Globalization, Trade Imbalances and Labor Market Adjustment *

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Abstract

We study the role of global trade imbalances in shaping both the adjustment dynamics, and the long-run consequences, of trade shocks. We build an estimable, general equilibrium, multi-country, multi-sector model of trade with three key ingredients: (a) representative households making consumption-saving decisions in each country; (b) costly reallocation of workers across sectors; and (c) matching frictions in segmented labor markets leading to unemployment dynamics. We estimate our model using data from the World Input-Output Database and household surveys and study the behavior of national labor markets in response to several globalization shocks, including shocks to: (1) technology; (2) trade costs; (3) inter-temporal tastes (e.g. savings gluts). We find that modeling trade imbalances changes both qualitatively and quantitatively the short- and long-run implications of globalization shocks for labor reallocation and unemployment dynamics. In a series of empirical applications, we study the labor market effects of shocks accrued to the global economy, their implications for the gains from trade, and we revisit the "China Shock" through the lens of our model. We show that the US enjoys a 2.2% gain in response to globalization shocks. These shocks would have been 73% larger in the absence of the global savings glut, but they would have been 40% smaller if we had lived in a balanced trade world.

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"One major contrast between most economic analyses of globalization's impact and those of the broader public ... is the focus, or lack thereof, on trade imbalances. The public tends to see trade surpluses or deficits as determining winners and losers; the general equilibrium trade models that underlay the 1990s' consensus gave no role to trade imbalances at all. The economists' approach is almost certainly right for the long run ... Yet in the long run we are all dead, and rapid changes in trade balances can cause serious problems of adjustment ..."

Paul Krugman, "Globalization: What Did We Miss?"[1]

1 Introduction

There is a large body of evidence indicating that globalization shocks can lead to important labor market disruptions. For instance, Autor et al. (2013) show that American workers in regions specialized in goods facing steeper competition from China are less likely to work in manufacturing, more likely to be unemployed, and more likely to rely on disability insurance. Similarly, Dix-Carneiro and Kovak (2017, 2019) show that Brazilian regions that were more exposed to foreign competition as a result of the 1990's trade liberalization experienced prolonged periods of lower formal employment and wages relative to less exposed regions, as well as increases in non-employment and informal employment.[2] This recent reduced-form literature has generated considerable interest in understanding, modelling and quantifying the adjustment process in response to globalization shocks. In particular, Artuç et al. (2010), Dix-Carneiro (2014), Traiberman (2019) and Caliendo et al. (2019) have estimated large mobility costs across sectors and occupations, long adjustment processes and sizable distributional consequences to trade shocks.

Yet, this literature has abstracted from modeling trade imbalances and has been silent on how they could influence the labor market adjustment process. This is puzzling in light of the increase in importance of trade imbalances in the last three decades, coupled with an increasing discomfort that many American policy makers manifest towards trade deficits.[3] Indeed, there is a pervasive concern among many policy makers and commentators that trade deficits are undesirable as they crowd out domestic production and are detrimental to jobs and workers. When trade is balanced, a contraction of import-competing sectors must be simultaneously met with an expansion of export-oriented sectors. On the other hand, if trade imbalances emerge in response to globalization shocks, these shifts are no longer synchronized, affecting the dynamics of reallocation. Specifically, the behavior of trade imbalances can influence the dynamics of job losses and gains, especially in the presence of labor market frictions.

[1] See Krugman (2019).

[2] Other recent papers tying globalization shocks to labor market disruptions include Pierce and Schott (2016), Costa et al. (2016), Dauth et al. (2018), Utar (2018), among many others.

[3] See, for example, Scott (1998), Bernanke (2005), and Navarro (2019).

In this paper, we study how endogenizing trade imbalances influence the labor market adjustment process in response to globalization. Does ignoring trade imbalances when we investigate the labor market consequences of trade shocks matter at all? How much insight do we lose in doing so? To address these questions, we build on existing models of globalization and labor market adjustment and develop an estimable, general equilibrium, multi-country, multi-sector model with three key ingredients: (i) Consumption-saving decisions in each country are commanded by a representative family, leading to endogenous trade imbalances; (ii) labor market frictions across and within sectors, leading to unemployment dynamics and sluggish transitions to shocks; and (iii) Ricardian comparative advantage forces promote trade but geographical barriers inhibit it.

In our model, trade imbalances arise from country-level representative agents making consumption and savings decisions.[4] This gives rise to an Euler Equation that dictates how countries smooth consumption over time, and allows us to decompose the observed dynamics of trade imbalances into (1) a consumption response to productivity and trade shocks, and to (2) a residual shock to country-level discount factors (inter-temporal preferences). Our approach relies neither on *ad hoc* rules for imbalances nor on specifying the path of imbalances exogenously, which are common in the international trade literature. Instead, our perspective builds on the workhorse model of imbalances in international macroeconomics, providing a natural benchmark for understanding how they interact with labor market frictions.[5]

Kletzer (2001) emphasizes that trade liberalization is often a focal point for anxiety about job losses and insecurity. Reinforcing this conclusion, data from the 2014 Pew Global Attitudes survey reveal that 50 percent of Americans believe that trade destroys jobs, whereas only 20 percent believe that it creates jobs (see Pavcnik, 2017). Not surprisingly, trade-induced job losses and unemployment are at the heart of policy makers' concerns with globalization. We directly speak to these concerns by incorporating labor market frictions across and within sectors into our model of globalization and trade imbalances. We model mobility frictions across sectors following Artuç et al. (2010) and search frictions and unemployment according to Mortensen and Pissarides (1994). In particular, our framework allows for job destruction and creation to respond to trade shocks, leading to potentially rich unemployment dynamics. Importantly, our framework is suitable for the analysis of out-of-steady state dynamics triggered by changes in the global economy.

We estimate our model using a simulated method of moments and data from the World Input Output Database and the United States Current Population Survey. To ensure tractability of the estimation procedure, we assume the economy is in steady state and we match data moments in year 2000. The procedure conditions on the observed trade shares and allows us to estimate our parameters country by country, greatly simplifying the process.

[4] See Obstfeld and Rogoff (1995) for a survey of this approach to imbalances in international macroeconomics.

[5] More recent work on global imbalances builds on the standard consumption savings model by adding financial frictions (e.g., Caballero et al. (2008) and Mendoza et al. (2009)), or demographics (e.g., Barany et al. (2018)).

To understand the main mechanisms in our model, we first compare its response to different sets of shocks to the response of the same model under balanced trade—an assumption that most of the literature on the topic makes. We find that modelling imbalances leads to significantly more reallocation across sectors in response to shocks to fundamental parameters of the global economy such as trade costs and technologies. Because, in our model, workers need to switch to unemployment before searching for a job in a new sector, reallocation is related to unemployent. Therefore, the response of unemployment to trade shocks is also amplified by the emergence of trade imbalances.

The pattern of reallocation can also be very different, both in the short and long runs, depending on the evolution of imbalances. For example, if China experiences a slow moving productivity increase before it reaches a plateau, it smooths consumption by running a trade deficit in the short run, heavily importing manufactured goods. In turn, the United States and the rest of the world lend to China and run trade surpluses in the short run, leading to an expansion of their manufacturing sectors. In the long run, China needs to pay its debt and runs a trade surplus. It does so by exporting manufactured goods, and therefore these sectors must eventually expand. On the other hand, the United States and the rest of the world enjoy manufacturing imports from China and downsize their manufacturing sectors. This leads to a non-monotonic pattern of adjustment that is absent if we abstract from trade imbalances. If we were to impose balanced trade, manufacturing employment would monotonically decline in all countries with the exception of China. In addition, the magnitude of the long-run contraction of manufacturing outside of China is of larger magnitude when we allow for trade imbalances.

When balanced trade is imposed, long-run allocations depend only on the final level of the shocks. In contrast, we find that the path of shocks matters for long-run allocations when trade imbalances are modelled. This dependence on the full path of shocks motivates us to conduct an empirical analysis where we extract the various technology and trade cost shocks that the global economy faced between 2000 and 2014. In this application, we show that the stark differences relative to a model without trade imbalances are quantitatively important. In addition, we study the implications of trade imbalances and labor market frictions for the gains from trade, which are typically computed using the sufficient-statistics approach of Arkolakis et al. (2012) and extended by Costinot and Rodríguez-Clare (2014). We find that our model leads to significant different gains compared to those predicted by Costinot and Rodríguez-Clare (2014), both in terms of magnitude and signs. Both imbalances and labor market frictions play important roles explaining the discrepancies. We also explore the "China shock" in the context of our model.[6] Aligned with previous findings, we corroborate that China contributed to the decline of the United States manufacturing between 2000 and 2014. However, this decline in manufacturing was quickly accompanied by job

[6]See Autor et al. (2016) and references therein for a discussion of the China shock.

creation in Services and Agriculture, leading to small unemployment effects. We find that shocks to the Chinese economy had a modest effect on the US trade deficit, highlighting that the evolution of the US trade deficit is a result of the full constellation of shocks accrued to the global economy. On the other hand, if we neutralize the Chinese savings glut, the trade deficit in the US would have shrunk in the short run, but it would have been amplified in the long run. Finally, we show that the US enjoys a 2.2% gain in response to globalization shocks. These shocks would have been 73% larger in the absence of the global savings glut, but they would have been 40% smaller if we had lived in a balanced trade world.

Our paper speaks to a large literature that investigates the labor market consequences of globalization both empirically and quantitatively. We make two contributions to this literature by incorporating both involuntary unemployment and trade imbalances into the state-of-the-art Ricardian trade model of Caliendo and Parro (2015). Broadly speaking, quantitative trade models based on Eaton and Kortum (2002) have only allowed for a non-employment option (i.e., voluntary unemployment) or have focused on steady-state analyses, ignoring transitional dynamics. Caliendo et al. (2019) is an important example of a dynamic quantitative trade model in which workers make a labor supply decision and face mobility frictions across sectors and regions. However, their model does not feature job losses and unemployment. On the other end, Carrère et al. (2020) and Guner et al. (2020) incorporate search frictions and unemployment into multi-sector extensions of Eaton and Kortum (2002), but do not study out-of-steady-steady dynamics. In a recent exception, Rodriguez-Clare et al. (2020) incorporates wage rigidity into the model of Caliendo et al. (2019) to investigate the unemployment effects of the China Shock on local labor markets in the United States.[7]

Importantly, though, none of these papers model trade imbalances. We do so by incorporating the workhorse model of imbalances used in the international macroeconomics literature allowing for savings decisions by means of an international bonds market as in Reyes-Heroles (2016).[8] In that regard, our paper is closely related to Kehoe et al. (2018) who explore the implications of the increase in the United States trade deficit for the secular decline in manufacturing labor over the last four decades. However, this paper does not incorporate sluggish labor market adjustment nor unemployment dynamics.[9]

[7] In addition to these papers based on the Eaton and Kortum model, Helpman and Itskhoki (2010) add labor market frictions to a two-country Melitz model, and Heid and Larch (2016) add labor market frictions to an Armington model of trade. Coşar et al. (2016) incorporate search frictions and unemployment to an estimable small open economy Melitz model with firm dynamics, but focus on steady-state analyses. Ruggieri (2019) extends that model to study the transition in response to trade shocks. Similarly, Helpman and Itskhoki (2015) also analyze the dynamic behavior of a two-country Melitz model with labor market frictions. Finally, Kambourov (2009), Artuç et al. (2010), Dix-Carneiro (2014), and Traiberman (2019) also study transitional dynamics, but from the lenses of small open economy models.

[8] A few papers have analyzed the consequences of current account rebalancing on labor reallocation and unemployment by considering changes in imbalances as exogenous, *e.g.* Obstfeld and Rogoff (2005), Dekle et al. (2007) and Eaton et al. (2013).

[9] In International Macroeconomics, Kehoe and Ruhl (2009) and Meza and Urrutia (2011) are examples of the

One limitation of our framework is that it does not incorporate endogenous capital accumulation decisions. Instead, imbalances arise as a result of consumption smoothing. Excluding capital from the model means that certain features of the data that are typically studied in International Real Business Cycles (IRBC) cannot be matched—for example, the countercyclicality of quarterly net exports observed for many developed economies (Backus et al., 1992, 1994).[10] Ultimately we ignore capital for two reasons. First, it helps maintain tractability of the model and allows us to focus on richer labor market dynamics than are present in typical dynamic models. This is in line with the trade literature more broadly which usually ignores dynamics in capital—treating it as a fixed factor, if modeling it at all. Second, capital is not necessary to derive our main results, which regard the quantitative importance of how trade shocks can drive imbalances, and how this feeds into the path of labor market outcomes. Nevertheless, capital dynamics may further interact with these channels, making it an important avenue for future research.

The rest of this paper is structured as follows. Section 2 outlines our model. Section 3 describes the data we use and our estimation procedure. In Section 4 we present our estimates and model fit analysis, but we also interpret the key mechanisms of the model by simulating a series of impulse response functions. Section 5 conducts two empirical applications of our model. We conclude and discuss future research in 6.

2 Model

Our model builds on existing workhorse models of globalization, imbalances and labor market adjustment. Trade imbalances are modelled according to the inter-temporal approach of Obstfeld and Rogoff (1995), and the trade bloc is based on Caliendo and Parro (2015). We adopt the framework in Artuç et al. (2010) to model labor mobility frictions across sectors and the structure in Mortensen and Pissarides (1994) to model search frictions and job creation and destruction. Sections 2.1 through 2.9 formalize our model showing how these different frameworks fit together.

2.1 Environment

There is no aggregate uncertainty, so that all agents have perfect foresight of aggregate variables. There are $i = 1, ..., N$ countries. Each country i has a constant labor force given by $\overline{L}_i$ workers/consumers. There are $k = 1, ..., K$ sectors. Each sector k is characterized by a continuum set of varieties $j \in [0, 1]$ that can be traded across countries. These varieties are then aggregated by perfectly competitive domestic firms, in each country, into non-tradable composite sector-specific

scarce work studying the effects of current account reversals on sluggish labor reallocation.

[10] Our model features vacancy posting decisions, which are a form of forward looking investment. This investment in vacancies to create jobs is procyclical and could in principle lead to countercyclical net exports in our model. However, this is not isomorphic to capital as a factor of production and we do not stress this feature.

intermediate goods according to:

$$Q_{k,i}^{I,t} = \left(\int_0^1 \left(Q_{k,i}^t (j) \right)^{\varepsilon} dj \right)^{\frac{1}{\varepsilon}}, \tag{1}$$

where $Q_{k,i}^t (j)$ is the quantity employed of variety j in sector k and country i at time t, and $\sigma = \frac{1}{1-\varepsilon}$ is the elasticity of substitution across varieties within sectors. These composite sector-specific goods are solely used as intermediate inputs for the production of a final good or for the production of varieties. The price of one unit of the composite good of sector k in country i is given by the price index associated with (1), which we denote by $P_{k,i}^{I,t}$.

A non-tradable final good is produced by perfectly competitive firms that aggregate sector-specific composite goods $Q_{k,i}^{I,t}$ according to:

$$Q_i^{F,t} = \prod_{k=1}^{K} \left(Q_{k,i}^{I,t} \right)^{\mu_{k,i}}, \tag{2}$$

where $\mu_{k,i} > 0$ and $\sum_{k=1}^{K} \mu_{k,i} = 1 \ \forall i$. The price of one unit of the final good is given by the price index associated with (2), which we denote by $P_i^{F,t}$.

2.2 Labor Markets

Workers and single-worker firms producing varieties engage in a costly search process. Firms post vacancies, but not all of them are filled. Workers search for a job, but not all of them are successful, leading to involuntary unemployment. Each variety j constitutes a different labor market. More precisely, the unemployment rate $u_{k,i}^t (j)$ is variety-specific, as is the vacancy rate $v_{k,i}^t (j)$. Both variables are expressed as a fraction of the labor force $L_{k,i}^t (j)$, measured as the sum of workers who are employed or unemployed and searching within sector k / variety j at time t. If $u_{k,i}^t (j)$ is the unemployment rate and $v_{k,i}^t (j)$ is the vacancy rate, we impose that $m_i \left(u_{k,i}^t (j), v_{k,i}^t (j) \right)$ matches are formed (as a fraction of the labor force $L_{k,i}^t (j)$), where the matching function m_i is increasing in both arguments, concave, and homogeneous of degree 1.

From now on, we drop the index j, but the reader should keep in mind that all labor market variables are country-sector-variety specific. Define labor market tightness as:

$$\theta_{k,i}^t \equiv \frac{v_{k,i}^t}{u_{k,i}^t}. \tag{3}$$

The fact that m_i is constant returns allows us to write the probability that a vacancy matches with a worker as $q_i(\theta_{k,i}^t)$, for a decreasing function q_i, and the probability that a worker matches with

a vacancy as $\theta^t_{k,i} q_i(\theta^t_{k,i})$. In addition to search frictions, workers face mobility costs across sectors, but there is free mobility across varieties j within a sector.

2.3 Households

Countries are organized into representative families, each with a household head that determines consumption, savings, and the allocation of workers across sectors maximizing aggregate utility. We first describe the utility of individual workers, then we show how household heads aggregate members' utilities. For ease of notation, we temporarily omit the country subscript i and let ℓ index individuals.

If a worker ended period $t-1$ unemployed in sector k she can either search in sector k at time t (at no additional cost) or incur a moving cost $C_{kk'}$ and search in sector k' at time t—so that $C_{kk}=0$. If a worker ℓ is not employed at the production stage at t, she receives preference shocks $\left\{\omega^t_{k,\ell}, k=1,...,K\right\}$ for each sector at time t. After unemployed workers receive these shocks, the household head decides whether to keep each worker in the same sector and restrict him to search there at t, or to incur a mobility cost and allow him to search in another sector. The $\omega^t_{k,\ell}$ shocks are iid across individuals, sectors and time, and are assumed to follow a Gumbel distribution with parameters $\left(-\gamma^{EM}\zeta, \zeta\right)$ where γ^{EM} is the Euler-Mascheroni constant and ζ its shape parameter. This structure follows closely Artuç et al. (2010).

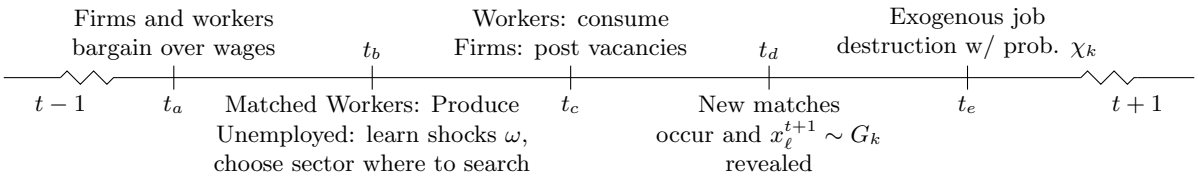
After being allocated to search in sector k' at t the unemployed worker receives unemployment utility $b_{k'}$ and matches with a firm with probability $\theta^t_{k'} q(\theta^t_{k'})$. We follow Mortensen and Pissarides (1994) and assume that once a worker and a firm match at t, a match-specific productivity for $t+1$ production, x^{t+1}_ℓ, is randomly drawn from a distribution $G_{k,i}$ with $[0,\infty)$ support. This productivity is constant over time from then on. At this point, the household head or the firm can break a match if keeping it active is not optimal. Finally, at the end of every period, and following the matching process, there is an exogenous probability χ_k of existing matches to dissolve (excluding new ones). Successful matches that occur at time t only start to produce at $t+1$, and workers employed in sector k are then paid wages denoted by $w^{t+1}_k\left(x^{t+1}_\ell\right)$. Finally, if a worker produces in sector k, she receives a non-pecuniary benefit of η_k. Figure 1 details the timing of the model. Section 2.5 describes the bargaining process that occurs at t_a and section 2.4.2 describes the decision of firms to post vacancies at time t_c.

Given this setup, the period utility for individual ℓ at time t is,

$$\mathcal{U}^t_\ell\left(e^t_\ell, k^{t+1}_\ell, k^t_\ell, \omega^t_\ell, c^t_\ell\right) = \left(1-e^t_\ell\right)\left(-C_{k^t_\ell, k^{t+1}_\ell} + b_{k^{t+1}_\ell} + \omega^t_{k^{t+1}_\ell,\ell}\right) + e^t_\ell \eta_{k^t_\ell} + u\left(c^t_\ell\right), \tag{4}$$

where k^t_ℓ is the sector where individual ℓ starts period t (that sector was determined at $t-1$), k^{t+1}_ℓ is the $t+1$ sector of choice (which is decided at time t, interim period t_b in Figure 1), e^t_ℓ is

Figure 1: Timing of the Model



the employment status at the production stage (interim period t_b), $\omega_\ell^t = \left(\omega_{1,\ell}^t, ..., \omega_{K,\ell}^t\right)$, and c_ℓ^t is individual consumption. If individual ℓ is unemployed in sector k at t ($e_\ell^t = 0, k_\ell^t = k$), the individual can switch sectors (from k to k_ℓ^{t+1}), so that mobility costs $C_{k_\ell^t, k_\ell^{t+1}}$, utility of unemployment $b_{k_\ell^{t+1}}$ and shock $\omega_{k_\ell^{t+1},\ell}^t$ are incurred (during period t). On the other hand, if individual ℓ is employed during the production stage at t, $e_\ell^t = 1$, the worker enjoys the non-pecuniary benefit of working in sector k_ℓ^t given by $\eta_{k_\ell^t}$. Finally, individual ℓ also enjoys the utility of consumption $u\left(c_\ell^t\right)$.

From the perspective of the household head, the allocation of workers follows a controlled stochastic process: while the head can choose workers' sectors given knowledge of switching costs and shocks, employment itself remains a probabilistic outcome. To this end, let $\widetilde{e}_k^t\left(x_\ell^{t+1}\right) \in \{0, 1\}$ indicate whether the household head continues on with a match at time t given a match productivity of x_ℓ^{t+1} in sector k. Then, the probability that worker ℓ is employed in sector k at time $t+1$, conditional on match productivity x_ℓ^{t+1} and time t information $\left(k_\ell^t, e_\ell^t\right)$ is given by:

$$\begin{aligned}\Pr\left(k_\ell^{t+1} = k, e_\ell^{t+1} = 1 | x_\ell^{t+1}, k_\ell^t, e_\ell^t\right) = {} & \mathcal{I}\left(k_\ell^t = k\right) e_\ell^t \left(1 - \chi_k\right) \widetilde{e}_k^t\left(x_\ell^{t+1}\right) \\ & + \left(1 - e_\ell^t\right) \mathcal{I}\left(k_\ell^{t+1} = k\right) \theta_k^t q\left(\theta_k^t\right) \widetilde{e}_k^t\left(x_\ell^{t+1}\right).\end{aligned} \tag{5}$$

In words, if $\mathcal{I}\left(k_\ell^t = k\right) e_\ell^t = 1$, then worker ℓ is employed in sector k at time t and the match survives with probability $(1 - \chi_k)$ if the family planner decides to keep the match ($\widetilde{e}_k^t\left(x_\ell^{t+1}\right) = 1$). If $e_\ell^t = 0$, that is, the worker is unemployed at t, and the planner chooses $k_\ell^{t+1} = k$, then the worker is employed in sector k at time $t+1$ with probability $\theta_k^t q\left(\theta_k^t\right) \widetilde{e}_k^t\left(x_\ell^{t+1}\right)$. Importantly, workers' sector and employment status at $t+1$, k_ℓ^{t+1} and e_ℓ^{t+1}, are determined by actions taken at t.

The head of the household aggregates (4) over family members and maximizes the net present value of utility subject to her budget constraint and the controlled process on employment (5). In addition to consumption and employment decisions, the household head has access to international financial markets by means of buying and selling one-period riskless bonds that are available in zero-net supply around the world. One can think of international bond markets in period t as spot markets in which a family buys a piece of paper with face value of B^{t+1} in exchange for a bundle of goods with the same value, and the piece of paper represents a promise to receive goods in period $t+1$ with a value equal to $R^{t+1}B^{t+1}$. International bond markets operate without frictions, so

that the nominal returns R^{t+1} are equalized across countries. Finally, the household collects and aggregates profits across all firms, Π^t, but takes this lump sum payment as given. The household head chooses the path of consumption, c_ℓ^t, the path of sectoral choices, k_ℓ^t, continuation decisions, $\widetilde{e}_k^t(x)$, and bonds, B^t, to solve:

$$\max_{\left\{k_\ell^t, \widetilde{e}_k^t(.), B^t, c_\ell^t\right\}} E_0 \left\{ \sum_{t=0}^{\infty} (\delta)^t \phi^t \int_0^{\overline{L}} \mathcal{U}_\ell^t d\ell \right\}, \tag{6}$$

subject to (5) and the budget constraint:

$$P^{F,t} \int_0^{\overline{L}} c_\ell^t d\ell + B^{t+1} - \Pi^t - R^t B^t - \int_0^{\overline{L}} \left(\sum_{k=1}^{K} \mathcal{I}\left(k_\ell^t = k\right) e_\ell^t w_k^t \left(x_\ell^t\right) \right) d\ell \leq 0. \tag{7}$$

E_0 denotes expectations with respect to the idiosyncratic shocks ω. The model has no aggregate uncertainty, so that households and firms have perfect foresight of aggregate variables. δ is the discount factor, common across all countries, and ϕ^t is an intertemporal preference shifter that the household head experiences in period t. These shifters can differ across countries. The budget constraint states the family can buy consumption or bonds for next period using profits and wage income, net of interest payments (or collections) on past bonds. Let $\widetilde{\lambda}^t$ be the Lagrange multiplier on the family's budget constraint (7).[11] The optimality condition with respect to c_ℓ^t is $u'\left(c_\ell^t\right) = \widetilde{\lambda}^t P^{F,t}$, so that individual consumption is equalized across individuals within the household: $c_\ell^t = c^t \ \forall \ell$. Henceforth, we will refer to c^t as per capita consumption. Armed with this observation, Appendix A shows that the labor supply decisions solving (6) subject to (7) and (5) can be decentralized and written recursively for unemployed and employed workers. We now turn to this recursive formulation.

Let us return to indexing countries by i. Since workers are symmetric up to x and η in each country, we stop indexing individual workers. We denote by $\widetilde{U}_{k,i}^t(\omega^t)$ the value of unemployment in sector k, country i at time t conditional on shocks ω^t, and by $W_{k,i}^t(x)$ the value of employment conditional on match-specific productivity x. If we define $\widehat{\phi}_i^{t+1} \equiv \frac{\phi_i^{t+1}}{\phi_i^t}$, the sector decision policy and the continuation rule $\widetilde{e}_k^t(.)$ must solve, conditional on shocks ω^t:

$$\widetilde{U}_{k,i}^t(\omega^t) = \max_{k'} \left(\begin{array}{c} -C_{kk',i} + \omega_{k'}^t + b_{k',i} \\ +\theta_{k',i}^t q\left(\theta_{k',i}^t\right) \delta \widehat{\phi}_i^{t+1} \int_0^{\infty} \max\left\{ W_{k',i}^{t+1}(x), U_{k',i}^{t+1} \right\} dG_{k',i}(x) \\ + \left(1 - \theta_{k',i}^t q\left(\theta_{k',i}^t\right)\right) \delta \widehat{\phi}_i^{t+1} U_{k',i}^{t+1}, \end{array} \right), \tag{8}$$

[11] In an abuse of terminology we will continue to refer to $\widetilde{\lambda}^t$ as the Lagrange multiplier. However, the correct shadow price associated with period's t budget constraint is given by $\delta^t \phi^t \widetilde{\lambda}^t$.

and

$$W_{k,i}^{t}\left(x\right)=\widetilde{\lambda}_{i}^{t}w_{k,i}^{t}\left(x\right)+\eta_{k,i}+\delta\widehat{\phi}_{i}^{t+1}\left(1-\chi_{k,i}\right)\max\left\{W_{k,i}^{t+1}\left(x\right),U_{k,i}^{t+1}\right\}+\delta\widehat{\phi}_{i}^{t+1}\chi_{k,i}U_{k,i}^{t+1}. \tag{9}$$

In equation (8), $U_{k,i}^{t}\equiv E_{\omega}\left(\widetilde{U}_{k,i}^{t}(\omega^{t})\right)$ is the expected value of $\widetilde{U}_{k,i}^{t}(\omega^{t})$ integrated over ω^{t}. The first line in this equation corresponds to the costs of switching sectors, $-C_{kk',i}+\omega_{k',i}^{t}$, as well as the sector-specific value of being unemployed in that sector $b_{k'i}$. The second line is the probability of finding a match $\theta_{k',i}^{t}q\left(\theta_{k',i}^{t}\right)$ multiplied by the discounted value of the match. Note that for low values of $W_{k',i}^{t+1}\left(x\right)$, the household head dissolves the match so that the worker obtains $U_{k',i}^{t+1}$. Finally, the third line is the discounted value of being unemployed next period if the worker fails to successfully match.

In equation (9), the first term on the left hand side is the wage function, multiplied by the household head's Lagrange multiplier on the budget constraint $\widetilde{\lambda}_{i}^{t}$. The second term is the non-pecuniary benefit of working in sector k in country i. The next terms are the continuation values: with probability $(1-\chi_{k,i})$ the match does not exogenously dissolve and the worker continues the match; with probability $\chi_{k,i}$ the match exogenously breaks and the worker receives the value of unemployment in k.

Since ω is Gumbel distributed, the policy rule for unemployed workers can be solved analytically, so that transition rates between t and $t+1$ can be written as:

$$s_{kk',i}^{t,t+1}=\frac{\exp\left\{\begin{array}{c}-C_{kk',i}+b_{k',i}+\\ \theta_{k',i}^{t}q(\theta_{k',i}^{t})\delta\widehat{\phi}_{i}^{t+1}\int_{0}^{\infty}\max\left\{W_{k',i}^{t+1}\left(x\right)-U_{k',i}^{t+1},0\right\}dG_{k',i}(x)+\delta\widehat{\phi}_{i}^{t+1}U_{k',i}^{t+1}\end{array}\right\}}{\sum_{k''=1}^{K}\exp\left\{\begin{array}{c}-C_{kk'',i}+b_{k'',i}+\\ \theta_{k'',i}^{t}q(\theta_{k'',i}^{t})\delta\widehat{\phi}_{i}^{t+1}\int_{0}^{\infty}\max\left\{W_{k'',i}^{t+1}\left(x\right)-U_{k'',i}^{t+1},0\right\}dG_{k'',i}(x)+\delta\widehat{\phi}_{i}^{t+1}U_{k'',i}^{t+1}\end{array}\right\}}. \tag{10}$$

2.4 Firms

2.4.1 Incumbents

Firms produce by combining the labor of one single worker with composite intermediate inputs purchased from all sectors. All firms producing variety j in sector k and country i have access to common productivity $z_{k,i}^{t}(j)$ and are paid $p_{k,i}^{t}\left(j\right)$ for each unit of production. A firm producing variety j in sector k with match-specific productivity x and employing composite intermediate inputs $\left\{M_{\ell,i}^{t}\right\}_{\ell=1}^{K}$ obtains revenue:

$$Y_{k,i}^{t}\left(j,x\right)=p_{k,i}^{t}\left(j\right)z_{k,i}^{t}\left(j\right)x^{\gamma_{k,i}}\left(\prod_{\ell=1}^{K}\left(M_{\ell,i}^{t}\right)^{\nu_{k\ell,i}}\right)^{\left(1-\gamma_{k,i}\right)}, \tag{11}$$

where $\gamma_{k,i} \in (0,1)$, $\nu_{k\ell,i} > 0$, and $\sum_{\ell=1}^{K} \nu_{k\ell,i} = 1$

Firms are price takers in product and input markets and decide on the usage of $\left\{M_{\ell,i}^t\right\}_{\ell=1}^K$ solving:

$$S_{k,i}^t(j,x) = \max_{\left\{M_{\ell,i}^t\right\}} p_{k,i}^t(j)\, z_{k,i}^t(j)\, x^{\gamma_{k,i}} \left(\prod_{\ell=1}^{K} \left(M_{\ell,i}^t\right)^{\nu_{k\ell,i}}\right)^{(1-\gamma_{k,i})} - \sum_{\ell=1}^{K} P_{\ell,i}^{I,t} M_{\ell,i}^t, \tag{12}$$

where $S_{k,i}^t(j,x)$ denotes the revenue (net of intermediate input payments) generated by the match between a firm and a worker with productivity x producing variety j. One can show that:

$$S_{k,i}^t(j,x) = \widetilde{w}_{k,i}^t(j)\, x, \tag{13}$$

where

$$\widetilde{w}_{k,i}^t(j) \equiv \gamma_{k,i}(1-\gamma_{k,i})^{\frac{1-\gamma_{k,i}}{\gamma_{k,i}}} \left(P_{k,i}^{M,t}\right)^{\frac{\gamma_{k,i}-1}{\gamma_{k,i}}} \left(p_{k,i}^t(j)\, z_{k,i}^t(j)\right)^{\frac{1}{\gamma_{k,i}}}, \tag{14}$$

$$\text{and } P_{k,i}^{M,t} \equiv \prod_{\ell=1}^{K} \left(\frac{P_{\ell,i}^t}{\nu_{k\ell,i}}\right)^{\nu_{k\ell,i}} \tag{15}$$

is the price of one unit of the intermediate input composite $im_{k,i}^t \equiv \prod_{\ell=1}^{K} \left(M_{\ell,i}^t\right)^{\nu_{k\ell,i}}$ in sector k and country i. We assume that following entry, switching across varieties within sector is costless for firms. Hence, no arbitrage across varieties will ensure that $\widetilde{w}_{k,i}^t(j) = \widetilde{w}_{k,i}^t(j')$ and $p_{k,i}^t(j) z_{k,i}^t(j) = p_{k,i}^t(j') z_{k,i}^t(j')\ \forall j, j'$. Therefore, $\widetilde{w}_{k,i}^t$ and $p_{k,i}^t z_{k,i}^t$ do not depend on the specific variety. This implies symmetry across varieties, allowing us to drop the index j identifying individual varieties. Given the expression of the surplus (13), we will henceforth refer to $\left\{\widetilde{w}_{k,i}^t\right\}$ as "sectoral surpluses". As will become clearer in Section 2.7 these sectoral surpluses will play the same role as wages do in Caliendo and Parro (2015).

Denote the wage paid by that a firm in sector k and country i with match-specific productivity x by $w_{k,i}^t(x)$. We can write the value function for incumbent firms, $J_{k,i}^t(x)$, as:

$$J_{k,i}^t(x) = \widetilde{\lambda}_i^t \left(\widetilde{w}_{k,i}^t x - w_{k,i}^t(x)\right) + (1-\chi_{k,i})\, \delta \widehat{\phi}_i^{t+1} \max\left\{J_{k,i}^{t+1}(x), 0\right\}. \tag{16}$$

The first term is the firm's current profit, and the second is the firm's continuation value of the match.[12] If $J_{k,i}^t(x) < 0$ the firm does not produce and exits.

[12] Firm profits are multiplied by the multiplier on the family's budget constraint in order to keep the units, utils, consistent between the firm's and worker's problem. However, if one divides $J_{k,i}^t(x)$ by $\widetilde{\lambda}_i^t$, then from the Euler Equation we derive below, it is clear that this formulation is equivalent to a risk neutral firm discounting profits using the nominal interest rate R^{t+1}.

2.4.2 New Entrants

Potential entrants can match with a worker by posting vacancies in k (and variety j). We assume that posting a vacancy costs $\kappa_{k,i}$ units of the final good, and so amounts to total cost $\kappa_{k,i}P_i^{F,t}$. Vacancies are posted at the interim period t_c as illustrated in Figure 1. As argued above, J is not variety specific, which means the variety distinction can also be ignored for new entrants. If a firm successfully matches with a worker at t, production starts at $t+1$. If we denote the expected value of an open vacancy by $V_{k,i}^t$, then:

$$V_{k,i}^t = -\widetilde{\lambda}_i^t \kappa_{k,i} P_i^{F,t} + \delta \widehat{\phi}_i^{t+1} \left[\begin{array}{c} q_i\left(\theta_{k,i}^t\right) \int_0^\infty \max\left\{J_{k,i}^{t+1}(s), 0\right\} dG_{k,i}(s) \\ + \left(1 - q_i\left(\theta_{k,i}^t\right)\right) \max\left\{V_{k,i}^{t+1}, 0\right\} \end{array} \right]. \tag{17}$$

The first term on the right hand side is the cost of posting vacancies, which is converted to utility units by the Lagrange multiplier $\widetilde{\lambda}_i^t$. The second term says that in the next period entrants match with probability $q_i\left(\theta_{k,i}^t\right)$ and obtain the expected value of $J_{k,i}^{t+1}$ starting in the next period, if the match is not exogenously destroyed. If they do not match (or the match is destroyed), they can post another vacancy.

There is an infinite mass of potential entrants, and there is free entry so that $V_{k,i}^t \leq 0 \ \forall k, i, t$. Throughout the paper, we impose that this holds with equality in equilibrium—both in steady state and in any transitions.

2.5 Wages

The surplus of a match between a worker and a firm is defined as the utility generated by the match in excess of the parties' outside options. Imposing the free entry condition, the outside option to the firm is 0, while to the worker it is $U_{k,i}^t$. Hence, the surplus of the match is given by $\mathcal{S}_{k,i}^t(x) \equiv J_{k,i}^t(x) + W_{k,i}^t(x) - U_{k,i}^t$. We assume that firms and workers engage in Nash bargaining over the surplus, with the workers' bargaining weight given by $\beta_{k,i}$. This leads to $J_{k,i}^t(x) = \beta_{k,i}\mathcal{S}_{k,i}^t(x)$, which combined with equations (9) and (16) imply that $\mathcal{S}_{k,i}^t$, $W_{k,i}^t$, and $J_{k,i}^t$ are increasing functions of x, implying that matches only remain active at t if $x > \underline{x}_{k,i}^t$ where $\underline{x}_{k,i}^t$ solve:

$$\mathcal{S}_{k,i}^t\left(\underline{x}_{k,i}^t\right) = J_{k,i}^t\left(\underline{x}_{k,i}^t\right) = W_{k,i}^t\left(\underline{x}_{k,i}^t\right) - U_{k,i}^t = 0 \tag{18}$$

The wage equation resulting from Nash bargaining is:

$$w_{k,i}^t(x) = \beta_{k,i}\widetilde{w}_{k,i}^t x + (1 - \beta_{k,i}) \frac{\left(U_{k,i}^t - \delta \widehat{\phi}_i^{t+1} U_{k,i}^{t+1} - \eta_{k,i}\right)}{\widetilde{\lambda}_i^t}. \tag{19}$$

This is similar to the standard wage equation in search models: the worker's wage is a weighted

average of output, and a function of their outside option. The only new term is the non-pecuniary benefit, which is subtracted from the outside option. By integrating wages across all individuals in the economy, one can solve for the family's total wage income.

