Consumer Inventory, Temporary Sales, and Price Indices

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Abstract

This paper examines consumers' inventory behavior and its implications. Using scanner data for Japan and a model of storable goods, we provide evidence that consumers' stockpiling during temporary sales generates a substantial bias in chained price indices. We then propose a method to infer consumption prices (rather than purchase prices) and quantities consumed (rather than quantities purchased). Our consumption-weighted price index eliminates the bias. Further, we discuss how consumer inventory influences the measurement of the intertemporal elasticity of substitution and how consumers' inventory behavior changes in response to an anticipated consumption tax hike and business cycles.

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1 Introduction

Storable goods are abundant in the real world. The COVID-19 outbreak in 2020 caused many products, such as pasta and toilet rolls, to disappear from supermarket shelves, which would not have happened if these products were not storable. Moreover, consumers' stockpiling occurs on a daily basis, especially during a temporary sale, and is known to have many important consequences.[1] However, most economic models deal with perishable goods for the sake of simplicity. A difficulty in observing consumer inventory prevents us from understanding consumers' stockpiling behavior and evaluating what we would miss by ignoring consumer inventory.

In this study, we aim to investigate consumers' inventory behavior and its implications. We employ retailer-side scanner data, which are daily and cover the past thirty years, and construct a model of storable goods, which have several testable model predictions. We then propose an approach to quantifying consumption, rather than purchases, and consumer inventory from the retailer-side scanner data.

Specifically, the first contribution of our study is to provide evidence that consumers' stockpiling during a temporary sale causes a substantial bias in conventional chained price indices by constructing a model of storable goods and applying it to Japanese data. It is often pointed out that an event causing price/quantity bouncing, particularly a temporary sale, generates a bias or a so-called "chain drift" in chained price indices.[2] Here, a chain drift is defined as a failure of a chained price index to return to a unity even if prices and quantities for all products in the current period return to their levels in a base period. The problem of chain drift is particularly serious when a price index is based on Laspeyres and Paasche. However, chain drift exists even in a Törnqvist index, and cumulatively, even a tiny bias becomes large over a long time horizon. Indeed, a number of empirical studies document sizable price changes when they calculate chained

[1] Hendel and Nevo (2013) show that consumer inventory is the source of intertemporal price discrimination between price sensitive consumers, who stockpile for future consumption, and price insensitive consumers, who do not stockpile, and that this price discrimination generates an incentive for firms to hold temporary sales. Baker, Johnson, and Kueng (2020) argue that consumers' inventory management influences their portfolio allocation and financial returns. Griffith et al. (2009) study implications on price indices. Coibion, Gorodnichenko, and Koustas (forthcoming) discuss the implications of consumers' inventory for the frequency of purchases and consumption inequality.

[2] See, e.g., Frisch (1936), Reinsdorf (1999), Feenstra and Shapiro (2003), ILO et al. (2004, 2020), Ivancic, Diewert and Fox (2011), and Diewert and Fox (2020). Throughout this study, we use a bias and chain drift interchangeably.

price indices using high-frequency price data.[3]

However, the mechanism of chain drift is not yet well understood either empirically or theoretically. Particularly, to the best of our knowledge, no empirical study investigates whether consumers' stockpiling during a sale generates a bias in a chained price index.[4] It is also worth noting that the directions of chain drifts documented in the earlier empirical studies are different, depending on countries, types of goods, index formulae, and the frequency of data, but the source of this discrepancy is hardly discussed.[5] We provide evidence that consumers' stockpiling during a temporary sale causes chain drift.

The second contribution of this study is to propose a method to infer both consumption and inventory relying only on retailer-side scanner data. Even if we confirm that consumers' stockpiling during a temporally sale generates a bias in chained price indices, constructing price indices without a bias is not easy so long as we calculate chained price indices based on purchases (purchase-weighted indices), which we can observe in retailer-side scanner data, rather than consumption (consumption-weighted indices), which is difficult to observe. This issue has become increasingly more important as national statistical offices use big data to measure price indices.[6] Meanwhile, several theoretical studies discuss a dynamic cost-of-living index (COLI), which is based on consumers' dynamic cost minimization problem, in a model incorporating consumer inventory; however, it is almost impossible to estimate the model and calculate a dynamic COLI in a simple and practical manner.

The method we propose is tractable. We construct a simple model incorporating

[3]See, e.g., Feenstra and Shapiro (2003), de Haan (2008), de Haan and van der Grient (2011), Ivancic, Diewert and Fox (2011), Nakamura, Nakamura, and Nakamura (2011), de Haan and Krsinich (2014), and Goolsbee and Klenow (2018).

[4]The empirical studies cited above often stop their empirical investigations by comparing price changes when they use chained price indices and when they use fixed base price indices and/or price changes when they use high-frequency data and when they do low-frequency data. Nakamura, Nakamura, and Nakamura (2011) investigate the effect of temporary sales on the bias by comparing price changes when they include temporary sales and when they exclude them.

[5]When an index formula is either Törnqvist or Fisher and the frequency of data is weekly, chain drift is positive for tuna in the United States (Feenstra and Shapiro 2003). Chain drift is negative for detergents in the Netherlands (de Haan 2008) and mostly so for many categories of products sold in supermarkets in Australia (Ivancic, Diewert and Fox 2011). It is positive for coffee and negative for soft drinks in the United States (Nakamura, Nakamura, and Nakamura 2011).

[6]For the United States and Europe, see Konny, Williams, Friedman (2019), Eurostat (2017), and EFTA (2020).

consumer inventory of storable goods. Notably, the model is quasi-dynamic in that it is household producers instead of consumers that hold inventories and solve a dynamic optimization problem. This helps simplify the calculation of a COLI because consumers' cost minimization problem is static. Using the model, we propose a method to infer consumers' inventory and consumption from retailer-side data on purchases and construct a COLI, which does not have a chain drift.

The third contribution of this study is to investigate implications of consumer inventory, other than price indices, in the following three points. First, we estimate the intertemporal price elasticity of consumption taking the effects of consumers' stockpiling behavior during a temporary sale into account.[7] Second, we investigate consumers' inventory behavior in response to an preannounced consumption tax hike. While some studies use a particular event to study implications of goods storability, we use frequently observed events, that is, temporary sales, to examine consumers' inventory behavior.[8] Nevertheless, it is worth investigating how an preannounced consumption tax increase influences consumers' inventory behavior in our data because this helps us check the validity of our model. Third, we investigate consumers' inventory behavior in response to business cycles. If consumers' inventory behavior change depending on business cycles, a number of things, such as chain drift, consumers' portfolio choice, the effects of monetary policy, and optimal monetary policy, are likely to depend on business cycles as well through endogenous changes in consumers' inventory behavior.[9] These studies on implications of consumer inventory become possible because we infer consumption and inventory.

The data we use are two sets of high-frequency (daily) scanner data for Japan. The first set consists of retailer-side data, namely, the point-of-sale (POS) scanner data of

[7]It is already well known that stockpiling matters for the measurement of price elasticity (Ogaki and Reinhart 1998, Erdem, Imai, and Keane 2003, Hendel and Nevo 2004, Cashin and Unayama 2016, and Baker, Johnson, and Kueng forthcoming).

[8]Cashin and Unayama (2016) examine the effects of preannounced increase in Japan's consumption tax rate on stockpiling to estimate the intertemporal elasticity of substitution. See also Hansman et al. (2020) and Baker, Johnson, and Kueng (forthcoming).

[9]Klenow and Willis (2007), Sudo et al. (2018), and Kryvtsov and Vincent (2020) document statistically significant relations beween temporary sales and business cycles. The latter two studies also show that the real effects of monetary policy are weakened by using a model. Sheremirov (2020) shows that a model with sales explains the actual price dispersion, which also influences implications on optimal inflation and the effects of monetary policy. However, these studies focus on temporary sales, and consumer inventory is not taken account of.

thirty years, whereas the second set consists of home scanner data. Although we use primarily the first data because they include information on purchase prices, the second data are unique in that they provide household- and product-level information on when households purchase individual products, when they consume them, and when they have used up all their inventory. We use the second data mainly to check the validity of our approach.

Our main results are as follows. First, using the retailer-side scanner data, we show that chained price indices entail extremely large biases, with the Paasche and Törnqvist indices having a downward and the Laspeyres index having an upward bias. Specifically, the chained Törnqvist price index compiled on a daily basis for the past thirty years implies that Japan experienced annual deflation of 46%. Such a bias decreases in a longer time horizon (e.g., on a weekly or monthly basis); however, it is still significantly negative even on a quarterly basis.

Second, we show that consumer stockpiling during a sale is likely to generate this substantial bias in chained price indices. We provide the evidence of a considerable difference in the quantities purchased before, during, and after a sale, which not only is consistent with the model we construct but also explains the bias in each of chained price indices. Moreover, both the frequency of sales and the magnitude of sale discounts are correlated with the bias in chained price indices across product categories, which is also consistent with our model. The model shows that the sign of chain drift based on Törnqvist can be both positive and negative, depending on whether the final date of a sale is known to consumers.

Furthermore, our model provides a new insight that using a purchase-weighted price index is not the sole reason for the bias. Simply using a consumption-weighted price index does not eliminate the bias so long as the price index is based on Törnqvist. Our model shows that a consumption path at the start and after a sale is asymmetric as a result of consumer inventory, or more fundamentally, nonnegative constraints on both inventory and purchases. This asymmetry causes a systematic higher-order approximation errors in price indices, although it is well known that the Törnqvist price index provides a good approximation to the COLI up to the second order (Diewert 1976). Thus, we need a better approximation of the COLI to resolve the bias in chained indices, such as a consumption-weighted order r superlative index. By inferring consumption and inventory from retailer scanner data, we find that the consumption-weighted order r superlative index succeeds in eliminating chained drift on a daily basis.

Finally, we show other results on consumer inventory than a price index. On the intertemporal price elasticity of consumption, we find that ignoring consumer inventory causes an upward bias in the estimate. By examining the event of an preannounced consumption tax increase in April 2014, we find that one month before the tax increase, the degree of stockpiling increased, and goods with a longer inventory duration were stockpiled more. Further, we find that consumers' inventory behavior during a sale depend on labor market conditions and the interest rate. Specifically, when hours worked are long, the degree of stockpiling tends to be small. This implies that longer hours worked decrease shopping time and prevent consumers from searching for products on sales or shops with sales. We also show that the degree of stockpiling decreases when the real interest rate increases, which suggests that the opportunity cost of stockpiling increases with the real interest rate.

Our study is related to three strands of research. The first strand concerns consumer inventory of storable goods. Boizot, Robin, and Visser (2001), Griffith et al. (2009), Hendel and Nevo (2006a), and Kano (2018) empirically document patterns of consumers' purchases and consumption. Among these studies, those by Boizot, Robin, and Visser (2001) and Kano (2018) are noteworthy in that they are part of a small group of studies that, like ours, use consumer inventory data. In terms of the theoretical approach, our model is closest to that developed by Hendel and Nevo (2006a). The novelty of our study is that it focuses on price indices and macroeconomic implications. Moreover, to do so, we develop a model that is original in that it is quasi-dynamic and separates households into consumers in a narrow sense and household producers that can hold inventories.[10]

The second strand of literature our study is related to is that on a method to resolve chain drift. Ivancic, Diewert, and Fox (2011), de Haan and van der Grient (2011), and de Haan and Krsinich (2014) propose using the GEKS index (originally proposed by Gini, Eltetö, Köves, and Szulc). This is constructed to satisfy the circularity test, which is one of several desirable axioms for price indices. A drawback of this approach is that it is not based on consumers' optimization problem.[11] Moreover, the GEKS index is complex and revised each time new data become available. Feenstra and Shapiro (2003) construct a

[10] Also see Hendel and Nevo (2004, 2013), Cashin and Unayama (2016), Erdem, Imai, and Keane (2003), Hendel and Nevo (2006b), and Osborne (2018).

[11] See, for example, Zhang, Johansen, and Nygaard (2019) and ILO et al. (2020). Zhang, Johansen, and Nygaard (2019) show that the GEKS index fails to satisfy four of five axiomatic tests, such as the identity test.

simple model incorporating storable goods and propose using a virtually fixed base price index. However, they introduce restrictive assumptions such as that households have perfect foresight and that households' expenditure is expressed in a specific functional form that is independent of inventories. Moreover, in calculating weights for the fixed base price index, they use a yearly mode price and average sales at that price, although we find that consumption is not constant, changing with inventories.

The third strand of literature that our study is related to is that on COLIs. This literature dates back to Könus (1924), and of more recent contributions, studies to which ours is most closely related include those by Feenstra and Shapiro (2003), Chevalier and Kashyap (2019), Reis (2009), Gowrisankaran and Rysman (2012), Wang (2013), Osborne (2018), and Ueda (2020). The studies by Feenstra and Shapiro (2003) and Chevalier and Kashyap (2019) propose a proxy for COLIs to deal with sales, in an environment where consumers' optimization problem is static. The other studies examine dynamic COLIs, which incorporate intertemporal substitution by households. For example, Osborne (2018) constructs a model that, like ours, incorporates both storable goods and sales. He then computes a dynamic COLI by calculating the sequence of taxes on or subsidies to households such that their period utility is kept constant over time. While Osborne's dynamic COLI is indeed a major contribution to the literature, a dynamic COLI is complex and difficult to calculate and, in addition, hard to interpret.[12] Because our model is quasi-dynamic and consumers' optimization problem is static, it is much simpler to define, interpret, and construct the COLI, which we believe is beneficial to practitioners working for national statistical offices as well as economists.[13]

The remainder of our study is organized as follows. Section 2 explains scanner data for Japan and provides stylized facts on a bias in chained price indices, consumer inventory, and relations between the two. Section 3 develops a quasi-dynamic model to incorporate stockpiling behavior, explain the stylized facts we show in Section 2, and derive a COLI. Section 4 shows the empirical results and Section 5 discusses implications of consumer inventory. Section 6 concludes.

[12]For example, Ueda (2020) considers two types of dynamic COLIs and shows that they do not satisfy monotonicity and the time reversal test simultaneously.

[13]Osborne (2018) finds that the price indices proposed by Feenstra and Shapiro (2003) and Chevalier and Kashyap (2018) approximate the dynamic COLI. However, his analysis is limited to particular products (canned tuna and canned soup) for particular periods, so there is a possibility that this finding is just a coincidence.

2 Stylized Facts Based on Japanese Scanner Data

In this section, we provide several evidence that shows the importance of consumer inventory for a bias in chained price indices. To this end, first, we explain two kinds of scanner data for Japan we use in this study. Second, using retailer-side scanner data, we show a bias in chained price indices. Third, we provide several facts that are associated with consumers' inventory behavior and the bias. In the following sections, we examine formally whether these facts can both be explained by consumer inventory and explain the bias in chained price indices by constructing a model of storable goods.

2.1 Two Kinds of Scanner Data

We use two sets of scanner data for Japan. The first set consists of retailer-side data, namely, the point-of-sale (POS) scanner data collected by Nikkei Inc. The data include the number of units sold and the sales amount (price times the number of units sold) for each product and retailer on a daily basis. The observation period is around thirty years, running from March 1, 1988 to December 31, 2019. Products recorded consist of processed food and daily necessities, covering 170 of the 588 categories in the CPI and making up about 20 percent of households' expenditure. See Appendix A as to how we aggregate variables of interest over days, products, and retailers, and Abe and Tonogi (2010), Sudo, Ueda, and Watanabe (2014), and Sudo et al. (2018) for a detailed description of the data.

