Determinacy without the Taylor Principle

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Outline

1. Introduction
2. A Simplified New Keynesian Model
3. The Standard Paradigm
4. Uniqueness with Fading Memory
5. The Generalized Model
6. Observing Past Outcomes
7. Discussion
8. Conclusion

The Equilibrium Selection Issue in the NK Model

- Can monetary policy regulate AD by adjusting interest rates?
- Important caveat (e.g., Sargent & Wallace):
  - Same nominal interest rate path consistent with **multiple bounded eq.**
  - Need for equilibrium selection
- Standard approach: **Taylor principle** (raise rates aggressively with inflation)
  - An off-eq. threat to trigger an explosion in π and y (Cochrane)
  - Or an off-eq. reversion to M regime for large enough deviations (Atkeson, Chari, & Kehoe)
- Alternative: **Fiscal Theory of the Price Level** (Leeper, Sims, Woodford)
  - An off-eq. threat to blow out the government budget (Kocherlakota & Phelan)
  - Or other off-eq. interpretations of non-Ricardian fiscal policy (Cochrane, Bassetto)
- Eq. selection debate is **a war of "religious beliefs"** (Kocherlakota & Phelan)
  - Cannot be guided by empirical evidence and are inherently untestable

This Paper: Determinacy without the Taylor Principle

- Sunspot eq. artifacts of **perfect intertemporal coordination ("infinite chain")**
  - Current agents respond to "irrelevant" sunspots only if future agents respond in a specific way
  - Future agents respond only if they expect agents further in the future respond; and so on.
- Small perturbations in memory/coordination $\Rightarrow$ breaks the infinite chain $\Rightarrow$ **determinacy**
- **Always selects the standard eq.** (minimum-state-variable eq.)
- Taylor principle perhaps less consequential than previously thought
- No room for FTPL as currently formalized (as an eq. selection device)
  - but **fiscal considerations can matter through the eq. conduct by MP**
- Eases the potential conflict between **stabilization** and **eq. selection**

Pause for Questions

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A Simplified Model

- Dynamic IS ($\bar{E}_t[\cdot] = \int E_{i,t}[\cdot]\, di$ is the average expectation)

$$c_t = -\sigma\left(i_t - \bar{E}_t[\pi_{t+1}]\right) + \bar{E}_t[c_{t+1}] + \rho_t$$

- Phillips curve (static for now, forward looking later)

$$\pi_t = \kappa c_t + \xi_t$$

- Monetary policy

$$i_t = z_t + \phi \pi_t$$

An Equivalent Representation

- Substituting monetary policy and Phillips curve in IS curve $\Rightarrow$

$$c_t = \theta_t + \delta \bar{E}_t [c_{t+1}]$$

where $\{\theta_t\}$ is a function of $\{\rho_t, \xi_t, z_t\}$ and

$$\delta = \delta(\phi) \equiv \frac{1 + \kappa\sigma}{1 + \phi\kappa\sigma}$$

- **Taylor principle** holds when

$$\phi > 1 \iff \delta < 1$$

- Equivalent formulation

$$\pi_t = \tilde{\theta}_t + \delta \bar{E}_t [\pi_{t+1}]$$

  - this nests the **flexible price case** ($i_t = \bar{E}_t [\pi_{t+1}]$) with $\kappa \to \infty$ ($\delta \to \frac{1}{\phi}$)

Fundamentals, Sunspots, and the Equilibrium Concept

- Fundamentals:

$$\theta_t = \rho\,\theta_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim_{\text{i.i.d}} \mathcal{N}(0,1)$$

  - In paper: generalization allowing generic state space representations

- Sunspots:

$$\eta_t \sim_{\text{i.i.d}} \mathcal{N}(0,1)$$

- State of nature, or (infinite) history, at t:

$$h^t = \{\theta_{t-k}, \eta_{t-k}\}_{k=0}^{\infty}$$

- Equilibrium concept: **REE (based on potentially limited information about h^t)**

$$c_t = \sum_{k=0}^{\infty} a_k \eta_{t-k} + \sum_{k=0}^{\infty} \gamma_k \theta_{t-k}$$

- Focus on **bounded** eq. ($Var(c_t)$ is finite). Can be justified by escape clauses by ACK.

