The Inference-Forecast Gap in Belief Updating*

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Abstract

Individual forecasts of economic variables show widespread overreaction to news, but laboratory experiments on belief updating typically find underinference from signals. We provide new experimental evidence to connect these two seemingly inconsistent phenomena. Building on a classic experimental paradigm, we study how people make inferences *and* revise forecasts in the same information environment. Subjects *underreact* to signals when inferring about underlying states, but *overreact* to signals when revising forecasts about future outcomes. This gap in belief updating is largely driven by the use of different simplifying heuristics for the two tasks. Additional treatments link our results to the difficulty of recognizing the conceptual connection between making inferences and revising forecasts.

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1 Introduction

When new information arrives, rational agents should update their beliefs according to Bayes' rule. Empirical research, however, has uncovered many instances in which agents' belief updating deviates from Bayes' rule. In surveys, individual forecasts of economic variables often overreact to recent news (e.g., Greenwood and Shleifer, 2014; Gennaioli et al., 2016; Conlon et al., 2018; Bordalo et al., 2020). Building on these findings, overreaction-based models have been proposed to explain many puzzling phenomena, such as excess volatility and return predictability in financial markets (e.g., Barberis et al., 2015) and the boom-bust cycles in macroeconomics (e.g., Maxted, 2020; Bordalo et al., 2021). In contrast, laboratory studies typically find that beliefs about underlying states underreact to new signals (Benjamin, 2019). Underreaction has been a key concept in understanding important financial puzzles, such as momentum and the post-earnings announcement drift (e.g., Barberis et al., 1998). It is also used to generate information rigidity, which can shed light on several puzzles in macroeconomics (e.g., Coibion and Gorodnichenko, 2015).

While both overreaction and underreaction have been widely documented and are useful concepts in economic analysis, the current discussion is not satisfying without addressing a central question about belief updating—what conditions prompt people to underreact in one case and overreact in another (Benjamin, 2019). Since the reaction to new information appears to change from setting to setting, neither underreaction nor overreaction should be considered a universal feature of belief updating. Instead, they may be better interpreted as conditional features with varying prominence under different contexts. Therefore, specifying conditions for overreaction and underreaction can bring more discipline to the modeling of belief biases.

In this paper, we propose one condition that determines underreaction or overreaction in a belief-updating task—namely, whether the task entails an inference process or a forecast-revision process. The differences between the two processes are illustrated in Figure 1. An inference process is one where a subject observes signals and learns about an underlying state that determines the distribution of signals. A forecast-revision process is one where a subject also observes signals but instead update beliefs about future outcomes whose distributions depend on the under-

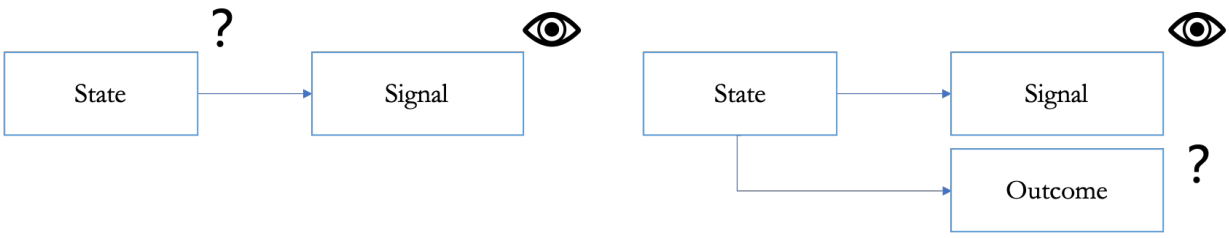


Figure 1: Inference problem (left panel) and forecast-revision problem (right panel)

Notes: In an inference problem, people observe a signal and then update their beliefs about the underlying states. In a forecast-revision problem, people revise their forecasts about outcomes in response to a realized signal.

lying state. In standard models, the forecast-revision process closely follows the inference process. However, we show, in the same experimental setting, a significant disconnect arises: subjects underreact to signals in the inference process but overreact in the forecast-revision process. This finding potentially explains the coexistence of overreaction in survey expectations and underreaction in laboratory studies: in most economic surveys, forecasters revise their forecasts about future outcomes (e.g., future GDP, inflation, stock market returns, job offers); in most lab experiments on belief updating, in contrast, subjects make inferences about the underlying states.

Our baseline treatment follows the "bookbag-and-poker-chip" paradigm[1] but phrase the relevant variables in economic terms. In each round of the experiment, there is a "firm" with a fixed state which is either good or bad. The firm generates signals, framed as its monthly stock price growth, which are informative of the state; good firms, on average, have a higher growth in stock price than bad firms. Subjects do not know the true state but are given the full data-generating process, including the prior distribution over the two states and the distributions of signals conditional on each state. In each month, the signal distribution is i.i.d. normal, with a mean of 100 if the state is good and 0 if it is bad.

The key to our experimental design is comparing belief updating about underlying states and about future outcomes in the same information environment. The baseline treatment has two main

[1] In a typical experiment under this paradigm, there is a bookbag that contains poker chips of several colors. Subjects do not know the bag's color composition, but are given the prior distribution of the composition. A random chip is then drawn from the bag and, upon observing its color, subjects are asked to report their posterior beliefs about the bag's color composition.

parts: *Inference* and *Forecast Revision*. In *Inference*, subjects observe one signal realization and then report their updated beliefs about *the states*—the likelihoods of the firm being good and being bad. In *Forecast Revision*, subjects also observe one signal realization, but instead report their updated expectations about *the next signal*—the expected stock price growth next month. In our environment, these two types of belief are tightly linked: if one believes that the firm is good with a $p\%$ chance, then by the Law of Iterated Expectations (LIE), the expectation about the next signal should be $p\% \times 100 + (1 - p\%) \times 0 = p$. The simplicity of this relation ensures that, for subjects who understand this link, the two problems pose a similar computational complexity.

Despite the straightforward connection between *Inference* and *Forecast Revision*, subjects' behaviors exhibit distinct patterns in the two tasks. In *Inference*, 60% of the answers underreact relative to the Bayesian benchmark while 25% overreact, a result that replicates the stylized fact of systematic underreaction in the bookbag-and-poker-chip literature. By contrast, in *Forecast Revision*, 43% of the answers underreact while 50% overreact. Similarly, when belief updates are measured using the difference between posterior and prior beliefs, the average size of belief updates is significantly larger for *Forecast Revision* than for *Inference*. We refer to this discrepancy in belief updating as the "inference-forecast gap."

To detect modes of behavior that could be driving the aggregate results, we examine the distributions of answers and find several interesting patterns. In *Inference*, the modal behavior is "non-updates"; that is, in 30% of the answers, the posteriors equal the priors. In *Forecast Revision*, the fraction of non-updates drops to 25%. Meanwhile, two other behaviors that rarely appear in *Inference* become modal. Under the first mode, which represents 20% of the answers, subjects answer 100 when the signal is good and 0 when it is bad. These subjects make forecasts as if they were 100% sure about being in the more representative state (the state more consistent with the signal)—a simplifying heuristic that we term "exact representativeness." The second mode, constituting 10% of the answers, is to report a forecast that equals the signal. That is, subjects directly use the past realization as their expectation of the next outcome—a simplifying heuristic we term "naive extrapolation." These three behavioral modes, we show, largely contribute to the existence

of the inference-forecast gap.

Additional analysis shows that the inference-forecast gap is robust across subsamples and under alternative framing of the signal and the outcome. Moreover, the main results persist in two alternative treatments, one with binary signals and one with an outcome that is dissimilar to the signal and completely determined by the state. By varying the property of the signal and the outcome variable, these treatments demonstrate that the inference-forecast gap is robust to alternative DGPs. They also help us rule out explanations, for example, that resort to signal-outcome similarity and misperceptions about signal autocorrelation (such as the hot-hand bias).

The documented inference-forecast gap, especially the different modal behaviors in the two problems, could not arise if, when solving a forecast-revision problem, subjects correctly implement the *infer-then-LIE* procedure by (a) first updating their beliefs about the states as in *Inference* and then (b) using these posterior beliefs to compute the expected value of the forecast outcome under the LIE. The rejection of this standard procedure of forecast revision prompts us to find alternative drivers for the gap. One possibility is that subjects intend to follow the infer-then-LIE procedure, but make errors or take shortcuts due to the procedure's complexity. To study this possibility, we run a treatment in which we show subjects their own inference answers when they solve the corresponding forecast-revision problems. This effectively reduces the two-step infer-then-LIE procedure to a one-step procedure of simply applying the LIE. The treatment, however, has little impact on the inference-forecast gap. Moreover, we confirm that subjects are largely capable of applying the LIE correctly when solving a standalone expectation-formation problem. Taken together, these results suggest that subjects do not appear to be following the infer-then-LIE procedure when solving forecast-revision problems—correctly or with errors. Instead, they resort to alternative procedures which contribute to the inference-forecast gap we document.

Why do subjects not use the infer-then-LIE procedure in *Forecast Revision*? We hypothesize that they do not recognize the conceptual link between inference and forecast revision. To test this hypothesis, we conduct a final treatment in which this link is made more transparent to the subjects. In the *Inference* part of the treatment, subjects still observe the firm's stock price growth

and then report their posterior beliefs about the states. In the *Forecast Revision* part, however, they are asked to predict the directional change of the firm's revenue next month, which is upward *if and only if* the firm is good. By equating the underlying states to the outcomes, this design makes it obvious that *Inference* and *Forecast Revision* are essentially asking the same question. In this treatment, subjects underreact to the same extent in both parts, eliminating the inference-forecast gap. Therefore, the difficulty of conceptually connecting the two tasks plays a key role in the inference-forecast gap.

One remaining question is why simplifying heuristics such as exact representativeness and naive extrapolation are more prevalent in *Forecast Revision* than in *Inference*, but non-updates are prevalent in both. We provide some speculative arguments that this finding may be explained by the theory of attribute substitution (Kahneman and Frederick, 2002).

Our work is related to an active body of experimental research on the conditions of overreaction and underreaction in belief updating (Afrouzi et al., 2020; Enke and Graeber, 2020; He and Kucinskas, 2020; Enke et al., 2021; Hartzmark et al., 2021; Liang, 2021).[2] We replicate the finding from the bookbag-and-poker-chip paradigm that people underreact to information when updating beliefs about underlying states (Phillips and Edwards, 1966; Benjamin, 2019). Importantly, we show that underreaction does not generalize to forecast-revision problems that ask subjects to predict future outcomes, even though the information environment does not change. We thus bring a new perspective to this literature; namely, that the direction of belief-updating biases depends on the type of belief elicited. The documented inference-forecast gap is largely due to the use of different simplifying heuristics in the two types of problems. This finding is consistent with recent papers on the roles of complexity and incorrect mental models in explaining belief-updating biases (Enke and Zimmermann, 2019; Enke, 2020; Esponda et al., 2020; Andre et al., 2021; Graeber, 2021).

