A Theory of Falling Growth and Rising Rents

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Abstract

Growth has fallen in the U.S. while firm concentration has risen. We propose a theory linking these trends in which the driving force is falling overhead costs of spanning multiple markets. In response, the most efficient firms (with higher markups) spread into new markets, thereby generating a temporary burst of growth. Eventually, due to greater competition from efficient firms, within-firm markups and incentives to innovate fall. When we calibrate our model, we find the rise in market share of more efficient firms outweighs the drop in long-run growth, leaving welfare modestly enhanced by the fall in overhead costs.

JEL classification: O31, O47, O51.

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1 Introduction

Recent studies have documented the following patterns in the U.S. economy over the past several decades:[1]

1. Slow growth interrupted by a temporary burst of growth
2. Rising firm concentration within industries at the national level
3. Reallocation of market share toward low labor share firms

Are these patterns linked? Is rising firm concentration slowing growth and driving down the aggregate labor share? Or are large firms more efficient so that their rising market share brings aggregate productivity benefits? These hypotheses are not mutually exclusive, so the answer could be that both are true. If so, there might be a tradeoff between level benefits and adverse longer run growth effects of rising concentration.

To contribute to this debate, we construct a model of endogenous growth and firm dynamics. There are two sources of firm heterogeneity in our model. The first is product quality, which differs across the product lines of a firm and improves endogenously through creative destruction. The second is process efficiency, which we assume to be common to all product lines of a firm. High process efficiency firms command a higher markup than low productivity firms, conditional on a given quality advantage over competitors.

A possible source of persistent heterogeneity in process efficiency across firms is their intangible capital. Firms such as Walmart and Amazon have established successful business models and logistics that are evidently hard to copy. Both firms experienced considerable expansion into new geographic and/or product markets over the past two decades. Similarly, Amazon and Microsoft have acquired dominant positions in cloud storage and computing

[1]See Fernald, Hall, Stock and Watson (2017) on productivity growth; Autor, Dorn, Katz, Patterson and Van Reenen (2020) on rising concentration; and Kehrig and Vincent (2020), De Loecker, Eeckhout and Unger (2020), and Baqaee and Farhi (2020) on reallocation to low labor share firms. We discuss this evidence in more detail in the next section.

due to their logistical advantage over potential competitors. Such firms have achieved a level of process efficiency which is arguably harder to reverse engineer and build upon than quality, which may be more observable.

The story we propose is that the IT (Information Technology) wave around 1995–2005 allowed high process efficiency firms to expand into a wider set of product lines. We model the IT wave as a downward shift in the overhead cost function $c(n)$ of running n product lines. This cost is assumed to be convex in n, which puts a brake on the quality innovation (creative destruction) efforts of high process efficiency firms. The downward shift in the overhead cost schedule induces high process efficiency firms to cover a larger fraction of lines. This expansion fuels a temporary surge in aggregate productivity growth — both because these firms innovate to take over more products and because they apply their superior process efficiency to those additional products.

Since high process efficiency firms have higher markups and lower labor shares on average across their product lines, their expansion into more markets is a force pushing the aggregate markup up and the aggregate labor share down. Within-firm markups eventually fall, however, as the quality leader on a product line is more likely to face a high process efficiency competitor. Competition from an efficient follower can limit the leader's markup whether the leader is a high or low process efficiency firm.

While the IT wave induces a burst of growth in the short run in our model, in the long run the fall in overhead cost may lead to a slowdown in productivity growth. The expansion of high productivity firms into more lines eventually deters innovation because innovating on a line where the incumbent firm has high productivity yields lower profits. Both high and low productivity firms eventually curtail their efforts at creative destruction, knowing they will face stiffer competition. This can outweigh the positive direct effect of a downward shift in the overhead cost on R&D incentives, such that long run innovation and productivity growth may fall. Thus falling growth can coincide with rising rents (profits net of R&D and overhead costs).

To gauge magnitudes, we choose parameter values to fit a pre-IT revolution period (1987–1995) in terms of productivity growth, the level of concentration, the aggregate markup, and the correlation across firms between their labor share and sales share. We then entertain shocks to three parameters to hit three targets. We allow the overhead cost, the process efficiency advantage of the best firms, and the scale of R&D costs to change in order to fit the post-2005 values of productivity growth, concentration, and the revenue per worker edge of the best firms. We then examine the effect of falling overhead costs on the path of productivity and consumption, and therefore welfare.

Our calibrated fall in overhead costs can explain a nontrivial portion of the temporary burst of productivity growth and the subsequent growth slowdown: about 10 basis points of the 1 percentage point acceleration, and 20 basis points of the 1.4 percentage point slowdown. Growth ends up 18 basis points lower after 2005 than before 1996, and lower overhead costs contribute 9 basis points to this, in our estimation. We find that the short run burst of growth outweighs the long run decline in growth, leaving consumption-equivalent welfare about 1/3 of a percentage point higher. Thus, in our calibrated model, welfare was enhanced by the fall in overhead costs and the resulting rise of superstar firms.

Our paper is complementary to a number of other recent studies on falling growth and rising concentration. These studies feature different driving forces than our IT-linked fall in overhead costs. The driving force is declining imitation rates in Akcigit and Ates (2019), declining population growth in Peters and Walsh (2020), and declining interest rates in Liu, Mian and Sufi (2020). In De Ridder (2020) some firms become particularly efficient at reducing their marginal costs through intangible inputs, which discourages other firms from innovating. Compared to these papers, Our study puts more emphasis on understanding the temporary burst in productivity growth and weighing this burst against the long run drop in growth. We also highlight that within-firm markups fell both in the data and in our model, offsetting the reallocation of market share toward high markup firms.

Like us, Hsieh and Rossi-Hansberg (2020) model how an IT shock can lead to rising national concentration and a burst of process improvements. They present evidence for Wholesale Trade, Retail Trade, and Services on the expansion of large firms into more geographic markets. They do not model long-run growth or changes in labor share due to markup dispersion. We follow them in focusing on these three sectors, which constitute about one-half of value added and two-thirds of employment in the nonfarm business sector.[2]

Our paper also relates to Hopenhayn, Neira and Singhania (2018) and Chatterjee and Eyigungor (2019), who study rising concentration. And to recent papers on forces behind the declining aggregate labor share such as Karabarbounis and Neiman (2013, 2019), Martinez (2019), Farhi and Gourio (2018), Kaymak and Schott (2020), Barkai (2020), Koh, Santaeulàlia-Llopis and Zheng (2020) and Eggertsson, Robbins and Wold (2020).

Kehrig and Vincent (2020) and Autor, Dorn, Katz, Patterson and Van Reenen (2020) look at labor share in U.S. Census data, while Baqaee and Farhi (2020) and De Loecker, Eeckhout and Unger (2020) estimate markups in Compustat firms. These papers decompose the evolution of the aggregate labor share (or markup) into within-firm and between-firm components. They find the dominant contributor to be the rising market share of low labor share (high markup) firms. We contribute to this literature by linking these trends to the slowdown in U.S. growth in recent decades.

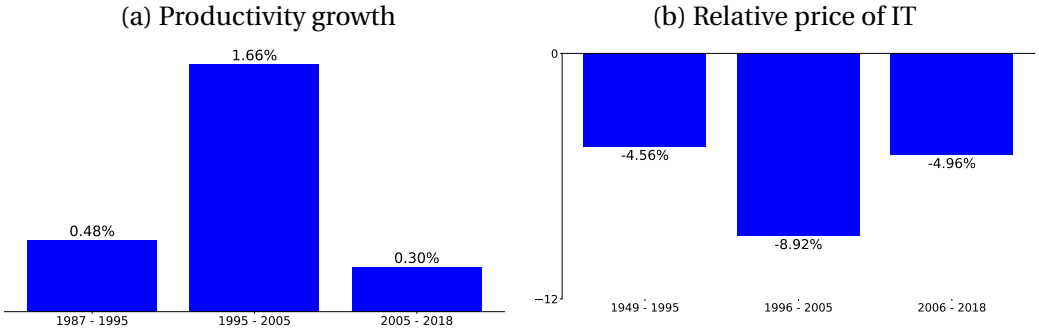
Section 2 describes the empirical patterns that motivate our modeling effort. Section 3 lays out our model. Section 4 solves for the steady state and performs some comparative statics. Section 5 calibrates the model to see how much a drop in overhead costs can contribute to the burst of growth and lower long-run growth. Section 6 concludes.

[2]Retail Trade, Wholesale Trade, and Services have the advantage of data going back further (before the 1990s) for productivity growth, concentration, and labor shares than for sectors such as Utilities and Transportation or Finance, Insurance and Real Estate. We also exclude Manufacturing since it may have been most affected by import competition from China — see Autor, Dorn and Hanson (2013) and Acemoglu and Restrepo (2018), for example.

2 Stylized facts

Fact 1: Slow growth interrupted by a burst of growth. Figure 1a presents U.S. annual TFP growth in Trade and Service industries from the Bureau of Labor Statistics (BLS) KLEMS data.[3] The BLS attempts to net out the contribution of both physical and human capital growth to output growth. The BLS sometimes subtracts contributions from R&D and other intellectual property investments; we consistently included this portion in TFP growth as part of what we are trying to explain. The Figure shows growth accelerating from its 1987–1995 average of 0.5% per year to 1.7% per year from 1995–2005, before falling to just 0.3% per year from 2005–2018.[4] Figure 1b shows that IT prices fell sharply at the same time that TFP growth accelerated.

Figure 1: Productivity growth and relative price of IT



The figures plot the average productivity growth and relative price of IT within each subperiod. The unit is percentage points. **Left panel:** Source: BLS KLEMS multifactor productivity series. We calculate yearly productivity growth in two digit NAICS trade and service industries by adding R&D and IP contribution to BLS MFP and then expressing the sum to labor augmenting form. We calculate trade and service growth by aggregating industry growth rates using industry share of labor costs. **Right panel:** Source: BEA. We calculate change per year in the price of IT relative to the GDP deflator.

