Mechanism and Priority Design for Distributional Objectives: An Application to Decrease Inequalities in Access to Experienced Teachers *

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Abstract

Centralized (re)assignment of workers to jobs is a feature in many countries for civil service sectors such as education, law enforcement, and medicine, and also in other contexts such as corporate-wide job rotations and inter- and intra-district school choice. Equipped with asymmetric information problems and complex constraints, these markets are ripe for design interventions to fulfill certain distributional objectives. To this end, we propose a model and two new (re)assignment mechanisms with the property that they improve both worker welfare and appropriately defined distributional measures over the status quo. While both mechanisms are strategy-proof, one achieves Pareto efficiency (and in particular worker optimality) and the other one achieves an appropriately defined stability property, targeted for applications that already use stability-based assignment schemes. As a concrete application, we address the unfair teacher distribution problem in education. In many countries that employ centralized (re)assignment mechanisms for teachers, public schools in disadvantaged districts have relatively fewer experienced (and good) teachers than those in more privileged districts. We empirically estimate teacher preferences using data from France and test our proposals' performance using several empirical metrics. We observe that imposing improvement of the distribution of teacher experience over the status quo together with a stability property can backfire: the experience gap between unattractive and attractive regions may be higher compared to what is achieved by mechanisms which do not impose such constraints. This issue does not occur under our efficient mechanism which successfully reduces the experience gap with respect to other proposals and the current French mechanism.

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1 Introduction

Centralized assignment and reassignment of workers to jobs, locales, tasks, managers are important features of many professions. In many countries, civil service jobs in a sector are initially assigned to new workers and then reassigned to senior workers through centralized procedures. Some examples include assignment and reassignment of medical doctors to public hospitals, police officers to precincts, teachers to public schools (Kamada and Kojima, 2015, Dur and Kesten, 2019, Combe et al., 2020). Similarly within large corporations, rotation and internal assignment procedures are used commonly to initially assign and then reassign workers to specific jobs and duties (Cheraskin and Campion, 1996). Similar examples exist in inter-district school choice in the US (Hafalir et al., 2019) as well as in intra-district school choice.

Especially, when different central vs. individual objectives exist and asymmetric information problems are involved regarding preferences, these markets induce complex matching problems, some of which we started to study and understand only recently as economists (Dur and Ünver, 2019b, Combe et al., 2020). Important differences set these markets apart from other centralized matching markets that have been studied or redesigned by economists (Roth, 1984a, 1991, Roth and Peranson, 1999, Balinski and Sönmez, 1999, Abdulkadiroğlu and Sönmez, 2003, Abdulkadiroğlu et al., 2005b,a), such as:

- There is a single employer.
- Assignment and reassignment have to be conducted for new and senior workers, respectively, either simultaneously or sequentially.
- Senior workers gain certain rights for their current jobs and sometimes for the jobs that they want to move to with respect to new workers.

Salary adjustments regarding these assignments are often not feasible as a tool either because of budgetary constraints, as in public sectors of many developing countries, or strong unionized labor forces, as for teachers or police officers, or the countries' laws.

A natural starting question for our current study is whether and how we can design transparent assignment mechanisms for such complex two-sided markets taking into account certain rights structures to fulfill distributional objectives, such as to help to remedy some of the perceived problems in these sectors. In this paper, we aim to answer this question through a concrete application in teacher (re)assignment to public schools.

Most education systems around the world suffer from a common problem: good teachers are not equally distributed between schools. In the United States, good teachers tend to work in schools that serve more affluent students (Hanushek et al., 2004, Jackson, 2009). In Norway, England, and France, good teachers prefer schools with a higher share of native and high-achieving students (Bonesrønning et al., 2005, Allen et al., 2018). Some countries try to solve this unequal distribution by making disadvantaged schools more attractive through higher salaries, lower class size, or better working conditions (Biasi, forthcoming, Falch, 2010, OECD, 2005). Yet, the scope to solve teachers' unequal distribution through higher salaries is often limited by a fundamental constraint: most

teachers are civil servants, which implies that their salary is regulated by a fixed pay scale that prevents policy makers from using it as a compensating factor. In such contexts, countries that use centralized assignment mechanisms benefit from an additional tool to mitigate this unequal distribution: the centralized assignment system itself.

In this paper, we design assignment mechanisms whose objective is to improve the distribution of teachers in schools (compared to an initial status-quo matching), and we empirically quantify the gains that these mechanisms would bring in a real-life teacher assignment setting. We start by introducing a two-sided matching framework in which we explicitly model schools' preferences as a proxy for the central authority's distributional objective. This objective could be to better balance inexperienced teachers across schools for instance. To help define a distribution, we introduce a new concept: Each teacher has a type which captures her observable characteristics that the central authority might want to better balance such as her experience, education, past performance, etc. In addition, in our model, teachers can either be tenured or new. Tenured teachers are initially assigned, which is captured by a *status-quo matching*. New teachers can be new graduates or the teachers who are employed by private schools. As in the standard setting, teachers have preferences over schools.

To incorporate distributional constraints in the model, a novelty of our approach is to consider that schools are part of a resource pool that is collectively managed by a central authority, whose objective is to improve the distribution of teachers in the school system. Schools have "preferences" over sets of teachers, and we think of these preferences as reflecting the central authority's objective. In many countries, this objective might be to assign experienced teachers to disadvantaged schools where inexperienced teachers are overrepresented and new graduates to schools where experienced teachers are overrepresented in order to improve the experience of new graduates. In that case, in our model, the disadvantaged schools would have a preference ordering over types that ranks teachers by decreasing levels of experience, i.e., the most experienced teachers would always be preferred to the least experienced teachers. Beyond this specific example, each school has a customized preference ordering over teachers' types. We assume that a school gets better-off if the distribution of types increases according to first-order stochastic dominance. In other words, given a set of teachers assigned to a school and given a schools' preference over teachers' types, a school gets better-off if, for each teacher's type, the fraction of teachers with this type or a more-preferred type increases after the match. By construction, when schools get better off compared to the status-quo matching, the distribution of teachers improves. We allow schools to have empty positions in the status-quo matching.

We impose that our mechanisms produce matchings that improve upon the status-quo for teachers but also for schools: Both sides should get weakly better-off compared to the initial matching. This defines the class of mechanisms that are *status-quo improving*. This is an important criterion that ensures that a better welfare of teachers is not achieved at the cost of a poorer distribution of teachers, or vice versa. Status-quo improvement and strategy-proofness (for teachers only as school or central preferences and objectives are known) are the two fundamental properties we require in our mechanisms. We then build on this by considering two additional criteria that matchings

should respect: efficiency (while welfare entities are both teachers and schools, in particular optimality for teachers) and stability. As these criteria are in conflict,[1] we introduce two mechanisms that are status-quo improving and strategy-proof. One will produce a matching that is (two-sided) Pareto efficient, and in particular teacher optimal among status-quo improving matchings; the other one achieves a stable matching where stability is appropriately defined for our environment with status-quo improvement constraint.

The efficient mechanism we propose, named the status-quo improving cycles and chains (hereafter, SI-CC), is related to top-trading cycles (TTC) mechanisms (in particular, inspired by Gale's TTC in Shapley and Scarf, 1974, YRMH-IGYT of Abdulkadiroğlu and Sönmez, 1999, and TTCC of Roth et al., 2004) but has one key feature different: While these TTC-type mechanisms ensure that only teachers necessarily get better off as we execute exchanges, under SI-CC both teachers and schools get better off with respect to the status quo, i.e., the initial matching. To reach this objective, we introduce two main innovations in the mechanism. First, we define the schools' pointing rule where the order in which the school would like to send out its status-quo employees. By pointing, the school effectively gives permission to one of its status-quo employees to be assigned to a different school. We define the pointing order such that less-preferred-type employees are pointed first (therefore, leaving first) and more-preferred-type employees are pointed later. The second innovation pertains to the teacher pointing rule, which lets teachers point to schools, and, therefore, determines which teachers can be assigned to a school. We only allow a teacher to point to a school if replacing the pointed teacher by that school with her does not make the school worse-off with respect to the status quo.

Next, we turn our attention to stability, noting that stability and status-quo improvment may in general be in conflict (e.g., see Compte and Jehiel, 2008, Pereyra, 2013). Indeed, due to the individual rationality constraint for teachers, a tenured teacher has the right to stay at her status-quo assignment if she does not obtain any of the schools she ranks. If this tenured teacher is disliked by all schools (including her initial school), blocking pairs may form. To overcome the conflict between stability and individual rationality, the standard approach consists in weakening stability by ignoring blocking pairs that where assigning the teacher to the school of the blocking pair would displace a status-quo employee. We show that when imposing status-quo improvement in contexts where schools may initially have vacant seats, this weakening alone does not resolve the conflict. We introduce a stability notion which implicitly gives new teachers rights over status-quo empty seats of a school. Under a mild overdemand assumption involving new teachers and schools with excess status-quo capacity, we show the existence of a strategy-proof and status-quo improving stable mechanism, the status-quo improving deferred acceptance mechanism (hereafter, SI-DA).[2] Although it may appear counter intuitive to give a new teacher priority for the empty seats of a school, we show by means of examples that this is necessary to sustain status-quo improvement.

[1] Even in the most basic Gale and Shapley (1962) setup, teacher optimality and stability are in general in conflict (see Roth, 1982, Balinski and Sönmez, 1999).

[2] The overdemand assumption and giving higher priority to new teachers over status-quo empty seats restrict currently employee teachers to flee away from their status-quo assignments without being replaced. In the absence of such a restriction some schools might be worse off compared to their status-quo assignment.

That is, the main reason for giving priority to new teachers for the empty seats is achieving status-quo improvement for the schools.

To define the SI-DA mechanism, we need to define choice rules for schools. To this end, we first distribute status-quo employees to individual positions of the school, and consider an order of precedence among these positions based on the desirability of the teacher occupying the position. Vacant status-quo positions of a school, if there are any, are placed at the very end of this order of precedence. We construct an auxiliary "preference" ranking for each position such that some teachers are unacceptable and each status-quo teacher is ranked first by her position. The rest of the preference ranking for each occupied position is determined by making only the teachers who are at least as good as the occupying teacher acceptable and ranked according to desirability just below this teacher. For the vacant positions, new teachers are ranked first in terms of their desirability and then the remaining teachers. When a set of applicants apply a school in the SI-DA mechanism, its choice rule fills positions according to the precedence order of positions: the first position gets the most desirable acceptable applicant in terms of its auxiliary preference ranking, the second position gets the most desirable acceptable applicant among the rest in terms of its auxiliary preference ranking, and so on.

As mentioned earlier, the applicability of our theoretical framework is not only limited to centralized teacher assignment. It can be applied to any centralized two-sided matching market in which status-quo matching is aimed to be improved for both sides of the market. Examples of such markets are, including, but not limited to, student exchange programs between colleges, public school districts targeting racial balances among schools, and corporate job rotations, besides other civil service sectors. We provide more concrete details for three applications in Appendix B. Moreover, our model does not restrict the way schools value the experience of the teachers. Hence, our results hold as long as each school has rankings based on a coarse metric of characteristics of teachers such that different schools use possibly different metrics.

In the second part of the paper, we quantify the gains that our mechanisms bring by using data on the annual assignment of teacher to regions in France. Like many other countries, France uses a centralized process to assign teachers to regions and then to schools.[3] This labor market is particularly appropriate to study our question because it suffers from severe imbalance in the distribution of teachers. About 50% of the tenured teachers who ask to change region come from two regions (out of 25)—called Créteil and Versailles—that are particularly disadvantaged and therefore unattractive. As a result of this imbalance in exiting request, every year, a majority of the new teachers are assigned one of these two regions to compensate for the large exit flows. This structural imbalance is a serious concern for policy makers. It is frequently raised as a reason for the lack of attractiveness of the teaching profession in France and it is seen as one of the structural determinants of the large achievement inequalities that France suffers from.[4] Reducing the unequal

[3]Countries that use a centralized process to assign teachers to schools include Germany, Czech Republic (Cechlárová et al., 2015), Italy (Barbieri et al., 2011), Turkey (Dur and Kesten, 2019), Mexico (Pereyra, 2013), Peru, Uruguay (Vegas et al., 2006), and Portugal.

[4]The PISA results show that, in OECD countries, a more socio-economically advantaged student scores 39 points higher in math than a less-advantaged student, which is equivalent to one year of schooling. There is a large variation

distribution of teachers across regions became one of the objectives of the French policy makers, who see this as a way to both reduce achievement inequalities and to improve the attractiveness of the teaching profession in the longer-run.

We start the empirical analysis by structurally estimating teachers' preferences over the French regions. A number of papers show that assuming that teachers truthfully report their preferences is a strong assumption, even when the mechanism is strategy-proof (as in France).[5] To avoid the potential bias generated by teachers untruthful reports, we estimate teachers' preferences under a weaker "stability assumption" developed by Fack et al. (2019) and applied to the teacher labor market by Combe et al. (2020). We estimate the preferences separately for 5,833 teachers who have a status-quo assignment—referred to as "tenured teachers"— and 4,627 teachers do not have a status-quo assignment—referred to as "new teachers". The estimations reveal interesting differences in the preferences of these two groups of teachers. While tenured teachers strongly dislike the Créteil and Versailles regions, these regions are more attractive for new teachers, who might see a first position in a disadvantaged school as a stepping stone for better positions in the future. This difference in preferences surely contributes to the unequal distribution of teachers denounced by policy makers. The counterfactual analysis shows that, above and beyond these preferences, the mechanism used also shapes the distribution of teachers in important ways.

We use the estimated preferences, along with data on regions' priorities and vacant positions, to run the two mechanisms we propose: SI-CC and SI-DA. We define a teacher type as her experience and assume that regions with relatively young teacher body have a preference ordering over types that ranks teachers by decreasing levels of experience, i.e., the most experienced teachers would always be preferred to the least experienced teachers. For regions with a relatively old teacher body, we assume on the contrary that they rank teachers by increasing levels of experience. Recall that SI-CC and SI-DA are both status-quo improving, i.e., they impose that the distribution of teacher experience must improve upon the status-quo distribution. One important goal behind this constraint is to produce a more equal distribution of teachers across regions. Of course, imposing status-quo improvement may have a cost in terms of teacher welfare (for instance, measured by the mobility of tenured teachers or the distribution of ranks in their preferences of the regions teachers obtain). To measure the effect of imposing status-quo improvement on both the distribution of teacher experience as well as on teacher welfare, we run two benchmark mechanisms, TTC* and DA*, which correspond to SI-CC and SI-DA when we do not impose status-quo improvement for schools (we only keep the status-quo improvement for teachers). As explained below, our results vastly differ when considering SI-CC or SI-DA.

When focusing on SI-CC we observe that, imposing status-quo improvement, improves the distribution of teacher types and, relatedly, reduces the experience gap between unattractive and attractive regions. Further, we show that mobility reduces when imposing status-quo improvement. More specifically, starting with unattractive regions, SI-CC assigns less inexperienced teachers to

between countries in how much a student social background predicts her achievement, and France is one of the worst countries on this inequality indicator, ranking in fourth position (starting from the bottom).

[5]The French Ministry of Education uses a modified version of the deferred acceptance mechanism.

these regions than its benchmark which does not require status-quo improvement. To illustrate the magnitudes, SI-CC only assigns 1,371 teachers with one or two years of experience to the three youngest regions, while TTC* assigns 1,844 of them to these three regions. We find a similar pattern for attractive regions, i.e., SI-CC assigns less experienced teachers to these regions compared to its benchmark TTC*. Finally, SI-CC reduces the gap in teacher experience between young disadvantaged regions and older regions compared to its benchmark mechanism which does not impose status-quo improvement. We then investigate whether achieving a better distribution is done at the cost of a lower welfare for teachers, as measured by their mobility and the rank of the region they obtain. In line with the existence of a distribution-efficiency trade-off, fewer tenured teachers manage to move under SI-CC than under the benchmark TTC*, but the difference is somewhat limited (1,598 versus 2,470 teachers). The distribution of ranks that tenured teachers obtain under the benchmark also stochastically dominates the one under SI-CC. Interestingly, the opposite is true for new teachers who obtain better ranks under SI-CC.[6] To conclude, while we are able to quantify the trade-off between teachers' mobility and the distribution of teacher experience empirically, qualitatively, the trade-off observed is conform to our expectations.

The picture is radically different for SI-DA. Indeed, we believe that one of the most interesting insight of our empirical exercise is the observation that imposing status-quo improvement to DA-inspired mechanisms can backfire. More specifically, we first show that imposing status-quo improvement has a tremendous mobility cost on SI-DA: only 7 tenured teachers move from their initial position under SI-DA, compared to 1,267 under its benchmark DA*. In addition, in the youngest three regions, SI-DA produces a distribution of teacher experience which does not dominate the distribution of DA*. In the three oldest regions, our results are even more striking: DA* produces a distribution of teacher experience which dominates the distribution under SI-DA. Put differently, under our DA-inspired mechanisms, status-quo improvement fails to achieve its goal: the distribution of teachers does not improve and the experience gap between unattractive and attractive regions does not reduce. To understand the phenomenon, recall our previous observation that imposing both status-quo improvement and stability reduces mobility of tenured teachers almost to zero. In addition, without status-quo improvement, we show that the mechanism only mildly violates status-quo improvement while significantly increasing mobility. These additional mobility gains yield better distribution of teachers in many regions which helps in reducing inequalities across regions.

Related literature. SI-CC mechanism has its roots in the top-trading cycles algorithms (see Shapley and Scarf, 1974, Abdulkadiroğlu and Sönmez, 1999, Roth et al., 2004, Dur and Ünver, 2019a) and SI-DA mechanism has its roots in the teacher proposing deferred acceptance algorithm (see Gale and Shapley, 1962, Abdulkadiroğlu and Sönmez, 2003, Hatfield and Milgrom, 2005, Kominers and Sönmez, 2016).

Our stability-based approach using SI-DA is related to the literature on stable matching un-

[6] We explain this by the fact that, due to larger mobility under the benchmark, tenured teachers are more likely to leave the unattractive regions of Créteil, which mechanically increases the need to assign new teachers to these regions. This lowers the rank of the region obtained because unattractive regions are often ranked relatively low by teachers.

der distributional constraints. In the school choice literature, Abdulkadiroğlu and Sönmez (2003) introduce assignment schemes imposing type-specific ceilings at schools. Other related papers are Abdulkadiroğlu (2005), Kojima (2012), Hafalir et al. (2013), Ehlers et al. (2014), Kamada and Kojima (2015), Dur et al. (2018), Sönmez and Yenmez (2019), and Dur et al. (2020). While the focus in these papers is on assignment schemes to achieve diversity and other distributional objectives mostly in school choice and government-mandated job allocation context, our work applies to a teacher assignment problem where there is a status-quo matching and concern of making both sides better off. This makes our model and analysis different from the existing ones. Our methodology for constructing the SI-DA mechanism and the choice rule is inspired by the choice rule constructions in slot-specific priorities model of Kominers and Sönmez (2016), which is also used in the latter three aforementioned papers. In these papers, the choice rules and its inputs are preliminaries of the problem. However, in our framework, neither school choice rules nor the inputs are given to us. In particular, our choice rule is defined in this way to satisfy desired properties under SI-DA mechanism.

The design of efficient mechanisms in two-sided matching markets with a possible status-quo matching constraint was previously studied by Dur and Ünver (2019a) in the context of student and worker exchange programs. The main difference between that model and the current model is that status-quo improvement was not a constraint in this previous paper. This substantially changes the modeling choices and mechanism design. Moreover, we have school preferences based on the first-order stochastic dominance relation over distributions of teacher types leading to a new class of pointing rules. Our design of efficient mechanisms in this domain gives in general higher welfare for the schools. We additionally focus on a stability-based approach with SI-DA in addition to efficient mechanism design and conduct a thorough empirical analysis.

It is through our notion of improvement with respect to status-quo that we achieve a better distribution of teachers in the school system. A related approach is followed in Combe et al. (2020) where a teacher assignment problem is also studied. In this paper, they introduce a class of TTC like mechanism, the teacher optimal block exchange (TO-BE) mechanisms. Their main focus is on two-sided efficiency but they show that a unique selection in this class of TO-BE mechanisms is teacher optimal. They show that it outperforms–in terms of distribution of teachers as well as in terms of efficiency–the assignment scheme used in France which is a variation on the DA mechanism. First, this result is shown by focusing mainly on teachers having a status-quo assignment, therefore, largely ignoring the imbalance issues that new teachers can create in terms of distribution of teachers. The generalization they propose to account for new teachers is a combination of TO-BE and the current French mechanism based on DA, which is further away from our SI-CC proposal. Instead of such a two-step approach, we propose a mechanism from scratch to account for both first-time assignment of new teachers and reassignment of tenured teachers. In addition, the preferences of schools we consider, based on the first-order stochastic dominance relation, can be seen as a generalization of the one they introduce. Interestingly, even within their framework of pure reassignment, we can show that our SI-CC mechanism is not in the class of TO-BE mechanisms and so can be viewed as another strategy-proof and teacher optimal mechanism (see Example 6 in Appendix C). Second, we

define SI-DA which requires improvement with respect to the status-quo matching that the current mechanism used in France does not impose. This mechanism can be viewed as the right benchmark to which one should compare SI-CC in our framework.

Despite these two previous studies and the current paper, the study of efficient mechanisms under distributional constraints is still rare. Suzuki et al. (2018) and its generalization by Hafalir et al. (2019) provide sufficient conditions on policy goals to get a version of TTC that take constraints into account and satisfies desirable properties. In particular, these sufficient conditions involve a notion of discrete convexity on the policy goals, namely, M-convexity. In our context with new teachers and vacant positions at schools, we show that M-convexity of the policy goals is not sufficient anymore to ensure a well-behaved version of TTC (see Example 7 in Appendix C).

Finally, our paper builds on a recent literature developing demand estimation methods in school choice environments (Abdulkadiroğlu et al., 2017; Agarwal and Somaini, 2018; Calsamiglia et al., 2020). In particular, we build on techniques based on discrete choice models with personalized choice sets which are relevant for preference estimation when reported preferences might fail to be truthfull even under strategy-proof mechanisms (Fack et al., 2019; Akyol and Krishna, 2017; Artemov et al., 2019).

2 Model

2.1 A Teacher (Re)assignment Market

Let T be a finite **set of teachers**. Each teacher t has a **type**. The type of a teacher captures her observable characteristics that matter for the schools, such as experience, education, past performance, etc, or only a subset of these.[7] Let $\Theta = (\theta_1, \theta_2, \ldots, \theta_n)$ be the finite type space. Let $\tau : T \to \Theta$ be the **type function** and $\tau(t)$ be the type of teacher t. For any $\hat{T} \subseteq T$, we denote type θ teachers in $\hat{T}$ with $\hat{T}^\theta$, i.e.,

$$\hat{T}^\theta \equiv \{t \in \hat{T} \ : \ \tau(t) = \theta\}.$$

Let S be a finite **set of schools**. Each school s has a **capacity** of q_s. Let $q = (q_s)_{s \in S}$. Each teacher t has a **strict preference relation**, which is a linear order and denoted by P_t, over the schools and **being unassigned option** denoted by $\emptyset$. Let $P = (P_t)_{t \in T}$. We denote the at least as good as relation related with P_t by R_t for all $t \in T$: $s \ R_t \ s'$ if and only if $s = s'$ or $s \ P_t \ s'$.

A **matching** $\mu : T \to S \cup \{\emptyset\}$ is a function such that $|\mu^{-1}(s)| \leq q_s$. With a slight abuse of notation, we use μ_t and μ_s instead of $\mu(t)$ and $\mu^{-1}(s)$, respectively.[8] We refer to μ_t as the **assignment** of teacher t and μ_s as the **assignment** of school s in matching μ. Also for a subset of teachers $\hat{T}$, we denote the set of their matches in μ, $\mu(\hat{T})$ as $\mu_{\hat{T}}$.

Initially some teachers are already employed by some schools. This is captured by a **status-quo matching** ω. If $\omega_t = s$, then teacher t is currently employed at school s. If $\omega_t = \emptyset$, then teacher t is called a **new teacher**. She is seeking employment for the first time and she is unemployed at

[7] For example, in the French application, the experience level of a teacher can be thought as the type of a teacher.
[8] Thus, μ_s^θ is the set of teachers of type θ that are assigned school s.

the status quo. By definition, $|\omega_s| \leq q_s$ for each school s. We denote the set of new teachers with N, i.e.,

$$N \equiv \{t \in T \ : \ \omega_t = \emptyset\}.$$

The rest of the teachers are refereed to as **status-quo employees**.

We make one assumption on the preferences of teachers: We assume that $\omega_t \ P_t \ \emptyset$ for each $t \in T \setminus N$, i.e., each employed teacher at the status quo matching finds her current school acceptable.

Finally, we define the preferences of schools over subsets of teachers. Unlike teacher preferences, these preferences are typically weak and allow indifferences. Typically $\succsim_s$ denotes the preferences of a school s over subsets of teachers. Let $\sim_s$ and $\succ_s$ be the associated indifference and strict preference relation with $\succsim_s$, respectively, denoting symmetric and asymmetric portions of the preferences.

In reality, are schools agents who have preferences over subsets of teachers or are they government-mandated entities or resources that have mandated priority orders over teachers, who are public servants, as in the French application? In our model, we are agnostic about this as either interpretation works. A school can be either seen as an agent that cares about its teacher quality or it can be seen as part of a resource pool that are collectively managed by a central authority, which has an objective of improving the quality of employed teachers in every school as much as possible with an eye on not making any school worse than its status quo. It is therefore only a semantic exercise to call them as agent or resource and these binary relations as preferences or priority orders of the school dictated by a centralized authority with a clear objective of improving education in mind. In the latter case, we are proposing these binary relations as a new priority design that can be used by a centralized authority to replace an existent one that solely relies on some other criteria, as in the French application.

To this end, each school s has a **type ranking**, which is a linear order and denoted by $\rhd$, over the types of teachers and an **individual rationality threshold** type denoted by $\theta_\emptyset$: $\theta \ \rhd_s \ \theta' \ \rhd_s \ \theta_\emptyset \ \rhd_s \ \theta''$ means school s ranks type θ teachers over type θ' teachers and finds both types of teachers **acceptable** to hire but it considers type θ'' teachers **unacceptable** to hire. Let $\theta \trianglerighteq_s \theta'$ if either $\theta \ \rhd_s \ \theta'$ or $\theta = \theta'$. We assume that if $\omega_s^\theta \neq \emptyset$, then $\theta \rhd_s \theta_\emptyset$, i.e., each school finds the types of its current teachers acceptable.

We make two assumptions on the preferences of schools.

1. We assume that when a school compares two subsets of teachers it uses **first-order stochastic dominance** (**FOSD**)[9] relation based on its type ranking. In particular, school s weakly prefers subset of teachers $\bar{T}$ to $\hat{T}$, i.e., $\bar{T} \succsim_s \hat{T}$, if
   (i) there does not exist an unacceptable teacher in $\bar{T}$, i.e., $\bar{T}^\theta = \emptyset$ for any $\theta \lhd_s \theta_\emptyset$, and

[9] Although FOSD relation is, in general, defined to compare statistical distribution functions, with a slight abuse of terminology we use the same name for the analogous binary relation that compares distributions of teacher types. Indeed, in our empirical analysis we use this language, comparing distributions of teachers using FOSD assigned to schools.

(ii) for any $\theta \rhd_s \theta_\emptyset$ we have

$$\sum_{\theta' \trianglerighteq_s \theta} |\bar{T}^{\theta'}| \geq \sum_{\theta' \trianglerighteq_s \theta} |\hat{T}^{\theta'}|.$$

Moreover, the preference is strict if at least one of the inequalities is strict.

If FOSD does not hold between two sets in either direction, then the school preferences do not compare them. Therefore, school preferences are incomplete. Thus, a school only unambiguously prefers groups whenever two groups of teachers can be ranked based on this FOSD comparison.

In the rest of our analysis, we compare outcome matchings with the status-quo matching. FOSD relation will be sufficient and we will achieve unambiguous comparisons for our purposes.

2. We assume that for any subset of teachers T', if $\theta_\emptyset \rhd_s \tau(t)$ for some $t \in T'$, then $\omega_s \succ_s T'$. That is, each school prefers its status-quo assignment to any teacher set with an unacceptable teacher in it.

We refer to the list $\langle T, S, q, \omega, P, \succsim \rangle$ as a **teacher (re)assignment market**. Typically, $T, S, q, \omega, \succsim$ are commonly known in our applications. Only teacher preferences are private information. For the rest of our analysis, we fix $T, S, q, \omega, \succsim$ and denote a market with teacher preferences P.

We are seeking a matching outcome given a market P.

The most basic property of outcome matchings we consider is *status-quo improvement*. Maybe surprisingly, this is sometimes in conflict with many other standard desiderata used in the literature for matching market design. A matching μ is **status-quo improving** if $\mu_t \; R_t \; \omega_t$ for all $t \in T$ and $\mu_s \succsim_s \omega_s$ for all $s \in S$. That is, each agent should be weakly better off in a status-quo improving matching with respect to the status-quo matching.