The finding of overreaction in forecast revisions provides experimental support for overreac-

[2]Empirical work using field or survey data, including Malmendier and Nagel (2011, 2016) and Wang (2020), also discusses the conditions under which people overreact and underreact to new information.

tion in survey expectations.[3] In this regard, our paper complements studies that find overextrapolation in autocorrelated time-series forecasts (Hey, 1994; Frydman and Nave, 2017; Afrouzi et al., 2020; He and Kucinskas, 2020).[4] DGPs in our experiment, unlike those in these previous studies, fully specify the underlying states, which in turn determine the signal and outcome distributions. This design brings the setting closer to standard models in macroeconomics and finance and lends several advantages for our analysis.[5] First, the explicit separation between states and outcomes makes it possible to design different questions targeting inference and forecast revision, respectively, thereby allowing us to pin down where a specific updating bias arises. Second, such a design allows us to separately identify different forms of overreaction, such as representativeness-based overreaction (Kahneman and Tversky, 1972; Bordalo et al., 2018) and mechanical extrapolation (Barberis et al., 2015, 2018). Indeed, both forms are prevalent in the data and contribute to the inference-forecast gap. Third, having a fully-specified DGP allows us to attribute biases in posterior beliefs to incorrect statistical reasoning rather than to misperceived DGPs. We also apply this design to show that overreaction in forecast revision generalizes to a setting in which signals and outcomes are of two different variables.

Overreaction in *Forecast Revision* is reminiscent of the hot-hand bias (Gilovich et al., 1985; Tversky and Gilovich, 1989; Suetens et al., 2016), which refers to the exaggeration of belief in an outcome after observing a long streak of the same outcomes. In contrast, overreaction occurs in our experiment after just *one* signal realization. Moreover, we find overreaction even when the forecast outcome is different from the signal variable and fully determined by the state, a setting in which misperceptions of outcome autocorrelation, such as the hot-hand bias, are irrelevant. Our underinference result is also inconsistent with the leading account of the hot-hand bias, which is based on overinference (Rabin, 2002; Rabin and Vayanos, 2010). On the design level, we use

[3]For example, see Greenwood and Shleifer (2014); Gennaioli et al. (2016); Conlon et al. (2018); Bordalo et al. (2020); Barrero (2021); and Kohlhas and Walther (2021).

[4]He and Kucinskas (2020) also finds that forecasts underreact to past observations of a different variable.

[5]In asset-pricing models, when investors are learning about firm quality (fundamentals), it is common to assume that they observe noisy signals of quality such as stock returns (e.g., Glaeser and Nathanson, 2017). In the mutual fund literature, investors learn about manager skills by observing past fund returns (e.g., Berk and Green, 2004; Rabin and Vayanos, 2010). In the labor literature, job seekers learn about their employabililty from the offers they receive (Burdett and Vishwanath, 1988).

explicit instructions and comprehension checks to make sure subjects do not commit the hot-hand fallacy. Overall, it is unlikely that our results are driven by or a manifestation of the hot-hand bias.

The rest of the paper proceeds as follows. Section 2 outlines our experimental design. Section 3 shows the existence of the inference-forecast gap. Section 4 studies the decision procedures used by subjects. Section 5 concludes and discusses the implications of our results.

2 Experimental Design

2.1 Environment

To compare belief updating between making inferences and revising forecasts for the same individual, we adopt a within-subject experimental design. For each inference problem a subject solves, there is a corresponding forecast-revision problem that shares the same information environment with an identical DGP and signal realization.

The *Baseline* treatment has five parts which are summarized in Table 1. Each part has eight rounds of problems. In each round, subjects are first presented with a "firm" randomly drawn from a new pool of 20 firms. A firm's state θ is either G(ood) or B(ad). Subjects do not know the state of the drawn firm, but are given the composition of the pool, which specifies the prior distribution over the states. The firm generates signals, s_t, which are framed as the firm's stock price growth in month t, and subjects are provided with their distribution: signals of a good firm follow an i.i.d. normal distribution of $N(100, \sigma^2)$ and signals of a bad firm follow i.i.d. $N(0, \sigma^2)$.[6] Because good firms are more likely to have higher stock price growth than bad firms, a signal of high stock price growth is diagnostic of the firm being good.

To sum up, in each round, the DGP is fully specified by two pieces of information: the prior distribution of states and the conditional distribution of signals. Both are presented to subjects using figures and text in a one-page display (see Figure 2 for an example), and we explain this

[6] In the actual implementation, we discretize the supports of normal distributions to multiples of 10 and truncate at both tails.

Table 1: Summary of variables elicited in each part of the experiment

Number	Part	Show signal?	Beliefs elicited
1	*Inference Prior*	No	$Pr(\theta)$
2	*Inference*	Yes	$Pr(\theta\|s_0)$
3	*Forecast Prior*	No	$\mathbb{E}(s_1)$
4	*Forecast Revision*	Yes	$\mathbb{E}(s_1\|s_0)$
5	*Expectation Formation*	No	$\mathbb{E}(s_1)$

Table 2: Parameter values for DGPs

Index	1	2	3	4	5	6	7	8
$Pr(G)$	50%	50%	50%	50%	50%	50%	80%	20%
σ	50	60	70	80	90	100	100	100

interface with detailed instructions.[7] Table 2 summarizes the parameter values for the eight DGPs. Each DGP is represented by one problem in each of the five parts (the DGP is modified in the *Expectation Formation* part, which we will explain later). As a result, each problem in any given part has a corresponding problem in each of the other four parts, which ensures that answers across parts are directly comparable. Unless mentioned otherwise, an observation is defined as a subject's answers to the five corresponding questions in the five parts.

The two main parts are *Inference* and *Forecast Revision*. In each round, subjects first observe the firm's stock price growth in the current month s_0. In *Inference*, after seeing the realized signal, subjects report their updated beliefs about the states $Pr(\theta|s_0)$. The beliefs are elicited in percentage, and henceforth we will refer to an inference answer as the reported belief about the Good state without the % sign.[8] In *Forecast Revision*, subjects instead report their updated expectations

[7]Screenshots of the experimental interface can be found at `http://yuchengliang.com/iegap/instructions.pdf`.

[8]In the experimental interface, there is one blank for the belief about the Good state and one for the Bad state. Once a subject types a number into one of the two blanks, the other blank will be automatically filled with 100 minus

There is a new pool of 20 firms.
The figure below describes the **stock price growth** of good firms and bad firms in any given month:
The green bar on top of each number is the chance (%) that a good firm's stock price grows by that number (in ¢) in any given month.
The orange bar on top of each number is the chance (%) that a bad firm's stock price grows by that number (in ¢) in any given month.



The pool of firms has the following composition.



Figure 2: An example of the interface for the DGP

about the firm's stock price growth next month $\mathbb{E}(s_1|s_0)$. To ensure an apples-to-apples comparison between the two parts, signal realization is set the same in any two corresponding rounds for the same subject, though it varies across subjects.

In the other three parts, subjects do not observe any signal realization before beliefs are elicited. In *Inference Prior*, they directly report their prior beliefs about the states $Pr(\theta)$ based on their knowledge about the DGP. Similarly, in *Forecast Prior*, they directly report their prior expectations about the signal $\mathbb{E}(s_1)$. These two parts test whether subjects can correctly form prior beliefs. The last part, *Expectation Formation*, is identical to *Forecast Prior*, except for the composition of firms in the pool. While the composition of firms in *Forecast Prior* is set exogenously according to Table 2, in *Expectation Formation* it is determined endogenously by subjects' reported posterior beliefs about the states in *Inference*. For example, if a subject reports a posterior belief of $Pr(G|s_0) = 40\%$ in a round in *Inference*, then the pool of firms in the corresponding round in *Expectation Formation* will have $40\% \times 20 = 8$ good firms and 12 bad ones.[9] *Expectation Formation* is designed to test whether subjects can correctly form expectations about the next signal when the states are distributed according to their own inference posteriors.

Subjects need to stay on each page for at least eight seconds before they can type in answers. This requirement aims to ensure that sufficient attention is paid to the problems and to prevent click-through behavior. For each subject, we further randomize (a) the order of different DGPs in each part and (b) the order of the five parts. For the latter randomization, we require that (a) priors are elicited before eliciting posteriors and (b) the *Expectation Formation* part comes after *Inference*. Hence, we are left with three orders of parts: 12345, 12534, and 34125.

After the five parts, the experiment ends with an unincentivized exit survey. At the end of the experiment, subjects may receive a \$5 bonus payment, the chance of which depends on their answer in one randomly selected round through a quadratic rule.[10]

that number. Only numbers in the range $[0, 100]$ are allowed.

[9]The numbers of good and bad firms in *Expectation Formation* are rounded to the nearest integer if the reported beliefs in *Inference* are not a multiple of 5%. Fourteen percent of the answers in *Inference* are not multiples of 5%, among which half are rounded up and the other half rounded down.

[10]If their answer in that round equals the rational benchmark according to standard probability theory, then they receive the bonus with certainty; otherwise, their chance of getting the bonus decreases quadratically in the difference

Building off the *Baseline* treatment, we implement several straightforward extensions as robustness checks. First, we frame the signal as revenue growth instead of stock price growth. Second, we ask subjects about their expectations of the *last* signal s_{-1} ("stock price/revenue growth in the previous month") instead of the *next* signal s_1. In Appendix A.5, we show that results are qualitatively similar across all these extensions. Therefore, we pool the data from all versions of the *Baseline* treatment for our main results.

2.2 The no inference-forecast gap benchmark

According to standard probability theory, answers in *Inference* and *Forecast Revision* should be tightly linked. Specifically, the Law of Iterated Expectation (LIE) implies the following equation:

$$\mathbb{E}(s_1|s_0) = Pr(G|s_0) \times \mathbb{E}(s_1|G, s_0) + Pr(B|s_0) \times \mathbb{E}(s_1|B, s_0). \tag{1}$$

In our experiment, s_1 and s_0 are independent conditional on the state θ, so $\mathbb{E}(s_1|G, s_0) = \mathbb{E}(s_1|G) = 100$ and $\mathbb{E}(s_1|B, s_0) = \mathbb{E}(s_1|B) = 0$. Therefore, Equation (1) simplifies to the following equation:

$$\mathbb{E}(s_1|s_0) = Pr(G|s_0) \times 100. \tag{2}$$

We term Equation (2) the *no inference-forecast gap* condition. It summarizes the theoretical link between the posterior belief about the underlying states and the updated expectation of the forecast outcome s_1. If an *Inference* answer and its corresponding *Forecast Revision* answer satisfy this condition, then there is no discrepancy between these two types of belief-updating problems: Bayesian inference would then translate to rational forecasts, and any deviation from Bayes' rule in the inference answer would imply the same deviation from rationality in the forecast-revision answer.

The computational simplicity of Equation (2) is an advantage of our experimental design. Un-

between their answer and the rational benchmark (see (Hartzmark et al., 2021) for a similar incentive structure). If the answer is p and the rational benchmark is q (in % for the two *Inference* parts), then the chance of receiving the bonus is $\max\{0, (100 - (p - q)^2)\%\}$.

der the no inference-forecast gap condition, if a signal leads to a belief that the good state has 40% probability, then the resulting expectation of the outcome should be 40. For subjects who understand this condition, the computational cost of solving a forecast-revision problem is very close to that of solving the corresponding inference problem. Therefore, computational complexity alone is unlikely to cause violations of the no inference-forecast gap condition.[11]

When subjects solve a forecast-revision problem, one simple and standard procedure that satisfies the no inference-forecast gap condition is the following *infer-then-LIE* procedure. In the first step, subjects update their beliefs about the states using the same (and possibly non-Bayesian) rule as in the corresponding inference problem. In the second step, they apply the LIE using the posteriors from the first step to obtain their expectations about the forecast outcome.

