Sufficient Statistics for Dynamic Spatial Economics

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Motivation

- How does the spatial distribution of economic activity respond to local shocks? (e.g. productivity, transport infrastructure, trade)
  - This response can be gradual because of migration frictions for mobile factors and the accumulation of immobile factors (capital structures)

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- We make four main contributions:

1. Incorporate forward-looking capital investments in a dynamic spatial model with migration and characterize existence / uniqueness
2. Linearize this model and derive a closed-form solution for the economy's entire transition path in terms of sufficient statistics
3. Use our linearization to analytically characterize of the properties of transition path in terms of the eigenvalues of a transition matrix
4. Apply our framework to examine the reallocation US economic activity from the "Rust Belt" to the "Sun Belt"

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  - Use an eigendecomposition of $\boldsymbol{P}$ to isolate the locations exposed to particular shocks and the shocks that impact particular locations
- Applications: US state data 1965-2015; state-industry data 1999-2015
  - Decline of the "Rust-Belt" and rise of the "Sun-Belt"
  - Slow convergence and heterogeneous impact of local shocks

Related Literature

- Theoretical work on economic geography
  - Krugman (1991, 1992), Helpman (1998), Fujita et al. (1999), Baldwin (2001)
- Static quantitative spatial trade models
  - Armington (1969), Eaton & Kortum (2002), Redding & Sturm (2008), Allen & Arkolakis (2014), Ramondo et al. (2016), Redding (2016), Donaldson (2018), Caliendo et al. (2018), Fajgelbaum et al. (2019), Fajgelbaum & Gaubert (2020)
- Dynamic Models of Trade and Geography
  - Artuç et al. (2010), Desmet & Rossi-Hansberg (2014), Desmet et al. (2018), Caliendo et al. (2019), Caliendo & Parro (2020), Peters (2019), Peters & Walsh (2019), Walsh (2019), Allen & Donaldson (2020), Greaney (2020)
- Research on sufficient statistics for welfare in international trade
  - Arkolakis et al. (2012), Adão et al. (2017), Adão et al. (2019), Baqaee & Farhi (2019), Galle et al. (2019), Huo et al. (2019), Barthelme et al. (2019), Kleinman et al. (2020)
- Empirical evidence on the U.S. Rust Belt and local labor market shocks
  - Blanchard & Katz (1992), Holmes (1998), Kovak (2013), Feyrer et al. (2007), Topolova (2010), Kovak (2013), Autor et al. (2013, 2014, 2020), Dix-Carneiro & Kovak (2017), Charles et al. (2018), Fort et al. (2018), Amior & Manning (2018), Pavcnik & McCaig (2018), Pierce & Schott (2019), Alder et al. (2019)

Outline

- Dynamic Spatial Model
- Extensions
- Data
- Empirical Results
- Conclusions

Model Setup

- Multi-location, single-sector Armington model (extensions later)
- Economy consists of a set of locations $i \in \{1, \ldots, N\}$
- Locations differ in productivity, amenities, bilateral goods trade costs, and bilateral migration costs
- Two types of agents: workers and landlords
- Continuum of workers
  - Endowed with one unit of labor
  - Geographically mobile subject to migration costs
  - No savings-investment technology ("hand to mouth")
  - Make dynamic forward-looking migration decisions to maximize intertemporal utility
- Continuum of landlords in each location
  - Own the stock of local capital
  - Geographically immobile
  - Make dynamic forward-looking consumption-investment choices to maximize intertemporal utility

Worker Migration (CDP)

- At the beginning of period t, mass of workers ℓ_{it} in location i:
  - Produce and consume
  - Observe extreme value idiosyncratic mobility shocks $\{\epsilon_{gt}\}$
  - Choose optimal location for period $t+1$ given mobility costs κ_{git}
- Expected value of living in location i in period t depends on wage (w_{it}), cost of living (p_{it}), amenities (b_{it}) and the expected value of optimal location choice

$$v_{it} = \ln\left(\frac{w_{it}}{p_{it}}\right) + \ln b_{it} + \rho \ln \sum_{g=1}^{N} \left(\exp\left(\beta v_{gt+1}\right) / \kappa_{git}\right)^{1/\rho}$$

- Location choice probabilities

$$D_{igt} = \frac{\left(\exp\left(\beta v_{gt+1}\right) / \kappa_{git}\right)^{1/\rho}}{\sum_{k=1}^{N} \left(\exp\left(\beta v_{kt+1}\right) / \kappa_{kit}\right)^{1/\rho}}$$

- Population flow condition

$$\ell_{gt+1} = \sum_{i=1}^{N} D_{igt} \ell_{it}$$

Trade and Production

- Armington differentiation of goods by location of origin

$$p_{nt} = \left[\sum_{i=1}^{N} p_{nit}^{-\theta} \right]^{-1/\theta}, \qquad \theta = \sigma - 1, \qquad \sigma > 1$$

- Competitive production and iceberg trade costs $\tau_{nit} \geq 1$
- Cost in location n of sourcing a variety from location i is

$$p_{nit} = \frac{\tau_{nit} w_{it}^{\lambda} r_{it}^{1-\lambda}}{z_{it}}, \qquad 0 < \lambda < 1$$

- Using profit maximization to substitute for equilibrium labor input, landlord income is linear in capital

$$\Pi_{it} = \lambda \left(p_{it} z_{it} \right)^{\frac{1}{\lambda}} \left(\frac{1-\lambda}{w_{it}} \right)^{\frac{1-\lambda}{\lambda}} k_{it}$$

Landlord Investment

- Landlords have the same preferences as workers but have access to an investment technology for local capital

$$v_{it}^k = \sum_{t=0}^{\infty} \beta^t \ln c_{it}^k$$

- Landlords in a location can produce one unit of capital in that location using one unit of the local consumption index
- Local capital is geographically immobile once installed and depreciates at constant rate δ
- Intertemporal budget constraint

$$r_{it} k_{it} = p_{it} c_{it}^k + p_{it} \left(k_{it+1} - (1 - \delta) k_{it} \right)$$

- Logarithmic utility and linear income in capital together imply a constant saving rate (as in Moll 2014) more

$$k_{it+1} = \beta \left(r_{it} / p_{it} + (1 - \delta) \right) k_{it}$$

Existence and Uniqueness

- Dynamic spatial model with many locations, rich geography of trade and migration costs, and two sources of dynamics

Proposition

There exists a unique steady-state equilibrium $\{w_i^*, v_i^*, \ell_i^*, k_i^*\}$ *(up to a numeraire) given time-invariant location characteristics* $\{z_i, b_i, \tau_{ni}, \kappa_{ni}\}$ *that is independent of the economy's initial conditions* $\{\ell_{i0}, k_{i0}\}$. ▸ Proof

