Child skill production: Accounting for parental and market-based time and goods investments*

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July 8, 2020

Abstract

This paper studies the multidimensional nature of investments in children within a dynamic framework. In particular, we examine the roles of parental time investments, purchased home goods/services inputs, and market-based child care services. We first document strong increases in total investment expenditures by maternal education; yet, expenditure shares, which skew heavily towards parental time, vary little with parental schooling. Second, we develop an intergenerational lifecycle model with multiple child investment inputs to study these patterns and the impacts of policies that alter the prices of different inputs. We analytically characterize investment behavior, focusing on the substitutability of different investment inputs and the way parental skills affect the productivity of family-based inputs. Third, we develop an estimation strategy that exploits intratemporal optimality conditions based on relative demand to estimate substitutability between inputs, the relative productivity of different inputs, and the role played by parental education. This approach requires no assumptions about the dynamics of skill investment, preferences, or credit markets. We also account for mis-measured inputs and wages, as well as unobserved heterogeneity in parenting skills. We further show how noisy measures of child achievement (measured several years apart) can also be incorporated in a GMM approach to additionally identify the dynamics of skill accumulation. Fourth, we use data from the Child Development Supplement of the Panel Study of Income Dynamics to estimate the skill production technology for children ages 12 and younger. Our estimates suggest complementarity between parental time and home goods/services inputs as well as between these family-based inputs and market-based child care, with elasticities of substitution ranging from 0.2 to 0.5. We find no systematic effects of parental education on the relative productivity of parental time and other home inputs. Finally, we use counterfactual simulations to explore the extent and sources of variation in investments across families, as well as investment responses to changes in input prices. We find that variation in prices explains 48% of the overall variance in investment expenditures, and differences in wages explain more than half of the investment expenditure gap between college and non-college educated parents. We further show that accounting for the degree of input complementarity implied by our estimates has important implications for the responses of individual inputs to any price change and for the responses in total investments and skill accumulation to large (but not small) price changes.

*For helpful suggestions, we thank Nail Kashaev, Roy Allen, Salvador Navarro, and David Rivers. For excellent research assistance, we thank Mauricio Torres Ferro, Tom Trivieri, and Han Yu. We are also grateful to Kristina Haynie of Child Care Aware of America for providing us with a digital compendium of child care prices from all annual reports. Caucutt and Lochner acknowledge generous research support from SSHRC. The views expressed in this paper are those of the authors and do not necessarily represent those of the Bank of Canada.

1 Introduction

Parents spend their own time and money at home investing in their children's human capital. Many also make substantial investments through market-based child care services. A wide range of government policies impact these investment decisions. For example, welfare policies with work requirements or that claw back gains from working, as well as the general structure of income taxation, distort parental time investment margins. Subsidies for sports and arts programs, along with publicly provided goods like libraries, favor family-based goods investments, while child care incentives favor market-based investments in children. The welfare and child development impacts of these policies depend on how families respond by adjusting their investment profiles across inputs within periods and over time. These adjustments depend critically on how family-based investments (time and goods/services) interact, how parental human capital affects the productivity of those investments, and how family-based investments interact with market-based child care.

Guryan, Hurst, and Kearney (2008) document that higher educated mothers spend more time caring for their children than do lower educated mothers. This is surprising at first, because more educated mothers work more and face a higher opportunity cost of time for other activities. However, the productivity of a mother's time in child investment may also increase with her human capital (Del Boca, Flinn, and Wiswall, 2014; Brilli, 2015). This not only suggests the importance of accounting for maternal skills in child production, but it also highlights the importance of accounting for leisure (or non-child care home production) in addition to time spent investing in children and working in the labor market.[1] Much of the literature abstracts from this additional margin, while other studies (Bernal, 2008; Bernal and Keane, 2010, 2011) assume that all of children's time is allocated to investment either at home or in child care facilities.

The literature estimating human capital production functions for children has either focused on dynamic interactions of investments over time (Cunha and Heckman, 2007; Cunha, Heckman, and Schennach, 2010; Cunha, Elo, and Culhane, 2013; Del Bono et al., 2016; Pavan, 2016; Agostinelli and Wiswall, 2020; Attanasio et al., 2017; Caucutt and Lochner, 2020), typically reducing investment each period to a single composite input, or has imposed assumptions about the substitutability between time and goods investments (Del Boca, Flinn, and Wiswall, 2014, 2016; Brilli, 2015; Lee and Seshadri, 2019; Mullins, 2019; Attanasio et al., 2020) or between home and child care environments (Griffen, 2019; Chaparro, Sojourner, and Wiswall, 2020). Exceptions include papers that estimate substitutability between time

[1] See Domeij and Klein (2013), Bick (2016), and Guner, Kaygusuz, and Ventura (Forthcoming) for related research that abstracts from children's human capital accumulation but examines the role child care policies play in parental labor supply.

and goods (Abbott, 2020) or time and child care (Moschini, 2020).[2] An important remaining challenge is to consider a technology that jointly characterizes patterns of substitution for time and purchased goods/services at home, as well as child care purchased from the market. Our theoretical and quantitative analyses demonstrate that all of these components are important for understanding cross-sectional patterns in the data as well as responses to changes in market prices and policies.

Accounting for multiple inputs in the production of children's human capital, this paper emphasizes two overarching aspects of the child development process. First, we study the degree to which parents' time with their child can be replaced by educational goods/services in the home and the degree to which these family-based investments can be substituted for with market-based child care services. Second, we consider how parental human capital influences the child production process through (i) the productivity of parental time investments, (ii) the productivity of home goods/services investments, and (iii) factor-neutral child productivity. Our approach explicitly recognizes that parental skills are more likely to directly impact the productivity of family-based investments than market-based child care services. This distinction is novel, yet conceptually important for understanding the types of investments made by different families.[3]

In considering the role of parental human capital in the child development process, it is important to distinguish between a parent's productivity in child investment and in the labor market. While these are both related to a parent's general human capital, they are not necessarily the same. A parent's skills may be more or less productive or may exhibit different degrees of diminishing returns in child-rearing relative to the labor market. Even conditional on parental skill levels, there may be many idiosyncratic factors that create a wedge between the value of skill at home and wage rates, including local labor market variation in wages or marginal tax rates, various forms of idiosyncratic wage shocks (e.g. worker-firm match quality), or wage growth due to long-term contracts or occupation- and firm-specific human capital accumulation. For a working parent, the opportunity cost of investing time in child production depends on all of the factors affecting his or her wage, including general human capital, while only the latter affects the productivity of child investments. This rich heterogeneity in time costs across families affects both labor market and child investment decisions and is important for understanding why policies

[2] Gayle, Golan, and Soytas (2014) estimate the importance of parental time inputs and required costs of child-rearing in a model with fertility, marriage, and divorce. Molnar (2020) estimates the elasticity between parental time and a Cobb-Douglas composite of home goods expenditures and market child care. Fiorini and Keane (2014) estimate the impacts of many different categories of children's time use on their cognitive and non-cognitive outcomes, concluding that educational activities with parents are most productive. Todd and Wolpin (2007) estimate the effects of both home inputs (i.e., an index of home environment quality) and school quality (e.g., local pupil-teacher ratios and teacher salaries) using value-added models of cognitive achievement. They conclude that differences in school quality account for a very small proportion of race/ethnicity gaps in achievement.

[3] Chaparro, Sojourner, and Wiswall (2020) consider a framework in which the productivity of parental time depends on parental skills and an endogenous (costly) effort choice; however, they abstract from other home goods/services inputs.

may have quite heterogeneous effects.

We begin our analysis by documenting cross-sectional patterns in investment using the Panel Study of Income Dynamics (PSID) and its Child Development Supplement (CDS), which provides the most comprehensive set of measures for the inputs we wish to study. We supplement this analysis using data from the American Time Use Survey (ATUS) and the Consumption Expenditure Survey (CEX), both of which suffer from a lack of joint measurements of all relevant inputs. We find that among working parents, total expenditures on child investments are strongly increasing in maternal education. Yet, the share of expenditures devoted to different inputs, including parental time, varies much less with parental education.

We next develop a dynamic theoretical framework in order to better understand these cross-sectional patterns and to explore the counterfactual impacts of policies that shape incentives to work and purchase child care. This model allows for multiple child investment inputs within periods and multiple periods of investment. Accordingly, we emphasize how the empirical content of this rich, multi-period model can be decomposed into *intratemporal* and *intertemporal* components.

In the intratemporal problem, families choose child input allocations (parental time, household investment goods/services, and market-based child care services) to minimize expenditures given a per-period level of total human capital investment. The optimal input allocation depends only on the technology for per-period human capital accumulation, relative input prices, and full family income. Importantly, it does not depend on the dynamics of skill production, credit markets, or preferences. We allow for varying degrees of substitutability across inputs and incorporate potential effects of parental human capital on the productivity of family-based time and goods/services investments. From this intratemporal problem, we characterize expenditure shares as a function of relative prices and parental human capital. In the intertemporal problem, families maximize lifetime utility by choosing consumption, savings, leisure, and per-period total/composite human capital investment, given the composite price of investment (from the intratemporal problem) and potentially binding borrowing constraints. From this problem, we characterize the dynamics of total investment expenditure as a function of the composite price of investment and family income. Drawing on results from both the intratemporal and intertemporal problems enables us to characterize the effects of input prices and parental human capital on each input over the lifecycle.

Our analysis demonstrates the critical role that substitutability across different inputs plays in the investment responses of families to changes in input prices, including parental wages. When inputs are sufficiently substitutable, families will substitute away from inputs whose price rises towards other inputs. This is not necessarily the case when inputs are complementary. A particularly interesting case concerns

the effects of wage changes, which not only affect the opportunity cost of time investment but also impact family income. We show that the positive income effects from higher wages create incentives to increase overall investments in children. If all investment inputs are sufficiently complementary, families may choose to increase all types of investments, including time investments, despite the increase in their opportunity costs. Thus, higher wages can cause parents to substitute leisure for time at work with little, or even positive, effects on time spent with children. Indeed, Bastian and Lochner (2020) estimate that wage subsidies implicit in expansions of the Earned Income Tax Credit (EITC) led single mothers to reduce their leisure time and home production but not their time spent investing in children.

Our analysis also highlights the many forces at work when comparing investments across parents with different levels of human capital. Parental human capital not only affects the price of time investment and family income levels, but it may also impact the productivity of investment inputs. We focus on the extent to which parental human capital raises the productivity of parents' time inputs and household goods/services inputs relative to the productivity of purchased child care. We show how these forces, along with the substitutability of different inputs, determine the relationship between parental human capital and investments in children. Most notably, we derive a useful "neutrality" result, which states that investment expenditure shares are independent of parental skills when (i) those skills raise the productivity of parental time with children and in the labor market at the same rate and (ii) parental skills have no effect on the productivity of other inputs.

Our theoretical analysis emphasizes two key sets of parameters that are critical to characterizing family investments in children: elasticities of substitution across inputs and the impacts of parental human capital on the productivity of family investment inputs. We develop two complementary strategies for estimating these parameters. First, we exploit intratemporal optimality alone to derive a series of relative demand relationships that can be estimated using the PSID-CDS data. We leverage the flexibility and simplicity of these estimating equations to explore the sensitivity of results to measures of parental human capital, inclusion of other covariates, endogeneity of input choices, and measurement error in wages. Second, we combine the intratemporal moment conditions with the intertemporal restrictions implied by different assumptions on credit markets to estimate all parameters of the production technology (including parameters related to the dynamics of skill accumulation), using a generalized method of moments (GMM) procedure that accounts for the fact that our data provide only noisy measures of skills collected several years apart. Our choice of specification and estimation strategy in this section is informed by the results in the previous stage of analysis.

In order to estimate the production technology, we augment our PSID-CDS data by compiling a novel

dataset of child care prices using annual reports on the cost of child care in the U.S. from Child Care Aware of America (2009–2019). These data provide state by year variation in child care prices, which we link with households in the PSID. Using this variation in prices (as well as variation in parental wages), we find robust evidence across empirical strategies that parental time and purchased goods/services are complements inside the home, while home investments are also complementary with child care inputs. In both cases, elasticities of substitution range from around 0.2 to 0.5. This suggests that the relatively constant expenditure shares across education categories that we document are not the consequence of a Cobb-Douglas technology, but rather that competing forces driving the effect of human capital on relative demand are in rough balance. In turn, the well-known parental education gradient in time spent with children (e.g., Guryan, Hurst, and Kearney, 2008) is driven by overall demand for child investment inputs, and not by factor augmentation of home inputs by parental human capital.

Finally, we explore some of the positive and normative implications of our estimates using counterfactual simulations. We begin by documenting considerable variation in the composite price of investment, driven largely by differences in parental wages. Roughly half of the variance in investment expenditures across families can be explained by differences in the prices paid for inputs. Differences in input prices (especially wages) are also responsible for some of the investment gaps by parental education. We next study the implications of tax and subsidy policies by simulating the effects of price reductions for each investment input. The moderate complementarity we estimate implies that all investment inputs move together in response to any input price change. As noted earlier, we find that a decline in parental wages leads to reductions in all types of investment, including small reductions in parental time investments, due to the decline in resources. The resulting declines in achievement growth are broadly consistent with previous evidence on the impacts of the EITC on child achievement (Dahl and Lochner, 2012; Agostinelli and Sorrenti, 2018). Reductions in the price of home goods/services inputs or child care services cause families to expand all types of investment, with much stronger own-price elasticities than cross-price elasticities. Given the complementarity of investments, we find that the welfare distortions of price changes on relative input allocations are quite modest; however, the full welfare distortions from wage changes are sizeable when also accounting for impacts on total investment levels, leisure, and consumption. To gauge the importance of estimating flexible patterns of substitution across inputs, we compare the policy responses based on our estimated nested CES within-period production function with the commonly assumed Cobb-Douglas specification (calibrated to match the same input expenditure shares). While the Cobb-Douglas case produces similar conclusions regarding responses in total investment (and achievement growth) to small input price changes, it performs much more poorly when it comes to measuring

the responses to larger price changes or in quantifying responses of different inputs to changes in relative prices.

This paper proceeds as follows. The next section documents investment expenditure patterns by maternal education for single mothers and two-parent households using data from the PSID-CDS, ATUS, and CEX. Section 3 develops our theoretical framework for studying investment behavior and characterizes the effects of parental human capital and input prices on three types of investment inputs: parental time, home goods/services inputs, and market child care services. In Sections 4 and 5, we describe our approach for estimating the technology of skill formation followed by the data used in estimation. Section 6 reports our estimation results, while Section 7 presents our counterfactual analysis based on those estimates. Section 8 concludes.

2 Basic Investment Patterns by Parental Education and Marital Status

This section gives an overview of child investment expenditures by parental education for single mothers and two-parent households. This not only provides a preview of the PSID-CDS data we use in estimating child production functions, but it also highlights several interesting patterns our framework is designed to better understand. Finally, due to the inherent difficulties of defining inputs like parental "investment time" or household "investment goods and services" and likely measurement error in these measures as well as expenditures on child care services, we also examine time investments in ATUS and household goods/services and child care expenditures in the CEX.

2.1 Investments in the PSID

We begin with some basic patterns in the PSID-CDS. As with our estimation sample below, this analysis focuses on families with only 1 or 2 children ages 12 or younger, parents ages 18–65, and mothers who were ages 16–45 when their youngest child was born. An advantage of the PSID is that it enables us to examine child-specific expenditures. Because the PSID only collected information on a broad set of household goods/services investments in the 2002 CDS, this analysis is limited to that year (with expenditures reported in year 2002 dollars) when nearly all of the children we study were ages 5–12. We will often refer to these as home (or household) "goods" investments; however, they not only include expenditures on school supplies, books, and toys, but they also include expenditures on services like tutoring, lessons, community groups, and sports activities. Consistent with previous studies using the PSID-CDS (e.g., Del Boca, Flinn, and Wiswall, 2014; Mullins, 2019), our measure of parental time

investments reflects time actively engaging with children as recorded on time diaries. See Section 5 for greater detail on these data.

Table 1 reports weekly expenditure amounts on parental time, household goods, and child care investments by household type and maternal education. In calculating the expenditures for parental time, we multiply active time with children by the parent's reported wage rate.[4] The amounts spent on children are sizeable, with single mothers spending, on average, about $250 per week on child investment and two-parent households spending more than double that amount, $608. Time expenditure amounts are similar for both married and single mothers; however, children from two-parent families also benefit from father's time expenditures, which are of roughly similar magnitude. Household goods input expenditures are about twice as high in two-parent homes, while child care expenditures are similar across family types. When looking across education categories, we observe that expenditures are increasing substantially with maternal education, with expenditures on each type of input roughly 2–3 times higher for children with mothers that are college graduates vs. high school dropouts.

Table 2 calculates average expenditure shares on each type of investment input. Two features are striking and could be anticipated by the expenditure amounts of the previous table. First, expenditures are dominated by time investments, with single mothers (both parents in two-parent households) contributing about 80% (90%) of their investment expenditures in the form of time. We see below that time investments still dominate (though not as much) even when using a much narrower measure of time investment from the ATUS. Second, the similarity of expenditure shares across maternal education levels is noteworthy. While expenditures increase dramatically with parental education, all types of investments increase at roughly similar rates. That said, we observe a modest increase in the expenditure share of single mother's time investment (beyond high school), coupled with a decline in household goods inputs. Expenditures on child care services represent a very small share of overall investment expenditures, which varies little with parental education or marital status.[5]

Because wages increase sharply with parental education, it is also interesting to look more directly at parental time spent with children. Table 3 reports average hours of active time spent with children each week in the 2002 PSID-CDS. Single and married mothers spend similar amounts of time investing in their children, with fathers spending about one-third less time than their spouses. Not surprisingly,

[4] We exclude non-working parents. We also trim the top/bottom 1% of reported wages to eliminate outliers.

[5] As discussed below, expenditures on child care services likely under-represent the full value of actual child care services received, because many families benefit from free care provided by grandparents and older siblings (Laughlin, 2013). Conditioning the sample on families with positive reported child care expenditures raises this share to 0.16 among all single mothers and 0.10 among all two-parent households. Among single mothers with positive child care spending, there is a more pronounced decline (increase) in the share of expenditures on child care services (maternal time) with education beyond high school. This is not the case for two-parent households where expenditure shares change little with maternal education beyond high school. See Appendix Table D-1.

differences in time spent with children by education are more muted than differences in the value of that time. Differences in the time married parents spend with their children (by mother's education) are quite small, while single mothers with a college degree spend up to 25% more time with their children than their less educated counterparts.

2.2 Investments in the ATUS and CEX

We turn now to the ATUS for an alternative set of parental time investment measures and the CEX for measures of household goods and child care expenditures. For comparability, we use the same sampling approach (notably, families with only 1 or 2 children, both ages 12 or less) as with the PSID; however, this analysis differs in several respects. First, these data are based on surveys from 2003–18; although, we denominate expenditures in year 2002 dollars to match with PSID amounts. Second, both ATUS and CEX only report total household (not child-specific) investments. About half of the families in our samples have one child, while the rest have two. Third, the PSID-CDS reference period for investment differs from those of ATUS and CEX. The PSID collects time diaries on one weekday and one weekend day for each child, recording all of their activities on those days, while ATUS records activities from a single household member (a parent, in our case) using a time diary from one day. The PSID asks respondents to report annual expenditures on the household investment measures we consider, while our primary measure of market child care spending comes from questions about the costs associated with current arrangements for the child. The CEX surveys respondents about expenditures at the quarterly level for four quarters. (In all cases, we adjust to weekly amounts.) Fourth, we do not observe all types of investment expenditures from any household in ATUS or CEX, so we cannot calculate expenditure shares at the household level. Finally, compared to the PSID, we consider a narrower set of time investment categories from ATUS and a slightly broader measure of household goods investment categories in the CEX. Regarding time investments in ATUS, we make an effort to include only parental time with children that is likely to directly reflect investments (e.g. reading to/with children, playing, helping with homework, talking with/listening to children, providing and obtaining medical care, attending museums or movies). Regarding household expenditures, in addition to expenses included in the PSID, the CEX also includes expenditures on computers and software, as well as admission fees and tickets for entertainment activities like movies, theater, concerts. See Appendix B for greater detail on the ATUS and CEX data.

Table 4 shows expenditure amounts on each type of investment from ATUS and the CEX. The most notable differences between these figures and those of Table 1 (based on the PSID) are the substantially lower estimated expenditures on parental time investment. Average time expenditures in ATUS are about one-half to one-third their counterparts in the PSID due to the narrower definition of invest-

ment.[6] Average expenditures on household goods investments are quite similar across the PSID and CEX; however, expenditures on child care services are 2–3 times higher in the CEX. Some of this gap is due to the fact that the CEX is measuring expenditures for families with 1.5 children, on average, rather than the child-specific amounts of the PSID. (This likely produces less of a discrepancy for time investments, because parents often spend time with more than one child at a time.) Altogether, combining expenditures for ATUS and CEX, we observe a lower share of total average expenditures coming from parental time (roughly 70% for single mothers and 80% for married couples) and a higher share from child care (roughly 20% for single mothers and 10% for married couples) relative to the PSID. Yet, even the narrow measure of parental time investment taken from ATUS suggests that it is the dominant form of investment expenditure for children ages 12 and under. More importantly for our analysis, the expenditure patterns by maternal education in the CEX and ATUS are quite similar to those of the PSID: total average expenditures increase substantially with mother's education, while the shares devoted to each form of investment are fairly stable.

3 Model

We develop an economic framework for understanding the investment and expenditure patterns of Section 2 and the impacts of policies that distort the prices of investment inputs. This analysis highlights the role of complementarity/substitutability of different inputs and the extent to which parental human capital raises the productivity of parental time and purchased home inputs.

Consider two-parent households that are made up of a mother, a father, and a child. These households may be ex ante heterogenous over the learning ability of the child, θ, initial human capital of the child, Ψ_1, and human capital of the mother and father, H_m and H_f, respectively.[7] (Single mother households are identical but without any "father" time, wages, etc.) In every period $t = 1, ..., T$, the household chooses consumption, c_t, mother and father's leisure, $l_{m,t}$ and $l_{f,t}$, respectively, and investments in children.

Child investments take place in the home or in the market. Home investments include time of the mother, $\tau_{m,t}$, time of the father, $\tau_{f,t}$, and goods, g_t. Market-based child care services are represented by

[6] See Appendix Table D-2 for average investment time by marital status and mother's education in ATUS.

[7] Because we abstract from schooling inputs, heterogeneity in θ would also reflect differences in school quality. We implicitly assume that school quality is equally substitutable with all of the inputs we consider.

$Y_{c,t}$.[8] Child skills evolve according to

$$\Psi_{t+1} = \mathcal{H}_t \left(f_t \left(\tau_{m,t}, \tau_{f,t}, g_t, Y_{c,t}; H_m, H_f \right), \theta, \Psi_t \right), \tag{1}$$

where $f_t \left(\tau_{m,t}, \tau_{f,t}, g_t, Y_{c,t}; H_m, H_f \right)$ is a homogenous of degree 1 function that represents the total human capital investment a child receives within a period as a function of all inputs that period.

Normalizing the time endowment to 1 for each parent, parental hours working are $h_{m,t} = 1 - l_{m,t} - \tau_{m,t}$ and $h_{f,t} = 1 - l_{f,t} - \tau_{f,t}$. A parent's period t wage is given by $W_{j,t} = w_{j,t} H_j$, $j = m, f$, where we distinguish between the part of wages related to skills used in child production (H_j) and an unrelated component ($w_{j,t}$). For expositional purposes, we assume the component related to child development is fixed over time (e.g. upon a parent finishing school or the child's birth), while the time-varying part, which we often refer to as the price of skill, incorporates wage differences across parents due to factors like labor market experience, discrimination in the labor market, or local wage variation.[9] Let y_t reflect income other than labor earnings (e.g. transfers) in period t. The price of home investment goods is given by p_t, and the price of market child care is given by $P_{c,t}$. Let $\Pi_t \equiv (W_{m,t}, W_{f,t}, p_t, P_{c,t})$ reflect the vector of all investment input prices faced by the household at time t. Assets at the start of period t are denoted by A_t, and households can borrow and save at interest rate r, subject to borrowing constraints requiring $A_{t+1} \geq A_{t,min}$.

