Mechanism and Priority Design for Distributional Objectives: An Application to Improve the Distribution of Teachers in Schools*

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Abstract

Centralized (re)assignment of workers to jobs is increasingly common both in the public sector (for teachers, doctors, police officers, judicial clerks, administrators,...) and in the private sector (for company-wide job rotations). Many of these markets suffer from distributional concerns (e.g., experienced teachers are not equally distributed across schools). This raises a new challenge: How to leverage key features of centralized (re)assignment systems—such as mechanism and priorities—to reach distributional objectives? We propose a model and two new (re)assignment mechanisms that improve both individual welfare and distributional welfare measures over an initial allocation. While both mechanisms are strategy-proof, one achieves two-sided Pareto efficiency (and in particular worker optimality) and the other achieves a novel stability property. We then quantify the performance of our mechanisms in a real-life application: teacher reassignment where the unequal distribution of experienced teachers in schools is a widespread concern around the world. Public schools in disadvantaged districts often have fewer experienced teachers than those in more privileged districts. After estimating teacher preferences using French data, we show that our efficient mechanism successfully reduces the teacher experience gap compared to other mechanisms and to the current French mechanism.

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1 Introduction

The centralized (re)assignment of workers, which involves first time assignment of new employees and reassignment of existing ones, to jobs, tasks, or managers is increasingly common in both the public and private sectors. In many countries, doctors are centrally assigned to hospitals (Agarwal, 2015), police officers to precincts (Sidibe et al., 2021), teachers to public schools (Combe et al., 2020, Bobba et al., 2021, Bates et al., 2021), and more generally civil servants to jobs (Thakur, 2020). Centralized (re)assignment also exists in the private sector. Within large corporations, rotation procedures are commonly used to (re)assign workers to jobs and duties (Cheraskin and Campion, 1996).[1]

From a market design perspective, these practices induce rich two-sided matching environments that differ in one (or more) ways from other centralized markets studied in the past, such as entry-level labor markets (Roth, 1984a, Roth and Peranson, 1999), housing assignment (Abdulkadiroğlu and Sönmez, 1999, Guillen and Kesten, 2008), refugee resettlement (Jones and Teytelboym, 2016, 2017, Andersson, 2019, Delacrétaz et al., 2020), or school choice (Balinski and Sönmez, 1999, Abdulkadiroğlu and Sönmez, 2003, Abdulkadiroğlu et al., 2005b,a). They differ, first, because these two-sided labor markets are characterized by the presence of *new workers* who need to be allocated their first job along with *existing workers* who hold a position and might therefore free up their initial position in the reassignment process. Second, existing workers who hold a position are usually allowed to keep their job if they do not obtain a new one, which might be at odds with employer preferences. Finally, employers have preferences over the workers who apply for a position. Such preferences may be commonly known or dictated by a central authority. Mechanism designers need to carefully consider these specificities.

These labor markets are also interesting from a policy and design perspective because many of them suffer from distributional problems. Rural hospitals have difficulties recruiting doctors (Agarwal, 2015). Police officers tend to shy away from urban city centers that are prone to violence (Sidibe et al., 2021). Public administrators prefer and are assigned jobs close to their home states, which is an obstacle against national integration objectives (Thakur, 2020). And good teachers are almost never equally distributed between schools (Hanushek et al., 2004, Jackson, 2009). Some countries and cities try to solve this unequal distribution by making disadvantaged jobs or locations more attractive through higher salaries or better working conditions (Bobba et al., 2021, Biasi, 2021, Falch, 2010).

The scope to solve workers unequal distribution through higher salaries is often limited by two fundamental constraints. In the public sector, teachers, doctors, police officers, and judges are generally public servants, which implies that their salary is regulated by a rigid pay schedule (usually based on experience) that prevents policy makers from using it as a compensating factor. In addition, in professions that attract workers based on intrinsic motivation, wage elasticities are often low, which makes wage policies ineffective or very costly (Bobba et al., 2021, Bates et al.,

[1] There are other examples of reassignments such as the reallocation of students to schools in inter-district school choice in the US (Hafalir et al., 2019) as well as in intra-district school choice.

2021). In such contexts, countries that use centralized assignment mechanisms benefit from an additional tool to mitigate the unequal distribution: the mechanism itself. This motivates the research question of this paper: In the rich two-sided environment we introduced, how can we design (re)assignment mechanisms that fulfill distributional objectives? This paper introduces a model and two mechanisms, and then empirically assesses how much the new mechanisms can improve the distribution of agents in a market.

We use the (re)assignment of public school teachers to regions in France as our empirical application. This market is particularly relevant because it suffers from large and persistent distributional problems. Around the world, good teachers tend to work in schools that serve more affluent students, and schools with a higher share of native and high-achieving students (Bobba et al., 2021, Bates et al., 2021, Biasi, 2021, Hanushek et al., 2004, Jackson, 2009, Bonesrønning et al., 2005, Allen et al., 2018). Rigid pay schedules have also limited schools' ability to mitigate their unattractiveness with higher wages, which explains the persistence of unequal distributions in countries as diverse as the US, Peru, Norway, England, and France. Despite a rich emerging literature that investigates how effective wage policies can be at attracting good teachers in disadvantaged schools, our understanding of the role played by mechanisms is still limited.[2,3] Our paper sheds light on this question. Although we present the theory with a focus on teachers, all theoretical results apply more broadly to the numerous applications mentioned earlier.[4]

Fundamental design desiderata. We start by introducing a two-sided matching framework in which we explicitly model each school's preferences to accommodate a distributional objective. This objective could be to better balance inexperienced teacher distribution across schools (or to have more senior police officers in urban cities) for instance. To help define a distribution, we introduce a new concept: Each teacher has a type that captures her observable characteristics such as her experience, education, past performance, etc. In addition, teachers can either be tenured or new. Tenured teachers are initially assigned to positions, which is captured by a *status-quo matching*. New teachers are new graduates. As in the standard matching settings, teachers have preferences over schools.

To incorporate distributional objectives, a novelty of our approach is to consider schools as part of a resource pool that is collectively managed by a central authority. Schools have "preferences" over sets of teachers, and we think of these preferences as reflecting the central authority's objective. This objective might be to assign more experienced teachers to disadvantaged schools where inexperienced teachers are overrepresented and new graduates to schools where more experienced teachers are overrepresented. In that case, in our model, the disadvantaged schools would have a preference ordering over types that ranks teachers by decreasing levels of experience while schools where

[2]Several recent papers have developed equilibrium models of the labor market for teachers, and used these models to look at the effect of compensation policies on the distribution of teacher quality (Biasi et al., 2021, Bobba et al., 2021, Bates et al., 2021, Tincani, 2021). We discuss at the end of the paper how our paper complements this fast-growing literature.

[3]Countries that use a centralized process to assign teachers to schools include Germany, Czech Republic (Cechlárová et al., 2015), Italy (Barbieri et al., 2011), Turkey (Dur and Kesten, 2019), Mexico (Pereyra, 2013), Peru, Uruguay (Vegas et al., 2006), and Portugal.

[4]We discuss some of these markets in more detail in Appendix B.

experienced teachers are overrepresented should rank teachers in the reverse order. Beyond this specific example, each school may have a customized preference ordering over teachers' types. We assume that a school gets better-off if the distribution of types first-order stochastically dominates the status quo. In other words, a school is better-off if, for each teacher's type, the fraction of teachers with this type or a more-preferred type increases after the match. We then say that the distribution of teachers improves over the status quo. Starting from an unequal distribution of teachers, we believe that a good design of schools' preferences together with status-quo improvement allows to achieve a more even distribution. One of the main goals of our empirical analysis, as we will discuss more below, is to provide support for this claim.

We impose that our mechanisms improve upon the status quo not only for teachers, but also for schools, i.e., to be *status-quo improving*. This is an important criterion that ensures that a better welfare of teachers is not achieved at the cost of a poorer distribution of teachers, or vice versa. In addition to status-quo improvement, we require strategy-proofness for teachers (as schools' preferences are customized by the designer and, hence, are known). We also consider two additional criteria: efficiency—where welfare entities are both teachers and schools—and fairness. The motivation for having teachers as welfare relevant entities is straightforward. The focus on schools' as welfare entities in our environment where schools' preferences are customized by the designer is similar to the motivation behind status-quo improvement: if one could Pareto-improve the assignment for schools, the distribution of teachers could be further improved. The motivation for a fairness-based axiom, such as stability, comes from the importance of this property in current teacher assignment, e.g., in France. However, the standard Gale and Shapley (1962) stability is not suitable for our desiderata: there may not be a status-quo improving and Gale-Shapley stable matching. We therefore introduce a novel stability notion that is neither weaker nor stronger than the standard Gale and Shapley (1962) stability notion.

Our efficiency and stability criteria are in conflict with each other (Example 8 in Appendix C).[5] We therefore introduce two mechanisms that fulfill the status-quo improvement property: one satisfies our novel efficiency refinement while the other one satisfies our novel stability concept.

The status-quo improving efficient mechanism. Together with two-sided efficiency and status-quo improvement, we focus on a stronger concept that we refer to as *status-quo improving teacher optimality* (or SI teacher optimality) (Proposition 1). These are the outcomes that are Pareto undominated for teachers among status-quo improving matchings. One rationale for this additional requirement is as follows: We show that no two-sided Pareto efficient and status-quo improving mechanism is strategy-proof unless it is also SI teacher optimal (Proposition 6 in Appendix A).

The SI teacher optimal mechanism we introduce, the *status-quo improving cycles and chains* (hereafter, SI-CC), is related to top-trading cycles (TTC) mechanisms (in particular, inspired by Gale's TTC in Shapley and Scarf, 1974, YRMH-IGYT of Abdulkadiroğlu and Sönmez, 1999, and TTCC of Roth et al., 2004) but has one key difference: While TTC-type mechanisms ensure that

[5]Even in the most basic Gale and Shapley (1962) setup, teacher optimality in efficiency and Gale-Shapley stability are in general in conflict (see Roth, 1982, Balinski and Sönmez, 1999).

only teachers necessarily get better off as we execute exchanges, under SI-CC both teachers and schools get better off with respect to the status quo. The SI-CC outcome is determined through an algorithm, which runs iteratively on directed graphs and assigns a group of teachers new positions in each round like TTC. When a cycle or an appropriately defined chain of a directed graph is encountered, we assign each teacher in it the school she is pointing.

Compared to TTC, we introduce two main innovations in the mechanism by endogenously defining pointing rules. First, we define the *schools' pointing rule* which determines the order in which the school would like to send out its status-quo employees. By pointing, the school effectively gives permission to one of its status-quo employees to be assigned to a different school. We define the pointing order in a way that less-preferred-type employees are pointed first (therefore, leaving first) and more-preferred-type employees are pointed later. The second innovation pertains to the *teacher pointing rule* which determines which schools a teacher can point and, therefore, be assigned. We allow a teacher to point to a school if replacing the teacher pointed by that school with her does not make the school worse-off with respect to the status quo or if there is a vacant position at the school that can be filled. The teacher points to the best school she is allowed to point.

We carefully tailor the pointing rules and the order in which cycles and chains are cleared. If we allowed substantial changes to these orderings, the mechanism might not be either two-sided Pareto efficient or strategy-proof or status-quo improving (e.g., see Example 1). Our precise formulation makes SI-CC satisfy these three properties (Theorem 1).[6,7,8]

The status-quo improving stable mechanism. Next, we turn our attention to fairness, noting that the standard fairness notion—Gale and Shapley (1962) stability—and status-quo improvement may in general be in conflict (e.g., see Compte and Jehiel, 2008, Pereyra, 2013). Indeed, due to the individual rationality constraint for teachers, a tenured teacher has the right to stay at her status-quo assignment if she does not obtain any of the schools she ranks. If this tenured teacher is the least favorite teacher for all schools (including her initial school), blocking pairs may form. To overcome the conflict between stability and individual rationality, the standard approach consists in weakening stability by ignoring blocking pairs in which assigning the teacher to the school of the blocking pair would displace a status-quo employee. This weakened stability notion is currently used in French teacher assignment for instance.

However, we show that when imposing status-quo improvement in contexts where schools may

[6]We earlier mentioned that for strategy-proofness and status-quo improvement, we need to focus on SI teacher optimal ones among two-sided Pareto efficient mechanisms. The key observation for SI teacher optimality is that a school does not need to improve from one round to the next to improve upon status quo, e.g., its welfare can sometimes decrease across rounds; yet, by using a buffer function, we make sure it is always weakly better off with respect to the status quo. Therefore, teachers can move more freely with less constraints, which in turn increases their welfare and achieves teacher optimality.

[7]The key observation for strategy-proofness that is different from TTC is the following. The assignment opportunity set of a teacher weakly shrinks through rounds as she may no longer be allowed to point at some schools. However, we show that no future assignment opportunity can be salvaged by her even if she points early to a school that will leave her pointing opportunity set in the future (through a preference manipulation).

[8]Status-quo improvement is ensured for schools through the coupling of their pointing rule and teacher pointing rule that uses the aforementioned buffer function, unlike in TTC. Status-quo improvement for teachers is ensured as they are guaranteed to be matched with their status-quo employer at the worst case.

initially have vacant positions, this weakening alone does not resolve the conflict. We introduce a novel notion, *status-quo improving stability* (or SI stability), which implicitly gives new teachers rights over vacant positions of a school. Under a mild overdemand assumption involving new teachers and schools with vacant positions, we prove that an SI stable matching exists and introduce a strategy-proof and status-quo improving stable mechanism, the *status-quo improving deferred acceptance mechanism* (hereafter, SI-DA).[9] Although it may appear counter intuitive to give a new teacher priority for the vacant positions of a school, we show by means of examples that this is necessary to sustain status-quo improvement.

To find the outcome of the SI-DA mechanism, we employ the teacher-proposing deferred acceptance algorithm of Gale and Shapley (1962) using novel auxiliary choice rules for schools. Such a rule is defined by treating each position of the school as having a different priority order over teachers as in Kominers and Sönmez (2016). To this end, we first distribute status-quo employees to individual positions of the school and consider an order of precedence among these positions based on the desirability of the teacher occupying the position. Vacant positions of a school, if there are any, are placed at the very end of this order of precedence. We construct an auxiliary preference ranking for each position such that some teachers are unacceptable as follows: First, for each occupied position, its status-quo occupying teacher is ranked first. Only the teachers who are at least as good as her for the school (based on their type) are deemed acceptable and ordered according to school's ranking over types just below her. Second, for each vacant position, new teachers are ranked first according to their type and remaining teachers are ranked below them according to their type.

When a set of teachers apply to a school in the SI-DA mechanism, its auxiliary choice rule fills positions according to the precedence order of positions: the first position gets the most desirable acceptable applicant in terms of its auxiliary preference ranking, the second position gets the most desirable acceptable applicant among the rest in terms of its auxiliary preference ranking, and so on.[10] We show that these rules possess substitutes and aggregate law of demand properties (Proposition 4), sufficient properties for deferred acceptance to converge to a Gale-Shapley stable outcome with respect to the designed auxiliary choice rules of the schools and be strategy-proof (see Hatfield and Milgrom, 2005). Finally, we prove that SI-DA is SI stable when our overdemand assumption is satisfied (Theorem 2).

Relaxing status-quo improvement. While status-quo improvement for schools ensures a better distribution of teachers, it may conflict with pre-existing, promised priorities of senior teachers over new teachers. For instance, it is a common practice for tenured teachers to accumulate priority points by working in undesirable regions for a while so that later they can move to better regions using these points. Status-quo improvement might prevent some of these tenured teachers from moving away if there are no more-experienced teachers to replace them. One way to mitigate

[9]The overdemand assumption and giving higher priority to new teachers for vacant positions restrict tenured teachers to flee away from their status-quo assignments without being replaced. In the absence of such a restriction some schools might be worse off compared to their status-quo assignment.

[10]We also show that a different sequencing of the order of precedence over positions can make teachers worse off (Proposition 5).

this conflict is to use improvement with respect to a lower-bound distribution instead of strict status-quo improvement. This enables more movement of tenured teachers as the tension between status-quo improvement and pre-existing priorities are resolved. We generalize our mechanisms to accommodate these practical concerns (Section 5). We call the mechanisms weak-SI-CC (wSI-CC) and weak-SI-DA (wSI-DA).

Empirical Application: Teacher unequal distribution in France. In the second part of the paper, we quantify the gains that our mechanisms bring by using data on the annual assignment of teacher to regions in France. Like many other countries, France uses a centralized process to assign teachers to regions and then to schools. This labor market is particularly appropriate to study our question because it suffers from severe imbalance in the distribution of teachers (see right map in Figure 1). About 50% of the tenured teachers who ask to change region come from two regions (out of 25) in the suburbs of Paris—called Créteil and Versailles—that are particularly disadvantaged (see left map in Figure 1) and therefore unattractive. As a result of this imbalance in exiting request, every year, a majority of the new teachers are assigned one of these two regions to compensate for the large exit flows. This structural imbalance is a serious concern for policy makers. It is frequently raised as a reason for the lack of attractiveness of the teaching profession in France and it is seen as one of the structural determinants of the large achievement inequalities that France suffers from.[11] Reducing the unequal distribution of teachers across regions became one of the objectives of the French policy makers, who see this as a way to both reduce achievement inequalities and improve the attractiveness of the teaching profession in the longer-run.

Teacher preference estimation. We start the empirical analysis by structurally estimating teachers' preferences over the French regions. A number of papers show that assuming that teachers truthfully report their preferences is a strong assumption, even when the mechanism is strategy-proof (as in France).[12] To avoid the potential bias generated by teachers untruthful reports, we estimate teachers' preferences under a weaker "stability assumption" developed by Fack et al. (2019) and applied to the teacher labor market by Combe et al. (2020).[13] We estimate the preferences separately for 5,833 teachers who have a status-quo assignment—referred to as *tenured teachers*—and 4,627 teachers who do not have a status-quo assignment—referred to as *new teachers*. The estimations reveal interesting differences in the preferences of these two groups of teachers. While tenured teachers strongly dislike the Créteil and Versailles regions, these regions are more attractive for new teachers, who might see a first position in a disadvantaged school as a stepping stone for better positions in the future. This difference in preferences surely contributes to the unequal distribution of teachers denounced by policy makers. The counterfactual analysis shows that, above and beyond these preferences, the mechanism used also shapes the distribution of teachers in important ways.

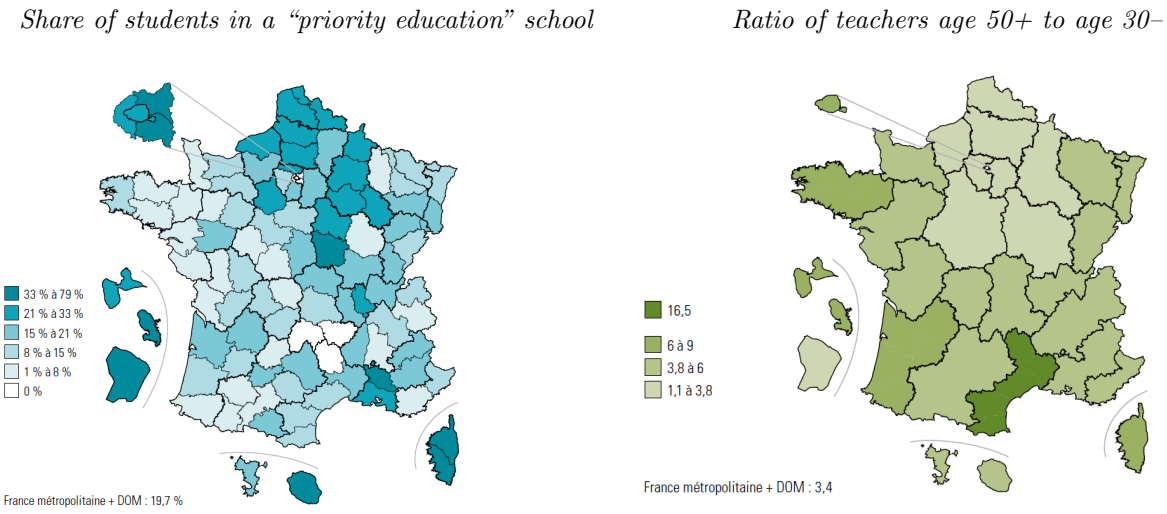
Counterfactuals. We use the estimated preferences, along with data on regions' priorities and

[11] The PISA results show that, in OECD countries, a more socio-economically advantaged student scores 39 points higher in Math than a less-advantaged student, which is equivalent to one year of schooling. There is a large variation between countries in how much a student social background predicts her achievement, and France is one of the worst countries on this inequality indicator, ranking in fourth position (starting from the bottom).

[12] The French Ministry of Education uses a modified version of the deferred acceptance mechanism.

[13] Related literature is discussed in depth in Section 7.

Figure 1: Share of Disadvantaged Students and Experienced to Inexperienced Teacher Ratio in France



Notes: The left map plots the share of students enrolled in a "priority education" school, a label given to the most disadvantaged schools in France. The right map plots the ratio of the number of teachers older than 50 to the number of teachers younger than 30. Column (3) of Table A.2 provides the underlying statistics. The ratio is equal to 1.1 and 1.6 in Créteil and Versailles, respectively. In contrast, the most attractive region, Rennes, had almost 7.4 times more teachers older than 50 than teachers younger than 30.

vacant positions, to run the two mechanisms we propose: SI-CC and SI-DA. We define a teacher type as her experience and assume that regions staffed with relatively young teachers—referred to as "young regions"—have a preference ordering over types that ranks teachers by decreasing levels of experience, i.e., the most experienced teachers would always be preferred to the least experienced teachers. For regions staffed with relatively old teachers—referred to as "old regions"— we assume on the contrary that they rank teachers by increasing levels of experience.

Recall that our motivation for designing preferences as above is to ensure that, under the status-quo improvement property, the distribution of teachers gets more even. SI-CC and SI-DA are both status-quo improving. Thus, verifying that these mechanisms produce a more equal distribution of teachers across regions is one important goal for us. This may also (empirically) justify our novel approach in which schools' preferences are part of the designers' instruments. This is the first outcome we consider. On the other hand, imposing status-quo improvement may have a cost in terms of teacher welfare for some teachers, which motivates looking at the mobility of tenured teachers and the rank distribution of the regions teachers obtain as additional outcomes. To measure the effect of imposing status-quo improvement on both the distribution of teacher experience as well as on teacher welfare, we run two benchmark mechanisms, TTC* and DA*, which correspond to SI-CC and SI-DA when we do not impose status-quo improvement for schools (we only keep the status-quo improvement for teachers). Our results vastly differ when considering SI-CC or SI-DA in comparison with their benchmarks.

Empirical performance of SI-CC versus its benchmark. When focusing on SI-CC, we

observe that imposing status-quo improvement makes the distribution of teacher types more equitable and, relatedly, reduces the experience gap between young and old regions with respect to its benchmark. We see this as one rationale for our new approach where schools' preferences can be used as instruments to reduce the inequalities in the teachers' distribution in a context where SI-CC is the mechanism at use. To illustrate the magnitudes, SI-CC only assigns 1,344 teachers with one or two years of experience to the three youngest regions, while TTC* assigns 1,844 of them to these three regions. We find a similar pattern for old regions, i.e., SI-CC assigns less experienced teachers to these regions compared to its benchmark TTC*.

We then investigate whether achieving a better distribution is done at the cost of a lower welfare for teachers, as measured by their mobility and the rank of the region they obtain. In line with the existence of a distribution-efficiency trade-off, fewer tenured teachers manage to move under SI-CC than under the benchmark TTC*, but the difference is somewhat limited (1,444 versus 2,470 teachers). The distribution of ranks that tenured teachers obtain under the benchmark also cumulatively dominates the one under SI-CC. Interestingly, the opposite is true for new teachers who obtain better ranks under SI-CC: It assigns less inexperienced teachers to the younger regions than its benchmark which does not require status-quo improvement. To conclude, our results show that SI-CC effectively improves teacher distribution. We empirically quantify the expected trade-off that exists between the distribution of teacher experience and teachers' mobility.

Empirical performance of SI-DA versus its benchmark. The picture is radically different for SI-DA. One of the most interesting insight of our empirical exercise is that imposing status-quo improvement to mechanisms designed to sustain fairness can backfire. We first show that imposing status-quo improvement has a tremendous mobility cost for SI-DA: no tenured teachers move from their initial position under SI-DA, compared to 894 under its benchmark DA*. In addition, in the three youngest regions of France, SI-DA produces a distribution of teacher experience which does not dominate the distribution of DA*. In the three oldest regions, our results are even more striking: DA* produces a distribution of teacher experience which dominates the distribution under SI-DA. Put differently, under SI-DA, status-quo improvement fails to achieve its goal: the distribution of teachers does not improve and the experience gap between young and old regions does not go down as targeted. To understand the phenomenon, recall our previous observation that imposing both status-quo improvement and stability reduces mobility of tenured teachers almost to zero. In addition, without status-quo improvement, we show that DA* only mildly violates status-quo improvement while significantly increasing mobility. DA*'s additional mobility gains yield better distribution of teachers in many regions which helps in reducing inequalities between regions.

Relaxing status-quo improvement: wSI-CC versus the current French mechanism. We finally compare our mechanisms' performance with that of the current French mechanism, which is similar to DA* with the exception that the preferences of regions are constructed using the Ministry's priority points (primarily based on teacher experience) rather than the FOSD preferences we introduced for schools.

Our empirical analysis identifies an important trade-off between teachers mobility in disadvantaged regions and the status-quo improvement requirement. If, for a policy maker, the trade-off

goes in favor of teachers mobility, that is, one is willing to allow tenured teachers to leave disadvantaged regions at the expense of status-quo improvement (for schools), our mechanisms can easily be modified to accommodate a more permissive requirement—as discussed at the end of the theory contributions above. Hence, we also implement a relaxation to status-quo improvement in which experienced teachers can leave the three youngest regions without being replaced. Apart from this modification, all other regions have their status-quo improvement constraints intact. Our results show that the wSI-CC mechanism leads to 5,554 teachers being assigned to a region different from their status-quo assignment versus 5,864 with the current French mechanism. While this is a small difference in favor of the current French mechanism, the average rank of the school teachers are assigned in their preferences is smaller (7.3) under wSI-CC than under the Current French mechanism (8.3), thus on average, there is a slight teacher welfare gain under wSI-CC on this dimension. Thus, taking different measures into account, the two mechanisms seem comparable in terms of teachers' welfare. On the other hand, wSI-CC yields a far more desirable distribution of teacher experience across regions than the current French mechanism. Younger regions get more experienced teachers and older regions get more inexperienced teachers. This distribution is very close to that of SI-CC, which is only slightly better for the younger regions. As a result, this version wSI-CC provides a convincing alternative to the current French mechanism that would improve the teacher experience distribution without hurting tenured teacher mobility.

2 Model

Let T be a finite **set of teachers**. Each teacher t has a **type**. The type of a teacher captures her observable characteristics that matter for the schools, such as experience, education, past performance, etc, or only a subset of these.[14] Let $\Theta = (\theta_1, \theta_2, \ldots, \theta_n)$ be the finite type space. Let $\tau : T \to \Theta$ be the **type function** and $\tau(t)$ be the type of teacher t. For any $\hat{T} \subseteq T$, we denote type θ teachers in $\hat{T}$ with $\hat{T}^\theta$, i.e.,

$$\hat{T}^\theta \equiv \{t \in \hat{T} \ : \ \tau(t) = \theta\}.$$

Let S be a finite **set of schools**. Each school s has a **capacity** of q_s. Let $q = (q_s)_{s \in S}$. Each teacher t has a **strict preference relation**, which is a linear order and denoted by P_t, over the schools and **being unassigned option** denoted by $\emptyset$. Let $P = (P_t)_{t \in T}$. We denote the at least as good as relation related with P_t by R_t for all $t \in T$: $s \ R_t \ s'$ if and only if $s = s'$ or $s \ P_t \ s'$.

A **matching** $\mu : T \to S \cup \{\emptyset\}$ is a function such that $|\mu^{-1}(s)| \leq q_s$. With a slight abuse of notation, we use μ_t and μ_s instead of $\mu(t)$ and $\mu^{-1}(s)$, respectively.[15] We refer to μ_t as the **assignment** of teacher t and μ_s as the **assignment** of school s in matching μ. Also for a subset of teachers $\hat{T}$, we denote the set of their matches in μ, $\mu(\hat{T})$ as $\mu_{\hat{T}}$.

Initially some teachers are already employed by some schools. This is captured by a **status-quo matching** ω. If $\omega_t = s$, then teacher t is currently employed at school s. If $\omega_t = \emptyset$, then teacher t is called a **new teacher**. She is seeking employment for the first time and she is unemployed at

[14] For example, in the French application, the experience level of a teacher can be thought as the type of a teacher.
[15] Thus, μ_s^θ is the set of teachers of type θ that are assigned school s.

the status quo. By definition, $|\omega_s| \leq q_s$ for each school s. We denote the set of new teachers with N, i.e.,

$$N \equiv \{t \in T \ : \ \omega_t = \emptyset\}.$$

The rest of the teachers are refereed to as **status-quo employees**.

We make one assumption on the preferences of teachers: We assume that $\omega_t \ P_t \ \emptyset$ for each $t \in T \setminus N$, i.e., each employed teacher at the status quo matching finds her current school acceptable.

Finally, we define the preferences of schools over subsets of teachers. Unlike teacher preferences, these preferences are typically weak and allow indifferences. Typically $\succsim_s$ denotes the preferences of a school s over subsets of teachers. Let $\sim_s$ and $\succ_s$ be the associated indifference and strict preference relation with $\succsim_s$, respectively, denoting symmetric and asymmetric portions of the preferences.

In reality, are schools agents who have preferences over subsets of teachers or are they government-mandated entities or resources that have mandated priority orders over teachers, who are public servants, as in the French application? In our model, we are agnostic about this as either interpretation works. A school can be either seen as an agent that cares about its teacher quality or it can be seen as part of a resource pool that are collectively managed by a central authority, which has an objective of improving the quality of employed teachers in every school as much as possible with an eye on not making any school worse than its status quo. It is therefore only a semantic exercise to call them as agent or resource and these binary relations as preferences or priority orders of the school dictated by a centralized authority with a clear objective of improving education in mind. In the latter case, we are proposing these binary relations as a new priority design that can be used by a centralized authority to replace an existent one that solely relies on some other criteria, as in the French application.

To this end, each school s has a **type ranking**, which is a linear order and denoted by $\triangleright$, over the types of teachers and an **individual rationality threshold** type denoted by $\theta_\emptyset$: $\theta \ \triangleright_s \ \theta' \ \triangleright_s \ \theta_\emptyset \ \triangleright_s \ \theta''$ means school s ranks type θ teachers over type θ' teachers and finds both types of teachers **acceptable** to hire but it considers type θ'' teachers **unacceptable** to hire. Let $\theta \trianglerighteq_s \theta'$ if either $\theta \ \triangleright_s \ \theta'$ or $\theta = \theta'$. We assume that if $\omega_s^\theta \neq \emptyset$, then $\theta \triangleright_s \theta_\emptyset$, i.e., each school finds the types of its current teachers acceptable.

We make two assumptions on the preferences of schools.

1. We assume that when a school compares two subsets of teachers it uses **first-order stochastic dominance** (**FOSD**)[16] relation based on its type ranking. In particular, school s weakly prefers subset of teachers $\bar{T}$ to $\hat{T}$, i.e., $\bar{T} \succsim_s \hat{T}$, if
   (i) there does not exist an unacceptable teacher in $\bar{T}$, i.e., $\bar{T}^\theta = \emptyset$ for any $\theta \triangleleft_s \theta_\emptyset$, and

[16] Although FOSD relation is, in general, defined to compare statistical distribution functions, with a slight abuse of terminology we use the same name for the analogous binary relation that compares distributions of teacher types. Indeed, in our empirical analysis we use this language, comparing distributions of teachers using FOSD assigned to schools.

(ii) for any $\theta \rhd_s \theta_\emptyset$ we have

$$\sum_{\theta' \trianglerighteq_s \theta} |\bar{T}^{\theta'}| \geq \sum_{\theta' \trianglerighteq_s \theta} |\hat{T}^{\theta'}|.$$

Moreover, the preference is strict if at least one of the inequalities is strict.

If FOSD does not hold between two sets in either direction, then the school preferences do not compare them. Therefore, school preferences are incomplete. Thus, a school only unambiguously prefers groups whenever two groups of teachers can be ranked based on this FOSD comparison.

In the rest of our analysis, we compare outcome matchings with the status-quo matching. FOSD relation will be sufficient and we will achieve unambiguous comparisons for our purposes.

2. We assume that for any subset of teachers T', if $\theta_\emptyset \rhd_s \tau(t)$ for some $t \in T'$, then $\omega_s \succ_s T'$. That is, each school prefers its status-quo assignment to any teacher set with an unacceptable teacher in it.

We refer to the list $\langle T, S, q, \omega, P, \succsim \rangle$ as a **teacher (re)assignment market**. Typically, $T, S, q, \omega, \succsim$ are commonly known in our applications. Only teacher preferences are private information. For the rest of our analysis, we fix $T, S, q, \omega, \succsim$ and denote a market with teacher preferences P.

We are seeking a matching outcome given a market P.

The most basic property of outcome matchings we consider is *status-quo improvement*. Maybe surprisingly, this is sometimes in conflict with many other standard desiderata used in the literature for matching market design. A matching μ is **status-quo improving** if $\mu_t \; R_t \; \omega_t$ for all $t \in T$ and $\mu_s \succsim_s \omega_s$ for all $s \in S$. That is, each agent should be weakly better off in a status-quo improving matching with respect to the status-quo matching.