- When we introduce agglomeration forces
  - Derive condition on parameters for the existence of unique equilibrium
  - Show this condition satisfied for sufficiently small agglomeration forces

Steady-state Sufficient Statistics

- Totally differentiating the general equilibrium conditions of the model and stacking them in matrix form

Proposition

The steady-state response of the endogenous variables to productivity and amenity shocks satisfies the linear system:

$$\begin{bmatrix} \mathrm{d}\ln \boldsymbol{\ell}^* \\ \mathrm{d}\ln \boldsymbol{k}^* \\ \mathrm{d}\ln \boldsymbol{w}^* \\ \mathrm{d}\ln \boldsymbol{v}^* \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}^{z*} \\ \boldsymbol{K}^{z*} \\ \boldsymbol{W}^{z*} \\ \boldsymbol{V}^{z*} \end{bmatrix} \mathrm{d}\ln \boldsymbol{z} + \begin{bmatrix} \boldsymbol{L}^{b*} \\ \boldsymbol{K}^{b*} \\ \boldsymbol{W}^{b*} \\ \boldsymbol{V}^{b*} \end{bmatrix} \mathrm{d}\ln \boldsymbol{b}$$

where the $N \times N$ matrices $\{\boldsymbol{L}^{z*}, \boldsymbol{K}^{z*}, \boldsymbol{W}^{z*}, \boldsymbol{V}^{z*}, \boldsymbol{L}^{b*}, \boldsymbol{K}^{b*}, \boldsymbol{W}^{b*}, \boldsymbol{V}^{b*}\}$ are functions of the four observed matrices of expenditure shares ($\boldsymbol{S}$), income shares ($\boldsymbol{T}$), outmigration shares ($\boldsymbol{D}$) and inmigration shares ($\boldsymbol{E}$) and the structural parameters of the model $\{\beta, \theta, \rho, \lambda, \delta\}$. ▸ more

- Element $\left[\boldsymbol{L}^{z*}\right]_{in} = \mathrm{d}\ln \ell_i^* / \mathrm{d}\ln z_n$
  - Elasticity of steady-state population in location i (ℓ_i^*) with respect to an increase in productivity in location n (z_n)

Transition Dynamics

- Suppose that the economy at time $t = 0$ is on a convergence path towards an initial steady-state with constant fundamentals $(\boldsymbol{z}, \boldsymbol{b}, \boldsymbol{\kappa}, \boldsymbol{\tau})$
- At time $t = 0$, agents learn about one-time, permanent shocks to productivity and amenities $(\widetilde{\boldsymbol{f}} = \begin{bmatrix} \widetilde{\boldsymbol{z}} \\ \widetilde{\boldsymbol{b}} \end{bmatrix})$ from time $t = 1$ onwards that are revealed under perfect foresight
- Use a tilde for log deviation from (unobserved) initial steady-state

$$\tilde{\boldsymbol{\ell}}_{\boldsymbol{t}} \equiv \ln \boldsymbol{\ell}_{\boldsymbol{t}} - \ln \boldsymbol{\ell}^{*}_{\text{initial}}$$

- Obtain a linear system of equations in terms of log deviations of the endogenous variables from the initial steady-state
- Reduces to 2nd-order difference equation in state variables $(\widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}}, \widetilde{\boldsymbol{k}}_{\boldsymbol{t}})$ that we solve using method of undetermined coefficients (Uhlig 1999)
- Generalize: (i) any convergent sequence of future shocks under perfect foresight; (ii) stochastic fundamentals and rational expectations

Transition Dynamics

Proposition

Suppose that the economy at time $t = 0$ is on a convergence path towards an initial steady-state with constant fundamentals $(\boldsymbol{z}, \boldsymbol{b}, \boldsymbol{\kappa}, \boldsymbol{\tau})$. At time $t = 0$, agents learn about one-time, permanent shocks to productivity and amenities $(\widetilde{\boldsymbol{f}} = \begin{bmatrix} \widetilde{\boldsymbol{z}} \\ \widetilde{\boldsymbol{b}} \end{bmatrix})$ from time $t = 1$ onwards. There exists a $2N \times 2N$ transition matrix $(\boldsymbol{P})$ and a $2N \times 2N$ impact matrix $(\boldsymbol{R})$ such that the second-order difference equation system has a closed-form solution of the form:

$$\widetilde{\boldsymbol{x}}_{t+1} = \boldsymbol{P}\widetilde{\boldsymbol{x}}_t + \boldsymbol{R}\widetilde{\boldsymbol{f}} \quad \text{for } t \geq 1.$$

where $\widetilde{\boldsymbol{x}}_{\boldsymbol{t}} \equiv \begin{bmatrix} \widetilde{\boldsymbol{\ell}}_{\boldsymbol{t}} \\ \widetilde{\boldsymbol{k}}_{\boldsymbol{t}} \end{bmatrix}$

▸ more ▸ convseq ▸ stochfund ▸ CDP

Transition Dynamics

- Use our linearization to decompose the dynamics for the spatial distribution of economic activity: more

$$\ln \boldsymbol{x}_t - \ln \boldsymbol{x}_{-1} = \underbrace{\sum_{s=0}^{t} \boldsymbol{P}^s \left(\ln \boldsymbol{x}_0 - \ln \boldsymbol{x}_{-1}\right)}_{\substack{\text{convergence given} \\ \text{initial fundamentals}}} + \underbrace{\sum_{s=0}^{t-1} \boldsymbol{P}^s \boldsymbol{R} \widetilde{\boldsymbol{f}}}_{\substack{\text{dynamics from} \\ \text{fundamental shocks}}} \qquad \text{for all } t \geq 1,$$

- With no shocks to productivity and amenities ($\widetilde{\boldsymbol{f}} = \boldsymbol{0}$), we have:

$$\ln \boldsymbol{x}^*_{\text{initial}} = \lim_{t \to \infty} \ln \boldsymbol{x}_t = \ln \boldsymbol{x}_{-1} + \left(\boldsymbol{I} - \boldsymbol{P}\right)^{-1} \left(\ln \boldsymbol{x}_0 - \ln \boldsymbol{x}_{-1}\right),$$

- Using only initial state variables (for $t = 0$ and $t = -1$) and trade and migration matrices, we can compute implied steady-states