Households have per period preferences over consumption (with price normalized to one) and leisure given by $u(c_t) + v_m(l_{m,t}) + v_f(l_{f,t})$ and discount across periods at the rate $\beta > 0$. In period $T+1$, households have a continuation value, $\tilde{U}(H_m, H_f, A_{T+1})$, that depends on parental human capital and assets. Households also care about the final human capital of children, $\tilde{V}(\Psi_{T+1})$. The household's problem for periods $t = 1, ..., T$, is given by:

$$\begin{aligned} &V_t(\theta, H_m, H_f, A_t, y_t, \Pi_t, \Psi_t) \\ &\quad = \max_{l_{m,t}, \tau_{m,t}, l_{f,t}, \tau_{f,t}, g_t, Y_{c,t}, A_{t+1}} u(c_t) + v_m(l_{m,t}) + v_f(l_{f,t}) + \beta V_{t+1}(\theta, H_m, H_f, A_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{t+1}) \end{aligned}$$

subject to non-negative inputs $(\tau_{m,t}, \tau_{f,t}, g_t, Y_{c,t})$, $l_{j,t} \geq 0$ and $l_{j,t} + \tau_{j,t} \leq 1$ for $j = m, f$, child human

[8] Without data on the quality of child care chosen by or available for families, we have chosen not to explicitly model a quantity-quality tradeoff for child care services, implicitly assuming that families purchase the optimal mix given available options. If care is priced according to its productivity in a competitive market, then families would generally be indifferent to the mix. In this case, our reference to a price of child care, $P_{c,t}$, can be thought of as the price for a fixed quality of care and unit of time.

[9] Our empirical analysis will explicitly incorporate some time-varying factors, like number of children or child's age, into the production of child skills.

capital production equation (1),

$$\begin{aligned} c_t + p_t g_t + W_{m,t}\tau_{m,t} + W_{f,t}\tau_{f,t} + P_{c,t}Y_{c,t} + A_{t+1} &= (1+r)A_t + y_t + W_{m,t}(1-l_{m,t}) + W_{f,t}(1-l_{f,t}), \\ A_{t+1} &\geq A_{min,t}, \\ V_{T+1}(\theta, H_m, H_f, A_{T+1}, y_{T+1}, \Pi_{T+1}, \Psi_{T+1}) &= \tilde{U}(H_m, H_f, A_{T+1}) + \tilde{V}(\Psi_{T+1}). \end{aligned}$$

We assume $u'(\cdot) > 0$, $u''(\cdot) < 0$, $v_j'(\cdot) > 0$, and $v_j''(\cdot) \leq 0$, $j = m, f$. We also assume standard Inada conditions for preferences over consumption and leisure.

The first order conditions for this problem are reported in Appendix A. During periods in which parents work (i.e., $h_{j,t} > 0$), it is straightforward and instructive to separate the problem into two distinct parts: (i) an *intratemporal* problem choosing child input allocations to minimize expenditures given child's per period total human capital investment $X_t = f_t(\tau_{m,t}, \tau_{f,t}, g_t, Y_{c,t}; H_m, H_f)$, and (ii) an *intertemporal* problem of maximizing lifetime utility by choosing savings A_{t+1} (or consumption c_t), leisure $l_{m,t}$ and $l_{f,t}$, and per period child's total human capital investment X_t. This approach highlights that assumptions about dynamics (e.g. credit markets, structure of $\mathcal{H}_t(\cdot)$) and the precise nature of preferences (over consumption, leisure, and child skill levels) are unimportant for the within-period allocation of child investment inputs.[10] Furthermore, when considering allocations over time, multiple investment inputs within a period can be collapsed into a composite per period total investment with an associated composite per period price that depends on all input prices and parameters of the within-period investment function $f_t(\cdot)$. Embedding complexity within a period does not complicate the dynamics of the model, and the dynamics do not complicate the within-period input allocation problem. Throughout the rest of this section, we consider the intratemporal and intertemporal problems assuming that parents are working, so the price of parents' time is reflected in their wages.

3.1 Intratemporal Problem

Given a level of total human capital investment in a period, X_t, the intratemporal problem minimizes its cost.[11] Assuming parents work in the market, the cost of investing time with their children is measured by their wages. The solution to this problem indicates how investment inputs depend on relative input prices and parental human capital for any given level of total investment X_t. It also determines the unit

[10] As discussed in Appendix A, the price of time investment depends on the marginal rate of substitution between consumption and leisure for parents that do not work. In this case, the investment cost is in terms of consumption (expenditures) and lost leisure, and cannot be written as a single expenditure minimization problem. If borrowing constraints bind, then the full problem can be trivially separated into a series of intratemporal problems.

[11] While we abstract from uncertainty about future prices and income to simplify the analysis of the intertemporal problem, the intratemporal problem described here is unaffected by uncertainty.

price of total investment X_t that is central to the dynamic decision problem. This unit price, $\bar{p}_t$, depends only on the parameters of the within-period investment function $f_t(\cdot)$ and input prices Π_t.

The intratemporal problem is given by:

$$\min_{g_t,\tau_{m,t},\tau_{f,t},Y_{c,t}} p_t g_t + P_{c,t}Y_{c,t} + W_{m,t}\tau_{m,t} + W_{f,t}\tau_{f,t}$$

subject to non-negative inputs $(\tau_{m,t}, \tau_{f,t}, g_t, Y_{c,t})$, $\tau_{m,t} \leq 1$, $\tau_{f,t} \leq 1$, and $X_t = f_t(\tau_{m,t}, \tau_{f,t}, g_t, Y_{c,t}; H_m, H_f)$.

Throughout our analysis, we consider a nested CES within-period investment function:

$$f_t = \left[\left(a_{m,t}(H_m)\tau_{m,t}^{\rho_t} + a_{f,t}(H_f)\tau_{f,t}^{\rho_t} + a_{g,t}(H_m, H_f)g_t^{\rho_t}\right)^{\gamma_t/\rho_t} + a_{Yct}Y_{c,t}^{\gamma_t}\right]^{1/\gamma_t}, \ \rho_t < 1, \gamma_t < 1. \tag{2}$$

We highlight three aspects of this specification. First, it allows parental human capital to affect the productivity of household time and goods investments through their respective share parameters. (We generally leave the conditioning on parental human capital implicit, except where it plays an important role.) Second, it accommodates flexible substitution patterns between parental time and goods within the household and between these inputs and market-based child care services. The elasticity of substitution between parental time and household goods inputs is constant and given by $\epsilon_{\tau,g,t} \equiv 1/(1-\rho_t)$. By contrast, the elasticity of substitution between market child care services and household goods or parental time investments varies with input levels; however, the elasticity between market child care $Y_{c,t}$ and "home composite investment" $X_{H,t} \equiv \left(a_{m,t}\tau_{m,t}^{\rho_t} + a_{f,t}\tau_{f,t}^{\rho_t} + a_{g,t}g_t^{\rho_t}\right)^{1/\rho_t}$ is given by $\epsilon_{Y,H,t} \equiv 1/(1-\gamma_t)$. We will generally refer to two inputs as substitutable if their elasticity of substitution is greater than one (e.g. $\epsilon_{\tau,g,t} > 1$ and $\rho_t > 0$) and complementary if their elasticity is less than one. The commonly employed Cobb-Douglas case assumes an elasticity of one across all inputs. Lastly, our specification for $f_t(\cdot)$ is homogenous of degree 1, which is essential for separating the full problem into intratemporal and intertemporal problems. See Appendix A.

Ratios of first order conditions for this expenditure minimization problem clarify the dependence of investment ratios on their relative prices, technology share parameters, elasticities of substitution between different inputs, and parental human capital:

$$\tilde{W}_{j,t} \equiv \frac{W_{j,t}}{p_t} = \frac{a_{j,t}\tau_{j_t}^{\rho_t - 1}}{a_{g,t}g_t^{\rho_t - 1}}, \qquad j = m, f, \tag{3}$$

$$\tilde{P}_{c,t} \equiv \frac{P_{c,t}}{p_t} = \frac{a_{Yc,t}Y_{c,t}^{\gamma_t - 1}}{(a_{m,t}\tau_{m,t}^{\rho_t} + a_{f,t}\tau_{f,t}^{\rho_t} + a_{g,t}g_t^{\rho_t})^{(\gamma_t - \rho_t)/\rho_t} a_{g,t}g_t^{\rho_t - 1}}. \tag{4}$$

From these expressions, we can see that parental time and market child care investments are proportional to household goods inputs: $\tau_{j,t} = \Phi_{j,t}g_t$, for $j = m, f$, and $Y_{c,t} = \Phi_{c,t}g_t$, where the factors of

proportionality depend on relative prices and production technology parameters:

$$\Phi_{j,t} = \left[\frac{a_{g,t}}{a_{j,t}}\tilde{W}_{j,t}\right]^{\frac{1}{\rho_t-1}}, \qquad j=m,f, \tag{5}$$

$$\Phi_{c,t} = \left(\frac{a_{g,t}}{a_{Yc,t}}\right)^{\frac{1}{\gamma_t-1}} \left(a_{m,t}\Phi_{m,t}^{\rho_t} + a_{f,t}\Phi_{f,t}^{\rho_t} + a_{g,t}\right)^{\frac{\gamma_t-\rho_t}{\rho_t(\gamma_t-1)}} \tilde{P}_{c,t}^{\frac{1}{\gamma_t-1}}. \tag{6}$$

These constants appear repeatedly throughout our analysis, including the econometric specifications below.

3.1.1 Unit Prices for Total/Composite Investment

Using Equations (5) and (6), it is straightforward to show that total investment X_t is proportional to each input. For example, we have the following relationship between X_t and household goods inputs:

$$X_t = g_t\left[\left(a_{m,t}\Phi_{m,t}^{\rho_t} + a_{f,t}\Phi_{f,t}^{\rho_t} + a_{g,t}\right)^{\frac{\gamma_t}{\rho_t}} + a_{Yc,t}\Phi_{c,t}^{\gamma_t}\right]^{\frac{1}{\gamma_t}}, \tag{7}$$

while total expenditures on investments in children are given by

$$E_t \equiv \bar{p}_t(\Pi_t; H_m, H_f)X_t = p_t g_t + P_{c,t}Y_{c,t} + W_{m,t}\tau_{m,t} + W_{f,t}\tau_{f,t}. \tag{8}$$

Equations (7) and (8) together imply that the unit price of period t investment is

$$\bar{p}_t(\Pi_t; H_m, H_f) = \frac{p_t + P_{c,t}\Phi_{c,t} + W_{m,t}\Phi_{m,t} + W_{f,t}\Phi_{f,t}}{\left[\left(a_{m,t}\Phi_{mt}^{\rho_t} + a_{f,t}\Phi_{f,t}^{\rho_t} + a_{g,t}\right)^{\frac{\gamma_t}{\rho_t}} + a_{Yc,t}\Phi_{c,t}^{\gamma_t}\right]^{\frac{1}{\gamma_t}}}, \tag{9}$$

where $\Phi_{j,t}$ for $j \in \{m,f\}$ and $\Phi_{c,t}$ depend on input prices and parental human capital as defined in Equations (5) and (6), respectively. The evolution of this unit price is central to the dynamics of investment as described in the intertemporal problem of Section 3.2.

3.1.2 Expenditure Shares

The analysis of the previous subsection makes clear that input choices, given any level of total investment expenditure, can be determined from the intratemporal problem alone. Based on this, we next characterize how the allocation of resources to each input depends on input prices and parental human capital. Focusing on expenditure shares, this analysis depends only on the within-period production function $f_t(\cdot)$.

For simplicity, we consider the case of single mothers and drop all time subscripts (as we focus on within-period relationships), so

$$f = \left[(a_m\tau_m^{\rho} + a_g g^{\rho})^{\gamma/\rho} + a_{Yc}Y_c^{\gamma}\right]^{1/\gamma}. \tag{10}$$

Total investment expenditures are $E = pg + P_c Y_c + W_m \tau_m = g\left(p + P_c \Phi_c + W_m \Phi_m\right)$, where the latter follows from Equations (5) and (6). We can write expenditure shares as:

$$S_g \equiv \frac{pg}{E} = \frac{p}{p + P_c \Phi_c + W_m \Phi_m}, \quad S_{\tau_m} \equiv \frac{W_m \tau_m}{E} = \frac{W_m \Phi_m}{p + P_c \Phi_c + W_m \Phi_m}, \quad S_{Yc} \equiv \frac{P_c Y_c}{E} = \frac{P_c \Phi_c}{p + P_c \Phi_c + W_m \Phi_m},$$

which are independent of total investment, X. This property is useful, because we can consider the effects of input prices or parental skills on input shares without concern for the level of total investment.

As is well-known, if $\rho = \gamma = 0$, the within-period production function $f(\cdot)$ is Cobb-Douglas, and expenditure shares are independent of input prices. More generally, impacts of price changes depend on the substitutability of inputs. Families facing a price increase for one input will tend to substitute away from that input and towards others that are sufficiently substitutable. By contrast, when inputs are sufficiently complementary, they will tend to co-move in response to any price change. The following results formally characterize these implications for expenditure shares.

Proposition 1. *If and only if $\gamma < 0$, then P_c has strictly positive own-price effects on S_{Y_c} and strictly negative cross-price effects on S_g and S_{τ_m}.*

Complementarity between market child care services and the home composite input ($\gamma < 0$) means that households do not substitute enough from external child care to household inputs in response to an increase in child care prices, so their share of investment expenditures on child care services increases, while expenditure shares on both household inputs (time and goods) decrease. With substitutability between home inputs and market child care ($\gamma > 0$), an increase in P_c causes households to substitute out of child care enough to offset the price increase, resulting in a lower share of expenditures on child care and higher shares for both household inputs.

Given the nested nature of $f(\cdot)$, the impacts of price changes on home inputs g and τ_m are slightly more complicated, though symmetric.

Proposition 2. *Expenditure shares on home inputs (g or τ_m) are strictly decreasing in their own price (p or w_m) if $\min\{\rho, \gamma\} > 0$ and strictly increasing in their own price if $\max\{\rho, \gamma\} < 0$. Expenditure shares on home inputs are strictly decreasing in the other home input price if $\rho < \min\{0, \gamma\}$, and strictly increasing in the other home input price if $\rho > \max\{0, \gamma\}$. The expenditure share on market child care services is strictly increasing in the price of both home inputs if and only if $\gamma > 0$.*

Complementarity between both home inputs ($\rho < 0$) and between the home composite input and market child care ($\gamma < 0$) ensures that substitution out of a home input whose price rises is insufficient to compensate for the higher price, leading to a greater expenditure share on that input. If home inputs are

not only complementary ($\rho < 0$) but also more complementary than home inputs with market child care ($\rho < \gamma$), then an increase in the price of one home input will cause the expenditure share of the other to fall. The converse of these statements applies when inputs are substitutes. Finally, substitutability between home and market inputs ($\gamma > 0$) implies that an increase in either home input will raise the share of expenditures on child care, while complementarity ($\gamma < 0$) implies the opposite.

Changes in parental human capital have still more complicated effects on expenditure shares, because they not only affect the price of parental time through wages, but they may also directly affect the production of child skills. Letting $a_m(H_m) = \bar{a}_m[\varphi_m(H_m)]^\rho$ and $a_g(H_m) = \bar{a}_g[\varphi_g(H_m)]^\rho$, with constants $\bar{a}_m > 0$ and $\bar{a}_g > 0$, we assume the following convenient functional forms:

$$\varphi_m(H_m) = H_m^{\bar{\varphi}_m} \quad \text{with } \bar{\varphi}_m \geq 0, \tag{11}$$

$$\varphi_g(H_m) = H_m^{\bar{\varphi}_g} \quad \text{with } \bar{\varphi}_g \geq 0. \tag{12}$$

The exponents $\bar{\varphi}_m$ and $\bar{\varphi}_g$ determine the returns to scale of parental human capital in the production of child skills. To isolate the role of productivity in mother's time investment for the relationship between expenditure shares and maternal human capital, we begin by considering the case of $\bar{\varphi}_g = 0$. We, then, discuss the implications of allowing maternal human capital to affect the productivity of home goods investments (i.e., $\bar{\varphi}_g > 0$) as well.

When $\bar{\varphi}_g = 0$, inspection of the factors of proportionality Φ_m and Φ_c (see Equations (5) and (6)) makes clear that $\frac{\partial \Phi_m}{\partial H_m} = (1 - \bar{\varphi}_m)\left(\frac{w_m}{H_m}\right)\frac{\partial \Phi_m}{\partial w_m}$ and $\frac{\partial \Phi_c}{\partial H_m} = (1 - \bar{\varphi}_m)\left(\frac{w_m}{H_m}\right)\frac{\partial \Phi_c}{\partial w_m}$. This implies that

$$\frac{\partial S_j}{\partial H_m} = (1 - \bar{\varphi}_m)\left(\frac{w_m}{H_m}\right)\frac{\partial S_j}{\partial w_m}, \quad \text{for } j = g, \tau_m, Y_c. \tag{13}$$

When $\bar{\varphi}_m = 1$, so $\varphi_m(H_m)$ exhibits constant returns to scale (CRS), parental human capital has no effect on investment expenditure shares, regardless of the substitutability of different inputs in skill production (i.e., ρ and γ). In this case, a mother's skills improve her labor market earnings and child productivity at the same rate, leaving her incentives to invest time in her child unaffected. This "neutrality" result is of particular interest, because it offers one potential explanation for the modest differences in expenditure shares by maternal education (despite substantial differences in investment expenditure levels) reported in Section 2.

When $\varphi_m(H_m)$ exhibits decreasing returns to scale (i.e., $\bar{\varphi}_m < 1$), the effects of mother's human capital on expenditure shares mirror the effects of changes in w_m as described in Proposition 2. She is relatively more productive working in the market and moves away from home time investments if inputs are substitutable.[12] By contrast, when $\varphi_m(H_m)$ exhibits increasing returns to scale (i.e., $\bar{\varphi}_m > 1$),

[12] In the special case of $\bar{\varphi}_m = 0$, maternal skills do not enter the child production technology, and changes in maternal

mothers are relatively more productive in child development than in the labor market. In this case, an increase in maternal skills raises her marginal benefit of investing relative to her marginal cost, which produces similar effects to a fall in the price of her time.

Now, consider $\bar{\varphi}_g > 0$, so the productivity of home goods investment is increasing in maternal human capital. This introduces two opposing forces. On one hand, goods inputs are more productive, creating an incentive for more skilled mothers to purchase more of them. On the other hand, maternal human capital is factor-augmenting so a more skilled mother can buy less goods inputs and still get more "effective" goods investment ($H_m^{\bar{\varphi}_g} g$), freeing up resources to be invested in terms of her time and market child care. If inputs are very substitutable, the first force dominates and it is beneficial to load investment into the more productive home goods as maternal human capital rises. By contrast, if inputs are very complementary, then the second force is stronger, and more educated mothers take advantage of the increased productivity of home goods input by shifting resources to other inputs. The more her productivity depends on her human capital, the stronger these two forces become. The overall implications of $\bar{\varphi}_g > 0$ on expenditure shares is most transparent when the effect of maternal skills on the productivity of time investment is neutralized by assuming $\bar{\varphi}_m = 1$. The following proposition formally characterizes this case. (Appendix A characterizes the relationship between maternal human capital and expenditure shares more generally.)

Proposition 3. *Suppose $\bar{\varphi}_m = 1$ and $\bar{\varphi}_g > 0$. (A) S_τ is strictly decreasing in H_m if $\rho > \max\{0, \gamma\}$, while it is strictly increasing in H_m if $\rho < \min\{0, \gamma\}$. (B) S_g is strictly decreasing in H_m if $\max\{\rho, \gamma\} < 0$, while it is strictly increasing in H_m if $\min\{\rho, \gamma\} > 0$. (C) S_{Y_c} is strictly decreasing in H_m if and only if $\gamma > 0$.*

While the intratemporal problem is sufficient to characterize expenditure shares, we next consider the intertemporal problem to study investment levels and how they respond to changes in input prices or parental skills.

3.2 Intertemporal Problem

Suppose in every period, $t = 1, ..., T$, along with leisure and assets, the household chooses an amount of total child investment X_t, given a per period composite price $\bar{p}_t$ (determined by the intratemporal problem). This problem can be written as follows:

$$V_t(\theta, H_m, H_f, A_t, y_t, \Pi_t, \Psi_t) = \max_{l_{m,t}, l_{f,t}, X_t, A_{t+1}} u(c_t) + v(l_{m,t}) + v(l_{f,t}) + \beta \left[V_{t+1}(\theta, H_m, H_f, A_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{t+1}) \right]$$

skills play the same role as changes in w_m with both simply raising the marginal cost of investment.

subject to $0 \leq l_{m,t}, l_{f,t} \leq 1$, $X_t \geq 0$,

$$\begin{aligned} c_t + \bar{p}_t(\Pi_t, H_m, H_f)X_t + A_{t+1} &= (1+r)A_t + y_t + W_{m,t}(1-l_{m,t}) + W_{f,t}(1-l_{f,t}), \\ \Psi_{t+1} &= \mathcal{H}_t\left(X_t, \theta, \Psi_t\right), \\ A_{t+1} &\geq A_{min,t}, \\ V_{T+1}(\theta, H_m, H_f, A_{T+1}, y_{T+1}, \Pi_{T+1}, \Psi_{T+1}) &= \tilde{U}(H_m, H_f, A_{T+1}) + \tilde{V}(\Psi_{T+1}). \end{aligned} \tag{14}$$

The first order conditions for parental leisure are:

$$v'(l_{j,t}) = u'(c_t)W_{j,t}, \ j = m, f, \tag{15}$$

while the standard Euler equation for consumption may be distorted by borrowing constraints:

$$u'(c_t) \geq \beta(1+r)u'(c_{t+1}) \quad \text{with strict inequality if and only if Equation (14) binds.}$$

As discussed in the literature (e.g., Becker and Tomes, 1986; Cunha and Heckman, 2007; Caucutt and Lochner, 2020), the presence of binding borrowing constraints can distort intertemporal consumption and child investment decisions.

Throughout the rest of this paper, we borrow two assumptions from Del Boca, Flinn, and Wiswall (2014) that facilitate an analytical characterization of investment behavior.

Assumption 1. $\tilde{V}(\Psi_{T+1}) = \alpha \ln(\Psi_{T+1})$.

Assumption 2. $\Psi_{t+1} = \theta X_t^{\delta_1} \Psi_t^{\delta_2}$.[13]

Under Assumptions 1 and 2, the first order condition for X_t is quite simple and can be written as:

$$\bar{p}_t u'(c_t) = \frac{\alpha\beta^{T-t+1}\delta_2^{T-t}\delta_1}{X_t}, \tag{16}$$

which has several useful properties. First, it implies that X_t depends only on past decisions (including past investments), current skills, and borrowing constraints through the marginal utility of consumption $u'(c_t)$. Second, X_t (and its dynamics) depends only on input prices through the unit price $\bar{p}_t$ (as determined by Equation (9) from the intratemporal problem). Third, if we define the constant $K_t \equiv \alpha\beta^{T-t+1}\delta_2^{T-t}\delta_1$, Equation (16) can be re-arranged to obtain a simple expression for total investment expenditures:

$$E_t = K_t / u'(c_t), \tag{17}$$

[13] This technology is consistent with self-productivity ($\delta_2 > 0$) and dynamic complementarity of investments (Cunha and Heckman, 2007).

which is a function of the marginal utility of consumption and parameters related to preferences and child skill production. This proves convenient for characterizing the effects of input prices and parental human capital on investment expenditures.[14]

We make standard assumptions about preferences for consumption and leisure throughout the rest of this section.