The other set consists of household-side data, namely, "*Shoku-map*"[14] scanner data collected by Lifescape Marketing Co. Respondents are mainly a female homemaker, and the data cover about 400 households in each period (about 4,000 households in total). The data record the number of units purchased and the date of consumption for each product and household on a daily basis. Moreover, they record when consumption ends (i.e., when a product is used up or has gone off, etc.) for each product and household. The data cover the period from September 1998 to February 2019. Note that products recorded are food only and there is no information on purchase prices. Another limitation is that there is no information on how much of a product (e.g., in terms of weight) is consumed each time it is consumed. The data record both the number of units purchased

[14] "*Shoku-map*" translates as "food map."

and consumed, which is sufficiently useful if products are consumed in discrete units, such as a cup of instant noodles or a can of beer. However, for products like salt, we do not know how much a household uses, although we do know the dates when they are used.[15] For example, Figure 2 shows the consumption pattern for salt of a particular household. In the figure, each vertical line represents a consumption flag. The figure indicates that the household purchased salt on day $t = 19$, started using it on day $t = 22$, and used it up on day $t = 144$. The inventory duration thus is $144 - 19 + 1 = 126$ days. See Appendix B for detailed explanations about the *Shoku-map* data.

In both sets of data, all products are identified by the Japanese Article Number (JAN) code, which enables us to merge the datasets. Further, we classify products into groups using the 3-digit product categories provided by Nikkei Inc. There are 218 categories in total, such as instant cup noodles, yogurt, beer, and toothbrushes.

In the following analyses using the POS data, we identify temporary sales by employing a sales filter. Specifically, we follow the procedure explained in Nakamura and Steinsson (2010) using their sales filter A with a window of $L = K = J = 42$ days.[16] Product k is classified as being on sale on date t if and only if its price p_t^k deviates from its regular price $\overline{p}_t^k$ by more than two yen.

When no sales are recorded for a particular product at a particular retailer on a particular date, earlier studies usually treated this simply as a missing observation, as if the product disappeared from shelves at the retailer on the date. However, the POS data quite often show no sales records for a particular product, retailer, and date after a sale, suggesting that stockpiling by households during a sale results in zero purchases after the sale ends. Since this conveys important information on consumer inventory, in this study, we interpolate missing observations by setting $p_t^k = \overline{p}_t^k$ and $x_t^k = 0$ if we have observations after t (i.e., unless the product permanently exits from the market). That is, we set the quantity purchased to zero and the price to the regular price on the nearest past date.

[15] If a household uses salt N times a day, the data record N, where N is an integer equal to or greater than zero.

[16] The window length is chosen following Eichenbaum, Jaimovich, and Rebelo (2011) and Kehoe and Midrigan (2015). For a detailed examination of the robustness of various filters to identify sales, see Sudo et al. (2018).

2.2 Four Stylized Facts

Using the two datasets, we present four stylized facts that are closely related to goods storability.

Fact 1. When weights are based on purchases, changes in price indices are characterized by the following inequality: $\pi^P < \pi^T < 0 < \pi^L$.

Let us denote the price and the quantity of product k in period t by p_t^k and x_t^k. Changes in chained price indices from $t - dt$ to t, π_t^X $(X = L, P, T)$, are defined by

$$\pi_t^L = \sum_{k \in K_{t-dt} \cap K_t} W_{t-dt}^k (K_{t-dt} \cap K_t) \log \left(\frac{p_t^k}{p_{t-dt}^k} \right), \tag{1}$$

$$\pi_t^P = \sum_{k \in K_{t-dt} \cap K_t} W_t^k (K_{t-dt} \cap K_t) \log \left(\frac{p_t^k}{p_{t-dt}^k} \right), \tag{2}$$

$$\pi_t^T = \sum_{k \in K_{t-dt} \cap K_t} \frac{W_{t-dt}^k (K_{t-dt} \cap K_t) + W_t^k (K_{t-dt} \cap K_t)}{2} \log \left(\frac{p_t^k}{p_{t-dt}^k} \right), \tag{3}$$

based on the Laspeyres, Paasche, and Törnqvist approach, respectively.[17] The weight share $W_t^k(K_{t-dt} \cap K_t)$ equals $p_t^k x_t^k / \sum_{k' \in K_{t-dt} \cap K_t} p_t^{k'} x_t^{k'}$, where $k \in K_{t-dt} \cap K_t$ represents a domain of products that exist both in $t-dt$ and t (such a common set is called a *matched sample*). When $dt = 1$, we can construct the chained price indices using the cumulative sum of the past price changes: $P_t^X = \exp\left(\sum_{s=1}^t \pi_s^X\right)$ for $X = L, P, T$.

It should be noted that the choices of p_t^k and x_t^k play a very important role. For the time being, we use conventional p_t^k and x_t^k that are observable from the POS data. That is, p_t^k and x_t^k represent the purchase price and quantity purchased, respectively, and the price indices are purchase-weighted. Later, we try to infer the consumption-weighted price r_t^k and consumption c_t^k and construct consumption-weighted chained indices .

Using the POS data, we calculate the time series of the price level (P_t^X), normalizing the initial price level to one. The results are striking. The left-hand panel of Figure 1 shows that the price level increases by almost 10^{92} over our 30-year observation period when based on the Laspeyres index, while it decreases to almost 10^{-111} when based on the Paasche index. The price decline is milder but nevertheless remains large when based on the Törnqvist index. The price level still decreases to almost 10^{-9}. Thus, the following inequality holds: $\pi^P < \pi^T < 0 < \pi^L$.

[17] There are various types of Laspeyres and Paasche indices. Here, we use the logarithmic Laspeyres and logarithmic Paasche indices.

Although the sizes of the changes in the price indices shown above are by far too large, they do not necessarily imply a bias because we do not observe a true price index such as COLI to compare with these price indices. Thus, we apply the following test for a bias.

Chain Drift Test As is argued by Ivancic, Diewert, and Fox (2011) among others, a chained price index should take the same value at $t = 0$ and τ if prices and quantities for all products are equal at $t = 0$ and τ (i.e., no chain drift). We formulate this argument by defining chain drift $d^X_{0,\tau,dt}$ as

$$d^X_{0,\tau,dt} = \sum_{s=1}^{(\tau-1)/dt} \pi^X_{(s-1)dt,sdt} + \pi^X_{\tau-1,0}, \tag{4}$$

where τ represents a time span to measure chain drift and set to 365 days, and $\pi^X_{t_1,t_2}$ represents a change in a chained price index from t_1 to t_2 for $X = L, P, T$.[18] The first term on the right-hand side of the equation represents the change in the chained price index when it is chained forward from 0 to $\tau - 1$ by the time span of dt gradually. The second term represents the change in the chained price index when it moves backward from $\tau - 1$ to 0 all at once.[19] If the chained price index has no chain drift, the second term should completely offset the first term, and thus, $d^X_{0,\tau,dt}$ should be zero. By definition, $d^T_{0,\tau,dt}$ is zero when $dt = 364$. We calculate $d^X_{0,\tau,dt}$ for each year of the observation period, each 3-digit product category, each price index (X), and each time span (dt).

The right-hand panel of Figure 1 shows the mean of $d^X_{0,\tau,dt}$ over years and 3-digit product categories for each price index (X) and time span (dt). This indicates considerable deviations from zero. Further, we find inequality given by $d^P < d^T < 0 < d^L$. The deviations are larger as dt is smaller (e.g., -40% and -1% for $dt = 1$ and 28, respectively, when the Törnqvist index is used).

After calculating $d^X_{0,\tau,dt}$, we conduct a test for chain drift. The test is a simple sign test for the null hypothesis that the probability to take positive and negative signs for $d^X_{0,\tau,dt}$ is equal to 0.5 each. The first column of Table 1 shows that the null hypothesis is rejected for $dt = 1$.

[18] $\pi^X_{t_1,t_2}$ is the same as $\pi^X_{t_2}$ defined in equations (1) to (3) if $t_1 = t_2 - dt$ and the domain of products is the same. Here, we use a domain of products that exist from $t = 0$ and τ to apply this formulation to the POS data. This implies that we ignore the effect of product turnover for this calculation.

[19] This is equal to a price change from $\tau - 1$ to τ because we assume that prices and quantities for all products are equal at $t = 0$ and τ.

Use of Lower Frequency Data One may recommend using lower frequency data (such as monthly or quarterly rather than daily data) or fixed base indices. They may help eliminate the bias in chained price indices because inventories of most products, particurarly those we analyze in this study, last for less than a month. However, the use of lower frequency data and/or fixed base indices has three shortcomings.

First, as pointed out in many previous studies, the use of lower frequency data alone does not completely eliminate the bias. The right-hand panel shows that chain drift decreases as a time span (dt) increases. However, it still takes about -1% and -0.5% for $dt = 28$ and 182, respectively, when the Törnqvist index is used, which is not negligble. Further, the chain drift is significantly different from zero for all dt as the first column of Table 1 shows. Given that in almost all industrial countries the consumer price index (CPI) is published on a monthly basis (around $dt = 28$), this suggests that using a frequency that is sufficiently low to avoid a bias is not a realistic solution (see also Ivancic, Diewert, and Fox 2011).

ILO et al. (2004) recommend that chained indices be replaced by fixed base indices when chain drift is serious. However, using fixed base indices also has shortcomings. The second shortcoming of using both lower frequency data and fixed base indices is that they ignore some products that are short lived or new. Szulc (1983) points out that a fixed base index is likely to be biased as the current period is distant from the base period. See also ILO et al. (2004, 2020) and Ivancic, Diewert, and Fox 2011).

The third shortcoming is that some of the high frequency (e.g., daily) fluctuations in prices and quantities may be closely related to business cycle fluctuations. Specifically, we will show that consumers change their stockpiling behavior depending on the macroeconomic environment such as hours worked and interest rates. This suggests that it may not be a good idea to discarding information from high frequency data in constructing price indices because this will yield biased price indices where the bias changes with the business cycle.

Reason for the Biases in Price Indices: Intertemporal Substitution Bias Theoretically, chain drift is frequently discussed in the context of a nonlinear correlation between changes in prices and changes in quantities purchased, which implies that a weight share for a product used to calculate a price index is asymmetric with respect to positive and negative price changes. Specifically, stockpiling causes the following price/quantity bouncing as a result of an intertemporal substitution. Let us consider the simple case

of sales shown in Table 2.[20] There are three periods: $t = 1$, 2, and 3, and two products: A and B. In period $t = 2$, the price of product A drops temporarily (due to a sale). Under these circumstances, and if the elasticity of substitutions is greater than one, the share of product A in households' total purchases will be greater in period $t = 2$ than that in period $t = 1$ ($W_1 < W_2$). Furthermore, because households stockpile, the share of product A in households' total purchases in period $t = 3$ should be smaller than that in period $t = 1$ ($W_3 < W_1$). That is, households have sufficient inventory on the day after a sale, so that they do not need to purchase as much of the product as before the sale. Thus, it can be immediately seen that $\pi^P < \pi^T < 0 < \pi^L$ from equations (1) to (3).[21] Further, by a similar reasoning and as we will discuss formally using a model in the next section, chain drift in the Törnqvist index occurs when a sale for product A is held for more than one day and its share in households' total purchases changes during the sale period.

This hypothetical example shows that stockpiling during sales is the fundamental reason for the bias in chained price indices.[22] However, we need to examine empirically whether this stockpiling pattern is observed in practice and how much it can explain the biases in price indices.

Fact 2. Asymmetry exists for the quantity purchased when price increases and decreases.

We examine whether the hypothetical pattern shown in Table 2 can be observed in practice using the POS data. To do this, we collect the following variables for each product k and sales event s: T denotes the number of days a product is on sale, P_H^1

[20] A similar argument was made, for example, by Triplett (2003), de Haan and van der Grient (2011), von Auer (2019).

[21] In the extreme case, product A is not purchased at all in period $t = 3$, that is, $W_3 = 0$. This causes the missing observation problem. If we do not interpolate the price and quantity in $t = 3$, for example, by setting $p^A_{t=3} = p^A$ and $x^A_{t=3} = 0$, the price increase is fully neglected from $t = 2$ to 3 because product A is not in the matched sample ($k \in K_{t=2} \cap K_{t=3}$) and, therefore, is excluded from the calculation of the changes in chained price indices in equations (1) to (3). Therefore, without interpolation, the downward bias of the Törnqvist price increases further.

[22] The relation between the Laspeyres and the Paasche indices, $\pi^P < 0 < \pi^L$, holds even when $W_1 = W_3$ if $W_1 < W_2$. Thus, one may argue that the chain drift does not necessarily occur due to stockpiling. However, the bias in the Törnqvist price index, $\pi^T < 0$ or the difference in the size of the bias, $|\pi^L| < |\pi^P|$, cannot be explained by the symmetric weight of $W_1 = W_3$.

and X_H^1 denote the price and the quantity purchased just before a sale (say, in period t), respectively, P_L^1 and X_L^1 denote the average price and quantity purchased during the first half of the sale, respectively (i.e., from $t+1$ to $t+\lfloor T/2 \rfloor - 1$), P_L^2 and X_L^2 denote the average price and quantity purchased during the second half of the sale, respectively (i.e., from $t+\lfloor T/2 \rfloor$ to $t+T$), and P_H^2 and X_H^2 denote the price and the quantity purchased just after the sale, respectively (period $t+T+1$).[23]

Next, we compare the quantities purchased just before a sale and just after a sale (denoted by Q_H), both of which correspond to when a product is not on sale. We calculate the difference of the inverse hyperbolic sine function, that is, $Q_H = \log(X_H^2 + \sqrt{1+(X_H^2)^2}) - \log(X_H^1 + \sqrt{1+(X_H^1)^2})$ for each product k and sales event s. This function is chosen because X_H may take zero. Then, we calculate the weighted average of Q_H over k and s within each 3-digit product category, where a weight is based on sales, that is, $P_H^1 X_H^1 + P_H^2 X_H^2$. Similarly, we compare the quantities purchased during the first half and the second half of a sale (denoted by Q_L), both of which correspond to when a product is on sale, as $Q_L = \log(X_L^2 + \sqrt{1+(X_L^2)^2}) - \log(X_L^1 + \sqrt{1+(X_L^1)^2})$. The weighted average is calculated based on sales given by $P_L^1 X_L^1 + P_L^2 X_L^2$. Both Q_H and Q_L indicate an asymmetry for the quantity purchased when price increases and decreases. That is, Q_H represents an asymmetry between just before price decreases (a sale starts) and just after price increases (a sale ends) for a non-sale event, while Q_L represents an asymmetry between just after price decreases (a sale starts) and just before price increases (a sale ends) for a sale event.

The left-hand panel of Figure 3 shows Q_H and Q_L for each year. It shows that Q_H tends to be negative particularly before 2000, suggesting that the quantity purchased just after a sale is smaller than that just before the sale. This is consistent with the hypothetical pattern shown in Table 2. However, Q_H increased, and being not much different from zero in the 2010s.

A similar result can be observed for Q_L. Particularly, the fact Q_L tends to be negative before 2000 suggests that the quantity purchased is larger at the beginning of a sale than at the end. This fact helps account for downward chain drift for the Törnqvist index. However, similar to Q_H, Q_L increased. Moreover, it turned to positive in the 2010s. This suggests that the quantity purchased is smaller at the beginning of a sale than at the end, which works to generate upward chain drift for the Törnqvist index.