Transition Dynamics

Proposition

Consider an economy that is initially in steady-state at time $t = 0$ *when agents learn about one-time, permanent shocks to productivity and amenities* $(\widetilde{\boldsymbol{f}} = \begin{bmatrix} \widetilde{\boldsymbol{z}} \\ \widetilde{\boldsymbol{b}} \end{bmatrix})$ *from time* $t = 1$ *onwards. Using the eigendecomposition* $\boldsymbol{P} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{V}$*, the transition path of the state variables can be written in terms of the eigenvalues* (λ_k) *and right and left eigenvectors* $(\boldsymbol{u}_k, \boldsymbol{v}_k')$ *of the transition matrix as follows:*

$$\widetilde{\boldsymbol{x}}_t \equiv \begin{bmatrix} \widetilde{\boldsymbol{\ell}}_t \\ \widetilde{\boldsymbol{k}}_t \end{bmatrix} = \sum_{s=0}^{t-1} \boldsymbol{P}^s \boldsymbol{R}\widetilde{\boldsymbol{f}} = \sum_{k=1}^{2N} \frac{1-\lambda_k^t}{1-\lambda_k} \boldsymbol{u}_k \boldsymbol{v}_k' \boldsymbol{R}\widetilde{\boldsymbol{f}}.$$

- We can use this result to characterize the economy's transition path in terms of lower-dimensional components

▸ more

Speed of Convergence

Proposition

Consider an economy that is initially in steady-state at $t = 0$ when agents learn about one-time, permanent shocks to productivity and amenities ($\widetilde{\boldsymbol{f}} = \begin{bmatrix} \widetilde{\boldsymbol{z}} \\ \widetilde{\boldsymbol{b}} \end{bmatrix}$) from $t = 1$ onwards. Suppose the initial impact of the shock to fundamentals on the state variables at $t = 1$ coincides with an eigenvector ($\boldsymbol{R}\widetilde{\boldsymbol{f}} = \boldsymbol{u}_k$) of the transition matrix ($\boldsymbol{P}$) (eigen-shock). The transition path of the state variables ($\widetilde{\boldsymbol{x}}_t \equiv \begin{bmatrix} \widetilde{\boldsymbol{\ell}}_t \\ \widetilde{\boldsymbol{k}}_t \end{bmatrix}$) reduces to:

$$\widetilde{\boldsymbol{x}}_t = \frac{1 - \lambda_k^t}{1 - \lambda_k} \boldsymbol{u}_k,$$

and the half-life is given by:

$$t_i^{(1/2)}\left(\widetilde{\boldsymbol{f}}\right) = -\left\lceil \frac{\ln 2}{\ln \lambda_k} \right\rceil$$

for all state variables $i = 1, \cdots, 2N$, where $\lceil \cdot \rceil$ is the ceiling function.

- We show in the paper that any shock to fundamentals ($\widetilde{\boldsymbol{f}}$) can be written as a linear combination of these eigen-shocks ▸ more

Outline

- Dynamic Spatial Model
- Extensions
  - Trade deficits ▸ more
  - Shocks to trade and migration costs ▸ more
  - Agglomeration and dispersion forces ▸ more ▸ existuniqagglom
  - Housing capital ▸ more
  - Multi-sector ▸ more
  - Multi-sector and input-output linkages ▸ more
- Data
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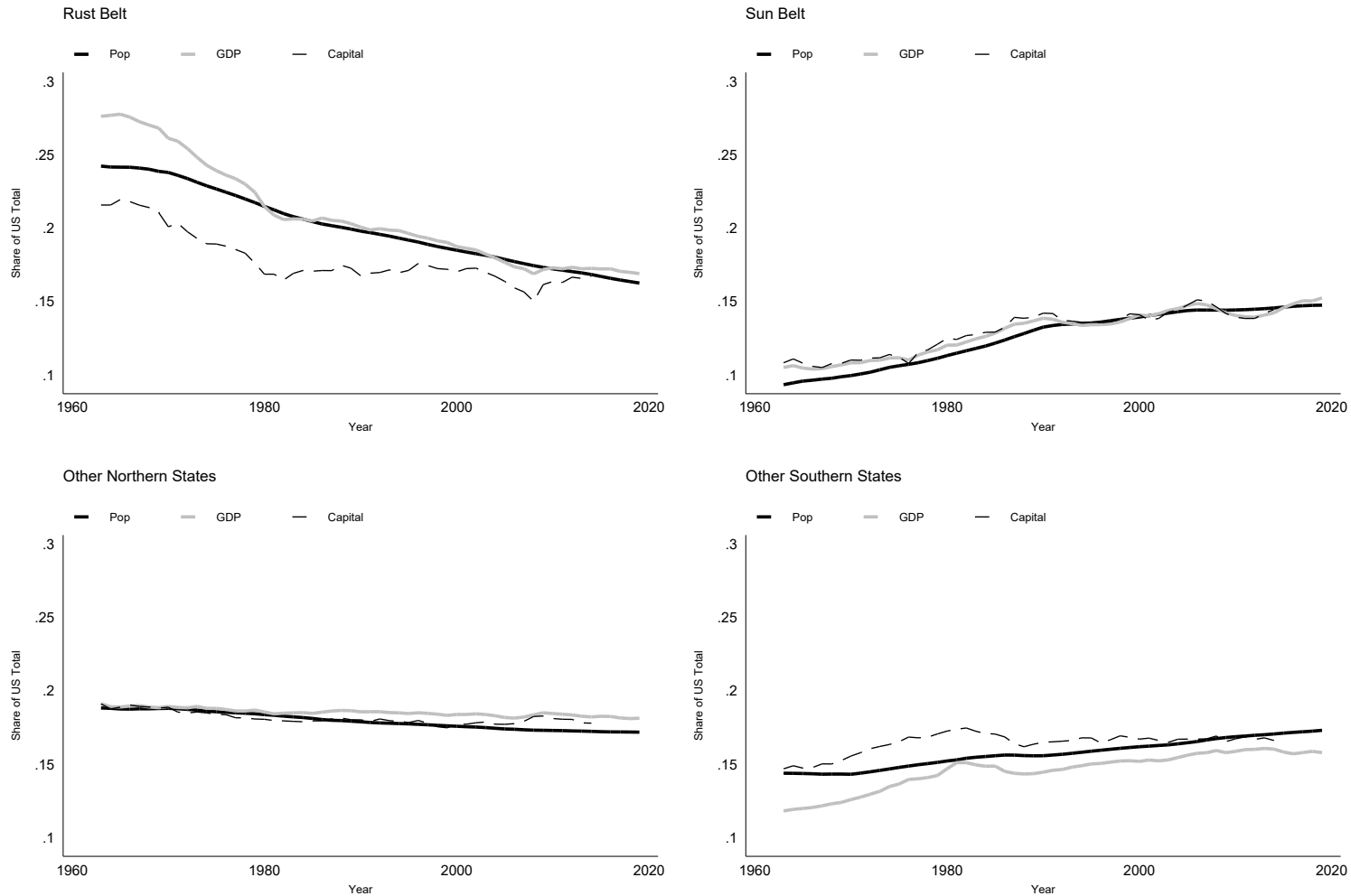
Data