2.6 Labor Market Dynamics

Since workers can switch sectors between periods t_a and t_c, the sector-specific unemployment rates actually differ at these two points in time within the same period. To this end, we first define the beginning of period sector-specific unemployment rate as $\widetilde{u}_{k,i}^{t-1}$, and labor force as $L_{k,i}^{t-1}$. After workers switch sectors, (measured before matching at t_d), we define $u_{k,i}^{t}$ to be the share of sector-k workers searching for a job. It is given by:

$$u_{k,i}^{t} = \frac{\sum_{\ell=1}^{K} L_{\ell,i}^{t-1} \widetilde{u}_{\ell,i}^{t-1} s_{\ell k,i}^{t,t+1}}{L_{k,i}^{t}}, \tag{20}$$

where $L_{k,i}^{t}$ is the number of workers in sector k at t (more precisely at t_c) and is equal to:

$$L_{k,i}^{t} = L_{k,i}^{t-1} + \underbrace{\sum_{\ell \neq k} L_{\ell,i}^{t-1} \widetilde{u}_{\ell,i}^{t-1} s_{\ell k,i}^{t,t+1}}_{\text{Inflow}} - \underbrace{L_{k,i}^{t-1} \widetilde{u}_{k,i}^{t-1} \left(1 - s_{kk,i}^{t,t+1}\right)}_{\text{Outflow}}, \tag{21}$$

where the second term in the right hand side is the flow of workers into sector k and the third terms is the flow of workers out of sector k.

Only firms with $x \geq \underline{x}_{k,i}^{t+1}$ produce at $t+1$. Therefore, the *number* of jobs created in sector k is given by:

$$JC_{k,i}^{t} = L_{k,i}^{t} u_{k,i}^{t} \theta_{k,i}^{t} q_i\left(\theta_{k,i}^{t}\right) \left(1 - G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right), \tag{22}$$

and the number of jobs destroyed is given by:

$$\begin{aligned} JD_{k,i}^{t} &\equiv \left(\chi_{k,i} + (1 - \chi_{k,i}) \Pr\left(\underline{x}_{k,i}^{t} \leq x < \underline{x}_{k,i}^{t+1} | \underline{x}_{k,i}^{t} \leq x\right)\right) L_{k,i}^{t-1} \left(1 - \widetilde{u}_{k,i}^{t-1}\right) \\ &= \left(\chi_{k,i} + (1 - \chi_{k,i}) \max\left\{\frac{G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right) - G_{k,i}\left(\underline{x}_{k,i}^{t}\right)}{1 - G_{k,i}\left(\underline{x}_{k,i}^{t}\right)}, 0\right\}\right) L_{k,i}^{t-1} \left(1 - \widetilde{u}_{k,i}^{t-1}\right), \end{aligned} \tag{23}$$

where $\Pr\left(\underline{x}_{k,i}^{t} \leq x < \underline{x}_{k,i}^{t+1} | \underline{x}_{k,i}^{t} \leq x\right)$ is the share of active firms above the productivity threshold at t but below at $t+1$ (endogenous exit). Consequently, the rate of unemployment at the end of period t, after job creation and job destruction, is given by:

$$\widetilde{u}_{k,i}^{t} = \frac{L_{k,i}^{t} u_{k,i}^{t} - JC_{k,i}^{t} + JD_{k,i}^{t}}{L_{k,i}^{t}}. \tag{24}$$

Equations (20)-(24) describe the evolution of labor market stocks over time. In any given period, these stocks are bound by the labor market clearing condition:

$$\sum_{k=1}^{K} L_{k,i}^{t} = \overline{L}_{i}. \tag{25}$$

2.7 International Trade

Our model of international trade closely follows Caliendo and Parro (2015). Varieties are traded across countries. Under the assumptions of perfect competition and iceberg trade costs, the cost of variety j from sector k produced in o can be purchased in country i at a price $p_{k,o}^{t}(j)\, d_{k,oi}^{t}$, where the first term is the domestic price of the variety and the second is the iceberg term. From (14) we can write:

$$p_{k,i}^{t}(j) = \frac{c_{k,i}^{t}}{z_{k,i}^{t}(j)}, \tag{26}$$

for each variety j and where

$$c_{k,i}^{t} \equiv \left(\frac{\widetilde{w}_{k,i}^{t}}{\gamma_{k,i}}\right)^{\gamma_{k,i}} \left(\frac{P_{k,i}^{M,t}}{1-\gamma_{k,i}}\right)^{1-\gamma_{k,i}}. \tag{27}$$

Recall that due to the no-arbitrage condition we impose across varieties, $c_{k,i}^{t}$ is equalized within sectors, so that price variation across varieties is dictated by $z_{k,i}^{t}(j)$.

We assume that in any country i, sector k and period t, the productivity component $z_{k,i}^{t}(j)$ is independently drawn from a Frechet distribution with scale parameter $A_{k,i}^{t}$—which is country, sector, and time specific—and shape parameter, λ, which is time invariant.[13] Consumers buy the lowest cost variety across countries, treating the same variety from different origins as perfect substitutes. Under these assumptions, it can be shown that the resulting price index for the composite sector-specific intermediate good 1) is given by:

$$P_{k,i}^{I,t} = \Gamma_{k,i} \left[\sum_{o=1}^{N} \frac{A_{k,o}^{t}}{\left(c_{k,o}^{t} d_{k,oi}^{t}\right)^{\lambda}}\right]^{-1/\lambda}, \tag{29}$$

where $\Gamma_{k,i}$ is a constant (see Caliendo and Parro, 2015). In turn, the price of the final good (2) is

[13] The CDF for the Frechet is given by,

$$F_{k,i}^{t}(z) = \exp\left(-A_{k,i}^{t} \times z^{-\lambda}\right). \tag{28}$$

given by:

$$P_i^{F,t} = \prod_{k=1}^{K} \left(\frac{P_{k,i}^{I,t}}{\mu_{k,i}} \right)^{\mu_{k,i}}. \tag{30}$$

If we define the object $\Phi_{k,i}^t = \sum_{o=1}^{N} \frac{A_{k,o}^t}{\left(c_{k,o}^t d_{k,oi}^t\right)^{\lambda}}$ (which is proportional to $(P_{k,i}^{I,t})^{-\lambda}$) then consumers in country i spend a share $\pi_{k,oi}^t$ of their sector-k expenditures on varieties from country o given by:

$$\pi_{k,oi}^t \equiv \frac{E_{k,oi}^t}{E_{k,i}^t} = \frac{A_{k,o}^t \left(c_{k,o}^t d_{k,oi}^t\right)^{-\lambda}}{\Phi_{k,i}^t} \tag{31}$$

where $E_{k,i}^t$ is total expenditure of country i on sector k varieties and $E_{k,oi}^t$ is total expenditure of country i on sector k varieties produced by country o: $E_{k,i}^t = \sum_{o=1}^{N} E_{k,oi}^t$. Market clearing requires that total revenue coming from the production of varieties in sector k and country o must be equal to sales to all countries $i = 1, ..., N$, and so:

$$Y_{k,o}^t = \sum_{i=1}^{N} E_{k,oi}^t = \sum_{i=1}^{N} \pi_{k,oi}^t E_{k,i}^t. \tag{32}$$

Let $C_i^t \equiv \overline{L}_i c_i^t$ denote total consumption of the final good. Define $E_{it}^C \equiv P_i^{F,t} C_i^t$ as total consumption expenditure with final goods, and let $E_{k,i}^{V,t}$ be the total expenditure of sector k in country i on vacancy posting costs. We can write $E_{k,i}^t$ as:

$$E_{k,i}^t = \mu_{k,i} E_i^{C,t} + \mu_{k,i} \sum_{\ell=1}^{K} E_{\ell,i}^{V,t} + \sum_{\ell=1}^{K} \left(1 - \gamma_{\ell,i}\right) \nu_{\ell k,i} Y_{\ell,i}^t. \tag{33}$$

In turn, let I_i^t denote total income in country i. Total income is given by the portion of revenue that is not devoted to intermediate good payments minus vacancy posting costs:

$$I_i^t = \sum_{\ell=1}^{K} \left(\gamma_{\ell,i} Y_{\ell,i}^t - E_{\ell,i}^{V,t} \right). \tag{34}$$

Net exports are then given by:

$$NX_i^t \equiv I_i^t - E_i^{C,t}, \tag{35}$$

and we can rewrite (33) as:

$$E_{k,i}^t = \mu_{k,i} \left(\sum_{\ell=1}^{K} \gamma_{\ell,i} Y_{\ell,i}^t - NX_i^t \right) + \sum_{\ell=1}^{K} \left(1 - \gamma_{\ell,i}\right) \nu_{\ell k,i} Y_{\ell,i}^t. \tag{36}$$

Finally, labor market clearing dictates that total revenues coming from the production of varieties in sector k and country i is given by:

$$Y_{k,i}^t = \frac{\widetilde{w}_{k,i}^t}{\gamma_{k,i}} L_{k,i}^{t-1} \left(1 - \widetilde{u}_{k,i}^{t-1}\right) \int_{\underline{x}_{k,i}^t}^{\infty} \frac{s}{1 - G_{k,i}\left(\underline{x}_{k,i}^t\right)} dG_{k,i}(s). \tag{37}$$

We now model how NX_i^t is determined in equilibrium.

2.8 Trade Imbalances

Bonds are in zero net supply, $\sum_i B_i^t = 0$, and initial conditions are given by $W_i^0 \equiv R^0 B_i^0$ with $\sum_{i=1}^{N} W_i^0 = 0$. The solution to the household head's problem (6) must satisfy the following Euler Equation:

$$\frac{u'(c_i^t)/P_i^{F,t}}{u'(c_i^{t+1})/P_i^{F,t+1}} = \delta \widehat{\phi}_i^{t+1} R^{t+1}. \tag{38}$$

If we assume log utility, $u(c) = \ln(c)$, the Euler equations become:

$$P_i^{F,t+1} C_i^{t+1} = \delta \widehat{\phi}_i^{t+1} R^{t+1} P_i^{F,t} C_i^t. \tag{39}$$

Therefore, bond returns are given by:

$$R^{t+1} = \frac{1}{\delta} \frac{\sum_{i=1}^{N} \frac{P_i^{F,t+1} C_i^{t+1}}{\widehat{\phi}_i^{t+1}}}{\sum_{i=1}^{N} P_i^{F,t} C_i^t} = \frac{1}{\delta} \frac{\sum_{i=1}^{N} \frac{E_i^{C,t+1}}{\widehat{\phi}_i^{t+1}}}{\sum_{i=1}^{N} E_i^{C,t}}. \tag{40}$$

Finally, we need a condition linking the financial and goods markets in each country:

$$NX_i^t = I_i^t - P_i^{F,t} C_i^t = B_i^{t+1} - R^t B_i^t. \tag{41}$$

It is worthwhile to note that the model implies that steady-state trade deficits depend on countries' initial levels of wealth. Denote T_{SS} as the time required for the steady state to be achieved, and use equation (7) to derive the intertemporal budget constraint. We obtain:

$$NX_i^{T_{SS}} = -\frac{1-\delta}{\delta} \left(\prod_{\tau=1}^{T_{SS}-1} (R^\tau)^{-1} \right)^{-1} \left(B_i^0 + \sum_{t=1}^{T_{SS}-1} \left(\prod_{\tau=1}^{t} (R^\tau)^{-1} \right) NX_i^t \right), \tag{42}$$

which implies that countries with high levels of initial wealth ($R^0 B_i^0 = \frac{1}{\delta} B_i^0$) are able to sustain trade deficits in steady state ($NX_i^{T_{SS}} < 0$).

2.9 Equilibrium

An equilibrium in this model is a set of initial allocations, $\{L^0_{k,i}, \underline{x}^0_{k,i}, B^0_i, \}$, a final steady state, $\{L^\infty_{k,i}, \underline{x}^\infty_{k,i}, B^\infty_i, \}$ and sequences of policy functions for workers/firms, $\{s^t_{kk',i}, \underline{x}^t_{k,i}, w^t_{k,i}(x)\}$, value functions for workers/firms, $\{U^t_{k,i}, W^t_{k,i}, J^t_{k,i}\}$, labor market tightnesses, $\{\theta^t_{k,i}\}$, bond decisions by the households, B^t_i, allocations, $\{L^t_{k,i}, u^t_{k,i}, c^t_{k,i}\}$, profits and household consumption, $\{\Pi^t_i, C^t_i\}$, trade shares, $\left\{\pi^t_{k,io}\right\}$, and sectoral surpluses, $\{\widetilde{w}^t_{k,i}\}$ such that:

1. Worker and firms' value functions solve (8), (9), and (16).
2. Prices and labor market tightness ensure the free entry condition holds in each country and sector:
$$V^t_{k,i} = 0 \ \forall k, i, t$$
3. The wage equation solves the Nash bargaining problem and is given by (19).
4. Allocations and unemployment rates evolve according to (20), (21), (24).
5. Consumption and bonds decisions solve the household's dynamic consumption-savings problem (6)-(7) + (5).
6. Goods markets clear: equations (32)-(34) are met.
7. Labor markets clear: $\sum_{k=1}^{K} L^t_{k,i} = \overline{L}_i$.
8. Bonds markets clear: $\sum_{i=1}^{N} B^t_i = 0$.
9. The initial and final steady state equilibria satisfy equations (A.15)-(A.36) in Appendix B.

We set world total gross production of varieties as the numeraire, that is, $\sum_{i=1}^{N}\sum_{k=1}^{K} Y^t_{k,i} = 1$ for all t. Therefore, all nominal aggregate variables are expressed in terms of shares of world gross production.

3 Data and Estimation

Our main source of cross-country data is the World Input Output Database (WIOD), which compiles data from individual National Accounts combined with bilateral international trade data. These data cover 56 sectors and 44 countries, including a Rest of the World aggregate, between 2000 and 2014. We divide the economy into six sectors and six countries. We consider a world comprised by the United States, China, Europe, Asia/Oceania, the Americas and a Rest of the

World aggregate constructed by the WIOD. Each country's economic activity consists of six sectors: Agriculture; Low-, Mid- and High-Tech Manufacturing; Low- and High-Tech Services. Tables 1 and 2 detail these divisions. We solve our model at the quarterly frequency.

Table 1: Country Definitions

1	USA
2	China
3	Europe
4	Asia/Oceania
5	Americas
6	Rest of the World (ROW)

Notes: Asia/Oceania = {Australia, Japan, South Korea, Taiwan}, Americas = {Brazil, Canada, Mexico}, Rest of the World ={Indonesia, India, Russia, Turkey, Rest of the World}

Table 2: Sector Definitions

1	Agriculture/Mining	Agriculture, Forestry and Fishing; Mining and quarrying
2	Low-Tech Manufacturing	Wood products; Paper, printing and publishing; Coke and refined petroleum; Basic and fabricated metals; Other manufacturing
3	Mid-Tech Manufacturing	Food, beverage and tobacco; Textiles; Leather and footwear; Rubber and plastics; Non-metallic mineral products
4	High-Tech Manufacturing	Chemical products; Machinery; Electrical and optical equipment; Transport equipment
5	Low Tech Services	Utilities; Construction; Wholesale and retail trade; Transportation; Accommodation and food service activities; Activities of households as employers
6	Hi Tech Services	Publishing; Media; Telecommunications; Financial, real estate and business services; Government, education, health

The WIOD dataset allow us to generate various moments and statistics that are used in the method of simulated moments procedure we use to estimate a subset of the parameters (more details below): (a) employment shares across sectors and countries; (b) average wages across sectors and countries; (c) trade shares; (d) net exports.

Other moments used in the estimation procedure are obtained from ILOSTAT and the United States Current Population Survey (CPS). From ILOSTAT, we extract country-specific unemployment rates. From the CPS, we compute yearly transition rates between sectors in the United States, as well as the dispersion of wages. Table 3 summarizes the statistics used in the estimation.

Table 3: Summary of Statistics Used in the MSM Procedure

Statistic	Source
Employment allocations across sectors and countries	WIOD
Average wages across sectors and countries	WIOD
Trade shares across country pairs and sectors	WIOD
Net exports across countries	WIOD
Unemployment rates across countries	ILOSTAT
Coefficient of variation of log-wages in the United States	CPS
Yearly transition rates between sectors and from and to unemployment for the United States	CPS

Table 4 summarizes the parameters we need to numerically solve the model. We split them into three categories: (i) parameters that are fixed at values previously reported in the literature because they are difficult or impossible to be identified given the available data (Panel A); (ii) parameters that can be determined outside of the model (Panel B); and (iii) parameters that are estimated by the method of simulated moments, where we target the moments shown in Table 3. [14]

We follow Artuç et al. (2010) and impose the dispersion of ω shocks, ζ_i, to be equal to 1.63 and common across countries. Given that we solve our model at the quarterly frequency, we impose the discount factor to be $\delta = 0.9924$, leading to an annual discount factor of 0.97. The matching function is parameterized according to Coşar et al. (2016), so we impose $\xi_i = 1.63$ and common across countries. The Frechet scale parameter $\lambda = 4$ comes from Simonovska and Waugh (2014).[15] Finally, we follow standard practice in the search literature and set $\beta_{k,i} = 0.5$ ((Mortensen and Pissarides, 1999)). As described in Panel B, the WIOD dataset allows us to directly extract various parameters of the model such as country-specific final expenditure shares $\mu_{k,i}$, labor expenditure shares $\gamma_{k,i}$ and input-output matrices $\nu_{k\ell,i}$; this is due to the Cobb-Douglas assumptions we imposed in both the production functions and how final consumption is aggregated.

We impose a few restrictions on the parameters estimated by the simulated method of moments. First, the identification of mobility costs $C_{k\ell,i}$ relies on good quality data on both wages and yearly inter-sectoral transition rates. Although these data can be found in household surveys across a few of the countries we consider, we opted to identify these parameters using transition rates for the United States only. Therefore, we impose mobility costs to be common across countries. Second, the dispersion of match productivities $\sigma^2_{k,i}$ is imposed to be the same across sectors and countries, and is identified by the cross-sectional coefficient of variation of log-wages in the CPS. Third, we

[14] To be more precise, we do not target trade shares and net exports, we impose them in the estimation, so that they are matched exactly.

[15] Caliendo and Parro (2015) find a similar value for λ when it is imposed to be equal across sectors: 4.55.

Table 4: Summary of Parameters

Panel A. Fixed According to the Literature			
Parameter	Value	Description	Source
δ	0.9924	Discount factor	Artuç et al. (2010)
ζ_i	1.63	Dispersion of ω shocks	Artuç et al. (2010)
ξ_i	1.84	Matching Function	Coşar et al. (2016)
λ	4	Frechet Scale Parameter	Simonovska and Waugh (2014)
$\beta_{k,i}$	0.5	Worker Bargaining Power	Mortensen and Pissarides (1999)

Panel B. Estimated Outside of the Model		
Parameter	Description	Source
$\mu_{k,i}$	Final Expenditure Shares	WIOD
$\gamma_{k,i}$	Labor Expenditure Shares	WIOD
$\nu_{k\ell,i}$	Input-Output Matrix	WIOD

Panel C. Estimated by Method of Simulated Moments	
Parameter	Description
$\widetilde{\kappa}_{k,i}$	Vacancy Costs
$\chi_{k,i}$	Exogenous Exit
$\sigma^2_{k,i}$	$G_{k,i}$
$\eta_{k,i}$	Sector-specific Utility
$C_{kk'}$	Mobility Costs
$b_{k,i}$	Unemployment Utility

Note: Artuç et al. (2010) use an annual discount factor of $\delta = 0.97$. Since we work at the quarterly frequency, we use $\delta = 0.97^{\frac{1}{4}}$.

impose that the exogenous exit rates $\chi_{k,i}$ are decomposed into a country component χ_i and a sector component χ_k (that is, $\chi_{k,i} = \chi_i + \chi_k$).[16] Fourth, the utility of unemployment $b_{k,i}$ is imposed to be country specific (that is $b_{k,i} = b_i$). Finally, to achieve identification, we need to impose $C_{kk} = 0$ for all k and $\eta_{k_0,i} = 0$ for some sector k_0 (we picked $k_0 = Agriculture$).

We estimate the parameters described in Panel C using the method of simulated moments. Let $\Theta = (\Theta_1, ..., \Theta_N)$ be the vector of these country-specific parameters. Our estimation procedure assumes that the economy is in steady state in 2000 and conditions on the observed 2000 trade shares $\pi^{Data}_{k,oi}$ and net exports NX^{Data}_i—so these moments are perfectly matched. Appendix B summarizes the equations that must be satisfied in a steady state equilibrium. For a given guess of Θ, we use our equilibrium equations, conditional on $\pi^{Data}_{k,oi}$ and NX^{Data}_i, to generate: (a) unemployment rates across countries; (b) employment allocations and average wages across sectors and countries; (c) yearly transition rates between sectors and from and to unemployment in the United States; and

[16] The sector-specific component χ_k is identified from US transitions from sector-specific employment to unemployment. The country specific components are then adjusted to better match the country-specific unemployment rates.

(d) cross-sectional wage dispersion in the United States. The estimation procedure searches for a vector of parameters Θ that minimizes the distance between the model generated moments and those we measure in the data.

A convenient aspect of our approach is that, by conditioning on the observed 2000 trade shares and trade imbalances, and normalizing world output $\sum_k \sum_i Y_{k,i} = 1$, we can solve for sector-country output levels $\{Y_{k,i}\}$ that are independent of Θ. In particular, equations (32) and (36) lead to a system of equations in Y that can be solved iteratively for a starting guess of Y. Conditional on gross output being known for each country-sector pair, the labor supply side in each country can be solved independently. To see this, notice that equation (37) contains gross output on the left hand side, and the right hand side variables *only* depend on country-specific prices, $\tilde{w}_{k,i}$ and the to-be-estimated labor supply side parameters outlined in Panel C of Table 4. This is to say, in steady state, observed trade flows and imbalances are a sufficient statistic for international linkages. This property allows us to estimate the model country by country, greatly simplifying the estimation procedure. Indeed, if all parameters were country specific and we had the same set of data for each country, estimation could be conducted in parallel for each country.

This property of our model also illuminates identification in our model. While we cannot offer a formal proof of how moments match to parameters, decoupling estimation by country and leaving only labor supply side parameters to be estimated allows us to make a heuristic case for identification. In particular, we are fixing gross output. The determinants of gross output for country k are the demand shifters for each country (the parameters in Panel B of Table 4), and national incomes. In this sense, fixing global output fixes the demand side of the model—thus, our estimation procedure solves for the supply parameters of the model that are consistent with (1) clearing demand and (2) the distribution of observed relative wages. Without time variation in wages, we are not able to separately identify ζ, and without data on firms, it is difficult to separately estimate the cost of vacancies and the matching function[17]—hence these parameters are fixed.

As a final point, a convenient aspect of conducting the estimation conditional on the observed trade shares is that we do not have to estimate the technology parameters $A_{k,i}$ and trade costs $d_{k,oi}$. This allows us to tractably integrate richer labor market dynamics, such as search, in a way that is consistent with the general equilibrium trade forces in the model. This feature of our setup extends to other frameworks, and the key assumption is that one can write the supply of k in country i as a function of i variables only. While this idea would hold with any trade matrix, not just our gravity setup, in the next section, we develop algorithms to perform counterfactual responses to shocks to A and d relying on the exact hat algebra approach in Dekle et al. (2007) and Caliendo

[17]For a discussion of identifying ζ see Artuç et al. (2010). See Flinn (2006) for a discussion of the difficulty in identifying vacancy costs and the matching function in a related set of search models.

and Parro (2015). Thus, the method we develop here allows us to estimate and simulate from our model, matching trade forces exactly but without needing to specify trade costs and sectoral productivities.

Although the model could be flexibly estimated country by country if we had data on yearly transitions across sectors for all countries, we impose, in this paper, $C_{k\ell,i}$ and $\sigma^2_{k,i}$ to be constant across countries and identified off the United States' CPS transition rates and wage dispersion data. Therefore, we proceed in two steps. First, we estimate the model for the United States, back out $C_{k\ell}$, σ^2_k and χ_k. In a second step, we estimate the rest of the parameters in parallel, country by country, conditional on $C_{k\ell}$, σ_k, and χ_k.

At this point, it is worthwhile mentioning that our estimation approach, which does not estimate $A_{k,i}$ and trade costs $d_{k,oi}$, cannot recover $\kappa_{k,i}$. Instead, we are able to recover $\widetilde{\kappa}_{k,i} \equiv \frac{\kappa_{k,i} P_i^F}{\widetilde{w}_{k,i}}$. Our counterfactual algorithms then allows $\widetilde{\kappa}_{k,i}$ to respond to shocks, using exact hat algebra. It is also important to clarify that identifying $\widetilde{\kappa}_{k,i}$ is difficult in practice. Even though the objective function is not flat with respect to $\widetilde{\kappa}_{k,i}$, it tends to be relatively flat for a wide range of values. For that reason, we follow Hagedorn and Manovskii (2008) and impose that the cost of posting one vacancy $\kappa_{k,i} P_i^F$ equals $0.58 \times \overline{w}_{k,i}$, where $\overline{w}_{k,i}$ is the average wage in sector k and in country i. Hagedorn and Manovskii (2008) base their calibration on a combination of the labor and capital costs of posting vacancies in a single-sector search model.[18] While their model and ours differ on several dimensions—e.g., multiple sectors and match heterogeneity—there are two reasons we turn to their calculation. First, Hagedorn and Manovskii combine micro and macro data from multiple sources, mapping their numbers to observable costs of recruitment, capital purchasing, and other costs; second, their calibration of vacancy posting costs does not rely on other parameters of the model, just observable data (e.g., the capital share of income and the unemployment rate), so that the estimate is not contaminated by the fact that our other parameters are not always estimated to be the same as Hagedorn and Manovskii's. We impose this constraint by adding the deviations between $\left(\widetilde{\kappa}_{k,i} \times \widetilde{w}_{k,i}\right) / \overline{w}_{k,i} = \left(\kappa_{k,i} \times P_i^F\right) / \overline{w}_{k,i}$ and 0.58 in our method of simulated moments objective function. The full estimation algorithm is described in Appendix C.1.

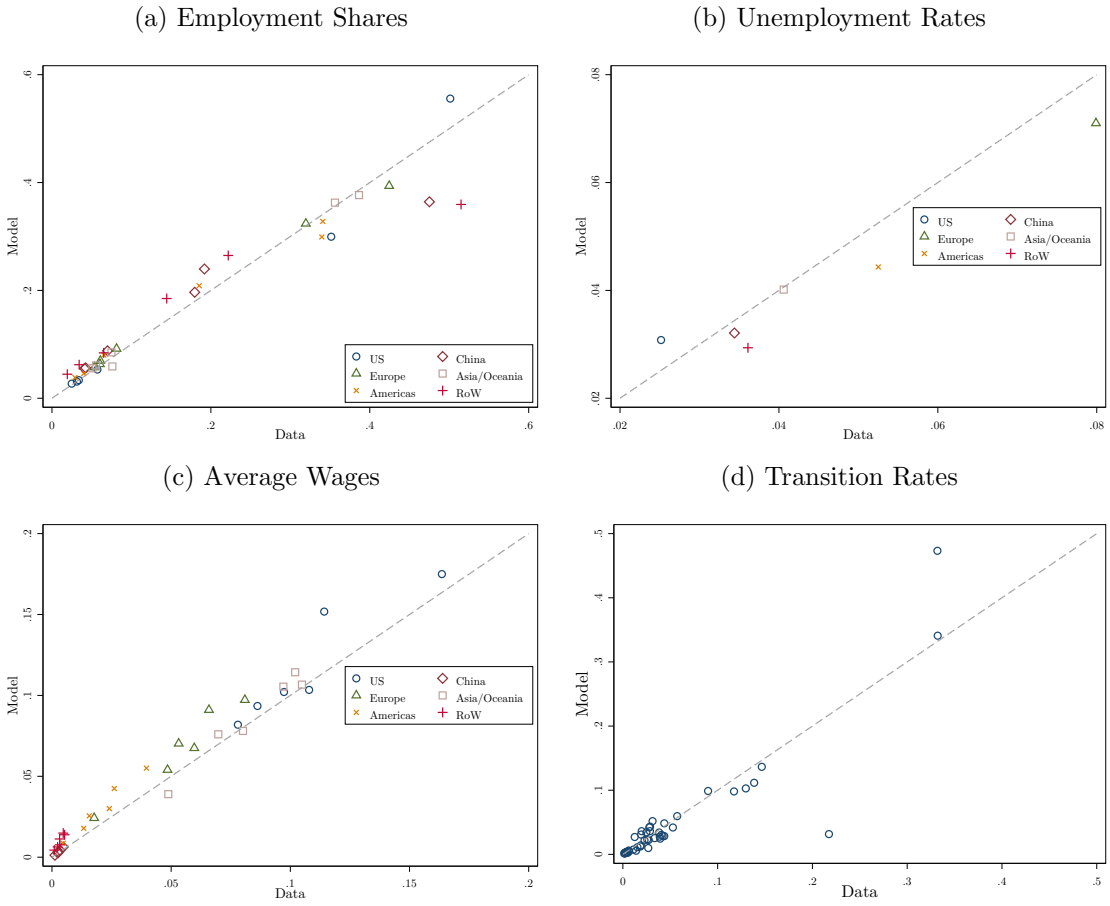
4 Estimation Results

4.1 Estimates and Model Fit

The estimated parameters can be found in the Appendix, in Tables A.1 through A.8. We first discuss the parameters that are obtained outside of the model. The online data Appendix details

[18] As the authors point out, in the steady state of a single sector search model, capital is equivalent to interpreting vacancies as capital and reinterpreting the productivity process. In this sense, a vacancy is a metaphor for the whole cost of hiring a worker—the flow cost of capital, HR efforts, etc.. We adopt a similar view in our paper, without modeling each piece.

Figure 2: Model Fit



how these are computed with data from the WIOD. Table A.1 displays the final expenditure shares $\mu_{k,i}$. We can separate the countries in this table in two groups with similar expenditure shares: (1) United States, Europe, and Americas; and (2) China and Rest of the World. The biggest difference between these two countries is that China and the Rest of the World spend a much larger share of their disposable income on Agricultural goods and significantly lower share on High-Tech Services.

Table A.2 displays the share of output devoted to labor payments. This share varies mostly within countries and across sectors and ranges between 0.24 (in High-Tech Manufacturing in China) and 0.68 (in High-Tech Services in RoW). Finally, Table A.3 displays the average across countries of the input-output matrices. As is well known, the diagonal elements tend to be larger than the off-diagonal elements.

Our mobility costs are displayed in Table A.4. Artuç et al. (2010) express their mobility costs estimates as multiples of average wages. In contrast, in our model, workers compute their value functions based on $\widetilde{\lambda}_i \times \widetilde{w}_{k,i}$, and not just based on sector-specific average wages. Therefore, to

make our estimates comparable to those in Artuç et al. (2010), we express mobility costs as a multiple of $\widetilde{\lambda}_i \times \overline{w}_{US}$, where $\overline{w}_{US}$ is the average wage in the US (in the data). We obtain that our estimated mobility costs across sectors (as a fraction of $\widetilde{\lambda}_i \times \overline{w}_{k,i}$) range from virtually zero to a maximum of 3.1. These numbers are of the same order of magnitude, but lower than what Artuç et al. (2010) estimate (in their preferred specification, mobility costs are 6 times average wages). We believe that two ingredients of our model absent from their, search frictions and sector-specific utility terms $\eta_{k,i}$, account for our lower estimates of mobility costs. Indeed, Artuç and McLaren (2015) extends ACM to allow for η terms and also find lower moving frictions. Finally, Table A.7 shows that the values of unemployment b_i tend to be negative across all countries. This is consistent with estimates in other models featuring search frictions and unemployment, such as Meghir et al. (2015).

Figure 2 shows the model fit for the various moments we target: (a) employment shares across sectors and countries; (b) unemployment rates across sectors and countries; (c) average wages across sectors and countries; and (d) inter-sectoral transition rates in the United States. As a general rule, the model matches the targeted moments well. Perhaps the main exception is the persistence in unemployment. 21% of unemployed workers remain unemployed the next year, whereas our model predicts that this share is much lower: 3%.

4.2 Impulse Response Functions

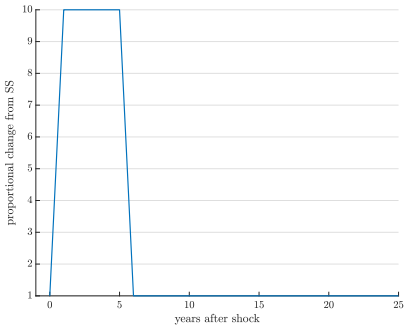
To understand the mechanisms in our model, we study the model's behavior in response to three sets of shocks. First, we subject the model to a tenfold increase in $A_{k,China}$, the productivity location parameter in China, uniform across sectors k and lasting for five years before turning back to its original level—the magnitude of this shock is in line with the size of actual changes in Chinese productivity that we recover in section 5.1. Next, we feed the model with a tenfold permanent increase in $A_{k,China}$ that happens once and for all at $t = 1$. Finally, we simulate a slow-moving and linear increase in $A_{k,China}$ that reaches a tenfold increase in 15 years. It remains at that level from there on. These shocks are illustrated in Figure 3. To highlight the quantitative and qualitative importance of modelling trade imbalances, we study the behavior of the complete model with bonds as well as the behavior of a model without bonds, where trade is balanced in every period—that is $NX_i = 0 \ \forall i$.[19]

We start with the temporary shock depicted in Figure 3a. The evolution of net exports in Figure 4 aligns with what we would have expected from standard macroeconomic models with inter-temporal decisions. The temporary boost in productivity, and therefore income, induces

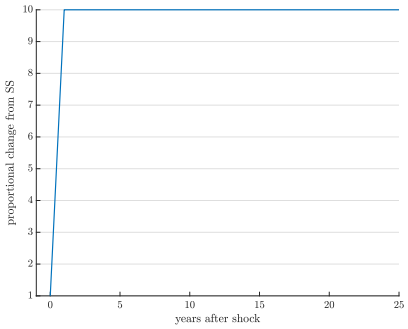
[19] To make these two cases comparable, we study the complete model responses (with trade imbalances) imposing that trade is balanced in the initial steady state. See Appendix C.3 for details on how we implement this procedure using exact hat algebra techniques.

Figure 3: Shocks to Chinese Productivity $\widehat{A}_{k,China}$ – Uniform Across Sectors

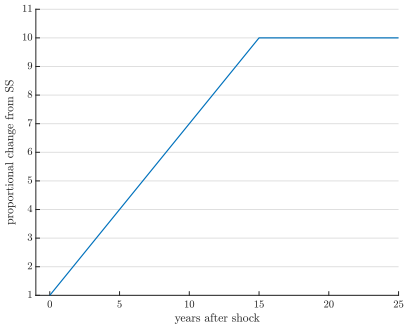
(a) Temporary



(b) Once-And-For-All



(c) Slow Moving



China to save in the short run by running a large trade surplus. As productivity reverts back to its initial level, China starts to sustain a small, but permanent trade deficit of 1.2% of GDP, as it consumes the return on its savings. On the other hand, the remaining countries borrow from China, running trade deficits in the short run. As the productivity increase in China vanishes, all countries start to sustain permanent trade surpluses, repaying their debts to China.

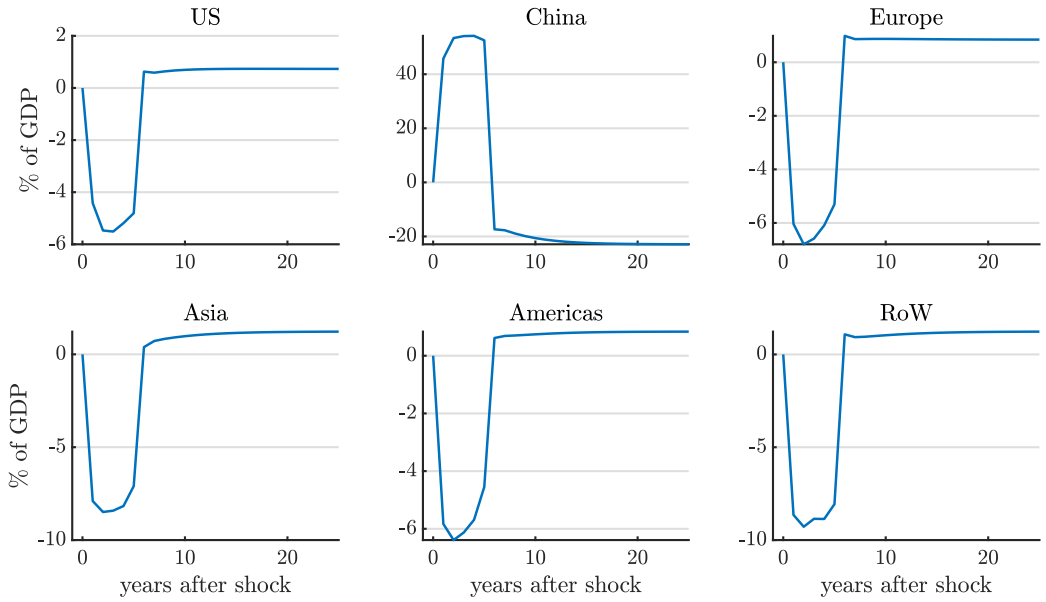
While these aggregate patterns appear standard, this productivity shock has varied impacts on the more disaggregated economy. Specifically, the rise in productivity induces substantial labor reallocation across sectors. We summarize reallocation patterns relying on the following measure::

$$Reallocation_i^t = \frac{1}{2}\sum_{s=1}^{t}\sum_{k=1}^{J}\left|\frac{L_{i,k}^s}{L_i^s} - \frac{L_{i,k}^{s-1}}{L_i^{s-1}}\right|, \tag{43}$$

which accumulates yearly changes in employment shares over time.