Since the correct implementation of the infer-then-LIE procedure satisfies the no inference-forecast gap condition, a gap can arise for two broad reasons. First, subjects may consciously follow the infer-then-LIE procedure, but in doing so make errors or take shortcuts that bias their expectations, resulting in a gap. Second, it may be that subjects do not use the infer-then-LIE procedure, but use alternative procedures in their forecast revisions.

2.3 Instructions and comprehension questions

Subjects receive extensive instructions, with the tasks and incentive structure explained in detailed and intuitive terms. In particular, we go to great lengths to ensure that subjects fully understand the DGP. First, we emphasize that the state of a firm is constant across months but the signals are i.i.d. conditional on the state. In doing so, we explicitly caution against incorrect beliefs that the signals are autocorrelated conditional on the state. Second, we use an example DGP to illustrate the discretized normal distributions of the signals. In particular, we highlight the conditional means (0 and 100) and the property that signals higher (lower) than 50 are good (bad) news about the firm's quality. Third, we present subjects with two explicit formulae, one for calculating the

[11] Moreover, because beliefs are equally incentivized across the two types of problems, rational tradeoff between monetary gains and computational costs, in the spirit of Sims (2003); Gabaix (2014); Caplin and Dean (2015); and Woodford (2020), cannot generate an inference-forecast gap.

prior distribution over states from the pool composition ($Pr(G) = \frac{\text{Number of Good Firms}}{20}$) and one for calculating the expectation about the signal from the belief about the states ($\mathbb{E}(s) = Pr(G) \times 100$). However, we do not mention or nudge subjects toward any specific belief-updating rule.

At the end of the instructions, subjects need to answer a set of comprehension questions to test their understanding of the DGP, the incentive structure, and the two formulae. Subjects can proceed only if they have answered all the comprehension questions correctly.[12]

2.4 Procedural details

We programmed our experiment using oTree (Chen et al., 2016). For *Baseline*, we recruited 202 subjects through Prolific, an online platform designed for social science research.[13] For 120 subjects, signals were framed as monthly revenue growth, and for 82 subjects, signals were framed as stock price growth. For 40 subjects, questions in the forecast parts—namely Parts 3, 4, and 5 in Table 1—asked about expectations of the *last* signal ("stock price or revenue growth in the previous month") instead of the *next* signal. There was also some variation across subjects in the order of parts: 72 subjects went through the experiment in the order of 12345, 73 in the order of 12534, and 57 in the order of 34125. A subject, on average, spent about 30 minutes on the experiment and earned a payment of $7.15, $5 of which was the base payment.

2.5 Other treatments

In addition to *Baseline*, we also implemented several other treatments that investigate the robustness of our results and the mechanisms behind. These treatments are summarized in Table 3. The details will be described in their respective sections.

[12] If there are mistakes, she will be asked to re-answer those questions.

[13] See Palan and Schitter (2018) on using Prolific as a subject pool. We recruited only US subjects who had completed more than 100 tasks on Prolific and who had an approval rate of at least 99%.

Table 3: Overview of additional treatments

Treatment	Section	Difference from *Baseline*
Cross-variable Forecast	3.3	Forecast outcome is a different variable; $= 100$ if $\theta = G$, $= 0$ if $\theta = B$
Binary Signal	3.4	Signals are binary; forecast questions ask about full distributions
Nudge	4.1	Beliefs about states and forecasts are elicited on the same page
Obvious Connection	4.2	Forecast outcome is a different variable; $= Up$ if $\theta = G$, $= Down$ if $\theta = B$; forecast questions ask about full distributions

3 Evidence for the Inference-Forecast Gap

In this section, we present results from our experiment to compare belief-updating in inference and forecast-revision problems. This comparison is carried out using three methods of analysis. First, we classify answers into *Near-rational*, *Overreact*, and *Underreact*, and compare the distributions of these three categories amongst inference and forecast-revision problems. Second, we compare the average belief movements from the priors in these two types of updating task. Third, we compare the distributions of individual answers and identify differences in modal behaviors. If the no inference-forecast gap condition in Equation (2) is met, then results from the two types of updating problems should exhibit identical patterns in all three kinds of analysis. Any systematic differences would imply the existence of an inference-forecast gap.

3.1 Aggregate patterns

For an inference problem in our experiment, the rational benchmark is given by Bayes' rule:

$$Pr^{Rational}(G|s_0) = \frac{Pr(G) \cdot Pr(s_0|G)}{Pr(G) \cdot Pr(s_0|G) + Pr(B) \cdot Pr(s_0|B)}. \tag{3}$$

For a forecast-revision problem in our experiment, the rational benchmark can be derived by applying LIE to the corresponding rational inference answer:

$$\begin{aligned}\mathbb{E}^{Rational}(s_1|s_0) &= Pr^{Rational}(G|s_0) \times \mathbb{E}(s_1|G) + Pr^{Rational}(B|s_0) \times \mathbb{E}(s_1|B) \\ &= Pr^{Rational}(G|s_0) \times 100. \end{aligned} \tag{4}$$

Note that the no inference-forecast gap condition in Equation (2) is satisfied by the rational benchmarks.

We classify answers in *Inference* and *Forecast Revision* by how they compare to the rational benchmarks. An answer is classified as *Near-rational* if its difference from the rational benchmark is no more than 2.5.[14] To introduce the categories of *Underreact* and *Overreact*, we first define "update" by how much an answer moves from its (objective) prior value in the direction of the realized signal:

$$\text{update} = \begin{cases} \text{answer} - \text{prior}, \text{ if } s_0 > 50 \\ \text{prior} - \text{answer}, \text{ if } s_0 < 50 \end{cases} \tag{5}$$

It is straightforward from equations (3) and (4) that rational updates between any two corresponding inference and forecast-revision problems are identical. We classify an answer as *Overreact* if its update is larger than the rational update by more than 2.5 and as *Underreact* if its update is smaller than the rational update by more than 2.5. We do not classify answers when $s_0 = 50$; that is, when the signal is uninformative.

Table 4 shows the aggregate patterns in the *Baseline* treatment (excluding observations with a realized signal of 50). Results from *Inference* replicate the key finding from the classic bookbag-and-poker-chip literature: subjects overwhelmingly underreact to new information and update too little about the firm's underlying state. Out of all the answers, 59.8% imply underreaction, 25.2% imply overreaction, and 15% are considered *Near-rational*. These patterns, however, flip in *Forecast Revision*: 49.5% of the answers indicate overreaction to new information—higher than the

[14] We choose the number 2.5 so that the interval for *Near-rational* covers at least one multiple of five, on which subjects' answers tend to cluster.

Table 4: Aggregate patterns in *Baseline*

N=202, Obs=1480	Classification			Update
	Underreact	*Near-rational*	*Overreact*	Mean (s.e.)
Inference	59.8%	15.0%	25.2%	15.1 (0.8)
Forecast Revision	43.1%	7.4%	49.5%	29.9 (2.3)
Rational				23.4 (0.3)

Notes: The first three columns present the percentages of answers classified as *Underreact*, *Near-rational*, and *Overreact*. The last column shows average belief movements in the signal direction from the (objective) priors and their rational benchmark. Observations with signal equal 50 are excluded. Standard errors are clustered by subject.

43.1% classified as underreaction.

The last column of Table 4 demonstrates that the inference-forecast gap also shows up in average updates. In *Inference*, the average update across all answers is 15.1, much smaller than the average rational update of 23.4. By contrast, in *Forecast Revision*, subjects update too much, leading to an average update of 29.9. Column (1) of Table A6 confirms the inference-forecast gap in updates in a regression.

The inference-forecast gap is highly robust in various cuts of the data (see Appendix A for detailed results). In a more "reasonable" subsample which only includes observations with a forecast-revision answer within $[0, 100]$ and updates that are nonnegative, forecast-revision answers no longer exhibit overreaction on average, but the inference-forecast gap remains highly significant. Moreover, the gap is present under all eight DGPs, which entail different priors and signal distributions. The gap increases in signal strength but exists even for the weakest signals. Our result also persists in a subsample that excludes observations with incorrect reported prior beliefs. In addition, the order of experimental parts, the framing of signals and outcomes, and subject characteristics have no qualitative impacts on the inference-forecast gap.

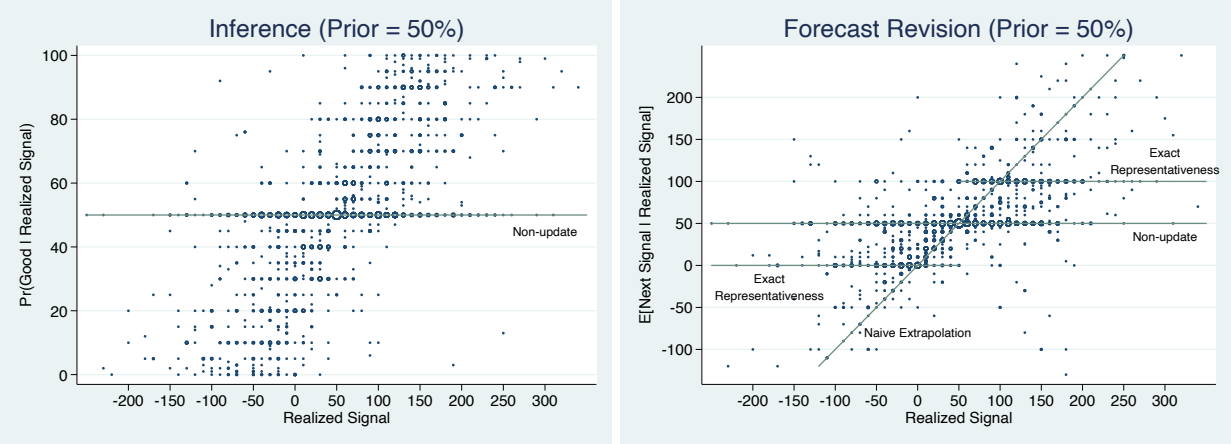


Figure 3: Scatterplots of answers against realized signals: subsample with symmetric priors

Notes: This figure plots the updated beliefs against the realized signals. The size of each circle represents the number of answers that equal the value on the y-axis given the realized signal on the x-axis.

3.2 Modes of behavior

To detect more nuanced patterns in individual answers, in this section we examine the distributions of posterior beliefs and explore modes of behavior that could be driving the inference-forecast gap. To illustrate, Figure 3 plots the answers against the realized signals for problems with symmetric objective priors in *Inference* and *Forecast Revision*.[15] Several behavioral modes appear salient in the plots. In *Inference*, a large fraction of answers equals the 50-50 prior. The prevalence of such non-updates replicates the stylized fact in previous inference experiments (e.g., Coutts, 2019; Graeber, 2021).