[3] https://www.bls.gov/mfp/special_requests/klemscombinedbymeasure.xlsx See Figure A1 in the Online Appendix A for U.S. annual TFP growth in all non-farm private industries.

[4] Fernald, Hall, Stock and Watson (2017) and Bergeaud, Cette and Lecat (2016) argue that the recent TFP growth slowdown is statistically significant and predates the Great Recession. Syverson (2017) and Aghion, Bergeaud, Boppart, Klenow and Li (2019) contend that the slowdown is unlikely to be fully attributable to growing measurement errors. Aghion et al. (2019) find that measurement error did not increase at all for trade and service industries.

Fact 2: Rising concentration. Table 1, presents the average change from 1982 to 2012 in top 20 firm concentration within 4-digit NAICS inside Retail Trade, Wholesale Trade, and Service industries, respectively.[5] These results are from firm-level data in U.S. Census years. Aggregating across the three sectors, top 20 concentration rose from 27% to 35%.

Table 1: Cumulative change in concentration 1982–2012 (ppt)

	RET	WHO	SRV	ALL 3
Top 20 firms sales share 1982	29	45	21	27
Top 20 firms sales share 2012	46	57	27	35
Change	17	12	6	8

Source: Figure 4 of Autor et al. (2020) and BLS KLEMS. Concentration in each industry are averages across 4-digit industries, with the industries weighted by industry sales shares. Concentration in ALL 3 (=RET+WHO+SRV) is the sales-weighted average share across all three sectors.

Table 2 displays the ratio of sales to employment of the top 20 firms relative to smaller firms in the three sectors. For the three sectors combined, the ratio was 1.48 for both 1982–1992 and 2007–2012, consistent with stable relative markups.

Table 2: Sales/employment of top 20 firms relative to remaining firms

	RET	WHO	SRV	ALL 3
1982–1992 average	1.19	2.76	1.27	1.48
2007–2012 average	1.18	2.73	1.32	1.48
Change	-0.01	-0.03	0.05	0.00

Source: Figure 4 of Autor et al. (2020). Sales over employment in ALL 3 (=RET+WHO+SRV) is the labor cost-weighted average across all three sectors.

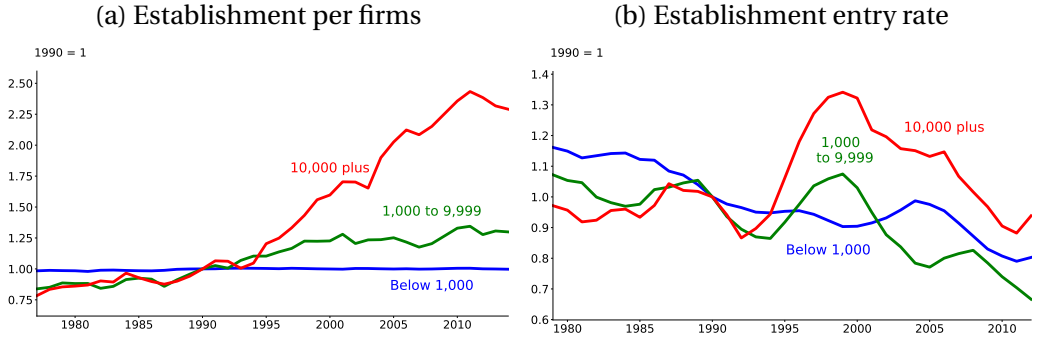
Figure 2a shows the number of establishments per firm from 1980 to 2014 in three size bins based on U.S. Census Bureau Business Dynamic Statistics.

[5]The Table 1 rise in national concentration contrasts with falling local concentration documented by Rossi-Hansberg, Sarte and Trachter (2020) and Rinz (2018). One explanation for the diverging trends is that the largest firms grew by adding establishments in new locations.

Firms with 10,000+ employees added establishments steadily starting in the early 1990s, when the relative price of IT plunged and TFP growth accelerated.[6]

Figure 2b shows the *rate* at which large firms added new establishments relative to their stock of establishments. This rate can be viewed as a crude proxy for their pace of product innovation. The largest firms experienced a burst of establishment entry in the 1990s, which receded from 2005 onward. Again, these broad trends dovetail with the acceleration and deceleration of TFP growth. In what follows, we offer the IT revolution as a driver of this comovement between aggregate productivity growth and the entry rate of establishments at the largest firms.

Figure 2: Establishments per firm by firm size in trade and service industries



Source: U.S. Census Bureau Business Dynamic Statistics. Left-hand side panel plots the number of establishments per firm relative to 1990 within employment bins. Right-hand side panel shows the number of new establishments over the total number of establishments for different firm size bins. The lines represent 5-year centered moving average, relative to 1990.

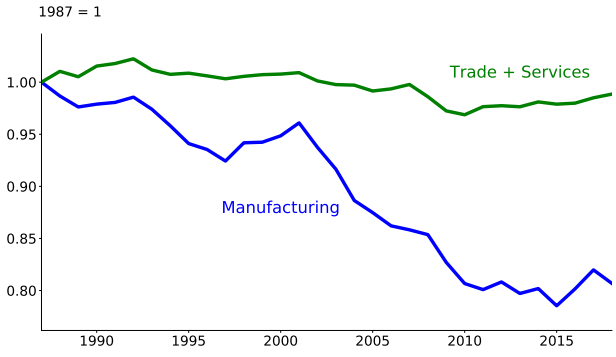
IT as a driving force We focus on changes in IT as a possible driver of the patterns described above for several reasons. First, price declines for IT goods accelerated sharply for a decade from the mid-1990s to the mid-2000s as shown in Figure 1b. Second, TFP growth surged from the mid-1990s to mid-2000s in IT-producing and IT-intensive sectors relative to

[6]Cao, Sager, Hyatt and Mukoyama (2019) document a similar pattern in the Quarterly Census of Employment and Wages data, and Rinz (2018) documents increasing number of markets with at least one establishment belonging to a top 5 firm.

non-IT-intensive sectors.[7] Third, Crouzet and Eberly (2019) and Lashkari, Bauer and Boussard (2019) document that bigger firms invest a higher share of their sales in intangibles and IT, respectively. The former evidence is for U.S. firms and the latter for French firms. Bessen (2019) provides evidence that industries with higher IT intensity experienced higher growth in the sales share of the largest firms. Babina, Fedyk, He and Hodson (2020) demonstrate that larger firms invested more in Artificial Intelligence in the last decade, and moved into more markets as a result.

Fact 3: Reallocation of market share toward low labor share firms. According to the BLS, the labor share of output in the nonfarm business sector fell about 6 percentage points since 1990. But Kehrig and Vincent (2020) stress that this decline was almost entirely driven by manufacturing. Autor et al. (2020) likewise find a declining labor share most sharply in manufacturing.

Figure 3: Labor share over time



Source: BLS KLEMS. Labor share is equal to Cost of Labor + Cost of Purchased Business Services divided by Cost of Labor + Cost of Purchased Business Services + Cost of Capital. Trade and Services consists of Retail Trade, Wholesale Trade and Service industries. The end values are 0.96 for Trade and Services and 0.75 for Manufacturing if labor costs and value added exclude purchased services.

As mentioned, manufacturing may have been more affected by automation, outsourcing, and import competition from China than other sectors. Thus our focus is on Retail Trade, Wholesale Trade, and Services. These sectors, make up around one-half of value added and two-thirds of

[7] Online Appendix Figure A2 plots TFP growth in these sectors, updating Fernald (2015).

Table 3: Cumulative change in labor share 1982–2012 (ppt)

	RET	WHO	SRV
$\Delta \frac{\text{Payroll}}{\text{Sales}}$	-0.85	-0.08	0.23
Within firm	4.39	4.66	1.73
Between	-5.44	-4.59	-0.76

Source: Table 5 in Autor et al. (2020). This is a Melitz-Polanec decomposition of the change in the labor share. The entry and exit margin is not reported. The unit is percentage points. RET, WHO, and SRV stand for retail, wholesale, and service.

employment in the nonfarm business sector. And, importantly for us, they have the requisite data from before the 1995–2005 growth burst. Figure 3 shows that the labor share in these three sectors as a whole ("Trade and Services") was fairly stable. Within these sectors, however, sales were reallocated to low labor share firms. Table 3 reproduces statistics from Autor et al. (2020) showing that the "between" firm component pushed labor share down from 1982–2012 in each of these sectors. Within-firm labor shares actually rose in all three sectors.

A complementary fact which Autor et al. (2020) document is that larger firms tend to have lower labor shares. Within four-digit industries, the elasticity of firm labor share with respect to firm sales averages -2.2 across these three Census sectors. The relationship is negative within each sector.

3 A model of innovation with heterogeneous firms

The above evidence leads us to seek a theory in which an IT shock leads to a burst of product innovation by large firms (with lower labor shares), thereby increasing their market share and bringing a burst of aggregate productivity growth. The rise in market share should eventually lower markups within firms, however, causing growth to fall in the long run.

3.1 Preferences

Time is discrete and the economy is populated by a representative household who chooses a path of consumption C and wealth a to maximize

$$U_0 = \sum_{t=0}^{\infty} \beta^t \log\left(C_t\right),$$

subject to $a_{t+1} = (1+r_t)a_t + w_t L - C_t$, a standard no-Ponzi game condition, and initial wealth $a_0 > 0$. Here r is the real interest rate, w is the real wage, and L is the endowment of labor, which is inelastically supplied to the labor market.