Figure 5a plots the evolution of the reallocation index (43) across countries. When we consider our complete model with bonds, there is an immediate and steep rise in reallocation in all countries following China's productivity jump. After productivity returns to its initial level, reallocation

Figure 4: Temporary Productivity Growth in China (Figure 3a) – Net Exports Over GDP



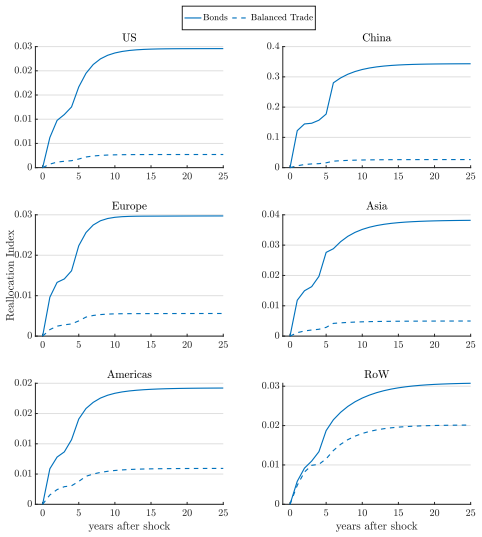
continues for several years before reaching its final steady-state level. Even though we consider a uniform productivity shock across sectors, we observe substantial labor reallocation within China and within other countries.[20] The reason that uniform productivity changes can have large impacts on labor reallocation is trade across countries, which adds asymmetries to how sectors respond to the same productivity shock. Our model features two sources of asymmetries. First, heterogeneity in trade costs implies that the tradability of goods differs across countries and sectors. As explained by Dornbusch et al. (1977), transfers (here, net exports) across countries alter the terms of trade and therefore the allocation of economic activity within each country. Second, countries differ in their final consumption bundles which implies that the global distribution of final consumption expenditure matters for the allocation of workers across sectors. Relatedly, heterogeneity in input-output linkages across countries also generate changes in countries' terms of trade and global expenditure patterns.

The increase in reallocation is directly tied to an increase in unemployment in all countries, as shown in Figure 5b. This highlights a key mechanism in our model: even positive labor demand shocks can increase unemployment through asymmetric shifts in labor demand. Once again, there are two reasons this can occur. First, contracting sectors will destroy low productivity jobs,

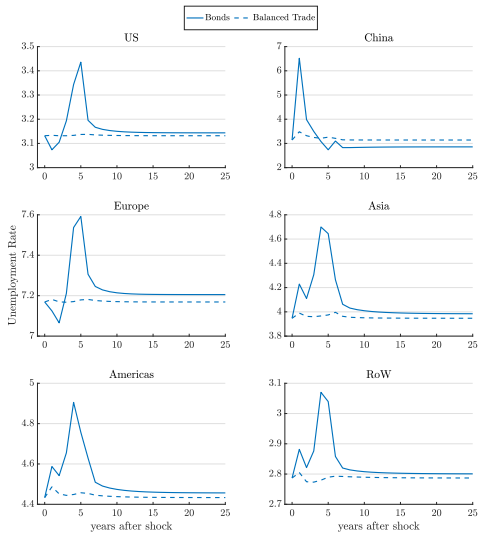
[20]Note that in a closed economy without mobility costs and search frictions, we would have seen no reallocation effects given homothetic preferences and our production structure as relative prices would not change. However, the presence of heterogeneous mobility costs across sectors could lead to some reallocation effects as the option value of search would change differentially across sectors. However, based on Shimer (2005) we expect this effect to be quantitatively small.

Figure 5: Temporary Productivity Growth in China (3a) – Reallocation and Unemployment Dynamics

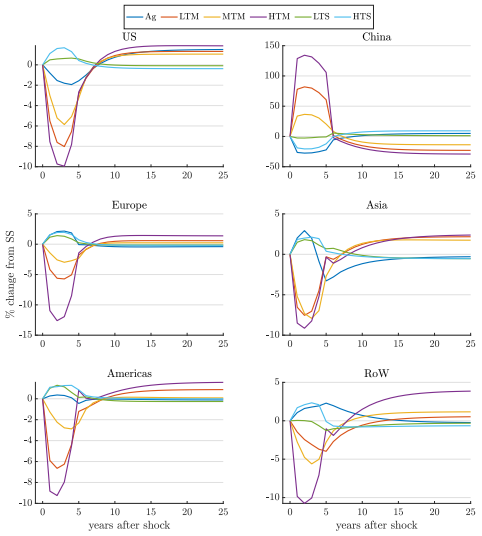
(a) Reallocation Index



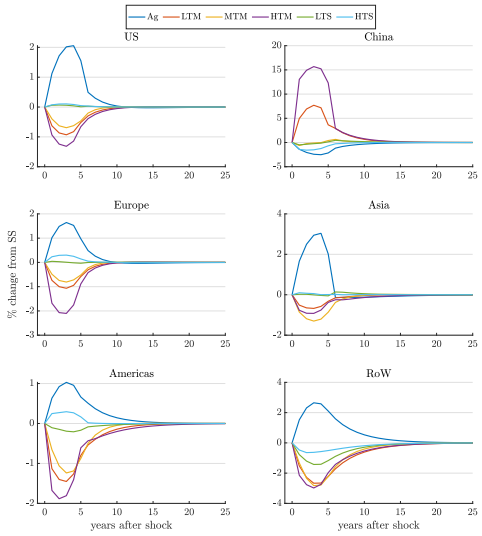
(b) Unemployment



(c) Labor Allocations - Full Model



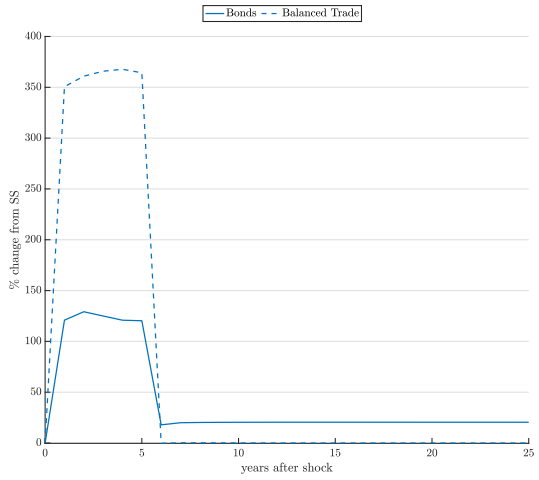
(d) Labor Allocations - Balanced Trade



dislocating workers who must switch sectors. Second, it takes time for workers who switch into expanding sectors to find new jobs.

We now turn to the role that trade imbalances play in our model and their interaction with trade shocks. First, imposing balanced trade leads not only to a dampening of the predicted short-run general equilibrium responses of labor markets to trade shocks, but it can also lead to opposite predictions. If bonds are not available and trade is balanced, China cannot smooth consumption in response to its temporary productivity boost. Therefore, short-run consumption spikes dramatically as illustrated in Figure 6. This leads to a low marginal value of reallocation—a concave (period) utility with respect to consumption implies that the marginal utility gain from moving is low relative to mobility costs when consumption is high. In addition, the shock we consider quickly reverts to its initial level, therefore any short-run reallocation must be undone in the long-run, contributing to large reallocation costs relative to benefits in a balanced-trade world.

Figure 6: Temporary Productivity Growth in China (Figure 3a) – Real Expenditure in China



Our model with bonds has implications not only for the magnitude of labor reallocation, but also for the pattern of reallocation as is shown in Figures 5c and 5d. When we consider the complete model with bonds, rising imports in the US following Chinese productivity growth do not need to be immediately met with rising exports. The US imports goods in the short run and reallocates labor to service sectors, which are less tradable and initially much larger in size. However, if we impose balanced trade in every period, exports from the US must immediately rise to offset the increase in imports. In doing so, labor reallocates to the sector in which it has highest comparative advantage (high relative productivity coupled with lower trade costs), which is also the sector in which China has higher relative demand: Agriculture. Reallocation when trade is balanced is driven by sectors with lower trade costs. Hence, reallocation across goods sectors is quite important, but reallocation

across service sectors is essentially unresponsive, therefore dampening aggregate reallocation.

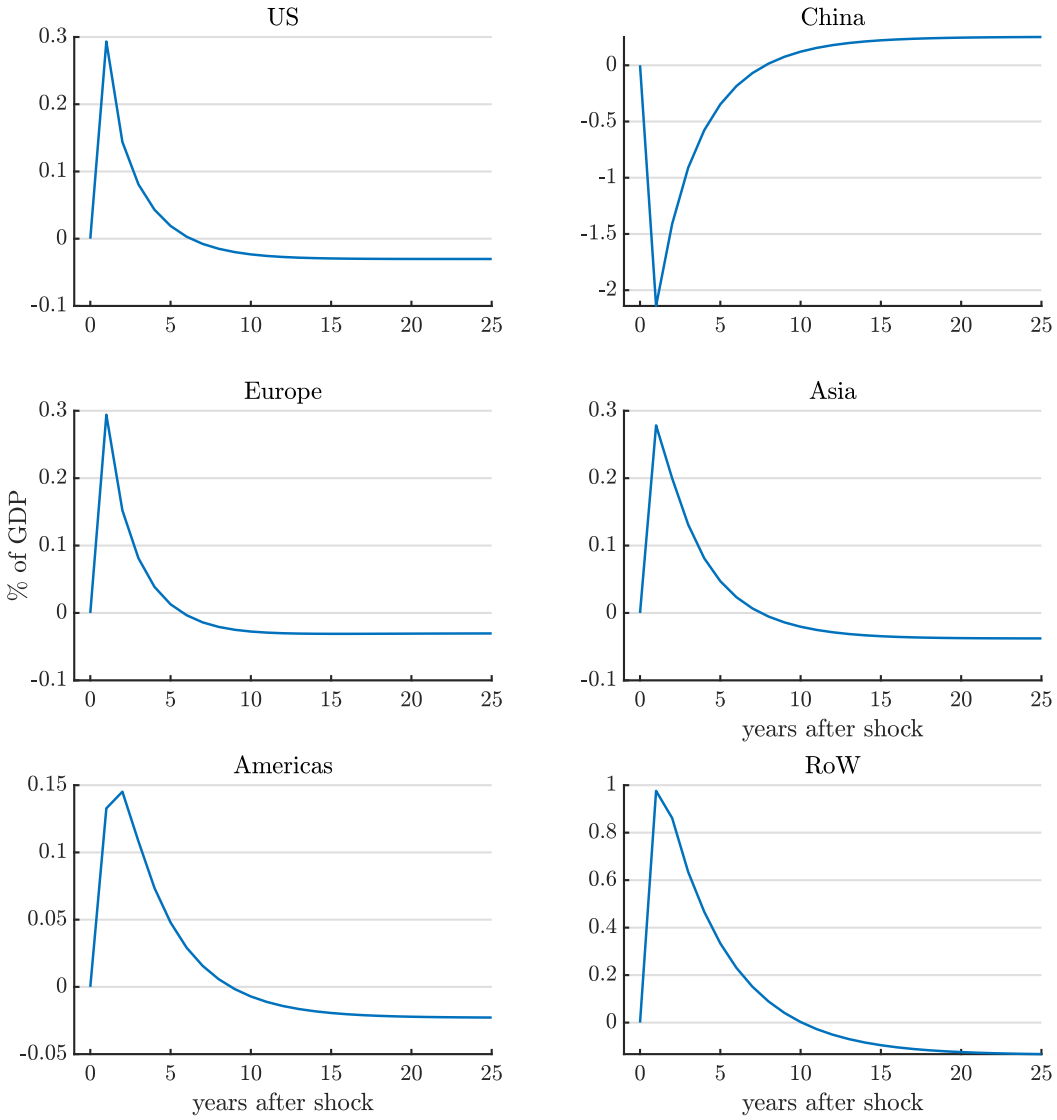
Turning to the long run, savings and borrowing behavior leads to permanent changes in final steady-state bond positions across countries. These permanent shifts in the long-run allocation of wealth carry implications for steady-state labor allocations and unemployment. The sources of long-run reallocation are similar to those operating in the short run, except that the effects of the temporary shock on the distribution of bonds positions (and therefore wealth) across countries is permanent. Specifically, the US runs a large deficit in the short run, which must be paid off in the form of long-run trade surpluses. To repay its debt with China, it shifts labor back to its goods sectors which can easily ship their production internationally. In the long run, the goods sectors in the US expand relative to the initial steady state, whereas its service sectors slightly contract. These long run effects in response to a temporary shock are in sharp contrast with those in the model with balanced trade, which predicts that long-run allocations are unchanged relative to initial ones.

We next turn to the analysis of the once-and-for-all shock in Chinese productivity depicted in Figure 3b. Figure 7 highlights that the effect of this shock on net exports across countries is of much smaller magnitude. Indeed, when an economy is subject to a permanent shock the importance of savings to smooth consumption is of second order. If labor were perfectly mobile across sectors, economies would instantaneously reallocate labor and achieve their unconstrained optimal levels of expenditure. Because labor is not perfectly mobile, we do observe imbalances arising, but these are modest. It is therefore not surprising that, in the event of once-and-for-all permanent shocks, the behavior of the model with imbalances is similar to one without imbalances.[21]

Finally, we study the behavior of our model when we feed it with a slow increase in Chinese productivity, as is illustrated in Figure 9. In this case, China heavily borrows in the short run in anticipation of its large increase in long-run productivity. All other countries lend to China by running large surpluses for over ten years. As with the temporary shock, trade imbalances greatly magnify the extent to whoch reallocation and unemployment respond relative to the trade balance benchmark. Figure 9 shows that the short-run labor market dynamics are very different across the two models. With trade imbalances, Figure 9c shows that the US reallocates resources away from its service sectors and towards its goods sectors so that their output can be shipped to China. However, in the long run, China needs to repay its debt and so ships goods back to the US. This leads to a long-run contraction of manufacturing in the US and an expansion of the goods sector. Compare this behavior with the monotonic decline in manufacturing in the US if trade is imposed to be balanced (see Figure 9d). The magnitude of the manufacturing contraction is also much more limited under trade balance.

[21] However, it is worth pointing out that this prediction of the model could be different for a shock that is not uniform across sectors.

Figure 7: Once-and-for-all Productivity Growth in China (Figure 3b) – Net Exports Over GDP



The main conclusion of this section is that, except for once-and-for-all permanent shocks, trade imbalances can significantly amplify the reallocation and unemployment responses to temporary shocks. They can also lead to very different patterns of inter-sectoral reallocation in the short and long run. Finally, our simulations highlight that, with trade imbalances, the path of shocks matter for the behavior of imbalances over time, but also for the determinantion of their long-run consequences. Given the importance of the path of shocks for trade imbalances and the adjustment process, we now turn to the analysis of the labor market consequences of the shocks the global economy endured between 2000 and 2014.

5 Counterfactuals

Section 4.2 showed that the exact path of shocks shape the magnitude and evolution of trade imbalances over time, directly influencing long-run outcomes through the accumulation of bonds. For this reason, we conduct an empirical exercise in which we extract the various shocks the global economy has actually experienced between 2000 and 2014. Armed with these shocks, we conduct a common exercise in the international trade literature: how do labor markets behave over time in response to the constellation of globalization shocks (shocks in productivities and trade costs)? In this exercise, we compare the labor markets responses that we obtain with our model of trade imbalances to the responses that we obtain imposing trade balance. Having established the quantitative importance of trade imbalances for both the adjustment process and long-run allocations, we compare the consumption gains in response to changes in trade costs in our model to those obtained in standard models of trade, as summarized by the sufficient statistic approach developped by Arkolakis et al. (2012). Finally, given the recent interest on the "China shock" on the US labor market, we use our extracted shocks to study the effects of the shocks to the Chinese economy on the behavior of unemployment and manufacturing employment in the US. We also investigate the contribution of the "China shock" for the US trade deficit and for the trade surplus in China.

5.1 Extracting Shocks from the Data

This section obtains the time series for three sets of shocks in the global economy between December of 2000 and December of 2014: changes in trade costs $\left\{\widehat{d}_{k,oi}^{t}\right\}$, inter-temporal preference shocks $\left\{\widehat{\phi}_{i}^{t}\right\}$ and productivity shocks $\left\{\widehat{A}_{k,i}^{t}\right\}$. We measure changes in trade costs and productivity relative to December of 2000 (which we label $t=0$): $\widehat{d}_{k,oi}^{t}=\frac{d_{k,oi}^{t}}{d_{k,oi}^{0}}$, $\widehat{A}_{k,i}^{t}=\frac{A_{k,i}^{t}}{A_{k,i}^{0}}$. On the other hand, shocks

Figure 8: Slow Productivity Growth in China (9) – Net Exports Over GDP

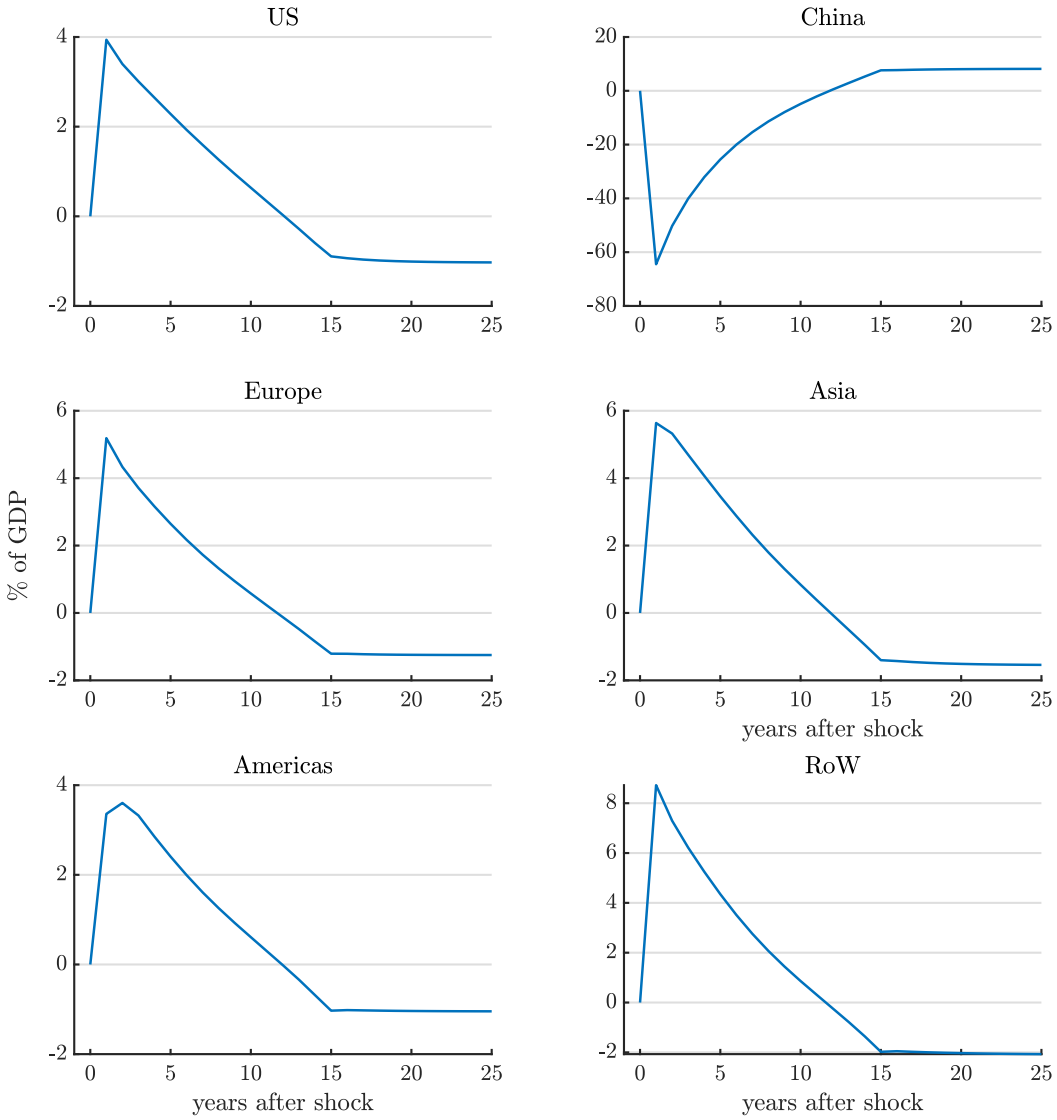
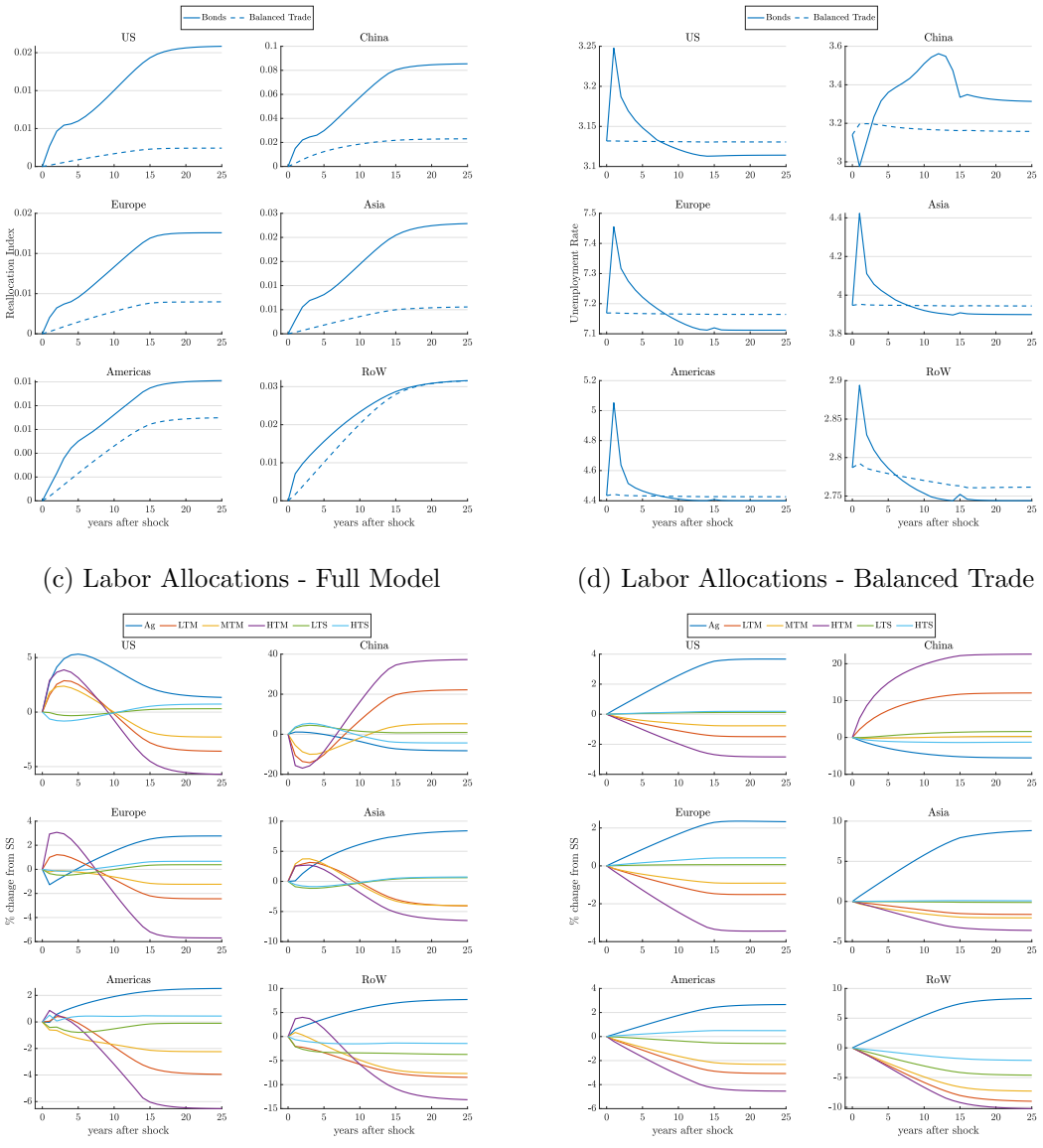


Figure 9: Slow Productivity Growth in China (Figure) – Reallocation and Unemployment Dynamics



to inter-temporal preferences are relative to the previous period: $\widehat{\phi}_i^{t+1} \equiv \frac{\phi_i^{t+1}}{\phi_i^t}$.[22]

We use WIOD data to construct time series of trade shares $\left\{\pi_{k,oi}^t\right\}$, sectoral price indices $\left\{P_{k,i}^{I,t}\right\}$ and final good expenditures $\left\{E_i^{C,t}\right\}$ between December of 2000 and December of 2014. Armed with these data, we can recover the changes in bilateral trade costs combining equations equations (29) and (31):

$$\widehat{d}_{k,oi}^t = \frac{\widehat{P}_{k,i}^{I,t}}{\widehat{P}_{k,o}^{I,t}} \left(\frac{\widehat{\pi}_{k,oo}^t}{\widehat{\pi}_{k,oi}^t}\right)^{1/\lambda}. \tag{44}$$

In turn, the Euler equation (39) allows us to recover the inter-temporal preference shocks, if we normalize $\widehat{\phi}_{US}^t = 1 \ \forall t$:

$$R^{t+1} = \frac{E_{US}^{C,t+1}}{\delta E_{US}^{C,t}} \text{ for } t = 1, ..., T-1, \tag{45}$$

$$\widehat{\phi}_i^{t+1} = \frac{E_i^{C,t+1}}{\delta E_i^{C,t} R^{t+1}} \text{ for } t = 1, ..., T-1, \tag{46}$$

where T is the last period for which we have data, which refers to December of 2014. Note that we still need to determine $\widehat{\phi}_i^{T+1}$, but we will need to use the structure of the model to do so, as this value will depend on the model-implied steady-state value for final good expenditures $E_i^{C,\infty}$.

Finally, we recover the productivity shocks $\left\{\widehat{A}_{k,i}^t\right\}$ using:

$$\widehat{A}_{k,i}^t = \frac{\widehat{\pi}_{k,ii}^t}{\left(\widehat{P}_{k,i}^{I,t}\right)^{\lambda}} \left(\widehat{c}_{k,i}^t\right)^{\lambda}. \tag{47}$$

Given that $\widehat{c}_{k,i}^t$ depends on $\widetilde{w}_{k,i}^t$, which has no data counterpart, we need to use the full structure of the model to recover the sequence of productivity shocks. Appendix C.6 details the algorithm to recover the shocks $\left\{\widehat{A}_{k,i}^t\right\}$ as well as $\left\{\widehat{\phi}_i^{T+1}\right\}$ using the full structure of the model.

We emphasize that the procedure we use to obtain the various shocks is based on standard practice in the international trade and macro literatures. Specifically, our approach to back out shocks to inter-temporal preferences is similar to those used by Reyes-Heroles (2016) and Eaton et al. (2016); and our calculation of changes in trade costs and productivities is common in trade models that generate a gravity equation, as in Eaton et al. (2016) and Head and Ries (2001).

Figure 10a shows an increase in productivity all over the world. In particular, China has experienced a large increase in productivity, especially in manufacturing sectors. While we plot changes in the productivity location parameters $\left\{\widehat{A}_{k,i}^t\right\}$, this is not directly comparable to productivity in the classic sense of a Solow Residual. In order to make sense of the magnitudes, note that TFP

[22] We impose $\widehat{A}_{k,i}^t = \widehat{A}_{k,i}^T$ and $\widehat{d}_{k,oi}^t = \widehat{d}_{k,oi}^T$ for all $t > T$, where T is the last period for which we have data ($T = 14$ and refers to December 2014). We also impose and $\widehat{\phi}_i^t = 1$ for all $t > T+1$—as explained below, the value of $\widehat{\phi}_i^{T+1} = 1$ is set to gauge the model-implied steady-state value for final consumption expenditures $E_i^{C,\infty}$.

growth, defined as $\widehat{c}_{k,i}^t/\widehat{P}_{k,i}^{I,t}$, can be expressed as $(\widehat{A}_{k,i}^t/\widehat{\pi}_{k,ii}^t)^{1/\lambda}$. Therefore, using our recovered values for $\widehat{A}_{k,i}^t$, data on changes in trade shares, and imposing $\lambda = 4$, the magnitude for actual annualized TFP growth in China ranges between 3 and 5% per year, depending on the sector—which is in line with growth accounting estimates discussed in Zhu (2012). Other emerging economies—which comprise the bulk of the Americas and the Rest of the World aggregate—also experienced impressive productivity growth, while growth was more muted for advanced economies.

Turning to trade costs, we first construct a summary statistic to capture this large object. We focus on the average import cost for each country-sector pair, weighted by their initial steady state import shares:

$$\overline{d}_{k,i}^t = \sum_{o \neq i} \frac{\pi_{k,oi}^0}{1 - \pi_{k,ii}^0} \widehat{d}_{k,oi}^t. \tag{48}$$

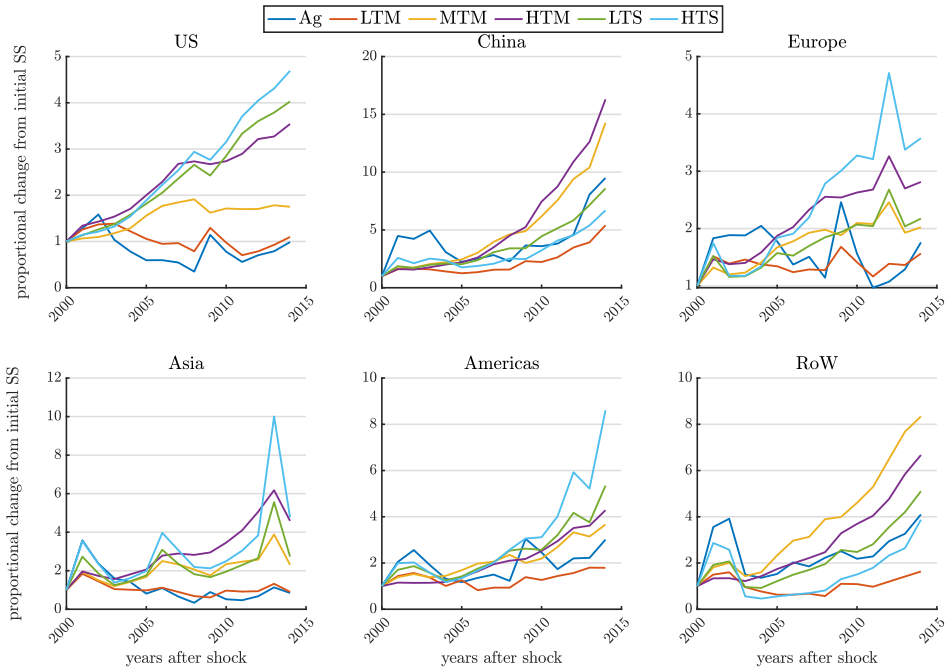
Figure 10b plots this index for each country and sector. In general, import trade costs are declining for the United States and Asia, and approximately flat in Europe (with some heterogeneity across sectors). Perhaps surprisingly, starting after the 2008 financial crisis and concurrent collapse in trade, initially falling import trade costs in China begin to revert and are actually larger by the end of the sample. This estimate of changes in trade costs reflects the fall in the share of trade in output, as documented in Bems et al. (2013). The sources for these increasing frictions are myriad, and include policy changes in countries like China, as well as changes in supply chain management, and other reasons. That said, our measures of frictions are a standard, straightforward, measure of the implied barriers to trade.

Finally, we turn to our measure of shocks to inter-temporal preferences, which are presented in Figure 11. The shocks in the US are normalized to 1 in every period. In Europe and Asia (except China), the discount factor shocks fluctuate around 1, suggesting little persistent deviations in consumption behavior from what would be expected with a simple consumption smoothing model. On the other hand, China, the Americas, and the aggregated remaining countries (Rest of the World) exhibit persistent shocks to their inter-temporal preferences, suggesting increased patience over the period we consider. These persistent deviations are often referred to as the "global savings glut."[23] It is important to recognize that there are rich dynamics to consumption in the real world, reflecting preferences, frictions, and other factors. We are agnostic on the exact theory, instead summarizing the effect of these channels with the $\widehat{\phi}_i^t$ shocks. This is useful because it allows us to ask counterfactual questions about the dynamics of globalization shocks without the global savings glut, without having to specify what policy or preference shifts to do this—a useful benchmark to compare against the usual assumption in trade of no consumption smoothing whatsoever.

[23]The large trade surplus that China has been running since the early 2000s is a puzzle for models in which the main driving forces are productivity shocks. For instance, as argued by Song et al. (2011), financial frictions within China are key drivers of the Chinese savings glut. Our inter-temporal preference shocks constitute a reduced-form way to allow the model to match the time series behavior of Chinese aggregate expenditures and the rest of the world.

Figure 10: Extracted Globalization Shocks

(a) Productivity Shocks $\widehat{A}^t_{k,i}$



(b) Trade-Weighted Import Costs $\overline{d}^t_{k,i}$ (See Equation (48))

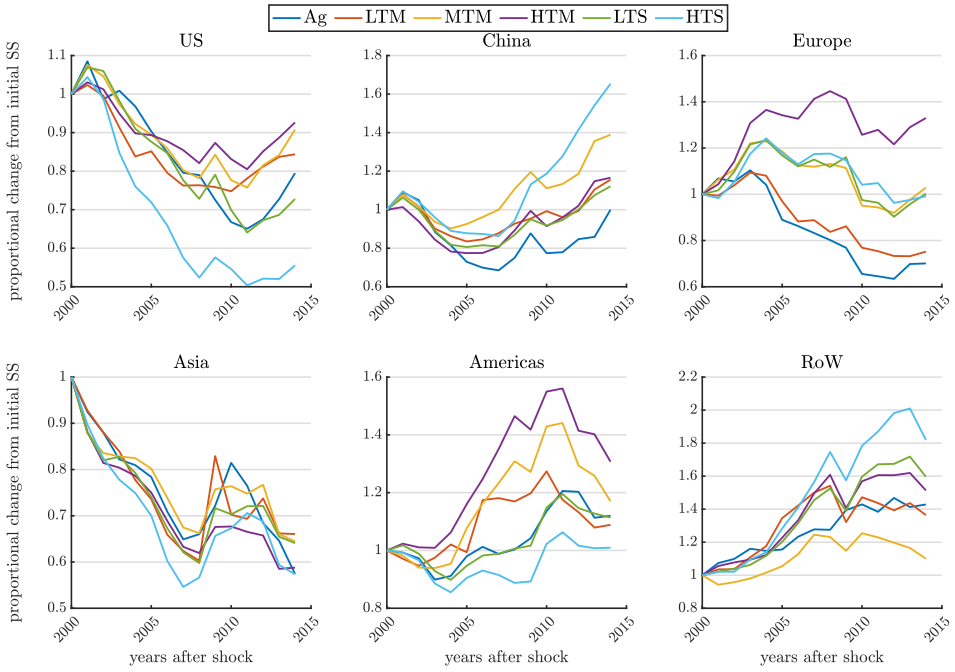


Figure 11: Extracted Inter-Temporal Preference Shocks $\widehat{\phi}_i^t$

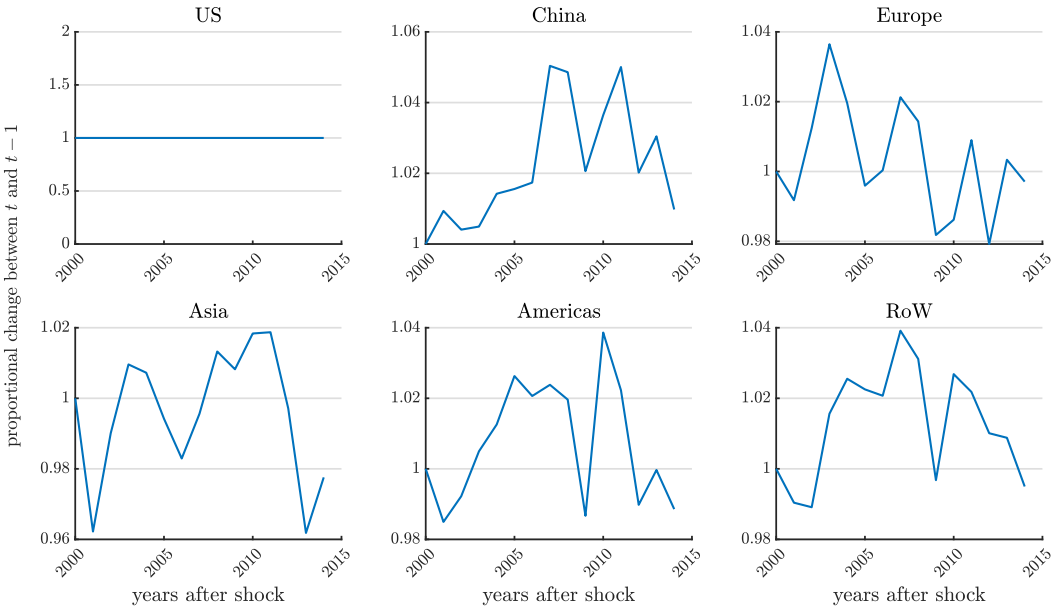
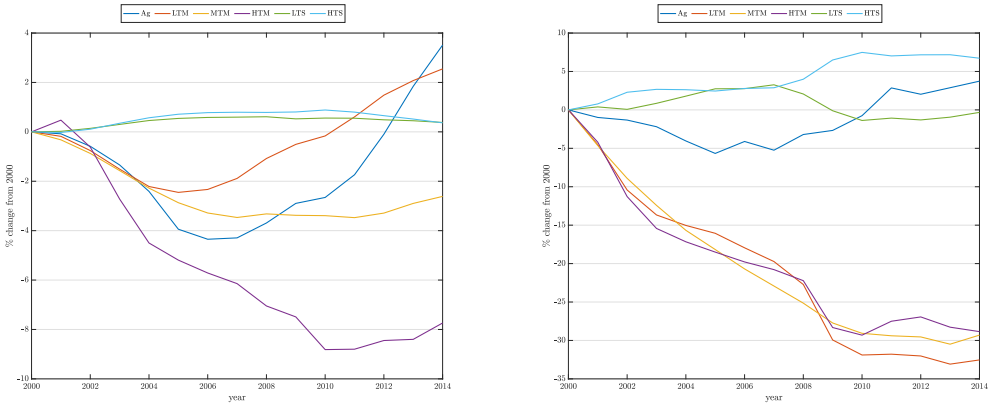


Figure 12: Comparing Labor Allocations in the Model and Data

(a) Model

(b) Data



To extract the shocks the global economy experienced between 2000 and 2014, we have solely relied on data on trade shares, sectoral prices, and aggregate final expenditures across countries. We have not used information on labor allocations or trade imbalances. It is therefore natural to ask how the model-implied behavior of labor allocations and trade imbalances compare to those in the data. A note of caution before we proceed with this comparison: given our perfect foresight assumption, once we feed the shocks into the model, responses at impact are large as agents (suddenly) fully anticipate the path of future shocks. That said, Figure 12 compares labor allocations in the United States in our model (Panel a) to labor allocations in the data (Panel b). Our model replicates the decline of manufacturing as a whole, the expansion of services, and the decline and rebound of agriculture. The magnitudes in our model are compressed, but we believe this is explained by pre-existing trends driving the decline of manufacturing as well as the fact that we impose that the inter-temporal shocks $\widehat{\phi}_i^t$ revert to 1 after 2014. The latter observation is important as individuals in the model anticipate the end of the savings glut years before the end of our sample period, leading to some rebalancing happening before 2014. This observation is also important to explain the strong rebound of Low-Tech Manufacturing, for which we do not find support in the data. To verify the plausibility of this explanation, we simulated the same shocks we extracted with one difference: we keep the $\widehat{\phi}_i^t$ shocks fixed at their 2012 to 2014 averages for 15 more years (that is, until 2029). Indeed, not only the magnitude of the downsize of manufacturing that we obtain is larger than what is depicted in Figure 12a, but also Low-Tech steadily contracts until 2014.