For *Forecast Revision*, non-updates also constitute a mode. However, two other modes emerge. First, a large number of forecast-revision answers cluster at 100 when $s_0 > 50$ and 0 when $s_0 < 50$. Subjects who give these answers behave as if they were certain about being in the representative state (the state consistent with the direction of the signal's realization) and base their forecasts solely on that state. We term this overreacting behavior "exact representativeness" because it is consistent with the representativeness heuristic (Kahneman and Tversky, 1972; Bordalo et al.,

[15] Distributions of answers in problems with asymmetric priors display similar patterns. See Appendix B for details.

Table 5: Modes of behavior in *Baseline*

Mode	Criterion for answer	*Inference*	*Forecast Revision*
Non-update	= prior	29.9%	25.1%
Exact Representativeness	= 100 if $s_0 > 50$, = 0 if $s_0 < 50$	3.9%	20.1%
Naive Extrapolation	$= s_0$	3.3%	10.3%
No Inference-Forecast Gap (excluding non-updates)	inference = forecast revision ($\neq$ prior)	5.3%	
Unclassified		59.8%	43.9%
Observations		1480	1480

Notes: The column "Criterion for answer" shows the criterion for an answer to be classified into a mode. Note that an answer may be classified into more than one mode. The percentages in the last two columns are the fractions of answers in each mode in *Inference* and *Forecast Revision*. Observations with signal equal to 50 are excluded.

2018).[16]

Second, a smaller yet still significant fraction of forecast-revision answers are anchored at the face value of the realized signal.[17] We term this behavior "naive extrapolation" because it suggests a particular form of extrapolative expectation formation (Barberis et al., 2015, 2018; Liao et al., 2021).[18] The face value of the realized signal is among the top three common answers for 19 out of 53 values of the realized signal. This behavior leads to overreaction in the problems with symmetric priors in our experiment.

In Table 5, we define the behavioral modes and quantify their prevalence in all inference and forecast-revision problems. Confirming the patterns in the scatterplots, non-updates are widespread in both types of problems, making up 29.9% and 25.1% of the answers in *Inference* and *Forecast*

[16]An alternative interpretation of this modal behavior is that subjects base their expectations solely on the ex-post more likely state, which can differ from the representative state when the prior is asymmetric. We can differentiate the two interpretations by examining problems with an asymmetric prior. In Appendix B, we study the distributions of forecast-revision answers under asymmetric priors and find evidence supporting the representativeness interpretation. However, this result should be interpreted as only suggestive due to a small sample size.

[17]For each x-axis value—that is the value of the realized signal—we rank answers by the frequency of their occurrence. For 19 out of the 53 x-axis values, anchoring on the signal value is among the top three most frequent answers. In comparison, non-updates and exact representativeness are each among the top two most frequent answers for 36 x-axis values.

[18]In general, extrapolation refers to people's tendency to rely heavily on past outcomes to forecast future outcomes.

Revision, respectively. The other two behavioral modes, exact representativeness and naive extrapolation, appear almost exclusively in *Forecast Revision*, making up 20.1% and 10.3% of the answers, respectively. In comparison, observations that meet the no inference-forecast gap condition and are not non-updates constitute only 5.3% of the answers. We conduct further analysis in Appendix B. In Table B2, we relax the classification criteria for the modes and find similar qualitative patterns. Table B3 shows similar patterns in a subject-part level classification exercise, where a subject is classified into a type for a given part (*Inference* or *Forecast Revision*) if more than half of her answers in that part are classified into the corresponding mode. Based on this subject-part-level classification, we also find a modest degree of consistency between a subject's types in the two parts. For example, many subjects are classified as non-updaters in both parts.

The difference in modal behaviors largely contributes to the inference-forecast gap. For example, for the more "reasonable" subsample in which all forecast-revision answers fall within [0, 100] and no answers update in the wrong direction, the inference-forecast gap is eliminated on the aggregate level if we exclude observations which have at least one answer in one of the three behavioral modes—non-updates, exact representativeness, and naive extrapolation (see Column (4) of Table A6).

3.3 *Cross-variable Forecast* treatment

In this and the next subsection, we investigate the inference-forecast gap in two additional treatments with alternative DGPs. Both generate similar patterns to those of the *Baseline* treatment. These results demonstrate the prevalence of the inference-forecast gap in various environments and help rule out several potential explanations for its emergence.

The forecast outcome in *Baseline* is the next signal, which is identical to the realized signal both in name and in distribution. We change this feature in an additional treatment (N=100) called *Cross-variable Forecast* in which the forecast outcome is a variable different from the signal. Specifically, the new outcome variable is framed as revenue growth when the signal is stock price growth, and vice versa. We also design the outcome variable to have a degenerate distribution

Table 6: Aggregate patterns in *Cross-variable Forecast*

N=100, Obs=748	Classification			Update
	Underreact	*Near-rational*	*Overreact*	Mean (s.e.)
Inference	63.8%	15.1%	21.1%	13.8 (1.3)
Forecast Revision	40.6%	8.7%	50.7%	32.9 (3.3)
Rational				23.3 (.5)

Notes: The first three columns present the percentages of answers classified as *Underreact*, *Near-rational*, and *Overreact*. The last column shows average belief movements in the signal direction from the (objective) priors and their rational benchmark. Observations with signal equal 50 are excluded. Standard errors are clustered by subject.

conditional on the state: it is 100 for sure in the Good state and 0 for sure in the Bad state. Thus, the outcome is different from the signal in both name and distribution and is similar to the state in distribution. Nevertheless, under this alternative DGP, the no inference-forecast gap condition remains the same as before: the forecast-revision answer equals the corresponding inference answer (minus the % sign).

Table 6 shows the results from *Cross-variable Forecast*. The inference-forecast gap, compared to that in *Baseline*, becomes even greater in magnitude. For example, only 21.1% of the inference posteriors are *Overreact*, while 50.7% of the forecast-revision answers are. Table A9 further shows, in a regression analysis, that statistically the gap is highly significant. In Table B4, the distribution of behavioral modes in *Cross-variable Forecast* is also similar to that in *Baseline*.

The results from *Cross-variable Forecast* help address four issues. First, they rule out the possibility that the inference-forecast gap is driven by similarity between signals and outcomes (Kahneman and Tversky, 1972). According to this explanation, because the signal and the outcome are represented by the same variable in *Forecast Revision*, signal-outcome similarity leads subjects to perceive the signal as more informative and therefore to overreact to it. The results in *Cross-variable Forecast* clearly demonstrate that subjects still overreact to signals when they are asked to make predictions about a different variable.

Second, the fact that the state fully determines the outcome indicates that the inference-forecast

gap is not due to the difference in distribution between the state and the outcome. Under this design, signals are equally diagnostic about the state and the outcome, thereby ruling out any explanations based on differential diagnosticity between inference and forecast-revision problems.

Third, the presence of overreaction in *Cross-variable Forecast* suggests that the overreaction in forecast-revision problems is not driven by misperceived signal autocorrelation. Because the forecast outcome is different from the signal and fully determined by the state, perception of signal autocorrelation is irrelevant to the expectation formation of the future outcome. This further differentiates our results from the hot-hand bias (Gilovich et al., 1985; Tversky and Gilovich, 1989; Suetens et al., 2016) and from overreaction in univariate forecasts (Hey, 1994; Frydman and Nave, 2017; Afrouzi et al., 2020) in which exaggerated autocorrelation is a key driving force.

Fourth, *Cross-variable Forecast* broadens the external relevance of the inference-forecast gap. In many empirical settings, the forecaster's information set is not limited to past observations of the variables to be predicted but also includes the past observations of other relevant variables. The results in *Cross-variable Forecast* suggest that the inference-forecast gap can be an explanation for overreactions in these settings as well (e.g., Bordalo et al., 2020; Roth and Wohlfart, 2020).

3.4 *Binary Signal* treatment

We implement a treatment (N=140) in which the signal s_t follows a binary distribution. The signal, framed as the direction of the firm's stock price movement, is either up or down, and the probability of an upward movement is higher if the firm's state is Good. The parameters for the DGPs are listed in Table 7. In the forecast-revision part of the treatment, the problem asks about the probability distribution $Pr(s_1)$ (instead of the outcome expectation $\mathbb{E}(s_1)$).

As in the *Baseline* treatment, the no inference-forecast gap condition for this treatment is given by the LIE:

$$Pr(s_1 = up|s_0) = Pr(G|s_0) \times Pr(up|G) + Pr(B|s_0) \times Pr(up|B). \tag{6}$$

Index	1	2	3	4	5	6	7	8
$Pr(G)$	50%	50%	50%	50%	50%	50%	80%	20%
$Pr(up\|G)$	60%	70%	80%	90%	70%	55%	70%	70%
$Pr(up\|B)$	40%	30%	20%	10%	45%	30%	30%	30%

Table 7: Parameter values for DGPs in the *Binary Signal* treatment

Substituting in $Pr(up) = Pr(up|G) \times Pr(G) + Pr(up|B) \times Pr(B)$, which is the LIE applied to the objective prior beliefs, we obtain the following equation:

$$\frac{Pr(s_1 = up|s_0) - Pr(up)}{Pr(up|G) - Pr(up|B)} = Pr(G|s_0) - Pr(G). \tag{7}$$

Equation (7) states that under the no inference-forecast gap condition, the inference update equals the *normalized* forecast-revision update, the latter defined by how much the forecast revision answer moves from the objective prior in the signal direction *divided by* the range of outcome probabilities, $Pr(up|G) - Pr(up|B)$. This equation is not as simple as Equation (2) in *Baseline*, so computational complexity could confound the comparison between inference and forecast revision answers. However, one advantage of the *Binary Signal* treatment is that it is closer to the common design in the bookbag-and-poker-chip paradigm.

In *Binary Signal*, the three categories—*Near-rational*, *Underreact*, and *Overreact*—are defined in the same way as in the *Baseline* treatment, except that the categories for forecast-revision answers are defined based on their *normalized* updates. Table 8 reports the results from the *Binary Signal* treatment. As in *Baseline*, more answers are classified as *Overreact* in *Forecast Revision* than in *Inference*, and the average update in the former part is also larger. (Table A10 shows in a regression that the gap in updates is significant at the 10% level.) However, answers in *Forecast Revision* do not exhibit overreaction on average. The modal behaviors are also similar to those in the *Baseline* treatment (see Table B5). Non-updates are prevalent in both *Inference* and *Forecast Revision*, making up 27.1% and 19.8% of answers in those two parts, respectively. In *Forecast*

Table 8: Aggregate patterns in *Binary Signal*

N=140, Obs=1120	Classification			Update
	Underreact	*Near-rational*	*Overreact*	Mean (s.e.)
Inference	61%	20.1%	18.9%	11 (0.9)
Forecast Revision	54.9%	6.7%	38.4%	14.2 (2.2)
Rational				18.7 (0)

Notes: The first three columns present the percentages of answers classified as *Underreact*, *Near-rational*, and *Overreact*. The last column shows average belief movements in the signal direction from the (objective) priors and their rational benchmark. The updates of forecast-revision answers are normalized by $Pr(up|G) - Pr(up|B)$ so that they are comparable to the inference updates. Observations with signal equal 50 are excluded. Standard errors are clustered by subject.