- Two empirical implementations
  - State-time data from 1965-2015 (decline Rust Belt and rise Sun Belt)
  - State-industry-time data from 1999-2015
- U.S. State GDP, population and capital stock
  - Bureau of Economic Analysis (BEA) 1965-2015
- Bilateral value of shipments between U.S. states
  - Commodity Flow Survey (CFS)
  - Commodity Transportation Survey (CTS)
- Bilateral migration flows between U.S. states
  - Population census and American Community Survey (ACS) 1960-2010
  - Five-year migration matrices
- Foreign imports and exports of U.S. states
  - Foreign exports by origin of movement (OM) state 1999-2015
  - Foreign imports by state of destination (SD) 1999-2015

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Shares of U.S. Economic Activity



Rust Belt: Illinois, Indiana, Michigan, New York, Ohio, Pennsylvania, West Virginia and Wisconsin. Sun Belt: Arizona, California, Florida, New Mexico and Nevada. Other Southern all other former members of the Confederacy. Other Northern all other Union states during the Civil War

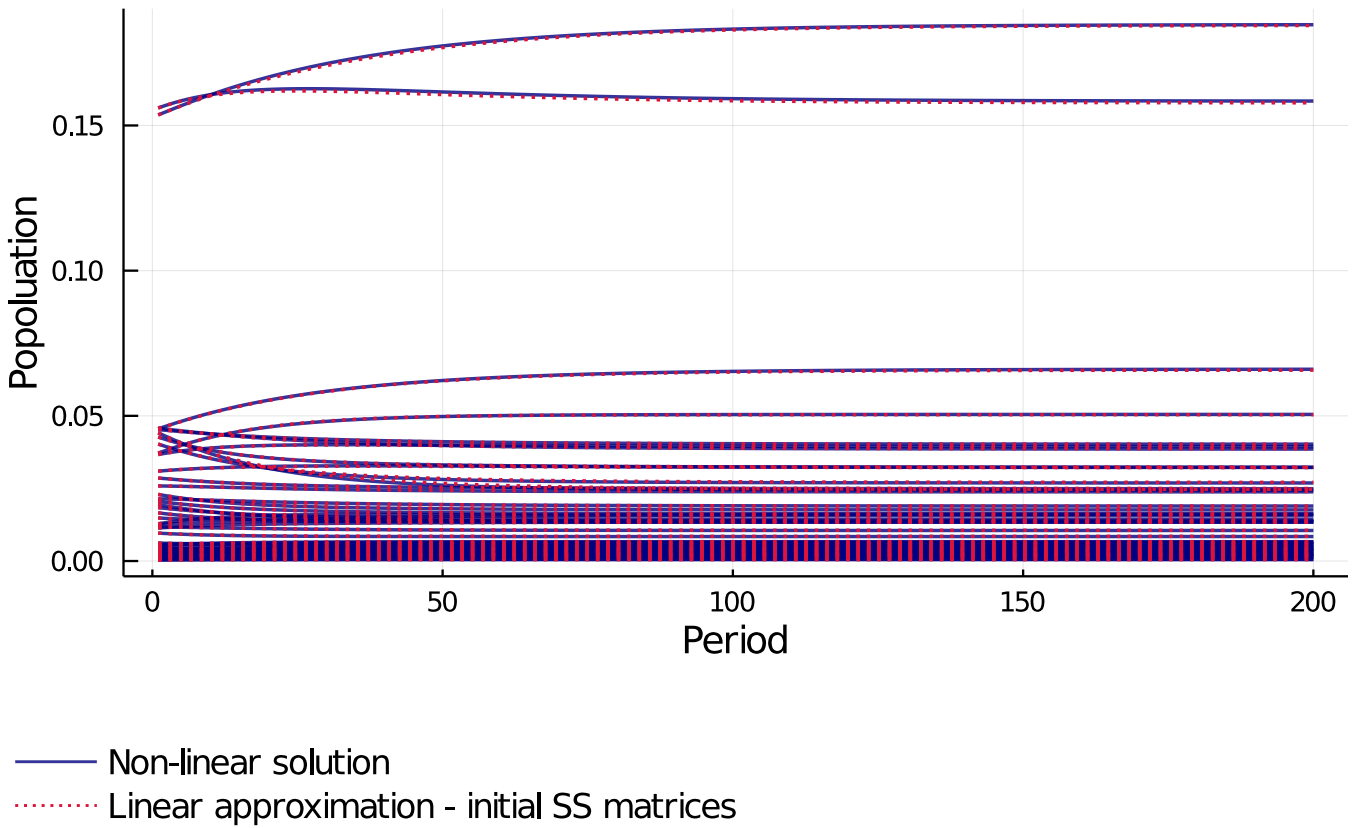
- Capital and GDP dynamics differ from population dynamics

migrants

Are we Missing Important Non-linearities?

Approximation Quality (Transition)