We inspect rules that select a matching for each market. Formally, a (**direct**) **mechanism** ϕ is a function that chooses an outcome matching for any market P. Let $\phi(P)$, $\phi_t(P)$, and $\phi_s(P)$ denote the matching selected by mechanism ϕ under market P, the assignment of teacher t, and the assignment of school s in that matching, respectively.

A mechanism ϕ is **strategy-proof** if truth telling is a weakly dominant strategy for all teachers, that is, for all markets P, for all teachers t, for all possible alternative preference reports P'_t,

$$\phi_t(P_t, P_{-t}) \; R_t \; \phi_t(P'_t, P_{-t}).$$

As we assume that schools' rankings over the types of teachers, and therefore, their preferences over the teachers are commonly known, schools do not need to report them.

In the next two sections, we provide two different mechanisms to achieve two different desiderata: a refinement of two-sided Pareto efficiency or an appropriate fairness concept for our applications together with status-quo improvement, respectively.[17]

[17] In Example 8 in Appendix C, we show that there does not exist a mechanism satisfying efficiency and stability.

3 Efficiency and Status-quo Improving Cycles and Chains

3.1 Status-quo Improving Teacher Optimality

Consider a market P. A matching μ **Pareto dominates** a matching ν **for teachers** if

$$\mu_t \; R_t \; \nu_t \quad \text{for all } t \in T, \text{ and} \tag{1}$$

$$\mu_{t'} \; P_{t'} \; \nu_{t'} \quad \text{for some } t' \in T. \tag{2}$$

Matching μ **Pareto dominates** a matching ν **for schools** if

$$\mu_s \; \succsim_s \; \nu_s \text{ for all } s \in S, \text{ and} \tag{3}$$

$$\mu_{s'} \; \succ_{s'} \; \nu_{s'} \text{ for some } s' \in S. \tag{4}$$

Finally, matching μ **Pareto dominates** a matching ν if (i) Equations 1 and 3 hold, and (ii) Equation 2 or Equation 4 holds. A matching is **two-sided Pareto efficient** if it is not Pareto dominated by any other matching.

A matching μ is **status-quo improving teacher optimal** (**SI teacher optimal** for short) if it is status-quo improving and not Pareto dominated for teachers by any other status-quo improving matching. While SI teacher optimality seems to care mostly about the welfare of teachers, any SI teacher optimal matching is also two-sided Pareto efficient, since teacher preferences are strict.

Proposition 1 *Any SI teacher optimal matching is two-sided Pareto efficient.*

All proofs are provided in Appendix A.

We will introduce a strategy-proof and SI teacher optimal mechanism. Why do we implement on SI teacher optimality rather than a dual concept such as SI school optimality or different two-sided Pareto efficient outcomes?[18] It has two reasons: First, in most of the applications we have in mind, the side we characterize as schools are quasi-agents rather than full agents unlike the side we characterize as teachers. Their welfare has been the main efficiency measure both in practice and in literature (for example, see Abdulkadiroğlu and Sönmez, 2003, Combe, Tercieux and Terrier, 2020).

Second, if we wanted to implement other status-quo improving two-sided Pareto efficient outcomes that are not SI teacher optimal, we would not be able to find a strategy-proof mechanism (Proposition 6 in Appendix A).

3.2 Pointing Rule Design and Status-quo Improving Cycles and Chains

Next, we will introduce a strategy-proof and SI teacher optimal mechanism. To achieve this goal, we introduce additional tools.

[18] Because of indifference classes in schools' FOSD preferences regarding same-type teachers, this concept defined as a simple dual notion of SI teacher optimality may not be two-sided Pareto efficient and has to be refined even further.

Our mechanism will iteratively construct a sequence of directed graphs in which teachers, schools, and being unassigned option are the nodes. Teachers can only point to schools or the being unassigned option and schools can only point to teachers in their status-quo assignment in each of these graphs. When node x points to node y, then a directed arc from x to y is activated.

Our mechanism relies on *executing* two types of multi-lateral exchanges based on the constructed directed graphs.

A **cycle** is a directed path of distinct teachers $\{t_m\}$ and distinct schools $\{s_m\}$, possibly being unassigned option $\emptyset$,

$$(s_1, t_1, s_2, t_2, \ldots, s_k, t_k)$$

such that $\omega_{t_m} = s_m$ for all m, each node points to the next node in the path, and t_k points back to s_1.

A **chain** is a directed path of distinct teachers $\{t_m\}$ and schools $\{s_m\}$

$$(t_0, s_1, t_1, \ldots, s_{k-1}, t_{k-1}, s_k)$$

such that $\omega_{t_m} = s_m$ for all $m = 1, \ldots k-1$, each node points to the next node in the path.[19] Here, we say the chain starts with t_0 and ends with s_k. In other words, s_k is the head of the chain and t_0 is the tail of the chain.

As certain cycles and chains are encountered in the constructed graph, we will **execute** the exchanges in them by assigning each teacher to the school she is pointing to and remove her.

Our main theoretical innovation relies on designing *pointing rules* that designate which possible directed arcs in a graph will be endogenously activated through the algorithm.

Pointing rule of teachers will be introduced within the definition of the mechanism below as it uses the endogenous working of the mechanism's algorithm. On the other hand, the pointing rule of schools relies on their type rankings and an exogenously given tie breaker.

Formally, a **tie breaker** is a linear order $\vdash$ over teachers.[20] It can be randomly determined or can be the mandated priority orders for a particular application, such as in the French case, or can be exogenously fixed in some other manner.

The tie breaker for the new teachers and tie breaker regarding teachers employed at the status quo are utilized differently in the algorithm. For each school s, using tie breaker $\vdash$ and its type ranking $\rhd_s$, we first construct a **pointing order** $\succcurlyeq_s$ over teachers in ω_s, which is a linear order as well: For any two distinct teachers $t, t' \in \omega_s$,

$$t \succcurlyeq_s t' \iff \tau(t) \lhd_s \tau(t') \quad \text{or} \quad \big[\tau(t) = \tau(t') \text{ and } t \vdash t'\big].$$

Note that a worse-type teacher is prioritized over a better-type teacher, and only when two same-type teachers are compared, we use the tie breaker to prioritize one over the other.

[19] Notice that, we allow a school to appear more than once in a chain.

[20] Technically, each tie breaker induces a new mechanism in our class.

As the mechanism will iteratively assign and remove teachers, the **pointing rule of schools** is "point to the highest remaining priority teacher in its priority order."

Now, we are ready to define our mechanism through an iterative algorithm:

Definition 1 ***Status-quo Improving Cycles and Chains (SI-CC) Mechanism***

We will construct a matching μ dynamically through the following algorithm. Initially, μ is the empty matching, in which no teacher is assigned to any school. In each step, as teachers are assigned in μ, they will be removed from the algorithm; similarly schools whose all positions are filled in μ and also some other schools chosen by the algorithm will be removed.

*For each school s and type θ, let b_s^θ track the **current balance** of type θ teachers at school s in **current matching** μ, which is the matching fixed until the beginning of the current step. The current balance is defined as the difference between the number of type θ teachers assigned to s in μ and the number of type θ teachers in its status-quo assignment assigned to any school in μ: $b_s^\theta \equiv |\mu_s^\theta| - |\{t \in \omega_s : \mu_t \neq \emptyset\}^\theta|$. Thus, we initialize $b_s^\theta = 0$.*

A general step k is defined as follows:

Step k:

- *Each remaining school s points to the highest priority remaining teacher in ω_s under $\succcurlyeq_s$, if not all students in ω_s are already assigned in μ; let t_s^k be the teacher pointed by school s in step k. Otherwise, school s does not point to any teacher.*
- *We define the **pointing rule of teachers** as follows: Any remaining teacher t is allowed to point to a remaining school s if at least one of the following two school **improvement conditions** hold for school s via teacher t:*
  1. *(Improvement for s by teacher trades) if the school points to a teacher t_s^k and*

$$\sum_{\theta' \trianglerighteq_s \theta} b_s^{\theta'} > 0 \text{ for all types } \theta \text{ such that } \tau(t_s^k) \trianglerighteq_s \ \theta \ \triangleright_s \ \tau(t),$$

  or[21]
  2. *(Improvement for s by only incoming teachers) $\tau(t) \triangleright_s \theta_\emptyset$, school s currently has an unfilled position, i.e., $q_s - |\mu_s| > |\{t' \in \omega_s : \mu_{t'} = \emptyset\}|$, and there are remaining new teachers.*

  Let A_t^k be the opportunity set for a remaining teacher t, i.e., the set of schools t can point in this step together with the being unassigned option $\emptyset$.[22]
  Each remaining teacher t points to her most preferred option in A_t^k.
- *Being unassigned option $\emptyset$ points to all teachers pointing to it.*

Due to finiteness, there exists either

(i) a cycle in which all schools in the cycle satisfy improvement Condition 1 or a cycle between a single teacher and the being unassigned option $\emptyset$, or

[21] Condition 1 is trivially satisfied if no such θ exists, i.e., if $\tau(t) \trianglerighteq_s \tau(t_s^k)$.

[22] Note that, $\omega_t \in A_t^k$ for all remaining teachers t who were employed at the status quo.

(ii) a chain.

Then:

- ***If Case (i) holds:*** *Each teacher can be in at most one cycle as she points at most to a single option. We execute exchanges in each cycle encountered in case (i) by assigning the teachers in that cycle to the school she points to, update current matching μ and current balances $\{b_s^\theta\}$ accordingly, remove assigned teachers and filled schools in μ, and go to step $k+1$.*
- ***If Case (i) does not hold:*** *Then case (ii) holds, i.e., there exists a chain. In particular, each remaining teacher initiates a chain. There are two subcases:*
  - ***If there exists a remaining new teacher****: Then we select a chain to be executed as follows:*
    - *Select as the tail of the chain the new teacher with the highest priority under tie breaker $\vdash$ and then include in the chain the school she points to.*[23] *If Improvement Condition 1 does not hold for this school via this teacher, but only Improvement Condition 2 holds, then we end the chain with this school; otherwise, we repeat the following:*
    - *Include to the chain the teacher pointed by the last school included.*[24] *If we include a teacher, we also include next in the chain the school she is pointing to. We repeat this iteratively until the Improvement Condition 1 does not hold for the next school via the included teacher, but only Improvement Condition 2 holds.*[25]

    The last school included is the head of the selected chain.
    We execute the exchanges in the selected chain by assigning each teacher in the chain to the school she points to, update current matching μ and current balances $\{b_s^\theta\}$ accordingly, remove assigned teachers and filled schools, and go to step $k+1$.
  - ***If there does not exist a remaining new teacher****: Then we remove each school s whose all status-quo employees in ω_s were already assigned in μ.*[26] *We continue with step $k+1$.*

The mechanism terminates when all teachers are removed. Its outcome is the final matching μ.

The name of the mechanism suggests that both teachers and schools become better off through the mechanism with respect to the status quo. Indeed, this is the case. We introduced several innovations in the mechanism that exploit different Pareto improvement possibilities for teachers and schools over the status-quo matching.

Pareto improvement of teachers is straightforward in the algorithm. Teachers who are employed at the status quo are eventually assigned to a school at least as good as their status-quo assignment. Moreover, all teachers are assigned the best option they can point in the step they are assigned.

[23] Such a school exists, because if she does not point to a school, then she pointed to being unassigned option $\emptyset$ and was removed previously.

[24] Such a teacher exists by Improvement Condition 1.

[25] This iterative procedure is guaranteed to terminate. Otherwise, we would have a cycle.

[26] There must exit a school which is still in the market and whose status-quo employees have already been assigned. Otherwise, each remaining school would point to one teacher and each remaining teacher would point to a school and so there would exist a cycle, a contradiction.

What is more delicate is the Pareto improvement of schools, i.e., how we make sure that they always weakly improve with respect to their status-quo assignment in every step. This is ensured through the introduction of both teacher and school pointing rules.

A school's pointing order designates in which order the school would like to *send out* its status-quo employees. By pointing, the school effectively gives permission to one of its status-quo employees to be assigned possibly to a different school. Thus, we make sure that this priority order is in reverse order of its preferences: Less preferred-type employees are pointed first and more preferred-type employees are pointed later. This is the first innovation.

On the other hand, the teacher pointing rule designates which teachers can be assigned to a school. Therefore, we only allow teachers who can improve the school's welfare with respect to its status-quo assignment after the currently pointed employee of the school is sent out.

The two school improvement conditions make sure of this.

Condition 1 has two cases: If the type of the possibly incoming teacher is at least as good as the type of the possibly outgoing teacher, the school has no danger of becoming worse off in this trade. The second case on the other hand is more delicate: As trades that strictly improve a school's welfare occur over steps, schools acquire new teachers who are actually of better types than the types of outgoing status-quo employees. Therefore, they may build up a *buffer*. If such a buffer exists, a worse-type teacher than its currently outgoing employee can still be assigned to the school, although this trade makes the school worse off with respect to the previous step. However, the school is still weakly better off with respect to the status quo thanks to the buffer. Only the buffer gets thinner. The existence of the buffer is tracked by checking whether the sums of the relevant type balances, b_s^θ's, are positive through Condition 1. The use of this buffer ensures teacher optimality.

While the first condition is about a trade the school will make by exchanging an outgoing teacher with an incoming teacher, Condition 2 is only relevant as long as new teachers remain in the algorithm. When Condition 2 holds for a school via some teacher, but not Condition 1, the school will not send out an employee as it has extra capacity: it will only hire one additional acceptable teacher.

We illustrate how the algorithm of the SI-CC mechanism works in Example 4 in Appendix C. We are ready to state our main result in this section.

Theorem 1 *The SI-CC mechanism is SI teacher optimal and strategy-proof.*

SI teacher optimality of the mechanism is delicate to show. Note that SI teacher optimality implies that the outcome matching is Pareto undominated for teachers among all status-quo improving matchings. However, the pointing rule of teachers has restrictions imposed by the school improvement conditions. That is, a teacher cannot arbitrarily point to the best school she likes. We show that the restrictions imposed by these conditions are the necessary and sufficient conditions for keeping status-quo improvement for schools without affecting the outcome being Pareto undominated for teachers. Therefore, implementing any further Pareto improvement for teachers would make the schools worse off with respect to the status quo. Moreover, imposing further restrictions

for teacher pointing would prevent SI teacher optimality.

Strategy-proofness of the mechanism relies on several observations: First, once a teacher is pointed by a school, she will continue to be pointed until she is assigned. We show that the opportunity set for each teacher t, A_k^t, weakly shrinks across steps k. Although Improvement Conditions 1 or 2 may stop holding for a school via a teacher t across steps, we show that teacher t cannot affect which schools leave and stay in A_k^t before she is assigned by submitting different preferences.

An immediate corollary to the theorem is that the SI-CC mechanism is also two-sided Pareto efficient by Proposition 1.

Under the pointing rule of schools, in each step of SI-CC, each school points to one of its employee who has the lowest ranked type. One can think whether Theorem 1 holds when we consider alternative school pointing rules under SI-CC mechanism. Example 1 shows that SI-CC can be manipulated by a teacher and it is not SI teacher optimal under an alternative pointing rule.

Example 1 *Let $S = \{s, s', s''\}$, $T = \{t_1, t_2, t_3, t_4\}$, the status-quo matching be*

$$\omega_s = \{t_1, t_2\}, \omega_{s'} = \{t_3\}, \omega_{s''} = \{t_4\},$$

$q_s = 2$, $q_{s'} = q_{s''} = 1$ and $\tau(t_1) \rhd_s \tau(t_2) = \tau(t_3) = \tau(t_4)$. The preferences of the teachers are

$$sP_{t_1}s'P_{t_1}s''P_{t_1}\emptyset, \qquad s'P_{t_2}sP_{t_2}s''P_{t_2}\emptyset,$$

$$sP_{t_3}s'P_{t_3}s''P_{t_3}\emptyset, \qquad s''P_{t_4}sP_{t_4}s'P_{t_4}\emptyset.$$

If in the first step of SI-CC school s points to t_1, the best school t_3 can point is s'. Therefore, she will be assigned to s'. In particular, under true preferences SI-CC assigns all employees to their status-quo schools. This outcome is not SI teacher optimal because it is Pareto dominated by another status-quo improving matching ν for teachers where $\nu_{t_1} = \nu_{t_3} = s$, $\nu_{t_2} = s'$ and $\nu_{t_4} = s''$. Moreover, if t_3 swaps the rankings of s' and s'', then SI-CC selects ν, i.e., t_3 manipulates SI-CC when s points t_1 in the first step.

We would like to emphasize one possible generalization of SI-CC through school pointing rule. We can easily use the first school improvement condition given in the definition of SI-CC to dynamically update the school pointing rule such that monotonicity of opportunity set for teachers is preserved.

4 Status-quo Improving Stability and Deferred Acceptance

4.1 Status-quo Improving Stability

Although two-sided Pareto efficiency is a very appealing property of matchings, many real-life applications use fairness or stability notions, which often conflict with SI teacher optimality and

in general with two-sided Pareto efficiency under incentive compatibility constraints. For example, in the French teacher (re)assignment application, the mechanism currently used is not two-sided Pareto efficient, while it satisfies a stability condition that is not necessarily status-quo improving.

To this end, we also introduce a stability concept that is consistent with status-quo improvement under a mild assumption about the number of new teachers in a market. Our notion has different requirements than Gale-Shapley stability (Gale and Shapley, 1962), which is extensively used in the literature, because in our setting we have a non-empty status-quo matching while most of the literature focuses on an empty matching as the status quo.

Consider a market P.

To introduce stability, we first start with blocking by an agent and a pair. A matching μ is **blocked by a teacher** t if $\emptyset \; P_t \; \mu_t$. A matching μ is **blocked by a school** s if there exists $t' \in \mu_s$ with $\theta_\emptyset \rhd_s \tau(t')$. Observe that a status-quo improving matching is not blocked by any agent, while a matching that is not blocked by any agent may not be status-quo improving.

Given a teacher t and school s, a matching μ is **blocked by pair** (t, s) **through** $t' \in \mu_s$ if (i) $s \; P_t \; \mu_t$ and (ii) $\tau(t) \rhd_s \tau(t')$. Similarly, a matching μ is **blocked by pair** (t, s) **through a vacant position** if (i) $s \; P_t \; \mu_t$, (ii) $\tau(t) \rhd_s \tau(t')$ and (iii) $|\mu_s| < q_s$.

A matching μ is **Gale-Shapley stable** if there is no blocking agent and no blocking pair. This classical concept potentially conflicts with our most basic property, status-quo improvement:

Proposition 2 *A Gale-Shapley stable matching always exists, however, it may not be status-quo improving. Thus, a Gale-Shapley stable and status-quo improving matching may not exist.*

One may think that the cause of incompability of status-quo improvement with Gale-Shapley stability is not giving employment rights to teachers at their status-quo schools. Indeed the current system in France uses a strategy-proof mechanism that satisfies the following stability concept implicitly.[27]

A matching μ is **teacher-status-quo-improving (teacher-SI) stable** if there is no blocking agent and no blocking pair through a vacant position, and if there is a blocking pair (t, s) through $t' \in \mu_s$, then $t' \in \omega_s$. This concept ignores blocking pairs as long as assigning the teacher to the school in the blocking pair would displace a status-quo employee of the school. This concept still does not resolve the main problem.

Proposition 3 *Even when there are no vacant positions at schools at status quo and there are no new teachers, the current French mechanism is teacher-SI stable but not status-quo improving, while the status-quo matching is both teacher-SI stable and status-quo improving. Moreover, if there are vacant positions at some schools at status quo, then a teacher-SI stable and status-quo improving matching may not exist.*

[27] To make the current French setup more consistent with ours one may think that each teacher has a different type and the type ranking of each school is given by the government-dictated strict priority order used in France.

We should strengthen no blocking by an agent to status-quo improvement. Given the above impossibility result, however, this remedy alone does not resolve our non-existence problem when there are vacant positions at the status quo. Instead, we introduce the following concept which implicitly gives new teachers rights over status-quo vacant positions of a school.

A matching μ is **status-quo improving stable** (**SI stable** for short) if

1. it is status-quo improving, i.e., $\mu_s \succsim_s \omega_s$ and $\mu_t \; R_t \; \omega_t$ for all $s \in S$ and $t \in T$;
2. there is no blocking pair (t, s) through a vacant position; and
3. there is no blocking pair (t, s) through t' such that either $t, t' \in N$ or $t, t' \in T \setminus (N \cup \omega_s)$.

SI stability requires status-quo improvement, which implies elimination of individual blocking, and elimination of blocking pairs through vacant positions. Moreover, it requires elimination of any blocking pair (t, s) through t' such that t and t' are either new teachers or are currently employed by another school s'.

As a result, this concept is neither weaker (because of the more stringent individual blocking condition) nor stronger (because of the less stringent pairwise blocking conditions) than both Gale-Shapley and teacher-SI stability concepts.

Although it may appear counter intuitive to allow certain blocking pairs, it turns out that this is necessary to sustain status-quo improvement. The solution provided in the following subsection eliminates further blocking pairs such as the ones including new teachers through an existing teacher. In Appendix C, by using examples we show that allowing the other blocking pairs not captured by Condition 3 is needed to guarantee existence of an SI stable matching.

4.2 Auxiliary Choice Rules and Status-quo Improving Deferred Acceptance

In this subsection, we introduce a strategy-proof mechanism that is SI stable under a mild assumption we will introduce below. Our main contribution here is to design this mechanism by introducing auxiliary choice rules for schools that will achieve SI stability and strategy-proofness when they are used in conjunction with the teacher-proposing deferred acceptance (DA) algorithm of Gale and Shapley (1962) adopted for complex matching terms by Roth and Sotomayor (1990) (which was itself adopted from more complex versions of such processes in Kelso and Crawford, 1982, Roth, 1984b, Blair, 1988).

Given a school s, an **auxiliary choice rule** is a function $C_s : 2^T \to 2^T$ such that for any $\hat{T} \subseteq T$, (i) $C_s(\hat{T}) \subseteq \hat{T}$ and $|C_s(\hat{T})| \leq q_s$.

Using the auxiliary choice rules we will design below we will employ the well-known teacher-proposing DA algorithm. We consider the sequential version of this algorithm also known as the cumulative offer process (Hatfield and Milgrom, 2005) for more complex contractual matching terms:

Definition 2 ***Teacher-Proposing Deferred Acceptance Algorithm (DA):***

Step 1***:*** *Some teacher t' proposes to her favorite acceptable school, denoted by s', if such a school exists. In this case, define $B^2_{s'} \equiv \{t'\}$ and $B^2_s \equiv \emptyset$ for each school $s \neq s'$. Otherwise, define*

$B_s^2 \equiv \emptyset$ *for each school* s.

Each school s *holds teachers in* $C_s(B_s^2)$ *and rejects all other teachers in* B_s^2.

In general,

Step $k > 1$***:*** *Some teacher* t'' *who is not currently held by any school proposes to her most favorite acceptable school that has not rejected her yet, denoted by* s''*, if such a school exists. In this case, define* $B_{s''}^{k+1} \equiv B_{s''}^k \cup \{t''\}$ *and* $B_s^{k+1} \equiv B_s^k$ *for each* $s \neq s''$*. Otherwise, define* $B_s^{k+1} \equiv B_s^k$ *for each school* s.

Each school s *holds* $C_s(B_s^{k+1})$ *and rejects all teachers in* $B_s^{k+1} \setminus C_s(B_s^{k+1})$.

The algorithm terminates when each teacher is either rejected by all of her acceptable schools or currently held by some school. We assign each school the students it is holding.

Our main contribution in this subsection is the construction of an auxiliary choice rule for each school. Fix a school s. First, we need some additional concepts.

A **tie breaker** is a linear order over teachers $\vdash$ as before. We construct a new linear order over the teachers in ω_s denoted by $\blacktriangleright_s$ as follows:

For any $t, t' \in \omega_s$,

$$t \ \blacktriangleright_s \ t' \iff \tau(t) \ \triangleright_s \ \tau(t') \text{ or } [\tau(t) = \tau(t') \text{ and } t \ \vdash \ t'].$$

Observe that a better-type teacher is prioritized over a worse-type teacher, and when two same-type teachers are compared, then we use the tie breaker to prioritize one over the other.[28]

The auxiliary choice rule will use a lexicographic decision structure within a school by dividing the school into independent slots where each **slot** eventually represents a position at the school. Such a model was previously introduced by Kominers and Sönmez (2016) in one-sided priority-based matching context for more complex contractual matching terms.

We fix a school s in this construction. Let $\mathcal{S}_s = \{s^1, s^2, \ldots, s^{q_s}\}$ be the set of slots at school s. Without loss of generality we label the types in Θ as $\theta_1, \ldots, \theta_{|\Theta|}$ based on the type ranking of the school such that $\theta_k \ \triangleright_s \ \theta_{k+1}$ for all $k \in \{1, \ldots, |\Theta| - 1\}$. We define a ranking for each slot over $T \cup \{\emptyset\}$ where $\emptyset$ denotes keeping the slot unfilled. The **ranking of slot** s^k, $\succ_s^k$, is defined separately for the slots representing the filled positions at the status-quo matching, i.e., for $k \leq |\omega_s|$, and slots representing the vacant positions at the status-quo matching, i.e., for $|\omega_s| < k \leq q_s$:

- For filled slots s^k at the status quo, i.e., all $k \leq |\omega_s|$:
  - the teacher $t \in \omega_s$ who is ranked k'th under $\blacktriangleright_s$ has the highest ranking under $\succ_s^k$,
  - any teacher t' with $\tau(t) \triangleright_s \tau(t')$ is ranked below $\emptyset$ under $\succ_s^k$, and
  - the rest of the ranking under $\succ_s^k$ is determined according to $\triangleright_s$ such that ties between same type teachers are broken according to tie breaker $\vdash$.

[28] Thus, linear order $\blacktriangleright_s$ effectively reverses the ordering of status-quo employees of different types in the pointing order $\gtrdot_s$ used in the SI-CC mechanism, while it respects the school's type ranking.

- For vacant slots s^k at the status quo, i.e., all k such that $|\omega_s| < k \leq q_s$:
  - a teacher t is ranked above $\emptyset$ under $\succ_s^k$ if and only if she is acceptable, i.e., $\tau(t) \rhd_s \theta_\emptyset$,
  - any acceptable new teacher t (i.e., $t \in N$ and $\tau(t) \rhd_s \theta_\emptyset$) is ranked under $\succ_s^k$ above any teacher t' employed at status quo by some school (i.e., $t' \notin N$), and
  - the rest of the ranking under $\succ_s^k$ is determined according to $\rhd_s$ such that ties between same type teachers are broken according to tie breaker $\vdash$.

Thus, the set of acceptable teachers for slot s^k is a superset of the set of acceptable teachers for slot s^{k-1}.

We will make the following mild overdemand assumption in the rest of this section involving new teachers and schools with excess status-quo capacity:

Assumption 1 *There exists a subset of new teachers $N' \subseteq N$ such that (i) there are at least as many new teachers in N' as vacant positions at status quo, i.e., $|N'| \geq \sum_{s \in S}(q_s - |\omega_s|)$, and (ii) each teacher $t \in N'$*

- *considers all schools with excess capacity acceptable, i.e., if $q_s > |\omega_s|$, then $s \, P_i \, \emptyset$, and*
- *is acceptable for all schools with excess capacity, i.e., if $q_s > |\omega_s|$, then $\tau(t) \rhd_s \theta_\emptyset$.*

In the absence of either part of Assumption 1, we can come up with examples such that some schools end up with fewer teachers than what they have under the status-quo matching and status-quo improvement is violated for schools (see Appendix C).

Since the auxiliary choice rule is defined through filling each slot one a time, we need to determine in which order the slots are processed. We process the slots in the natural order[29]

$$s^1, \; s^2, \; \ldots, s^{q_s}.$$

Definition 3 *The **auxiliary choice rule** C_s of school s is defined through an iterative procedure. The chosen set from the set of teachers $\hat{T}$ by school s, denoted by $C_s(\hat{T})$, is determined as follows:*

- ***Step** 1: The most preferred acceptable teacher under $\succ_s^1$ in $\hat{T}_1 = \hat{T}$ is assigned to slot s^1 and she is removed. If there is no such teacher, then s^1 remains vacant. Denote the remaining teachers with $\hat{T}_2$.*
  In general,
- ***Step** $k \geq 2$: The most preferred acceptable teacher under $\succ_s^k$ in $\hat{T}_k$ is assigned to slot s^k and she is removed. If there is no such teacher, then s^k remains vacant. Denote the remaining teachers with $\hat{T}_{k+1}$.*

The process terminates when all slots are processed, i.e., step q_s is the last step. Chosen set $C_s(\hat{T})$ is the set of teachers assigned to the slots of school s.

We illustrate how a chosen set is found in the Example 5 in Appendix C.

[29] Later we will explain why this precedence order is chosen.

We define the following notions for the auxiliary choice rules that will be crucial for our mechanism to be both strategy-proof and SI stable.

The auxiliary choice rule C_s satisfies **substitutes** (Kelso and Crawford, 1982) if for all $\bar{T} \subseteq T$ and distinct teachers $t, t' \in \bar{T}$,

$$t \in C_s(\bar{T}) \implies t \in C_s\big(\bar{T} \setminus \{t'\}\big).$$

The auxiliary choice rule C_s satisfies the **law of aggregate demand** (Alkan and Gale, 2003, Hatfield and Milgrom, 2005) if for all $\bar{T}, \hat{T} \subseteq T$,

$$\bar{T} \subseteq \hat{T} \implies \big|C_s(\bar{T})\big| \leq \big|C_s(\hat{T})\big|.$$

Next, we show that C_s satisfies these two properties.

Proposition 4 *The auxiliary choice rule C_s satisfies the substitutes and law of aggregate demand conditions.*

We refer to the mechanism that selects the outcome of the DA algorithm using the auxiliary choice rules $(C_s)_{s\in S}$ that we designed as the **status-quo improving deferred acceptance** (**SI-DA** for short) mechanism. The logic behind naming will be clear with the following result:

Theorem 2 *SI-DA mechanism is strategy-proof, and under Assumption 1, it is also SI stable.*

Notice that, when there is no new teacher, i.e., $N = \emptyset$, Theorem 2 holds without Assumption 1.

In the proof of Proposition 4 the processing of slots does not play any role. Hence, the Proposition 4 holds for any order we use in the calculation of chosen teachers. As a result, SI-DA mechanism continues to be strategy-proof independent of the processing order of the slots. Moreover, the proof of SI-Stability of SI-DA does not rely on the processing order. Hence, SI-DA mechanism continues to be SI-Stable independent of the processing order of the slots. However, the processing order has an impact on the mobility and the welfare of the teachers.

Let $\blacktriangleright_s$ and $\hat{\blacktriangleright}_s$ be arbitrary two processing orders of positions at school s such that $\hat{\blacktriangleright}_s$ is obtained from $\blacktriangleright_s$ by swapping two adjacent slots s^k and s^ℓ where $k < \ell \leq |\omega_s|$. Let $\blacktriangleright_{\bar{s}} = \hat{\blacktriangleright}_{\bar{s}}$ for any $\bar{s} \neq s$. Let $D_{s'}$ and $\hat{D}_{s'}$ be the auxiliary choice rules induced by $\blacktriangleright_{s'}$ and $\hat{\blacktriangleright}_{s'}$ by using the procedure defined in Definition 3 for all $s' \in S$. Let μ and $\hat{\mu}$ be the outcome of DA algorithm by using auxiliary choice rules $D_{s'}$ and $\hat{D}_{s'}$ for all $s' \in S$. Then, the following proposition holds.

Proposition 5 *Each teacher t (weakly) prefers μ_t to $\hat{\mu}_t$.*

Proposition 5 implies that the processing order that we use for the first $|\omega_s|$ positions at each school s increases the welfare of the teachers compared to the alternative processing orders. Moreover, it also implies that the processing order that we use increases the teacher mobility. However,

Proposition 5 does not say anything about the relative order the last $q_s - |\omega_s|$ positions at each school s. In the following example, we illustrate that we cannot find an optimal processing order for the last $q_s - |\omega_s|$ positions.

Example 2 *Let $S = \{s, s', s''\}$, $T = \{t_1, t_2, t_3, t_4\}$, the status-quo matching be*

$$\omega_s = \{t_4\},\ \omega_{s'} = \{t_2\},\ \omega_{s''} = \emptyset,$$

$q_s = 2$, $q_{s'} = q_{s''} = 1$, $\tau(t_1) \rhd_s \tau(t_2) \rhd_s \tau(t_3) \rhd_s \tau(t_4)$, $\tau(t_1) \rhd_{s'} \tau(t_3) \rhd_{s'} \tau(t_4) \rhd_{s'} \tau(t_2)$, and $\tau(t_1) \rhd_{s''} \tau(t_2) \rhd_{s''} \tau(t_3) \rhd_{s''} \tau(t_4)$. The preferences of the teachers are

$$s\ P_{t_1}\ s'\ P_{t_1}\ s''\ P_{t_1}\ \emptyset, \qquad s\ P_{t_2}\ s''\ P_{t_2}\ s'\ P_{t_2}\ \emptyset,$$

$$s\ P_{t_3}\ s''\ P_{t_3}\ s'\ P_{t_3}\ \emptyset, \qquad s'\ P_{t_4}\ s''\ P_{t_4}\ s\ P_{t_4}\ \emptyset.$$

If s^1 is filled before s^2, then under DA t_1 and t_3 are assigned to s, t_2 is assigned to s'' and t_4 is assigned to s'. If s^2 is filled before s^1, then under DA t_1 and t_2 are assigned to s, t_3 is assigned to s'' and t_4 is assigned to s'. Hence, we cannot have the same conclusion as in Proposition 5.

We use tie-breaking in the construction of the slot priorities. Inclusion of this exogenous tool causes efficiency loss: SI-DA does not select a Pareto-undominated SI stable matching (and therefore, it is not two-sided Pareto efficient either). We illustrate this situation in the following example.