The usual Euler equation resulting from household optimization is given by

$$\frac{C_{t+1}}{C_t} = \beta(1+r_{t+1}).$$

3.2 Production of final output

A final output good is produced competitively using a unit continuum of intermediate inputs according to a Cobb-Douglas technology:

$$Y = \exp\left(\int_0^1 \log\left[q(i)y(i)\right]di\right).$$

Here $y(i)$ denotes the quantity and $q(i)$ the quality of product i. This structure yields demand for each product i as

$$y(i) = \frac{YP}{p(i)}, \tag{1}$$

where the aggregate price index (which we normalize to 1 in each period) is

$$P \equiv \exp\left(\int_0^1 \log\left[p(i)/q(i)\right]di\right).$$

3.3 Production and market structure for intermediate inputs

There are J firms indexed by j. J is "large" such that firms take P as given. Each firm j has the knowledge to produce quality $q(i,j) \geq 0$ in a specific market $i \in [0,1]$. There are two sources of heterogeneity across firms: (i) product-specific quality $q(i,j)$ which evolves endogenously with innovation; and (ii) permanent heterogeneity in firm-specific process efficiency.

We denote firm-specific process efficiency by $\varphi(j)$. A firm with process efficiency $\varphi(j)$ can produce in any line i with the linear technology

$$y(i,j) = \varphi(j) \cdot l(i,j), \tag{2}$$

where $l(i,j)$ is labor used by firm j to produce output $y(i,j)$ in product line i. We assume the heterogeneity in process efficiency is permanent. This heterogeneity in process efficiency will translate into persistent differences in markups and labor share across firms. The linear technology in (2) applies irrespective of the specific quality $q(i,j)$ at which firm j produces in line i.

We explain below how product-specific quality changes endogenously due to innovations. For the static firm problem here we take the line-specific quality $q(i,j)$ of a firm in a period t as given. Labor is fully mobile such that the wage rate is equal across firms. Hence, the marginal cost of firm j in line i is $w/\varphi(j)$.

3.4 Pricing

In each market i firms engage in Bertrand competition. This implies that only the firm with the highest quality-adjusted productivity $q(i,j) \cdot \varphi(j)$ will be active in equilibrium in a given market. We denote the leading firm in line i by $j(i)$ and the second-highest quality producer by $j'(i)$. Hence the quality-adjusted productivity of the leader in line i is $q(i,j(i)) \cdot \varphi(j(i))$, whereas it is $q(i,j'(i)) \cdot \varphi(j'(i))$ for the second-best firm. Under Bertrand competition, price setting of the leading firm is constrained by the second-best producer. The leader will

set its quality-adjusted price equal to the quality-adjusted marginal cost of the second-best firm. Formally, we then have

$$\frac{p(i,j(i),j'(i))}{q(i,j(i))} = \frac{w}{q(i,j'(i)) \cdot \varphi(j'(i))}.$$

Note that the equilibrium price in line i depends on the process efficiency of the *second-best* firm as well as the quality difference between them.

The markup in line i, the price of a unit divided by the marginal cost, is

$$\mu(i,j(i),j'(i)) \equiv \frac{p(i,j(i),j'(i))}{w/\varphi(j(i))} = \frac{q(i,j(i)) \cdot \varphi(j(i))}{q(i,j'(i)) \cdot \varphi(j'(i))}.$$

The markup is increasing in the quality gap $q(i,j(i))/q(i,j'(i))$ and the process efficiency gap $\varphi(j(i))/\varphi(j'(i))$ between the leading and the second-best firm. Operating profits of the leader in line i are $Y\left[1-1/\mu(i,j(i),j'(i))\right]$. This follows from the demand function (1) with P normalized to one.

3.5 Innovation and productivity growth

The quality distribution evolves endogenously over time as a result of innovation. Any firm j can engage in R&D to acquire a patent to produce a product at higher than existing quality. More specifically, by investing $x_t(j) \cdot \psi_r \cdot Y_t$ units of final output in R&D in period t, $x_t(j)$ product lines are randomly drawn among the lines in which firm j is currently not actively producing. In these randomly drawn lines the highest existing quality is multiplied by a factor $\gamma > 1$ and the innovating firm j obtains a perpetual patent to produce at this higher quality level from the next period $t+1$ onward.

We assume that a period is short enough such that no two innovations arrive on the same line in a given period. As we denote the innovation rate of firm j in period t by $x_t(j)$, the aggregate rate of creative destruction is given by

$$z_{t+1} = \sum_{j=1}^{J} x_t(j).$$

That is, for any given line, an innovation arrives in $t+1$ with probability z_{t+1}. These quality improvements are the source of long-run growth.

3.6 Boundary of the firm

Given the constant cost of acquiring a line through innovation and the fact that firms with higher process efficiency make higher expected operating profits in an additional line, more productive firms have a stronger incentive to invest in R&D. To prevent the firm with the highest productivity from taking over all lines, we assume that firms have to pay a per-period overhead cost which is a convex function of the number of markets they span. More specifically, we assume a quadratic per-period overhead cost

$$\frac{1}{2}\psi_o\, n(j)^2\, Y,$$

with $\psi_o > 0$, where $n(j)$ denotes the number of lines in which firm j owns the highest quality patent. The convexity of the overhead cost in $n(j)$ gives rise to a natural boundary of the firm. High process efficiency will want to operate more lines than low process efficiency firms, but no firm type will operate all lines.

It may be helpful to compare our model to Klette and Kortum (2004), a benchmark model in the firm dynamics and growth literature. We assume a linear cost of innovating on a new line and convex overhead costs. By contrast, Klette and Kortum (2004) assume a convex cost of acquiring extra product lines through creative destruction, and a non-diminishing value of additional lines (the firm's value function is linear in n in their steady state).[8]

Our model allows us to do comparative statics with respect to the scalar ψ_o without altering the technology for undertaking innovations. With IT improvements in mind, we lower ψ_o permanently for all firms and study the effect on concentration, labor share, and growth during the transition as well

[8]Our model shares some features with Luttmer (2011), in which more efficient firms endogenously expand into more products because their efficiency extends across product lines.

as in the new steady state. Another difference with Klette and Kortum (2004) is that we assume that each firm operates a continuum of lines, so that there is no firm exit in our model.

3.7 Markups with binary process efficiency levels

For simplicity we assume in the following two types of firms. A fraction ϕ of all firms has high process efficiency φ_H whereas the remaining fraction $1-\phi$ has a low process efficiency φ_L. We denote their efficiency ratio by $\Delta \equiv \varphi_H/\varphi_L > 1$. We further assume $\gamma > \Delta$ so that the firm with the highest quality is the active producer irrespective of its type or the type of the second-best firm.

Given the two process efficiency levels (high and low) there are four potential cases of markups $\mu(i)$ and operating profits $\pi(i)$ in a given line i:[9]

1. A high productivity leader $\varphi(j(i)) = \varphi_H$ facing a high productivity second-best firm $\varphi(j'(i)) = \varphi_H$ in line i. In this case we have

$$\mu(i) = \gamma \text{ and } \pi(i) = Y\left(1 - \frac{1}{\gamma}\right)$$

2. A high productivity leader $\varphi(j(i)) = \varphi_H$ facing a low productivity second-best firm $\varphi(j'(i)) = \varphi_L$ in line i.

$$\mu(i) = \Delta\gamma \text{ and } \pi(i) = Y\left(1 - \frac{1}{\Delta\gamma}\right)$$

3. A low productivity leader $\varphi(j(i)) = \varphi_L$ facing a high productivity second-best firm $\varphi(j'(i)) = \varphi_H$ in line i.

$$\mu(i) = \frac{\gamma}{\Delta} \text{ and } \pi(i) = Y\left(1 - \frac{\Delta}{\gamma}\right)$$

4. A low productivity leader $\varphi(j(i)) = \varphi_L$ facing a low productivity second-

[9] To ease notation we denote the markup in line i, $\mu(i, j(i), j'(i))$, by $\mu(i)$.

best firm $\varphi(j'(i)) = \varphi_L$ in line i.

$$\mu(i) = \gamma \text{ and } \pi(i) = Y\left(1 - \frac{1}{\gamma}\right)$$

3.8 Labor income shares

Our baseline model abstracts from physical capital; labor is the only factor of production. Furthermore, both R&D expenditure and overhead costs are denominated in final output and are treated as investment as opposed to intermediate inputs. These last two assumptions are made to avoid a mechanical effect of overhead and R&D costs on the labor income share. Hence in our baseline framework the aggregate labor income share is simply determined by the distribution of markups across product lines.

Because of the Cobb-Douglas technology in final good production, revenue for each product is equal to Y. Labor costs in a line i equal $wl(i) = Y/\mu(i)$. Integrating both sides over i yields $wL = Y\int_0^1 \mu(i)^{-1}di$. Dividing $wl(i)$ by wL, the wage bill (or employment) share of product line i is

$$\frac{l(i)}{L} = \frac{1}{\mu(i)}\frac{1}{\int_0^1 \frac{1}{\mu(\iota)}d\iota}.$$

The employment share on a line, $l(i)/L$, is inversely proportional to the markup on the line. This comes from revenue being equalized across lines.

Finally, the aggregate labor income share λ is given by the inverse of the cost-weighted markup:

$$\lambda \equiv \frac{wL}{Y} = \frac{1}{\int_0^1 \mu(i)l(i)/L\, di} = \int_0^1 \mu(i)^{-1}\, di.$$

Because there is no physical capital in the model the profit share and the labor income share add up to one. However, the aggregate labor share depends non-trivially upon the full distribution of markups across lines. This distribution is determined by the types of the leader and second-best firm across lines.

