Globalization, Trade Imbalances and Labor Market Adjustment *

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Abstract

Recent literature has shown that globalization can substantially disrupt labor markets. At the same time, the U.S. economy has sustained large trade deficits for decades. While this has led some policy makers to regard trade deficits as a threat to workers, research in International Trade typically disregards imbalances in studies of labor market adjustment. We fill this gap and study the role of global imbalances in shaping both the adjustment dynamics, and the long run consequences, of trade shocks. We build an estimable, general equilibrium, multi-country, multi-sector model of trade with three key ingredients: (a) representative households making consumption-saving decisions in each country; (b) costly reallocation of workers across sectors; and (c) matching frictions in segmented labor markets leading to unemployment dynamics. We estimate a steady-state of our model using data from the World Input-Output Database and household surveys and study the behavior of national labor markets in response to several globalization shocks, including shocks to: (1) technology; (2) trade costs; (3) inter-temporal tastes (e.g. savings gluts). We find that modeling trade imbalances changes both qualitatively and quantitatively the short and long run implications of globalization shocks for labor reallocation and unemployment dynamics. In an empirical application, we revisit the China Shock through the lens of our model. In line with the reduced-form evidence, the model predicts a precipitous decline in manufacturing; however, there is little to no unemployment response as job destruction in manufacturing is met with job creation in other sectors.

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1 Introduction

There is a large body of evidence indicating that globalization shocks can lead to important labor market disruptions. For instance, Autor et al. (2013) show that American workers in regions specialized in goods facing steeper competition from China are less likely to work in manufacturing, more likely to be unemployed, and more likely to rely on disability insurance. Similarly, Dix-Carneiro and Kovak (2017, 2019) show that Brazilian regions that were more exposed to foreign competition as a result of the 1990's trade liberalization experienced prolonged periods of lower formal employment and wages relative to less exposed regions, as well as increases in non-employment and informal employment.[1] This recent reduced-form literature has generated considerable interest in understanding, modelling and quantifying the adjustment process in response to globalization shocks. In particular, Artuç et al. (2010), Dix-Carneiro (2014), Traiberman (2019) and Caliendo et al. (2019) have estimated large mobility costs across sectors and occupations, long adjustment processes and sizable distributional consequences to trade shocks.

Yet, this literature has abstracted from modeling trade imbalances and has been silent on how they could influence the labor market adjustment process. This is puzzling in light of the increase in importance of trade imbalances in the last three decades, coupled with an increasing discomfort that many American policy makers manifest towards trade deficits.[2] Indeed, there is a pervasive concern among many policy makers and commentators that trade deficits are undesirable as they crowd out domestic production and are detrimental to jobs and workers. When trade is balanced, a contraction of import-competing sectors must be simultaneously met with an expansion of export-oriented sectors. On the other hand, if trade imbalances emerge in response to globalization shocks, these shifts are no longer synchronized, affecting the dynamics of reallocation. Specifically, the behavior of trade imbalances can influence the dynamics of job losses and gains, especially in the presence of labor market frictions.

In this paper, we study how endogenizing trade imbalances influence the labor market adjustment process in response to globalization. Does ignoring trade imbalances when we investigate the labor market consequences of trade shocks matter at all? How much insight do we lose in doing so? To address these questions, we build on existing models of globalization and labor market adjustment and develop an estimable, general equilibrium, multi-country, multi-sector model with three key ingredients: (i) Consumption-saving decisions in each country are commanded by a representative family, leading to endogenous trade imbalances; (ii) labor market frictions across and within sectors, leading to unemployment dynamics and sluggish transitions to shocks; and (iii) Ricardian comparative advantage forces promote trade but geographical barriers inhibit it.

[1] Other recent papers tying globalization shocks to labor market disruptions include Pierce and Schott (2016), Costa et al. (2016), Dauth et al. (2018), Utar (2018), among many others.

[2] See, for example, Scott (1998), Bernanke (2005), and Navarro (2019).