[23] Variable T is the smallest integer that is greater than or equal to one and satisfies $|p_{t+T+1} - \overline{p_{t+T+1}}| \leq 2$, where $\overline{p_{t+T+1}}$ represents the regular price.

In summary, we find that Q_H is negative and that Q_L can take both a negative and positive value, although it used to be negative. This finding cannot be explained by a standard model of perishable goods because the same price should lead to the same quantity purchased, and thus, Q_H and Q_L should have mean zero (i.e., no asymmetry). By contrast, a model of storable goods can account for the asymmetry, as we will show in the next section.

Fact 3. Chain drift is associated with consumers' stockpiling during temporary sales.

The next question is whether the asymmetry of the quantity purchased, which we observed above, causes chain drift. We aim to answer this question by investigating whether Q_H and Q_L are correlated with chain drift, although causal inference is difficult because Q_H and Q_L are endogenous. Then, we investigate correlations between variables associated with temporary sales, which are more exogenous than Q_H and Q_L, and chain drift.

First, we examine correlations between the asymmetry of the quantity purchased (Q_H and Q_L) and the chain drift ($d^T_{0,\tau,dt=1}$) based on the Törnqvist index in the dimension of time series. Observing increasing Q_H and Q_L, one may anticipate the mitigation of downward chain drift. The right-hand panel of Figure 3 shows that this is indeed the case. The extent of downward chain drift has been decreasing. However, the chain drift remains negative.

Second, we investigate whether chain drift is positively associated with the asymmetry of the quantity purchased. We run a regression for $d^T_{0,\tau,dt=1}$ using Q_H and Q_L at the 3-digit product category level after we calculate the mean of each variable over years. The first column of Table 3 shows the estimation result. Both of the coefficients on Q_H and Q_L are significantly positive at the 5% level.

Third, to examine if chain drift is driven by stockpiling especially during temporary sales, we run a regression for $d^T_{0,\tau,dt=1}$ using variables associated with sales: q_j, $\overline{q_j}$, $\underline{q_j}$, and $(P_L/P_H)_j$, representing the probability that a product is on sale, the probability that a product will go on sale on the following day given that it is not currently on sale, the probability that a product will continue to be on sale on the following day given that it is currently on sale, and the ratio of the sale price to the regular price. respectively, for category j.

The second and third columns of Table 3 show the estimation results. The coefficient on $(P_L/P_H)_j$ is significantly positive, showing that the size of deflation increases as the size of sale discount is larger. The coefficient on q_j is significantly negative, suggesting that the size of deflation increases as the probability of a sale increases.[24] Furthermore, the adjusted R^2 is around 0.4, suggesting that temporary sales can account for the bias in chained price indices to a considerable extent.

Fact 4. Consumption tends to decrease as household inventories decrease.

While earlier studies on household inventories often assume that consumption is constant, Hendel and Nevo (2006a) and Kano (2018) highlight that consumption is state dependent. To examine whether consumption is indeed state dependent, we investigate whether consumption decreases until the next purchase as inventories decrease, using the *Shoku-map* data. Suppose that household i uses product k on date $t_l \in t_1, t_2, \cdots, t_{n_{ikt}}$ and t represents the purchase date. If the household uses product k twice on date t_{l_0}, we record t_{l_0} twice. Thus, n_{ikt} represents the number of times product k is used. We define the inventory on date $t^{'}$ $(t \leq t^{'} \leq t_{n_{ikt}})$ as $\lambda_{ikt'} = n_{ikt} - n^*$, where n^* is the maximum integer n that satisfies $t_n < t^{'}$. Note that $\lambda_{ikt'}$ is an integer between 0 and n_{ikt}. For example, $\lambda_{ikt'} = n_{ikt}$ for $t \leq t^{'} \leq t_1$. We further define the sum of inventory Λ_{ijt} for household i in product category j at the beginning of date t as

$$\Lambda_{ijt} = \sum_{k \in j} \lambda_{ikt}. \tag{5}$$

We then estimate the following linear probability model when $\Lambda_{ijt} > 0$:

$$y_{ijt} = c_i + d_j + \alpha \Lambda_{ijt} + \varepsilon_{ijt}, \tag{6}$$

where y_{ijt} is a binary variable for consumption and takes a value of one if household i uses products in product category j on date t and zero otherwise. As an alternative, we also use y_{ijt} defined as the sum of the times that products in product category j are used by household i on date t.

As Table 4 shows, the coefficient on inventories is significantly positive, regardless of which dependent variable is used. This indicates that consumption is state dependent

[24]The third column shows that the coefficient on $\overline{q_j}$ is insignificant and that on $\underline{q_j}$ is significantly negative. This result changes when we control a degree of stockpiling, m, which will be introduced in the next section. The fourth column shows that the coefficient on $\overline{q_j}$ is significantly negative and that on $\underline{q_j}$ is insignificant.

and decreases as inventories decrease.[25] Further, it can be pointed out that the size of the coefficient is small and significanly lower than one. This suggets consumption smoothing, that is, households do not consume as much as they purchase or hold as inventories. Thus, consumption-weighted chained indices yield more stable price change than purchase-weighted chained indices.

3 Quasi-Dynamic Model

In this section, we propose a simple partial-equilibrium model for the following three purposes. The first purpose is to examine whether the stylized facts described in the previous section can be explained by consumers' stockpiling during temporary sales. This examination helps not only confirm the importance of stockpiling but also validate the model by checking the consistency between the data and the model. The second purpose is to infer inventories and consumption, which are often unobservable (e.g., the retailer-side POS scanner data we use). This helps not only calculate the COLI incorporating consumer inventories but also deepen our understandings on the implications of stockpiling for the measurement of the intertemporal elasticity of substitution, consumers' inventory behavior in response to an preannounced consumption tax increase, and the macroeconomy. Finally, we aim to construct the COLI in a tractable manner, which we explain in more detail soon below.

3.1 Setup

We assume that product $k \in K_t$ is storable and, for simplicity, that it does not depreciate. Time t is a discrete day.

The novelty of our model is that we assume that households comprise consumers and household producers and distinguish between the two. That is, household producers are a special type of household member that has technology to hold inventories, although

[25] For the estimation presented in the table, we did not use instrumental variables but employed ordinary least squares (OLS). This means that the estimates are biased if, for example, $\mathrm{Cor}(\varepsilon_{ijt}, \varepsilon_{ijt-1}) > 0$, that is, if high consumption demand at time $t-1$ not only decreases inventories Λ_{ijt} at time t but also increases consumption at time t. If such endogeneity exists, the coefficient on inventories is underestimated when using OLS. Thus, the fact that we obtained a significantly positive estimate using OLS indicates that the state dependency of consumption continues to hold or is stronger than our estimates suggest. See Appendix B for the estimation results at a product category level.

there is a cost associated. The household producer provides inventory services consisting of purchasing storable goods from manufacturers (the quantity purchased denoted by x_t^k), holding inventory (denoted by i_t^k), and selling the goods to consumers (the quantity sold denoted by y_t^k). Market entry is free, so that the expected firm value is zero. In this respect, there is no loss of consumer surplus. On the other hand, consumers in a narrow sense cannot hold inventory: their purchases always equal their consumption, c_t^k. They purchase goods from household producers and/or manufacturers at the *consumption price* r_t^k.

This framework enables us to solve the COLI in a conventional static manner. All we need to know is two variables: c_t^k and r_t^k. We do not need to use a complex dynamic COLI. Although the economy is hypothetical, many key properties continue to hold. For example, the stylized facts can be explained not only by a dynamic model like the one used by Hendel and Nevo (2006a) but also by our quasi-dynamic model. On the other hand, it would be difficult to explain the stylized facts using a completely static model ignoring storability.[26]

We consider the optimization problems of household producers and consumers. The price of storable goods, p_t^k, can take one of two different values, a high (regular) value and a low (sales) value, which are determined stochastically and exogenously.

Consumers

There are a unit mass of consumers. Consumers' cost minimization problem is given by

$$\min_{c_t^k} \left\{ \sum_{k\in K_t} r_t^k c_t^k + \lambda_t \left\{ U - \left[\sum_{k\in K_t} b^k \left(c_t^k\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right\} \right\}, \tag{7}$$

where U is the target utility, $\sigma(>0)$ denotes the elasticity of substitution, and b^k is a taste or quality parameter for product k in period t.[27] We extensively use the following

[26] A caveat of this framework is that consumers do not have a consumption-smoothing motive. If the consumption price jumps, consumption also jumps. However, this caveat applies to all the conventional framework that considers consumers' static optimization problem when discussing price indices. The caveat emerges even if goods are perishable. To incorporate consumption smoothing, a dynamic model and dynamic COLIs are needed, but as we discussed in Introduction, interpreting the short-run movements of dynamic COLIs is often difficult.

[27] There is no consumer heterogeneity in the model. Many previous studies, such as Boizot, Robin, and Visser (2001) and Hendel and Nevo (2004, 2006a), investigate the frequency of purchases conditional on past purchase behavior. To obtain quantitatively plausible values for this, we need household heterogeneity, because otherwise the frequency of purchases would be either zero or one. In our study, we

relation with respect to the optimal quantity purchased:

$$c_t^k = \left(r_t^k / r_{t'}^k\right)^{-\sigma} c_{t'}^k. \tag{8}$$

Household producers

Household producers maximize their "firm" value:

$$V(i_{t-1}, p_t) = \mathrm{E}_t \left[\sum_{j=0}^{\infty} \beta^j \left\{ \sum_{k \in K_{t+j}} \left(r_{t+j}^k y_{t+j}^k - p_{t+j}^k x_{t+j}^k - C(i_{t+j}^k) \right) \right\} \right], \tag{9}$$

subject to the cost of inventory, $C(0) > 0$, $C(i)' > 0$, $C(i)'' \geq 0$, and the evolution of inventory:

$$i_t^k = i_{t-1}^k - y_t^k + x_t^k. \tag{10}$$

Household producers sell amount y_t^k of product k to consumers at consumption price r_t^k. Furthermore, purchases and inventories must be nonnegative:

$$x_t^k,\ i_t^k \geq 0. \tag{11}$$

The first-order conditions with respect to x_t^k and i_t^k are

$$0 = r_t^k - p_t^k + \psi_t^k, \tag{12}$$

$$C'(i_t^k) = \beta \mathrm{E}_t[r_{t+1}^k] - r_t^k + \mu_t^k, \tag{13}$$

where ψ_t^k and μ_t^k represent the Lagrange multipliers with respect to x_t^k and i_t^k, respectively. Note that ψ_t^k is strictly positive when x_t^k is zero, and zero when x_t^k is positive. Likewise, μ_t^k is strictly positive when i_t^k is zero, and zero when i_t^k is positive.

The free entry condition leads to a nonpositive value for an entering household producer with zero inventory holdings: $V(i_{t-1} = 0, p_t) \leq 0$.

Prices

The prices of storable goods follow a Markov process. They take one of the following two values: P_H when there is no sale and P_L ($P_H > P_L$) during a sale. Moreover:

$$\begin{aligned} \mathrm{Prob}(P_L | P_H) &= \overline{q} \\ \mathrm{Prob}(P_L | P_L) &= \underline{q}. \end{aligned} \tag{14}$$

The unconditional frequency of sales $\mathrm{Prob}(P_L)$ is $f = \overline{q}/(1 + \overline{q} - \underline{q})$.

ignore consumer heterogeneity to focus on macroeconomic implications rather than individual behavior.

Market clearing

Goods market clearing is given by $\int_0^{N_t} y_{t,j}^k dj + \int_0^{M_t} z_{t,j}^k dj = \int_0^1 c_{t,j}^k dj$, where $z_{t,j}^k$ represents the direct supply of storable product k by manufacturers to consumers. Household consumption c_t equals consumers' purchases from household producers, y_t, and manufacturers, z_t. In the market, there are a unit mass of consumers, N_t represents household producers, and M_t represents manufacturers.

Note that the household producers we consider in the model are still part of the households, although we separate them to simplify our analysis. Thus, the quantity purchased that is recorded in the POS data, X_t, should equal the sum of the quantity purchased by household producers $\int_0^{N_t} x_{t,j}^k dj$ and the quantity purchased directly by consumers $\int_0^{N_t} z_{t,j}^k dj$. Clearly, this is not equal to aggregate consumption $\int_0^1 c_{t,j}^k dj$.

The COLI

As highlighted, consumers' optimization problem is static, so our COLI is identical to the conventional COLI. Consumers' cost minimization problem subject to constant utility yields the following equation for the optimal quantity consumed:

$$c_t^k = \left(r_t^k / b^k\right)^{-\sigma} \lambda_t^{\sigma} U. \tag{15}$$

The unit cost function, $\lambda_t = \mathcal{C}(r_t)$ for $U = 1$, is given by

$$\mathcal{C}(r_t) = \sum_{k \in K_t} r_t^k c_t^k = \left[\sum_{k \in K_t} \left(b^k\right)^{\sigma} \left(r_t^k\right)^{1-\sigma} \right]^{1/(1-\sigma)}. \tag{16}$$

Although b^k is unobservable and even if it is time varying, equation (8) tells us that we need to know only two variables for each period, the consumption price r_t^k and the consumption share $r_t^k c_t^k$, to calculate the change in the COLI between period t and period t', $C(r_t)/C(r_{t'})$.[28]

3.2 Equilibrium Properties

We discuss the equilibrium properties of the model in relation to the aforementioned stylized facts. In the following discussion, we omit the superscript for product k for

[28] Given that household producers form part of households, it would clearly be desirable to incorporate the effect of changes in their "firm" value on consumers' utility and the COLI. However, this effect likely is small, because the free entry condition means that the firm value is close to zero.

simplicity. Further, we denote aggregate inventories at the end of period $t-1$, by $I_{t-1} \equiv \int_0^{N_t} i_{t-1,j} dj$.

The first lemma states the property of the consumption price, the price at which consumers make their purchase, r_t. When p_t equals P_L (during a sale), r_t equals P_L. That is, consumers purchase goods directly from manufacturers at $r_t = P_L$. When $p_t = P_H$ (during a non-sale), consumers may purchase goods from household producers at a price below P_H. Price r_t is lower the larger inventories I_{t-1} are.

Lemma 1 *Consumption price r_t satisfies $P_L \leq r_t \leq P_H$. When $p_t = P_L$, $r_t = P_L$. Furthermore, $r_t = r(I_{t-1}, p_t, b)$ is nondecreasing in p_t and b and nonincreasing in I_{t-1}.*

The proofs of this and the lemmas that follow are provided in Appendix C. The next lemma states the stockpiling behavior of household producers. Only when $p_t = P_L$ do household producers purchase goods and hold inventories with the aim of selling the goods at a higher price after the sale has ended.