Consider a firm j with $n(j)$ lines that faces a fraction $h(j)$ of high type second-best firms and a remaining fraction $1-h(j)$ of low productivity second-best firms. If firm j is itself of high type, its overall labor income share is given by

$$\lambda_H(h(j)) = h(j)\frac{1}{\gamma} + (1-h(j))\frac{1}{\gamma\Delta}. \tag{3}$$

In contrast, if firm j is low type its overall labor income share is given by

$$\lambda_L(h(j)) = h(j)\frac{\Delta}{\gamma} + (1-h(j))\frac{1}{\gamma}. \tag{4}$$

Faced with the same share of high type competitors $h(j)$, high productivity firms have a lower labor income share as they can charge higher markups on average. Hence the model generates persistent differences in labor shares across firms.[10] As the composition of competitors $h(j)$ is endogenous, the model can generate changes in the labor share within firms over time.

3.9 Dynamic firm problem

There are two individual state variables in the firm's problem: the number of lines firm j operates, $n(j)$, and the fraction of high productivity second-best producers, $h(j)$, the firm faces in its lines. Each firm then chooses how many new lines to innovate upon, $x_t(j)$, to maximize the net present value of its flow of profits. Denoting per-period profits after overhead costs and relative to aggregate output Y by π_H and π_L, respectively, we have

$$\pi_H(n(j),h(j)) = n(j) - \frac{n(j)h(j)}{\gamma} - \frac{n(j)\left(1-h(j)\right)}{\gamma\Delta} - \frac{1}{2}\psi_o n(j)^2, \tag{5}$$

and

$$\pi_L(n(j),h(j)) = n(j) - \frac{n(j)h(j)\Delta}{\gamma} - \frac{n(j)\left(1-h(j)\right)}{\gamma} - \frac{1}{2}\psi_o n(j)^2. \tag{6}$$

[10] See Hsieh and Klenow (2009) and David and Venkateswaran (2019) for evidence of persistent differences in revenue per worker across firms.

These scaled by Y_t profits only depend on the individual states $n(j)$ and $h(j)$ and are otherwise time invariant. Letting S_t denote the aggregate fraction of lines operated by high productivity firms, the problem of a firm of type $k = H, L$ can be written as

$$V_{0,k} = \max_{\{x_t, n_{t+1}, h_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} Y_t \left[\pi_k(n_t, h_t) - x_t \psi_r\right] \prod_{s=0}^{t} \left(\frac{1}{1+r_s}\right)$$

subject to

$$n_{t+1} = n_t(1 - z_{t+1}) + x_t, \tag{7}$$

$$h_{t+1} n_{t+1} = h_t n_t (1 - z_{t+1}) + S_t x_t, \tag{8}$$

and a given initial n_0 and h_0. For completeness there are also non-negativity constraints $x_t \geq 0$. Equation (7) states that the number of product lines of a firm next period is equal to the newly added lines x plus the number of lines today times one minus the rate of creative destruction in the economy, z. Equation (8) states that the number of lines in which the firm faces a high type second-best firm tomorrow is equal to the number of such lines today times $1 - z$ plus the number of newly added lines times the aggregate fraction of lines currently operated by high type firms S. The firm takes the path of output Y_t, the interest rate r_t, the rate of creative destruction z_{t+1}, and the aggregate fraction of lines operated by high productivity firms S_t as given.

3.10 Market clearing and resource constraints

We close the model with the following market clearing conditions that hold each period. First, final output will be used for consumption C, total overhead costs O, and total R&D expenditures Z:

$$Y = C + O + Z, \tag{9}$$

where

$$O = \sum_{j=1}^{J} \frac{1}{2}\psi_o n(j)^2 Y \quad \text{and} \quad Z = \sum_{j=1}^{J} x(j)\psi_r Y.$$

Labor is used as a variable input by the producers of different intermediate product lines. Labor and asset market clearing conditions imply

$$L = \sum_{j=1}^{J} \int_0^1 l(j,i)\, di \quad \text{and} \quad \sum_{j=1}^{J} V_t(j) = a_t,$$

where $l(j,i)$ denotes labor used by firm j on line i.

In addition, we have the equations defining the aggregate share of lines operated by high types and an accounting equation that states that all lines are operated by some firm:[11]

$$S_t = \sum_{j=1}^{\phi J} n_t(j) \text{ and } 1 = \sum_{j=1}^{J} n_t(j). \tag{10}$$

Finally, there is an equation relating aggregate output to the distribution of process efficiency, quality levels, and markups

$$Y_t = Q_t \frac{\varphi_L \Delta^{S_t} \exp\left[-\int_0^1 \log\left(\mu_t(i)\right) di\right]}{\int_0^1 \left(\mu_t(i)\right)^{-1} di} L. \tag{11}$$

Here $Q_t = \exp\left[\int_0^1 \log\left(q_t(i,j)\right) di\right]$ denotes the geometric average quality level.

An equilibrium in this economy is a path of allocations and prices that jointly solve the household and firm problems and is consistent with the market clearing and accounting equations stated above.

There is no free entry and the number of firms is fixed. Hence total firm profits from selling at a markup over marginal cost may exceed the total investments in R&D and overhead costs. We call such net profits "rents."

Since output is a function of the full distribution of markups across product

[11] Here we assume that the high productivity type firms are indexed by $j = 1, 2, \ldots, \phi J$.

lines, the equilibrium path is a function of the initial joint distribution of product lines $n(j)$ and level of competition $h(j)$ across firms. We assume that all firms of the same type $k = H, L$ start out with the same level of n_0 and h_0.[12] Using the law of large numbers, firms of the same type will then be identical along the entire equilibrium path. Therefore only two firm problems — one for a high type and one for a low type — need to be solved. The aggregate state vector can then be summarized by S and the shares of high second-best firms h_H and h_L in lines operated by high and low productivity firms.

With the two "representative" firms, aggregate labor productivity can be expressed in terms of these aggregate state variables (S, h_L, h_H) and the level of average quality Q as[13]

$$\frac{Y_t}{L} = Q_t \cdot \varphi_L \Delta^{S_t} \cdot \frac{\Delta^{(1-S_t)h_{Lt} - S_t(1-h_{Ht})}}{S_t h_{Ht} + (1-S_t)(1-h_{Lt}) + S_t(1-h_{Ht})\frac{1}{\Delta} + (1-S_t)h_{Lt}\Delta}$$

Aggregate labor productivity is the product of three terms. The first term Q_t captures the geometric average level of quality across product lines. The second term, $\varphi_L \Delta^{S_t}$, captures the aggregate level of process efficiency. If $S_t = 0$ then aggregate process efficiency is just the level of the low type φ_L, whereas if $S_t = 1$ aggregate process efficiency is equal to the high level $\varphi_H = \varphi_L \Delta$. The third and final term, which we call allocative efficiency, captures the output distortion due to markup dispersion. If $S_t = h_{Ht} = 1$ or $S_t = h_{Lt} = 0$ this final term is equal to 1 (no dispersion of markups since all markups are equal to γ in all lines). In all other cases the third term is smaller than one.

In section 4 below we show that the steady state takes a tractable form that can be solved analytically. We then discuss how a permanent drop in ψ_o (say triggered by improvements in IT) affects market concentration, labor income shares (within firms as well as in the aggregate), and productivity growth in the long run. In Section 5 we calibrate the model and numerically solve for the transition path of the economy.

[12] This assumption will automatically be fulfilled if the economy starts in steady state.

[13] The derivation of this expression can be found in Online Appendix B.2.

4 Solving for the steady-state

4.1 Steady state definition

We define a steady state equilibrium in the following way:

Definition 1 *A steady state is an equilibrium path along which the real interest rate and the gross rate of output remain constant, equal to $r^\star$ and $g^\star$, and along which a constant fraction of lines, $S^\star$, is provided by high productivity producers.*

In a steady state all high productivity firms have the same constant number of products $n_H^\star$, whereas all low productivity firms have a different number of products $n_L^\star$. For the number of lines within a firm to be constant, its R&D activity must be proportional to its number of products, i.e., $x(j)^\star = n(j)^\star z^\star$, where $z^\star$ is the aggregate rate of creative destruction in steady state. Since all firms draw new lines from a stationary distribution, they all face the same share of high productivity second-best firms in their lines:

$$h(j)^\star = S^\star \; \forall j. \tag{12}$$

As the markup distribution is stationary in steady state, from (11) aggregate output Y_t grows at the same rate as average quality Q_t:

$$\frac{Y_{t+1}}{Y_t} = \frac{Q_{t+1}}{Q_t} = \gamma^{z^\star} \equiv g^\star.$$

Finally, since total overhead O and total R&D Z each grow at the gross rate $g^\star$, given (9) consumption has to grow at this rate as well. Using the Euler equation this pins down the steady state real interest rate as

$$r^\star = \frac{g^\star}{\beta} - 1.$$

Next, we show that solving for the steady state boils down to solving for the quadruple $S^\star, n_L^\star, n_H^\star$, and $z^\star$.

4.2 Steady state characterization

With $h(j)^\star = S^\star$, (5) and (6) yield for the period profits of high and low type firms (relative to total output) as

$$\pi_H(n, S^\star) = n\left(1 - \frac{S^\star}{\gamma} - \frac{1 - S^\star}{\gamma\Delta}\right) - \frac{1}{2}\psi_o n^2, \tag{13}$$

and

$$\pi_L(n, S^\star) = n\left(1 - \frac{S^\star\Delta}{\gamma} - \frac{1 - S^\star}{\gamma}\right) - \frac{1}{2}\psi_o n^2. \tag{14}$$

Denote v as the value of a firm relative to total output, $v \equiv V/Y$. The number of products per firm n becomes the only individual state variable in the firm problem, so we can write $v_k = v_k(n)$, $k = H, L$. High and low productivity firms then solve the following Bellman equations:

$$v_H(n) = \max_{n' \geq n(1-z^\star)} \{\pi_H(n, S^\star) - (n' - n(1 - z^\star))\psi_r + \beta v_H(n')\}, \tag{15}$$

$$v_L(n) = \max_{n' \geq n(1-z^\star)} \{\pi_L(n, S^\star) - (n' - n(1 - z^\star))\psi_r + \beta v_L(n')\}. \tag{16}$$

We denote their solutions as $n' = f_H(n)$ and $n' = f_L(n)$.