Assumption 3. *$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, $\sigma > 0$, and $v_j(l) = \psi_j \frac{l^{1-\nu}}{1-\nu}$, $\nu > 0$, for $j = m, f$.*

With Assumptions 1–3, equations (15) and (17) can be written as

$$W_{j,t} l_{j,t} = \psi_j^{1/\nu} W_{j,t}^{(\nu-1)/\nu} c_t^{\sigma/\nu} \tag{18}$$

$$E_t = K_t c_t^{\sigma}. \tag{19}$$

While these conditions for leisure and investment behavior do not depend on the ability to smooth consumption, the dynamics of investment choices depend on whether or not credit constraints bind. We begin by examining the simpler case in which constraints bind, followed by the slightly more complicated case of non-binding constraints.

3.2.1 Binding Borrowing Constraints

Suppose a family is borrowing constrained in period t with $A_{t+1} = A_{min,t}$. The first order conditions for leisure and total investment continue to hold, and the budget constraint for this period is given by:

$$c_t = (1+r)A_t + W_{m,t}(1 - l_{m,t}) + W_{f,t}(1 - l_{f,t}) + y_t - E_t - A_{min,t}. \tag{20}$$

Substituting in for $W_{j,t} l_{j,t}$ and E_t using Equations (18) and (19) yields,

$$c_t + \psi_m^{1/\nu} c_t^{\sigma/\nu} W_{m,t}^{(\nu-1)/\nu} + \psi_f^{1/\nu} c_t^{\sigma/\nu} W_{f,t}^{(\nu-1)/\nu} + K_t c_t^{\sigma} = (1+r)A_t + W_{m,t} + W_{f,t} + y_t - A_{min,t}.$$

From this, we apply the implicit function theorem to obtain the effects of input prices, non-labor income, and parental human capital on consumption: $\partial c_t / \partial p_t = \partial c_t / \partial P_{c,t} = 0$,

$$\frac{\partial c_t}{\partial w_{j,t}} = \frac{\left(1 - (1 - \frac{1}{\nu})\psi_j^{1/\nu} W_{j,t}^{-1/\nu} c_t^{\sigma/\nu}\right) H_j}{1 + \left(\psi_m^{1/\nu} W_{m,t}^{-1/\nu} + \psi_f^{1/\nu} W_{f,t}^{-1/\nu}\right)\left(\frac{\sigma}{\nu}\right) c_t^{(\sigma-\nu)/\nu} + K_t \sigma c_t^{\sigma-1}} > 0, \quad j \in \{m, f\},$$

$$\frac{\partial c_t}{\partial y_t} = \frac{1}{1 + \left(\psi_m^{1/\nu} W_{m,t}^{-1/\nu} + \psi_f^{1/\nu} W_{f,t}^{-1/\nu}\right)\left(\frac{\sigma}{\nu}\right) c_t^{(\sigma-\nu)/\nu} + K_t \sigma c_t^{\sigma-1}} > 0,$$

[14] A fourth implication is that child learning ability θ does not affect investment behavior (or any other decisions). This derives from Assumption 1 and multiplicative separability between θ and (X_t, Ψ_t) in skill production (Assumption 2). Together, these imply that θ is additively separable from all other choice and state variables in discounted lifetime utility.

and $\frac{\partial c_t}{\partial H_j} = \frac{\partial c_t}{\partial w_{j,t}} \frac{w_{j,t}}{H_j} > 0$ for $j \in \{m, f\}$.[15] Because $\frac{\partial E_t}{\partial \pi} = K_t \sigma c_t^{\sigma-1} \frac{\partial c_t}{\partial \pi}$ for $\pi \in \{p_t, P_{c,t}, y_t, w_{m,t}, w_{f,t}, H_m, H_f\}$, we have the following result.

Proposition 4. *Suppose borrowing constraints bind in period t. Then, total investment expenditures in period t are strictly increasing in parental human capital, skill prices in period t, and non-labor income in period t, with $\frac{\partial E_t}{\partial H_j} = \frac{\partial E_t}{\partial w_{j,t}} \frac{w_{j,t}}{H_j} > 0$ for $j \in \{m, f\}$. Total investment expenditures are independent of all future prices and non-labor income, as well as period t prices for household goods inputs and child care.*

Only current income, current wages, and parental human capital affect total investment expenditures. Notably, any increase in the unit price of investment $\bar{p}_t$ caused by changes in the prices of home goods inputs or child care is perfectly offset by adjustments in input quantities. (We further discuss the impacts of input price changes on each of the inputs quantities below.)

Finally, if we follow Del Boca, Flinn, and Wiswall (2014) (and several subsequent papers) by assuming log preferences for consumption and leisure (i.e., $u(c) = \ln(c)$ and $v_j(l_j) = \psi_j \ln(l_j)$, $\psi_j \geq 0$, for $j \in \{m, f\}$), then we obtain a closed form expression for total investment:

$$X_t = \frac{K_t \left[(1+r)A_t + W_{m,t} + W_{f,t} + y_t - A_{min,t}\right]}{\bar{p}_t \left[1 + \psi_m + \psi_f + K_t\right]}. \tag{21}$$

From this, we see that the dynamics of constrained investment depend on both the dynamics of input prices through $\bar{p}_t$ and the dynamics of "full" family income, $W_{m,t} + W_{f,t} + y_t$. Furthermore, changes in input prices or parental human capital only affect total investment expenditures, $E_t = \bar{p}_t X_t$, through their effects on "full" income that period.

3.2.2 Non-binding borrowing constraints

If a family is not borrowing constrained in period t, then $u'(c_t) = \beta(1+r)u'(c_{t+1})$. Combining this with the first order conditions for X_t and X_{t+1} implies the following dynamics for total child investment:

$$X_t = \left[\frac{\bar{p}_{t+1}\delta_2}{\bar{p}_t(1+r)}\right] X_{t+1}. \tag{22}$$

This condition does not depend on functional forms for period utility, $u(\cdot)$ or $v(\cdot)$, and highlights that in the absence of credit frictions, the dynamics of total investment depend only on δ_2 and relative composite input prices $\bar{p}_t$ — current income levels and the dynamics of income are irrelevant.[16]

We now consider the case of non-binding constraints throughout the remaining parents' lifetime as the polar opposite case to binding constraints above. In this case, lifetime family income matters. The

[15] Note that the first order conditions for leisure imply $l_{j,t} = \psi_j^{1/\nu} W_{j,t}^{-1/\nu} c_t^{\sigma/\nu} < 1$, so $\left(1 - \frac{1}{\nu}\right) \psi_j^{1/\nu} W_{j,t}^{-1/\nu} c_t^{\sigma/\nu} < 1$.

[16] As noted in Abbott (2020), uninsured wage risk for parents introduces an intertemporal investment wedge due to covariation between the stochastic discount factor and future investment. We abstract from this uncertainty.

lifecycle budget constraint for unconstrained families (from t to $T+1$) is given by:[17]

$$\begin{aligned}\sum_{j=0}^{T-t}(1+r)^{-j}c_{t+j} &= (1+r)A_t - (1+r)^{-(T-t)}A_{T+1} \\ &+ \sum_{j=0}^{T-t}(1+r)^{-j}\left[W_{m,t+j}(1-l_{m,t+j}) + W_{f,t+j}(1-l_{f,t+j}) + y_{t+j} - \bar{p}_{t+j}X_{t+j}\right].\end{aligned}$$

To characterize investment expenditures when constraints are non-binding throughout parents' lives, we make a simplifying assumption on the continuation value function $\tilde{U}$.

Assumption 4. *$\tilde{U}(H_m, H_f, A) = \hat{U}(A + D_m H_m + D_f H_f)$ where the constants D_m and D_f are non-negative and $\hat{U}(\cdot)$ is strictly increasing and strictly concave.*

Assumption 4 represents the case where parents at date $T+1$ value their remaining lifetime wealth as defined by current assets plus the discounted present value of all future earnings represented by $D_j H_j$.[18] It is useful to define $\Delta(x) \equiv \hat{U}'(x)$, which is a strictly decreasing function given strict concavity of $\hat{U}(\cdot)$.

Assumptions 1–3 imply Equations (18) and (19), while Assumption 4 implies $A_{T+1} = \Delta^{-1}(\beta^{-1}c_T^{-\sigma}) - DH_m$. We next make the convenient assumption that $\beta(1+r) = 1$. This implies $c_t = c$ for all t, which simplifies expressions that follow without altering any important conclusions. As with the approach in the binding constraint case, we can now substitute these expressions into the lifecycle budget constraint, and collect consumption terms to obtain:

$$\begin{aligned}&\Upsilon_{T-t}c + \sum_{j=0}^{T-t}(1+r)^{-j}\left[\psi_m^{1/\nu}W_{m,t+j}^{(\nu-1)/\nu} + \psi_f^{1/\nu}W_{f,t+j}^{(\nu-1)/\nu}\right]c^{\sigma/\nu} + \bar{K}_t c^{\sigma} + (1+r)^{-(T-t)}\Delta^{-1}\left(\beta^{-1}c^{-\sigma}\right) \\ &= (1+r)A_t + \sum_{j=0}^{T-t}(1+r)^{-j}\left[W_{m,t+j} + W_{f,t+j} + y_{t+j}\right] + (1+r)^{-(T-t)}\left[D_m H_m + D_f H_f\right], \quad (23)\end{aligned}$$

where the constants $\Upsilon_{T-t} \equiv \sum_{j=0}^{T-t}(1+r)^{-j} > 0$ and $\bar{K}_t \equiv \sum_{j=0}^{T-t}(1+r)^{-j}K_{t+j} > 0$. This implicitly defines consumption as a function of current and future wages, non-labor income, parental human capital, period t assets, and other preference/technology parameters. We then use the implicit function theorem to determine how prices, non-labor income, and maternal human capital affect consumption. Because $E_t = K_t c^{\sigma}$ (see Equation (19)), we then characterize total investment expenditures (see Appendix A for details).

[17] Intermediate cases in which constraints are non-binding for a subperiod of parents' remaining lives would be quite similar, with the relevant budget constraint determining period t investment behavior covering period t through the period when constraints first bind.

[18] For example, $D_j = \sum_{k=0}^{T_R-(T+1)}(1+r)^{-k}w_{T+1+k}$, assuming individuals retire at date T_R.

Proposition 5. *Suppose borrowing constraints are non-binding from year t to T. Total investment expenditures, E_t, are strictly increasing in current and future skill prices, current and future non-labor income, and parental human capital. E_t is independent of current and future prices for home goods inputs and child care services.*

As with the case when borrowing constraints bind, total investment expenditures are increasing in current non-labor income, the current skill price and parental human capital, while they are independent of home goods inputs and child care prices. Contrary to the constrained case, the ability to smooth income across periods means that investment expenditures also depend on all future levels of non-labor income and skill prices. Thus, a permanent increase in skill prices will have greater impacts on current investment expenditures than a one-time increase in the price.

Finally, if we assume log preferences for consumption and leisure (i.e., $u(c) = \ln(c)$ and $v_j(l_j) = \psi_j \ln(l_j)$, $\psi_j \geq 0$, for $j \in \{m, f\}$), as well as a log continuation utility (i.e., $\tilde{U}(H_m, H_f, A) = D_0 \ln(A + D_m H_m + D_f H_f)$, with D_0, D_m, and D_f all non-negative), then we obtain a closed form expression for total investment much like Equation (21) of the borrowing constrained case:

$$X_t = \frac{K_t \left[(1+r)A_t + \sum_{j=0}^{T-t} (1+r)^{-j} \left(W_{m,t+j} + W_{f,t+j} + y_{t+j} \right) + (1+r)^{-(T-t)} \left(D_m H_m + D_f H_f \right) \right]}{\bar{p}_t \left[(1+\psi_m+\psi_f)\Upsilon_{T-t} + (1+r)^{-(T-t)} \beta D_0 + \bar{K}_t \right]}.$$

Compared to the constrained case, we see that total investment for unconstrained families depends on the discounted present value of lifetime (rather than current) "full" income as well as the continuation value of parental human capital. Also, note that the denominator reflects discounted lifetime sums of $(1 + \psi_m + \psi_f)$ and K_t rather than only their current values. As a result, a single period change in wages or non-labor income in period t will have much smaller effects on investment that period when constraints are not binding compared to when they bind. This is not surprising, because any change in income is spread across all periods (in terms of investment and consumption) when families are unconstrained.

3.3 Effects of Input Prices and Parental Human Capital on Investment Inputs

It is possible to fully characterize investment behavior analytically if Assumptions 1 and 2 hold, preferences for consumption and leisure are given by $u(c) = \ln(c)$ and $v_j(l_j) = \psi_j \ln(l_j)$, $j \in \{m, f\}$, and $\tilde{U}(H_m, H_f, A) = D_0 \ln(A + D_m H_m + D_f H_f)$.[19] We make these assumptions here to facilitate an analysis of the effects of input price changes on investment input allocations, as well as the relationship

[19] Unlike Del Boca, Flinn, and Wiswall (2014) and several subsequent studies, we do not rely on strong assumptions about the within-period production function $f_t(\cdot)$ nor do we limit ourselves to the case of no borrowing/saving.

between parental human capital and investments in children. To simplify the exposition, we characterize investment behavior for single mothers, assuming the within-period production function of Equation (10).

As above, we consider families that are currently borrowing constrained or that are always unconstrained. Combining the intertemporal solution for X_t in Equation (21) with the intratemporal solutions from Section 3.1 (Equations (7) and (9)), for a constrained family, we obtain:

$$g_t = \left(\frac{(1+r)A_t + y_t - A_{min,t} + W_{m,t}}{p_t + P_{c,t}\Phi_{c,t} + W_{m,t}\Phi_{m,t}} \right) \left(\frac{K_t}{1 + \psi_m + K_t} \right), \tag{24}$$

while we can similarly obtain the following for always unconstrained families:

$$g_t = \left(\frac{(1+r)A_t + \sum_{j=0}^{T-t} (1+r)^{-j} \left[W_{m,t+j} + y_{t+j} \right] + (1+r)^{t-T} D_m H_m}{p_t + P_{c,t}\Phi_{c,t} + W_{m,t}\Phi_{m,t}} \right) \left(\frac{K_t}{(1+\psi_m)\Upsilon_{T-t} + (1+r)^{-(T-t)}\beta D_0 + \bar{K}_t} \right). \tag{25}$$

From g_t, we can recover maternal time investment $\tau_{m,t} = \Phi_{m,t} g_t$ and market child care investment $Y_{c,t} = \Phi_{c,t} g_t$, where $\Phi_{m,t}$ and $\Phi_{c,t}$ are given by Equations (5) and (6), respectively.

Because $\Phi_{m,t}$ and $\Phi_{c,t}$ are independent of non-labor income, it is clear that this income positively affects all investment inputs. We are more interested in the extent to which input prices and parental education affect different types of investments in children, much as we studied their impacts on input expenditure shares in Section 3.1.2. The additional assumptions on preferences make it feasible to account for the effects of price changes on the levels of investment in our fully dynamic framework. Note that current prices enter the two expressions for g_t, Equations (24) and (25), in the same manner. Mother's skill level and skill prices impact input levels via current and all future wages when a family is unconstrained, while they only affect input levels through current wages when a family is constrained.

3.3.1 Input Price Effects on Investment Inputs

To emphasize the roles played by the intratemporal and intertemporal problems and to link to previous results, it is useful to decompose the optimal quantity of an input into three parts: its expenditure share, total expenditures, and its own price. Consider, for example, the identity for home goods inputs:

$$g_t = \frac{E_t S_{g,t}}{p_t}.$$

The expenditure share is determined in the intratemporal problem, with its response to changes in input prices and maternal skills described in Section 3.1.2. The level of expenditure comes out of the intertemporal problem, and its dependence on input prices and maternal skills is characterized in Propositions 4 and 5. Lastly, for each input amount, there is a direct negative impact of its own-price change.

Total expenditures are invariant to the prices of home goods and market child care services (see Propositions 4 and 5). As such, changes in these prices have exactly the same impact on home investment time as they do on home investment time shares (in percentage terms). This is also the case for cross-price effects of changes in p on Y_c and changes in P_c on g. For the impacts of changes in p on g or P_c on Y_c, we also need to account for the negative effects of own-price changes. A key difference with our expenditure share results is that the quantities of purchased home investment goods and market child services always fall with an own-price increase, while their shares increase if there is complementarity across inputs (see Propositions 1 and 2).

Increases in skill prices and parental human capital not only affect expenditure shares, but they also raise total investment expenditures (see Propositions 4 and 5). Consequently, an increase in skill prices or parental skills may increase home goods investments or child care services even if their expenditure shares decline. Additionally, the value of parental time devoted to child investment increases more (in percentage terms) than does its expenditure share; however, the amount of time parents spend with their children may decline. The following proposition discusses the role of input substitutability in determining investment input responses to changes in the skill price. Like the propositions that follow, it applies equally to the case of borrowing constrained and always unconstrained families.[20]

Proposition 6. *(A) If* $\min\{\gamma, \rho\} > 0$*, then parental time investment is strictly decreasing in* $w_{m,t}$*. (B) If* $\rho > \max\{0, \gamma\}$*, then home goods inputs are strictly increasing in* $w_{m,t}$*. (C) If* $\gamma > 0$*, then market child care is strictly increasing in* $w_{m,t}$*.*

Proposition 6 considers the case of a change in the current skill price, $w_{m,t}$.[21] Recall that a rise in the current skill price increases both family income and the price of time, where the former leads to greater total investment expenditures (Propositions 4 and 5). Despite this increase, the higher price of time leads to a reduction in parental time investment when all inputs are substitutes. If parental time and goods investments are more substitutable than home inputs and market child care, then the reduction in time investment is compensated for with an increase in home goods inputs, while child care services also increase as long as they are substitutes with home inputs.

[20] This proposition and the two that follow require that current family debts ($-A_t$) are not too large. In the constrained case, the required conditions are always satisfied if borrowing constraints are not growing in discounted present value (i.e., $A_{t-1,min} \geq (1+r)^{-1} A_{t,min}$). In the unconstrained case, the conditions are always satisfied if the current value of debts does not exceed the discounted present value of all future non-labor income (and spousal earnings for two-parent households); however, several results rely on much weaker conditions. For example, Proposition 6 only requires that the value of current debt not exceed the discounted present value of all future family wages and non-labor income, as well as the value of human capital after period T. See Appendix A for details.

[21] A permanent change in the skill price has the same qualitative impacts on inputs as does a change in only the current skill price.

Complementarity can lead to a potentially surprising response by families. Specifically, it can cause families to increase parental time investments even when the opportunity cost of time (i.e., the skill price) rises. Intuitively, the increase in family income associated with higher wages can spur families to increase total investments, and if investments are sufficiently complementary, families will want to increase all investment inputs, including parental time. Indeed, simulations based on the elasticities (and other parameters) we estimate below suggest that this is the case (for borrowing constrained families, at least).

3.3.2 Effects of Mother's Human Capital on Investment Inputs

Finally, we study the effects of maternal human capital on child investment input decisions, continuing to assume $a_m(H_m) = \bar{a}_m[\varphi_m(H_m)]^\rho$ and $a_g(H_m) = \bar{a}_g[\varphi_g(H_m)]^\rho$ with $\varphi_m(H_m)$ and $\varphi_g(H_m)$ defined in Equations (11) and (12).

The relationship between both home goods and time investments and mother's human capital depends quite generally on $\bar{\varphi}_m$ and $\bar{\varphi}_g$ as well as the substitutability of different inputs. These conditions are difficult to interpret on their own, so we discuss two special cases that provide greater intuition. (See Appendix A for the general case.)

Begin by assuming that a mother's human capital does not impact the productivity of home goods inputs. Whether time invested in children rises or falls with mother's human capital depends on the substitutability of investments and the returns to her skill associated with time investment in child development as described in the next proposition.

Proposition 7. *Suppose $\bar{\varphi}_g = 0$. Home goods inputs, g, are strictly increasing in H_m and maternal time investment, τ_m, is strictly decreasing in H_m if any of the following conditions are met: (i) $\bar{\varphi}_m < 1$ and $\rho \geq \gamma \geq 0$, (ii) $\bar{\varphi}_m = 1$, or (iii) $\bar{\varphi}_m > 1$ and $\rho \leq \gamma \leq 0$.*

Compared to what we see with expenditure shares, there is an additional positive effect of maternal human capital on input levels from the increase in family income. Of course, the own-price effect tends to reduce the amount of time investment even though its expenditure share may rise. When maternal skills are equally productive in child-rearing and the labor market (i.e., $\varphi_m(\cdot)$ is CRS), an increase in maternal human capital causes families to substitute more goods investments for less time investments, leaving expenditure shares unchanged (see Section 3.1.2). We observe a qualitatively similar shift from parental time to home goods inputs when $\varphi_m(\cdot)$ exhibits increasing (decreasing) returns to scale and inputs are substitutable (complementary) with stronger substitutability (complementarity) between parental time and home goods inputs than between the composite home input and market child care.

Next, consider $\bar{\varphi}_g > 0$, so the productivity of home goods investment is increasing in maternal human capital. Recall from Section 3.1.2 that the increase in marginal productivity encourages more skilled mothers to shift their investment portfolio towards home goods if inputs are sufficiently substitutable; otherwise, the factor-augmenting nature of H_m can cause them to turn more to other inputs. To focus on the productivity effects of maternal human capital on home goods investment, we consider the case of $\bar{\varphi}_m = 1$, which implies equal productivity of H_m at home and in the labor market.

Proposition 8. *Suppose $\bar{\varphi}_m = 1$ (φ_m is CRS) and $\bar{\varphi}_g > 0$. If $\rho \geq \gamma \geq 0$, then home goods investment is strictly increasing in H_m and parental time investment is strictly decreasing in H_m.*

Comparing this result to Proposition 7, we see that a positive effect of H_m on the productivity of goods inputs tends to dampen the substitution we see from parental time investment towards home goods investments: with $\bar{\varphi}_g > 0$, we can only be assured of this substitution if $\rho \geq \gamma \geq 0$ even when $\varphi_m(\cdot)$ is CRS. This same condition ensures an analogous shift in expenditure shares (see Proposition 3), with the marginal productivity effect dominating the factor-augmenting effect under sufficient substitutability.

Throughout this section, we have analytically characterized the impacts of both input prices and parental human capital on family investments in children, emphasizing empirically quantifiable relationships for specific input amounts, input expenditure shares, and total investment expenditures. These relationships depend critically on the substitutability of different inputs and on the role of parental human capital in the production of child skills. We next turn to estimation strategies aimed at identifying these characteristics of the production process from rich data on investment choices, parental skills and other factors that might affect the productivity of investments, and measures of child skills.

4 Estimation Approach

Our empirical analysis adopts a revealed preference approach that exploits relative demand for inputs to estimate the within-period production function $f_t(\cdot)$ described in Section 3.1. An important advantage of this approach is that it requires no assumptions about the dynamics of skill production (as given by $\mathcal{H}_t(\cdot)$) nor about credit markets. An implicit assumption is that families are knowledgeable about the within-period skill production process. More generally, this approach identifies individual beliefs about the skill production function, which is important for understanding how families might react to different policies.

Molnar (2020) and Moschini (2020) also use this revealed preference approach to estimate intratemporal features of a more limited production technology with a single unknown elasticity of substitution between inputs. Our richer child production technology with three distinct types of inputs and flexible

substitution patterns across those inputs introduces additional challenges when inputs are measured with error. We address this measurement error, as well as measurement error in wages, and employ a few different approaches to address unobserved heterogeneity in parenting skills and selection into work.[22]

After estimating the within-period skill production process $f_t(\cdot)$ using relative demands, we then impose additional structure on the intertemporal production process, combined with assumptions about credit markets, to estimate both $f_t(\cdot)$ and $\mathcal{H}_t(\cdot)$ simultaneously.