Example 3 *Let $S = \{s, s', s''\}$, $T = \{t_1, t_2, t_3\}$, the status-quo matching be*

$$\omega_s = \{t_1\},\ \omega_{s'} = \{t_2\},\ \omega_{s''} = \{t_3\},$$

$q_s = q_{s'} = q_{s''} = 1$, $\tau(t_1) = \tau(t_2) = \tau(t_3)$, and all teachers are acceptable for all schools. The preferences of the teachers are

$$s'\ P_{t_1}\ s\ P_{t_1}\ s''\ P_{t_1}\ \emptyset, \qquad s\ P_{t_2}\ s'\ P_{t_2}\ s''\ P_{t_2}\ \emptyset, \qquad s'\ P_{t_3}\ s''\ P_{t_3}\ s\ P_{t_3}\ \emptyset.$$

Let $t_2 \vdash t_3 \vdash t_1$ be the tie breaker, then the rankings of the slots are given as:

$$t_1 \succ_s^1 t_2 \succ_s^1 t_3, \qquad t_2 \succ_{s'}^1 t_3 \succ_{s'}^1 t_1, \qquad t_3 \succ_{s''}^1 t_2 \succ_{s''}^1 t_1.$$

SI-DA assigns t_1 to s, t_2 to s' and t_3 to s''. However, this outcome is Pareto dominated by another status-quo improving stable matching in which t_1, t_2 and t_3 are assigned to s', s, and s'', respectively.

5 Generalizing Lower Bound for Welfare Improvements for Schools

In this section, we introduce a relaxation of the status-quo improvement requirement for schools in our mechanisms because this requirement can sometimes be too stringent in applications. For

example, senior teachers may have pre-existing priorities granted to them over new teachers and status-quo improvement for schools may prevent them from exercising these rights. This could be interpreted as an *ex post facto* legislation and may be undesirable for its effects immediately after adoption of design changes. Instead of status quo, we consider improvements with respect to a *hypothetical teacher distribution*. Consider a school $s \in S$. Let school s's ranking over teacher types be $\theta_1 \rhd_s \theta_2 \rhd_s \ldots \rhd_s \theta_m \rhd_s \theta_\emptyset$. That is, any type $\theta \in \Theta \setminus \{\theta_1, \theta_2, \ldots, \theta_m\}$ is unacceptable for school s. We refer to a vector $d_s = (d_s^{\theta_1}, d_s^{\theta_2}, \ldots, d_s^{\theta_n}) \in \mathbb{R}_+^n$ as a **threshold acceptability distribution for school** s if

- $d_s^{\theta_1} \leq \left|\omega_s^{\theta_1}\right|$,
- $d_s^{\theta_1} + d_s^{\theta_2} \leq \left|\omega_s^{\theta_1} \cup \omega_s^{\theta_2}\right|$,
- $\ldots$
- $d_s^{\theta_1} + d_s^{\theta_2} + \ldots + d_s^{\theta_m} = \left|\omega_s^{\theta_1} \cup \omega_s^{\theta_2} \cup \ldots \cup \omega_s^{\theta_m}\right| = |\omega_s|$.

Notice that threshold values are relaxation over the number of status-quo teachers from best type to the worst in a cumulative sense. A matching μ is **d_s-improving for school s** if types of teachers in μ_s are acceptable for school s and

- $d_s^{\theta_1} \leq \left|\mu_s^{\theta_1}\right|$,
- $d_s^{\theta_1} + d_s^{\theta_2} \leq \left|\mu_s^{\theta_1} \cup \mu_s^{\theta_2}\right|$,
- $\ldots$
- $d_s^{\theta_1} + d_s^{\theta_2} + \ldots + d_s^{\theta_m} \leq \left|\mu_s^{\theta_1} \cup \mu_s^{\theta_2} \cup \ldots \cup \mu_s^{\theta_m}\right| = |\mu_s|$.

Let a profile $d_S = (d_s)_{s \in S}$ be such that for each school s, d_s is a threshold acceptability distribution. A matching μ is **(d_S, ω_T)-improving** if it is d_s-improving for each school $s \in S$ and for each teacher $t \in T$, $\mu_t \; R_t \; \omega_t$. Observe that if matching μ is status-quo improving, then μ is (d_S, ω_T)-improving. Thus, this notion is a relaxation of status-quo improvement.

Under this relaxation, we can use the SI-CC and SI-DA mechanisms after we relabel the types of current employees for each school as follows: For each school $s \in S$, we construct a strict order, $\preccurlyeq_s$, using a tie breaker and its type ranking $\rhd_s$, such that a better type teacher is prioritized over a worse-type teacher, and teachers with the same type are ranked by using the tie breaker. When we determine the pointing rule of school s and the pointing rule of teachers under SI-CC and priority rankings of seats under SI-DA, we treat the first $d_s^{\theta_1}$ teachers under $\preccurlyeq_s$ as type θ_1 and teachers ranked between $d_s^{\theta_1} + \ldots + d_s^{\theta_{\hat{m}-1}} + 1$ and $d_s^{\theta_1} + \ldots + d_s^{\theta_{\hat{m}}}$ under $\preccurlyeq_s$ as type $\theta_{\hat{m}}$ for all $\hat{m} > 1$. For the rest of the steps of the mechanisms, each teacher is treated with her real type (see Appendix H for formal definitions).

These mechanisms, that we call **weak-SI-CC (wSI-CC)** and **weak-SI-DA (wSI-DA)**, respectively, inherit their desired properties mentioned in Sections 3 and 4 under the modification that each status-quo improvement regarding schools should be replaced with d_S-improvement in the property definitions. We use wSI-CC in our empirical simulations besides the original SI-CC in Section 6.7.

6 Empirical Analysis

This section provides empirical evidence on the changes that the mechanisms we suggest would bring in a real world setting: teacher (re)assignment to regions in France. After presenting the institutional context and the data, we structurally estimate teachers' preferences over regions, and run counterfactuals to quantify the improvements that our mechanisms may yield.

6.1 Institutional Background

Teacher recruitment and assignment. Teacher certification and recruitment are highly centralized in France. Anyone who wishes to become a teacher has to pass a competitive examination. Those who succeed are assigned a teaching position by the Ministry for a probation period of one year, at the end of which they get tenure or not. Once they get tenure, teachers in public schools become civil servants. The government manages both the first assignment of newly tenured teachers to a school, and the mobility between schools of tenured teachers who previously received an assignment but wish to change.[30] In the rest of the empirical analysis, we refer to newly tenured teachers who ask for their first assignment as *new teachers* and tenured teachers with a status-quo assignment as *tenured teachers*, with a slight abuse of terminology.

The two-step assignment process. The assignment procedure takes place in two successive steps.[31] During the first step, which is managed centrally by the Ministry, teachers are assigned to one of the 31 French regions using a mechanism to which tenured teachers who wish to change regions and new teachers submit a preference list over regions. In the second step, teachers who are newly assigned to a region and tenured teachers who wish to change schools within their region submit a preference list over schools and the same first step mechanism is run in the second step for each region with the new inputs. Since 1999, this step is managed directly by local administrations within the regions. Our empirical analysis focuses on the first regional assignment because of potential strategic reports of preferences during the second phase.[32] Participation to the assignment mechanism is compulsory for all new teachers as they do not have status-quo position. On the other hand, participation is optional for tenured teachers as they are never forced to change region or school.

Transfer requests, vacant positions, and new teachers. We use data on the assignment of teachers to one of the 31 French regions in 2013 for our empirical analysis. There were 700,000 secondary public school teachers in France that year, a number that fluctuates from year to year due to both entries and exits from the profession. Exits are mainly due to teachers retiring — 9,793

[30] This centralized assignment process is used for public school teachers only. Private schools employ 16% of teachers. For them, the recruitment process is similar but public and private school teachers face completely different rules for their mobility—between regions and between schools. In private schools, teachers apply directly to schools—as would be the case in usual labor markets.

[31] Before 1999, teacher assignment to schools was managed centrally by running an algorithm once, which assigned teachers directly to schools. This highly centralized process was argued to be at odds with the regional nature of most demands: the majority of teachers asking for a transfer ranked schools within their current regions.

[32] Preferences reported during the second phase of the assignment are more difficult to interpret for two reasons. First, teachers can only rank up to 20 or 30 schools, depending on the region. Second, in addition to ranking schools, teachers can also rank larger geographic areas, such as cities for instance.

public secondary school teachers retired in 2013 — while entries correspond to new teachers who have passed the recruitment exam and validated their probation year. As a result, when organizing the annual mobility process, the central administration has to take into account not only a large pool of tenured teachers who wish to change positions, but also some vacant positions that need to be filled and new teachers who wish to be assigned to their first jobs. In 2013, the year for which we have data, about 25,100 teachers took part in the centralized regional mobility process. Among them 17,200 are tenured teachers and 7,900 are new teachers.

6.2 Data and Descriptive Statistics

We use data on the assignment of teachers to one of the 25 French regions in 2013.[33] We have information on teachers' reported preferences and status-quo assignment (if any), Ministry-mandated regions' priorities,[34] and the number of vacant positions in each region. We keep all teachers from the 8 largest subjects, such as French, Math, English, and Sports. We discard couples from the sample because they benefit from a special treatment in the assignment process.[35] Finally, in order to keep the market structure balanced, we drop one seat for each teacher we omit.[36] Our final sample contains 10,460 teachers: 5,833 tenured teachers (55.8%) and 4,627 new teachers. Table A.1 shows the decomposition by three of the main subjects, French, Math, and English.

A central motivation of our analysis is to rebalance the unequal distribution of teachers across regions. Part of this large imbalance stems from differences in regions' attractiveness. Table 1 reports descriptive statistics on teachers (Panel A), their status-quo assignment (Panel B), and the region they rank first (Panel C).[37] Two regions surrounding Paris, called Créteil and Versailles, are particularly unattractive. The imbalance is blatant when comparing the number of teachers asking to leave the region and the number asking to enter. For instance, in Math, 52.3% of the tenured teachers who ask to change region come from Créteil or Versailles, but only 3.3% rank one of these two regions as their first choice.[38] Table A.2 in Appendix E provides additional evidence on attractiveness differences and their potential determinants for the three most attractive regions (Rennes, Bordeaux, and Toulouse), the three least attractive regions (Créteil, Versailles, and Amiens), and three of the intermediate regions (Paris, Aix-Marseille, and Grenoble).[39]

[33] We discard the 6 overseas regions because of their specificities in terms of (i) teacher preferences — in contrast to what we find in our estimates, distance from the current location often becomes an attractive feature — and (ii) regions' Ministry-mandated priorities — some of these regions, like Mayotte, give teachers who grew up in these regions bonus points when they rank it first.

[34] The Ministry-mandated priorities are determined by a point system which is mainly based on teachers' experience, whether they ask for a spousal reunification in a region and whether their current school is a disadvantaged one. This is presented in details by Combe et al. (2020). Since this is not key for understanding our results, we refer the reader to their paper for more details.

[35] Spouses in two different subjects can submit joint mobility applications (by submitting two identical lists). This creates dependencies across markets for different fields.

[36] For each tenured teacher we discard, we drop her corresponding position as well. For new teachers, we find the share of new teachers discarded among all new teachers and denote it as $S\%$; then we delete $S\%$ of vacant positions in each region.

[37] Appendix F provides a detailed description of each variable.

[38] This rate is a bit larger for new teachers. Between 10% and 15% of them rank Créteil or Versailles as their first choice across the 8 subjects we consider in our analysis.

[39] Attractiveness is measured as the ratio of the number of tenured teachers asking to enter a region over the number

The large share of transfer requests that originate from unattractive regions have a direct consequence on the annual mobility flows: Under the current assignment system, a large number of teachers exit these regions, which results in numerous vacant positions that need to be filled. About 50% of the new teachers get their first assignment in one of the three least attractive regions (Créteil, Versailles, and Amiens). This structural imbalance is a serious concern for policy makers. It is frequently raised as a reason for the lack of attractiveness of the teaching profession in France.[40] In addition, it creates large differences in the age profile of teachers across regions. As reported in column (3) of Table A.2, the ratio of the number of teachers older than 50 to the number of teachers younger than 30 is equal to 1.1 and 1.6 in Créteil and Versailles, respectively. In contrast, the most attractive region, Rennes, had almost 7.4 times more teachers older than 50 than teachers younger than 30. In Bordeaux and Toulouse, this ratio was 6.5 and 5.3, respectively.

Several papers have found that teachers tend to help with their students' progress less during the first years of their career than when they have more experience (Bates et al., 2021, Chetty et al., 2014, Rockoff, 2004). Reducing the unequal distribution of teachers across regions and reducing the chances of assignment to a disadvantaged region in the first place became one of the objectives of the French policy makers, who see this as a way to both reduce achievement inequalities between students and improve the attractiveness of the teaching profession in the longer run.

6.3 Specifications of the Empirical Analysis

Mechanisms. Our theoretical results regarding the SI-CC and SI-DA mechanisms can be used to achieve policy maker objectives to alleviate the unequal distribution of teachers in regions. Our counterfactual analysis aims to both formally define possible inputs and quantify the performance of these mechanisms. We also benchmark them with variants of two widely used mechanisms:

- **Benchmark for SI-CC:** A variant of SI-CC, which we refer to as **TTC***, that accounts for teacher types as defined in the theoretical analysis but relaxes the mechanism features that ensure status-quo improvement. More precisely, this mechanism differs from SI-CC in two respects: (1) we lift the restrictions on the set of schools that a teacher can point to, and (2) tenured teachers can now start a chain (and potentially leave their position without being replaced) (see Appendix H for a precise definition). This benchmark is close to the well-known TTC-variant mechanism "you request my house – I get your turn" (YRMH-IGYT) (Abdulkadiroğlu and Sönmez, 1999, Sönmez and Ünver, 2010) which is strategy-proof, Pareto efficient, and individually rational for teachers, but not status-quo improving. Intuitively, TTC* might, therefore, be expected to generate more mobility than our mechanisms at the cost of a potentially more unequal distribution.
- **Benchmark for SI-DA:** A variant of SI-DA, which we refer to as **DA***, that accounts for teacher types but relaxes the mechanism features that ensure status-quo improvement. More

of tenured teachers asking to leave the region. This ratio ranges from 15.5 in Rennes to 0.03 in Créteil.

[40] The most common recruitment exam in France is called the CAPES. Every year, the Ministry decides on the number of teaching positions it opens for the exam. In 2014, 24% of the CAPES positions remained vacant because of both a shortage of applicants and the poor quality of those applying. The shortage situation has not improved since.

precisely, this mechanism is the standard DA mechanism where teachers are moved at the top of the priority of their status-quo school. A school's priority follows its preferences over teachers' types and the same tie-breaker as the one used under SI-DA. This mechanism does not use slot specific priorities so that it differs from SI-DA in two respects: (1) we lift the restrictions on schools' rankings over teacher types (i.e, an applicant teacher with a less-preferred type than a status-quo teacher will no longer be considered as unacceptable by a school), and (2) vacant positions in a region no longer prioritize new teachers over tenured teachers (see Appendix H for a precise definition). If the Ministry-mandated priorities are used for regions, this mechanism becomes equivalent to the mechanism used in each step of the current French assignment process. However, we use our regional FOSD preferences to account for priorities, making DA* different from the one step of the current French process. Incorporating teacher types as schools' preferences provides an interesting benchmark that targets teacher-SI stability, potentially at the cost of efficiency and distributional objectives.

In Section 6.7, we further consider counterfactual comparisons regarding two more mechanisms based on SI-CC and DA*, respectively. We explain these mechanisms in that section.

Teacher Types. We run our different mechanisms using teachers' preferences, teachers' types, and regions' type rankings as inputs. To illustrate our theoretical contribution, we define for a teacher her type as her experience and we classify teachers into 12 experience bins, where the first bin corresponds to teachers with 1 or 2 years of experience, the second bin to teachers with 3 to 4 years of experience, and so on. Each bin represents a *teacher experience type*. To distinguish teachers with the least experience within the system (those in the first experience bin), we further assume that new teachers have a smaller experience type than any tenured teachers, hence resulting in 13 effective experience bins. Figure A.1 shows a distribution of teacher experience type.

Regions' Type Rankings. To define regions' rankings over teacher types, we start by identifying which regions would benefit most from receiving more experienced teachers. To do so, we compute average teacher type in each region at status quo (see Figure A.2) and we classify regions into two groups based on whether their average type is above or below the median.

- The first group contains all regions whose average types are strictly below the median, i.e, *younger regions* that could benefit from receiving more experienced teachers. We set the type rankings of these regions so that they rank types by decreasing level of experience, i.e, the most experienced teacher types are always preferred to the least experienced teacher types.
- The second group contains all regions whose average types are above the median, i.e., *older regions*. These regions rank types by increasing level of experience.

Our ultimate goal in setting schools' preferences is to ensure that the matching outcomes of our mechanisms eventually yield more equal distributions of teachers across regions. Both SI-DA and SI-CC, which are status-quo improving, achieve this goal through these type rankings. Note that the way we define type rankings for younger and older regions is tailored to the objectives of French policy makers (as discussed in Section 6.2). Evidence from the US also supports our assumption that

young regions value teacher experience more than old regions. By estimating principals preferences for teachers, Bates et al. (2021) find that, when hiring teachers, Title I principals have a stronger preference for high-value-added teachers than principals in non-Title I schools. Combined with the very large increase in teacher value-added they observe during teachers first year of experience (of 0,09 student standard deviations), this suggests a larger preference of disadvantaged schools for experienced teachers. To further support our choice of ranking, we also conduct robustness tests and show (in Appendix G) results in which all regions have the same type rankings.[41]

We further assume that all regions find all types acceptable. This means that any teacher is always preferred to a vacant position. Indeed, in the French system, by law, all teachers are acceptable for all regions.[42] Finally, running SI-CC (and its variants) requires an additional ordering over teachers to determine which chain will be selected.[43] For the main results presented in the paper, we use the true point system of the French Ministry and sort teachers by decreasing level of the maximum points they obtain over all regions. However, we show in Appendix E that the performance of the SI-CC mechanism is sometimes sensitive to the ordering chosen. We report robustness results in which we flip the ordering to rank the teachers by increasing level of their maximum priority points.

6.4 Structural Estimation of Teacher Preferences

Teachers can rank all regions when they submit their preference list in the first step of the assignment process and the Ministry uses a modified version of the DA mechanism to assign teachers to regions, as mentioned earlier. If the whole assignment were done in a single step, this mechanism would induce being truthful as a dominant strategy for teachers. In our setting, if teachers have lexicographic preferences over regions first and schools second or, more generally, if preferences over regions are well-defined objects, the mechanism assigning teachers to regions in the first step is strategy-proof.[44] Yet, even under strategy-proof mechanisms, a number of experimental and empirical papers show that truthfulness is a strong assumption (Chen and Sönmez, 2006, Pais and Pinter, 2008, Rees-Jones, 2018, Chen and Pereyra, 2019, Hassidim et al., 2017). In our context, French teachers have reasonably accurate information on their admission probabilities to each region, which might encourage some teachers to discard from their preference list the regions

[41] Assuming that all regions prefer experienced teachers over inexperienced teachers, we obtain similar improvements in terms of teacher experience in the three youngest regions. However, unsurprisingly, allowing old regions to prefer experienced teachers over inexperienced teachers leads to fewer inexperienced teachers being assigned these regions.

[42] For tie-breaking, we need an additional ordering over teachers for the SI-CC, SI-DA, and two benchmark mechanisms. We use the tie-breaking rule used by the French Ministry which uses the date of birth of teachers and some extra conditions for the rare cases with the same date of birth.

[43] This ordering only applies to new teachers under SI-CC. It applies to all teachers under TTC* because chains can start from any teacher.

[44] In the end what teachers primarily care about is the school they obtain within a region, which questions whether preferences over regions are well-defined objects. Combe et al. (2020) show evidence that teachers' preferences seem to be lexicographic, i.e., that teachers primarily care about the region in which the school is located and about a school within that region. The paper also shows that changes in a number of mechanisms in the first phase (the regional assignment) only marginally impact the pool of participants in the second phase (the school assignment) so that teachers' assessments of their school admission probabilities (within a region) should not vary much between mechanisms. This implies that teachers' preferences over regions may be relatively insensitive to the mechanism used and so are well-defined through backward induction.

where their chances to be accepted are too low.[45] These omissions could introduce a bias in any counterfactual analysis done using teachers' reported preferences. Combe et al. (2020) previously rejected truth telling among French teachers. To avoid this potential bias, instead of using the reported preferences, we estimate teachers' preferences using an identifying assumption (presented below) that does not require teachers to be fully truthful.

Model. We estimate teachers' preferences over regions using the following utility function:

$$u_{t,r} = \delta_r + Z'_{t,r}\beta + \varepsilon_{t,r} \tag{5}$$

Teacher t's utility for region r is a function of region fixed effect δ_r, teacher-region-specific observables $Z_{t,r}$ (with coefficients β) and a random shock $\varepsilon_{t,r}$ which is i.i.d. over t and r and follows a type-I extreme value distribution, Gumbel$(0, 1)$. The region fixed effect captures region characteristics such as average socio-economic and academic level of students in the region, cultural activities, housing prices, facilities, etc. We estimate preferences separately for tenured teachers and new teachers. This allows us to include a richer set of variables for the former group. The vector $Z_{t,r}$ includes a dummy specifying if region r is the birth region of teacher t. If teacher t is tenured, it also includes a dummy showing whether r is the status-quo region of teacher t, as well as the distance between region r and the status-quo region of teacher t. Vector $Z_{t,r}$ additionally includes interaction terms between teacher t's and region r schools' characteristics (that are presented in Panels A and B of Table 1). We apply standard scale and position normalization, setting the scale parameter of the Gumbel distribution to 1 and the fixed effect of the Paris region to 0.

Identifying assumptions. To avoid the potential bias generated by teachers omitting regions they consider as infeasible, we estimate teachers' preferences under a weaker "stability assumption" developed by Fack et al. (2019) and applied to the teacher assignment by Combe et al. (2020).[46] We start by defining the *feasible set* of each teacher as the set of regions that has a cutoff — that is, the lowest priority of the teacher assigned to a region — smaller than her own priority. These are the regions a teacher could be assigned to if she ranked the region first in her preference list. The key identifying assumption is that, for each teacher, the region obtained is her most preferred region among all regions that are in her feasible set.[47] Hence, we estimate a discrete choice model with personalized choice sets. Choice probabilities have closed form solutions and we estimate parameters using maximum likelihood.

[45] Cutoffs for entry in each region are published every year. Combe et al. (2020) show that these cutoffs are relatively persistent over time, so they provide reasonably accurate information to teachers on their chances to be admitted to each region.

[46] Combe et al. (2020) provide an in-depth discussion of the two alternative identifying assumptions (truthfulness versus stability), as well as statistical tests in favor of the later. For more references on estimations that do not require truth telling, see Akyol and Krishna (2017), Artemov et al. (2019), Agarwal and Somaini (2018), Calsamiglia et al. (2020). They mainly focus on the estimation of the preferences of tenured teachers and we use the same estimation in our analysis. Here, we provide an additional detailed discussion on the estimation of the preferences of new teachers.

[47] This assumption is theoretically founded: Artemov et al. (2019) show that, in a large market environment, any (regular) equilibrium outcome of DA* must have this property. Since a variant of DA* with the Ministry-mandated priorities is the current mechanism for the regional assignment step, this result also applies to our setup.

Table 1: Descriptive Statistics on Teachers and Regions

	Tenured Teachers			New Teachers		
	French	Math	English	French	Math	English
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A. Characteristics of teachers						
Female (%)	76.1	47.0	85.4	80.3	41.7	80.4
Married (%)	48.5	45.0	46.8	41.1	39.4	40.9
Is in priority education school (%)	10.4	13.2	4.4	-	-	-
Experience (yrs)	7.48	7.23	7.18	2.76	2.24	2.30
Has advanced teaching qualifications (%)	7.9	29.1	8.8	16.8	31.7	15.2
Panel B. Characteristics of the regions to which teachers are assigned at status quo						
Is the teacher's birth region (%)	8.7	8.6	9.3	-	-	-
Is Créteil or Versailles (%)	37.7	52.3	35.6	-	-	-
Is in South of France (%)	5.6	9.3	12.7	-	-	-
Students in urban areas (%)	61.7	67.4	64.0	-	-	-
Disadvantaged students (%)	52.5	54.0	53.5	-	-	-
Students in priority education (%)	26.0	24.5	22.7	-	-	-
Students in private school (%)	15.2	16.3	17.4	-	-	-
Teachers younger than 30 yrs (%)	11.9	13.3	11.3	-	-	-
Panel C. Characteristics of the regions teachers rank first						
Distance to status-quo region (km)	2,148.7	1,316.9	1,521.9	-	-	-
Is the teacher's birth region (%)	36.5	35.8	40.0	35.8	38.4	39.3
Is in South of France (%)	25.2	25.2	25.4	20.1	18.6	20.0
Is Créteil or Versailles (%)	2.8	3.3	3.4	14.3	11.9	12.5
Students in urban area (%)	60.2	56.2	51.7	61.9	58.6	59.2
Disadvantaged students (%)	52.8	53.1	53.5	53.3	53.1	53.0
Students in priority education (%)	20.3	19.9	17.9	21.8	20.6	21.0
Students in private school (%)	23.7	22.9	25.8	22.2	21.8	21.7
Teachers younger than 30 yrs (%)	6.5	6.4	6.0	8.2	8.1	8.2
Observations (#)	859	605	628	786	958	746

Notes: This table reports descriptive statistics for teachers and regions in three subjects: French, Math, and English. Statistics are reported for the sample of teachers we use for the demand estimations. Columns (1) to (3) report statistics for tenured teachers. Columns (4) to (6) report statistics for new teachers. New teachers have missing values for statistics related to the region of status-quo assignment. We discard teachers who submit a joint list with their partner, teachers who are from one of the six regions that are overseas, and teachers for whom one of the individual characteristics is missing. The last row reports the number of teachers in each subject. Panels A, B, and C, respectively, present descriptive statistics on teachers, on the region to which they are assigned at status quo, and on the region they rank first. Appendix F provides a detailed description of each variable.

Table 2: Teacher Preference Estimates

	Tenured Teachers				New Teachers			
	French		Math		French		Math	
	coef.	s.e.	coef.	s.e.	coef.	s.e.	coef.	s.e.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Region BESANCON	-3.88***	(0.99)	0.37	(0.8)	-1.52*	(0.65)	-0.32	(0.78)
Region BORDEAUX	-1.36	(0.95)	1.12	(0.66)	1.14	(0.62)	-0.40	(0.64)
Region DIJON	-5.08***	(0.97)	-2.88***	(0.73)	-1.39*	(0.59)	-1.88**	(0.61)
Region LILLE	-5.09***	(0.95)	-1.55	(0.81)	-1.7**	(0.62)	-1.26	(0.67)
Region REIMS	-6.22***	(1.00)	-3.60***	(0.74)	-2.39***	(0.61)	-3.04***	(0.65)
Region AMIENS	-6.44***	(1.06)	-3.31***	(0.75)	-1.88**	(0.59)	-2.35***	(0.62)
Region ROUEN	-5.96***	(0.97)	-2.17**	(0.69)	-1.64**	(0.59)	-1.53*	(0.62)
Region CRÉTEIL	-6.66***	(1.00)	-3.65***	(0.71)	-0.34	(0.60)	-1.33*	(0.58)
Region VERSAILLES	-5.12***	(0.89)	-2.13***	(0.60)	-0.50	(0.55)	-1.14*	(0.56)
Status-quo region	4.97	(6.72)	-15.72	(8.27)				
Birth region	10.21**	(3.41)	14.89***	(3.53)	10.46***	(2.48)	8.31**	(2.69)
Distance to status-quo region	-23.33***	(4.61)	-23.52***	(5.47)				
% stud. urban x Status-quo region	-5.82***	(0.85)	-5.40***	(1.15)				
% stud. urban x Teach. from CV	2.81***	(0.71)	0.12	(0.68)				
% stud. in priority educ. x Married	-7.61***	(1.60)	-3.89*	(1.65)	-7.13***	(1.58)	-4.93***	(1.31)
% stud. in priority educ. x Status-quo region	11.26***	(2.99)	0.67	(3.80)				
% stud. in private sch. x Teach. in disadv. sch.	5.48**	(2.01)	6.58***	(1.83)				
% teach. younger than 30 x Advanced qualif.	10.59**	(3.65)	0.24	(3.06)	10.80**	(2.77)	5.58**	(1.99)
% teach. younger than 30 x Status-quo region	52.42***	(5.19)	54.15***	(6.47)				
% teach. younger than 30 x Birth region	-22.08***	(3.73)	-19.10***	(4.79)	-8.42**	(2.79)	-9.26***	(2.61)
Region in South of France x Teach. from CV	-1.27***	(0.37)	0.35	(0.36)				
Number of teachers	859		605		786		958	
Fit measure	0.669		0.674		0.632		0.624	

Notes: This table reports selected coefficients from estimations of teachers' preferences for regions' characteristics based on Equation 5. We set the fixed effect of the Paris region to 0. The last row reports our goodness of fit measure that we compute by looking at the top two regions that a teacher has included in her submitted preference list. We measure, for each teacher, the probability of observing this particular preference ordering in the preference list predicted with our estimations. We then average these probabilities across teachers. Stars correspond to the following p-values: * p< 0.05; ** p< 0.01; *** p< 0.001. Variable "Teach. from CV" refers to whether the status-quo region of the teacher is Créteil or Versailles.

Estimation results. Table 2 reports preference estimates for tenured teachers and new teachers for a selected group of coefficients. We run the estimations in each of the 8 subjects separately and report results for Math and French teachers in the table. The first 9 rows report coefficients for a selected set of region fixed effects. They reveal an interesting difference between the preferences of tenured and new teachers. While the Créteil and Versailles regions are very unattractive for tenured teachers (as indicated by the negative coefficient of their fixed effect relative to the Paris region), these regions are less unattractive for new teachers, who often see a first position in a disadvantaged school as a stepping stone for better positions in the future.[48] The fact that Créteil and Versailles are more attractive for new teachers than for tenured teachers surely contributes to the unequal distribution of teachers denounced by policy makers. Yet, this is not the only explanation for teachers' unequal distribution. The counterfactual analysis we present in the next section shows that the assignment mechanism also shapes the distribution of teachers in important ways. The fact that preferences alone are not driving the unequal distribution is fundamental for our ability to improve both teacher distribution and teacher welfare.

Goodness of fit measures. Our main fit measure (also reported in Table 2) considers the top two regions that a teacher has included in her submitted preference list. We then compute the probability of observing this particular relative ordering in the preference list predicted by our estimations. This fit measure based on relative ranking (instead of the characteristics of the school ranked first for instance) is particularly suitable for our environment in which some teachers might not rank regions that they consider as infeasible.[49] In addition to the overall fit quality, we also compute fit measures for the tenured teachers who are employed in the two least attractive regions, namely, Créteil and Versailles, at the status quo. Inspecting the fit quality for this sub-group of teachers is particularly important because teachers from Créteil and Versailles represent a large share of the tenured teachers who submit a transfer request every year and they are more likely to stay in their positions. These two facts could affect the preference estimation for these teachers under our stability assumption. Across the 8 subjects, our fit measures range from 0.62 to 0.72 for tenured teachers and from 0.56 to 0.69 for new teachers, which compare favorably to the ones obtained by Fack et al. (2019) (between 0.553 and 0.615).

Simulations. We use our estimates of utility coefficients to draw teachers' preferences 1,000 times using Equation 5. After having drawn them, we keep the entire set of regions without imposing any truncation for their simulated preference lists: tenured teachers find all regions ranked as high as their status-quo assignment acceptable while new teachers find all regions acceptable.[50] In each of the 8 subjects and for each draw, we use these simulated preferences to run the mechanisms. The

[48] Teachers who stay in a disadvantaged school for at least 5 years benefit from additional priority when they ask to change region or school.

[49] When teachers skip regions perceived as infeasible, the first region they report might not be their most preferred region — and indeed, the tests we perform reject truth telling — but conditional on ranking schools, the order in which a teacher ranks the schools might correspond to teacher's true relative preferences. This is why we prefer to use a fit measure that is based on relative ranking rather than on the characteristics of the school ranked first.

[50] This implicit assumption about new teachers is in line with the policy of the Ministry. Teachers are indeed not required to rank all regions when they submit their lists, but the Ministry fills the incomplete lists of new teachers to make sure that all of them get an assignment even those who ranked few regions.

results reported in the next section correspond to averages over these 1,000 draws, aggregated over 8 subjects.

6.5 Relative Performance of the SI-CC and SI-DA Mechanisms

We start by discussing the relative performance of SI-CC and SI-DA. Recall that while SI-CC is SI teacher optimal (i.e., Pareto undominated for teachers among all status-quo improving matchings) and two-sided Pareto efficient (incorporating the welfare of both regions and teachers), SI-DA is SI stable (under Assumption 1) and not two-sided Pareto efficient. Comparing the performance of these two mechanisms that target teacher welfare (and Pareto efficiency) and stability is important for various reasons. First, neither mechanism satisfies any obvious optimality property for the distribution of teacher experience at outcome matchings, which is the measure of regional welfare.[51] Therefore, it is important to understand which one leads to a more desirable teacher experience distribution. Second, the mechanism that is currently used by the French Ministry of Education is teacher-SI stable (with respect to their Ministry-mandated priorities). This suggests that policy makers consider stability as an important feature of the assignment process. As there is no mechanism that is both SI teacher optimal and SI stable, and there is even no Pareto comparison regarding teacher welfare between these two mechanisms, understanding the tradeoff between these properties is important.