In our model, trade imbalances arise from country-level representative agents making consumption and savings decisions.[3] This gives rise to an Euler Equation that dictates how countries smooth consumption over time, and allows us to decompose the observed dynamics of trade imbalances into (1) a consumption response to productivity and trade shocks, and to (2) a residual shock to country-level discount factors (inter-temporal preferences). Our approach relies neither on *ad hoc* rules for imbalances nor on specifying the path of imbalances exogenously, which are common in the international trade literature. Instead, our perspective builds on the workhorse model of imbalances in international macroeconomics, providing a natural benchmark for understanding how they interact with labor market frictions.[4]

Kletzer (2001) emphasizes that trade liberalization is often a focal point for anxiety about job losses and insecurity. Reinforcing this conclusion, data from the 2014 Pew Global Attitudes survey reveal that 50 percent of Americans believe that trade destroys jobs, whereas only 20 percent believe that it creates jobs (see Pavcnik, 2017). Not surprisingly, trade-induced job losses and unemployment are at the heart of policy makers' concerns with globalization. We directly speak to these concerns by incorporating labor market frictions across and within sectors into our model of globalization and trade imbalances. We model mobility frictions across sectors following Artuç et al. (2010) and search frictions and unemployment according to Mortensen and Pissarides (1994). In particular, our framework allows for job destruction and creation to respond to trade shocks, leading to potentially rich unemployment dynamics. Importantly, our framework is suitable for the analysis of out-of-steady state dynamics triggered by changes in the global economy.

We estimate our model using a simulated method of moments and data from the World Input Output Database and the United States Current Population Survey. To ensure tractability of the estimation procedure, we assume the economy is in steady state and we match data moments in year 2000. The procedure conditions on the observed trade shares and allows us to estimate our parameters country by country, greatly simplifying the process.

To understand the main mechanisms in our model, we first compare its response to different sets of shocks to the response of the same model under balanced trade – an assumption that most of the literature on the topic makes. We find that modelling imbalances leads to significantly more reallocation across sectors in response to shocks to fundamental parameters of the global economy such as trade costs and technologies. Because, in our model, workers need to switch to unemployment before searching for a job in a new sector, reallocation is related to unemployent. Therefore, the response of unemployment to trade shocks is also amplified by the emergence of trade imbalances.

The pattern of reallocation can also be very different, both in the short and long runs, depending

[3] See Obstfeld and Rogoff (1995) for a survey of this approach to imbalances in international macroeconomics.

[4] More recent work on global imbalances builds on the standard consumption savings model by adding financial frictions (e.g., Caballero et al. (2008) and Mendoza et al. (2009)), or demographics (e.g.,Barany et al. (2018)).

on the evolution of imbalances. For example, if China experiences a slow moving productivity increase before it reaches a plateau, it smooths consumption by running a trade deficit in the short run, heavily importing manufactured goods. In turn, the United States and the rest of the world lend to China and run trade surpluses in the short run, leading to an expansion of their manufacturing sectors. In the long run, China needs to pay its debt and runs a trade surplus. It does so by exporting manufactured goods, and therefore these sectors must eventually expand. On the other hand, the United States and the rest of the world enjoy manufacturing imports from China and downsize their manufacturing sectors. This leads to a non-monotonic pattern of adjustment that is absent if we abstract from trade imbalances. If we were to impose balanced trade, manufacturing employment would monotonically decline in all countries with the exception of China. In addition, the magnitude of the long-run contraction of manufacturing outside of China is of larger magnitude when we allow for trade imbalances.

When balanced trade is imposed, long-run allocations depend only on the final level of the shocks. In contrast, we find that the path of shocks matters for long-run allocations when trade imbalances are modelled. This dependence on the full path of shocks motivates us to conduct an empirical analysis where we extract the various technology and trade cost shocks that the global economy endured between 2000 and 2014. In this application, we show that the stark differences relative to a model without trade imbalances are quantitatively important. We also explore the "China shock" in the context of our model.[5] Aligned with previous findings, we corroborate that China contributed to the decline of US manufacturing between 2000 and 2014. However, this decline in manufacturing was quickly accompanied by job creation in Services and Agriculture. As a matter of fact, our model suggests that at most 93,100 jobs were lost at a given point in time over this period, and only 10,000 jobs were lost over the whole period. Interestingly, we find that the large trade surplus China sustained over this time would have occurred even in the absence of China-specific shocks. In other words, global shocks greatly contributed to the large Chinese trade surplus between 2000 and 2014. The same is true for the US trade deficit. Finally, we find that shocks to the Chinese economy affected welfare around the world only modestly. Most countries gained from these shocks, including the US.