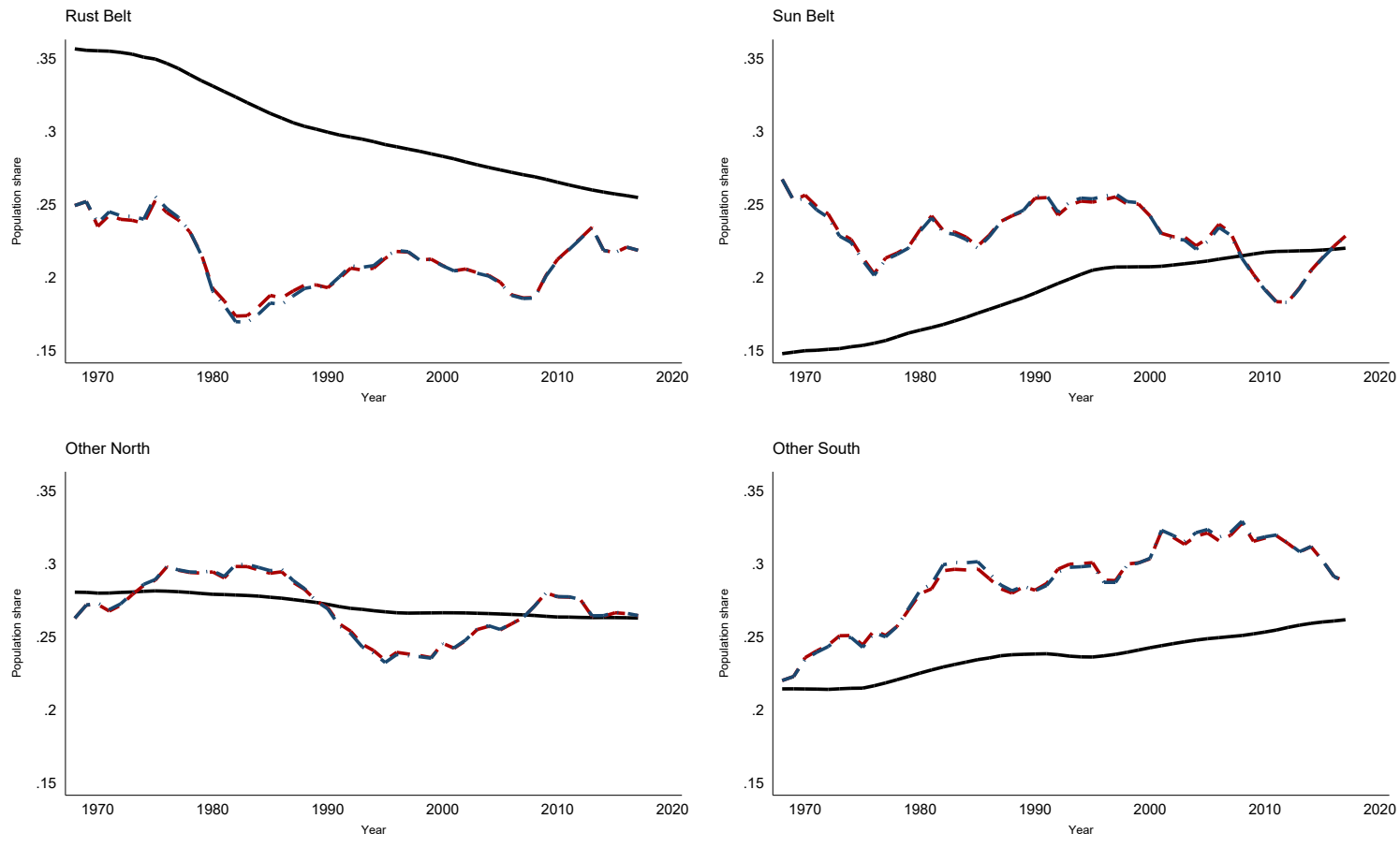
- Invert non-linear model (prod., amenities, trade & migration costs)
- Start from steady-state implied by these 1990 fundamentals
- Shock by vector of productivity shocks 1990-2000
- Compare transition paths in our linearization and non-linear model