We inspect rules that select a matching for each market. Formally, a (**direct**) **mechanism** ϕ is a function that chooses an outcome matching for any market P. Let $\phi(P)$, $\phi_t(P)$, and $\phi_s(P)$ denote the matching selected by mechanism ϕ under market P, the assignment of teacher t, and the assignment of school s in that matching, respectively.

A mechanism ϕ is **strategy-proof** if truth telling is a weakly dominant strategy for all teachers, that is, for all markets P, for all teachers t, for all possible alternative preference reports P'_t,

$$\phi_t(P_t, P_{-t}) \; R_t \; \phi_t(P'_t, P_{-t}).$$

As we assume that schools' rankings over the types of teachers, and therefore, their preferences over the teachers are commonly known, schools do not need to report them.

In the next two sections, we provide two different mechanisms to achieve two different desiderata: a refinement of Pareto efficiency or an appropriate stability concept for our applications together with status-quo improvement, respectively.[10]

[10] In Example 8 in Appendix C, we show that there does not exist a mechanism satisfying efficiency and stability.

3 Efficiency, Pointing Rule Design, and Status-quo Improving Cycles and Chains

3.1 Status-quo Improving Teacher Optimality

Consider a market P. A matching μ **Pareto dominates** a matching ν **for teachers** if

$$\mu_t \; R_t \; \nu_t \quad \text{for all } t \in T, \text{ and} \tag{1}$$

$$\mu_{t'} \; P_{t'} \; \nu_{t'} \quad \text{for some } t' \in T. \tag{2}$$

Matching μ **Pareto dominates** a matching ν **for schools** if

$$\mu_s \; \succsim_s \; \nu_s \text{ for all } s \in S, \text{ and} \tag{3}$$

$$\mu_{s'} \; \succ_{s'} \; \nu_{s'} \text{ for some } s' \in S. \tag{4}$$

Finally, matching μ **Pareto dominates** a matching ν if (i) Equations 1 and 3 hold, and (ii) Equation 2 or Equation 4 holds. A matching is **(two-sided) Pareto efficient** if it is not Pareto dominated by any other matching.

A matching μ is **status-quo improving – teacher optimal** (**SI teacher optimal** for short) if it is status-quo improving and not Pareto dominated for teachers by any other status-quo improving matching. While SI teacher optimality seems to care mostly about the welfare of teachers, any SI teacher optimal matching is also Pareto efficient, since teacher preferences are strict.

Proposition 1 *Any SI teacher optimal matching is Pareto efficient.*

All proofs are provided in Appendix A.

We will introduce a strategy-proof and SI teacher optimal mechanism. Why do we implement on SI teacher optimality rather than a dual concept such as SI school optimality or different Pareto efficient outcomes?[11] It has two reasons: First, in most of the applications we have in mind, the side we characterize as schools are quasi-agents rather than full agents unlike the side we characterize as teachers. Their welfare has been the main efficiency measure both in practice and in literature (for example, see Abdulkadiroğlu and Sönmez, 2003, Combe, Tercieux and Terrier, 2020). Second, if we wanted to implement other Pareto efficient outcomes that are not SI teacher optimal, we would not be able to find a strategy-proof mechanism.[12]

3.2 Status-quo Improving Cycles and Chains Mechanism

Next, we will introduce a strategy-proof and SI teacher optimal mechanism. To achieve this goal, we introduce additional tools.

[11] Because of indifference classes in schools' FOSD preferences regarding same-type teachers, this concept defined as a simple dual notion of SI teacher optimality may not be Pareto efficient and has to be refined even further.

[12] In Appendix A, Proposition 6 shows that if a status-quo improving mechanism selects a Pareto efficient matching other than SI teacher optimal whenever such a matching exists, then this mechanism can be manipulated by teachers.

Our mechanism will iteratively construct a sequence of directed graphs in which teachers, schools, and being unassigned option are the nodes. Teachers can only point to schools or the being unassigned option and schools can only point to teachers in their status-quo assignment in each of these graphs. When node x points to node y, then a directed arc from x to y is activated.

Our mechanism relies on *executing* two types of multi-lateral exchanges based on the constructed directed graphs.

A **cycle** is a directed path of distinct teachers $\{t_m\}$ and distinct schools $\{s_m\}$, possibly being unassigned option $\emptyset$,

$$(s_1, t_1, s_2, t_2, \ldots, s_k, t_k)$$

such that $\omega_{t_m} = s_m$ for all m, each node points to the next node in the path, and t_k points back to s_1.

A **chain** is a directed path of distinct teachers $\{t_m\}$ and schools $\{s_m\}$

$$(t_0, s_1, t_1, \ldots, s_{k-1}, t_{k-1}, s_k)$$

such that $\omega_{t_m} = s_m$ for all $m = 1, \ldots k-1$, each node points to the next node in the path.[13] Here, we say the chain starts with t_0 and ends with s_k. In other words, s_k is the head of the chain and t_0 is the tail of the chain.

As certain cycles and chains are encountered in the constructed graph, we will **execute** the exchanges in them by assigning each teacher to the school she is pointing to and remove her.

Our main theoretical innovation relies on designing *pointing rules* that designate which possible directed arcs in a graph will be endogenously activated through the algorithm.

Pointing rule of teachers will be introduced within the definition of the mechanism below as it uses the endogenous working of the mechanism's algorithm. On the other hand, the pointing rule of schools relies on their type rankings and an exogenously given tie breaker.

Formally, a **tie breaker** is a linear order $\vdash$ over teachers.[14] It can be randomly determined or can be the mandated priority orders for a particular application, such as in the French case, or can be exogenously fixed in some other manner.

The tie breaker for the new teachers and tie breaker regarding teachers employed at the status quo are utilized differently in the algorithm. For each school s, using tie breaker $\vdash$ and its type ranking $\rhd_s$, we first construct a **pointing order** $\succcurlyeq_s$ over teachers in ω_s, which is a linear order as well: For any two distinct teachers $t, t' \in \omega_s$,

$$t \succcurlyeq_s t' \iff \tau(t) \lhd_s \tau(t') \quad \text{or} \quad \big[\tau(t) = \tau(t') \text{ and } t \vdash t'\big].$$

Note that a worse-type teacher is prioritized over a better-type teacher, and only when two same-type teachers are compared, we use the tie breaker to prioritize one over the other.

[13] Notice that, we allow a school to appear more than once in a chain.

[14] Technically, each tie breaker induces a new mechanism in our class.

As the mechanism will iteratively assign and remove teachers, the **pointing rule of schools** is "point to the highest remaining priority teacher in its priority order."

Now, we are ready to define our mechanism through an iterative algorithm:

Definition 1 ***Status-quo Improving Cycles and Chains (SI-CC) Mechanism***

We will construct a matching μ dynamically through the following algorithm. Initially, μ is the empty matching, in which no teacher is assigned to any school. In each step, as teachers are assigned in μ, they will be removed from the algorithm; similarly schools whose all seats are filled in μ and also some other schools chosen by the algorithm will be removed.

For each school s and type θ, let b_s^θ track the ***current balance*** *of type θ teachers at school s in* ***current matching*** *μ, which is the matching fixed until the beginning of the current step. The current balance is defined as the difference between the number of type θ teachers assigned to s in μ and the number of type θ teachers in its status-quo assignment assigned to any school in μ: $b_s^\theta \equiv |\mu_s^\theta| - |\{t \in \omega_s : \mu_t \neq \emptyset\}^\theta|$. Thus, we initialize $b_s^\theta = 0$.*

A general step k is defined as follows:

Step k:

- *Each remaining school s points to the highest priority remaining teacher in ω_s under $\succcurlyeq_s$, if not all students in ω_s are already assigned in μ; let t_s^k be the teacher pointed by school s in step k. Otherwise, school s does not point to any teacher.*
- *We define the* ***pointing rule of teachers*** *as follows: Any remaining teacher t is allowed to point to a remaining school s if at least one of the following two school* ***improvement conditions*** *hold for school s via teacher t:*
  1. *(Improvement for s by teacher trades) if the school points to a teacher t_s^k and*

$$\sum_{\theta' \trianglerighteq_s \theta} b_s^{\theta'} > 0 \text{ for all types } \theta \text{ such that } \tau(t_s^k) \trianglerighteq_s \ \theta \ \triangleright_s \ \tau(t),$$

  or[15]
  2. *(Improvement for s by only incoming teachers) $\tau(t) \triangleright_s \theta_\emptyset$, school s currently has an unfilled seat, i.e., $q_s - |\mu_s| > |\{t' \in \omega_s : \mu_{t'} = \emptyset\}|$, and there are remaining new teachers.*

  Let A_t^k be the opportunity set for a remaining teacher t, i.e., the set of schools t can point in this step together with the being unassigned option $\emptyset$.[16]
  Each remaining teacher t points to her most preferred option in A_t^k.
- *Being unassigned option $\emptyset$ points to all teachers pointing to it.*

Due to finiteness, there exists either

(i) a cycle in which all schools in the cycle satisfy improvement Condition 1 or a cycle between a

[15]Condition 1 is trivially satisfied if no such θ exists, i.e., if $\tau(t) \trianglerighteq_s \tau(t_s^k)$. An alternative condition to this one is provided in Appendix C.

[16]Note that, $\omega_t \in A_t^k$ for all remaining teachers t who were employed at the status quo.

single teacher and the being unassigned option $\emptyset$, or

(ii) a chain.

Then:

- ***If Case (i) holds:*** *Each teacher can be in at most one cycle as she points at most to a single option. We execute exchanges in each cycle encountered in case (i) by assigning the teachers in that cycle to the school she points to, update current matching μ and current balances $\{b_s^\theta\}$ accordingly, remove assigned teachers and filled schools in μ, and go to step $k+1$.*
- ***If Case (i) does not hold:*** *Then case (ii) holds, i.e., there exists a chain. In particular, each remaining teacher initiates a chain. There are two subcases:*
  - ***If there exists a remaining new teacher****: Then we select a chain to be executed as follows:*
    - *Select as the tail of the chain the new teacher with the highest priority under tie breaker $\vdash$ and then include in the chain the school she points to.*[17] *If Improvement Condition 1 does not hold for this school via this teacher, but only Improvement Condition 2 holds, then we end the chain with this school; otherwise, we repeat the following:*
    - *Include to the chain the teacher pointed by the last school included.*[18] *If we include a teacher, we also include next in the chain the school she is pointing to. We repeat this iteratively until the Improvement Condition 1 does not hold for the next school via the included teacher, but only Improvement Condition 2 holds.*[19]

    The last school included is the head of the selected chain.
    We execute the exchanges in the selected chain by assigning each teacher in the chain to the school she points to, update current matching μ and current balances $\{b_s^\theta\}$ accordingly, remove assigned teachers and filled schools, and go to step $k+1$.
  - ***If there does not exist a remaining new teacher****: Then we remove each school s whose all status-quo employees in ω_s were already assigned in μ.*[20] *We continue with step $k+1$.*

The mechanism terminates when all teachers are removed. Its outcome is the final matching μ.

The name of the mechanism suggests that both teachers and schools become better off through the mechanism with respect to the status quo. Indeed, this is the case. We introduced several innovations in the mechanism that exploit different Pareto improvement possibilities for teachers and schools over the status-quo matching.

Pareto improvement of teachers is straightforward in the algorithm. Teachers who are employed at the status quo are eventually assigned to a school at least as good as their status-quo assignment.

[17] Such a school exists, because if she does not point to a school, then she pointed to being unassigned option $\emptyset$ and was removed previously.

[18] Such a teacher exists by Improvement Condition 1.

[19] This iterative procedure is guaranteed to terminate. Otherwise, we would have a cycle.

[20] There must exit a school which is still in the market and whose status-quo employees have already been assigned. Otherwise, each remaining school would point to one teacher and each remaining teacher would point to a school and so there would exist a cycle, a contradiction.

Moreover, all teachers are assigned the best option they can point in the step they are assigned.

What is more delicate is the Pareto improvement of schools, i.e., how we make sure that they always weakly improve with respect to their status-quo assignment in every step. This is ensured through the introduction of both teacher and school pointing rules.

A school's pointing order designates in which order the school would like to *send out* its status-quo employees. By pointing, the school effectively gives permission to one of its status-quo employees to be assigned possibly to a different school. Thus, we make sure that this priority order is in reverse order of its preferences: Less preferred-type employees are pointed first and more preferred-type employees are pointed later. This is the first innovation.

On the other hand, the teacher pointing rule designates which teachers can be assigned to a school. Therefore, we only allow teachers who can improve the school's welfare with respect to its status-quo assignment after the currently pointed employee of the school is sent out.

The two school improvement conditions make sure of this.

Condition 1 has two cases: If the type of the possibly incoming teacher is at least as good as the type of the possibly outgoing teacher, the school has no danger of becoming worse off in this trade. The second case on the other hand is more delicate: As trades that strictly improve a school's welfare occur over steps, schools acquire new teachers who are actually of better types than the types of outgoing status-quo employees. Therefore, they may build up a *buffer*. If such a buffer exists, a worse-type teacher than its currently outgoing employee can still be assigned to the school, although this trade makes the school worse off with respect to the previous step. However, the school is still weakly better off with respect to the status quo thanks to the buffer. Only the buffer gets thinner. The existence of the buffer is tracked by checking whether the sums of the relevant type balances, b_s^θ's, are positive through Condition 1. The use of this buffer ensures teacher optimality.

While the first condition is about a trade the school will make by exchanging an outgoing teacher with an incoming teacher, Condition 2 is only relevant as long as new teachers remain in the algorithm. When Condition 2 holds for a school via some teacher, but not Condition 1, the school will not send out an employee as it has extra capacity: it will only hire one additional acceptable teacher.

Before stating our main result, we illustrate how the SI-CC mechanism works using Example 1.

Example 1 *Let* $S = \{s_1, s_2, s_3, s_4\}$, $T = \{t_1, t_1', t_2, t_2', t_3, t, t'\}$, *the status-quo matching be*

$$\omega_{s_1} = \{t_1, t_1'\},\ \omega_{s_2} = \{t_2, t_2'\}\ \omega_{s_3} = \{t_3\},\ \omega_{s_4} = \emptyset,$$

$q_{s_1} = 3$, $q_{s_2} = q_{s_4} = 2$, *and* $q_{s_3} = 1$. *The preferences of teachers are:*

$$s_2\ P_{t_1}\ s_1\ P_{t_1}\ \emptyset\ P_{t_1}\ s_3\ P_{t_1}\ s_4$$

$$s_4\ P_{t_1'}\ s_1\ P_{t_1'}\ \emptyset\ P_{t_1'}\ s_2\ P_{t_1'}\ s_3$$

$$s_3 \; P_{t_2} \; s_2 \; P_{t_2} \; \emptyset \; P_{t_2} \; s_1 \; P_{t_2} \; s_4$$

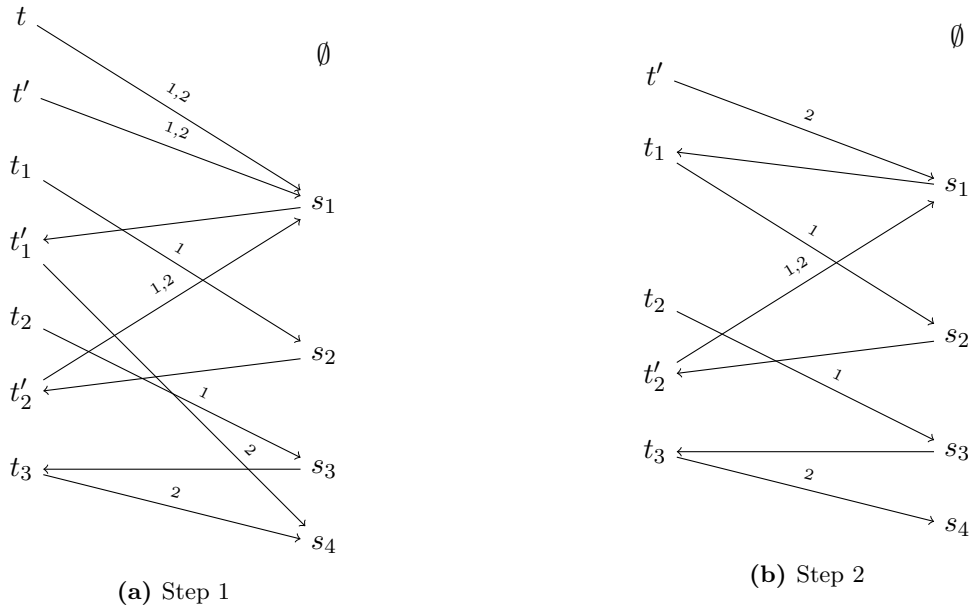
$$s_1 \; P_{t'_2} \; s_2 \; P_{t'_2} \; \emptyset \; P_{t'_2} \; s_3 \; P_{t'_2} \; s_4$$

$$s_4 \; P_{t_3} \; s_2 \; P_{t_3} \; s_3 \; P_{t_3} \; \emptyset \; P_{t_3} \; s_4$$

$$s_1 \; P_t \; \emptyset \; P_t \; s_2 \; P_t \; s_3 \; P_t \; s_4$$

$$s_1 \; P_{t'} \; \emptyset \; P_{t'} \; s_2 \; P_{t'} \; s_3 \; P_{t'} \; s_4$$

There are only two types of teachers: $\bar{\theta}$ and $\underline{\theta}$ for respectively "High" and "Low" experience. All schools prefer high experience teachers to low experience teachers so that $\bar{\theta} \rhd_{s_i} \underline{\theta}$ for $i = 1, 2, 3, 4$. We assume that $\tau(t'_i) = \tau(t_3) = \underline{\theta}$ for $i = 1, 2$, $\tau(t_i) = \bar{\theta}$ for $i = 1, 2$ and $\tau(t) = \tau(t') = \underline{\theta}$. Since each initial teacher of a school has a different type, the pointing order can be arbitrary. For new teachers, assume that the tie-breaker ranks t above t' so that $t \vdash t'$. At the beginning of Step 1 of SI-CC, using the pointing behaviors of the definition, we obtain the graph in Figure 1a.



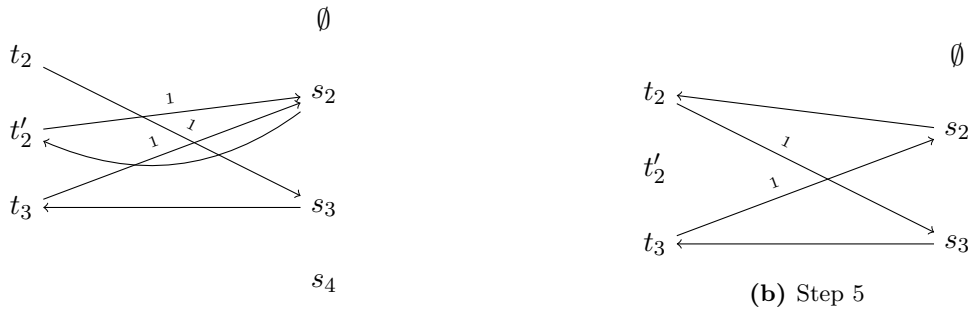
(a) Step 1

(b) Step 2

Figure 1: Graph of the steps 1 and 2 of SI-CC

For each arrow going from a teacher to a school, we report the improvement conditions, i.e. 1 and/or 2 in the definition of pointing rule for teachers, which hold for that arrow. One can note that there is no cycle in this graph. Thus there are only two possible chains: one starting at t or one starting at t'. Since $t \vdash t'$, we pick the one starting with t and, following the procedure described, implement the chain $\{t, s_1, t'_1, s_4\}$ since t'_1 points to s_4 only because of the improvement condition 2. At the beginning of Step 2, the graph becomes the one in Figure 1b.

In that case, one can check that the cycle $\{s_1, t_1, s_2, t'_2\}$ is implemented and that, at Step 3, the chain $\{t', s_1\}$ is implemented. At the beginning of Step 4, the graph of SI-CC is the one in Figure 2a. Note that even though teacher t_3 prefers s_4 to s_2, she cannot point to the former because even though it has an empty seat left, there is no remaining new teacher.



(a) Step 4 **(b)** Step 5

Figure 2: Graph of the steps 4 and 6 of SI-CC

In that step, we implement the cycle $\{s_2, t_2'\}$*. At the next step, we obtain the graph in Figure 2b. Note that even though* t_3 *has a low experience and* t_2 *a high experience, the former can still point to* s_2 *since it has accepted* t_1*, a high experience teacher, at step 2 so that the improvement condition 1 in the pointing rule of teachers is satisfied. So we implement the cycle* $\{s_2, t_2, s_3, t_3\}$ *and the algorithm stops.*

Now we are ready to state our main result in this section.

Theorem 1 *The SI-CC mechanism is SI teacher optimal and strategy-proof.*

SI teacher optimality of the mechanism is delicate to show. Note that SI teacher optimality implies that the outcome matching is Pareto undominated for teachers among all status-quo improving matchings. However, the pointing rule of teachers has restrictions imposed by the school improvement conditions. That is, a teacher cannot arbitrarily point to the best school she likes. We show that the restrictions imposed by these conditions are the necessary and sufficient conditions for keeping status-quo improvement for schools without affecting the outcome being Pareto undominated for teachers. Therefore, implementing any further Pareto improvement for teachers would make the schools worse off with respect to the status quo. Moreover, imposing further restrictions for teacher pointing would prevent SI teacher optimality.

Strategy-proofness of the mechanism relies on several observations: First, once a teacher is pointed by a school, she will continue to be pointed until she is assigned. We show that the opportunity set for each teacher t, A_k^t, weakly shrinks across steps k. Although Improvement Conditions 1 or 2 may stop holding for a school via a teacher t across steps, we show that teacher t cannot affect which schools leave and stay in A_k^t before she is assigned by submitting different preferences.

An immediate corollary to the theorem is that the SI-CC mechanism is also (two-sided) Pareto efficient by Proposition 1.

Under the pointing rule of schools, in each step of SI-CC, each school points to one of its employee who has the lowest ranked type. One can think whether Theorem 1 holds when we consider alternative school pointing rules under SI-CC mechanism. Example 2 shows that SI-CC

can be manipulated by a teacher and it is not SI teacher optimal under an alternative pointing rule.

Example 2 *Let* $S = \{s, s', s''\}$, $T = \{t_1, t_2, t_3, t_4\}$, *the status-quo matching be*

$$\omega_s = \{t_1, t_2\}, \omega_{s'} = \{t_3\}, \omega_{s''} = \{t_4\},$$

$q_s = 2$, $q_{s'} = q_{s''} = 1$ *and* $\tau(t_1) \rhd_s \tau(t_2) = \tau(t_3) = \tau(t_4)$. *The preferences of the teachers are*

$$sP_{t_1}s'P_{t_1}s''P_{t_1}\emptyset,$$

$$s'P_{t_2}sP_{t_2}s''P_{t_2}\emptyset,$$

$$sP_{t_3}s'P_{t_3}s''P_{t_3}\emptyset,$$

$$s''P_{t_4}sP_{t_4}s'P_{t_4}\emptyset.$$

If in the first step of SI-CC school s *points to* t_1, *the best school* t_3 *can point is* s'. *Therefore, she will be assigned to* s'. *In particular, under true preferences SI-CC assigns all employees to their status-quo schools. This outcome is not SI teacher optimal because it is Pareto dominated by another status-quo improving matching* ν *for teachers where* $\nu(t_1) = \nu(t_3) = s$, $\nu(t_2) = s'$ *and* $\nu(t_4) = s''$. *Moreover, if* t_3 *swaps the rankings of* s' *and* s'', *then SI-CC selects* ν, *i.e.,* t_3 *manipulates SI-CC when* s *points* t_1 *in the first step.*

We would like to emphasize one possible generalization of SI-CC through school pointing rule. We can easily use the first school improvement condition given in the definition of SI-CC to dynamically update the school pointing rule such that monotonicity of opportunity set for teachers is preserved.

4 Stability, Choice Rule Design, and Status-quo Improving Deferred Acceptance

4.1 Status-quo Improving Stability

Although Pareto efficiency is a very appealing property of matchings, many real-life applications use fairness or stability notions, which often conflict with SI teacher optimality and in general with Pareto efficiency under incentive compatibility constraints. For example, in the French teacher (re)assignment application, the mechanism currently used is not Pareto efficient, while it satisfies a stability condition that is not necessarily status-quo improving.

To this end, we also introduce a stability concept that is consistent with status-quo improvement under a mild assumption about the number of new teachers in a market. Our notion has different requirements than Gale-Shapley stability (Gale and Shapley, 1962), which is extensively used in the literature, because in our setting we have a non-empty status-quo matching while most of the literature focuses on an empty matching as the status quo.

Consider a market P.

To introduce stability, we first start with blocking by an agent and a pair. A matching μ is **blocked by a teacher** t if $\emptyset \; P_t \; \mu_t$. A matching μ is **blocked by a school** s if there exists $t' \in \mu_s$ with $\theta_\emptyset \rhd_s \tau(t')$. Observe that a status-quo improving matching is not blocked by any agent, while a matching that is not blocked by any agent may not be status-quo improving.

Given a teacher t and school s, a matching μ is **blocked by pair** (t, s) **through** $t' \in \mu_s$ if (i) $s \; P_t \; \mu_t$ and (ii) $\tau(t) \rhd_s \tau(t')$. Similarly, a matching μ is **blocked by pair** (t, s) **through an empty slot** if (i) $s \; P_t \; \mu_t$, (ii) $\tau(t) \rhd_s \tau(t')$ and (iii) $|\mu_s| < q_s$.

A matching μ is **Gale-Shapley stable** if there is no blocking agent and no blocking pair. This classical concept potentially conflicts with our most basic property, status-quo improvement:

Proposition 2 *A Gale-Shapley stable matching always exists, however, it may not be status-quo improving. Thus, a Gale-Shapley stable and status-quo improving matching may not exist.*

One may think that the cause of incompability of status-quo improvement with Gale-Shapley stability is not giving employment rights to teachers at their status-quo schools. Indeed the current system in France uses a strategy-proof mechanism that satisfies the following stability concept implicitly.[21]

A matching μ is **teacher-status-quo-improving (teacher-SI) stable** if there is no blocking agent and no blocking pair through an empty slot, and if there is a blocking pair (t, s) through $t' \in \mu_s$, then $t' \in \omega_s$. This concept ignores blocking pairs as long as assigning the teacher to the school in the blocking pair would displace a status-quo employee of the school. This concept still does not resolve the main problem.

Proposition 3 *Even when there are no empty seats at schools at status quo and there are no new teachers, the current French mechanism is teacher-SI stable but not status-quo improving, while the status-quo matching is both teacher-SI stable and status-quo improving. Moreover, if there are empty seats at some schools at status quo, then a teacher-SI stable and status-quo improving matching may not exist.*

We should strengthen no blocking by an agent to status-quo improvement. Given the above impossibility result, however, this remedy alone does not resolve our non-existence problem when there are empty seats at the status quo. Instead, we introduce the following concept which implicitly gives new teachers rights over status-quo empty seats of a school.

A matching μ is **status-quo improving stable** (**SI stable** for short) if

1. it is status-quo improving, i.e., $\mu_s \succ_s \omega_s$ and $\mu_t \; P_t \; \omega_t$ for all $s \in S$ and $t \in T$;
2. there is no blocking pair (t, s) through an empty slot; and

[21] To make the current French setup more consistent with ours one may think that each teacher has a different type and the type ranking of each school is given by the government-dictated strict priority order used in France.

3. there is no blocking pair (t, s) through t' such that either $t, t' \in N$ or $t, t' \in T \setminus (N \cup \omega_s)$.

SI stability requires status-quo improvement, which implies elimination of individual blocking, and elimination of blocking pairs through empty slots. Moreover, it requires elimination of any blocking pair (t, s) through t' such that t and t' are either new teachers or are currently employed by another school s'.

As a result, this concept is neither weaker (because of the more stringent individual blocking condition) nor stronger (because of the less stringent pairwise blocking conditions) than both Gale-Shapley and teacher-SI stability concepts.

Although it may appear counter intuitive to allow certain blocking pairs, it turns out that this is necessary to sustain status-quo improvement. The solution provided in the following subsection eliminates further blocking pairs such as the ones including new teachers through an existing teacher. In Appendix C, by using examples we show that allowing the other blocking pairs not captured by Condition 3 is needed to guarantee existence of an SI stable matching.

4.2 Choice Rule Design and Status-quo Improving Deferred Acceptance Mechanism

In this section, we introduce a strategy-proof mechanism that is SI stable under a mild assumption we will introduce below. Our main contribution here is to introduce a choice rule for schools that will achieve SI stability and strategy-proofness when it is used in conjunction with the teacher proposing deferred acceptance (DA) algorithm of Gale and Shapley (1962) adopted for complex matching terms by Roth and Sotomayor (1990) (which was itself adopted from more complex versions of such processes in Kelso and Crawford, 1982, Roth, 1984b, Blair, 1988).