Lemma 2 *If $p_t = P_H$, household producers do not purchase goods, that is, $x_t = 0$. If $p_t = P_L$, household producers purchase goods and hold inventories. Inventories are independent of i_{t-1}, I_{t-1}, and b.*

The left-hand panel of Figure 4 illustrates the pattern of price and quantity changes during a sales event when inventories are held just for one period. The top and bottom panels show prices and quantities, respectively. The sales event takes place in periods $t = 2$ and 3, when the price is lower than during other periods. The bottom panel shows that, in period $t = 2$, quantities purchased by household producers and consumers, X_t, represented by the solid dot, increase. The reason is not only that households consume more but also that they stockpile. Thus, X_t is higher than consumption, c_t, represented by the circle, with the difference representing stockpiling. In period $t = 3$, X_t coincides with c_t, since there is no additional need for stockpiling. Then, in period $t = 4$ when the sale ends, household producers sell their inventories to consumers. The consumption price r_t, at which the household producers and consumers transact, lies between P_L and P_H. Since consumption decreases with the consumption price, its level in period $t = 4$ is lower than in periods $t = 2$ and 3 but higher than in periods $t = 1$ and 5. At the beginning of $t = 5$, household producers hold no inventories, so consumers purchase goods at price P_H and consume less.

Stylized Fact 2

The following lemma explains Stylized Fact 2.

Lemma 3 *The quantity purchased by household producers and consumers just before a sale is greater than or equal to that just after a sale.*

The quantity purchased by household producers and consumers on the first day of a sale is greater than or equal to that on the final day of a sale.

In the previous section, we actually find that the latter part of the lemma is not necessarily robust; that is, in the past few years, the quantity purchased during the first half of a sale tends to be smaller than that purchased during the second half of a sale. Whether the quantity purchased during the first half of a sale is smaller than that purchased during the second half of a sale or not depends on whether prices are stochastic. In the above model, prices are stochastic, and neither household producers nor consumers can accurately predict future prices. This is why household producers hold inventories at the end of the first day of a sale. Suppose instead that the duration of a sale is known *ex ante*. In this case, there is no incentive for household producers to stockpile except for the final day of the sale. Thus, the quantity purchased by household producers and consumers on the first day of a sale should be smaller than or equal to that on the final day of a sale.[29]

Stylized Fact 4

Consumption c_t is clearly state dependent; specifically, it depends on consumption price r_t.

Lemma 4 *c_t decreases in r_t. After a sale ends, r_t and c_t are nondecreasing and nonincreasing over time, respectively, until the next sale begins.*

Stylized Fact 1

Our model succeeds in explaining the bias in chained indices. Changes in chained price indices from $t - dt$ to t, π_t^X $(X = L, P, T)$, are based on the Laspeyres, Paasche, and

[29]Feenstra and Shapiro (2003) document upward chain drift for the Törnqvist index and conjecture that temporary sales "attract high purchases only when they are accompanied by advertising, and this tends to occur in the final weeks of a sale." If the advertisement specifies when the sale ends, their conjecture is consistent with our model.

Törnqvist approach, respectively. We denote the change in purchase-weighted chained price indices by π^X $(X = L, P, T)$, that in the consumption-weighted chained Törnqvist price index by π^{T*}, and that in the COLI by π^{COLI}. Moreover, we introduce the degree of stockpiling $m(\geq 0)$, which indicates how many days' worth of inventories remain in the hands of household producers. If $m = 0$, no stockpiling occurs during a sale and hence no bias (i.e., no chain drift) due to sales arises. Therefore, what we are interested in is the case where $m > 0$. In the next section, we will specify how m is determined. In Appendix C, we discuss how equilibrium is determined in the special case of $m = 1$, that is, when inventories are cleared in just one period after a sale.

Lemma 5 *Consider one sales event for product k such that $p_t = p_{t+T+1} = P_H$ and $p_{t+j} = P_L$ for $j = 1, \cdots, T$ $(T \geq 1)$. Suppose that the prices and quantities of other goods remain unchanged; $\sum_{k' \in K_0 \cap K_t} p_t^{k'} x_t^{k'} = 1$; and the price before and after the sale is P_H for a sufficiently long duration (i.e., $p_{t-j} = p_{t+T+1+j} = P_H$ for $j = 0, 1, \cdots, T_H$, where T_H is sufficiently large compared with m).*

If $\sigma > (<)1$ and $m > 0$, the change in the price index from t to $t+T+1+T_H$ satisfies $\pi^{COLI} = 0$; $\pi^L > (<)0$; $\pi^P < (>)0$; $\pi^T < 0$; and $\pi^{T*} < (>)0$.

If $\sigma = 1$ or $m = 0$, then $\pi^{COLI} = \pi^L = \pi^P = \pi^{T*} = 0$ and $\pi^T < 0$.

If $\sigma > 1$, $\pi^P < \pi^T < 0 < \pi^L$. If $\sigma > 1$ and $m = 1$, $\pi^T < \pi^{T*} < 0$.

This lemma shows that, when $\sigma > 1$, purchase-weighted chained price indices entail biases, with the Paasche and Törnqvist indices having a downward and the Laspeyres index having an upward bias. It is well known that the Törnqvist index is a good approximation of the COLI up to the second order (Diewert 1976). However, as pointed out by Feenstra and Shapiro (2003) and Ivancic, Diewert, and Fox (2011) among others, the purchase-weighted Törnqvist price index entails a bias, which stems from using quantities purchased rather than quantities consumed.

The above lemma also suggests that using purchase weights is not the sole reason for the bias. Using consumption weights does not eliminate the bias from the Törnqvist index (i.e., $\pi^{T*} < 0$ if $\sigma > 1$) because the path of consumption prices is asymmetric. Specifically, the model shows that the consumption price decreases quickly when the purchase price drops at the start of a sale but then increases again only gradually when the purchase price returns to the regular price after the sale ends, as a result of consumer inventories. Such asymmetry is not a coincidence but a natural outcome of the non-negative constraints on inventories and purchases. This asymmetric response of the

consumption price and the fact that a temporary price change almost always consists of a decrease (i.e., a sale) mean that although the Törnqvist price index is a good approximation of the COLI up to the second order, the third-order approximation error cannot be eliminated if $\sigma > 1$, so that the Törnqvist price index continues to entail a downward bias even when it is constructed based on consumption weights.

The elasticity of substitution σ influences the sign of chain drift for purchase-weighted indices. When $\sigma < 1$, the sign of chain drift changes for the Paasche and Laspeyres indices changes, entailing positive and negative biases, respectively. Interestingly, the sign of a bias in the Törnqvist index is negative irrespective of σ.

Stylized Fact 3

The above Lemma 5 and its proof suggest that the asymmetry of the quantity purchased, given by Q_H and Q_L defined in Section 2.2, causes chain drift. On the one hand, consumer inventory makes Q_H always negative, which decreases a chained price index. On the other hand, Q_L can take both a positive and negative value depending on whether agents know when a sale will end. If Q_L is positive, it increases a chained price index, and vice versa. Thus, when we run a regression for chain drift using Q_H and Q_L, both of the coefficients should be positive.

Furthermore, this implies that the sign of chain drift can be positive. If $Q_L < 0$, both negative Q_H and Q_L act in the same direction, generating downward chain drift. If $Q_L > 0$, negative Q_H and positive Q_L act in the opposite direction, generating upward chain drift if the degree of stockpiling (m) is sufficiently strong. The following lemma shows that the sign of a bias in the Törnqvist index can be positive in the following case.

Lemma 6 *Suppose that households know when a sale will end once it begins. Then, π^T is positive if m exceeds a certain threshold and negative otherwise.*

Next, we investigate how chain drift in the purchase-weighted Törnqvist index depends on variables associated with temporary sales. We consider a price change over a sufficiently long period (e.g., 30 years), which we denote by $\overline{\pi^T}$. The following lemma shows that $\overline{\pi^T}$ depends on the probability of sales and the ratio of the sale price to the regular price, as $\overline{\pi^T} = \overline{\pi^T}(\overline{q}, \underline{q}, P_L/P_H)$.

Lemma 7 *Suppose $\sigma > 1$ and $m > 0$. Then, $\overline{\pi^T} = \overline{\pi^T}(\overline{q}, \underline{q}, P_L/P_H)$ is decreasing with $\overline{q}$, increasing with $\underline{q}$, and increasing with P_L/P_H.*

The result that $\overline{\pi^T}$ increases with $\underline{q}$ may appear counterintuitive. The reason for this is that an increase in $\underline{q}$ lengthens the duration of a sale. This decreases the number of sales event, and thus, increases $\overline{\pi^T}$.

COLI

To resolve the bias in chained indices and derive a better approximation of the COLI, we need a superlative index that takes the elasticity of substitution σ into account. One candidate is the order r superlative index, where we define P_r as

$$P_r(r_0, r_1, c_0, c_1) = \frac{\left\{\sum_{k\in K_0\cap K_1} s_0^k \left(r_1^k/r_0^k\right)^{(1-\sigma)}\right\}^{1/\{2(1-\sigma)\}}}{\left\{\sum_{k\in K_0\cap K_1} s_1^k \left(r_0^k/r_1^k\right)^{(1-\sigma)}\right\}^{1/\{2(1-\sigma)\}}}, \tag{17}$$

where s_t^k represents the consumption share of product k in period t.

The following lemma shows that P_r serves as a COLI if the unit cost function is expressed as

$$\mathcal{C}(r_t) = \left[\sum_{i\in K_0\cap K_1}\sum_{k\in K_0\cap K_1} \alpha^{ik}\left(r_t^i\right)^{(1-\sigma)}\left(r_t^k\right)^{(1-\sigma)}\right]^{1/\{2(1-\sigma)\}} \tag{18}$$

where $\alpha^{ik} = \alpha^{ki}$. This cost function is based on a more generalized form of utility than that given by equation (7).[30]

Lemma 8 *Given the unit cost function of (18), P_r equals $\mathcal{C}(r_1)/\mathcal{C}(r_0)$.*

3.3 Inference of Consumption and Consumption Prices and Calculation of the COLI

3.3.1 Motivation

Many important variables such as consumption, inventories, and the consumption price are often unobservable, although we can observe the quantity purchased and posted price using retailer-side POS scanner data. From a practical perspective, consumption and the consumption price are essential for constructing the COLI. For macroeconomists, it is of great interest to see whether any changes in stockpiling behavior can be observed over time and, if so, what the determinants are. Therefore, in this section, we propose

[30] Another index that can serve as a COLI is the Lloyd–Moulton Index. See ILO et al. (2004).

a simple and tractable methodology to infer these variables using retailer-side scanner data.

Some of the previous studies on households' inventory behavior, including Erdem, Imai, and Keane (2003) and Hendel and Nevo (2006b), structurally estimate the parameters associated with inventory cost functions, which would be helpful in identifying the paths of consumption, inventories, and consumption prices. These studies employ household-side scanner data, where the quantities and prices of the goods purchased by individual households are recorded, to conduct their empirical exercises. One of the datasets we employ in the present study, the *Shoku-map* data, is similar to the datasets they use, and it contains information on purchases and consumption at the household level. However, it lacks information on purchase prices, making it difficult for us to take an approach similar to theirs. More importantly, few practitioners at national statistical offices have access to household-side scanner data, so that an approach for constructing a COLI based on consumption and consumption prices that requires household-side scanner data would not be very useful for them. Furthermore, the empirical exercises conducted in the previous studies focus on a particular product (e.g., ketchup in Erdem, Imai, and Keane (2003) and detergent in Hendel and Nevo (2006b)) to estimate the parameters associated with the inventory cost function. The task we have set ourselves in this study is quite different, in that we seek to estimate a COLI which covers numerous products that are heterogeneous in terms of their storability and the form of their inventory cost function. However, reliably estimating inventory cost functions for all products likely is next to impossible, so that we use a different approach.

Specifically, we employ retailer-side scanner data rather than household-side scanner data to construct a COLI. Retailer-side scanner data, which record transactions at the store level rather than at the household level, are widely used by national statistical offices in an attempt to construct COLIs using alternative data sources (see, for example, chapter 10 of ILO et al., 2020). Because it is almost impossible to obtain reliable estimates for the inventory cost functions of all products, we introduce an assumption about the path of consumption prices after a sale ends; that is, we assume that the increase in consumption prices after a sale ends is *linear* with time. This assumption allows us to estimate consumption, consumption prices, and inventories from retailer-side scanner data and, as explained later, is consistent with the model developed in the previous section. Note that previous studies on household inventories often assume that consumption is constant over time (e.g., Boizot, Robin, and Visser, 2001 and the

empirical part of Hendel and Nevo, 2006a). The assumption of constant consumption may allow us to infer consumption, consumption prices, and inventories. However, this implies that consumption prices are constant in our model. This is inconsistent with our model, where consumption prices fluctuate taking a value between P_L and P_H. Moreover, this assumption is not consistent with the fact we showed in Section 2, that is, people consume more when they have more inventories.

3.3.2 Methodology

The consumption price at $p_t = P_L$ equals P_L. The key variable is the consumption price at $p_t = P_H$ after a sale ends, that is, $r_H(I_{t-1}) \equiv r(I_{t-1}, P_H)$, where $r_H(0) = P_H$. Knowing this variable enables us to obtain consumption c_t after a sale using equation (8).

When $p_t = P_H$, household producers' optimization problem is given by

$$C'(i_H; I_{t-1}) = \beta \left\{(1 - \overline{q}) r_H(I_t) + \overline{q} P_L\right\} - r_H(I_{t-1}) + \mu_t \tag{19}$$

from equation (13). This equation shows that household producers strike a balance between the benefits of a future consumption-price increase (the right-hand side) and the costs of holding inventories (the left-hand side). Note that μ_t is the Lagrange multiplier associated with i_t and equals zero if $i_t > 0$.

We note that when inventory cost function $C(\cdot)$ is written in a certain form (see Appendix D), $r_H(I_t) - r_H(I_{t-1})$ becomes a positive constant. In other words, the expectation of a *linear* consumption-price increase prevents household producers from selling all of their inventories instantaneously or from selling none at all. Household producers gradually sell off their inventories to consumers.

In the following analysis, we assume this linearity holds. While this admittedly is a restrictive assumption, it can be interpreted as an intermediate of the following two scenarios. The first is when $C'' \to 0$. In this scenario, $r_H(I_t) - r_H(I_{t-1})$ increases in t. Put differently, if inventory costs are not convex, household producers require a greater consumption-price increase as time goes by, because they discount the future ($\beta < 1$) and expect another sale to come at some point ($\overline{q} > 0$). As for the second scenario, suppose $\beta = 1$ and $\overline{q} = 0$. Then $r_H(I_t) - r_H(I_{t-1})$ decreases in t. The cost of holding inventories decreases as household producers' inventories decrease because of $C'' > 0$, which makes household producers require a smaller consumption-price increase as time goes by.

The benefit of this linearity assumption is that it greatly simplifies our analysis. Given the path of $r_H(I_{t-1})$, we compute consumption c_t as $(r_H(I_{t-1})/P_L)^{-\sigma} c_L^*$, where c_L^* represents consumption during a sale. Briefly put, c_L^* is the lower value of the quantity purchased (observable in the POS data) during a sale because according to the model, consumption during a sale is equal to the lowest amount of purchases during the sale. Furthermore, we can calculate the degree of stockpiling, m. More precisely, we define m to denote how long inventories last after a sale ends. In continuous time, we can derive the following equation:

$$m_{cont} = \frac{P_H - P_L}{P_L} \frac{\sigma - 1}{1 - (P_H/P_L)^{-\sigma+1}} \frac{I_L}{c_L^*}. \tag{20}$$

The proof is provided in Appendix D. Simply put, the equation can be derived because cumulative consumption for m_{cont} periods equals the initial inventories outstanding just after a sale ends, I_L, and the consumption price linearly increases from P_L to P_H in m_{cont} periods. Inventories I_L equal the cumulative amount of purchases during a sale minus the cumulative amount of consumption, Tc_L^*, in the same period.