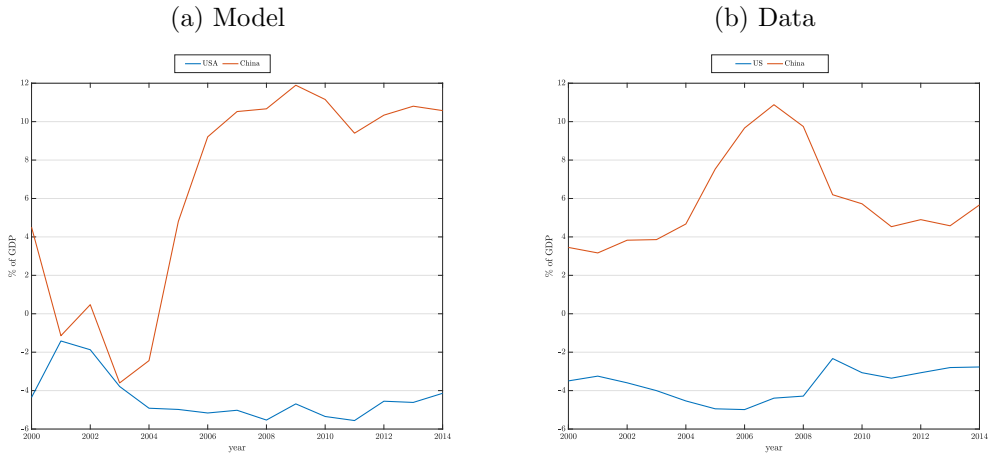
Figure 13 compares the model-implied trade imbalances in the US and China to those in the data. The model is able to capture the large surplus in China as well as the persistent deficit in the US. However, the model misses the behavior of Chinese imbalances in the beginning of the period. This happens as China anticipates its massive shocks that are coming (both large increases in productivity and shocks to inter-temporal preferences), which leads to large unemployment in the short run. In turn, this implies an initial production shortage and trade deficits.

5.2 Global Technology and Trade Shocks

Our first counterfactual exercise focuses on the general equilibrium effects of changes in productivity and trade costs since 2000, with no shocks to inter-temporal preferences. In particular, we focus on the consequences of our extracted series of $\left\{\widehat{A}_i^t\right\}$ and $\left\{\widehat{d}_{k,oi}^t\right\}$ above. Our goal is to isolate the quantitative significance of globalization on labor market outcomes in a setting where workers smooth their consumption over time. As we will show, the shocks we recover from the data have significant impacts on global imbalances, which has implications for adjustment dynamics, unemployment, wages, and, ultimately, consumption.

Before discussing results, it is helpful to contrast our approach with standard practice. There are two typical approaches to dealing with imbalances in the quantitative trade literature. The

Figure 13: Comparing Net Exports in the Model and Data



first approach involves fixing imbalances in the data, or assuming balanced trade period by period. This method has been employed in both static models (e.g., Eaton and Kortum (2002) and Dekle et al. (2007)) and dynamic models (e.g., Dix-Carneiro (2014) and Traiberman (2019)). The use of this method in dynamic models is a particularly strong assumption, since it implicitly imposes that workers and governments have *no* access to borrowing or savings mechanisms. The second approach, involves assigning ownership shares of capital or fixed factors to agents at an initial point, and fixing these shares over time and in counterfactuals. Although this procedure does not follow from optimizing behavior, this method has been used in Caliendo et al. (2019), allowing for imbalances to change as returns to fixed factors change. If returns to capital increase while those to labor decline, this implies that workers have access to some social insurance. Nevertheless, agents are prevented from buying or selling shares in response to the shocks they face.[24]

To be able to compare the implications of the globalization shocks in our model of trade imbalances relative to a model without imbalances, we start both models from a steady steady with trade balance—that is $NX_i = 0 \ \forall i$, see Appendix C.3 for details on our procedure, which is based on exact hat algebra. Figure 14 shows that the globalization shocks we feed into the model can lead to substantial imbalances in the short run. These range from a trade *deficit* in China of up to 65% of GDP to a trade surplus in Asia close to 20%. The US runs a short-run surplus amounting to close to 2.5% of GDP. As expected from previous literature, if we purge the model of the intertemporal preference shocks, we see China running a large trade deficit in the short run. This is

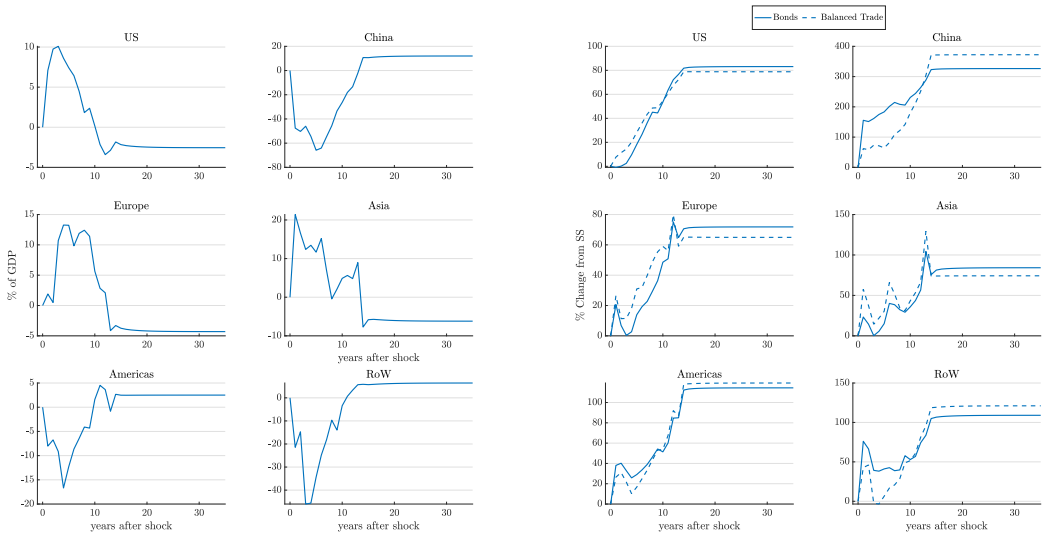
[24]These approximations may be reasonable in a situation where imbalances are small along the transition path from one equilibrium to the next. This would be the case, for example, if the primary force for changes in imbalances were shocks to aggregate consumption, and orthogonal to trade shocks (i.e., if imbalances were driven by $\widehat{\phi}_i^t$). Nevertheless, these are imperfect if the path and magnitude of trade shocks lead to large changes in consumption and savings, which we show is the case below.

not surprising in light of our discussion in section 4.2 as Chinese productivity strongly and steadily increases between 2000 and 2014, more than in other countries.

Figure 14: Aggregate Trade and Consumption Dynamics Following Globalization Shocks

(a) NX Over GDP

(b) Consumption



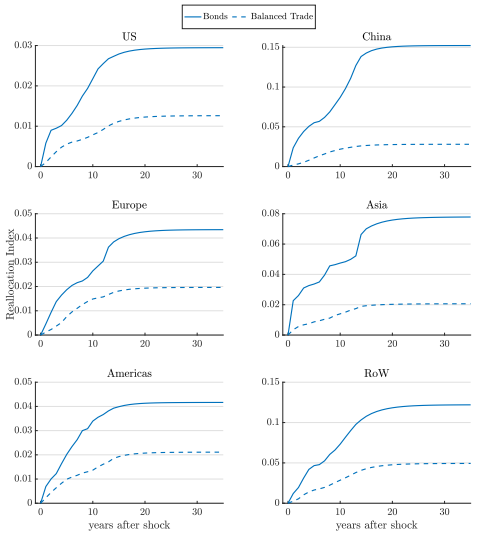
We learned in our analysis of section 4.2 that trade imbalances can amplify the amount of reallocation in the economy relative to a model where trade is balanced. Our empirical exercise corroborates this finding, as we illustrate in Figure 15a. The behavior of unemployment is also quite different, and driven by a larger amount of reallocation. Importantly, Figures 15c and 15d highlight the importance of modelling trade imbalances in understanding the adjustment process in response to globalization shocks. A model that imposes trade balance often predicts opposing patterns of short-run reallocation. For example, our model with imbalances predicts that the globalization shocks would lead to an expansion of all manufacturing sectors in the US in the short run, but a contraction in the long run. On the other hand, trade balance would lead to a monotonic decline of High-Tech Manufacturing and a long-run expansion of Low-Tech Manufacturing. Other countries also display drastically different patterns of adjustment in both the short and long runs.

5.3 Trade Costs and Imbalances

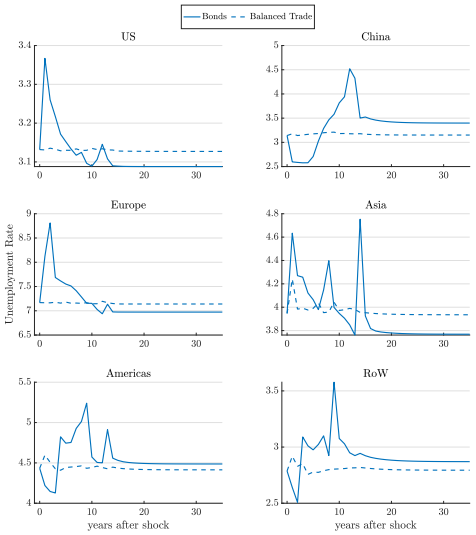
The previous section established that empirically-extracted shocks can lead to significant trade imbalances, and that accounting for these imbalances can substantially alter the adjustment process relative to a balanced-trade world. This section studies the implications of both trade imbalances and labor market frictions for the consumption gains from trade, and for how these compare with

Figure 15: Reallocation and Unemployment Dynamics Following Globalization Shocks

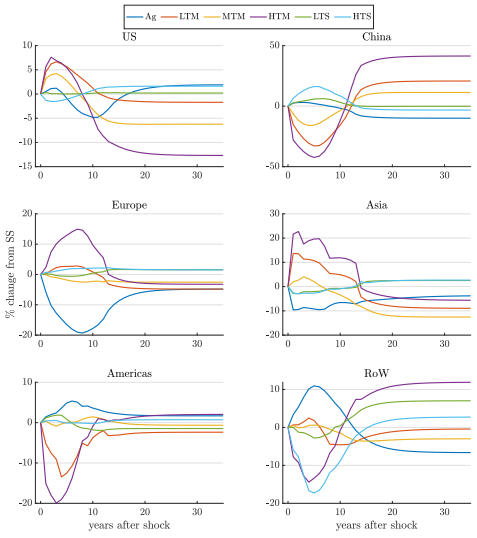
(a) Reallocation Index



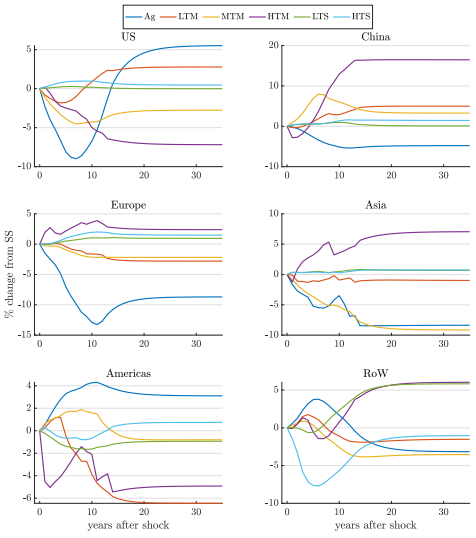
(b) Unemployment



(c) Labor Allocations - Full Model



(d) Labor Allocations - Balanced Trade



the widely used sufficient-statistics approach based on Arkolakis et al. (2012), henceforth ACR. Our model nests the Ricardian model considered in ACR, but violates two of their key assumptions: (a) no labor market frictions, no unemployment, no wage differences within and across sectors; and (b) no trade imbalances. In addition, the sufficient-statistics approach consider static models, with the interpretation that they capture the long-run effects of trade shocks.

Concretely, we consider the changes in trade costs between 2000 and 2014 we obtain in Section 5.1, purging the model of shocks to inter-temporal preferences and of productivity shocks. In this case, the implied gains from trade following Costinot and Rodríguez-Clare (2014), who extend the ACR formula to allow for input-output linkages is given by:A

$$\widehat{W}_i = \prod_{j=1}^{K}\prod_{k=1}^{K} \widehat{\pi}_{k,ii}^{-\mu_{j,i}\aleph_{jk,i}/\lambda}, \tag{49}$$

where all of the changes are between final and initial steady states and $\aleph_{jk,i}$ is the j,k^{th} element of the Leontief Inverse of the input-output matrix in country i. We obtain $\widehat{\pi}_{k,ii}$ solving our full model.

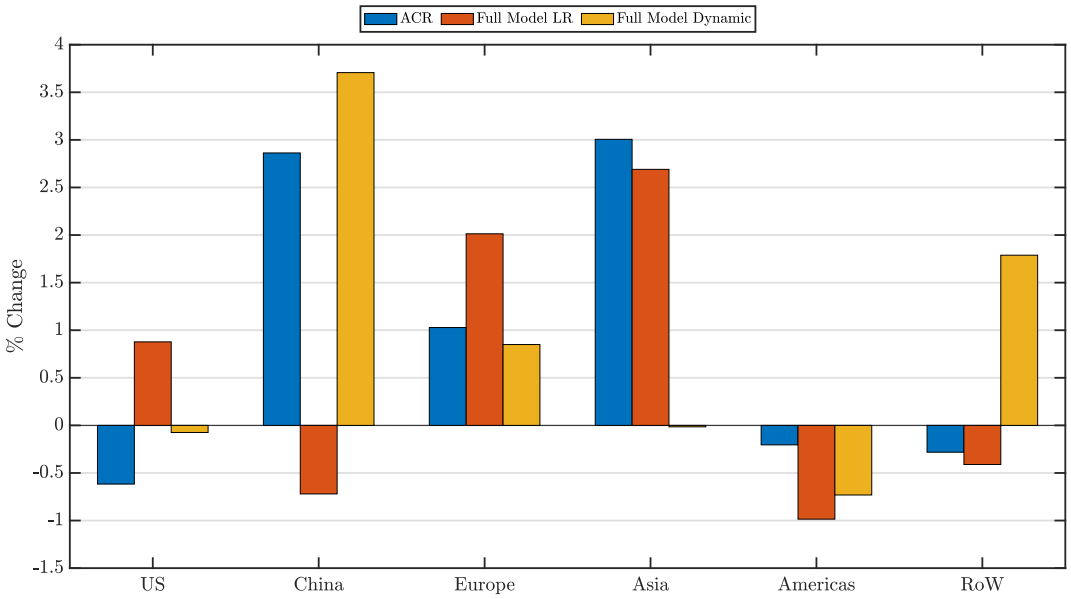
To perform the comparison between consumption gains in our model and those using the ACR formula, we will compute two measures of gains in the context of our model. First, we focus on the change in steady-state consumption given by our model. Given the static nature of the ACR model, we believe this is a more direct comparison between our predictions for the gains from trade. Second, given the dynamic nature of our model, we define the dynamic consumption gains from trade as the ratio between the level of constant consumption that would yield the same net present value utility that unfolds along the transition path and the initial steady-state consumption. Mathematically:

$$\widehat{W}_i \equiv \exp\left\{(1-\delta)\sum_{t=0}^{\infty}\delta^t \log(C_i^t) - \log(C_i^{SS_0})\right\}. \tag{50}$$

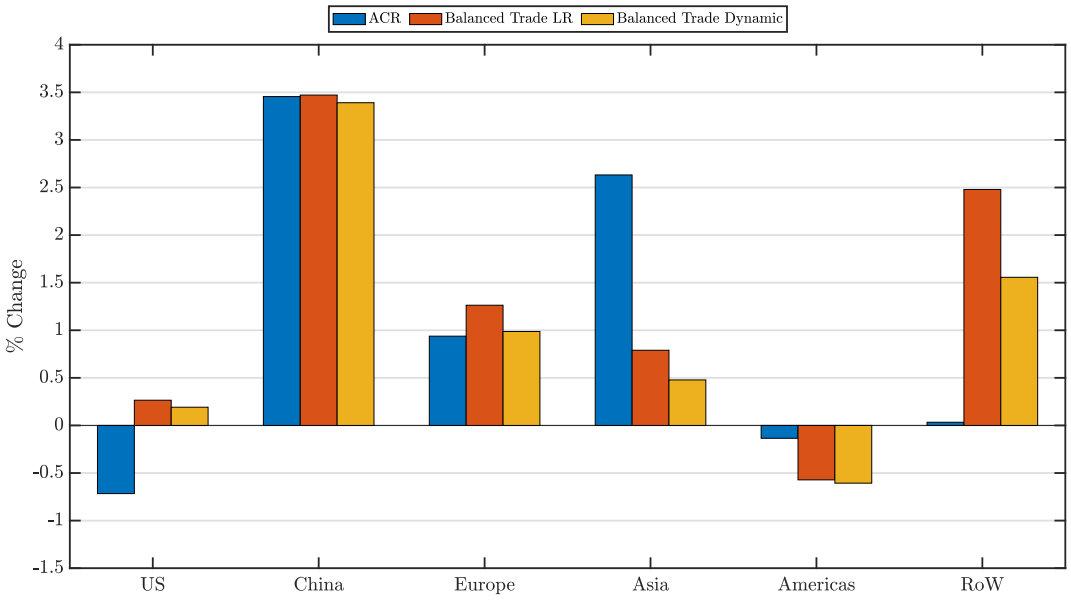
Figure 16 displays the comparison between the ACR gains and the gains we obtain in our model. We first compare the welfare changes predicted by the extended ACR formula (blue bars) to the long-run gains obtained in our full model, which focuses on differences in final and initial steady-state consumption (red bars). Overall, these gains are quite different. The ACR formula predicts that the US endures a 0.6% loss in response to the changes in trade costs the global economy experiences, whereas our model predicts that the US experiences a gain of 0.9%. Our conclusions differ starkly in China, where the ACR formula predicts a gain of almost 3%, but our model predicts a long-run loss of 0.7%. Next, we turn to the comparison between the ACR formula to the full dynamic gains in our model (equation (50)). Differences in predictions are, again, quite significant. For example, the ACR formula predicts a 3% gain in Asia, whereas our model assigns an essentially zero gain. Finally, the ACR formula predicts a loss of 0.25% in the Rest of the World, whereas our model predicts a gain of 1.8%.

Figure 16: Trade Costs and the Consumption Gains from Trade

(a) Benchmark: Full Dynamic Model w/ Imbalances

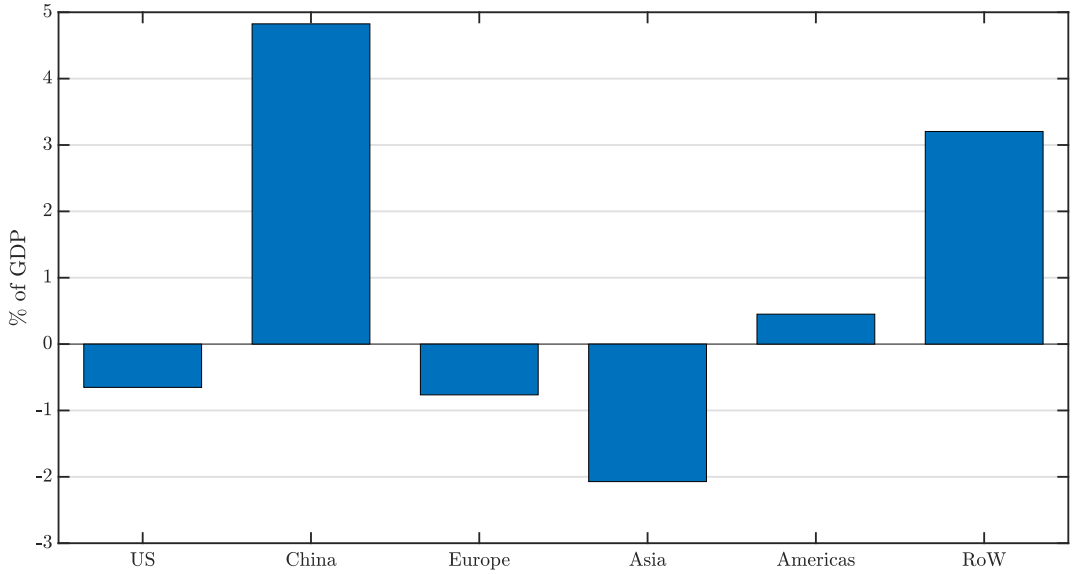


(b) Benchmark: Full Dynamic Model w/ Balanced Trade



Notes:

Figure 17: Long-Run Net Exports when Full Model is Subject to Changes in Trade Costs Only



Differences in the predictions of the ACR formula and those implied by our model and equation (50) were expected: the former computes static gains, whereas the latter computes the present value of gains over the transition. However, the predictions between the ACR formula and the long-run consumption gains in our model are more directly comparable. These numbers differ because of both labor market frictions and long-run trade imbalances that arise in our model. The latter figures depend on the full path of shocks fed into the model, and not just the initial and final levels of trade costs. Figure 17 shows the long-run trade imbalances that arise in our model. They are particularly important in China, Asia, and the Rest of the World.

We now investigate the role of labor market frictions in the discrepancy between the ACR formula and our model's predictions. To that aim, we compute changes in the long-run change in the share of expenditure on domestic goods, $\widehat{\pi}_{k,ii}$, based on our dynamic model restricted to balanced trade. In this case, $\widehat{\pi}_{k,ii}$ does not depend on the full path of shocks, only on the initial and final levels of trade costs. The only reason why the ACR formula calculation will differ from the long-run consumption gains implied by our model relies on labor market frictions. Figure 16b shows that, even in this case, the ACR formula leads to conclusions that are quite different from those of our model. For example, the ACR formula predicts a *loss* of 0.7% in the US, but our model predicts a slight *gain* of 0.25%. Perhaps more starkly, the ACR formula predicts a virtually zero effect on Rest of the World consumption, but our model predicts a gain of 2.5%. We conclude this section highlighting that both labor market frictions and trade imbalances can lead to quantitatively important discrepancies between gains predicted by our model and those obtained

using the widely-used ACR formula.

5.4 The China Shock

As a final application of our model, we investigate the role of China on the adjustment of the labor market in the US. This topic has attracted much academic interest since the work of Autor et al. (2013) and Pierce and Schott (2016). We compare the behavior of our model when we feed it with *all* of the shocks we recovered in Section 5.1 (which we label as the Benchmark counterfactual) to the behavior of the model when (a) shocks to China are set to be equal to the average of shocks experienced by the remaining countries; and when (b) we neutralize China's savings glut by setting $\widehat{\phi}^t_{China} = 1$ for all t.[25]

We first analyze the situation where shocks to China between 2000 and 2014 are imposed to be equal to the average of shocks across all of the remaining countries.[26] The objective is to understand the behavior of the global economy in a counterfactual world where Chinese fundamentals evolved like those of an average country. The red dashed line in Figure 18a shows that China runs a much more modest trade surplus over time compared to the Benchmark illustrated by the blue solid line. Interestingly, Figure 18b shows that the behavior of the US trade deficit is barely affected if shocks to China are set to other countries' averages. This result suggests that the evolution of the US trade deficit between 2000 and 2014 was not greatly influenced by the extraordinary shocks China experienced over this period. Instead, the evolution of the trade deficit in the US was primarily dictated by the full constellation of shocks that the global economy experienced over the 2000's. This result highlights an important advantage of our multi-country model relative to two-country models, where changes in the trade balance in one country are mirrored in the other.

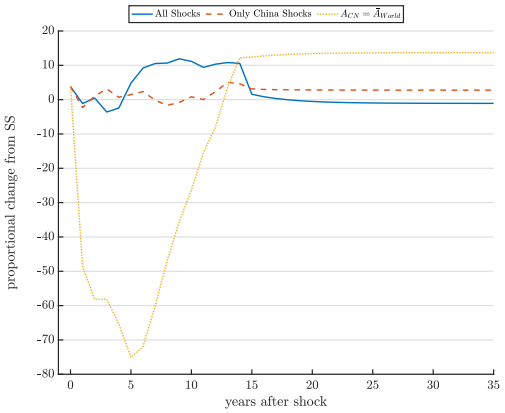
Figure 19 describes the impact of the China shock on sectoral reallocation in the US. The red dashed line shows that if shocks to China had behaved in an ordinary way between 2000 and 2014, the contraction in Mid- and High-Tech Manufacturing would have been less pronounced. The effect of the China shock on employment in Low-Tech Manufacturing is not as visible, but the dashed red line is consistently above the solid blue line, showing that shocks to the Chinese economy led to mild contractions in employment in the sector (or prevented stronger growth in the long run). In contrast, employment in Agriculture and Services (particularly Low-Tech Services) moved in opposite directions, quickly absorbing workers displaced from Manufacturing. Figure 20 shows that the behavior of unemployment with the China shock (solid blue line) or without it (dashed red line) is very similar. Indeed, in either case, the unemployment rate reaches a maximum of 3.14%.

[25] Shocks to inter-temporal preferences and their interaction with standard shocks in productivities and trade costs differentiate these counterfactual experiments from Caliendo et al. (2019) and Adão et al. (2020).

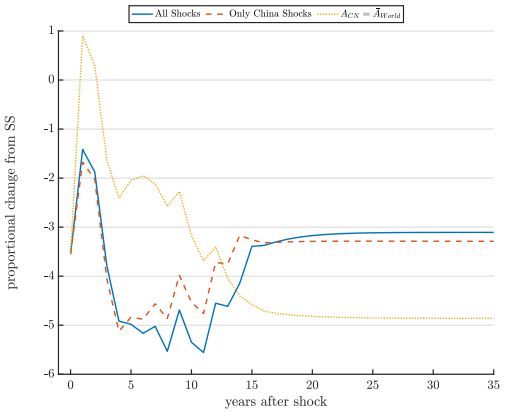
[26] Shocks to trade costs from country i to China are set to be equal to a weighted average of shocks between country i and all other countries—weights are given by country sizes $\overline{L}_i$. Shocks to trade costs from China to country i are set to be equal to the weighted average of shocks between all remaining countries and country i.

Figure 18: The China Shock: Net Exports

(a) NX / GDP in the China



(b) NX / GDP in the US

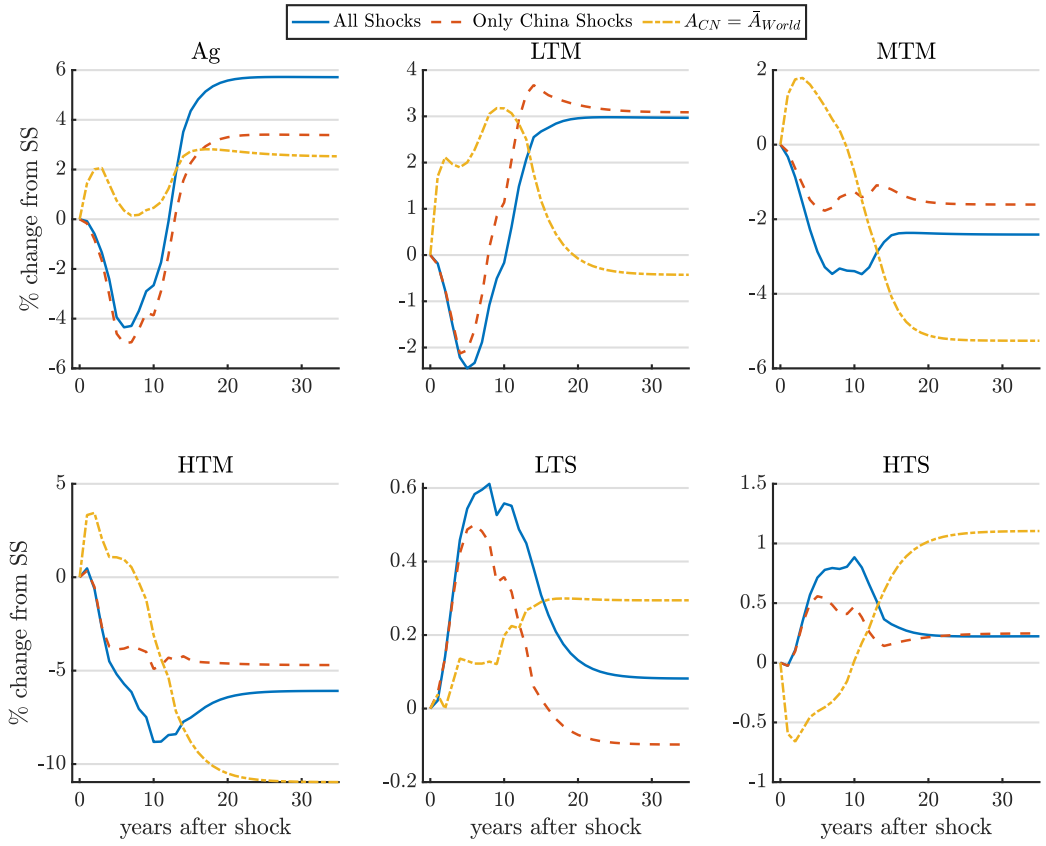


Notes: "All Shocks": model is fed with all shocks recovered in section 5.1. "China Receives World Avg. Shocks": model is fed with all shocks recovered in section 5.1, with the exception of China. For China, productivity and inter-temporal productivity shocks between 2000 and 2014 are imposed to be equal to the average of these shocks across all the remaining countries. Shocks to trade costs from country i to China are set to be equal to a weighted average of shocks between country i and all other countries—weights are given by country sizes $\overline{L}_i$. Shocks to trade costs from China to country i are set to be equal to the weighted average of shocks between all remaining countries and country i. "All Shocks but $\widehat{\phi}_{China} = 1$": model is fed with all shocks recovered in section 5.1 but China's savings glut by setting $\widehat{\phi}^t_{China} = 1$ for all t. $t = 0$ corresponds to year 2000. $t = 14$ corresponds to 2014, the last year of data we employed to extract the shocks.

Compared to the baseline of 3.08%, this accounts for a total of $\overline{L}_{US} \times (3.14\% - 3.08\%) \approx 93,100$ lost jobs, but only for a short period of time.

The small response of unemployment in the United States to the China Shock can be understood by breaking things down to the sectoral level. Figure 20b plots these for the "All Shocks Except China" counterfactual. Clearly, the response of sectoral unemployment is much larger—declining or rising by over 15% in some sectors. However, the composition of worker reallocation explains the aggregate response: workers exiting manufacturing enter services, which feature much tighter labor markets, and thus lower unemployment rates. This composition effect mutes the effect in the US. Under different circumstances, the composition effect may exacerbate the shock by pushing workers into less tight labor markets. According to our model, this happened, for example, in Europe, which sees a dramatic spike in unemployment.

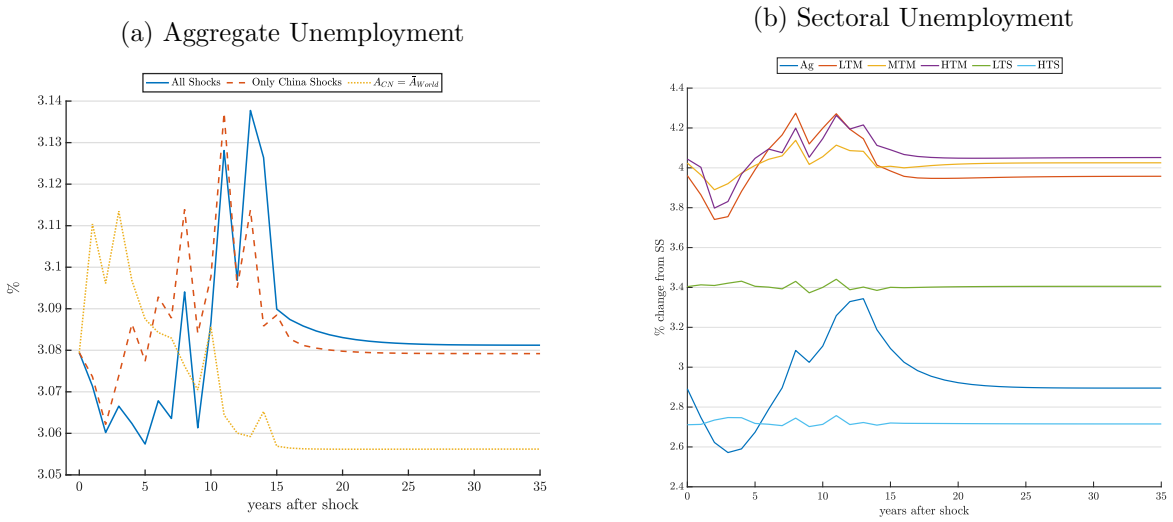
Figure 19: The China Shock: Labor Allocations in the US



Notes: "All Shocks": model is fed with all shocks recovered in section 5.1. "China Receives World Avg. Shocks": model is fed with all shocks recovered in section 5.1, with the exception of China. For China, productivity and inter-temporal productivity shocks between 2000 and 2014 are imposed to be equal to the average of these shocks across all the remaining countries. Shocks to trade costs from country i to China are set to be equal to a weighted average of shocks between country i and all other countries—weights are given by country sizes $\overline{L}_i$. Shocks to trade costs from China to country i are set to be equal to the weighted average of shocks between all remaining countries and country i. "All Shocks but $\widehat{\phi}_{China} = 1$": model is fed with all shocks recovered in section 5.1 but China's savings glut by setting $\widehat{\phi}^t_{China} = 1$ for all t. $t = 0$ corresponds to year 2000. $t = 14$ corresponds to 2014, the last year of data we employed to extract the shocks.

It is also instructive to compare the behavior of the model when it is subjected to all the global shocks we extracted in Section 5.1 (the Benchmark) to its behavior when we subject it with the same shocks but remove China's saving glut (that is, we set $\widehat{\phi}^t_{China} = 1$ for all t). The yellow dotted line in Figure 18a shows that, in the absence of China's savings glut, China would have run a massive trade deficit in the short run, reaching 75% of GDP. These large short-run deficits are then accompanied by large permanent trade surpluses in the long run, surpassing 10% of GDP. The evolution of the dotted yellow line is unsurprising in light of our our discussion in Section 4.2, and, more specifically, given Figure 8. In anticipation of a much higher level of productivity in the future (see Figure 10a), China borrows greatly in the short run to smooth consumption over time.

Figure 20: The China Shock: Unemployment in the US



Notes: "All Shocks": model is fed with all shocks recovered in section 5.1. "China Receives World Avg. Shocks": model is fed with all shocks recovered in section 5.1, with the exception of China. For China, productivity and inter-temporal productivity shocks between 2000 and 2014 are imposed to be equal to the average of these shocks across all the remaining countries. Shocks in trade costs from country i to China are set to be equal to the simple average of shocks between country i and all other countries. Shocks to trade costs from country i to China are set to be equal to a weighted average of shocks between country i and all other countries—weights are given by country sizes $\overline{L}_i$. Shocks to trade costs from China to country i are set to be equal to the weighted average of shocks between all remaining countries and country i. "All Shocks but $\widehat{\phi}_{China} = 1$": model is fed with all shocks recovered in section 5.1 but China's savings glut by setting $\widehat{\phi}^t_{China} = 1$ for all t. $t = 0$ corresponds to year 2000. $t = 14$ corresponds to 2014, the last year of data we employed to extract the shocks.

This debt must be repaid in the form of large surpluses in the long run. This result illustrates the quantitatively important role of the inter-temporal shocks to preferences $\widehat{\phi}^t_{China}$ not only in offsetting these large short-run trade deficits, but also in turning them into surpluses. Neutralizing the Chinese savings glut also has important effects on the dynamics of the US trade deficit.