Revision, 17.4% of the answers equal the outcome probability of the representative state, which constitutes the behavioral mode of exact representativeness.

Overall, the *Binary Signal* treatment shows that the inference-forecast gap extends to environments with alternative signal distributions. It also shows that this phenomenon can persist when the elicited object in *Forecast Revision* is the outcome distribution instead of its expected value.

4 Decision Procedures

4.1 Implementation errors or nonstandard procedures?

As discussed in Section 2.2, the inference-forecast gap should not arise if subjects, in answering a forecast-revision question, correctly implement the infer-then-LIE procedure by: (a) first updating their beliefs about the states as in the corresponding inference problem and (b) then applying the LIE to form expectations about the forecast outcome. The evidence we have documented so far on the inference-forecast gap clearly rejects the correct implementation of this procedure, prompting us to look for alternative explanations.

One possible explanation for the inference-forecast gap is that subjects intend to follow the

infer-then-LIE procedure when revising forecasts, but make errors or take shortcuts because the procedure is complex. For instance, a decision-maker may be capable of forming probabilistic beliefs about the states when making inference is the only task. But when implementing the two-step infer-then-LIE procedure for the forecast-revision problem, she may have only enough cognitive bandwidth to form a binary belief ("the firm is good" or "the firm is bad") in the first step. This error can lead to overreacting behaviors that look like exact representativeness.

We run an additional treatment, *Nudge*, with 99 subjects to test the above hypothesis. For parts that provide signals, after observing the signal realization, subjects are first asked to report their beliefs about the states and then, while their answers are still on the screen, they are asked to report their expectations about the next signal.[19] With this design, from the point of view of a subject intending to follow the infer-then-LIE procedure, a forecast-revision problem is made no more complex than a standalone expectation-formation problem. Indeed, one need only multiply the inference posterior by 100 to complete the infer-then-LIE procedure.[20] This reduction in complexity should mitigate any implementation errors in the procedure and reduce the inference-forecast gap according to the hypothesis.

Table 9 shows the aggregate patterns in *Nudge*. In this treatment, subjects overwhelmingly underreact in *Inference* and on average overreact in *Forecast Revision*. In fact, the inference-forecast gap in *Nudge* is even larger than in *Baseline*, according to the regression analysis in Table A9. Table B6 further examines the modal behaviors in *Nudge*. The fraction of non-updates in *Inference* is 53.4%, a notable increase from the 29.9% in *Baseline*. However, the fraction of non-updates in *Forecast Revision* remains almost the same as in *Baseline*, as does the fraction of answers classified as exact representativeness and naive extrapolation. In addition, the fraction of answers that satisfy the no inference-forecast gap condition increases to 11.3% from the 5.3% in the *Baseline* treatment, suggesting that the *Nudge* treatment induces a greater tendency to give internally con-

[19] More specifically, subjects have to stay on the page for eight seconds before answering each question. The forecast-revision question appears only after the answer to the inference question has been submitted. Subjects can revise their answers to the inference question before they submit their answers to the forecast-revision question.

[20] In fact, because answers to the *Inference* problems are given in the unit of percentage, the infer-then-LIE procedure implies that subjects should type in the same number in the corresponding *Forecast Revision* problems.

Table 9: Aggregate patterns in *Nudge*

N=99, Obs=715	Classification			Update
	Underreact	*Near-rational*	*Overreact*	Mean (s.e.)
Inference	70.6%	10.2%	19.2%	10.3 (1.3)
Forecast Revision	42.2%	6.7%	51%	28.9 (2.9)
Expectation Formation	60.6%	6.9%	32.6%	13.7 (2.1)
Rational				22.6 (.5)

Notes: The first three columns present the percentages of answers classified as *Underreact*, *Near-rational*, and *Overreact*. The last column shows average belief movements in the signal direction from the (objective) priors and their rational benchmark. The expectation-formation answers are analyzed in the same way as the corresponding forecast-revision answers: the update of an expectation-formation answer is defined as the answer minus the (objective) prior in the corresponding forecast-revision problem if the signal in the latter problem is greater than 50 and the reverse if the signal is smaller than 50. The classification of an expectation-formation answer is conducted against the rational benchmark for the corresponding forecast-revision problem. Observations with signal equal to 50 are excluded. Standard errors are clustered by subject.

sistent answers to the two types of updating questions. However, this small increase does not have material impact in the aggregate. Taken together, displaying the inference answer when subjects revise their forecasts does not change the overall pattern of the inference-forecast gap.

How can one explain the ineffectiveness of the *Nudge* treatment? One possibility is that while it indeed makes the infer-then-LIE procedure no more complex than solving a standalone expectation-formation problem, even the latter is too complex for subjects and their resulting errors lead to overreaction. To test this possibility, in another part of the *Nudge* treatment called *Expectation Formation*, we ask subjects to report their beliefs about the state and then their expectations of the next signal *without* showing them any signal realization. In addition, for each subject, we set the distribution over states in an expectation-formation problem to match the posterior belief the subject reported in the corresponding inference problem. For example, if a subject reports $Pr(G|s_0) = 40\%$ in a round in *Inference*, then the pool of firms in the corresponding *Expectation Formation* round will have $40\% \times 20 = 8$ good firms and 12 bad ones. This design enables us to directly quantify how much of the inference-forecast gap in *Nudge* can be attributed to mistakes in

expectation formation.

Figure C2 in Appendix C shows the average deviations from LIE in the expectation-formation problems by the prior probability of the Good state; the deviations are small across the board. Moreover, in the last row of Table 9, we classify expectation-formation answers and calculate their updates by treating them in the same way as their corresponding forecast-revision answers. Specifically, the update of an expectation-formation answer is defined as the answer minus the (objective) prior in the corresponding forecast-revision problem if the signal in the latter problem is greater than 50 and the reverse if the signal is smaller than 50. The classification of an expectation-formation answer is conducted against the rational benchmark for the corresponding forecast-revision problem. Comparing the average updates in the inference, forecast-revision, and expectation-formation problems, we find that mistakes in expectation formation can account for only 18% of the inference-forecast gap. These results indicate that mistakes in standalone expectation-formation problems do not explain the null effect of the *Nudge* treatment on the inference-forecast gap.

Taken together, results from the *Nudge* treatment reject the hypothesis that the inference-forecast gap stems from complexity-induced errors or shortcuts when subjects try to implement the infer-then-LIE procedure in forecast-revision problems. Rather, the gap is likely a result of the use of other procedures altogether.

4.2 Obviousness of the connection between inference and forecast revision

Why do subjects not use the infer-then-LIE procedure for forecast-revision problems? One hypothesis is that they do not recognize the conceptual connection between the two kinds of problems. To investigate this hypothesis, we implement an additional treatment, *Obvious Connection*, with 30 subjects. In this treatment, the realized signal is still the stock price growth of the firm in the current month, but the forecast question concerns the probability that the firm's revenue will go up next month. Moreover, subjects are informed that a firm's revenue goes up *if and only if* the state is Good. As the name of the treatment suggests, by equating the underlying states to

Table 10: Aggregate patterns in *Obvious Connection*

N=30, Obs=238	Classification			Update
	Underreact	*Near-rational*	*Overreact*	Mean (s.e.)
Inference	69.7%	10.1%	20.2%	12.5 (2)
Forecast Revision	66.4%	9.7%	23.9%	11.9 (2.6)
Rational				22.6 (.8)

Notes: The first three columns present the percentages of answers classified as *Underreact*, *Near-rational*, and *Overreact*. The last column shows average belief movements in the signal direction from the (objective) priors and their rational benchmark. Observations with signal equal 50 are excluded. Standard errors are clustered by subject.

the outcomes, we design the treatment to make it obvious that the inference and forecast-revision questions are asking about the same event.

Table 10 shows the results in the *Obvious Connection* treatment. With the predicted outcome obviously connected to the state, the inference-forecast gap almost completely vanishes, and we obtain the familiar underreaction pattern in the forecast-revision problems. Table B7 shows the breakdown of answers into different types in this treatment. Exact representativeness is still more prevalent in *Forecast Revision* than in *Inference*, though only slightly. However, the other differences in modal behaviors between *Inference* and *Forecast Revision* disappear. Moreover, 16.4% of answers satisfy the no inference-forecast gap condition, which is much higher than the 5.3% in the *Baseline* treatment. These results confirm the hypothesis that many subjects do not follow the infer-then-LIE procedure when revising forecasts because they fail to recognize the conceptual connection between forecast revision and inference.

5 Concluding Remarks

In this paper, we present new experimental evidence to show that people overreact more to new information when they revise forecasts about future outcomes than when they make inferences about underlying states. This inference-forecast gap in belief updating is largely driven by the

use of different heuristics in the two types of problems. Through a series of subsample analyses and additional treatments, we show that the gap is robust to order effects, framing effects, subject characteristics, and alternative data-generating processes. Moreover, it cannot be explained by existing theories of belief-updating biases. We further examine the underlying mechanism and show that the gap does not stem from the implementation errors subjects make when they try to follow a standard decision procedure. Rather, the discrepancy may result from people not recognizing the conceptual connection between inference and forecast revision. This inability to link the two processes, in turn, motivates them to follow nonstandard decision heuristics when revising forecasts.

In the remainder of this paper, we discuss the theoretical implications of our finding and its relationship to empirical evidence on survey forecasts.

5.1 Implications for theory

Our finding that subjects overreact more in forecast-revision problems than in inference problems cannot be directly accounted for by existing theories of belief updating. Indeed, most existing theories do not allow updating biases to depend on the type of belief elicited. While we do not have a formal model in this paper to explain the inference-forecast gap, we provide some speculation about the underlying mechanism.

The inference-forecast gap in our experiment is in a large part driven by distinct heuristics used in the two types of updating problems. While non-updates are prevalent in both types, exact representativeness and naive extrapolation emerge only as modal behaviors in forecast-revision problems. One potential explanation for why subjects use these specific heuristics comes from the psychological theory of "attribute substitution" (Kahneman and Frederick, 2002), which proposes that when people are asked a complex question, they often substitute it with a related question that has an easily accessible answer. Both types of belief-updating questions in our experiment are complex, so one can imagine subjects asking which easily accessible information can reasonably substitute for answers to these questions.

For example, the expected outcome conditional on the representative state is a value easily accessible to the subjects. Furthermore, this variable is conceptually similar to the expected outcome conditional on the signal that the forecast-revision question actually asks for. Therefore, it is reasonable, though not accurate, to use this value as a substitute answer to the forecast-revision question. In contrast, this variable is conceptually very different from the probability over the states, which the inference problem asks for. The fact that this variable fits the answer to a forecast-revision problem better than it fits the answer to an inference problem may explain why exact representativeness is a behavioral mode in the former but not in the latter.

A similar argument can explain why a significant fraction of subjects use the face value of the realized signal as the answer to the forecast-revision question, but not to the inference question. Because past realizations are conceptually closer to future outcomes rather than to underlying states, subjects tend to use this value in answering forecast-revision questions. The theory can also explain why non-updates are prevalent in both types of problems. Because prior beliefs about the states and prior expectations of the outcome are conceptually similar to their posterior counterparts, sticking to the priors is a reasonable heuristic for both inference and forecast-revision problems. For future research, it would be important to test this and other hypotheses for why certain heuristics are prevalent in decision-making.

