# Household Inventory, Temporary Sales, and Price Indices

Kozo Ueda* Kota Watanabe† Tsutomu Watanabe‡

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**Abstract**

Large-scale household inventory buildups occurred in Japan five times over the last decade, including those triggered by the Tohoku earthquake in 2011, the spread of COVID-19 infections in 2020, and the consumption tax hikes in 2014 and 2019. Each of these episodes was accompanied by considerable swings in GDP, suggesting that fluctuations in household inventories are one of the sources of macroeconomic fluctuations in Japan. In this paper, we focus on changes in household inventories associated with temporary sales and propose a methodology to estimate changes in household inventories at the product level using retail scanner data. We construct a simple model on household stockpiling and derive equations for the relationships between the quantity consumed and the quantity purchased and between consumption and purchase prices. We then use these relationships to make inferences about quantities consumed, consumption prices, and inventories. Next, we test the validity of this methodology by calculating price indexes and check whether the intertemporal substitution bias we find in the price indexes is consistent with theoretical predictions. We empirically show that there exists a surprisingly large bias in Laspeyres, Paasche, and Törnqvist price indexes, which is smaller at lower frequencies but non-trivial even at a quarterly frequency and that intertemporal substitution bias disappears for a particular type of price index if we switch from purchase-based data to consumption-based data.

*Waseda University (E-mail: kozo.ueda@waseda.jp)

†Canon Institute for Global Studies and University of Tokyo (E-mail: watanabe.kota@canon-igs.org)

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# 1 Introduction

In the first week of March 2020, when the first wave of COVID-19 infections hit Japan, supermarket sales went up more than 20% over the previous year. This was due to hoarding by consumers stemming from an increase in uncertainty regarding the spread of the virus. Similar hoarding occurred during the third wave, which struck Japan in October 2020. Such hoarding has occurred not only during the COVID-19 pandemic but also after the Tohoku earthquake in March 2011 and the subsequent nuclear power plant accident in Fukushima, when residents of Tokyo and other areas that were spared serious damage went on a buying spree for food and other necessities. Consumer hoarding also occurred due to policy shocks: when the consumption tax rate was raised in April 2014 and in October 2019, people hoarded large amounts of goods just before the tax rate was raised, and a prolonged consumption slump occurred thereafter. Each of these episodes was accompanied by considerable swings in GDP, suggesting that fluctuations in household inventories are one of the sources of macroeconomic fluctuations in Japan.

There is a considerable literature on household inventories, addressing a variety of aspects. For instance, Boizot et al. (2001), Griffith et al. (2009), Hendel and Nevo (2006a, 2006b), and Kano (2018) investigate intertemporal patterns of household purchases and consumption from a theoretical and/or empirical perspective. Hendel and Nevo (2013) focus on heterogeneity in households' ability to store goods and show that temporary sales are driven by price discrimination. More recently, the responses of household inventories to various shocks have been studied by Baker, Johnson, and Kueng (forthcoming), Cashin and Unayama (2021), and Hansman et al. (2020).[1] Meanwhile, Baker, Johnson, and Kueng (2020) regard inventories as part of households' non-financial wealth and show that households can obtain significant financial returns from the strategic manage-

[1]Baker, Johnson, and Kueng (forthcoming) examine consumers' responses to changes in the sales tax rate in the United States, showing that households stock up on storable goods before taxes rise and increase online and cross-border shopping. Meanwhile, comparing the responses of expenditure on durable and nondurable goods as well as storable and nonstorable goods to Japan's consumption tax hike in 1997, Cashin and Unayama (2021) show that spending on durables and storables surged in the months prior to the tax rate increase and fell sharply upon implementation, while expenditure on nondurables and nonstorables increased only slightly in the month prior to the tax hike. They argue that households engaged in a significant amount of stockpiling just prior to the tax rate increase. Hansman et al. (2020) examine stockpiling by households during disasters such as the COVID-19 pandemic, focusing on the implications of sticky store prices for stockpiling. They show that sticky prices exacerbated hoarding during the 2008 Global Rice Crisis as households realized that prices would rise more in the future.

ment of inventories. Coibion, Gorodnichenko, and Koustas (forthcoming) focus on the increase in US households' stockpiling due to a lower frequency of shopping trips since 1980 and provide evidence that under these circumstances inequality across households is overestimated when measured based on expenditure rather than on consumption.

In this study, we focus on changes in household inventories associated with temporary sales. Whereas the five episodes we outlined above are noteworthy for consumer hoarding, it occurs on a daily basis, especially during a temporary sale, which brings about many important consequences. The objective of this study is twofold. First, we propose a methodology to estimate changes in household inventories at the product level using retail scanner data. Suppose that a temporary sale takes place unexpectedly: a product is sold at the regular price in period 0, at the sale price in period 1, and again at the regular price in period 2. A typical response to this temporary sale would be that the quantity purchased increases in period 1 and exceeds the quantity consumed, resulting in an increase in household inventories. The quantity purchased falls below the quantity consumed in period 2 and subsequent periods, so that inventories are gradually drawn down in these periods. Importantly, the price consumers face in each period also deviates from retailers' selling price. In this process, the quantity and price of purchases can be observed with scanner data, but the quantity consumed, the consumption price, and changes in inventory are not observable. We construct a simple model on household stockpiling and derive equations for the relationships between the quantity consumed and the quantity purchased and between consumption and purchase prices. We then use these relationships to make inference about quantities consumed, consumption prices, and inventories.

The second objective is to empirically test the validity of the methodology described above, focusing on the intertemporal substitution bias in consumer price indexes, which has been extensively discussed in the field of price index theory. Existing studies have regarded the intertemporal substitution bias as stemming from changes in household inventories associated with temporary sales (see, for example, Nakamura, Nakamura, and Nakamura 2011). Taking the Törnqvist index, which is known to be a second-order approximation of the cost of living index (COLI), as an example, the weight attached to the price decline from period 0 to period 1 is the sum of the shares of the amount purchased of this product in period 0 and period 1 divided by 2, while the weight attached to the price increase from period 1 to period 2 is the sum of the shares of the amount purchased in the corresponding periods divided by 2. Since the amount purchased in

period 2 tends to be smaller than the amount purchased in period 0, the latter exceeds the former. Therefore, the inflation rate from period 0 to period 2 takes a negative value, even though the price has returned to its original level. To eliminate this bias, it is necessary to replace the purchase quantity with the consumption quantity and the purchase price with the consumption price. In this study, we test the validity of our methodology by calculating consumption-based (rather than purchase-based) price indexes using the consumption quantity and consumption price estimated based on the model and check whether the intertemporal substitution bias we find in the price indexes is consistent with theoretical predictions.

The contributions of this study are as follows. First, we construct a simple partial equilibrium model on stockpiling by consumers, in which goods prices switch between regular and sale prices following a Markov process. Our model is close to that by Hendel and Nevo (2006a) but differs from it in that our model is a quasi-dynamic one. Specifically, we assume that a household consists of a household producer and a household consumer, and that household inventory is held only by the household producer. The household producer is like an inventory trader. On the other hand, the household consumer holds no inventory: all he/she does is to purchase something today to consume today. This setting allows us to divide household decisions into two parts, making it easier to distinguish between purchasing and consumption. That is, the household consumer has nothing to do with intertemporal optimization, and the optimization problem he/she faces is a static one. All decisions related to intertemporal changes in inventory are made by the household producer. We refer to the price associated with transactions between the household consumer and the household producer as the "consumption price," while the price associated with transactions between the household producer and firms is referred to as the "purchase price." Similarly, the quantity associated with transactions between the household consumer and the household producer is referred to as the "consumption quantity" in our analysis, while the quantity associated with transactions between the household producer and firms is referred to as "purchase quantity." Optimization by the household producer and the household consumer determines the relationships between the consumption price and quantity on the one hand and the purchase price and quantity on the other. Using these relationships, we estimate the price and quantity for consumption.

The second contribution of our study is that we apply our methodology for estimating consumption prices and quantities to the issue of intertemporal substitution bias in price

indexes stemming from temporary sales.[2] Using our model, we first show theoretically that consumer stockpiling driven by temporary sales results in intertemporal substitution bias and then provide empirical evidence supporting this prediction. Specifically, based on retail scanner data, we show that (1) quantities purchased before, during, and after a temporary sale differ substantially, as predicted by the model; (2) there exists a surprisingly large bias in Laspeyres, Paasche, and Törnqvist price indexes, and their sign and magnitude relationships are consistent with model prediction. Further, the bias is smaller at lower frequencies but non-trivial even at a quarterly frequency; (3) the intertemporal substitution bias is closely correlated with the frequency and magnitude of sales at the product category level, which is again consistent with the model.

We then replace purchase prices and quantities with the consumption prices and quantities estimated using our methodology and construct price indexes. We empirically show that the intertemporal substitution bias of these price indexes is much smaller than in price indexes using purchase-based data; however, non-trivial bias still remains. We show that this is also consistent with our model, which predicts that while the consumption price declines at the start of a sale, closely following the purchase price, after the sale ends, the consumption price returns to the regular price level only gradually, even though the purchase price returns to it immediately. Due to this asymmetric evolution of the consumption price, the consumption-based Törnqvist price index deviates downward

[2]Intertemporal substitution bias is often referred to as a reason of chain drift, as the bias arises typically in chained price indexes. Notable empirical studies on chain drift include Frisch (1936), Reinsdorf (1999), Feenstra and Shapiro (2003), de Haan (2008), de Haan and van der Grient (2011), Ivancic et al. (2011), Nakamura et al. (2011), de Haan and Krsinich (2014), Goolsbee and Klenow (2018), and Diewert and Fox (2020). The direction of the chain drift documented in these studies differs by country and type of goods examined and the index formula and frequency of data used. Feenstra and Shapiro (2003) show that chain drift associated with a weekly Törnqvist/Fisher index is positive for tuna in the United States, while de Haan (2008) and Ivancic et al. (2011) show that it is negative for detergents in the Netherlands, and for a wide range of products sold in Australia. Meanwhile, Nakamura et al. (2011) show that it is positive for coffee and negative for soft drinks in the United States. Furthermore, the mechanism of the chain drift is not yet well understood either empirically or theoretically. Particularly, to the best of our knowledge, no empirical study investigates whether consumers' stockpiling during a sale generates the chain drift. These studies often stop their empirical investigations on chain drift simply by comparing price changes when they use chained price indices and when they use fixed base price indices and/or price changes when they use high-frequency data and when they do low-frequency data, although Nakamura, Nakamura, and Nakamura (2011) investigate the effect of temporary sales on the bias by comparing price changes when they include temporary sales and when they exclude them.

from the COLI. However, using the model, we show theoretically that intertemporal substitution bias is completely eliminated for the consumption-weighted order $r$ superlative index and confirm this empirically.

The third contribution of our study is that we apply our methodology to a range of other issues, such as the estimation of the elasticity of substitution, the examination of the response of consumer stockpiling to consumption tax hikes, and the analysis of the extent to which stockpiling is affected by business cycle fluctuations. For instance, previous studies including Ogaki and Reinhart (1998), Erdem et al. (2003), Hendel and Nevo (2004), Cashin and Unayama (2016, 2021), and Baker et al. (forthcoming) have shown that stockpiling matters for the estimation of the elasticity of substitution. Specifically, Hendel and Nevo (2004) show that the response of purchase quantities to a price reduction consists of the consumption effect (i.e., consumers consume more as the good is cheap now) and the stockpiling effect (i.e., consumers stockpile as the good is cheap now), suggesting that estimation based on purchase-based data leads to an upward bias. We empirically compare estimates of the elasticity of substitution based on purchase-based data and those based on consumption-based data and find that the elasticity of substitution is indeed overestimated when it is estimated using purchase-based data.

As for consumers' response to consumption tax hikes, we show that Japan's consumption tax hike in April 2014 led households to increase stockpiling one month before the tax hike, and that stockpiling tended to be more pronounced for goods with longer storability.[3] Finally, regarding the extent to which stockpiling is affected by the business cycle, we empirically find that the extent of stockpiling during a sale depends on labor market conditions as well as financial conditions (which we represent by the interest rate level). Specifically, we find that (1) the extent of stockpiling tends to be smaller when hours worked are longer, implying that, due to longer working hours, consumers have less time to spend on searching for temporary sales, and (2) consumers tend to stockpile less when financial conditions are tighter (interest rates higher), presumably due to the higher costs of financing stockpiling.[4]

[3]Another study using consumer stockpiling in response to a consumption tax hike in Japan (the hike in April 1997) to estimate the intertemporal elasticity of substitution is that by Cashin and Unayama (2016, 2021). For related studies focusing on sales tax increases in the United States, see Baker et al. (forthcoming).

[4]These results are in line with studies by Klenow and Willis (2007), Sudo et al. (2018), and Kryvtsov and Vincent (2020) showing that there is a statistically significant link between temporary sales and

Our study is related to three strands of research. The first strand consists of research on household inventories of storable goods. Boizot et al. (2001), Griffith et al. (2009), Hendel and Nevo (2006a), and Kano (2018) empirically document patterns of consumer purchases and consumption.[5] Among these studies, those by Boizot et al. (2001) and Kano (2018) are noteworthy in that they are part of a small group of studies that, like ours, use household inventory data. In terms of the theoretical approach, our model is closest to that developed by Hendel and Nevo (2006a) but differs from it in that our model is a quasi-dynamic one with producer households, who hold and sell inventories, and consumer households, who do not hold inventories. Moreover, while we focus on a methodology to estimate consumption, consumption prices, and changes in inventories, they focus on intertemporal price discrimination.

The second strand of literature our study is related to is that on methodologies for eliminating chain drift. Ivancic et al. (2011), de Haan and van der Grient (2011), and de Haan and Krsinich (2014) propose using the GEKS index (originally proposed by Gini, Elteto, Koves, and Szulc). This is constructed to satisfy the circularity test, which is one of several desirable axioms for price indexes. An important difference from our approach is that the GEKS index is not based on intertemporal optimization by consumers. Moreover, the GEKS index is revised each time new data become available. Meanwhile, Feenstra and Shapiro (2003) construct a simple model incorporating storable goods and propose using a virtually fixed base price index. However, their approach is based on restrictive assumptions such as that households have perfect foresight and that households' expenditure takes a specific functional form that is independent of inventories. Moreover, the weights associated with the price index are assumed to be constant over time, which is inconsistent with our finding that consumption fluctuates over time, depending on the level of inventories consumers hold.

The third strand of literature that our study is related to is that on COLIs. This literature dates back to Könus (1924), and of more recent contributions, studies to which ours is most closely related include those by Feenstra and Shapiro (2003), Chevalier and

business cycles. Similarly, Sheremirov (2020) shows that temporary sales account for price dispersion. These studies show that temporary sales are important for considering business cycles even if they are highly short-run (e.g., daily or weekly) events like household stockpiling; however, they focus on temporary sales and do not explicitly discuss household stockpiling associated with them.

