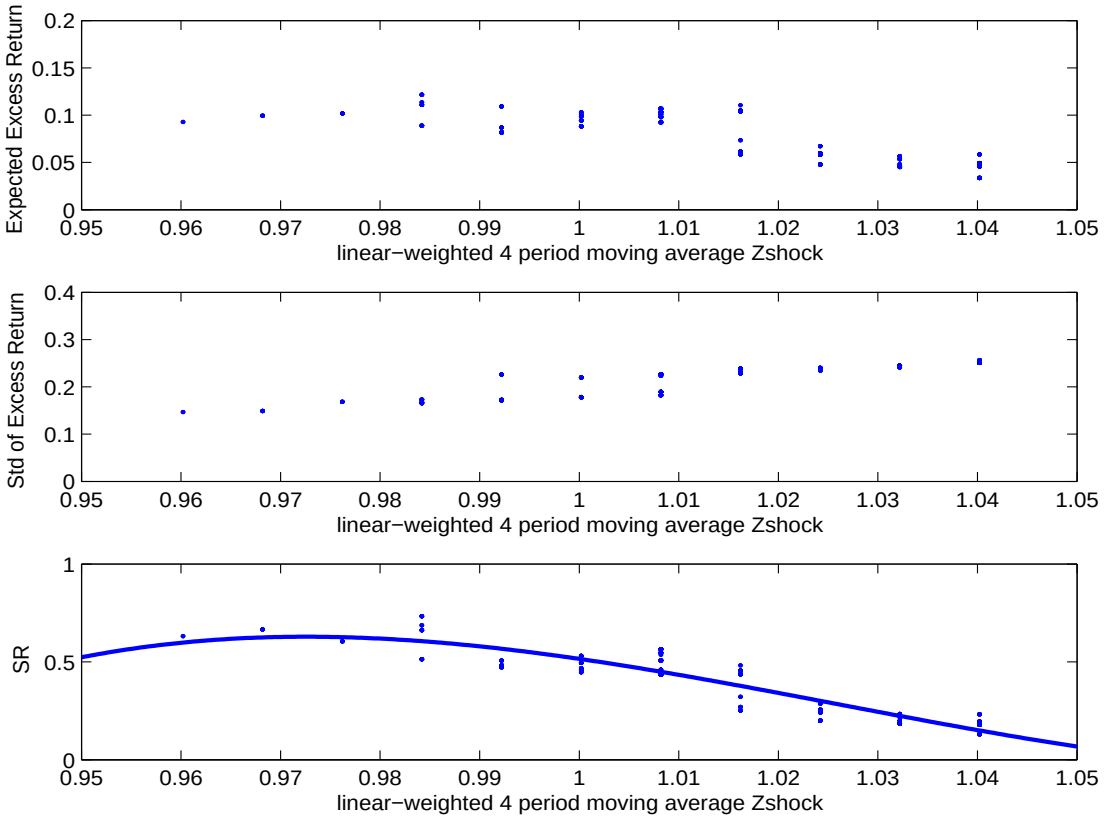


Figure 4: Counter-Cyclical Time Variation: Complete Traders

The dashed line is a 4-period moving average of aggregate consumption growth with linearly decreasing weights. This calibration has 50 % non-participants, 5% complete and 45 % either in *crb* or *irb* traders. The target equity share is 33 %. The *irb* traders re-balance every three periods in a staggered fashion (1/3 each year). Storesletten et al. (2007) of idiosyncratic shocks; Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks. Parameters: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 10%. The results are generated by simulating an economy with $3,000$ agents and $10,000$ periods.

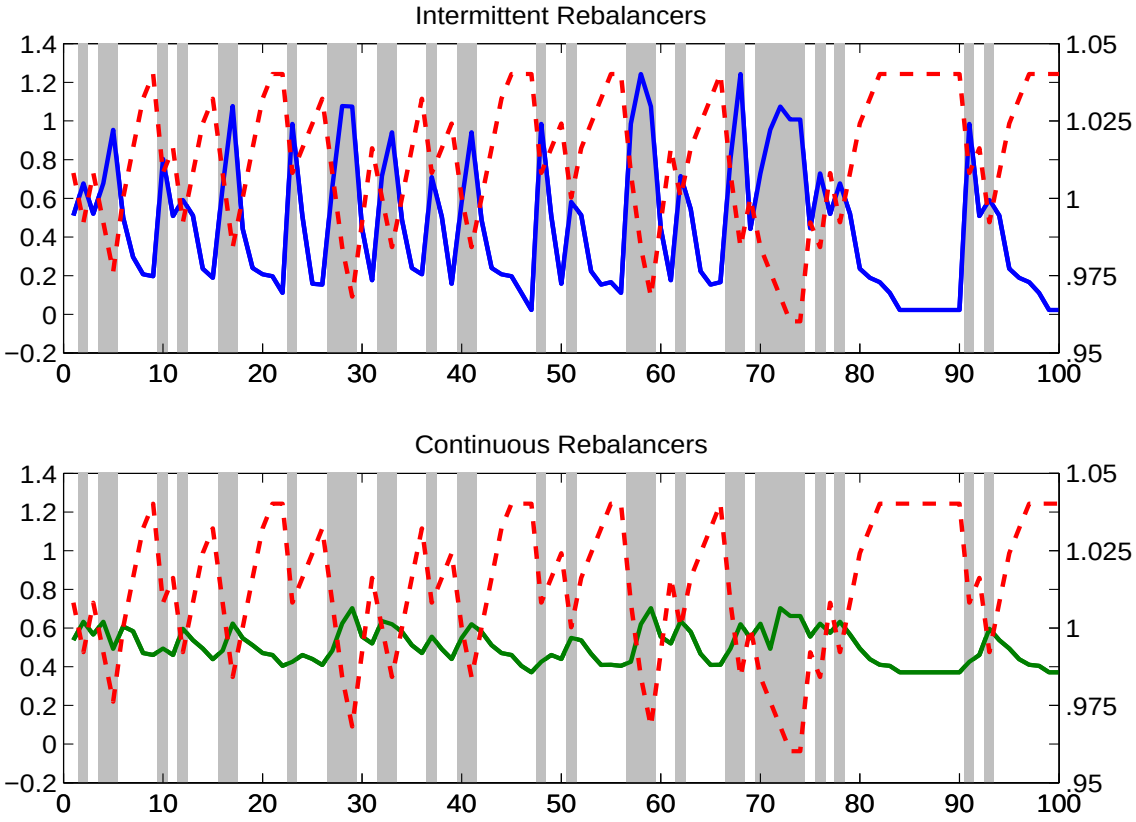


Figure 5: Benchmark case with Complete Active Traders and *irb* Passive Traders.

Scatter plot of the 100 data points in figure 4. On the x-axis is a 4-period moving average of aggregate consumption growth with linearly decreasing weights. On the y-axis is the conditional expected excess return on equity (top panel), the conditional standard deviation (middle panel), the conditional SR (bottom panel). This calibration has 50 % non-participants, 5% complete and 45 % *irb* traders. The target equity share is 33 %. The *irb* traders re-balance every three periods in a staggered fashion (1/3 each year). Storesletten et al. (2007) of idiosyncratic shocks; Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks. Parameters: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 10%. The results are generated by simulating an economy with $3,000$ agents and $10,000$ periods.