Given a school s, a **choice rule** is a function $C_s : 2^T \to 2^T$ such that for any $\hat{T} \subseteq T$, (i) $C_s(\hat{T}) \subseteq \hat{T}$ and $|C_s(\hat{T})| \leq q_s$.

Using the choice rules we will design below we will employ the well-known teacher proposing DA algorithm. We consider the sequential version of this algorithm also known as the cumulative offer process (Hatfield and Milgrom, 2005) for more complex contractual matching terms:

Definition 2 ***Teacher Proposing Deferred Acceptance Algorithm (DA):***

Step 1***:*** *Some teacher t' proposes to her favorite acceptable school, denoted by s', if such a school exists. In this case, define $B^2_{s'} \equiv \{t'\}$ and $B^2_s \equiv \emptyset$ for each school $s \neq s'$. Otherwise, define $B^2_s \equiv \emptyset$ for each school s.*

Each school s holds teachers in $C_s(B^2_s)$ and rejects all other teachers in B^2_s.

In general,

Step $k > 1$***:*** *Some teacher t'' who is not currently held by any school proposes to her most favorite acceptable school that has not rejected her yet, denoted by s'', if such a school exists. In this case, define $B^{k+1}_{s''} \equiv B^k_{s''} \cup \{t''\}$ and $B^{k+1}_s \equiv B^k_s$ for each $s \neq s''$. Otherwise, define $B^{k+1}_s \equiv B^k_s$ for each school s.*

Each school s holds $C_s(B_s^{k+1})$ and rejects all teachers in $B_s^{k+1} \setminus C_s(B_s^{k+1})$.

The algorithm terminates when each teacher is either rejected by all of her acceptable schools or currently held by some school. We assign each school the students it is holding.

Our main contribution in this subsection is the construction of a choice rule for each school. Fix a school s. First, we need some additional concepts.

A **tie breaker** is a linear order over teachers $\vdash$ as before. We construct a new linear order over the teachers in ω_s denoted by $\blacktriangleright_s$ as follows:

For any $t, t' \in \omega_s$,

$$t \ \blacktriangleright_s \ t' \iff \tau(t) \ \triangleright_s \ \tau(t') \text{ or } [\tau(t) = \tau(t') \text{ and } t \ \vdash \ t'].$$

Observe that a better-type teacher is prioritized over a worse-type teacher, and when two same-type teachers are compared, then we use the tie breaker to prioritize one over the other.[22]

The auxiliary choice rule will use a lexicographic decision structure within a school by dividing the school into independent slots where each **slot** eventually represents a seat at the school. Such a model was previously introduced by Kominers and Sönmez (2016) in one-sided priority-based matching context for more complex contractual matching terms.

We fix a school s in this construction. Let $\mathcal{S}_s = \{s^1, s^2, \ldots, s^{q_s}\}$ be the set of slots at school s. Without loss of generality we label the types in Θ as $\theta_1, \ldots, \theta_{|\Theta|}$ based on the type ranking of the school such that $\theta_k \ \triangleright_s \ \theta_{k+1}$ for all $k \in \{1, \ldots, |\Theta| - 1\}$. We define a ranking for each slot over $T \cup \{\emptyset\}$ where $\emptyset$ denotes keeping the slot unfilled. The **ranking of slot** s^k, $\succ_s^k$, is defined separately for the slots representing the filled seats at the status-quo matching, i.e., for $k \leq |\omega_s|$, and slots representing the empty seats at the status-quo matching, i.e., for $|\omega_s| < k \leq q_s$:

- For filled slots s^k at the status quo, i.e., all $k \leq |\omega_s|$:
  - the teacher $t \in \omega_s$ who is ranked k'th under $\blacktriangleright_s$ has the highest ranking under $\succ_s^k$,
  - any teacher t' with $\tau(t) \triangleright_s \tau(t')$ is ranked below $\emptyset$ under $\succ_s^k$, and
  - the rest of the ranking under $\succ_s^k$ is determined according to $\triangleright_s$ such that ties between same type teachers are broken according to tie breaker $\vdash$.
- For empty slots s^k at the status quo, i.e., all k such that $|\omega_s| < k \leq q_s$:
  - a teacher t is ranked above $\emptyset$ under $\succ_s^k$ if and only if she is acceptable, i.e., $\tau(t) \triangleright_s \theta_\emptyset$,
  - any acceptable new teacher t (i.e., $t \in N$ and $\tau(t) \triangleright_s \theta_\emptyset$) is ranked under $\succ_s^k$ above any teacher t' employed at status quo by some school (i.e., $t' \notin N$), and
  - the rest of the ranking under $\succ_s^k$ is determined according to $\triangleright_s$ such that ties between same type teachers are broken according to tie breaker $\vdash$.

Thus, the set of acceptable teachers for slot s^k is a superset of the set of acceptable teachers for slot s^{k-1}.

[22] Thus, linear order $\blacktriangleright_s$ effectively reverses the ordering of status-quo employees of different types in the pointing order $\succcurlyeq_s$ used in the SI-CC mechanism, while it respects the school's FOSD preferences.

We will make the following mild overdemand assumption in the rest of this section involving new teachers and schools with excess status-quo capacity:

Assumption 1 *There exists a subset of new teachers $N' \subseteq N$ such that (i) there are at least as many new teachers in N' as empty seats at status quo, i.e., $|N'| \geq \sum_{s \in S}(q_s - |\omega_s|)$, and (ii) each teacher $t \in N'$*

- *considers all schools with excess capacity acceptable, i.e., if $q_s > |\omega_s|$, then $s \; P_i \; \emptyset$, and*
- *is acceptable for all schools with excess capacity, i.e., if $q_s > |\omega_s|$, then $\tau(t) \rhd_s \theta_\emptyset$.*

In the absence of either part of Assumption 1, we can come up with examples such that some schools end up with fewer teachers than what they have under the status-quo matching and status-quo improvement is violated for schools (see Appendix C).

Since the choice rule is defined through filling each slot one a time, we need to determine in which order the slots are processed. We process the slots in the natural order[23]

$$s^1, \; s^2, \; \ldots, s^{q_s}.$$

Definition 3 *The **choice rule** C_s of school s is defined through an iterative procedure. The chosen set from the set of teachers $\hat{T}$ by school s, denoted by $C_s(\hat{T})$, is determined as follows:*

- ***Step** 1**:** The most preferred acceptable teacher under $\succ_s^1$ in $\hat{T}_1 = \hat{T}$ is assigned to slot s^1 and she is removed. If there is no such teacher, then s^1 remains empty. Denote the remaining teachers with $\hat{T}_2$.*
  In general,
- ***Step** $k \geq 2$**:** The most preferred acceptable teacher under $\succ_s^k$ in $\hat{T}_k$ is assigned to slot s^k and she is removed. If there is no such teacher, then s^k remains empty. Denote the remaining teachers with $\hat{T}_{k+1}$.*

The process terminates when all slots are processed, i.e., step q_s is the last step. Chosen set $C_s(\hat{T})$ is the set of teachers assigned to the slots of school s.

We illustrate how a chosen set is found in the following example.

Example 3 *Suppose there are five teachers one of whom is new: $T = \{t_1, t_2, t_3, t_4, t_5\}$ and $t_5 \in N$. The status-quo assignment of school s, which has capacity $q_s = 3$ is $\omega_s = \{t_1, t_2\}$. The type ranking of school s is*

$$\tau(t_1) = \tau(t_3) \rhd_s \tau(t_2) \rhd_s \tau(t_4) \rhd_s \tau(t_5) \rhd_s \theta_\emptyset.$$

The slot set of s is $\mathcal{S}_s = \{s^1, s^2, s^3\}$ such that s^1 and s^2 correspond to filled seats at status quo and s^3 corresponds to the empty seat.

Let the tie breaker $\vdash$ be such that $t_1 \vdash t_3$.

[23] Later we will explain why this precedence order is chosen.

We construct the rankings for each slot as follows:

$$t_1 \succ_s^1 t_3 \succ_s^1 \emptyset \succ_s^1 t \quad \text{for any } t \notin \{t_1, t_3\},$$
$$t_2 \succ_s^2 t_1 \succ_s^2 t_3 \succ_s^2 \emptyset \succ_s^2 t \quad \text{for any } t \notin \{t_1, t_2, t_3\},$$
$$t_5 \succ_s^3 t_1 \succ_s^3 t_3 \succ_s^3 t_2 \succ_s^3 t_4 \succ_s^3 \emptyset.$$

Suppose $\hat{T} = \{t_2, t_3, t_4, t_5\}$. Then, the set of chosen teachers $C_s(\hat{T})$ is found as follows:

- ***Step** 1: Teacher t_3 is the most preferred for slot s^1 among the teachers in $\hat{T}_1 = \hat{T}$. Hence, t_3 is assigned to slot s^1 and she is removed. We set $\hat{T}_2 = \hat{T}_1 \setminus \{t_3\}$.*
- ***Step** 2: Teacher t_2 is the most preferred for slot s^2 among the teachers in $\hat{T}_2$. Hence, t_2 is assigned to slot s^2 and she is removed. We set $\hat{T}_3 = \hat{T}_2 \setminus \{t_2\}$.*
- ***Step** 3: Teacher t_5 has the highest priority for slot s^3 among the teachers in $\hat{T}_3$. Hence, t_5 is assigned to slot s^3 and she is removed. We set $\hat{T}_4 = \hat{T}_3 \setminus \{t_5\}$.*

Hence, $C_s(\hat{T}) = \{t_3, t_2, t_5\}$.

We define the following notions for the choice rules that will be crucial for our mechanism to be both strategy-proof and SI stable.

The choice rule C_s satisfies **substitutes** (Kelso and Crawford, 1982) if for all $\bar{T} \subseteq T$ and distinct teachers $t, t' \in \bar{T}$,

$$t \in C_s(\bar{T}) \implies t \in C_s\big(\bar{T} \setminus \{t'\}\big).$$

The choice rule C_s satisfies the **law of aggregate demand** (Alkan and Gale, 2003, Hatfield and Milgrom, 2005) if for all $\bar{T}, \hat{T} \subseteq T$,

$$\bar{T} \subseteq \hat{T} \implies \big|C_s(\bar{T})\big| \leq \big|C_s(\hat{T})\big|.$$

Next, we show that C_s satisfies these two properties.

Proposition 4 *The choice rule C_s satisfies the substitutes and law of aggregate demand conditions.*

We refer to the mechanism that selects the outcome of the DA algorithm using the choice rules $(C_s)_{s \in S}$ that we designed as the **status-quo improving deferred acceptance** (**SI-DA** for short) mechanism. The logic behind naming will be clear with the following result:

Theorem 2 *SI-DA mechanism is strategy-proof, and under Assumption 1, it is also SI stable.*

Notice that, when there is no new teacher, i.e., $N = \emptyset$, Theorem 2 holds without Assumption 1.

In the proof of Proposition 4 the processing of slots does not play any role. Hence, the Proposition 4 holds for any order we use in the calculation of chosen teachers. As a result, SI-DA mechanism continues to be strategy-proof independent of the processing order of the slots. Moreover, the proof

of SI-Stability of SI-DA does not rely on the processing order. Hence, SI-DA mechanism continues to be SI-Stable independent of the processing order of the slots. However, the processing order has an impact on the mobility and the welfare of the teachers.

Let $\blacktriangleright_s$ and $\hat{\blacktriangleright}_s$ be arbitrary two processing orders of seats at school s such that $\hat{\blacktriangleright}_s$ is obtained from $\blacktriangleright_s$ by swapping two adjacent slots s^k and s^ℓ where $k < \ell \leq |\omega_s|$. Let $\blacktriangleright_{\bar{s}} = \hat{\blacktriangleright}_{\bar{s}}$ for any $\bar{s} \neq s$. Let $D_{s'}$ and $\hat{D}_{s'}$ be the choice rules induced by $\blacktriangleright_{s'}$ and $\hat{\blacktriangleright}_{s'}$ by using the procedure defined in Definition 3 for all $s' \in S$. Let μ and $\hat{\mu}$ be the outcome of DA algorithm by using choice rules $D_{s'}$ and $\hat{D}_{s'}$ for all $s' \in S$. Then, the following proposition holds.

Proposition 5 *Each teacher t (weakly) prefers μ_t to $\hat{\mu}_t$.*

Proposition 5 implies that the processing order that we use for the first $|\omega_s|$ seats at each school s increases the welfare of the teachers compared to the alternative processing orders. Moreover, it also implies that the processing order that we use increases the teacher mobility. However, Proposition 5 does not say anything about the relative order the last $q_s - |\omega_s|$ seats at each school s. In the following example, we illustrate that we cannot find an optimal processing order for the last $q_s - |\omega_s|$ seats.

Example 4 *Let $S = \{s, s', s''\}$, $T = \{t_1, t_2, t_3, t_4\}$, the status-quo matching be*

$$\omega_s = \{t_4\},\ \omega_{s'} = \{t_2\},\ \omega_{s''} = \emptyset,$$

$q_s = 2$, $q_{s'} = q_{s''} = 1$, $\tau(t_1) \rhd_s \tau(t_2) \rhd_s \tau(t_3) \rhd_s \tau(t_4)$, $\tau(t_1) \rhd_{s'} \tau(t_3) \rhd_{s'} \tau(t_4) \rhd_{s'} \tau(t_2)$, and $\tau(t_1) \rhd_{s''} \tau(t_2) \rhd_{s''} \tau(t_3) \rhd_{s''} \tau(t_4)$. The preferences of the teachers are

$$s\ P_{t_1}\ s'\ P_{t_1}\ s''\ P_{t_1}\ \emptyset,$$

$$s\ P_{t_2}\ s''\ P_{t_2}\ s'\ P_{t_2}\ \emptyset,$$

$$s\ P_{t_3}\ s''\ P_{t_3}\ s'\ P_{t_3}\ \emptyset,$$

$$s'\ P_{t_4}\ s''\ P_{t_4}\ s\ P_{t_4}\ \emptyset.$$

If s_1^1 is filled before s_1^2, then under DA t_1 and t_3 are assigned to s, t_2 is assigned to s'' and t_4 is assigned to s'. If s_1^2 is filled before s_1^1, then under DA t_1 and t_2 are assigned to s, t_3 is assigned to s'' and t_4 is assigned to s'. Hence, we cannot have the same conclusion as in Proposition 5.

We use tie-breaking in the construction of the slot priorities. Inclusion of this exogenous tool causes efficiency loss: SI-DA does not select a Pareto-undominated SI stable matching (and therefore, it is not Pareto efficient either). We illustrate this situation in the following example.

Example 5 *Let $S = \{s, s', s''\}$, $T = \{t_1, t_2, t_3\}$, the status-quo matching be*

$$\omega_s = \{t_1\},\ \omega_{s'} = \{t_2\},\ \omega_{s''} = \{t_3\},$$

$q_s = q_{s'} = q_{s''} = 1$, $\tau(t_1) = \tau(t_2) = \tau(t_3)$, *and all teachers are acceptable for all schools. The preferences of the teachers are*

$$s' \; P_{t_1} \; s \; P_{t_1} \; s'' \; P_{t_1} \; \emptyset,$$

$$s \; P_{t_2} \; s' \; P_{t_2} \; s'' \; P_{t_2} \; \emptyset,$$

$$s' \; P_{t_3} \; s'' \; P_{t_3} \; s \; P_{t_3} \; \emptyset.$$

Let $t_2 \vdash t_3 \vdash t_1$ *be the tie breaker, then the rankings of the slots are given as:*

$$t_1 \succ_s^1 t_2 \succ_s^1 t_3,$$

$$t_2 \succ_{s'}^1 t_3 \succ_{s'}^1 t_1,$$

$$t_3 \succ_{s''}^1 t_2 \succ_{s''}^1 t_1.$$

SI-DA assigns t_1 *to* s, t_2 *to* s' *and* t_3 *to* s''. *However, this outcome is Pareto dominated by another status-quo improving stable matching in which* t_1, t_2 *and* t_3 *are assigned to* s', s, *and* s'', *respectively.*

5 Generalizing Lower Bound for Welfare Improvements for Schools

In this section, we introduce a relaxation of the status-quo improvement requirement for schools in our mechanisms as this requirement can sometimes be stringent for applications. Instead of status quo, we consider improvements with respect to a *hypothetical teacher distribution.* Consider a school $s \in S$. Let school s's ranking over teacher types be $\theta_1 \rhd_s \theta_2 \rhd_s \ldots \rhd_s \theta_m \rhd_s \theta_\emptyset$. That is, any type $\theta \in \Theta \backslash \{\theta_1, \theta_2, \ldots, \theta_m\}$ is unacceptable for school s. We refer to a vector $d_s = (d_s^{\theta_1}, d_s^{\theta_2}, \ldots, d_s^{\theta_n}) \in \mathbb{R}_+^n$ as a **threshold acceptability distribution for school** s if

- $d_s^{\theta_1} \leq \left|\omega_s^{\theta_1}\right|$,
- $d_s^{\theta_1} + d_s^{\theta_2} \leq \left|\omega_s^{\theta_1} \cup \omega_s^{\theta_2}\right|$,
- $\ldots$
- $d_s^{\theta_1} + d_s^{\theta_2} + \ldots + d_s^{\theta_m} = \left|\omega_s^{\theta_1} \cup \omega_s^{\theta_2} \cup \ldots \cup \omega_s^{\theta_m}\right| = |\omega_s|$.

Notice that threshold values are relaxation over the number of status-quo teachers from best type to the worst in a cumulative sense. A matching μ is **d_s-improving for school s** if types of teachers in μ_s are acceptable for school s and

- $d_s^{\theta_1} \leq \left|\mu_s^{\theta_1}\right|$,
- $d_s^{\theta_1} + d_s^{\theta_2} \leq \left|\mu_s^{\theta_1} \cup \mu_s^{\theta_2}\right|$,
- $\ldots$
- $d_s^{\theta_1} + d_s^{\theta_2} + \ldots + d_s^{\theta_m} \leq \left|\mu_s^{\theta_1} \cup \mu_s^{\theta_2} \cup \ldots \cup \mu_s^{\theta_m}\right| = |\mu_s|$.

Let a profile $d_S = (d_s)_{s \in S}$ be such that for each school s, d_s is a threshold acceptability distribution. A matching μ is **(d_S, ω_T)-improving** if it is d_s-improving for each school $s \in S$ and for each teacher $t \in T$, $\mu_t \; R_t \; \omega_t$. Observe that if matching μ is status-quo improving, then μ is (d_S, ω_T)-improving. Thus, this notion is a relaxation of status-quo improvement.

Under this relaxation, we can use the SI-CC and SI-DA mechanisms after we relabel the types of current employees for each school as follows: For each school $s \in S$, let $\bar{m}_s \in \{1, \ldots, n\}$ such that $d_s^{\theta_1} + \ldots + d_s^{\theta_{\bar{m}-1}} < |\omega_s| = d_s^{\theta_1} + \ldots + d_s^{\theta_{\bar{m}}}$. For each better type $\theta \in \Theta$ than $\theta_s^{\bar{m}}$, i.e., with $\theta \triangleright_s \theta_s^{\bar{m}}$, we select $|\omega_s^\theta| - d_s^\theta$ teachers from ω_s and treat them as if they were of type $\theta_s^{\bar{m}}$ when we determine the pointing rule of school s and the pointing rule of teachers under SI-CC and priority rankings of seats under SI-DA. For the rest of the steps of the mechanisms, each teacher is treated with her real type (see Appendix H for formal definitions) UU: ADD wSI-DA definition to this Appendix.

These mechanisms, that we call **weak-SI-CC (wSI-CC)** and **weak-SI-DA (wSI-DA)**, respectively, inherit their desired properties mentioned in Sections 3 and 4 under the modification that each status-quo improvement regarding schools should be replaced with d_S-improvement in the property definitions. We use wSI-CC in our empirical simulations besides the original SI-CC. We explain our rationale for using this relaxation in Section 6.7.

UU: Minor comment: we used she/her/her for teachers; I tried to update he/his/him below with female pronouns, but I may have missed above.

6 Empirical Analysis

This section provides empirical evidence on the changes that the mechanisms we suggest would bring in a real world setting: teacher (re)assignment to regions in France. After presenting the institutional context and the data, we structurally estimate teachers' preferences over regions, and run counterfactuals to quantify the improvements that our mechanisms may yield.

6.1 Institutional Background

Teacher recruitment and assignment. Teacher certification and recruitment is highly centralized in France. Anyone who wishes to become a teacher has to pass a competitive examination. Those who succeed are allocated a teaching position by the Ministry for a probation period of one year, at the end of which they get tenure or not. Once they get tenure, teachers in public schools become civil servants. The government manages both the first assignment of newly tenured teachers to a school, and the mobility between schools of tenured teachers who previously received an assignment but wish to change.[24] In the rest of the empirical analysis, we refer to newly tenured teachers who ask for their first assignment as *new teachers* and tenured teachers with a status-quo assignment as *tenured teachers*, with a slight abuse of terminology.

The two-step assignment process. The assignment procedure takes place in two successive steps.[25] During the first step, which is managed centrally by the Ministry, teachers are assigned to one of the 31 French regions using a mechanism to which tenured teachers who wish to change

[24] This centralized assignment process is used for public school teachers only. Private schools make up 16% of teachers. For them, the recruitment process is similar but public and private school teachers face completely different rules for their mobility—between regions and between schools. In private schools, teachers apply directly to schools—as would be the case in usual labor markets.

[25] Before 1999, teacher assignment to schools was managed centrally by running an algorithm once, which assigned teachers directly to schools. This highly centralized process was argued to be at odds with the regional nature of most demands: the majority of teachers asking for a transfer ranked schools within their current regions.

regions and new teachers submit a preference list over regions. In the second step, teachers who are newly assigned to a region and tenured teachers who wish to change schools within their region submit a preference list over schools and the same first step mechanism is run in the second step for each region with the new inputs. Since 1999, this step is managed directly by local administrations within the regions. Our empirical analysis focuses on the first regional assignment because of potential strategic reports of preferences during the second phase.[26] Participation to the assignment mechanism is compulsory for all new teachers as they do not have status-quo position. On the other hand, participation is optional for tenured teachers as they are never forced to change region or school.

Mobility requests, vacant positions, and new teachers. We use data on the assignment of teachers to one of the 31 French regions in 2013 for our empirical analysis. There were 700,000 secondary public school teachers in France that year, a number that fluctuates from year to year due to both entries and exits from the profession. Exits are mainly due to teachers retiring — 9,793 public secondary school teachers retired in 2013 — while entries correspond to new teachers who have passed the recruitment exam and validated their probation year. As a result, when organizing the annual mobility process, the central administration has to take into account not only a large pool of tenured teachers who wish to change positions, but also some vacant positions that need to be filled and new teachers who wish to be assigned to their first jobs. In 2013, the year for which we have data, about 25,100 teachers took part in the centralized regional mobility process. Among them 17,200 are tenured teachers and 7,900 are new teachers.

6.2 Data and Descriptive Statistics

We use data on the assignment of teachers to one of the 25 French regions in 2013.[27] We have information on teachers' reported preferences and initial assignment (if any), regions' priorities, and the number of vacant positions in each region. We keep all teachers from the 8 largest subjects, such as French, math, English, and sports. We discard couples from the sample because they benefit from a special treatment in the assignment process.[28] Finally, in order to keep the market structure balanced, we drop one seat for each teacher we omit.[29] Our final sample contains 10,460 teachers: 5,833 tenured teachers (55.8%) and 4,627 new teachers. Table A.1 shows the decomposition by three of the main subjects, French, math, and English.

[26]Preferences reported during the second phase of the assignment are more difficult to interpret for two reasons. First, teachers can only rank up to 20 or 30 schools, depending on the region. Second, in addition to ranking schools, teachers can also rank larger geographic areas, such as cities for instance.

[27]We discard the 6 overseas regions because of their specificities in terms of (i) teacher preferences — in contrast to what we find in our estimates, distance from the current location often becomes an attractive feature — and (ii) regions' priorities — some of these regions, like Mayotte, give teachers who grew up in these regions bonus points when they rank it first.

[28]Spouses in two different subjects can submit joint mobility applications (by submitting two identical lists). This creates dependencies across markets for different fields.

[29]For each tenured teacher we discard, we drop her corresponding position as well. For new teachers, we find the share of new teachers discarded among all new teachers and denote it as $S\%$; then we delete $S\%$ of vacant positions in each region.

Table 1: Descriptive Statistics on Teachers and Regions

	Tenured Teachers			New Teachers		
	French	Math	English	French	Math	English
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A. Characteristics of teachers						
Female (%)	76.1	47.0	85.4	80.3	41.7	80.4
Married (%)	48.5	45.0	46.8	41.1	39.4	40.9
Is in disadvantaged school (%)	10.4	13.2	4.4	-	-	-
Experience (yrs)	7.48	7.23	7.18	2.76	2.24	2.30
Has advanced teaching qualifications (%)	7.9	29.1	8.8	16.8	31.7	15.2
Panel B. Characteristics of the regions to which teachers are assigned at status quo						
Is the teacher's birth region (%)	8.7	8.6	9.3	-	-	-
Is Créteil or Versailles (%)	37.7	52.3	35.6	-	-	-
Is in South of France (%)	5.6	9.3	12.7	-	-	-
Students in urban areas (%)	61.7	67.4	64.0	-	-	-
Disadvantaged students (%)	52.5	54.0	53.5	-	-	-
Students in priority education (%)	26.0	24.5	22.7	-	-	-
Students in private school (%)	15.2	16.3	17.4	-	-	-
Teachers younger than 30 yrs (%)	11.9	13.3	11.3	-	-	-
Panel C. Characteristics of the regions teachers rank first						
Distance to status-quo region (km)	2,148.7	1,316.9	1,521.9	-	-	-
Is the teacher's birth region (%)	36.5	35.8	40.0	35.8	38.4	39.3
Is in South of France (%)	25.2	25.2	25.4	20.1	18.6	20.0
Is Créteil or Versailles (%)	2.8	3.3	3.4	14.3	11.9	12.5
Students in urban area (%)	60.2	56.2	51.7	61.9	58.6	59.2
Disadvantaged students (%)	52.8	53.1	53.5	53.3	53.1	53.0
Students in priority education (%)	20.3	19.9	17.9	21.8	20.6	21.0
Students in private school (%)	23.7	22.9	25.8	22.2	21.8	21.7
Teachers younger than 30 yrs (%)	6.5	6.4	6.0	8.2	8.1	8.2
Observations (#)	859	605	628	786	958	746

Notes: This table reports descriptive statistics for teachers and regions in three subjects: French, math and English. Statistics are reported for the sample of teachers we use for the demand estimations. Columns (1) to (3) report statistics for tenured teachers. Columns (4) to (6) report statistics for new teachers. New teachers have missing values for statistics related to the region of status-quo assignment. We discard teachers who submit a joint list with their partner, teachers who are from one of the six regions that are overseas, and teachers for whom one of the individual characteristics is missing. The last row reports the number of teachers in each subject. Panels A, B, and C, respectively, present descriptive statistics on teachers, on the region to which they are assigned at status quo, and on the region they rank first. UU: Define disadvantaged school and student, and priority education; MAYBE in the main text only.

A central motivation of our analysis is to rebalance the unequal distribution of teachers across regions. Part of this large imbalance stems from differences in regions' attractiveness. Table 1 reports descriptive statistics on teachers (Panel A), their status-quo assignment (Panel B), and the

region they rank first (Panel C).[30] Two regions surrounding Paris, called Créteil and Versailles, are particularly unattractive. The imbalance is blatant when comparing the number of teachers asking to leave the region and the number asking to enter. For instance, in math, 52.3% of the tenured teachers who ask to change region come from Créteil or Versailles, but only 3.3% rank one of these two regions as their first choice.[31] Table A.2 provides additional evidence on attractiveness differences and their potential determinants for the three most attractive regions (Rennes, Bordeaux, and Toulouse), the three least attractive regions (Créteil, Versailles, and Amiens), and three of the intermediate regions (Paris, Aix-Marseille, and Grenoble).[32]

The large share of mobility requests that originate from unattractive regions have a direct consequence on the annual mobility flows: Under the current assignment system, a large number of teachers exit these regions, which results in numerous vacant positions that need to be filled. About 50% of the new teachers get their first assignment in one of the three least attractive regions (Créteil, Versailles, and Amiens). This structural imbalance is a serious concern for policy makers. It is frequently raised as a reason for the lack of attractiveness of the teaching profession in France.[33] In addition, it creates large differences in the age profile of teachers across regions. As reported in column (3) of Table A.2, the ratio of the number of teachers older than 50 to the number of teachers younger than 30 is equal to 1.1 and 1.6 in Créteil and Versailles, respectively. In contrast, the most attractive region, Rennes, had almost 7.4 times more teachers older than 50 than teachers younger than 30. In Bordeaux and Toulouse, this ratio was 6.5 and 5.3, respectively.