[5]Also see Hendel and Nevo (2004, 2013), Cashin and Unayama (2016, 2021), Erdem et al. (2003), Hendel and Nevo (2006b), and Osborne (2018).

Kashyap (2019), Reis (2009), Gowrisankaran and Rysman (2012), Wang (2013), Osborne (2018), and Ueda (2020). The studies by Feenstra and Shapiro (2003) and Chevalier and Kashyap (2019) propose a proxy for COLIs to deal with sales in an environment where consumers' optimization problem is static. The other studies examine dynamic COLIs rather than static COLIs by incorporating intertemporal substitution by households. For instance, Osborne (2018) constructs a model that, like ours, incorporates both storable goods and sales. He then computes a dynamic COLI by calculating the sequence of taxes on or subsidies to households that would keep their period utility constant over time. While Osborne's dynamic COLI is an important contribution to the literature, a dynamic COLI is complex and difficult to calculate and, in addition, hard to interpret.[6] Meanwhile, Ueda (2020) considers two types of dynamic COLI and shows that they do not simultaneously satisfy monotonicity and the time reversal test. In contrast to these studies, our model is a quasi-dynamic one in which consumers solve a static problem. Using our model makes it straightforward to define, construct, and interpret a COLI, so that our approach is useful to practitioners at national statistics offices who seek to construct scanner data-based price indexes that are free from intertemporal substitution bias.

The remainder of this study is organized as follows. Section 2 develops a quasi-dynamic model on household stockpiling behavior. Section 3 provides an overview of the scanner data for Japan that we use and presents stylized facts on intertemporal substitution bias in chained price indexes, on household inventory, and on the relationship between the two. Section 4 shows the empirical results, while Section 5 discusses the application of our methodology to other areas involving consumer stockpiling. Section 6 concludes.

# 2 Quasi-Dynamic Model

## 2.1 Setup

In this section, we construct a simple partial-equilibrium model of household inventory. We assume that product $k \in K_t$ is storable and, for simplicity, that it does not depreciate.

[6] Osborne (2018) empirically shows that the price indexes proposed by Feenstra and Shapiro (2003) and Chevalier and Kashyap (2018) approximate his dynamic COLI. However, his analysis is limited to particular products (canned tuna and canned soup) for particular periods.

Time $t$ is a discrete day.

The novelty of our model is that we assume that households comprise consumers and household producers and distinguish between the two. That is, household producers are a special type of household member that has technology to hold inventories, although there is a cost associated. The household producer provides inventory services consisting of purchasing storable goods from manufacturers (the quantity purchased denoted by $x_t^k$), holding inventory (denoted by $i_t^k$), and selling the goods to consumers (the quantity sold denoted by $y_t^k$). Market entry is free, so that the expected firm value is zero. In this respect, there is no loss of consumer surplus. On the other hand, consumers in a narrow sense cannot hold inventory: their purchases always equal their consumption, $c_t^k$. They purchase goods from household producers and/or manufacturers at the *consumption price* $r_t^k$.

This framework enables us to infer inventories, consumption and consumption price, which are often unobservable (e.g., in the retailer-side POS scanner data we use) and distinct from the observable quantity and price purchased. Furthermore, we can derive a COLI in a conventional static manner. All we need to know is two variables: $c_t^k$ and $r_t^k$. We do not need to use a complex dynamic COLI. Although the economy is hypothetical, many key properties continue to hold.[7]

We consider the optimization problems of household producers and consumers. The price of storable goods, $p_t^k$, can take one of two different values, a high (regular) value and a low (sales) value, which are determined stochastically and exogenously.

### Consumers

There are a unit mass of consumers. Consumers' cost minimization problem is given by

$$\min_{c_t^k} \left\{ \sum_{k\in K_t} r_t^k c_t^k + \lambda_t \left\{ U - \left[ \sum_{k\in K_t} b^k \left(c_t^k\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right\} \right\}, \tag{1}$$

[7]For example, stylized facts we show in the next section can be explained not only by a dynamic model like the one used by Hendel and Nevo (2006a) but also by our quasi-dynamic model. On the other hand, it would be difficult to explain the stylized facts using a completely static model ignoring storability. A caveat of this framework is that consumers do not have a consumption-smoothing motive. If the consumption price jumps, consumption also jumps. However, this caveat applies to all the conventional framework that considers consumers' static optimization problem when discussing price indices. The caveat emerges even if goods are perishable. To incorporate consumption smoothing, a dynamic model and dynamic COLIs are needed, but as we discussed in Introduction, interpreting the short-run movements of dynamic COLIs is often difficult.

where $U$ is the target utility, $\sigma(>0)$ denotes the elasticity of substitution, and $b^k$ is a taste or quality parameter for product $k$ in period $t$.[8] We extensively use the following relation with respect to the optimal quantity purchased:

$$c_t^k = \left(r_t^k / r_{t'}^k\right)^{-\sigma} c_{t'}^k. \tag{2}$$

**Household producers**

Household producers maximize their "firm" value:

$$V(i_{t-1}, p_t) = \mathrm{E}_t \left[ \sum_{j=0}^{\infty} \beta^j \left\{ \sum_{k \in K_{t+j}} \left( r_{t+j}^k y_{t+j}^k - p_{t+j}^k x_{t+j}^k - C(i_{t+j}^k) \right) \right\} \right], \tag{3}$$

subject to the cost of inventory, $C(0) > 0$, $C(i)' > 0$, $C(i)'' \geq 0$, and the evolution of inventory:

$$i_t^k = i_{t-1}^k - y_t^k + x_t^k. \tag{4}$$

Household producers sell amount $y_t^k$ of product $k$ to consumers at consumption price $r_t^k$. Furthermore, purchases and inventories must be nonnegative:

$$x_t^k,\ i_t^k \geq 0. \tag{5}$$

The first-order conditions with respect to $x_t^k$ and $i_t^k$ are

$$0 = r_t^k - p_t^k + \psi_t^k, \tag{6}$$

$$C'(i_t^k) = \beta \mathrm{E}_t[r_{t+1}^k] - r_t^k + \mu_t^k, \tag{7}$$

where $\psi_t^k$ and $\mu_t^k$ represent the Lagrange multipliers with respect to $x_t^k$ and $i_t^k$, respectively. Note that $\psi_t^k$ is strictly positive when $x_t^k$ is zero, and zero when $x_t^k$ is positive. Likewise, $\mu_t^k$ is strictly positive when $i_t^k$ is zero, and zero when $i_t^k$ is positive.

The free entry condition leads to a nonpositive value for an entering household producer with zero inventory holdings: $V(i_{t-1} = 0, p_t) \leq 0$.

[8] There is no consumer heterogeneity in the model. Many previous studies, such as Boizot, Robin, and Visser (2001) and Hendel and Nevo (2004, 2006a), investigate the frequency of purchases conditional on past purchase behavior. To obtain quantitatively plausible values for this, we need household heterogeneity, because otherwise the frequency of purchases would be either zero or one. In our study, we ignore consumer heterogeneity to focus on macroeconomic implications rather than individual behavior.

### Prices

The prices of storable goods follow a Markov process. They take one of the following two values: $P_H$ when there is no sale and $P_L$ ($P_H > P_L$) during a sale. Moreover:

$$\begin{aligned} \text{Prob}(P_L|P_H) &= \overline{q} \\ \text{Prob}(P_L|P_L) &= \underline{q}. \end{aligned} \tag{8}$$

The unconditional frequency of sales $\text{Prob}(P_L)$ is $f = \overline{q}/(1 + \overline{q} - \underline{q})$.

### Market clearing

Goods market clearing is given by $\int_0^{N_t} y_{t,j}^k dj + \int_0^{M_t} z_{t,j}^k dj = \int_0^1 c_{t,j}^k dj$, where $z_{t,j}^k$ represents the direct supply of storable product $k$ by manufacturers to consumers. Household consumption $c_t$ equals consumers' purchases from household producers, $y_t$, and manufacturers, $z_t$. In the market, there are a unit mass of consumers, $N_t$ represents household producers, and $M_t$ represents manufacturers.

Note that the household producers we consider in the model are still part of the households, although we separate them to simplify our analysis. Thus, the quantity purchased that is recorded in the POS data, $X_t$, should equal the sum of the quantity purchased by household producers $\int_0^{N_t} x_{t,j}^k dj$ and the quantity purchased directly by consumers $\int_0^{N_t} z_{t,j}^k dj$. Clearly, this is not equal to aggregate consumption $\int_0^1 c_{t,j}^k dj$.

### The COLI

As highlighted, consumers' optimization problem is static, so our COLI is identical to the conventional COLI. Consumers' cost minimization problem subject to constant utility yields the following equation for the optimal quantity consumed:

$$c_t^k = \left(r_t^k / b^k\right)^{-\sigma} \lambda_t^\sigma U. \tag{9}$$

The unit cost function, $\lambda_t = \mathcal{C}(r_t)$ for $U = 1$, is given by

$$\mathcal{C}(r_t) = \sum_{k \in K_t} r_t^k c_t^k = \left[ \sum_{k \in K_t} \left(b^k\right)^\sigma \left(r_t^k\right)^{1-\sigma} \right]^{1/(1-\sigma)}. \tag{10}$$

Although $b^k$ is unobservable and even if it is time varying, equation (2) tells us that we need to know only two variables for each period, the consumption price $r_t^k$ and the

consumption share $r_t^k c_t^k$, to calculate the change in the COLI between period $t$ and period $t'$, $C(r_t)/C(r_{t'})$.[9]

## 2.2 Equilibrium Properties

We discuss the equilibrium properties of the model. In the following discussion, we omit the superscript for product $k$ for simplicity. Further, we denote aggregate inventories at the end of period $t-1$, by $I_{t-1} \equiv \int_0^{N_t} i_{t-1,j} dj$.

The first lemma states the property of the consumption price, the price at which consumers make their purchase, $r_t$. When $p_t$ equals $P_L$ (during a sale), $r_t$ equals $P_L$. That is, consumers purchase goods directly from manufacturers at $r_t = P_L$. When $p_t = P_H$ (during a non-sale), consumers may purchase goods from household producers at a price below $P_H$. Price $r_t$ is lower the larger inventories $I_{t-1}$ are.

**Lemma 1** *Consumption price $r_t$ satisfies $P_L \leq r_t \leq P_H$. When $p_t = P_L$, $r_t = P_L$. Furthermore, $r_t = r(I_{t-1}, p_t, b)$ is nondecreasing in $p_t$ and $b$ and nonincreasing in $I_{t-1}$.*

The proofs of this and the lemmas that follow are provided in Appendix A. The next lemma states the stockpiling behavior of household producers. Only when $p_t = P_L$ do household producers purchase goods and hold inventories with the aim of selling the goods at a higher price after the sale has ended.

**Lemma 2** *If $p_t = P_H$, household producers do not purchase goods, that is, $x_t = 0$. If $p_t = P_L$, household producers purchase goods and hold inventories. Inventories are independent of $i_{t-1}$, $I_{t-1}$, and $b$.*

The left-hand panel of Figure 1 illustrates the pattern of price and quantity changes during a sales event when inventories are held just for one period. The top and bottom panels show prices and quantities, respectively. The sales event takes place in periods $t = 2$ and 3, when the price is lower than during other periods. The bottom panel shows that, in period $t = 2$, quantities purchased by household producers and consumers, $X_t$, represented by the solid dot, increase. The reason is not only that households consume more but also that they stockpile. Thus, $X_t$ is higher than consumption, $c_t$, represented

[9] Given that household producers form part of households, it would clearly be desirable to incorporate the effect of changes in their "firm" value on consumers' utility and the COLI. However, this effect likely is small, because the free entry condition means that the firm value is close to zero.

by the circle, with the difference representing stockpiling. In period $t = 3$, $X_t$ coincides with $c_t$, since there is no additional need for stockpiling. Then, in period $t = 4$ when the sale ends, household producers sell their inventories to consumers. The consumption price $r_t$, at which the household producers and consumers transact, lies between $P_L$ and $P_H$. Since consumption decreases with the consumption price, its level in period $t = 4$ is lower than in periods $t = 2$ and 3 but higher than in periods $t = 1$ and 5. At the beginning of $t = 5$, household producers hold no inventories, so consumers purchase goods at price $P_H$ and consume less.

### 2.2.1 Asymmetry for the quantity purchased (intertemporal substitution)

The next lemma shows that an asymmetry for the quantity purchased exists between price increase and decrease.

**Lemma 3** *The quantity purchased by household producers and consumers just before a sale is greater than or equal to that just after a sale.*

*The quantity purchased by household producers and consumers on the first day of a sale is greater than or equal to that on the final day of a sale.*

This asymmetry reflects the motive of intertemporal substitution. When a price is low, household producers stockpile, so that on the day after a sale, households do not need to purchase as much of the product as before the sale.

It should be noted that whether the quantity purchased on the first day of a sale is smaller than that purchased on the final day of a sale depends on whether prices are stochastic. In the model, prices are stochastic, and neither household producers nor consumers can accurately predict future prices. This is why household producers hold inventories at the end of the first day of a sale. Suppose instead that the duration of a sale is known *ex ante*. In this case, there is no incentive for household producers to stockpile except for the final day of the sale. Thus, the quantity purchased by household producers and consumers on the first day of a sale should be smaller than or equal to that on the final day of a sale.[10]

[10]Feenstra and Shapiro (2003) document upward chain drift for the Törnqvist index and conjecture that temporary sales "attract high purchases only when they are accompanied by advertising, and this tends to occur in the final weeks of a sale." If the advertisement specifies when the sale ends, their conjecture is consistent with our model.

### 2.2.2 State-dependent consumption

Consumption $c_t$ is state dependent; specifically, it depends on consumption price $r_t$.