Our paper speaks to a large literature that investigates the labor market consequences of globalization both empirically and quantitatively. We make two contributions to this literature by incorporating both involuntary unemployment and trade imbalances into the state-of-the-art Ricardian trade model of Caliendo and Parro (2015). Broadly speaking, quantitative trade models based on Eaton and Kortum (2002) have only allowed for a non-employment option (i.e., voluntary unemployment) or have focused on steady-state analyses, ignoring transitional dynamics. Caliendo et al. (2019) is an important example of a dynamic quantitative trade model in which workers

[5] See Autor et al. (2016) and references therein for a discussion of the China shock.

make a labor supply decision and face mobility frictions across sectors and regions. However, their model does not feature job losses and unemployment. On the other end, Carrère et al. (2020) and Guner et al. (2020) incorporate search frictions and unemployment into multi-sector extensions of Eaton and Kortum (2002), but do not study out-of-steady-steady dynamics. In a recent exception, Rodriguez-Clare et al. (2020) incorporates wage rigidity into the model of Caliendo et al. (2019) to investigate the unemployment effects of the China Shock on US local labor markets.[6]

Importantly, though, none of these papers model trade imbalances. We do so by incorporating the workhorse model of imbalances used in the international macroeconomics literature allowing for savings decisions by means of an international bonds market as in Reyes-Heroles (2016).[7] In that regard, our paper is closely related to Kehoe et al. (2018) who explore the implications of the increase in the U.S trade deficit for the secular decline in manufacturing labor over the last four decades. However, this paper does not incorporate sluggish labor market adjustment nor unemployment dynamics.[8]

One limitation of our framework is that it does not incorporate endogenous capital accumulation decisions. Instead, imbalances arise as a result of consumption smoothing. Excluding capital from the model means that certain features of the data that are typically studied in International Real Business Cycles (IRBC) cannot be matched—for example, the countercyclicality of quarterly net exports observed for many developed economies (Backus et al., 1992, 1994).[9] Ultimately we ignore capital for two reasons. First, it helps maintain tractability of the model and allows us to focus on richer labor market dynamics than are present in typical dynamic models. This is in line with the trade literature more broadly which usually ignores dynamics in capital—treating it as a fixed factor, if modeling it at all. Second, capital not necessary to derive our main results, which regard the quantitative importance of how trade shocks can drive imbalances, and how this feeds into the path of labor market outcomes. Nevertheless, capital dynamics may further interact with these channels, making it an important avenue for future research.

The rest of this paper is structured as follows. Section 2 outlines our model. Section 3 describes the data we use and our estimation procedure. In Section 4 we present our estimates and model fit

[6] In addition to these papers based on the Eaton and Kortum model, Helpman and Itskhoki (2010) add labor market frictions to a two-country Melitz model, and Heid and Larch (2016) add labor market frictions to an Armington model of trade. Coşar et al. (2016) incorporate search frictions and unemployment to an estimable small open economy Melitz model with firm dynamics, but focus on steady-state analyses. Ruggieri (2019) extends that model to study the transition in response to trade shocks. Finally, Kambourov (2009), Artuç et al. (2010), Dix-Carneiro (2014), and Traiberman (2019) also study transitional dynamics, but from the lenses of small open economy models.

[7] A few papers have analyzed the consequences of current account rebalancing on labor reallocation and unemployment by considering changes in imbalances as exogenous, *e.g.* Obstfeld and Rogoff (2005), Dekle et al. (2007) and Eaton et al. (2013).

[8] In International Macroeconomics, Kehoe and Ruhl (2009) and Meza and Urrutia (2011) are examples of the scarce work studying the effects of current account reversals on sluggish labor reallocation.