5.2 Relationship to survey evidence

Recent field studies find widespread overreaction in survey forecasts of variables such as stock returns, macroeconomic indicators, firm performances, housing prices, and job offers. This is in stark contrast with the prevalent underreaction documented in lab experiments on belief updating. While our experimental evidence is consistent with both sets of facts, we do not claim that the inference-forecast gap explains the entire discrepancy between these two literatures. After all, field settings are different from the lab in many other aspects that could be driving overreaction in survey forecasts. First, the DGP can be much more complex in reality than in simple experimental settings. The underlying state may be time-variant and the forecast outcome may be correlated with

past signals even conditional on the state. The DGP itself may even be unknown. Second, survey takers may come from a different pool than experimental subjects. For instance, many financial survey participants are professional forecasters or investors who are more likely to possess a good understanding of the financial market.

Despite the caveats, we believe our evidence can still speak—at least partially—to what is going on in the field for the following reasons. First, with more complex DGPs in reality, it could be all the more likely that people revise their forecasts using heuristics that are detached from their beliefs about fundamentals. Second, while survey takers may be more sophisticated, most market participants are households and closer to the subjects we study. It is also worth noting that even professionals' forecasts have subjective inputs (Stark, 2013) and are highly correlated with household expectations (Greenwood and Shleifer, 2014).[21]

[21] Robert Shiller's United States Stock Market Confidence Indices also show that U.S. institutions and individuals exhibit highly correlated beliefs over time; see https://som.yale.edu/faculty-research-centers/centers-initiatives/international-center-for-finance/data/stock-market-confidence-indices/united-states-stock-market-confidence-indices

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A Robustness of the Inference-Forecast Gap

In this section, we examine the properties of the inference-forecast gap in various subsamples of the data.

A.1 A more "reasonable" subsample

We start by examining the inference-forecast gap in a subsample of the *Baseline* treatment that satisfies two basic rationality criteria. In this subsample, we only keep observations whose forecast revision answer falls within $[0, 100]$, the range marked by the expected outcome of the Good state and of the Bad state. Furthermore, we exclude observations in which either the inference update or the forecast revision update is negative; these observations indicate that the subjects' reactions to signals are in the wrong direction.

Table A1: Aggregate patterns in *Baseline*: subsample with "reasonable" updates

N=202, Obs=978	Classification			Update
	Underreact	*Near-rational*	*Overreact*	Mean (s.e.)
Inference	55.8%	17.3%	26.9%	17.7 (1)
Forecast Revision	45.6%	10.1%	44.3%	23.1 (1.4)
Rational				23.5 (0.4)

Notes: The first three columns present the percentages of answers classified as *Underreact*, *Near-rational*, and *Overreact*. The last column shows average belief movements in the signal direction from the (objective) priors and their rational benchmark. Observations with signal equal to 50, forecast revision answers that are outside [0,100], or updates in the wrong direction are excluded. Standard errors are clustered by subject.

Table A1 shows the results of this subsample. Although the average update in *Forecast Revision* is close to rational, there is still more overreaction and less underreaction in *Forecast Revision* than in *Inference*. The gap in updates between these two parts is significant, as is shown in a regression analysis in Column (2) of Table A6.

A.2 Priors and signals

The inference-forecast gap exists in all eight problems (see Table A2). Notably, the eight problems include DGPs with symmetric and asymmetric priors, indicating that our result persists with and without the potential influence of base-rate neglect.

For the subsample with symmetric (objective) priors, we further examine how the inference-forecast gap depends on the strength of the signal. We measure signal strength by the Bayesian update it induces; the more a Bayesian decision-maker moves her belief in response to the signal, the more diagnostic it is about the underlying state. Table A3 shows the results. Overall, there is a larger inference-forecast gap when the signal is more diagnostic. But the gap exists even for the weakest signals.

Most subjects report correct prior beliefs about the states and about the outcome in *Inference Prior* and *Forecast Prior*, but small errors sometimes occur (see Figure C1). To control for the impact of errors in priors on our result, we repeat the classification exercise for a subsample in which the reported inference prior and forecast prior are both correct. The pattern in this sample, shown in Table A4 and in Column (3) of Table A6, is similar: there is more overreaction and less underreaction in *Forecast Revision* than in *Inference*.

A.3 Order between parts

The gap is also robust to different ordering of the parts. Table A5 compares the gap across different orders and shows that there is a large and statistically significant gap for all three. Comparing the inference answers under orders 12345 and 12534 with the forecast revision answers under order 34125, our results also indicate that the gap persists in a between-subject analysis.

A.4 Subject characteristics

Finally, we examine the heterogeneity of the gap across subject characteristics, such as gender, education, investment experience, familiarity with statistics and economics, and performance in

Table A2: Aggregate patterns in *Baseline* (by problem)

		Classification			Update
		Underreact	*Near-rational*	*Overreact*	Mean (s.e.)
$Pr(G)$ = 50%	*Inference*	69.3%	18.8%	12%	20 (1.5)
σ = 50	*Forecast Revision*	45.8%	13.5%	40.6%	30.7 (2.9)
(Obs = 192)	Rational				36.8 (0.9)
$Pr(G)$ = 50%	*Inference*	64.5%	18.8%	16.7%	17.9 (1.5)
σ = 60	*Forecast Revision*	48.4%	7%	44.6%	28.9 (3.4)
(Obs = 186)	Rational				32.4 (1)
$Pr(G)$ = 50%	*Inference*	64.7%	12.6%	22.6%	15.6 (1.4)
σ = 70	*Forecast Revision*	43.2%	7.9%	48.9%	28.1 (3.2)
(Obs = 190)	Rational				26.7 (0.9)
$Pr(G)$ = 50%	*Inference*	65.1%	11.6%	23.3%	12.5 (1.4)
σ = 80	*Forecast Revision*	45%	5.3%	49.7%	29.5 (3.6)
(Obs = 189)	Rational				22.8 (0.9)
$Pr(G)$ = 50%	*Inference*	50.5%	17.9%	31.6%	17 (1.3)
σ = 90	*Forecast Revision*	40.5%	5.8%	53.7%	33.9 (3.8)
(Obs = 190)	Rational				21.2 (0.9)
$Pr(G)$ = 50%	*Inference*	52.6%	15.1%	32.3%	13.7 (1.4)
σ = 100	*Forecast Revision*	36.5%	7.8%	55.7%	33.6 (3.6)
(Obs = 192)	Rational				19.7 (0.9)
$Pr(G)$ = 80%	*Inference*	55.7%	10.9%	33.3%	12 (1.8)
σ = 100	*Forecast Revision*	44.8%	3.4%	51.7%	27.2 (4.6)
(Obs = 174)	Rational				13 (0.8)
$Pr(G)$ = 20%	*Inference*	55.1%	13.8%	31.1%	11 (2)
σ = 100	*Forecast Revision*	40.7%	7.8%	51.5%	26.3 (4.2)
(Obs = 167)	Rational				12.7 (0.8)

Notes: The first three columns present the percentages of answers classified as *Underreact*, *Near-rational*, and *Overreact*. The last column shows average belief movements in the signal direction from the (objective) priors and their rational benchmark. Observations with signal equal to 50 are excluded. Standard errors are clustered by subject.

Table A3: Aggregate patterns in *Baseline* (by signal strength)

Signal Strength		Classification			Update
		Underreact	*Near-rational*	*Overreact*	Mean (s.e.)
Weakest	*Inference*	50.3%	21.2%	28.6%	4.9 (1)
(Obs = 189)	*Forecast Revision*	49.7%	13.8%	36.5%	10.2 (1.8)
	Rational				6.3 (0.2)
Weak	*Inference*	64.7%	12.7%	22.6%	8.8 (1.1)
(Obs = 252)	*Forecast Revision*	42.1%	5.6%	52.4%	20.5 (2.3)
	Rational				16 (0.2)
Medium	*Inference*	60.9%	7.9%	31.2%	15.6 (1.4)
(Obs = 202)	*Forecast Revision*	41.1%	4%	55%	31.4 (3.3)
	Rational				25.1 (0.2)
Strong	*Inference*	64.2%	11.2%	24.7%	21.1 (1.6)
(Obs = 215)	*Forecast Revision*	36.7%	4.2%	59.1%	46.4 (4.9)
	Rational				34.3 (0.2)
Strongest	*Inference*	63%	24.2%	12.8%	26.8 (1.6)
(Obs = 281)	*Forecast Revision*	46.3%	11.7%	42%	41.5 (4.2)
	Rational				44.9 (0.2)

Notes: The first three columns present the percentages of answers classified as *Underreact*, *Near-rational*, and *Overreact*. The last column shows average belief movements in the signal direction from the (objective) priors and their rational benchmark. Observations with signal equal to 50 or asymmetric (objective) priors are excluded. The five categories for signal strength correspond to five intervals of rational updates: $[0, 10)$, $[10, 20)$, $[20, 30)$, $[30, 40)$, and $[40, 50]$. Standard errors are clustered by subject.

Table A4: Aggregate patterns in *Baseline*: subsample with correct priors

N=202, Obs=1095	Classification			Update
	Underreact	*Near-rational*	*Overreact*	Mean (s.e.)
Inference	57.7%	17.2%	25.1%	15.6 (1)
Forecast Revision	45.8%	8.8%	45.5%	26.3 (2.5)
Rational				23.8 (.4)

Notes: The first three columns present the percentages of answers classified as *Underreact*, *Near-rational*, and *Overreact*. The last column shows average belief movements in the signal direction from the (objective) priors and their rational benchmark. Observations with signal equal to 50 or with incorrect priors are excluded. Standard errors are clustered by subject.

Table A5: Aggregate patterns in *Baseline* (by order between parts)

		Classification			Update
		Underreact	*Near-rational*	*Overreact*	Mean (s.e.)
Order: 12345	*Inference*	58.3%	16.1%	25.6%	14.9 (1.3)
(N = 72)	*Forecast Revision*	41%	8.7%	50.3%	30.1 (3.6)
(Obs = 527)	Rational				23.3 (0.5)
Order: 12534	*Inference*	58.6%	16.4%	25%	15.2 (1.3)
(N = 73)	*Forecast Revision*	42.9%	5.8%	51.2%	33 (3.8)
(Obs = 531)	Rational				23.5 (0.5)
Order: 34125	*Inference*	63.3%	11.8%	24.9%	15 (1.9)
(N = 57)	*Forecast Revision*	46%	7.6%	46.4%	25.5 (4.6)
(Obs = 422)	Rational				23.5 (0.6)

Notes: The first three columns present the percentages of answers classified as *Underreact*, *Near-rational*, and *Overreact*. The last column shows average belief movements in the signal direction from the (objective) priors and their rational benchmark. Observations with signal equal to 50 are excluded. Standard errors are clustered by subject.

the comprehension questions. Table A7 show regression results by interacting variables for these characteristics with a *Forecast Revision* dummy. One notable result is that subjects who pass all comprehension checks in one pass exhibit less underreaction in *Inference* and less overreaction in *Forecast Revision*, which leads to an inference-forecast gap that is only half as that of the other subjects. In addition, subjects who report being familiar with economics or finance also exhibit a smaller gap. These results suggest that better comprehension of the subject matter is associated with a smaller inference-forecast gap.