**Lemma 4** *$c_t$ decreases in $r_t$. After a sale ends, $r_t$ and $c_t$ are nondecreasing and nonincreasing over time, respectively, until the next sale begins.*

### 2.2.3 Intertemporal substitution bias in chained price indices

Let us denote the price and quantity purchased for product $k$ in period $t$ by $p_t^k$ and $x_t^k$. Changes in chained price indices from $t-dt$ to $t$, $\pi_t^X$ $(X = L, P, T)$, are defined by

$$\pi_t^L = \sum_{k \in K_{t-dt} \cap K_t} W_{t-dt}^k (K_{t-dt} \cap K_t) \log \left( \frac{p_t^k}{p_{t-dt}^k} \right), \tag{11}$$

$$\pi_t^P = \sum_{k \in K_{t-dt} \cap K_t} W_t^k (K_{t-dt} \cap K_t) \log \left( \frac{p_t^k}{p_{t-dt}^k} \right), \tag{12}$$

$$\pi_t^T = \sum_{k \in K_{t-dt} \cap K_t} \frac{W_{t-dt}^k (K_{t-dt} \cap K_t) + W_t^k (K_{t-dt} \cap K_t)}{2} \log \left( \frac{p_t^k}{p_{t-dt}^k} \right), \tag{13}$$

based on the Laspeyres, Paasche, and Törnqvist approach, respectively.[11] The weight share $W_t^k(K_{t-dt} \cap K_t)$ equals $p_t^k x_t^k / \sum_{k' \in K_{t-dt} \cap K_t} p_t^{k'} x_t^{k'}$, where $k \in K_{t-dt} \cap K_t$ represents a domain of products that exist both in $t-dt$ and $t$ (such a common set is called a *matched sample*). Here, we consider only the case of $dt = 1$, although we calculate price indices for several values of $dt$ in the empirical part that follows. Further, we define a change in the consumption-weighted chained Törnqvist price index by $\pi^{T*}$, which we use consumption quantity $c_t^k$ and price $r_t^k$ instead of $x_t^k$ and $p_t^k$, and that in the COLI by $\pi^{COLI}$.

Moreover, we introduce the degree of stockpiling $m(\geq 0)$, which indicates how many days' worth of inventories remain in the hands of household producers. If $m = 0$, no stockpiling occurs during a sale and hence no bias (i.e., no chain drift) due to sales arises. Therefore, what we are interested in is the case where $m > 0$. In the next subsection, we will specify how $m$ is determined. In Appendix A, we discuss how equilibrium is determined in the special case of $m = 1$, that is, when inventories are cleared in just one period after a sale.

[11] There are various types of Laspeyres and Paasche indices. Here, we use the logarithmic Laspeyres and logarithmic Paasche indices.

**Lemma 5** *Consider one sales event for product $k$ such that $p_t = p_{t+T+1} = P_H$ and $p_{t+j} = P_L$ for $j = 1, \cdots, T$ ($T \geq 1$). Suppose that the prices and quantities of other goods remain unchanged; $\sum_{k' \in K_0 \cap K_t} p_t^{k'} x_t^{k'} = 1$; and the price before and after the sale is $P_H$ for a sufficiently long duration (i.e., $p_{t-j} = p_{t+T+1+j} = P_H$ for $j = 0, 1, \cdots, T_H$, where $T_H$ is sufficiently large compared with $m$).*

*If $\sigma > (<) 1$ and $m > 0$, the cumulative sum of changes in the chained price index from $t$ to $t + T + 1 + T_H$ satisfies $\pi^{COLI} = 0$; $\pi^L > (<) 0$; $\pi^P < (>) 0$; $\pi^T < 0$; and $\pi^{T*} < (>) 0$.*

*If $\sigma = 1$ or $m = 0$, then $\pi^{COLI} = \pi^L = \pi^P = \pi^{T*} = 0$ and $\pi^T < 0$.*

*If $\sigma > 1$, $\pi^P < \pi^T < 0 < \pi^L$. If $\sigma > 1$ and $m = 1$, $\pi^T < \pi^{T*} < 0$.*

This lemma shows that, when $\sigma > 1$, purchase-weighted chained price indices entail biases, with the Paasche and Törnqvist indices having a downward and the Laspeyres index having an upward bias. It is well known that the Törnqvist index is a good approximation of the COLI up to the second order (Diewert 1976). However, as pointed out by Feenstra and Shapiro (2003) and Ivancic, Diewert, and Fox (2011) among others, the purchase-weighted Törnqvist price index entails a bias, which stems from using quantities purchased rather than quantities consumed.

This bias is caused by the asymmetry for the quantity purchased (Lemma 3) for the following two reasons. First, the quantity purchased before a sale is greater than or equal to that just after a sale. This decreases the share of the product in households' total purchases when price increases at the end of a sale more than that when price decreases at the beginning of the sale. Thus, the purchase-weighted Törnqvist price index decreases. Second, the quantity purchased on the first day of a sale is greater than or equal to that on the final day of a sale. For the same reason, this also decreases purchase-weighted Törnqvist price index.[12]

[12]Chain drift is frequently discussed in the context of a nonlinear correlation between changes in prices and changes in quantities purchased (price/quantity bouncing), which implies that a weight share for a product used to calculate a price index is asymmetric with respect to positive and negative price changes. Our model shows that stockpiling causes such price/quantity bouncing as a result of an intertemporal substitution. A similar argument was made, for example, by Triplett (2003), de Haan and van der Grient (2011), von Auer (2019). Also note that the relation between the Laspeyres and the Paasche indices, $\pi^P < 0 < \pi^L$, holds even when no stockpiling occurs. Thus, one may argue that the chain drift does not necessarily occur due to stockpiling. However, the bias in the Törnqvist price index, $\pi^T < 0$ cannot be explained without stockpiling.

The above lemma also suggests that using purchase weights is not the sole reason for the bias. Using consumption weights does not eliminate the bias from the Törnqvist index (i.e., $\pi^{T*} < 0$ if $\sigma > 1$) because the path of consumption prices is asymmetric. Specifically, the model shows that the consumption price decreases quickly when the purchase price drops at the start of a sale but then increases again only gradually when the purchase price returns to the regular price after the sale ends, as a result of household inventories. Such asymmetry is not a coincidence but a natural outcome of the non-negative constraints on inventories and purchases. This asymmetric response of the consumption price and the fact that a temporary price change almost always consists of a decrease (i.e., a sale) mean that although the Törnqvist price index is a good approximation of the COLI up to the second order, the third-order approximation error cannot be eliminated if $\sigma > 1$, so that the Törnqvist price index continues to entail a downward bias even when it is constructed based on consumption weights.

The elasticity of substitution $\sigma$ influences the sign of chain drift for purchase-weighted indices. When $\sigma < 1$, the sign of chain drift changes for the Paasche and Laspeyres indices changes, entailing positive and negative biases, respectively. Interestingly, the sign of a bias in the Törnqvist index is negative irrespective of $\sigma$.

### 2.2.4 The sign and magnitude of the bias in chained price

The next lemma shows how the bias in the purchase-weighted Törnqvist index depends on variables associated with temporary sales. We consider a price change over a sufficiently long period (e.g., 30 years), which we denote by $\overline{\pi^T}$. The following lemma shows that $\overline{\pi^T}$ depends on the probability of sales and the ratio of the sale price to the regular price, as $\overline{\pi^T} = \overline{\pi^T}(\overline{q}, \underline{q}, P_L/P_H)$.

**Lemma 6** *Suppose $\sigma > 1$ and $m > 0$. Then, $\overline{\pi^T} = \overline{\pi^T}(\overline{q}, \underline{q}, P_L/P_H)$ is decreasing with $\overline{q}$, increasing with $\underline{q}$, and increasing with $P_L/P_H$.*

The result that $\overline{\pi^T}$ increases with $\underline{q}$ may appear counterintuitive. The reason for this is that an increase in $\underline{q}$ lengthens the duration of a sale. This decreases the number of sales event, and thus, increases $\overline{\pi^T}$.

The following lemma shows that the sign of a bias in the Törnqvist index can be positive in the following case.

**Lemma 7** *Suppose that households know when a sale will end once it begins. Then, $\pi^T$ is positive if $m$ exceeds a certain threshold and negative otherwise.*

As we noted, the asymmetry for the quantity purchased causes the bias in chained price indices. When the asymmetry is in the opposite sign, that is, when the quantity purchased on the first day of a sale is smaller than or equal to that on the final day of a sale, the sign of a bias in the Törnqvist index can be positive.

### 2.2.5 COLI

To resolve the bias in chained indices and derive a better approximation of the COLI, we need a superlative index that takes the elasticity of substitution $\sigma$ into account. One candidate is the order $r$ superlative index, where we define $P_r$ as

$$P_r(r_0, r_1, c_0, c_1) = \frac{\left\{\sum_{k \in K_0 \cap K_1} s_0^k \left(r_1^k / r_0^k\right)^{(1-\sigma)}\right\}^{1/\{2(1-\sigma)\}}}{\left\{\sum_{k \in K_0 \cap K_1} s_1^k \left(r_0^k / r_1^k\right)^{(1-\sigma)}\right\}^{1/\{2(1-\sigma)\}}}, \tag{14}$$

where $s_t^k$ represents the consumption share of product $k$ in period $t$.

The following lemma shows that $P_r$ serves as a COLI if the unit cost function is expressed as

$$\mathcal{C}(r_t) = \left[\sum_{i \in K_0 \cap K_1} \sum_{k \in K_0 \cap K_1} \alpha^{ik} \left(r_t^i\right)^{(1-\sigma)} \left(r_t^k\right)^{(1-\sigma)}\right]^{1/\{2(1-\sigma)\}} \tag{15}$$

where $\alpha^{ik} = \alpha^{ki}$. This cost function is based on a more generalized form of utility than that given by equation (1).[13]

**Lemma 8** *Given the unit cost function of (15), $P_r$ equals $\mathcal{C}(r_1)/\mathcal{C}(r_0)$.*

## 2.3 Inference of Consumption and Consumption Prices and Calculation of the COLI

### 2.3.1 Motivation

In Section 3, we examine the validity of the model predictions regarding prices and quantities purchased, which were shown in Lemmas 3 to 6, using scanner data for Japan. Before doing this, we should note that many important variables such as consumption, inventories, and the consumption price are often unobservable, although we can observe the quantity purchased and posted price using retailer-side POS scanner data. From a

[13] Another index that can serve as a COLI is the Lloyd–Moulton Index. See ILO et al. (2004).

practical perspective, consumption and the consumption price are essential for constructing the COLI.[14] For macroeconomists, it is of great interest to see whether any changes in stockpiling behavior can be observed over time and, if so, what the determinants are. Therefore, in this subsection, we propose a simple and tractable methodology to infer these variables using retailer-side scanner data.

Some of the previous studies on households' inventory behavior, including Erdem, Imai, and Keane (2003) and Hendel and Nevo (2006b), structurally estimate the parameters associated with inventory cost functions, which would be helpful in identifying the paths of consumption, inventories, and consumption prices. These studies employ household-side scanner data, where the quantities and prices of the goods purchased by individual households are recorded, to conduct their empirical exercises. However, these data often lack information on purchase prices, making it difficult for us to take an approach similar to theirs. More importantly, few practitioners at national statistical offices have access to household-side scanner data, so that an approach for constructing a COLI based on consumption and consumption prices that requires household-side scanner data would not be very useful for them. Furthermore, the empirical exercises conducted in the previous studies focus on a particular product (e.g., ketchup in Erdem, Imai, and Keane (2003) and detergent in Hendel and Nevo (2006b)) to estimate the parameters associated with the inventory cost function. The task we have set ourselves in this study is quite different, in that we seek to estimate a COLI which covers numerous products that are heterogeneous in terms of their storability and the form of their inventory cost function. However, reliably estimating inventory cost functions for all products likely is next to impossible, so that we use a different approach.

Specifically, we employ retailer-side scanner data rather than household-side scanner data to construct a COLI. Retailer-side scanner data, which record transactions at the store level rather than at the household level, are widely used by national statistical offices in an attempt to construct COLIs using alternative data sources (see, for example, chapter 10 of ILO et al., 2020). Because it is almost impossible to obtain reliable estimates for the inventory cost functions of all products, we introduce an assumption about the path of consumption prices after a sale ends; that is, we assume that the

[14]How to deal with chain drift including intertemporal substitution bias is an important issue for national statistics offices, especially when they employ high frequency scanner data in constructing consumer price indexes. See Konny et al. (2019), Eurostat (2017), and EFTA (2020) for more on this practical aspect of chain drift.

increase in consumption prices after a sale ends is *linear* with time. This assumption allows us to estimate consumption, consumption prices, and inventories from retailer-side scanner data and, as explained later, is consistent with the model developed in the previous section. Note that previous studies on household inventories often assume that consumption is constant over time (e.g., Boizot, Robin, and Visser, 2001 and the empirical part of Hendel and Nevo, 2006a). The assumption of constant consumption may allow us to infer consumption, consumption prices, and inventories. However, this implies that consumption prices are constant in our model. This is inconsistent with our model, where consumption prices fluctuate taking a value between $P_L$ and $P_H$.

### 2.3.2 Methodology

The consumption price at $p_t = P_L$ equals $P_L$. The key variable is the consumption price at $p_t = P_H$ after a sale ends, that is, $r_H(I_{t-1}) \equiv r(I_{t-1}, P_H)$, where $r_H(0) = P_H$. Knowing this variable enables us to obtain consumption $c_t$ after a sale using equation (2).

When $p_t = P_H$, household producers' optimization problem is given by

$$C'(i_H; I_{t-1}) = \beta \left\{(1 - \overline{q}) r_H(I_t) + \overline{q} P_L\right\} - r_H(I_{t-1}) + \mu_t \tag{16}$$

from equation (7). This equation shows that household producers strike a balance between the benefits of a future consumption-price increase (the right-hand side) and the costs of holding inventories (the left-hand side). Note that $\mu_t$ is the Lagrange multiplier associated with $i_t$ and equals zero if $i_t > 0$.

We note that when inventory cost function $C(\cdot)$ is written in a certain form (see Appendix B), $r_H(I_t) - r_H(I_{t-1})$ becomes a positive constant. In other words, the expectation of a *linear* consumption-price increase prevents household producers from selling all of their inventories instantaneously or from selling none at all. Household producers gradually sell off their inventories to consumers.

In the following analysis, we assume this linearity holds. While this admittedly is a restrictive assumption, it can be interpreted as an intermediate of the following two scenarios. The first is when $C'' \to 0$. In this scenario, $r_H(I_t) - r_H(I_{t-1})$ increases in $t$. Put differently, if inventory costs are not convex, household producers require a greater consumption-price increase as time goes by, because they discount the future ($\beta < 1$) and expect another sale to come at some point ($\overline{q} > 0$). As for the second scenario, suppose $\beta = 1$ and $\overline{q} = 0$. Then $r_H(I_t) - r_H(I_{t-1})$ decreases in $t$. The cost of holding inventories decreases as household producers' inventories decrease because of $C'' > 0$,

which makes household producers require a smaller consumption-price increase as time goes by.