[9] Our model features vacancy posting decisions, which are a form of forward looking investment. This investment in vacancies to create jobs is procyclical and could in principle lead to countercyclical net exports in our model. However, this is not isomorphic to capital as a factor of production and we do not stress this feature.

analysis, but we also interpret the key mechanisms of the model by simulating a series of impulse response functions. Section 5 conducts two empirical applications of our model. We conclude and discuss future research in 6.

2 Model

Our model builds on existing workhorse models of globalization, imbalances and labor market adjustment. Trade imbalances are modelled according to the inter-temporal approach of Obstfeld and Rogoff (1995), and the trade bloc is based on Caliendo and Parro (2015). We adopt the framework in Artuç et al. (2010) to model labor mobility frictions across sectors and the structure in Mortensen and Pissarides (1994) to model search frictions and job creation and destruction. Sections 2.1 through 2.9 formalize our model showing how these different frameworks fit together.

2.1 Environment

There is no aggregate uncertainty, so that all agents have perfect foresight of aggregate variables. There are $i = 1, ..., N$ countries. Each country i has a constant labor force given by $\overline{L}_i$ workers/consumers. There are $k = 1, ..., K$ sectors. Each sector k is characterized by a continuum set of varieties $j \in [0, 1]$ that can be traded across countries. These varieties are then aggregated by perfectly competitive domestic firms, in each country, into non-tradable composite sector-specific intermediate goods according to:

$$Q_{k,i}^{I,t} = \left(\int_0^1 \left(Q_{k,i}^t (j) \right)^{\varepsilon} dj \right)^{\frac{1}{\varepsilon}}, \tag{1}$$

where $Q_{k,i}^t (j)$ is the quantity employed of variety j in sector k and country i at time t, and $\sigma = \frac{1}{1-\varepsilon}$ is the elasticity of substitution across varieties within sectors. These composite sector-specific goods are solely used as intermediate inputs for the production of a final good or for the production of varieties. The price of one unit of the composite good of sector k in country i is given by the price index associated with (1), which we denote by $P_{k,i}^{I,t}$.

A non-tradable final good is produced by perfectly competitive firms that aggregate sector-specific composite goods $Q_{k,i}^{I,t}$ according to:

$$Q_i^{F,t} = \prod_{k=1}^{K} \left(Q_{k,i}^{I,t} \right)^{\mu_{k,i}}, \tag{2}$$

where $\mu_{k,i} > 0$ and $\sum_{k=1}^{K} \mu_{k,i} = 1 \ \forall i$. The price of one unit of the final good is given by the price index associated with (2), which we denote by $P_i^{F,t}$.

2.2 Labor Markets

Workers and single-worker firms producing varieties engage in a costly search process. Firms post vacancies, but not all of them are filled. Workers search for a job, but not all of them are successful, leading to involuntary unemployment. Each variety j constitutes a different labor market. More precisely, the unemployment rate $u_{k,i}^t(j)$ is variety-specific, as is the vacancy rate $v_{k,i}^t(j)$. Both variables are expressed as a fraction of the labor force $L_{k,i}^t(j)$, measured as the sum of workers who are employed or unemployed and searching within sector k / variety j at time t. If $u_{k,i}^t(j)$ is the unemployment rate and $v_{k,i}^t(j)$ is the vacancy rate, we impose that $m_i\left(u_{k,i}^t(j), v_{k,i}^t(j)\right)$ matches are formed (as a fraction of the labor force $L_{k,i}^t(j)$), where the matching function m_i is increasing in both arguments, concave, and homogeneous of degree 1.

From now on, we drop the index j, but the reader should keep in mind that all labor market variables are country-sector-variety specific. Define labor market tightness as:

$$\theta_{k,i}^t \equiv \frac{v_{k,i}^t}{u_{k,i}^t}. \tag{3}$$

The fact that m_i is constant returns allows us to write the probability that a vacancy matches with a worker as $q_i(\theta_{k,i}^t)$, for a decreasing function q_i, and the probability that a worker matches with a vacancy as $\theta_{k,i}^t q_i(\theta_{k,i}^t)$. In addition to search frictions, workers face mobility costs across sectors, but there is free mobility across varieties j within a sector.