In steady state, the two accounting equations in (10) become

$$S^\star = n_H^\star \phi J$$

$$n_H^\star \phi J + n_L^\star (1 - \phi) J = 1.$$

Finally, we must have

$$n_H^\star = f_H(n_H^\star), \quad n_L^\star = f_L(n_L^\star).$$

These equations fully characterize the steady state. The two dynamic programming problems (15) and (16) are very simple since π_H and π_L are quadratic functions of n from (13) and (14).

In the following we focus on an interior steady state wherein $S^\star \in (0,1)$ and $z^\star \in (0,1)$. When such a steady state exists, the policy and value functions can be characterized in closed form.[14] Next we impose parameter restrictions that ensure the existence of an interior steady state solution.

Assumption 1 *To ensure an interior steady state where both firm types are active and long-run growth is positive, we assume*

$$\frac{\Delta-1}{\gamma} < \frac{\psi_o}{\phi J}, \tag{17}$$

and

$$0 < \frac{1}{\psi_r} - \frac{1-\beta}{\beta} - \frac{1}{\psi_r}\frac{\frac{\psi_o}{J}+\frac{1}{\gamma}}{1-(1-\phi)\phi\frac{(\Delta-1)^2}{\gamma\Delta}\frac{J}{\psi_o}} < 1. \tag{18}$$

Restriction (17) ensures that the low type firms are active in steady state, $S^\star < 1$.[15] This is fulfilled as long as neither the productivity differential Δ nor the number of high productivity firms ϕJ are too large. Restriction (18) ensures a positive but less than certain rate of creative destruction, $0 < z^\star < 1$. It is fulfilled as long as ψ_r relative to β is neither too small nor too large.

The next two propositions characterize the interior steady state solution and prove that Assumption 1 is sufficient for the existence of such a steady state.

Proposition 1 *If an interior steady state exists, it is given by a quadruple* $(n_H^\star, n_L^\star, S^\star, z^\star)$ *that fulfills*

$$\phi J n_H^\star = S^\star \text{ and } (1-\phi)Jn_L^\star + \phi J n_H^\star = 1, \tag{19}$$

[14] Let us denote the marginal steady state profits per line before overhead cost of firms by $\widetilde{\pi}_H = 1 - S^\star/\gamma - (1-S^\star)/(\Delta\gamma)$ and $\widetilde{\pi}_L = 1 - \Delta S^\star/\gamma - (1-S^\star)/\gamma$. Then, for any $n \leq \bar{n}_k/(1-z^\star)$, where $\bar{n}_k \equiv (\widetilde{\pi}_k + (1-z^\star)\psi_r - \psi_r/\beta)/\psi_o$ we have the policy function $f_k(n) = \bar{n}_k$ and the value function $v_k(n) = \widetilde{\pi}_k n - \frac{1}{2}\psi_o n^2 - \psi_r(\bar{n}_k - (1-z^\star)n) + \beta(\widetilde{\pi}_k\bar{n}_k - \frac{1}{2}\psi_o\bar{n}_k^2 - \psi_r z^\star\bar{n}_k)/(1-\beta)$, for $k = H, L$. See Online Appendix B.1 for details.

[15] With $\frac{\Delta-1}{\gamma} \geq \frac{\psi_o}{\phi J}$ there exists a trivial steady state with $n_L^\star = 0$, $n_H^\star = 1/(\phi J)$, $S^\star = 1$, and $z^\star = (1 - 1/\gamma - \psi_o/(\phi J))/\psi_r + 1 - 1/\beta$, where $0 < (1 - 1/\gamma - \psi_o/(\phi J))/\psi_r + 1 - 1/\beta < 1$ needs to be imposed to ensure that the high type firms invest strictly positive amounts and that the rate of creative destruction is less than 100%, i.e., $z^\star \in (0,1)$.

as well as the following research arbitrage equations for high and low productivity firms:

$$\psi_r = \frac{1 - S^\star/\gamma - (1 - S^\star)/(\gamma\Delta) - \psi_o n_H^\star}{1/\beta - 1 + z^\star} \tag{20}$$

$$\psi_r = \frac{1 - S^\star\Delta/\gamma - (1 - S^\star)/\gamma - \psi_o n_L^\star}{1/\beta - 1 + z^\star}. \tag{21}$$

Proof. By definition $S^\star \in (0,1)$ in an interior steady state. This implies that $n_k^\star$ and $x_k^\star$ are positive for $k = H, L$. Thus both firm policy functions satisfy the first-order condition for the Bellman equation. For the high type this is

$$\psi_r = \beta \frac{\partial v_H(n')}{\partial n'}.$$

Using the envelope theorem we have

$$\frac{\partial v_H(n')}{\partial n'} = 1 - S^\star/\gamma - (1 - S^\star)/(\gamma\Delta) - \psi_o n' + (1 - z^\star)\psi_r.$$

Using the fact that $n' = n_H^\star$ in steady state then yields the research arbitrage equation of the high type firm. The research arbitrage equation of the low type firm is derived in an analogous way. ■

The intuition for the two research arbitrage equations is straightforward. In steady state the marginal cost of innovating in a line ψ_r equals the marginal (expected) value of having an additional line. For the high type firm, this marginal value is proportional to the marginal profit $1 - S^\star/\gamma - (1 - S^\star)/(\gamma\Delta)$ minus the marginal overhead cost $\psi_o n_H^\star$. These terms are are divided by $1/\beta - 1 + z^\star$ (due to pure time discounting and the probability $z^\star$ losing the additional line in each future period) to arrive at the marginal value.

Equations (19)–(21) are four equations in the four unknowns $(n_H^\star, n_L^\star, S^\star, z^\star)$ that can be solved explicitly. We use them to derive conditions that guarantee an interior solution, and to solve for all the other endogenous variables.

Proposition 2 *Assumption 1 implies that the steady state is interior and is characterized by Proposition 1. Furthermore, this interior steady state has the following properties:*

(i) The share of lines operated by high productivity firms is equal to

$$S^\star = \frac{\frac{1}{1-\phi} + \frac{\Delta-1}{\gamma\Delta}\frac{J}{\psi_o}}{\frac{1}{(1-\phi)\phi} - \frac{(\Delta-1)^2}{\gamma\Delta}\frac{J}{\psi_o}}, \tag{22}$$

and the rate of creative destruction is given by

$$z^\star = \frac{1}{\psi_r} - \frac{1-\beta}{\beta} - \frac{1}{\psi_r}\frac{\frac{\psi_o}{J} + \frac{1}{\gamma}}{1-(1-\phi)\phi\frac{(\Delta-1)^2}{\gamma\Delta}\frac{J}{\psi_o}}. \tag{23}$$

(ii) High productivity firms operate more lines than low productivity firms:

$$n_H^\star > n_L^\star.$$

(iii) The labor income share of a high type firm is given by

$$\lambda_H^\star = S^\star\frac{1}{\gamma} + (1-S^\star)\frac{1}{\gamma\Delta}, \tag{24}$$

which is strictly smaller than the labor income share of a low type firm

$$\lambda_L^\star = S^\star\frac{\Delta}{\gamma} + (1-S^\star)\frac{1}{\gamma}. \tag{25}$$

Finally, the aggregate labor income share is given by

$$\lambda^\star = S^\star\lambda_H^\star + (1-S^\star)\lambda_L^\star. \tag{26}$$

Proof. Replacing $n_H^\star$ and $n_L^\star$ in (20) and (21) by $S^\star/(\phi J)$ and $(1-S^\star)/(J(1-\phi))$, respectively, and solving the two equations for $S^\star$ and $z^\star$ yields the unique solution in part (i). Note that restriction (17) ensures $S^\star < 1$ and restriction

(18) ensures $0 < z^\star < 1$. Finally, note that $S^\star > 0$ is always guaranteed since (17) implies $\frac{\psi_o}{\phi J} > \frac{\Delta-1}{\gamma} > \frac{\Delta-1}{\gamma}\frac{\Delta-1}{\Delta}(1-\phi)$, as $\frac{\Delta-1}{\Delta}(1-\phi) < 1$ — Assumption 1 is sufficient to ensures the existence of an interior steady state.

For part (ii), by combining (20) and (21) the difference in the number of products can be expressed as

$$n_H^\star - n_L^\star = \frac{S^\star(\Delta-1)^2}{\gamma\Delta\psi_o} + \frac{\Delta-1}{\gamma\Delta\psi_o} > 0.$$

The labor income shares follow from (3), (4) and (12). This proves part (iii). ■

In steady state, $S^\star$ can be viewed as a summary statistic of market concentration, whereas $z^\star$ pins down the long-run growth rate of the economy. Note that all of the endogenous steady state values depend only on the ratio ψ_o/J and not on the individual level of ψ_o or J.

The intuition for (ii) in Proposition 2 is that high process efficiency firms can (on average) charge higher markups. Consequently their incentive to undertake R&D is higher and they push up into a steeper area of the convex overhead cost schedule, as they operate more lines than low process efficiency firms do in steady state. A corollary is that we have $S^\star > \phi$ since high productivity firms are larger (in terms of sales per firm) than than low productivity firms. High and low productivity firms also differ in their employment, but the employment difference is smaller than the sales difference as high productivity firms charge higher markups — see part (iii) of Proposition 2.