The benefit of this linearity assumption is that it greatly simplifies our analysis. Given the path of $r_H(I_{t-1})$, we compute consumption $c_t$ as $(r_H(I_{t-1})/P_L)^{-\sigma} c_L^*$, where $c_L^*$ represents consumption during a sale. Briefly put, $c_L^*$ is the lower value of the quantity purchased (observable in the POS data) during a sale because according to the model, consumption during a sale is equal to the lowest amount of purchases during the sale. Furthermore, we can calculate the degree of stockpiling, $m$. More precisely, we define $m$ to denote how long inventories last after a sale ends. In continuous time, we can derive the following equation:

$$m_{cont} = \frac{P_H - P_L}{P_L} \frac{\sigma - 1}{1 - (P_H/P_L)^{-\sigma+1}} \frac{I_L}{c_L^*}. \tag{17}$$

The proof is provided in Appendix B. Simply put, the equation can be derived because cumulative consumption for $m_{cont}$ periods equals the initial inventories outstanding just after a sale ends, $I_L$, and the consumption price linearly increases from $P_L$ to $P_H$ in $m_{cont}$ periods. Inventories $I_L$ equal the cumulative amount of purchases during a sale minus the cumulative amount of consumption, $Tc_L^*$, in the same period. Because the variables $P_H$, $P_L$, $c_L^*$, $\sigma$, and $I_L$ are observable/estimable from purchase data, we can estimate $m_{cont}$, the path of consumption prices, and the path of quantities consumed after a sale ends. In Section 4, we apply our methodology to infer consumption from retail scanner data and verify the methodology empirically by showing that the consumption-weighted Törnqvist price index and order $r$ superlative index have small but still downward bias and no bias, respectively, as shown in Lemmas 5 and 8.

The right-hand panel of Figure 1 illustrates the pattern of price and quantity changes in the case of $m = 5$. The consumption price, depicted by the circles in the top panel, increases at a constant rate from $t = 3$ to 8. The bottom panel shows that while the quantity purchased falls to zero from $t = 4$ to 7, consumption does not fall to zero, but decreases gradually.

In Appendix C, we employ numerical simulations to depict a typical path of the price and quantity and calculate biases in chained price indices. It shows that the COLI and the chained order $r$ superlative index do not show any bias and asymptotically return to their original level. The chained consumption-weighted Törnqvist index has a downward bias, while the purchase-weighted price indices have a much larger bias. The size of the bias in the purchase-weighted Törnqvist index is comparable to the actual size of the

bias in the Törnqvist index for Japan.

### 2.3.3 Robustness of Our Approach

It should be noted that the fact that goods are storable means that price indices will always be subject to the intertemporal substitution bias. This is the case regardless of whether a simplifying assumption such as the one we introduce is employed. In fact, the model in the previous section succeeded in qualitatively explaining the bias in various chained price indices, including its sign, without relying on this simplifying assumption.

However, additional assumptions are needed for quantitative purposes such as investigating the size of the bias. Since the linearity assumption is only one of many possible assumptions, in Appendix D, we examine how much the size of the chain drift changes in response to different assumptions about the path of the consumption price after a sale ends. We conclude that the alternative approaches leave our quantitative results more or less unchanged.

# 3 Stylized Facts Based on Japanese Scanner Data

In this section, we examine whether the model we have constructed in the previous section is valid. Specifically, using retailer-side scanner data, we show that chained price indices have sizable biases and that the sign and magnitude of the biases are consistent with the model predictions. This examination helps not only validate the model but also confirm the importance of household inventory for explaining the biases in price indices, because the biases are not necessarily driven solely by household inventory.

## 3.1 Two Kinds of Scanner Data

We use two sets of scanner data for Japan. The first set consists of retailer-side data, namely, the point-of-sale (POS) scanner data collected by Nikkei Inc. The data include the number of units sold and the sales amount (price times the number of units sold) for each product and retailer on a daily basis. The observation period is around thirty years, running from March 1, 1988 to December 31, 2019. Products recorded consist of processed food and daily necessities, covering 170 of the 588 categories in the CPI

and making up about 20 percent of households' expenditure. See Appendix E as to how we aggregate variables of interest over days, products, and retailers, and Abe and Tonogi (2010), Sudo, Ueda, and Watanabe (2014), and Sudo et al. (2018) for a detailed description of the data.

The other set consists of household-side data, namely, "*Shoku-map*"[15] scanner data collected by Lifescape Marketing Co. Respondents are mainly a female homemaker, and the data cover about 400 households in each period (about 4,000 households in total). The data record the number of units purchased and the date of consumption for each product and household on a daily basis. Moreover, they record when consumption ends (i.e., when a product is used up or has gone off, etc.) for each product and household. The data cover the period from September 1998 to February 2019. Note that products recorded are food only and there is no information on purchase prices. Another limitation is that there is no information on how much of a product (e.g., in terms of weight) is consumed each time it is consumed. The data record both the number of units purchased and consumed, which is sufficiently useful if products are consumed in discrete units, such as a cup of instant noodles or a can of beer. However, for products like salt, we do not know how much a household uses, although we do know the dates when they are used.[16] For example, Figure 2 shows the consumption pattern for salt of a particular household. In the figure, each vertical line represents a consumption flag. The figure indicates that the household purchased salt on day $t = 19$, started using it on day $t = 22$, and used it up on day $t = 144$. The inventory duration thus is $144 - 19 + 1 = 126$ days. See Appendix F for detailed explanations about the *Shoku-map* data.

In both sets of data, all products are identified by the Japanese Article Number (JAN) code, which enables us to merge the datasets. Further, we classify products into groups using the 3-digit product categories provided by Nikkei Inc. There are 218 categories in total, such as instant cup noodles, yogurt, beer, and toothbrushes.

In the following analyses using the POS data, we identify temporary sales by employing a sales filter. Specifically, we follow the procedure explained in Nakamura and Steinsson (2010) using their sales filter A with a window of $L = K = J = 42$ days.[17]

[15] "*Shoku-map*" translates as "food map."

[16] If a household uses salt $N$ times a day, the data record $N$, where $N$ is an integer equal to or greater than zero.

[17] The window length is chosen following Eichenbaum, Jaimovich, and Rebelo (2011) and Kehoe and Midrigan (2015). For a detailed examination of the robustness of various filters to identify sales, see Sudo et al. (2018).

Product $k$ is classified as being on sale on date $t$ if and only if its price $p_t^k$ deviates from its regular price $\overline{p}_t^k$ by more than two yen.

When no sales are recorded for a particular product at a particular retailer on a particular date, earlier studies usually treated this simply as a missing observation, as if the product disappeared from shelves at the retailer on the date. However, the POS data quite often show no sales records for a particular product, retailer, and date after a sale, suggesting that stockpiling by households during a sale results in zero purchases after the sale ends. Since this conveys important information on household inventory, in this study, we interpolate missing observations by setting $p_t^k = \overline{p}_t^k$ and $x_t^k = 0$ if we have observations after $t$ (i.e., unless the product permanently exits from the market). That is, we set the quantity purchased to zero and the price to the regular price on the nearest past date.[18]

## 3.2 Four Stylized Facts

Using the two datasets, we present four stylized facts that are closely related to the predictions of the model that incorporates goods storability, namely, Lemma 3, 4, 5, and 6.

**Fact 1 (Lemma 3). Asymmetry for the quantity purchased exists between when price increases and decreases.**

Using the POS data, we collect the following variables for each product $k$ and sales event $s$: $T$ denotes the number of days a product is on sale, $P_H^1$ and $X_H^1$ denote the price and the quantity purchased just before a sale (say, in period $t$), respectively, $P_L^1$ and $X_L^1$ denote the average price and quantity purchased during the first half of the sale, respectively (i.e., from $t+1$ to $t+\lfloor T/2 \rfloor - 1$), $P_L^2$ and $X_L^2$ denote the average price and quantity purchased during the second half of the sale, respectively (i.e., from $t+\lfloor T/2 \rfloor$ to $t+T$), and $P_H^2$ and $X_H^2$ denote the price and the quantity purchased just after the sale, respectively (period $t+T+1$).[19]

[18] If we do not interpolate the price and quantity, the price increase after a sale is fully neglected because the product is not in the matched sample and, therefore, is excluded from the calculation of the changes in chained price indices in equations (11) to (13). Therefore, without interpolation, the downward bias of the Törnqvist price increases further.

[19] Variable $T$ is the smallest integer that is greater than or equal to one and satisfies $|p_{t+T+1} - \overline{p_{t+T+1}}| \leq 2$, where $\overline{p_{t+T+1}}$ represents the regular price.

Next, we compare the quantities purchased just before a sale and just after a sale (denoted by $Q_H$), both of which correspond to when a product is not on sale. We calculate the difference of the inverse hyperbolic sine function, that is, $Q_H = \log(X_H^2 + \sqrt{1 + (X_H^2)^2}) - \log(X_H^1 + \sqrt{1 + (X_H^1)^2})$ for each product $k$ and sales event $s$. This function is chosen because $X_H$ may take zero. Then, we calculate the weighted average of $Q_H$ over $k$ and $s$ within each 3-digit product category, where a weight is based on sales, that is, $P_H^1 X_H^1 + P_H^2 X_H^2$. Similarly, we compare the quantities purchased during the first half and the second half of a sale (denoted by $Q_L$), both of which correspond to when a product is on sale, as $Q_L = \log(X_L^2 + \sqrt{1 + (X_L^2)^2}) - \log(X_L^1 + \sqrt{1 + (X_L^1)^2})$. The weighted average is calculated based on sales given by $P_L^1 X_L^1 + P_L^2 X_L^2$. Both $Q_H$ and $Q_L$ indicate an asymmetry for the quantity purchased when price increases and decreases. That is, $Q_H$ represents an asymmetry between just before price decreases (a sale starts) and just after price increases (a sale ends) for a non-sale event, while $Q_L$ represents an asymmetry between just after price decreases (a sale starts) and just before price increases (a sale ends) for a sale event.

The left-hand panel of Figure 3 shows $Q_H$ and $Q_L$ for each year. It shows that $Q_H$ tends to be negative particularly before 2000, suggesting that the quantity purchased just after a sale is smaller than that just before the sale. However, $Q_H$ increased, and being not much different from zero in the 2010s.

A similar result can be observed for $Q_L$. Particularly, the fact $Q_L$ tends to be negative before 2000 suggests that the quantity purchased is larger at the beginning of a sale than at the end. This fact helps account for downward chain drift for the Törnqvist index. However, similar to $Q_H$, $Q_L$ increased. Moreover, it turned to positive in the 2010s. This suggests that the quantity purchased is smaller at the beginning of a sale than at the end, which works to generate upward chain drift for the Törnqvist index. This can occur when the duration of a sale is known *ex ante* because household producers do not need to stockpile except for the final day of the sale.

In summary, we find that $Q_H$ is negative and that $Q_L$ can take both a negative and positive value, although it used to be negative. This finding cannot be explained by a standard model of perishable goods because the same price should lead to the same quantity purchased, and thus, $Q_H$ and $Q_L$ should have mean zero (i.e., no asymmetry). By contrast, our model of storable goods can account for this asymmetry.

**Fact 2 (Lemma 4). Consumption tends to decrease as household inventories decrease.**

While earlier studies on household inventories often assume that consumption is constant, Hendel and Nevo (2006a) and Kano (2018) highlight that consumption is state dependent. To examine whether consumption is indeed state dependent, we investigate whether consumption decreases until the next purchase as inventories decrease, using the *Shoku-map* data. Suppose that household $i$ uses product $k$ on date $t_l \in t_1, t_2, \cdots, t_{n_{ikt}}$ and $t$ represents the purchase date. If the household uses product $k$ twice on date $t_{l_0}$, we record $t_{l_0}$ twice. Thus, $n_{ikt}$ represents the number of times product $k$ is used. We define the inventory on date $t^{'}$ $(t \leq t^{'} \leq t_{n_{ikt}})$ as $\lambda_{ikt'} = n_{ikt} - n^*$, where $n^*$ is the maximum integer $n$ that satisfies $t_n < t^{'}$. Note that $\lambda_{ikt'}$ is an integer between 0 and $n_{ikt}$. For example, $\lambda_{ikt'} = n_{ikt}$ for $t \leq t^{'} \leq t_1$. We further define the sum of inventory $\Lambda_{ijt}$ for household $i$ in product category $j$ at the beginning of date $t$ as

$$\Lambda_{ijt} = \sum_{k \in j} \lambda_{ikt}. \tag{18}$$

We then estimate the following linear probability model when $\Lambda_{ijt} > 0$:

$$y_{ijt} = c_i + d_j + \alpha \Lambda_{ijt} + \varepsilon_{ijt}, \tag{19}$$

where $y_{ijt}$ is a binary variable for consumption and takes a value of one if household $i$ uses products in product category $j$ on date $t$ and zero otherwise. As an alternative, we also use $y_{ijt}$ defined as the sum of the times that products in product category $j$ are used by household $i$ on date $t$.

As Table 1 shows, the coefficient on inventories is significantly positive, regardless of which dependent variable is used. This indicates that consumption is state dependent and decreases as inventories decrease.[20] Further, it can be pointed out that the size of the coefficient is small and significantly lower than one. This suggests consumption smoothing, that is, households do not consume as much as they purchase or hold as

[20]For the estimation presented in the table, we did not use instrumental variables but employed ordinary least squares (OLS). This means that the estimates are biased if, for example, $\mathrm{Cor}(\varepsilon_{ijt}, \varepsilon_{ijt-1}) > 0$, that is, if high consumption demand at time $t-1$ not only decreases inventories $\Lambda_{ijt}$ at time $t$ but also increases consumption at time $t$. If such endogeneity exists, the coefficient on inventories is underestimated when using OLS. Thus, the fact that we obtained a significantly positive estimate using OLS indicates that the state dependency of consumption continues to hold or is stronger than our estimates suggest. See Appendix B for the estimation results at a product category level.

inventories. Thus, consumption-weighted chained indices yield more stable price change than purchase-weighted chained indices.

**Fact 3 (Lemma 5). When weights are based on purchases, changes in price indices are characterized by the following inequality:** $\pi^P < \pi^T < 0 < \pi^L$.

We calculate changes in purchase-weighted chained price indices from $t - dt$ to $t$, $\pi_t^X$ $(X = L, P, T)$, defined by equations (11) to (13) based on the Laspeyres, Paasche, and Törnqvist approach, respectively. When $dt = 1$, we can construct the chained price indices using the cumulative sum of the past price changes: $P_t^X = \exp\left(\sum_{s=1}^{t} \pi_s^X\right)$ for $X = L, P, T$.

Using the POS data, we calculate the time series of the price level $(P_t^X)$, normalizing the initial price level to one. The results are striking. The left-hand panel of Figure 4 shows that the price level increases by almost $10^{92}$ over our 30-year observation period when based on the Laspeyres index, while it decreases to almost $10^{-111}$ when based on the Paasche index. The price decline is milder but nevertheless remains large when based on the Törnqvist index. The price level still decreases to almost $10^{-9}$. Thus, the following inequality holds: $\pi^P < \pi^T < 0 < \pi^L$, which is consistent with Lemma 5.