Validation of Assumption 1. We show in Section 4 that the SI-DA mechanism is SI stable under Assumption 1. When all regions are acceptable by new teachers, all new teachers are acceptable by regions, and there are at least as many new teachers as empty seats then this assumption is satisfied. These conditions are met in each of the 8 subjects we consider (see Table A.1). This means that the matching obtained under SI-DA is SI stable.

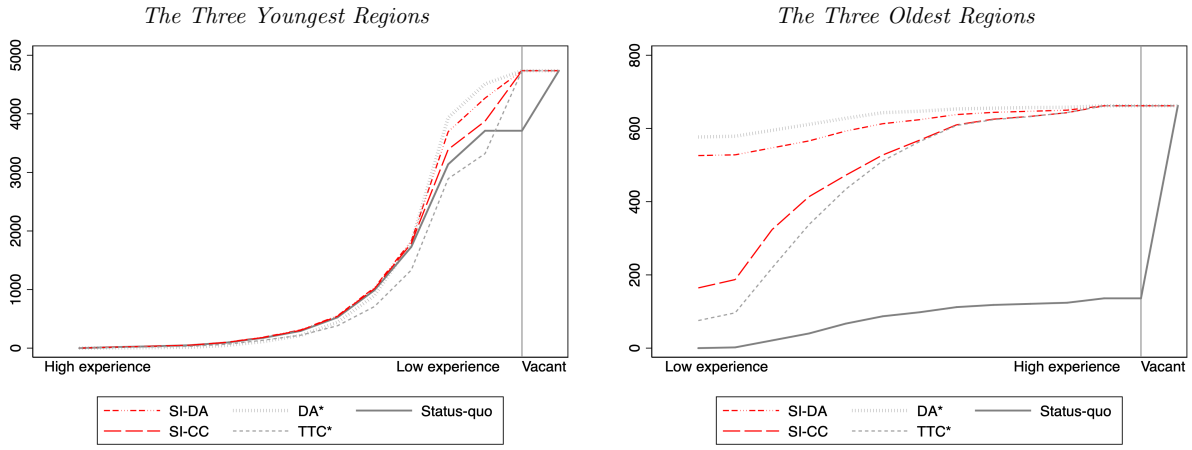
Distribution of teacher experience. We start by comparing, for different regions, the cumulative distribution of teacher experience under SI-CC and SI-DA. We classify the assigned teachers to regions into 13 types based on experience as explained before. The left panel of Figure 1 shows the cumulative distribution of teacher experience in the three youngest regions of France (Créteil, Versailles, and Amiens). Teachers in these regions represent 78% of teachers in disadvantaged regions.[52] SI-DA slightly outperforms SI-CC in these regions, i.e., it assigns fewer inexperienced teachers: 1,041 teachers with up to two years of experience are assigned to one of the three regions under SI-DA versus 1,344 under SI-CC. SI-DA also improves the experience distribution in the three oldest regions by assigning them a larger number of inexperienced teachers (see the right panel of Figure 1).

Stability – teacher welfare trade-off. However, SI-DA's better distributional performance comes at a large cost in terms of teacher mobility. Panel A of Table 3 shows that only 3,912 teachers obtain a new assignment under SI-DA, compared to 5,356 under SI-CC. The lack of mobility under SI-DA is particularly striking for tenured teachers: none of them move from their status-quo

[51] In this whole section while talking about regional welfare comparing different outcome matchings, we will talk about comparisons of teacher experience type distributions (at these matchings), with a slight abuse of terminology.

[52] See Figure A.2 for the type distribution in regions.

Figure 1: Cumulative Distribution of Teacher Experience Types



Notes: This Figure shows the cumulative distribution of teacher experience types. The left panel reports the distribution in the three youngest regions of France (Créteil, Versailles, and Amiens), and the right panel the distribution in the three oldest regions of France (Rennes, Bordeaux, and Lyon). The horizontal axis reports the 13 experience types of teachers, ordered from most experienced to least experienced (left panel) and from least experienced to most experienced (right panel). The 14^{th} type corresponds to vacant positions. The mechanisms that satisfy status-quo improvement are plotted in red. Those that do not are in grey. The thick black line (marked as "Status-quo" in the legend) corresponds to the cumulative distribution of teacher types at the status-quo matching.

assignment, compared to 1,444 under SI-CC. The very low level of mobility under SI-DA is due to the strength of the requirements imposing status-quo improvement and SI stability. Indeed, status-quo improvement implies that many tenured teachers from unattractive regions will be unable to leave the region together with the low demand from tenured teachers for these regions. Given this, any teacher entering a region almost automatically generates justified envy from a tenured teacher stuck in an unattractive region (in particular, since the least attractive regions are also the youngest, the relatively low experience of the tenured teachers stuck in these regions are preferred by the older regions like Bordeaux). SI stability requirement prevents such assignments from happening and, thus, blocks mobility of tenured teachers.

This simple example and the results from our counterfactual analysis show that, under DA-inspired mechanisms satisfying different notions of stability, imposing a status-quo improvement can have the unintended consequence of dramatically reducing mobility. Said differently, prioritizing SI stability and status-quo improvement (under SI-DA) over SI teacher optimality (under SI-CC) entails a very large efficiency cost for teachers in our context.

Concerning stability measures, SI-DA is, by construction, SI stable contrary to SI-CC. As reported in Panel E of Table 3, the latter leads to 8,122 teachers being involved in at least one blocking pair not authorized in the definition of SI stability that we introduced in Section 4. Note that neither SI-DA nor SI-CC are Gale-Shapley stable, which forbids all blocking pairs: SI-CC leads to 8,206 teachers being involved in at least one blocking pair compared to 8,438 teachers under SI-DA. This reduction can be explained by the important teacher welfare gains that SI-CC has achieved, compared to SI-DA. Since many more tenured teachers move under SI-CC, the number

of blocking pairs caused by tenured teachers staying at their status-quo assignment decreases. The small differences between the number of unauthorized blocking pairs in our three stability notions under SI-CC mean that the vast majority of blocking pairs are caused by less preferred teachers being assigned to a new region despite more preferred teachers requesting that region. For SI-DA, only 2,171 teachers are blocking due to this last reason while the remaining 6,267 teachers block because of a tenured teacher staying at her status-quo region. We conclude this subsection with the following summary of our main findings so far:

Fact 1 *Despite a slightly better distributional performance and obtaining an SI stable matching, SI-DA has a tremendous mobility cost compared to SI-CC. No tenured teacher moves from her status-quo region under SI-DA, compared to 1,444 under SI-CC. Imposing the status-quo improvement constraint together with SI stability comes at a large teacher welfare cost.*

6.6 Benefits and Costs of Status-quo Improvement Constraints

We now turn to a discussion of the benefits and costs of adding status-quo improvement constraints to assignment mechanisms. To do so, we compare SI-CC and TTC*, the benchmark mechanism which uses the same type rankings for teachers as SI-CC, but is not status-quo improving. We also compare SI-DA and DA*, although we devote less time to this comparison due to the relatively poor performance of SI-DA identified in the previous section.

Better distribution of teacher experience. The left panel of Figure 1 shows the cumulative distribution of teacher experience in the three youngest regions of France.[53] Every year a very large number of teachers with a few years of experience leave Créteil, Versailles, and Amiens. They are replaced by an equally large number of inexperienced teachers. That structural imbalance means that the status-quo improvement requirement is unlikely to be respected by mechanisms such as DA* or TTC*. Our counterfactual analysis confirms this (see the left panel of Figure 1). Our main finding for the youngest regions is as follows:

Fact 2 *In the three youngest regions of France, the distribution of teacher experience under SI-CC FOSDs the one under TTC*. SI-CC assigns only 1,344 teachers with one or two years of experience to the three youngest regions, while TTC* assigns 1,844 of them to these three regions. On the contrary, the distribution of teacher experience under SI-DA does not FOSD the one under DA*.*

The distributions under the benchmark mechanisms need not FOSD the status-quo distribution as they do not require status-quo improvement. Indeed, the distribution of teacher experience under DA* and TTC* do not FOSD the status-quo distribution.[54,55]

[53] See Figure A.2 for the type distribution in all regions at status quo.

[54] Figure A.5 shows that the distributional performance of TTC* depends on the tie breaker used. For instance, when the teachers starting a chain are selected by increasing order of their maximum Ministry-mandated priority points, the resulting distribution FOSDs the ones when they are ordered randomly or by decreasing order of their maximum Ministry-mandated priority points. These distributions of teacher experience under TTC* do not FOSD the status-quo distribution.

[55] Appendix Figures A.3 and A.4 show that these results persist for TTC* when we consider the entire group of

Interestingly, the distributional benefits of status-quo improvement we find for SI-CC do not hold for SI-DA. In the three youngest regions of France, the SI-DA mechanism produces a distribution of teacher experience which does not FOSD the distribution under DA*. SI-DA also assigns more teachers with one or two years of experience (1,041) to the three youngest regions than DA* (801). This finding confirms that imposing status-quo improvement to DA-inspired mechanisms can backfire. In general, when Pareto efficiency is satisfied, we expect that an increase in mobility upon a status-quo improving matching can only be done at the expense of the distribution of teacher experience. This is indeed what we observe when comparing SI-CC and TTC*. However, when Pareto efficiency is not satisfied and mobility is extremely low, as under SI-DA, this trade-off may disappear. In essence, SI-DA just assigns new teachers to vacant positions and leaves all tenured teachers at their status-quo regions. The improvement upon the status-quo distribution in terms of teacher types is thus minimal among tenured teachers under SI-DA and further movement may help improving these distributions in many regions. Indeed, even though DA* does not impose status-quo improvement, the higher mobility it creates improves the distribution of teacher experience in these three youngest regions in France. Finally, note that the two DA-inspired mechanisms assign fewer inexperienced teachers to Créteil, Versailles, and Amiens than the two TTC-inspired mechanisms (1,041 for SI-DA and 801 for DA*, while 1,344 for SI-CC and 1,844 for TTC*). Again, this is due to the severe lack of mobility from these regions.

To complement the results for the three youngest regions of France, we also report the results for the three oldest regions of France. The objective is now to assign younger teachers.

Fact 3 *In the three oldest regions of France, the distribution under SI-CC FOSDs the one under TTC* (see the right panel of Figure 1). SI-CC assigns 187 teachers with one or two years of experience to these regions, while TTC* only assigns 96 of them. On the contrary, the distribution of teacher experience under DA* FOSDs the one under SI-DA.*

Two channels, that work in opposite directions, could explain SI-CC's better performance compared to TTC*. For example, take one of the youngest and least attractive regions, Créteil. On one hand, for tenured teachers, SI-CC prevents them leaving Créteil, which limits the possibility of assigning these (relatively inexperienced but tenured) teachers to attractive regions. On the other hand, we might expect SI-CC to prevent new teachers replacing tenured teachers in Créteil (due to new teachers' even lower experience), which would redirect these new teachers to attractive regions. Our results confirm that the second channel dominates the first one in terms of magnitude. Indeed, by preventing tenured teachers to leave Créteil for attractive regions, SI-CC lowers competition for vacant seats in the attractive regions. New teachers, who prefer these regions to Créteil, are able to get assigned to these vacant seats. (As we discuss below, the new teachers are on average and in distribution assigned to more preferred regions under SI-CC than under TTC*.)

A salient fact emerges when comparing distributional performances in the youngest and oldest regions: In the oldest regions, all mechanisms easily produce a distribution of teacher experience

regions whose average experience is below the median.

that FOSDs the status quo, while in the youngest regions only mechanisms that respect status-quo improvement do. This finding reflects the very different levels of attractiveness of these regions. As the oldest regions are highly demanded by teachers, it is easier to improve their teacher experience distribution. This is much more difficult in the youngest regions as they are not as highly demanded, especially by experienced teachers. The ratio of entry to exit requests is equal to 15.5 in Rennes but 0.03 in Créteil (see Table A.2). This large difference in demand also explains why the distributions of teacher experience are very compressed in the youngest regions, but not in the oldest ones (see Figure 1). Due to the limited room for improvement in disadvantaged regions, most mechanisms have a similar *capped* performance. Last, the performance of DA* is very good for the oldest regions since it produces a distribution of teacher experience which FOSDs those of all other mechanisms. For these regions, the mechanism gives priority to the youngest teachers among those applying. Since these regions are overdemanded, the regions accept the youngest teachers, and, hence, status-quo improvement for these regions is fulfilled. Note that, under SI-DA, many young (tenured) teachers cannot apply to these regions since they are stuck in the youngest regions such as Créteil. Thus, the effective demand for these oldest regions under SI-DA is lower. This high performance of DA* in the oldest regions is achieved by accepting tenured teachers from other regions at the expense of these other regions. As we can see in Figure 1, DA* violates status-quo improvement in the three youngest regions.

Lower inequality between regions. The status-quo improvement requirement makes sure that regions are not harmed by the reassignment of teachers. Older regions become younger, and younger regions become older, reducing the initial differences in teacher experience between regions. While the previous paragraph discussed the distributional performance of the mechanisms for the three youngest and oldest regions, we now consider their performance across all regions. Figure 2 plots, for each region, the change in tenured teacher experience (proxied by the average type) between SI-CC and the status-quo matching (top left figure) and between SI-DA and the status-quo matching (bottom left figure).[56] Regions are ordered, along the horizontal axis, by average experience of their teachers at the status quo, so that all regions on the left are the younger regions that need more experienced teachers.

The top left panel of Figure 2 shows that, compared to the status-quo matching, SI-CC increases the average experience of tenured teachers in the younger regions (by 0.07 experience types on average, i.e., about 0.14 years) and reduces the average experience of tenured teachers in the older regions (by 0.13 experience types on average, i.e., 0.26 years). Therefore, SI-CC effectively lowers existing inequalities between young and old regions by about 0.4 years of experience. The top right panel of Figure 2 shows that SI-CC reduces inequality compared to not only the status-quo matching, but also TTC*.[57] As discussed when comparing SI-DA and its benchmark DA* for the oldest and youngest regions, the same conclusion may not hold for those mechanisms. Indeed, relaxing the

[56]Since after the assignment mechanisms are run new teachers are assigned for the first time, the average experience falls in all regions with respect to status quo. Therefore, first we only inspect the distribution of tenured teachers after reassignment across regions under different mechanisms and status quo.

[57]We reach similar (if not better) conclusions regarding SI-CC's better performance than its benchmark TTC* when considering both new and tenured teachers together (See Figure A.6).

status-quo improvement requirement creates a more equal distribution of teacher experience. In the bottom right panel of Figure 2, we observe that SI-DA does not reduce the average experience gap between the younger and older regions compared to DA* for tenured teachers. We summarize these findings as follows:

Fact 4 *SI-CC reduces the large gap in average tenured teacher experience that exists at the status-quo matching between the younger regions and the older regions. This gap goes down by 0.4 years of experience. SI-CC also reduces the gap by 0.3 years compared to TTC*. In contrast, SI-DA is less effective than DA* at reducing the gap in average tenured teacher experience between young and old regions.*

Limited trade-off between teacher distribution and teacher welfare. Next, we investigate whether the better distributional performance of SI-CC comes at the cost of a poorer welfare for teachers, as measured by the number of teachers who obtain a new region and the rank of assigned region in teachers' preference list. Table 3 shows that the number of tenured teachers who move is larger under the benchmark mechanisms. Under SI-CC 1,444 tenured teachers move and under TTC* 2,470 move, a difference of 1,026 teachers. For DA-inspired mechanisms, the difference is lower: 894 additional tenured teachers move under DA* compared to SI-DA.

The cost in terms of teacher mobility is larger in the three youngest region (Créteil, Versailles, and Amiens) than in the three oldest regions (Rennes, Bordeaux, and Lyon). Moreover, similar numbers, 116 vs 117, tenured teachers leave Rennes, Bordeaux, and Lyon under SI-CC and TTC*, respectively, while more tenured teachers leave Créteil, Versailles, and Amiens under TTC* (1,018) than under SI-CC (239). The lower demand for these three youngest and least attractive regions from tenured teachers compared to the oldest regions explains the huge difference in outflow under TTC*. Status-quo improvement requirement considerably bounds the outflow from the youngest regions under SI-CC.

Welfare differences between tenured and new teachers. The differences we observe between the youngest and oldest regions might explain an interesting finding: despite a larger movement under TTC*, the rank distribution of the region that new teachers are assigned is FOSDed by the distribution under SI-CC (see Panel C of Table 3).[58] It confirms our prior explanations when comparing the mechanisms with their respective benchmarks.

Fact 5 *The distribution of ranks of the regions that tenured teachers are assigned under TTC* FOSDs the one under SI-CC. The opposite FOSD comparison holds for new teachers.*

On average, new teachers are assigned their 8.2^{th} ranked region under SI-CC and their 10.2^{th} ranked region under TTC* (see Panel D of Table 3). This is because a much larger number of tenured teachers leave the youngest regions of Créteil, Versailles, and Amiens under TTC* (1,018)

[58] Although we defined FOSD relation for groups of teachers regarding preferences of regions, it is trivial to extend it to any arbitrary cumulative distribution obtained from histograms of statistics, and this is what we use here.

than under SI-CC (239). These exiting teachers have to be replaced, and new teachers are the most likely substitutes due to lower demand from other tenured teachers. This is because very few tenured teachers ask to enter younger regions and because, under TTC*, new teachers can replace tenured teachers in younger regions, even if they have a lower experience. In practice, we see that 1,415 new teachers are assigned to Amiens, Créteil, or Versailles under TTC* versus 862 under SI-CC. The large share of new teachers being assigned unattractive regions under TTC* explains why these teachers are assigned to lower ranked regions than under SI-CC.[59]

Effects on stability measures. Last, we investigate whether imposing the status-quo improvement requirement has an impact on the number of blocking pairs under different stability notions. For DA-inspired mechanisms, imposing the status-quo improvement requirement can only create more blocking pairs since it forbids certain teachers to move from their initial position and gives a top priority to new teachers over empty slots. This is indeed what we observe since SI-DA has 385 more teachers involved in at least one blocking pair (in the sense of Gale-Shapley stability) compared to its benchmark. The latter being teacher-SI stable, the only blocking pairs it has are caused by teachers staying at their initial position. In addition, SI-DA has 2,171 teachers blocking because of an empty slot.

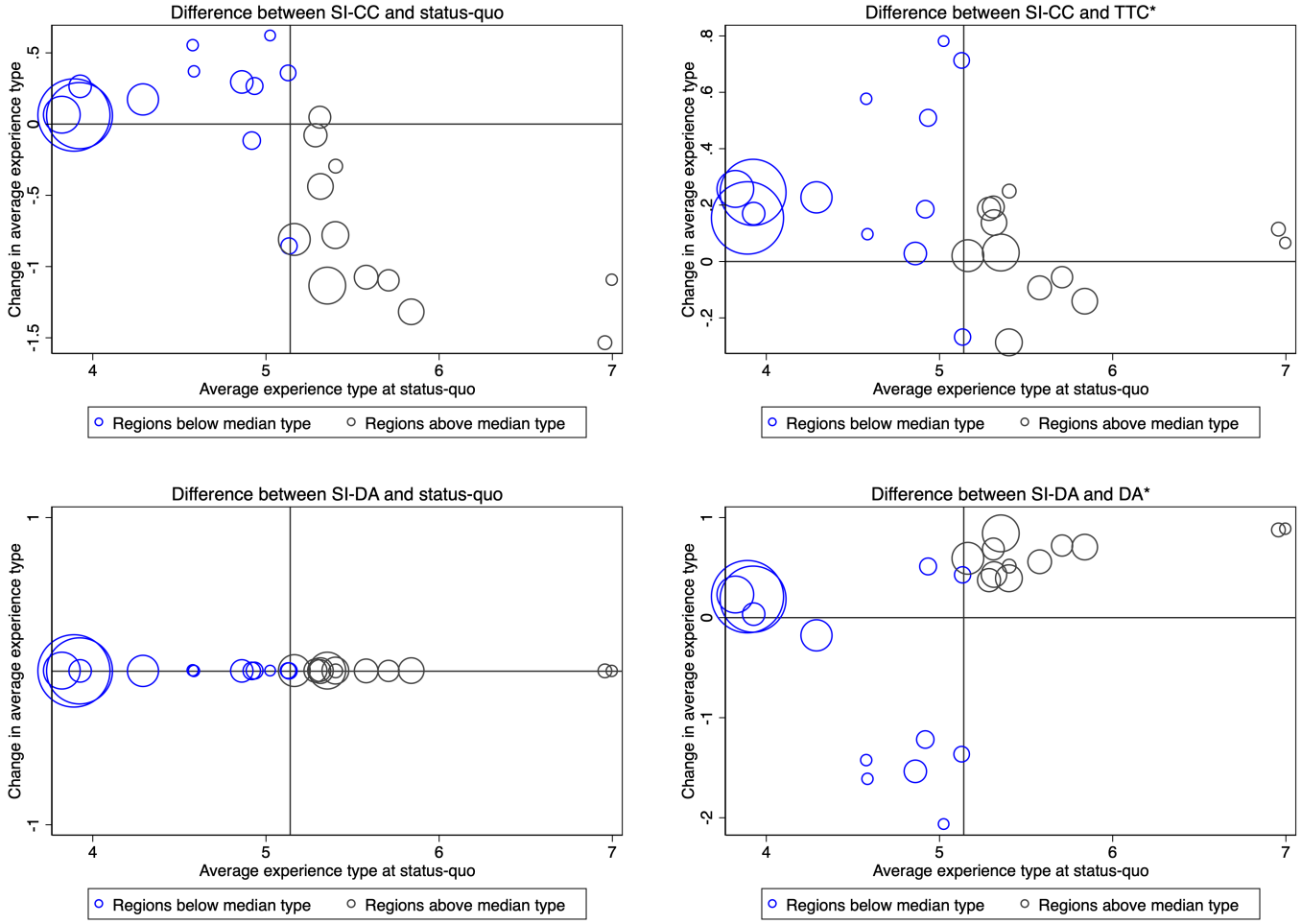
For SI-CC, the opposite happens. Indeed, since the latter and its benchmark do not impose any stability condition, the additional mobility created by relaxing the status-quo improvement constraint is done at the further expense of stability. TTC* has 268 additional teachers involved in a blocking pair compared to SI-CC. The smaller differences between the three stability notions show that blocking pairs of the TTC* and SI-CC matchings are mainly driven by their high mobility rates which imply that less preferred teachers are assigned to a new region at the expense of more preferred ones who also requested that region.

6.7 Relaxing Distributional Constraints

This last empirical subsection investigates by how much we can increase mobility if we relax the status-quo improvement distributional objectives. The results we have presented so far show that SI-CC's better distributional performance (compared to its benchmark mechanism TTC*) comes at the cost of a lower mobility, especially for teachers in the three youngest regions. We show in this section that the wSI-CC mechanism presented in Section 5, which has weaker distributional objectives than SI-CC, can increase mobility, while preserving good distributional results.

[59] Our preference estimates reveal that new teachers dislike unattractive regions less than tenured teachers. Yet, only 11.9% of Math teachers rank Créteil or Versailles as their first choice and 14.3% of French teachers. That mild preference for unattractive regions is not large enough to justify that assigning a large share of the new teachers to these two regions will improve the ranking of the region they obtain.

Figure 2: Change in Average Tenured Teacher Experience Types Across Regions: SI-CC and SI-DA



Notes: This figure shows the difference in the average teacher experience types between the matching obtained by SI-CC between the status-quo matching (top left figure) and its benchmark TTC* (top right figure). It reports the same difference between the matching obtained by SI-DA between the status-quo matching (bottom left figure) and its benchmark DA* (bottom right figure). Each observation represents a French region. Circle size reflects region size. Regions are ordered (on the horizontal axis) by average experience type of their teachers at status quo. The vertical line represents the median type. All regions on the left have an average type that is strictly below the median. This is the group of regions we identified as *younger regions*. All regions on right of the vertical line are regions whose average type is above the median, i.e., *older regions*. In regions above the horizontal line, average experience of tenured teachers after reassignment is larger than the one under the benchmark matching to which it is compared.

Table 3: Teacher Welfare and Teacher Blocking

	Suggested mech.		Benchmark mech.		Relaxed distrib. constraint	
	SI-CC	SI-DA	TTC*	DA*	wSI-CC	Current French
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A. Teacher mobility						
Tenured teachers moved and new teachers assigned	5,356	3,912	6,382	4,806	5,554	5,864
Tenured teachers moved - from the 3 youngest regions	239	0	1,018	316	464	980
Tenured teachers moved - from the 3 oldest regions	116	0	117	117	116	72
Tenured teachers moved - from all regions	1,444	0	2,470	894	1,642	1,952
New teachers assigned	3,912	3,912	3,912	3,912	3,912	3,912
New teachers unassigned	715	715	715	715	715	715
Panel B. Cumulative distribution of ranks of regions that tenured teachers are assigned						
Rank $= 1$	749	250	1,112	338	761	833
Rank ≤ 2	1,604	919	2,203	1,050	1,632	1,775
Rank ≤ 3	2,080	1,352	2,758	1,531	2,113	2,300
Rank ≤ 4	2,485	1,745	3,168	1,967	2,522	2,737
Rank any	5,833	5,833	5,833	5,833	5,833	5,833
Panel C. Cumulative distribution of ranks of regions that new teachers are assigned						
Rank $= 1$	1,243	967	645	803	1,207	621
Rank ≤ 2	1,780	1,557	1,010	878	1,723	990
Rank ≤ 3	2,128	1,942	1,289	1,397	2,059	1,263
Rank ≤ 4	2,404	2,242	1,535	1,735	2,327	1,502
Rank any	4,627	4,627	4,627	4,627	4,627	4,627
Panel D. Average rank of region assigned						
Average rank - All teachers	7.3	8.5	7.5	8.4	7.3	8.3
Average rank - Teachers from the 3 youngest regions	7.2	7.5	5.8	7.1	6.9	5.8
Average rank - Teachers from the 3 oldest regions	2.2	9.2	2.0	6.6	2.2	5.0
Average rank - Tenured teachers	6.6	8.5	5.3	7.5	6.5	6.0
Average rank - New teachers	8.2	8.6	10.2	9.5	8.4	11.1
Panel E. Number of teachers blocking under different stability notions						
Gale-Shapley stability	8,206	8,438	8,474	8,126	8,233	8,697
Teacher-SI stability	8,205	2,171	8,473	0	8,232	8,665
SI stability	8,122	0	8,408	0	8,146	8,627

Notes: Panel A of this table reports statistics on teacher mobility: numbers of tenured teachers who moved to a new region and assigned and unassigned new teachers. Panels B and C present the cumulative distribution of the ranks of the regions teachers are assigned to in their preferences. Panel D reports statistics on the average rank of the region teachers obtain. Panel E reports the numbers of teachers causing unauthorized blocks under different stability notions.

From a policy and market design perspective, currently tenured teachers accumulate priority points during their tenure at a region, and these priority points are utilized as higher priorities for all regions in the current mechanism. Therefore, a design intervention should be sensitive to these accumulated points, as the abrupt change of the mechanism from one that does not require status-quo improvement to one that does may have undesirable consequences such as retroactively eliminating earned rights and leading to an undesirable ex post facto change from a legal point of view. In that regard, adopting a milder distributional objective, at least initially, by partially relaxing the status-quo improvement requirement for these regions meets this requirement. Such a policy can be adopted temporarily in a transition period from the old mechanism to the new until these priority points are fully used up. Or if desired, it can be adopted permanently, as we show below that the distributional improvement of such a weaker requirement would still be very good.

As explained in Section 5, wSI-CC relaxes the distributional constraint on status-quo teachers by allowing some of them to leave their school while being replaced by a teacher with a potentially lower experience type. The number of teachers replaced by a less experienced teacher is finely controlled at the school level by the threshold acceptability distribution concept presented in Section 5. In this section, we explore what teacher allocations would look like if schools in the three youngest regions (Créteil, Versailles, and Amiens) were setting the vector of thresholds $d_s = (d_s^{\theta_1}, d_s^{\theta_2}, \ldots, d_s^{\theta_n})$ to $(0, \ldots, 0, |\omega_s|)$. In other words, in three regions (out of 25) we allow all status-quo teachers to be replaced by any teacher, irrespective of her experience type.[60]

There are two natural benchmarks to which we compare the wSI-CC teacher assignment. First, we compare the matchings under wSI-CC and SI-CC to quantify the mobility benefits and the potential distributional costs of relaxing the distributional objectives. Another interesting benchmark is the matching obtained with the DA* mechanism and Ministry-mandated priorities, i.e., priorities that uses Ministry's formula in determining priorities (see Footnote 34) and ignores school preferences based on teacher type rankings (where this latter approach only uses the Ministry priorities as a tie-breaker for the same type teachers in most of our empirical analysis). We refer to this mechanism as **Current French** (see also Appendix H). This second benchmark allows us to check whether partially relaxing SI-CC's distributional constraints boosts mobility while maintaining a better distribution of teachers than the one that prevails under the current French mechanism.

The results are reported in the last two columns of Table 3. Several findings stand out. First, partially relaxing the distributional objectives allows significantly more teachers to move away from the three youngest regions compared to the original SI-CC studied in the previous sections. Mobility from these regions goes up from 239 under SI-CC to 464 under wSI-CC. Second, as expected, partially relaxing distributional objectives leads to a smaller improvement in the distribution of teachers. While SI-CC and wSI-CC produce almost the same distributions in the three oldest regions—in which distributional objectives have not changed— a small difference emerges in the three youngest regions (Figure 3). wSI-CC assigns 96 more teachers with one or to years of expe-

[60]Note, however, that in contrast to TTC*, a teacher cannot leave her position without being replaced. The relaxation only implies that the incoming teacher can have a less preferred experience type. This is an important difference which explains the mobility gap that we observe for the three youngest regions between wSI-CC and TTC*.

rience to the three youngest regions than SI-CC. As a result, wSI-CC leads to a reduction of the average teacher experience in the three youngest regions (see top right results of Figure 4 in which the three youngest regions are represented by the three large blue circles on the left). The mobility gain we observe under wSI-CC in these three regions is mainly driven by newcomers who replace tenured teachers. They would have been prevented from doing so under SI-CC due to their lower experience type.

Finally, note that, for all distributional metrics considered, wSI-CC has a much better performance than the current French mechanism. In both young and old regions, the cumulative distribution of teacher exprience types clearly dominates the distribution under the current French mechanism (Figure 3). As a result, wSI-CC better fulfills the twofold objective of (i) making young regions older and (ii) making old regions younger (bottom panel of Figure 4). This is true when considering tenured teachers only but also when considering all teachers, i.e., tenured and newcomer teachers. These last results are important as they show that there is significant room to improve upon the distribution that the Ministry of Education reaches every year after the annual assignment process. This improvement comes at a small cost in terms of overall teacher mobility, 5,864 teachers move under the current French mechanism versus 5,554 under wSI-CC. For tenured teachers in the three youngest regions, 980 teachers move under the current French mechanism versus 464 under wSI-CC.

Role of tie breaking. The tie breaker rule that determines the order in which the newcomer chains are implemented influences the final allocations (as mentioned in Footnote 54).[61] All the results presented so far order teachers by decreasing order of their maximum Ministry-mandated priority points. This choice is conservative. It leads to lower mobility under both SI-CC and wSI-CC than the mobility we obtain under alternative tie breaker rules. For instance, reversing the order and ranking teachers by increasing order of their maximum Ministry-mandated priority points increases mobility under wSI-CC from 5,554 to 5,687 (and from 464 to 597 in the three young regions), which mitigates the mobility cost of wSI-CC compared to the Current French mechanism while preserving the good distributional results. This highlights that, in practice, policy makers have multiple tools that can be used to tailor the mechanism to their objectives. Investigating precisely how the different tools can impact the outcomes would be of practical interest for future research.

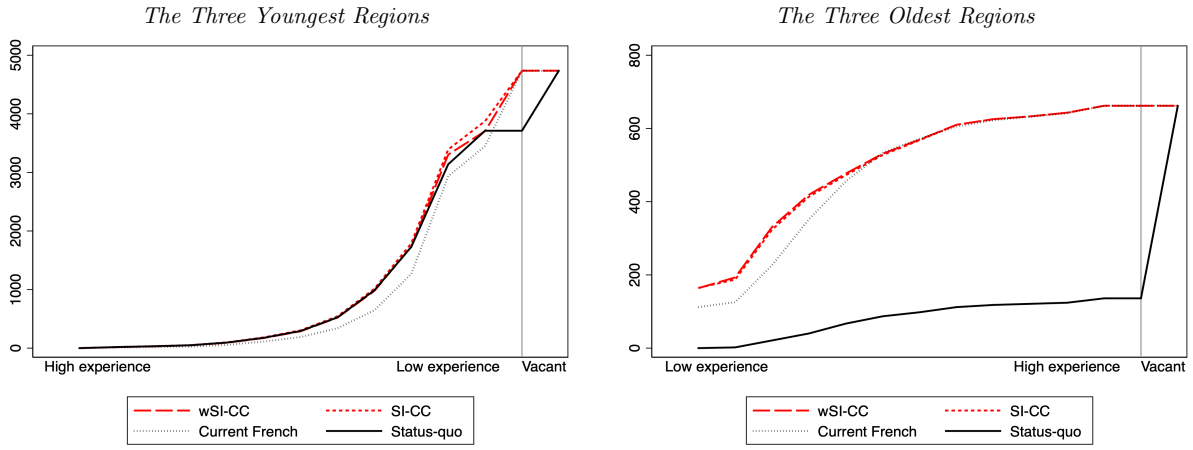
7 Related Literature

SI-CC mechanism has its roots in the top-trading cycles algorithms (see Shapley and Scarf, 1974, Abdulkadiroğlu and Sönmez, 1999, Roth et al., 2004, Dur and Ünver, 2019) and SI-DA mechanism has its roots in the teacher-proposing deferred acceptance algorithm (see Gale and Shapley, 1962, Abdulkadiroğlu and Sönmez, 2003, Hatfield and Milgrom, 2005, Kominers and Sönmez, 2016).