4.3 Steady state effects of a decrease in overhead costs (ψ_o)

We hypothesize that the IT revolution may have contributed to lower costs of managing multiple product lines within a firm. See Aghion and Tirole (1997). In this section we consider how the steady state in our model changes following a permanent reduction in the overhead cost parameter ψ_o. We are particularly interested in how the following endogenous variables respond: (i)

market concentration, $S^\star$; (ii) the labor income share at the aggregate level as well as within firms; and (iii) the long-run growth rate.

Proposition 3 *Concentration $S^\star$ increases monotonically as ψ_o decreases.*

Proof. Taking derivatives of (22) with respect to ψ_o yields

$$\frac{\partial S^\star}{\partial \psi_o} = -\frac{[1+\phi(\Delta-1)]\frac{\Delta-1}{\gamma\Delta}\frac{J}{\psi_o^2(1-\phi)\phi}}{\left(\frac{1}{(1-\phi)\phi}-\frac{(\Delta-1)^2}{\gamma\Delta}\frac{J}{\psi_o}\right)^2} < 0.$$

■

The intuition that a fall in ψ_o increases $S^\star$ is the following: with a lower ψ_o a larger size gap $n_H^\star - n_L^\star$ is needed to yield the same difference in the marginal overhead cost between high and low productivity firms. Consequently, high process efficiency firms will operate more lines as ψ_o decreases, whereas low productivity firms will shrink in size. Therefore market concentration goes up.

Proposition 4 *As ψ_o decreases (i) the labor income share within firms increases, (ii) market shares are reallocated toward low labor share firms, and (iii) the aggregate labor income share increases (decreases) if initial $S^\star$ is larger (smaller) than 1/2.*

Proof. For (i) note that both (24) and (25) are monotonically increasing in $S^\star$ (and $S^\star$ increases as ψ_o falls as demonstrated in Proposition 3). For (ii), as $S^\star$ increases the sales share of high productivity firms (with higher average markups and lower labor shares) goes up. For (iii), we obtain from (26) that:

$$\frac{\partial \lambda^\star}{\partial S^\star} = \lambda_H^\star + S^\star\frac{\Delta-1}{\gamma\Delta} - \lambda_L^\star + (1-S^\star)\frac{\Delta-1}{\gamma}.$$

Replacing the expression for $\lambda_H^\star$ and $\lambda_L^\star$ by (24) and (25) and simplifying gives

$$\frac{\partial \lambda^\star}{\partial \psi_o} = \frac{\partial \lambda^\star}{\partial S^\star}\frac{\partial S^\star}{\partial \psi_o} = \frac{(\Delta-1)^2}{\gamma\Delta}(1-2S^\star)\frac{\partial S^\star}{\partial \psi_o}.$$

Since $S^\star$ is decreasing in ψ_o (see Proposition 3) this implies that the aggregate labor income share decreases as ψ_o falls if and only if $S^\star > 1/2$. ■

The model makes sharp predictions about the labor income shares at the aggregate vs. micro level. As $S^\star$ increases due to the drop in ψ_o, all firms are more likely to face a high productivity second-best competitor on any given line. As a consequence *within* firm markups decrease and *within* firm labor shares increase — see (24) and (25). The sales reallocation across firms goes in the opposite direction. As $S^\star$ increases the high productivity firms with their lower labor shares expand and the low productivity firms contract. This between firm effect pushes the aggregate labor income share downwards. As emphasized in Section 2, within and between firm labor shares going in opposite directions is a salient feature of the U.S. micro data.

Whether the within or between firm effect on the labor share dominates in our model depends on the initial level of $S^\star$. Specifically, the aggregate labor income share falls as ψ_o decreases if and only if $S^\star > 1/2$.

Proposition 5 *We have $\partial z^\star/\partial\psi_o > 0$ such that long-run growth decreases as ψ_o falls if and only if*

$$\frac{J(\Delta-1)^2}{\gamma\Delta\psi_o}\left(\frac{J}{\gamma\psi_o}+2\right) > \frac{1}{\phi(1-\phi)}. \tag{27}$$

Proof. Taking derivatives of (23) with respect to ψ_o gives

$$\frac{\partial z^\star}{\partial\psi_o} = \frac{2\phi(1-\phi)\frac{(\Delta-1)^2}{\gamma\Delta\psi_o} + \phi(1-\phi)\frac{(\Delta-1)^2}{\gamma\Delta\psi_o}\frac{J}{\gamma\psi_o} - \frac{1}{J}}{\psi_r\left(1-(1-\phi)\phi\frac{(\Delta-1)^2}{\gamma\Delta}\frac{J}{\psi_o}\right)^2}.$$

This expression is positive if and only if (27) holds. ■

The long-run growth rate is affected by a drop in ψ_o in two ways. First, there is a direct positive effect on growth: at a given $S^\star$, a lower overhead cost raises the marginal value of operating an additional line and therefore stimulates R&D investment and growth. Second, there is a general equilibrium effect that

goes in the opposite direction. As ψ_o decreases $S^\star$ rises. This reduces the expected markup in an additional line (as the probability of facing a high productivity second-best firm went up). This general equilibrium effect decreases the incentive to undertake R&D and consequently long-run growth can potentially fall as ψ_o decreases. Whether the direct or indirect effect dominates depends on the precise parameter values. Proposition 5 states the parameter range that guarantees that long-run growth falls as ψ_o decreases. The condition holds as long as $\psi_o/\left[(\Delta-1)J\right]$ is not too large.[16] We find this condition holds in our calibrations below.

To recap, our theory can generate a productivity slowdown, rising concentration, and opposite changes in the labor income shares within firms and between firms as the outcome of a drop in ψ_o. In Section 5 below we will gauge the quantitative size of these effects in a simple calibration. We can also explore the effect of changes in other parameters, such as the R&D cost parameter ψ_r and the process efficiency advantage of high type firms Δ.

4.4 Theoretical extensions

We kept our baseline model parsimonious to show the minimum ingredients needed to speak to the empirical facts in Section 2 This tractable model can be augmented in various ways to explore the same mechanisms in richer environments. In the Online Appendix, we consider various extensions of the model. We replace the Cobb-Douglas aggregation across goods with a CES final good aggregator in section (C). This introduces a finite monopoly markup and hence leads to less markup dispersion. We generalize from two to an arbitrary number of firm types in terms of their permanent process efficiency in section (D). Finally, we allow mergers and acquisitions in section (E). In all of these extensions, we derive the conditions under which the qualitative results of our baseline model are still valid.

[16] Restriction (27) is consistent with Assumption 1 for a non-empty set of parameters as long as $1/\phi + 2(\Delta-1) > \Delta/(1-\phi)$.

5 Calibration

We now investigate how much a decline in overhead costs might contribute quantitatively to a burst and then slowdown of productivity growth. We define the initial steady state period as 1987–1995 and the ending steady state period as 2005–2018. We calibrate five parameters to match five moments in the initial steady state. And we infer changes in three of the parameters — ψ_o, ψ_r, and Δ — to match three moments in the ending steady state. We then assess the importance of the overhead cost channel by calculating changes in growth in the model when only the overhead parameter ψ_o changes.

5.1 Initial and ending steady states

The five moments we match are: 1) concentration (share of sales going to the largest 0.137% of firms) within industries in 1987 from Autor et al. (2020);[17] 2) the average annual rate of productivity growth over 1987–1995 in Trade and Services from the BLS KLEMS dataset; 3) average markups in Trade and Service industries as estimated by Hall (2018) over 1988–2015; 4) the real interest rate from Farhi and Gourio (2018) for 1980–1995; 5) the semi-elasticity of firm labor share with respect to firm sales within four-digit industries divided by the aggregate labor share from Autor et al. (2020).[18] The calibrated parameters are: 1) the initial overhead cost parameter ψ_o^0; 2) the initial R&D cost parameter ψ_r^0; 3) the proportional process efficiency gap $\Delta > 1$ between high and low type firms; 4) the quality step size $\gamma > 1$; and 5) the discount factor $0 < \beta < 1$. We set the share of high type firms ϕ to 0.137% to match the fraction of top 20 firms.

[17] Autor et al. (2020) report the average sales shares of the top 20 firms and the average number of firms within 4-digit industries in Trade and Service sectors. We use this information to infer that the top 20 firms are approximately the top 0.137% of firms.

[18] We aggregate concentration and the semi-elasticity from Autor et al. (2020) for Retail Trade, Wholesale Trade, and Services using value of production weights from KLEMS. To calculate the price/cost markup for Trade and Services, we aggregate industry level Lerner indices (one minus the inverse of markups) estimated by Hall (2018) using sector output shares provided by Hall (2018) and then convert the resulting aggregate index to markups.

Tables 4 and 5 display the targets and the calibrated parameter values. First, concentration $S^\star$ is monotonically increasing in ψ_o in the model. Hence the level of concentration helps to pin down ψ_o^0 to 0.050%. Next, the semi-elasticity of labor shares with respect to size becomes more negative with higher Δ, helping to set Δ to 1.134. That is, the high-type firms enjoy about 13% higher process efficiency. Given Δ and $S^\star$, the average markup increases with the quality step γ. To match the average markup in the data, the model asks for a 25% increase in quality upon innovation (γ = 1.249). Given γ and concentration, the growth rate in the model decreases with ψ_r, which scales the cost of R&D. We obtain ψ_r = 2.201. Finally, for a given growth rate of the economy, the real interest rate decreases with the discount factor β. Matching the real interest rate requires $\beta = 0.947$. As shown in Table 4, the model is able to fit all of the moments despite its simplicity.

Table 4: Baseline Calibration Targets

Targeted	Years	Data	Model
1. top 0.137% concentration	1987	26.7	26.7
2. productivity growth	1987–1995	0.48	0.48
3. price/cost markup	1988-2015	1.25	1.25
4. real interest rate	1980–1995	6.10	6.10
5. semi-elasticity of labor share wrt sales	1987	-2.18	-2.18

Source: 1 and 5: Autor et al. (2020). 5 is relative to aggregate labor share. 2: BLS KLEMS series. 3: Hall (2018). 4: Farhi and Gourio (2018).