Although the sizes of the changes in the price indices shown above are by far too large, they do not necessarily imply a bias because we do not observe a true price index such as COLI to compare with these price indices. Thus, we apply the following test for a bias.

**Chain Drift Test** As is argued by Ivancic, Diewert, and Fox (2011) among others, a chained price index should take the same value at $t = 0$ and $\tau$ if prices and quantities for all products are equal at $t = 0$ and $\tau$ (i.e., no chain drift). We formulate this argument by defining chain drift $d_{0,\tau,dt}^X$ as

$$d_{0,\tau,dt}^X = \sum_{s=1}^{(\tau-1)/dt} \pi_{(s-1)dt,sdt}^X - \pi_{0,\tau-1}^X, \tag{20}$$

where $\tau$ represents a time span to measure chain drift and set to 365 days, and $\pi_{t_1,t_2}^X$ represents a change in a chained price index from $t_1$ to $t_2$ for $X = L, P, T$.[21] The first

[21] $\pi_{t_1,t_2}^X$ is the same as $\pi_{t_2}^X$ defined in equations (11) to (13) if $t_1 = t_2 - dt$ and the domain of products is the same. Here, we use a domain of products that exist from $t = 0$ and $\tau$ to apply this formulation to the POS data. This implies that we ignore the effect of product turnover for this calculation.

term on the right-hand side of the equation represents the change in the chained price index when it is chained forward from 0 to $\tau - 1$ by the time span of $dt$ gradually. The second term represents the change in the chained price index when it moves from 0 to $\tau - 1$ all at once.[22] If the chained price index has no chain drift, the second term should completely offset the first term, and thus, $d^X_{0,\tau,dt}$ should be zero. By definition, $d^T_{0,\tau,dt}$ is zero when $dt = 364$. We calculate $d^X_{0,\tau,dt}$ for each year of the observation period, each 3-digit product category, each price index ($X$), and each time span ($dt$).

The right-hand panel of Figure 4 shows the mean of $d^X_{0,\tau,dt}$ over years and 3-digit product categories for each price index ($X$) and time span ($dt$). This indicates considerable deviations from zero. Further, we find inequality given by $d^P < d^T < 0 < d^L$. The deviations are larger as $dt$ is smaller (e.g., $-40\%$ and $-1\%$ for $dt = 1$ and 28, respectively, when the Törnqvist index is used).

After calculating $d^X_{0,\tau,dt}$, we conduct a test for chain drift. The test is a simple sign test for the null hypothesis that the probability to take positive and negative signs for $d^X_{0,\tau,dt}$ is equal to 0.5 each. The first column of Table 2 shows that the null hypothesis is rejected for $dt = 1$.

**Use of Lower Frequency Data** One may recommend using lower frequency data (such as monthly or quarterly rather than daily data) or fixed base indices. They may help eliminate the bias in chained price indices because inventories of most products, particularly those we analyze in this study, last for less than a month. However, the use of lower frequency data and/or fixed base indices has three shortcomings.

First, as pointed out in many previous studies, the use of lower frequency data alone does not completely eliminate the bias. The right-hand panel shows that chain drift decreases as a time span ($dt$) increases. However, it still takes about $-1\%$ and $-0.5\%$ for $dt = 28$ and 182, respectively, when the Törnqvist index is used, which is not negligible. Further, the chain drift is significantly different from zero for all $dt$ as the first column of Table 2 shows. Given that in almost all industrial countries the consumer price index (CPI) is published on a monthly basis (around $dt = 28$), this suggests that using a frequency that is sufficiently low to avoid a bias is not a realistic solution (see also Ivancic, Diewert, and Fox 2011).

ILO et al. (2004) recommend that chained indices be replaced by fixed base indices

[22] This is equal to a price change from $\tau - 1$ to $\tau$ because we assume that prices and quantities for all products are equal at $t = 0$ and $\tau$.

when chain drift is serious. However, using fixed base indices also has shortcomings. The second shortcoming of using both lower frequency data and fixed base indices is that they ignore some products that are short lived or new. Szulc (1983) points out that a fixed base index is likely to be biased as the current period is distant from the base period. See also ILO et al. (2004, 2020) and Ivancic, Diewert, and Fox 2011).

The third shortcoming is that some of the high frequency (e.g., daily) fluctuations in prices and quantities may be closely related to business cycle fluctuations. Specifically, we will show that consumers change their stockpiling behavior depending on the macroeconomic environment such as hours worked and interest rates. This suggests that it may not be a good idea to discarding information from high frequency data in constructing price indices because this will yield biased price indices where the bias changes with the business cycle.[23]

**Fact 4 (Lemma 6). Chain drift is associated with consumers' stockpiling during temporary sales.**

Household inventory, particularly, the asymmetry of the quantity purchased, is not necessarily the sole reason for the chain drift. Thus, we next examine whether the asymmetry of the quantity purchased can account for chain drift. To this end, we investigate whether the extent of the asymmetry, $Q_H$ and $Q_L$, are correlated with chain drift, although causal inference is difficult because $Q_H$ and $Q_L$ are endogenous.

First, we examine correlations between the asymmetry of the quantity purchased ($Q_H$ and $Q_L$) and the chain drift ($d^T_{0,\tau,dt=1}$) based on the Törnqvist index in the dimension of time series. Observing increasing $Q_H$ and $Q_L$, one may anticipate the mitigation of downward chain drift. The right-hand panel of Figure 3 shows that this is indeed the case. The extent of downward chain drift has been decreasing. However, the chain drift remains negative.

Second, we investigate whether chain drift is positively associated with the asymmetry of the quantity purchased. We run a regression for $d^T_{0,\tau,dt=1}$ using $Q_H$ and $Q_L$ at the 3-digit product category level after we calculate the mean of each variable over years. The

[23] An analogical issue is whether we can ignore temporary sales for a monetary policy analysis. Since temporary sales are high frequency events lasting only around a week, most models ignore temporary sales despite the fact that they decrease price stickiness considerably. However, Klenow and Willis (2007), Sudo et al. (2018), and Kryvtsov and Vincent (2020) document statistically significant relations between temporary sales and business cycles, arguing that the real effect of monetary policy decreases.

first column of Table 3 shows the estimation result. Both of the coefficients on $Q_H$ and $Q_L$ are significantly positive at the 5% level.

Third, we investigate correlations between chain drift and variables associated with temporary sales, which are more exogenous than $Q_H$ and $Q_L$. We run a regression for $d^T_{0,\tau,dt=1}$ using variables associated with sales: $q_j$, $\overline{q_j}$, $\underline{q_j}$, and $(P_L/P_H)_j$, representing the probability that a product is on sale, the probability that a product will go on sale on the following day given that it is not currently on sale, the probability that a product will continue to be on sale on the following day given that it is currently on sale, and the ratio of the sale price to the regular price. respectively, for category $j$.

The second to fourth columns of Table 3 show the estimation results. The coefficient on $(P_L/P_H)_j$ is significantly positive, showing that the size of deflation increases as the size of sale discount is larger. The coefficient on $q_j$ is significantly negative, suggesting that the size of deflation increases as the probability of a sale increases. The third column shows that the coefficient on $\overline{q_j}$ is insignificant and that on $\underline{q_j}$ is significantly negative. This result changes when we control a degree of stockpiling, $m$. As the fourth column shows, the coefficient on $\overline{q_j}$ is significantly negative and that on $\underline{q_j}$ is insignificant. These estimation results are consistent with Lemma 7 except that the coefficient on $\underline{q_j}$ is insignificant. By simulations, we find that an increase in $\underline{q}$ increases the price change but that the size of the effect of $\underline{q}$ is small compared with that of $\overline{q}$ and $P_L/P_H$. Furthermore, the adjusted $R^2$ is around 0.5, suggesting that temporary sales can account for the bias in chained price indices to a considerable extent.

# 4 Application of Our Methodology to Japanese Retailer Scanner Data

Using the POS data, we calculate key variables such as consumption price $r_t = r_H(I_{t-1})$, consumption $c_t$, consumption during a sale $c^*_L$, the degree of stockpiling during a sale $m$, and inventory $I_L$ for each sales event for each product at each retailer, while we calculate the elasticity of substitution $\sigma$ for each 3-digit product category. Then, we check the validity of our approach in calculating these unobservable variables in several dimensions and calculate price indices based on consumption we infer.

## 4.1 Summary of Our Approach

We take the following four steps based on the model we have constructed. First, for each product and retailer, we identify all sales events using the sales filter.

Second, we record the price and the quantity purchased just before and after a sale. Further, we record the average price and the quantity purchased in the first and second half of a sale. This enables us to obtain consumption prices and the quantities consumed just before and during a sale (but not after a sale).

Third, we calculate the elasticity of substitution, $\sigma$, for each 3-digit product category. We apply the consumption prices and the quantities consumed before and during a sale obtained in the previous step to equation (2). We will come back to this point in Section 5.1.

Fourth, we calculate $r_t = r_H(I_{t-1})$, $c_t$, $c_L^*$, $m$, and $I_L$ for each sales event for each product at each retailer. We employ the methodology discussed in the previous subsection to calculate $m$. Once we obtain $m$, we can calculate the path of consumption prices $r_t = r_H(I_{t-1})$ and consumption $c_t$ after a sale ends.

In Appendix B, we provide a detailed explanation of how we calculate these variables.

## 4.2 The Degree of Stockpiling

We apply the approach explained in Section 4.1 to Japanese POS scanner data. Figure 5 presents a histogram of the degree of stockpiling $m_{cont}$ at the 3-digit product category level (hereafter we simplify the notation by dropping *cont* from $m_{cont}$). The distribution of $m$ ranges from one to five, with the mode being around two. Table 4 lists the top and bottom five product categories with the largest and smallest $m$. The top three categories are instant cup noodles, diluted beverages, and frozen meals, in that order. These products can indeed be stored for a long time. However, the storability of products does not necessarily imply a high degree of stockpiling. In fact, of the five bottom categories, razors, cosmetic accessories (e.g., hand cream and sunscreen lotion), home medical supplies, and batteries are also highly storable. What distinguishes them from storable goods such as cup noodles is that once they are opened, they can continue to be used for a long time. That is, whereas cup noodles, for example, are consumed more or less immediately after they are opened, razors, cosmetic accessories, and batteries can often be used for longer than a week. As a result, consumers tend not to purchase more than two units even when they are on sale. This suggests that the degree of stockpiling,

$m$, may be negatively correlated with the length that a product lasts once it is opened.[24]

## 4.3 Validity Checks of Our Approach

### 4.3.1 Comparison of Inflation Rates: Data and Simulation

To check the validity of our approach, we first examine whether the simulated bias in chained indices is comparable to the actual price change. The actual price change is the average of price changes based on the purchase-weighted Törnqvist index for each 3-digit product category $j$ (denoted by $\pi_j$) over the observation period. At the same time, we record the following variables: $\overline{q_j}$, $\underline{q_j}$, and $(P_L/P_H)_j$. Furthermore, we calculate the average of the degree of stockpiling, $m_j$. Then, using these values, we simulate the model and calculate the average price change in the purchase-weighted Törnqvist index (see Appendix C for the simulation method). The elasticity of substitution $\sigma$ is set to 3 for all product categories.

Figure 6 shows that our approach does a reasonably good job in explaining the actual change in chained price indices. In the figure, each circle represents a product category. The figure shows that product categories that have a large price change in the data tend to exhibit a large bias in the simulation as well. The circles tend to lie below the 45 degree line, suggesting that the actual price change is smaller than the simulated bias.[25]

### 4.3.2 Comparison with the *Shoku-map* Data

To further check the validity of our approach, we compare the degree of stockpiling ($m$) obtained from the POS data with related variables in the *Shoku-map* data. Specifically, we look at the duration of inventories defined as the difference between the date of purchase ($t_p$) and the date on which the item is used up ($t_l$). Using the cross-sectional dispersion of $m$ at the 3-digit product category level, we examine if there is any significant correlation. The left-hand panel of Figure 7 shows that there is no significant correlation between $\log(m)$ in the POS data and $\log(t_l - t_p + 1)$ in the *Shoku-map* data for the period of $1988 - 2013$. This suggests that $m$ does not necessarily represent storability.

[24]The degree of stockpiling is highly heterogeneous even within products belonging to the same product category. See Appendix C.

[25]The fourth column of Table 3 shows that chain drift is negatively correlated with $m_j$. In Appendix C, we investigate by simulations how the inflation rate, $\pi_j$, depends on $m_j$, $\overline{q_j}$, $\underline{q_j}$, and $(P_L/P_H)_j$. We find that $\pi_j$ depends negatively on $m_j$.

If $m$ captures stockpiling, it should incorporate not only storability but also the quantity purchased. For this reason, the average quantity purchased in the *Shoku-map* data, $\log(q)$, may correlate with $\log(m)$ in the POS data. The right-hand panel of Figure 7 shows that this is indeed the case. A positive correlation is observed between $\log(q)$ and $\log(m)$ for the average of the entire observation period with the correlation coefficient of $+0.32$.

## 4.4 Price Indices

Using the POS data, we calculate the time-series of the price level based on the following three definitions. The first is the Törnqvist index based on the purchase weight. This is the conventional approach and we showed the results in Figure 4. The second definition is the Törnqvist index based on the consumption weight. Here, we use the consumption price as well as consumption to calculate the price index. The third definition is the order $r$ superlative index, where we use the estimated elasticity of substitution $\sigma$ at the category level.

Table 2 displays the average chain drift and inflation rate based on the three price indices. The second column of the table shows that chain drift is significantly negative for $dt \leq 52$ when we use the consumption-weighted Törnqvist index. By contrast, the third column shows that chain drift is insignificant when we use the consumption-weighted order $r$ superlative index for $dt = 1$. In this respect, our approach that uses the consumption-weighted order $r$ superlative index is effective in eliminating chain drift, although it is significantly negative for intermediate $dt$ (i.e., 7 or 14).

The bottom row shows that the average of annualized inflation rates are negative and still sizable for all the three indices. However, the size of deflation decreases as we take account of household inventory more appropriately, that is, $-46\%$ for the purchase-weighted Törnqvist index, $-13\%$ for the consumption-weighted Törnqvist index, and $-11\%$ for the consumption-weighted order $r$ superlative index. Appendix C compares the time-series changes in the inflation rates.[26]

[26]Other factors may contribute to a bias in chained price indices. The most important factor would be product turnover. Especially in Japan, Ueda, Watanabe, and Watanabe (2019) show that product prices tend to decline over the life-span of a product. In this case, the price indices that use only a matched sample by ignoring price difference between old and new products will be biased downward. This downbard bias is indeed consistent with the result shown in Table 2 that the inflation rate based on the Törnqvist index is 6% lower than its chain drift in the POS data. A cross-sectional substitution

# 5 Further Issues of Household Inventory

The importance of household inventory is not limited to price indices. In this section, we investigate implications of household inventory for the measurement of the elasticity of substitution. Then, we study how consumers' inventory behavior changes in response to an preannounced consumption tax hike and business cycles.