Several papers have found that teachers tend to help with their students' progress less during the first years of their career than when they have more experience UU: Move these references to bibtex file and site using \citealp{} command while in parentheses→ (Chetty, Friedman and Rockoff (2014) Rockoff (2004)). Reducing the unequal distribution of teachers across regions and reducing the chances to be assigned to a disadvantaged region in the first place became one of the objectives of the French policy makers, who see this as a way to both reduce achievement inequalities between students and improve the attractiveness of the teaching profession in the longer run.

6.3 Specifications of the Empirical Analysis

Mechanisms. Our theoretical results regarding the SI-CC and SI-DA mechanisms can be used to achieve policy maker objectives to alleviate the unequal distribution of teachers in regions. Our counterfactual analysis aims to both formally define possible inputs and quantify the performance of these mechanisms. We also benchmark them with variants of two widely used mechanisms:

- **Benchmark for SI-CC:** A variant of TTC, which we refer to as **TTC***, that accounts for

[30] Appendix F.1 provides a detailed description of each variable.

[31] This rate is a bit larger for new teachers. Between 10% and 15% of them rank Créteil or Versailles as their first choice across the 8 subjects we consider in our analysis.

[32] Attractiveness is measured as the ratio of the number of tenured teachers asking to enter a region over the number of tenured teachers asking to leave the region. This ratio ranges from 15.5 in Rennes to 0.03 in Créteil.

[33] The most common recruitment exam in France is called the CAPES. Every year, the Ministry decides on the number of teaching positions it opens for the exam. In 2014, 24% of the CAPES positions remained vacant because of both a shortage of applicants and the poor quality of those applying. The shortage situation has not improved since.

teacher types as defined in the theoretical analysis but relaxes the mechanism features that ensure status-quo improvement. More precisely, this mechanism differs from SI-CC in two respects: (1) we lift the restrictions on the set of schools that a teacher can point to, and (2) tenured teachers can now start a chain (and potentially leave their position without being replaced) (see Appendix H for a precise definition). This benchmark is close to the well-known TTC-variant mechanism "you request my house – I get your turn" (YRMH-IGYT) (Abdulkadiroğlu and Sönmez, 1999, Sönmez and Ünver, 2010) which is strategy-proof, Pareto efficient, and individually rational for teachers, but is not status-quo improving. Intuitively, TTC* might therefore be expected to generate more mobility than our mechanisms at the cost of a potentially more unequal distribution.

- **Benchmark for SI-DA:** A variant of DA, which we refer to as **DA***, that accounts for teacher types but relaxes the mechanism features that ensure status-quo improvement.[34] More precisely, this mechanism differs from SI-DA in two respects: (1) we lift the restrictions on schools' rankings over teacher types (i.e, an applicant teacher with a less-preferred type than a status-quo teacher will no longer be considered as unacceptable by a school), and (2) vacant positions in a region no longer prioritize new teachers over tenured teachers (see Appendix H for a precise definition). If the Ministry's current priorities are used for regions, this mechanism becomes equivalent to the mechanism used in each step of the current French assignment process. However, we use our regional FOSD preferences to account for priorities, making DA* different from the one step of the current French process. Incorporating teacher types into schools' priorities provides an interesting benchmark that targets teacher-SI stability, potentially at the cost of efficiency and distributional objectives.

In Section 6.7, we further consider counterfactual comparisons regarding two more mechanisms based on SI-CC and DA*, respectively. We explain these mechanisms in that section.

Teacher Types and Regional Type Rankings. We run our different mechanisms using teachers' preferences, teachers' types, and regions' type rankings as inputs. To illustrate our theoretical contribution, we define for a tenured teacher her type as her experience and we classify tenured teachers into 12 experience bins, where the first bin corresponds to teachers with 1 or 2 years of experience, the second bin to teachers with 3 to 4 years of experience, and so on. Each bin represents a *teacher experience type*. To distinguish teachers with the least experience within the system, we further assume that new teachers have a smaller experience type than any tenured teachers. UU: Give statistic of the new teacher distribution in the first bin and other bins. or now, in terms of their experience. We need to have 13 bins in the graphs. To define regions' rankings over teacher types, we start by identifying which regions would benefit most from receiving more experienced teachers. To do so, we compute average teacher type in each region at status quo (see Figure A.1) and we classify regions into two groups based on whether their average type is above or below the median.

[34] We borrow the notation DA* to Combe et al. (2020) who study a modified version of DA which ensures status-quo improvement for teachers. We kept the same logic for our notation when dealing with TTC-based mechanisms using TTC*.

- The first group contains all regions whose average types are strictly below the median, i.e, *younger regions* that could benefit from receiving more experienced teachers. We set the type rankings of these regions so that they rank types by decreasing level of experience, i.e, the most experienced teacher types are always preferred to the least experienced teacher types.
- The second group contains all regions whose average types are above the median, i.e., *older regions*. These regions rank types by increasing level of experience.

Our ultimate goal in setting schools' preferences is to ensure that the matching outcomes of our mechanisms eventually yield more equal distributions of teachers across regions. Both SI-DA and SI-CC, which are status-quo improving, achieve this goal through these type rankings.

Observe that the way we define type rankings of younger and older regions is tailored to the objectives of French policy makers (as discussed in Section 6.2). We also conduct robustness tests for this definition and show (in Appendix G) results in which all regions have the same type rankings. UU: Are they substantially different? similar? A sentence in this manner especially if really they are similar

We also assume that all regions find all types acceptable. This means that any teacher is always preferred to a vacant position. Indeed, in the French system, by law, all teachers are acceptable for all regions.[35] Finally, running SI-CC (and its variants) requires an additional ordering over teachers to determine which chain will be selected.[36] For the main results presented in the paper, we use the true point system of the French Ministry and sort teachers by decreasing level of the maximum points they obtain over all regions. However, we show in Appendix E that the performance of the SI-CC mechanism is sometimes sensitive to the ordering chosen. We report robustness results in which we flip the ordering to rank the teachers by increasing level of their maximum priority points.

6.4 Structural Estimation of Teacher Preferences

Teachers can rank all regions when they submit their preference list in the first step of the assignment process and the Ministry uses a modified version of the DA mechanism to assign teachers to regions, as mentioned earlier. If the whole assignment were done in a single step, this mechanism would induce being truthful as a dominant strategy for teachers. In our setting, if teachers have lexicographic preferences over regions first and schools second or, more generally, if preferences over regions are well-defined objects, the mechanism assigning teachers to regions in the first step is strategy-proof.[37] Yet, even under strategy-proof mechanisms, a number of experimental

[35]For tie-breaking, we need an additional ordering over teachers for the SI-CC, SI-DA, and two benchmark mechanisms. We use the tie-breaking rule used by the French Ministry which uses the date of birth of teachers and some extra conditions for the rare cases with the same date of birth.

[36]This ordering only applies to new teachers under SI-CC. It applies to all teachers under TTC* because chains can start from any teacher.

[37]In the end what teachers primarily care about is the school they obtain within a region, which questions whether preferences over regions are well-defined objects. Combe et al. (2020) show evidence that teachers' preferences seem to be lexicographic, i.e that teachers primarily care about the region in which the school is located and about a school within that region. The paper also shows that changes in a number of mechanisms in the first phase (the regional assignment) only marginally impact the pool of participants in the second phase (the school assignment) so that teachers' assessments of their school admission probabilities (within a region) should not vary much between

and empirical papers show that truthfulness is a strong assumption (Chen and Sönmez, 2006, Pais and Pinter, 2008, Rees-Jones, 2018, Chen and Pereyra, 2019, Hassidim et al., 2017). In our context, French teachers have reasonably accurate information on their admission probabilities to each region, which might encourage some teachers to discard from their preference list the regions where their chances to be accepted are too low.[38] These omissions could introduce a bias in any counterfactual analysis done using teachers' reported preferences. Combe et al. (2020) previously rejected truth telling among French teachers. To avoid this potential bias, instead of using the reported preferences, we estimate teachers' preferences using an identifying assumption (presented below) that does not require teachers to be fully truthful.

Model. We estimate teachers' preferences over regions using the following utility function:

$$u_{t,R} = \delta_R + Z'_{t,R}\beta + \varepsilon_{t,R} \tag{5}$$

Teacher t's utility for region R is a function of region fixed effect δ_R, teacher-region-specific observables $Z_{t,R}$ (with coefficients β) and a random shock $\varepsilon_{t,R}$ which is i.i.d. over t and R and follows a type-I extreme value distribution, Gumbel$(0, 1)$. The region fixed effect captures region characteristics such as average socio-economic and academic level of students in the region, cultural activities, housing prices, facilities, etc. We estimate preferences separately for tenured teachers and new teachers. This allows us to include a richer set of variables for the former group. The vector $Z_{t,R}$ includes a dummy specifying if region R is the birth region of teacher t. If teacher t is tenured, it also includes a dummy showing whether R is the status-quo region of teacher t, as well as the distance between region R and the status-quo region of teacher t. Vector $Z_{t,R}$ additionally includes interaction terms between teacher t's and region R schools' characteristics (that are presented in Panels A and B of Table 1). We apply standard scale and position normalization, setting the scale parameter of the Gumbel distribution to 1 and the fixed effect of the Paris region to 0.

Identifying assumptions. To avoid the potential bias generated by teachers omitting regions they consider as infeasible, we estimate teachers' preferences under a weaker "stability assumption" developed by Fack et al. (2019) and applied to the teacher assignment by Combe et al. (2020).[39] We start by defining the *feasible set* of each teacher as the set of regions that has a cutoff — that is, the lowest priority of the teacher assigned to a region — smaller than her own priority. These are the regions a teacher could be assigned to if she ranked the region first in her preference list. The key identifying assumption is that, for each teacher, the region obtained is her most preferred region

mechanisms. This implies that teachers' preferences over regions may be relatively insensitive to the mechanism used and so are well-defined through backward induction.

[38] Cutoffs for entry in each region are published every year. Combe et al. (2020) show that these cutoffs are relatively persistent over time, so they provide reasonably accurate information to teachers on their chances to be admitted to each region.

[39] Combe et al. (2020) provide an in-depth discussion of the two alternative identifying assumptions (truthfulness versus stability), as well as statistical tests in favor of the later. For more references on estimations that do not require truth telling, see Akyol and Krishna (2017), Artemov et al. (2019), Agarwal and Somaini (2018), Calsamiglia et al. (2020). They mainly focus on the estimation of the preferences of tenured teachers and we use the same estimation in our analysis. Here, we provide an additional detailed discussion on the estimation of the preferences of new teachers.

among all regions that are in her feasible set.[40] Hence, we estimate a discrete choice model with personalized choice sets. Choice probabilities have closed form solutions and we estimate parameters using maximum likelihood.

Estimation results. Table 2 reports preference estimates for tenured teachers and new teachers for a selected group of coefficients. We run the estimations in each of the 8 subjects separately and report results for Math and French teachers in the table. The first 9 rows report coefficients for a selected set of region fixed effects. They reveal an interesting difference between the preferences of tenured and new teachers. While the Créteil and Versailles regions are very unattractive for tenured teachers (as indicated by the negative coefficient of their fixed effect relative to the Paris region), these regions are less unattractive for new teachers, who often see a first position in a disadvantaged school as a stepping stone for better positions in the future.[41] The fact that Créteil and Versailles are more attractive for new teachers than for tenured teachers surely contributes to the unequal distribution of teachers denounced by policy makers. Yet, this is not the only explanation for teachers' unequal distribution. The counterfactual analysis we present in the next section shows that the assignment mechanism also shapes the distribution of teachers in important ways. The fact that preferences alone are not driving the unequal distribution is fundamental for our ability to improve both teacher distribution and teacher welfare.

Goodness of fit measures. Our main fit measure (also reported in Table 2) considers the top two regions that a teacher has included in her submitted preference list. We then compute the probability of observing this particular relative ordering in the preference list predicted by our estimations. This fit measure based on relative ranking (instead of the characteristics of the school ranked first for instance) is particularly suitable for our environment in which some teachers might not rank regions that they consider as infeasible.[42] In addition to the overall fit quality, we also compute fit measures for the tenured teachers who are employed in the two least attractive regions, namely, Créteil and Versailles, at the status quo. Inspecting the fit quality for this sub-group of teachers is particularly important because teachers from Créteil and Versailles represent a large share of the tenured teachers who submit a mobility request every year and they are more likely to stay in their positions. These two facts could affect the preference estimation for these teachers under our stability assumption. Across the 8 subjects, our fit measures range from 0.62 to 0.72 for tenured teachers and from 0.56 to 0.69 for new teachers, which compare favorably to the ones obtained by Fack et al. (2019) (between 0.553 and 0.615).

Simulations. We use our estimates of utility coefficients to draw teachers' preferences 1,000 times using Equation 5. After having drawn them, we keep the entire set of regions without imposing

[40] This assumption is theoretically founded: Artemov et al. (2019) show that, in a large market environment, any (regular) equilibrium outcome of DA* must have this property. Since a variant of DA* with the Ministry-mandated priorities is the current mechanism for the regional assignment step, this result also applies to our setup.

[41] Teachers who stay in a disadvantaged school for at least 5 years benefit from additional priority when they ask to change region or school.

[42] When teachers skip regions perceived as infeasible, the first region they report might not be their most preferred region — and indeed, the tests we perform reject truth telling — but conditional on ranking schools, the order in which a teacher ranks the schools might correspond to teacher's true relative preferences. This is why we prefer to use a fit measure that is based on relative ranking rather than on the characteristics of the school ranked first.

any truncation for their simulated preference lists: tenured teachers find all regions ranked as high as their status-quo assignment acceptable while new teachers find all regions acceptable.[43] In each of the 8 subjects and for each draw, we use these simulated preferences and the priorities from our data to run the mechanisms. The results reported in the next section correspond to averages over these 1,000 draws, aggregated over 8 subjects.

6.5 Relative Performance of the SI-CC and SI-DA Mechanisms

We start by discussing the relative performance of SI-CC and SI-DA. Recall that while SI-CC is SI teacher optimal (i.e., Pareto undominated for teachers among all status-quo improving matchings) and Pareto efficient (incorporating the welfare of both regions and teachers), SI-DA is SI stable (under Assumption 1) and not Pareto efficient. Comparing the performance of these two mechanisms that target teacher welfare (and Pareto efficiency) and stability is important for various reasons. First, neither mechanism satisfies any obvious optimality property for the distribution of teacher experience at outcome matchings, which is the measure of regional welfare.[44] Therefore, it is important to understand which one leads to a more desirable teacher experience distribution. Second, the mechanism that is currently used by the French Ministry of Education is teacher-SI stable (with respect to their mandated priorities). This suggests that policy makers consider stability as an important feature of the assignment process. As there is no mechanism that is both SI teacher optimal and SI stable, and there is even no Pareto comparison regarding teacher welfare between these two mechanisms, understanding the tradeoff between these properties is important.

Validation of Assumption 1. We show in Section 4 that the SI-DA mechanism is SI stable under Assumption 1. When all regions are acceptable by new teachers and all new teachers are acceptable by regions, this assumption is satisfied when there are at least as many new teachers as empty seats. Table A.1 shows that this assumption holds in each of the 8 subjects we consider. This means that the matching obtained under SI-DA is SI stable.

Distribution of teacher experience. We start by comparing, for different regions, the cumulative distribution of teacher experience under SI-CC and SI-DA. We classify the assigned teachers to regions into 12 types based on experience as explained before.[45] The left panel of Figure 1 shows the cumulative distribution of teacher experience in the three youngest regions of France (Créteil, Versailles, and Amiens). Teachers in these regions represent 78% of teachers in disadvantaged regions.[46] SI-DA slightly outperforms SI-CC in these regions, i.e., it assigns fewer inexperienced teachers: 1,041 teachers with up to two years of experience are assigned to one of the three regions under SI-DA versus 1,344 under SI-CC. SI-DA also improves the experience distribution in the three oldest regions by assigning them a larger number of inexperienced teachers (see the right panel of

[43] This implicit assumption about new teachers is in line with the policy of the Ministry. Teachers are indeed not required to rank all regions when they submit their lists, but the Ministry fills the incomplete lists of new teachers to make sure that all of them get an assignment even those who ranked few regions.

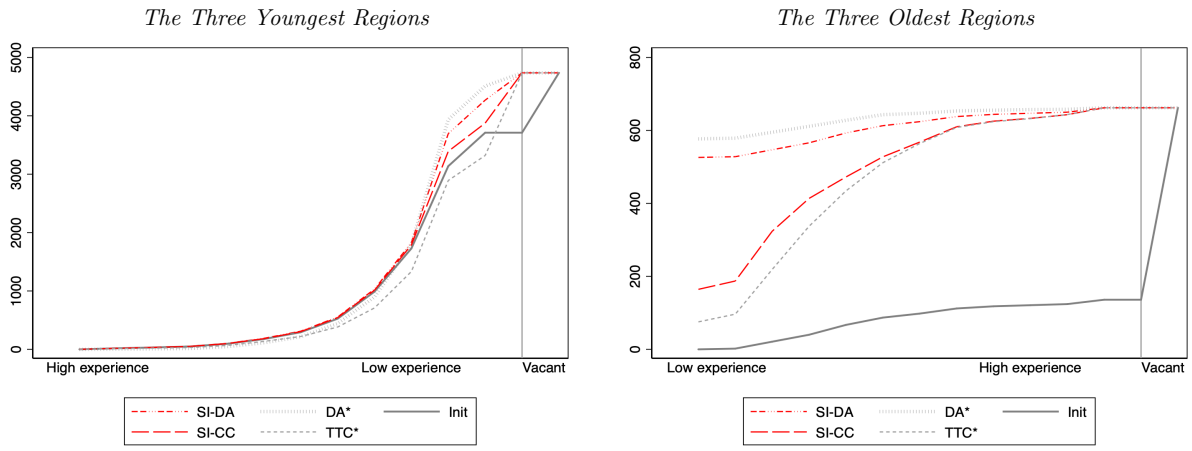
[44] In this whole section while talking about regional welfare comparing different outcome matchings, we will talk about comparisons of teacher experience type distributions (at these matchings), with a slight abuse of terminology.

[45] In addition to the 12 teacher types, we also consider vacant positions as the least preferred type for every region (as shown on the right of the horizontal axis in Figure 1).

[46] See Figure A.1 for the type distribution in regions.

Figure 1).

Figure 1: Cumulative Distribution of Teacher Experience Types



Notes: This Figure shows the cumulative distribution of teacher experience types. The left panel reports the distribution in the three youngest regions of France (Créteil, Versailles, and Amiens), and the right panel the distribution in the three oldest regions of France (Rennes, Bordeaux, and Lyon). The horizontal axis reports the 12 experience types of teachers, ordered from most experienced to least experienced (left panel) and from least experienced to most experienced (right panel). The 13^{th} type corresponds to vacant positions. The mechanisms that satisfy status-quo improvement are plotted in red. Those that do not are in grey. The thick black line (marked as Init in the legend) corresponds to the cumulative distribution of teacher types at the status-quo matching.

Stability – teacher welfare trade-off. However, SI-DA's better distributional performance comes at a large cost in terms of teacher mobility. Panel A of Table 3 shows that only 3,912 teachers obtain a new assignment under SI-DA, compared to 5,356 under SI-CC. The lack of mobility under SI-DA is particularly striking for tenured teachers: none of them move from their status-quo assignment, compared to 1,444 under SI-CC. The very low level of mobility under SI-DA is due to the strength of the requirements imposing status-quo improvement and SI stability. Indeed, status-quo improvement implies that many tenured teachers from unattractive regions will be unable to leave the region together with the low demand from tenured teachers for these regions. Given this, any teacher entering a region almost automatically generates justified envy from a tenured teacher stuck in an unattractive region (in particular, since the least attractive regions are also the youngest, the relatively low experience of the tenured teachers stuck in these regions are preferred by the older regions like Bordeaux). SI stability requirement prevents such assignments from happening and, thus, blocks mobility of tenured teachers.

This simple example and the results from our counterfactual analysis show that, under DA-inspired mechanisms satisfying different notions of stability, imposing a status-quo improvement can have the unintended consequence of dramatically reducing mobility. Said differently, prioritizing SI stability and status-quo improvement (under SI-DA) over SI teacher optimality (under SI-CC) entails a very large efficiency cost for teachers in our context.

Concerning stability measures, SI-DA is, by construction, SI stable contrary to SI-CC. As reported in Panel E of Table 3, the latter leads to 8,122 teachers being involved in at least one

blocking pair not authorized in the definition of SI stability that we introduced in Section 4. Note that neither SI-DA nor SI-CC are Gale-Shapley stable, which forbids all blocking pairs: SI-CC leads to 8,206 teachers being involved in at least one blocking pair compared to 8,438 teachers under SI-DA. This reduction can be explained by the important teacher welfare gains that SI-CC has achieved, compared to SI-DA. Since many more tenured teachers move under SI-CC, the number of blocking pairs caused by tenured teachers staying at their status-quo assignment decreases. The small differences between the number of unauthorized blocking pairs in our three stability notions under SI-CC mean that the vast majority of blocking pairs are caused by less preferred teachers being assigned to a new region despite more preferred teachers requesting that region. For SI-DA, only 2,171 teachers are blocking due to this last reason while the remaining 6,267 teachers block because of a tenured teacher staying at her status-quo region. We conclude this subsection with the following summary of our main findings so far:

Fact 1 *Despite a slightly better distributional performance and obtaining an SI stable matching, SI-DA has a tremendous mobility cost compared to SI-CC. No tenured teacher moves from her status-quo region under SI-DA, compared to 1,444 under SI-CC. Imposing the status-quo improvement constraint together with SI stability comes at a large teacher welfare cost.*

6.6 Benefits and Costs of Status-quo Improvement Constraints

We now turn to a discussion of the benefits and costs of adding status-quo improvement constraints to assignment mechanisms. To do so, we compare SI-CC and TTC*, the benchmark mechanism which uses the same type rankings for teachers as SI-CC, but is not status-quo improving. We also compare SI-DA and DA*, although we devote less time to this comparison due to the relatively poor performance of SI-DA identified in the previous section.

Better distribution of teacher experience. The left panel of Figure 1 shows the cumulative distribution of teacher experience in the three youngest regions of France.[47] Every year a very large number of teachers with a few years of experience leave Créteil, Versailles, and Amiens. They are replaced by an equally large number of inexperienced teachers. That structural imbalance means that the status-quo improvement requirement is unlikely to be respected by mechanisms such as DA* or TTC*. Our counterfactual analysis confirms this (see the left panel of Figure 1). Our main finding for the youngest regions is as follows:

Fact 2 *In the three youngest regions of France, the distribution of teacher experience under SI-CC FOSDs the one under TTC*. SI-CC assigns only 1,344 teachers with one or two years of experience to the three youngest regions, while TTC* assigns 1,844 of them to these three regions. On the contrary, the distribution of teacher experience under SI-DA does not FOSD the one under DA*.*

The distributions under the benchmark mechanisms need not FOSD the status-quo distribution

[47] See Figure A.1 for the type distribution in all regions at status quo.

as they do not require status-quo improvement. Indeed, the distribution of teacher experience under DA* and TTC* do not FOSD the status-quo distribution.[48,49]

Interestingly, the distributional benefits of status-quo improvement we find for SI-CC do not hold for SI-DA. In the three youngest regions of France, the SI-DA mechanism produces a distribution of teacher experience which does not FOSD the distribution under DA*. SI-DA also assigns more teachers with one or two years of experience (1,041) to the three youngest regions than DA* (801). This finding confirms that imposing status-quo improvement to DA-inspired mechanisms can backfire. In general, when Pareto efficiency is satisfied, we expect that an increase in mobility upon a status-quo improving matching can only be done at the expense of the distribution of teacher experience. This is indeed what we observe when comparing SI-CC and TTC*. However, when Pareto efficiency is not satisfied and mobility is extremely low, as under SI-DA, this trade-off may disappear. In essence, SI-DA just assigns new teachers to vacant positions and leaves all tenured teachers at their status-quo regions. The improvement upon the status-quo distribution in terms of teacher types is thus minimal among tenured teachers under SI-DA and further movement may help improving these distributions in many regions. Indeed, even though DA* does not impose status-quo improvement, the higher mobility it creates improves the distribution of teacher experience in these three youngest regions in France. Finally, note that the two DA-inspired mechanisms assign fewer inexperienced teachers to Créteil, Versailles, and Amiens than the two TTC-inspired mechanisms (1,041 for SI-DA and 801 for DA*, while 1,344 for SI-CC and 1,844 for TTC*). Again, this is due to the severe lack of mobility from these regions.

To complement the results for the three youngest regions of France, we also report the results for the three oldest regions of France. The objective is now to assign younger teachers.

Fact 3 *In the three oldest regions of France, the distribution under SI-CC FOSDs the one under TTC* (see the right panel of Figure 1). SI-CC assigns 187 teachers with one or two years of experience to these regions, while TTC* only assigns 96 of them. On the contrary, the distribution of teacher experience under DA* FOSDs the one under SI-DA.*

Two channels, that work in opposite directions, could explain SI-CC's better performance compared to TTC*. For example, take one of the youngest and least attractive regions, Créteil. On one hand, for tenured teachers, SI-CC prevents them leaving Créteil, which limits the possibility of assigning these (relatively inexperienced but tenured) teachers to attractive regions. On the other hand, we might expect SI-CC to prevent new teachers replacing tenured teachers in Créteil (due to new teachers' even lower experience), which would redirect these new teachers to attractive regions. Our results confirm that the second channel dominates the first one in terms of magnitude. Indeed,

[48] Figure A.4 shows that the distributional performance of TTC* depends on the chain selection rule. For instance, when the teachers starting a chain are selected by increasing order of their maximum Ministry-mandated priority points, the resulting distribution FOSDs the ones when they are ordered randomly or by decreasing order of their maximum Ministry-mandated priority points. These distributions of teacher experience under TTC* do not FOSD the status-quo distribution.

[49] Appendix Figures A.2 and A.3 show that these results persist for TTC* when we consider the entire group of regions whose average experience is below the median.

by preventing tenured teachers to leave Créteil for attractive regions, SI-CC lowers competition for vacant seats in the attractive regions. New teachers, who prefer these regions to Créteil, are able to get assigned to these vacant seats. (As we discuss below, the new teachers are on average and in distribution assigned to more preferred regions under SI-CC than under TTC*.)

A salient fact emerges when comparing distributional performances in the youngest and oldest regions: In the oldest regions, all mechanisms easily produce a distribution of teacher experience that FOSDs the status quo, while in the youngest regions only mechanisms that respect status-quo improvement do. This finding reflects the very different levels of attractiveness of these regions. As the oldest regions are highly demanded by teachers, it is easier to improve their teacher experience distribution. This is much more difficult in the youngest regions as they are not as highly demanded, especially by experienced teachers. The ratio of entry to exit requests is equal to 15.5 in Rennes but 0.03 in Créteil (see Table A.2). This large difference in demand also explains why the distributions of teacher experience are very compressed in the youngest regions, but not in the oldest ones (see Figure 1). Due to the limited room for improvement in disadvantaged regions, most mechanisms have a similar *capped* performance. Last, the performance of DA* is very good for the oldest regions since it produces a distribution of teacher experience which FOSDs those of all other mechanisms. For these regions, the mechanism gives priority to youngest teachers among those applying. Since these regions are overdemanded, the regions accept the youngest teachers, and, hence, status-quo improvement for these regions is fulfilled. Note that, under SI-DA, many young (tenured) teachers cannot apply to these regions since they are stuck in the youngest regions such as Créteil. Thus, the effective demand for these oldest regions under SI-DA is lower. This high performance of DA* in the oldest regions is achieved by accepting tenured teachers from other regions at the expense of these other regions. As we can see in Figure 1, DA* violates status-quo improvement in the three youngest regions.

Lower inequality between regions. The status-quo improvement requirement makes sure that regions are not harmed by the reassignment of teachers. Older regions become younger, and younger regions become older, reducing the initial differences in teacher experience between regions. While the previous paragraph discussed the distributional performance of the mechanisms for the three youngest and oldest regions, we now consider their performance across all regions. Figure 2 plots, for each region, the change in tenured teacher experience (proxied by the average type) between SI-CC and the status-quo matching (top left figure) and between SI-DA and the status-quo matching (bottom left figure).[50] Regions are ordered, along the horizontal axis, by average experience of their teachers at the status quo, so that all regions on the left are the younger regions that need more experienced teachers.