The right-hand panel of Figure 4 illustrates the pattern of price and quantity changes in the case of $m = 5$. The consumption price, depicted by the circles in the top panel, increases at a constant rate from $t = 3$ to 8. The bottom panel shows that while the quantity purchased falls to zero from $t = 4$ to 7, consumption does not fall to zero, but decreases gradually.

In Appendix E, we employ numerical simulations to depict a typical path of the price and quantity and calculate biases in chained price indices. It shows that the COLI and the chained order r superlative index do not show any bias and asymptotically return to their original level. The chained consumption-weighted Törnqvist index has a downward bias, while the purchase-weighted price indices have a much larger bias. The size of the bias in the purchase-weighted Törnqvist index is comparable to the actual size of the bias in the Törnqvist index for Japan.

3.3.3 Robustness of Our Approach

It should be noted that the fact that goods are storable means that price indices will always be subject to the intertemporal substitution bias. This is the case regardless of whether a simplifying assumption such as the one we introduce is employed. In fact, the model in the previous section succeeded in qualitatively explaining the bias in various chained price indices, including its sign, without relying on this simplifying assumption.

However, additional assumptions are needed for quantitative purposes such as investigating the size of the bias. Since the linearity assumption is only one of many possible assumptions, in Appendix F, we examine how much the size of the chain drift changes in response to different assumptions about the path of the consumption price after a sale ends. We conclude that the alternative approaches leave our quantitative results more or less unchanged.

3.4 Summary of Our Approach

Using the POS data, we calculate key variables such as $r_t = r_H(I_{t-1})$, c_t, c_L^*, m, and I_L for each sales event for each product at each retailer, while we calculate σ for each 3-digit product category. Specifically, we take the following four steps. First, for each product and retailer, we identify all sales events using the sales filter.

Second, we record the price and the quantity purchased just before and after a sale. Further, we record the average price and the quantity purchased in the first and second half of a sale. This enables us to obtain consumption prices and the quantities consumed just before and during a sale (but not after a sale).

Third, we calculate the elasticity of substitution, σ, for each 3-digit product category. We apply the consumption prices and the quantities consumed before and during a sale obtained in the previous step to equation (8). We will come back to this point in Section 5.1.

Fourth, we calculate $r_t = r_H(I_{t-1})$, c_t, c_L^*, m, and I_L for each sales event for each product at each retailer. We employ the methodology discussed in the previous subsection to calculate m. Once we obtain m, we can calculate the path of consumption prices $r_t = r_H(I_{t-1})$ and consumption c_t after a sale ends.

In Appendix D, we provide a detailed explanation of how we calculate these variables.

4 Application to Japanese Retailer Scanner Data

4.1 The Degree of Stockpiling

We apply the approach explained in Section 3.4 to Japanese POS scanner data. Figure 5 presents a histogram of the degree of stockpiling m_{cont} at the 3-digit product category level (hereafter we simplify the notation by dropping *cont* from m_{cont}). The distribution of m ranges from one to five, with the mode being around two. Table 5 lists the top and

bottom five product categories with the largest and smallest m. The top three categories are instant cup noodles, diluted beverages, and frozen meals, in that order. These products can indeed be stored for a long time. However, the storability of products does not necessarily imply a high degree of stockpiling. In fact, of the five bottom categories, razors, cosmetic accessories (e.g., hand cream and sunscreen lotion), home medical supplies, and batteries are also highly storable. What distinguishes them from storable goods such as cup noodles is that once they are opened, they can continue to be used for a long time. That is, whereas cup noodles, for example, are consumed more or less immediately after they are opened, razors, cosmetic accessories, and batteries can often be used for longer than a week. As a result, consumers tend not to purchase more than two units even when they are on sale. This suggests that the degree of stockpiling, m, may be negatively correlated with the length that a product lasts once it is opened.[31]

4.2 Validity Checks of Our Approach

4.2.1 Comparison of Inflation Rates: Data and Simulation

To check the validity of our approach, we first examine whether the simulated bias in chained indices is comparable to the actual price change. The actual price change is the average of price changes based on the purchase-weighted Törnqvist index for each 3-digit product category j (denoted by π_j) over the observation period. At the same time, we record the following variables: $\overline{q_j}$, $\underline{q_j}$, and $(P_L/P_H)_j$. Furthermore, we calculate the average of the degree of stockpiling, m_j. Then, using these values, we simulate the model and calculate the average price change in the purchase-weighted Törnqvist index (see Appendix E for the simulation method). The elasticity of substitution σ is set to 3 for all product categories.

Figure 6 shows that our approach does a reasonably good job in explaining the actual change in chained price indices. In the figure, each circle represents a product category. The figure shows that product categories that have a large price change in the data tend to exhibit a large bias in the simulation as well. The circles tend to lie below the 45 degree line, suggesting that the actual price change is smaller than the simulated bias.[32]

[31] The degree of stockpiling is highly heterogeneous even within products belonging to the same product category. See Appendix E.

[32] The fourth column of Table 3 shows that chain drift is negatively correlated with m_j. In Appendix E, we investigate by simulations how the inflation rate, π_j, depends on m_j, $\overline{q_j}$, $\underline{q_j}$, and $(P_L/P_H)_j$. We

4.2.2 Comparison with the *Shoku-map* Data

To further check the validity of our approach, we compare the degree of stockpiling (m) obtained from the POS data with related variables in the *Shoku-map* data. Specifically, we look at the duration of inventories defined as the difference between the date of purchase (t_p) and the date on which the item is used up (t_l). Using the cross-sectional dispersion of m at the 3-digit product category level, we examine if there is any significant correlation. The left-hand panel of Figure 7 shows that there is no significant correlation between $\log(m)$ in the POS data and $\log(t_l - t_p + 1)$ in the *Shoku-map* data for the period of $1988 - 2013$. This suggests that m does not necessarily represent storability.

If m captures stockpiling, it should incorporate not only storability but also the quantity purchased. For this reason, the average quantity purchased in the *Shoku-map* data, $\log(q)$, may correlate with $\log(m)$ in the POS data. The right-hand panel of Figure 7 shows that this is indeed the case. A positive correlation is observed between $\log(q)$ and $\log(m)$ for the average of the entire observation period with the correlation coefficient of $+0.32$.

4.3 Price Indices

Using the POS data, we calculate the time-series of the price level based on the following three definitions. The first is the Törnqvist index based on the purchase weight. This is the conventional approach and we showed the results in Figure 1. The second definition is the Törnqvist index based on the consumption weight. Here, we use the consumption price as well as consumption to calculate the price index. The third definition is the order r superlative index, where we use the estimated elasticity of substitution σ at the category level.

Table 1 displays the average chain drift and inflation rate based on the three price indices. The second column of the table shows that chain drift is significantly negative for $dt \leq 52$ when we use the consumption-weighted Törnqvist index. By contrast, the third column shows that chain drift is insignificant when we use the consumption-weighted order r superlative index for $dt = 1$. In this respect, our approach that uses the consumption-weighted order r superlative index is effective in eliminating chain drift, although it is significantly negative for intermediate dt (i.e., 7 or 14).

The bottom row shows that the average of annualized inflation rates are negative

find that π_j depends negatively on m_j.

and still sizable for all the three indices. However, the size of deflation decreases as we take account of consumer inventory more appropriately, that is, -46% for the purchase-weighted Törnqvist index, -13% for the consumption-weighted Törnqvist index, and -11% for the consumption-weighted order r superlative index. Appendix E compares the time-series changes in the inflation rates.[33]

5 Further Issues of Consumer Inventory

The importance of consumer inventory is not limited to price indices. In this section, we investigate implications of consumer inventory for the measurement of the elasticity of substitution. Then, we study how consumers' inventory behavior changes in response to an preannounced consumption tax hike and business cycles.

5.1 The Elasticity of Substitution

5.1.1 Our Approach to Measuring the Elasticity of Substitution

Our approach is also useful for obtaining the value of the elasticity of substitution, σ.[34] Note that the estimate of the elasticity of substitution is biased unless we properly take account of stockpiling. Even if there is a clear negative relationship between observable

[33]Oher factors may contribute to a bias in chained price indices. The most important factor would be product turnover. Especially in Japan, Ueda, Watanabe, and Watanabe (2019) show that product prices tend to decline over the life-span of a product. In this case, the price indices that use only a matched sample by ignoring price difference between old and new products will be biased downward. This downbard bias is indeed consistent with the result shown in Table 1 that the inflation rate based on the Törnqvist index is 6% lower than its chain drift in the POS data. A cross-sectional substitution might also cause a bias, if, for example, the timing of a sale is correlated across retailers or products. This may amplify or reduce consumers' stockpiling behaviors, affecting the bias in chained price indices. Using scanner data for Australia, Ivancic, Diewert, and Fox (2011) calculate how much changes in price indices change when they use the set of products which are sold in both the first and last periods to eliminate the effect of product turnover or when they aggregate the same product over stores to examine the effect of cross-sectional aggregation.

[34]The elasticity we measure in this study represents the short-run intertemporal own price elasticity of consumption. By short-run intertemporal, we mean the elasticity that incorporates stockpiling. Ogaki and Reinhart (1998) distinguish the elasticities of intertemporal substitution of spending and consumption when goods are storable. It is the own elasticity in that we do not estimate a cross-price elasticity, which compares substitution among different products.

purchases and posted prices, this does not necessarily indicate the elasticity of substitution. As argued by Hendel and Nevo (2004), price reductions influence the quantity purchased not only via the consumption effect (consumption is price sensitive) but also via the stockpiling effect (i.e., consumers stockpile for future consumption). Owing to the latter effect, which is often larger than the former, the elasticity of substitution is likely to be overestimated when we ignore stockpiling.

Our approach makes it possible to obtain the value of σ. For each 3-digit product category, for each sales event of product k and retailer r, we collect the records of the log ratio of the quantity consumed during a sale to the quantity consumed when the product is sold at the regular price divided by the log ratio of the sale price to the regular price, that is, $\Gamma \equiv -\log(c_L/c_H)/\log(r_L/r_H)$. Equation (8) suggests that Γ equals σ on average.[35] We calculate the unweighted average of Γ across sales events, products, and retailers for each 3-digit product category, which we define as σ.

For comparison, a simple calculation of σ is possible if we ignore storability. Hendel and Nevo (2004) argue that neglecting stockpiling leads to an overestimation of σ by a factor of two to six. To confirm this, we obtain σ by simply calculating the slope of $-\Delta \log X_t / \Delta \log p_t$ using the observed series of purchases X_t and posted prices p_t, where Δ is the difference from the previous date. In this simple calculation, we use all observations as long as the posted price changes by more than two yen from the previous date. Note that, according to equation (8), the simple calculation would be valid if c_t and r_t equal X_t and p_t, respectively.

5.1.2 Results

Figure 8 shows the value of the elasticity of substitution σ for 3-digit product categories, calculated from $\Gamma = -\log(c_L/c_H)/\log(r_L/r_H)$. The left-hand panel displays the histogram of σ and shows that σ is distributed smoothly around the mode of three and most of the values are positive.

The dotted line in the left-hand panel shows the histogram of the simple estimate of σ, which is distributed to the right of the solid line (our estimate). This result is in line with Hendel and Nevo's (2004) result, suggesting that σ is overestimated if we ignore storability. The right-hand panel shows the scatter plot of the value of σ, where

[35] It can be assumed that prices are exogenous for households, especially over such a short time horizon. Thus, any bias from endogeneity is unlikely to be a major problem.

each dot represents a 3-digit product category. In most categories, the simple estimates are larger than our estimates. Nevertheless, the two measures are not independent of each other and exhibit a positive correlation. Finally, it should be noted that the estimates of σ are negative for some product categories (although they are not necessarily significant), whereas from a theoretical perspective we would expect them to be positive. We aggregate the variables of interest at the 3-digit category level only when σ is greater than zero.

5.2 Effect of the Consumption Tax Rate Hike

On April 1st, 2014, the consumption tax rate was raised from 5% to 8%. This was a preannounced event, which prompted households to stockpile before the tax hike.[36] To examine the effect of the consumption tax hike, we calculate the change in $\log(m)$ in March 2014 from the same month in the previous year.

The left-hand panel of Figure 9 shows that the changes in $\log(m)$ in March 2014 are positive for most product categories. Furthermore, the correlation coefficient between this variable and $\log(t_l - t_p + 1)$ based on the *Shoku-map* data is high at +0.48, although the correlation was insignificant for the period of $1988 - 2013$. This evidence supports our interpretation that m captures the degree of stockpiling. The degree of stockpiling increases in response to an anticipated price increase and that goods with a longer inventory duration tend to be stockpiled more. Furthermore, the right-hand panel of Figure 9 shows that a positive correlations continues to be observed between $\log(q)$ and $\log(m)$ in March 2014, where the correlation coefficient is +0.40.

5.3 Developments in Households' Stockpiling Behavior

5.3.1 Changes in Stockpiling Behavior

The degree of stockpiling m is not only heterogeneous but also time-varying. The line with dots in Figure 10 shows the time-series developments in aggregate $\log(m_t)$, given by the unweighted mean of $\log(m_j)$ for 3-digit product category j, from January 1989

[36] In Japan, increases in the consumption tax rate—from 3 to 5% in April 1997, from 5 to 8% in April 2014, and from 8 to 10% in October 2019—have always been a major political and economic event, because they have been accompanied by large demand increases before the tax hike and persistent weak demand (recessions) after the hike. For this reason, Prime Minister Shinzo Abe postponed the latest hike, from 8% to 10% twice, once in 2014 and once in 2016.

to December 2018. The line indicates that there has been a secular decrease in the last two decades.

It should be noted that m can change as a result of changes not only in households' intrinsic behavior but also in prices, which are exogenous to households. To show this formally, we use equation (13) when $p_t = P_L$. Household producers optimize their inventories during a sale to satisfy

$$C'(i_L; I_{t-1}) = \beta \left\{ (1 - \underline{q}) r_H(I_t) + \underline{q} P_L \right\} - P_L + \mu_t. \tag{21}$$

If a sale ends in period t, the consumption price increases in period $t+1$. This provides household producers with a profit if they hold inventories, as the right-hand side of the equation shows. However, household producers incur a cost when holding inventories, as shown in the left-hand side of the equation. Suppose $C'(i_t; I_{t-1}) = C > 0$ and linear consumption-price increases. Then we have $C = \beta(1-\underline{q}) \left\{ \frac{1}{m} \frac{P_H - P_L}{P_L} + 1 \right\} P_L - (1 - \beta \underline{q}) P_L$, and, in turn,

$$m = \beta(1 - \underline{q}) \frac{P_H - P_L}{P_L} \left(1 - \beta + \frac{C}{P_L} \right)^{-1}. \tag{22}$$

This equation suggests that the degree of stockpiling m negatively depends on the probability that a sale will continue to occur at $t+1$ given that a sale occurs at t, positively depends on the size of the sale discount, and negatively depends on the cost of holding inventories. In other words, more stockpiling occurs the sooner a sale is expected to end and the larger the sale discount is. More generally, $\overline{q}$ is also likely to influence r_H and, in turn, m. Thus, $\overline{q}$, $\underline{q}$, and $(P_H - P_L)/P_H$ should constitute explanatory variables for m.