The yellow dotted line in Figure 18b shows that the US trade deficit would be significantly smaller in the short run. However, the trade deficit would stabilize at a larger value in the long run (5% of GDP) as China runs a permanent large trade surplus. This result highlights that, even in the absence of the China's savings glut, consumption smoothing motives in the global economy would lead to a large trade surplus in China in the long run (north of 10% of GDP) as the trade deficit in the US deteriorates to 5% of GDP.

The dotted yellow and solid blue lines in Figure 19 show interesting consequences of China's savings glut on the size of the manufacturing sector in the US. The Chinese savings glut was responsible for large declines in US manufacturing in the short run. This behavior is consistent across all three manufacturing sectors. However, absent the Chinese savings glut, manufacturing employment in the US would be substantially lower in the long run as China runs a permanently large trade surplus, and the US runs a permanently large trade deficit.

We now turn to the welfare consequences of the China shock. Specifically, we use equation (50) to compute the welfare effects of the shocks we recover in Section 5.1 (Benchmark), relative to the counterfactuals where (a) shocks to China are set to be equal to the average shocks experienced by the remaining countries: $\widehat{W}_i^{Benchmark}/\widehat{W}_i^{(a)}$; and where (b) we neutralize China's savings glut by setting $\widehat{\phi}_{China}^t = 1$ for all t: $\widehat{W}_i^{Benchmark}/\widehat{W}_i^{(b)}$. By doing so, we are capturing the effect of China's shocks relative to a situation where the Chinese fundamentals behaved similarly to the average global economy (excluding China), and relative to a situation where China did not experience a savings glut.

Table 5: Global Consumption Gains of the China Shock (2000-2014)

	Panel A. $\widehat{W}_i^{Benchmark}/\widehat{W}_i^{(a)}$	
Country	Complete Model	Balanced Trade
US	1.001	1.012
Europe	1.001	1.003
Asia	1.004	0.994
Americas	1.001	1.001
Rest of the World	1.005	1.031

	Panel B. $\widehat{W}_i^{Benchmark}/\widehat{W}_i^{(b)}$	
Country	Complete Model	Balanced Trade
US	0.997	1.000
Europe	0.998	1.000
Asia	0.998	1.000
Americas	1.003	1.000
Rest of the World	1.010	1.000

Notes: Changes in welfare obtained using equation (50). $\widehat{W}_i^{Benchmark}$ refers to all shocks extracted in Section 5.1 fed into the model. $\widehat{W}_i^{(a)}$ refers to these same shocks, but China's shocks at set to the average of shocks in remaining countries. $\widehat{W}_i^{(b)}$ refers to all shocks extracted in Section 5.1 fed into the model, but $\widehat{\phi}_{China}^t = 1$ for all t.

Panel A of Table 5 shows the effect of the China shock relative to a situation where China experienced shocks set to the average of the remaining countries. The first column displays the consumption effects of the China shock in our complete model with trade imbalances. The second column shows the consumption effects if we impose trade balance period by period. First, note that all countries benefitted from the extraordinary shocks accrued to China. However, these consumption effects were all very small. Interestingly, the welfare effects of the China shock in the US and the Rest of the World would be significantly higher if trade balanced every period. Panel B shows the effect of the Chinese savings glut. Our model predicts that the US, Europe, and Asia

were negatively affected by the large shocks to inter-temporal preferences in China, but that these effects were, again, very mild. The exception is the Rest of the World, that actually benefits from the Chinese savings glut. The Chinese savings glut would have had virtually no consumption effects around the world under balanced trade.

We end this section asking by how much countries benefitted from globalization during the period we consider. For each country at a time, we neutralize the shocks it experienced, while feeding the model with the path of shocks faced by the remaining countries. For example, to assess the effects of globalization on US consumption, we set all US shocks to 1, but we keep shocks to remaining countries at the values we recovered in Section 5.1. Table 6 displays the results. The first column shows that all countries benefitted from globalization. China benefitted the most, with consumption gains reaching 6.6%. The Americas and Asia/Oceania gained the least: 0.03%. The magnitude of gains can be significantly affected if we impose trade balance: the second column shows that gains in the US are reduced from 2.2% to 1.3%. Finally, we investigate the effect of inter-temporal shocks in the third and fourth columns (with and without trade imbalances, respectively). If we shut down the global savings glut, welfare in the US is significantly higher, reaching 3.8%. We conclude observing that even though our results are consistent with the belief that the global savings glut was detrimental to the US, balancing trade would lead to an even worse situation.

Table 6: Globalization Consumption Gains Over 2000-2014

Country	Complete Model	Balanced Trade	Complete Model $\widehat{\phi}_i^t = 1\ \forall i,t$	Balanced Trade $\widehat{\phi}_i^t = 1\ \forall i,t$
United States	1.022	1.013	1.038	1.013
China	1.066	1.067	1.056	1.067
Europe	1.015	1.015	1.025	1.016
Asia/Oceania	1.003	1.021	0.997	1.021
Americas	1.003	1.006	1.002	1.006
Rest of the World	1.052	1.045	1.037	1.045

Notes: Each row displays $\widehat{W}_i$ (see equation (50)) in response to extracted shocks to all other countries when own country shocks are neutralized. Third and fourth columns impose $\widehat{\phi}_i^t = 1\ \forall i,t$, neutralizing all inter-temporal preference shocks.

6 Conclusion

There is a widespread concern among policy makers that, as globalization brings disruption to the labor market, growing trade deficits can accentuate job losses and the decline of manufacturing. Given how persistent these concerns have been in the past four decades, it is surprising that trade economists typically abstract from trade imbalances when they study the labor market consequences

of globalization shocks. This paper aims to fill this gap by extending the workhorse model of trade (Eaton and Kortum, 2002, and Caliendo and Parro, 2015) to allow for inter-sectoral mobility costs (as in Artuç et al., 2010), search frictions (as in Mortensen and Pissarides, 1999), and trade imbalances (as in Reyes-Heroles, 2016). In short, we extend the state-of-the-art model of trade with the workhorse models for mobility frictions, unemployment and trade imbalances. We estimate the model using data from the WIOD, ILOSTAT, and the US CPS. Our estimation method conditions on trade shares and net exports and can be performed country by country. Even though the parameter space is large, the country by country estimation allows us to break a large optimization problem into smaller ones.

We find that: (a) incorporating trade imbalances has the potential to significantly magnify the effect of globalization shocks on inter-sectoral reallocation and unemployment; (b) reallocation paths can be substantially different if we allow for trade imbalances compared to balanced trade—specifically, sectors that expand in the short run with trade imbalances can instead contract with balanced trade; (c) the behavior of trade imbalances and of the labor market depends on the whole path of shocks. In particular, temporary shocks can have important persistent long-run effects in our model of trade imbalances, but none if we impose balanced trade. This motivates us to conduct an empirical application and extract the path of shocks the global economy actually experienced between 2000 and 2014 using standard methods from the international trade and macro literatures.

We conduct a standard exercise in the international trade literature and subject the model to extracted global technology and trade cost shocks. Aligned with our impulse response function analyses, we find that allowing for trade imbalances have a first order effect on the behavior of labor markets around the world. Next, we investigate how the gains from trade in our model compare with those obtained using the influential sufficient-statistics approach pioneered by Arkolakis et al. (2012). We find that both trade imbalances and labor market frictions can lead to substantial discrepancies between the gains from trade in our model relative to those in the literature on quantitative models of trade, which is summarized in Costinot and Rodríguez-Clare (2014). Not only there are differences in the magnitude of the gains, but also on their signs. We also use our model to investigate the impact of the "China Shock" on the US trade deficit and labor market. Consonant with previous literature, we find that shocks to the Chinese economy contributed to the decline of manufacturing in the US, especially Mid-Tech and High-Tech Manufacturing. However, other sectors of the economy quickly absorbed workers from these sectors, leading to small effects on unemployment. We also find that shocks to the Chinese economy had only a modest effect on the evolution of the US trade deficit, highlighting that the evolution of the US trade deficit was mostly driven by the full constellation of shocks the global economy experienced. Interestingly, if China had not experienced its savings glut, the US trade deficit would have been smaller in the short run, but larger in the long run. Finally, we assessed, for each country, the gains from

globalization between 2000 and 2014. Our model predicts that the US has enjoyed a gain of 2.2% in response to globalization shocks over this period. Our estimates suggest that these gains would have been 73% larger in the absence of the global savings glut. However, in a world where balanced trade is imposed, the US would have experienced 40% smaller gains.

Our work shows that carefully modeling imbalances can have quantitatively important implications for the adjustment process in response to globalization shocks and opens important questions for future work. Given the importance of imbalances for the reallocation process, a natural question to address next is to quantitatively characterize how trade imbalances shape the inequality effects of trade. Our model can be easily extended to allow heterogeneous workers, so this is a natural next step. In addition, we also hope to be able to add endogenous capital accumulation decisions and capital-skill complementarity, generating rich mechanisms linking globalization to the skill premium. Finally, an interesting extension of our framework would allow workers to make borrowing and savings decisions, which will aggregate into global imbalances. Even though this is a hard problem, especially regarding estimation, we believe that our method of simulated moments that can be performed country by country (conditional on trade shares and imbalances) can be applied to this situation.

References

ao, Rodrigo Ad Costas Arkolakis, and Federico Esposito, "General Equilibrium Effects in Space: Theory and Measurement," Technical Report 2020.

Arkolakis, Costas, Arnaud Costinot, and Andrés Rodríguez-Clare, "New trade models, same old gains?," *American Economic Review*, 2012, *102* (1), 94–130.

Artuç, Erhan, Shubham Chaudhuri, and John McLaren, "Trade Shocks and Labor Adjustment: A Structural Empirical Approach," *American Economic Review*, June 2010, *100* (3), 1008–45.

Artuç, Erhan and John McLaren, "Trade policy and wage inequality: A structural analysis with occupational and sectoral mobility," *Journal of International Economics*, 2015, *97* (2), 278–294.

Autor, David H., David Dorn, and Gordon H. Hanson, "The China Syndrome: Local Labor Market Effects of Import Competition in the United States," *American Economic Review*, October 2013, *103* (6), 2121–68.

_ , _ , and _ , "The China Shock: Learning from Labor-Market Adjustment to Large Changes in Trade," *Annual Review of Economics*, 2016, *8* (1), 205–240.

Backus, David K, Patrick J Kehoe, and Finn E Kydland, "International real business cycles," *Journal of political Economy*, 1992, *100* (4), 745–775.

Backus, David K., Patrick J. Kehoe, and Finn E. Kydland, "Dynamics of the Trade Balance and the Terms of Trade: The J-Curve?," *The American Economic Review*, 1994, *84* (1), 84–103.

Barany, Zsofia, Nicholas Coeurdacier, and Stéphane Guibaud, "Capital Flows in an Aging World," 2018.

Bems, Rudolfs, Robert C. Johnson, and Kei-Mu Yi, "The Great Trade Collapse," *Annual Review of Economics*, 2013, *5* (1), 375–400.

Bernanke, Benjamin S., "The Global Saving Glut and the U.S. Current Account Deficit," Speech presented at the Sandridge Lecture, Virginia Association of Economists, Richmond, VA, March 10 2005.

Caballero, Ricardo J., Emmanuel Farhi, and Pierre-Olivier Gourinchas, "An Equilibrium Model of "Global Imbalances" and Low Interest Rates," *American Economic Review*, March 2008, *98* (1), 358–93.

Caliendo, Lorenzo and Fernando Parro, "Estimates of the Trade and Welfare Effects of NAFTA," *Review of Economic Studies*, 2015, *82* (1), 1–44.

_ , Maximiliano Dvorkin, and Fernando Parro, "Trade and Labor Market Dynamics: General Equilibrium Analysis of the China Trade Shock," *Econometrica*, 2019, *87* (3), 741–835.

Carrère, Céline, Anja Grujovic, and Frédéric Robert-Nicoud, "Trade and Frictional Unemployment in the Global Economy," *Journal of the European Economic Association*, 01 2020. jvz074.

Coşar, A. Kerem, Nezih Guner, and James Tybout, "Firm Dynamics, Job Turnover, and Wage Distributions in an Open Economy," *American Economic Review*, March 2016, *106* (3), 625–63.

Costa, Francisco, Jason Garred, and João Paulo Pessoa, "Winners and losers from a commodities-for-manufactures trade boom," *Journal of International Economics*, 2016, *102*, 50 – 69.

Costinot, Arnaud and Andrés Rodríguez-Clare, "Trade Theory with Numbers: Quantifying the Consequences of Globalization," in "Handbook of International Economics," Vol. 4, Elsevier, 2014, pp. 197–261.

Dauth, Wolfgang, Sebastian Findeisen, and Jens Suedekum, "Adjusting to Globalization in Germany," IZA DP No. 11299, IZA 2018.

Dekle, Robert, Jonathan Eaton, and Samuel Kortum, "Unbalanced Trade," *American Economic Review*, May 2007, *97* (2), 351–355.

Dix-Carneiro, Rafael, "Trade Liberalization and Labor Market Dynamics," *Econometrica*, 2014, *82* (3), 825–885.

_ and Brian K. Kovak, "Trade Liberalization and Regional Dynamics," *American Economic Review*, October 2017, *107* (10), 2908–46.

_ and _, "Margins of labor market adjustment to trade," *Journal of International Economics*, 2019, *117*, 125 – 142.

Dornbusch, R., S. Fischer, and P. A. Samuelson, "Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods," *The American Economic Review*, 1977, *67* (5), 823–839.

Eaton, Jonathan and Samuel Kortum, "Technology, Geography, and Trade," *Econometrica*, 2002, *70* (5), 1741–1779.

_ , _ , and Brent Neiman, "On Deficits and Unemployment," *Revue économique*, 2013, *64* (3), 405–420.

_ , _ , _ , and John Romalis, "Trade and the Global Recession," *American Economic Review*, November 2016, *106* (11), 3401–38.

Flinn, Christopher J., "Minimum Wage Effects on Labor Market Outcomes under Search, Matching, and Endogenous Contact Rates," *Econometrica*, 2006, *74* (4), 1013–1062.

Guner, Nezih, Alessandro Ruggieri, and James Tybout, "Training, Offshoring, and the Job Ladder," Technical Report, Pennsylvania State University 2020.

Hagedorn, Marcus and Iourii Manovskii, "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited," *American Economic Review*, September 2008, *98* (4), 1692–1706.

Head, Keith and John Ries, "Increasing Returns versus National Product Differentiation as an Explanation for the Pattern of U.S.-Canada Trade," *American Economic Review*, September 2001, *91* (4), 858–876.

Heid, Benedikt and Mario Larch, "Gravity with unemployment," *Journal of International Economics*, 2016, *101*, 70 – 85.

Helpman, Elhanan and Oleg Itskhoki, "Labour Market Rigidities, Trade and Unemployment," *The Review of Economic Studies*, 07 2010, *77* (3), 1100–1137.

_ **and** _ , "Trade Liberalization and Labor Market Dynamics with Heterogeneous Firms," Technical Report 2015.

Kambourov, Gueorgui, "Labour Market Regulations and the Sectoral Reallocation of Workers: The Case of Trade Reforms," *The Review of Economic Studies*, 10 2009, *76* (4), 1321–1358.

Kehoe, Timothy J and Kim J Ruhl, "Sudden stops, sectoral reallocations, and the real exchange rate," *Journal of Development Economics*, 2009, *89* (2), 235–249.

Kehoe, Timothy J., Kim J. Ruhl, and Joseph B. Steinberg, "Global Imbalances and Structural Change in the United States," *Journal of Political Economy*, 2018, *126* (2), 761–796.

Kletzer, Lori, *Job Loss from Imports: Measuring the Costs*, Washington, DC: Institute for International Economics, 2001.

Krugman, Paul, "Globalization: What Did We Miss?," in Luis Catao and Maury Obstfeld, eds., *Meeting Globalization's Challenges*, Princeton University Press, 2019, pp. 113–120.

Meghir, Costas, Renata Narita, and Jean-Marc Robin, "Wages and Informality in Developing Countries," *American Economic Review*, April 2015, *105* (4), 1509–46.

Mendoza, Enrique G., Vincenzo Quadrini, and José-Víctor Ríos-Rull, "Financial Integration, Financial Development, and Global Imbalances," *Journal of Political Economy*, 2009, *117* (3), 371–416.

Meza, Felipe and Carlos Urrutia, "Financial liberalization, structural change, and real exchange rate appreciations," *Journal of International Economics*, 2011, *85* (2), 317–328.

Mortensen, Dale T. and Christopher A. Pissarides, "Job Creation and Job Destruction in the Theory of Unemployment," *The Review of Economic Studies*, 07 1994, *61* (3), 397–415.

_ **and** _ , "Unemployment Responses to 'Skill-Biased' Technology Shocks: The Role of Labour Market Policy," *The Economic Journal*, 1999, *109* (455), 242–265.

Navarro, Peter, "A Tariff Issue on Which Free and Fair Traders Can Agree," *Wall Street Journal*, May 2019.

Obstfeld, Maurice and Kenneth Rogoff, "The Intertemporal Approach to the Current Account," *Handbook of International Economics*, 1995, *3*, 1731–99.

_ **and** _ , "Global current account imbalances and exchange rate adjustments," *Brookings papers on economic activity*, 2005, *2005* (1), 67–146.

Pavcnik, Nina, "The Impact of Trade on Inequality in Developing Countries," Working Paper 23878, National Bureau of Economic Research September 2017.

Pierce, Justin R. and Peter K. Schott, "The Surprisingly Swift Decline of US Manufacturing Employment," *American Economic Review*, July 2016, *106* (7), 1632–62.

Reyes-Heroles, Ricardo, "The Role of Trade Costs in the Surge of Trade Imbalances," Technical Report 2016.

Rodriguez-Clare, Andres, Mauricio Ulate, and Juan Pablo Vasquez, "New Keynesian Trade: Understanding the Employment and Welfare Effects of Sectoral Shocks," Technical Report, University of California at Berkeley 2020.

Ruggieri, Alesandro, "Trade and labour market institutions: A tale of two liberalizations," Research Paper 2019/15, University of Nottingham 2019.

Scott, Robert E., "Causes of the Trade Deficit and Its Implications for the U.S. Economy: Hearing Before the Committee on Finance, US Senate," 105th Congress. Testimony of Robert E. Scott 1998.

Shimer, Robert, "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, March 2005, *95* (1), 25–49.

Simonovska, Ina and Michael E Waugh, "The elasticity of trade: Estimates and evidence," *Journal of International Economics*, 2014, *92* (1), 34–50.

Song, Zheng, Kjetil Storesletten, and Fabrizio Zilibotti, "Growing Like China," *American Economic Review*, February 2011, *101* (1), 196–233.

Traiberman, Sharon, "Occupations and Import Competition: Evidence from Denmark," *American Economic Review*, December 2019, *109* (12), 4260–4301.

Utar, Hale, "Workers beneath the Floodgates: Low-Wage Import Competition and Workers' Adjustment," *The Review of Economics and Statistics*, 2018, *100* (4), 631–647.

Zhu, Xiaodong, "Understanding China's Growth: Past, Present, and Future," *Journal of Economic Perspectives*, November 2012, *26* (4), 103–24.

Appendix

A Decentralizing the Labor Supply Decision in the Household Problem

This Section shows that the labor supply decision solving (6) subject to (7) and (5) can be decentralized to individual workers solving equations (5) and (9).
The Lagrangian of problem (6), (7) and (5) is

$$\mathcal{L}=E_0\left\{\begin{array}{c}\sum_{t=0}^{\infty}\left[\delta^t\phi^t\left(\overline{L}u\left(c_\ell^t\right)-\widetilde{\lambda}^t\left(\overline{L}P^{F,t}c_\ell^t+B^{t+1}-\Pi^t-R^tB^t\right)\right)\right]+\\ \int_0^{\overline{L}}\left[\sum_{t=0}^{\infty}\left[\delta^t\phi^t\left(\begin{array}{c}\left(1-e_\ell^t\right)\left(-C_{k_\ell^t,k_\ell^{t+1}}+\omega^t_{\ell,k_\ell^{t+1}}+b_{k_\ell^{t+1}}\right)+\\ e_\ell^t\eta_{k_\ell^t}+\widetilde{\lambda}^t\left(\sum_{k=1}^{K}\mathcal{I}\left(k_\ell^{t+1}=k\right)e_\ell^tw_k^t\left(x_\ell^t\right)\right)\end{array}\right)\right]\right]d\ell\end{array}\right\} \tag{A.1}$$

Because each worker is infinitesimal, and the allocation of one worker does not interfere with the allocation/utility of other individual workers (conditional on aggregates), maximizing

$$\int_0^{\overline{L}}\left[\sum_{t=0}^{\infty}\delta^t\phi^t\left(\begin{array}{c}\left(1-e_\ell^t\right)\left(-C_{k_\ell^t,k_\ell^{t+1}}+\omega^t_{\ell,k_\ell^{t+1}}+b_{k_\ell^{t+1}}\right)+\\ e_\ell^t\eta_{k_\ell^t}+\widetilde{\lambda}^t\left(\sum_{k=1}^{K}\mathcal{I}\left(k_\ell^{t+1}=k\right)e_\ell^tw_k^t\left(x_\ell^t\right)\right)\end{array}\right)\right]d\ell \tag{A.2}$$

means maximizing each individual term. Therefore, the planner solves, for each individual, the recursive problem:

$$\mathcal{L}_W^t\left(k_\ell^t,e_\ell^t,x_\ell^t,\omega_\ell^t\right)=\max_{k_\ell^{t+1},\widetilde{e}_k^{t+1}(.)}\left\{\begin{array}{c}\left(1-e_\ell^t\right)\left(-C_{k_\ell^t,k_\ell^{t+1}}+\omega^t_{\ell,k_\ell^{t+1}}+b_{k_\ell^{t+1}}\right)+e_\ell^t\eta_{k_\ell^t}\\ +\widetilde{\lambda}_t\sum_{k=1}^{K}\mathcal{I}\left(k_\ell^t=k\right)e_\ell^tw_k^t\left(x_\ell^t\right)+\\ \delta\widehat{\phi}^{t+1}E_t\mathcal{L}_W^{t+1}\left(k_\ell^{t+1},e_\ell^{t+1},x_\ell^{t+1},\omega_\ell^{t+1}\right)\end{array}\right\}, \tag{A.3}$$

where $\widehat{\phi}^{t+1}\equiv\frac{\phi^{t+1}}{\phi^t}$.

Denote by $\mathcal{F}^t$ the set of information at t. So, $E_t\left(.\right)=E\left(.|\mathcal{F}^t\right)$. For an unemployed worker in sector k at time t, $k_\ell^t=k$, $e_\ell^t=0$:

$$\mathcal{L}_W^t\left(k_\ell^t=k,e_\ell^t=0,x_\ell^t,\omega_\ell^t\right)=\max_{k',\{\widetilde{e}_k^{t+1}(.)\}}-C_{kk'}+\omega^t_{\ell,k'}+b_{k'}+\delta\widehat{\phi}^{t+1}E_t\mathcal{L}_W^{t+1}\left(k',e_\ell^{t+1},x_\ell^{t+1},\omega_\ell^{t+1}\right). \tag{A.4}$$

Using the law of iterated expectations we obtain:

$$\begin{aligned}
&\mathcal{L}_W^t\left(k_\ell^t=k,e_\ell^t=0,x_\ell^t,\omega_\ell^t\right)=\max_{k',\{\widetilde{e}_k^{t+1}(.)\}}-C_{kk'}+\omega_{\ell,k'}^t+b_{k'}\\
&+\delta\widehat{\phi}^{t+1}E\left\{E\left[\mathcal{L}_W^{t+1}\left(k',1,x_\ell^{t+1},\omega_\ell^{t+1}\right)|x_\ell^{t+1},\mathcal{F}^t\right]\times\Pr\left(k_\ell^{t+1}=k',e_\ell^{t+1}=1|x_\ell^{t+1},\mathcal{F}^t\right)|\mathcal{F}^t\right\}\\
&+\delta\widehat{\phi}^{t+1}E\left\{E\left[\mathcal{L}_W^{t+1}\left(k',0,x_\ell^{t+1},\omega_\ell^{t+1}\right)|x_\ell^{t+1},\mathcal{F}^t\right]\times\Pr\left(k_\ell^{t+1}=k',e_\ell^{t+1}=0|x_\ell^{t+1},\mathcal{F}^t\right)|\mathcal{F}^t\right\}\\
&=\max_{k',\{\widetilde{e}_k^{t+1}(.)\}}-C_{kk'}+\omega_{\ell,k'}^t+b_{k'}\\
&+\delta\widehat{\phi}^{t+1}\theta_{k'}^t q\left(\theta_{k'}^t\right)E\left\{\mathcal{L}_W^{t+1}\left(k',1,x_{\ell'}^{t+1},\omega_\ell^{t+1}\right)\widetilde{e}_{k'}^{t+1}\left(x_\ell^{t+1}\right)|\mathcal{F}^t\right\}\\
&+\delta\widehat{\phi}^{t+1}E\left\{\left(1-\theta_k^t q\left(\theta_{k'}^t\right)\widetilde{e}_{k'}^{t+1}\left(x_\ell^{t+1}\right)\right)\mathcal{L}_W^{t+1}\left(s',0,x_\ell^{t+1},\omega_\ell^{t+1}\right)|\mathcal{F}^t\right\}
\end{aligned}\tag{A.5}$$

For an employed worker in sector k, $k_\ell^t=k$, $e_\ell^t=1$:

$$\begin{aligned}
\mathcal{L}_W^t\left(k_\ell^t=k,e_\ell^t=1,x_\ell^t,\omega_\ell^t\right)&=\max_{\{\widetilde{e}_k^{t+1}(.)\}}\widetilde{\lambda}^t w_k^t\left(x_\ell^t\right)+\eta_k+\delta\widehat{\phi}^{t+1}E_t\mathcal{L}_W^{t+1}\left(k,e_\ell^{t+1},x_\ell^t,\omega_\ell^{t+1}\right)\\
&=\max_{\{\widetilde{e}_k^{t+1}(.)\}}\widetilde{\lambda}^t w_k^t\left(x_\ell^t\right)+\eta_k\\
&+\delta\widehat{\phi}^{t+1}E\left\{\begin{array}{c}E\left[\mathcal{L}_W^{t+1}\left(k,1,x_\ell^t,\omega_\ell^{t+1}\right)|k_\ell^{t+1}=k,,e_\ell^{t+1}=1,x_\ell^{t+1},\mathcal{F}^t\right]\times\\ \Pr\left(k_\ell^{t+1}=k,e_\ell^{t+1}=1|x_\ell^{t+1},\mathcal{F}^t\right)|\mathcal{F}^t\end{array}\right\}\\
&+\delta\widehat{\phi}^{t+1}E\left\{\begin{array}{c}E\left[\mathcal{L}_W^{t+1}\left(k,0,x_\ell^t,\omega_\ell^{t+1}\right)|k_\ell^{t+1}=k,e_\ell^{t+1}=0,x_\ell^{t+1},\mathcal{F}^t\right]\times\\ \Pr\left(k_\ell^{t+1}=k,e_\ell^{t+1}=0|x_\ell^{t+1},\mathcal{F}^t\right)|\mathcal{F}^t\end{array}\right\}\\
&=\max_{\{\widetilde{e}_k^{t+1}(.)\}}\widetilde{\lambda}^t w_k^t\left(x_\ell^t\right)+\eta_k\\
&+\delta\widehat{\phi}^{t+1}\left(1-\chi_k\right)E\left[\begin{array}{c}\widetilde{e}_k^{t+1}\left(x_\ell^t\right)\mathcal{L}_W^{t+1}\left(k,1,x_\ell^t,\omega_\ell^{t+1}\right)\\ +\left(1-\widetilde{e}_k^{t+1}\left(x_\ell^t\right)\right)\mathcal{L}_W^{t+1}\left(k,0,x_\ell^t,\omega_\ell^{t+1}\right)|\mathcal{F}^t\end{array}\right]\\
&+\delta\widehat{\phi}^{t+1}\chi_k E\left[\mathcal{L}_W^{t+1}\left(k,0,x_\ell^t,\omega_\ell^{t+1}\right)|\mathcal{F}^t\right]
\end{aligned}\tag{A.6}$$

Make the following definitions

$$\begin{aligned}
\widetilde{U}_k^t\left(\omega_\ell^t\right)&\equiv\mathcal{L}_W^t\left(k_\ell^t=k,e_\ell^t=0,x_\ell^t,\omega_\ell^t\right),\text{ and}\\
W_k^t\left(x\right)&\equiv\mathcal{L}_W^t\left(k_\ell^t=k,e_\ell^t=1,x,\omega_\ell^t\right).
\end{aligned}\tag{A.7}$$

$\widetilde{U}_k^t\left(\omega_\ell^t\right)$ is the value of unemployment in sector k, conditional on the preference shocks ω_ℓ^t, and $W_k^t\left(x\right)$ is the value of a job with match productivity x. Note that $\mathcal{L}_W^t\left(k_\ell^t=k,e_\ell^t=0,x_\ell^t,\omega_\ell^t\right)$ does not depend on x_ℓ^t and $\mathcal{L}_W^t\left(k_\ell^t=k,e_\ell^t=1,x,\omega_\ell^t\right)$ does not depend on ω_ℓ^t. Rewrite $\widetilde{U}_k^t\left(\omega_\ell^t\right)$ as

$$\begin{aligned}
\widetilde{U}_k^t\left(\omega_\ell^t\right)&=\max_{k',\{\widetilde{e}_k^{t+1}(.)\}}-C_{kk'}+\omega_{\ell,k'}^t+b_{k'}\\
&+\delta\widehat{\phi}^{t+1}\theta_{k'}^t q\left(\theta_{k'}^t\right)\int W_{k'}^{t+1}\left(x\right)\widetilde{e}_{k'}^{t+1}\left(x\right)dG_{k'}\left(x\right)\\
&+\delta\widehat{\phi}^{t+1}\left(1-\theta_{k'}^t q\left(\theta_{k'}^t\right)\Pr\left(\widetilde{e}_{k'}^{t+1}\left(x_\ell^{t+1}\right)=1\right)\right)E_\omega\left(\widetilde{U}_{k'}^{t+1}\left(\omega_\ell^{t+1}\right)\right),
\end{aligned}\tag{A.8}$$

and so:

$$\widetilde{U}_k^t\left(\omega_\ell^t\right) = \max_{k',\left\{\widetilde{e}_k^{t+1}(.)\right\}} -C_{kk'} + \omega_{\ell,k'}^t + b_{k'} + \delta\widehat{\phi}^{t+1}\theta_{k'}^t q\left(\theta_{k'}^t\right)\int\left(\begin{array}{c} W_{k'}^{t+1}(x)\,\widetilde{e}_{k'}^{t+1}(x) + \\ E_\omega\left(\widetilde{U}_{k'}^{t+1}\left(\omega_\ell^{t+1}\right)\right)\left(1-\widetilde{e}_{k'}^{t+1}(x)\right)\end{array}\right)dG_{k'}(x) + \delta\widehat{\phi}^{t+1}\left(1-\theta_{k'}^t q\left(\theta_{k'}^t\right)\right)E_\omega\left(\widetilde{U}_{k'}^{t+1}\left(\omega_\ell^{t+1}\right)\right). \tag{A.9}$$

Now, wewrite $W_k^t(x)$ as:

$$\begin{aligned} W_k^t(x) = &\max_{\left\{\widetilde{e}_k^{t+1}(.)\right\}} \widetilde{\lambda}^t w_k^t(x) + \eta_k \\ &+ \delta\widehat{\phi}^{t+1}\left(1-\chi_k\right)\widetilde{e}_k^{t+1}(x)\,W_k^{t+1}(x) \\ &+ \delta\widehat{\phi}^{t+1}\left(1-\left(1-\chi_k\right)\widetilde{e}_k^{t+1}(x)\right)E\left(\widetilde{U}_k^{t+1}\left(\omega_\ell^{t+1}\right)\right), \end{aligned} \tag{A.10}$$

and so

$$\begin{aligned} W_k^t(x) = &\max_{\left\{\widetilde{e}_k^{t+1}(.)\right\}} \widetilde{\lambda}^t w_k^t\left(x_\ell^t\right) + \eta_k \\ &+ \delta\widehat{\phi}^{t+1}\left(1-\chi_k\right)\left(\widetilde{e}_k^{t+1}(x)\,W_k^{t+1}(x) + \left(1-\widetilde{e}_k^{t+1}(x)\right)E_\omega\left(\widetilde{U}_k^{t+1}\left(\omega_\ell^{t+1}\right)\right)\right) \\ &+ \delta\widehat{\phi}^{t+1}\chi_k E_\omega\left(\widetilde{U}_k^{t+1}\left(\omega_\ell^{t+1}\right)\right). \end{aligned} \tag{A.11}$$

It is now clear that the optimal policy $\widetilde{e}_k^{t+1}(.)$ is:

$$\widetilde{e}_k^{t+1}(x) = \left\{\begin{array}{c} 1 \text{ if } W_k^{t+1}(x) > E_\omega\left(\widetilde{U}_k^{t+1}\left(\omega_\ell^{t+1}\right)\right) \\ 0 \text{ otherwise}\end{array}\right\}. \tag{A.12}$$

Define $U_k^t \equiv E_\omega\left(\widetilde{U}_k^t\left(\omega_\ell^t\right)\right)$. We therefore have the following Bellman equations:

$$U_k^t = E_\omega\left(\begin{array}{c} \max_{k'} -C_{kk'} + b_{k'} + \omega_{\ell,k'}^t \\ +\delta\widehat{\phi}^{t+1}\theta_{k'}^t q\left(\theta_{k'}^t\right)\int\max\left\{W_{k'}^{t+1}(x), U_{k'}^{t+1}\right\}dG_{k'}(x) \\ +\delta\widehat{\phi}^{t+1}\left(1-\theta_{k'}^t q\left(\theta_{k'}^t\right)\right)U_{k'}^{t+1}\end{array}\right) \tag{A.13}$$

$$W_k^t(x) = \widetilde{\lambda}^t w_k^t(x) + \eta_k + \delta\widehat{\phi}^{t+1}\left(1-\chi_k\right)\left(\max\left\{W_k^{t+1}(x), U_k^{t+1}\right\}\right) + \delta\widehat{\phi}^{t+1}\chi_k U_k^{t+1} \tag{A.14}$$

B Steady State Equilibrium

In this section we derive the equations characterizing the steady state equilibrium. The key conditions that we impose is that variables are constant over time, inflows of workers into each sector equal outflows, and job destruction rates equal job creation rates. We also impose that the preference shifters $\left\{\phi_i^t\right\}$ are constant and equal to 1 in the long run.

Wage Equation

$$w_{k,i}(x) = \beta_{k,i}\widetilde{w}_{k,i}x + \frac{(1-\beta_{k,i})(1-\delta)U_{k,i} - (1-\beta_{k,i})\eta_{k,i}}{\widetilde{\lambda}_i} \tag{A.15}$$

Firms' value function

$$J_{k,i}(x) = \frac{1-\beta_{k,i}}{1-(1-\chi_{k,i})\delta}\widetilde{\lambda}_i\widetilde{w}_{k,i}\left(x - \underline{x}_{k,i}\right) \tag{A.16}$$

Probability of filling a vacancy

$$q_i(\theta_{k,i}) = \frac{\kappa_{k,i}P_i^F}{\widetilde{w}_{k,i}} \times \frac{1-\delta(1-\chi_{k,i})}{\delta(1-\beta_{k,i})I_{k,i}\left(\underline{x}_{k,i}\right)} \tag{A.17}$$

where

$$I_{k,i}\left(\underline{x}_{k,i}\right) \equiv \int_{\underline{x}_{k,i}}^{x_{\max}} \left(s - \underline{x}_{k,i}\right) dG_{k,i}(s) \tag{A.18}$$

Unemployed workers' Bellman equation

$$U_{k,i} = \zeta_i \log\left(\sum_{k'} \exp\left\{\frac{-C_{kk',i} + b_{k',i} + \theta_{k',i}\frac{\kappa_{k',i}P_i^F}{\widetilde{w}_{k',i}} \times \widetilde{\lambda}_i\widetilde{w}_{k',i}\frac{\beta_{k',i}}{\left(1-\beta_{k',i}\right)} + \delta U_{k',i}}{\zeta_i}\right\}\right) \tag{A.19}$$

Transition rates

$$s_{k\ell,i} = \frac{\exp\left\{\frac{-C_{k\ell,i}+b_{\ell,i}+\theta_{\ell,i}\frac{\kappa_{\ell,i}P_i^F}{\widetilde{w}_{\ell,i}}\times\widetilde{\lambda}_i\widetilde{w}_{\ell,i}\frac{\beta_{\ell,i}}{\left(1-\beta_{\ell,i}\right)}+\delta U_{\ell,i}}{\zeta_i}\right\}}{\sum_{\overline{k}}\exp\left\{\frac{-C_{k\overline{k},i}+b_{\overline{k},i}+\theta_{\overline{k},i}\frac{\kappa_{\overline{k},i}P_i^F}{\widetilde{w}_{\overline{k},i}}\times\widetilde{\lambda}_i\widetilde{w}_{\overline{k},i}\frac{\beta_{\overline{k},i}}{\left(1-\beta_{\overline{k},i}\right)}+\delta U_{\overline{k},i}}{\zeta_i}\right\}} \tag{A.20}$$

Steady-state unemployment rates

$$u_{k,i} = \frac{\chi_{k,i}}{\theta_{k,i}q_i(\theta_{k,i})\left(1 - G_{k,i}\left(\underline{x}_{k,i}\right)\right) + \chi_{k,i}} \tag{A.21}$$

Trade + Price System

Input Bundle Price

$$P_{k,i}^{M}=\prod_{\ell=1}^{K}\left(\frac{P_{\ell,i}^{I}}{\nu_{k\ell,i}}\right)^{\nu_{k\ell,i}} \tag{A.22}$$

Domestic Sectoral Output Price

$$c_{k,i}=\left(\frac{\widetilde{w}_{k,i}}{\gamma_{k,i}}\right)^{\gamma_{k,i}}\left(\frac{P_{k,i}^{M}}{1-\gamma_{k,i}}\right)^{1-\gamma_{k,i}} \tag{A.23}$$

Price of Composite Sector-Specific Intermediate Good

$$P_{k,i}^{I}=\Gamma_{k,i}\left[\sum_{j=1}^{N}\frac{A_{k,j}}{\left(c_{k,j}d_{k,ji}\right)^{\lambda}}\right]^{-1/\lambda} \tag{A.24}$$

where $\Gamma_{k,i}$ is a sector and country specific constant.