4.1 Within-Period Production Function, $f_t(\cdot)$

We begin by describing our relative demand approach for estimating $f_t(\cdot)$. Let $Z_{i,t}$ reflect a set of observed household characteristics for child i at date t, including parental characteristics (e.g., marital status, education, age, race), child characteristics (e.g. age), and other household demographic factors (e.g. number of children in the household). We also consider unobserved heterogeneity in the productivity of parent's time with children, $\eta_{m,i}$ and $\eta_{f,i}$.

We estimate the following nested CES within-period production function:

$$f(\tau_{m,i}, \tau_{f,i}, g_i, Y_i | Z_{i,t}) = \left[\left(a_m(Z_{i,t}, \eta_{m,i}) \tau_{m,i,t}^{\rho} + a_f(Z_{i,t}, \eta_{f,i}) \tau_{f,i,t}^{\rho} + a_g(Z_{i,t}) g_{i,t}^{\rho} \right)^{\frac{\gamma}{\rho}} + a_{Y_c}(Z_{i,t}) Y_{c,i,t}^{\gamma} \right]^{\frac{1}{\gamma}},$$

assuming $a_j(Z, \eta_j) = \exp(Z\phi_j + \eta_j)$ for $j = m, f$ and $a_g(Z) = \exp(Z\phi_g)$ (allowing ϕ_m and ϕ_g to differ for single and married mothers).[23] We assume $a_f(Z) = 0$ (and exclude father characteristics from $Z_{i,t}$) for single mother households, because we do not generally observe much, if anything, about fathers in these cases. Additionally, we impose $\phi_j = 0$ for coefficients on some characteristics (e.g. one parent's age or education does not affect the productivity of the other parent's time). Finally, a normalization is required on the share constants (a_m, a_f, a_{Y_c}), because the scale of $f(\cdot)$ is not pinned down. In estimation, we normalize $a_{Y_c}(Z) = 1$ as discussed further below.[24]

To link our assumptions on $a_m(Z, \eta_m)$ and $a_f(Z, \eta_f)$ to our theoretical analysis of Section 3, suppose human capital for parent $j \in \{m, f\}$ is given by $H_{j,i,t} = \exp(Z_{i,t}\Gamma_j + \lambda^{-1}\eta_{j,i})$. This implies $\ln(W_{j,i,t}) =$

[22] Moschini (2020) takes a time fixed effects approach to address unobserved heterogeneity (and selection into work among parents). To address endogeneity concerns, Molnar (2020) exploits the introduction of a universal child care subsidy in Quebec as an instrument for relative price changes. In related work, Abbott (2020) specifies a similar relative demand function (for his two inputs, time and goods), but he addresses unobserved heterogeneity through estimation of a full dynamic lifecycle model. Unfortunately, this forfeits an important strength of the revealed preference approach, which does not require assumptions about preferences or dynamic features of the environment.

[23] One could also allow production parameters ρ and γ to vary with time or household characteristics (e.g. child age); however, we refrain from this given the PSID-CDS sample sizes and limited child age range used in our analysis.

[24] This normalization is natural, because most household characteristics (e.g. parental age or education, number of children in the household) are unlikely to directly effect the productivity of market child care services, while they are more likely to directly influence the productivity of household inputs. Regardless, with a full child production function $\mathcal{H}_t(f_t(\cdot), \theta, \Psi_t)$ that is multiplicatively separable in θ and $f_t(\cdot)$, as we will assume, the normalization $a_{Y_c}(Z) = 1$ means that any factors affecting the productivity of child care services will come through θ.

$\ln(w_{j,i,t}) + Z_{i,t}\Gamma_j + \lambda^{-1}\eta_{j,i}$, where $w_{j,i,t}$ is the price of skill in parent (j,i)'s labor market. For $\phi_j/\Gamma_j = \lambda = \rho\bar{\varphi}_j$, we can write $a_j(Z,\eta_j) = \bar{a}(\varphi_j(H_j))^\rho = H_j^{\rho\bar{\varphi}_j}$, consistent with our theoretical analysis. Thus, the importance of parental education for child production scaled by ρ (i.e., ϕ_j/ρ) relative to log wages (Γ_j) is the empirical counterpart to the "returns to scale" parameter $\bar{\varphi}_j$ central to several results in Section 3.

Our empirical analysis recognizes that investment inputs, as well as parental wage rates, may be measured with error. We use an o superscript to reflect observed measures of these variables, assuming $\ln(x^o_{i,t}) = \ln(x_{i,t}) + \xi_{x,i,t}$ for $x \in \{\tau_m, \tau_f, g, Y_c, W_m, W_f\}$. We assume that all idiosyncratic measurement errors are mean zero and independent of all "true" variables (inputs, prices, as well as $Z_{i,t}$ characteristics), unobserved heterogeneity $(\eta_{m,i}, \eta_{f,i})$, and other measurement errors.

Next, define wages and child care prices relative to the price of investment goods: $\tilde{W}_{j,i,t} \equiv W_{j,i,t}/p_{i,t}$, $\tilde{W}^o_{j,i,t} \equiv W^o_{j,i,t}/p_{i,t}$, $\tilde{P}_{i,t} \equiv P_{c,i,t}/p_{i,t}$. It is also convenient to define the ratio of observed expenditures on parental time and child care relative to observed expenditures on household goods:

$$R_{j,i,t} \equiv \frac{W^o_{j,i,t}\tau^o_{j,i,t}}{p_{i,t}g^o_{i,t}}, \quad \text{for } j \in \{m,f\}, \qquad \text{and} \qquad R_{Y_c,i,t} \equiv \frac{P_{c,i,t}Y^o_{c,i,t}}{p_{i,t}g^o_{i,t}}.$$

4.1.1 Relative demand for parental time vs. household goods

Based on Equation (5), relative demand for parental time vs. household goods (for working parents) is given by

$$\ln\left(\frac{\tau_{j,i,t}}{g_{i,t}}\right) = \left(\frac{1}{1-\rho}\right)\ln\left(\frac{a_j(Z_{i,t},\eta_{j,i})}{a_g(Z_{i,t})}\right) + \left(\frac{1}{\rho-1}\right)\ln\tilde{W}_{j,i,t}, \quad j = \{m,f\}.$$

Substituting in our assumptions for $a_j(\cdot)$ and $a_g(\cdot)$, incorporating measurement error, and adding $\ln\tilde{W}^o_{j,i,t}$ to both sides implies the following estimating equation for relative time vs. goods expenditures:

$$\ln(R_{j,i,t}) = Z'_{i,t}\tilde{\phi}_{jg} + \left(\frac{\rho}{\rho-1}\right)\ln\tilde{W}^o_{j,i,t} + \tilde{\eta}_{j,i} + \xi_{\tau_j/g,i,t} + \tilde{\xi}_{W_j,i,t}, \tag{26}$$

where $\tilde{\phi}_{jg} \equiv \left(\frac{1}{1-\rho}\right)(\phi_j - \phi_g)$, $\tilde{\eta}_{j,i} \equiv \left(\frac{1}{1-\rho}\right)\eta_{j,i}$, $\xi_{\tau_j/g,i,t} \equiv \xi_{\tau_j,i,t} - \xi_{g,i,t}$, and $\tilde{\xi}_{W_j,i,t} \equiv \left(\frac{1}{1-\rho}\right)\xi_{W_j,i,t}$. This shows how relative time vs. goods expenditures depend on their relative prices, as well as characteristics that affect their relative productivity. Because $\rho < 1$, household characteristics that raise the productivity of time relative to goods inputs (i.e., $Z_{i,t}$ for which $\phi_j > \phi_g$) will lead to greater relative time investment expenditures, where the effect also depends on the elasticity of substitution between time and goods. Note that this elasticity can be obtained easily from the coefficient on log relative wages, because $\epsilon_{\tau,g} = 1 - \left(\frac{\rho}{\rho-1}\right)$.

Three potential econometric challenges arise in estimation of Equation (26). First, unobserved differences in parenting skills $\eta_{j,i}$ may be correlated with wages $W_{j,i,t}$.[25] This would be the case if skills valued in the labor market are also productive in child-rearing as discussed above. Second, measurement error in wages is correlated with observed wages. OLS estimation of Equation (26) for any period t will produce estimates of $\left(\frac{\rho}{\rho-1}\right)$ with an asymptotic bias of $\left(\frac{1}{1-\rho}\right)\left(\frac{Cov(\eta_{j,i},\ln(\tilde{W}_{j,i,t})|Z_{i,t})-Var(\xi_{W_j,i,t}|Z_{i,t})}{Var(\ln(\tilde{W}^o_{j,i,t})|Z_{i,t})}\right)$. Notice that measurement error in log wages does not necessarily produce the standard attenuation bias towards zero, because we use relative expenditures, which are functions of potentially mismeasured wages, as our dependent variable.[26] Instead, measurement error produces a negative OLS bias for $\rho/(\rho-1)$ (upward bias for $\epsilon_{\tau,g}$) while a positive correlation between market and child skill production produces an opposing bias. A third challenge also arises due to unobserved heterogeneity in $\eta_{j,i}$ due to selection into work, because we do not observe wages for those who do not work at all during the year.

The first two estimation concerns can be addressed using standard instrumental variables techniques.[27] Below, we use state of residence indicators as instruments, assuming that unobserved differences in parental child production abilities are the same across states (conditional on other observed factors like parental age, education, and race). From consistent estimation of Equation (26), we can obtain estimates of ρ (and elasticity $\epsilon_{\tau,g}$) as well as $(\phi_j - \phi_g)$ for $j \in \{m, f\}$.

Concerns about selection into work arise if unobserved parental child-rearing skills are correlated with labor supply decisions. To address this issue, one could consider a control function approach (Heckman and Robb Jr, 1985), modeling the expected value of $\eta_{i,j}$ conditional on log relative wages, $Z_{i,t}$ characteristics, and other exogenous factors that impact labor supply behavior. Based on our model, these additional excluded variables could include factors like state of residence (determining the price of skill in the market), family assets or non-wage income, or factors affecting the child productivity parameter θ. All would potentially affect labor supply, $h_{j,i,t}$, but are excluded from the relative demand for parental time vs. household goods inputs. Unfortunately, without strong assumptions, it is difficult to derive a simple single index equation that would make a propensity score approach practical, especially if there are any additional unobserved factors affecting θ or preferences for leisure. We, instead, address selection concerns in three ways. First, we estimate Equation (26) conditioning on parents with a high predicted probability of work as described further below. As this predicted probability approaches one, such esti-

[25] We assume throughout that unobserved parenting skills, $\eta_{i,j}$ are independent of household goods and child care prices $(p_{i,t}, P_{c,i,t})$.

[26] Measurement error would lead to attenuation bias if we regressed log relative inputs $\ln(\tau^o_{j,i,t}/g^o_{i,t})$ rather than log relative expenditures $R_{j,i,t}$ on observed log relative wages.

[27] With at least two periods of data, a time fixed effects strategy can be used to address unobserved heterogeneity $\eta_{j,i}$ as in Moschini (2020); however, this would likely exacerbate concerns about measurement error in wages. Because we estimate Equation (26) using a single period of data, we do not discuss fixed effects strategies further.

mates should be consistent. Second, we use the panel nature of our data to estimate log wage fixed effects for each individual parent. This provides an estimate of unobserved parental skills, which we include in our set of observed factors affecting relative demand. Third, our estimation of relative demand for child care vs. household goods inputs, discussed next, is not confounded by selection into work. This provides an additional set of estimates for ρ that can be compared against those from Equation (26).

4.1.2 Relative demand for child care vs. household goods

Based on Equation (6), relative demand for child care vs. household goods implies the following for single mothers:

$$\ln(R_{Y_c,i,t}) = \left(\frac{1}{1-\gamma}\right)\ln a_{Y_c}(Z_{i,t}) + Z'_{i,t}\tilde{\phi}_g + \left[\frac{\gamma-\rho}{\rho(\gamma-1)}\right]\ln\left(1+R_{m,i,t}e^{-\xi_{W_m\tau_m/g,i,t}}\right) + \left(\frac{\gamma}{\gamma-1}\right)\ln\tilde{P}_{c,i,t} + \xi_{Y_c/g,i,t}, \tag{27}$$

where $\xi_{\tau_m W_m/g,i,t} \equiv \xi_{\tau_m,i,t} + \xi_{W_m,i,t} - \xi_{g,i,t}$, and $\xi_{Y_c/g,i,t} \equiv \xi_{Y_c,i,t} - \xi_{g,i,t}$, and

$$\tilde{\phi}_g \equiv \underbrace{\left[\frac{\gamma}{\rho(\gamma-1)}\right]}_{<0 \text{ if } \max\{\rho,\gamma\}<0} \phi_g.$$

As noted above, we normalize $a_{Y_c}(Z) = 1$. This implies that when both γ and ρ are negative (as our estimates below suggest), family characteristics that raise the productivity of household goods inputs will lead to reductions in expenditures on child care relative to household goods, because $\tilde{\phi}_g < 0$.[28] The elasticity of substitution between the composite home input and market child care can be obtained from the coefficient on log relative child care prices, because $\epsilon_{Y,H} = 1 - \left(\frac{\gamma}{\gamma-1}\right)$. In the rest of this subsection, we set $a_{Y_c} = 1$ and drop t subscripts to simplify expressions.

In the absence of measurement error in $(W_{m,i}\tau_{m,i}/g_i)$, $\xi_{W_m\tau_m/g,i} = 0$ and Equation (27) becomes

$$\ln(R_{Y_c,i}) = Z'_i\tilde{\phi}_g + \left[\frac{\gamma-\rho}{\rho(\gamma-1)}\right]\ln(1+R_{m,i}) + \left(\frac{\gamma}{\gamma-1}\right)\ln\tilde{P}_{c,i} + \xi_{Y_c/g,i}, \tag{28}$$

which can be estimated via OLS.

A two-step estimation approach that accounts for measurement error in all child investment inputs is possible if (i) wages are not measured with error (i.e., $\xi_{W_m,i} = 0$) and (ii) there is no unobserved

[28] While it is natural to think that parental characteristics and household demographic factors do not directly affect the productivity of child care services, it is possible that child care productivity depends on child characteristics like age. Given our specification for $\mathcal{H}_t(f_t(\cdot),\theta,\Psi_t)$ is multiplicative in θ, normalizing $a_{Y_c}(Z)$ to a constant means that these productivity effects would appear in θ, offset by adjustments in ϕ_j and ϕ_g. Alternatively, we could normalize θ to a constant invariant to Z, allowing $a_{Y_c}(Z) = Z'\phi_{Y_c}$. In this case, Equation (27) would identify $\tilde{\phi}_g = \frac{1}{1-\gamma}\left(\phi_{Y_c} - \frac{\gamma}{\rho}\phi_g\right)$, so the estimated effects of characteristics like child's age on relative demand for child care services vs. household goods identify the effects of child's age on the productivity of child care relative to goods inputs (the latter scaled by γ/ρ).

heterogeneity in maternal child production ability (i.e., $\eta_{m,i} = 0$). Under these assumptions, we obtain a similar specification:

$$\ln(R_{Y_c,i}) = Z_i'\tilde{\phi}_g + \left[\frac{\gamma-\rho}{\rho(\gamma-1)}\right]\ln\left(1+e^{\ln(\tilde{\Phi}_{m,i})}\right) + \left(\frac{\gamma}{\gamma-1}\right)\ln\tilde{P}_{c,i} + \xi_{Y_c/g,i}, \tag{29}$$

where we define $\tilde{\Phi}_{j,i} \equiv \frac{W_{j,i}\tau_{j,i}}{p_i g_i}$ generally for both mothers and fathers $j = m, f$. Absent measurement error in wages and unobserved heterogeneity in maternal child productivity, the predicted values from OLS estimation of Equation (26), $\widehat{\ln(R_{m,i})}$, provide consistent estimates of $\ln(\tilde{\Phi}_{m,i}) = Z_i'\tilde{\phi}_{mg} + \left(\frac{\rho}{\rho-1}\right)\ln\tilde{W}_{m,i}$. Thus, we can substitute these predicted values in for $\ln(\tilde{\Phi}_{m,i})$ in Equation (29) and estimate it using OLS.

Consistent estimates of γ, ρ, and ϕ_g can be obtained from estimation of Equations (28) or (29) under the stated assumptions. Combining these estimates from those of Equation (26), estimates of ϕ_m can also be obtained.

Measurement error in wages as well as inputs complicates estimation. However, taking expectations of Equation (27) conditional on observed data produces

$$\begin{aligned} &E\left[\ln(R_{Y_c,i})\middle| Z_i, R_{m,i}, \tilde{P}_{c,i}, g_i^o\right] \\ &= Z_i'\tilde{\phi}_g + \left[\tfrac{\gamma-\rho}{\rho(\gamma-1)}\right] E\left[\ln\left(1+R_{m,i}e^{-\xi_{W_m\tau_m/g,i}}\right)\middle| R_{m,i}\right] + \left(\tfrac{\gamma}{\gamma-1}\right)\ln\tilde{P}_{c,i} - E[\xi_{g,i}|g_i^o]. \end{aligned}$$

If the distribution of measurement error in $(W_{m,i}, \tau_{m,i}, g_i)$ is fully known, we can simply calculate the expectations on the right hand side of the expression and use GMM to estimate (γ, ρ, ϕ_g). In some cases, we only need to know the type of distribution for measurement errors, not all parameters of the distributions. For example, if $\xi_{g,i}$ and $\ln(g_i)$ are both normally distributed (and independent), then

$$E[\xi_{g,i}|g_i^o] = Var(\xi_{g,i})\left(\frac{\ln(g_i^o) - E[\ln(g_i^o)]}{Var(\ln(g_i^o))}\right)$$

and

$$\begin{aligned} &E\left[\ln(R_{Y_c,i})\middle| Z_i, R_{m,i}, \tilde{P}_{c,i}, g_i^o\right] \\ &= Z_i'\tilde{\phi}_g + \left[\tfrac{\gamma-\rho}{\rho(\gamma-1)}\right] E\left[\ln\left(1+R_{m,i}e^{-\xi_{W_m\tau_m/g,i}}\right)\middle| R_{m,i}\right] + \left(\tfrac{\gamma}{\gamma-1}\right)\ln\tilde{P}_{c,i} + \lambda\Big(\ln g_i^o - E[\ln g_i^o]\Big), \end{aligned} \tag{30}$$

where $\lambda \equiv -Var(\xi_{g,i})/Var(\ln g_i^o)$. Further assuming that both $\xi_{W_m,i}$ and $\xi_{\tau_m,i}$ are also normally distributed implies that $\xi_{W_m\tau_m/g,i} \sim N\left(0, \sigma^2_{W_m\tau_m/g}\right)$. In this case, we could integrate over this measurement error to calculate the expectation term on the right hand side of Equation (30) as a function of $\left(R_{m,i}, \sigma^2_{W_m\tau_m/g}\right)$.[29] Equation (30) could then be estimated using GMM where λ and $\sigma^2_{W_m\tau_m/g}$ must be estimated along with (γ, ρ, ϕ_g).

[29] In general, $\xi_{W_m,i}$ and $\xi_{\tau_m,i}$ need not be normally distributed. Knowledge of their distributions should be sufficient to calculate the expectation term.

Unfortunately, $E\left[\ln\left(1+R_{m,i}e^{-\xi_{W_m\tau_m/g,i}}\right)\middle|R_{m,i}\right]$ in Equation (30) does not have a closed form expression. Using a second order Taylor approximation to integrate over measurement error produces

$$E\left[\ln(R_{Y_c,i})\middle|Z_i,R_{m,i},\tilde{P}_{c,i},g_i^o\right] \approx Z_i'\tilde{\phi}_g+\left(\frac{\gamma-\rho}{\rho(\gamma-1)}\right)\ln\left(1+R_{m,i}\right)+\sigma^2_{W_m\tau_m/g}\left(\frac{\gamma-\rho}{\rho(\gamma-1)}\right)\left(\frac{R_{m,i}}{2(1+R_{m,i})^2}\right)$$
$$+\left(\frac{\gamma}{\gamma-1}\right)\ln(\tilde{P}_{c,i})+\lambda\left(\ln(g_i^o)-E[\ln(g_i^o)]\right), \quad (31)$$

where $\sigma^2_{W_m\tau_m/g}\equiv Var(\xi_{\tau_m W_m/g,i})$. While this expression is only an approximation, it does not require any knowledge of the distribution for $(\xi_{W_m,i},\xi_{\tau_m,i})$. A GMM approach can be applied to Equation (31) to estimate technology parameters (γ,ρ,ϕ_g) as well as $(\sigma^2_{W_m\tau_m/g},\lambda)$. Notice that OLS regression of $\ln(R_{Y_c,i})$ on Z_i, $\ln\left(1+R_{m,i}\right)$, $\left(\frac{R_{m,i}}{2(1+R_{m,i})^2}\right)$, $\ln(\tilde{P}_{c,i,t})$, and $(\ln(g_i^o)-E[\ln(g_i^o)])$ can also be used to obtain consistent estimates. It is noteworthy that $\sigma^2_{W_m\tau_m/g}$ is only identified when $\gamma\neq\rho$. For very similar γ and ρ, we would expect imprecise estimates of this variance in practice.

An analogous set of results applies for two-parent households; however, the estimating equations are slightly more complicated due to the roles of both father's and mother's time inputs. Continuing to normalize $a_{Y_c}=1$, relative demand for child care vs. goods in two-parent families implies

$$\ln R_{Y_c,i}=Z_i'\tilde{\phi}_g+\left[\frac{\gamma-\rho}{\rho(\gamma-1)}\right]\ln\left(1+R_{f,i}e^{-\xi_{W_f\tau_f/g,i}}+R_{m,i}e^{-\xi_{W_m\tau_m/g,i}}\right)+\left(\frac{\gamma}{\gamma-1}\right)\ln\tilde{P}_{c,i}+\xi_{Y_c/g,i}, \quad (32)$$

where $\xi_{\tau_f W_f/g,i}\equiv\xi_{\tau_f,i}+\xi_{W_f,i}-\xi_{g,i}$ and other variables are defined earlier.

No measurement error in $(W_{j,i}\tau_{j,i}/g_i)$ for $j\in\{m,f\}$ implies

$$\ln(R_{Y_c,i})=Z_i'\tilde{\phi}_g+\left[\frac{\gamma-\rho}{\rho(\gamma-1)}\right]\ln\left(1+R_{f,i}+R_{m,i}\right)+\left(\frac{\gamma}{\gamma-1}\right)\ln\tilde{P}_{c,i}+\xi_{Y_c/g,i}, \quad (33)$$

which can be estimated via OLS.

Alternatively, incorporating measurement error in all child investment inputs but assuming (i) wages for both parents are measured without error (i.e., $\xi_{W_m,i}=\xi_{W_f,i}=0$) and (ii) no unobserved heterogeneity in either parent's child production ability (i.e., $\eta_{m,i}=\eta_{f,i}=0$) yields the following:

$$\ln(R_{Y_c,i})=Z_i'\tilde{\phi}_g+\left[\frac{\gamma-\rho}{\rho(\gamma-1)}\right]\ln\left(1+e^{\ln(\tilde{\Phi}_{fi})}+e^{\ln(\tilde{\Phi}_{m,i})}\right)+\left(\frac{\gamma}{\gamma-1}\right)\ln\tilde{P}_{c,i}+\xi_{Y_c/g,i}. \quad (34)$$

As with single mothers, the stated assumptions enable a two-step approach for estimating Equation (34), using predicted values from OLS estimation of Equation (26) for both fathers and mothers, $\widehat{\ln(R_{j,i})}$, in place of $\ln(\tilde{\Phi}_{j,i})$ for $j\in\{m,f\}$.