Table 6 displays the moments in the new steady state when ψ_o, ψ_r and Δ change to match the "long run" (post-2005 vs. pre-1996) empirical changes in concentration, growth, and relative markups of the top firms. In the model, the relative markup of the top firms is equal to Δ. According to Autor et al. (2020), both employment and sales shares of the largest firms increased for Trade and Services such that relative markups did not changed significantly (see Table 2). Hence we calibrate the change in Δ to 0. With an unchanged Δ, overhead cost parameter ψ_o must decline by 23% to generate the rise in concentration seen in

Table 5: Baseline Parameter Values

Calibrated	Parameter	Value
1. overhead costs	ψ_o^0	0.050%
2. R&D costs	ψ_r^0	2.201
3. productivity gap	Δ	1.134
4. quality step	γ	1.249
5. discount factor	β	0.947
Assigned	Parameter	Value
6. share of H-type firms	ϕ	0.137%

the data.[19] While the decline in ψ_o alone leads to lower steady state growth, it does not generate the entire growth decline observed in the data. As a consequence, the data asks for the R&D cost parameter ψ_r to increase by about 6%. This is reminiscent of Bloom, Jones, Van Reenen and Webb (2020), who argue that growth is held down by ever-rising research costs.

Table 6: Calibrated change in parameters to fit the ending steady state

	Change	Targeted change	Data	Model
1. overhead costs ψ_o	-23.1%	concentration	8.3	8.3
2. R&D costs ψ_r	5.78%	productivity growth	-0.18	-0.18
3. efficiency gap Δ	0%	relative markup	0	0

Source: 1: Autor et al. (2020), change in the sales share of the top 0.137% firms between 1987 and 2012. 2: BLS KLEMS. 3: Autor et al. (2020), change in revenue per worker of the top 0.137% firms relative to the rest of the firms. Columns 'Data' and 'Model' are in percentage points.

Since both ψ_o and ψ_r contribute to the decline in steady state growth, in Table 7 we isolate the contribution of ψ_o alone. This contribution can be calculated in two ways: 1) the change in growth when only ψ_o changes, and 2) the change in growth when ψ_o does not change relative to when all parameters change. These two methods differ because the model is nonlinear. The average

[19]The parameter ψ_r does not affect concentration. The 23% decline in ψ_o compares to a 35% decline in the relative price of IT goods over 1996–2005 in Figure 1b.

Table 7: Contribution of overhead costs to the decline in steady state growth

	1. ψ_o, ψ_r	2. only ψ_r	1. minus 2.	3. only ψ_o	ψ_o contribution
change in g	-0.180	-0.094	-0.086	-0.089	**-0.088**

Note: Each column displays the percentage points change in steady state growth rate when the parameters in the column header changes as in Table 6. Column 1. matches the change in the data. Column 'ψ_o contribution' equals the average of columns 3. and 4.

of the two says the decline in ψ_o contributed 9 basis points out of the 18 basis point decline in steady state productivity growth.

Table 8 displays selected endogenous variables in the initial and ending steady states. Lower overhead costs increase the share of products and sales at the high efficiency firms (higher $S^\star$). Employment concentration increases by less than sales concentration because the high efficiency firms charge higher markups ($\mu_H > \mu_L$). With the rise in $S^\star$, within-firm markups *decline* for both firm types because the next best producer is more likely to be a high type firm. Despite the decline in within-firm markups, the aggregate markup changes little because of the rising market share of high-type firms. I.e., the between effect roughly cancels out the within effect. Table 9 shows that the model generates about 10% of the between and within changes in labor share seen for Trade and Services in Autor et al. (2020).

The decline in within-firm markups discourages innovation by both firm types, lowering the rate of creative destruction and growth. R&D spending as a share of total output declines.[20] Meanwhile, the rise in concentration leads to a rise in overhead costs as a share of output despite the downward shift in the overhead cost curve. The decline in R&D share exceeds the rise in overhead cost share. This, combined with a stable aggregate labor share (the inverse of the aggregate markup), implies a higher share of rents in GDP by about one percentage point. Finally, the 18 basis point decline in the growth rate generates a decline in the real interest from 6.1% in the initial steady state to

[20]Note that Trade and Services report little R&D, so this is probably not captured well by R&D data, which is predominantly in manufacturing and software.

Table 8: Initial vs. ending steady state

	$z^\star$	$S^\star$	μ	μ_H	μ_L	r	Z/Y	O/Y	$Rent/Y$
Initial	2.2	26.7	1.25	1.37	1.21	6.1	4.7	1.3	13.6
Ending	1.4	35.0	1.25	1.35	1.19	5.9	3.2	1.7	14.8

Note: μ, μ_H and μ_L denote the aggregate and firm-level markups. Aggregate labor share equals $1/\mu$. Aggregate labor income, total R&D expenditure Z, total overhead cost O and rents sum up to GDP Y. Creative destruction rate ($z^\star$), concentration ($S^\star$), interest rate ($r^\star$), and shares of GDP are in percent.

Table 9: Steady state change in labor share

	Total	Within	Between
Data change over 1987–2012	0.55	10.55	-10.01
Model	0.05	1.11	-1.06

Source: Autor et al. (2020) trade and service industries. All entries are in percentage points.

5.9% in the ending steady state. This is in the direction of the decline estimated by Farhi and Gourio (2018) from 6.1% for 1980–1995 to 4.5% from 2000–2016.

5.2 Transition dynamics

Our analysis so far has been based on steady state comparative statics. Yet in Section 2 we described a ten-year burst in U.S. productivity growth from 1995 to 2005. So here we compute our model's transition dynamics in response to the ψ_o decline shown in Table 6 to see its potential contribution to the acceleration and deceleration of growth.[21]

It is easy to show that, as ψ_o falls, our model will generate a surge in productivity growth along the transition. The reason for this temporary burst is twofold: 1) The general equilibrium force that decreases the incentive to innovate — stiffer competition as S_t increases — is only realized over time. Hence on impact, as ψ_o decreases, the incentive to do R&D increases and therefore quality growth will increase initially; and 2) the new steady state with a higher $S^\star$ exhibits higher average process efficiency as the efficient firms

[21] See Online Appendix F for a description of the computation method.

operate a larger fraction of the product lines. This static efficiency gain is realized along the transition and contributes to the burst of growth.

Figures 4a-4c display the transition dynamics for the share of lines operated by the high type firms (S_t), the rate of creative destruction (z_{t+1}), and productivity growth after the overhead cost parameter ψ_o declines in year 0. S_t rises sharply and converges to the new steady state after around 8 years. On impact, there is a spike in the rate of creative destruction ($z^\star$) that generates higher productivity growth for about 8 years.[22]

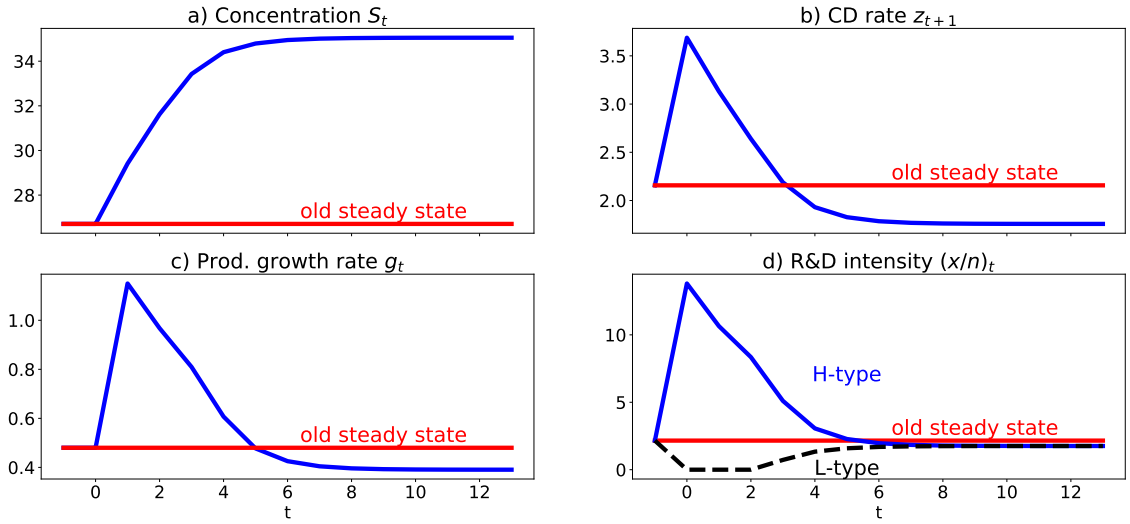
The rise in creative destruction comes from a jump in innovation by high productivity firms, which in turns lifts concentration over time. Figure 4d shows the innovation rate (x/n) by firm type. The Innovation by low type firms actually falls. Eventually the innovation rate for both types converges to a level below the initial steady state. If one makes the strong assumption that new product lines are associated with plant entry, then this behavior qualitatively matches the pattern in Figure 2b above, wherein only the largest firms experienced a burst of plant entry rate during the high growth period from 1995–2005.

Finally, Figure 5 compares the path of consumption following the reduction in ψ_o with the initial steady state path associated with no change in ψ_o. Following the decline in ψ_o, consumption drops sharply in the first period as high efficiency firms increase R&D and overhead investments. Consumption then recovers and is above the initial steady state path for about three decades. Eventually the long run slowdown in innovation and growth takes its toll and consumption falls below its old steady state trajectory.

Table 10 displays the contribution of the decline in ψ_o alone to productivity growth. We find that the decline in overhead costs might account for 11% of the rise in productivity growth from 1995–2005 and 16% of the subsequent decline in productivity growth.