The top left panel of Figure 2 shows that, compared to the status-quo matching, SI-CC increases the average experience of tenured teachers in the younger regions and reduces the average experience of tenured teachers in the older regions. Therefore, SI-CC effectively lowers existing inequalities

[50]Since after the assignment mechanisms are run new teachers are assigned for the first time, the average experience falls in all regions with respect to status quo. Therefore, first we only inspect the distribution of tenured teachers after reassignment across regions under different mechanisms and status quo.

between regions. The top right panel of Figure 2 shows that SI-CC reduces inequality compared to not only the status-quo matching, but also TTC*.[51] As discussed when comparing SI-DA and its benchmark DA* for the oldest and youngest regions, the same conclusion may not hold for those mechanisms. Indeed, relaxing the status-quo improvement requirement creates a more equal distribution of teacher experience. In the bottom right panel of Figure 2, we observe that SI-DA does not reduce the average experience gap between the younger and older regions compared to DA* for tenured teachers. We summarize these findings as follows:

Fact 4 *SI-CC reduces the large gap in average tenured teacher experience that exists at the status-quo matching between the younger regions and the older regions. SI-CC also reduces the gap compared to TTC*. In contrast, SI-DA slightly reduces this gap in average tenured teacher experience that exists at the status-quo matching, and DA* further reduces it compared to SI-DA.*

Limited trade-off between teacher distribution and teacher welfare. Next, we investigate whether the better distributional performance of SI-CC comes at the cost of a poorer welfare for teachers, as measured by the number of teachers who obtain a new region and the rank of assigned region in teachers' preference list. Table 3 shows that the number of tenured teachers who move is larger under the benchmark mechanisms. Under SI-CC 1,444 tenured teachers move and under TTC* 2,470 move, a difference of 1,026 teachers. For DA-inspired mechanisms, the difference is lower: 894 additional tenured teachers move under DA* compared to SI-DA.

The cost in terms of teacher mobility is larger in the three youngest region (Créteil, Versailles, and Amiens) than in the three oldest regions (Rennes, Bordeaux, and Lyon). Moreover, similar numbers, 116 vs 117, tenured teachers leave Rennes, Bordeaux, and Lyon under SI-CC and TTC*, respectively, while more tenured teachers leave Créteil, Versailles, and Amiens under TTC* (1,018) than under SI-CC (239). The lower demand for these three youngest and least attractive regions from tenured teachers compared to the oldest regions explains the huge difference in outflow under TTC*. Status-quo improvement requirement considerably bounds the outflow from the youngest regions under SI-CC.

Welfare differences between tenured and new teachers. The differences we observe between the youngest and oldest regions might explain an interesting finding: despite a larger movement under TTC*, the rank distribution of the region that new teachers are assigned is FOSDed by the distribution under SI-CC (see Panel C of Table 3).[52] It confirms our prior explanations when comparing the mechanisms with their respective benchmarks.

Fact 5 *The distribution of ranks of the regions that tenured teachers are assigned under TTC* FOSDs the one under SI-CC. The opposite FOSD comparison holds for new teachers.*

[51] We reach similar (if not better) conclusions regarding SI-CC's better performance than its benchmark TTC* when considering both new and tenured teachers together (See Figure A.5).

[52] Although we defined FOSD relation for groups of teachers regarding preferences of regions, it is trivial to extend it to any arbitrary cumulative distribution obtained from histograms of statistics, and this is what we use here.

On average, new teachers are assigned their 8.2^{th} ranked region under SI-CC and their 10.2^{th} ranked region under TTC* (see Panel D of Table 3). This is because a much larger number of tenured teachers leave the youngest regions of Créteil, Versailles, and Amiens under TTC* (1,018) than under SI-CC (239). These exiting teachers have to be replaced, and new teachers are the most likely substitutes due to lower demand from other tenured teachers. This is because very few tenured teachers ask to enter younger regions and because, under TTC*, new teachers can replace tenured teachers in younger regions, even if they have a lower experience. In practice, we see that 1,415 new teachers are assigned to Amiens, Créteil, or Versailles under TTC* versus 862 under SI-CC. The large share of new teachers being assigned unattractive regions under TTC* explains why these teachers are assigned to lower ranked regions than under SI-CC.[53]

Effects on stability measures. Last, we investigate whether imposing the status-quo improvement requirement has an impact on the number of blocking pairs under different stability notions. For DA-inspired mechanisms, imposing the status-quo improvement requirement can only create more blocking pairs since it forbids certain teachers to move from their initial position and gives a top priority to new teachers over empty slots. This is indeed what we observe since SI-DA has 385 more teachers involved in at least one blocking pair (in the sense of Gale-Shapley stability) compared to its benchmark. The latter being teacher-SI stable, the only blocking pairs it has are caused by teachers staying at their initial position. In addition, SI-DA has 2,171 teachers blocking because of an empty slot.

For SI-CC, the opposite happens. Indeed, since the latter and its benchmark do not impose any stability condition, the additional mobility created by relaxing the status-quo improvement constraint is done at the further expense of stability. However, this increase is limited since TTC* has only UU: Where did this number come from?→ 36 additional teachers involved in a blocking pair compared to SI-CC. The small differences between the three stability notions show that blocking pairs of the TTC* and SI-CC matchings are mainly driven by their high mobility rates which imply that less preferred teachers are assigned to a new region at the expense of more preferred ones who also requested that region.

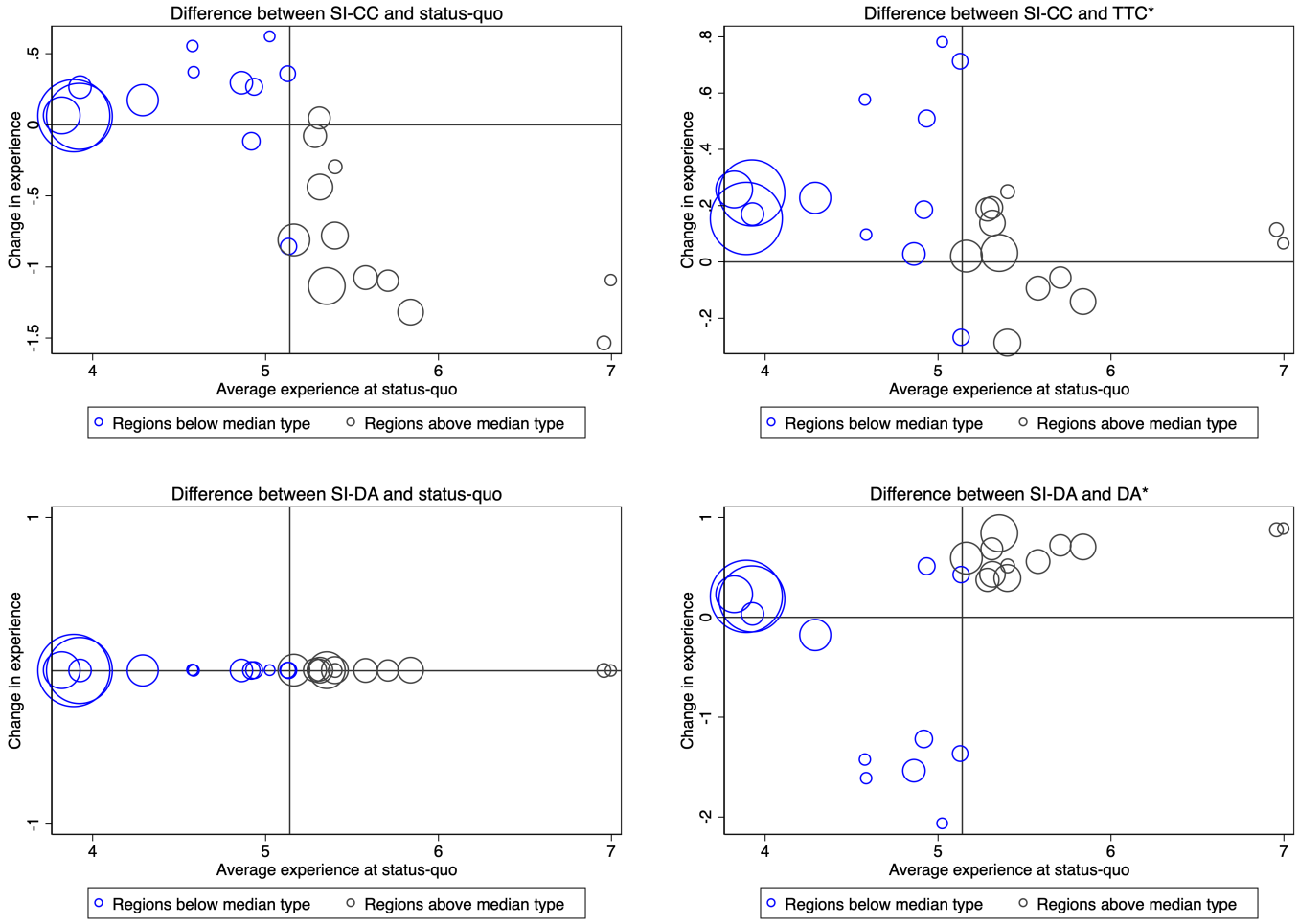
[53] Our preference estimates reveal that new teachers dislike unattractive regions less than tenured teachers. Yet, only 11.9% of the math teachers rank Créteil or Versailles as their first choice and 14.3% of French teachers. That mild preference for unattractive regions is not large enough to justify that assigning a large share of the new teachers to these two regions will improve the ranking of the region they obtain.

Table 2: Teacher Preference Estimates

	Tenured Teachers				New Teachers			
	French		Math		French		Math	
	coef.	s.e.	coef.	s.e.	coef.	s.e.	coef.	s.e.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Region BESANCON	-3.88***	(0.99)	0.37	(0.8)	-1.52*	(0.65)	-0.32	(0.78)
Region BORDEAUX	-1.36	(0.95)	1.12	(0.66)	1.14	(0.62)	-0.40	(0.64)
Region DIJON	-5.08***	(0.97)	-2.88***	(0.73)	-1.39*	(0.59)	-1.88**	(0.61)
Region LILLE	-5.09***	(0.95)	-1.55	(0.81)	-1.7**	(0.62)	-1.26	(0.67)
Region REIMS	-6.22***	(1.00)	-3.60***	(0.74)	-2.39***	(0.61)	-3.04***	(0.65)
Region AMIENS	-6.44***	(1.06)	-3.31***	(0.75)	-1.88**	(0.59)	-2.35***	(0.62)
Region ROUEN	-5.96***	(0.97)	-2.17**	(0.69)	-1.64**	(0.59)	-1.53*	(0.62)
Region CRÉTEIL	-6.66***	(1.00)	-3.65***	(0.71)	-0.34	(0.60)	-1.33*	(0.58)
Region VERSAILLES	-5.12***	(0.89)	-2.13***	(0.60)	-0.50	(0.55)	-1.14*	(0.56)
Status-quo region	4.97	(6.72)	-15.72	(8.27)				
Birth region	10.21**	(3.41)	14.89***	(3.53)	10.46***	(2.48)	8.31**	(2.69)
Distance to status-quo region	-23.33***	(4.61)	-23.52***	(5.47)				
% stud. urban x Status-quo region	-5.82***	(0.85)	-5.40***	(1.15)				
% stud. urban x Teach. from CV	2.81***	(0.71)	0.12	(0.68)				
% stud. in priority educ. x Married	-7.61***	(1.60)	-3.89*	(1.65)	-7.13***	(1.58)	-4.93***	(1.31)
% stud. in priority educ. x Status-quo region	11.26***	(2.99)	0.67	(3.80)				
% stud. in private sch. x Teach. in disadv. sch.	5.48**	(2.01)	6.58***	(1.83)				
% teach. younger than 30 x Advanced qualif.	10.59**	(3.65)	0.24	(3.06)	10.80**	(2.77)	5.58**	(1.99)
% teach. younger than 30 x Status-quo region	52.42***	(5.19)	54.15***	(6.47)				
% teach. younger than 30 x Birth region	-22.08***	(3.73)	-19.10***	(4.79)	-8.42**	(2.79)	-9.26***	(2.61)
Region in South of France x Teach. from CV	-1.27***	(0.37)	0.35	(0.36)				
Number of teachers	859		605		786		958	
Fit measure	0.669		0.674		0.632		0.624	

Notes: This table reports selected coefficients from estimations of teachers' preferences for regions' characteristics based on Equation 5. We set the fixed effect of the Paris region to 0. The last row reports our goodness of fit measure that we compute by looking at the top two regions that a teacher has included in her submitted preference list. We measure, for each teacher, the probability of observing this particular preference ordering in the preference list predicted with our estimations. We then average these probabilities across teachers. Stars correspond to the following p-values: * p< 0.05; ** p< 0.01; *** p< 0.001. Variable "Teach. from CV" refers to whether the status-quo region of the teacher is Créteil or Versailles.

Figure 2: Change in Average Tenured Teacher Experience Types Across Regions



UU: Horizontal axis title: "average experience type" instead of "average experience" UU: Vertixalaxis title: "Change in average experience type" instead of "Chain in experience" Notes: This figure shows the difference in the average teacher experience types between the matching obtained by SI-CC between the status-quo matching (top left figure) and its benchmark TTC* (top right figure). It reports the same difference between the matching obtained by SI-DA between the status-quo matching (bottom left figure) and its benchmark DA* (bottom right figure). Each observation represents a French region. Circle size reflects region size. Regions are ordered (on the horizontal axis) by average experience type of their teachers at status quo. The vertical line represents the median type. All regions on the left have an average type that is strictly below the median. This is the group of regions we identified as *younger regions*. All regions on right of the vertical line are regions whose average type is above the median, i.e., *older regions*. In regions above the horizontal line, average experience of tenured teachers after reassignment is larger than the one under the benchmark matching to which it is compared.

Table 3: Teacher Welfare and Teacher Blocking

	Suggested mech.		Benchmark mech.		Policy mech.	
	SI-CC	SI-DA	TTC*	DA*	wSI-CC	Current French
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A. Teacher mobility						
Tenured teachers moved and new teachers assigned	5,356	3,912	6,382	4,806	6,206	5,864
Tenured teachers moved - from the 3 youngest regions	239	0	1,018	316	1,118	980
Tenured teachers moved - from the 3 oldest regions	116	0	117	117	114	72
Tenured teachers moved - from all regions	1,444	0	2,470	894	2,294	1,952
UU: Tenured teachers stayed at status-quo region						
New teachers assigned	3,912	3,912	3,912	3,912	3,912	3,912
New teachers unassigned	715	715	715	715	715	715
Panel B. Cumulative distribution of ranks of regions that tenured teachers are assigned						
Rank $= 1$	749	250	1,112	338	1,029	833
Rank ≤ 2	1,604	919	2,203	1,050	2,086	1,775
Rank ≤ 3	2,080	1,352	2,758	1,531	2,614	2,300
Rank ≤ 4	2,485	1,745	3,168	1,967	3,013	2,737
Rank any	5,833	5,833	5,833	UU: Not correct? 2,393	5,833	5,833
Panel C. Cumulative distribution of ranks of regions that new teachers are assigned						
Rank $= 1$	1,243	967	645	803	731	621
Rank ≤ 2	1,780	1,557	1,010	878	1,113	990
Rank ≤ 3	2,128	1,942	1,289	1,397	1,392	1,263
Rank ≤ 4	2,404	2,242	1,535	1,735	1,632	1,502
Rank any	4,627	4,627	4,627	4,627	4,627	4,627
Panel D. Average rank of region assigned						
Average rank - All teachers	7.3	8.5	7.5	8.4	7.6	8.3
Average rank - Teachers from the 3 youngest regions	7.2	7.5	5.8	7.1	5.5	5.8
Average rank - Teachers from the 3 oldest regions	2.2	9.2	2.0	6.6	2.5	5.0
Average rank - Tenured teachers	6.6	8.5	5.3	7.5	5.6	6.0
Average rank - New teachers	8.2	8.6	10.2	9.5	10.0	11.1
Panel E. Number of teachers blocking under different stability notions						
Gale-Shapley stability	8,206	8,438	8,474	8,126	8,469	8,688
Teacher-SI stability	8,205	2,171	8,473	0	8,468	8,655
SI stability	8,122	0	8,408	0	8,418	8,626

Notes: Panel A of this table reports statistics on teacher mobility: numbers of tenured teachers who moved to a new region and assigned and unassigned new teachers. Panels B and C present the cumulative distribution of the ranks of the regions teachers are assigned to in their preferences. Panel D reports statistics on the average rank of the region teachers obtain. Panel E reports the numbers of teachers causing unauthorized blocks under different stability notions.

6.7 Relaxing Distributional Constraints: Towards Policy Maker Objectives

This last section investigates whether we can increase mobility—and by how much—if we relax the status-quo improvement distributional objectives in order to target weaker objectives that match the current ones of the French Ministry of Education. The results we have presented so far show that SI-CC's better distributional performance comes at the cost of a lower mobility, especially for teachers in the three youngest regions. Limiting mobility away from disadvantaged regions might have the undesirable consequence of reinforcing the low attractiveness of these regions. We show in this section that low mobility can be avoided. Two simple modifications of SI-CC that weaken the status-quo improvment criterion allow to maintain mobility away from young regions at about the same level as it is currently.

From a policy perspective, the motivation for relaxing the distributional objective is clear: the mechanism currently used by the French Ministry of Education does not require that teachers leaving disadvantaged regions are replaced by more experienced teachers. In that regard, partially relaxing our demanding status-quo improvement distributional objective would still match the current objectives of policy makers. To relax our distributional objectives, we slightly modify SI-CC to allow teachers from the three youngest regions to leave without being replaced by a higher-type teacher. We call this mechanism **wSI-CC**, where "wSI" refers to a weaker version of status-quo improvement.

A natural benchmark for these versions of SI-CC with weaker distributional objectives is the matching obtained with the mechanism and priorities currently used by the French Ministry, i.e., priorities that ignore types. We refer to this mechanism as **Current French**. This will allow us to check whether mobility under the modified SI-CC manages to reach the mobility levels of the current mechanism.[54]

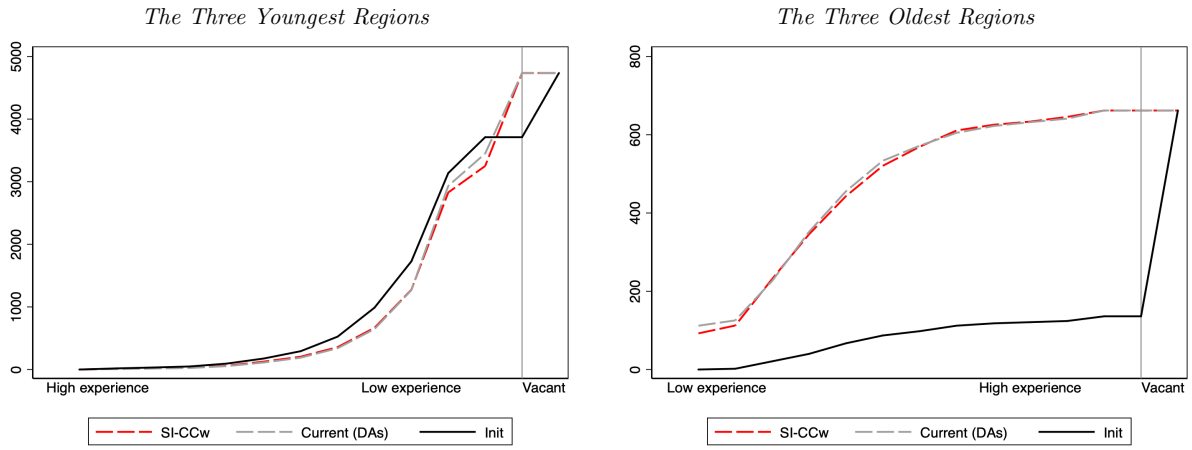
The results (reported in the last two columns of Table 3) show that partially relaxing the distributional objectives allows significantly more teachers to move away from the three youngest regions compared to the original SI-CC studied in the previous sections. Mobility from these regions goes up from 239 under SI-CC to 1118 under wSI-CC. As a benchmark, note that 980 move away from Créteil, Versailles, and Amiens under the current mechanism. These results show that relaxing SI-CC's distributional constraint in young regions improves mobility in these regions up to the levels currently observed, and even exceeds these levels. This improved mobility results in very similar performances between wSI-CC and the current mechanism in terms of distributions. Figures 3 report the cumulative distributions of teacher types and confirms that the distribution are almost identical. Figure 4 shows, for each region, the difference between teacher experience under wSI-CC and under the current mechanism. Both Figures show that wSI-CC tends to better fulfill the twofold objective of (i) making young regions older and (ii) making old regions younger.

[54] We previously mentioned that the DA* mechanism used so far as benchmark is equivalent to the mechanism currently used by the French Ministry of Education, except that we replaced the schools priorities by those that account for teacher types. In contrast, the priorities used in this section ignore teacher types and correspond to the priorities currently used by the Ministry.

UU: Comments:

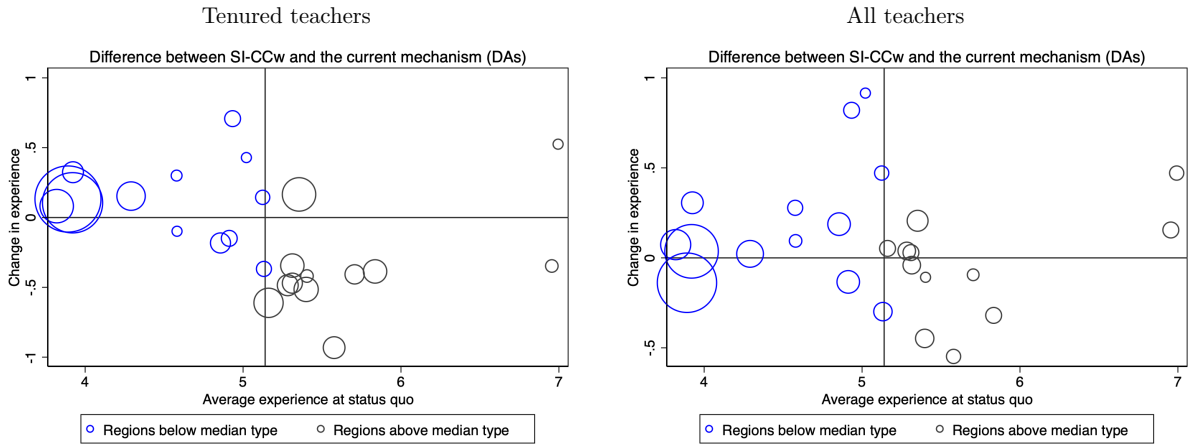
- Legend below: "Current (DAs)" should be "Current French" below
- In the all paper: Replace everywhere "SI-CCw" with "wSI-CC"
- Make all Table and Figure titles CONSISTENT in the same font and either capitalize all first letters of words or just the first word in the title for all

Figure 3: Cumulative Distribution of Teacher Experience Types - Policy Objectives



Notes: This figure shows the cumulative distribution of teacher experience types. The left panel reports the distribution in the three youngest regions of France (Créteil, Versailles, and Amiens), and the right panel the distribution in the three oldest regions of France (Rennes, Bordeaux, and Lyon). The horizontal axis reports the 12 experience types of teachers, ordered from most experienced to least experienced (left panel) and from least experienced to most experienced (right panel). The 13^{th} type corresponds to the vacant positions. The mechanisms that respect status-quo improvement are plotted in red. Those that do not are in grey. The thick black line (with legend Init) corresponds to the cumulative distribution of teacher types at the status-quo matching.

Figure 4: Change in Average Tenured Teacher Experience Type Across Regions - Policy Objectives



Notes: This figure shows, for each region, the difference in the average experience of teachers under wSI-CC and under the current mechanism. The left column reports the experience of tenured teachers, while the right column reports the experience of all teachers. Each observation represents a French region. Circle size reflects region size. Regions are ordered (on the horizontal axis) by average experience of their teachers at status quo. The vertical line represents the median type. All regions on the left have an average type that is strictly below the median. This is the group of regions we identified as inexperienced regions. All regions on the right of the vertical line are regions whose average type is above the median. In regions above the horizontal line, teachers average experience post reassignment is larger than the one of the benchmark matching to which it is compared.

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Appendix A Omitted Result and Proofs

Proposition 6 *Let ψ be a status-quo improving and Pareto efficient mechanism which selects a Pareto efficient matching other than SI teacher optimal whenever such a matching exists. Then, ψ is not strategy-proof.*

Proof: On the contrary suppose ψ is strategy-proof. Let $S = \{s_1, s_2, s_3\}$, $T = \{t_1, t_2, t_3\}$, $\omega_{s_1} = \{t_1\}$, $\omega_{s_2} = \{t_2\}$, $\omega_{s_3} = \{t_3\}$ and $q_{s_1} = q_{s_2} = q_{s_3} = 1$. Let $\tau(t_3) \rhd_{s_1} \tau(t_2) \rhd_{s_1} \tau(t_1) \rhd_{s_1} \theta_\emptyset$, $\tau(t_1) \rhd_{s_2} \tau(t_3) \rhd_{s_2} \tau(t_2) \rhd_{s_2} \theta_\emptyset$ and $\tau(t_2) \rhd_{s_3} \tau(t_1) \rhd_{s_2} \tau(t_3) \rhd_{s_3} \theta_\emptyset$, $s_3\ P_{t_1}\ s_2 P_{t_1}\ s_1 P_{t_1}\ \emptyset$, $s_1\ P_{t_2}\ s_3 P_{t_2}\ s_2 P_{t_2}\ \emptyset$, and $s_2\ P_{t_3}\ s_1 P_{t_3}\ s_3 P_{t_3}\ \emptyset$.

There exists a unique SI teacher optimal matching, denoted by ν, in which each teacher is assigned to her top choice. In any other status-quo improving Pareto efficient matching at most one teacher is assigned to her top choice and at least one teacher is assigned to her second choice. Suppose ψ selects Pareto efficient and status-quo improving matching μ in which t_1 is assigned to her second choice s_2. If t_1 reports only s_3 and s_1 acceptable, then ν is the unique SI teacher optimal matching and ψ assigns t_1 to s_1, t_2 to s_3 and t_3 to s_2. In this updated market, if t_2 reports only s_1 and s_2 acceptable, then ν is the unique SI teacher optimal matching and ψ assigns t_1 to s_3, t_2 to s_2 and t_3 to s_1. Finally, in this updated market, if t_3 reports only s_2 and s_3 acceptable, then ν is the Pareto efficient and status-quo improving matching and ψ needs to select it. However, this contradicts with strategy-proofness of ψ, i.e., it is manipulated by t_3.

We can show the same result for any Pareto efficient and status-quo improving matching in which at least one teacher is assigned to her second choice under the original market. ■

Proof of Proposition 1: On the contrary, suppose μ is SI teacher optimal and it is Pareto dominated by ν. Since μ status-quo improves ω so does ν. Hence, ν is status-quo improving. In addition, since ν Pareto-dominates μ, all teachers weakly prefer ν to μ. Because, $\nu \neq \mu$ and teachers' preferences are strict, some teachers strictly prefers ν to μ. However, this violates SI teacher optimality of μ. This is a contradiction. ■

Proof of Theorem 1: SI teacher optimality: Recall that the requirement of status-quo improvement is embedded in the definition of SI teacher optimality. Consider an arbitrary market P. Let $\hat{\mu}$ be the outcome of SI-CC under this market. We proceed in two parts.

1. We first show that $\hat{\mu}$ is status-quo improving.
   First, consider teachers. Under SI-CC, each school s points to all teachers in ω_s one by one. When a teacher $t \in \omega_s$ is pointed by s in some Step k, then $s \in A_t^k$ and she can always form a one-school cycle (s, t) whenever she points to s. Similarly, any new teacher $t \in N$ can form a cycle with $\emptyset$ in any step of SI-CC. Hence, $\hat{\mu}_t\ R_t\ \omega_t$ for all $t \in T$.
   Next, consider schools. The tail of any executed chain is a new teacher. Hence, if in some step of SI-CC, a school s is sending out a teacher then it is simultaneously acquiring another

teacher; as a result $|\hat{\mu}_s| \geq |\omega_s|$. In any step k of SI-CC, when we consider the set of remaining status-quo employees and teachers assigned in the first $k-1$ steps, because of the positive balance requirement in Improvement Condition 1, the previous observation and the fact that a teacher cannot be assigned a school if she is unacceptable, each school s is weakly better off compared to ω_s. Hence, $\hat{\mu}_s \succsim_s \omega_s$ for all $s \in S$.

We showed that $\hat{\mu}$ is status-quo improving.

2. Before proving $\hat{\mu}$ cannot be Pareto dominated by another status-quo improving matching for teachers, we first state a claim that will be used in the proof.

Claim 1: For a school s, suppose step K is the final step in which school s is assigned a teacher in an executed chain such that s is the head of this chain. Let the set of remaining status-quo employees of s at the end of step K be denoted as ω_s^K and all assigned teachers to s from the beginning of step 1 until the end of step K as μ_s^K. Let ν be a matching such that all teachers assigned in the first K steps of SI-CC are matched with their assignment under SI-CC and $|\nu_s| < |\omega_s^K \cup \mu_s^K|$. Then, ν is not a status-quo improving matching.