Therefore, we regress the following equation: $\log(m_{jt}) = c_j + d_t + AX_{jt} + \varepsilon_{jt}$, using the vector of explanatory variables $X_{jt} = \{\log(1 - \overline{q_{jt}}), \log\left(1 - \underline{q_{jt}}\right), \log((P_H - P_L)/P_H)_{jt}\}$, where j and t represent the 3-digit product category and the month (January 1989 to December 2018), respectively. Time fixed effect d_t captures the aggregate, demand-side, time-varying component of m_{jt}. Some of the variables we use are persistent and close to an I(1) process. Thus, to avoid spurious regression, we also estimate the above equation using the time differences of the variables such as $\Delta\log(m_{jt}) \equiv \log(m_{jt}) - \log(m_{jt-1})$.

Table 6 presents the estimation results. In the table, column (1) shows the result when we use $\log(m_{jt})$ as the dependent variable and do not include the time fixed effect. Column (2) shows the result when we use $\log(m_{jt})$ as the dependent variable and include the time fixed effect. Column (3) shows the result when we use the time difference of $\log(m_{jt})$ as the dependent variable and do not include the time fixed effect.

The coefficient on $\log\left((P_H - P_L)/P_H\right)$ is significantly positive in all columns. The positive relationship between $(P_H - P_L)/P_H$ and m is consistent with the relationship derived from equation (22). The coefficient on $\log\left(1 - \overline{q}\right)$ is insignificant at the five percent level in columns (1) and (2) but significantly negative in the regression that uses time differences (column (3)).

The coefficient on $\log\left(1 - \underline{q}\right)$ is significantly negative in all columns, even though equation (22) suggests it should be positive. One possible reason is that $\underline{q}$ might be endogenous. It should be noted that in this regression, the cost of inventories is not controlled for. For product categories with low inventory costs, m is likely to be high. If firms hold longer sales (high $\underline{q}$) for these products, we would expecte to observe a negative coefficient on $\log\left(1 - \underline{q}\right)$ rather than a positive one. Another reason is that in equation (20) m is proportional to I_L, which tends to increase as the duration of a sale T increases. Owing to this construction, our measure of m tends to increase as $\underline{q}$ increases.

The solid line in Figure 10 shows the time-series of time fixed effect d_t (column 2 in the table), which represents changes in the degree of stockpiling after controlling for the effects of price changes. Developments in d_t differ from those in aggregate $\log(m_t)$. Specifically, d_t exhibits an increase in the 2000s, while aggregate $\log(m_t)$ does not.[37]

5.3.2 Effects of Macroeconomic Variables on Stockpiling Behavior

What brought about the secular decrease in household stockpiling behavior (m_t and d_t) in the 1990s and then the reversal in the 2000s (d_t, but not m_t)? Household stockpiling behavior likely is influenced by a number of factors, which we consider in this subsection.

Possible Channels First, according to equation (22), an increase in β increases stockpiling because households put greater weight on future consumption. Possible factors that may bring about changes in β are preference shocks, which are often incorporated in dynamic stochastic general equilibrium models as part of demand shocks and also cause changes in real interest rates. Specifically, preference shocks generate a negative relationship between stockpiling and real interest rates. Furthermore, it is also thought that higher inflation expectations promote stockpiling, which also yields a negative correlation between real interest rates and stockpiling. Thus, investigating developments

[37] We find that this deviation is explained by both the decrease in the probability of sales ($\underline{q}$) and the decrease in the size of sale discounts ($\log\left((P_H - P_L)/P_H\right)$). For the detailed changes in these variables associated with sales, see Sudo, Ueda, and Watanabe (2014).

in real interest rates should provide a clue as to how and why stockpiling has changed in Japan.[38] Over the last two decades, Japan has seen successive waves of monetary accommodation, leading to a decline in nominal interest rates, although, due to the zero lower bound on nominal interest rates, it is debatable whether real interest rates have declined as a result of monetary policy.

Second, stockpiling behavior may be influenced by factors that change the cost of holding inventories C. According to equation (22), an increase in C decreases the incentive for stockpiling. One possible factor that may influence C is an interest rate. For example, a higher interest rate increases borrowing costs, which may prevent households from stockpiling.[39]

The third factor concerns labor market conditions. Consider the following two opposing hypotheses. Suppose that labor market conditions are unfavorable for households, that is, low labor demand brings about high unemployment, low hours worked, and low income. One hypothesis is that households face stricter financial (liquidity) constraints and are therefore unable to purchase as much as they would like when prices are low. In that case, unemployment has a negative effect on stockpiling, while hours worked have a positive effect. The other hypothesis is that when unemployment is high and hours worked are low, households have more time for shopping, which allows them to find products that are on sale and stockpile inventories. Also, a decrease in income may make households more price-sensitive. In that case, unemployment has a positive effect on stockpiling and hours worked have a negative effect.

Regression Bearing these factors in mind, we examine whether the degree of stockpiling m_t depends on the macroeconomic environment. We estimate the following equation: $\Delta\log(m_{jt}) = c_j + B\Delta Z_t + A\Delta X_{jt} + \mu_{jt}$ or $\Delta\log(m_{jt}) = c_j + B\Delta Z_t + A\widetilde{\nu_{jt}} + \mu_{jt}$. Here,

[38] Similarly, interest rates and the cost of holding inventories are also important determinants of firm inventory investment (see, e.g., Kahn, 2016).

[39] Another possible factor is the size of houses and Japan's demographic structure (population aging). However, looking at data for the size of houses from the Housing and Land Survey conducted by the Ministry of Internal Affairs and Communications every five years shows that there has been a steady increase in both the average housing area and the average housing area per household member from 1993 to 2013 from $88.4m^2$ to $93.0m^2$ and from $29.8m^2$/person to $38.5m^2$/person, respectively. This suggests that C should have decreased monotonically and d_t increased monotonically. However, such a monotonic decrease did not occur, as shown in Figure 10. Population aging is also a monotonic development in Japan.

ΔX_t is the time difference of the price variables used above from month $t-1$ to t, and ΔZ_t is the time difference of exogenous variables consisting of the unemployment rate, log hours worked, and the real interest rate from month $t-1$ to t and $t-2$ to $t-1$. The real interest rate in period t is defined as the overnight call rate in period t minus the actual inflation rate based on the CPI from t to $t+12$ (all in percent). We estimate the equation using the time difference to avoid spurious regression. Lagged variables for Z_t are added to incorporate the possibility that it takes time for labor market conditions and the real interest rate to influence household stockpiling behavior.[40]

In the first regression, we estimate the degree of stockpiling using ΔX_t as the independent variable. In the second regression, we use $\widetilde{\nu_{jt}}$, the residuals of the estimated equation of $\Delta X_{jt} = e_j + D\Delta Z_t + \nu_{jt}$, as the independent variable. Labor market conditions and the real interest rate ΔZ_t likely influence firms' pricing ΔX_{jt} as well. Since $\widetilde{\nu_{jt}}$ is orthogonal to ΔZ_t, by using $\widetilde{\nu_{jt}}$, we aims to evaluate the overall effect of ΔZ_t on $\Delta \log(m_{jt})$, which incorporates the indirect effect through ΔX_{jt}.

Table 7, particularly column (5), shows the main estimation results, while column (1) shows the estimation results when we simply use ΔX_{jt}. The effect of the unemployment rate on $\Delta \log(m_{jt})$ is small, because the two coefficients on the unemployment rate at t and $t-1$ more or less cancel each other out. The two coefficients on hours worked at t and $t-1$ are significantly negative, suggesting that longer hours worked decrease the degree of stockpiling. This result supports the hypothesis that longer hours worked decrease households' time for shopping, which prevents them from stockpiling inventories during sales, rather than the hypothesis focusing on households' financial constraints and predicting the opposite effect. The coefficient on the real interest rate at t is significantly negative, suggesting that a higher real interest rate decreases the degree of stockpiling. This is consistent with the reasoning mentioned above.

Columns (2) to (4) show that, in response to longer hours worked, firms change their pricing so that the frequency of sales decreases (low $\overline{q}$), the duration of sales increases (high $\underline{q}$), and the size of sale discounts increases (high $(P_H - P_L)/P_H$). The effects of the changes in the unemployment rate and the real interest rate on pricing are unclear, however.

These results suggest that using lower frequency data to calculate price indices has the risk of ignoring the effects of business cycle fluctuations. This may yield biased price

[40]Note that m_{jt} is not likely to influence Z_t because the former is a variable at the product category level.

indices where the bias changes with the business cycle. In Appendix G, we investigate implications of this endogeneity of stockpiling quantitatively.

6 Conclusion

We investigated consumer inventory from the perspective of a bias in chained price indices. Goods storability, especially stockpiling during temporary sales, causes a tremendous degree of bias when price indices are based on purchases, since consumers tend to stockpile when prices are low (i.e., during a sale) and purchases exceed consumption. We constructed a model to explain the facts we observe, proposed a tractable approach to infer consumption and consumption prices using data on purchases and purchase prices, and applied it to Japanese data. We showed that consumers' stockpiling behavior can be conveniently summarized by a single variable: the degree of stockpiling during a sale, which expresses how long inventories last after a sale ends. Applying the approach to POS data for Japan, we found that our approach of using a consumption-weighted index succeeds in eliminating the chain drift. Furthermore, we showed that the degree of stockpiling depends on the macroeconomy.

Tasks for the future include, first, a more careful consideration of heterogeneity at the product and household levels. We found that there exists sizable heterogeneity in the degree of stockpiling across products. A more detailed investigation might shed new light on consumer inventory behavior. Equally important is the heterogeneity at the household level. Considering the possibility that stockpiling behavior depends on the size of the family and home, income, age, etc., could provide new insights.

Second, we should apply our approach to a wider range of product categories than those covered in our data, processed food and daily necessities, which make up only about 20 percent of households' expenditure. For instance, prices for some storable goods (e.g., gasoline and fresh food), durable goods (e.g., clothing and personal computers), and services (e.g., travel) occasionally change substantially just like in a temporary sale, which seems to cause demand fluctuations similar to stockpiling. It is worth testing whether our approach is useful for the analysis of these product categories.

References

[1] Abe, Naohito, and Akiyuki Tonogi (2010) "Micro and Macro Price Dynamics in Daily Data," *Journal of Monetary Economics*, 57(6), 716–728.

[2] Baker, Scott R., Stephanie Johnson, and Lorenz Kueng (forthcoming) "Shopping for Lower Sales Tax Rates," *American Economic Journal: Macroeconomics*.

[3] Baker, Scott R, Stephanie G Johnson, and Lorenz Kueng (2020) "Financial Returns to Household Inventory Management." NBER Working Paper 27740.

[4] Blattberg, Robert, and Scott Neslin (1989) "Sales Promotion: The Long and the Short of It," *Marketing Letters*, 1(1), 81–97.

[5] Boizot, Christine, Jean-Marc Robin, and Michael Visser (2001) "The Demand for Food Products. An Analysis of Interpurchase Times and Purchased Quantities," *Economic Journal*, 111, 391–419.

[6] Cashin, David, and Takashi Unayama (2016) "Measuring Intertemporal Substitution in Consumption: Evidence from a VAT Increase in Japan," *Review of Economics and Statistics*, 98(2), 285–297.

[7] Chevalier, Judith, and Anil Kashyap (2019) "Best Prices: Price Discrimination and Consumer Substitution," *American Economic Journal: Economic Policy*, 11(1), 126–159.

[8] Coibion, Olivier, Yuriy Gorodnichenko, and Dmitri Koustas (forthcoming) "Consumption Inequality and the Frequency of Purchases." *American Economic Journal: Macroeconomics*.

[9] de Haan, Jan (2008) "Reducing Drift in Chained Superlative Price Indexes for Highly Disaggregated Data," presented at the EMG Workshop.

[10] Diewert, W. Erwin (1976) "Exact and Superlative Index Numbers," *Journal of Econometrics*, 4(2), 115–145.

[11] Diewert, W. Erwin and Kevin J. Fox (2020) "Substitution Bias in Multilateral Methods for CPI Construction," *Journal of Business & Economic Statistics*, forthcoming.

[12] Eichenbaum, Martin, Nir Jaimovich, and Sergio Rebelo (2011) "Reference Prices, Costs and Nominal Rigidities," *American Economic Review* , 101, 234–262.

[13] Erdem, Tulin, Susumu Imai, and Michael Keane (2003) "A Model of Consumer Brand and Quantity Choice Dynamics under Price Uncertainty," *Quantitative Marketing and Economics*, 1(1), 5–64.

[14] European Free Trade Association (EFTA) (2020) "Transaction Data in the Consumer Price Index", https://www.efta.int/Statistics/news/Transaction-data-consumer-price-index-518411.

[15] Eurostat (2017) "Practical Guide for Processing Supermarket Scanner Data", https://circabc.europa.eu/sd/a/8e1333df-ca16-40fc-bc6a-1ce1be37247c/Practical-Guide-Supermarket-Scanner-Data-September-2017.pdf.

[16] Feenstra, Robert, and Matthew Shapiro (2003) "High-Frequency Substitution and the Measurement of Price Indexes," in Robert Feenstra and Matthew Shapiro (eds.), *Scanner Data and Price Indexes*, University of Chicago and NBER.

[17] Frisch, Ragnar (1936) "Annual Survey of General Economic Theory: The Problem of Index Numbers," *Econometrica*, 4(1), 1–38.

[18] Goolsbee, Austan D., and Peter J. Klenow. (2018) "Internet Rising, Prices Falling: Measuring Inflation in a World of E-Commerce." *AEA Papers and Proceedings*, 108, 488–492.

[19] Gowrisankaran, Gautam, and Marc Rysman (2012) "Dynamics of Consumer Demand for New Durable Goods," *Journal of Political Economy*, 120, 1173–1219.

[20] Griffith, Rachel, Aviv Nevo, Ephraim Leibtag, and Andrew Leicester (2009) "Consumer Shopping Behavior: How Much Do Consumers Save?" *Journal of Economic Perspectives*, 23, 99–120.

[21] de Haan, Jan, and Heymerik A. van der Grient (2011) "Eliminating Chain Drift in Price Indexes Based on Scanner Data," *Journal of Econometrics*, 161, 36–46.

[22] de Haan, Jan, and Frances Krsinich (2014) "Scanner Data and the Treatment of Quality Change in Nonrevisable Price Indexes," *Journal of Business & Economic Statistics*, 32(3), 341–358.

[23] Hansman, Christopher, Harrison Hong, Aureo de Paula, and Vishal Singh (2020) "A Sticky-Price View of Hoarding," NBER Working Paper No. 27051.

[24] Hendel, Igal, Alessandro Lizzeri, and Nikita Roketskiy (2014) "Nonlinear Pricing of Storable Goods," *American Economic Journal: Microeconomics*, 6(3), 1–34.

[25] Hendel, Igal, and Aviv Nevo (2003) "The Post-Promotion Dip Puzzle: What Do the Data Have to Say?" *Quantitative Marketing and Economics*, 1, 409–424.

[26] Hendel, Igal, and Aviv Nevo (2004) "Intertemporal Substitution and Storable Products," *Journal of the European Economic Association*, 2(2/3), 536–547.

[27] Hendel, Igal, and Aviv Nevo (2006a) "Sales and Consumer Inventory," *RAND Journal of Economics*, 37, 543–561.

[28] Hendel, Igal, and Aviv Nevo (2006b) "Measuring the Implications of Sales and Consumer Inventory Behavior," *Econometrica*, 74, 1637–1673.