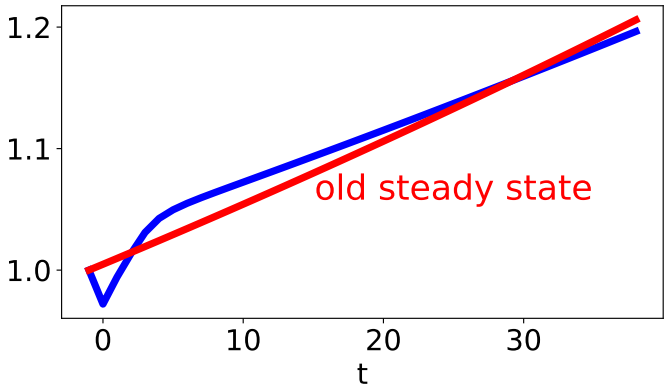
[22] In addition to the rise in innovation, aggregate process efficiency rises by about 1% as more products are produced by the high productivity firms. Allocative efficiency declines slightly due to higher markup dispersion.

Figure 4: Transition dynamics



Note: The blue line plots the transition dynamics when overhead cost ψ_o declines by 23.1% (as in Table 6) in period 0 while other parameters stay the same. The unit is percent.

Figure 5: Transition dynamics for consumption



Note: The blue line plots consumption relative to period -1 when overhead cost ψ_o declines by 23.1% (as in Table 6) in period 0 while other parameters stay the same.

5.3 Welfare analysis

The drop in ψ_o raises consumption growth in the short run but reduces consumption growth in the long run. Hence it is natural to ask whether welfare is higher or lower because of the drop in ψ_o. Recall that utility from a

Table 10: Contribution of decline in overhead costs to growth burst

	Data	1. ψ_o, ψ_r	2. ψ_r	3. 1. minus 2.	4. ψ_o	5. ψ_o contribution
Acceleration	1.18	0.04	-0.09	0.13	0.12	**0.13 (10.9%)**
Deceleration	1.36	0.22	0.00	0.22	0.21	**0.22 (15.9%)**

Note: For data, 'Acceleration' = growth over 1995–2005 minus growth over 1987–1995. 'Deceleration' = growth over 1995–2005 minus growth over 2005–2018. In the model, 'Acceleration' = growth over period 1 to 10 minus initial steady state growth. 'Deceleration' = growth over period 1 to 10 minus new steady state growth. Each column displays the acceleration and deceleration in growth when the parameters in the column header changes as in Table 6. Column 'ψ_o contribution' equals the average of columns 3. and 4. Entries are in percentage points.

consumption path is given by

$$U_0 = \sum_{t=0}^{\infty} \beta^t \log C_t = U(\{C_t\}_{t=0}^{\infty}).$$

The change in welfare can therefore be evaluated in (permanent) consumption-equivalent terms, ξ, using

$$U\left(\{(1+\xi)C_t^{old}\}_{t=0}^{\infty}\right) = \frac{\log(1+\xi)}{1-\beta} + U(\{C_t^{old}\}_{t=0}^{\infty}) = U(\{C_t^{new}\}_{t=0}^{\infty}),$$

where $\{C_t^{new}\}_{t=0}^{\infty}$ and $\{C_t^{old}\}_{t=0}^{\infty}$ are paths of consumption with and without a change in ψ_o and/or ψ_r. When we allow both ψ_o to fall and ψ_r to rise, welfare is *lower* by the same amount as a permanent 0.57% decrease in consumption (ξ = -0.57%). As shown in Table 11, however, when only ψ_o declines, welfare improves (ξ = 0.28%). When only ψ_r rises, welfare declines even more (ξ = -1.01%). Averaging these two ways (0.28% and 1.01%-0.57%), the decline in ψ_o raises welfare by the same amount as a permanent 0.36% increase in consumption. Even though the decline in overhead costs reduces long-run innovation, overall it increases welfare through a combination of a permanent boost in process efficiency and a temporary surge in innovation.

Table 11: Contribution of the decline in overhead costs to welfare

	1. ψ_o and ψ_r	2. only ψ_r	3. 1. minus 2.	4. only ψ_o	5. ψ_o contribution
ξ%	-0.57	-1.01	0.43	0.28	**0.36%**

Note: Each column displays the welfare change in consumption-equivalent percentage terms when the parameters in the column header change to the value in Table 6. Column '5. ψ_o contribution' equals the average of columns 3 and 4.

5.4 Discussion

According to our calibration, the decline in ψ_o explains only a fraction of the growth burst and slowdown as well as the long run decline in growth. In this sub-section we present potential ways to amplify the growth effect of ψ_o.

5.4.1 Source of ψ_r increase

In our benchmark calibration, a 6% increase in the R&D cost parameter, ψ_r, accounts for half of the modest decline in long run growth. We can endogenize this increase in R&D costs as stemming from diminishing returns with respect to n in research. Suppose the cost of innovating on x lines is given by $\psi_r n^\nu x/nY$. In a steady state where $x/n = z^\star$ for all firms, the aggregate cost of R&D as a share of output can be written as $\widetilde{\psi}_r z^\star$ where $\widetilde{\psi}_r = \psi_r[S^\star n_H^{\nu-1} + (1-S^\star)n_L^{\nu-1}]$. Higher $\widetilde{\psi}_r$ means lower aggregate R&D efficiency. Our baseline model features $\nu = 1$ and $\widetilde{\psi}_r = \psi_r$. When $\nu > 1$, R&D intensity increases with firm size and higher $S^\star$ endogenously raises $\widetilde{\psi}_r$ and lowers R&D efficiency. For example, when $\nu = 1.5$ as in De Ridder (2020), the observed rise in concentration raises $\widetilde{\psi}_r$ by 21%, leading to a much bigger decline in growth. We did not go this route because it would make R&D intensity increase markedly with firm size, contrary to available evidence.[23]

[23] According to the 2016 Business R&D and Innovation Survey (BRDIS) Table 17, R&D intensity of firms reporting R&D declines with firm employment (3.5% for firms with 10K or more employees vs. 5.2% for other firms). We combine BRDIS with Business Dynamics Statistics (BDS) to estimate the share of firms that report R&D and find that unconditional R&D intensity is 0.63% for 10K+ firms and 0.43% for the other firms. This translates to $\nu \approx 1$ because 10K+ firms are about 1600 times larger than the rest of the firms (2016 BDS).

5.4.2 Sensitivity of the growth effects in Δ

In Proposition 3 and 5, the effect of ψ_o on concentration $S^\star$ and growth $z^\star$ is increasing in the firm process efficiency gap Δ. For example, suppose the constant level of Δ is 1.168 rather than its baseline value of 1.134, because the semi-elasticity target for labor share with respect to firm size is -2.7 rather than -2.2. Then the steady state growth decline due to lower ψ_o becomes 11.3 basis points per year vs. our baseline of 8.8 basis points..

Alternatively, suppose the IT revolution enhanced the process efficiency advantage of high-type firms, increasing Δ over time.[24] In our model an increase in Δ can lower the long-run growth rate by raising market concentration and increasing the efficiency of second-best producers confronting innovators (see Online Appendix B.3). If we fix ψ_o and calibrate the change in Δ and ψ_r to fit the rise in concentration and decline in steady state growth, the model asks Δ to increase by 3.9% and ψ_r to *decline* by 2.5%. Thus a rise in Δ alone could explain more than 100% of the decline in steady state growth. We did not make this alternative our baseline because of evidence for stable markups at large relative to small firms in Autor et al. (2020).

6 Conclusion

We developed a model of innovation-led growth with intrinsic firm heterogeneity. We solved for the steady state and transition dynamics and analyzed the extent to which the model can potentially account for a significant portion of the U.S. growth experience over the past 30 years: (i) a productivity slowdown (after a burst in productivity growth); (ii) rising concentration at the national level; and (iii) opposing between and within firm changes in labor share.

[24] This channel is emphasized by Lashkari et al. (2019) and De Ridder (2020). Large firms may be more likely than small firms to make fixed investments in IT and intangibles to reduce marginal production costs.

We argued that a significant part of these phenomena can be explained by IT improvements in the mid-1990s to mid-2000s which allowed the most efficient firms to expand their boundaries. In our story, these firms enjoy higher markups, so when they expand their reach into more markets this pushes down the aggregate labor share. High productivity firms expand by innovating on more product lines, bringing a temporary surge of productivity growth. Within-firm markups eventually fall for both high and low productivity firms, as they are more likely to face high productivity competitors. This force ultimately reduces within-firm markups, and drags down innovation and growth. We find that welfare increases despite the lower long run growth.

We focused our analysis on the overhead cost parameter ψ_o. However, the model lends itself to richer comparative static and transition analyses. In particular it is straightforward to explore the steady state effects of changes in the efficiency gap Δ, the innovation size γ, the innovation cost ψ_r or the share of high productivity firms ϕ. We see it as a virtue of our model that the within vs. between firm effects of such changes can be studied easily.

Our baseline framework is based on leapfrogging innovations. In other words, our model does not feature a positive *escape competition effect* as in Akcigit and Ates (2019) or Liu, Mian and Sufi (2020). One can introduce such an effect into our model via step-by-step innovation.

One could also explore optimal tax and subsidy policies in our quantitative framework. The decentralized equilibrium is suboptimal due to markup dispersion across products as well as knowledge spillovers across firms. Falling overhead costs may increase welfare more strongly in the presence of an optimal R&D subsidy.

Finally, our framework is well suited for discussing competition policy and its relation with the productivity slowdown. We analyzed the implications of allowing mergers and acquisitions, but other dimensions of competition policy such as data access or firm breakup can be naturally considered through the lens of our model. We leave these extensions of our analysis for future research.

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