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The Standard Paradigm

- **FIRE (full information rational expectations)/perfect recall benchmark:**

$$c_t = \theta_t + \delta E_t [c_{t+1}]$$

  - $E_t[\cdot]$ is rational expectation conditional on entire history h^t

- **The MSV (minimum state variable) solution:**

$$c_t = c_t^F \equiv \frac{1}{1-\delta\rho}\theta_t$$

  - guess and verify $c_t = \gamma\theta_t$

- **Is MSV the only solution?**
  - Taylor principle holds when $\phi > 1 \iff \delta < 1$
  - If it does not hold $\delta > 1$, solve backward $\Longrightarrow$ sunspot and backward looking eq.

The Standard Paradigm

Proposition 1. Perfect Recall Benchmark

- When the Taylor principle is satisfied ($|\delta| < 1$), the MSV equilibrium is the unique one
- When this principle is violated $|\delta| > 1$), there exist **a continuum of equilibria**

$$c_t = (1-b)c_t^F + bc_t^B + ac_t^\eta,$$

where

- **Sunspot equilibria** (non-zero solution to $c_t = \delta E_t[c_{t+1}]$)

$$c_t^\eta \equiv \sum_{k=0}^{\infty} \delta^{-k} \eta_{t-k}$$

- **Backward fundamental equilibria**

$$c_t^B \equiv -\sum_{k=1}^{\infty} \delta^{-k} \theta_{t-k}$$

Understanding the Multiplicity

Using the sunspot eq. as an example:

$$c_t^\eta = \delta E_t \left[c_{t+1}^\eta \right]$$

Infinite chain of perfect intertemporal coordination:

- Current agents respond **against their intrinsic interest** because they **expect to be rewarded by future agents**
- Future agents themselves respond based on a similar expectation
- ...

What's Next: Breaking the Infinite Chain

What's next: two perturbations **breaking the infinite chain of perfect coordination**

Two equivalent representations of the sunspot equilibrium

$$\text{Sequential}: \quad c_t^\eta = \sum_{k=0}^{\infty} \delta^{-k} \eta_{t-k}$$

$$\text{Recursive}: \quad c_t^\eta = \delta^{-1} c_{t-1}^\eta + \eta_t$$

- c_t^η needs to **respond to distant-past sunspots** (directly or indirectly)

First perturbation motivated by the sequential representation

- Fading social memory about $\eta_{t-k} \Longrightarrow$ determinacy

Second perturbation motivated by the recursive representation

- Bounded social memory what drives (a tiny part of) $c_{t-1}^\eta \Longrightarrow$ determinacy

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The First Perturbation

Memory:

- In each period, a randomly $\lambda \in [0,1]$ of agents are replaced by newborn agents.
- Agents **know fundamentals & sunspots during their lives** but **not before**
- The period-t information set of an agent born s periods ago is given by

$$I_t^s \equiv \{(\theta_t, \eta_t), ..., (\theta_{t-s}, \eta_{t-s})\}$$

The First Perturbation

$$I_t^s \equiv \{(\theta_t, \eta_t), ..., (\theta_{t-s}, \eta_{t-s})\}$$

Interpretation:

- **OLG with "fading" social memory**
  - Consistent with perfect individual recall & standard rational expectations solution concept
  - Equivalent behavioral interpretation: agents are infinitely-lived but have bounded recall

Standard paradigm:

- Perfect social memory, nested by $\lambda = 0$

Properties:

- For any $\lambda > 0$, zero mass of agents has *infinite* memory
  - But as $\lambda \to 0$, **almost all agents have *arbitrarily long* memory**
- Prevent direct knowledge about history of endogenous $\{c_{t-k}\}$
  - But as $\lambda \to 0$, **arbitrarily well informed long histories of** $\{c_{t-k}\}$

Determinacy without the Taylor Principle

Proposition 2. Determinacy without the Taylor Principle

With fading social memory, the **unique equilibrium** is the **MSV solution**, $c_t = c_t^F$

- **Regardless** of the value of δ, or **equivalently monetary policy** ϕ.
- No matter how slow the memory decay is (how small λ is).

Proof sketch: focusing on responses to η_0 (a_t).