Our fairness-based approach using SI-DA is related to the literature on stable matching under distributional constraints. In the school choice literature, Abdulkadiroğlu and Sönmez (2003) in-

[61] In contrast, the tie breaker used for tenured teachers does not lead to significant differences in the outcomes.

Figure 3: Cumulative Distribution of Teacher Experience Types — Weaker Distributional Objectives



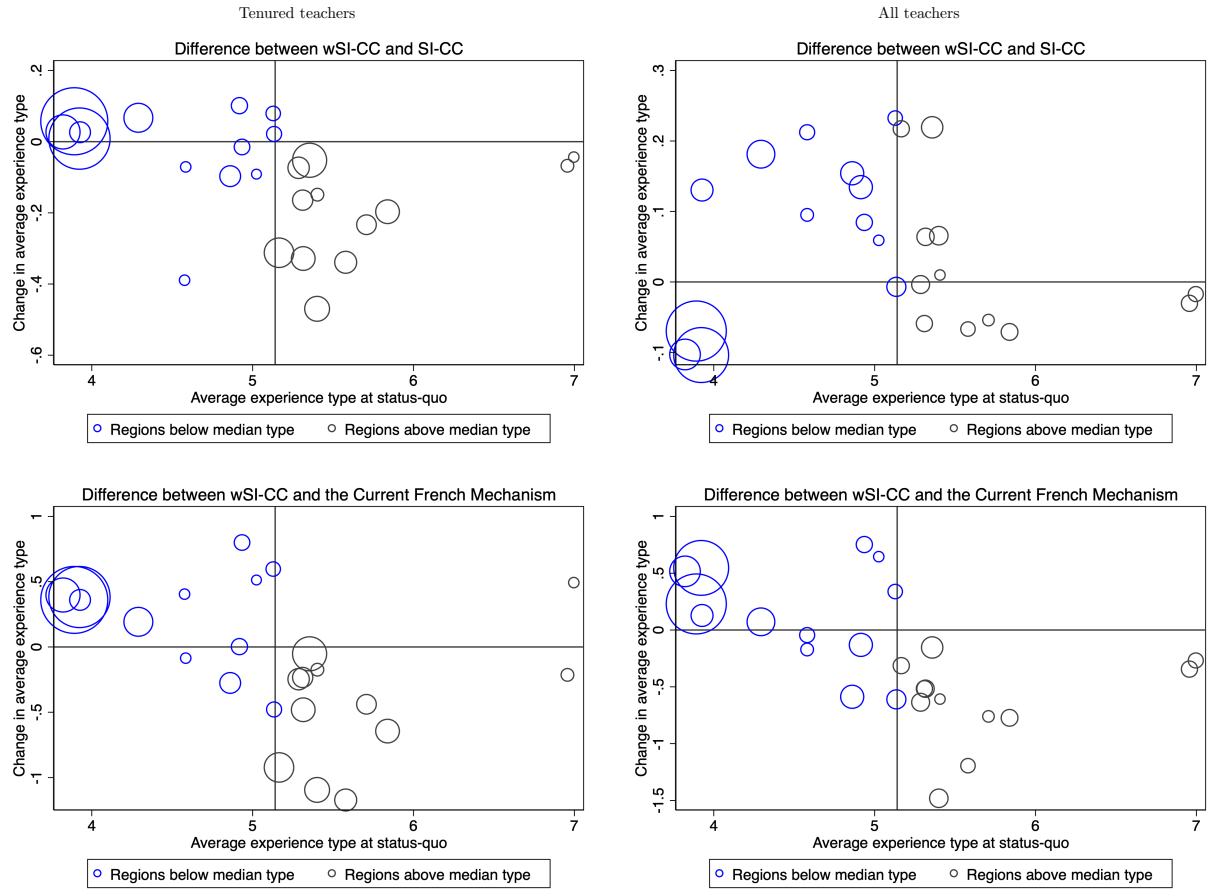
Notes: This figure shows the cumulative distribution of teacher experience types. The left panel reports the distribution in the three youngest regions of France (Créteil, Versailles, and Amiens), and the right panel the distribution in the three oldest regions of France (Rennes, Bordeaux, and Lyon). The horizontal axis reports the 13 experience types of teachers, ordered from most experienced to least experienced (left panel) and from least experienced to most experienced (right panel). The 14^{th} type corresponds to the vacant positions. The mechanisms that respect status-quo improvement are plotted in red. Those that do not are in grey. The thick black line (with legend Status-quo) corresponds to the cumulative distribution of teacher types at the status-quo matching.

troduce assignment schemes imposing type-specific ceilings at schools. Other related papers are Abdulkadiroğlu (2005), Kojima (2012), Hafalir et al. (2013), Ehlers et al. (2014), Kamada and Kojima (2015, 2016), Kojima et al. (2018), Dur et al. (2018), Sönmez and Yenmez (2019), Dur et al. (2020). While the focus in these papers is on assignment schemes to achieve diversity and other distributional objectives mostly in school choice and government-mandated job assignment context, our work applies to a teacher assignment problem where there is a status-quo matching and concern of making both sides better off. This makes our model and analysis different from the existing ones. Our methodology for constructing the SI-DA mechanism and the auxiliary choice rule is inspired by the choice rule constructions in slot-specific priorities model of Kominers and Sönmez (2016), which is also used in the latter three aforementioned papers.[62] In these papers, the choice rules and its inputs are preliminaries of the problem. However, in our framework, neither school auxiliary choice rules nor the inputs are given to us. In particular, our auxiliary choice rule is defined in this way to satisfy desired properties regarding status-quo improvement and SI stability under SI-DA mechanism, besides standard Gale-Shapley stability induced by the substitutes choice rules.

The design of efficient mechanisms in two-sided matching markets with a balanced exchange constraint was previously studied by Dur and Ünver (2019) in the context of student and worker exchange programs. The main difference between that model and the current model is that status-quo improvement was not a constraint in this previous paper. This substantially changes the

[62]Two notable exceptions in the above papers is Kamada and Kojima (2016), Kojima et al. (2018). By denoting which hospitals got acceptable assignments and which ones did not, the first paper defines distributional constraints over matchings. The second paper introduces a unique choice function selecting contracts for all hospitals at once. It then provides a sufficient condition, M-concavity from discrete convex analysis, on this choice function which guarantees the existence of stable matchings.

Figure 4: Change in Average Tenured Teacher Experience Type Across Regions — Weaker Distributional Objectives



Notes: This figure shows, for each region, the difference in the average experience of teachers under wSI-CC and under SI-CC. The left column reports the experience of tenured teachers, while the right column reports the experience of all teachers. Each observation represents a French region. Circle size reflects region size. Regions are ordered (on the horizontal axis) by average experience of their teachers at status quo. The vertical line represents the median type. All regions on the left have an average type that is strictly below the median. This is the group of regions we identified as inexperienced regions. All regions on the right of the vertical line are regions whose average type is above the median. In regions above the horizontal line, teachers average experience post reassignment is larger than the one of the benchmark matching to which it is compared.

modeling choices and mechanism design. Moreover, we have school preferences based on the first-order stochastic dominance relation over distributions of teacher types leading to a new class of pointing rules. Our design of efficient mechanisms in this domain gives in general higher welfare for the schools. We additionally focus on a fairness-based approach with SI-DA in addition to efficient mechanism design and conduct a thorough empirical analysis.

It is through our notion of improvement with respect to status-quo that we achieve a better distribution of teachers in the school system. A related approach is followed in Combe et al. (2020) where a teacher reassignment problem is studied. In this paper, they introduce a class of mechanisms, the teacher optimal block exchange (TO-BE). Their main focus is on two-sided Pareto efficiency but they show that a unique selection in this class of TO-BE mechanisms is teacher

optimal. They show that it outperforms the assignment scheme used in France which is a variation on the DA mechanism. There are four main differences between our paper and this one. First, their main theoretical and empirical results are shown by focusing mainly on teachers having a status-quo assignment, therefore, largely ignoring the imbalance issues that new teachers can create in terms of distribution of teachers.[63] The generalization they propose to account for new teachers is a two step procedure, which is further away from our SI-CC proposal. In a first step, an extension of their TO-BE mechanisms to account for new teachers is used but these new teachers are forced to only point to an employed teacher and not to vacant positions. In a second step, the unassigned new teachers from the first step are assigned to the remaining vacant positions using the DA mechanism. Instead of such a two-step approach, we propose a mechanism from scratch to account for both first-time assignment of new teachers and reassignment of tenured teachers. Indeed this difference is quite important as many of our desired properties do not automatically translate with ease (such as stability notions and status-quo improvement) to markets with vacancies and new teachers. For instance, they show that their two step procedure is not two-sided Pareto efficient and is strategy-proof only if new teachers are required to rank all the schools. Second, even with their two-sided model within their pure reassignment framework, which is a special case of our framework, we show that our SI-CC mechanism is not in the class of TO-BE mechanisms and vice-versa and so can be viewed as another strategy-proof and teacher optimal mechanism (see Example 6 in Appendix C). Third, we introduce a novel stability property that satisfies status-quo improvement, that is neither weaker nor stronger than Gale-Shapley stability and the stability notion satisfied by the current French mechanism. Note that these latter notions are not compatible with status-quo improvement. We also introduce a new mechanism SI-DA that implements this property. This mechanism can be viewed as the right benchmark to which one should compare SI-CC in our framework. Moreover, in other applications of our framework, SI-DA may play an important role in practice. Finally and foremost, their paper's theoretical motivation and empirical study are not on distributional issues. Our foremost motivation is to improve distribution of workers according to some distributional measure while status-quo improvement and incentive properties are in place. Two-sided Pareto efficiency and fairness arise as two important side objectives to fulfill in our work and our empirical work is tailored in this way. On the other hand, the primary focus in this other study is efficiency-based design and efficiency gains and documenting possible gains emprically.

Despite these two previous studies and the current paper, the study of efficient mechanisms under distributional constraints is still rare. Suzuki et al. (2018) and its generalization by Hafalir et al. (2019) provide sufficient conditions on policy goals to get a version of TTC that take constraints into account and satisfies desirable properties. In particular, these sufficient conditions involve a notion of discrete convexity on the policy goals, namely, M-convexity. In our context with new teachers and vacant positions at schools, we show that M-convexity of the policy goals is not sufficient anymore to ensure a well-behaved version of TTC (see Example 7 in Appendix C).

[63]Similarly, because Combe et al. (2020) mostly focus on markets without new teachers, the estimation of teacher preference is primarily carried out on tenured teachers, and the discussion of teachers preferences ignores the interesting difference between the preferences of tenured and new teachers, which is central in this paper.

On the empirical side, our paper also complements a fast-growing literature that explores wage-based solutions to the unequal distribution of quality teachers in schools. Building on the widespread observation of large sorting of (good) teachers in (good) schools, several recent papers have developed equilibrium models of the labor market for teachers, and used these models to look at the effect of compensation policies on the distribution of teacher quality (Biasi et al., 2021, Bobba et al., 2021, Bates et al., 2021, Tincani, 2021). Despite the tremendous progress done by these papers to shed light on price-based solutions to distribution concerns, much less is known on solutions for labor markets that do not rely on prices. Yet, understanding how to address distributional concerns in these regulated markets is of first-order importance for at least two reasons. First, because in practice many teacher labor markets do not rely on prices, or imperfectly do so. Several countries use a centralized process to assign teachers to schools, like Germany, Italy (Barbieri et al., 2011), Turkey (Dur and Kesten, 2019), Mexico (Pereyra, 2013), Peru (Bobba et al., 2021), Uruguay (Vegas et al., 2006), Portugal, and Czech Republic (Cechlárová et al., 2015).[64] In addition, the evidence points to a large cost of wage-based policies to attract good teachers in disadvantaged schools, which might encourage some countries to rely on centralized solutions to better distribute teachers across schools (Bobba et al., 2021).[65] Thakur (2020) investigates the distributional consequences of centralized assignment for Indian Administrative Service jobs, the top-tier government jobs located across the country before and after a mechanism change adapted.

Finally, our paper builds on a recent literature developing demand estimation methods in school choice environments (Abdulkadiroğlu et al., 2017; Agarwal and Somaini, 2018; Calsamiglia et al., 2020). In particular, we build on techniques based on discrete choice models with personalized choice sets which are relevant for preference estimation when reported preferences might fail to be truthfull even under strategy-proof mechanisms (Fack et al., 2019; Akyol and Krishna, 2017; Artemov et al., 2019).

8 Conclusions

Besides our novel design axioms such as SI teacher optimality, SI stability, status-quo improvement and its relaxation, we introduced two novel mechanisms that satisfy the two possibly conflicting properties, SI teacher optimality and SI stability, respectively, in addition to strategy-proofness and status-quo improvement. An important feature of the first mechanism that is aimed at efficient assignment, SI-CC, is the first time, as far as we know, a TTC-like mechanism takes preferences of both sides of the market into account in its design besides status-quo improvement. This approach relies on novel *pointing rule* design for both schools and teachers. The second mechanism, motivated by the current French teacher (re)assignment market, uses a fairness-based solution. Our novel concept, SI stability overcomes non-existence of Gale-Shapley stable and status-quo improving

[64] Beside these fully centralized markets, in most teacher labor markets (like in the US), wage variations are strongly limited by rigid pay scales that determine teachers salary as a function of their experience. Biasi et al. (2021) provides insightful discussions on non-flexible wage policies in the US: "Most US public school districts pay teachers according to "steps-and-lanes" schedules, which express a teacher's salary as a function of their experience and education."

[65] Bobba et al. (2021) finds that "it would take six times the current budget to equalize access to teacher quality across Peru".

matchings under a mild overdemand assumption that is satisfied by our application. This concept is neither weaker not stronger than Gale-Shapley stability. The SI-DA mechanism utilizes novel auxiliary choice rules for schools that are not fundamental to our problem domain unlike in other DA-like mechanisms, but introduced for defining the mechanism.

We also test these solutions against various benchmarks using field data from France. An important finding of our empirical analysis for the French teacher (re)assignment market is that SI-DA that satisfies SI stability decreases the movement of tenured teachers considerably in spite of decreasing the experience gap, and thus, fairness-based approaches, like SI-DA or the one employed by the French Ministry of Education, may be less suitable for decreasing teacher experience gap between regions. On the other hand, SI-CC that satisfies our two-sided Pareto efficiency refinement, SI teacher optimality, performs much better than the current French mechanism and other benchmarks in decreasing the experience gap of teachers while facilitating mobility. The weaker version of SI-CC leads to even more movement of tenured teachers, helping them to exercise their pre-existing accumulated priority points better. Therefore, for decreasing the experience gap, SI-CC or its wSI-CC variant are ideal mechanisms.

The applicability of our theoretical and empirical framework is not only limited to centralized teacher (re)assignment. It can be applied to any centralized two-sided matching market in which status-quo matching is aimed to be improved for both sides of the market. Examples of such markets are, including, but not limited to, student exchange programs between colleges, public school districts targeting racial balances among schools, and corporate job rotations, besides other civil service sectors. We provide more concrete details for three applications in Appendix B. Moreover, our model does not restrict the way schools value the experience of the teachers. Hence, our results hold as long as each school has rankings based on a coarse metric of characteristics of teachers such that different schools use possibly different metrics.[66] It remains as a future policy and research question to explore whether SI-DA and SI stability are more suitable for these other applications.

References

Abdulkadiroğlu, Atila, "College Admission with Affirmative Action," *International Journal of Game Theory*, 2005, *33*, 535–549.

_ and Tayfun Sönmez, "House Allocation with Existing Tenants," *Journal of Economic Theory*, 1999, *88*, 233–260.

_ and _ , "School Choice: A Mechanism Design Approach," *American Economic Review*, 2003, *93*, 729–747.

_ , Nikhil Agarwal, and Parag A. Pathak, "The welfare effects of coordinated assignment: Evidence from the New York City high school match," *American Economic Review*, 2017, *107* (12), 3635–89.

[66] Although in teacher (re)assignment, the central authority can successfully rank teachers based on their observable features and previous records, asymmetric information can be important in some of these other settings. In Appendix D, we show that only when there are two teacher types, an efficient and strategy-proof (for teachers) mechanism would also be immune to type ranking manipulation by schools, and SI-CC satisfies this property. We also explore the consequences of a weaker preference restriction than the FOSD relation for schools in the same appendix. We show that if schools have general responsive preferences that do not satisfy FOSD then an SI teacher optimal and strategy-proof mechanism does not exist (recall that SI teacher optimality is needed for strategy-proofness, two-sided Pareto efficiency, and status-quo improvement).

_ , Parag A. Pathak, Alvin E. Roth, and Tayfun Sönmez, "The Boston Public School Match," *American Economic Review Papers and Proceedings*, 2005, *95*, 368–372.

_ , _ , and _ , "The New York City High School Match," *American Economic Review Papers and Proceedings*, 2005, *95*, 364–367.

Agarwal, Nikhil, "An Empirical Model of the Medical Match," *American Economic Review*, 2015, *105* (7), 1939–78.

_ and Paulo Somaini, "Demand Analysis Using Strategic Reports: An Application to a School Choice Mechanism," *Econometrica*, 2018, *86* (2), 391–444.

AISD, "District Demographics," 2021. Austin Independent School District, Available at: `https://www.austinisd.org/planning-asset-management/district-demographics`.

Akyol, Pelin and Kala Krishna, "Preferences, selection, and value added: A structural approach," *European Economic Review*, 2017, *91*, 89–117.

Alkan, Ahmet and David Gale, "Stable Schedule Matching Under Revealed Preferences," *Journal of Economic Theory*, 2003, *112*, 289–306.

Allen, Rebecca, Simon Burgess, and Jennifer Mayo, "The teacher labour market, teacher turnover and disadvantaged schools: new evidence for England," *Education Economics*, 2018, *26* (1), 4–23.

Andersson, Tommy, *Refugee Matching as a Market Design Application*, Springer, 2019.

Artemov, Georgy, Yeon-Koo Che, and Yinghua He, "Strategic 'Mistakes': Implications for Market Design Research," 2019. mimeo.

Balinski, Michel and Tayfun Sönmez, "A tale of two mechanisms: student placement," *Journal of Economic Theory*, 1999, *84*, 73–94.

Barbieri, Gianna, Claudio Rossetti, and Paolo Sestito, "The determinants of teacher mobility: Evidence using Italian teachers' transfer applications," *Economics of Education Review*, 2011, *30* (6), 1430 – 1444. Special Issue: Economic Returns to Education.

Bates, Michael, Michael Dinerstein, Andrew C. Johnston, and Isaac Sorkin, "Teacher Preferences and Student Achievement," *Working paper*, 2021.

Biasi, Barbara, "The Labor Market for Teachers Under Different Pay Schemes," *American Economic Journal: Economic Policy*, 2021, *13* (3).

_ , Chao Fu, and John Stromme, "Equilibrium in the Market for Public School Teachers: District Wage Strategies and Teacher Comparative Advantage," *Working paper*, 2021.

Blair, Charles, "The Lattice Structure of the Set of Stable Matchings with Multiple Partners," *Mathematics of Operations Research*, 2020/01/22 1988, *13* (4), 619–628.

Bobba, Matteo, Tim Ederer, Gianmarco Leon-Ciliotta, Christopher Neilson, and Marco G. Nieddu, "Teacher Compensation and Structural Inequality: Evidence from Centralized Teacher School Choice in Peru," *Working paper*, 2021.

Bonesrønning, Hans, Torberg Falch, and Bjarne Strøm, "Teacher sorting, teacher quality, and student composition," *European Economic Review*, 2005, *49* (2), 457 – 483.

Calsamiglia, Caterina, Chao Fu, and Maia Güell, "Structural Estimation of a Model of School Choices: The Boston Mechanism versus Its Alternatives," *Journal of Political Economy*, 2020, *128* (2), 642–680.

Cechlárová, Katarína, Tamás Fleiner, David F. Manlove, Iain McBride, and Eva Potpinková, "Modelling practical placement of trainee teachers to schools," *Central European Journal of Operations Research*, 2015, *23* (3), 547–562.

Chen, Li and Juan Sebastian Pereyra, "Self-selection in school choice," *Games and Economic Behavior*, 2019, *117*, 59 – 81.

Chen, Yan and Tayfun Sönmez, "School choice: an experimental study," *Journal of Economic Theory*, 2006, *127* (1), 202 – 231.

Cheraskin, Lisa and Michael A Campion, "Study clarifies job-rotation benefits.," *Personnel Journal*, 1996, *75* (11), 31–36.

Chetty, Raj, John N. Friedman, and Jonah E. Rockoff, "Measuring the Impacts of Teachers I: Evaluating Bias in Teacher Value-Added Estimates," *American Economic Review*, 2014, *104*, 2593–2632.

Combe, Julien, Olivier Tercieux, and Camille Terrier, "The Design of Teacher Assignment: Theory and Evidence," 2020. mimeo.

Compte, Olivier and Philippe Jehiel, "Voluntary Participation and Re-assignment in Two-sided Matching," 2008. Paris School of Economics, Unpublished mimeo.

DCS, "Transfer Option Form/Diversity Plan," 2019. Davenport Community Schools, Available at: https://www.davenportschools.org/pdf/district/Diversity-Plan_Chart_19-20.pdf.

Delacrétaz, David, Scott Duke Kominers, and Alexander Teytelboym, "Matching Mechanisms for Refugee Resettlement," 2020. Working paper.

Direction de l'Evaluation de la Prospective et de la Performance, "Géographie de l'école 2014," 2014, *20*.

Dur, Umut and M. Utku Ünver, "Two-Sided Matching via Balanced Exchange," *Journal of Political Economy*, 2019, *127*, 1156–1177.

_ and Onur Kesten, "Sequential versus Simultaneous Assignment Systems and Two Applications," *Economic Theory*, 2019, *68*, 251–283.

_ , Parag A. Pathak, and Tayfun Sönmez, "Explicit vs. Statistical Targeting in Affirmative Action: Theory and Evidence from Chicago's Exam Schools," *Journal of Economic Theory*, 2020, *187*.

_ , Scott Duke Kominers, Parag A. Pathak, and Tayfun Sönmez, "Reserve Design: Unintended Consequences and the Demise of Boston's Walk Zones," *Journal of Political Economy*, 2018, *126(6)*.

ED, "Guidance on the Voluntary Use of Race to Achieve Diversity and Avoid Racial Isolation in Elementary and Secondary Schools," 2011. The United States Department of Education, Available at: https://www2.ed.gov/programs/charter/diversityguidance.pdf.

Ehlers, Lars, Isa E. Hafalir, M. Bumin Yenmez, and Muhammed A. Yildirim, "School Choice with Controlled Choice Constraints: Hard Bounds versus Soft Bounds," *Journal of Economic Theory*, 2014, *153*, 648–683.

F1S, "Majority to Minority Transfers 2019-2020," 2019. Florence 1 Schools, Available at: https://www.f1s.org/cms/lib/SC02209534/Centricity/Domain/1786/MajoritytoMinorityForm2019b.pdf.

Fack, Gabrielle, Julien Grenet, and Yinghua He, "Beyond Truth-Telling: Preference Estimation with Centralized School Choice and College Admissions," *American Economic Review*, 2019, *109* (4), 1486–1529.

Falch, Torberg, "The Elasticity of Labor Supply at the Establishment Level," *Journal of Labor Economics*, 2010, *28* (2), 237–266.

Gale, David and Lloyd S. Shapley, "College Admissions and the Stability of Marriage," *American Mathematical Monthly*, 1962, *69*, 9–15.

Guillen, Pablo and Onur Kesten, "On-Campus Housing: Theory vs. Experiment," 2008. mimeo.

_ and _ , "Matching Markets with Mixed Ownership: The Case for a Real-life Assignment Mechanism," *International Economic Review*, 2012, *53* (3), 1027–1046.

Hafalir, Isa E., Fuhito Kojima, and M. Bumin Yenmez, "Interdistrict School Choice: A Theory of Student Assignment," 2019. mimeo.

_ , M. Bumin Yenmez, and Muhammed A. Yildirim, "Effective Affirmative Action in School Choice," *Theoretical Economics*, 2013, *8*, 325–363.

Hanushek, Eric A., John F. Kain, and Steven G. Rivkin, "Why Public Schools Lose Teachers," *The Journal of Human Resources*, 2004, *39* (2), 326–354.

Hassidim, Avinatan, Déborah Marciano, Assaf Romm, and Ran I. Shorrer, "The Mechanism Is Truthful, Why Aren't You?," *American Economic Review*, May 2017, *107* (5), 220–24.

Hatfield, John William and Paul Milgrom, "Matching with Contracts," *American Economic Review*, 2005, *95* (4), 913–935.

HCS, "Majority-to-Minority Student Transfer Application," 2020. Huntsville City Schools, Available at: https://www.huntsvillecityschools.org/sites/default/files/2021%202022%20M2M%20FAQ.pdf.

Jackson, Kirabo C., "Student Demographics, Teacher Sorting, and Teacher Quality: Evidence from the End of School Desegregation," *Journal of Labor Economics*, 2009, *27* (2), 213–256.

Jones, Will and Alexander Teytelboym, "Choices, preferences and priorities in a matching system for refugees," *Forced Migration Review*, January 2016, *51*, 80–82.

_ and _ , "The Local Refugee Match: Aligning Refugees' Preferences with the Capacities and Priorities of Localities," *Journal of Refugee Studies*, 2017, *31*, 152–158.

Kamada, Yuichiro and Fuhito Kojima, "Efficient Matching under Distributional Constraints: Theory and Applications," *American Economic Review*, 2015, *105(1)*, 67–99.

_ and _ , "Stability Concepts in Matching under Distributional Constraints," *JET*, 2016.

Kelso, Alexander and Vincent Crawford, "Job matching, coalition formation, and gross substitutes," *Econometrica*, 1982, *50*, 1483–1504.

Kojima, Fuhito, "School choice: Impossibilities for affirmative action," *Games and Economic Behavior*, 2012, *75*, 685–693.

_ , Akihisa Tamura, and Makoto Yokoo, "Designing matching mechanisms under constraints: An approach from discrete convex analysis," *Journal of Economic Theory*, 2018, *176*, 803–833.

Kominers, Scott D. and Tayfun Sönmez, "Matching with Slot-Specific Priorities: Theory," *Theoretical Economics*, 2016, *11*, 683–710.

Pais, Joana and Agnes Pinter, "School choice and information: An experimental study on matching mechanisms," *Games and Economic Behavior*, 2008, *64* (1), 303 – 328.

Pereyra, Juan Sebastián, "A Dynamic School Choice Model," *Games and Economic Behavior*, 2013, *80*, 100–114.

Rees-Jones, Alex, "Suboptimal behavior in strategy-proof mechanisms: Evidence from the residency match," *Games and Economic Behavior*, 2018, *108*, 317 – 330.

Rockoff, Jonah E., "The Impact of Individual Teachers on Student Achievement: Evidence from Panel Data," *American Economic Review*, 2004, *94*, 247–252.

Roth, Alvin E., "The Economics of Matching: Stability and Incentives," *Mathematics of Operations Research*, 1982, *7*, 617–628.

_ , "The Evolution of the Labor Market for Medical Interns and Residents: A Case Study in Game Theory," *Journal of Political Economy*, 1984, *92*, 991–1016.

_ , "Stability and Polarization of Interests in Job Matching," *Econometrica*, 1984, *52* (1), 47–57.

_ **and Elliot Peranson**, "The Redesign of the Matching Market for American Physicians: Some Engineering Aspects of Economic Design," *American Economic Review*, 1999, *89* (4), 748–780.

_ **and Marilda A. O. Sotomayor**, *Two-sided matching: a study in game-theoretic modeling and analysis*, Cambridge: Cambridge University Press, 1990.

_ **, Tayfun Sönmez, and M. Utku Ünver**, "Kidney Exchange," *Quarterly Journal of Economics*, 2004, *119*, 457–488.

SCPS, "Elementary School Transfer Options," 2021. Seminole County Public Schools, Available at: `https://www.scps.k12.fl.us/district/departments/assignment-access/step-2/capacity-transfer-options.stml`.

Shapley, Lloyd and Herbert Scarf, "On Cores and Indivisibility," *Journal of Mathematical Economics*, 1974, *1*, 23–37.

Sidibe, Modibo, Bocar Ba, Patrick Bayer, Nayoung Rim, and Roman Rivera, "Police Officer Assignment and Neighborhood Crime," 2021.

Sönmez, Tayfun and M. Bumin Yenmez, "Affirmative Action in India via Vertical and Horizontal Reservations," 2019. mimeo.

_ **and M. Utku Ünver**, "Matching, Allocation, and Exchange of Discrete Resources," in Jess Benhabib, Alberto Bisin, and Matthew O. Jackson, eds., *Social Economics*, Vol. 1A of *Handbooks in Economics*, Elsevier, 2010, pp. 781–852.

SPSK12, "Majority-to-Minority Transfer Option," 2021. Suffolk Public Schools, Available at: `https://www.spsk12.net/departments/curriculum___instruction/majority-to-minority_transfer_option`.

Suzuki, Takamasa, Akihisa Tamura, and Makoto Yokoo, "Efficient allocation mechanism with endowments and distributional constraints," *Proceedings of the 17th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2018) M. Dastani, G. Sukthankar, E. André, S. Koenig (eds.)*, 2018, *July 10-15, Stockholm, Sweden.*

Thakur, Ashutosh, "Matching in the Civil Service: A Market Design Approach to Public Administration and Development," 2020. Stanford GSB Working Paper.

Tincani, M M., "Teacher labor markets, school vouchers and student cognitive achievement: Evidence from Chile," *Quantitative Economics*, 2021, *12* (1), 173–216.

Vegas, Emiliana, Miguel Urquiola, and Pedro Cerdàn-Infantes, "Teacher Assignment, Mobility and their Impact on Equity and Quality of Education in Uruguay," 2006. Washington, DC: The World Bank, Working Paper.

Appendices

Appendix A Omitted Result and Proofs

Proposition 6 *Let ψ be a status-quo improving and two-sided Pareto efficient mechanism which selects a matching that is not SI teacher optimal whenever such a matching exists. Then, ψ is not strategy-proof.*

Proof: On the contrary suppose ψ is strategy-proof. Let $S = \{s_1, s_2, s_3\}$, $T = \{t_1, t_2, t_3\}$, $\omega_{s_1} = \{t_1\}$, $\omega_{s_2} = \{t_2\}$, $\omega_{s_3} = \{t_3\}$ and $q_{s_1} = q_{s_2} = q_{s_3} = 1$. Let $\tau(t_3) \rhd_{s_1} \tau(t_2) \rhd_{s_1} \tau(t_1) \rhd_{s_1} \theta_\emptyset$, $\tau(t_1) \rhd_{s_2} \tau(t_3) \rhd_{s_2} \tau(t_2) \rhd_{s_2} \theta_\emptyset$ and $\tau(t_2) \rhd_{s_3} \tau(t_1) \rhd_{s_2} \tau(t_3) \rhd_{s_3} \theta_\emptyset$, $s_3\ P_{t_1}\ s_2 P_{t_1}\ s_1 P_{t_1}\ \emptyset$, $s_1\ P_{t_2}\ s_3 P_{t_2}\ s_2 P_{t_2}\ \emptyset$, and $s_2\ P_{t_3}\ s_1 P_{t_3}\ s_3 P_{t_3}\ \emptyset$.

There exists a unique SI teacher optimal matching, denoted by ν, in which each teacher is assigned to her top choice. In any other status-quo improving two-sided Pareto efficient matching at most one teacher is assigned to her top choice and at least one teacher is assigned to her second choice. Suppose ψ selects two-sided Pareto efficient and status-quo improving matching μ in which t_1 is assigned to her second choice s_2. If t_1 reports only s_3 and s_1 acceptable, then ν is the unique SI teacher optimal matching and ψ assigns t_1 to s_1, t_2 to s_3 and t_3 to s_2. In this updated market, if t_2 reports only s_1 and s_2 acceptable, then ν is the unique SI teacher optimal matching and ψ assigns t_1 to s_3, t_2 to s_2 and t_3 to s_1. Finally, in this updated market, if t_3 reports only s_2 and s_3 acceptable, then ν is the two-sided Pareto efficient and status-quo improving matching and ψ needs to select it. However, this contradicts with strategy-proofness of ψ, i.e., it is manipulated by t_3.

We can show the same result for any two-sided Pareto efficient and status-quo improving matching in which at least one teacher is assigned to her second choice under the original market. ■

Proof of Proposition 1: On the contrary, suppose μ is SI teacher optimal and it is Pareto dominated by ν. Since μ status-quo improves ω so does ν. Hence, ν is status-quo improving. In addition, since ν Pareto-dominates μ, all teachers weakly prefer ν to μ. Because, $\nu \neq \mu$ and teachers' preferences are strict, some teachers strictly prefers ν to μ. However, this violates SI teacher optimality of μ. This is a contradiction. ■

Proof of Theorem 1: SI teacher optimality: Recall that the requirement of status-quo improvement is embedded in the definition of SI teacher optimality. Consider an arbitrary market P. Let $\hat{\mu}$ be the outcome of SI-CC under this market. We proceed in two parts.

1. We first show that $\hat{\mu}$ is status-quo improving.
   First, consider teachers. Under SI-CC, each school s points to all teachers in ω_s one by one. When a teacher $t \in \omega_s$ is pointed by s in some Step k, then $s \in A_t^k$ and she can always form a one-school cycle (s, t) whenever she points to s. Similarly, any new teacher $t \in N$ can form a cycle with $\emptyset$ in any step of SI-CC. Hence, $\hat{\mu}_t\ R_t\ \omega_t$ for all $t \in T$.