## 5.1 The Elasticity of Substitution

### 5.1.1 Our Approach to Measuring the Elasticity of Substitution

Our approach is also useful for obtaining the value of the elasticity of substitution, $\sigma$.[27] Note that the estimate of the elasticity of substitution is biased unless we properly take account of stockpiling. Even if there is a clear negative relationship between observable purchases and posted prices, this does not necessarily indicate the elasticity of substitution. As argued by Hendel and Nevo (2004), price reductions influence the quantity purchased not only via the consumption effect (consumption is price sensitive) but also via the stockpiling effect (i.e., consumers stockpile for future consumption). Owing to the latter effect, which is often larger than the former, the elasticity of substitution is likely to be overestimated when we ignore stockpiling.

Our approach makes it possible to obtain the value of $\sigma$. For each 3-digit product category, for each sales event of product $k$ and retailer $r$, we collect the records of the log ratio of the quantity consumed during a sale to the quantity consumed when the product is sold at the regular price divided by the log ratio of the sale price to the regular price, that is, $\Gamma \equiv -\log\left(c_L/c_H\right)/\log\left(r_L/r_H\right)$. Equation (2) suggests that $\Gamma$ equals $\sigma$ on

might also cause a bias, if, for example, the timing of a sale is correlated across retailers or products. This may amplify or reduce consumers' stockpiling behaviors, affecting the bias in chained price indices. Using scanner data for Australia, Ivancic, Diewert, and Fox (2011) calculate how much changes in price indices change when they use the set of products which are sold in both the first and last periods to eliminate the effect of product turnover or when they aggregate the same product over stores to examine the effect of cross-sectional aggregation.

[27] The elasticity we measure in this study represents the short-run intertemporal own price elasticity of consumption. By short-run intertemporal, we mean the elasticity that incorporates stockpiling. Ogaki and Reinhart (1998) distinguish the elasticities of intertemporal substitution of spending and consumption when goods are storable. It is the own elasticity in that we do not estimate a cross-price elasticity, which compares substitution among different products.

average.[28] We calculate the unweighted average of $\Gamma$ across sales events, products, and retailers for each 3-digit product category, which we define as $\sigma$.

For comparison, a simple calculation of $\sigma$ is possible if we ignore storability. Hendel and Nevo (2004) argue that neglecting stockpiling leads to an overestimation of $\sigma$ by a factor of two to six. To confirm this, we obtain $\sigma$ by simply calculating the slope of $-\Delta \log X_t / \Delta \log p_t$ using the observed series of purchases $X_t$ and posted prices $p_t$, where $\Delta$ is the difference from the previous date. In this simple calculation, we use all observations as long as the posted price changes by more than two yen from the previous date. Note that, according to equation (2), the simple calculation would be valid if $c_t$ and $r_t$ equal $X_t$ and $p_t$, respectively.

### 5.1.2 Results

Figure 8 shows the value of the elasticity of substitution $\sigma$ for 3-digit product categories, calculated from $\Gamma = -\log(c_L/c_H)/\log(r_L/r_H)$. The left-hand panel displays the histogram of $\sigma$ and shows that $\sigma$ is distributed smoothly around the mode of three and most of the values are positive.

The dotted line in the left-hand panel shows the histogram of the simple estimate of $\sigma$, which is distributed to the right of the solid line (our estimate). This result is in line with Hendel and Nevo's (2004) result, suggesting that $\sigma$ is overestimated if we ignore storability. The right-hand panel shows the scatter plot of the value of $\sigma$, where each dot represents a 3-digit product category. In most categories, the simple estimates are larger than our estimates. Nevertheless, the two measures are not independent of each other and exhibit a positive correlation. Finally, it should be noted that the estimates of $\sigma$ are negative for some product categories (although they are not necessarily significant), whereas from a theoretical perspective we would expect them to be positive. We aggregate the variables of interest at the 3-digit category level only when $\sigma$ is greater than zero.

[28] It can be assumed that prices are exogenous for households, especially over such a short time horizon. Thus, any bias from endogeneity is unlikely to be a major problem.

## 5.2 Effect of the Consumption Tax Rate Hike

On April 1st, 2014, the consumption tax rate was raised from 5% to 8%. This was a preannounced event, which prompted households to stockpile before the tax hike.[29] To examine the effect of the consumption tax hike, we calculate the change in $\log(m)$ in March 2014 from the same month in the previous year.

The left-hand panel of Figure 9 shows that the changes in $\log(m)$ in March 2014 are positive for most product categories. Furthermore, the correlation coefficient between this variable and $\log(t_l - t_p + 1)$ based on the *Shoku-map* data is high at +0.48, although the correlation was insignificant for the period of $1988 - 2013$. This evidence supports our interpretation that $m$ captures the degree of stockpiling. The degree of stockpiling increases in response to an anticipated price increase and that goods with a longer inventory duration tend to be stockpiled more. Furthermore, the right-hand panel of Figure 9 shows that a positive correlations continues to be observed between $\log(q)$ and $\log(m)$ in March 2014, where the correlation coefficient is +0.40.

## 5.3 Developments in Households' Stockpiling Behavior

### 5.3.1 Changes in Stockpiling Behavior

The degree of stockpiling $m$ is not only heterogeneous but also time-varying. The line with dots in Figure 10 shows the time-series developments in aggregate $\log(m_t)$, given by the unweighted mean of $\log(m_j)$ for 3-digit product category $j$, from January 1989 to December 2018. The line indicates that there has been a secular decrease in the last two decades.

It should be noted that $m$ can change as a result of changes not only in households' intrinsic behavior but also in prices, which are exogenous to households. To show this formally, we use equation (7) when $p_t = P_L$. Household producers optimize their inventories during a sale to satisfy

$$C'(i_L; I_{t-1}) = \beta \left\{ (1 - \underline{q}) r_H(I_t) + \underline{q} P_L \right\} - P_L + \mu_t. \tag{21}$$

[29] In Japan, increases in the consumption tax rate—from 3 to 5% in April 1997, from 5 to 8% in April 2014, and from 8 to 10% in October 2019—have always been a major political and economic event, because they have been accompanied by large demand increases before the tax hike and persistent weak demand (recessions) after the hike. For this reason, Prime Minister Shinzo Abe postponed the latest hike, from 8% to 10% twice, once in 2014 and once in 2016.

If a sale ends in period $t$, the consumption price increases in period $t+1$. This provides household producers with a profit if they hold inventories, as the right-hand side of the equation shows. However, household producers incur a cost when holding inventories, as shown in the left-hand side of the equation. Suppose $C'(i_t; I_{t-1}) = C > 0$ and linear consumption-price increases. Then we have $C = \beta(1-\underline{q}) \left\{ \frac{1}{m} \frac{P_H - P_L}{P_L} + 1 \right\} P_L - (1-\beta\underline{q}) P_L$, and, in turn,

$$m = \beta(1-\underline{q}) \frac{P_H - P_L}{P_L} \left( 1 - \beta + \frac{C}{P_L} \right)^{-1}. \tag{22}$$

This equation suggests that the degree of stockpiling $m$ negatively depends on the probability that a sale will continue to occur at $t+1$ given that a sale occurs at $t$, positively depends on the size of the sale discount, and negatively depends on the cost of holding inventories. In other words, more stockpiling occurs the sooner a sale is expected to end and the larger the sale discount is. More generally, $\overline{q}$ is also likely to influence $r_H$ and, in turn, $m$. Thus, $\overline{q}$, $\underline{q}$, and $(P_H - P_L)/P_H$ should constitute explanatory variables for $m$.

Therefore, we regress the following equation: $\log(m_{jt}) = c_j + d_t + AX_{jt} + \varepsilon_{jt}$, using the vector of explanatory variables $X_{jt} = \{\log(1-\overline{q_{jt}}), \log\left(1-\underline{q_{jt}}\right), \log\left((P_H - P_L)/P_H\right)_{jt}\}$, where $j$ and $t$ represent the 3-digit product category and the month (January 1989 to December 2018), respectively. Time fixed effect $d_t$ captures the aggregate, demand-side, time-varying component of $m_{jt}$. Some of the variables we use are persistent and close to an I(1) process. Thus, to avoid spurious regression, we also estimate the above equation using the time differences of the variables such as $\Delta\log(m_{jt}) \equiv \log(m_{jt}) - \log(m_{jt-1})$.

Table 5 presents the estimation results. In the table, column (1) shows the result when we use $\log(m_{jt})$ as the dependent variable and do not include the time fixed effect. Column (2) shows the result when we use $\log(m_{jt})$ as the dependent variable and include the time fixed effect. Column (3) shows the result when we use the time difference of $\log(m_{jt})$ as the dependent variable and do not include the time fixed effect.

The coefficient on $\log\left((P_H - P_L)/P_H\right)$ is significantly positive in all columns. The positive relationship between $(P_H - P_L)/P_H$ and $m$ is consistent with the relationship derived from equation (22). The coefficient on $\log(1-\overline{q})$ is insignificant at the five percent level in columns (1) and (2) but significantly negative in the regression that uses time differences (column (3)).

The coefficient on $\log\left(1-\underline{q}\right)$ is significantly negative in all columns, even though equation (22) suggests it should be positive. One possible reason is that $\underline{q}$ might be

endogenous. It should be noted that in this regression, the cost of inventories is not controlled for. For product categories with low inventory costs, $m$ is likely to be high. If firms hold longer sales (high $\underline{q}$) for these products, we would expect to observe a negative coefficient on $\log\left(1-\underline{q}\right)$ rather than a positive one. Another reason is that in equation (17) $m$ is proportional to $I_L$, which tends to increase as the duration of a sale $T$ increases. Owing to this construction, our measure of $m$ tends to increase as $\underline{q}$ increases.

The solid line in Figure 10 shows the time-series of time fixed effect $d_t$ (column 2 in the table), which represents changes in the degree of stockpiling after controlling for the effects of price changes. Developments in $d_t$ differ from those in aggregate $\log(m_t)$. Specifically, $d_t$ exhibits an increase in the 2000s, while aggregate $\log(m_t)$ does not.[30]

### 5.3.2 Effects of Macroeconomic Variables on Stockpiling Behavior

What brought about the secular decrease in household stockpiling behavior ($m_t$ and $d_t$) in the 1990s and then the reversal in the 2000s ($d_t$, but not $m_t$)? Household stockpiling behavior likely is influenced by a number of factors, which we consider in this subsection.

**Possible Channels** First, according to equation (22), an increase in $\beta$ increases stockpiling because households put greater weight on future consumption. Possible factors that may bring about changes in $\beta$ are preference shocks, which are often incorporated in dynamic stochastic general equilibrium models as part of demand shocks and also cause changes in real interest rates. Specifically, preference shocks generate a negative relationship between stockpiling and real interest rates. Furthermore, it is also thought that higher inflation expectations promote stockpiling, which also yields a negative correlation between real interest rates and stockpiling. Thus, investigating developments in real interest rates should provide a clue as to how and why stockpiling has changed in Japan.[31] Over the last two decades, Japan has seen successive waves of monetary accommodation, leading to a decline in nominal interest rates, although, due to the zero lower bound on nominal interest rates, it is debatable whether real interest rates have declined as a result of monetary policy.

[30] We find that this deviation is explained by both the decrease in the probability of sales ($\underline{q}$) and the decrease in the size of sale discounts ($\log\left((P_H - P_L)/P_H\right)$). For the detailed changes in these variables associated with sales, see Sudo, Ueda, and Watanabe (2014).

[31] Similarly, interest rates and the cost of holding inventories are also important determinants of firm inventory investment (see, e.g., Kahn, 2016).

Second, stockpiling behavior may be influenced by factors that change the cost of holding inventories $C$. According to equation (22), an increase in $C$ decreases the incentive for stockpiling. One possible factor that may influence $C$ is an interest rate. For example, a higher interest rate increases borrowing costs, which may prevent households from stockpiling.[32]

The third factor concerns labor market conditions. Consider the following two opposing hypotheses. Suppose that labor market conditions are unfavorable for households, that is, low labor demand brings about high unemployment, low hours worked, and low income. One hypothesis is that households face stricter financial (liquidity) constraints and are therefore unable to purchase as much as they would like when prices are low. In that case, unemployment has a negative effect on stockpiling, while hours worked have a positive effect. The other hypothesis is that when unemployment is high and hours worked are low, households have more time for shopping, which allows them to find products that are on sale and stockpile inventories. Also, a decrease in income may make households more price-sensitive. In that case, unemployment has a positive effect on stockpiling and hours worked have a negative effect.

**Regression** Bearing these factors in mind, we examine whether the degree of stockpiling $m_t$ depends on the macroeconomic environment. We estimate the following equation: $\Delta\log(m_{jt}) = c_j + B\Delta Z_t + A\Delta X_{jt} + \mu_{jt}$ or $\Delta\log(m_{jt}) = c_j + B\Delta Z_t + A\widetilde{\nu_{jt}} + \mu_{jt}$. Here, $\Delta X_t$ is the time difference of the price variables used above from month $t-1$ to $t$, and $\Delta Z_t$ is the time difference of exogenous variables consisting of the unemployment rate, log hours worked, and the real interest rate from month $t-1$ to $t$ and $t-2$ to $t-1$. The real interest rate in period $t$ is defined as the overnight call rate in period $t$ minus the actual inflation rate based on the CPI from $t$ to $t+12$ (all in percent). We estimate the equation using the time difference to avoid spurious regression. Lagged variables for $Z_t$ are added to incorporate the possibility that it takes time for labor market conditions

[32] Another possible factor is the size of houses and Japan's demographic structure (population aging). However, looking at data for the size of houses from the Housing and Land Survey conducted by the Ministry of Internal Affairs and Communications every five years shows that there has been a steady increase in both the average housing area and the average housing area per household member from 1993 to 2013 from $88.4m^2$ to $93.0m^2$ and from $29.8m^2$/person to $38.5m^2$/person, respectively. This suggests that $C$ should have decreased monotonically and $d_t$ increased monotonically. However, such a monotonic decrease did not occur, as shown in Figure 10. Population aging is also a monotonic development in Japan.

and the real interest rate to influence household stockpiling behavior.[33]

In the first regression, we estimate the degree of stockpiling using $\Delta X_t$ as the independent variable. In the second regression, we use $\widetilde{\nu_{jt}}$, the residuals of the estimated equation of $\Delta X_{jt} = e_j + D\Delta Z_t + \nu_{jt}$, as the independent variable. Labor market conditions and the real interest rate $\Delta Z_t$ likely influence firms' pricing $\Delta X_{jt}$ as well. Since $\widetilde{\nu_{jt}}$ is orthogonal to $\Delta Z_t$, by using $\widetilde{\nu_{jt}}$, we aims to evaluate the overall effect of $\Delta Z_t$ on $\Delta \log(m_{jt})$, which incorporates the indirect effect through $\Delta X_{jt}$.