- "Twin" economy with perfect memory but modified best response:

$$c_t = \theta_t + \delta \bar{E}_t [c_{t+1}] \quad \Longrightarrow \quad c_t = \delta \mu_t E_t [c_{t+1}],$$

  where $\mu_t = (1-\lambda)^t \to 0$ is the proportion of agents remembering η_0 at t.

- But $\delta \mu_t < 1$ eventually, so always determinacy.

Logic

- I can see the current sunspot very clearly
- It would make sense to react if all future agents will keep responding to it **in perpetuity**
- But I worry that agents **far in the future will fail to do so**
  - either because they will have forgotten it
  - or because they may worry that agents further into the future will not react to it
- It therefore makes sense to ignore the sunspot

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A Micro-funded NK Model

- A micro-founded IS curve robust to incomplete information

$$c_t = -\beta\omega\sigma\left\{\sum_{k=0}^{+\infty}(\beta\omega)^k\,\bar{E}_t\left[i_{t+k}-\pi_{t+k+1}\right]\right\}+(1-\beta\omega)\left\{\sum_{k=0}^{+\infty}(\beta\omega)^k\,\bar{E}_t\left[c_{t+k}\right]\right\}+\rho_t$$

  - $\omega = 1-\lambda$ is the survival probability (as the OLG structure above)
  - embeds individual optimality + market clearing + budgets
  - reduces to the RA Euler equation (plus transversality) when $\bar{E}_t[\cdot] = E_t[\cdot]$

- Standard dynamic NKPC

$$\pi_t = \kappa c_t + \beta E_t\left[\pi_{t+1}\right] + \xi_t$$

- Monetary policy

$$i_t = z_t + \phi_c c_t + \phi_\pi \pi_t$$

The Generalized Model and Nesting

- The generalized model

$$c_t = \theta_t + \bar{E}_t \left[\sum_{k=0}^{+\infty} \delta_k c_{t+k} \right]$$

  - only requires that the sum $\sum_{k=0}^{\infty} |\delta_k|$ is finite

- Nests the previous micro-founded NK with

$$\delta_k = (1 - \beta\omega - \beta\omega\sigma\phi_c)(\beta\omega)^k + \omega\sigma\kappa \left(-\phi_\pi\beta + (1 - \omega\phi_\pi\beta) \frac{1 - \omega^k}{1 - \omega} \right) \beta^k.$$

The Generalized Results mu

Proposition 3. Fading Memory Rules out Sunspot Volatility

With fading social memory ($\lambda > 0$), the equilibrium is unique and is given by the MSV solution.

Proof sketch: focusing on response to η_0 (a_t).

- "Twin" economy with perfect memory but modified best response:

$$c_t = \theta_t + \bar{E}_t\left[\sum_{k=0}^{+\infty} \delta_k c_{t+k}\right] \quad \Longrightarrow \quad c_t = \mu_t E_t\left[\sum_{k=0}^{+\infty} \delta_k c_{t+k}\right],$$

  where $\mu_t \to 0$ is the proportion of agents remembering η_0 at t.

- But $\mu_t\left(\sum_{k=0}^{\infty} |\delta_k|\right) < 1$ eventually, so always determinacy
- Effective complementary < 1, uniquely pinned down by iterating of best responses

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Observing Past Outcomes

- Baseline: preclude *direct* observation of past outcomes, such as c_{t-1}
- But note: agents have *almost perfect* knowledge of past outcomes
  - for any T, almost all agents learn $\{c_{t-1}, ..., c_{t-T}\}$ nearly perfectly as $\lambda \to 0$
- Still, what if perfectly observing past outcomes?
  - Could **long memory of sunspots and past fundamentals** be efficiently "stored" in **short memory of past outcomes**?
- For example, the recursive formulation of the sunspot equilibrium (turn off θ_t briefly)

$$c_t = \eta_t + \delta^{-1} c_{t-1}$$

- Perfect memory of c_{t-1} suffice as the memory of the history of sunspots
  - sunspot equilibria strike back?