Next, consider schools. The tail of any executed chain is a new teacher. Hence, if in some step of SI-CC, a school s is sending out a teacher then it is simultaneously acquiring another teacher; as a result $|\hat{\mu}_s| \geq |\omega_s|$. In any step k of SI-CC, when we consider the set of remaining status-quo employees and teachers assigned in the first $k-1$ steps, because of the positive balance requirement in Improvement Condition 1, the previous observation and the fact that a teacher cannot be assigned a school if she is unacceptable, each school s is weakly better off compared to ω_s. Hence, $\hat{\mu}_s \succsim_s \omega_s$ for all $s \in S$.

We showed that $\hat{\mu}$ is status-quo improving.

2. Before proving $\hat{\mu}$ cannot be Pareto dominated by another status-quo improving matching for teachers, we first state a claim that will be used in the proof.

Claim 1: For a school s, suppose step K is the final step in which school s is assigned a teacher in an executed chain such that s is the head of this chain. Let the set of remaining status-quo employees of s at the end of step K be denoted as ω_s^K and all assigned teachers to s from the beginning of step 1 until the end of step K as μ_s^K. Let ν be a matching such that all teachers assigned in the first K steps of SI-CC are matched with their assignment under SI-CC and $|\nu_s| < |\omega_s^K \cup \mu_s^K|$. Then, ν is not a status-quo improving matching.

Proof of Claim 1: Suppose teacher t is assigned to s in this chain in step K and t is pointing s under Condition 2. First observe that $\mu_s^K \subseteq \nu_s$. Also notice that, if $\omega_s^K = \emptyset$, then $|\nu_s| = |\omega_s^K \cup \mu_s^K|$. Hence, $\omega_s^K \neq \emptyset$. Since Condition 1 does not hold for school s via teacher t, there exists some type θ such that $\tau(t_s^K) \trianglerighteq_s \theta \triangleright_s \tau(t)$ and

$$\sum_{\theta' \trianglerighteq_s \theta} b_s^{\theta'} \leq 0$$

where $b_s^{\theta'}$ is the current balance of type θ' at step K of SI-CC.[67] That is, the number of teachers with weakly better type than θ in $\mu_s^K \cup \omega_s^K$ cannot be more than what it is in ω_s. Moreover, all teachers in ω_s^K have weakly better type than θ. Hence, $|\nu_s| < |\omega_s^K \cup \mu_s^K|$ and $\mu_s^K \subseteq \nu_s$ imply that the number of teachers with weakly better type than θ in ν_s is strictly less than this number in ω_s. Therefore, ν_s is not preferred to ω_s by school s. $\diamond$

Next, we show that $\hat{\mu}$ cannot be Pareto dominated by another status-quo improving matching for teachers.

On the contrary, suppose there exists a status-quo improving matching ν that Pareto dominates $\hat{\mu}$ for teachers. By considering the teachers assigned in each step of SI-CC inductively, we show that such a matching cannot exist, in particular we should have $\nu = \hat{\mu}$.

We denote the set of teachers assigned in step k of SI-CC under market P with T_k and union of these sets up to step k as $\bar{T}_k \equiv \cup_{k'=1}^{k} T_{k'}$.

Step 1: Each teacher $t \in T_1$ is assigned in $\hat{\mu}_t$ to the best school in A_t^1. If $\nu_t P_t \hat{\mu}_t$ for some $t \in T_1$,

[67] Actually, sum of balances $\sum_{\theta' \trianglerighteq_s \theta} b_s^{\theta'}$ never becomes negative in the mechanism for any type θ, as the sum starts at zero at the beginning of Step 1, and whenever it is zero, we do not admit a teacher with a type worse than θ by sending out a teacher with a type better than θ by Improvement Condition 1.

then $\nu_t \notin A_t^1$. Thus, for school $s \equiv \nu_t$ both improvement conditions are violated via teacher t. Since this is Step 1, the current matching satisfies $\mu = \emptyset$, and hence, the current balances $b_s^\theta = 0$ for all schools s and types θ. The violation of Condition 1 implies that

- if there exists a teacher $t_s^1 \in \omega_s$ that s is pointing, then it has type $\tau(t_s^1) \triangleright_s \tau(t)$: thus, t has a worse type than the worst type status-quo employees of this school; and
- if such a teacher does not exist, then $\omega_s = \emptyset$.

Thus, in either case, $\nu_s \setminus \{t\} \succsim_s \omega_s$ and $|\nu_s| > |\omega_s|$ as otherwise ν is not status-quo improving for s by FOSD preferences. The violation of Condition 2 for s via t, on the other hand, implies one of the following conditions to hold:

- t is not acceptable for s: in this case status-quo improvement for s under ν would be violated; or
- there are no new teachers: in this case, as we showed $|\nu_s| > |\omega_s|$ implies that there exists some schools s' such that $|\omega_{s'}| > |\nu_{s'}|$; as a result in this case status-quo improvement for s under ν would be violated by FOSD preferences; or
- $q_s = |\omega_s|$: in this case, as we showed $|\nu_s| > |\omega_s|$, $|\nu_s| > q_s$ contradicting the feasibility of ν as matching.

Then, Condition 2 cannot be violated as none of these conditions hold, which is a contradiction. Hence, such a teacher t cannot exist with $\nu_t P_t \hat{\mu}_t$. Since $\nu_t R_t \hat{\mu}_t$ for all t then for all $t \in T_1$, $\nu_t = \hat{\mu}_t$.
Inductive assumption: For any $k > 1$, Assume that for all $k' < k$ and $t \in T_{k'}$, $\nu_t = \hat{\mu}_t$. We show that the same holds for teachers in T_k:
Step k: Each teacher $t \in T_k$ is assigned in $\hat{\mu}_t$ to the best school in A_t^k. If $\nu_t P_t \hat{\mu}_t$ for some $t \in T_k$, then $\nu_t \notin A_t^k$. Thus, for school $s \equiv \nu_t$ both improvement conditions are violated via teacher t. Noting μ is the current matching determined until the end of step $k-1$, the violation of Condition 1 implies that

- if there exists a teacher $t_s^k \in \omega_s$ that s is pointing, then it has type $\tau(t_s^k) \triangleright_s \tau(t)$ and there exists an intermediate type θ such that $\tau(t_s^k) \trianglerighteq_s \theta \triangleright_s \tau(t)$ with

$$\sum_{\theta' \trianglerighteq_s \theta} |\mu_s^{\theta'}| - |\{t' \in \omega_s : \mu_{t'} \neq \emptyset\}^{\theta'}| \leq 0.$$

  By the inductive assumption for the current matching $\mu_{t'} = \nu_{t'}$ for all t' assigned until this step (i.e., those in $\bar{T}_{k-1}$), and hence we also have

$$\sum_{\theta' \trianglerighteq_s \theta} \left|\left(\nu_s \cap \bar{T}_{k-1}\right)^{\theta'}\right| - \left|\left(\omega_s \cap \bar{T}_{k-1}\right)^{\theta'}\right| \leq 0.$$

  Teacher t has a worse type than the remaining worst-type status-quo employee of this school i.e., those in $\omega_s \setminus \bar{T}_{k-1}$. Thus, in ν replacing any of these employees with t would violate status-quo improvement for s in ν, as this would have led to an FOSD violation for type θ:

$$\sum_{\theta' \trianglerighteq_s \theta} \left|\nu_s^{\theta'}\right| - \left|\omega_s^{\theta'}\right| < 0.$$

Then t does not replace any of the remaining status-quo employees, but she is an additional teacher acquired: $|\nu_s \setminus \bar{T}_{k-1}| > |\omega_s \setminus \bar{T}_{k-1}|$.

- if such a teacher does not exist, then $\omega_s \setminus \bar{T}_{k-1} = \emptyset$, and hence, as $t \in \nu_s \setminus \bar{T}_{k-1}$ we have $|\nu_s \setminus \bar{T}_{k-1}| > |\omega_s \setminus \bar{T}_{k-1}|$.

Observe that in the algorithm at each step we make sure that each school acquires at least as many teachers as it sends out and hence, for the current matching $|\mu_s| \geq |\omega_s \cap \bar{T}_{k-1}|$. Since $\mu_s = \nu_s \cap \bar{T}_{k-1}$ by the inductive assumption, we have $|\nu_s \cap \bar{T}_{k-1}| \geq |\omega_s \cap \bar{T}_{k-1}|$. Therefore, as we also showed that $|\nu_s \setminus \bar{T}_{k-1}| > |\omega_s \setminus \bar{T}_{k-1}|$ above, we obtain $|\nu_s| > |\omega_s|$.

The violation of Condition 2 for s via t, on the other hand, implies one of the following conditions to hold:

- t is not acceptable for s: in this case status-quo improvement for s under ν would be violated; or
- there are no remaining new teachers: Claim 1 implies that there exists at least one school s' such that $\nu_{s'}$ does not status-quo improve upon $\omega_{s'}$; or
- $q_s = |\omega_s|$: in this case, as we showed $|\nu_s| > |\omega_s|$, $|\nu_s| > q_s$ contradicting the feasibility of ν as matching.

Then, Condition 2 cannot be violated as none of these conditions hold, which is a contradiction. Hence, such a teacher $t \in T_k$ with $\nu_t P_t \hat{\mu}_t$ cannot exist.

Since $\nu_t R_t \hat{\mu}_t$ for all t then for all $t \in T_k$, $\nu_t = \hat{\mu}_t$, completing the induction and showing that $\nu = \hat{\mu}$.

Strategy-proofness: We state two claims that we will use in the proof.

Claim 2: Suppose teacher t is assigned in step K of SI-CC. For any $k < K$, then $A_t^{k+1} \subseteq A_t^k$.

Proof of Claim 2: Let $s \notin A_t^k$. We will show that $s \notin A_t^{k+1}$. We consider two possible cases.

Case 1: s does not have an unfilled position at step k: First notice that, if there is no remaining status-quo employee of s in step k, then it should have been removed in an earlier step of SI-CC. Then, there exists some type θ such that $\tau(t_s^k) \trianglerighteq_s \theta \triangleright_s \tau(t)$ with

$$\sum_{\theta' \trianglerighteq_s \theta} b_s^{\theta'} \leq 0$$

where $b_s^{\theta'}$ is the current balance of type θ' at step k of SI-CC. If school s is part of the executed cycle or chain in step k, then the teacher assigned to s has a type weakly better than type θ under $\triangleright_s$ and similarly, the teacher leaving school s, namely, t_s^k also has a type weakly better than type θ. Hence, after executing the cycle in step k relation above still holds. Moreover, s cannot send out a status-quo employee without getting a new one by the definition of SI-CC. Similarly, s cannot get a teacher without sending a status-quo employee. If school s is not part of the executed cycle or chain in step k, equation above still holds. In either case, $s \notin A_t^{k+1}$.

Case 2: s has an unfilled position at step k: Either t is unacceptable for s or t is acceptable for s but there does not exist a remaining new teacher in step k. If the former case holds, then $s \notin A_t^{k+1}$

by definition.

If the latter case holds, then either s does not have remaining status-quo employee or there exists some type θ such that $\tau(t_s^k) \trianglerighteq_s \theta \triangleright_s \tau(t)$ and

$$\sum_{\theta' \trianglerighteq_s \theta} b_s^{\theta'} \leq 0$$

where $b_s^{\theta'}$ is the current balance of type θ' at step k of SI-CC. If the former subcase holds, then neither Condition 1 nor Condition 2 holds for t in step $k+1$. For the later condition, we refer to Case 1 above. Hence, $s \notin A_t^{k+1}$. $\diamond$

Claim 3: Consider a step k of SI-CC mechanism such that there exists a path of schools and teachers $(s_1, t_1, s_2, t_2, \ldots, s_\ell, t_\ell)$ in which school $s_{\ell'}$ points to teacher $t_{\ell'}$ and teacher $t_{\ell'-1}$ points to school $s_{\ell'}$ for each $\ell' < \ell$ and $s_1 \in A_{t_\ell}^k$. If none of the schools in this path are assigned a teacher in this step, the same path forms in step $k+1$ and $s_1 \in A_{t_\ell}^{k+1}$.

Proof of Claim 3: As no teacher is assigned to the schools of the path in step k, the teachers in the path remain in the step $k+1$. Since $t_{\ell'} \equiv t_{s_{\ell'}}^k$ is the highest priority remaining status-quo employee in step k of school $s_{\ell'}$, she continues to be so in step $k+1$, thus, school $s_{\ell'}$ points to $t_{\ell'}$ in step $k+1$. No other status quo employee of these schools is assigned to any other school in step k, either, because the assignment of status-quo employees requires the school pointing to them and each school points to at most one teacher in this step. Thus, as Condition 1 or Condition 2 holds for each school $s_{\ell'}$ via teacher $t_{\ell'-1}$ (in modulo ℓ, thus $t_0 \equiv t_\ell$) in step k, the same condition continues to hold in step $k+1$ via the same teacher. Hence, $s_{\ell'} \in A_{t_{\ell'-1}}^{k+1}$ for each ℓ'. Since $A_{t_{\ell'-1}}^{k+1} \subseteq A_{t_{\ell'-1}}^k$ by Claim 2, and $s_{\ell'}$ is the favorite school of teacher $t_{\ell'-1}$ in the opportunity set in step k, we still have $s_{\ell'}$ as the favorite school of teacher $t_{\ell'-1}$ in step $k+1$ and she continues to point to $s_{\ell'}$ in Step $k+1$. $\diamond$

We are ready to finish the proof for the strategy-proofness of SI-CC. Recall that we denote the set of teachers assigned in step k of SI-CC with T_k. First, notice that a teacher t' cannot change the schools in $A_{t'}^1$ by misreporting her preferences since $A_{t'}^1$ does not depend on the submitted preferences. Moreover, by Claim 2, $\{A_t^k\}$, the opportunity sets for teacher t, weakly shrink in through out SI-CC. Hence, a teacher t cannot be assigned to a school $s \notin A_t^1$ under SI-CC. We first consider the teachers in T_1. Each $t \in T_1$ is assigned to her best choice in A_t^1. Hence, any teacher $t \in T_1$ cannot benefit from misreporting her preferences.

Next, we consider a teacher $t \in T_2$. As explained above, teacher t cannot be assigned to school $s \notin A_t^1$ under SI-CC. Teacher $t \in T_2$ is assigned to best school in A_t^2 when she submits her true preferences. We denote the best school in A_t^2 according to P_t with s'. By Claim 2, $A_t^2 \subseteq A_t^1$. Hence, if $t \in T_2$ can benefit from misreporting her preferences, then she is assigned to some school $s \in A_t^1 \setminus A_t^2$. If $A_t^1 \equiv A_t^2$, then t cannot benefit from misreporting her preferences. Suppose $A_t^1 \setminus A_t^2 \neq \emptyset$. We will show that t cannot be assigned to a school $s \in A_t^1 \setminus A_t^2$ such that $s \; P_t s'$ by misreporting. Particularly, we show t cannot prevent the cycle or chain executed in step 1 without

hurting herself.

First notice that, if t forms a cycle in step 1 by misreporting and pointing to some school $s'' \in A_t^1$, then by Claim 3, $s'' \in A_t^2$ and the path leading to t in this cycle starting with school s'' forms again when she submits P_t, which does not match her in step 1. Hence, any such school s'' cannot be preferred to s', i.e., t's assignment under truthtelling.

If a chain is executed in step 1, teacher t cannot be a part of that chain by misreporting and pointing some other school in A_t^1. This follows from the fact that the executed chain starts with a specific new teacher and a teacher $\bar{t}$, who is pointed by her status-quo school $\bar{s}$, can only be added to the executed chain if a previously included teacher points to $\bar{s}$, independent of $\bar{t}$'s preference. Teacher t can prevent the executed chain by only forming a cycle by misreporting. However, as explained above, under such a cycle t will be assigned to a school weakly worse than s'.

Moreover, with a similar reasoning to a chain, teacher t cannot affect the executed cycles in step 1 by submitting a different preference without being matched in step 1 in a new cycle (and therefore, making her weakly worse off as we showed above).

By using similar arguments, we can show that any teacher in T_k where $k > 2$ cannot benefit from misreporting her preferences. ■

Proof of Proposition 2: We first show the existence of Gale-Shapley stable matching. Consider a market P. We construct a strict rank order list, $\hat{\succ}_s$, for each school s over the teachers as follows: for any $t, t' \in T$

- if $\tau(t) \triangleright_s \tau(t')$ then $t \hat{\succ}_s t'$;
- if $\tau(t) = \tau(t')$, then the relative order between t and t' is determined arbitrarily;
- $\tau(t) \triangleright_s \theta_\emptyset$ if and only if $t \hat{\succ}_s \emptyset$.

It is easy to verify that the outcome of teacher-proposing DA algorithm (Abdulkadiroğlu and Sönmez, 2003) under $(P, \hat{\succ})$ is Gale-Shapley stable.

Next, we show that for some market there does not exist a Gale-Shapley stable and status-quo improving matching. Let $S = \{s, s'\}$, $T = \{t_1, t_2\}$, the status-quo matching be

$$\omega_s = \{t_1\},\ \omega_{s'} = \{t_2\},$$

with quotas $q_s = q_{s'} = 1$, type rankings $\tau(t_1) \triangleright_s \tau(t_2) \triangleright_s \theta_\emptyset$, and $\tau(t_1) \triangleright_{s'} \tau(t_2) \triangleright_{s'} \theta_\emptyset$. The preferences of the teachers are

$$s' \ P_{t_1} \ s \ P_{t_1} \ \emptyset,$$

$$s' \ P_{t_2} \ s \ P_{t_2} \ \emptyset,$$

Under this market, unique status-quo improving matching is ω. However, ω is blocked by (t_1, s'). ■

Proof of Proposition 3: Under the current French mechanism, when there are no vacant

positions, each school fills its capacity and only the status-quo employees are assigned to the schools. This follows from the fact that teachers in ω_s have the q_s highest priority at school s and they are considering their status-quo school acceptable. Hence, if there is a blocking pair (t, s) and $\tau(t) \ \triangleright_s \ \tau(t')$ and t' is assigned to s, then $t' \in \omega_s$. Teacher-SI stability and status-quo improving property of ω directly follows from the definition.

Next, by slightly modifying the example in the proof of Proposition 2, we show that when there are no vacant positions at schools the current French mechanism is not status-quo improving. Consider the example in the proof of Proposition 2 such that teacher t_2 prefers school s most. Then, the French mechanism assigns t_1 and t_2 to s' and s, respectively. This matching is not status-quo improving for school s.

Finally, via an example, we show that when there are vacant positions at some school, then there does not exist a teacher-SI stable and status-quo improving matching. Let $S = \{s, s'\}$, $T = \{t_1\}$, $\omega_s = \{t_1\}$, $\omega_{s'} = \emptyset$. Each school has one available position and t_1 prefers s' to s. The unique status-quo improving matching is ω but it is blocked by (t_1, s'). Hence, it is not teacher-SI stable. ■

Proof of Proposition 4: Substitutes: On the contrary, we suppose there exist $\bar{T} \subseteq T$ and distinct $t, t' \in \bar{T}$ such that $t \in C_s(\bar{T})$ and $t \notin C_s(\bar{T} \setminus \{t'\})$. There exists some other teacher t'' who was assigned to s^k under $C_s(\bar{T} \setminus \{t'\})$, where s^k is t's slot under $C_s(\bar{T})$. Consider the execution of the algorithm to determine $C_s(\bar{T} \setminus \{t'\})$ in step k when t'' is assigned to s^k: as t is not assigned in $C_s(\bar{T} \setminus \{t'\})$, she is still available and is not picked by slot s^k; thus, $t'' \succ_s^k t$. As a consequence, when the algorithm was executed to determine $C_s(\bar{T})$, teacher t'' was already assigned to a slot $s^{k''}$ such that $k'' < k$ so that she was not available when t was assigned s^k.

We will show that such a teacher t'' cannot exist, leading to a contradiction and completing the proof for the substitutes condition.

Claim: There is no teacher $\bar{t}$ such that she is assigned to a slot $s^{\hat{k}}$ in $C_s(\bar{T} \setminus \{t'\})$ and to a slot $s^{\bar{k}}$ in $C_s(\bar{T})$ such that $\bar{k} < \hat{k}$.

Proof of Claim: Suppose to the contrary such a teacher $\bar{t}$ exists. Let $\bar{t}$ be chosen such that $\bar{k}$ is the smallest such index among the indexes of slots filled by such teachers in $C_s(\bar{T})$.

If slot $s^{\bar{k}}$ is unfilled in $C_s(\bar{T} \setminus \{t'\})$, then as $\bar{t}$ is still available when slot $s^{\bar{k}}$ is filled in determining $C_s(\bar{T})$ by the supposition, we should have $\emptyset \succ_s^{\bar{k}} \bar{t}$. But then teacher $\bar{t}$ cannot be assigned to $s^{\bar{k}}$ in $C_s(\bar{T})$.

If a teacher $\hat{t}$ is assigned to $s^{\bar{k}}$ in $C_s(\bar{T} \setminus \{t'\})$, then as $\bar{t}$ is still available when slot $s^{\bar{k}}$ is filled in determining $C_s(\bar{T})$ by the supposition. Therefore, by the choice of $\bar{k}$, teacher $\hat{t}$ is not assigned a slot preceding $s^{\bar{k}}$ in $C_s(\bar{T})$. Therefore, she is available when $s^{\bar{k}}$ is filled in $C_s(\bar{T})$. Yet she is not picked even though $\hat{t} \succ_s^{\bar{k}} \bar{t} \succ_s^{\bar{k}} \emptyset$, a contradiction. Thus, such a teacher $\bar{t}$ cannot exist. ◇

Law of Aggregate Demand: On the contrary, we suppose there exists $\bar{T} \subseteq T$, $t \notin \bar{T}$ and $|C_s(\bar{T})| > |C_s(\bar{T} \cup \{t\})|$. Then, there exists a slot s^k which is filled under $C_s(\bar{T})$ but not under

$C_s(\bar{T} \cup \{t\})$. However, due to the above Claim in the proof for the substitutes condition, the teacher who was assigned s^k in $C_s(\bar{T})$ is available when s^k is being filled in $C_s(\bar{T} \cup \{t\})$. Then this slot cannot be vacant in $C_s(\bar{T} \cup \{t\})$ as this teacher is acceptable for the slot, which is a contradiction. We showed that $|C_s(\bar{T})| \leq |C_s(\bar{T} \cup \{t\})|$.

By the repeated application of this argument, we conclude that whenever $\bar{T} \subseteq \hat{T}$, $|C_s(\bar{T})| \leq |C_s(\hat{T})|$. ■

Proof of Theorem 2: Strategy-proofness: It was shown by Hatfield and Milgrom (2005) that whenever the choice rules of schools satisfy the substitutes and law of aggregate demand conditions, the resulting mechanism through DA is strategy-proof for teachers. Since for each school s, auxiliary choice rule C_s satisfies these conditions and only incomplete information is about the preferences of teachers, SI-DA is strategy-proof.

SI Stability: Suppose Assumption 1 holds. Let SI-DA outcome be μ. We will show that it is status-quo improving first. By our construction of the slot priorities, a teacher t will be accepted by ω_t whenever she applies and she will never be rejected in the further steps. Hence, it is status-quo improving for teachers. Consider the schools. First, we prove the following claim.

Claim: Each school fills all its positions in μ.

Proof of Claim: To see this, notice that no teacher t is assigned to a school s that is less preferred to ω_t in μ. Therefore, all teachers who were employed at the status quo are assigned to some school in μ. Moreover, we claim that exactly $\sum_{s \in S}(q_s - |\omega_s|)$ new teachers are assigned in μ. On the contrary, suppose this claim does not hold. Then, at least one position of a school s is unfilled in μ and this matching leaves at least one new teacher $t \in N$ unmatched such that she considers all schools with vacant position acceptable and is acceptable at all schools with vacant positions at status quo. In determining $C_s(B_s^{K+1})$, where K is the final step of the DA algorithm, if the slot corresponding to this vacant position is one of slots s^k that was unfilled at the status quo, then an unassigned new teacher would have applied to that school and have been assigned to that slot by Assumption 1. Thus, this slot is filled at the status quo.

Then as all employed teachers at status quo are assigned to some school in μ, there exists a teacher $\hat{t} \notin N$ assigned in μ to a slot $\hat{s}^{\hat{k}}$ that was unfilled at the status quo at some school $\hat{s}$.

Since new teacher t is unassigned in μ, she should have applied to all schools with vacant positions at status quo (which she considers acceptable by Assumption 1) including $\hat{s}$. Since $\hat{s}$ has an unfilled position at status quo, by Assumption 1, it considers t acceptable. Moreover, at the slots that are unfilled at the status quo, acceptable new teachers have higher ranking than employed teachers at the status quo by construction of the slot rankings: $t \succ_{\hat{s}}^{\hat{k}} \hat{t}$. Thus, slot $\hat{s}^{\hat{k}}$ should have held t instead of $\hat{t}$, a contradiction.

Hence, all positions are filled in μ. ◇

Since all positions are filled in μ, by our construction of the rankings of the slots, the matching

FOSDs the status-quo matching ω. Hence, μ is also status-quo improving for schools.

Next, we will show that there is no blocking pair of μ that is not allowed. By construction of slot rankings for a school s, for teachers neither in ω_s nor in N, the type ranking of the school is respected in rankings of its slots.

Suppose there exists a blocking teacher-school pair (t, s) of μ, i.e., t prefers s to μ_t and there exists a teacher $\hat{t} \in \mu(s)$ such that $\tau(t) \rhd_s \tau(\hat{t}) \rhd_s \theta_\emptyset$ for some $\hat{t} \in \mu(s)$, as all positions of s are filled in μ.

Then t should have made an offer to s that was rejected in the DA algorithm. Then all teachers assigned to all slots of s have higher ranking in that slot than t. If $\hat{t}$ was assigned in μ to a filled slot at status quo then $\hat{t} \in \omega_s$ and $t \notin \omega_s$ as $\tau(t) \rhd_s \tau(\hat{t})$. If $\hat{t}$ was assigned in μ to an unfilled slot at status quo then $\hat{t} \in N$ and $t \notin N$ as $\tau(t) \rhd_s \tau(\hat{t})$. By definition of SI stability, then (t, s) is an allowed blocking pair of μ. As such a blocking pair (t, s) and such a teacher $\hat{t}$ is arbitrary, μ is SI stable. ■

Proof of Proposition 5: We use the following lemma in our proof.

Lemma 1 *For any $\bar{T} \subseteq T$, $\hat{D}_s(\bar{T}) \subseteq D_s(\bar{T})$.*

Proof: Let s^k and s^ℓ (s^ℓ and s^k) be m^{th} and $(m+1)^{th}$ positions under $\blacktriangleright_s$ ($\hat{\blacktriangleright}_s$), respectively. Since the relative positions of the first $(m-1)$ positions are the same under $\blacktriangleright_s$ and $\hat{\blacktriangleright}_s$, the same teachers are assigned to the first $(m-1)$ positions by D_s and $\hat{D}_s$. Therefore, we consider the same set of teachers for the m^{th} position under both $\blacktriangleright_s$ and $\hat{\blacktriangleright}_s$. Let $\tilde{T}$ be the set of teachers considered for the m^{th} position under both $\blacktriangleright_s$ and $\hat{\blacktriangleright}_s$.

Recall that, by our construction, the set of teachers acceptable for position s^ℓ is a (weak) superset of the teachers acceptable for position s^k. Hence, if there does not exist an acceptable teacher in $\tilde{T}$ for position s^ℓ, then there does not exist an acceptable teacher in $\tilde{T}$ for position s^k. If there is no acceptable teacher in $\tilde{T}$ for position s^k but there is some acceptable teacher for position s^ℓ, then that teacher is assigned to s^ℓ under both auxiliary choice rules. Since the relative positions of the remaining positions are the same under $\blacktriangleright_s$ and $\hat{\blacktriangleright}_s$, we have $D_s(\bar{T}) = \hat{D}_s(\bar{T})$ whenever the set of acceptable teachers in $\tilde{T}$ for either s^k or s^ℓ is vacant.

Now suppose there exist acceptable teachers in $\tilde{T}$ for positions s^k and s^ℓ. Let t^k and t^ℓ be the highest ranked teachers for positions s^k and s^ℓ among the ones in $\tilde{T}$, respectively.

If $t^k \neq t^\ell$, then under both auxiliary choice rules D_s and $\hat{D}_s$ t^k and t^ℓ are assigned to positions s^k and s^ℓ, respectively. Since the relative positions of the remaining positions are the same under $\blacktriangleright_s$ and $\hat{\blacktriangleright}_s$, we have $D_s(\bar{T}) = \hat{D}_s(\bar{T})$.

If $t^k = t^\ell = t'$, then t' is assigned to positions s^k and s^ℓ under auxiliary choice rules D_s and $\hat{D}_s$, respectively. Next, we consider the teachers in $\tilde{T} \setminus t'$. First notice that, the status-quo employees who have the highest priority among all teachers in T for s^k and s^ℓ cannot be in $\tilde{T} \setminus t'$. This would conflict with the fact that t' has the highest priority for both positions among the teachers in $\tilde{T}$.

Then, there is one teacher in $\tilde{T} \setminus t'$ who has highest priority for both s^k and s^ℓ. We denote such teacher with t''. If t'' is acceptable for both s^k and s^ℓ, then t'' is assigned to s^ℓ and s^k under both auxiliary choice rules D_s and $\hat{D}_s$, respectively. If t'' is unacceptable for both s^k and s^ℓ, then no teacher is assigned to s^ℓ and s^k under both auxiliary choice rules D_s and $\hat{D}_s$, respectively. Under both cases, since the relative positions of the remaining positions are the same under $\blacktriangleright_s$ and $\hat{\blacktriangleright}_s$, we have $D_s(\bar{T}) = \hat{D}_s(\bar{T})$. We are left with one remaining case: t'' is acceptable for s^ℓ but not for s^k. Then, t'' is assigned to s^ℓ under D_s but s^k is not filled under $\hat{D}_s$. Then, when we consider the remaining positions under both auxiliary choice rules and the remaining teachers, we can treat the assignment is done via DA mechanism where each teacher ranks the positions according to their positions under $\blacktriangleright_s$ and $\hat{\blacktriangleright}_s$. Since DA is population monotonic and individually rational, any teacher assigned under $\hat{D}_s$ is assigned under D_s. But the other way is not always true. ■

Now, consider a sequential application of DA algorithm in which we allow teachers one by one as long as they do not apply to school s (see Dur et al., 2018 for details). Then, eventually, we will have a set of teachers $\bar{T}$ who have been rejected from their all choices better than s. Once all teachers apply to s, Lemma 1 implies that the rejected teachers under D is a subset of the rejected teachers under $\hat{D}$. Then, we allow only the rejected teachers under both auxiliary choice rules from s and all other teachers who have not applied to s to apply one by one. Following this procedure will give us matching μ assignment for all schools except s under both auxiliary choice rules. Moreover, DA algorithm terminates under auxiliary choice rule D. However, by Lemma 1, there might be teachers rejected from s and have not applied to their next best choice under $\hat{D}$. That is, we may observe some teachers to be rejected from their assignment under μ. Hence, no teacher t prefers $\hat{\mu}_t$ to μ_t. ■

Appendix B Other Applications

In this appendix, we explain other applications of our mechanisms in more detail. We give three concrete applications.

Intra-district school choice after a status-quo assignment. In the US, Austin Independent School District (AISD) of Texas assigns students to the schools through an address based matching procedure.[68,69] Unfortunately, address based assignment ends up with segregated schools.[70] In order to eliminate the segregation and fill the empty seats at the under-demanded schools, AISD runs a transfer procedure in which a student who is in relative demographic majority in her assigned school can apply to the schools in which she belongs to the minority demographic group. Moreover, new students who arrive at the district after the matching procedure is run can

[68] Although we mentioned earlier possible application of our approach for inter-district-school choice (cf. Hafalir et al., 2019), a direct application of our methodology exists in intra-district school choice, which we discuss here.

[69] There are 128 school programs in AISD. In 2020-2021 school year, the total enrolment in AISD is more than 75,000 (AISD, 2021).

[70] In 2019, student body at 15% and 63% of the elementary schools were composed of more than 60% white and hispanic-black students, respectively.

also participate in this transfer procedure.[71]

In addition to the transfer programs to achieve racial diversity at schools, many school districts, including Davenport, IA (DCS, 2019), and Seminole, FL (SCPS, 2021), run transfer programs to achieve diversity in terms of socioeconomic status (SES) of the students. Diversity transfer programs based on SES are also suggested to the school district by The United States Department of Education and the United States Department of Justice (ED, 2011).

Job rotation. Job rotation is defined as the horizontal movement of employees among different positions in a company. It is a well-established and commonly practiced human resource management program in many settings. It benefits companies through employee enrichment and success of developing future managers as well as decreased worker turn-over due to increased job satisfaction of the participants (Cheraskin and Campion, 1996).

Job rotation programs can also be used as a mean to obtain certain distributional goals a company such as achieving gender balance across different departments of the company and retention of female employees and increasing the development of more female leaders through rotation programs.[72]

Other civil services. There are other centralized matching procedures for civil servants from different professions. For example, police officers are assigned to neighborhoods by centralized procedures in several US cities such as Chicago; doctors are assigned to government hospitals in some countries such as Turkey. Our procedures can be used in these domains as well to achieve different distributional objectives. A concrete example in this domain is the Indian Administrative Services, the top-tier government jobs in India (Thakur, 2020). This selective service conducts first time assignment of officials to regional government jobs in states of India every year, while reassignment is conducted separately. The state has distributional objectives based on spread of talent across states, constitutional affirmative action, and respect of preferences for home states.