more

Closed-form Solutions for Transition Path

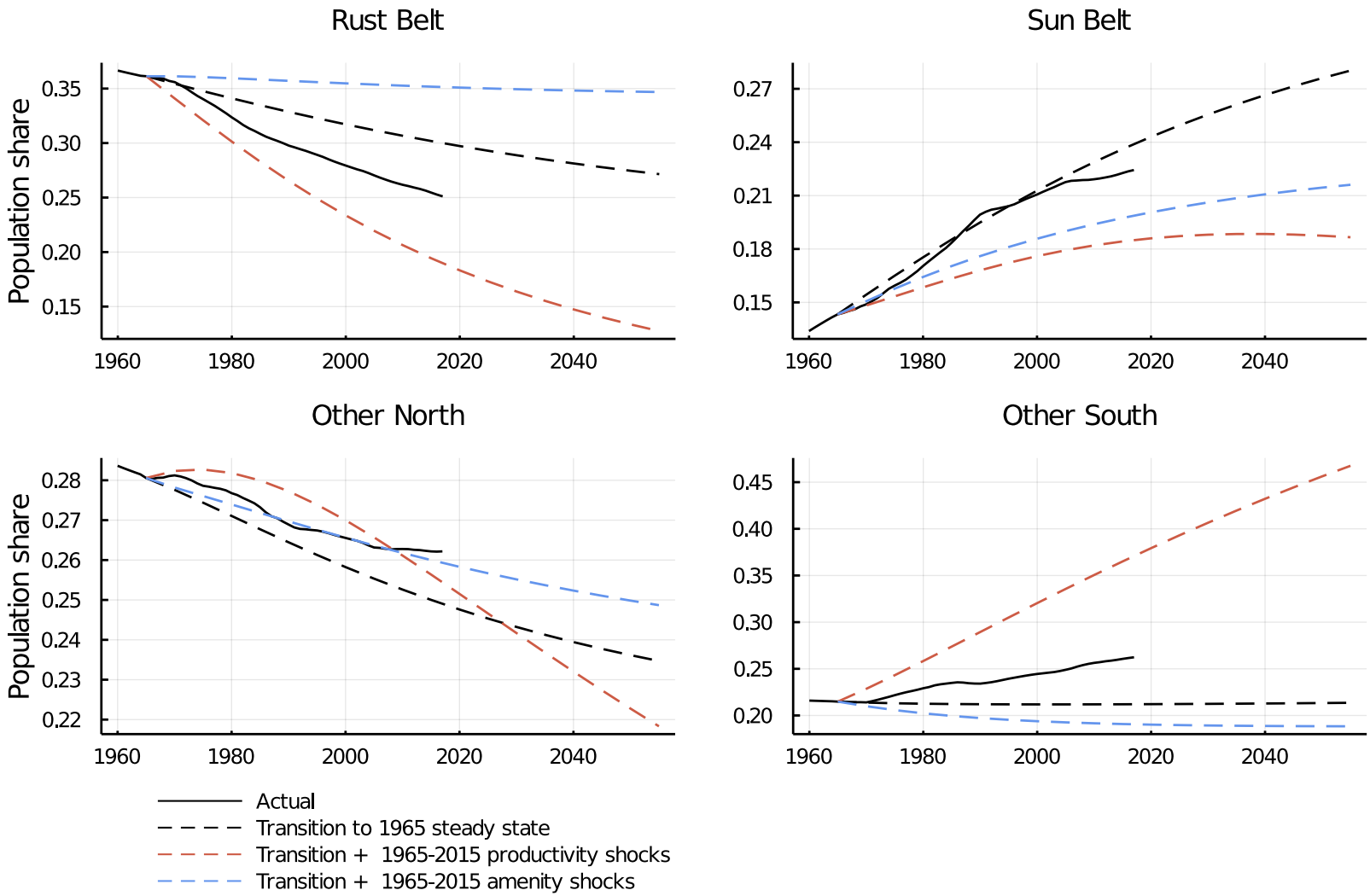
Population Gap from Steady-State



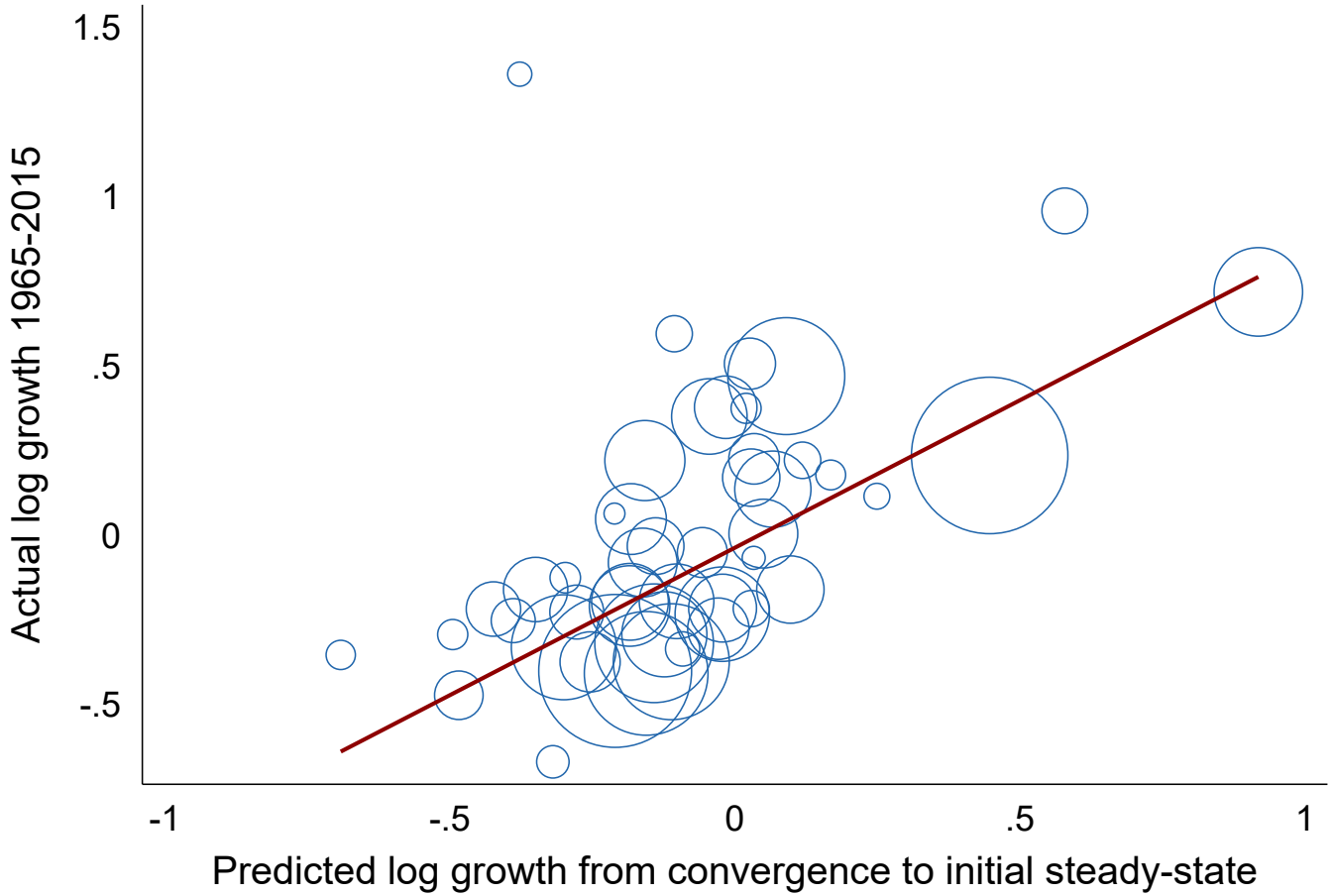
Rust Belt: Illinois, Indiana, Michigan, New York, Ohio, Pennsylvania, West Virginia and Wisconsin. Sun Belt: Arizona, California, Florida, New Mexico and Nevada. North and South definitions based on Federal and Confederacy states