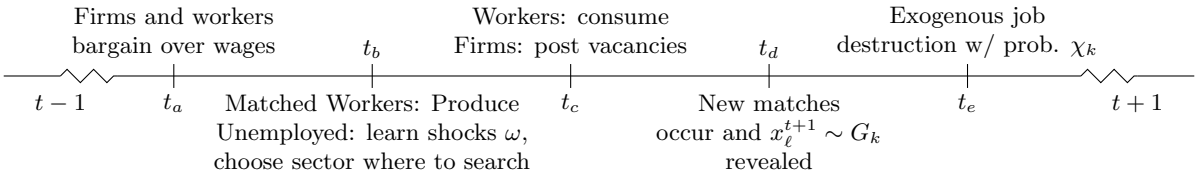
2.3 Households

Countries are organized into representative families, each with a household head that determines consumption, savings, and the allocation of workers across sectors maximizing aggregate utility. We first describe the utility of individual workers, then we show how household heads aggregate members' utilities. For ease of notation, we temporarily omit the country subscript i and let ℓ index individuals.

If a worker ended period $t-1$ unemployed in sector k she can either search in sector k at time t (at no additional cost) or incur a moving cost $C_{kk'}$ and search in sector k' at time t—so that $C_{kk}=0$. If a worker ℓ is not employed at the production stage at t, she receives preference shocks $\left\{\omega_{k,\ell}^t, k=1,...,K\right\}$ for each sector at time t. After unemployed workers receive these shocks, the household head decides whether to keep each worker in the same sector and restrict him to search there at t, or to incur a mobility cost and allow him to search in another sector. The $\omega_{k,\ell}^t$ shocks are iid across individuals, sectors and time, and are assumed to follow a Gumbel distribution with parameters $\left(-\gamma^{EM}\zeta,\zeta\right)$ where γ^{EM} is the Euler-Mascheroni constant and ζ its shape parameter. This structure follows closely Artuç et al. (2010).

After being allocated to search in sector k' at t the unemployed worker receives unemployment utility $b_{k'}$ and matches with a firm with probability $\theta^t_{k'} q(\theta^t_{k'})$. We follow Mortensen and Pissarides (1994) and assume that once a worker and a firm match at t, a match-specific productivity for $t+1$ production, x^{t+1}_ℓ, is randomly drawn from a distribution $G_{k,i}$ with $[0,\infty)$ support. This productivity is constant over time from then on. At this point, the household head or the firm can break a match if keeping it active is not optimal. Finally, at the end of every period, and following the matching process, there is an exogenous probability χ_k of existing matches to dissolve (excluding new ones). Successful matches that occur at time t only start to produce at $t+1$, and workers employed in sector k are then paid wages denoted by $w^{t+1}_k\left(x^{t+1}_\ell\right)$. Finally, if a worker produces in sector k, she receives a non-pecuniary benefit of η_k. Figure 1 details the timing of the model. Section 2.5 describes the bargaining process that occurs at t_a and section 2.4.2 describes the decision of firms to post vacancies at time t_c.

Figure 1: Timing of the Model



Given this setup, the period utility for individual ℓ at time t is,

$$\mathcal{U}^t_\ell\left(e^t_\ell, k^{t+1}_\ell, k^t_\ell, \omega^t_\ell, c^t_\ell\right) = \left(1-e^t_\ell\right)\left(-C_{k^t_\ell, k^{t+1}_\ell} + b_{k^{t+1}_\ell} + \omega^t_{k^{t+1}_\ell,\ell}\right) + e^t_\ell \eta_{k^t_\ell} + u\left(c^t_\ell\right), \tag{4}$$

where k^t_ℓ is the sector where individual ℓ starts period t (that sector was determined at $t-1$), k^{t+1}_ℓ is the $t+1$ sector of choice (which is decided at time t, interim period t_b in Figure 1), e^t_ℓ is the employment status at the production stage (interim period t_b), $\omega^t_\ell = \left(\omega^t_{1,\ell}, ..., \omega^t_{K,\ell}\right)$, and c^t_ℓ is individual consumption. If individual ℓ is unemployed in sector k at t ($e^t_\ell = 0, k^t_\ell = k$), the individual can switch sectors (from k to k^{t+1}_ℓ), so that mobility costs $C_{k^t_\ell, k^{t+1}_\ell}$, utility of unemployment $b_{k^{t+1}_\ell}$ and shock $\omega^t_{k^{t+1}_\ell,\ell}$ are incurred (during period t). On the other hand, if individual ℓ is employed during the production stage at t, $e^t_\ell = 1$, the worker enjoys the non-pecuniary benefit of working in sector k^t_ℓ given by $\eta_{k^t_\ell}$. Finally, individual ℓ also enjoys the utility of consumption $u\left(c^t_\ell\right)$.