Table 6, particularly column (5), shows the main estimation results, while column (1) shows the estimation results when we simply use $\Delta X_{jt}$. The effect of the unemployment rate on $\Delta \log(m_{jt})$ is small, because the two coefficients on the unemployment rate at $t$ and $t-1$ more or less cancel each other out. The two coefficients on hours worked at $t$ and $t-1$ are significantly negative, suggesting that longer hours worked decrease the degree of stockpiling. This result supports the hypothesis that longer hours worked decrease households' time for shopping, which prevents them from stockpiling inventories during sales, rather than the hypothesis focusing on households' financial constraints and predicting the opposite effect. The coefficient on the real interest rate at $t$ is significantly negative, suggesting that a higher real interest rate decreases the degree of stockpiling. This is consistent with the reasoning mentioned above.

Columns (2) to (4) show that, in response to longer hours worked, firms change their pricing so that the frequency of sales decreases (low $\overline{q}$), the duration of sales increases (high $\underline{q}$), and the size of sale discounts increases (high $(P_H - P_L)/P_H$). The effects of the changes in the unemployment rate and the real interest rate on pricing are unclear, however.

These results suggest that using lower frequency data to calculate price indices has the risk of ignoring the effects of business cycle fluctuations. This may yield biased price indices where the bias changes with the business cycle. In Appendix G, we investigate implications of this endogeneity of stockpiling quantitatively.

# 6 Conclusion

Goods storability, especially stockpiling during temporary sales, causes a tremendous degree of bias when price indices are based on purchases, since consumers tend to stock-

[33]Note that $m_{jt}$ is not likely to influence $Z_t$ because the former is a variable at the product category level.

pile when prices are low (i.e., during a sale) and purchases exceed consumption. We constructed a model to explain the facts we observe, proposed a tractable approach to infer consumption and consumption prices using data on purchases and purchase prices, and applied it to Japanese data. We showed that consumers' stockpiling behavior can be conveniently summarized by a single variable: the degree of stockpiling during a sale, which expresses how long inventories last after a sale ends. Applying the approach to POS data for Japan, we found that our approach of using a consumption-weighted index succeeds in eliminating the chain drift. Furthermore, we showed that the degree of stockpiling depends on the macroeconomy.

Tasks for the future include, first, a more careful consideration of heterogeneity at the product and household levels. We found that there exists sizable heterogeneity in the degree of stockpiling across products. A more detailed investigation might shed new light on household inventory behavior. Equally important is the heterogeneity at the household level. Considering the possibility that stockpiling behavior depends on the size of the family and home, income, age, etc., could provide new insights.

Second, we should apply our approach to a wider range of product categories than those covered in our data, processed food and daily necessities, which make up only about 20 percent of households' expenditure. For instance, prices for some storable goods (e.g., gasoline and fresh food), durable goods (e.g., clothing and personal computers), and services (e.g., travel) occasionally change substantially just like in a temporary sale, which seems to cause demand fluctuations similar to stockpiling. It is worth testing whether our approach is useful for the analysis of these product categories.

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Table 1: State-dependent Consumption

| Dependent variable | 1 if household uses product | Number of times a product is used |
|---|---|---|
| Inventory | 0.0006895*** | 0.00410*** |
| | (0.0000171) | (0.000901) |
| Observations | 90,545,020 | 90,545,020 |
| No. of HH | 3,602 | 3,602 |
| $R^2$ | 0.12 | 0.12 |
| Fixed effects | HH/category | HH/category |

Note: Figures in parentheses represent robust standard errors. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Table 2: Chain Drift and Inflation Rate

| | *Price indices* | | |
|---|---|---|---|
| *dt* | Törnqvist (purchase-weighted) | Törnqvist (consumption-weighted) | Order r superlative (consumption-weighted) |
| | | *Annual chain drift* | |
| 1 | $-40.44^{***}$ | -5.69*** | $-1.20$ |
| 7 | $-7.24^{***}$ | $-2.28^{***}$ | $-1.23^{***}$ |
| 14 | $-2.43^{***}$ | $-1.10^{***}$ | $-0.59^{***}$ |
| 28 | $-0.97^{***}$ | $-0.46^{***}$ | $-0.04$ |
| 52 | $-0.66^{***}$ | $-0.20^{**}$ | 0.23 |
| 91 | $-0.61^{***}$ | 0.06 | 0.17 |
| 182 | $-0.49^{***}$ | 0.01 | 0.14 |
| | | *Annualized inflation rate* | |
| 1 | -46.34 | -13.06 | -10.71 |

Note: Chain drift is the average from 1989 to 2019. See the main text for the calculation of chain drift, $d_{0,365,dt}$, where $dt$ represetns a time span. We transform $d_{0,365,dt}$, which is on a daily basis, to an annualized rate by $\exp(365 \times d_{0,365,dt}) - 1$. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively, when we apply the sign test for the null hypothesis that the probability to take positive/negative chain drift is equal to 0.5. The annualized inflation rate is calculated as $\exp(365 \times \overline{x_{dt=1}}) - 1$, where $\overline{x_{dt=1}}$ is the mean of daily log inflation from 1990 to 2018.

Table 3: Relation between Chain Drift and Stockpiling

| | *Dependent variable:* | | | |
|---|---|---|---|---|
| | Chain drift for purchase-weighted Törnqvist | | | |
| | (1) | (2) | (3) | (4) |
| $Q_H$ | $0.698^{**}$ | | | |
| | (0.315) | | | |
| $Q_L$ | $1.781^{***}$ | | | |
| | (0.463) | | | |
| $q$ | | $-1.916^{***}$ | | |
| | | (0.483) | | |
| $\overline{q}$ | | | $-0.807$ | $-2.190^{**}$ |
| | | | (1.039) | (1.089) |
| $\underline{q}$ | | | $-0.708^{***}$ | 0.014 |
| | | | (0.154) | (0.259) |
| $\log(P_L/P_H)$ | | $3.979^{***}$ | $3.929^{***}$ | $2.899^{***}$ |
| | | (0.644) | (0.618) | (0.673) |
| $\log m$ | | | | $-0.344^{***}$ |
| | | | | (0.101) |
| Constant | $-0.320^{***}$ | $0.178^{***}$ | $0.418^{***}$ | $0.223^{**}$ |
| | (0.039) | (0.062) | (0.075) | (0.093) |
| Adjusted $R^2$ | 0.102 | 0.400 | 0.440 | 0.470 |
| Observations | 189 | 195 | 195 | 195 |

Note: Figures in parentheses represent standard errors. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. The explanatory variables are as follows: $Q_H$ and $Q_L$ represent a degree of an asymmetry for the quantity purchased when price increases and decreases, $q$ is the probability that a product is on sale, $\overline{q}$ is the probability that a product will go on sale on the following day given that it is not currently on sale, $\underline{q}$ is the probability that a product will continue to be on sale on the following day given that it is currently on sale, $P_L/P_H$ is the ratio of the sale price to the regular price, and $m$ is the degree of stockpiling.

Table 4: Top and Bottom Five Categories for the Degree of Stockpiling

| | Product category | m | | Product category | m |
|---|---|---|---|---|---|
| Top 5 | Instant cup noodles | 4.98 | Bottom 5 | Razors | 1.15 |
| | Diluted beverages | 3.74 | | Prepared bread meals | 1.19 |
| | Frozen meals | 3.58 | | Cosmetic accessories | 1.26 |
| | Packaged instant noodles | 3.23 | | Home medical supplies | 1.27 |
| | Coffee beverages | 3.08 | | Batteries | 1.30 |

Note: The degree of stockpiling $m$ is inferred using the POS data.

Table 5: Regression of the Degree of Stockpiling

| | (1) | (2) | | (3) |
|---|---|---|---|---|
| | $\log(m)$ | $\log(m)$ | | $\Delta\log(m)$ |
| $\log(1-\overline{q})$ | 0.6254 | -0.9757* | $\Delta\log(1-\overline{q})$ | -2.4682*** |
| | (0.428) | (0.547) | | (0.607) |
| $\log(1-\underline{q})$ | -0.6270*** | -0.7785*** | $\Delta\log(1-\underline{q})$ | -0.5104*** |
| | (0.035) | (0.034) | | (0.036) |
| $\log(1-P_L/P_H)$ | 0.3511*** | 0.3057*** | $\Delta\log(1-P_L/P_H)$ | 0.2546*** |
| | (0.044) | (0.046) | | (0.032) |
| Fixed effects | category | category/month | | category |
| Observations | 54,000 | 54,000 | | 53,850 |
| Within $R^2$ | 0.650 | 0.753 | | 0.283 |
| # of categories | 150 | 150 | | 150 |

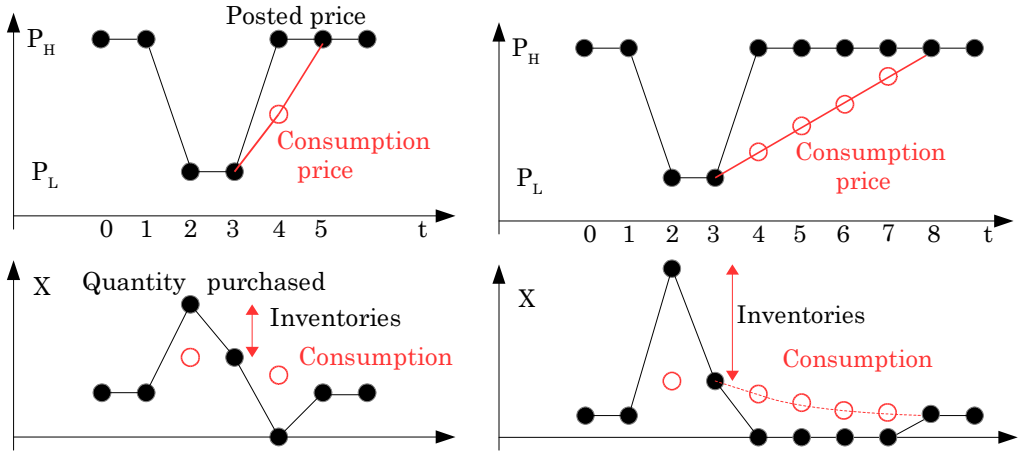
Note: Variable $m$ is the degree of stockpiling, $\overline{q}$ is the probability that a product will go on sale on the following day given that it is not currently on sale, $\underline{q}$ is the probability that a product will continue to be on sale on the following day given that it is currently on sale, and $P_L/P_H$ is the ratio of the sale price to the regular price. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Table 6: Effects of Macroeconomic Variables on Stockpiling Behavior

| | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| | $\Delta\log(m)$ | $\Delta\log(1-\overline{q})$ | $\Delta\log(1-\underline{q})$ | $\Delta\log(1-P_L/P_H)$ | $\Delta\log(m)$ |
| $\Delta$(unemp rate) | 0.0026 | -0.0017*** | -0.0338*** | 0.0277*** | 0.0312*** |
| | (0.004) | (0.000) | (0.003) | (0.003) | (0.003) |
| $\Delta$(unemp rate(-1)) | -0.0106*** | 0.0007*** | 0.0244*** | -0.0300*** | -0.0327*** |
| | (0.004) | (0.000) | (0.003) | (0.003) | (0.004) |
| $\Delta\log$(hours worked) | -0.2940*** | 0.0210*** | 0.1341*** | 0.1989*** | -0.3612*** |
| | (0.043) | (0.002) | (0.030) | (0.033) | (0.043) |
| $\Delta\log$(hours worked(-1)) | -0.6677*** | 0.0077*** | -0.6106*** | 0.4365*** | -0.2602*** |
| | (0.046) | (0.001) | (0.044) | (0.040) | (0.044) |
| $\Delta$(real r) | -0.0133*** | 0.0009*** | 0.0147*** | -0.0052*** | -0.0243*** |
| | (0.003) | (0.000) | (0.002) | (0.002) | (0.002) |
| $\Delta$(real r(-1)) | -0.0036* | -0.0002*** | -0.0086*** | 0.0037** | 0.0024 |
| | (0.002) | 0.000 | (0.002) | (0.002) | (0.002) |
| $\Delta\log(1-\overline{q})$ | -2.4013*** | | | | -2.4013*** |
| | (0.615) | | | | (0.615) |
| $\Delta\log(1-\underline{q})$ | -0.5117*** | | | | -0.5117*** |
| | (0.036) | | | | (0.036) |
| $\Delta\log(1-P_L/P_H)$ | 0.2604*** | | | | 0.2604*** |
| | (0.032) | | | | (0.032) |
| Fixed effects | category | category | category | category | category |
| Observations | 53,700 | 53,700 | 53,700 | 53,700 | 53,700 |
| Within $R^2$ | 0.289 | 0.008 | 0.008 | 0.005 | 0.289 |
| # of categories | 150 | 150 | 150 | 150 | 150 |

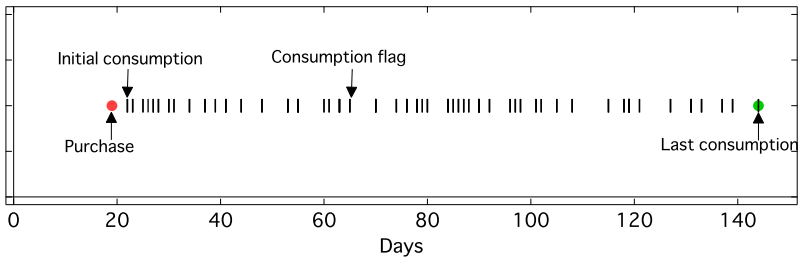
Note: Variable $m$ is the degree of stockpiling, $\overline{q}$ is the probability that a product will go on sale on the following day given that it is not currently on sale, $\underline{q}$ is the probability that a product will continue to be on sale on the following day given that it is currently on sale, $P_L/P_H$ is the ratio of the sale price to the regular price, and real r is the real interest rate. In column (5), the explanatory variables corresponding to $\Delta\log(1-\overline{q})$, $\Delta\log(1-\underline{q})$, and $\Delta\log(1-P_L/P_H)$ are the residuals of the estimation for columns (2), (3), and (4), respectively. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Figure 1: Pattern of Price and Quantity Changes during a Sales Event



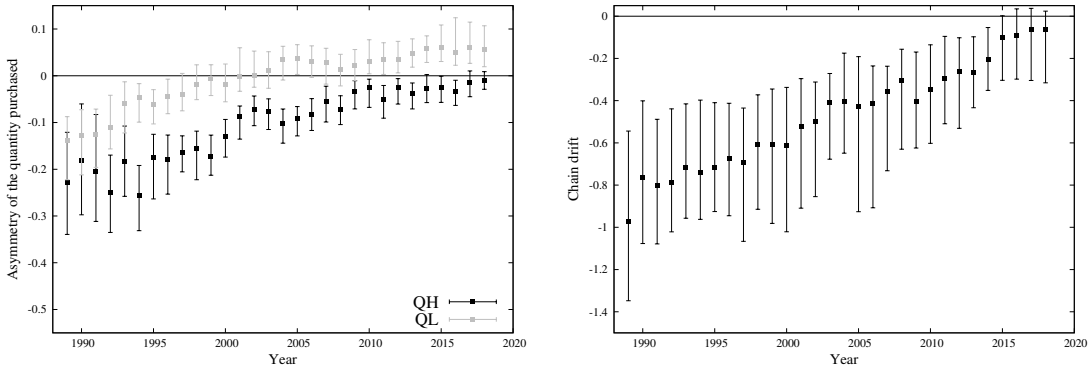

Note: The solid dots represent observable posted prices (top) and quantities purchased (bottom). The circles represent unobservable consumption prices (top) and quantities consumed (bottom).