Price of Final Consumption Good

$$P_{i}^{F}=\prod_{k=1}^{K}\left(\frac{P_{k,i}^{I}}{\mu_{k,i}}\right)^{\mu_{ki}} \tag{A.25}$$

Trade Shares

$$\pi_{k,oi}=\frac{A_{k,o}\left(c_{k,o}d_{k,oi}\right)^{-\lambda}}{\Phi_{k,i}}, \tag{A.26}$$

where

$$\Phi_{k,i}=\sum_{o=1}^{N}A_{k,o}\left(c_{k,o}d_{k,oi}\right)^{-\lambda}. \tag{A.27}$$

Zero net flows condition

$$\left(L_{i}.u_{i}\right)=\left(\sum_{\ell=1}^{K}s_{\ell k,i}L_{\ell,i}u_{\ell,i}\right)_{k=1}^{K}=s_{i}'\left(L_{i}.u_{i}\right) \tag{A.28}$$

Product market clearing

Gross Output

$$\gamma_{k,o}Y_{k,o}=\widetilde{w}_{k,o}L_{k,o}\left(1-u_{k,o}\right)\int_{\underline{x}_{k,o}}^{x_{\max}}\frac{s}{1-G_{k,i}\left(\underline{x}_{k,o}\right)}dG_{k,o}\left(s\right) \tag{A.29}$$

$$=\widetilde{w}_{k,o}\widetilde{L}_{k,o} \tag{A.30}$$

Expenditure with Vacancies

$$E_{k,o}^{V}=\kappa_{k,o}P_{o}^{F}\theta_{k,o}u_{k,o}L_{k,o} \tag{A.31}$$

Market Clearing System

$$Y_{k,o} = \sum_{i=1}^{N} \pi_{k,oi} E_{k,i} \tag{A.32}$$

$$E_{k,i} = \mu_{k,i} \left(\sum_{\ell=1}^{K} \gamma_{\ell,i} Y_{\ell,i} \right) + \sum_{\ell=1}^{K} \left(1 - \gamma_{\ell,i}\right) \nu_{\ell k,i} Y_{\ell,i} - \mu_{k,i} N X_i \tag{A.33}$$

Normalization: World total output is the numeraire

$$\sum_{i=1}^{N} \sum_{k=1}^{K} Y_{k,i} = 1 \tag{A.34}$$

Final Good Consumption Expenditure

$$E_i^C = \sum_{k=1}^{K} \gamma_{k,i} Y_{k,i} - \sum_{k=1}^{K} E_{k,i}^V - N X_i \tag{A.35}$$

Lagrange multipliers

$$\widetilde{\lambda}_i = \frac{\overline{L}_i}{E_i^C} \tag{A.36}$$

C Solution Methods

This Section presents the different algorithms we developped to estimate the model and to perform counterfactual simulations. Section C.1 details the estimation algorithm and Section C.2 obtains expressions for simulated moments. Section C.3 outlines an exact hat alebra algorithm to compute changes in the steady state equilibrium in response to shocks in trade costs, productivities or net exports. Section C.4 develops the algorithm solving for the transition path of our complete model with trade imbalances. Section C.5 adapts this algorithm to the case where we have exogenous deficits. Finally, Section C.6 outlines the procedure we use in Section 5.1 to extract the shocks in trade costs, productivities and inter-temporal shocks.

C.1 Estimation Algorithm

Define $I_{k,i}(x) \equiv \int_x^{x_{\max}} (s-x)\, dG_{k,i}(s)$. Imposing $G_{k,i} \sim \log \mathcal{N}\left(0, \sigma_{k,i}^2\right)$ and a bit of algebra leads to:

- $G_{k,i}(x) = \Phi\left(\frac{\ln x}{\sigma_{k,i}}\right)$
- $I_{k,i}(x) = \exp\left(\frac{\sigma_{k,i}^2}{2}\right)\Phi\left(\sigma_{k,i} - \frac{\ln x}{\sigma_{k,i}}\right) - x\Phi\left(-\frac{\ln x}{\sigma_{k,i}}\right)$
- $I_{k,i}(0) = \exp\left(\frac{\sigma_{k,i}^2}{2}\right)$
- $\int_{\underline{x}_{k,i}}^{x_{\max}} \frac{s}{1-G_{k,i}\left(\underline{x}_{k,i}\right)} dG_{k,i}(s) = \exp\left(\frac{\sigma_{k,i}^2}{2}\right) \frac{\Phi\left(\sigma_{k,i} - \frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}$

Note: The estimation procedure we describe takes trade shares $\pi_{k,oi}^{Data}$ as given.
Step 1: Solve for $\{Y_{k,i}\}$ using:

$$Y_{k,o} = \sum_{i=1}^{N}\sum_{\ell=1}^{K} \pi_{k,oi}^{Data}\left(\mu_{k,i}\gamma_{\ell,i} + (1-\gamma_{\ell,i})\,\nu_{\ell k,i}\right) Y_{\ell,i} - \sum_{i=1}^{N} \pi_{k,oi}^{Data}\mu_{k,i} N X_i$$

$$\sum_{o=1}^{N}\sum_{k=1}^{K} Y_{k,o} = 1$$

The rest of the procedure conditions on these values of $\{Y_{k,i}\}$.

Step 2: Guess model parameters Ω. We treat $\widetilde{\kappa}_{k,i} \equiv \frac{\kappa_{k,i} P_i^F}{\widetilde{w}_{k,i}}$ as parameters to be estimated.

Step 3: Define

$$\varpi_{k,i} \equiv \frac{\left(1-(1-\chi_{k,i})\,\delta\right)\widetilde{\kappa}_{k,i}}{\delta\left(1-\beta_{k,i}\right)}$$

If $\frac{\left(1-\left(1-\chi_{k,i}\right)\delta\right)\widetilde{\kappa}_{k,i}}{\delta\left(1-\beta_{k,i}\right)I_{k,i}(0)} = \frac{\varpi_{k,i}}{I_{k,i}(0)} \geq 1$, the free entry condition cannot be satisfied—$I_{k,i}$ is decreasing. Abort the procedure and highly penalize the objective function.

Step 4: Find $\underline{x}_{k,i}^{ub}$ such that $\frac{\left(1-\left(1-\chi_{k,i}\right)\delta\right)\widetilde{\kappa}_{k,i}}{\delta\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}_{k,i}^{ub}\right)} = 1 \Longleftrightarrow I_{k,i}\left(\underline{x}_{k,i}^{ub}\right) = \varpi_{k,i}$. If along the algorithm $\underline{x}_{k,i}$ goes above $\underline{x}_{k,i}^{ub}$, we update it to be equal to $\underline{x}_{k,i}^{ub}$ (minus a small number).

Step 5: Guess $\{L_{k,i}\}$, and $\left\{\underline{x}_{k,i}\right\}$

Step 6: Compute $I_{k,i}\left(\underline{x}_{k,i}\right)$, $G_{k,i}\left(\underline{x}_{k,i}\right)$, $\theta_{k,i}$ and $u_{k,i}$.

- $\theta_{k,i} = q_i^{-1}\left(\frac{\varpi_{k,i}}{I_{k,i}\left(\underline{x}_{k,i}\right)}\right)$ where $q_i^{-1}(y) = \left(\frac{1-y^{\xi_i}}{y^{\xi_i}}\right)^{1/\xi_i}$
- $u_{k,i} = \frac{\chi_{k,i}}{\theta_{k,i}q_i\left(\theta_{k,i}\right)\left(1-G_{k,i}\left(\underline{x}_{k,i}\right)\right)+\chi_{k,i}}$

Step 7: Compute $\left\{\widetilde{L}_{k,i}\right\}$

$$\begin{aligned}\widetilde{L}_{k,i} &\equiv L_{k,i}\left(1-u_{k,i}\right)\int_{\underline{x}_{k,i}}^{x_{\max}}\frac{s}{1-G_{k,i}\left(\underline{x}_{k,i}\right)}dG_{k,i}(s)\\ &= L_{k,i}\left(1-u_{k,i}\right)\exp\left(\frac{\sigma_{k,i}^2}{2}\right)\frac{\Phi\left(\sigma_{k,i}-\frac{\ln\underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln\underline{x}_{k,i}}{\sigma_{k,i}}\right)}\end{aligned}$$

Step 8: Compute $\{\widetilde{w}_{k,i}\}$

$$\widetilde{w}_{k,i} = \frac{\gamma_{k,i}Y_{k,i}}{\widetilde{L}_{k,i}}$$

Step 9: Compute $\left\{E_{k,i}^V\right\}$

$$E_{k,i}^V = \widetilde{\kappa}_{k,i}\widetilde{w}_{k,i}\theta_{k,i}u_{k,i}L_{k,i}$$

Step 10: Compute $\left\{E_i^C\right\}$

$$E_i^C = \sum_{k=1}^{K}\gamma_{k,i}Y_{k,i} - \sum_{k=1}^{K}E_{k,i}^V - NX_i$$

Step 11: Compute $\left\{\widetilde{\lambda}_i\right\}$

$$\widetilde{\lambda}_i = \frac{\overline{L}_i}{E_i^C}$$

Step 12: Obtain $\{U_{k,i}\}$.

- Step 12a: Guess $\left\{U_{ki}^0\right\}$

- Step 12b: Compute until convergence

$$U_{k,i}^{g+1} = \zeta_i \log\left(\sum_{\ell=1}^{K} \exp\left\{\frac{-C_{k\ell,i} + b_{\ell,i} + \theta_{\ell,i}\widetilde{\kappa}_{\ell,i}\widetilde{\lambda}_i\widetilde{w}_{\ell,i}\frac{\beta_{\ell,i}}{(1-\beta_{\ell,i})} + \delta U_{\ell,i} - \delta U_{k,i}^{g}}{\zeta_i}\right\}\right) + \delta U_{k,i}^{g}$$

Step 13: Update $\{L_{k,i}\}$.

- Step 13a: Given knowledge of $\{U_{k,i}\}$, compute transition rates $s_{k\ell,i}$.

$$s_{k\ell,i} = \frac{\exp\left\{\frac{-C_{k\ell,i}+b_{\ell,i}+\theta_{\ell,i}\widetilde{\kappa}_{\ell,i}\widetilde{\lambda}_i\widetilde{w}_{\ell,i}\frac{\beta_{\ell,i}}{1-\beta_{\ell,i}}+\delta U_{\ell,i}}{\zeta_i}\right\}}{\sum_{\overline{k}}\exp\left\{\frac{-C_{k\overline{k},i}+b_{\overline{k},i}+\theta_{\overline{k},i}\widetilde{\kappa}_{\overline{k},i}\widetilde{\lambda}_i\widetilde{w}_{\overline{k},i}\frac{\beta_{\overline{k},i}}{1-\beta_{\overline{k},i}}+\delta U_{\overline{k},i}}{\zeta_i}\right\}}$$

- Step 13b: Find y_i such that

$$\left(I - s_i'\right) y_i = 0$$

- Step 13c: Find allocations $L_{k,i}$

$$L_{k,i}u_{k,i} = \varphi y_{k,i}$$

$$\Rightarrow L_{k,i} = \varphi \underbrace{y_{k,i}/u_{k,i}}_{\widetilde{y}_{k,i}}$$

$$\Rightarrow L_i' 1_{K\times 1} = \varphi \widetilde{y}_{k,i}' 1_{K\times 1} = \overline{L}_i$$

$$\Rightarrow \varphi = \frac{\overline{L}_i}{\widetilde{y}_{k,i}' 1_{K\times 1}}$$

$$\left(L_{k,i}\right)' = \varphi \widetilde{y}_{k,i}$$

$$L_{k,i}^{new} = \left(1-\lambda_L\right) L_{k,i} + \lambda_L \left(L_{k,i}\right)'$$

Step 14: Update $\left\{\underline{x}_{k,i}\right\}$.

Note that in equilibrium:

$$\widetilde{\lambda}_i\widetilde{w}_{k,i}\underline{x}_{k,i} = \left(1-\delta\right) U_{k,i} - \eta_{k,i} \tag{A.37}$$

So, we update $\underline{x}_{k,i}$ according to:

$$\left(\underline{x}_{k,i}\right)' = \frac{\left(1-\delta\right) U_{k,i} - \eta_{k,i}}{\widetilde{\lambda}_i\widetilde{w}_{k,i}}$$

$$\underline{x}_{k,i}^{new} = \min\left\{\left(1-\lambda_x\right)\underline{x}_{k,i} + \lambda_x\left(\underline{x}_{k,i}\right)', \underline{x}_{k,i}^{ub}\right\}$$

Step 15: Armed with $L_{k,i}^{new}$ and $\underline{x}_{k,i}^{new}$ go to Step 6 until $\left\|\left\{L_{k,i}^{new} - L_{k,i}\right\}\right\| \to 0$ and $\left\|\left\{\underline{x}_{k,i}^{new} - \underline{x}_{k,i}\right\}\right\| \to 0$.

Note that $\left\|\left\{\underline{x}_{k,i}^{new} - \underline{x}_{k,i}\right\}\right\| \to 0$ does not imply that (A.37) is satisfied. Therefore, we penalize deviations from (A.37) in the objective function.

Step 16: Generate moments, compute Loss Function, guess new parameter set Ω and go to Step 3, until objective function is minimized.

Note: Given that we condition on the trade shares $\pi_{k,oi}^{Data}$, we can estimate the model country by country, separately. However, in practice, we will first estimate the model for the US and obtain all of the US specific parameters. Next, armed with US-specific mobility costs $C_{kk'}$ and sector-specific exogenous exit components χ_k (we will impose $\chi_{k,i} = \chi_i + \chi_k$), we estimate the remaining countries' parameters separately, in parallel.

C.2 Expressions for Simulated Moments

C.2.1 Employment Shares

$$emp_{k,i} = \frac{L_{k,i}\left(1 - u_{k,i}\right)}{\sum_{k=1}^{K} L_{k,i}\left(1 - u_{k,i}\right)}$$

C.2.2 National Unemployment Rate

$$unemp_i = \frac{\sum_{k=1}^{K} L_{k,i} u_{k,i}}{\sum_{k=1}^{K} L_{k,i}}$$

C.2.3 Sector-Specific Average Wages

$$w_{k,i}(x) = \left(1 - \beta_{k,i}\right)\widetilde{w}_{k,i}\underline{x}_{k,i} + \beta_{k,i}\widetilde{w}_{k,i}x$$

$$\begin{aligned}
\overline{w}_{k,i} &= \frac{\int_{\underline{x}_{k,i}}^{x_{\max}} w_{k,i}(s)\, dG_{k,i}(s)}{1 - G_{k,i}\left(\underline{x}_{k,i}\right)} \\
&= \left(1 - \beta_{k,i}\right)\widetilde{w}_{k,i}\underline{x}_{k,i} + \beta_{k,i}\widetilde{w}_{k,i}\int_{\underline{x}_{k,i}}^{x_{\max}} \frac{s}{1 - G_{k,i}\left(\underline{x}_{k,i}\right)} dG_{k,i}(s) \\
&= \left(1 - \beta_{k,i}\right)\widetilde{w}_{k,i}\underline{x}_{k,i} + \beta_{k,i}\widetilde{w}_{k,i}\exp\left(\frac{\sigma_{k,i}^2}{2}\right)\frac{\Phi\left(\sigma_{k,i} - \frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}
\end{aligned}$$

C.2.4 Sector-Specific Variance of Wages

$$\begin{aligned}\sigma_{w,k,i}^2 &= \frac{\int_{\underline{x}_{k,i}}^{\infty}\left(w_{k,i}\left(s\right)-\overline{w}_{k,i}\right)^2 dG_{k,i}\left(s\right)}{1-G_{k,i}\left(\underline{x}_{k,i}\right)} \\ &= \left(\beta_{k,i}\widetilde{w}_{k,i}\right)^2 \times \frac{\int_{\underline{x}_{k,i}}^{\infty}\left(s-\exp\left(\frac{\sigma_{k,i}^2}{2}\right)\frac{\Phi\left(\sigma_{k,i}-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}\right)^2 dG_{k,i}\left(s\right)}{1-G_{k,i}\left(\underline{x}_{k,i}\right)} \\ &= \left(\beta_{k,i}\widetilde{w}_{k,i}\right)^2 \times \left(\exp\left(2\sigma_{k,i}^2\right)\frac{\Phi\left(2\sigma_{k,i}-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}-\exp\left(\sigma_{k,i}^2\right)\left(\frac{\Phi\left(\sigma_{k,i}-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}\right)^2\right)\end{aligned}$$

C.2.5 Transition Rates

Note that the transition rates in equation (10) are transitions from unemployment in sector k to search in sector k' within period t. There are no data counterfactuals for this variable. However, we can construct a matrix with transition rates between all possible (model) states between time t and time $t+N$ (where N is even)—where variables are measured at the t_a stage (which is the production stage). From this matrix, we can obtain N-period transition rates between all states observed in the data (employment in each of the sectors and unconditional unemployment). First, we obtain the one-year transition matrix $\widetilde{s}^{t,t+1}$ between states $\{\widetilde{u}_1,...,\widetilde{u}_K,1,...,K\}$. Here, we abuse notation to mean $\widetilde{u}_k$ as sector-k unemployment at the very beginning of a period.

The one-year transition rate between sector-ℓ unemployment and sector-k unemployment is given by:

$$\widetilde{s}_{\widetilde{u}_\ell\widetilde{u}_k,i}^{t,t+1} = s_{\ell k,i}^{t,t+1}\left(1-\theta_{k,i}^t q_i\left(\theta_{k,i}^t\right)\left(1-G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right)\right), \tag{A.38}$$

that is, a share $s_{\ell k,i}^t$ of individuals starting period t unemployed in sector ℓ choose to search in sector k. A fraction $\left(1-\theta_{k,i}^t q_i\left(\theta_{k,i}^t\right)\left(1-G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right)\right)$ of those do not find a match that survives until $t+1$. Similarly, the one-year transition rate between sector-ℓ unemployment and sector-k employment is given by:

$$\begin{aligned}\widetilde{s}_{\widetilde{u}_\ell k,i}^{t,t+1} &= s_{\ell k,i}^{t,t+1}\theta_{k,i}^t q_i\left(\theta_{k,i}^t\right)\left(1-G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right) \\ &= s_{\ell k,i}^{t,t+1} - \widetilde{s}_{\widetilde{u}_\ell\widetilde{u}_k,i}^{t,t+1}.\end{aligned} \tag{A.39}$$

According to the timing assumptions of the model, the one-year transition rate between employment in sector k and employment in sector k' is zero if $k \neq k'$. However, the persistence rate of employment in sector k is given by the probability that a match does not receive a death shock times the probability that the match is not dissolved because the threshold for production increases in the following period:

$$\widetilde{s}_{kk',i}^{t,t+1} = \begin{cases} 0 \text{ if } k \neq k' \\ \left(1-\chi_{k,i}\right)\Pr\left(x \geq \underline{x}_{k,i}^{t+1} | x \geq \underline{x}_{k,i}^t\right) \text{ if } k = k' \end{cases}. \tag{A.40}$$

Finally, the one-year transition rate between sector-k employment and unemployment in sector ℓ

is given by:

$$\widetilde{s}^{t,t+1}_{k\widetilde{u}_\ell,i} = \begin{cases} 0 \text{ if } k \neq \ell \\ \chi_{k,i} + (1-\chi_{k,i}) \Pr\left(x < \underline{x}^{t+1}_{k,i} | x \geq \underline{x}^{t}_{k,i}\right) \text{ if } k = \ell \end{cases}. \tag{A.41}$$

That is, if a worker is employed in sector k at t, she cannot start next period unemployed in sector ℓ if $k \neq \ell$. Otherwise, workers transition between sector k employment to sector k unemployment if their match is hit with a death shock or if their employer's productivity goes below the threshold for production at $t+1$.

We can now write the N-period transition matrix as:

$$\widetilde{s}^{t,t+N} = \widetilde{s}^{t+k-1,t+k} \times ... \times \widetilde{s}^{t+1,t+2} \times \widetilde{s}^{t,t+1}, \tag{A.42}$$

and we can write transition rates between unemployment $\widetilde{u}$ and sector-k employment between t and $t+N$ as:

$$\widetilde{s}^{t,t+N}_{\widetilde{u},k,i} = \frac{\sum_{\ell=1}^{K} L^{t-1}_{\ell,i} \widetilde{u}^{t-1}_{\ell,i} \widetilde{s}^{t,t+N}_{\widetilde{u}_\ell,k}}{\sum_{\ell=1}^{K} L^{t-1}_{\ell,i} \widetilde{u}^{t-1}_{\ell,i}}. \tag{A.43}$$

Finally, we can write transition rates between sector-k employment and unemployment $\widetilde{u}$ as:

$$\widetilde{s}^{t,t+N}_{k,\widetilde{u},i} = 1 - \sum_{k'=1}^{K} \widetilde{s}^{t,t+N}_{k,k',i}. \tag{A.44}$$

1-period transition rates

$$\widetilde{s}_{\widetilde{u}_\ell \widetilde{u}_k,i} = s_{\ell k,i} \left(1 - \theta_{k,i} q_i\left(\theta_{k,i}\right)\left(1 - G_{k,i}\left(\underline{x}_{k,i}\right)\right)\right)$$

$$\widetilde{s}_{\widetilde{u}_\ell k,i} = s_{\ell k,i} \theta_{k,i} q_i\left(\theta_{k,i}\right)\left(1 - G_{k,i}\left(\underline{x}_{k,i}\right)\right)$$

$$\widetilde{s}_{\ell k,i} = \begin{cases} 0 \text{ if } \ell \neq k \\ (1-\chi_{\ell,i}) \text{ if } \ell = k \end{cases}$$

$$\widetilde{s}_{\ell \widetilde{u}_k,i} = \begin{cases} 0 \text{ if } \ell \neq k \\ \chi_{k,i} \text{ if } \ell = k \end{cases}$$

N-period transition rates from and to unconditional unemployment: $\widetilde{s}^N$

$$\widetilde{s}^{N}_{\widetilde{u},k,i} = \frac{\sum_{\ell=1}^{K} L_{\ell,i} u_{\ell,i} \widetilde{s}^{N}_{\widetilde{u}_\ell,k,i}}{\sum_{\ell=1}^{K} L_{\ell,i} u_{\ell,i}}$$

$$\widetilde{s}^{N}_{k,\widetilde{u},i} = 1 - \sum_{\ell=1}^{K} \widetilde{s}^{N}_{k,\ell,i}.$$

C.3 Algorithm: Steady-State Equilibrium Following Shock

Consider shocks $\left\{A_{k,i}^0\right\} \to \left\{A_{k,i}^1\right\}$, $\left\{d_{k,oi}^0\right\} \to \left\{d_{k,oi}^1\right\}$, $\{NX^0\} \to \{NX_i^1\}$

We will be using 0 superscripts to denote the initial steady state, and 1 superscripts to denote the final steady state.

Start from estimated Steady State: $\left\{L_{k,i}^0\right\}$, $\left\{\underline{x}_{k,i}^0\right\}$, $\left\{\widetilde{w}_{k,i}^0\right\}$, $\left\{\pi_{k,oi}^0\right\}$

Note that $\pi_{k,oi}^0 = \pi_{k,oi}^{Data}$

We also have $\widetilde{\kappa}_{k,i}^0 = \frac{\kappa_{k,i} P_i^{F,0}}{\widetilde{w}_{k,i}^0}$, but we do not know $\left\{P_i^{F,0}\right\}$

Denote relative changes in variable a by $\widehat{a} = \frac{a^1}{a^0}$

Step 1: Guess $\left\{L_{k,i}^1\right\}$ and $\left\{\underline{x}_{k,i}^1\right\}$

Step 2: Guess $\left\{\widetilde{w}_{k,i}^1\right\}$

- Step 2a: Compute $\widehat{\widetilde{w}}_{k,i} = \frac{\widetilde{w}_{k,i}^1}{\widetilde{w}_{k,i}^0}$ and iteratively solve for $\widehat{P}_{k,i}^I$ and $\widehat{c}_{k,i}$ using the system

$$\widehat{c}_{k,i} = \left(\widehat{\widetilde{w}}_{k,i}\right)^{\gamma_{k,i}} \prod_{\ell=1}^{K} \left(\widehat{P}_{\ell,i}^I\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}}$$

$$\widehat{P}_{k,i}^I = \left(\sum_{o=1}^{N} \pi_{k,oi}^0 \widehat{A}_{k,o} \left(\widehat{c}_{k,o}\widehat{d}_{k,oi}\right)^{-\lambda}\right)^{-1/\lambda}$$

- Step 2b: Compute $\widehat{P}_{k,i}^F$:

$$\widehat{P}_i^F = \prod_{k=1}^{K} \left(\widehat{P}_{k,i}^I\right)^{\mu_{ki}}$$

- Step 2c: Compute $\widehat{\pi}_{k,oi}$:

$$\widehat{\pi}_{k,oi} = \widehat{A}_{k,o} \left(\frac{\widehat{c}_{k,o}\widehat{d}_{k,oi}}{\widehat{P}_{k,i}^I}\right)^{-\lambda}$$

- Step 2d: Compute

- $\pi^1_{k,oi} = \pi^0_{k,oi}\widehat{\pi}_{k,oi}$
- $\widetilde{\kappa}^1_{k,i} \equiv \frac{\kappa_{k,i}P^{F,1}_i}{\widetilde{w}^1_{k,i}} = \frac{\kappa_{k,i}P^{F,0}_i}{\widetilde{w}^0_{k,i}}\frac{P^{F,1}_i}{P^{F,0}_i}\frac{\widetilde{w}^0_{k,i}}{\widetilde{w}^1_{k,i}} = \widetilde{\kappa}^0_{k,i}\frac{\widehat{P}^F_i}{\widehat{\widetilde{w}}_{k,i}}$

Step 3: If $\widetilde{\kappa}^1_{k,i} \times \frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}^1_{k,i}\right)} \geq 1$ abort, set $\underline{x}^1_{k,i}$ such that $\widetilde{\kappa}^1_{k,i} \times \frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}^1_{k,i}\right)} = 1-\varepsilon$ and go back to Step 1 with this new guess. If $\widetilde{\kappa}^1_{k,i} \times \frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}^1_{k,i}\right)} < 1$, proceed to Step 4.

Step 4: Compute

$$q_i\left(\theta^1_{k,i}\right) = \widetilde{\kappa}^1_{k,i} \times \frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}^1_{k,i}\right)}$$

$$\theta^1_{k,i} = q_i^{-1}\left(\widetilde{\kappa}^1_{k,i} \times \frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}^1_{k,i}\right)}\right)$$

$$u^1_{k,i} = \frac{\chi_{k,i}}{\theta^1_{k,i}q_i\left(\theta^1_{k,i}\right)\left(1-G_{k,i}\left(\underline{x}^1_{k,i}\right)\right)+\chi_{k,i}}$$

Step 5: Solve system in $\left\{Y^1_{k,o}\right\}$

$$Y^1_{k,o} = \sum_{i=1}^N \pi^1_{k,oi}\left(\mu_{k,i}\left(\sum_{\ell=1}^K \gamma_{\ell,i}Y^1_{\ell,i}\right) + \sum_{\ell=1}^K\left(1-\gamma_{\ell,i}\right)\nu_{\ell k,i}Y^1_{\ell,i}\right) - \sum_{i=1}^N \pi^1_{k,oi}\mu_{k,i}NX^1_i$$

$$\sum_{i=1}^N\sum_{k=1}^K Y^1_{k,i} = 1$$

Step 6: Compute $\left\{\widetilde{L}^1_{k,i}\right\}$

$$\widetilde{L}^1_{k,i} \equiv L^1_{k,i}\left(1-u^1_{k,i}\right)\int_{\underline{x}^1_{k,i}}^{x_{\max}} \frac{s}{1-G_{k,i}\left(\underline{x}^1_{k,i}\right)}dG_{k,i}\left(s\right)$$

$$= L^1_{k,i}\left(1-u^1_{k,i}\right)\exp\left(\frac{\sigma^2_{k,i}}{2}\right)\frac{\Phi\left(\sigma_{k,i}-\frac{\ln \underline{x}^1_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}^1_{k,i}}{\sigma_{k,i}}\right)}$$

Step 7: Update $\left\{\widetilde{w}^1_{k,i}\right\}$

$$\left(\widetilde{w}^1_{k,i}\right)^{new} = \frac{\gamma_{k,i}Y^1_{k,i}}{\widetilde{L}^1_{k,i}}$$

Go back to Step 2a and repeat until converegence of $\left\{\widetilde{w}^1_{k,i}\right\}$.

Step 8: Compute

$$E_{k,i}^{V,1} = \widetilde{\kappa}_{k,i}^{1} \widetilde{w}_{k,i}^{1} \theta_{k,i}^{1} u_{k,i}^{1} L_{k,i}^{1}$$

$$E_{i}^{C,1} = \sum_{k=1}^{K} \gamma_{k,i} Y_{k,i}^{1} - \sum_{k=1}^{K} E_{k,i}^{V,1} - NX_{i}^{1}$$

Step 9: Obtain Lagrange Multipliers

$$\widetilde{\lambda}_{i}^{1} = \frac{\overline{L}_{i}}{E_{i}^{C,1}}$$

Step 10: Compute Bellman Equations

$$U_{k,i}^{1} = \zeta_{i} \log \left(\sum_{k'} \exp \left\{ \frac{-C_{kk',i} + b_{k',i} + \theta_{k',i}^{1} \widetilde{\kappa}_{k',i}^{1} \widetilde{\lambda}_{i}^{1} \widetilde{w}_{k',i}^{1} \frac{\beta_{k',i}}{\left(1-\beta_{k',i}\right)} + \delta U_{k',i}^{1}}{\zeta_{i}} \right\} \right)$$

Step 11: Update $\left\{ L_{k,i}^{1} \right\}$.

- Step 11a: Given knowledge of $\left\{ U_{k,i}^{1} \right\}$, compute transition rates $s_{k\ell,i}^{1}$.

$$s_{k\ell,i}^{1} = \frac{\exp \left\{ \frac{-C_{k\ell,i} + b_{\ell,i} + \theta_{\ell,i}^{1} \widetilde{\kappa}_{\ell,i}^{1} \widetilde{\lambda}_{i}^{1} \widetilde{w}_{\ell,i}^{1} \frac{\beta_{\ell,i}}{\left(1-\beta_{\ell,i}\right)} + \delta U_{\ell,i}^{1}}{\zeta_{i}} \right\}}{\sum_{\overline{k}} \exp \left\{ \frac{-C_{k\overline{k},i} + b_{\overline{k},i} + \theta_{\overline{k},i}^{1} \widetilde{\kappa}_{\overline{k},i}^{1} \widetilde{\lambda}_{i}^{1} \widetilde{w}_{\overline{k},i}^{1} \frac{\beta_{\overline{k},i}}{\left(1-\beta_{\overline{k},i}\right)} + \delta U_{\overline{k},i}^{1}}{\zeta_{i}} \right\}}$$

- Step 11b: Find y_i such that

$$\left(I - \left(s_{i}^{1} \right)^{T} \right) y_{i} = 0$$

- Step 11c: Find allocations $L_{k,i}$

$$L_{k,i}^{1} u_{k,i}^{1} = \varphi y_{k,i}$$

$$\Rightarrow L_{k,i}^{1} = \varphi \underbrace{y_{k,i} / u_{k,i}^{1}}_{\widetilde{y}_{k,i}}$$

$$\Rightarrow \left(L_{i}^{1} \right)^{T} 1_{K \times 1} = \varphi \widetilde{y}_{k,i}^{T} 1_{K \times 1} = \overline{L}_{i}$$

$$\Rightarrow \varphi = \frac{\overline{L}_{i}}{\widetilde{y}_{k,i}^{T} 1_{K \times 1}}$$

$$\left(L_{k,i}^{1}\right)' = \varphi \widetilde{y}_{k,i}$$

$$\left(L_{k,i}^{1}\right)^{new} = \left(1-\lambda_L\right) L_{k,i}^{1} + \lambda_L \left(L_{k,i}^{1}\right)'$$

Step 12: Update $\left\{\underline{x}_{k,i}^{1}\right\}$.

Note that in equilibrium:

$$\widetilde{\lambda}_i^1 \widetilde{w}_{k,i}^1 \underline{x}_{k,i}^1 = \left(1-\delta\right) U_{k,i}^1 - \eta_{k,i}$$

So, we update $\underline{x}_{k,i}^{1}$ according to:

$$\left(\underline{x}_{k,i}^{1}\right)' = \frac{\left(1-\delta\right) U_{k,i}^1 - \eta_{k,i}}{\widetilde{\lambda}_i^1 \widetilde{w}_{k,i}^1}$$

$$\left(\underline{x}_{k,i}^{1}\right)^{new} = \min\left\{\left(1-\lambda_x\right)\underline{x}_{k,i}^{1} + \lambda_x \left(\underline{x}_{k,i}^{1}\right)', \underline{x}_{k,i}^{ub}\right\}$$

Step 13: Armed with $\left(L_{k,i}^{1}\right)^{new}$ and $\left(\underline{x}_{k,i}^{1}\right)^{new}$ go to Step 2 until $\left\|\left\{\left(L_{k,i}^{1}\right)^{new} - L_{k,i}^{1}\right\}\right\| \to 0$ and $\left\|\left\{\left(\underline{x}_{k,i}^{1}\right)^{new} - \underline{x}_{k,i}^{1}\right\}\right\| \to 0$.

C.4 Algorithm: Out-of-Steady-State Transition

Inner Loop: conditional on paths for expenditures $\left\{E_i^{C,t}\right\}$—determined in the Outer Loop below.

Consider paths $\left\{A_{k,i}^t\right\}_{t=0}^{T_{SS}}$ and $\left\{d_{o,i,k}^t\right\}_{t=0}^{T_{SS}}$ with $A_{k,i}^0 = 1$ and $d_{o,i,k}^0 = 1$. Also, consider paths $\left\{\phi_i^t\right\}_{t=0}^{T_{SS}}$ with $\phi_i^0 = 1$ and $\widehat{\phi}_i^t = 1$ for $T \leq t \leq T_{SS}$, for some $T << T_{SS}$.

Step 0: Given paths $\left\{E_i^{C,t}\right\}$, compute paths $\left\{\widetilde{\lambda}_i^t\right\}$: $\widetilde{\lambda}_i^t = \frac{\overline{L}_i}{E_i^{C,t}}$

Step 1: Guess paths $\left\{\widetilde{w}_{k,i}^t\right\}_{t=1}^{T_{SS}}$ for each sector k and country i.

Step 2: Compute $\underline{x}_{k,i}^{T_{SS}}$ consistent with $\widetilde{w}_{k,i}^{T_{SS}}$ and $\widetilde{\lambda}_i^{T_{SS}}$. Obtain $\theta_{k,i}^{T_{SS}}$, $U_{k,i}^{T_{SS}}$, $s_{k\ell,i}^{T_{SS},T_{SS}+1}$ and $\pi_{k,oi}^{T_{SS}}$.

- Step 2a: Compute $\widehat{\widetilde{w}}_{k,i} = \frac{\widetilde{w}_{k,i}^{T_{SS}}}{\widetilde{w}_{k,i}^0}$, $\widehat{A}_{k,i} = \frac{A_{k,i}^{T_{SS}}}{A_{k,i}^0}$ and $\widehat{d}_{k,i} = \frac{d_{o,i,k}^{T_{SS}}}{d_{o,i,k}^0}$. Iteratively solve for $\widehat{P}_{k,i}^I$ and $\widehat{c}_{k,i}$ using the system

$$\widehat{c}_{k,i} = \left(\widehat{\widetilde{w}}_{k,i}\right)^{\gamma_{k,i}} \prod_{\ell=1}^{K} \left(\widehat{P}_{\ell,i}^I\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}}$$

$$\widehat{P}_{k,i}^I = \left(\sum_{o=1}^{N} \pi_{k,oi}^0 \widehat{A}_{k,o} \left(\widehat{c}_{k,o}\widehat{d}_{k,oi}\right)^{-\lambda}\right)^{-1/\lambda}$$

- Step 2b: Compute $\widehat{P}_{k,i}^F$:

$$\widehat{P}_i^F = \prod_{k=1}^{K} \left(\widehat{P}_{k,i}^I\right)^{\mu_{ki}}$$

- Step 2c: Compute

$$\widehat{\pi}_{k,oi} = \widehat{A}_{k,o} \left(\frac{\widehat{c}_{k,o}\widehat{d}_{k,oi}}{\widehat{P}_{k,i}^I}\right)^{-\lambda}$$

  And obtain $\pi_{k,oi}^{T_{SS}} = \pi_{k,oi}^0 \widehat{\pi}_{k,oi}$

- Step 2d: Compute
  - $\widetilde{\kappa}_{k,i}^{T_{SS}} = \widetilde{\kappa}_{k,i}^0 \frac{\widehat{P}_i^F}{\widehat{\widetilde{w}}_{k,i}}$

- Step 2e: Guess $\left\{\underline{x}_{k,i}^{T_{SS}}\right\}$

- Step 2f: Compute

$$\theta_{k,i}^{T_{SS}} = q_i^{-1} \left(\widetilde{\kappa}_{k,i}^{T_{SS}} \times \frac{1 - \delta\left(1 - \chi_{k,i}\right)}{\delta\left(1 - \beta_{k,i}\right) I_{k,i}\left(\underline{x}_{k,i}^{T_{SS}}\right)}\right)$$

- Step 2g: Compute Bellman Equations

$$U_{k,i}^{T_{SS}} = \zeta_i \log\left(\sum_{k'} \exp\left\{\frac{-C_{kk',i} + b_{k',i} + \theta_{k',i}^{T_{SS}} \widetilde{\kappa}_{k',i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{k',i}^{T_{SS}} \frac{\beta_{k',i}}{(1-\beta_{k',i})} + \delta U_{k',i}^{T_{SS}}}{\zeta_i}\right\}\right)$$

- Step 2h: Compute

$$\left(\underline{x}_{k,i}^{T_{SS}}\right)' = \frac{(1-\delta)\, U_{k,i}^{T_{SS}} - \eta_{k,i}}{\widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{k,i}^{T_{SS}}}$$

- Step 2i: Update $\underline{x}_{k,i}^{T_{SS}} = (1-\lambda_x)\, \underline{x}_{k,i}^{T_{SS}} + \lambda_x \left(\underline{x}_{k,i}^{T_{SS}}\right)'$ and go back to Step 2d until convergence.