Proof of Claim 1: Suppose teacher t is assigned to s in this chain in step K and t is pointing s under Condition 2. First observe that $\mu_s^K \subseteq \nu_s$. Also notice that, if $\omega_s^K = \emptyset$, then $|\nu_s| = |\omega_s^K \cup \mu_s^K|$. Hence, $\omega_s^K \neq \emptyset$. Since Condition 1 does not hold for school s via teacher t, there exists some type θ such that $\tau(t_s^K) \trianglerighteq_s \theta \triangleright_s \tau(t)$ and

$$\sum_{\theta' \trianglerighteq_s \theta} b_s^{\theta'} \leq 0$$

where $b_s^{\theta'}$ is the current balance of type θ' at step K of SI-CC.[55] That is, the number of teachers with weakly better type than θ in $\mu_s^K \cup \omega_s^K$ cannot be more than what it is in ω_s. Moreover, all teachers in ω_s^K have weakly better type than θ. Hence, $|\nu_s| < |\omega_s^K \cup \mu_s^K|$ and $\mu_s^K \subseteq \nu_s$ imply that the number of teachers with weakly better type than θ in ν_s is strictly less than this number in ω_s. Therefore, ν_s is not preferred to ω_s by school s. $\diamond$

Next, we show that $\hat{\mu}$ cannot be Pareto dominated by another status-quo improving matching for teachers.

On the contrary, suppose there exists a status-quo improving matching ν that Pareto dominates $\hat{\mu}$ for teachers. By considering the teachers assigned in each step of SI-CC inductively, we show that such a matching cannot exist, in particular we should have $\nu = \hat{\mu}$.

We denote the set of teachers assigned in step k of SI-CC under market P with T_k and union of these sets up to step k as $\bar{T}_k \equiv \cup_{k'=1}^{k} T_{k'}$.

Step 1: Each teacher $t \in T_1$ is assigned in $\hat{\mu}_t$ to the best school in A_t^1. If $\nu_t P_t \hat{\mu}_t$ for some $t \in T_1$, then $\nu_t \notin A_t^1$. Thus, for school $s \equiv \nu_t$ both improvement conditions are violated via teacher t. Since this is Step 1, the current matching satisfies $\mu = \emptyset$, and hence, the current balances

[55] Actually, sum of balances $\sum_{\theta' \trianglerighteq_s \theta} b_s^{\theta'}$ never becomes negative in the mechanism for any type θ, as the sum starts at zero at the beginning of Step 1, and whenever it is zero, we do not admit a teacher with a type worse than θ by sending out a teacher with a type better than θ by Improvement Condition 1.

$b_s^\theta = 0$ for all schools s and types θ. The violation of Condition 1 implies that

- if there exists a teacher $t_s^1 \in \omega_s$ that s is pointing, then it has type $\tau(t_s^1) \triangleright_s \tau(t)$: thus, t has a worse type than the worst type status-quo employees of this school; and
- if such a teacher does not exist, then $\omega_s = \emptyset$.

Thus, in either case, $\nu_s \setminus \{t\} \succsim_s \omega_s$ and $|\nu_s| > |\omega_s|$ as otherwise ν is not status-quo improving for s by FOSD preferences. The violation of Condition 2 for s via t, on the other hand, implies one of the following conditions to hold:

- t is not acceptable for s: in this case status-quo improvement for s under ν would be violated; or
- there are no new teachers: in this case, as we showed $|\nu_s| > |\omega_s|$ implies that there exists some schools s' such that $|\omega_{s'}| > |\nu_{s'}|$; as a result in this case status-quo improvement for s under ν would be violated by FOSD preferences; or
- $q_s = |\omega_s|$: in this case, as we showed $|\nu_s| > |\omega_s|$, $|\nu_s| > q_s$ contradicting the feasibility of ν as matching.

Then, Condition 2 cannot be violated as none of these conditions hold, which is a contradiction. Hence, such a teacher t cannot exist with $\nu_t P_t \hat{\mu}_t$. Since $\nu_t R_t \hat{\mu}_t$ for all t then for all $t \in T_1$, $\nu_t = \hat{\mu}_t$.
Inductive assumption: For any $k > 1$, Assume that for all $k' < k$ and $t \in T_{k'}$, $\nu_t = \hat{\mu}_t$. We show that the same holds for teachers in T_k:
Step k: Each teacher $t \in T_k$ is assigned in $\hat{\mu}_t$ to the best school in A_t^k. If $\nu_t P_t \hat{\mu}_t$ for some $t \in T_k$, then $\nu_t \notin A_t^k$. Thus, for school $s \equiv \nu_t$ both improvement conditions are violated via teacher t. Noting μ is the current matching determined until the end of step $k-1$, the violation of Condition 1 implies that

- if there exists a teacher $t_s^k \in \omega_s$ that s is pointing, then it has type $\tau(t_s^k) \triangleright_s \tau(t)$ and there exists an intermediate type θ such that $\tau(t_s^k) \trianglerighteq_s \theta \triangleright_s \tau(t)$ with

$$\sum_{\theta' \trianglerighteq_s \theta} |\mu_s^{\theta'}| - |\{t' \in \omega_s : \mu_{t'} \neq \emptyset\}^{\theta'}| \leq 0.$$

  By the inductive assumption for the current matching $\mu_{t'} = \nu_{t'}$ for all t' assigned until this step (i.e., those in $\bar{T}_{k-1}$), and hence we also have

$$\sum_{\theta' \trianglerighteq_s \theta} \left| \left(\nu_s \cap \bar{T}_{k-1}\right)^{\theta'} \right| - \left| \left(\omega_s \cap \bar{T}_{k-1}\right)^{\theta'} \right| \leq 0.$$

  Teacher t has a worse type than the remaining worst-type status-quo employee of this school i.e., those in $\omega_s \setminus \bar{T}_{k-1}$. Thus, in ν replacing any of these employees with t would violate status-quo improvement for s in ν, as this would have led to an FOSD violation for type θ:

$$\sum_{\theta' \trianglerighteq_s \theta} \left|\nu_s^{\theta'}\right| - \left|\omega_s^{\theta'}\right| < 0.$$

  Then t does not replace any of the remaining status-quo employees, but she is an additional

teacher acquired: $|\nu_s \setminus \bar{T}_{k-1}| > |\omega_s \setminus \bar{T}_{k-1}|$.

- if such a teacher does not exist, then $\omega_s \setminus \bar{T}_{k-1} = \emptyset$, and hence, as $t \in \nu_s \setminus \bar{T}_{k-1}$ we have $|\nu_s \setminus \bar{T}_{k-1}| > |\omega_s \setminus \bar{T}_{k-1}|$.

Observe that in the algorithm at each step we make sure that each school acquires at least as many teachers as it sends out and hence, for the current matching $|\mu_s| \geq |\omega_s \cap \bar{T}_{k-1}|$. Since $\mu_s = \nu_s \cap \bar{T}_{k-1}$ by the inductive assumption, we have $|\nu_s \cap \bar{T}_{k-1}| \geq |\omega_s \cap \bar{T}_{k-1}|$. Therefore, as we also showed that $|\nu_s \setminus \bar{T}_{k-1}| > |\omega_s \setminus \bar{T}_{k-1}|$ above, we obtain $|\nu_s| > |\omega_s|$.

The violation of Condition 2 for s via t, on the other hand, implies one of the following conditions to hold:

- t is not acceptable for s: in this case status-quo improvement for s under ν would be violated; or
- there are no remaining new teachers: Claim 1 implies that there exists at least one school s' such that $\nu_{s'}$ does not status-quo improve upon $\omega_{s'}$; or
- $q_s = |\omega_s|$: in this case, as we showed $|\nu_s| > |\omega_s|$, $|\nu_s| > q_s$ contradicting the feasibility of ν as matching.

Then, Condition 2 cannot be violated as none of these conditions hold, which is a contradiction. Hence, such a teacher $t \in T_k$ with $\nu_t P_t \hat{\mu}_t$ cannot exist.

Since $\nu_t R_t \hat{\mu}_t$ for all t then for all $t \in T_k$, $\nu_t = \hat{\mu}_t$, completing the induction and showing that $\nu = \hat{\mu}$.

Strategy-proofness: We state two claims that we will use in the proof.

Claim 2: Suppose teacher t is assigned in step K of SI-CC. For any $k < K$, then $A_t^{k+1} \subseteq A_t^k$.

Proof of Claim 2: Let $s \notin A_t^k$. We will show that $s \notin A_t^{k+1}$. We consider two possible cases.

Case 1: s does not have an unfilled seat at step k: First notice that, if there is no remaining status-quo employee of s in step k, then it should have been removed in an earlier step of SI-CC. Then, there exists some type θ such that $\tau(t_s^k) \trianglerighteq_s \theta \triangleright_s \tau(t)$ with

$$\sum_{\theta' \trianglerighteq_s \theta} b_s^{\theta'} \leq 0$$

where $b_s^{\theta'}$ is the current balance of type θ' at step k of SI-CC. If school s is part of the executed cycle or chain in step k, then the teacher assigned to s has a type weakly better than type θ under $\triangleright_s$ and similarly, the teacher leaving school s, namely, t_s^k also has a type weakly better than type θ. Hence, after executing the cycle in step k relation above still holds. Moreover, s cannot send out a status-quo employee without getting a new one by the definition of SI-CC. Similarly, s cannot get a teacher without sending a status-quo employee. If school s is not part of the executed cycle or chain in step k, equation above still holds. In either case, $s \notin A_t^{k+1}$.

Case 2: s has an unfilled seat at step k: Either t is unacceptable for s or t is acceptable for s but there does not exist a remaining new teacher in step k. If the former case holds, then $s \notin A_t^{k+1}$ by definition.

If the latter case holds, then either s does not have remaining status-quo employee or there exists some type θ such that $\tau(t_s^k) \trianglerighteq_s \theta \triangleright_s \tau(t)$ and

$$\sum_{\theta' \trianglerighteq_s \theta} b_s^{\theta'} \leq 0$$

where $b_s^{\theta'}$ is the current balance of type θ' at step k of SI-CC. If the former subcase holds, then neither Condition 1 nor Condition 2 holds for t in step $k+1$. For the later condition, we refer to Case 1 above. Hence, $s \notin A_t^{k+1}$. $\diamond$

Claim 3: Consider a step k of SI-CC mechanism such that there exists a path of schools and teachers $(s_1, t_1, s_2, t_2, \ldots, s_\ell, t_\ell)$ in which school $s_{\ell'}$ points to teacher $t_{\ell'}$ and teacher $t_{\ell'-1}$ points to school $s_{\ell'}$ for each $\ell' < \ell$ and $s_1 \in A_{t_\ell}^k$. If none of the schools in this path are assigned a teacher in this step, the same path forms in step $k+1$ and $s_1 \in A_{t_\ell}^{k+1}$.

Proof of Claim 3: As no teacher is assigned to the schools of the path in step k, the teachers in the path remain in the step $k+1$. Since $t_{\ell'} \equiv t_{s_{\ell'}}^k$ is the highest priority remaining status-quo employee in step k of school $s_{\ell'}$, she continues to be so in step $k+1$, thus, school $s_{\ell'}$ points to $t_{\ell'}$ in step $k+1$. No other status quo employee of these schools is assigned to any other school in step k, either, because the assignment of status-quo employees requires the school pointing to them and each school points to at most one teacher in this step. Thus, as Condition 1 or Condition 2 holds for each school $s_{\ell'}$ via teacher $t_{\ell'-1}$ (in modulo ℓ, thus $t_0 \equiv t_\ell$) in step k, the same condition continues to hold in step $k+1$ via the same teacher. Hence, $s_{\ell'} \in A_{t_{\ell'-1}}^{k+1}$ for each ℓ'. Since $A_{t_{\ell'-1}}^{k+1} \subseteq A_{t_{\ell'-1}}^k$ by Claim 2, and $s_{\ell'}$ is the favorite school of teacher $t_{\ell'-1}$ in the opportunity set in step k, we still have $s_{\ell'}$ as the favorite school of teacher $t_{\ell'-1}$ in step $k+1$ and she continues to point to $s_{\ell'}$ in Step $k+1$. $\diamond$

We are ready to finish the proof for the strategy-proofness of SI-CC. Recall that we denote the set of teachers assigned in step k of SI-CC with T_k. First, notice that a teacher t' cannot change the schools in $A_{t'}^1$ by misreporting her preferences since $A_{t'}^1$ does not depend on the submitted preferences. Moreover, by Claim 2, $\{A_t^k\}$, the opportunity sets for teacher t, weakly shrink in through out SI-CC. Hence, a teacher t cannot be assigned to a school $s \notin A_t^1$ under SI-CC. We first consider the teachers in T_1. Each $t \in T_1$ is assigned to her best choice in A_t^1. Hence, any teacher $t \in T_1$ cannot benefit from misreporting her preferences.

Next, we consider a teacher $t \in T_2$. As explained above, teacher t cannot be assigned to school $s \notin A_t^1$ under SI-CC. Teacher $t \in T_2$ is assigned to best school in A_t^2 when she submits her true preferences. We denote the best school in A_t^2 according to P_t with s'. By Claim 2, $A_t^2 \subseteq A_t^1$. Hence, if $t \in T_2$ can benefit from misreporting her preferences, then she is assigned to some school $s \in A_t^1 \setminus A_t^2$. If $A_t^1 \equiv A_t^2$, then t cannot benefit from misreporting her preferences. Suppose $A_t^1 \setminus A_t^2 \neq \emptyset$. We will show that t cannot be assigned to a school $s \in A_t^1 \setminus A_t^2$ such that $s \; P_t s'$ by misreporting. Particularly, we show t cannot prevent the cycle or chain executed in step 1 without hurting herself.

First notice that, if t forms a cycle in step 1 by misreporting and pointing to some school $s'' \in A_t^1$, then by Claim 3, $s'' \in A_t^2$ and the path leading to t in this cycle starting with school s'' forms again when she submits P_t, which does not match her in step 1. Hence, any such school s'' cannot be preferred to s', i.e., t's assignment under truthtelling.

If a chain is executed in step 1, teacher t cannot be a part of that chain by misreporting and pointing some other school in A_t^1. This follows from the fact that the executed chain starts with a specific new teacher and a teacher $\bar{t}$, who is pointed by her status-quo school $\bar{s}$, can only be added to the executed chain if a previously included teacher points to $\bar{s}$, independent of $\bar{t}$'s preference. Teacher t can prevent the executed chain by only forming a cycle by misreporting. However, as explained above, under such a cycle t will be assigned to a school weakly worse than s'.

Moreover, with a similar reasoning to a chain, teacher t cannot affect the executed cycles in step 1 by submitting a different preference without being matched in step 1 in a new cycle (and therefore, making her weakly worse off as we showed above).

By using similar arguments, we can show that any teacher in T_k where $k > 2$ cannot benefit from misreporting her preferences. ■

Proof of Proposition 2: We first show the existence of Gale-Shapley stable matching. Consider a market P. We construct a strict rank order list, $\hat{\succ}_s$, for each school s over the teachers as follows: for any $t, t' \in T$

- if $\tau(t) \rhd_s \tau(t')$ then $t \hat{\succ}_s t'$;
- if $\tau(t) = \tau(t')$, then the relative order between t and t' is determined arbitrarily;
- $\tau(t) \rhd_s \theta_\emptyset$ if and only if $t \hat{\succ}_s \emptyset$.

It is easy to verify that the outcome of teacher-proposing DA mechanism (Abdulkadiroğlu and Sönmez, 2003) under $(P, \hat{\succ})$ is Gale-Shapley stable.

Next, we show that for some market there does not exist a Gale-Shapley stable and status-quo improving matching. Let $S = \{s, s'\}$, $T = \{t_1, t_2\}$, the status-quo matching be

$$\omega_s = \{t_1\},\ \omega_{s'} = \{t_2\},$$

with quotas $q_s = q_{s'} = 1$, type rankings $\tau(t_1) \rhd_s \tau(t_2) \rhd_s \theta_\emptyset$, and $\tau(t_1) \rhd_{s'} \tau(t_2) \rhd_{s'} \theta_\emptyset$. The preferences of the teachers are

$$s' \ P_{t_1} \ s \ P_{t_1} \ \emptyset,$$

$$s' \ P_{t_2} \ s \ P_{t_2} \ \emptyset,$$

Under this market, unique status-quo improving matching is ω. However, ω is blocked by (t_1, s'). ■

Proof of Proposition 3: Under the current French mechanism, when there are no empty seats, each school fills its capacity and only the status-quo employees are assigned to the schools.

This follows from the fact that teachers in ω_s have the q_s highest priority at school s and they are considering their status-quo school acceptable. Hence, if there is a blocking pair (t, s) and $\tau(t) \ \triangleright_s \ \tau(t')$ and t' is assigned to s, then $t' \in \omega_s$. Teacher-SI stability and status-quo improving property of ω directly follows from the definition.

Next, by slightly modifying the example in the proof of Proposition 2, we show that when there are no empty seats at schools the current French mechanism is not status-quo improving. Consider the example in the proof of Proposition 2 such that teacher t_2 prefers school s most. Then, the French mechanism assigns t_1 and t_2 to s' and s, respectively. This matching is not status-quo improving for school s.

Finally, via an example, we show that when there are empty seats at some school, then there does not exist a teacher-SI stable and status-quo improving matching. Let $S = \{s, s'\}$, $T = \{t_1\}$, $\omega_s = \{t_1\}$, $\omega_{s'} = \emptyset$. Each school has one available seat and t_1 prefers s' to s. The unique status-quo improving matching is ω but it is blocked by (t_1, s'). Hence, it is not teacher-SI stable. ■

Proof of Proposition 4: Substitutes: On the contrary, we suppose there exist $\bar{T} \subseteq T$ and distinct $t, t' \in \bar{T}$ such that $t \in C_s(\bar{T})$ and $t \notin C_s(\bar{T} \setminus \{t'\})$. There exists some other teacher t'' who was assigned to s^k under $C_s(\bar{T} \setminus \{t'\})$, where s^k is t's slot under $C_s(\bar{T})$. Consider the execution of the algorithm to determine $C_s(\bar{T} \setminus \{t'\})$ in step k when t'' is assigned to s^k: as t is not assigned in $C_s(\bar{T} \setminus \{t'\})$, she is still available and is not picked by slot s^k; thus, $t'' \succ_s^k t$. As a consequence, when the algorithm was executed to determine $C_s(\bar{T})$, teacher t'' was already assigned to a slot $s^{k''}$ such that $k'' < k$ so that she was not available when t was assigned s^k.

We will show that such a teacher t'' cannot exist, leading to a contradiction and completing the proof for the substitutes condition.

Claim: There is no teacher $\bar{t}$ such that she is assigned to a slot $s^{\hat{k}}$ in $C_s(\bar{T} \setminus \{t'\})$ and to a slot $s^{\bar{k}}$ in $C_s(\bar{T})$ such that $\bar{k} < \hat{k}$.

Proof of Claim: Suppose to the contrary such a teacher $\bar{t}$ exists. Let $\bar{t}$ be chosen such that $\bar{k}$ is the smallest such index among the indexes of slots filled by such teachers in $C_s(\bar{T})$.

If slot $s^{\bar{k}}$ is unfilled in $C_s(\bar{T} \setminus \{t'\})$, then as $\bar{t}$ is still available when slot $s^{\bar{k}}$ is filled in determining $C_s(\bar{T})$ by the supposition, we should have $\emptyset \succ_s^{\bar{k}} \bar{t}$. But then teacher $\bar{t}$ cannot be assigned to $s^{\bar{k}}$ in $C_s(\bar{T})$.

If a teacher $\hat{t}$ is assigned to $s^{\bar{k}}$ in $C_s(\bar{T} \setminus \{t'\})$, then as $\bar{t}$ is still available when slot $s^{\bar{k}}$ is filled in determining $C_s(\bar{T})$ by the supposition. Therefore, by the choice of $\bar{k}$, teacher $\hat{t}$ is not assigned a slot preceding $s^{\bar{k}}$ in $C_s(\bar{T})$. Therefore, she is available when $s^{\bar{k}}$ is filled in $C_s(\bar{T})$. Yet she is not picked even though $\hat{t} \succ_s^{\bar{k}} \bar{t} \succ_s^{\bar{k}} \emptyset$, a contradiction. Thus, such a teacher $\bar{t}$ cannot exist. ◇

Law of Aggregate Demand: On the contrary, we suppose there exists $\bar{T} \subseteq T$, $t \notin \bar{T}$ and $|C_s(\bar{T})| > |C_s(\bar{T} \cup \{t\})|$. Then, there exists a slot s^k which is filled under $C_s(\bar{T})$ but not under $C_s(\bar{T} \cup \{t\})$. However, due to the above Claim in the proof for the substitutes condition, the teacher who was assigned s^k in $C_s(\bar{T})$ is available when s^k is being filled in $C_s(\bar{T} \cup \{t\})$. Then this slot

cannot be empty in $C_s(\bar{T} \cup \{t\})$ as this teacher is acceptable for the slot, which is a contradiction. We showed that $|C_s(\bar{T})| \leq |C_s(\bar{T} \cup \{t\})|$.

By the repeated application of this argument, we conclude that whenever $\bar{T} \subseteq \hat{T}$, $|C_s(\bar{T})| \leq |C_s(\hat{T})|$. ■

Proof of Theorem 2: Strategy-proofness: It was shown by Hatfield and Milgrom (2005) that whenever the choice rules of schools satisfy the substitutes and law of aggregate demand conditions, the resulting mechanism through DA is strategy-proof for teachers. Since for each school s, choice rule C_s satisfies these conditions and only incomplete information is about the preferences of teachers, SI-DA is strategy-proof.

SI Stability: Suppose Assumption 1 holds. Let SI-DA outcome be μ. We will show that it is status-quo improving first. By our construction of the slot priorities, a teacher t will be accepted by ω_t whenever she applies and she will never be rejected in the further steps. Hence, it is status-quo improving for teachers. Consider the schools. First, we prove the following claim.

Claim: Each school fills all its seats in μ.

Proof of Claim: To see this, notice that no teacher t is assigned to a school s that is less preferred to ω_t in μ. Therefore, all teachers who were employed at the status quo are assigned to some school in μ. Moreover, we claim that exactly $\sum_{s \in S}(q_s - |\omega_s|)$ new teachers are assigned in μ. On the contrary, suppose this claim does not hold. Then, at least one seat of a school s is unfilled in μ and this matching leaves at least one new teacher $t \in N$ unmatched such that she considers all schools with empty seat acceptable and is acceptable at all schools with empty seats at status quo. In determining $C_s(B_s^{K+1})$, where K is the final step of the DA algorithm, if the slot corresponding to this empty seat is one of slots s^k that was unfilled at the status quo, then an unassigned new teacher would have applied to that school and have been assigned to that slot by Assumption 1. Thus, this slot is filled at the status quo.

Then as all employed teachers at status quo are assigned to some school in μ, there exists a teacher $\hat{t} \notin N$ assigned in μ to a slot $\hat{s}^{\hat{k}}$ that was unfilled at the status quo at some school $\hat{s}$.

Since new teacher t is unassigned in μ, she should have applied to all schools with empty seats at status quo (which she considers acceptable by Assumption 1) including $\hat{s}$. Since $\hat{s}$ has an unfilled seat at status quo, by Assumption 1, it considers t acceptable. Moreover, at the slots that are unfilled at the status quo, acceptable new teachers have higher ranking than employed teachers at the status quo by construction of the slot rankings: $t \succ_{\hat{s}}^{\hat{k}} \hat{t}$. Thus, slot $\hat{s}^{\hat{k}}$ should have held t instead of $\hat{t}$, a contradiction.

Hence, all seats are filled in μ. ◇

Since all seats are filled in μ, by our construction of the rankings of the slots, the matching FOSDs the status-quo matching ω. Hence, μ is also status-quo improving for schools.

Next, we will show that there is no blocking pair of μ that is not allowed. By construction of

slot rankings for a school s, for teachers neither in ω_s nor in N, the type ranking of the school is respected in rankings of its slots.

Suppose there exists a blocking teacher-school pair (t, s) of μ, i.e., t prefers s to μ_t and there exists a teacher $\hat{t} \in \mu(s)$ such that $\tau(t) \rhd_s \tau(\hat{t}) \rhd_s \theta_\emptyset$ for some $\hat{t} \in \mu(s)$, as all seats of s are filled in μ.

Then t should have made an offer to s that was rejected in the DA algorithm. Then all teachers assigned to all slots of s have higher ranking in that slot than t. If $\hat{t}$ was assigned in μ to a filled slot at status quo then $\hat{t} \in \omega_s$ and $t \notin \omega_s$ as $\tau(t) \rhd_s \tau(\hat{t})$. If $\hat{t}$ was assigned in μ to an unfilled slot at status quo then $\hat{t} \in N$ and $t \notin N$ as $\tau(t) \rhd_s \tau(\hat{t})$. By definition of SI stability, then (t, s) is an allowed blocking pair of μ. As such a blocking pair (t, s) and such a teacher $\hat{t}$ is arbitrary, μ is SI stable. ■

Proof of Proposition 5: We use the following lemma in our proof.

Lemma 1 *For any $\bar{T} \subseteq T$, $\hat{D}_s(\bar{T}) \subseteq D_s(\bar{T})$.*

Proof: Let s^k and s^ℓ (s^ℓ and s^k) be m^{th} and $(m+1)^{th}$ seats under $\blacktriangleright_s$ ($\hat{\blacktriangleright}_s$), respectively. Since the relative positions of the first $(m-1)$ seats are the same under $\blacktriangleright_s$ and $\hat{\blacktriangleright}_s$, the same teachers are assigned to the first $(m-1)$ seats by D_s and $\hat{D}_s$. Therefore, we consider the same set of teachers for the m^{th} seat under both $\blacktriangleright_s$ and $\hat{\blacktriangleright}_s$. Let $\tilde{T}$ be the set of teachers considered for the m^{th} seat under both $\blacktriangleright_s$ and $\hat{\blacktriangleright}_s$.

Recall that, by our construction, the set of teachers acceptable for seat s^ℓ is a (weak) superset of the teachers acceptable for seat s^k. Hence, if there does not exist an acceptable teacher in $\tilde{T}$ for seat s^ℓ, then there does not exist an acceptable teacher in $\tilde{T}$ for seat s^k. If there is no acceptable teacher in $\tilde{T}$ for seat s^k but there is some acceptable teacher for seat s^ℓ, then that teacher is assigned to s^ℓ under both choice rules. Since the relative positions of the remaining seats are the same under $\blacktriangleright_s$ and $\hat{\blacktriangleright}_s$, we have $D_s(\bar{T}) = \hat{D}_s(\bar{T})$ whenever the set of acceptable teachers in $\tilde{T}$ for either s^k or s^ℓ is empty.

Now suppose there exist acceptable teachers in $\tilde{T}$ for seats s^k and s^ℓ. Let t^k and t^ℓ be the highest ranked teachers for seats s^k and s^ℓ among the ones in $\tilde{T}$, respectively.

If $t^k \neq t^\ell$, then under both choice rules D_s and $\hat{D}_s$ t^k and t^ℓ are assigned to seats s^k and s^ℓ, respectively. Since the relative positions of the remaining seats are the same under $\blacktriangleright_s$ and $\hat{\blacktriangleright}_s$, we have $D_s(\bar{T}) = \hat{D}_s(\bar{T})$.

If $t^k = t^\ell = t'$, then t' is assigned to seats s^k and s^ℓ under choice rules D_s and $\hat{D}_s$, respectively. Next, we consider the teachers in $\tilde{T} \setminus t'$. First notice that, the status-quo employees who have the highest priority among all teachers in T for s^k and s^ℓ cannot be in $\tilde{T} \setminus t'$. This would conflict with the fact that t' has the highest priority for both seats among the teachers in $\tilde{T}$. Then, there is one teacher in $\tilde{T} \setminus t'$ who has highest priority for both s^k and s^ℓ. We denote such teacher with t''. If t'' is acceptable for both s^k and s^ℓ, then t'' is assigned to s^ℓ and s^k under both choice rules D_s and $\hat{D}_s$, respectively. If t'' is unacceptable for both s^k and s^ℓ, then no teacher is assigned to s^ℓ and s^k

under both choice rules D_s and $\hat{D}_s$, respectively. Under both cases, since the relative positions of the remaining seats are the same under $\blacktriangleright_s$ and $\hat{\blacktriangleright}_s$, we have $D_s(\bar{T}) = \hat{D}_s(\bar{T})$. We are left with one remaining case: t'' is acceptable for s^ℓ but not for s^k. Then, t'' is assigned to s^ℓ under D_s but s^k is not filled under $\hat{D}_s$. Then, when we consider the remaining seats under both choice rules and the remaining teachers, we can treat the assignment is done via DA mechanism where each teacher ranks the seats according to their positions under $\blacktriangleright_s$ and $\hat{\blacktriangleright}_s$. Since DA is population monotonic and individually rational, any teacher assigned under $\hat{D}_s$ is assigned under D_s. But the other way is not always true. ■

Now, consider a sequential application of DA algorithm in which we allow teachers one by one as long as they do not apply to school s (see Dur et al., 2018 for details). Then, eventually, we will have a set of teachers $\bar{T}$ who have been rejected from their all choices better than s. Once all teachers apply to s, Lemma 1 implies that the rejected teachers under D is a subset of the rejected teachers under $\hat{D}$. Then, we allow only the rejected teachers under both choice rules from s and all other teachers who have not applied to s to apply one by one. Following this procedure will give us matching μ assignment for all schools except s under both choice rules. Moreover, DA algorithm terminates under choice rule D. However, by Lemma 1, there might be teachers rejected from s and have not applied to their next best choice under $\hat{D}$. That is, we may observe some teachers to be rejected from their assignment under μ. Hence, no teacher t prefers $\hat{\mu}_t$ to μ_t. ■

Appendix B Other Applications

In this appendix, we explain other applications of our mechanisms in more detail. We give three concrete applications.