[29] Hendel, Igal, and Aviv Nevo (2013) "Intertemporal Price Discrimination in Storable Goods Markets." *American Economic Review*, 103(7), 2722–2251.

[30] Hong, Pilky, Preston McAfee, and Ashish Nayyar (2002) "Equilibrium Price Dispersion with Consumer Inventories," *Journal of Economic Theory*, 105, 503–517.

[31] ILO, IMF, OECD, Eurostat, United Nations, and World Bank (2004) "Consumer Price Index Manual: Theory and Practice," ILO Publications, Geneva.

[32] ILO, IMF, OECD, Eurostat, United Nations, and World Bank (2020) "Consumer Price Index Manual: Concepts and Methods," draft version (January), https://www.imf.org/en/Data/Statistics/cpi-manual.

[33] Ivancic, Lorraine, W. Erwin Diewert, and Kevin J. Fox (2011) "Scanner Data, Time Aggregation and the Construction of Price Indexes," *Journal of Econometrics*, 161, 24–35.

[34] Kahn, James A. (2016) "Inventory Investment," *New Palgrave Dictionary of Economics*, Palgrave Macmillan, https://doi.org/10.1057/978-1-349-95121-5_2283-1.

[35] Kano, Kazuko (2018) "Consumer Inventory and Demand for Storable Goods: New Evidence from a Consumer Survey," *Japanese Economic Review*, 69(3), 284–305.

[36] Kehoe, Patrick J., and Virgiliu Midrigan (2015) "Prices Are Sticky After All," *Journal of Monetary Economics*, 75, 35–53.

[37] Klenow, Peter J., and Jonathan L. Willis (2007) "Sticky Information and Sticky Prices," *Journal of Monetary Economics*, 54(Supplement), 79–99.

[38] Könus A. A. (1924) "The Problem of the True Index of the Cost of Living," in *Economic Bulletin of the Institute of Economic Conjuncture* (in Russian), 9–10, 64–71; published in English (1939) in *Econometrica*, 7, 10–29.

[39] Konny, Crystal G., Brendan K. Williams, and David M. Friedman (2019) "Big Data in the U.S. Consumer Price Index: Experiences and Plans," NBER Chapters, in *Big Data for Twenty-First Century Economic Statistics*, NBER.

[40] Kryvtsov, Oleksiy, and Nicolas Vincent (2020) "The Cyclicality of Sales and Aggregate Price Flexibility," *Review of Economic Studies*, forthcoming.

[41] Nakamura, Alice O., Emi Nakamura, and Leonard I. Nakamura (2011) "Price Dynamics, Retail Chains and Inflation Measurement," *Journal of Econometrics*, 161, 47–55.

[42] Nakamura, Emi, and Jon Steinsson (2010) "More Facts about Prices." Columbia University. https://eml.berkeley.edu/~jsteinsson/papers/fivefactssupplement.pdf. Accessed April 30, 2020.

[43] Neslin, Scott A., and Linda G. Schneider Stone (1996) "Consumer Inventory Sensitivity and the Postpromotion Dip," *Marketing Letters*, 7, 77–94.

[44] Ogaki, Masao, and Carmen M. Reinhart (1998) "Measuring Intertemporal Substitution: The Role of Durable Goods," *Journal of Political Economy*, 106(5), 1078–1098.

[45] Osborne, Matthew (2018) "Approximating the Cost-of-Living Index for a Storable Good," *American Economic Journal: Microeconomics*, 10(2), 286–314.

[46] Reinsdorf, Marshall B. (1999) "Using Scanner Data to Construct CPI Basic Component Indexes," *Journal of Business and Economic Statistics*, 17(2), 152–160.

[47] Reis, Ricardo (2009) "A Dynamic Measure of Inflation," Working Paper.

[48] Salop, Steven C., and Joseph E. Stiglitz (1982) "The Theory of Sales: A Simple Model of Equilibrium Price Dispersion with Identical Agents," *American Economic Review*, 72(5), 1121–1130.

[49] Sheremirov, Viacheslav (2020) "Price Dispersion and Inflation: New Facts and Theoretical Implications." *Journal of Monetary Economics*, 114, 59–70.

[50] Sudo, Nao, Kozo Ueda, and Kota Watanabe (2014) "Micro Price Dynamics during Japan's Lost Decades," *Asian Economic Policy Review*, 9(1), 44–64.

[51] Sudo, Nao, Kozo Ueda, Kota Watanabe, and Tsutomu Watanabe (2018) "Working Less and Bargain Hunting More: Macroimplications of Sales during Japan's Lost Decades," *Journal of Money, Credit and Banking*, 50(2–3), 449–478.

[52] Szulc, Bohdan J. (1983) "Linking Price Index Numbers." In *Price Level Measurement*, W. E. Diewert and C. Montmarquette (eds). Ottawa: Statistics Canada.

[53] Triplett, Jack E. (2003) "Using Scanner Data in the Consumer Price Index: Some Neglected Conceptual Considerations," in Robert Feenstra and Matthew Shapiro (eds.), *Scanner Data and Price Indexes*, University of Chicago and NBER.

[54] Ueda, Kozo, Kota Watanabe, and Tsutomu Watanabe (2019) "Product Turnover and the Cost of Living Index: Quality vs. Fashion Effects," *American Economic Journal: Macroeconomics*, 11(2), 310–347.

[55] Ueda, Kozo (2020) "Dynamic Cost of Living Index for Storable Goods," *Economics Letters*, 189, 109013.

[56] von Auer, Ludvig (2019) "The Nature of Chain Drift," presented at the Ottawa Group Meeting.

[57] Wang, Emily Yucai (2013) "The Effect of More Accurate Cost-of-Living Indexes on Welfare Recipients," Working Paper.

[58] Watanabe, Kota, and Tsutomu Watanabe (2014) "Estimating Daily Inflation Using Scanner Data: A Progress Report," CARF Working Paper F342, University of Tokyo.

[59] Zhang, Li-Chun, Ingvild Johansen, and Ragnhild Nygaard (2019) "Tests for Price Indices in a Dynamic Item Universe," *Journal of Official Statistics*, 35(3), 683–697.

Table 1: Chain Drift and Inflation Rate

	Price indices		
dt	Törnqvist (purchase-weighted)	Törnqvist (consumption-weighted)	Order r superlative (consumption-weighted)
		Annual chain drift	
1	-40.44^{***}	-5.69***	-1.20
7	-7.24^{***}	-2.28^{***}	-1.23^{***}
14	-2.43^{***}	-1.10^{***}	-0.59^{***}
28	-0.97^{***}	-0.46^{***}	-0.04
52	-0.66^{***}	-0.20^{**}	0.23
91	-0.61^{***}	0.06	0.17
182	-0.49^{***}	0.01	0.14
		Annualized inflation rate	
1	-46.34	-13.06	-10.71

Note: Chain drift is the average from 1989 to 2019. See the main text for the calculation of chain drift, $d_{0,365,dt}$, where dt represetns a time span. We transform $d_{0,365,dt}$, which is on a daily basis, to an annualized rate by $\exp(365 \times d_{0,365,dt}) - 1$. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively, when we apply the sign test for the null hypothesis that the probability to take positive/negative chain drift is equal to 0.5. The annualized inflation rate is calculated as $\exp(365 \times \overline{x_{dt=1}}) - 1$, where $\overline{x_{dt=1}}$ is the mean of daily log inflation from 1990 to 2018.

Table 2: Reason for the Bias

		$t=1$	$t=2$	$t=3$
Product A	Price	p_A	$(1-r)p_A$	p_A
	Share	W_1	W_2	W_3
Product B	Price	p_B	p_B	p_B
	Share	$1-W_1$	$1-W_2$	$1-W_3$

Note: $0 < r < 1$. If the elasticity of substitution is greater than one, we would expect to observe $W_3 < W_1 < W_2$.

Table 3: Relation between Chain Drift and Stockpiling

	Dependent variable:			
	Chain drift for purchase-weighted Törnqvist			
	(1)	(2)	(3)	(4)
Q_H	0.698^{**}			
	(0.315)			
Q_L	1.781^{***}			
	(0.463)			
q		-1.916^{***}		
		(0.483)		
$\overline{q}$			-0.807	-2.190^{**}
			(1.039)	(1.089)
$\underline{q}$			-0.708^{***}	0.014
			(0.154)	(0.259)
$\log(P_L/P_H)$		3.979^{***}	3.929^{***}	2.899^{***}
		(0.644)	(0.618)	(0.673)
$\log m$				-0.344^{***}
				(0.101)
Constant	-0.320^{***}	0.178^{***}	0.418^{***}	0.223^{**}
	(0.039)	(0.062)	(0.075)	(0.093)
Adjusted R^2	0.102	0.400	0.440	0.470
Observations	189	195	195	195

Note: Figures in parentheses represent standard errors. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. The explanatory variables are as follows: Q_H and Q_L represent a degree of an asymmetry for the quantity purchased when price increases and decreases, q is the probability that a product is on sale, $\overline{q}$ is the probability that a product will go on sale on the following day given that it is not currently on sale, $\underline{q}$ is the probability that a product will continue to be on sale on the following day given that it is currently on sale, P_L/P_H is the ratio of the sale price to the regular price, and m is the degree of stockpiling.

Table 4: State-dependent Consumption

Dependent variable	1 if household uses product	Number of times a product is used
Inventory	0.0006895***	0.00410***
	(0.0000171)	(0.000901)
Observations	90,545,020	90,545,020
No. of HH	3,602	3,602
R^2	0.12	0.12
Fixed effects	HH/category	HH/category

Note: Figures in parentheses represent robust standard errors. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Table 5: Top and Bottom Five Categories for the Degree of Stockpiling

	Product category	m		Product category	m
Top 5	Instant cup noodles	4.98	Bottom 5	Razors	1.15
	Diluted beverages	3.74		Prepared bread meals	1.19
	Frozen meals	3.58		Cosmetic accessories	1.26
	Packaged instant noodles	3.23		Home medical supplies	1.27
	Coffee beverages	3.08		Batteries	1.30

Note: The degree of stockpiling m is inferred using the POS data.

Table 6: Regression of the Degree of Stockpiling

	(1)	(2)		(3)
	$\log(m)$	$\log(m)$		$\Delta\log(m)$
$\log(1-\overline{q})$	0.6254	-0.9757*	$\Delta\log(1-\overline{q})$	-2.4682***
	(0.428)	(0.547)		(0.607)
$\log(1-\underline{q})$	-0.6270***	-0.7785***	$\Delta\log(1-\underline{q})$	-0.5104***
	(0.035)	(0.034)		(0.036)
$\log(1-P_L/P_H)$	0.3511***	0.3057***	$\Delta\log(1-P_L/P_H)$	0.2546***
	(0.044)	(0.046)		(0.032)
Fixed effects	category	category/month		category
Observations	54,000	54,000		53,850
Within R^2	0.650	0.753		0.283
# of categories	150	150		150

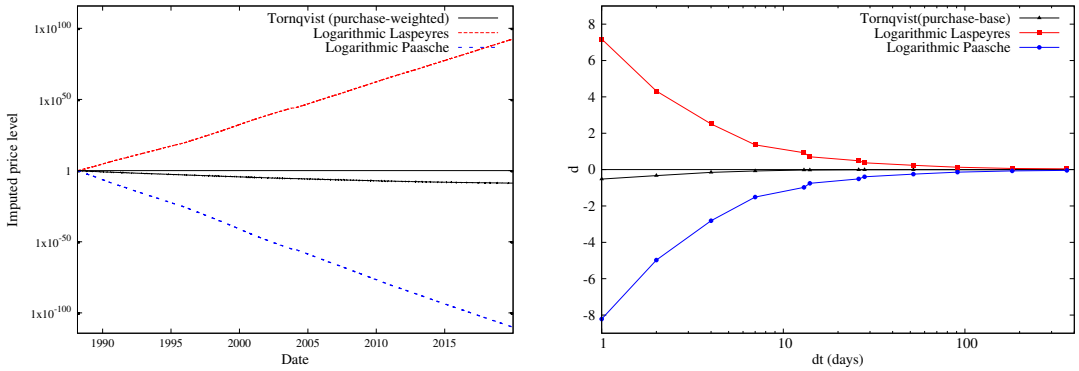
Note: Variable m is the degree of stockpiling, $\overline{q}$ is the probability that a product will go on sale on the following day given that it is not currently on sale, $\underline{q}$ is the probability that a product will continue to be on sale on the following day given that it is currently on sale, and P_L/P_H is the ratio of the sale price to the regular price. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Table 7: Effects of Macroeconomic Variables on Stockpiling Behavior

	(1) $\Delta\log(m)$	(2) $\Delta\log(1-\overline{q})$	(3) $\Delta\log(1-\underline{q})$	(4) $\Delta\log(1-P_L/P_H)$	(5) $\Delta\log(m)$
Δ(unemp rate)	0.0026	-0.0017***	-0.0338***	0.0277***	0.0312***
	(0.004)	(0.000)	(0.003)	(0.003)	(0.003)
Δ(unemp rate(-1))	-0.0106***	0.0007***	0.0244***	-0.0300***	-0.0327***
	(0.004)	(0.000)	(0.003)	(0.003)	(0.004)
$\Delta\log$(hours worked)	-0.2940***	0.0210***	0.1341***	0.1989***	-0.3612***
	(0.043)	(0.002)	(0.030)	(0.033)	(0.043)
$\Delta\log$(hours worked(-1))	-0.6677***	0.0077***	-0.6106***	0.4365***	-0.2602***
	(0.046)	(0.001)	(0.044)	(0.040)	(0.044)
Δ(real r)	-0.0133***	0.0009***	0.0147***	-0.0052***	-0.0243***
	(0.003)	(0.000)	(0.002)	(0.002)	(0.002)
Δ(real r(-1))	-0.0036*	-0.0002***	-0.0086***	0.0037**	0.0024
	(0.002)	0.000	(0.002)	(0.002)	(0.002)
$\Delta\log(1-\overline{q})$	-2.4013***				-2.4013***
	(0.615)				(0.615)
$\Delta\log(1-\underline{q})$	-0.5117***				-0.5117***
	(0.036)				(0.036)
$\Delta\log(1-P_L/P_H)$	0.2604***				0.2604***
	(0.032)				(0.032)
Fixed effects	category	category	category	category	category
Observations	53,700	53,700	53,700	53,700	53,700
Within R^2	0.289	0.008	0.008	0.005	0.289
# of categories	150	150	150	150	150

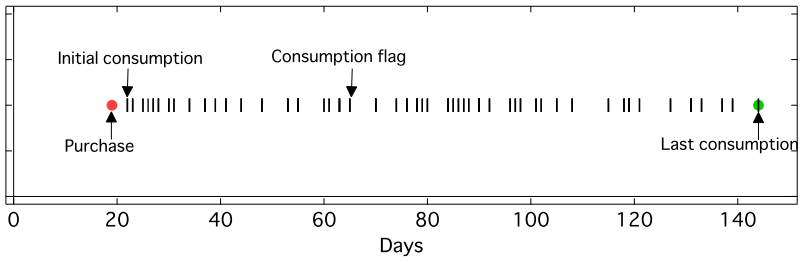
Note: Variable m is the degree of stockpiling, $\overline{q}$ is the probability that a product will go on sale on the following day given that it is not currently on sale, $\underline{q}$ is the probability that a product will continue to be on sale on the following day given that it is currently on sale, P_L/P_H is the ratio of the sale price to the regular price, and real r is the real interest rate. In column (5), the explanatory variables corresponding to $\Delta\log(1-\overline{q})$, $\Delta\log(1-\underline{q})$, and $\Delta\log(1-P_L/P_H)$ are the residuals of the estimation for columns (2), (3), and (4), respectively. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Figure 1: Price Changes in Chained Indices



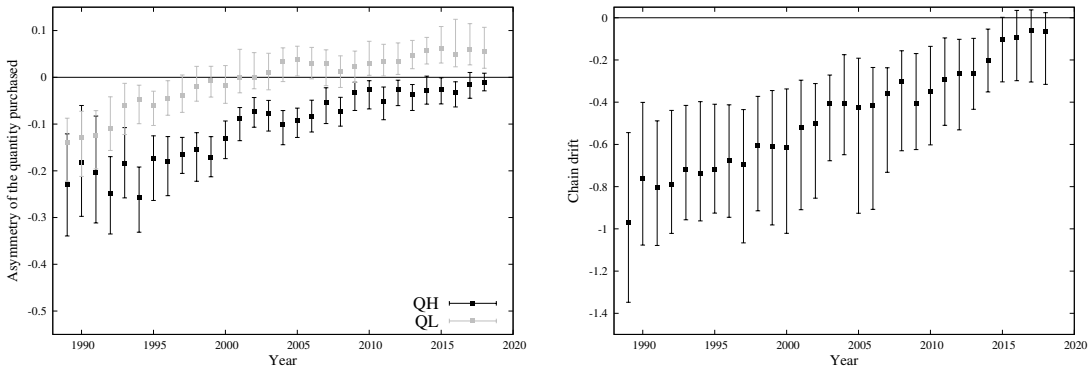
Note: The left-hand figure shows the time-series of price levels based on the purchase-weighted Laspeyres, Paasche, and Törnqvist indices using the POS data. The initial price level is normalized to one. The right-hand figure shows a degree of chain drift, $d_{0,365,dt}$, based on the purchase-weighted Laspeyres, Paasche, and Törnqvist indices, where we employ different time spans dt from 1 day to 364 days.