- Step 2j: Compute $s_{kk'}^{T_{SS}, T_{SS}+1}$

$$s_{k\ell,i}^{T_{SS},T_{SS}+1} = \frac{\exp\left\{\frac{-C_{k\ell,i} + b_{\ell,i} + \theta_{\ell,i}^{T_{SS}} \widetilde{\kappa}_{\ell,i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{\ell,i}^{T_{SS}} \frac{\beta_{\ell,i}}{1-\beta_{\ell,i}} + \delta U_{\ell,i}^{T_{SS}}}{\zeta_i}\right\}}{\sum_{\overline{k}} \exp\left\{\frac{-C_{k\overline{k},i} + b_{\overline{k},i} + \theta_{\overline{k},i}^{T_{SS}} \widetilde{\kappa}_{\overline{k},i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{\overline{k},i}^{T_{SS}} \frac{\beta_{\overline{k},i}}{1-\beta_{\overline{k},i}} + \delta U_{\overline{k},i}^{T_{SS}}}{\zeta_i}\right\}}$$

Step 3: Obtain series $\left\{\pi_{k,oi}^t\right\}_{t=0}^{T_{SS}}$, $\left\{\widetilde{\kappa}_{k,i}^t\right\}_{t=0}^{T_{SS}}$. Define $\widehat{x}^t \equiv \frac{x^t}{x^0}$.

- Step 3a: For $t = 1, ..., T_{SS} - 1$ compute $\widehat{\widetilde{w}}_{k,i}^t = \frac{\widetilde{w}_{k,i}^t}{\widetilde{w}_{k,i}^0}$ and iteratively solve for $\widehat{P}_{k,i}^{I,t}$ and $\widehat{c}_{k,i}^t$ using the system

$$\widehat{c}_{k,i}^t = \left(\widehat{\widetilde{w}}_{k,i}^t\right)^{\gamma_{k,i}} \prod_{\ell=1}^{K} \left(\widehat{P}_{\ell,i}^{I,t}\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}}$$

$$\widehat{P}_{k,i}^{I,t} = \left(\sum_{o=1}^{N} \pi_{k,oi}^0 \widehat{A}_{k,o}^t \left(\widehat{c}_{k,o}^t \widehat{d}_{k,oi}^t\right)^{-\lambda}\right)^{-1/\lambda}$$

- Step 3b: Compute $\widehat{P}_{k,i}^{F,t}$ for $t = 1, ..., T_{SS} - 1$:

$$\widehat{P}_i^{F,t} = \prod_{k=1}^{K} \left(\widehat{P}_{k,i}^{I,t}\right)^{\mu_{ki}}$$

- Step 3c: Compute $\widehat{\pi}_{k,oi}^t$ for $t = 1, ..., T_{SS} - 1$:

$$\widehat{\pi}_{k,oi}^t = \widehat{A}_{k,o}^t \left(\frac{\widehat{c}_{k,o}^t \widehat{d}_{k,oi}^t}{\widehat{P}_{k,i}^{I,t}}\right)^{-\lambda}$$

- Step 3d: Compute or $t = 1, ..., T_{SS} - 1$:

- $\pi^t_{k,oi} = \pi^0_{k,oi}\widehat{\pi}^t_{k,oi}$
- $\widetilde{\kappa}^t_{k,i} \equiv \frac{\kappa_{k,i}P^{F,t}_i}{\widetilde{w}^t_{k,i}} = \frac{\kappa_{k,i}P^{F,0}_i}{\widetilde{w}^0_{k,i}}\frac{P^{F,t}_i}{P^{F,0}_i}\frac{\widetilde{w}^0_{k,i}}{\widetilde{w}^t_{k,i}} = \widetilde{\kappa}^0_{k,i}\frac{\widehat{P}^{F,t}_i}{\widehat{\widetilde{w}}^t_{k,i}}$

Step 4: Given knowledge of $\widetilde{w}^{T_{SS}}_{k,i}$, $\widetilde{\lambda}^{T_{SS}}_i$ and $\underline{x}^{T_{SS}}_{k,i}$ (and therefore $J^{T_{SS}}_{k,i}(s)$), start at $t = T_{SS}-1$ and sequentially compute (backwards) for each $t = T_{SS}-1, ..., 1$

- Step 4a: Given $\widetilde{w}^t_{k,i}$, $\underline{x}^{t+1}_{k,i}$, $\widetilde{\kappa}^t_{k,i}$, $\widetilde{\lambda}^t_i$ and $J^{t+1}_{k,i}(s)$ compute $\theta^t_{k,i}$.

If $\frac{\widetilde{\lambda}^t_i\widetilde{\kappa}^t_{k,i}\widetilde{w}^t_{k,i}}{\delta\widehat{\phi}^{t+1}_i\int^{x^{\max}}_{\underline{x}^{t+1}_{k,i}}J^{t+1}_{k,i}(s)dG_{k,i}(s)} \leq 1$ then

$$\theta^t_{k,i} = q_i^{-1}\left(\frac{\widetilde{\lambda}^t_i\widetilde{\kappa}^t_{k,i}\widetilde{w}^t_{k,i}}{\delta\widehat{\phi}^{t+1}_i\int^{x^{\max}}_{\underline{x}^{t+1}_{k,i}}J^{t+1}_{k,i}(s)\,dG_{k,i}(s)}\right)$$

If $\frac{\widetilde{\lambda}^t_i\widetilde{\kappa}^t_{k,i}\widetilde{w}^t_{k,i}}{\delta\widehat{\phi}^{t+1}_i\int^{x^{\max}}_{\underline{x}^{t+1}_{k,i}}J^{t+1}_{k,i}(s)dG_{k,i}(s)} > 1$, it is not possible to satisfy $V^t_{k,i} = 0$, so that $V^t_{k,i} < 0$ and

$$\theta^t_{k,i} = 0.$$

- Step 4b: Given $\underline{x}^{t+1}_{k,i}$, $W^{t+1}_{k,i}(x) = \frac{\beta_{k,i}}{1-\beta_{k,i}}J^{t+1}_{k,i}(x) + U^{t+1}_{k,i}$ (for $x \geq \underline{x}^{t+1}_{k,i}$), $\theta^t_{k,i}$, $U^{t+1}_{k,i}$ compute $U^t_{k,i}$. Notice that $\int^{x^{\max}}_{\underline{x}^{t+1}_{k,i}}W^{t+1}_{k,i}(s)\,dG_{k,i}(s) = \frac{\beta_{k,i}}{1-\beta_{k,i}}\int^{x^{\max}}_{\underline{x}^{t+1}_{k,i}}J^{t+1}_{k,i}(s)\,dG_{k,i}(s) + \left(1-G_{k,i}\left(\underline{x}^{t+1}_{k,i}\right)\right)U^{t+1}_{k,i}$ so that:

$$U^t_{k,i} = \zeta_i\log\left(\sum_{k'}\exp\left\{\frac{\begin{array}{c}-C_{kk',i}+b_{k',i}\\ +\delta\widehat{\phi}^{t+1}_i\theta^t_{k',i}q_i\left(\theta^t_{k',i}\right)\frac{\beta_{k,i}}{1-\beta_{k,i}}\int^{x^{\max}}_{\underline{x}^{t+1}_{k,i}}J^{t+1}_{k,i}(s)\,dG_{k,i}(s) + \delta\widehat{\phi}^{t+1}_i U^{t+1}_{k',i}\end{array}}{\zeta_i}\right\}\right)$$

- Step 4c: Given $J^{t+1}_{k,i}(x)$, $\widetilde{w}^t_{k,i}$, $U^t_{k,i}$, $U^{t+1}_{k,i}$ and $\underline{x}^{t+1}_{k,i}$ compute $J^t_{k,i}(x)$

$$\begin{aligned}J^t_{k,i}(x) &= (1-\beta_{k,i})\widetilde{\lambda}^t_i\widetilde{w}^t_{k,i}x + (1-\beta_{k,i})\eta_{k,i}\\ &\quad-(1-\beta_{k,i})\left(U^t_{k,i}-\delta\widehat{\phi}^{t+1}_i U^{t+1}_{k,i}\right) + (1-\chi_{k,i})\delta\widehat{\phi}^{t+1}_i\max\left\{J^{t+1}_{k,i}(x),0\right\}\end{aligned}$$

- Step 4d: Solve for $\underline{x}^t_{k,i}$: $J^t_{k,i}\left(\underline{x}^t_{k,i}\right) = 0$

Step 5: Compute transition rates $\left\{s^{t,t+1}_{kk',i}\right\}_{t=1}^{T_{SS}-1}$ for all countries i according to:

$$s^{t,t+1}_{kk',i} = \frac{\exp\left\{\begin{array}{c}-C_{kk',i}+b_{k',i}+\\ \delta\widehat{\phi}^{t+1}_i\theta^t_{k',i}q(\theta^t_{k',i})\frac{\beta_{k',i}}{1-\beta_{k',i}}\int^{x^{\max}}_{\underline{x}^{t+1}_{k',i}}J^{t+1}_{k',i}(x)\,dG_{k',i}(x) + \delta\widehat{\phi}^{t+1}_i U^{t+1}_{k',i}\end{array}\right\}}{\sum_{k''}\exp\left\{\begin{array}{c}-C_{kk'',i}+b_{k'',i}+\\ \delta\widehat{\phi}^{t+1}_i\theta^t_{k'',i}q(\theta^t_{k'',i})\frac{\beta_{k'',i}}{1-\beta_{k'',i}}\int^{x^{\max}}_{\underline{x}^{t+1}_{k'',i}}J^{t+1}_{k'',i}(x)\,dG_{k'',i}(x) + \delta\widehat{\phi}^{t+1}_i U^{t+1}_{k'',i}\end{array}\right\}}.$$

Step 6: Start loop over t going forward ($t = 0$ to $t = T_{SS} - 1$)

Initial conditions: we know $\widetilde{u}_{k,i}^{t=-1} = u_{k,i}^{t=0}$, $L_{k,i}^{t=-1} = L_{k,i}^{t=0}$, and $\theta_{k,i}^{t=0}$ from the initial steady state computation. Obtain $\widetilde{u}_{k,i}^{t}$ and $L_{k,i}^{t}$ using flow conditions and sequences $\left\{\theta_{k,i}^{t}\right\}$, $\left\{\underline{x}_{k,i}^{t}\right\}$.

- Step 6a: Compute

$$JC_{k,i}^{t} = L_{k,i}^{t} u_{k,i}^{t} \theta_{k,i}^{t} q_i\left(\theta_{k,i}^{t}\right)\left(1 - G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right)$$

$$JD_{k,i}^{t} = \left(\chi_{k,i} + (1 - \chi_{k,i}) \max\left\{\frac{G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right) - G_{k,i}\left(\underline{x}_{k,i}^{t}\right)}{1 - G_{k,i}\left(\underline{x}_{k,i}^{t}\right)}, 0\right\}\right) L_{k,i}^{t-1}\left(1 - \widetilde{u}_{k,i}^{t-1}\right)$$

$$\widetilde{u}_{k,i}^{t} = \frac{L_{k,i}^{t} u_{k,i}^{t} - JC_{k,i}^{t} + JD_{k,i}^{t}}{L_{k,i}^{t}}$$

- Step 6b: Compute

$$L_{k,i}^{t+1} = L_{k,i}^{t} + IF_{k,i}^{t+1} - OF_{k,i}^{t+1},$$

where

$$IF_{k,i}^{t+1} = \sum_{\ell \neq k} L_{\ell,i}^{t} \widetilde{u}_{\ell,i}^{t} s_{\ell k,i}^{t+1,t+2},$$

and

$$OF_{k,i}^{t+1} = L_{k,i}^{t} \widetilde{u}_{k,i}^{t}\left(1 - s_{kk,i}^{t+1,t+2}\right).$$

- Step 6c: Compute

$$u_{k,i}^{t+1} = \frac{\sum_{\ell=1}^{K} L_{\ell,i}^{t} \widetilde{u}_{\ell,i}^{t} s_{\ell k,i}^{t+1,t+2}}{L_{k,i}^{t+1}}$$

- Step 6d: Compute

$$\begin{aligned}\widetilde{L}_{k,i}^{t+1} &= L_{k,i}^{t}\left(1 - \widetilde{u}_{k,i}^{t}\right) \int_{\underline{x}_{k,i}^{t+1}}^{\infty} \frac{s}{1 - G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)} dG_{k,i}(s) \\ &= L_{k,i}^{t}\left(1 - \widetilde{u}_{k,i}^{t}\right) \exp\left(\frac{\sigma_{k,i}^{2}}{2}\right) \frac{\Phi\left(\sigma_{k,i} - \frac{\ln \underline{x}_{k,i}^{t+1}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}^{t+1}}{\sigma_{k,i}}\right)}\end{aligned}$$

- Step 6e: Compute expenditure with vacancies

$$E_{k,i}^{V,t+1} = \widetilde{\kappa}_{k,i}^{t+1} \widetilde{w}_{k,i}^{t+1} \theta_{k,i}^{t+1} u_{k,i}^{t+1} L_{k,i}^{t+1}$$

- Step 6f: Solve for $\left\{Y_{k,i}^{t+1}\right\}$ in the system

$$E_{k,i}^{t+1} = \mu_{k,i} E_i^{C,t+1} + \sum_{\ell=1}^{K} \left(\mu_{k,i} E_{\ell,i}^{V,t+1} + (1 - \gamma_{\ell,i}) \nu_{\ell k,i} Y_{\ell,i}^{t+1} \right).$$

$$Y_{k,o}^{t+1} = \sum_{i=1}^{N} \pi_{k,oi}^{t+1} E_{k,i}^{t+1}.$$

- Step 6g: Compute $\left(\widetilde{w}_{k,i}^{t+1}\right)' = \frac{\gamma_{k,i} Y_{k,i}^{t+1}}{\widetilde{L}_{k,i}^{t+1}}$

Step 7: Compute distance $dist\left(\left\{\widetilde{w}_{k,i}^{t}\right\}, \left\{\left(\widetilde{w}_{k,i}^{t}\right)'\right\}\right)$

- Step 7b: Update $\widetilde{w}_{k,i}^{t} = (1 - \lambda_w)\, \widetilde{w}_{k,i}^{t} + \lambda_w \left(\widetilde{w}_{k,i}^{t}\right)'$ $t = 1, ..., T_{SS}$
- Step 7c: At this point, we have a new series for $\left\{\widetilde{w}_{k,i}^{t}\right\}$ – go back to Step 2 until convergence of $\left\{\widetilde{w}_{k,i}^{t}\right\}$.

Step 8: Compute disposable income $\left\{I_i^t\right\}_{t=1}^{T_{SS}}$

$$I_i^t = \sum_{\ell=1}^{K} \left(\gamma_{\ell,i} Y_{\ell,i}^{t} - E_{\ell,i}^{V,t} \right)$$

Outer Loop: iteration on $\left\{NX_i^t\right\}$

Step 0: Impose a change in a subset of parameters that happens at $t = 0$, but between t_c and t_d. That is, the shock occurs **after** production, workers' decisions of where to search and after firms post vacancies at $t = 0$. Impose a large value for T_{SS}. Assume that for $t \geq T_{SS}$ the system will have converged to a new steady state. World expenditure with final goods $\sum_{i=1}^{I} E_i^{C,t}$ is normalized to 1 for every t.

Step 1: Start with estimated state equilibrium at $t = 0$. Remember that we used the normalization $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$ during the estimation procedure. **Change the normalization** from $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$ to $\sum_{i=1}^{I} E_i^C = 1$. Nominal variables to be renormalized: $\left\{Y_{k,i}^0\right\}$, $\left\{\widetilde{w}_{k,i}^0\right\}$, $\left\{E_i^{C,0}\right\}$, $\left\{NX_i^0\right\}$.

Step 2: Obtain B_i^0 with respect to the normalization $\sum_{i=1}^{I} E_i^C = 1$. Equation (XX) gives us:

$$B_i^0 = \frac{NX_i^0}{\left(1 - \frac{1}{\delta}\right)}$$

Step 3: Make initial guess for $NX_i^{T_{SS}}$ (with respect to the normalization $\sum_{i=1}^{I} E_i^C = 1$).

Step 4: Compute steady state equilibrium at T_{SS}, conditional on $NX_i^{T_{SS}}$, and the change in parameter values.

- Step 4a: Notice that steady algorithm uses the normalization $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$. Normalize $NX_i^{T_{SS}}$ with respect to normalization $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$. To perform such normalization, use output $\left\{Y_{k,i}^{T_{SS}}\right\}$ obtained in the initial steady state if this is the first outer loop iteration, otherwise use output $\left\{Y_{k,i}^{T_{SS}}\right\}$ obtained in Step 6 below.

- Step 4b: After computing the final steady state, **change the normalization** from $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$ to $\sum_{i=1}^{I} E_i^C = 1$ using $\left\{E_i^C\right\}$ obtained in Step 3a. Nominal variables to be renormalized: $\left\{Y_{k,i}^{T_{SS}}\right\}$, $\left\{\widetilde{w}_{k,i}^{T_{SS}}\right\}$, $\left\{E_i^{C,T_{SS}}\right\}$, $\left\{NX_i^{T_{SS}}\right\}$.

Step 5: Start at $t = T_{SS} - 1$ and go backward until $t = 1$ and sequentially compute:

$$R^{t+1} = \frac{1}{\delta}\frac{\sum_{i=1}^{N}\frac{E_i^{C,t+1}}{\widehat{\phi}_i^{t+1}}}{\sum_{i=1}^{N} E_i^{C,t}} = \frac{1}{\delta}\sum_{i=1}^{N}\frac{E_i^{C,t+1}}{\widehat{\phi}_i^{t+1}},$$

$$E_i^{C,t} = \frac{E_i^{C,t+1}}{\delta\widehat{\phi}_i^{t+1}R^{t+1}}$$

to obtain paths for $\left\{R^t\right\}$ and $\left\{E_i^{C,t}\right\}$. Note that, because B_i^1 is decided at $t = 0$, before the shock, $R^1 = R^0 = \frac{1}{\delta}$.

Step 6: Solve for the out-of-steady-state dynamics conditional on aggregate expenditures $\left\{E_i^{C,t}\right\}$.

Step 7: Using the path for disposable income $\left\{I_i^t\right\}_{t=1}^{T_{SS}}$ obtained in Step 6 and equation (7) compute:

$$\left(NX_i^t\right)' = I_i^t - E^{C,t} \text{ for } 1 \leq t < T_{SS}$$

$$\left(NX_i^{T_{SS}}\right)' = -\frac{1-\delta}{\delta}\frac{1}{\left(\prod_{\tau=1}^{T_{SS}-1}(R^\tau)^{-1}\right)}\left(B_i^0 + \sum_{t=1}^{T_{SS}-1}\left(\prod_{\tau=1}^{t}(R^\tau)^{-1}\right)\left(NX_i^t\right)'\right)$$

Step 8: Compute

$$dist\left(\left\{NX_i^{T_{SS}}\right\}, \left\{\left(NX_i^{T_{SS}}\right)'\right\}\right)$$

Step 9: Update $NX_i^{T_{SS}}$

$$NX_i^{T_{SS}} = (1 - \lambda_o)\, NX_i^{T_{SS}} + \lambda_o \left(NX_i^{T_{SS}} \right)'$$

Go back to Step 4 until convergence of $\left\{ NX_i^{T_{SS}} \right\}$.

C.5 Algorithm: Out-of-Steady-State Transition, Exogenous Deficits (No Bonds)

Consider paths $\left\{A_{k,i}^t\right\}_{t=0}^{T_{SS}}$ and $\left\{d_{o,i,k}^t\right\}_{t=0}^{T_{SS}}$ with $A_{k,i}^0 = 1$ and $d_{o,i,k}^0 = 1$. Also, consider paths $\left\{\phi_i^t\right\}_{t=0}^{T_{SS}}$ with $\phi_i^0 = 1$ and $\widehat{\phi}_i^t = 1$ for $T \leq t \leq T_{SS}$, for some $T << T_{SS}$.

We condition on an exogenous path for $\left\{NX_i^t\right\}_{t=1}^{T_{SS}}$.

Step 1: Guess paths $\left\{\widetilde{\lambda}_i^t\right\}_{t=1}^{T_{SS}}$ for each country i.

Step 2: Guess paths $\left\{\widetilde{w}_{k,i}^t\right\}_{t=1}^{T_{SS}}$ for each sector k and country i.

Step 3: Compute $\underline{x}_{k,i}^{T_{SS}}$ consistent with $\widetilde{w}_{k,i}^{T_{SS}}$ and $\widetilde{\lambda}_i^{T_{SS}}$. Obtain $\theta_{k,i}^{T_{SS}}$, $U_{k,i}^{T_{SS}}$, $s_{k\ell,i}^{T_{SS},T_{SS}+1}$ and $\pi_{k,oi}^{T_{SS}}$.

- Step 3a: Compute $\widehat{\widetilde{w}}_{k,i} = \frac{\widetilde{w}_{k,i}^{T_{SS}}}{\widetilde{w}_{k,i}^0}$, $\widehat{A}_{k,i} = \frac{A_{k,i}^{T_{SS}}}{A_{k,i}^0}$ and $\widehat{d}_{k,i} = \frac{d_{o,i,k}^{T_{SS}}}{d_{o,i,k}^0}$. Iteratively solve for $\widehat{P}_{k,i}^I$ and $\widehat{c}_{k,i}$ using the system

$$\widehat{c}_{k,i} = \left(\widehat{\widetilde{w}}_{k,i}\right)^{\gamma_{k,i}} \prod_{\ell=1}^{K} \left(\widehat{P}_{\ell,i}^I\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}}$$

$$\widehat{P}_{k,i}^I = \left(\sum_{o=1}^{N} \pi_{k,oi}^0 \widehat{A}_{k,o} \left(\widehat{c}_{k,o}\widehat{d}_{k,oi}\right)^{-\lambda}\right)^{-1/\lambda}$$

- Step 3b: Compute $\widehat{P}_{k,i}^F$:

$$\widehat{P}_i^F = \prod_{k=1}^{K} \left(\widehat{P}_{k,i}^I\right)^{\mu_{ki}}$$

- Step 3c: Compute

$$\widehat{\pi}_{k,oi} = \widehat{A}_{k,o} \left(\frac{\widehat{c}_{k,o}\widehat{d}_{k,oi}}{\widehat{P}_{k,i}^I}\right)^{-\lambda},$$

  and obtain $\pi_{k,oi}^{T_{SS}} = \pi_{k,oi}^0 \widehat{\pi}_{k,oi}$

- Step 3d: Compute

  - $\widetilde{\kappa}_{k,i}^{T_{SS}} = \widetilde{\kappa}_{k,i}^0 \frac{\widehat{P}_i^F}{\widehat{\widetilde{w}}_{k,i}}$

- Step 3e: Guess $\left\{\underline{x}_{k,i}^{T_{SS}}\right\}$

- Step 3f: Compute

$$\theta_{k,i}^{T_{SS}} = q_i^{-1} \left(\widetilde{\kappa}_{k,i}^{T_{SS}} \times \frac{1 - \delta\left(1 - \chi_{k,i}\right)}{\delta\left(1 - \beta_{k,i}\right) I_{k,i}\left(\underline{x}_{k,i}^{T_{SS}}\right)}\right)$$

- Step 3g: Compute Bellman Equations

$$U_{k,i}^{T_{SS}} = \zeta_i \log\left(\sum_{k'} \exp\left\{\frac{-C_{kk',i} + b_{k',i} + \theta_{k',i}^{T_{SS}} \widetilde{\kappa}_{k',i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{k',i}^{T_{SS}} \frac{\beta_{k',i}}{(1-\beta_{k',i})} + \delta U_{k',i}^{T_{SS}}}{\zeta_i}\right\}\right)$$

- Step 3h: Compute

$$\left(\underline{x}_{k,i}^{T_{SS}}\right)' = \frac{(1-\delta)\, U_{k,i}^{T_{SS}} - \eta_{k,i}}{\widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{k,i}^{T_{SS}}}$$

- Step 3i: Update $\underline{x}_{k,i}^{T_{SS}} = (1-\lambda_x)\, \underline{x}_{k,i}^{T_{SS}} + \lambda_x \left(\underline{x}_{k,i}^{T_{SS}}\right)'$ and go back to Step 2d until convergence.

- Step 3j: Compute $s_{kk'}^{T_{SS}, T_{SS}+1}$

$$s_{k\ell,i}^{T_{SS},T_{SS}+1} = \frac{\exp\left\{\frac{-C_{k\ell,i} + b_{\ell,i} + \theta_{\ell,i}^{T_{SS}} \widetilde{\kappa}_{\ell,i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{\ell,i}^{T_{SS}} \frac{\beta_{\ell,i}}{1-\beta_{\ell,i}} + \delta U_{\ell,i}^{T_{SS}}}{\zeta_i}\right\}}{\sum_{\overline{k}} \exp\left\{\frac{-C_{k\overline{k},i} + b_{\overline{k},i} + \theta_{\overline{k},i}^{T_{SS}} \widetilde{\kappa}_{\overline{k},i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{\overline{k},i}^{T_{SS}} \frac{\beta_{\overline{k},i}}{1-\beta_{\overline{k},i}} + \delta U_{\overline{k},i}^{T_{SS}}}{\zeta_i}\right\}}$$

Step 4: Obtain series $\left\{\pi_{k,oi}^t\right\}_{t=0}^{T_{SS}}$, $\left\{\widetilde{\kappa}_{k,i}^t\right\}_{t=0}^{T_{SS}}$. Define $\widehat{x}^t \equiv \frac{x^t}{x^0}$.

- Step 4a: For $t = 1, ..., T_{SS}-1$ compute $\widehat{\widetilde{w}}_{k,i}^t = \frac{\widetilde{w}_{k,i}^t}{\widetilde{w}_{k,i}^0}$ and iteratively solve for $\widehat{P}_{k,i}^{I,t}$ and $\widehat{c}_{k,i}^t$ using the system

$$\widehat{c}_{k,i}^t = \left(\widehat{\widetilde{w}}_{k,i}^t\right)^{\gamma_{k,i}} \prod_{\ell=1}^{K} \left(\widehat{P}_{\ell,i}^{I,t}\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}}$$

$$\widehat{P}_{k,i}^{I,t} = \left(\sum_{o=1}^{N} \pi_{k,oi}^0 \widehat{A}_{k,o}^t \left(\widehat{c}_{k,o}^t \widehat{d}_{k,oi}^t\right)^{-\lambda}\right)^{-1/\lambda}$$

- Step 4b: Compute $\widehat{P}_{k,i}^{F,t}$ for $t = 1, ..., T_{SS}-1$:

$$\widehat{P}_i^{F,t} = \prod_{k=1}^{K} \left(\widehat{P}_{k,i}^{I,t}\right)^{\mu_{ki}}$$

- Step 4c: Compute $\widehat{\pi}_{k,oi}^t$ for $t = 1, ..., T_{SS}-1$:

$$\widehat{\pi}_{k,oi}^t = \widehat{A}_{k,o}^t \left(\frac{\widehat{c}_{k,o}^t \widehat{d}_{k,oi}^t}{\widehat{P}_{k,i}^{I,t}}\right)^{-\lambda}$$

- Step 4d: Compute or $t = 1, ..., T_{SS}-1$:

  - $\pi_{k,oi}^t = \pi_{k,oi}^0 \widehat{\pi}_{k,oi}^t$

– $\widetilde{\kappa}_{k,i}^{t} \equiv \frac{\kappa_{k,i} P_i^{F,t}}{\widetilde{w}_{k,i}^{t}} = \frac{\kappa_{k,i} P_i^{F,0}}{\widetilde{w}_{k,i}^{0}} \frac{P_i^{F,t}}{P_i^{F,0}} \frac{\widetilde{w}_{k,i}^{0}}{\widetilde{w}_{k,i}^{t}} = \widetilde{\kappa}_{k,i}^{0} \frac{\widehat{P}_i^{F,t}}{\widehat{\widetilde{w}}_{k,i}^{t}}$

Step 5: Given knowledge of $\widetilde{w}_{k,i}^{T_{SS}}$, $\widetilde{\lambda}_i^{T_{SS}}$ and $\underline{x}_{k,i}^{T_{SS}}$ (and therefore $J_{k,i}^{T_{SS}}(s)$), start at $t = T_{SS} - 1$ and sequentially compute (backwards) for each $t = T_{SS} - 1, ..., 1$

- Step 5a: Given $\widetilde{w}_{k,i}^{t}$, $\widetilde{\lambda}_i^{t}$, $\underline{x}_{k,i}^{t+1}$, $\widetilde{\kappa}_{k,i}^{t}$ and $J_{k,i}^{t+1}(s)$ compute $\theta_{k,i}^{t}$.

If $\frac{\widetilde{\lambda}_i^t \widetilde{\kappa}_{k,i}^t \widetilde{w}_{k,i}^t}{\delta \widehat{\phi}_i^{t+1} \int_{\underline{x}_{k,i}^{t+1}}^{x^{\max}} J_{k,i}^{t+1}(s) dG_{k,i}(s)} \leq 1$ then

$$\theta_{k,i}^{t} = q_i^{-1} \left(\frac{\widetilde{\lambda}_i^t \widetilde{\kappa}_{k,i}^t \widetilde{w}_{k,i}^t}{\delta \widehat{\phi}_i^{t+1} \int_{\underline{x}_{k,i}^{t+1}}^{x^{\max}} J_{k,i}^{t+1}(s)\, dG_{k,i}(s)} \right)$$

If $\frac{\widetilde{\lambda}_i^t \widetilde{\kappa}_{k,i}^t \widetilde{w}_{k,i}^t}{\delta \widehat{\phi}_i^{t+1} \int_{\underline{x}_{k,i}^{t+1}}^{x^{\max}} J_{k,i}^{t+1}(s) dG_{k,i}(s)} > 1$, it is not possible to satisfy $V_{k,i}^{t} = 0$, so that $V_{k,i}^{t} < 0$ and

$$\theta_{k,i}^{t} = 0.$$

- Step 5b: Given $\underline{x}_{k,i}^{t+1}$, $W_{k,i}^{t+1}(x) = \frac{\beta_{k,i}}{1-\beta_{k,i}} J_{k,i}^{t+1}(x) + U_{k,i}^{t+1}$ (for $x \geq \underline{x}_{k,i}^{t+1}$), $\theta_{k,i}^{t}$, $U_{k,i}^{t+1}$ compute $U_{k,i}^{t}$. Notice that $\int_{\underline{x}_{k,i}^{t+1}}^{x^{\max}} W_{k,i}^{t+1}(s)\, dG_{k,i}(s) = \frac{\beta_{k,i}}{1-\beta_{k,i}} \int_{\underline{x}_{k,i}^{t+1}}^{x^{\max}} J_{k,i}^{t+1}(s)\, dG_{k,i}(s) + \left(1 - G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right) U_{k,i}^{t+1}$ so that:

$$U_{k,i}^{t} = \zeta_i \log \left(\sum_{k'} \exp \left\{ \frac{\begin{array}{c} -C_{kk',i} + b_{k',i} \\ +\delta \widehat{\phi}_i^{t+1} \theta_{k',i}^{t} q_i \left(\theta_{k',i}^{t}\right) \frac{\beta_{k,i}}{1-\beta_{k,i}} \int_{\underline{x}_{k,i}^{t+1}}^{x^{\max}} J_{k,i}^{t+1}(s)\, dG_{k,i}(s) + \delta \widehat{\phi}_i^{t+1} U_{k',i}^{t+1} \end{array}}{\zeta_i} \right\} \right)$$

- Step 5c: Given $\widetilde{\lambda}_i^{t}$, $J_{k,i}^{t+1}(x)$, $\widetilde{w}_{k,i}^{t}$, $\theta_{k,i}^{t}$, δ, $U_{k,i}^{t}$, $U_{k,i}^{t+1}$ and $\underline{x}_{k,i}^{t+1}$ compute $J_{k,i}^{t}(x)$

$$\begin{aligned} J_{k,i}^{t}(x) = {} & (1-\beta_{k,i}) \widetilde{\lambda}_i^t \widetilde{w}_{k,i}^t x + (1-\beta_{k,i}) \eta_{k,i} \\ & - (1-\beta_{k,i}) \left(U_{k,i}^{t} - \delta \widehat{\phi}_i^{t+1} U_{k,i}^{t+1} \right) + (1 - \chi_{k,i}) \delta \widehat{\phi}_i^{t+1} \max \left\{ J_{k,i}^{t+1}(x), 0 \right\} \end{aligned}$$

- Step 5d: Solve for $\underline{x}_{k,i}^{t}$: $J_{k,i}^{t}\left(\underline{x}_{k,i}^{t}\right) = 0$

Step 6: Compute transition rates $\left\{ s_{kk',i}^{t,t+1} \right\}_{t=1}^{T_{SS}-1}$ for all countries i according to:

$$s_{kk',i}^{t,t+1} = \frac{\exp \left\{ \begin{array}{c} -C_{kk',i} + b_{k',i} + \\ \delta \widehat{\phi}_i^{t+1} \theta_{k',i}^{t} q(\theta_{k',i}^{t}) \frac{\beta_{k',i}}{1-\beta_{k',i}} \int_{\underline{x}_{k',i}^{t+1}}^{x^{\max}} J_{k',i}^{t+1}(x)\, dG_{k',i}(x) + \delta \widehat{\phi}_i^{t+1} U_{k',i}^{t+1} \end{array} \right\}}{\sum_{k''} \exp \left\{ \begin{array}{c} -C_{kk'',i} + b_{k'',i} + \\ \delta \widehat{\phi}_i^{t+1} \theta_{k'',i}^{t} q(\theta_{k'',i}^{t}) \frac{\beta_{k'',i}}{1-\beta_{k'',i}} \int_{\underline{x}_{k'',i}^{t+1}}^{x^{\max}} J_{k'',i}^{t+1}(x)\, dG_{k'',i}(x) + \delta \widehat{\phi}_i^{t+1} U_{k'',i}^{t+1} \end{array} \right\}}.$$

Step 7: Start loop over t going forward ($t = 0$ to $t = T_{SS} - 1$)

Initial conditions: we know $\widetilde{u}_{k,i}^{t=-1} = u_{k,i}^{t=0}$, $L_{k,i}^{t=-1} = L_{k,i}^{t=0}$, and $\theta_{k,i}^{t=0}$ from the initial steady state computation. Obtain $\widetilde{u}_{k,i}^{t}$ and $L_{k,i}^{t}$ using flow conditions and sequences $\left\{\theta_{k,i}^{t}\right\}$, $\left\{\underline{x}_{k,i}^{t}\right\}$.

- Step 7a: Compute

$$JC_{k,i}^{t} = L_{k,i}^{t} u_{k,i}^{t} \theta_{k,i}^{t} q_i \left(\theta_{k,i}^{t}\right) \left(1 - G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right)$$

$$JD_{k,i}^{t} = \left(\chi_{k,i} + (1 - \chi_{k,i}) \max\left\{\frac{G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right) - G_{k,i}\left(\underline{x}_{k,i}^{t}\right)}{1 - G_{k,i}\left(\underline{x}_{k,i}^{t}\right)}, 0\right\}\right) L_{k,i}^{t-1}\left(1 - \widetilde{u}_{k,i}^{t-1}\right)$$

$$\widetilde{u}_{k,i}^{t} = \frac{L_{k,i}^{t} u_{k,i}^{t} - JC_{k,i}^{t} + JD_{k,i}^{t}}{L_{k,i}^{t}}$$

- Step 7b: Compute

$$L_{k,i}^{t+1} = L_{k,i}^{t} + IF_{k,i}^{t+1} - OF_{k,i}^{t+1},$$

where

$$IF_{k,i}^{t+1} = \sum_{\ell \neq k} L_{\ell,i}^{t} \widetilde{u}_{\ell,i}^{t} s_{\ell k,i}^{t+1,t+2},$$

and

$$OF_{k,i}^{t+1} = L_{k,i}^{t} \widetilde{u}_{k,i}^{t} \left(1 - s_{kk,i}^{t+1,t+2}\right).$$

- Step 7c: Compute

$$u_{k,i}^{t+1} = \frac{\sum_{\ell=1}^{K} L_{\ell,i}^{t} \widetilde{u}_{\ell,i}^{t} s_{\ell k,i}^{t+1,t+2}}{L_{k,i}^{t+1}}$$

- Step 7d: Compute

$$\widetilde{L}_{k,i}^{t+1} = L_{k,i}^{t}\left(1 - \widetilde{u}_{k,i}^{t}\right) \int_{\underline{x}_{k,i}^{t+1}}^{\infty} \frac{s}{1 - G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)} dG_{k,i}(s)$$

$$= L_{k,i}^{t}\left(1 - \widetilde{u}_{k,i}^{t}\right) \exp\left(\frac{\sigma_{k,i}^{2}}{2}\right) \frac{\Phi\left(\sigma_{k,i} - \frac{\ln \underline{x}_{k,i}^{t+1}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}^{t+1}}{\sigma_{k,i}}\right)}$$

$$\text{and } Y_{k,i}^{t+1} = \widetilde{w}_{k,i}^{t+1} \widetilde{L}_{k,i}^{t+1}$$

- Step 7e: Compute expenditure with vacancies

$$E_{k,i}^{V,t+1} = \widetilde{\kappa}_{k,i}^{t+1} \widetilde{w}_{k,i}^{t+1} \theta_{k,i}^{t+1} u_{k,i}^{t+1} L_{k,i}^{t+1}$$

- Step 7f: Compute $E_i^{C,t+1} = \frac{\overline{L}_i}{\widetilde{\lambda}_i^{t+1}}$

- Step 7g: Solve for $\left\{ Y_{k,i}^{t+1} \right\}$ in the system

$$E_{k,i}^{t+1} = \mu_{k,i} E_i^{C,t+1} + \sum_{\ell=1}^{K} \left(\mu_{k,i} E_{\ell,i}^{V,t+1} + (1 - \gamma_{\ell,i}) \nu_{\ell k,i} Y_{\ell,i}^{t+1} \right).$$

$$Y_{k,o}^{t+1} = \sum_{i=1}^{N} \pi_{k,oi}^{t+1} E_{k,i}^{t+1}.$$

- Step 7h: Normalize $\left\{ Y_{k,i}^{t+1} \right\}$ to make sure it sums to 1 across sectors and countries.