As with single mothers, we can address measurement error in wages and inputs, as well as unobserved heterogeneity in maternal and paternal child productivity, by taking expectations of Equation

(32) conditional on observed data:

$$E\left[\ln R_{Y_c,i}\middle| Z_i, R_{f,i}, R_{m,i}, \tilde{P}_{c,i}, g_i^o\right]$$
$$= Z_i'\tilde{\phi}_g + \left[\tfrac{\gamma-\rho}{\rho(\gamma-1)}\right] E\left[\ln\left(1 + R_{f,i}e^{-\xi_{W_f\tau_f/g,i}} + R_{m,i}e^{-\xi_{W_m\tau_m/g,i}}\right)\middle| R_{f,i}, R_{m,i}\right] + \left(\tfrac{\gamma}{\gamma-1}\right)\ln\tilde{P}_{c,i} - E[\xi_{g,i}|g_i^o].$$

Knowledge of measurement error distributions would allow for direct calculation of the conditional expectation terms on the right hand side. Alternatively, a second order Taylor approximation to integrate over measurement error and $\xi_{g,i} \sim N(0, \sigma_g^2)$ yields:

$$\begin{aligned} E\left[\ln R_{Y_c,i}\middle| Z_i, R_{f,i}, R_{m,i}, \tilde{P}_{c,i}, g_i^o\right] &\approx Z_i'\tilde{\phi}_g + \left[\frac{\gamma-\rho}{\rho(\gamma-1)}\right]\ln\left(1 + R_{f,i} + R_{m,i}\right) \\ &+ \sigma^2_{W_f\tau_f}\left[\tfrac{\gamma-\rho}{\rho(\gamma-1)}\right]\left(\tfrac{R_{f,i}(1+R_{m,i})}{2(1+R_{f,i}+R_{m,i})^2}\right) + \sigma^2_{W_m\tau_m}\left[\tfrac{\gamma-\rho}{\rho(\gamma-1)}\right]\left(\tfrac{R_{m,i}(1+R_{f,i})}{2(1+R_{f,i}+R_{m,i})^2}\right) \\ &+ \sigma_g^2\left[\tfrac{\gamma-\rho}{\rho(\gamma-1)}\right]\left(\frac{R_{f,i}+R_{m,i}}{2\left(1+R_{f,i}+R_{m,i}\right)^2}\right) - \sigma_g^2\left(\tfrac{\ln(g_i)^o - E[\ln(g_i^o)]}{Var(\ln(g_i^o))}\right) + \left(\tfrac{\gamma}{\gamma-1}\right)\ln(\tilde{P}_{c,i}), \end{aligned} \tag{35}$$

where $\sigma^2_{W_j\tau_j} \equiv Var(\xi_{W_j} + \xi_{\tau_j})$ for $j \in \{m, f\}$.[30] Based on this moment condition, GMM can be used to efficiently estimate the technology parameters (γ, ρ, ϕ_g) and measurement error variances $\left(\sigma^2_{W_m\tau_m/g}, \sigma^2_{W_m\tau_m/g}, \sigma_g^2\right)$. OLS can also be used; however, there may be some efficiency loss by not imposing parameter restrictions across terms.[31]

It is important to note that unobserved parenting skill $\eta_{i,j}$ does not appear in any of the estimating equations for child care vs. household goods relative demand. As a consequence, these estimates are not subject to concerns about unobserved heterogeneity or parental selection into work. Several specifications, therefore, provide a set of consistent estimates for ρ that can be compared against those obtained from estimation of relative demand for parental time vs. household goods (i.e., Equation (26)).

4.2 Intertemporal Skill Production Function, $\mathcal{H}_t(\cdot)$

To jointly estimate all parameters of the child production function (i.e., those of both $f_t(\cdot)$ and $\mathcal{H}_t(\cdot)$), we combine moment conditions implied by intratemporal optimality, intertemporal optimality, and the intertemporal relationship between investment inputs and child skill accumulation. The additional use of intertemporal relationships is necessary for estimation of dynamic productivity parameters in $\mathcal{H}_t(\cdot)$. Given our data, this requires additional assumptions on preferences and the structure of $\mathcal{H}_t(\cdot)$ (i.e., Assumptions 1–3), as well as assumptions about credit markets. These additional moments not only allow for identification of parameters defined in $\mathcal{H}_t(\cdot)$, but they also provide additional information about parameters in $f_t(\cdot)$, which can improve efficiency. For this analysis, we consider measurement error only

[30] As with the case for single mothers, these time expenditure measurement error variances are only identified when $\gamma \neq \rho$.

[31] Specifically, OLS would produce two separate estimates of σ_g^2, while GMM would take advantage of the cross-term restrictions on parameters.

in inputs (not wages) and abstract from unobserved heterogeneity in factor shares. Thus, our expressions for factor shares reduce to $a_j(Z_{i,t}) = \exp(Z_{i,t}\phi_{j,it}),\ j \in \{m, f, g\}$, while we continue to normalize $a_{Y_c} = 1$.

4.2.1 Intratemporal Moments

As before, we use the intratemporal conditions based on Equations (5) and (6) to express the ratio of any two observed inputs x_1 and x_2, given prices $\Pi_{i,t}$ and parental marital status, $M_{i,t} \in \{0, 1\}$:

$$\ln\left(\frac{x_{1,i,t}}{x_{2,i,t}}\right) = \ln\left(\Phi_{x_1,x_2}(\Pi_{i,t}, M_{i,t})\right) + \xi_{x_1,i,t} - \xi_{x_2,i,t}, \qquad x_1, x_2 \in \{\tau_m^o, \tau_f^o, Y_c^o, g^o\}.$$

Assuming measurement error is independently distributed across individuals, we define the first set of moments as:

$$E\left(\left[\ln\left(\frac{x_{1,i,t}}{x_{2,i,t}}\right) - \ln(\Phi_{x_1,x_2}(\Pi_{i,t}, M_{i,t}))\right] Z_{\Phi,x_1,x_2,i,t}\right) = 0, \tag{36}$$

where the pair of inputs (x_1, x_2) is chosen from the set of available comparisons at time t, the time period t is from 1997 ($t = 0$) or 2002 ($t = 5$), and the vector $Z_{\Phi,x_1,x_2,i,t}$ is the set of instruments chosen for this equation as described below in Section 6.2. For each child i, we stack moments from Equation (36) for each input ratio and year.

4.2.2 Intertemporal Moments

We address several practical challenges in using intertemporal moments related to child skill accumulation: (i) child skill measures are not observed every year, (ii) investment inputs are not observed every year, and (iii) inputs and child skill levels are measured with error.

Our data allow the comparison of child human capital measures 5 years apart. We denote $\tilde{\Psi}_{i,t} \equiv \ln(\Psi_{i,t})$ and iterate on the Cobb-Douglas dynamic specification for human capital production (see Assumption 2) to obtain:

$$\tilde{\Psi}_{i,5} = \sum_{t=0}^{4} \delta_2^{4-t}\left[\delta_1 \ln(X_{i,t}) + \ln(\theta_{i,t})\right] + \delta_2^5 \tilde{\Psi}_{i,0},$$

where we embed age variation in $\mathcal{H}_t(\cdot)$ in the individual productivity parameter $\theta_{i,t}$. Because total investment $X_{i,t}$ is not directly observed, we first use the results of Section 3.1 to derive an expression for the ratio of time inputs τ_j to composite input X, denoted $\Phi_{j,X}$, which is a known function of prices, $\Pi_{i,t}$, marital status, $M_{i,t}$, and technology parameters.[32] Allowing for measurement error in $\tau_{j,i,t}$ gives the relationship:

$$\tau_{j,i,t}^o = \Phi_{j,X}(\Pi_{i,t}, M_{i,t}) X_{i,t} \exp(\xi_{j,i,t}),\ j \in \{m, f\}.$$

[32]The approach freely generalizes to the use of other inputs, but our preferred estimates use time use as the investment proxy given its prominence in the share of investment expenditures.

This would, in principle, allow us to estimate the outcome equation using time investment as a proxy for total investment (subject to accounting for measurement error). However, because inputs are only observed five years apart, we must impute them for the intervening periods. To do this, we use the solution for optimal investment based on the two cases described in Section 3.2: (i) non-binding borrowing constraints and (ii) no savings or borrowing.

Using the intertemporal optimality condition for total investments when borrowing constraints do not bind from Equation (22), we write period $t+5$ outcomes in terms of period t investments:

$$\tilde{\Psi}_{i,5} = \sum_{t=0}^{4} \delta_2^{4-t} \left[\delta_1 t \ln\left(\frac{1+r}{\delta_2}\right) + Z_{i,t}\phi_\theta \right] + \sum_{t=0}^{4} \delta_2^{4-t}\delta_1 \left[\ln\left(\frac{\overline{p}_{i,0}\tau^o_{j,i,0}}{\overline{p}_{i,t}\Phi_{j,X}(\Pi_{i,0}, M_{i,0})}\right) - \xi_{j,i,0} \right] + \delta_2^5 \tilde{\Psi}_{i,0}, \quad (37)$$

where we assume $\theta_{i,t} = \exp(Z_{i,t}\phi_\theta)$.[33] Similarly, when no borrowing or saving is permitted, Equation (21) implies that period $t+5$ outcomes can be written as:

$$\begin{aligned} \tilde{\Psi}_{i,5} \quad = \quad & \sum_{t=1}^{4} \delta_2^{4-t} \left[\delta_1 \ln\left(\frac{K_t}{1+\psi_m+\psi_f+K_t}\right) + Z_{i,t}\phi_\theta \right] + \sum_{t=1}^{4} \delta_2^{4-t}\delta_1 \ln\left(\frac{W_{m,i,t}+W_{f,i,t}+y_{i,t}}{\overline{p}_{i,t}}\right) \\ & + \delta_2^4\delta_1 \left[\ln\left(\frac{\tau^0_{j,i,0}}{\Phi_{j,X}(\Pi_{i,0}, M_{i,0})}\right) - \xi_{j,i,0} \right] + \delta^5 \tilde{\Psi}_{i,0}. \end{aligned} \quad (38)$$

To finish, the estimation procedure must address measurement error in child human capital. In the PSID-CDS, we use two age-normalized measures of cognitive ability from the *Letter-Word* ($LW_{i,t}$) and *Applied Problems* ($AP_{i,t}$) modules of the Woodcock-Johnson aptitude test. We write the measurement equations as:

$$S_{i,t} = \lambda_S \tilde{\Psi}_{i,t} + \tilde{\xi}_{S,i,t}, \quad S \in \{LW, AP\}, \ t \in \{0, 5\},$$

where the periods of measurement t correspond to the 1997 and 2002 waves of the PSID-CDS. These measurement assumptions require a normalization on the factor loading for one measure, as in Cunha, Heckman, and Schennach (2010). We set $\lambda_{LW} = 1$, leaving the factor loading on the Applied Problems score (λ_{AP}) to be identified. Collecting error terms, we can write the final outcome equations using both scores $S \in \{LW, AP\}$ and the time investment of parent $j \in \{m, f\}$ under non-binding constraints as:

$$\lambda_S^{-1} S_{i,5} = Z_{i,0}\tilde{\phi}_\theta + \sum_{t=0}^{4} \delta_2^{4-t}\delta_1 \ln\left(\frac{\overline{p}_{i,0}\tau^o_{i,0}}{\overline{p}_{i,t}\Phi_{j,X}(\Pi_{i,0}, M_{i,0})}\right) + \delta_2^5 \lambda_S^{-1} S_{i,0} + \xi_{S,j,i} \quad (39)$$

and alternatively for the no borrowing/saving case as:

$$\lambda_S^{-1} S_{i,5} = Z_{i,0}\tilde{\phi}_\theta + \sum_{t=1}^{4} \delta_2^{4-t}\delta_1 \ln\left(\frac{W_{m,i,t}+W_{f,i,t}+y_{i,t}}{\overline{p}_{i,t}}\right) + \delta_2^4\delta_1 \ln\left(\frac{\tau^0_{j,i,0}}{\Phi_{j,X}(\Pi_{i,0}, M_{i,0})}\right) + \delta^5 \lambda_S^{-1} S_{i,0} + \xi_{S,j,i}. \quad (40)$$

[33] Our strategy naturally accommodates (potentially stochastic) unobserved variation in $\theta_{i,t}$ that is independent of $(Z_{i,0}, ..., Z_{i,4})$ characteristics and mean independent of $(\tilde{\Psi}_{i,0}, \tau_{i,0})$, measurement errors, and input prices (as well as parental wages and non-labor income in the case with no borrowing/saving) conditional on $(Z_{i,0}, ..., Z_{i,4})$.

In each case, the term $Z_{i,0}\tilde{\phi}_\theta$ absorbs the first summation term in Equations (37) and (38).[34] We assume that the full vector of error terms $\xi_{i,t}$ is independent of $\xi_{i,t'}$ for $t \neq t'$, and that within a time period, the measurement error in Letter-Word scores ($\tilde{\xi}_{LW,i,t}$) is independent of the measurement error in Applied Problems ($\tilde{\xi}_{AP,i,t}$).[35] No other restrictions on covariances are imposed here. For notational convenience, we write the outcome Equations (39) and (40) as:

$$S_{i,5} = G_{S,j}(\tau_{j,i,0}, \Pi_i, M_{i,t}) + \xi_{S,j,i}, \quad j \in \{m, f\},\ S \in \{LW, AP\}$$

and use the following vector of moments for estimation:

$$E\left(\left[S_{i,5} - G_{S,j}(\tau_{j,i,0}, \Pi_i, M_{i,t})\right] Z_{S,i}\right) = 0, \quad j \in \{m, f\},\ \forall S \in \{AP, LW\}, \tag{41}$$

where the instrument set $Z_{S,i}$ is described in Section 6.2.

In order to identify the factor loading λ_{AP}, we use the assumption that measurement error is independent over time to write:

$$\lambda_{AP} = \frac{Cov(AP_{i,5}, LW_{i,0})}{Cov(LW_{i,5}, LW_{i,0})}, \qquad \lambda_{AP}^2 = \frac{Cov(AP_{i,5}, AP_{i,0})}{Cov(LW_{i,5}, LW_{i,0})}.$$

Because we normalize our measurements to have mean zero, these two identifying conditions can be written as the following pair of moments:

$$E\left[(AP_{i,5} - \lambda_{AP} LW_{i,5}) LW_{i,0}\right] = 0 \qquad \text{and} \qquad E\left[AP_{i,5} AP_{i,0} - \lambda_{AP}^2 LW_{i,5} LW_{i,0}\right] = 0. \tag{42}$$

We estimate the model by stacking the vector of moment conditions on input ratios described in Equation (36), the vector of moment conditions implied by outcomes in Equation (41), and the moment conditions derived from measurement assumptions in Equation (42).

5 Data Sources and Construction

We construct a panel dataset on family work behavior, investment in children, and child outcomes from the PSID-CDS. The PSID is a dynastic, longitudinal survey taken annually from 1968 to 1997, and biennially since 1997. The main interview of this survey collects household level data on economic and demographic variables. The CDS consists of three waves, collected in 1997, 2002, and 2007. The youngest two children in a PSID household between the ages of 0 and 12 at the time of the 1997 survey

[34] The only time-varying $Z_{i,t}$ affecting $\theta_{i,t}$ in our empirical analysis is the child's age, which allows us to write the entire first term in Equation (37) as a linear function of $Z_{i,0}$. In Equation (38), the first term depends on additional structural parameters $(\alpha, \beta, r, \psi_m, \psi_f)$; however, it is only necessary to estimate age-specific intercept terms that absorb all of these expressions. We use a linear term in age as a first order approximation.

[35] This assumption permits our instrumenting strategy described below, which addresses the correlation between measurement error $\xi_{S,j,i}$ and $S_{i,0}$ due to $\tilde{\xi}_{S,i,0}$.

were considered eligible for interview in the supplement. We summarize the sources and methods of variable construction from this dataset below, while Table 5 provides an overview.

Our estimation approach also requires merging these data with price variables for home-based goods and child care inputs. We construct these variables by combining several data sources, as described in the final section of this discussion.

5.1 Parental Investment and Child Outcomes from the PSID-CDS

In each wave of the CDS, a *Primary Caregiver* (PCG) and *Other Caregiver* (OCG) are identified in the household. The Child, the PCG, and the OCG each complete a module of the survey.

Cognitive Outcomes In all three waves of the survey, several assessments of cognitive and socioemotional development are collected for children. We use the *Letter-Word* (LW) and *Applied Problems* scores from the Woodcock-Johnson battery of tests, which are completed by children ages 3 and older. We use the age-normed scores provided by the PSID from the 1997 and 2002 waves.

Time Investment Measures on time investment come from time diaries completed by CDS children, with assistance from the PCG when necessary. This portion of the survey requires participants to record a detailed, minute by minute timeline of their activities for one random weekday and one random day of the weekend. Activities were subsequently coded at a fine level of detail. For each recorded activity, an indicator is provided for whether the mother and/or the father are actively participating in the activity. Our chosen measure of time investment for each parent is the weighted sum of the time each parent actively participates in activities with the child (Del Boca, Flinn, and Wiswall, 2014), with the weekday receiving weight 5/7 and the weekend day receiving weight 2/7. We construct these measures from the 1997 and 2002 time diaries.

Child care Expenditure In the 1997 and 2002 PCG interviews, respondents for children older than age 5 answer questions about current child care arrangements, costs, and time spent in each arrangement. For children younger than age 5, a retrospective history of arrangements is collected, from which we collect all arrangements that are reported as ongoing. We construct a measure of weekly expenditures from these answers.[36] When this variable is either missing or unavailable, we use total household expenditures on child care from the main interview, divided by the number of children ages 12 or younger.

[36] Costs are not reported in uniform units, and weekly expenditures are imputed from answers to questions on price per time unit and the usual amounts of time in care.

Goods Expenditures In the 2002 PCG interview, respondents answer questions on annual expenditures for the child on food, clothing, vacation, school supplies, and toys. Additionally, respondents answer questions on whether the child participates in private lessons, sports, tutoring, or community groups, along with questions on costs of these activities. We determine weekly expenditures in all of these categories, choosing the sum of spending on school supplies, toys, sports, tutoring, lessons, and community groups as our preferred measure of market goods expenditure.

5.2 Household Variables from the PSID Main Interview

For each child in the CDS, we use the PSID's childbirth record to link children with mothers, and the PSID's individual file to link mothers with their corresponding household interview in each year. From the main interview, information is collected on household structure, annual household expenditures on child care, state of residence, as well as the hours of work, earnings, race, and education of household members. We use mothers' childbirth history to construct the number and age of children in the household. While the PSID is available only biennially after 1997, earnings and hours for individuals in "missing" years is made available through supplemental interviews in years after 1997. We combine these data to construct a panel of each mother's marital status, race, education, state of residence, work behavior, and wages.[37] When the mother is married, we use her spouse's wages and education as the father's wage and education in our analysis below.

Using the large panel of wage data, we estimate parents' log wage fixed effects from (gender-specific) panel regressions of log wages on individual fixed effects, potential experience and experience-squared, number of children ages 0–12, and state dummies.[38] This effectively nets out differences in average wage rates across states and provides a measure of a parent's value in the labor market at the time he or she leaves school.

5.3 Sample Selection

We limit the sample to mothers of working age (between 18 and 65), who were aged between 16 and 45 in the year of the child's birth. We consider only child-year observations for children ages 12 or younger, and for households with no more than 2 children ages 12 or younger. We exclude children whose birth

[37] We treat cohabiting couples as "married". Wages are imputed as annual earnings divided by annual hours, with the bottom and top 1% of observations dropped.

[38] Potential experience is given by age - education - 6. Available wage observations from all PSID survey years up to 2002 are used. Whenever we control for these fixed effects in our analysis below, we only include parents with at least 3 wage observations available to estimate the fixed effect. After this restriction, the average number of wage observations used in estimating the fixed effects for both mothers and fathers in our sample is slightly less than 11 (roughly 85% have 7 or more observations).

records indicate that they are adopted. The characteristics of families in our main sample are reported in Table 6.

5.4 Price Variables

The estimation procedure also requires prices of child care, P_c, and prices of home-based goods, p. For child care prices, we draw from annual reports on the cost of child care in the U.S. compiled by Child Care Aware of America (2009–2019) to construct a state-level panel of hourly prices for 4 year-old family- and center-based care from 2006 to 2018. We impute these prices back to 1997 using the average earnings of child care workers in each state and year from the Current Population Survey. More details describing this imputation procedure are provided in Appendix B.

To construct the price of home-based goods, p, we combine data on *Regional Price Parities by State* provided by the Bureau of Economic Analysis, and the *Consumer Price Index* from the Bureau of Labor Statistics. Further details on the construction of this variable can also be found in Appendix B.

Our data collection results in a pair of prices, (p, P_c), for each state and year, which are merged with our PSID-CDS panel using state of residence and calendar year for each PSID household.

6 Child Production Functions Estimates

This section presents estimates of child production functions for children from single- and two-parent homes based on the approaches described in Section 4. Our analysis focuses on children ages 0–12 from families with only one or two children in that age range.

Much of our analysis requires wage measures for parents; however, some parents do not work. To alleviate concerns about selection into work, we limit our main estimation sample to parents with a relatively high predicted probability of working. To obtain predicted probabilities of work, we estimate separate linear probability models for working during the year (in 1997 and 2002) for single mothers, married mothers, and married fathers. We also estimate the probability that both parents (in two-parent households) are working. These regressions control for parental age and education, number of children and young children in the household, age of youngest child, age of CDS child, and survey year. (See Appendix Table D-3 for estimates.) The median (first quartile) predicted probability of work is 0.77 (0.69) for mothers and 0.91 (0.86) for fathers. Among married couples, the median (first quartile) predicted probability that both worked was 0.71 (0.64). Except where noted, we restrict our analysis of relative demand for parental time vs. household goods to women (men) with a predicted probability of work no less than 0.7 (0.85). When estimating relative demand for child care vs. household goods, we

restrict our samples to single women with a predicted probability no less than 0.7 and married couples with a predicted probability that both work of no less than 0.65; however, selection is not generally a concern for these specifications for reasons discussed in Section 4.

Before examining the role of parental education in the child development process, we first document the relationship between education and wages among parents in our sample. Appendix Table D-4 reports estimates from regressions of log wages on educational attainment, parental age and age-squared, and race separately for mothers and fathers.[39] These estimates are broadly consistent with the literature (see, e.g., Heckman, Lochner, and Todd, 2006), suggesting that parents with a college degree earn roughly 40–50% more than those with only a high school degree.

6.1 Relative Demand Estimates

Because the CDS did not collect (adequate) information on household goods expenditures on children in 1997, most of our analysis uses data from the 2002 CDS survey.

Table 7 reports OLS estimates of Equation (26) for all mothers accounting for potential determinants of the productivity of mother's time with children and/or household goods inputs. Several specifications are shown. As discussed in Section 4, the coefficient on mother's relative log wages provides an estimate of $\rho/(\rho-1)$, which equals $1-\epsilon_{\tau,g}$. Near the bottom of the table, we also report estimates of ρ as implied by the coefficients on $\ln(\tilde{W}_{m,i})$. Coefficients on all other variables provide estimates of $\tilde{\phi}_{mg} = (\phi_m - \phi_g)/(1-\rho)$.

In addition to log relative wages, column (1) only controls for the mother's marital status, while all other columns also control for child's age, whether the mother is white, the number of young children (ages 0–5) and number of children in the household. Columns (2)–(4) also control for the mother's age and educational attainment. Instead of controlling for factors indirectly related to a mother's human capital (e.g. age and education), column (5) conditions directly on her log wage fixed effect as a measure of her labor market productivity. (As discussed in Section 5.2, parent log wage fixed effects are based on all available wage measures for the parent from 1968–2002.) These estimates identify the elasticity of substitution between mother's time and goods from the effects of current log wages conditional on long-run average wages (reflecting both observed and unobserved parental skills). Thus, they should minimize concerns about endogeneity bias due to unobserved heterogeneity.