Storing Memory in Endogenous Outcomes

- Still takes a strong, **fragile**, form of **intertemporal coordination**
  - Current agents respond because they expect future **respond in a perfect way**
  - Infinite chain of coordination $\cdots$

- Add i.i.d. fundamental shocks $\zeta_t \in [-\varepsilon, \varepsilon]$ (arbitrarily small) known only to t

$$c_t = \zeta_t + \delta \bar{E}_t [c_{t+1}]$$

- For a sunspot eq, requires **perfect knowledge of ζ_t at $t+1$**

$$c_{t+1} = \eta_{t+1} + \delta^{-1} (c_t - \zeta_t)$$

- But if ζ_t unknown to agents at $t+1$, the sunspot equilibrium collapses

The Second Perturbation

- Bring back fundamentals θ_t with arbitrarily small. i.i.d. perturbations $\zeta_t \in [-\varepsilon, \varepsilon]$

$$c_t = \theta_t + \zeta_t + \delta \mathbb{E}[c_{t+1}|I_t]$$

- A representative agent in each period, with info set

$$I_t = \{\zeta_t\} \cup \{\theta_t ..., \theta_{t-K}\} \cup \{\eta_t ..., \eta_{t-K}\} \cup \{c_{t-1}, \cdots, c_{t-K}\}$$

  - Long memory of past sunspots, fundamentals, & outcomes **for arbitrarily large but finite** K
  - But knowledge of only current ζ_t & no memory of past ζs

Proposition 5. Storing Memory in Endogenous Outcomes

With above info. structure, regardless of δ, there is a **unique equilibrium** and is given by $c_t = c_t^F + \zeta_t$, where c_t^F is the same **MSV solution** as before.

- Break the infinite chain $\Longrightarrow$ MSV as the unique eq

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Fiscal Theory of Price Level (FTPL)

- Essence of the FTPL: **non-Ricardian fiscal policy**
  - primary surplus do respond enough to public debt level
  - An off-equilibrium threat to blow out the government budget (Kocherlakota & Phelan)
  - Or other interpretations (Cochrane, Bassetto)
- Standard paradigm: FTPL perfectly logical with "passive MP" ($\phi < 1$)
  - concur with **passive-monetary and active-fiscal regime in Leeper (1991)**
- Our contribution: no need/space for eq selection from FTPL
  - **underscores the fragility of existing formalization of FTPL**
  - but allow fiscal considerations to matter on eq. through conduct of MP

Feedback Rules and the Ramsey Implementation

- Consider the **Ramsey optimum**. How can monetary policy uniquely implement it?
- If the monetary authority **observes the underlying shocks**, uniquely implemented with:

$$i_t = i_t^o + \phi(\pi_t - \pi_t^o),$$

  where i_t^o and π_t^o are rates and inflation in the optimum and $\phi > 1$.

- What if the monetary authority **does not observe the underlying shocks?**
  - implemented through feedback rules?

$$i_t = \phi \pi_t$$

- Two conflicting roles
  - **Stabilization** ($\phi < 1$ possible in the Ramsey optimum)
  - **Eq. selection** ($\phi > 1$ necessary in the standard paradigm)
- Here: Liberates the **stabilization role** of monetary policy from **its eq. selection role**

Alternative Boundedly-Rational Solution Concepts

- Group 1: **relax REE but maintain a "fix point" between expectations & actual eq.**
  - e.g., Cognitive discounting in Gabaix (20); Diagnostic expectations in Bordalo et. al (20)
  - may shrink the determinacy region but the indeterminacy problem remains
- Group 2: completely **shuts down the "fix point"**
  - e.g. level-k thinking (Garcia-Schmidt & Woodford, 19; Farhi & Werning, 19)
  - produces a unique solution but opens a new issue
  - whenever $\phi < 1$, **Level-k solution becomes infinitely sensitive to Level-0 behavior**

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Conclusion

- Main lesson: NK indeterminacy/FTPL hinge on **strong info assumptions**
- **A small friction in memory & intertemporal coordination** can result in **determinacy**
- Taylor principle perhaps less consequential than previously thought
  - more crucial: boundedness (commitment to rule out large deviations)
- No room for FTPL as currently formalized (as an eq. selection device)
  - but **fiscal considerations can matter if internalized by MP**
  - Model MP-FP interaction as a game of between monetary & fiscal authority?