Appendix C Examples

We Illustrate how SI-CC works with the following example:

Example 4 *Let* $S = \{s_1, s_2, s_3, s_4\}$, $T = \{t_1, t_1', t_2, t_2', t_3, t, t'\}$, *the status-quo matching be*

$$\omega_{s_1} = \{t_1, t_1'\},\ \omega_{s_2} = \{t_2, t_2'\}\ \omega_{s_3} = \{t_3\},\ \omega_{s_4} = \emptyset,$$

$q_{s_1} = 3$, $q_{s_2} = q_{s_4} = 2$, *and* $q_{s_3} = 1$. *The preferences of teachers are:*

$$s_2\ P_{t_1}\ s_1\ P_{t_1}\ \emptyset\ P_{t_1}\ s_3\ P_{t_1}\ s_4$$

$$s_4\ P_{t_1'}\ s_1\ P_{t_1'}\ \emptyset\ P_{t_1'}\ s_2\ P_{t_1'}\ s_3$$

[71] Majority minority transfer program is used in many school district in US including Huntsville, AL (HCS, 2020), Suffolk, VA (SPSK12, 2021), and Florence, SC (F1S, 2019).

[72] Observe that companies use professionally designed centralized matching software for job rotations, for example see `https://www.tws-partners.com/corporate-functions/hr/`.

$$s_3 \; P_{t_2} \; s_2 \; P_{t_2} \; \emptyset \; P_{t_2} \; s_1 \; P_{t_2} \; s_4$$

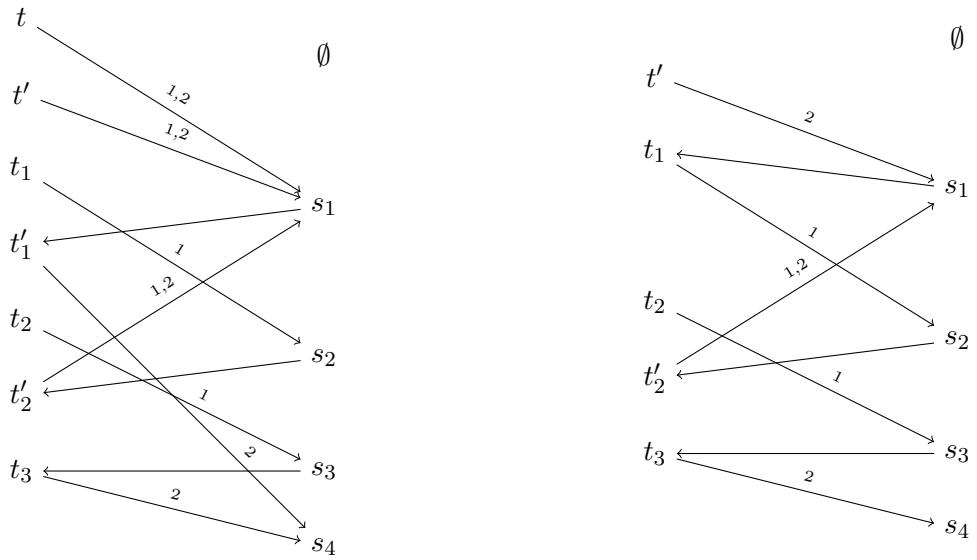
$$s_1 \; P_{t'_2} \; s_2 \; P_{t'_2} \; \emptyset \; P_{t'_2} \; s_3 \; P_{t'_2} \; s_4$$

$$s_4 \; P_{t_3} \; s_2 \; P_{t_3} \; s_3 \; P_{t_3} \; \emptyset \; P_{t_3} \; s_4$$

$$s_1 \; P_t \; \emptyset \; P_t \; s_2 \; P_t \; s_3 \; P_t \; s_4$$

$$s_1 \; P_{t'} \; \emptyset \; P_{t'} \; s_2 \; P_{t'} \; s_3 \; P_{t'} \; s_4$$

There are only two types of teachers: $\bar{\theta}$ and $\underline{\theta}$ for respectively "High" and "Low" experience. All schools prefer high experience teachers to low experience teachers so that $\bar{\theta} \rhd_{s_i} \underline{\theta}$ for $i = 1, 2, 3, 4$. We assume that $\tau(t'_i) = \tau(t_3) = \underline{\theta}$ for $i = 1, 2$, $\tau(t_i) = \bar{\theta}$ for $i = 1, 2$ and $\tau(t) = \tau(t') = \underline{\theta}$. Since each initial teacher of a school has a different type, the pointing order can be arbitrary. For new teachers, assume that the tie-breaker ranks t above t' so that $t \vdash t'$. At the beginning of Step 1 of SI-CC, using the pointing behaviors of the definition, we obtain the graph in Figure 5a.



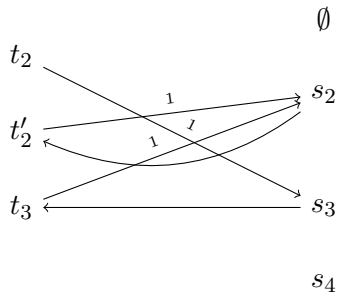
(a) Step 1

(b) Step 2

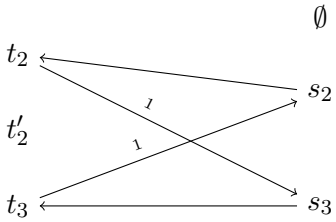
Figure 5: Graph of the steps 1 and 2 of SI-CC

For each arrow going from a teacher to a school, we report the improvement conditions, i.e. 1 and/or 2 in the definition of pointing rule for teachers, which hold for that arrow. One can note that there is no cycle in this graph. Thus there are only two possible chains: one starting at t or one starting at t'. Since $t \vdash t'$, we pick the one starting with t and, following the procedure described, implement the chain $\{t, s_1, t'_1, s_4\}$ since t'_1 points to s_4 only because of the improvement condition 2. At the beginning of Step 2, the graph becomes the one in Figure 5b.

In that case, one can check that the cycle $\{s_1, t_1, s_2, t'_2\}$ is implemented and that, at Step 3, the chain $\{t', s_1\}$ is implemented. At the beginning of Step 4, the graph of SI-CC is the one in Figure 6a. Note that even though teacher t_3 prefers s_4 to s_2, she cannot point to the former because even though it has a vacant position left, there is no remaining new teacher.



(a) Step 4



(b) Step 5

Figure 6: Graph of the steps 4 and 6 of SI-CC

In that step, we implement the cycle $\{s_2, t_2'\}$*. At the next step, we obtain the graph in Figure 6b. Note that even though* t_3 *has a low experience and* t_2 *a high experience, the former can still point to* s_2 *since it has accepted* t_1*, a high experience teacher, at step 2 so that the improvement condition 1 in the pointing rule of teachers is satisfied. So we implement the cycle* $\{s_2, t_2, s_3, t_3\}$ *and the algorithm stops.*

Example 5 *Suppose there are five teachers one of whom is new:* $T = \{t_1, t_2, t_3, t_4, t_5\}$ *and* $t_5 \in N$*. The status-quo assignment of school* s*, which has capacity* $q_s = 3$ *is* $\omega_s = \{t_1, t_2\}$*. The type ranking of school* s *is*

$$\tau(t_1) = \tau(t_3) \rhd_s \tau(t_2) \rhd_s \tau(t_4) \rhd_s \tau(t_5) \rhd_s \theta_\emptyset.$$

The slot set of s *is* $\mathcal{S}_s = \{s^1, s^2, s^3\}$ *such that* s^1 *and* s^2 *correspond to filled positions at status quo and* s^3 *corresponds to the vacant position.*

Let the tie breaker $\vdash$ *be such that* $t_1 \vdash t_3$*.*

We construct the rankings for each slot as follows:

$$\begin{aligned}
&t_1 \succ_s^1 t_3 \succ_s^1 \emptyset \succ_s^1 t \qquad \text{for any } t \notin \{t_1, t_3\},\\
&t_2 \succ_s^2 t_1 \succ_s^2 t_3 \succ_s^2 \emptyset \succ_s^2 t \qquad \text{for any } t \notin \{t_1, t_2, t_3\},\\
&t_5 \succ_s^3 t_1 \succ_s^3 t_3 \succ_s^3 t_2 \succ_s^3 t_4 \succ_s^3 \emptyset.
\end{aligned}$$

Suppose $\hat{T} = \{t_2, t_3, t_4, t_5\}$*. Then, the set of chosen teachers* $C_s(\hat{T})$ *is found as follows:*

- ***Step*** 1***:*** *Teacher* t_3 *is the most preferred for slot* s^1 *among the teachers in* $\hat{T}_1 = \hat{T}$*. Hence,* t_3 *is assigned to slot* s^1 *and she is removed. We set* $\hat{T}_2 = \hat{T}_1 \setminus \{t_3\}$*.*
- ***Step*** 2***:*** *Teacher* t_2 *is the most preferred for slot* s^2 *among the teachers in* $\hat{T}_2$*. Hence,* t_2 *is assigned to slot* s^2 *and she is removed. We set* $\hat{T}_3 = \hat{T}_2 \setminus \{t_2\}$*.*
- ***Step*** 3***:*** *Teacher* t_5 *has the highest priority for slot* s^3 *among the teachers in* $\hat{T}_3$*. Hence,* t_5 *is assigned to slot* s^3 *and she is removed. We set* $\hat{T}_4 = \hat{T}_3 \setminus \{t_5\}$*.*

Hence, $C_s(\hat{T}) = \{t_3, t_2, t_5\}$*.*

In Example 6, we show that, in the same setting as Combe et al. (2020), SI-CC is not equivalent to the teacher optimal selection of TO-BE they propose.[73]

Example 6 Let $S = \{s_1, s_2\}$, $T = \{t_1, t_2, t_2'\}$, $\omega_{s_1} = \{t_1\}$, $\omega_{s_2} = \{t_2, t_2'\}$, $q_{s_1} = 1$ and $q_{s_2} = 2$. Let $\tau(t_1) = \theta_1$, $\tau(t_2) = \theta_2$, $\tau(t_2') = \theta_2'$. Finally, the preferences of the schools over types are: $\theta_2 \rhd_{s_1} \theta_2' \rhd_{s_1} \theta_1$ and $\theta_1 \rhd_{s_2} \theta_2 \rhd_{s_2} \theta_2'$. One can check that the matching returned by the teacher optimal selection of TO-BE matches[74] t_1 to s_1 and t_2 to s_1 while SI-CC matches t_1 to s_1 but t_2' to s_1.

In Example 7, we show that M-convexity of the policy goals is not sufficient anymore to ensure existence of SI teacher optimal and strategy-proof mechanism.

Example 7 *Let $S = \{s_1, s_2\}$, $T = N = \{t_1, t_2\}$, $\omega_{s_1} = \omega_{s_2} = \emptyset$, $q_{s_1} = q_{s_2} = 1$ and $\tau(s_1) = \tau(s_2) = \theta$. Suppose the constraint over the distribution of teachers require that a teacher of type θ is assigned to s_1. This is a constraint fixing a floor which is known to be M-convex. Suppose both teachers rank s_2 ahead of s_1 and s_1 ahead of $\emptyset$. If teachers report their true preferences, then there will be a teacher assigned to s_1 under any SI teacher optimal (or two-sided Pareto efficient) matching. Then, the teacher assigned to s_1, say t_1, has an incentive to claim that s_1 is unacceptable to her. Indeed, any SI teacher optimal mechanism must then assign t_2 to s_1 and t_1 to s_2.*

In Example 8, we show that in some market there does not exist an SI teacher optimal and SI stable matching.

Example 8 *Let $S = \{s_1, s_2, s_3\}$, $T = \{t_1, t_2, t_3\}$, $\omega_{s_1} = \{t_1\}$, $\omega_{s_2} = \{t_2\}$, $\omega_{s_3} = \{t_3\}$ and $q_{s_1} = q_{s_2} = q_{s_3} = 1$. Let $\tau(t_3) \rhd_{s_1} \tau(t_2) \rhd_{s_1} \tau(t_1) \rhd_{s_1} \theta_\emptyset$, $\tau(t_3) \rhd_{s_2} \tau(t_1) \rhd_{s_2} \tau(t_2) \rhd_{s_2} \theta_\emptyset$ and $\tau(t_1) \rhd_{s_3} \tau(t_2) \rhd_{s_2} \tau(t_3) \rhd_{s_3} \theta_\emptyset$, $s_2\ P_{t_1}\ s_1 P_{t_1}\ s_3 P_{t_1}\ \emptyset$, $s_1\ P_{t_2}\ s_2 P_{t_2}\ s_3 P_{t_2}\ \emptyset$, and $s_1\ P_{t_3}\ s_2 P_{t_3}\ s_3 P_{t_3}\ \emptyset$.*

Let $\mu_{s_1} = t_2$, $\mu_{s_2} = t_1$ and $\mu_{s_3} = t_3$. Notice that, μ is SI teacher optimal. Under this market, ω is the unique SI stable matching and μ Pareto dominates ω for teachers.

In Examples 9 - 11, we inspect possible relaxations in the definition of SI stability. First, we show that if we exclude the first condition from the definition of SI stability, then for some market there does not exist an SI stable matching.

Example 9 *Let $S = \{s_1, s_2\}$, $T = \{t_1, t_2\}$, $\omega_{s_1} = \{t_1\}$, $\omega_{s_2} = \{t_2\}$ and $q_{s_1} = q_{s_2} = 1$. Let $\tau(t_2) \rhd_s \tau(t_1) \rhd_s \theta_\emptyset$ for both $s \in S$ and $s_1\ P_t\ s_2\ P_t\ \emptyset$ for all $t \in T$.*

In this market, the unique status-quo improving matching is ω. However, it is blocked by (t_2, s_1). Hence, any SI stable matching does not exist in this market when the first condition is excluded.

Next, via example we show that if we exclude the second condition from the definition of SI stability, then for some market there does not exist an SI stable matching.

[73] Combe et al. (2020) already noted that their class of TO-BE mechanisms did not entirely defined the class of statu-quo improving, strategy-proof and two-sided Pareto efficient mechanisms. However, they did not investigate it further. Our example suggests that other non-trivial mechanisms, such as SI-CC, exist outside their class.

[74] One can easily check that this example is well defined in their setting. Just set the preferences of the schools over the teachers being equivalent to the schools' ranking over their corresponding types.

Example 10 *Let* $S = \{s_1, s_2\}$, $T = \{t_1, t_2\}$, $\omega_{s_1} = \{t_1\}$, $\omega_{s_2} = \emptyset$ *and* $q_{s_1} = q_{s_2} = 1$. *Let* $\tau(t_1) \rhd_s \tau(t_2) \rhd_s \theta_\emptyset$ *for both* $s \in S$, $s_2 \; P_{t_1} \; s_1 \; P_{t_1} \; \emptyset$ *and* $s_2 \; P_{t_2} \; \emptyset \; P_{t_2} \; s_1$.

Under this market, in any status-quo improving matching t_1 *is assigned to* s_1. *However, any such matching is blocked by* (t_1, s_2). *Hence, any SI stable matching does not exist in this market when the second condition is excluded.*

One can wonder if there exists a strategy-proof mechanism which selects a matching which is stable when one of the conditions is excluded whenever such a matching exists and selects a stable matching under both conditions, otherwise. In the following example, we show that such a mechanism does not exist.

Example 11 *Let* $S = \{s_1, s_2\}$, $T = \{t_1, t_2\}$, $\omega_{s_1} = \{t_1\}$, $\omega_{s_2} = \emptyset$ *and* $q_{s_1} = q_{s_2} = 1$. *Let* $\tau(t_2) \rhd_{s_1} \tau(t_1) \rhd_{s_1} \theta_\emptyset$, $\tau(t_1) \rhd_{s_2} \tau(t_2) \rhd_{s_2} \theta_\emptyset$ *for both* $s \in S$, $s_2 \; P_{t_1} \; s_1 \; P_{t_1} \; \emptyset$ *and* $s_2 \; P_{t_2} \; s_1 \; P_{t_2} \; \emptyset$.

Under this market, there exists a unique stable matching when condition 2 is excluded: t_1 *is assigned to* s_2 *and* t_2 *is assigned to* s_1. *Hence, it will be selected.*

Suppose teacher t_2 *reports* $s_2 P'_{t_2} \emptyset P'_{t_2} s_1$. *Then, we have the same problem as in Example 10. Since there does not exists a stable matching where condition 2 is excluded, we consider stable matchings when condition 2 is included. There exists a unique stable matching in which* t_1 *is assigned to* s_1 *and* t_2 *is assigned to* s_2. *Hence,* t_2 *is better off by manipulating.*

In Examples 12 -14, we relax the conditions of Assumption 1 one by one and show that the existence of an SI stable outcome may not be guaranteed.

Example 12 *We consider a market in which there does not exist* $N' \subseteq N$ *such that* $|N'| \geq \sum_{s \in S}(q_s - |\omega_s|)$.

Let $S = \{s, s'\}$, $T = \{t_1\}$, *the status-quo matching be*

$$\omega_s = \{t_1\}, \; \omega_{s'} = \emptyset,$$

$q_s = q_{s'} = 1$, *and teacher* t_1 *is acceptable for both schools. The preferences of the teacher* t_1 *is*

$$s' \; P_{t_1} \; s \; P_{t_1} \; \emptyset.$$

In this market, ω_s *is the unique status-quo improving matching but it is blocked by* (t_1, s').

Example 13 *We consider a market in which there exists* $N' \subseteq N$ *such that* $|N'| \geq \sum_{s \in S}(q_s - |\omega_s|)$ *and each teacher in* N' *is acceptable for all schools with excess capacity but not all teacher in* N' *consider all schools with excess capacity acceptable.*

Let $S = \{s, s'\}$, $T = \{t_1, t_2\}$, *the status-quo matching be*

$$\omega_s = \{t_1\}, \; \omega_{s'} = \emptyset,$$

$q_s = q_{s'} = 1$, $\tau(t_1) \rhd_s \tau(t_2) \rhd_s \theta_\emptyset$ *and* $\tau(t_1) \rhd_{s'} \tau(t_2) \rhd_{s'} \theta_\emptyset$. *The preferences of the teachers are*

$$s' \; P_{t_1} \; s \; P_{t_1} \; \emptyset,$$

$$s \; P_{t_2} \; \emptyset \; P_{t_2} \; s'.$$

In this market, ω *is the unique status-quo improving matching but it is blocked by* (t_1, s').

Example 14 *We consider a market in which there exists* $N' \subseteq N$ *such that* $|N'| \geq \sum_{s \in S}(q_s - |\omega_s|)$ *and all teacher in* N' *consider all schools with excess capacity acceptable but some teacher in* N' *is not acceptable for some school with excess capacity.*

Let $S = \{s, s'\}$, $T = \{t_1, t_2\}$, *the status-quo matching be*

$$\omega_s = \{t_1\}, \; \omega_{s'} = \emptyset,$$

$q_s = q_{s'} = 1$, $\tau(t_1) \rhd_s \tau(t_2) \rhd_s \theta_\emptyset$ *and* $\tau(t_1) \rhd_{s'} \theta_\emptyset \rhd_{s'} \tau(t_2)$. *The preferences of the teachers are*

$$s' \; P_{t_1} \; s \; P_{t_1} \; \emptyset,$$

$$s \; P_{t_2} \; s' \; P_{t_2} \; \emptyset.$$

In this market, ω_s *is the unique status-quo improving matching but it is blocked by* (t_1, s').

Appendix D Extensions

D.1 Responsive Preferences and Impossibility

In this section, we weaken restrictions on school preferences over teachers. Instead of the FOSD relation, we assume schools ranking over the types of teachers are responsive.

Each school s has strict ranking over the types and no type option denoted by $\theta_\emptyset$ denoted with $\rhd_s$. For school s, type θ teachers are acceptable if and only if $\theta \rhd_s \emptyset$. Given $\rhd_s$, the preference order of school s over $T \cup \{\emptyset\}$ is given as:

- $\tau(t) \rhd_s \tau(t')$ if and only if $t \succ_s t'$;
- $\tau(t) = \tau(t')$ if and only if $t \sim_s t'$;
- $\tau(t) \rhd_s \theta_\emptyset$ if and only if $t \succ_s \emptyset$.

For any $|\bar{T}| < q_s$ responsiveness implies that for any $t, t' \in T \setminus \bar{T}$

- $\bar{T} \cup \{t\} \succ_s \bar{T}$ if and only if $t \succ_s \emptyset$;
- $\bar{T} \cup \{t\} \succ_s \bar{T} \cup \{t'\}$ if and only if $t \succ_s t'$.

Note that, responsive preferences is more general than FOSD. In particular, if μ_s first-order stochastically dominates matching ω_s, then $\mu_s \succsim_s \omega_s$. However, the other way may not be true. We illustrate this in the following example.

Example 15 *Let $\omega_s = \{t_1, t_2, t_3, t_4\}$ such that $\tau(t_1) \rhd_s \tau(t_2) = \tau(t_3) \rhd_s \tau(t_4)$. Consider the following matching $\mu_s = \{t_1, t'_1, t_4, t'_4\}$ such that $\tau(t_1) = \tau(t'_1) \rhd_s \tau(t_4) = \tau(t'_4)$. Matching μ_s does not FOSD ω_s. However, it is possible that $\mu_s \succ_s \omega_s$.*

The following example shows that, with responsive preferences, there is no mechanism that is SI teacher optimal and strategy-proof.

Example 16 *There are 6 teachers, $T = \{t_1, t'_1, t_2, t'_2, t, t'\}$, and 4 schools, $S = \{s_1, s_2, s, s'\}$. Let $\omega_{s_1} = \{t_1, t'_1\}$, $\omega_{s_2} = \{t_2, t'_2\}$, $\omega_s = \{t\}$ and $\omega_{s'} = \{t'\}$. Schools s_1 and s_2's ranking over teacher types are:*

$$\tau(t) \rhd_{s_1} \tau(t_1) \rhd_{s_1} \tau(t'_1) \rhd_{s_1} \tau(t')$$
$$\tau(t') \rhd_{s_2} \tau(t_2) \rhd_{s_2} \tau(t'_2) \rhd_{s_2} \tau(t).$$

Moreover, we assume that $\{t, t'\} \succ_{s_k} \{t_k, t'_k\}$ for $k = 1, 2$. Notice that, this relation is consistent with responsive orders. Preferences of the teachers are:

$$s_2 P_t s_1 P_t s P_t \emptyset$$
$$s_1 P_{t'} s_2 P_{t'} s' P_{t'} \emptyset$$
$$s P_{t_1} s_1 P_{t_1} \emptyset$$
$$s' P_{t'_1} s_1 P_{t'_1} \emptyset$$
$$s P_{t_2} s_2 P_{t_1} \emptyset$$
$$s' P_{t'_2} s_2 P_{t'_2} \emptyset$$

First note that under any status-quo improving matching, if t is assigned to her first ranked school s_2, then t' must also be assigned to s_2. Indeed, let μ be a status-quo improving matching such that $\mu_t = s_2$. Since $\{t, t'\} \succ_{s_2} \{t_2, t'_2\} \succ_{s_2} \{t_2, t\}, \{t'_2, t\}$, status-quo improvement implies that $\mu_{t'} = s_2$. With a similar argument, if $\mu_{t'} = s_1$, then $\mu_t = s_1$. So it implies that there are only three possible SI teacher optimal matchings:

$$\mu^1 := \begin{pmatrix} t & t' & t_1 & t'_1 & t_2 & t'_2 \\ s_1 & s_1 & s & s' & s_2 & s_2 \end{pmatrix}$$
$$\mu^2 := \begin{pmatrix} t & t' & t_1 & t'_1 & t_2 & t'_2 \\ s_2 & s_2 & s_1 & s_1 & s & s' \end{pmatrix}$$
$$\mu^3 := \begin{pmatrix} t & t' & t_1 & t'_1 & t_2 & t'_2 \\ s_1 & s_2 & s & s_1 & s_2 & s' \end{pmatrix}$$

Let φ be an SI teacher optimal mechanism. Assume that $\varphi(P) = \mu^1$. In that case, let P'_t : $s_2 P'_t s P'_t \emptyset$. Under (P'_t, P_{-t}), the only SI teacher optimal matching is μ^2 so that $\varphi(P'_t, P_{-t}) = \mu^2$ and

the manipulation of t is successful. If $\varphi(P) = \mu^2$, then t' can report $P'_{t'} : s_1 P'_{t'} s' P'_{t'} \emptyset$ so that the only SI teacher optimal matching under $(P_{t'}, P_{-t'})$ is μ^1 and $\varphi(P_{t'}, P_{-t'}) = \mu^1$, a successful manipulation for t'. If $\varphi(P) = \mu^3$ then t or t' can manipulate in reporting the same profile as before. We conclude that φ cannot be strategy-proof.

D.2 Immunity to Type Ranking Manipulation

In this section, we investigate whether there exists a status-quo improving, strategy-proof and efficient mechanism which is immune to possible type ranking manipulations. As we explained in our model, we assume schools' preferences, and therefore their type rankings, are commonly known, specifically by policy makers. However, policy makers may choose to report some school s's type ranking differently to the mechanism in order to improve its assignment. We say a mechanism ϕ is **immune to type ranking manipulation** if for any P and $\triangleright$ there does not exist a school s and a type ranking $\triangleright'_s$ such that

$$\phi_s(\succsim', P) \ \succ_s \ \phi_s(\succsim, P)$$

where $\succsim$ and $\succsim'$ are preferences induced by type ranking profiles $\triangleright$ and $(\triangleright'_s, \triangleright_{-s})$, respectively. If a mechanism is not immune to type ranking manipulation, then we say it is **vulnerable to type ranking manipulation**.

We first show that there does not exist an SI teacher optimal and strategy-proof mechanism which is immune to type ranking manipulation.

Proposition 7 *Any SI teacher optimal and strategy-proof mechanism is vulnerable to type ranking manipulation.*

Proof: We prove this result by means of an example. On the contrary, suppose there exists an SI teacher optimal and strategy-proof mechanism which is immune to type ranking manipulation. Let ϕ be that mechanism. Let $S = \{s, s', s''\}$, $T = \{t_1, t_2, t_3\}$, the status-quo matching be

$$\omega_s = \{t_1\},\ \omega_{s'} = \{t_2\},\ \omega_{s''} = \{t_3\},$$

and $q_s = q'_s = q''_s = 1$. Let $\tau(t_2) \triangleright_s \tau(t_3) \triangleright_s \tau(t_1)$, $\tau(t_1) \triangleright_{s'} \tau(t_2)$, and $\tau(t_1) \triangleright_{s''} \tau(t_3)$. Let $\succsim$ be the school proference profile which is induced by $\triangleright$. The preferences of the teachers are

$$s'' \ P_{t_1} \ s' \ P_{t_1} \ s$$

$$s \ P_{t_2} \ s' \ P_{t_2} \ s''$$

$$s \ P_{t_3} \ s'' \ P_{t_3} \ s'.$$

In this market, there exist two SI teacher optimal matchings:

$$\mu_t = s',\ \mu_{t'} = s,\ \mu_{t''} = s''$$

$$\nu_t = s'',\ \nu_{t'} = s',\ \nu_{t''} = s.$$

Suppose $\phi(\succsim, P) = \mu$. Let $\tau(t_2) \rhd'_s \tau(t_1) \rhd'_s \tau(t_3)$ and $\succsim'$ be the school preference profile induced by $(\rhd'_s, \rhd_{-s})$. Then, under market $(\succsim', P)$ ν is the unique SI teacher optimal matching.

Suppose $\phi(\succsim, P) = \nu$. Let $s''\ P'_{t_1}\ s\ P'_{t_1}\ s'$. Then, under market $(\succsim, P'_{t_1}, P_{-t_1})$ μ is the unique SI teacher optimal matching. ■

Notice that, we prove Proposition 7 by using a market in which there are at least three types. In many applications, agents are characterized based on two types based on race or gender. In the following proposition, we show that when there are only two types, SI-CC is immune to type ranking manipulation.[75]

Proposition 8 *When $|\Theta| = 2$ and $|\omega_s| = q_s$ for all $s \in S$, SI-CC is immune to type ranking manipulation.*

Proof: On the contrary, suppose there exists a problem $(\succsim, P)$ such that school s can be better off when its type ranking is changed. Let $\rhd$ induce $\succsim$ and $\rhd'_s$ be type ranking resulting into improvement for school s. Let $\Theta = \{\theta_1, \theta_2\}$. Without loss of generality, suppose $\theta_1\ \rhd_s\ \theta_2$. Since teachers in ω_s are with acceptable types, $\theta_1\ \rhd_s\ \theta_\emptyset$. We consider the following cases.

Case 1: $\theta_2\ \rhd_s\ \theta_\emptyset$. Then, SI-CC weakly increases the number of θ_1 teachers compared to the one under ω_s. Moreover, all positions will be filled. Under any other type ranking, the number of assigned θ_1 type teachers is at most $|\omega_s^{\theta_1}|$.

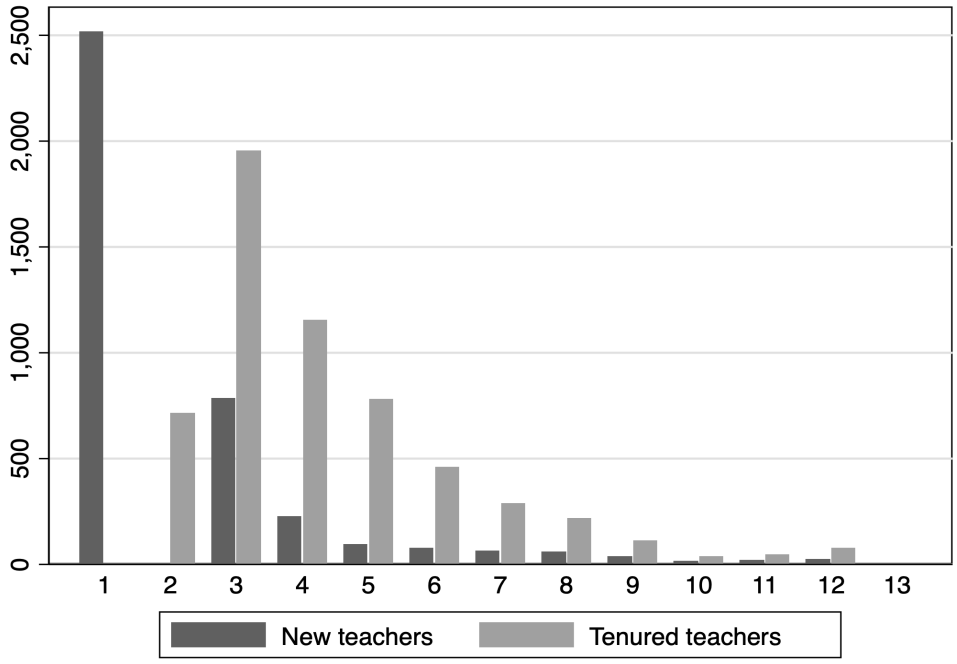
Case 2: $\theta_\emptyset\ \rhd_s\ \theta_2$. Then, under SI-CC all assigned teachers to s are with type θ_1.

In either case, we cannot improve school s by changing its type ranking. ■

[75] If there are at least two new teachers and a school with a vacant position, then by ranking one type as unacceptable a school's assignment can be improved under any SI teacher optimal mechanism.

Appendix E Additional Figures and Tables

Figure A.1: Distribution of Teacher Experience Types

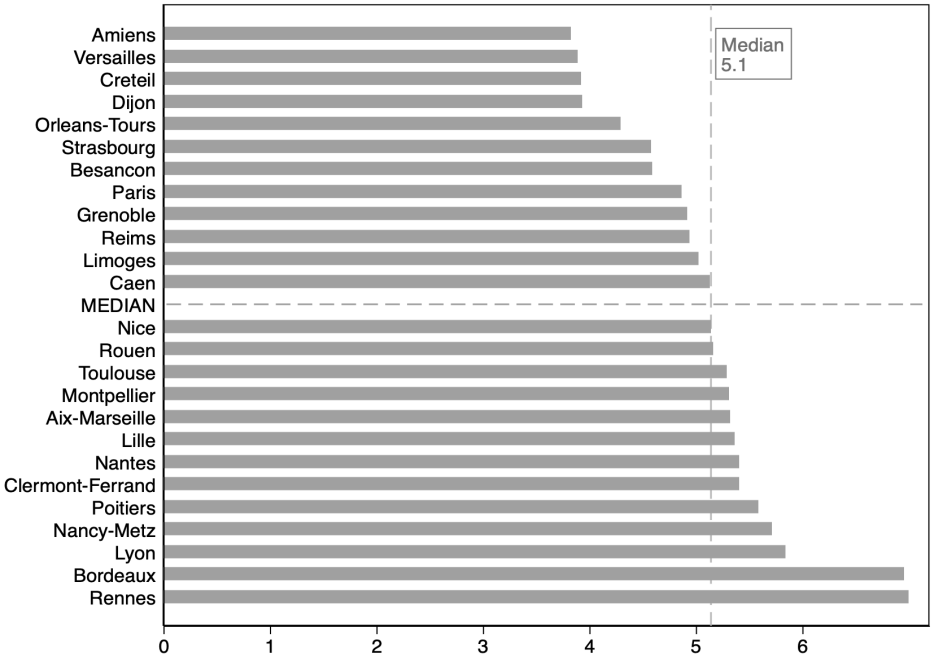


Notes: This figure shows the number of teachers with each experience type. We define a teacher type as her experience and we classify teachers into 12 experience bins, where the first bin corresponds to teachers with 1 or 2 years of experience, the second bin to teachers with 3 to 4 years of experience, and so on. Because a large number of teachers belong to the first bin, we further use a tie-breaker for the first bin by ordering new teachers above tenured teachers, hence effectively generating 13 experience bins. The first bin corresponds to new teachers with 1 or 2 years of experience, the second bin corresponds to tenured teachers with 1 or 2 years of experience, the third bin to new and tenured teachers with 3 to 4 years of experience, and so on ...