▶ NJ

Transition Dynamics and Shocks



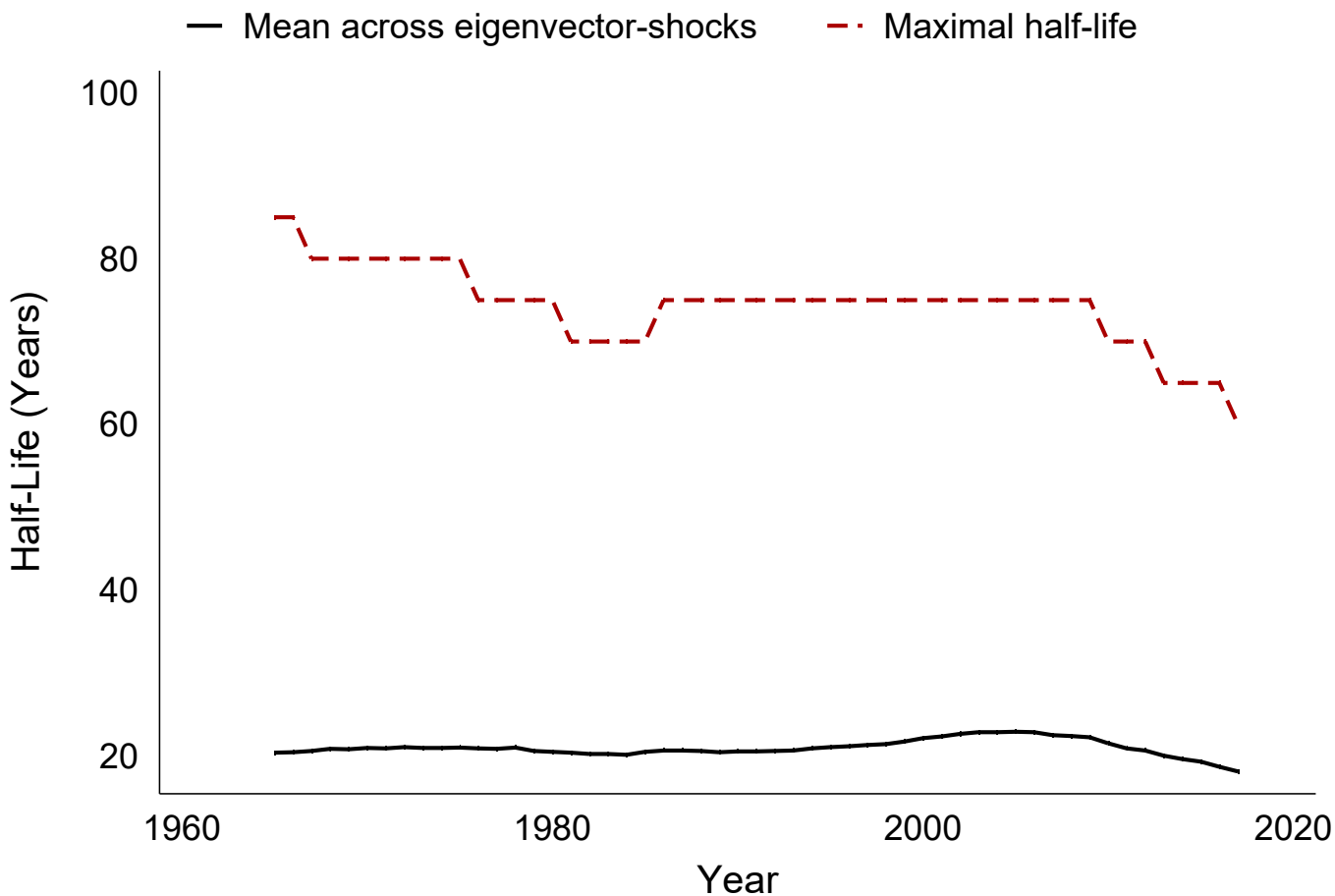
Predictive Power Initial Steady-State



Note: Slope coefficient: 0.8709; standard error: 0.1081; R-squared: 0.5035.

- Robust to controlling for initial log population and capital stock and initial log population growth more

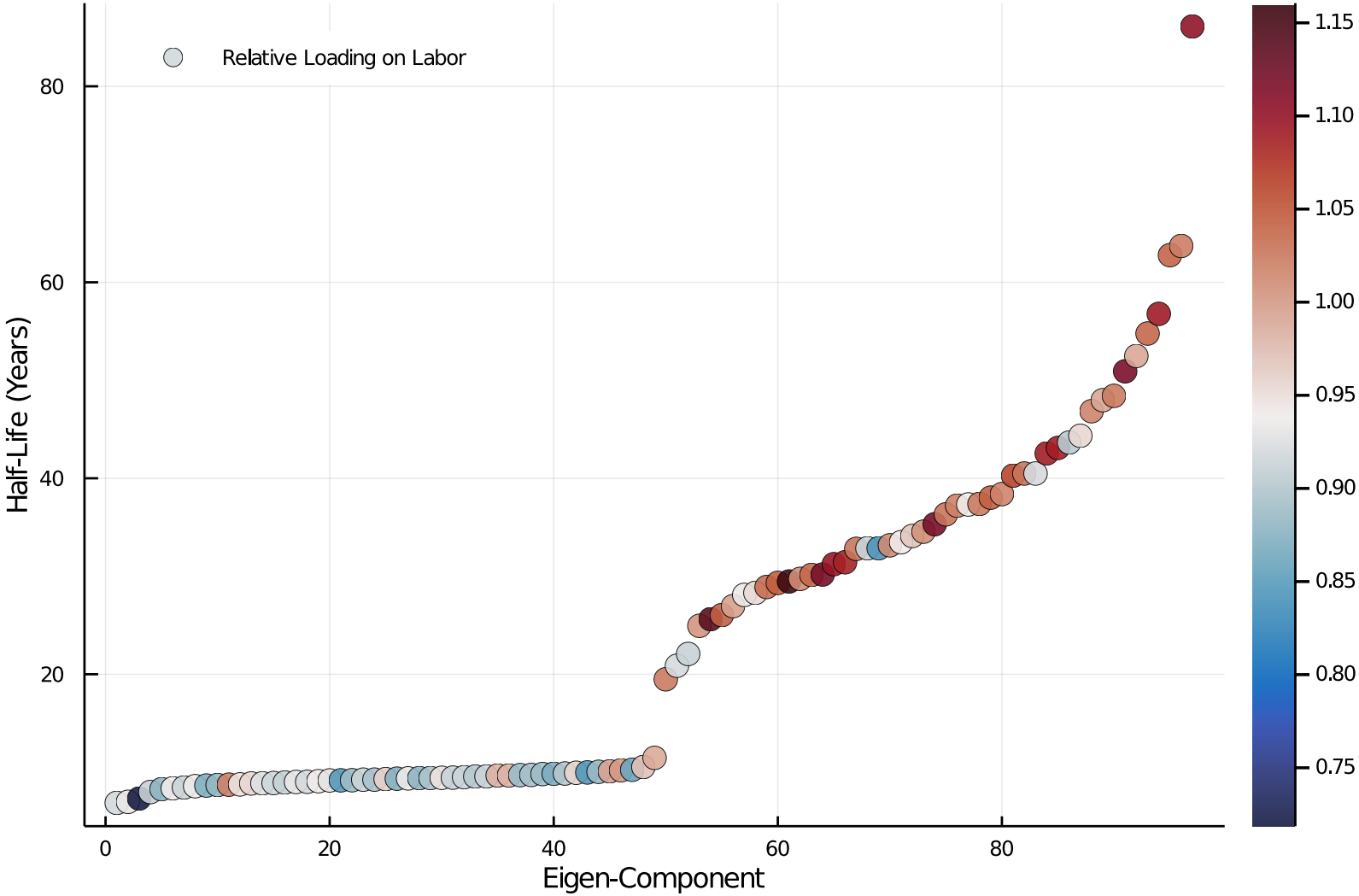
Half-lifes



Note: Half-life Corresponds to the time in years for the state variables to converge half of the way towards steady-state for a shock to productivity and amenities for which its initial impact on the state variables ($\boldsymbol{R}\widetilde{\boldsymbol{f}}$) corresponds to an eigenvector ($\boldsymbol{u_k}$) of the transition matrix ($\boldsymbol{P}$); figure shows mean and maximum half-life across eigenvectors of the transition matrix in each year from 1965-2015.