Specifications that condition on maternal age and education (columns (2)–(4)) all indicate an elasticity of substitution between maternal time and household goods inputs of roughly $\epsilon_{\tau,g} = 0.45$, with estimates of ρ ranging from -1.13 to -1.31. In all of these cases, the estimates suggest that maternal time and

[39] Estimates for single and for married mothers do not include indicators for high school completion, because there are very few high school dropout mothers once we restrict our sample to those with a high predicted probability of work.

household goods are more complementary (elasticity statistically significantly different from 1) in child production than the Cobb-Douglas case.[40] Column (5), which controls for mother's log wage fixed effects (rather than age and education) suggests an even smaller elasticity of around 0.25.

Next, consider estimates of $\tilde{\phi}_{mg}$, the (scaled) effects of maternal and child characteristics on the productivity of mother's time relative to household goods inputs. The very general specification in column (3) suggests that mother's education (particularly finishing high school) increases the productivity of her time relative to other goods inputs, while older children and those with white mothers have a lower productivity of mother's time relative to goods inputs compared to their younger counterparts and those with non-white mothers. While the productivity of mother's time relative to goods appears to be increasing in the number of children in the household, these estimates are not statistically significant. The number of young children has comparatively small and statistically insignificant negative effects.[41] Unfortunately, our sample contains very few mothers that are high school dropouts once we condition on a high predicted probability of work. (Among married mothers, we have none.) Column (4), therefore, reports a specification identical to that of column (3) but drops the indicator for high school graduate. Most estimates are nearly identical, except now we see no effect of education — consistent with column (3), only completing high school appears to affect the relative productivity of time to goods. Finally, column (5) continues to show that older children with white mothers have lower relative productivity of maternal time relative to goods. Perhaps surprisingly, mothers with higher log wage fixed effects (i.e., higher labor market productivity), have a much lower relative productivity of time compared to goods inputs than do lower wage women. It is important to note that this need not imply lower productivity of mother's time, because higher wage women may have a higher productivity of household goods inputs.

Table 8 reports analogous estimates of Equation (26) using state dummies as instruments for relative log wages, assuming that the relative productivity of mother's time vs. household goods inputs are the same across states (or at least uncorrelated with state-level wage average rates), conditional on available measures of mother's human capital (e.g. age, education). The estimated elasticity of substitution between maternal time and goods in columns (2)–(5) range from 0.22 to 0.33. While the standard errors are roughly three times as large as their OLS counterparts, the elasticities are still significantly different from 1 (Cobb-Douglas) once we control for maternal education or log wage fixed effects. The effects of maternal and child characteristics are similar to their OLS counterparts reported in Table 7.

[40] Throughout the paper, we use a statistical significance level of 0.05 when indicating whether an estimate is statistically significant.

[41] We have also estimated the specification in column (3) while including a measure of the mother's cognitive ability (based on a paragraph comprehension test administered to primary child caretakers in 1997). This measure had negligible effects on relative demand and had little impact on other estimated coefficients. Because over 15% of our sample did not have a reported score, we exclude it more generally from our analysis.

To explore potential sample selection concerns, Appendix Table D-5 reports OLS and IV estimates for different samples based on parents' predicted probabilities of work. Estimates are quite similar whether we restrict our sample to women with a higher predicted probability of work (no less than 0.8) or whether we use the full sample of women. These results, plus the estimates from column (5) of Tables 7 and 8, which controls for log wage fixed effects, suggest that problems related to sample selection are likely to be minor.

In Table 9, we report estimates for the specification in column (4) of Tables 7 and 8 separately for single and married mothers, as well as married fathers. Note that specifications for mothers (fathers) condition on mother's (father's) log relative wages, education, and age. Except for the IV estimates for married fathers, which are imprecise, the elasticity estimates are remarkably similar across the different parent types, always suggesting greater complementarity than Cobb-Douglas. The effects of child's age on the productivity of parental time vs. household goods are generally negative for all parent types; however, they are strongest and only significantly different from zero for married mothers. Married mothers with at least some college appear to have a lower relative productivity of time vs. goods compared to mothers with only a high school degree. Estimated differences in relative productivity of parental time vs. household goods by parental post-secondary attainment are much smaller and insignificant for single mothers and married fathers. While the total number of children in the household appears to raise the relative productivity of parental time for all parents, the number of young children (ages 0–5) has strong negative effects on the productivity of single mother's time relative to goods inputs. No such effects of young children are observed for married mothers or fathers. Finally, we find mixed evidence on the effect of father's age, with modest and statistically significant negative effects from OLS and weaker, insignificant effects from IV estimation.

We note that standard Hausman tests fail to reject equality of OLS and IV estimates for all specifications and parent types in Tables 7–9. This, in part, reflects that state dummies are not particularly strong instruments for relative log wages in our sample, with first-stage F-statistics generally in the range of 1-3. Because weak/many instrument concerns would tend to bias IV estimates towards their OLS counterparts, our findings that IV estimated elasticities $\epsilon_{\tau,g}$ are typically even smaller (and further from 1) than our OLS estimates strongly suggests that parental time and household goods are more complementary than a Cobb-Douglas assumption implies. We are more reassured by the fact that our estimated elasticities (from both OLS and IV) generally range from 0.2 to 0.5 whether we control for parental education and age or for parental log wage fixed effects, where the latter estimates are unlikely to be biased from unobserved heterogeneity in parental skills. (See Appendix Table D-6 for specifications with

log wage fixed effects estimated separately for all parent types.)

We next turn to relative demand for child care services vs. household goods inputs, Equations (27) and (32). An unfortunate practical problem arises here, because child care expenditures are frequently unreported or zero, even among families with parents working significant hours. (By contrast, parental time and household goods inputs are nearly always reported and positive.) Zero expenditures pose a challenge for our estimation approach, which relies on log expenditure amounts. To better understand who reports spending on child care, we estimate the effects of household characteristics $Z_{i,t}$ and the price of child care on the probability of reporting positive expenditures. As shown in Appendix Table D-7, most household characteristics are not predictive of who reports positive child care spending. More importantly, child care prices have negligible effects on whether someone reports positive expenditures, despite the fact that these prices significantly affect the amount families spend on child care (among those who report spending) as we show below. These findings suggest that families who report spending positive amounts on child care are fairly representative of the full sample of parents, at least based on factors we can observe. This is consistent with many families receiving some form of free child care from family or friends, with few household characteristics helping predict which families benefit from this support.[42] With no measures of that care, we omit these families when estimating relative demand for child care vs. household goods. However, these families can be (and still are) used in estimating the relative demand for parental time vs. household goods, because that tradeoff is unaffected by the availability of free (but presumably limited) external child care.

We begin our analysis of the relative demand for child care vs. household goods inputs with simple OLS specifications that only condition on the relative price of child care and on parent and child characteristics. These specifications, reported in columns (1)–(3) of Tables 10 and 11, ignore the potential influence of relative parental time vs. goods expenditures, $R_{m,i}$ and $R_{f,i}$, on the relative demand for child care vs. goods. While these specifications are not generally valid unless $\gamma = \rho$, they provide a useful benchmark. Table 10 reports estimates for single mothers, while Table 11 reports estimates for two-parent households. Unfortunately, sample sizes are small (and estimates often imprecise). Still, all estimated elasticities of substitution between child care services and the home composite input, $\epsilon_{H,Y}$, are less than one, several significantly so (recall that the coefficient on $\ln(\tilde{P}_{c,i})$ estimates $\gamma/(1-\gamma) = 1 - \epsilon_{Y,H}$). Elasticities from specifications controlling for parental education or log wage fixed effects (columns (2) or (3)) range from 0.55 to 0.79 for single mothers and 0.36 to 0.46 for two-parent households. The elasticities are quite

[42] In 2011, 42% of children ages 0–5 whose mother was employed received child care from a grandparent, sibling, or other relative (Laughlin, 2013). In Appendix Table D-7, the number and ages of children in the household are among the few characteristics affecting the probability of positive child care spending, consistent with the role older siblings play in providing child care.

similar to estimated elasticities of substitution between parental time and household goods, suggesting that failure to account for parental time relative to goods expenditures may not be very problematic. The significant negative coefficients on child's age (i.e., $\tilde{\phi}_g$) on the relative demand for child care, indicate that the relative productivity of household goods is greater among older children.[43] The effects of post-secondary schooling levels among parents are mixed and noisily estimated.[44] Most other coefficients are modest in size and all are statistically insignificant.

The last two columns of Tables 10 and 11 account for the effects of relative parental time vs. goods expenditures on the relative demand for child care vs. goods when $\gamma \neq \rho$. Column (4) reports results from estimation of Equations (28) and (33), which assume that parental time and goods inputs, as well as wages, are not measured with error. Column (5) shows results from estimation of Equations (29) and (34), which allow for measurement error in all child investment inputs but assume no unobserved heterogeneity in parenting skills. Estimated elasticities of substitution between child care and the household composite investment are similar in both cases, around 0.85 for single mothers and 0.45 for two-parent households.[45] The estimated effects of child and parent characteristics on the productivity of goods inputs are similar to those from columns (1)–(3).

Table 12 presents estimates from our most general specification of relative demand for child care services vs. household goods inputs, Equations (31) and (35), which accounts for measurement error in inputs and wages, as well as unobserved heterogeneity in parenting productivity. For two-parent households, we consider a case with "Restricted Measurement Error", which assumes $\sigma^2_{W_m\tau_m} = \sigma^2_{W_f\tau_f}$.[46] We reduce the set of household characteristics that may affect a_g to child's age and parental education based on the findings from the previous two tables and our interest in the role of parental human capital. Estimates of the elasticity of substitution between child care services and the household composite input are quite similar to those of Tables 10 and 11: 0.77 for single mothers and about 0.32 for two-parent households. The estimated $\epsilon_{Y,H}$ for two-parent households are statistically less than one. (Again, estimates for ρ are quite noisy, though always negative, suggesting $\epsilon_{\tau,g} < 1$.) Turning to estimates of $\tilde{\phi}_g$, we observe more muted (and statistically insignificant) effects of child's age compared to previous estimates. Coefficient

[43] Because estimated γ and ρ are both negative, ϕ_g has the opposite sign of $\tilde{\phi}_g$.

[44] There is some indication of a U-shaped effect of father's education on the relative productivity of home goods investments, which becomes more evident when an indicator for father high school completion is included.

[45] As indicated near the bottom of Tables 10 and 11, it is possible to obtain estimates of ρ (and the elasticity of substitution between parental time and goods), because the coefficients on $\ln(1+R_{m,i})$ or $\ln(1+e^{\tilde{\Phi}_{m,i}})$ (and their counterparts for two-parent households) yield estimates of $(\gamma-\rho)/[\rho(\gamma-1)]$. These can be combined with the estimates of γ obtained from coefficients on $\ln(\tilde{P}_{c,i})$. Unfortunately, these estimates are quite noisy, especially for two-parent households.

[46] Estimated measurement error variances are not shown but are available upon request. They are very noisy, sometimes negative, and largely uninformative (with t-statistics typically less than one). Only the coefficient on the de-meaned/scaled measure of goods investment is fairly precisely estimated, suggesting that nearly two-thirds of the variation in $\ln(g_i^o)$ may reflect measurement error.

estimates on maternal education appear to be more positive (suggesting negative ϕ_g) than in Tables 10 and 11; however, they are far from statistically significant. Estimated effects of father's education are quite similar to those reported in Table 11.

An important challenge is the lack of precision for key parameters. Sample sizes in the PSID are small, especially when restricting observations to parents with a high probability of working and to households reporting positive expenditures on child care services. In an effort to improve efficiency from cross-equation restrictions (i.e., ρ appears in all relative demand equations), we use GMM to estimate both sets of relative demand equations simultaneously.[47] Unfortunately, the lack of information about ρ apparent in estimates of child care vs. goods relative demand (see Table 12) means that there is little benefit from joint estimation of Equations (26) and (31) for single mothers.[48]

GMM estimates for two-parent households, reported in Table 13, are more informative. The improved precision (relative to Tables 9 and 12) not only derives from joint estimation of both sets of relative demand equations, but it also stems from estimation of time vs. goods relative demand for both fathers and mothers. Estimated elasticities of substitution are reported near the bottom of Table 13. The estimates are quite robust across specifications (whether or not we instrument for log relative wages in Equation (26) or restrict measurement error variances to be the same for both parents) and suggest that elasticities $\epsilon_{\tau,g}$ and $\epsilon_{Y,H}$ are both around 0.34 to 0.41.[49] The estimated effects of child's age on the relative productivity of household goods, ϕ_g, are strongly positive if not quite statistically significant. Because the estimated effects of child's age on the relative productivity of maternal time to goods, $\phi_m - \phi_g$ is quite similar but the opposite sign, it suggests that ϕ_m is roughly zero for child's age. Estimated effects of mother's education on the productivity of her time relative to goods inputs and on goods relative to

[47] For single mothers, we use GMM to estimate the following moments based on Equations (26) and (31): $E(u_{m,i}J_{m,i}) = 0$ and $E(u_{Y,i}J_{Y,i}) = 0$, where

$$u_{m,i} = \ln(R_{m,i}) - Z_i'\tilde{\phi}_{mg} - \left(\frac{\rho}{\rho-1}\right)\ln \tilde{W}^o_{m,i} \tag{43}$$

$$u_{Y,i} = \ln(R_{Y,i}) - Z_i'\tilde{\phi}_g - \left(\frac{\gamma-\rho}{\rho(\gamma-1)}\right)\ln\left(1+R_{m,i}\right) - \sigma^2_{W_m\tau_m/g}\left(\frac{\gamma-\rho}{\rho(\gamma-1)}\right)\left(\frac{R_{m,i}}{2(1+R_{m,i})^2}\right)$$
$$- \left(\frac{\gamma}{\gamma-1}\right)\ln(\tilde{P}_{c,i}) - \lambda\left(\ln(g^o_i) - E[\ln(g^o_i)]\right), \tag{44}$$

$J_{m,i}$ includes all Z_i characteristics in Equation (43) and either $\ln \tilde{W}^o_{m,i}$ or state dummies, and $J_{Y,i}$ includes all Z_i characteristics in Equation (44), $\ln(1+R_{m,i})$, $\frac{R_{m,i}}{2(1+R_{m,i})^2}$, $\ln(\tilde{P}_{c,i})$, and $\ln(g^o_i) - E[\ln(g^o_i)]$. For two-parent households, an analogous approach is taken based on Equation (26) for both parents and Equation (35). We use a one-step estimator with the identity weighting matrix when using state dummies as instruments; otherwise, we use a two-step estimator with an identity matrix for the initial weighting matrix.

[48] When we do not instrument for mother's wages in the time vs. goods relative demand equation, we obtain an estimate for ρ of -1.07 (SE=0.70) and for γ of -0.22 (SE=0.27). Instrumenting for mother's wages produced extremely imprecise and implausible estimates. See Appendix Table D-8.

[49] Because ρ and γ are estimated to be quite similar, estimated measurement error variances for time investment expenditures, $\sigma^2_{W_j\tau_j}$, are extremely noisy and not reported.

child care both appear to be negative, while estimated effects of father's education are mixed.

Summarizing all of our estimates based only on relative demand, we find remarkable consistency across specifications in the estimated elasticities of substitution between inputs: the elasticity between parental time and home goods inputs and the elasticity between the composite home input and child care both tend to range from 0.2 to 0.5, implying a moderate degree of complementarity. Most specifications suggest that child's age raises the productivity of household goods inputs relative to both child care services and maternal time inputs. Perhaps surprisingly, we find no consistent patterns for the effects of parental post-secondary schooling on the relative productivity of parental time or home goods inputs. Finally, we note that the similarity of our estimates, regardless of whether or how we account for unobserved heterogeneity, suggests that unmeasured differences across parents have little impact on the relative productivity of different investment inputs. We also find little evidence to suggest that measurement error in wages confounds our estimation approaches that abstract from it.

6.2 GMM Estimation based on Relative Demand and the Dynamics of Achievement

To estimate the full model, including both $f(\cdot)$ and $\mathcal{H}(\cdot)$, we combine moments for the individual input ratios in 1997 ($t = 0$) and 2002 ($t = 5$), described in Equation (36), as well as moments for child achievement scores, defined in Equation (41), using measured household prices for all years 1997–2001.[50] To calculate the moments involving test scores, we only include observations for children ages 3–8 in 1997, whose mothers are observed working in each year 1997–2001.[51] The full vector of moments takes all available comparisons for input shares across years, as well as outcome equations for both Letter-Word and Applied Problems scores using both mother's time and father's time (when available) as proxies for investment.

Table 14 summarizes the input ratios used. With complete data, some of these ratios would be implied by combinations of the others; however, there are a substantial number of cases in which only some of the inputs are measured, so using additional combinations exploits the available information.

Instruments for Input Ratios Consider the moment condition on the ratio of input x_1 to input x_2. The vector of instruments $Z_{\Phi,x_1,x_2,i,t}$ contains the full set of observables that are permitted to affect either a_{x_1} or a_{x_2}, in addition to the relative price of input x_1 to input x_2. For example, for the input ratio Y_c/τ_m, this relative price ratio would be $P_{c,i,t}/W_{m,i,t}$. We evaluate these ratios separately by marital status and, hence, exclude the marital status indicator from the set of instruments.

[50] We also include the moments for achievement measurements given in Equation (42).

[51] We continue to restrict samples to those whose parents have a high predicted probability of work when moments include parental time; however, the main conclusions are unchanged when this restriction is dropped.

Instruments for Outcome Equations The vector of instruments used for the Applied Problems score, $Z_{AP,i}$, includes all observables that are permitted to affect θ_i, the pair $(\ln(\tau^o_{m,i,5}), \ln(\tau^o_{f,i,5}))$, which instrument for time investment in 1997, and the Letter-Word score in 1997, $LW_{i,0}$, which instruments for the noisy measure of skills, $AP_{i,0}$, in the outcome equation. Construction of $Z_{LW,i}$ is identical, except that $AP_{i,0}$ now instruments for $LW_{i,0}$ in the outcome equation. This strategy allows us to account for measurement error in inputs and in the current stock of child human capital.

Missing Data We interact each moment with a binary variable that indicates when there is sufficient data to evaluate the moment. This does not affect the validity of the moment conditions as long as data is not missing in a way that systematically varies with the vector of measurement errors.

6.2.1 Results

Table 15 reports the estimates from the GMM procedure using all observations of relative demand reported in Table 14, in addition to the moment conditions for achievement scores. For comparison, estimates are also provided for the case in which only the moments for relative demand are used. Increases in precision for these estimates are largely achieved from a combination of (i) additional observations coming from 1997 incorporated with the use of Y_c/τ_m input ratios and (ii) cross-equation restrictions on elasticities and factor shares, which appear in all moment equations.

Most parameter estimates are quite similar, regardless of our assumptions on borrowing constraints; although, the estimated effects of total investment on skill accumulation as determined by δ_1 is greater (0.11 vs. 0.05) in the case of no borrowing/saving. Because our achievement scores are normalized to have standard deviation of 1, δ_1 can be interpreted as the fraction of a standard deviation increase in Letter-Word scores resulting from a log point increase in investment.[52] Estimates of δ_2 are both about 0.95 and suggest strong persistence in skills (i.e., self-productivity) over ages 5–12.

Estimates of ρ and γ imply similar degrees of complementarity to estimates reported earlier; however, they are generally more precise. Estimates of ρ imply an elasticity of substitution between parental time and home goods inputs of around 0.4 to 0.5, while estimates of γ imply slightly stronger complementarity between the home composite input and child care services with an elasticity of around 0.3.

Finally, we highlight several points regarding the estimated share parameters (ϕ_m, ϕ_f, ϕ_g).[53] First,

[52] Because we estimate a factor loading on Applied Problems (λ_{AP}) around 1.2, the fraction of a standard deviation increase in AP scores is $1.2 \times \delta_1$.

[53] Appendix Table D-9 reports estimates of λ_{AP} and $\tilde{\phi}_\theta$ for marital status, parental education, and child's age. Estimated $\tilde{\phi}_\theta$ imply greater skill growth among children in two-parent households, but relatively modest differences by parental education. There is some indication that older children have greater skill growth, where the difference is significant for the estimates assuming families are not borrowing constrained. While these differences are important for the accumulation of human

our estimates suggest similar relative productivity of parental time and goods investments for married vs. single parents. Second, we observe little systematic relationship between parental education and the productivity of parental time or home goods inputs; although, there is some indication that the relative productivity of father's time is lower among those with some college education than those with either more or less education. Third, older children have a lower relative productivity of maternal time and higher relative productivity of home goods inputs. Fourth, more young children in the household reduce the relative productivity of parental time (especially for mothers) and goods inputs.

7 Counterfactual Analysis: Explaining Investment Behavior

In this section, we use our model and the GMM estimates for the case of no borrowing/saving (reported in column (1) of Table 15) to study key factors driving family investment decisions. In particular, we investigate the sources of investment gaps across families and the role of technology in determining investment responses to price changes.[54]

7.1 Variation in Investment

It is common in the recent literature on the dynamics of skill accumulation to assume a single price of "investment"; however, wages vary considerably across families and, as we demonstrate, parental time inputs are a major form of investment. This suggests that the actual price of composite investment may vary considerably across families. We explore this issue by first investigating the extent to which variation in investment expenditures, $E_t = \bar{p}_t X_t$, derives from variation in the choice of investment quantity X_t vs. variation in composite input prices faced by families, $\bar{p}_t$. Using our estimated technology and input prices to construct $\bar{p}_t$ for each child in the 2002 PSID, Table 16 shows that while the variance of log expenditures is higher for single-parent households (0.70 vs. 0.57), the variance in log composite prices is also higher. For both single mothers and two-parent households, 48% of the variance of log investment expenditures is explained by the variance of log composite prices.

We next explore the sources of this price variation. The composite price of investment is a function of input prices and parameters of the per-period investment technology (see Equation 9), with share parameters of that technology (a_m, a_f, a_g) all depending on parental education and other family characteristics. Thus, variance in the composite price arises from differences in input prices and differences in these family characteristics. Table 17 decomposes this price into different sources by computing counterfactual

capital, they play no role in investment behavior as discussed in Section 3.2.

[54] In these counterfactual exercises, we exclude families with zero child care expenditures from 2002 PSID, because they are not used for estimation when we rely on log expenditure ratios.

composite prices under different equalization scenarios for input prices.[55] When all prices are equalized in the last column, the remaining variation is due to differences in technology (i.e., share parameters). It is clear from the table that variation in parental wages is the most important source of variation in the composite price of investment, especially for single mothers, while variation in the price of goods and market child care play little role. Variation in the productivity of investments across families accounts for about 40% of the price variation for two-parent households and less than one-quarter of the variation for single-parent households.

As documented in Section 2, more educated parents spend much more on investments in their children than less educated parents. We next explore the extent to which these gaps are driven by systematic differences in preferences for child skills, parental wages, and the productivity of investments. This exercise requires us to calibrate preference parameters (α, ψ_m, ψ_f), which we allow to vary by parental education in order to fit average maternal time investment levels and parental hours worked depending on whether mothers had attended college or not. (See Appendix C for details.) We note that preference parameters only affect the levels of investments, not relative input shares or the composite price of investment. Because more educated parents earn higher wages, on average, they face higher investment prices but also have more available household resources. As such, differences in parental wages have competing effects on investment. Lastly, productivity differences in a_m, a_f, and a_g arising from differences in parental human capital affect the price and quantity of investment, but these effects are offsetting such that total expenditures are invariant to these differences (see Equation (21)).

The first column of Table 18 shows total investment expenditures, prices, and quantities for families with college-educated mothers, relative to those with non-college mothers, as implied by the model. College-educated single mothers spend 58% more on child investments compared to their non-college-educated counterparts. The discrepancy is even greater (87%) for two-parent households. For two-parent households, the difference in expenditures is almost entirely driven by the higher prices they face (due to higher wages). For single mothers, differences in investment quantities also account for some of the expenditure difference. The second column presents these same gaps when preference parameters for college-educated mothers are equalized to those for non-college mothers. While this has no impact on prices (as mentioned above), it reduces investment expenditure gaps by substantially narrowing differences in investment quantities.[56] For single mothers, the gap in quantities is reduced by about 21 percentage

[55] When equalizing wages, we set wages for all parents to the average wage conditional on gender and marital status. When we equalize goods and market child care prices, we set them equal to the unconditional average prices for all families.