Intra-District School Choice after A Status-quo Assignment: In the US, Austin Independent School District (AISD) of Texas assigns students to the schools through an address based matching procedure.[56,57] Unfortunately, address based assignment ends up with segregated schools.[58] In order to eliminate the segregation and fill the empty seats at the under-demanded schools, AISD runs a transfer procedure in which a student who is in relative demographic majority in her assigned school can apply to the schools in which she belongs to the minority demographic group. Moreover, new students who arrive at the district after the matching procedure is run can also participate in this transfer procedure.[59]

In addition to the transfer programs to achieve racial diversity at schools, many school districts,

[56]Although we mentioned earlier possible application of our approach for inter-district-school choice (cf. Hafalir et al., 2019), a direct application of our methodology exists in intra-district school choice, which we discuss here.

[57]There are 128 school programs in AISD. In 2020-2021 school year, the total enrolment in AISD is more than 75,000 (AISD, 2021).

[58]In 2019, student body at 15% and 63% of the elementary schools were composed of more than 60% white and hispanic-black students, respectively.

[59]Majority minority transfer program is used in many school district in US including Huntsville, AL (HCS, 2020), Suffolk, VA (SPSK12, 2021), and Florence, SC (F1S, 2019).

including Davenport, IA (DCS, 2019), and Seminole, FL (SCPS, 2021), run transfer programs to achieve diversity in terms of socioeconomic status (SES) of the students. Diversity transfer programs based on SES are also suggested to the school district by The United States Department of Education and the United States Department of Justice (ED, 2011).

Job Rotation: Job rotation is defined as the horizontal movement of employees among different positions in a company. It is a well-established and commonly practiced human resource management program in many settings. It benefits companies through employee enrichment and success of developing future managers as well as decreased worker turn-over due to increased job satisfaction of the participants (Cheraskin and Campion, 1996).

Job rotation programs can also be used as a mean to obtain certain distributional goals a company such as achieving gender balance across different departments of the company and retention of female employees and increasing the development of more female leaders through rotation programs.[60]

Other Civil Services: There are other centralized matching procedures for civil servants from different professions. For example, police officers are assigned to neighborhoods by centralized procedures in several US cities such as Chicago; doctors are assigned to government hospitals in some countries such as Turkey. Our procedures can be used in these domains as well to achieve different distributional objectives.

Appendix C Examples

In Example 6, we show that, in the same setting as Combe et al. (2020), SI-CC is not equivalent to the teacher optimal selection of TO-BE they propose.[61]

Example 6 Let $S = \{s_1, s_2\}$, $T = \{t_1, t_2, t_2'\}$, $\omega_{s_1} = \{t_1\}$, $\omega_{s_2} = \{t_2, t_2'\}$, $q_{s_1} = 1$ and $q_{s_2} = 2$. Let $\tau(t_1) = \theta_1$, $\tau(t_2) = \theta_2$, $\tau(t_2') = \theta_2'$. Finally, the preferences of the schools over types are: $\theta_2 \rhd_{s_1} \theta_2' \rhd_{s_1} \theta_1$ and $\theta_1 \rhd_{s_2} \theta_2 \rhd_{s_2} \theta_2'$. One can check that the matching returned by the teacher optimal selection of TO-BE matches[62] t_1 to s_1 and t_2 to s_1 while SI-CC matches t_1 to s_1 but t_2' to s_1.

In Example 7, we show that M-convexity of the policy goals is not sufficient anymore to ensure existence of SI teacher optimal and strategy-proof mechanism.

Example 7 *Let $S = \{s_1, s_2\}$, $T = N = \{t_1, t_2\}$, $\omega_{s_1} = \omega_{s_2} = \emptyset$, $q_{s_1} = q_{s_2} = 1$ and $\tau(s_1) = \tau(s_2) = \theta$. Suppose the constraint over the distribution of teachers require that a teacher of type θ is assigned to s_1. This is a constraint fixing a floor which is known to be M-convex. Suppose both teachers rank s_2 ahead of s_1 and s_1 ahead of $\emptyset$. If teachers report their true preferences, then there*

[60] Observe that companies use professionally designed centralized matching software for job rotations, for example see `https://www.tws-partners.com/corporate-functions/hr/`.

[61] Combe et al. (2020) already noted that their class of TO-BE mechanisms did not entirely defined the class of statu-quo improving, strategy-proof and two-sided efficient mechanisms. However, they did not investigate it further. Our example suggests that other non-trivial mechanisms, such as SI-CC, exist outside their class.

[62] One can easily check that this example is well defined in their setting. Just set the preferences of the schools over the teachers being equivalent to the schools' ranking over their corresponding types.

will be a teacher assigned to s_1 under any SI teacher optimal (or Pareto efficient) matching. Then, the teacher assigned to s_1, say t_1, has an incentive to claim that s_1 is unacceptable to her. Indeed, any SI teacher optimal mechanism must then assign t_2 to s_1 and t_1 to s_2.

In Example 8, we show that in some market there does not exist an SI teacher optimal and SI stable matching.

Example 8 *Let $S = \{s_1, s_2, s_3\}$, $T = \{t_1, t_2, t_3\}$, $\omega_{s_1} = \{t_1\}$, $\omega_{s_2} = \{t_2\}$, $\omega_{s_3} = \{t_3\}$ and $q_{s_1} = q_{s_2} = q_{s_3} = 1$. Let $\tau(t_3) \rhd_{s_1} \tau(t_2) \rhd_{s_1} \tau(t_1) \rhd_{s_1} \theta_\emptyset$, $\tau(t_3) \rhd_{s_2} \tau(t_1) \rhd_{s_2} \tau(t_2) \rhd_{s_2} \theta_\emptyset$ and $\tau(t_1) \rhd_{s_3} \tau(t_2) \rhd_{s_2} \tau(t_3) \rhd_{s_3} \theta_\emptyset$, $s_2\ P_{t_1}\ s_1 P_{t_1}\ s_3 P_{t_1}\ \emptyset$, $s_1\ P_{t_2}\ s_2 P_{t_2}\ s_3 P_{t_2}\ \emptyset$, and $s_1\ P_{t_3}\ s_2 P_{t_3}\ s_3 P_{t_3}\ \emptyset$.*

Let $\mu_{s_1} = t_2$, $\mu_{s_2} = t_1$ and $\mu_{s_3} = t_3$. Notice that, μ is SI teacher optimal. Under this market, ω is the unique SI stable matching and μ Pareto dominates ω for teachers.

In Examples 9 - 11, we show that if we exclude the first condition from the definition of SI stability, then for some market there does not exist an SI stable matching.

Example 9 *Let $S = \{s_1, s_2\}$, $T = \{t_1, t_2\}$, $\omega_{s_1} = \{t_1\}$, $\omega_{s_2} = \{t_2\}$ and $q_{s_1} = q_{s_2} = 1$. Let $\tau(t_2) \rhd_s \tau(t_1) \rhd_s \theta_\emptyset$ for both $s \in S$ and $s_1\ P_t\ s_2\ P_t\ \emptyset$ for all $t \in T$.*

In this market, the unique status-quo improving matching is ω. However, it is blocked by (t_2, s_1). Hence, any SI stable matching does not exist in this market when the first condition is excluded.

Next, via example we show that if we exclude the second condition from the definition of SI stability, then for some market there does not exist an SI stable matching.

Example 10 *Let $S = \{s_1, s_2\}$, $T = \{t_1, t_2\}$, $\omega_{s_1} = \{t_1\}$, $\omega_{s_2} = \emptyset$ and $q_{s_1} = q_{s_2} = 1$. Let $\tau(t_1) \rhd_s \tau(t_2) \rhd_s \theta_\emptyset$ for both $s \in S$, $s_2\ P_{t_1}\ s_1\ P_{t_1}\ \emptyset$ and $s_2\ P_{t_2}\ \emptyset\ P_{t_2}\ s_1$.*

Under this market, in any status-quo improving matching t_1 is assigned to s_1. However, any such matching is blocked by (t_1, s_2). Hence, any SI stable matching does not exist in this market when the second condition is excluded.

One can wonder if there exists a strategy-proof mechanism which selects a matching which is stable when one of the conditions is excluded whenever such a matching exists and selects a stable matching under both conditions, otherwise. In the following example, we show that such a mechanism does not exist.

Example 11 *Let $S = \{s_1, s_2\}$, $T = \{t_1, t_2\}$, $\omega_{s_1} = \{t_1\}$, $\omega_{s_2} = \emptyset$ and $q_{s_1} = q_{s_2} = 1$. Let $\tau(t_2) \rhd_{s_1} \tau(t_1) \rhd_{s_1} \theta_\emptyset$, $\tau(t_1) \rhd_{s_2} \tau(t_2) \rhd_{s_2} \theta_\emptyset$ for both $s \in S$, $s_2\ P_{t_1}\ s_1\ P_{t_1}\ \emptyset$ and $s_2\ P_{t_2}\ s_1\ P_{t_2}\ \emptyset$.*

Under this market, there exists a unique stable matching when condition 2 is excluded: t_1 is assigned to s_2 and t_2 is assigned to s_1. Hence, it will be selected.

Suppose teacher t_2 reports $s_2 P'_{t_2} \emptyset P'_{t_2} s_1$. Then, we have the same problem as in Example 10. Since there does not exists a stable matching where condition 2 is excluded, we consider stable matchings

when condition 2 is included. There exists a unique stable matching in which t_1 is assigned to s_1 and t_2 is assigned to s_2. Hence, t_2 is better off by manipulating.

In Examples 12 -14, we relax the conditions of Assumption 1 one by one and show that the existence of an SI stable outcome may not be guaranteed.

Example 12 *We consider a market in which there does not exist $N' \subseteq N$ such that $|N'| \geq \sum_{s\in S}(q_s - |\omega_s|)$.*

Let $S = \{s, s'\}$, $T = \{t_1\}$, the status-quo matching be

$$\omega_s = \{t_1\},\ \omega_{s'} = \emptyset,$$

$q_s = q_{s'} = 1$, and teacher t_1 is acceptable for both schools. The preferences of the teacher t_1 is

$$s'\ P_{t_1}\ s\ P_{t_1}\ \emptyset.$$

In this market, ω_s is the unique status-quo improving matching but it is blocked by (t_1, s').

Example 13 *We consider a market in which there exists $N' \subseteq N$ such that $|N'| \geq \sum_{s\in S}(q_s - |\omega_s|)$ and each teacher in N' is acceptable for all schools with excess capacity but not all teacher in N' consider all schools with excess capacity acceptable.*

Let $S = \{s, s'\}$, $T = \{t_1, t_2\}$, the status-quo matching be

$$\omega_s = \{t_1\},\ \omega_{s'} = \emptyset,$$

$q_s = q_{s'} = 1$, $\tau(t_1) \rhd_s \tau(t_2) \rhd_s \theta_\emptyset$ and $\tau(t_1) \rhd_{s'} \tau(t_2) \rhd_{s'} \theta_\emptyset$. The preferences of the teachers are

$$s'\ P_{t_1}\ s\ P_{t_1}\ \emptyset,$$

$$s\ P_{t_2}\ \emptyset\ P_{t_2}\ s'.$$

In this market, ω is the unique status-quo improving matching but it is blocked by (t_1, s').

Example 14 *We consider a market in which there exists $N' \subseteq N$ such that $|N'| \geq \sum_{s\in S}(q_s - |\omega_s|)$ and all teacher in N' consider all schools with excess capacity acceptable but some teacher in N' is not acceptable for some school with excess capacity.*

Let $S = \{s, s'\}$, $T = \{t_1, t_2\}$, the status-quo matching be

$$\omega_s = \{t_1\},\ \omega_{s'} = \emptyset,$$

$q_s = q_{s'} = 1$, $\tau(t_1) \rhd_s \tau(t_2) \rhd_s \theta_\emptyset$ and $\tau(t_1) \rhd_{s'} \theta_\emptyset \rhd_{s'} \tau(t_2)$. The preferences of the teachers are

$$s'\ P_{t_1}\ s\ P_{t_1}\ \emptyset,$$

$$s \; P_{t_2} \; s' \; P_{t_2} \; \emptyset.$$

In this market, ω_s is the unique status-quo improving matching but it is blocked by (t_1, s').

Appendix D Extensions

D.1 Responsive Preferences and Impossibility

In this section, we extend the school rankings. Instead of FOSD relation, we assume schools ranking over the types of teachers are responsive.

Each school s has strict ranking over the types and no type option denoted by $\theta_\emptyset$ denoted with $\rhd_s$. For school s, type θ teachers are acceptable if and only if $\theta \rhd_s \emptyset$. Given $\rhd_s$, the preference order of school s over $T \cup \{\emptyset\}$ is given as:

- $\tau(t) \rhd_s \tau(t')$ if and only if $t \succ_s t'$;
- $\tau(t) = \tau(t')$ if and only if $t \sim_s t'$;
- $\tau(t) \rhd_s \theta_\emptyset$ if and only if $t \succ_s \emptyset$.

For any $|\bar{T}| < q_s$ responsiveness implies that for any $t, t' \in T \setminus \bar{T}$

- $\bar{T} \cup \{t\} \succ_s \bar{T}$ if and only if $t \succ_s \emptyset$;
- $\bar{T} \cup \{t\} \succ_s \bar{T} \cup \{t'\}$ if and only if $t \succ_s t'$.

Note that, responsive preferences is more general than FOSD. In particular, if μ_s first-order stochastically dominates matching ω_s, then $\mu_s \succsim_s \omega_s$. However, the other way may not be true. We illustrate this in the following example.

Example 15 *Let $\omega_s = \{t_1, t_2, t_3, t_4\}$ such that $\tau(t_1) \rhd_s \tau(t_2) = \tau(t_3) \rhd_s \tau(t_4)$. Consider the following matching $\mu_s = \{t_1, t'_1, t_4, t'_4\}$ such that $\tau(t_1) = \tau(t'_1) \rhd_s \tau(t_4) = \tau(t'_4)$. Matching μ_s does not FOSD ω_s. However, it is possible that $\mu_s \succ_s \omega_s$.*

The following example shows that, with responsive preferences, there is no mechanism that is SI teacher optimal and strategy-proof.

Example 16 *There are 6 teachers, $T = \{t_1, t'_1, t_2, t'_2, t, t'\}$, and 4 schools, $S = \{s_1, s_2, s, s'\}$. Let $\omega_{s_1} = \{t_1, t'_1\}$, $\omega_{s_2} = \{t_2, t'_2\}$, $\omega_s = \{t\}$ and $\omega_{s'} = \{t'\}$. Schools s_1 and s_2's ranking over teacher types are:*

$$\tau(t) \rhd_{s_1} \tau(t_1) \rhd_{s_1} \tau(t'_1) \rhd_{s_1} \tau(t')$$
$$\tau(t') \rhd_{s_2} \tau(t_2) \rhd_{s_2} \tau(t'_2) \rhd_{s_2} \tau(t).$$

Moreover, we assume that $\{t, t'\} \succ_{s_k} \{t_k, t'_k\}$ for $k = 1, 2$. Notice that, this relation is consistent

with responsive orders. Preferences of the teachers are:

$$s_2P_ts_1P_tsP_t\emptyset$$
$$s_1P_{t'}s_2P_{t'}s'P_{t'}\emptyset$$
$$sP_{t_1}s_1P_{t_1}\emptyset$$
$$s'P_{t'_1}s_1P_{t'_1}\emptyset$$
$$sP_{t_2}s_2P_{t_1}\emptyset$$
$$s'P_{t'_2}s_2P_{t'_2}\emptyset$$

First note that under any status-quo improving matching, if t is assigned to her first ranked school s_2, then t' must also be assigned to s_2. Indeed, let μ be a status-quo improving matching such that $\mu_t = s_2$. Since $\{t, t'\} \succ_{s_2} \{t_2, t'_2\} \succ_{s_2} \{t_2, t\}, \{t'_2, t\}$, status-quo improvement implies that $\mu_{t'} = s_2$. With a similar argument, if $\mu_{t'} = s_1$, then $\mu_t = s_1$. So it implies that there are only three possible SI teacher optimal matchings:

$$\mu^1 := \begin{pmatrix} t & t' & t_1 & t'_1 & t_2 & t'_2 \\ s_1 & s_1 & s & s' & s_2 & s_2 \end{pmatrix}$$
$$\mu^2 := \begin{pmatrix} t & t' & t_1 & t'_1 & t_2 & t'_2 \\ s_2 & s_2 & s_1 & s_1 & s & s' \end{pmatrix}$$
$$\mu^3 := \begin{pmatrix} t & t' & t_1 & t'_1 & t_2 & t'_2 \\ s_1 & s_2 & s & s_1 & s_2 & s' \end{pmatrix}$$

Let φ be an SI teacher optimal mechanism. Assume that $\varphi(P) = \mu^1$. In that case, let $P'_t : s_2P'_tsP'_t\emptyset$. Under (P'_t, P_{-t}), the only SI teacher optimal matching is μ^2 so that $\varphi(P'_t, P_{-t}) = \mu^2$ and the manipulation of t is successful. If $\varphi(P) = \mu^2$, then t' can report $P'_{t'} : s_1P'_{t'}s'P'_{t'}\emptyset$ so that the only SI teacher optimal matching under $(P_{t'}, P_{-t'})$ is μ^1 and $\varphi(P_{t'}, P_{-t'}) = \mu^1$, a successful manipulation for t'. If $\varphi(P) = \mu^3$ then t or t' can manipulate in reporting the same profile as before. We conclude that φ cannot be strategy-proof.

D.2 Immunity to Type Ranking Manipulation

In this section, we investigate whether there exists a status-quo improving, strategy-proof and efficient mechanism which is immune to possible type ranking manipulations. As we explained in our model, we assume schools' preferences, and therefore their type rankings, are commonly known, specifically by policy makers. However, policy makers may choose to report some school s's type ranking differently to the mechanism in order to improve its assignment. We say a mechanism ϕ is **immune to type ranking manipulation** if for any P and $\triangleright$ there does not exist a school s and a type ranking $\triangleright'_s$ such that

$$\phi_s(\succsim', P) \ \succ_s \ \phi_s(\succsim, P)$$

where $\succsim$ and $\succsim'$ are preferences induced by type ranking profiles $\triangleright$ and $(\triangleright'_s, \triangleright_{-s})$, respectively. If a mechanism is not immune to type ranking manipulation, then we say it is **vulnerable to type ranking manipulation**.

We first show that there does not exist an SI teacher optimal and strategy-proof mechanism which is immune to type ranking manipulation.

Proposition 7 *Any SI teacher optimal and strategy-proof mechanism is vulnerable to type ranking manipulation.*

Proof: We prove this result by means of an example. On the contrary, suppose there exists an SI teacher optimal and strategy-proof mechanism which is immune to type ranking manipulation. Let ϕ be that mechanism. Let $S = \{s, s', s''\}$, $T = \{t_1, t_2, t_3\}$, the status-quo matching be

$$\omega_s = \{t_1\},\ \omega_{s'} = \{t_2\},\ \omega_{s''} = \{t_3\},$$

and $q_s = q'_s = q''_s = 1$. Let $\tau(t_2) \triangleright_s \tau(t_3) \triangleright_s \tau(t_1)$, $\tau(t_1) \triangleright_{s'} \tau(t_2)$, and $\tau(t_1) \triangleright_{s''} \tau(t_3)$. Let $\succsim$ be the school proference profile which is induced by $\triangleright$. The preferences of the teachers are

$$s'' \ P_{t_1} \ s' \ P_{t_1} \ s$$

$$s \ P_{t_2} \ s' \ P_{t_2} \ s''$$

$$s \ P_{t_3} \ s'' \ P_{t_3} \ s'.$$

In this market, there exist two SI teacher optimal matchings:

$$\mu_t = s',\ \mu_{t'} = s,\ \mu_{t''} = s''$$

$$\nu_t = s'',\ \nu_{t'} = s',\ \nu_{t''} = s.$$

Suppose $\phi(\succsim, P) = \mu$. Let $\tau(t_2) \triangleright'_s \tau(t_1) \triangleright'_s \tau(t_3)$ and $\succsim'$ be the school preference profile induced by $(\triangleright'_s, \triangleright_{-s})$. Then, under market $(\succsim', P)$ ν is the unique SI teacher optimal matching.

Suppose $\phi(\succsim, P) = \nu$. Let $s'' \ P'_{t_1} \ s \ P'_{t_1} \ s'$. Then, under market $(\succsim, P'_{t_1}, P_{-t_1})$ μ is the unique SI teacher optimal matching. ■

Notice that, we prove Proposition 7 by using a market in which there are at least three types. In many applications, agents are characterized based on two types based on race or gender. In the following proposition, we show that when there are only two types, SI-CC is immune to type ranking manipulation.[63]

[63] If there are at least two new teachers and a school with empty seat, then by ranking one type as unacceptable a school's assignment can be improved under any SI teacher optimal mechanism.

Proposition 8 *When* $|\Theta| = 2$ *and* $|\omega_s| = q_s$ *for all* $s \in S$*, SI-CC is immune to type ranking manipulation.*

Proof: On the contrary, suppose there exists a problem $(\succsim, P)$ such that school s can be better off when its type ranking is changed. Let $\triangleright$ induce $\succsim$ and $\triangleright'_s$ be type ranking resulting into improvement for school s. Let $\Theta = \{\theta_1, \theta_2\}$. Without loss of generality, suppose $\theta_1 \triangleright_s \theta_2$. Since teachers in ω_s are with acceptable types, $\theta_1 \triangleright_s \theta_\emptyset$. We consider the following cases.

Case 1: $\theta_2 \triangleright_s \theta_\emptyset$. Then, SI-CC weakly increases the number of θ_1 teachers compared to the one under ω_s. Moreover, all seats will be filled. Under any other type ranking, the number of assigned θ_1 type teachers is at most $|\omega_s^{\theta_1}|$.

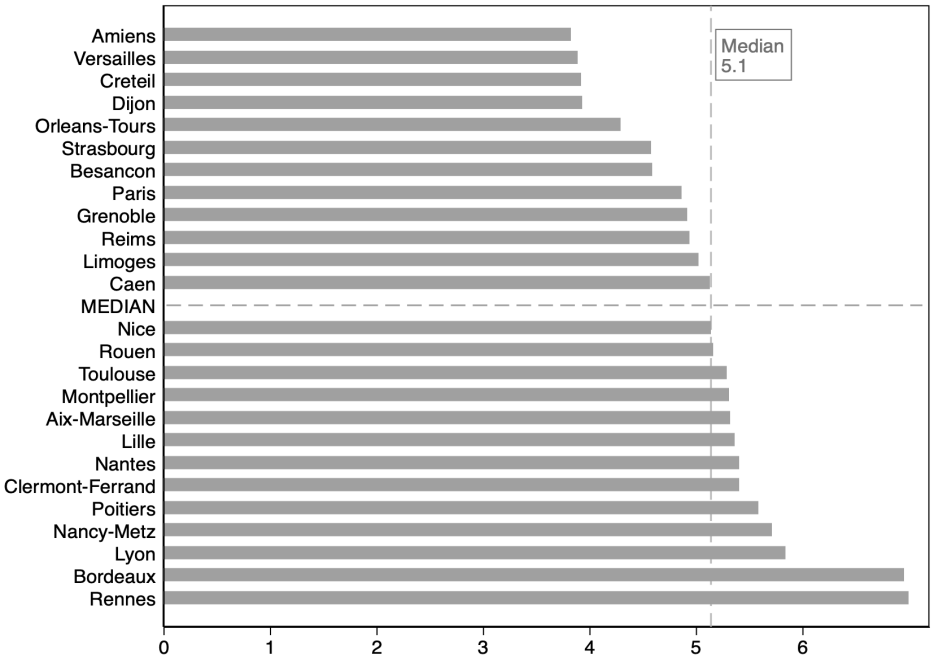
Case 2: $\theta_\emptyset \triangleright_s \theta_2$. Then, under SI-CC all assigned teachers to s are with type θ_1.

In either case, we cannot improve school s by changing its type ranking. ■

UU: Comment for all paper: Instead of "initial", we should use "status-quo"

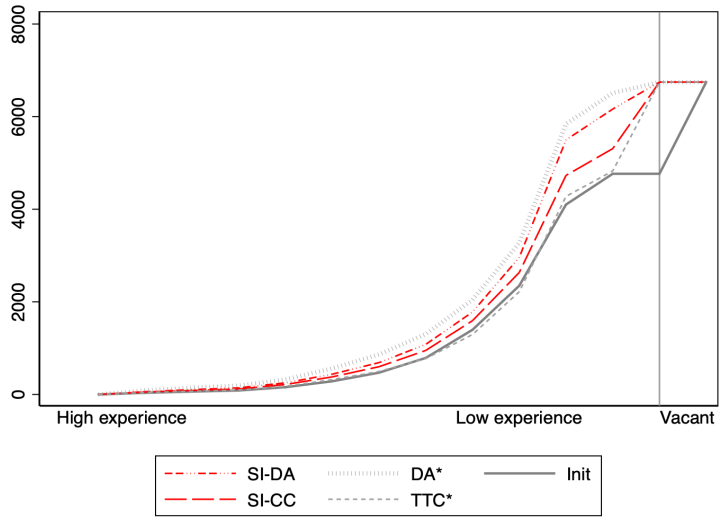
Appendix E Additional Figures and Tables

Figure A.1: Average Teacher Experience Types at Status Quo



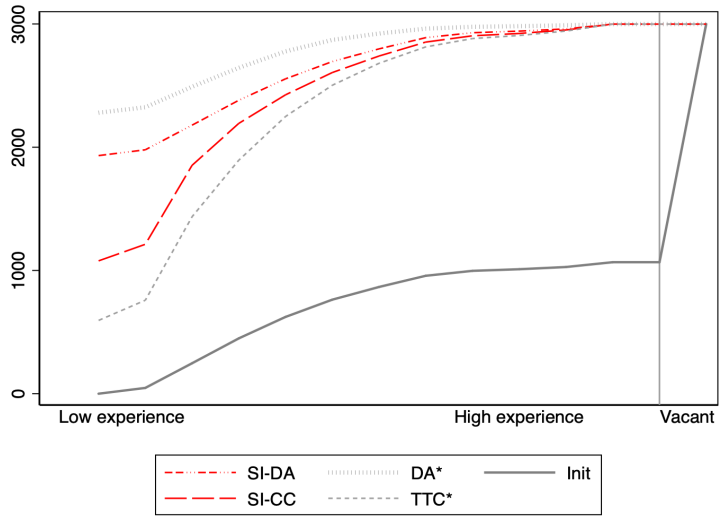
Notes: This figure shows the average teacher type at status quo (lower types correspond to lower experience levels). The *younger regions* are listed above the median and the *older regions* are listed below the median.

Figure A.2: Cumulative Distribution of Teacher Experience Types in the *Younger Regions*



Notes: This figure shows the cumulative distribution of teacher experience types in younger regions of France. These are the regions whose average teacher type is strictly lower than the median of average teacher type distribution at status quo. The horizontal axis reports the 12 types of teachers, ordered from the most experienced to the least experienced. The 13^{th} type corresponds to the vacant positions. The mechanisms that respect status-quo improvement are plotted in red. Those that do not are in grey. The thick black line (Init) corresponds to the cumulative distribution of teacher types at the status-quo matching.

Figure A.3: Cumulative Distribution of Teacher Experience Types in the *Older Regions*



Notes: This figure shows the cumulative distribution of teacher experience types in older regions of France. These are the regions whose average teacher type is higher than the median of average teacher type distribution at status quo. The horizontal axis reports the 12 types of teachers, ordered from the least experienced to the most experienced. The 13^{th} type corresponds to the vacant positions. The mechanisms that respect status-quo improvement are plotted in red. Those that do not are in grey. The thick black line (Init) corresponds to the cumulative distribution of teacher types at the status-quo matching.