nocap

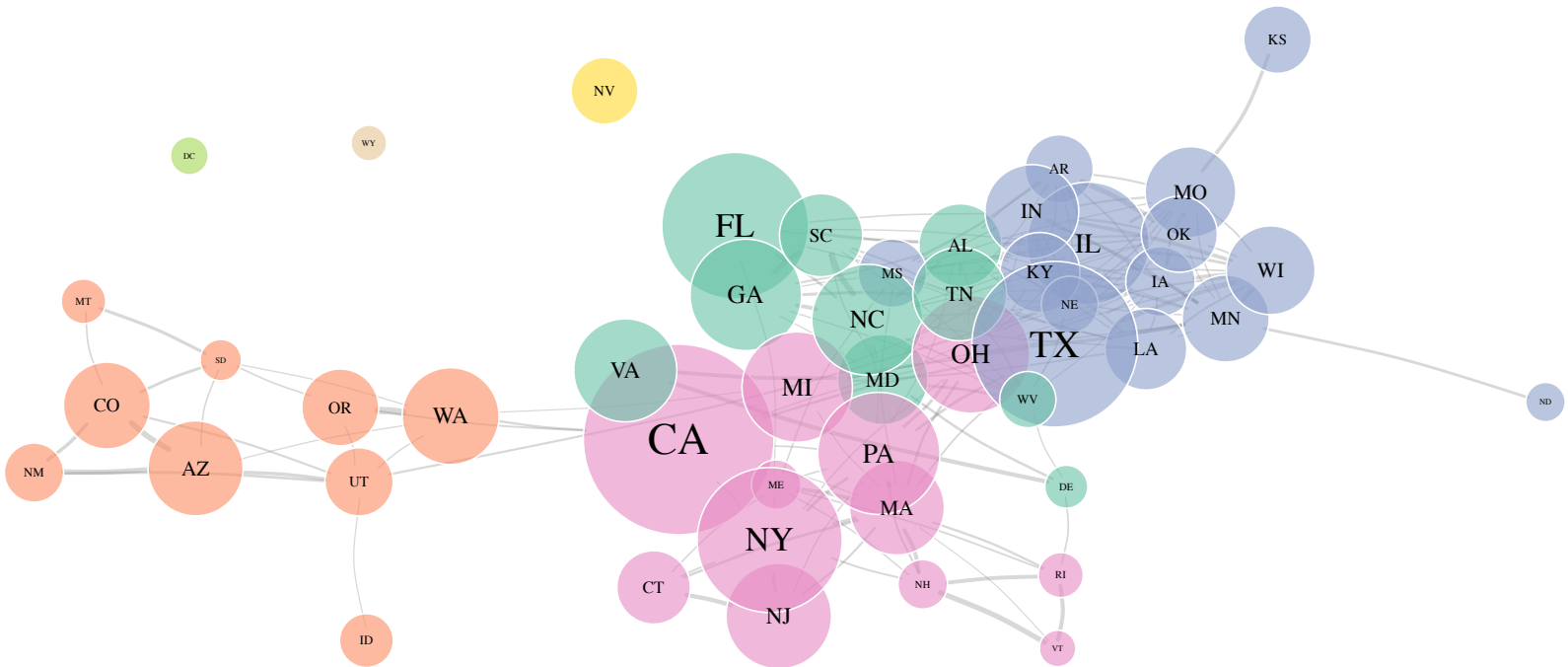
Relative Loading on Labor



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Network of State Exposure 2015

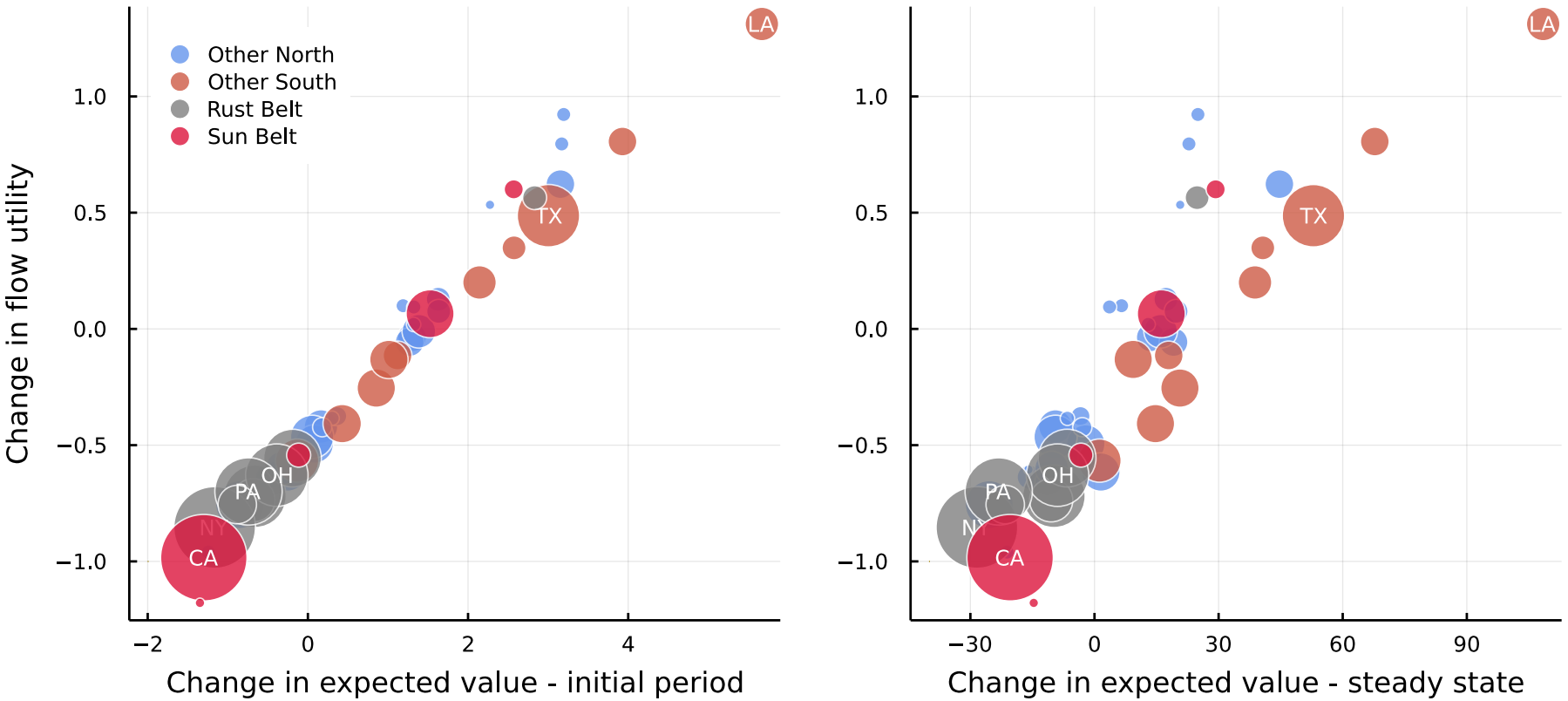
Panel B: Year 2015



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Distributional Effects

- Compare time path of welfare effects *by location*
- Start from the observed data in 1965
- Shock with vector of productivity shocks from 1965-2015



Conclusions

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- Empirical setting features both capital dynamics and migration
- Use our linearization to provide new evidence on slow convergence, labor v. capital dynamics, and heterogeneous impact of local shocks

Thank You