[56] In practice, preferences for child skills, α, and for leisure, ψ_m and ψ_f, are calibrated to explain gaps in average investment levels and hours worked once all other sources of variation in the model (i.e., input prices and technology parameters) have been taken into account. Table 18 simply decomposes these gaps in a different order from that used in calibrating these parameters.

points, while for two-parent households the gap is removed entirely. The next column shows that accounting for parental wage differences eliminates nearly all of the price gaps and some of the remaining investment quantity gaps (after already accounting for differences in preferences). The impacts on investment quantity gaps show that the additional family income associated with higher wages dominates the effects of higher investment prices when it comes to investments in children. (This is also the case for single mother's time investments, which are also shown in the table.) The final column also equalizes other input prices, which has little additional effect on prices or quantities. All remaining differences are quite modest, consistent with our finding that estimated technology share parameters are not systematically related to parental education. Altogether, Table 18 shows that sizeable investment expenditure gaps by parental education are largely driven by differences in the price of investment, which are, in turn, driven by the higher wage rates faced by more educated parents; however, more educated parents also appear to have a stronger preference for child skills than less-educated parents which accounts for some of the expenditure gaps (more so for single mothers).

7.2 Price Changes

Because many policies designed to encourage investments in children (e.g. child care subsidies, publicly provided goods like libraries and community activities), as well as many tax and welfare policies, primarily influence family investment decisions through changes in input prices (or their shadow prices), we next consider the impacts of reducing these prices. In doing so, we consider changes in prices when children are ages 5–12, the ages covered by most children in our sample. Using our estimates for the case of no borrowing/saving, we simulate effects on behavior under that same assumption.

Tables 19 and 20 report the effects of separately reducing each input price by 10%. Start by focusing on the first three columns, which report results for our estimated nested CES production function. Due to the complementarity we estimate, a change in the price of any input causes all inputs to adjust in the same direction as seen in Panel A. Except for changes in the price of parental time (i.e., wages), cross-price elasticities are substantially weaker than own-price elasticities, but not negligible. For example, a 10% reduction in the price of child care, leads to a 4–5% increase in child care inputs and a 0.5–1.1% increase in parental time and home goods inputs. The effects of a decline in wages are notably different, because this not only lowers the price of investment but it also directly impacts family income. Proposition 4 tells us investment expenditures must decline with the reduction in family resources, but we see an even stronger result in that the levels of investment decline. Even parental time investments decline slightly due to the complementarity of inputs.

In Panel B of Tables 19 and 20, we calculate the effects of input price changes on child achievement

measured at age 13. The first row reports changes in scaled log achievement, which is equivalent to percentages of a standard deviation in Letter-Word scores ($LW_{i,t}$). The second row reports the consumption equivalent value of the changes in achievement, measured as the percent increase in consumption over ages 5–12 that would make a family indifferent to the change in achievement. Due to well-known issues regarding interpretability of test score scales (Cunha, Heckman, and Schennach, 2010), this consumption equivalent measure is our preferred method of interpreting changes in child outcomes. We see that declines in investment associated with a 10% reduction in wages would lead to a reduction in achievement at age 13 of 1.6% of a standard deviation, which is valued at nearly 2% of consumption over ages 5–12 for the children of single mothers. A 10% reduction in the price of child care would raise achievement at age 13 by an amount valued at about 1.2% of ages 5–12 consumption (for children of single mothers), while a 10% reduction in the price of home goods inputs would raise achievement by 40% less. The impacts of price changes on log achievement are notably smaller for children from two-parent households. For example, a 10% reduction in wages for two-parent households produces a decline in child achievement equivalent to 0.6% of consumption, just over a quarter of the effect for single-mother households.

Panel C in Tables 19 and 20 reports the welfare implications of price changes in monetary terms. In particular, we show the present value of welfare changes from price reductions over ages 5–12 discounted back to age 5. We first report the average welfare gain (i.e., equivalent variation, EV), which suggests gains of more than $2,000 for a 10% reduction in child care costs and gains of $1,350–1,521 for a 10% reduction in the price of home investment goods and services. Not surprisingly, a 10% reduction in wages would substantially lower family welfare due to income losses. Perhaps more interesting, we also report on the distortionary effects of the price changes on behavior (implicitly assuming current prices are socially optimal), netting out the standard income effects of price changes (as well as direct effects on income in the case of wage reductions). Intuitively, we measure distortions as the amount families are willing to pay in order to eliminate the price distortion and instead receive a lump-sum monetary transfer equal to any changes in their budget.[57] A convenient property of this welfare measure is that it enables a decomposition into the distortion caused by input misallocation (i.e., relative investment effects) and distortions in the total level of investment, consumption, and leisure. Given the complementarity of inputs, we observe small distortions (resulting in willingness to pay measures of at most $67) due to misallocation of resources across different inputs. However, the total distortions associated with wage reductions are sizeable, amounting to over $2,000 for two-parent households. Total welfare distortions due to changes in the prices of other inputs are quite small, largely because they represent a small share

[57]The hypothetical lump-sum transfer is based on choices under the new prices. Our distortion measure equivalently represents the welfare change for families if given the lump-sum transfer less the standard EV measure of welfare changes.

of family investment.

To better understand the extent to which complementarity of inputs plays in the response to policies, the last three columns of Tables 19 and 20 report results from the same set of price changes using a Cobb-Douglas production function instead of our nested CES. For comparability, the share parameters of the Cobb-Douglas function are calibrated to generate the same expenditure shares as our estimated specification. Looking at Panel A, we observe dramatic differences in the incidence of price changes on different inputs. With changes in home goods or child care prices, there are no cross-price effects and input quantities adjust one-for-one with price changes to maintain constant expenditure shares and total expenditure levels. Wage reductions lead to one-for-one reductions in total expenditures due to the income reduction with no adjustments in the amount of parental time invested. Yet, if one is only interested in the effects on total investment X_t (and, consequently, skill growth as reported in Panel B), the Cobb-Douglas specification produces very similar results to our nested CES specification. Looking at Panel C, we see that the Cobb-Douglas case over-states the distortions due to misallocation of inputs (by factors of 2–4). Intuitively, the stronger complementarity implied by our estimates makes it optimal for families to maintain a similar bundle of inputs regardless of the relative prices. This is not without cost, however. As is evident from the EV welfare measure, the total welfare benefits from reductions in the price of home goods inputs or child care services are smaller under the complementarity we estimate than the standard Cobb-Douglas case would suggest.

The similar effects of price changes on total investment for very different input substitutability is an artefact of our analysis of small price changes. Indeed, as shown in Appendix A.5, the impacts of marginal changes in input prices on total investment depend on expenditure shares but not elasticities of substitution. This is not the case for larger price changes. Appendix Table D-10 shows that the Cobb-Douglas specification over-predicts total investment responses by about 20% for larger (i.e., 50%) reductions in the price of home goods or child care services, while it under-predicts (by 13–18%) the decline in total investment associated with a 50% wage decline. Together, these results imply that if one is interested in the investment response to small price changes, it is possible to rely only on expenditure shares, without taking a stand on technology; however, the complementarity of investments must be accounted for when studying the effects of larger price changes. A second lesson can be garnered from Appendix Table D-10. For both the nested CES and Cobb-Douglas cases, the price elasticities of total investment are stronger for larger price changes, suggesting that one cannot simply "scale up" the impacts estimated from small price changes to obtain accurate predictions about larger price changes. For larger price changes, it is essential to compute the elasticity of such a change directly based on estimates of

the full technology. These lessons provide a cautionary tale for research approaches looking to assemble global policy analysis from local elasticities.

8 Conclusions

Parents spend considerable sums investing in their children in terms of their time, purchased home goods/services inputs, and purchased market child care services. Of these different types of investments, parental time is the most costly investment made by the majority of parents.

We document a strong increase in total investment expenditures with maternal education in the PSID-CDS, ATUS, and CEX. Despite this strong increase, we find that the allocation of expenditures across different investment inputs — parental time, household goods and services, and purchased market child care — changes little with parental schooling. To understand these patterns and to study the impacts of policies that act on input prices, we develop a dynamic model of investments in children with multiple inputs each period, flexibility in substitution patterns across those inputs, and several channels through which parental skills may affect the productivity of those inputs. We analytically characterize investment behavior, showing how the substitutability of different inputs determines the qualitative responses to input price changes. We also show how the relationship between parental skills and investments in children depends on both the substitutability of inputs and the extent to which parental skills raise the productivity of both parental time and home goods inputs.

We then develop an estimation strategy based on intratemporal optimality alone to identify elasticities of substitution across inputs and the impacts of parental skills on the productivity of home investment inputs. This relative demand estimation approach not only avoids assumptions about preferences, credit markets, and the dynamics of skill production, but it also enables us to address unobserved heterogeneity in parental skills and measurement error in all investment inputs and wages. In order to estimate the full technology of skill production, including parameters governing the dynamics of skill accumulation, we incorporate intertemporal moments based on noisy measures of child skills, addressing the fact that our data do not contain measures of skills or inputs in consecutive years.

Exploiting PSID-CDS data, along with novel measures of child care prices from Child Care Aware of America (2009–2019), we find robust evidence across empirical strategies that parental time and purchased goods inputs are complements inside the home, while home investments are also complementary with market child care services. In both cases, elasticities of substitution range from 0.2 to 0.5, which suggests that the relatively constant expenditure shares across parental education are not the consequence of a Cobb-Douglas technology. Instead, the patterns of complementarity and relative input productivity imply

a rough balance in the process of skill production such that parental human capital does not strongly favor one type of investment over others. Among other demographic factors that affect child development, we find that the presence of more young children in the household reduces the productivity of both parental time and goods inputs relative to market child care, while maternal time becomes relatively less productive and home goods inputs relatively more productive for older children.

Our analysis suggests that there is considerable investment price variation across families, driven primarily by differences in parental wages. This variation directly accounts for about half of all variation in total investment expenditures across families. Unlike other prices, wages play two competing roles within the family, determining the price of time investment as well as available family resources. The extent of complementarity we estimate suggests that the latter role slightly dominates, so an increase in parental wages leads to an increase in all forms of investment, including small increases in parental time despite its higher opportunity cost. Because we estimate little systematic effect of parental education on the productivity of home investments, the positive parental education gradient for time spent with children and expenditures on child investments is driven by overall demand for investments (partly from greater family resources and partly from stronger preferences for child skills) and not factor augmentation.

Our analysis of price change effects on constrained households sheds light on the likely impacts of a wide array of policies that distort incentives to invest in various forms, from welfare and tax policies to child care subsidies. Perhaps the most important lesson from this analysis is that the estimated patterns of complementarity for investment inputs implies that all inputs move together with any price change; although, cross-price elasticities are generally modest except for wage changes which substantially impact investments due to changes in family resources. For small price changes, own-price elasticities for home goods inputs and child care services are roughly -0.5, while the cross-price elasticities for these inputs range from -0.02 to -0.10. Our estimates suggest modest distortionary impacts of changes in relative input prices on the allocation of inputs conditional on a given level of total investment; however, total welfare distortions from changes in wages are sizeable due to changes in the level of total investment, leisure, and consumption allocations. We also measure the impacts of price changes (over child ages 5–12) on child skill levels at age 13. Due to enhanced family resources, we find that policies which raise parental wage rates would lead to sizeable positive impacts on child achievement, consistent with studies that estimate positive effects of EITC expansions on child achievement (Dahl and Lochner, 2012; Agostinelli and Sorrenti, 2018). A 10% subsidy for child care would lead to sizeable improvements in the skills of children with single mothers valued at about 1.2% of consumption over child ages 5–12, while a 10% subsidy for home goods/services inputs would produce only about two-thirds that benefit. Across all

policies, the achievement impacts are smaller for children in two-parent households.

To evaluate the importance of accounting for general patterns of substitution across inputs, we conduct a similar policy analysis assuming a Cobb-Douglas within-period production function calibrated to fit the same expenditure shares as we obtain with our estimates. Comparing the predictions from our more general nested CES function (with elasticities of substitution of roughly 1/2 and 1/3) with the Cobb-Douglas function (with elasticities of 1) is revealing. Perhaps unsurprisingly, the Cobb-Douglas specification produces results that are highly misleading in terms of specific input responses to relative price changes. However, many researchers may be primarily interested in quantifying the effects of policy on total investments and child achievement, regardless of effects on specific child inputs. In this case, our conclusions are nuanced. For small price changes, only input expenditure shares are needed to accurately predict total investment and skill accumulation, so the Cobb-Douglas specification performs well. This is not the case for larger price changes. When price changes are larger, the substitutability of inputs becomes important and our nested CES production function predicts substantially different responses to those obtained from the comparable Cobb-Douglas case. An additional lesson can be gleaned from our comparison of small vs. large price changes: "scaling up" effects on investment estimated from small price changes is inappropriate. With complementary inputs, price elasticities for total investment are substantially stronger for large price changes.

This paper has aimed to clarify several key issues and challenges that arise when attempting to understand the complex set of decisions parents must make regarding how and when to invest in their children. A key limitation of this, as well as other papers in this literature, is the dearth of rich data. The PSID-CDS may be the only single data set with available measures of all three types of inputs we consider, repeated measures of achievement, and other key family measures. Yet, the useable sample sizes are frustratingly small and both achievement and investment inputs are infrequently measured. Future research in this area should endeavor to make better use of multiple data sets that may specialize in subsets of needed measures but which contain much larger samples, richer measures of specific inputs or outcomes, and which share rough sampling frames and important family characteristics like household composition, parental wages, family income, and labor supply (e.g. ATUS and CEX). Another path forward would be to combine the results from several natural or actual experiments, connecting marginal policy effects to primitive parameters of the child production function and/or preferences. Chaparro, Sojourner, and Wiswall (2020) and Mullins (2020) take productive steps in this direction. Richer data may also enable researchers to combine the relative demand approach we take with very flexible production function estimation using data on both inputs and outcomes (e.g., Cunha, Heckman, and Schennach,

2010; Agostinelli and Wiswall, 2016, 2020) to learn more about parental beliefs about vs. actual relative productivity of different inputs.[58]

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[58] See Cunha, Elo, and Culhane (2013) for innovative research on this issue using elicited beliefs from parents.

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Table 1: Weekly Child Investment Expenditures by Mother's Education (PSID, 2002)

		Mother's Education			
Expenditure Amount	All	HS dropout	HS graduate	Some College	College+
A. Single Mothers					
Mother's time	217.97	126.07	161.20	227.39	362.80
	(13.45)	(17.99)	(15.00)	(20.38)	(47.94)
	271	*26*	*99*	*97*	*49*
HH goods	12.71	7.72	11.30	13.63	17.71
	(0.78)	(1.22)	(1.20)	(1.29)	(2.54)
	356	*40*	*139*	*120*	*57*
Child care	8.99	2.16	5.90	12.45	14.36
	(2.47)	(1.24)	(1.42)	(7.02)	(3.26)
	389	*44*	*154*	*130*	*61*
Total	249.25	136.80	181.73	262.42	417.75
	(15.30)	(19.15)	(16.57)	(23.02)	(54.66)
	253	*25*	*89*	*94*	*45*
B. Two-Parent Households					
Mother's time	285.79	134.12	228.89	273.51	364.18
	(10.34)	(25.65)	(14.83)	(17.86)	(19.68)
	518	*26*	*144*	*167*	*181*
Father's time	294.67	121.75	219.73	255.94	417.71
	(14.75)	(27.06)	(21.50)	(22.57)	(30.57)
	612	*38*	*174*	*183*	*217*
Total Parental time	572.43	290.90	457.09	513.67	749.83
	(21.39)	(58.83)	(35.28)	(30.88)	(41.15)
	480	*23*	*133*	*151*	*173*
HH goods	19.77	9.70	15.96	18.93	25.54
	(0.75)	(2.01)	(1.16)	(1.47)	(1.34)
	685	*41*	*205*	*203*	*236*
Child care	10.98	6.44	4.57	9.93	18.37
	(2.98)	(3.05)	(1.05)	(1.63)	(8.65)
	730	*45*	*215*	*224*	*246*
Total	608.18	298.13	482.53	544.25	797.43
	(23.12)	(64.02)	(36.38)	(33.07)	(44.84)
	456	*21*	*129*	*139*	*167*

Notes: Samples restricted to children ages 0–12 from families with only 1 or 2 children ages 0–12, parents ages 18–65, mothers ages 16–45 when youngest child was born. Table reports means (std. errors) and *number of obs.* Expenditures in 2002 dollars.

Table 2: Child Investment Expenditure Shares by Mother's Education (PSID, 2002)

		Mother's Education			
Expenditure Shares	All	HS dropout	HS graduate	Some College	College+
A. Single Mothers					
Mother's time	0.81	0.88	0.79	0.78	0.86
	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)
HH goods	0.16	0.11	0.17	0.18	0.11
	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)
Child care	0.04	0.02	0.04	0.04	0.03
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Sample size	*253*	*25*	*89*	*94*	*45*
B. Two-Parent Households					
Mother's time	0.51	0.48	0.53	0.52	0.50
	(0.01)	(0.05)	(0.02)	(0.02)	(0.02)
Father's time	0.41	0.44	0.40	0.41	0.42
	(0.01)	(0.05)	(0.02)	(0.02)	(0.02)
Total Parental time	0.92	0.91	0.92	0.92	0.92
	(0.01)	(0.03)	(0.01)	(0.01)	(0.01)
HH goods	0.06	0.05	0.06	0.06	0.05
	(0.00)	(0.02)	(0.01)	(0.01)	(0.01)
Child care	0.02	0.03	0.01	0.02	0.02
	(0.00)	(0.02)	(0.00)	(0.00)	(0.01)
Sample size	*456*	*21*	*129*	*139*	*167*

Notes: Samples restricted to children ages 0–12 from families with only 1 or 2 children ages 0–12, parents ages 18–65, mothers ages 16–45 when youngest child was born. Table reports means (std. errors).

Table 3: Weekly Hours of Child Investment Time by Mother's Education (PSID, 2002)

Time with Children (hours)	All	Mother's Education			
		HS dropout	HS graduate	Some College	College+
A. Single Mothers					
Mother's time	17.79	18.47	16.02	17.79	21.63
	(0.66)	(2.06)	(1.05)	(1.14)	(1.51)
Sample size	*347*	*38*	*135*	*118*	*56*
B. Two-Parent Households					
Mother's time	19.57	15.38	20.77	18.63	20.18
	(0.43)	(1.72)	(0.88)	(0.69)	(0.73)
Father's time	13.00	11.41	12.19	12.57	14.33
	(0.43)	(1.65)	(0.80)	(0.78)	(0.75)
Total parental time	32.57	26.79	32.96	31.20	34.50
	(0.74)	(2.69)	(1.46)	(1.24)	(1.30)
Sample size	*664*	*42*	*190*	*202*	*230*

Notes: Samples restricted to children ages 0–12 from families with only 1 or 2 children ages 0–12, parents ages 18–65, mothers ages 16–45 when youngest child was born. Table reports means (std. errors).

Table 4: Weekly Child Investment Expenditures by Mother's Education (2003–18 ATUS & CEX)

		Mother's Education			
Expenditure Amount	All	HS dropout	HS graduate	Some College	College+
A. Single Mothers					
Mother's time	65.69	27.23	46.04	61.13	128.93
	(2.21)	(3.15)	(2.68)	(2.82)	(7.38)
	4,309	*321*	*1,197*	*1,655*	*1,136*
HH goods	10.20	4.50	5.67	9.69	18.71
	(0.48)	(0.55)	(0.52)	(0.90)	(1.16)
	1,152	*147*	*315*	*410*	*280*
Child care	19.98	7.99	11.67	16.87	39.47
	(1.19)	(1.73)	(1.55)	(1.37)	(3.74)
	1,152	*147*	*315*	*410*	*280*
Total Avg. Expenditures	95.87	39.71	63.39	87.68	187.10
B. Two-Parent Households					
Mother's time	131.68	37.40	72.38	89.98	181.55
	(2.71)	(6.48)	(4.36)	(3.66)	(4.35)
	6,959	*217*	*1,018*	*1,836*	*3,888*
Father's time	127.13	40.53	81.78	94.58	165.42
	(3.14)	(7.79)	(5.89)	(4.70)	(4.93)
	6,026	*167*	*918*	*1,590*	*3,351*
HH goods	20.46	6.59	12.43	17.51	27.68
	(0.34)	(0.56)	(0.47)	(0.52)	(0.59)
	6,663	*508*	*1,200*	*1,866*	*3,089*
Child care	33.60	5.59	12.65	23.39	52.63
	(0.85)	(0.76)	(0.85)	(1.23)	(1.56)
	6,663	*508*	*1,200*	*1,866*	*3,089*
Total Avg. Expenditures	312.88	90.11	179.24	225.46	427.28

Notes: Samples restricted to families with only 1 or 2 children ages 0–12, parents ages 18–65, mothers ages 16–45 when youngest child was born. Table reports means (std. errors) and *number of obs.* Expenditures in 2002 dollars.

Table 5: Summary of PSID-CDS Variables

Model Variable	Name	Measurement Details	Years	Number child-year observations
$W_{m,it}, W_{f,it}$	Mother's and father's wages	Earnings/hours from PSID Main Interview	1985–2002	21,892 & 17,148
$P_{c,it}Y_{c,it}$	Expenditures on child care	PCG interview from CDS, with supplement from Main Interview	1997–2002	800
$\tau_{m,it}$,$\tau_{f,it}$	Mother's and father's time investment	Sum of active time from CDS time diaries	1997, 2002	1,586 & 1,167
$p_{i,t}g_{i,t}$	Household expenditure on market goods	PCG interview from CDS	2002	709
$\Psi_{i,t}$	Child human capital	Letter-Word and Applied Problems scores from CDS	1997, 2002	470 & 467
$Z_{i,t}$	Mother's and father's education, race, household composition, mother's marital status, child's age	PSID Main Interviews, childbirth file	1997–2002	8,148

Table 6: Descriptive Statistics (1997 & 2002 PSID-CDS)

	Num. Obs.	Mean	Std. Deviation
Mother's wage (W_m)	2,349	12.60	8.10
Father's wage (W_f)	1,905	20.14	14.59
Child's age	3,469	7.17	3.43
Mother complete high school	3,469	0.34	0.47
Mother some college	3,469	0.32	0.46
Mother college+	3,469	0.25	0.43
Mother's age	3,469	34.48	7.04
Father complete high school	2,319	0.38	0.49
Father some college	2,319	0.22	0.41
Father college+	2,319	0.30	0.46
Father's age	2,311	37.61	7.44
Mother white	3,431	0.57	0.50
Num. children ages 0–5 in household	3,469	0.56	0.69
Num. children in household	3,469	1.99	0.73

Notes: Samples from 1997 and 2002 PSID CDS include children ages 0–12 from families with no more than 2 children ages 0–12.