Figure 2: Consumption Pattern of Salt



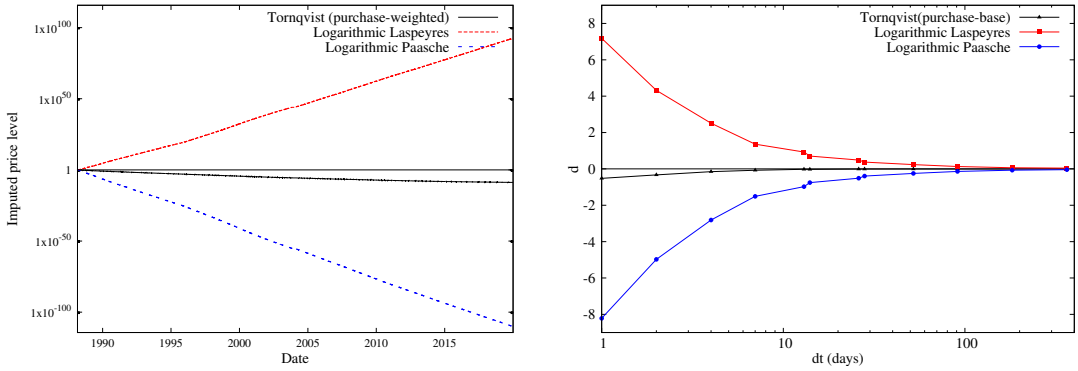

Note: The figure shows the consumption pattern for salt of a particular household in the *Shoku-map* data. Each vertical line represents a consumption flag.

Figure 3: Asymmetry of the Quantity Purchased When Price Increases and Decreases




Note: In the left-hand figure, $Q_H$ and $Q_L$ represent a degree of an asymmetry for the quantity purchased when price increases and decreases. Specifically, $Q_H$ indicates an asymmetry for the quantity purchased between just before a sale starts and just after a sale ends, while $Q_L$ indicates an asymmetry for the quantity purchased between just after a sale starts and just before a sale ends. The circles and two ends represent the median and 25 and 75 percentiles for products and sale events in each year. The right-hand figure shows a degree of chain drift, $d_{0,365,1}$, for each year.

Figure 4: Price Changes in Chained Indices




Note: The left-hand figure shows the time-series of price levels based on the purchase-weighted Laspeyres, Paasche, and Törnqvist indices using the POS data. The initial price level is normalized to one. The right-hand figure shows a degree of chain drift, $d_{0,365,dt}$, based on the purchase-weighted Laspeyres, Paasche, and Törnqvist indices, where we employ different time spans $dt$ from 1 day to 364 days.

Figure 5: Histogram of the Degree of Stockpiling

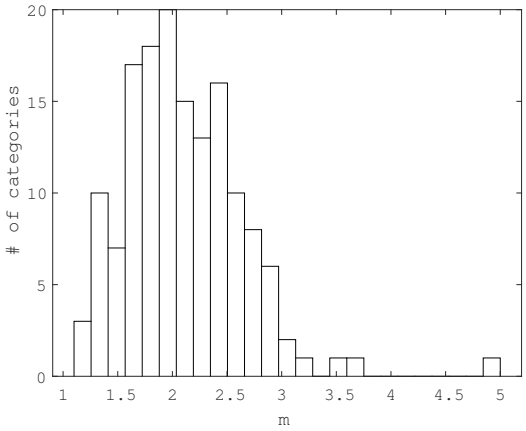
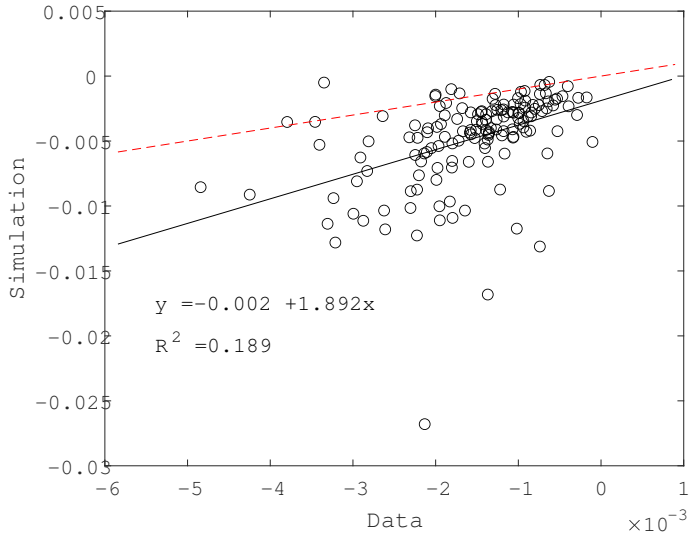
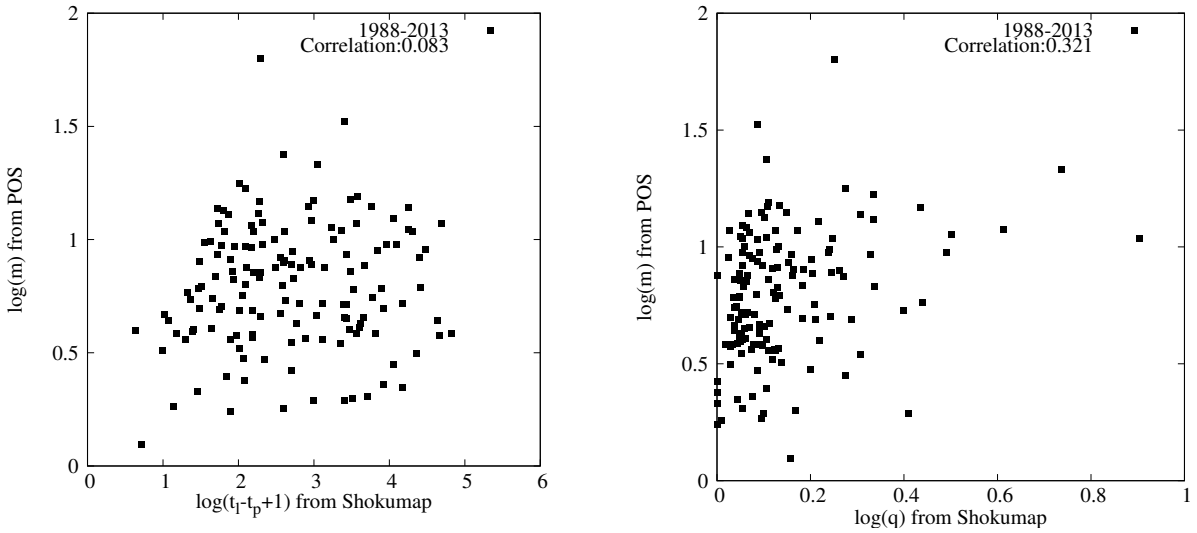



Figure 6: Inflation Rates Based on the Törnqvist Index: Data and Simulation



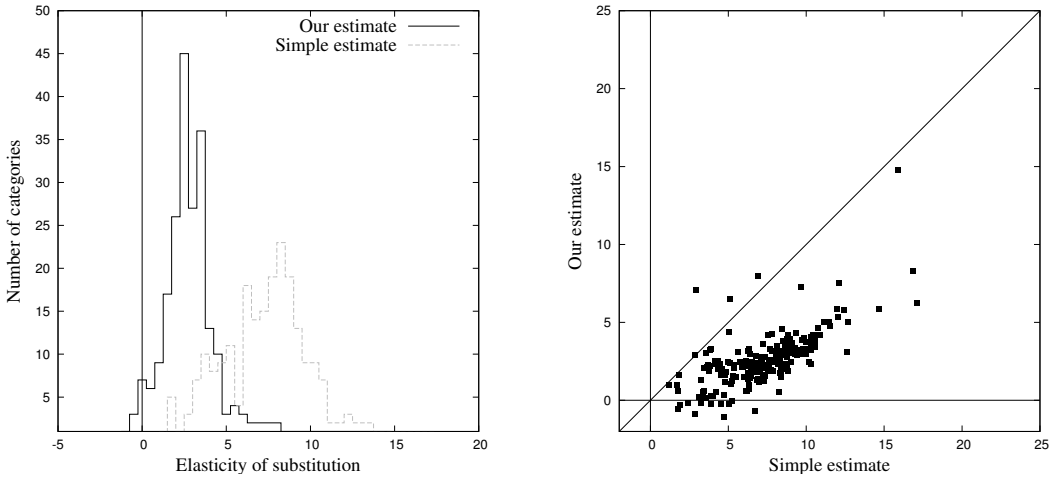

Note: Each circle represents a 3-digit product category. The inflation rates are the daily averages and are based on the purchase-weighted Törnqvist index. The red dashed line represents the 45 degree line.

Figure 7: The Consumption Period and the Quantity Purchased (*Shoku-map*) versus the Degree of Stockpiling (POS)



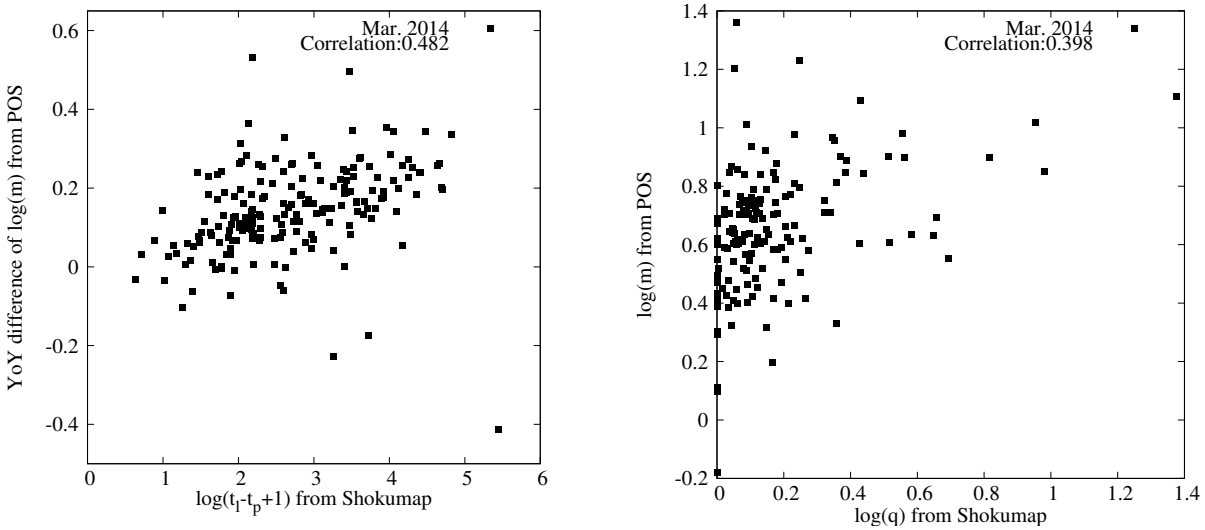

Note: Each dot represents a 3-digit product category. In the left-hand panel, the horizontal axis represents the log consumption period, where the consumption period is defined as the difference between the date of purchase ($t_p$) and the date the household finishes the product ($t_l$) plus one. In the right-hand panel, the horizontal axis represents the log quantity purchased.

Figure 8: Elasticity of Substitution



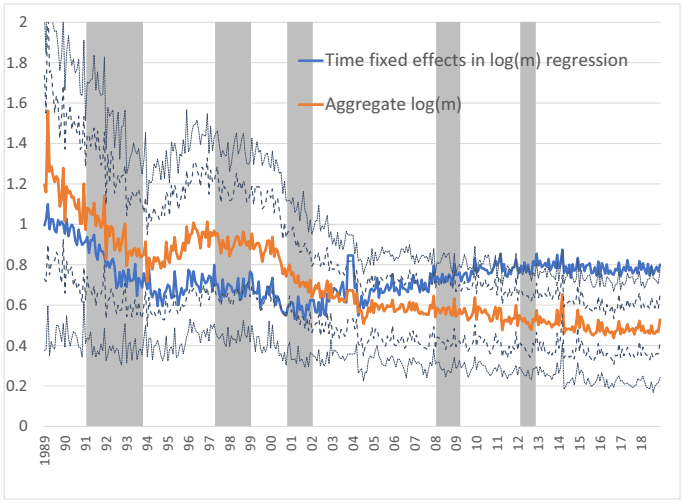

Note: "Our estimate" represents our calculation of the elasticity of substitution $\sigma$ from $\Gamma \equiv -\log\left(c_L/c_H\right)/\log\left(r_L/r_H\right)$ using the inferred series of consumption $c$ and consumption price $r$. "Simple estimate" represents the calculation of $\sigma$ from $-\Delta\log X_t/\Delta\log p_t$, where $X$ and $p$ represent the quantity purchased and posted price, respectively. The left-hand panel shows the histogram of the values of $\sigma$ for 3-digit product categories, while the right-hand panel shows a scatter plot where each dot represents a 3-digit product category.

Figure 9: The Consumption Period and the Quantity Purchased (*Shoku-map*) versus the Degree of Stockpiling (POS): One Month Before the Consumption Tax Increase




Note: See the note for the previous figure.

Figure 10: Aggregate Time-Series of the Degree of Stockpiling




Note: The solid thick line shown as aggregate log$m$ indicates the mean of log$m$ for the 3-digit product categories, while the thin lines show 10, 25, 75, and 90 percentiles of log$m$. The shaded areas indicate recession periods determined by the Cabinet Office.