- Step 7i: Compute $\left(\widetilde{w}_{k,i}^{t+1} \right)' = \frac{\gamma_{k,i} Y_{k,i}^{t+1}}{\widetilde{L}_{k,i}^{t+1}}$

Step 8: Compute $dist \left(\left\{ \widetilde{w}_{k,i}^{t} \right\}_{t=1}^{T_{SS}}, \left\{ \left(\widetilde{w}_{k,i}^{t} \right)' \right\}_{t=1}^{T_{SS}} \right)$

Step 9: Update $\widetilde{w}_{k,i}^{t} = (1 - \alpha_w) \widetilde{w}_{k,i}^{t} + \alpha_w \left(\widetilde{w}_{k,i}^{t} \right)'$ for $t = 1, ..., T_{SS}$ and go back to Step 3 until convergence of $\left\{ \widetilde{w}_{k,i}^{t} \right\}$

Step 10: Compute disposable income $\left\{ I_i^t \right\}_{t=1}^{T_{SS}}$

$$I_i^t = \sum_{\ell=1}^{K} \left(\gamma_{\ell,i} Y_{\ell,i}^{t} - E_{\ell,i}^{V,t} \right)$$

Step 11: Update $\left\{ E_i^{C,t} \right\}_{t=1}^{T_{SS}}$ using

$$E_i^{C,t} = I_i^t - N X_i^t$$

Step 12a: Compute $\left(\widetilde{\lambda}_i^t \right)' = \frac{\overline{L}_i}{E_i^{C,t}}$ for all $t = 1, ..., T_{SS}$

- Step 12b: Compute $dist \left(\left\{ \widetilde{\lambda}_i^t \right\}_{t=1}^{T_{SS}}, \left\{ \left(\widetilde{\lambda}_i^t \right)' \right\}_{t=1}^{T_{SS}} \right)$

- Step 12c: Update $\widetilde{\lambda}_i^t = (1 - \alpha_\lambda) \widetilde{\lambda}_i^t + \alpha_\lambda \left(\widetilde{\lambda}_i^t \right)'$ for $t = 1, ..., T_{SS}$ and go back to Step 2 until convergence of $\left\{ \widetilde{\lambda}_i^t \right\}$.

C.6 Algorithm: Recovering Shocks

Inner Loop: conditional on paths for expenditures $\left\{E_i^{C,t}\right\}_{t=1}^{T_{SS}}$ and shocks $\left\{\widehat{\phi}_i^t\right\}_{t=2}^{T_{SS}}$, $\left\{\widehat{d}_{k,i}^t\right\}_{t=1}^{T_{SS}}$ and $\widehat{A}_{k,i} \equiv \frac{A_{k,i}^{T_{SS}}}{A_{k,i}^0}$ – determined in the Outer Loop below.

As before, we denote changes relative to $t = 0$ by $\widehat{x}^t = \frac{x^t}{x^0}$. This loop **conditions** on data on $\left\{\widehat{\pi}_{k,oi}^t\right\}_{t=1}^{T}$ and $\left\{\widehat{P}_{k,i}^{I,t}\right\}_{t=1}^{T}$, where T is the last period we have data on these variables. We assume $t = 0$ is the estimated steady state. Define $\widehat{d}_{k,i} \equiv \frac{d_{o,i,k}^{T_{SS}}}{d_{o,i,k}^0}$.

Step 1: Given paths $\left\{E_i^{C,t}\right\}$, compute paths $\left\{\widetilde{\lambda}_i^t\right\}$: $\widetilde{\lambda}_i^t = \frac{\overline{L}_i}{E_i^{C,t}}$.

Step 2: Guess paths $\left\{\widetilde{w}_{k,i}^t\right\}_{t=1}^{T_{SS}}$ for each sector k and country i.

Step 3: Compute $\underline{x}_{k,i}^{T_{SS}}$ consistent with $\widetilde{w}_{k,i}^{T_{SS}}$ and $\widetilde{\lambda}_i^{T_{SS}}$. Obtain $\theta_{k,i}^{T_{SS}}$, $U_{k,i}^{T_{SS}}$, $s_{k\ell,i}^{T_{SS},T_{SS}+1}$ and $\pi_{k,oi}^{T_{SS}}$.

- Step 3a: Compute $\widehat{\widetilde{w}}_{k,i} = \frac{\widetilde{w}_{k,i}^{T_{SS}}}{\widetilde{w}_{k,i}^0}$. Iteratively solve for $\widehat{P}_{k,i}^I$ and $\widehat{c}_{k,i}$ using the system

$$\widehat{c}_{k,i} = \left(\widehat{\widetilde{w}}_{k,i}\right)^{\gamma_{k,i}} \prod_{\ell=1}^{K} \left(\widehat{P}_{\ell,i}^I\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}}$$

$$\widehat{P}_{k,i}^I = \left(\sum_{o=1}^{N} \pi_{k,oi}^0 \widehat{A}_{k,o} \left(\widehat{c}_{k,o}\widehat{d}_{k,oi}\right)^{-\lambda}\right)^{-1/\lambda}$$

- Step 3b: Compute $\widehat{P}_{k,i}^F$:

$$\widehat{P}_i^F = \prod_{k=1}^{K} \left(\widehat{P}_{k,i}^I\right)^{\mu_{ki}}$$

- Step 3c: Compute

$$\widehat{\pi}_{k,oi} = \widehat{A}_{k,o} \left(\frac{\widehat{c}_{k,o}\widehat{d}_{k,oi}}{\widehat{P}_{k,i}^I}\right)^{-\lambda}$$

  And obtain $\pi_{k,oi}^{T_{SS}} = \pi_{k,oi}^0 \widehat{\pi}_{k,oi}$

- Step 3d: Compute

  - $\widetilde{\kappa}_{k,i}^{T_{SS}} = \widetilde{\kappa}_{k,i}^0 \frac{\widehat{P}_i^F}{\widehat{\widetilde{w}}_{k,i}}$

- Step 3e: Guess $\left\{\underline{x}_{k,i}^{T_{SS}}\right\}$

- Step 3f: Compute

$$\theta_{k,i}^{T_{SS}} = q_i^{-1}\left(\widetilde{\kappa}_{k,i}^{T_{SS}} \times \frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right) I_{k,i}\left(\underline{x}_{k,i}^{T_{SS}}\right)}\right)$$

- Step 3g: Compute Bellman Equations

$$U_{k,i}^{T_{SS}} = \zeta_i \log\left(\sum_{k'} \exp\left\{\frac{-C_{kk',i} + b_{k',i} + \theta_{k',i}^{T_{SS}} \widetilde{\kappa}_{k',i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{k',i}^{T_{SS}} \frac{\beta_{k',i}}{\left(1-\beta_{k',i}\right)} + \delta U_{k',i}^{T_{SS}}}{\zeta_i}\right\}\right)$$

- Step 3h: Compute

$$\left(\underline{x}_{k,i}^{T_{SS}}\right)' = \frac{\left(1-\delta\right) U_{k,i}^{T_{SS}} - \eta_{k,i}}{\widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{k,i}^{T_{SS}}}$$

- Step 3i: Update $\underline{x}_{k,i}^{T_{SS}} = \left(1-\lambda_x\right) \underline{x}_{k,i}^{T_{SS}} + \lambda_x \left(\underline{x}_{k,i}^{T_{SS}}\right)'$ and go back to Step 2d until convergence.

- Step 3j: Compute $s_{kk'}^{T_{SS},T_{SS}+1}$

$$s_{k\ell,i}^{T_{SS},T_{SS}+1} = \frac{\exp\left\{\frac{-C_{k\ell,i} + b_{\ell,i} + \theta_{\ell,i}^{T_{SS}} \widetilde{\kappa}_{\ell,i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{\ell,i}^{T_{SS}} \frac{\beta_{\ell,i}}{1-\beta_{\ell,i}} + \delta U_{\ell,i}^{T_{SS}}}{\zeta_i}\right\}}{\sum_{\overline{k}} \exp\left\{\frac{-C_{k\overline{k},i} + b_{\overline{k},i} + \theta_{\overline{k},i}^{T_{SS}} \widetilde{\kappa}_{\overline{k},i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{\overline{k},i}^{T_{SS}} \frac{\beta_{\overline{k},i}}{1-\beta_{\overline{k},i}} + \delta U_{\overline{k},i}^{T_{SS}}}{\zeta_i}\right\}}$$

Step 4: Obtain series $\left\{\pi_{k,oi}^t\right\}_{t=T+1}^{T_{SS}}$ and $\left\{\widetilde{\kappa}_{k,i}^t\right\}_{t=1}^{T_{SS}}$.

- Step 4a: For $t = T+1, ..., T_{SS}$ do:

Compute $\widehat{\widetilde{w}}_{k,i}^t = \frac{\widetilde{w}_{k,i}^t}{\widetilde{w}_{k,i}^0}$ and iteratively solve for $\widehat{P}_{k,i}^{I,t}$ and $\widehat{c}_{k,i}^t$ using the system

$$\widehat{c}_{k,i}^t = \left(\widehat{\widetilde{w}}_{k,i}^t\right)^{\gamma_{k,i}} \prod_{\ell=1}^{K} \left(\widehat{P}_{\ell,i}^{I,t}\right)^{\left(1-\gamma_{k,i}\right)\nu_{k\ell,i}},$$

$$\widehat{P}_{k,i}^{I,t} = \left(\sum_{o=1}^{N} \pi_{k,oi}^0 \widehat{A}_{k,o} \left(\widehat{c}_{k,o}^t \widehat{d}_{k,oi}\right)^{-\lambda}\right)^{-1/\lambda}.$$

- Step 4b: Compute $\widehat{P}_{k,i}^{F,t}$ for $t = 1, ..., T_{SS}-1$ (remember $\widehat{P}_{k,i}^{I,t}$ is data for $t = 1, ..., T$):

$$\widehat{P}_i^{F,t} = \prod_{k=1}^{K} \left(\widehat{P}_{k,i}^{I,t}\right)^{\mu_{ki}}$$

- Step 4c: Compute $\widehat{\pi}_{k,oi}^{t}$ and $\pi_{k,oi}^{t}$ for $t = T+1, ..., T_{SS}-1$.

For $t = 1, ..., T_{SS}-1$ do:
First Case: If $t \leq T$ then $\widehat{\pi}_{k,oi}^{t}$ is data, so do:

$$\pi_{k,oi}^{t} = \pi_{k,oi}^{0} \widehat{\pi}_{k,oi}^{t}$$

End of First Case
Second Case if $t \geq T+1$ do:

$$\widehat{\pi}_{k,oi}^{t} = \left(\widehat{A}_{k,o}^{t}\right)' \left(\frac{\widehat{c}_{k,o}^{t} \widehat{d}_{k,oi}^{t}}{\widehat{P}_{k,i}^{I,t}}\right)^{-\lambda}$$

$$\pi_{k,oi}^{t} = \pi_{k,oi}^{0} \widehat{\pi}_{k,oi}^{t}$$

End of Second Case

- Step 4d: Compute for $t = 1, ..., T_{SS}-1$

  - $\widetilde{\kappa}_{k,i}^{t} \equiv \frac{\kappa_{k,i} P_i^{F,t}}{\widetilde{w}_{k,i}^{t}} = \frac{\kappa_{k,i} P_i^{F,0}}{\widetilde{w}_{k,i}^{0}} \frac{P_i^{F,t}}{P_i^{F,0}} \frac{\widetilde{w}_{k,i}^{0}}{\widetilde{w}_{k,i}^{t}} = \widetilde{\kappa}_{k,i}^{0} \frac{\widehat{P}_i^{F,t}}{\widehat{\widetilde{w}}_{k,i}^{t}}$

Step 5: Given knowledge of $\widetilde{w}_{k,i}^{T_{SS}}$, $\widetilde{\lambda}_i^{T_{SS}}$ and $\underline{x}_{k,i}^{T_{SS}}$ (and therefore $J_{k,i}^{T_{SS}}(s)$), start at $t = T_{SS}-1$ and sequentially compute (backwards) for each $t = T_{SS}-1, ..., 1$

- Step 5a: Given $\widetilde{w}_{k,i}^{t}$, $\underline{x}_{k,i}^{t+1}$, $\widetilde{\kappa}_{k,i}^{t}$, $\widetilde{\lambda}_i^{t}$ and $J_{k,i}^{t+1}(s)$ compute $\theta_{k,i}^{t}$.

If $\frac{\widetilde{\lambda}_i^{t} \widetilde{\kappa}_{k,i}^{t} \widetilde{w}_{k,i}^{t}}{\delta \widehat{\phi}_i^{t+1} \int_{\underline{x}_{k,i}^{t+1}}^{x^{\max}} J_{k,i}^{t+1}(s) dG_{k,i}(s)} \leq 1$ then

$$\theta_{k,i}^{t} = q_i^{-1} \left(\frac{\widetilde{\lambda}_i^{t} \widetilde{\kappa}_{k,i}^{t} \widetilde{w}_{k,i}^{t}}{\delta \widehat{\phi}_i^{t+1} \int_{\underline{x}_{k,i}^{t+1}}^{x^{\max}} J_{k,i}^{t+1}(s)\, dG_{k,i}(s)}\right)$$

If $\frac{\widetilde{\lambda}_i^{t} \widetilde{\kappa}_{k,i}^{t} \widetilde{w}_{k,i}^{t}}{\delta \widehat{\phi}_i^{t+1} \int_{\underline{x}_{k,i}^{t+1}}^{x^{\max}} J_{k,i}^{t+1}(s) dG_{k,i}(s)} > 1$, it is not possible to satisfy $V_{k,i}^{t} = 0$, so that $V_{k,i}^{t} < 0$ and

$$\theta_{k,i}^{t} = 0.$$

- Step 5b: Given $\underline{x}_{k,i}^{t+1}$, $W_{k,i}^{t+1}(x) = \frac{\beta_{k,i}}{1-\beta_{k,i}} J_{k,i}^{t+1}(x) + U_{k,i}^{t+1}$ (for $x \geq \underline{x}_{k,i}^{t+1}$), $\theta_{k,i}^{t}$, $U_{k,i}^{t+1}$ compute $U_{k,i}^{t}$. Notice that $\int_{\underline{x}_{k,i}^{t+1}}^{x^{\max}} W_{k,i}^{t+1}(s)\, dG_{k,i}(s) = \frac{\beta_{k,i}}{1-\beta_{k,i}} \int_{\underline{x}_{k,i}^{t+1}}^{x^{\max}} J_{k,i}^{t+1}(s)\, dG_{k,i}(s) + \left(1 - G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right) U_{k,i}^{t+1}$ so that:

$$U_{k,i}^{t} = \zeta_i \log\left(\sum_{k'} \exp\left\{\frac{\begin{array}{c} -C_{kk',i} + b_{k',i} \\ +\delta \widehat{\phi}_i^{t+1} \theta_{k',i}^{t} q_i \left(\theta_{k',i}^{t}\right) \frac{\beta_{k,i}}{1-\beta_{k,i}} \int_{\underline{x}_{k,i}^{t+1}}^{x^{\max}} J_{k,i}^{t+1}(s)\, dG_{k,i}(s) + \delta \widehat{\phi}_i^{t+1} U_{k',i}^{t+1} \end{array}}{\zeta_i}\right\}\right)$$

- Step 5c: Given $J_{k,i}^{t+1}(x)$, $\widetilde{w}_{k,i}^{t}$, $U_{k,i}^{t}$, $U_{k,i}^{t+1}$ and $\underline{x}_{k,i}^{t+1}$ compute $J_{k,i}^{t}(x)$

$$\begin{aligned} J_{k,i}^{t}(x) = & (1-\beta_{k,i})\,\widetilde{\lambda}_{i}^{t}\widetilde{w}_{k,i}^{t}x + (1-\beta_{k,i})\,\eta_{k,i} \\ & - (1-\beta_{k,i})\left(U_{k,i}^{t} - \delta\widehat{\phi}_{i}^{t+1}U_{k,i}^{t+1}\right) + (1-\chi_{k,i})\,\delta\widehat{\phi}_{i}^{t+1}\max\left\{J_{k,i}^{t+1}(x),0\right\} \end{aligned}$$

- Step 5d: Solve for $\underline{x}_{k,i}^{t}$: $J_{k,i}^{t}\left(\underline{x}_{k,i}^{t}\right)=0$

Step 6: Compute transition rates $\left\{s_{kk',i}^{t,t+1}\right\}_{t=1}^{T_{SS}-1}$ for all countries i according to:

$$s_{kk',i}^{t,t+1} = \frac{\exp\left\{\begin{array}{c} -C_{kk',i} + b_{k',i} + \\ \delta\widehat{\phi}_{i}^{t+1}\theta_{k',i}^{t}q(\theta_{k',i}^{t})\frac{\beta_{k',i}}{1-\beta_{k',i}}\int_{\underline{x}_{k',i}^{t+1}}^{x_{\max}}J_{k',i}^{t+1}(x)\,dG_{k',i}(x) + \delta\widehat{\phi}_{i}^{t+1}U_{k',i}^{t+1} \end{array}\right\}}{\sum_{k''}\exp\left\{\begin{array}{c} -C_{kk'',i} + b_{k'',i} + \\ \delta\widehat{\phi}_{i}^{t+1}\theta_{k'',i}^{t}q(\theta_{k'',i}^{t})\frac{\beta_{k'',i}}{1-\beta_{k'',i}}\int_{\underline{x}_{k'',i}^{t+1}}^{x_{\max}}J_{k'',i}^{t+1}(x)\,dG_{k'',i}(x) + \delta\widehat{\phi}_{i}^{t+1}U_{k'',i}^{t+1} \end{array}\right\}}.$$

Step 7: Start loop over t going forward ($t=0$ to $t=T_{SS}-1$)

Initial conditions: we know $\widetilde{u}_{k,i}^{t=-1} = u_{k,i}^{t=0}$, $L_{k,i}^{t=-1} = L_{k,i}^{t=0}$, and $\theta_{k,i}^{t=0}$ from the initial steady state computation. Obtain $\widetilde{u}_{k,i}^{t}$ and $L_{k,i}^{t}$ using flow conditions and sequences $\left\{\theta_{k,i}^{t}\right\}$, $\left\{\underline{x}_{k,i}^{t}\right\}$.

- Step 7a: Compute

$$JC_{k,i}^{t} = L_{k,i}^{t}u_{k,i}^{t}\theta_{k,i}^{t}q_{i}\left(\theta_{k,i}^{t}\right)\left(1-G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right)$$

$$JD_{k,i}^{t} = \left(\chi_{k,i} + (1-\chi_{k,i})\max\left\{\frac{G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right) - G_{k,i}\left(\underline{x}_{k,i}^{t}\right)}{1-G_{k,i}\left(\underline{x}_{k,i}^{t}\right)},0\right\}\right)L_{k,i}^{t-1}\left(1-\widetilde{u}_{k,i}^{t-1}\right)$$

$$\widetilde{u}_{k,i}^{t} = \frac{L_{k,i}^{t}u_{k,i}^{t} - JC_{k,i}^{t} + JD_{k,i}^{t}}{L_{k,i}^{t}}$$

- Step 7b: Compute

$$L_{k,i}^{t+1} = L_{k,i}^{t} + IF_{k,i}^{t+1} - OF_{k,i}^{t+1},$$

where

$$IF_{k,i}^{t+1} = \sum_{\ell\neq k}L_{\ell,i}^{t}\widetilde{u}_{\ell,i}^{t}s_{\ell k,i}^{t+1,t+2},$$

and

$$OF_{k,i}^{t+1} = L_{k,i}^{t}\widetilde{u}_{k,i}^{t}\left(1-s_{kk,i}^{t+1,t+2}\right).$$

- Step 7c: Compute

$$u_{k,i}^{t+1} = \frac{\sum_{\ell=1}^{K} L_{\ell,i}^{t} \widetilde{u}_{\ell,i}^{t} s_{\ell k,i}^{t+1,t+2}}{L_{k,i}^{t+1}}$$

- Step 7d: Compute

$$\begin{aligned}\widetilde{L}_{k,i}^{t+1} &= L_{k,i}^{t}\left(1-\widetilde{u}_{k,i}^{t}\right)\int_{\underline{x}_{k,i}^{t+1}}^{\infty}\frac{s}{1-G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)}dG_{k,i}\left(s\right)\\ &= L_{k,i}^{t}\left(1-\widetilde{u}_{k,i}^{t}\right)\exp\left(\frac{\sigma_{k,i}^{2}}{2}\right)\frac{\Phi\left(\sigma_{k,i}-\frac{\ln \underline{x}_{k,i}^{t+1}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}^{t+1}}{\sigma_{k,i}}\right)}\end{aligned}$$

- Step 7e: Compute expenditure with vacancies

$$E_{k,i}^{V,t+1} = \widetilde{\kappa}_{k,i}^{t+1}\widetilde{w}_{k,i}^{t+1}\theta_{k,i}^{t+1}u_{k,i}^{t+1}L_{k,i}^{t+1}$$

- Step 7f: Solve for $\left\{Y_{k,i}^{t+1}\right\}$ in the system

$$E_{k,i}^{t+1} = \mu_{k,i}E_{i}^{C,t+1} + \sum_{\ell=1}^{K}\left(\mu_{k,i}E_{\ell,i}^{V,t+1} + \left(1-\gamma_{\ell,i}\right)\nu_{\ell k,i}Y_{\ell,i}^{t+1}\right).$$

$$Y_{k,o}^{t+1} = \sum_{i=1}^{N}\pi_{k,oi}^{t+1}E_{k,i}^{t+1}.$$

- Step 7g: Compute $\left(\widetilde{w}_{k,i}^{t+1}\right)' = \frac{\gamma_{k,i}Y_{k,i}^{t+1}}{\widetilde{L}_{k,i}^{t+1}}$

Step 8: Compute distance $dist\left(\left\{\widetilde{w}_{k,i}^{t}\right\}, \left\{\left(\widetilde{w}_{k,i}^{t}\right)'\right\}\right)$

- Step 8b: Update $\widetilde{w}_{k,i}^{t} = \left(1-\lambda_{w}\right)\widetilde{w}_{k,i}^{t} + \lambda_{w}\left(\widetilde{w}_{k,i}^{t}\right)'$ $t = 1, ..., T_{SS}$
- Step 8c: At this point, we have a new series for $\left\{\widetilde{w}_{k,i}^{t}\right\}$ – go back to Step 3 until convergence of $\left\{\widetilde{w}_{k,i}^{t}\right\}$.

Step 9: Compute disposable income $\left\{I_{i}^{t}\right\}_{t=1}^{T_{SS}}$

$$I_{i}^{t} = \sum_{\ell=1}^{K}\left(\gamma_{\ell,i}Y_{\ell,i}^{t} - E_{\ell,i}^{V,t}\right)$$

Step 10: Compute $\left\{\left(\widehat{A}_{k,i}^{t}\right)'\right\}$.

For $t=1,...,T$ compute $\widehat{\widetilde{w}}_{k,i}^{t}=\frac{\widetilde{w}_{k,i}^{t}}{\widetilde{w}_{k,i}^{0}}$, obtain $\widehat{c}_{k,i}^{t}$:

$$\widehat{c}_{k,i}^{t}=\left(\widehat{\widetilde{w}}_{k,i}^{t}\right)^{\gamma_{k,i}}\prod_{\ell=1}^{K}\left(\widehat{P}_{\ell,i}^{I,t}\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}},$$

and compute $\left(\widehat{A}_{k,i}^{t}\right)'$:

$$\left(\widehat{A}_{k,i}^{t}\right)'=\frac{\widehat{\pi}_{k,ii}^{t}}{\left(\widehat{P}_{k,i}^{I,t}\right)^{\lambda}}\left(\widehat{c}_{k,i}^{t}\right)^{\lambda}.$$

For $t\geq T+1$ set:

$$\left(\widehat{A}_{k,i}^{t}\right)'=\left(\widehat{A}_{k,i}^{T}\right)'$$

Feed outer loop with $\left\{\left(\widehat{A}_{k,i}^{T_{SS}}\right)'\right\}$.

Outer Loop: iteration on $\left\{NX_{i}^{t}\right\}$

Step 0: Compute changes in trade costs $\left\{\widehat{d}_{k,oi}^{t}\right\}_{t=1}^{T}$:

$$\widehat{d}_{k,oi}^{t}=\left(\frac{\widehat{\pi}_{k,oo}^{t}}{\widehat{\pi}_{k,oi}^{t}}\right)^{1/\lambda}\frac{\widehat{P}_{k,i}^{I,t}}{\widehat{P}_{k,o}^{I,t}}$$

Set $\widehat{d}_{k,oi}^{t}=\widehat{d}_{k,oi}^{T}$ for $t>T$.

Step 1: Start with estimated state equilibrium at $t=0$. Remember that we used the normalization $\sum_{i=1}^{I}\sum_{k=1}^{K}Y_{k,i}=1$ during the estimation procedure. **Change the normalization** from $\sum_{i=1}^{I}\sum_{k=1}^{K}Y_{k,i}=1$ to $\sum_{i=1}^{I}E_{i}^{C}=1$. Nominal variables to be renormalized: $\left\{Y_{k,i}^{0}\right\}$, $\left\{\widetilde{w}_{k,i}^{0}\right\}$, $\left\{E_{i}^{C,0}\right\}$, $\left\{NX_{i}^{0}\right\}$.

Step 2: Compute $E_{i}^{C,t}=\frac{E_{i}^{C,0}\left(\widehat{E}_{i}^{C,t}\right)_{Data}}{\sum_{i=1}^{N}E_{i}^{C,0}\left(\widehat{E}_{i}^{C,t}\right)_{Data}}$ for $t=1,...,T$ where $E_{i}^{C,0}$ is aggregate consumption expenditure in the estimated steady state, and $\left(\widehat{E}_{i}^{C,t}\right)_{Data}$ comes from the data.

Step 3: Normalize $\widehat{\phi}_{US}^{t}=1$ for $t=2,...,T-1$. This yields

$$R^{t+1}=\frac{E_{US}^{C,t+1}}{\delta E_{US}^{C,t}}\text{ for }t=1,...,T-1$$

Obtain remaining shocks $\left\{\widehat{\phi}_{i}^{t}\right\}_{t=2}^{T}$ using

$$\widehat{\phi}_{i}^{t+1}=\frac{E_{i}^{C,t+1}}{\delta E_{i}^{C,t}R^{t+1}}\text{ for }t=1,...,T-1$$

Set $\widehat{\phi}_i^t = 1$ for $t \geq T+2$. The value of $\widehat{\phi}_i^{T+1}$ will depend on $E_i^{C,T_{SS}}$ and will be recovered in Step 7. Note: the value of $\widehat{\phi}_i^{t=1}$ does not matter. Individuals made decisions at $t=0$ assuming $\widehat{\phi}_i^{t=1} = 1$, as the economy was assumed to be in steady state at $t=0$.

Step 4: Obtain B_i^0 with respect to the normalization $\sum_{i=1}^{I} E_i^C = 1$. Equation (XX) gives us:

$$B_i^0 = \frac{NX_i^0}{\left(1 - \frac{1}{\delta}\right)}$$

Step 5: Make initial guess for $NX_i^{T_{SS}}$ (with respect to the normalization $\sum_{i=1}^{I} E_i^C = 1$) and for $\widehat{A}_{k,i}^{T_{SS}} = \widehat{A}_{k,i}^{T} \equiv \frac{A_{k,i}^T}{A_{k,i}^0}$.

Step 6: Compute steady state equilibrium at T_{SS}, conditional on $NX_i^{T_{SS}}$, $\widehat{A}_{k,i}^{T_{SS}}$ and $\widehat{d}_{k,oi}^{T_{SS}}$.

- Step 6a: Notice that steady algorithm uses the normalization $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$. Normalize $NX_i^{T_{SS}}$ with respect to normalization $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$. To perform such normalization, use output $\left\{Y_{k,i}^{T_{SS}}\right\}$ obtained in the initial steady state if this is the first outer loop iteration, otherwise use output $\left\{Y_{k,i}^{T_{SS}}\right\}$ obtained in Step 9 below.

- Step 6b: After computing the final steady state, **change the normalization** from $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$ to $\sum_{i=1}^{I} E_i^C = 1$ using $\left\{E_i^C\right\}$ obtained in Step 3a. Nominal variables to be renormalized: $\left\{Y_{k,i}^{T_{SS}}\right\}$, $\left\{\widetilde{w}_{k,i}^{T_{SS}}\right\}$, $\left\{E_i^{C,T_{SS}}\right\}$, $\left\{NX_i^{T_{SS}}\right\}$.

Step 7: Set $E_i^{C,t} = \begin{cases} E_i^{C,0}\left(\frac{E_i^{C,2000+t}}{E_i^{C,2000}}\right)_{Data} & \text{for } t = 0,...,T, \text{ as in Step 0} \\ E_i^{C,T_{SS}} \text{ for } t = T+1,...,T_{SS} \end{cases}$

Note: $\widehat{\phi}_i^t = 1$ for $t \geq T+2 \Rightarrow E_i^{C,t} = E_i^{C,T_{SS}}$ for at $t \geq T+1$ (Euler Equation)

Step 8: Compute $\widehat{\phi}_i^{T+1}$, imposing $\widehat{\phi}_{US}^{T+1} = 1$:

$$\widehat{\phi}_i^{T+1} = \frac{E_i^{C,T_{SS}}}{\delta R^{T+1} E_i^{C,T}},$$

$$R^{T+1} = \frac{E_{US}^{C,T_{SS}}}{\delta E_{US}^{C,T}}.$$

We now have a full path for $\left\{\widehat{\phi}_i^t\right\}_{t=2}^{T_{SS}}$.

Step 9: Solve for the out-of-steady-state dynamics conditional on aggregate expenditures $\left\{E_i^{C,t}\right\}_{t=0}^{T_{SS}}$, on preference shifters $\left\{\widehat{\phi}_i^t\right\}_{t=2}^{T_{SS}}$ and steady-state productivity shocks $\left\{\widehat{A}_{k,i}^{T_{SS}}\right\}$.

Step 10: Using the path for disposable income $\left\{I_i^t\right\}_{t=1}^{T_{SS}}$ obtained in Step 9 and equation (7) compute:

$$\left(NX_i^t\right)' = I_i^t - E^{C,t} \text{ for } 1 \leq t < T_{SS}$$

$$\left(NX_i^{T_{SS}}\right)' = -\frac{1-\delta}{\delta}\frac{1}{\left(\prod_{\tau=1}^{T_{SS}-1}\left(R^\tau\right)^{-1}\right)}\left(B_i^0 + \sum_{t=1}^{T_{SS}-1}\left(\prod_{\tau=1}^{t}\left(R^\tau\right)^{-1}\right)\left(NX_i^t\right)'\right)$$

Step 11: Compute

$$dist\left(\left\{NX_i^{T_{SS}}\right\}, \left\{\left(NX_i^{T_{SS}}\right)'\right\}\right) \text{ and}$$

$$dist\left(\left\{\widehat{A}_{k,i}^{T_{SS}}\right\}, \left\{\left(\widehat{A}_{k,i}^{T_{SS}}\right)'\right\}\right),$$

using the values of $\left\{\left(\widehat{A}_{k,i}^{T_{SS}}\right)'\right\}$ obtained in Step 9.

Step 12: Update $NX_i^{T_{SS}}$

$$NX_i^{T_{SS}} = (1-\lambda_o)\, NX_i^{T_{SS}} + \lambda_o \left(NX_i^{T_{SS}}\right)'$$

and $\widehat{A}_{k,i}^{T_{SS}}$

$$\widehat{A}_{k,i}^{T_{SS}} = (1-\lambda_A)\, \widehat{A}_{k,i}^{T_{SS}} + \lambda_A \left(\widehat{A}_{k,i}^{T_{SS}}\right)'$$

Go back to Step 6 until convergence of $\left\{NX_i^{T_{SS}}\right\}$ and $\left\{\widehat{A}_{k,i}^{T_{SS}}\right\}$.

C.7 Parameter Estimates

In this section, we display the complete set of parameter estimates.

Table A.1: Final Expenditure Shares $\mu_{k,i}$

Sector ↓ Country →	USA	China	Europe	Asia/Oceania	Americas	ROW
Agr.	0.01	0.12	0.02	0.01	0.02	0.09
LT Manuf.	0.03	0.02	0.04	0.03	0.04	0.03
MT Manuf.	0.05	0.11	0.08	0.07	0.1	0.11
HT Manuf.	0.1	0.15	0.11	0.1	0.11	0.1
LT Serv.	0.3	0.35	0.34	0.38	0.33	0.39
HT Serv.	0.51	0.25	0.41	0.41	0.4	0.29

Table A.2: Labor Shares in Production $\gamma_{k,i}$

Sector ↓ Country →	USA	China	Europe	Asia/Oceania	Americas	ROW
Agr.	0.45	0.58	0.56	0.54	0.62	0.67
LT Manuf.	0.37	0.25	0.32	0.35	0.28	0.27
MT Manuf.	0.33	0.28	0.31	0.37	0.32	0.28
HT Manuf.	0.39	0.24	0.33	0.32	0.31	0.25
LT Serv.	0.61	0.37	0.49	0.54	0.56	0.48
HT Serv.	0.62	0.55	0.63	0.67	0.67	0.68

Table A.3: Input-Output Table – Average Across Countries $\frac{1}{N}\sum_i \nu_{k\ell,i}$

User ↓ Supplier →	Agr.	LT Manuf.	MT Manuf.	HT Manuf.	LT Serv.	HT Serv.
Agr.	0.267	0.079	0.118	0.138	0.26	0.139
	(0.056)	(0.019)	(0.036)	(0.029)	(0.06)	(0.07)
LT Manuf.	0.195	0.376	0.043	0.081	0.223	0.082
	(0.047)	(0.064)	(0.009)	(0.015)	(0.041)	(0.04)
MT Manuf.	0.222	0.066	0.287	0.106	0.225	0.095
	(0.034)	(0.017)	(0.04)	(0.022)	(0.052)	(0.044)
HT Manuf.	0.022	0.157	0.067	0.463	0.184	0.106
	(0.018)	(0.022)	(0.015)	(0.052)	(0.044)	(0.05)
LT Serv.	0.057	0.136	0.105	0.101	0.343	0.259
	(0.042)	(0.03)	(0.027)	(0.034)	(0.073)	(0.11)
HT Serv.	0.007	0.076	0.033	0.11	0.267	0.507
	(0.005)	(0.023)	(0.019)	(0.071)	(0.069)	(0.171)

Note: Standard Deviation of $\nu_{k\ell}$ across countries.

Table A.4: Mobility Costs Estimates $C_{k\ell}$

From ↓ / To →	Agr.	LT Manuf.	MT Manuf.	HT Manuf.	LT Serv.	HT Serv.
Agriculture	0	0.825	1.560	0.454	0.189	1.676
LT Manufacturing	0.414	0	0.005	0.000	0.799	2.034
MT Manufacturing	2.033	0.000	0	0.002	0.866	2.646
HT Manufacturing	0.015	0.001	0.003	0	0.276	0.917
LT Services	0.268	0.972	1.221	0.466	0	0.002
HT Services	0.790	1.826	2.150	1.201	0.004	0

Table A.5: Sector-Specific Utility and Variance Estimates $\eta_{k,i}, \sigma^2_{k,i}$

Sector ↓ / Country →	USA	China	Europe	Asia/Oceania	Americas	ROW
Agriculture	0	0	0	0	0	0
LT Manuf.	-0.383	-0.943	-0.428	-0.521	-0.588	-0.316
MT Manuf.	0.026	-0.566	-0.245	-0.229	-0.195	-0.072
HT Manuf.	-0.551	-1.557	-0.743	-0.607	-1.292	-0.934
LT Services	0.085	-0.454	-0.382	-0.236	-0.405	-0.588
HT Services	-0.052	-0.467	-0.603	-0.518	-0.894	-1.011
σ^2_{USA}				0.727		

Table A.6: Exogenous Death Rates Estimates $\chi_{k,} = \chi_i + \chi_k$

Country Component χ_i	USA	0.026
	China	0.022
	Europe	0.054
	Asia/Oceania	0.021
	Americas	0.027
	ROW	0.020
Sector Component χ_k	Agriculture	0
	Low Tech Man	0.005
	Med Tech Man	0.014
	High Tech Man	0.002
	Other Services	0.008
	Hi Tech Services	-0.001

Table A.7: Unemployment Utility $b_{k,i} = b_i$

USA	-14.4
China	-12.7
Europe	-7.1
Asia/Oceania	-8.5
Americas	-8.6
ROW	-14.9

Table A.8: Vacancy Posting Costs Estimates $\widetilde{\kappa}_{k,i}$

Sector $\downarrow$ / Country $\rightarrow$	USA	China	Europe	Asia/Oceania	Americas	ROW
Agriculture	0.539	0.450	0.451	0.664	0.451	0.452
LT Manuf.	0.632	0.810	0.681	0.865	0.809	0.583
MT Manuf.	0.472	0.615	0.549	0.660	0.540	0.447
HT Manuf.	0.698	0.984	0.821	0.922	1.076	0.830
LT Services	0.449	0.617	0.592	0.660	0.649	0.639
HT Services	0.511	0.674	0.672	0.840	0.872	0.813