Table 7: OLS estimates for mother time/goods relative demand (all mothers)

	(1)	(2)	(3)	(4)	(5)
$\ln(\tilde{W}_{m,i})$	0.467*	0.530*	0.558*	0.567*	0.758*
	(0.078)	(0.083)	(0.084)	(0.084)	(0.098)
Married	-0.276*	-0.256*	-0.183	-0.173	-0.176
	(0.095)	(0.096)	(0.104)	(0.104)	(0.103)
Child's age		-0.084*	-0.093*	-0.096*	-0.115*
		(0.023)	(0.024)	(0.024)	(0.024)
Mother HS grad.		1.598*	1.649*		
		(0.773)	(0.771)		
Mother some coll.		1.495	1.534*	-0.101	
		(0.774)	(0.772)	(0.108)	
Mother coll+		1.389	1.460	-0.185	
		(0.779)	(0.778)	(0.119)	
Mother's age		-0.010	-0.011	-0.010	
		(0.008)	(0.008)	(0.008)	
Mother white			-0.209*	-0.201*	-0.216*
			(0.099)	(0.099)	(0.097)
Num. children ages 0–5 in HH			-0.046	-0.037	0.061
			(0.120)	(0.121)	(0.116)
Num. children in HH			0.133	0.134	0.120
			(0.068)	(0.068)	(0.068)
Mother's log wage fixed effect					-0.425*
					(0.101)
Constant	1.753*	1.247	1.048	2.633*	1.934*
	(0.206)	(0.819)	(0.831)	(0.378)	(0.336)
Implied ρ	-0.877	-1.129	-1.262	-1.312	-3.132
	(0.275)	(0.376)	(0.428)	(0.447)	(1.676)
R-squared	0.062	0.104	0.116	0.109	0.126
N	628	628	628	628	618

Notes: Sample from 2002 PSID CDS includes children ages 0–12 from families with no more than 2 children ages 0–12. Sample is limited to all mothers with predicted probability of work at least 0.7. Standard errors in parentheses. * statistically sig. at 0.05 level.

Table 8: IV estimates (instruments: state dummies) for mother time/goods relative demand (all mothers)

	(1)	(2)	(3)	(4)	(5)
$\ln(\tilde{W}_{m,i})$	0.386	0.667*	0.779*	0.778*	0.752*
	(0.222)	(0.256)	(0.262)	(0.263)	(0.258)
Married	-0.266*	-0.258*	-0.187	-0.177	-0.175
	(0.098)	(0.095)	(0.104)	(0.104)	(0.102)
Child's age		-0.083*	-0.093*	-0.095*	-0.115*
		(0.023)	(0.024)	(0.024)	(0.025)
Mother HS grad.		1.530*	1.542*		
		(0.779)	(0.778)		
Mother some coll.		1.394	1.374	-0.152	
		(0.791)	(0.790)	(0.124)	
Mother coll+		1.258	1.252	-0.281	
		(0.810)	(0.810)	(0.164)	
Mother's age		-0.012	-0.014	-0.013	
		(0.008)	(0.009)	(0.009)	
Mother white			-0.216*	-0.208*	-0.216*
			(0.099)	(0.099)	(0.097)
Num. children ages 0–5 in HH			-0.051	-0.042	0.060
			(0.120)	(0.120)	(0.120)
Num. children in HH			0.159*	0.158*	0.119
			(0.074)	(0.074)	(0.069)
Mother's log wage fixed effect					-0.422*
					(0.183)
Constant	1.953***	1.056	0.709	2.210*	1.947*
	(0.550)	(0.884)	(0.912)	(0.627)	(0.642)
Implied ρ	-0.628	-1.999	-3.517	-3.499	-3.036
	(0.590)	(2.302)	(5.340)	(5.317)	(4.205)
N	628	628	628	628	618

Notes: Sample from 2002 PSID CDS includes children ages 0–12 from families with no more than 2 children ages 0–12. Sample is limited to all mothers with predicted probability of work at least 0.7. Standard errors in parentheses. * statistically sig. at 0.05 level.

Table 9: Estimates for parental time vs. goods relative demand

	OLS				Instrumental Variables			
	All Mothers	Single Mothers	Married Mothers	Married Fathers	All Mothers	Single Mothers	Married Mothers	Married Fathers
$\ln(\tilde{W}_{j,i})$	0.567*	0.514*	0.588*	0.683*	0.778*	0.872*	0.556	0.129
	(0.084)	(0.173)	(0.094)	(0.101)	(0.263)	(0.298)	(0.284)	(0.350)
Married	-0.173				-0.177			
	(0.104)				(0.104)			
Child's age	-0.096*	-0.087	-0.109*	-0.054	-0.095*	-0.083	-0.109*	-0.051
	(0.024)	(0.048)	(0.028)	(0.030)	(0.024)	(0.047)	(0.028)	(0.031)
Parent some college	-0.101	0.125	-0.255*	-0.147	-0.152	0.033	-0.248	-0.011
	(0.108)	(0.195)	(0.130)	(0.145)	(0.124)	(0.202)	(0.143)	(0.170)
Parent coll+	-0.185	0.160	-0.383**	-0.044	-0.281	0.006	-0.369*	0.251
	(0.119)	(0.241)	(0.137)	(0.142)	(0.164)	(0.260)	(0.183)	(0.230)
Parent's age	-0.010	-0.022	-0.001	-0.018*	-0.013	-0.024	-0.001	-0.012
	(0.008)	(0.015)	(0.010)	(0.009)	(0.009)	(0.015)	(0.011)	(0.010)
Mother white	-0.201*	-0.316	-0.167	-0.076	-0.208*	-0.346	-0.167	0.007
	(0.099)	(0.192)	(0.115)	(0.134)	(0.099)	(0.191)	(0.114)	(0.145)
Num. children ages 0–5 in HH	-0.037	-0.494*	0.169	0.199	-0.042	-0.480*	0.171	0.196
	(0.121)	(0.246)	(0.137)	(0.139)	(0.120)	(0.243)	(0.136)	(0.142)
Num. children in HH	0.134	0.102	0.170*	0.131	0.158*	0.125	0.166	0.118
	(0.068)	(0.123)	(0.083)	(0.091)	(0.074)	(0.123)	(0.091)	(0.093)
Constant	2.633*	3.145*	2.189*	1.656*	2.210*	2.337*	2.250*	2.844*
	(0.378)	(0.759)	(0.445)	(0.475)	(0.627)	(0.929)	(0.682)	(0.866)
Implied ρ	-1.312	-1.058	-1.425	-2.151	-3.499	-6.809	-1.252	-0.148
	(0.447)	(0.734)	(0.554)	(0.999)	(5.317)	(18.142)	(1.439)	(0.461)
R-squared	0.109	0.098	0.132	0.121				
N	628	197	431	486	628	197	431	486

Notes: Sample from 2002 PSID CDS includes children ages 0–12 from families with no more than 2 children ages 0–12. Samples examining mother (father) time are limited to those with predicted probability of work at least 0.7 (0.85). Specifications for mothers (fathers) include mother's (father's) log relative wage, education indicators, and age. Standard errors in parentheses. * statistically sig. at 0.05 level.

Table 10: OLS estimates for child care/goods relative demand (single mothers)

	(1)	(2)	(3)	(4)	(5)
$\ln(\tilde{P}_{c,i})$	0.439	0.449	0.214	0.150	0.165
	(0.380)	(0.402)	(0.402)	(0.384)	(0.399)
Child's age		-0.196*	-0.174*	-0.135	-0.178*
		(0.073)	(0.069)	(0.073)	(0.088)
Mother some coll.		0.158		0.014	0.175
		(0.309)		(0.326)	(0.323)
Mother coll+		-0.239		-0.406	-0.258
		(0.336)		(0.338)	(0.415)
Mother's age		0.005		0.015	0.008
		(0.020)		(0.021)	(0.022)
Mother white		-0.452	-0.503	-0.473	-0.404
		(0.293)	(0.282)	(0.280)	(0.317)
Num. children		-0.247	-0.515	0.073	-0.087
ages 0–5 in HH		(0.338)	(0.318)	(0.329)	(0.456)
Num. children		0.030	0.096	0.124	0.071
in HH		(0.211)	(0.204)	(0.194)	(0.207)
Mother's log wage			0.206		
fixed effect			(0.246)		
$\ln(1+R_{m,i})$				0.550*	
				(0.138)	
$\ln\left(1+e^{\tilde{\Phi}_{m,i}}\right)$					0.228
					(0.627)
Constant	0.236	1.891*	2.032*	-0.477	1.133
	(0.363)	(0.900)	(0.798)	(1.073)	(2.703)
Implied γ	-0.781	-0.815	-0.272	-0.177	-0.197
	(1.206)	(1.324)	(0.650)	(0.531)	(0.572)
Implied ρ				-0.500	-0.271
				(1.903)	(0.827)
R-squared	0.013	0.126	0.108	0.299	0.136
N	104	103	98	83	97

Notes: Sample from 2002 PSID CDS includes children ages 0–12 from families with no more than 2 children ages 0–12. Sample is limited to single mothers with predicted probability of work at least 0.7. Standard errors in parentheses. * statistically sig. at 0.05 level.

Table 11: OLS estimates for child care/goods relative demand (two-parent households)

	(1)	(2)	(3)	(4)	(5)
$\ln(\tilde{P}_{c,i})$	0.478	0.637	0.540	0.521	0.603
	(0.322)	(0.330)	(0.333)	(0.361)	(0.357)
Child's age		-0.151*	-0.168*	-0.119	-0.146*
		(0.054)	(0.055)	(0.063)	(0.068)
Mother some coll.		0.079		0.210	0.080
		(0.256)		(0.290)	(0.278)
Mother coll+		-0.329		0.010	-0.346
		(0.265)		(0.301)	(0.283)
Mother's age		0.004		-0.028	0.012
		(0.028)		(0.034)	(0.031)
Father some coll.		0.248		0.194	0.257
		(0.244)		(0.274)	(0.263)
Father coll+		-0.377		-0.621*	-0.397
		(0.239)		(0.262)	(0.271)
Father's age		-0.018		0.013	-0.021
		(0.024)		(0.028)	(0.026)
Mother white		-0.242	-0.347	-0.262	-0.203
		(0.198)	(0.213)	(0.213)	(0.218)
Num. children		-0.110	0.090	-0.046	-0.065
ages 0–5 in HH		(0.197)	(0.202)	(0.223)	(0.246)
Num. children		0.028	0.038	0.124	0.042
in HH		(0.185)	(0.189)	(0.205)	(0.215)
Mother's log wage			-0.363		
fixed effect			(0.208)		
Father's log wage			-0.103		
fixed effect			(0.214)		
$\ln(1 + R_{f,i} + R_{m,i})$				0.583*	
				(0.114)	
$\ln\left(1 + e^{\tilde{\Phi}_{f,i}} + e^{\tilde{\Phi}_{m,i}}\right)$					0.136
					(0.492)
Constant	0.049	2.025*	1.648*	-0.595	1.331
	(0.332)	(0.852)	(0.702)	(1.033)	(2.116)
Implied γ	-0.914	-1.757	-1.173	-1.090	-1.521
	(1.179)	(2.507)	(1.572)	(1.575)	(2.272)
implied ρ				4.997	-2.312
				(15.107)	(6.089)
R-squared	0.011	0.152	0.101	0.312	0.145
N	203	203	191	149	179

Notes: Samples from 2002 PSID CDS include children ages 0–12 from families with no more than 2 children ages 0–12. Sample is limited to two-parent households with predicted probability that both parents work at least 0.65. Standard errors in parentheses. * statistically sig. at 0.05 level.

Table 12: OLS estimates for time/goods and child care/goods relative demand accounting for measurement error & independent unobserved heterogeneity

	Single Mothers	Two Parent Households	
		Unrestricted Meas. Error	Restricted Meas. Error
$\ln(\tilde{P}_{c,i})$	0.231	0.677*	0.687*
	(0.376)	(0.319)	(0.319)
Child's age	-0.074	-0.090	-0.088
	(0.062)	(0.053)	(0.053)
Mother some coll.	0.288	0.299	0.256
	(0.299)	(0.258)	(0.256)
Mother coll+	0.280	0.335	0.311
	(0.355)	(0.267)	(0.267)
Father some coll.		0.134	0.171
		(0.238)	(0.236)
Father coll+		-0.377	-0.374
		(0.235)	(0.236)
$\ln(1+R_{m,i})$ or	0.647	-0.064	-0.113
$\ln(1+R_{f,i}+R_{m,i})$	(0.399)	(0.272)	(0.270)
Constant	-1.731	1.018	1.065
	(1.671)	(1.272)	(1.274)
Implied γ	-0.300	-2.097	-2.190
	(0.635)	(3.056)	(3.248)
Implied ρ	-1.893	-1.751	-1.608
	(8.601)	(2.440)	(2.086)
R-squared	0.378	0.438	0.432
N	83	149	149

Notes: Samples from 2002 PSID CDS include children ages 0–12 from families with no more than 2 children ages 0–12. Sample for single mothers (two-parent households) is limited to those with predicted probability that the mother (both parents) work at least 0.7 (0.65). Estimated coefficients related to measurement error in Equations (31) and (35) not shown. Restricted measurement error case assumes $\sigma^2_{W_m\tau_m} = \sigma^2_{W_f\tau_f}$. Standard errors in parentheses. * statistically sig. at 0.05 level.

Table 13: GMM estimates for time/goods and child care/goods relative demand accounting for measurement error and unobserved heterogeneity (two-parent households)

	General Measurement Error		Restricted Measurement Error	
	No Instruments	Instruments: State	No Instruments	Instruments: State
γ	-1.905*	-1.827	-1.738*	-1.461
	(0.909)	(1.812)	(0.829)	(1.292)
ρ	-1.762*	-1.428	-1.765*	-1.481
	(0.504)	(1.325)	(0.506)	(1.299)
$(\phi_m - \phi_g)$:				
Constant	5.594*	2.985*	5.608*	3.009*
	(1.267)	(1.306)	(1.270)	(1.321)
Child's age	-0.284*	-0.204	-0.284*	-0.209
	(0.093)	(0.139)	(0.093)	(0.138)
Mother some coll.	-0.766	-0.543	-0.757	-0.561
	(0.405)	(0.599)	(0.404)	(0.598)
Mother coll+	-1.116*	-0.992	-1.102*	-1.024
	(0.471)	(0.894)	(0.469)	(0.883)
Mother's age	-0.003	0.035	-0.004	0.035
	(0.027)	(0.028)	(0.027)	(0.028)
Mother white	-0.321	-0.170	-0.314	-0.174
	(0.305)	(0.293)	(0.306)	(0.298)
Num. children	0.600	0.865	0.595	0.882
ages 0–5 in HH	(0.354)	(0.511)	(0.355)	(0.509)
Num. children	0.394	0.435	0.398	0.447
in HH	(0.261)	(0.379)	(0.261)	(0.377)
$(\phi_f - \phi_g)$:				
Constant	5.039*	3.457*	4.992*	3.488*
	(1.320)	(1.470)	(1.318)	(1.479)
Child's age	-0.128	-0.080	-0.128	-0.081
	(0.089)	(0.092)	(0.089)	(0.093)
Father some coll.	-0.456	-0.344	-0.454	-0.356
	(0.414)	(0.495)	(0.414)	(0.499)
Father coll+	-0.047	-0.062	-0.040	-0.074
	(0.402)	(0.471)	(0.402)	(0.475)
Father's age	-0.052*	-0.028	-0.051	-0.029
	(0.026)	(0.031)	(0.026)	(0.031)
Mother white	-0.249	-0.083	-0.248	-0.089
	(0.385)	(0.351)	(0.385)	(0.357)
Num. children	0.503	0.622	0.499	0.636
ages 0–5 in HH	(0.393)	(0.470)	(0.394)	(0.472)
Num. children	0.263	0.312	0.265	0.318
in HH	(0.246)	(0.259)	(0.246)	(0.261)
ϕ_g:				
Constant	-2.446	-3.464	-1.766	-2.169
	(1.570)	(5.859)	(1.550)	(2.523)
Child's age	0.249	0.204	0.261	0.258
	(0.132)	(0.184)	(0.137)	(0.160)
Mother some coll.	-0.784	-0.834	-0.628	-0.572
	(0.646)	(0.732)	(0.651)	(0.720)
Mother coll+	-0.887	-0.831	-0.808	-0.691
	(0.661)	(0.651)	(0.673)	(0.772)
Father some coll.	-0.387	-0.115	-0.549	-0.418
	(0.598)	(1.198)	(0.600)	(0.634)
Father coll+	0.962	0.882	0.903	0.897
	(0.724)	(0.736)	(0.745)	(0.823)
implied $\epsilon_{\tau,g}$	0.363	0.412	0.362	0.403
	(0.066)	(0.225)	(0.066)	(0.211)
implied $\epsilon_{Y,g}$	0.344	0.354	0.365	0.406
	(0.108)	(0.227)	(0.111)	(0.213)
Objective Fun.	0.0006	0.0056	0.0012	0.0056
N	547	547	547	547

Notes: Samples from 2002 PSID CDS include children ages 0–12 from families with no more than 2 children ages 0–12. Sample is limited to two-parent households with predicted probability that both parents work at least 0.65. Estimated coefficients related to measurement error in Equation (35) not shown. Restricted measurement error cases assume $\sigma^2_{W_m\tau_m} = \sigma^2_{W_f\tau_f}$. Standard errors in parentheses. * statistically sig. at 0.05 level.

Table 14: Input Ratios used in Estimation

Marital Status	Year	Ratios
Married ($M_i = 1$)	1997 ($t = 0$)	$\frac{Y_c}{\tau_m}$
Married ($M_i = 1$)	2002 ($t = 5$)	$\frac{Y_c}{\tau_m}, \frac{\tau_m}{g}, \frac{Y_c}{g}, \frac{\tau_f}{g}$
Single ($M_i = 0$)	1997 ($t = 0$)	$\frac{Y_c}{\tau_m}$
Single ($M_i = 0$)	2002 ($t = 5$)	$\frac{Y_c}{\tau_m}, \frac{\tau_m}{g}, \frac{Y_c}{g}$

Table 15: GMM estimates for full child production function

	Includes Achievement Moments		Without Achievement Moments
	No Borrowing/Saving	Unconstrained	
δ_1	0.11*	0.05*	
	(0.04)	(0.02)	
δ_2	0.94*	0.95*	
	(0.02)	(0.02)	
γ	-2.05*	-2.13*	-2.36*
	(0.94)	(1.01)	(1.18)
ρ	-1.14*	-1.39*	-1.39*
	(0.30)	(0.38)	(0.40)
ϕ_m:			
Married	1.90	2.43	2.30
	(1.03)	(1.29)	(1.25)
Single	2.22*	2.62*	2.60*
	(0.81)	(1.00)	(0.99)
Mother some coll.	-0.22	-0.36	-0.25
	(0.24)	(0.30)	(0.28)
Mother coll+	0.17	0.15	0.10
	(0.24)	(0.29)	(0.27)
Child's age	-0.11*	-0.13*	-0.14*
	(0.04)	(0.05)	(0.07)
Num. children	-0.83*	-0.94*	-1.06*
ages 0–5 in HH	(0.26)	(0.30)	(0.34)
ϕ_f:			
Married	0.95	1.20	1.05
	(0.98)	(1.19)	(1.14)
Father some coll.	-0.88*	-0.94*	-0.78
	(0.39)	(0.44)	(0.44)
Father coll+	0.36	0.36	0.43
	(0.33)	(0.37)	(0.38)
Child's age	0.03	0.04	0.03
	(0.06)	(0.07)	(0.07)
Num. children	-0.34	-0.30	-0.48
ages 0–5 in HH	(0.30)	(0.34)	(0.36)
ϕ_g:			
Married	-2.26*	-2.03	-2.10*
	(0.87)	(1.06)	(1.02)
Single	-2.36*	-2.31*	-2.35*
	(0.67)	(0.78)	(0.77)
Mother some coll.	0.13	0.09	0.12
	(0.30)	(0.34)	(0.34)
Mother coll+	0.34	0.43	0.32
	(0.36)	(0.42)	(0.41)
Father some coll.	-0.49	-0.50	-0.46
	(0.34)	(0.38)	(0.39)
Father coll+	0.49	0.52	0.62
	(0.39)	(0.44)	(0.45)
Child's age	0.15*	0.17*	0.17*
	(0.06)	(0.07)	(0.07)
Num. children	-0.56	-0.64	-0.83*
ages 0–5 in HH	(0.30)	(0.35)	(0.39)

Notes: Sample from PSID CDS includes children ages 0–12 from families with no more than 2 children ages 0–12. Moments using mother (father) time are limited to those with predicted probability of work at least 0.7 (0.85). Standard errors in parentheses. * statistically sig. at 0.05 level.

Table 16: Variance of Log Investment Expenditure and Price

	$Var(\ln E_t^o)$	$Var(\ln \overline{p}_t)$	$\frac{Var(\ln \overline{p}_t)}{Var(\ln E_t^o)}$
Single Mothers	0.70	0.34	0.48
Two-Parent Households	0.57	0.27	0.48

Table 17: Actual and Counterfactual Variance of Log Investment Price

	Actual	Equalizing:	
		Wages	Wages and Other Prices
Single Mothers	0.34	0.09	0.08
Two-Parent Households	0.27	0.11	0.11

Table 18: Gaps in Investment (% Difference between Non-College and College)

	Baseline	Equalizing:		
		Preferences	Preferences and Wages	All but Technology
A. Single Mothers				
Total Investment:				
Expenditure (E)	58.22	33.27	2.42	0.00
Price ($\bar{p}$)	24.45	24.45	-6.57	-4.61
Quantity (X)	37.58	15.94	10.79	6.38
Mother's Time Investment (τ_m)	24.91	5.26	1.58	-2.28
B. Two-Parent Households				
Total Investment:				
Expenditure (E)	86.50	71.51	-0.12	0.00
Price ($\bar{p}$)	86.45	86.45	15.20	14.81
Quantity (X)	8.05	-0.64	-5.76	-4.73
Mother's Time Investment (τ_m)	2.56	-5.69	-0.70	-0.40

Table 19: Effects of 10% Reduction in Prices: Single Mothers

	Nested CES			Cobb-Douglas		
	Wages	Goods	Child Care	Wages	Goods	Child Care
A. Change in Investment at Age 5 (%)						
Total Expenditure (E)	-10.00	0.00	0.00	-10.00	0.00	0.00
Composite Price ($\bar{p}$)	-8.09	-0.50	-1.47	-7.94	-0.56	-1.68
Investment Quantity:						
Mother's Time (τ_m)	-1.42	0.28	1.09	0.00	0.00	0.00
Goods (g)	-6.10	5.32	1.03	-10.00	11.11	0.00
Child Care (Y_c)	-4.82	0.36	4.60	-10.00	0.00	11.11
Total (X)	-2.27	0.55	1.64	-2.23	0.57	1.70
B. Effects on Age 13 Achievement						
100×Log Achievement at age 13	-1.64	0.58	0.98	-1.61	0.59	1.01
Value (% Cons. over Ages 5–12)	-1.94	0.70	1.18	-1.90	0.71	1.22
C. Welfare and Distortions over Ages 5–12 (Dollars Discounted to Age 5)						
Welfare Gain (EV)	-26054	1350	2259	-26354	1536	2588
Distortions:						
Relative Investment	59	32	35	163	77	122
Total	753	35	46	851	82	136

Table 20: Effects of 10% Reduction in Prices: Two-Parent Households

	Nested CES			Cobb-Douglas		
	Wages	Goods	Child Care	Wages	Goods	Child Care
A. Change in Investment at Age 5 (%)						
Total Expenditure (E)	-10.00	0.00	0.00	-10.00	0.00	0.00
Composite Price ($\bar{p}$)	-9.00	-0.31	-0.71	-8.95	-0.34	-0.82
Investment Quantity:						
Mother's Time (τ_m)	-0.77	0.17	0.56	0.00	0.00	0.00
Father's Time (τ_f)	-0.75	0.17	0.54	0.00	0.00	0.00
Goods (g)	-5.49	5.22	0.51	-10.00	11.11	0.00
Child Care (Y_c)	-4.16	0.22	4.07	-10.00	0.00	11.11
Total (X)	-1.23	0.33	0.84	-1.17	0.34	0.84
B. Effects on Age 13 Achievement						
100×Log Achievement at Age 13	-0.82	0.37	0.50	-0.82	0.35	0.49
Value (% Cons. over Ages 5–12)	-0.60	0.25	0.34	-0.59	0.25	0.35
C. Welfare and Distortions over Ages 5–12 (Dollars Discounted to Age 5)						
Welfare Gain (EV)	-73257	1521	2052	-73537	1650	2280
Distortions:						
Relative Investment	67	37	34	177	86	117
Total	2015	39	39	2124	89	123