A.5 Framing

In different versions of the *Baseline* treatment, we show that the gap is robust to several changes in the framing of the signal and forecast outcome. First, we frame the signal as the firm's revenue growth (rather than stock price growth); we find the same gap. Second, in the three forecast parts, we ask subjects to make predictions about the *previous* signal instead of the next signal; we find an inference-forecast gap that is quantitatively smaller but still significant at 5% level. Table A8 show these results in regressions.

A.6 Regression analysis

Table A6: The inference-forecast gap under various sample restrictions

	Update			
	Full sample	"Reasonable" updates	Correct priors	"Reasonable" updates & exclude modal behaviors
	(1)	(2)	(3)	(4)
Forecast Revision	14.801***	5.398***	10.642***	0.593
	(2.429)	(1.403)	(2.683)	(1.409)
Rational Update	1.012***	0.561***	0.923***	0.777***
	(0.078)	(0.049)	(0.077)	(0.102)
Problem FE	Yes	Yes	Yes	Yes
Subject FE	Yes	Yes	Yes	Yes
Observations	2960	1956	2190	438
R^2	0.339	0.474	0.366	0.513

Notes: *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively. Standard errors are clustered by subject. This table presents results for our *Baseline* treatment. Each observation corresponds either to an inference answer or a forecast-revision answer. We define the dependent variable, *Update*, as the answer minus the (objective) prior if the signal is greater than 50, the opposite if it is smaller than 50. *Rational Update* is the update prescribed by Bayes' rule (and the Law of Iterated Expectation). Observations with the signal equal to 50 are excluded. In Column (2), based on the full sample, we further drop observations with the forecast revision answer outside the [0, 100] range and observations with at least one update that is in the opposite direction as the signal. In Column (3), based on the full sample, we further drop observations with an incorrect answer for *Inference Prior* or *Forecast Prior*. In Column (4), based on the subsample in Column (2), we further exclude observations in which the inference answer or the forecast revision answer is classified into one of the three modes: non-updates, exact representativeness, and perfect extrapolation.

Table A7: Heterogeneity of the inference-forecast gap across demographics

	Update
Forecast Revision	21.464***
	(4.658)
Male × Forecast Revision	-1.579
	(4.783)
College × Forecast Revision	2.835
	(4.690)
Investor × Forecast Revision	-1.445
	(5.123)
Familiar with Stats × Forecast Revision	-2.290
	(4.640)
Familiar with Econ × Forecast Revision	-9.176*
	(5.314)
High Comprehension × Forecast Revision	-9.705**
	(4.540)
Male	2.006
	(1.689)
College	-1.348
	(1.852)
Investor	3.548*
	(1.958)
Familiar with Stats	3.157
	(1.978)
Familiar with Econ	-3.139
	(2.326)
High Comprehension	5.006**
	(1.925)
Rational Update	0.987***
	(0.074)
Problem FE	Yes
Observations	2960
R^2	0.149

Notes: *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively. Standard errors are clustered by subject. This table presents results for our baseline treatment. Each observation corresponds either to an inference answer or a forecast-0revision answer. We define the dependent variable, *Update*, as the answer minus the (objective) prior if the signal is greater than 50, the opposite if it is smaller than 50. *Rational Update* is the update prescribed by Bayes' rule (and the Law of Iterated Expectation). Observations with the signal equal to 50 are excluded. We define *Male* as 1 if the subject indicates their gender as Male; the base group is thus Female or Others. We define *College* as 1 if the subject has a bachelor's or postgraduate degree. We define *Investor* as 1 if the subject indicates that they have investments in stocks or mutual funds. We define *Familiar with Stats* as 1 if the subject indicates that they are familiar with probability theory and statistics. We define *Familiar with Econ* as 1 if the subject indicates that they are familiar with economics or finance. We define *High Comprehension* as 1 if the subject correctly answers all comprehension questions in one pass.

Table A8: Heterogeneity of the inference-forecast gap across alternative framing

	Update	
	Stock price vs. revenue	Next vs. last signal
	(1)	(2)
Stock Price × Forecast Revision	14.858***	
	(3.465)	
Revenue × Forecast Revision	14.761***	
	(3.161)	
Revenue	4.316**	
	(1.726)	
Next × Forecast Revision		15.998***
		(2.651)
Last × Forecast Revision		9.779**
		(4.875)
Last		1.804
		(2.380)
Rational Update	0.991***	0.994***
	(0.075)	(0.074)
Problem FE	Yes	Yes
Observations	2960	2960
R^2	0.138	0.136

Notes: *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively. Standard errors are clustered by subject. This table presents results for our *Baseline* treatment. Each observation corresponds either to an inference answer or a forecast-revision answer. We define the dependent variable, *Update*, as the answer minus the (objective) prior if the signal is greater than 50, the opposite if it is smaller than 50. *Rational Update* is the update prescribed by Bayes' rule (and the Law of Iterated Expectation). Observations with the signal equal to 50 are excluded. In the first two columns, we explore heterogeneity of the effects depending on whether we frame the signal as stock price growth or revenue growth. In the last two columns, we explore heterogeneity of the effects depending on whether we ask about the expectation of the *next* signal or the *last* signal in Forecast Revision.

Table A9: The inference-forecast gap across different treatments

	Update
Baseline × Forecast Revision	14.801***
	(2.341)
Cross-variable × Forecast Revision	19.198***
	(3.304)
Nudge × Forecast Revision	18.640***
	(2.962)
Obvious Connection × Forecast Revision	-0.644
	(2.932)
Cross-variable	-1.167
	(1.555)
Nudge	-4.218***
	(1.552)
Obvious Connection	-1.881
	(2.308)
Rational Update	0.942***
	(0.051)
Problem FE	Yes
Observations	6362
R^2	0.148

Notes: *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively. Standard errors are clustered by subject. In this table, we pool the data from our *Baseline* treatment, *Cross-variable Forecast* treatment, *Nudge* treatment, and *Obvious Connection* treatment. Each observation corresponds either to an inference posterior or an extrapolation posterior. Each observation corresponds either to an inference answer or a forecast-revision answer. We define the dependent variable, *Update*, as the answer minus the (objective) prior if the signal is greater than 50, the opposite if it is smaller than 50. *Rational Update* is the update prescribed by Bayes' rule (and the Law of Iterated Expectation). Observations with the signal equal to 50 are excluded.

Table A10: The inference-forecast gap in *Binary Signal* treatment

	Update
Forecast Revision	3.632*
	(1.992)
Rational Update	0.532***
	(0.074)
Problem FE	Yes
Subject FE	Yes
Observations	2240
R^2	0.204

Notes: *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively. Standard errors are clustered by subject. This table presents results for the *Binary Signal* treatment. Each observation corresponds either to an inference posterior or an forecast-revision posterior. Each observation corresponds either to an inference answer or a forecast-revision answer. We define the dependent variable, *Update*, as the answer minus the (objective) prior if the signal is up, the opposite if it is $down$. The updates of forecast revision answers are normalized by $Pr(up|G) - Pr(up|B)$ so that they are comparable to the inference updates. *Rational Update* is the update prescribed by Bayes' rule.

B Additional Analysis on Modes of Behavior

In this section, we provide additional analysis on the modes of behavior in *Inference* and *Forecast Revision* in the baseline treatment.

B.1 Problems with asymmetric priors

Table B1 quantifies the prevalence of the modal behaviors in problems with asymmetric priors. The overall pattern is similar to that for problems with symmetric priors: non-updates are prevalent in both *Inference* and *Forecast Revision*, while exact representativeness and naive extrapolation show up almost exclusively in the latter.

Table B1: Modes of behavior in *Baseline* treatment: subsample with asymmetric priors

Mode	Criterion for answer	*Inference*	*Forecast Revision*
Non-update	= prior	29.9%	23.2%
Exact Representativeness	= 100 if $s_0 > 50$, = 0 if $s_0 < 50$	4.4%	15%
Naive Extrapolation	= s_0	3.5%	9.1%
No Inference-Forecast Gap (excluding non-updates)	inference = forecast revision ($\neq$ prior)	3.8%	
Unclassified		60.7%	51.9%
Observations		341	341

Notes: The column titled "Criterion for answer" shows the criterion for an answer to be classified into a given mode. Note that an answer may be classified into more than one mode. The percentages in the last two columns are the fractions of answers in each mode in *Inference* and *Forecast Revision*. Observations with signal equal to 50 are excluded.

In forecast revision problems with symmetric priors, an alternative interpretation of answers classified as exact representativeness is that subjects form expectations solely based on the *ex-post more likely* state. This interpretation is distinguishable from the representativeness interpretation in problems with asymmetric priors. To illustrate, consider a forecast revision problem in which the prior belief $Pr(G)$ is 20% and the realized signal s_0 is only slightly above 50. Because the signal

is good news, the representative state is G. However, because the signal contradicts the prior and is relatively weak, the ex-post more likely state (judged from the subject's own inference) could still be B. Therefore, this problem allows us to tell whether subjects, when revising forecasts, are more likely to focus exclusively on the representative state or the ex-post more likely state.

We focus on a subsample of observations in which the objective prior is asymmetric, the reported inference prior and forecast prior are both correct, the signal direction is opposite to the prior direction, and both the inference answer and its rational benchmark are between the prior and 50. Within this subsample, five forecast revision answers equal the expected outcome of the representative state, whereas none equal the expected outcome of the ex-post more likely state. While the sample size is too small to draw any definitive conclusion, the result nevertheless suggests that subjects are more likely to focus on the representative state when they revise forecasts.

B.2 Relaxing criteria for classification

Table B2 shows the prevalence of behavioral modes when we relax the classification criteria to allow for errors within $[-4, 4]$. Compared to the results with strict classification criteria (Table 5), the fraction of answers in each mode increases only slightly, and the overall qualitative pattern remains the same.

B.3 Subject-part–level classification

To study the consistency of behavior within subjects, we conduct a classification exercise on the subject-part level. Specifically, a subject is classified into a type in a part (*Inference* or *Forecast Revision*) if more than half of her answers in that part are classified into the corresponding mode. Table B3 shows the joint distribution of types across the two parts. The numbers of subjects classified in the two parts are 73 and 81, and the marginal distribution of types in each part resembles that of the answer-level classification. On the relationship between types in the two parts, many subjects are non-updaters in both parts. Meanwhile, subjects classified as exact representativeness and naive extrapolation in *Forecast Revision* are mostly unclassified in *Inference*.