Table A.1: Number of teachers and vacant positions

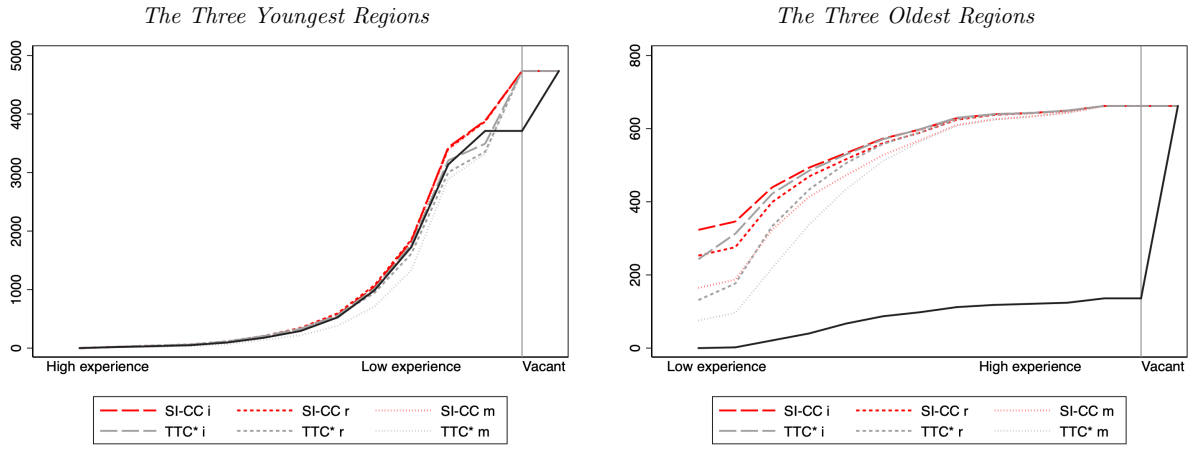
Subjects	All teachers (1)	New teachers (2)	Tenured teachers (3)	Vacant positions (4)
All subjects	10,460	4,627	5,833	3,912
Sports	2,066	568	1,498	475
French	1,645	786	859	663
Math	1,563	958	605	824
English	1,374	746	628	640
History-Geography	1,230	657	573	562
Spanish	999	316	683	248
Physics-Chemistry	837	310	527	254
Biology	746	286	460	246

Table A.2: Statistics on regions

Regions	Ratio: # of tenured teachers asking to enter / exit the region (1)	% of teachers asking for a new assignment coming from each region (2)	Ratio: # of teachers aged more than 50 / less than 30 (3)	% of students enrolled in priority education (4)	% of students whose reference parent has no diploma (5)	% of students obtaining their baccalaureate (6)
Rennes	15.55	0.5	8.10	7.9	14.18	91.54
Bordeaux	8.95	0.8	6.56	14.6	19.22	86.25
Toulouse	6.56	1.5	5.29	8.9	17.38	88.57
Paris	3.02	2.8	6.90	25.5	21.54	85.48
Aix-Marseille	2.54	1.9	5.08	30.1	27.20	81.77
Grenoble	1.74	2.3	3.91	16.5	19.80	88.17
Amiens	0.08	6.2	1.89	23.9	27.71	82.41
Créteil	0.03	22.7	1.14	35.5	31.62	83.94
Versailles	0.05	25.7	1.62	24.9	21.88	87.92

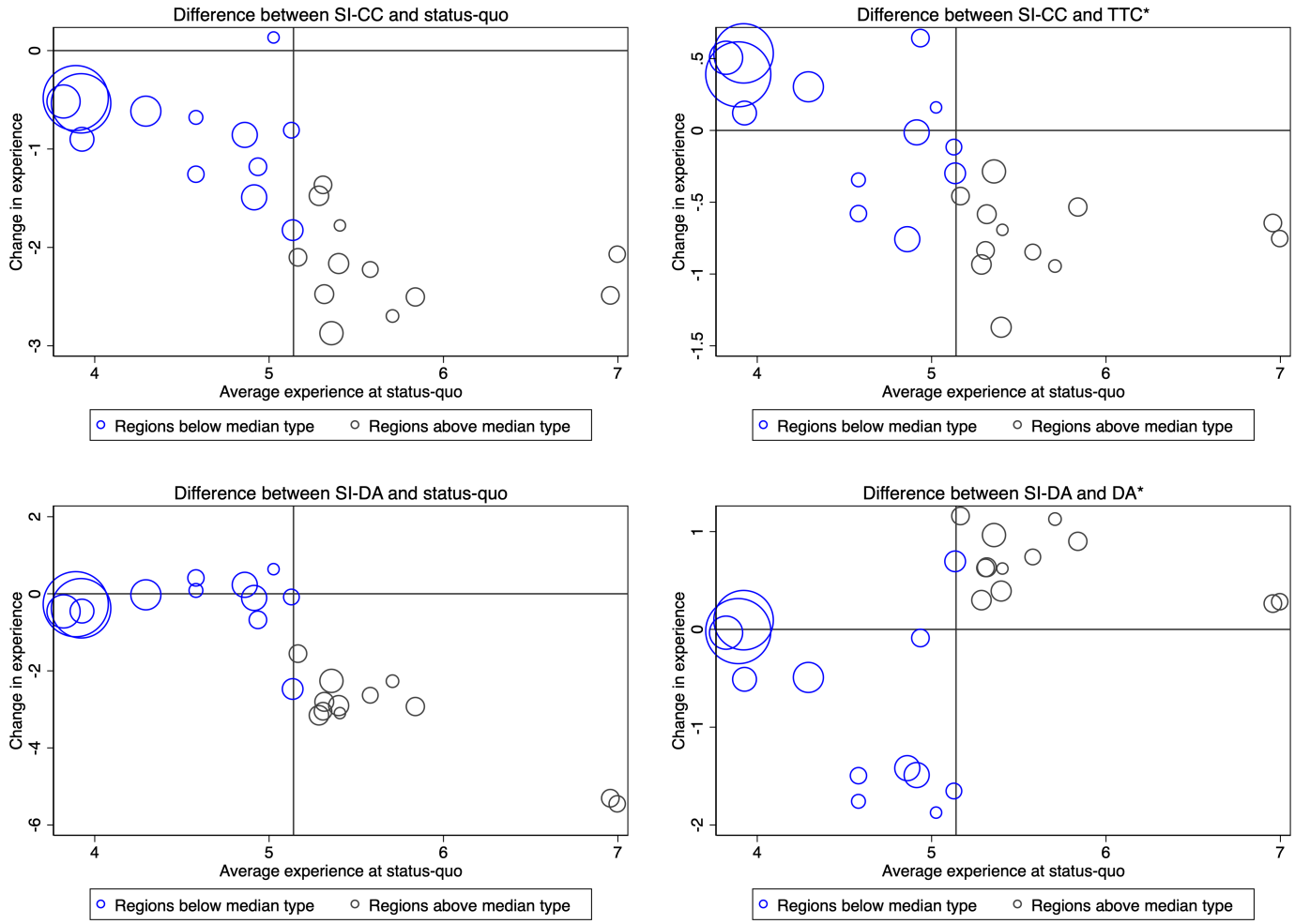
Notes: This table reports descriptive statistics for the three most attractive (Rennes, Bordeaux, and Toulouse), the three least attractive regions (Amiens, Créteil, and Versailles), and three intermediate regions (Paris, Aix-Marseille, and Grenoble). Attractiveness is measured by the ratio of the number of tenured teachers asking to enter a region to the number of teachers asking to leave the region (reported in column 2). All statistics reported in this table come from the following reference: Direction de l'Evaluation de la Prospective et de la Performance (2014). UU: WHAT DOES WANT TO ENTER,LEAVE MEAN? EVER RANK IN THE ROL for ENTER, APPLY FOR REASSIGNMENT for LEAVE? DEFINE HERE MORE FORMALLY

Figure A.4: Cumulative Distribution of Teacher Experience Types



Notes: This Figure shows the cumulative distribution of teacher experience types. The left panel reports the distribution in the three youngest regions of France (Amiens, Versailles, and Créteil), and the right panel the distribution in the three oldest regions of France (Rennes, Bordeaux, and Lyon). The horizontal axis reports the 12 experience types of teachers, ordered from most experienced to least experienced (left panel) and from least experienced to most experienced (right panel). The 13^{th} type corresponds to the vacant positions. The mechanisms that respect status-quo improvement are plotted in red. Those that do not are in grey. The thick black line (Init) corresponds to the cumulative distribution of teacher types at the status-quo matching.

Figure A.5: Change in Average Experience Type for All Teachers Across Regions



Notes: This figure shows the difference in the average experience of teachers (among both tenured and newcomers) between the matching obtained with SI-CC against the status-quo matching (top left figure) and its benchmark TTC* (top right figure). It reports the same difference between the matching obtained with SI-DA against the status-quo matching (bottom left figure) and its benchmark DA* (bottom right figure). Each observation represents a French region. Circle size reflects region size. Regions are ordered (on the horizontal axis) by average experience of their teachers at status quo (the status-quo matching). The vertical line represents the median type. All regions on the left have an average type that is strictly below the median. This is the group of regions we identified as inexperienced regions. All regions on right of the vertical line are regions whose average type is above the median. In regions above the horizontal line, teachers average experience post reassignment is larger than at the status-quo matching. The name of the three least experienced regions (Créteil, Versailles, and Amiens) and most experienced regions (Rennes, Bordeaux, and Lyon) are reported on the graphs.

Appendix F Teacher Preference Estimations

F.1 Variables Used for Teacher Preference Estimations

This Appendix describes the variables we use for teacher preference estimations.

We use the following regions' characteristics:

- Share of students classified as disadvantaged (labeled as "% disadv stud").
- Share of students living in an urban area (labeled as "% stud urban").
- Share of students who attend a school classified as "priority education" (labeled as "% stud in priority educ"). Priority education is a label given to the most disadvantaged schools in France.
- Share of students who attend a private school (labeled as "% stud in private school").
- Share of teachers who are younger than 30 (labeled as "% teachers younger than 30")
- Region is in South of France (labeled as "Region in South of France"). The following 5 regions are classified as being in the South of France: Aix-Marseille, Bordeaux, Montpellier, Toulouse, and Nice.

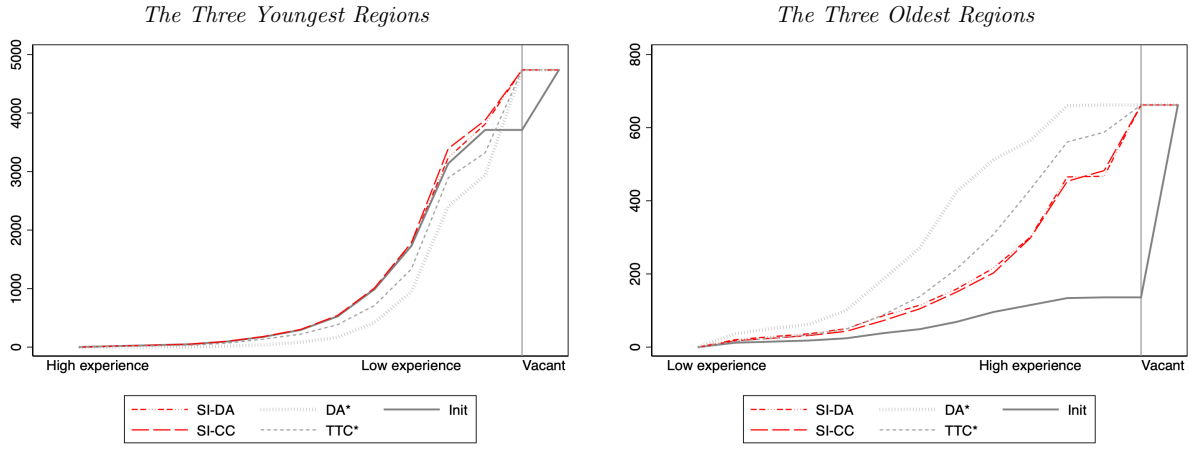
We use the following teachers characteristics:

- Current region of the teacher (labeled as "Current region"). This is the region a teacher is initially assigned to.
- Region where a teacher was born (labeled as "Birth region").
- Distance between the region ranked and the current region of a teacher (labeled as "Distance current region").
- Number of years of teaching experience (labeled as "Teach exp").
- Squared number of years of teaching experience (labeled as "Teach exp sq").
- Teacher's current region is Créteil or Versailles, which are the two least attractive regions (labeled as "Teach from CV"). The attractiveness of a region is measured by the ratio of the number of teachers who rank the region divided by the number of teachers who ask to leave the region.
- Teacher is married (labeled as "Married").
- Teacher has spent at least 5 years in a school labelled as priority education (labeled as "Teach in disadv sch").
- Teacher has an advanced teaching qualification (labeled as "Advanced qualif").

Appendix G Empirical Results when All Regions Have the Same Preferences over Teachers

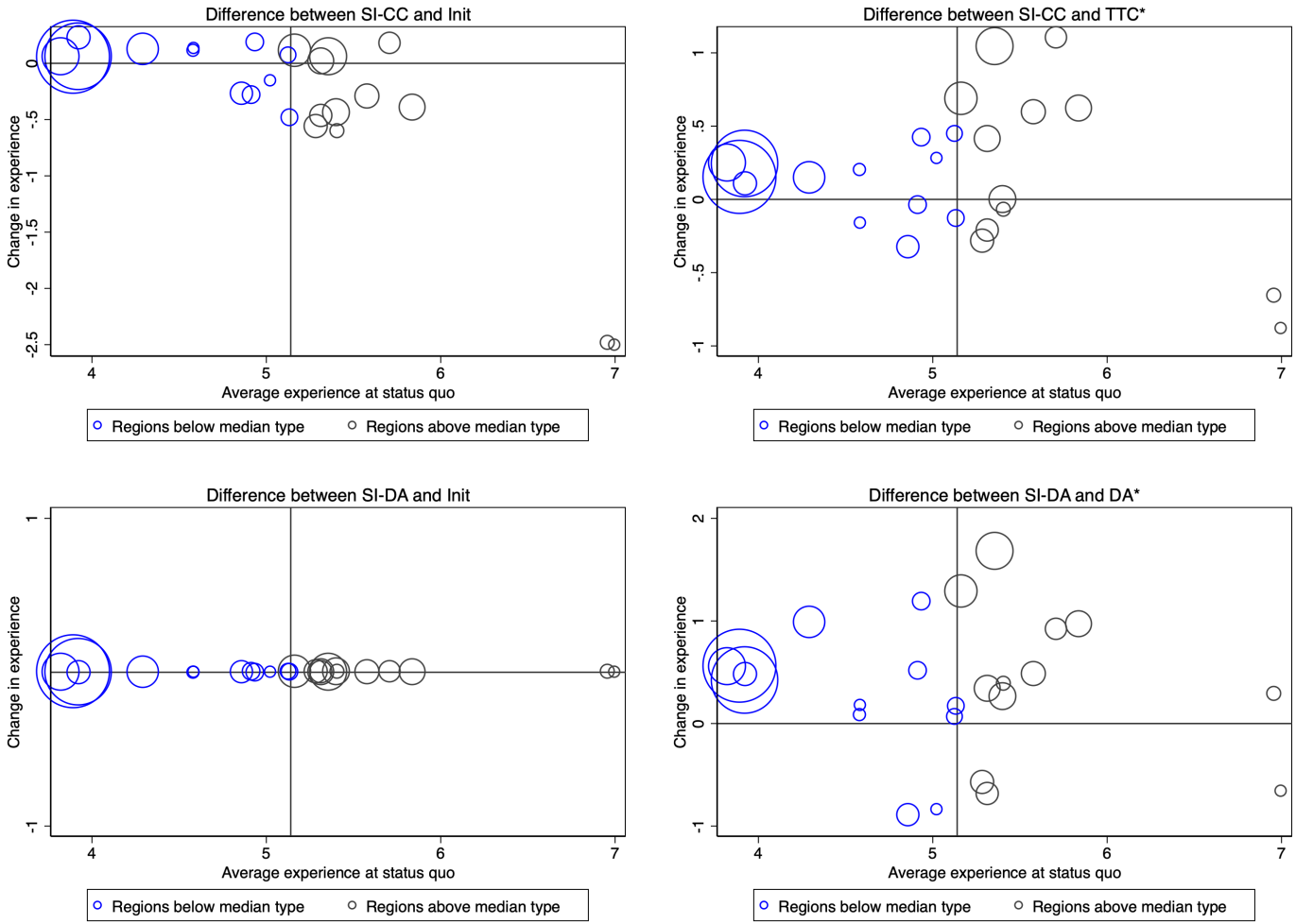
This section reports results in which all regions have the same preferences over teachers: Each region ranks types by decreasing level of experience, i.e the most experienced teachers are always preferred to the least experienced teachers.

Figure A.6: Cumulative Distribution of Teacher Experience Types - Same Preferences for all Regions



Notes: This Figure shows the cumulative distribution of teacher experience types. The left panel reports the distribution in the three youngest regions of France (Amiens, Versailles, and Créteil), and the right panel the distribution in the three oldest regions of France (Rennes, Bordeaux, and Lyon). The horizontal axis reports the 12 experience types of teachers, ordered from most experienced to least experienced (left panel) and from least experienced to most experienced (right panel). The 13^{th} type corresponds to the vacant positions. The mechanisms that are status-quo improving are plotted in red. Those that do not are in grey. The thick black line (Init) corresponds to the cumulative distribution of teacher types at the status-quo matching.

Figure A.7: Change in Tenured Teacher Experience (type) Across Regions - Same Preferences for all Regions



Notes: This figure shows the difference in the average experience of teachers between the matching obtained with SI-CC against the status-quo matching (top left figure) and its benchmark TTC* (top right figure). It reports the same difference between the matching obtained with SI-DA against the status-quo matching (bottom left figure) and its benchmark DA* (bottom right figure). Each observation represents a French region. Circle size reflects region size. Regions are ordered (on the horizontal axis) by average experience of their teachers at status quo. The vertical line represents the median type. All regions on the left have an average type that is strictly below the median. This is the group of regions we identified as inexperienced regions. All regions on right of the vertical line are regions whose average type is above the median. In regions above the horizontal line, teachers average experience post reassignment is larger than the one of the benchmark matchings to which it is compared. The name of the three least experienced regions (Créteil, Versailles, and Amiens) and most experienced regions (Rennes, Bordeaux, and Lyon) are reported on the graphs.

UU: Highlight the Difference Between DA* and Current Frenchalso Explain wSI-CC in the new Appendix

Appendix H Descriptions of Benchmark Mechanisms

H.1 Descriptions of wSI-CC wSI-DA

As explained in Section 5, under wSI-CC and wSI-DA, we relabel the type of the teachers based on d_S. We call the relabeled type of a teacher as **pseudo type**. Let $\tau^p : T \to \Omega$ be the **pseudo type** function. For each school s, using tie breaker $\vdash$ and d_s, we determine the **pseudo type** of teachers in ω_s as follows: For each $\theta \rhd_s \theta^{\bar{m}}$ and $t \in \omega_s^\theta$, if $|\{t' \in \omega_s^\theta \ : \ t' \vdash t\}| \geq d_s^\theta$, then $\tau^p(t) = \theta_{\bar{m}}$; otherwise, $\tau^p(t) = \theta$. wSI-CC and wSI-DA mechanisms are almost identical to wSI-CC (see Section 6.6) and wSI-DA (see Section 4), respectively. They differ from these base mechanisms due to the usage of the pseudo type function which helps us to weaken the status-quo improvement constraint. For the sake of completeness, we define wSI-CC and wSI-DA below.

Definition 4 ***Weak Status-quo Improving Cycles and Chains (wSI-CC) Mechanism***

We construct a ***pointing order*** $\succcurlyeq_s$ *over teachers in* ω_s*: For any two distinct teachers* $t, t' \in \omega_s$

$$t \succcurlyeq_s t' \iff \tau^p(t) \lhd_s \tau^p(t') \text{ or } [\tau^p(t) = \tau^p(t') \text{ and } t \vdash t'].$$

We will construct a matching μ *dynamically through the following algorithm. Initially,* μ *is the empty matching, in which no teacher is assigned to any school. In each step, as teachers are assigned in* μ*, they will be removed from the algorithm; similarly schools whose all seats are filled in* μ *and also some other schools chosen by the algorithm will be removed.*

For each school s *and type* θ*, let* b_s^θ *track the* ***current balance*** *of type* θ *teachers at school* s *in* ***current matching*** μ*, which is the matching fixed until the beginning of the current step. The current balance is defined as the difference between the number of type* θ *teachers assigned to* s *in* μ *and the number of type* θ *teachers in its status-quo assignment assigned to any school in* μ*:* $b_s^\theta \equiv |\mu_s^\theta| - |\{t \in \omega_s : \mu_t \neq \emptyset \text{ and } \tau^p(t) = \theta\}|$*. Thus, we initialize* $b_s^\theta = 0$*.*

A general step k *is defined as follows:*

Step k:

- *Each remaining school* s *points to the highest priority remaining teacher in* ω_s *under* $\succcurlyeq_s$*, if not all students in* ω_s *are already assigned in* μ*; let* t_s^k *be the teacher pointed by school* s *in step* k*. Otherwise, school* s *does not point to any teacher.*
- *We define the* ***pointing rule of teachers*** *as follows: Any remaining teacher* t *is allowed to point to a remaining school* s *if at least one of the following two school* ***improvement conditions*** *hold for school* s *via teacher* t*:*

1. *(Improvement for s by teacher trades) if the school points to a teacher t_s^k and*

$$\sum_{\theta' \trianglerighteq_s \theta} b_s^{\theta'} > 0 \text{ for all types } \theta \text{ such that } \tau^p(t_s^k) \trianglerighteq_s \ \theta \ \triangleright_s \ \tau(t),$$

or

2. *(Improvement for s by only incoming teachers) $\tau(t) \triangleright_s \theta_\emptyset$, school s currently has an unfilled seat, i.e., $q_s - |\mu_s| > |\{t' \in \omega_s : \mu_{t'} = \emptyset\}|$, and there are remaining new teachers.*

Let A_t^k be the opportunity set for a remaining teacher t, i.e., the set of schools t can point in this step together with the being unassigned option $\emptyset$.[64]

Each remaining teacher t points to her most preferred option in A_t^k.

- *Being unassigned option $\emptyset$ points to all teachers pointing to it.*

Due to finiteness, there exists either

(i) a cycle in which all schools in the cycle satisfy improvement Condition 1 or a cycle between a single teacher and the being unassigned option $\emptyset$, or

(ii) a chain.

Then:

- ***If Case (i) holds:** Each teacher can be in at most one cycle as she points at most to a single option. We execute exchanges in each cycle encountered in case (i) by assigning the teachers in that cycle to the school she points to, update current matching μ and current balances $\{b_s^\theta\}$ accordingly, remove assigned teachers and filled schools in μ, and go to step $k+1$.*
- ***If Case (i) does not hold:** Then case (ii) holds, i.e., there exists a chain. In particular, each remaining teacher initiates a chain. There are two subcases:*
  - ***If there exists a remaining new teacher**: Then we select a chain to be executed as follows:*
    - *Select as the tail of the chain the new teacher with the highest priority under tie breaker $\vdash$ and then include in the chain the school she points to. If Improvement Condition 1 does not hold for this school via this teacher, but only Improvement Condition 2 holds, then we end the chain with this school; otherwise, we repeat the following:*
    - *Include to the chain the teacher pointed by the last school included. If we include a teacher, we also include next in the chain the school she is pointing to. We repeat this iteratively until the Improvement Condition 1 does not hold for the next school via the included teacher, but only Improvement Condition 2 holds.*

    The last school included is the head of the selected chain.

    We execute the exchanges in the selected chain by assigning each teacher in the chain to the school she points to, update current matching μ and current balances $\{b_s^\theta\}$ accordingly, remove assigned teachers and filled schools, and go to step $k+1$.
  - ***If there does not exist a remaining new teacher**: Then we remove each school s*

[64] Note that, $\omega_t \in A_t^k$ for all remaining teachers t who were employed at the status quo.

whose all status-quo employees in ω_s were already assigned in μ. We continue with step $k+1$.

The mechanism terminates when all teachers are removed. Its outcome is the final matching μ.

Definition 5 ***Weak Status-quo Improving Deferred Acceptance (wSI-DA) Mechanism*** *For each school s, we construct a linear order over the teachers in ω_s denoted by $\blacktriangleright_s$ as follows:*

For any $t, t' \in \omega_s$,

$$t \blacktriangleright_s t' \iff \tau^p(t) \rhd_s \tau^p(t') \text{ or } [\tau^p(t) = \tau^p(t') \text{ and } t \vdash t'].$$

We fix a school s in this construction. Let $\mathcal{S}_s = \{s^1, s^2, \ldots, s^{q_s}\}$ be the set of slots at school s. Without loss of generality we label the types in Θ as $\theta_1, \ldots, \theta_{|\Theta|}$ based on the type ranking of the school such that $\theta_k \rhd_s \theta_{k+1}$ for all $k \in \{1, \ldots, |\Theta| - 1\}$. We define a ranking for each slot over $T \cup \{\emptyset\}$ where $\emptyset$ denotes keeping the slot unfilled. The ***ranking of slot*** *s^k, $\succ_s^k$, is defined separately for the slots representing the filled seats at the status-quo matching, i.e., for $k \leq |\omega_s|$, and slots representing the empty seats at the status-quo matching, i.e., for $|\omega_s| < k \leq q_s$:*

- *For filled slots s^k at the status quo, i.e., all $k \leq |\omega_s|$:*
  - *the teacher $t \in \omega_s$ who is ranked k'th under $\blacktriangleright_s$ has the highest ranking under $\succ_s^k$,*
  - *any teacher t' with $\tau^p(t) \rhd_s \tau(t')$ is ranked below $\emptyset$ under $\succ_s^k$, and*
  - *the rest of the ranking under $\succ_s^k$ is determined according to $\rhd_s$ such that ties between same type teachers are broken according to tie breaker $\vdash$.*
- *For empty slots s^k at the status quo, i.e., all k such that $|\omega_s| < k \leq q_s$:*
  - *a teacher t is ranked above $\emptyset$ under $\succ_s^k$ if and only if she is acceptable, i.e., $\tau(t) \rhd_s \theta_\emptyset$,*
  - *any acceptable new teacher t (i.e., $t \in N$ and $\tau(t) \rhd_s \theta_\emptyset$) is ranked under $\succ_s^k$ above any teacher t' employed at status quo by some school (i.e., $t' \notin N$), and*
  - *the rest of the ranking under $\succ_s^k$ is determined according to $\rhd_s$ such that ties between same type teachers are broken according to tie breaker $\vdash$.*

We refer to the mechanism that selects the outcome of the DA algorithm using the choice rules $(C_s)_{s \in S}$ (see Definition 3), which use the slot priorities constructed above, that we designed as wSI-DA mechanism.

H.2 Description of TTC*

Let $\vdash$ be a tie breaker over teachers. We construct a **pointing order** $\succcurlyeq_s$ over teachers in ω_s: For any two distinct teachers $t, t' \in \omega_s$

$$t \succcurlyeq_s t' \iff \tau(t) \lhd_s \tau(t') \text{ or } [\tau(t) = \tau(t') \text{ and } t \vdash t'].$$

A general step k is defined as follows:

Step k:

- Each remaining school s points to the highest priority remaining teacher in ω_s under $\succcurlyeq_s$, if not all students in ω_s are already assigned in μ. Otherwise, school s does not point to any teacher.
- Each remaining teacher t points to her most preferred remaining option.
- Being unassigned option $\emptyset$ points to all teachers pointing to it.

Due to finiteness, there exists either

(i) a cycle, or
(ii) a chain.

Then:

- **If Case (i) holds:** Each teacher can be in at most one cycle as she points at most to a single option. We execute exchanges in each cycle by assigning the teachers in that cycle to the school she points to, remove assigned teachers and filled schools, and go to step $k+1$.
- **If Case (i) does not hold:** Then case (ii) holds, i.e., there exists a chain. In particular, each remaining teacher initiates a chain. Then we select a chain to be executed as follows:
  - Select as the tail of the chain the teacher with the highest priority under tie breaker $\vdash$ and then include in the chain the school she points to.
  - Include to the chain the teacher pointed by the last school included. If we include a teacher, we also include next in the chain the school she is pointing to. We repeat this iteratively until the last included school points a teacher.

  The last school included is the head of the selected chain.
  We execute the exchanges in the selected chain by assigning each teacher in the chain to the school she points to, remove assigned teachers and filled schools, and go to step $k+1$.

  The mechanism terminates when all teachers are removed.

H.3 Descriptions of DA* and the Current French Mechanism

Mechanism **DA*** uses a version of the DA algorithm that is modified to ensure status-quo improvement for teachers. School preferences are modified such that each teacher t, with a status-quo assignment s, is ranked in the (modified) ranking of her status-quo school $s := \omega_t$, above any teacher $t' \notin \omega_s$. Other than this modification, the schools' preference relations remain unchanged among the status-quo teachers and among the the non-status-quo teachers. That is, the school uses the FOSD preferences over teacher experience types introduced in the empirical section. Then it runs the DA algorithm using these modified school preferences and the submitted teacher preferences. Also see Guillen and Kesten (2012), Pereyra (2013), Compte and Jehiel (2008) regarding the use of this algorithm in another context and teacher assignment context.

The French Ministry of Education's current mechanism, which we refer to as **Current French** in the empirical analysis, uses the same algorithm as DA*. However, instead of the FOSD preferences for regions to rank status-quo teachers among themselves and non-status-quo teachers among

themselves, it uses the Ministry-mandated priorities of regions over teachers. See Combe et al. (2020) for a more detailed presentation of this mechanism and its properties.

By construction, DA* and Current French are both status-quo improving for teachers. They do not satisfy status-quo improvement for schools and is not Gale-Shapley stable. However, DA* and Current French are teacher-SI stable using regional FOSD preferences for region priorities and Ministry-mandated regional priorities, respectively.