Table A.1: Number of teachers and vacant positions

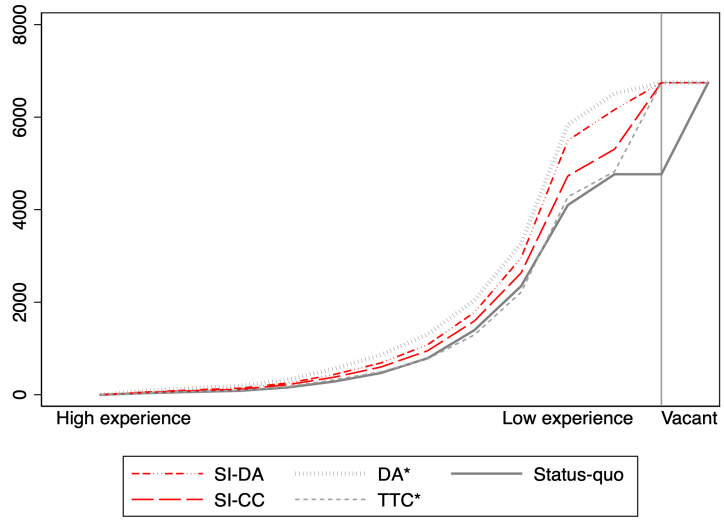
Subjects	All teachers (1)	New teachers (2)	Tenured teachers (3)	Vacant positions (4)
All subjects	10,460	4,627	5,833	3,912
Sports	2,066	568	1,498	475
French	1,645	786	859	663
Math	1,563	958	605	824
English	1,374	746	628	640
History-Geography	1,230	657	573	562
Spanish	999	316	683	248
Physics-Chemistry	837	310	527	254
Biology	746	286	460	246

Figure A.2: Average Teacher Experience Types at Status Quo



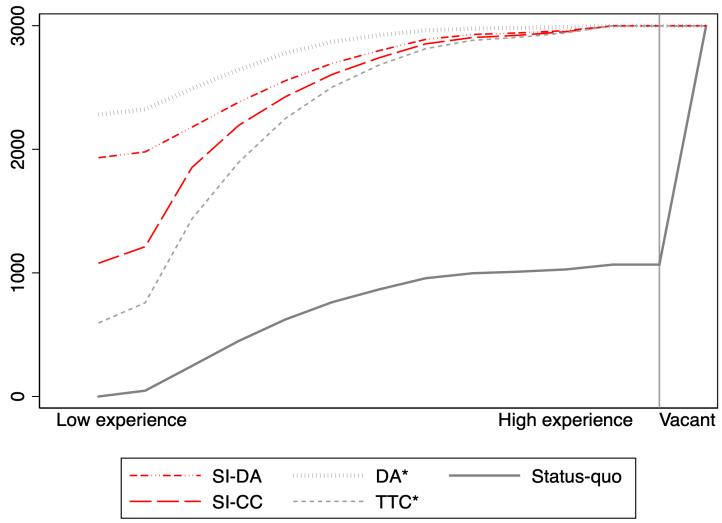
Notes: This figure shows the average teacher experience type at status quo (lower types correspond to lower experience levels). The *younger regions* are listed above the median and the *older regions* are listed below the median.

Figure A.3: Cumulative Distribution of Teacher Experience Types in the *Younger Regions*



Notes: This figure shows the cumulative distribution of teacher experience types in younger regions of France. These are the regions whose average teacher type is strictly lower than the median of average teacher type distribution at status quo. The horizontal axis reports the 13 types of teachers, ordered from the most experienced to the least experienced. The 14^{th} type corresponds to the vacant positions. The mechanisms that respect status-quo improvement are plotted in red. Those that do not are in grey. The thick black line ("Status-quo") corresponds to the cumulative distribution of teacher types at the status-quo matching.

Figure A.4: Cumulative Distribution of Teacher Experience Types in the *Older Regions*



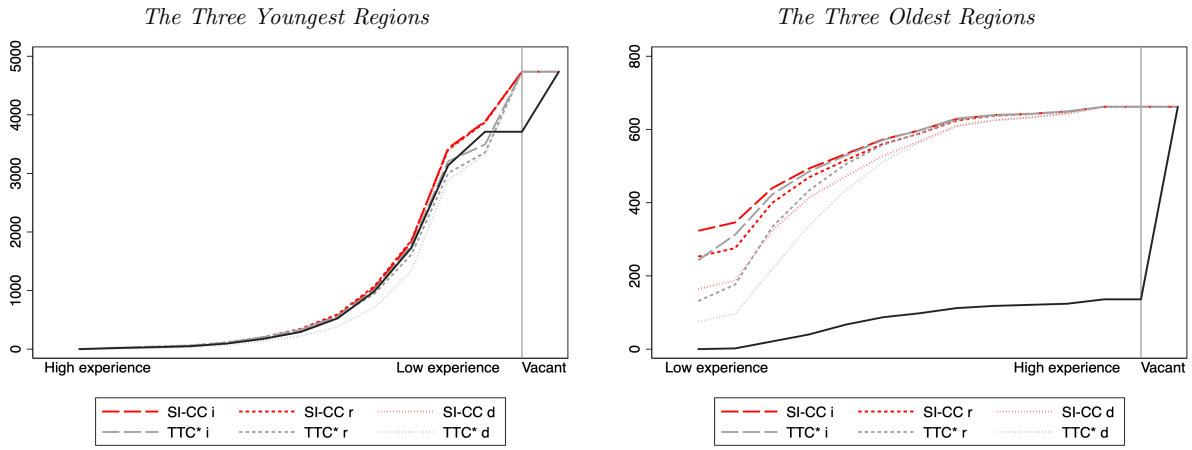
Notes: This figure shows the cumulative distribution of teacher experience types in older regions of France. These are the regions whose average teacher type is higher than the median of average teacher type distribution at status quo. The horizontal axis reports the 13 types of teachers, ordered from the least experienced to the most experienced. The 14^{th} type corresponds to the vacant positions. The mechanisms that respect status-quo improvement are plotted in red. Those that do not are in grey. The thick black line ("Status-quo") corresponds to the cumulative distribution of teacher types at the status-quo matching.

Table A.2: Statistics on Regions

Regions	Ratio: # of tenured teachers asking to enter / exit the region	% of teachers asking for a new assignment coming from each region	Ratio: # of teachers aged more than 50 / less than 30	% of students enrolled in priority education	% of students whose reference parent has no diploma	% of students obtaining their baccalaureate
	(1)	(2)	(3)	(4)	(5)	(6)
Rennes	15.55	0.5	8.10	7.9	14.18	91.54
Bordeaux	8.95	0.8	6.56	14.6	19.22	86.25
Toulouse	6.56	1.5	5.29	8.9	17.38	88.57
Paris	3.02	2.8	6.90	25.5	21.54	85.48
Aix-Marseille	2.54	1.9	5.08	30.1	27.20	81.77
Grenoble	1.74	2.3	3.91	16.5	19.80	88.17
Amiens	0.08	6.2	1.89	23.9	27.71	82.41
Créteil	0.03	22.7	1.14	35.5	31.62	83.94
Versailles	0.05	25.7	1.62	24.9	21.88	87.92

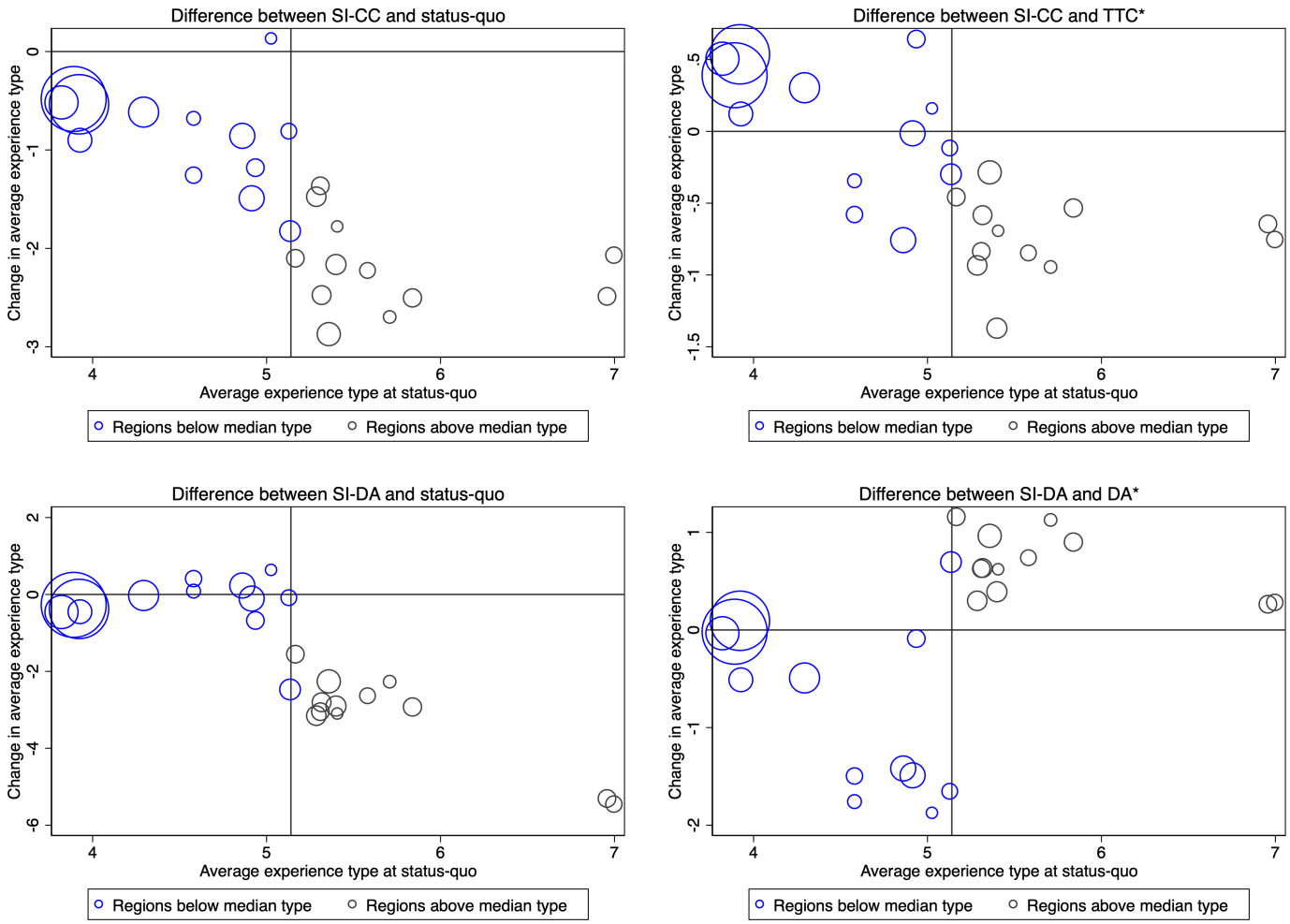
Notes: This table reports descriptive statistics for the three most attractive (Rennes, Bordeaux, and Toulouse), the three least attractive regions (Amiens, Créteil, and Versailles), and three intermediate regions (Paris, Aix-Marseille, and Grenoble). Attractiveness is measured by the ratio of the number of tenured teachers asking to enter a region to the number of teachers asking to leave the region (reported in column 2). All statistics reported in this table come from the following reference: Direction de l'Evaluation de la Prospective et de la Performance (2014). In column (1), the number of teachers asking to enter the region corresponds to the number of teachers who rank the region as their first choice in their preference list, while the number of teachers asking to leave the region corresponds to the number of teachers who are initially assigned the region and submit a preference list to move to another region.

Figure A.5: Cumulative Distribution of Teacher Experience Types with Different Chain Selection Rules in SI-CC and TTC*



Notes: This Figure shows the cumulative distribution of teacher experience types. For each mechanism (SI-CC and TTC*), we report results for three different chain selection rules. The subscripts "i", "r", and "d" respectively stand for *increasing*, *random*, and *decreasing*. These ordering mean that the teachers starting a chain are respectively selected by increasing, random, and decreasing order of their maximum Ministry-mandated priority points. The left panel reports the distribution in the three youngest regions of France (Amiens, Versailles, and Créteil), and the right panel the distribution in the three oldest regions of France (Rennes, Bordeaux, and Lyon). The horizontal axis reports the 13 experience types of teachers, ordered from most experienced to least experienced (left panel) and from least experienced to most experienced (right panel). The 14^{th} type corresponds to the vacant positions. The mechanisms that respect status-quo improvement are plotted in red. Those that do not are in grey. The thick black line ("Status-quo") corresponds to the cumulative distribution of teacher types at the status-quo matching.

Figure A.6: Change in Average Experience Type for All Teachers Across Regions: SI-CC and SI-DA



Notes: This figure shows the difference in the average experience of teachers (among both tenured and newcomers) between the matching obtained with SI-CC against the status-quo matching (top left figure) and its benchmark TTC* (top right figure). It reports the same difference between the matching obtained with SI-DA against the status-quo matching (bottom left figure) and its benchmark DA* (bottom right figure). Each observation represents a French region. Circle size reflects region size. Regions are ordered (on the horizontal axis) by average experience of their teachers at status quo (the status-quo matching). The vertical line represents the median type. All regions on the left have an average type that is strictly below the median. This is the group of regions we identified as inexperienced regions. All regions on right of the vertical line are regions whose average type is above the median. In regions above the horizontal line, teachers average experience post reassignment is larger than at the status-quo matching. The name of the three least experienced regions (Créteil, Versailles, and Amiens) and most experienced regions (Rennes, Bordeaux, and Lyon) are reported on the graphs.

Appendix F Variables Used for Teacher Preference Estimations

This appendix describes the variables we use for teacher preference estimation (the way they are abbreviated in Table 2 is written in parentheses):

We use the following region characteristics:

- Share of students classified as disadvantaged.
- Share of students living in an urban area as % (labeled as "% stud. urban").
- Share of students who attend a school classified as priority education (labeled as "% stud. in priority educ."). Priority education is a label given to the most disadvantaged schools in France.
- Share of students who attend a private school (labeled as "% stud. in private sch.").
- Share of teachers who are younger than 30 (labeled as "% teach. younger than 30")
- Region is in South of France (labeled as "Region in South of France"). The following 5 regions are classified as being in the South of France: Aix-Marseille, Bordeaux, Montpellier, Toulouse, and Nice.

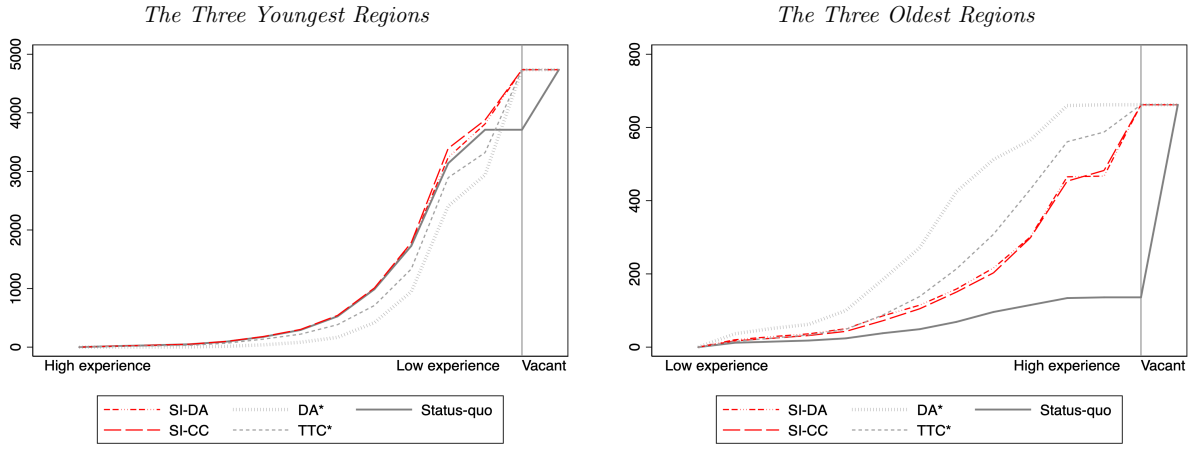
We use the following teacher characteristics:

- Current region of the teacher (labeled as "Status-quo region"). This is the region a teacher is initially assigned to.
- Region where a teacher was born (labeled as "Birth region").
- Distance between the region ranked and the status-quo region of a teacher (labeled as "Distance to status-quo region").
- Teacher's current region is Créteil or Versailles, which are the two least attractive regions (labeled as "Teach. from CV"). The attractiveness of a region is measured by the ratio of the number of teachers who rank the region as their first choice divided by the number of teachers who ask to leave the region.
- Teacher is married (labeled as "Married").
- Teacher has spent at least 5 years in a school labelled as priority education (labeled as "Teach. in priority education").
- Teacher has an advanced teaching qualification (labeled as "Advanced qualif.").

Appendix G Empirical Results when All Regions Have the Same Preferences over Teachers

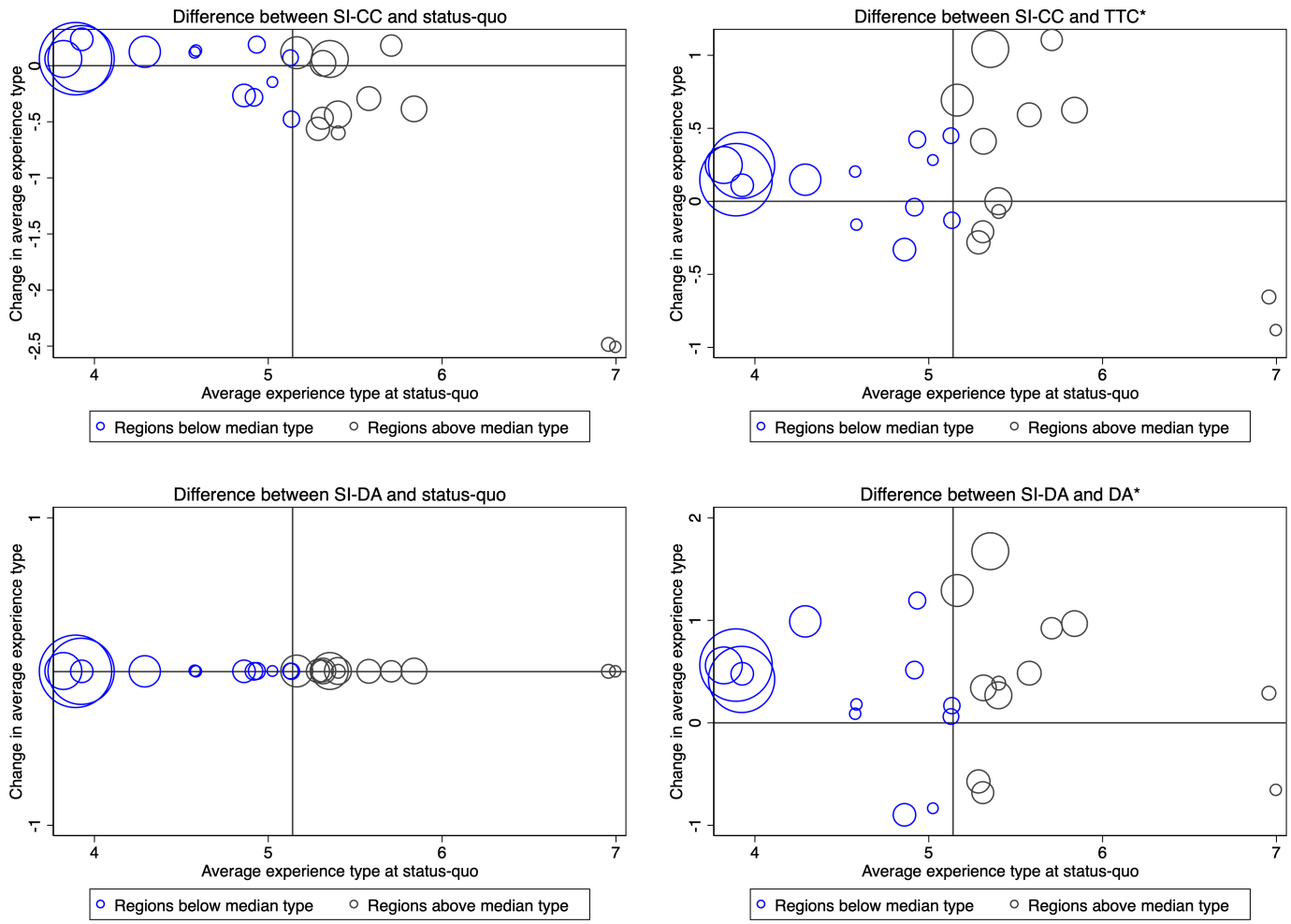
This appendix reports results in which all regions have the same preferences over teachers: Each region ranks types by decreasing level of experience, i.e., the most experienced teachers are always preferred to the least experienced teachers.

Figure A.7: Cumulative Distribution of Teacher Experience Types - Same Preferences for all Regions



Notes: This Figure shows the cumulative distribution of teacher experience types. The left panel reports the distribution in the three youngest regions of France (Amiens, Versailles, and Créteil), and the right panel the distribution in the three oldest regions of France (Rennes, Bordeaux, and Lyon). The horizontal axis reports the 13 experience types of teachers, ordered from most experienced to least experienced (left panel) and from least experienced to most experienced (right panel). The 14^{th} type corresponds to the vacant positions. The mechanisms that are status-quo improving are plotted in red. Those that do not are in grey. The thick black line ("Status-quo") corresponds to the cumulative distribution of teacher types at the status-quo matching.

Figure A.8: Change in Tenured Teacher Experience (type) Across Regions - Same Preferences for all Regions



Notes: This figure shows the difference in the average experience of teachers between the matching obtained with SI-CC against the status-quo matching (top left figure) and its benchmark TTC* (top right figure). It reports the same difference between the matching obtained with SI-DA against the status-quo matching (bottom left figure) and its benchmark DA* (bottom right figure). Each observation represents a French region. Circle size reflects region size. Regions are ordered (on the horizontal axis) by average experience of their teachers at status quo. The vertical line represents the median type. All regions on the left have an average type that is strictly below the median. This is the group of regions we identified as inexperienced regions. All regions on right of the vertical line are regions whose average type is above the median. In regions above the horizontal line, teachers average experience post reassignment is larger than the one of the benchmark matchings to which it is compared. The name of the three least experienced regions (Créteil, Versailles, and Amiens) and most experienced regions (Rennes, Bordeaux, and Lyon) are reported on the graphs.

Appendix H Descriptions of Benchmark Mechanisms

H.1 Formal Descriptions of wSI-CC and wSI-DA

As explained in Section 5, under wSI-CC and wSI-DA, we relabel the type of the teachers based on d_S. We call the relabeled type of a teacher as **pseudo type**. Let $\tau^p : T \to \Omega$ be the **pseudo type** function. For each school s, using $\preccurlyeq_s$ and d_s, we determine the **pseudo type** of teachers in ω_s as follows: If $|\{t' \in \omega_s \ : \ t' \preccurlyeq_s t\}| \leq d_s^{\theta_1}$, then $\tau^p(t) = \theta_1$; otherwise, $\tau^p(t) = \theta_{\hat{m}}$ where $(d_s^{\theta_1} + \ldots + d_s^{\theta_{\hat{m}-1}} + 1) \geq |\{t' \in \omega_s \ : \ t' \preccurlyeq_s t\}| \leq (d_s^{\theta_1} + \ldots + d_s^{\theta_{\hat{m}}})$. wSI-CC and wSI-DA mechanisms are almost identical to wSI-CC (see Section 3) and wSI-DA (see Section 4), respectively. They differ from these base mechanisms due to the usage of the pseudo type function which helps us to weaken the status-quo improvement constraint. For the sake of completeness, we define wSI-CC and wSI-DA below.

Definition 4 ***Weak Status-quo Improving Cycles and Chains (wSI-CC) Mechanism***

We construct a ***pointing order*** $\succcurlyeq_s$ *over teachers in* ω_s*: For any two distinct teachers* $t, t' \in \omega_s$

$$t \succcurlyeq_s t' \iff \tau^p(t) \lhd_s \tau^p(t') \text{ or } [\tau^p(t) = \tau^p(t') \text{ and } t \vdash t'].$$

We will construct a matching μ *dynamically through the following algorithm. Initially,* μ *is the empty matching, in which no teacher is assigned to any school. In each step, as teachers are assigned in* μ*, they will be removed from the algorithm; similarly schools whose all seats are filled in* μ *and also some other schools chosen by the algorithm will be removed.*

For each school s *and type* θ*, let* b_s^θ *track the* ***current balance*** *of type* θ *teachers at school* s *in* ***current matching*** μ*, which is the matching fixed until the beginning of the current step. The current balance is defined as the difference between the number of type* θ *teachers assigned to* s *in* μ *and the number of type* θ *teachers in its status-quo assignment assigned to any school in* μ*:* $b_s^\theta \equiv |\mu_s^\theta| - |\{t \in \omega_s : \mu_t \neq \emptyset \text{ and } \tau^p(t) = \theta\}|$*. Thus, we initialize* $b_s^\theta = 0$*.*

A general step k *is defined as follows:*

Step k:

- *Each remaining school* s *points to the highest priority remaining teacher in* ω_s *under* $\succcurlyeq_s$*, if not all students in* ω_s *are already assigned in* μ*; let* t_s^k *be the teacher pointed by school* s *in step* k*. Otherwise, school* s *does not point to any teacher.*
- *We define the* ***pointing rule of teachers*** *as follows: Any remaining teacher* t *is allowed to point to a remaining school* s *if at least one of the following two school* ***improvement conditions*** *hold for school* s *via teacher* t*:*
  1. *(Improvement for* s *by teacher trades) if the school points to a teacher* t_s^k *and*

$$\sum_{\theta' \trianglerighteq_s \theta} b_s^{\theta'} > 0 \text{ for all types } \theta \text{ such that } \tau^p(t_s^k) \trianglerighteq_s \theta \triangleright_s \tau(t),$$

or

2. *(Improvement for s by only incoming teachers) $\tau(t) \rhd_s \theta_\emptyset$, school s currently has an unfilled seat, i.e., $q_s - |\mu_s| > |\{t' \in \omega_s : \mu_{t'} = \emptyset\}|$, and there are remaining new teachers.*

Let A_t^k be the opportunity set for a remaining teacher t, i.e., the set of schools t can point in this step together with the being unassigned option $\emptyset$.[76]

Each remaining teacher t points to her most preferred option in A_t^k.

- *Being unassigned option $\emptyset$ points to all teachers pointing to it.*

Due to finiteness, there exists either

(i) a cycle in which all schools in the cycle satisfy improvement Condition 1 or a cycle between a single teacher and the being unassigned option $\emptyset$, or

(ii) a chain.

Then:

- ***If Case (i) holds:** Each teacher can be in at most one cycle as she points at most to a single option. We execute exchanges in each cycle encountered in case (i) by assigning the teachers in that cycle to the school she points to, update current matching μ and current balances $\{b_s^\theta\}$ accordingly, remove assigned teachers and filled schools in μ, and go to step $k+1$.*
- ***If Case (i) does not hold:** Then case (ii) holds, i.e., there exists a chain. In particular, each remaining teacher initiates a chain. There are two subcases:*
  - ***If there exists a remaining new teacher**: Then we select a chain to be executed as follows:*
    - *Select as the tail of the chain the new teacher with the highest priority under tie breaker $\vdash$ and then include in the chain the school she points to. If Improvement Condition 1 does not hold for this school via this teacher, but only Improvement Condition 2 holds, then we end the chain with this school; otherwise, we repeat the following:*
    - *Include to the chain the teacher pointed by the last school included. If we include a teacher, we also include next in the chain the school she is pointing to. We repeat this iteratively until the Improvement Condition 1 does not hold for the next school via the included teacher, but only Improvement Condition 2 holds.*

    The last school included is the head of the selected chain.

    We execute the exchanges in the selected chain by assigning each teacher in the chain to the school she points to, update current matching μ and current balances $\{b_s^\theta\}$ accordingly, remove assigned teachers and filled schools, and go to step $k+1$.
  - ***If there does not exist a remaining new teacher**: Then we remove each school s whose all status-quo employees in ω_s were already assigned in μ. We continue with step $k+1$.*

The mechanism terminates when all teachers are removed. Its outcome is the final matching μ.

[76] Note that, $\omega_t \in A_t^k$ for all remaining teachers t who were employed at the status quo.

Definition 5 ***Weak Status-quo Improving Deferred Acceptance (wSI-DA) Mechanism*** *For each school s, we construct a linear order over the teachers in ω_s denoted by $\blacktriangleright_s$ as follows: For any $t, t' \in \omega_s$,*

$$t \blacktriangleright_s t' \iff \tau^p(t) \rhd_s \tau^p(t') \text{ or } [\tau^p(t) = \tau^p(t') \text{ and } t \vdash t'].$$

We fix a school s in this construction. Let $\mathcal{S}_s = \{s^1, s^2, \ldots, s^{q_s}\}$ be the set of slots at school s. Without loss of generality we label the types in Θ as $\theta_1, \ldots, \theta_{|\Theta|}$ based on the type ranking of the school such that $\theta_k \rhd_s \theta_{k+1}$ for all $k \in \{1, \ldots, |\Theta| - 1\}$. We define a ranking for each slot over $T \cup \{\emptyset\}$ where $\emptyset$ denotes keeping the slot unfilled. The ***ranking of slot*** *s^k, $\succ_s^k$, is defined separately for the slots representing the filled seats at the status-quo matching, i.e., for $k \leq |\omega_s|$, and slots representing the empty seats at the status-quo matching, i.e., for $|\omega_s| < k \leq q_s$:*

- *For filled slots s^k at the status quo, i.e., all $k \leq |\omega_s|$:*
  - *the teacher $t \in \omega_s$ who is ranked k'th under $\blacktriangleright_s$ has the highest ranking under $\succ_s^k$,*
  - *any teacher t' with $\tau^p(t) \rhd_s \tau(t')$ is ranked below $\emptyset$ under $\succ_s^k$, and*
  - *the rest of the ranking under $\succ_s^k$ is determined according to $\rhd_s$ such that ties between same type teachers are broken according to tie breaker $\vdash$.*
- *For empty slots s^k at the status quo, i.e., all k such that $|\omega_s| < k \leq q_s$:*
  - *a teacher t is ranked above $\emptyset$ under $\succ_s^k$ if and only if she is acceptable, i.e., $\tau(t) \rhd_s \theta_\emptyset$,*
  - *any acceptable new teacher t (i.e., $t \in N$ and $\tau(t) \rhd_s \theta_\emptyset$) is ranked under $\succ_s^k$ above any teacher t' employed at status quo by some school (i.e., $t' \notin N$), and*
  - *the rest of the ranking under $\succ_s^k$ is determined according to $\rhd_s$ such that ties between same type teachers are broken according to tie breaker $\vdash$.*

We refer to the mechanism that selects the outcome of the DA algorithm using the choice rules $(C_s)_{s \in S}$ (see Definition 3), which use the slot priorities constructed above, that we designed as wSI-DA mechanism.

H.2 Description of TTC*

Definition 6 ***TTC* Mechanism***

Let $\vdash$ be a tie breaker over teachers. For each school s, we construct a ***pointing order*** *$\succcurlyeq_s$ over teachers in ω_s: For any two distinct teachers $t, t' \in \omega_s$*

$$t \succcurlyeq_s t' \iff \tau(t) \lhd_s \tau(t') \text{ or } [\tau(t) = \tau(t') \text{ and } t \vdash t'].$$

A general step k is defined as follows:

Step k:

- *Each remaining school s points to the highest priority remaining teacher in ω_s under $\succcurlyeq_s$, if not all students in ω_s are already removed. Otherwise, school s does not point to any teacher.*

- *Each remaining teacher t points to her most preferred remaining option.*
- *Being unassigned option $\emptyset$ points to all teachers pointing to it.*

Due to finiteness, there exists either

(i) a cycle, or
(ii) a chain.

Then:

- ***If Case (i) holds:*** *Each teacher can be in at most one cycle as she points at most to a single option. We execute exchanges in each cycle by assigning the teachers in that cycle to the school she points to, remove assigned teachers and filled schools, and go to step $k+1$.*
- ***If Case (i) does not hold:*** *Then case (ii) holds, i.e., there exists a chain. In particular, each remaining teacher initiates a chain. Then we select the chain such that the tail of the chain is the highest priority teacher under tie breaker $\vdash$ and the head of the chain is a school which does not point to a teacher.*
  We execute the exchanges in the selected chain by assigning each teacher in the chain to the school she points to, remove assigned teachers and filled schools, and go to step $k+1$.

  The mechanism terminates when all teachers are removed.

H.3 Descriptions of DA* and the Current French Mechanism

Mechanism **DA*** uses a version of the DA algorithm that is modified to ensure status-quo improvement for teachers. School preferences are modified such that each teacher t, with a status-quo assignment s, is ranked in the (modified) ranking of her status-quo school $s := \omega_t$, above any teacher $t' \notin \omega_s$. Other than this modification, the schools' preference relations remain unchanged among the status-quo teachers and among the the non-status-quo teachers. That is, the school uses the FOSD preferences over teacher experience types introduced in the empirical section. Then it runs the DA algorithm using these modified school preferences and the submitted teacher preferences. Also see Guillen and Kesten (2012), Pereyra (2013), Compte and Jehiel (2008) regarding the use of this algorithm in another context and teacher assignment context.

The French Ministry of Education's current mechanism, which we refer to as **Current French** in the empirical analysis, uses the same algorithm as DA*. However, instead of the FOSD preferences for regions to rank status-quo teachers among themselves and non-status-quo teachers among themselves, it uses the Ministry-mandated priorities of regions over teachers. See Combe et al. (2020) for a more detailed presentation of this mechanism and its properties.

By construction, DA* and Current French are both status-quo improving for teachers. They do not satisfy status-quo improvement for schools and is not Gale-Shapley stable. However, DA* and Current French are teacher-SI stable using regional FOSD preferences for region priorities and Ministry-mandated regional priorities, respectively.