Table B2: Modes of behavior in *Baseline* with relaxed criteria for mode classification

Mode	Criterion for answer	*Inference*	*Forecast Revision*
Non-update	$\approx$ prior	35.7%	23.3%
Exact Representativeness	≈ 100 if $s_0 > 50$, ≈ 0 if $s_0 < 50$	5.3%	20.6%
Naive Extrapolation	$\approx s_0$	3.9%	13.5%
No Inference-Forecast Gap (excluding non-updates)	inference $\approx$ forecast revision ($\not\approx$ prior)	7.8%	
Unclassified		51.3%	41%
Observations		1480	1480

Notes: The column titled "Criterion for answer" shows the criterion for an answer to be classified into a given mode. The $\approx$ sign means that the criterion allows for errors within $[-4, 4]$. Note that an answer may be classified into more than one mode. The percentages in the last two columns are the fractions of answers in each mode in *Inference* and *Forecast Revision*. Observations with signal equal to 50 are excluded.

Table B3: Joint distribution of *Inference* types and *Forecast Revision* types in *Baseline*

Forecast Revision type \ *Inference* type	Non-update	Exact Repre-sentativeness	Naive Extrapolation	No Inference-Forecast Gap	Unclassified	Total
Non-update	23	1	1	0	17	41
Exact Representativeness	2	2	0	1	22	26
Naive Extrapolation	4	0	0	0	9	13
No Inference-Forecast Gap	0	1	0	2	0	2
Unclassified	19	0	1	0	101	121
Total	48	3	2	2	149	202

Notes: This table shows the number of subjects that are classified into each type in *Inference* and *Forecast Revision*. Note that a subject may be classified into more than one type in a part.

B.4 Modes of behavior in other treatments

This subsection presents results on the modal behaviors in four treatments: *Cross-variable Forecast*, *Binary Signal*, *Nudge*, and *Obvious Connection*.

Table B4: Modes of behavior in *Cross-variable Forecast*

Mode	Criterion for answer	*Inference*	*Forecast Revision*
Non-update	= prior	35.7%	23.3%
Exact Representativeness	= 100 if $s_0 > 50$, = 0 if $s_0 < 50$	5.3%	20.6%
Naive Extrapolation	= s_0	3.9%	13.5%
No Inference-Forecast Gap (excluding non-updates)	inference = forecast revision ($\neq$ prior)	7.8%	
Unclassified		51.3%	41%
Observations		748	748

Notes: The percentages in the last two columns are the fractions of answers in each mode in *Inference* and *Forecast Revision*. Observations with signal equal to 50 are excluded.

Table B5: Modes of behavior in *Binary Signal*

Part	Mode	Criterion for answer	% of answers
Both	No Inference-Forecast Gap (excluding non-updates)	Equation (7)	3.5%
Inference	Non-update	$Pr(\theta\|s_0) = Pr(\theta)$	27.1%
	Exact Representativeness	$Pr(G\|s_0) = 100\%$ if $s_0 = up$ $Pr(G\|s_0) = 0$ if $s_0 = down$	3.1%
	Unclassified		67.6%
Forecast Revision	Non-update	$Pr(s_1\|s_0) = Pr(s_1)$	19.8%
	Exact Representativeness	$Pr(s_1\|s_0) = Pr(s_1\|G)$ if $s_0 = up$ $Pr(s_1\|s_0) = Pr(s_1\|B)$ if $s_0 = down$	17.4%
	Unclassified		60.6%
Observations			1120

Notes: The percentages in the last column are the fractions of answers in each mode for each part.

Table B6: Modes of behavior in *Nudge*

Mode	Criterion for answer	*Inference*	*Forecast Revision*
Non-update	= prior	53.4%	22%
Exact Representativeness	= 100 if $s_0 > 50$, = 0 if $s_0 < 50$	2.7%	17.9%
Naive Extrapolation	= s_0	3.6%	9.1%
No Inference-Forecast Gap (excluding non-updates)	inference = forecast revision ($\neq$ prior)	11.3%	
Unclassified		32.6%	44.2%
Observations		715	715

Notes: The percentages in the last two columns are the fractions of answers in each mode in *Inference* and *Forecast Revision*. Observations with signal equal to 50 are excluded.

Table B7: Modes of behavior in *Obvious Connection*

Mode	Criterion for answer	*Inference*	*Forecast Revision*
Non-update	= prior	35.3%	34%
Exact Representativeness	= 100 if $s_0 > 50$, = 0 if $s_0 < 50$	5.5%	13%
Naive Extrapolation	= s_0	4.6%	4.6%
No Inference-Forecast Gap (excluding non-updates)	inference = forecast revision ($\neq$ prior)	16.4%	
Unclassified		42.9%	37.4%
Observations		238	238

Notes: The percentages in the last two columns are the fractions of answers in each mode in *Inference* and *Forecast Revision*. Observations with signal equal to 50 are excluded.

C Beliefs without signal realization

In this section, we present results from the parts of our experiment in which subjects do not see any signal realization: *Inference Prior*, *Forecast Prior*, and *Expectation Formation*. Figure C1 shows the distribution of answers in *Inference Prior* and *Forecast Prior*. The majority of answers are correct, with the fraction of correct answers larger under symmetric priors. Subjects are more likely to report incorrect priors in *Forecast Prior* than in *Inference Prior*. The distribution of errors is mostly unsystematic.

Like *Forecast Prior*, the experimental part *Expectation Formation* asks about subjects' expectations of the outcome without seeing any signal realization. The unique feature of this part, however, is that the distribution over states in an expectation-formation problem for each subject is set to match the posterior over states reported by this subject in the corresponding inference problem. Figure C2 shows how much expectation-formation answers deviate from the correct answers prescribed by the LIE in the *Baseline* treatment and the *Nudge* treatment. The errors are mostly small and unsystematic.

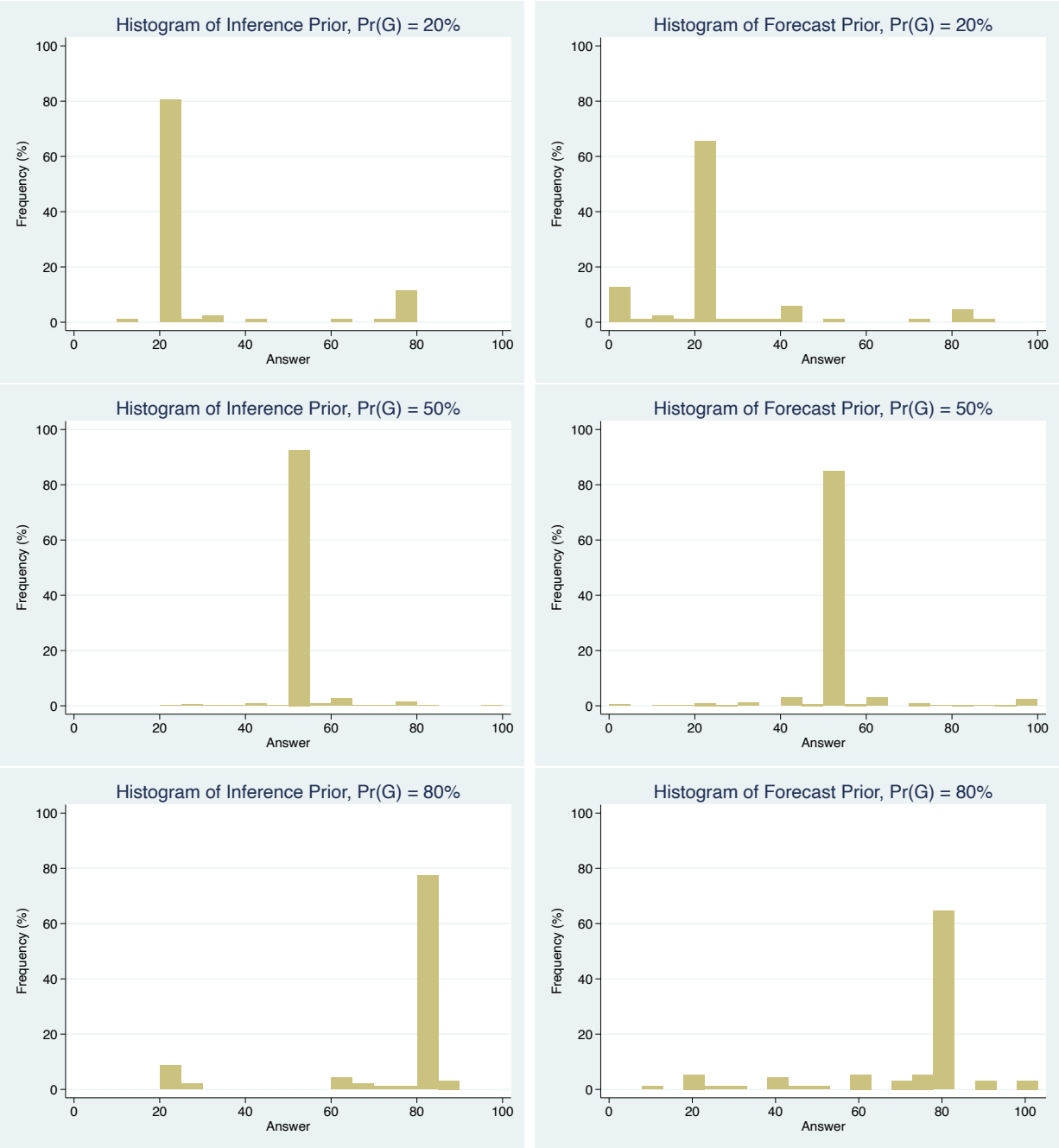


Figure C1: Distributions of answers in *Inference Prior* and *Forecast Prior* in *Baseline*

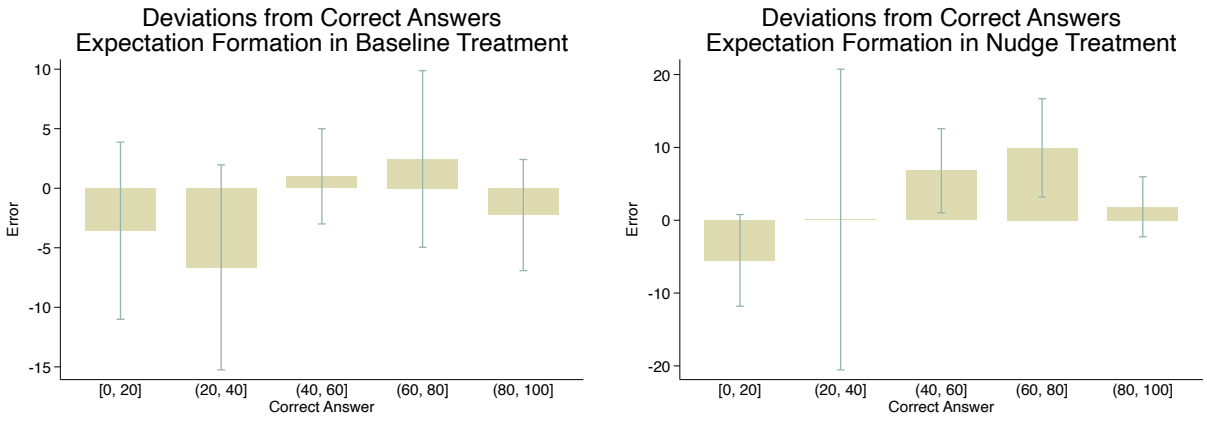


Figure C2: Deviations from LIE in expectation-formation problems

Notes: Standard errors are clustered by subject.