Figure 2: Consumption Pattern of Salt



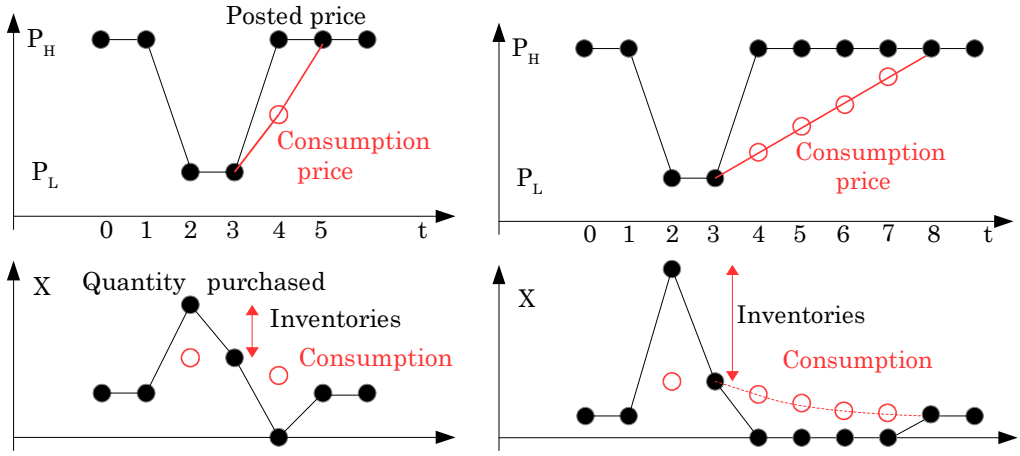
Note: The figure shows the consumption pattern for salt of a particular household in the *Shoku-map* data. Each vertical line represents a consumption flag.

Figure 3: Asymmetry of the Quantity Purchased When Price Increases and Decreases



Note: In the left-hand figure, Q_H and Q_L represent a degree of an asymmetry for the quantity purchased when price increases and decreases. Specifically, Q_H indicates an asymmetry for the quantity purchased between just before a sale starts and just after a sale ends, while Q_L indicates an asymmetry for the quantity purchased between just after a sale starts and just before a sale ends. The circles and two ends represent the median and 25 and 75 percentiles for products and sale events in each year. The right-hand figure shows a degree of chain drift, $d_{0,365,1}$, for each year.

Figure 4: Pattern of Price and Quantity Changes during a Sales Event



Note: The solid dots represent observable posted prices (top) and quantities purchased (bottom). The circles represent unobservable consumption prices (top) and quantities consumed (bottom).

Figure 5: Histogram of the Degree of Stockpiling

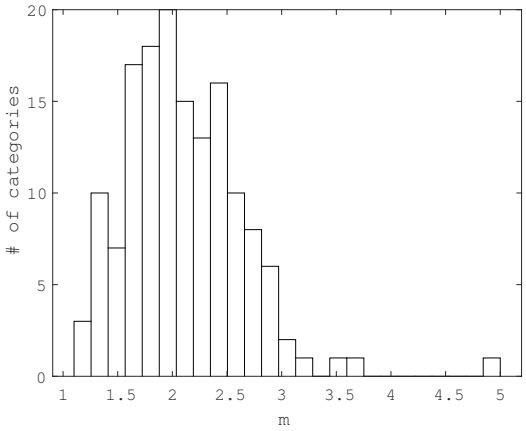
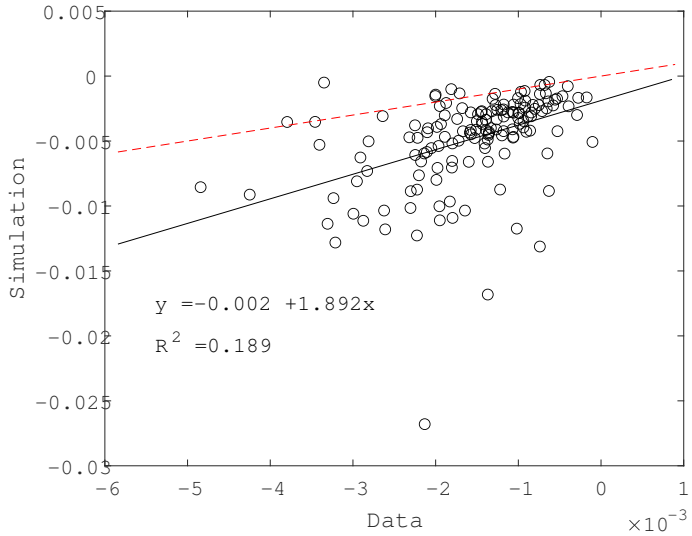
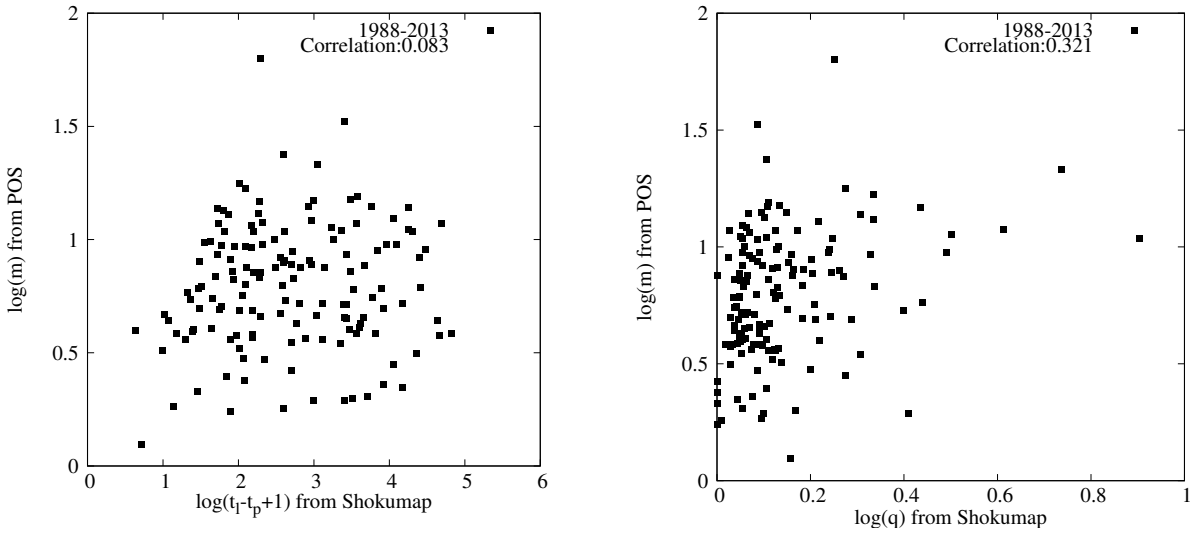


Figure 6: Inflation Rates Based on the Törnqvist Index: Data and Simulation



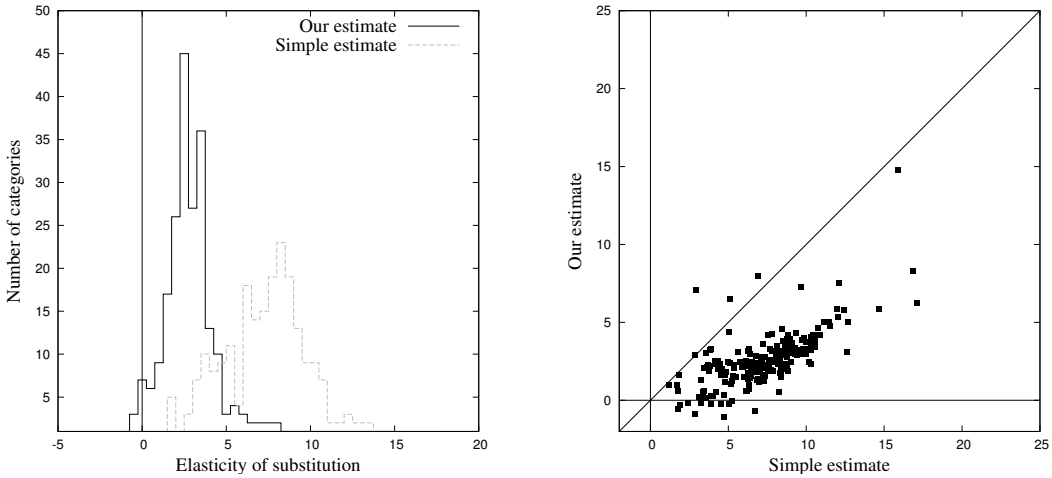
Note: Each circle represents a 3-digit product category. The inflation rates are the daily averages and are based on the purchase-weighted Törnqvist index. The red dashed line represents the 45 degree line.

Figure 7: The Consumption Period and the Quantity Purchased (*Shoku-map*) versus the Degree of Stockpiling (POS)



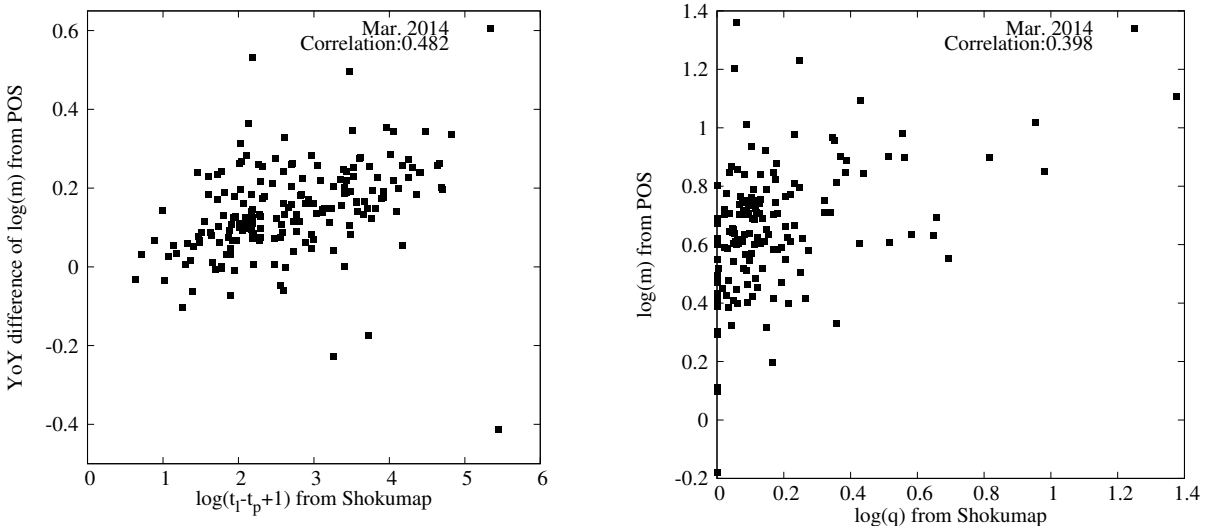
Note: Each dot represents a 3-digit product category. In the left-hand panel, the horizontal axis represents the log consumption period, where the consumption period is defined as the difference between the date of purchase (t_p) and the date the household finishes the product (t_l) plus one. In the right-hand panel, the horizontal axis represents the log quantity purchased.

Figure 8: Elasticity of Substitution



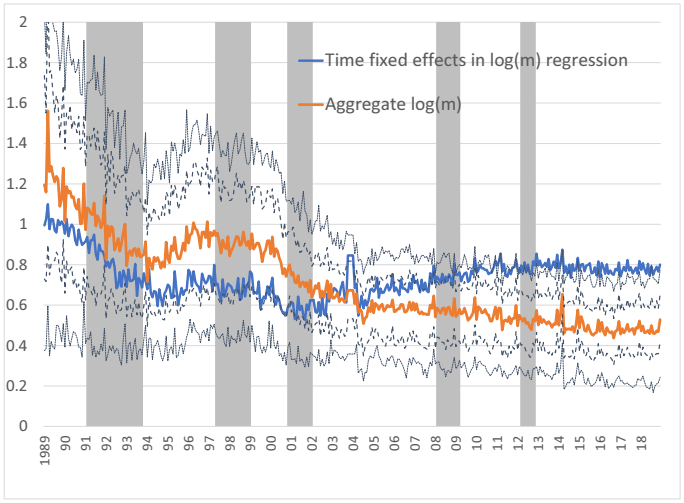
Note: "Our estimate" represents our calculation of the elasticity of substitution σ from $\Gamma \equiv -\log\left(c_L/c_H\right)/\log\left(r_L/r_H\right)$ using the inferred series of consumption c and consumption price r. "Simple estimate" represents the calculation of σ from $-\Delta\log X_t/\Delta\log p_t$, where X and p represent the quantity purchased and posted price, respectively. The left-hand panel shows the histogram of the values of σ for 3-digit product categories, while the right-hand panel shows a scatter plot where each dot represents a 3-digit product category.

Figure 9: The Consumption Period and the Quantity Purchased (*Shoku-map*) versus the Degree of Stockpiling (POS): One Month Before the Consumption Tax Increase



Note: See the note for the previous figure.

Figure 10: Aggregate Time-Series of the Degree of Stockpiling



Note: The solid thick line shown as aggregate log$m$ indicates the mean of log$m$ for the 3-digit product categories, while the thin lines show 10, 25, 75, and 90 percentiles of log$m$. The shaded areas indicate recession periods determined by the Cabinet Office.