From the perspective of the household head, the allocation of workers follows a controlled stochastic process: while the head can choose workers' sectors given knowledge of switching costs and shocks, employment itself remains a probabilistic outcome. To this end, let $\widetilde{e}^t_k\left(x^{t+1}_\ell\right) \in \{0,1\}$ indicate whether the household head continues on with a match at time t given a match productivity of x^{t+1}_ℓ in sector k. Then, the probability that worker ℓ is employed in sector k at time $t+1$,

conditional on match productivity x_ℓ^{t+1} and time t information $\left(k_\ell^t, e_\ell^t\right)$ is given by:

$$\begin{aligned}\Pr\left(k_\ell^{t+1}=k, e_\ell^{t+1}=1 | x_\ell^{t+1}, k_\ell^t, e_\ell^t\right) = & \mathcal{I}\left(k_\ell^t=k\right) e_\ell^t \left(1-\chi_k\right) \widetilde{e}_k^t\left(x_\ell^{t+1}\right) \\ & + \left(1-e_\ell^t\right) \mathcal{I}\left(k_\ell^{t+1}=k\right) \theta_k^t q\left(\theta_k^t\right) \widetilde{e}_k^t\left(x_\ell^{t+1}\right).\end{aligned} \tag{5}$$

In words, if $\mathcal{I}\left(k_\ell^t=k\right) e_\ell^t = 1$, then worker ℓ is employed in sector k at time t and the match survives with probability $(1-\chi_k)$ if the family planner decides to keep the match ($\widetilde{e}_k^t\left(x_\ell^{t+1}\right)=1$). If $e_\ell^t = 0$, that is, the worker is unemployed at t, and the planner chooses $k_\ell^{t+1}=k$, then the worker is employed in sector k at time $t+1$ with probability $\theta_k^t q\left(\theta_k^t\right) \widetilde{e}_k^t\left(x_\ell^{t+1}\right)$. Importantly, workers' sector and employment status at $t+1$, k_ℓ^{t+1} and e_ℓ^{t+1}, are determined by actions taken at t.

The head of the household aggregates (4) over family members and maximizes the net present value of utility subject to her budget constraint and the controlled process on employment (5). In addition to consumption and employment decisions, the household head has access to international financial markets by means of buying and selling one-period riskless bonds that are available in zero-net supply around the world. One can think of international bond markets in period t as spot markets in which a family buys a piece of paper with face value of B^{t+1} in exchange for a bundle of goods with the same value, and the piece of paper represents a promise to receive goods in period $t+1$ with a value equal to $R^{t+1}B^{t+1}$. International bond markets operate without frictions, so that the nominal returns R^{t+1} are equalized across countries. Finally, the household collects and aggregates profits across all firms, Π^t, but takes this lump sum payment as given. The household head chooses the path of consumption, c_ℓ^t, the path of sectoral choices, k_ℓ^t, continuation decisions, $\widetilde{e}_k^t(x)$, and bonds, B^t, to solve:

$$\max_{\left\{k_{\ell,}^t \widetilde{e}_k^t(.), B^t, c_\ell^t\right\}} E_0\left\{\sum_{t=0}^{\infty} (\delta)^t \phi^t \int_0^{\overline{L}} \mathcal{U}_\ell^t d\ell\right\}, \tag{6}$$

subject to (5) and the budget constraint:

$$P^{F,t}\int_0^{\overline{L}} c_\ell^t d\ell + B^{t+1} - \Pi^t - R^t B^t - \int_0^{\overline{L}}\left(\sum_{k=1}^{K} \mathcal{I}\left(k_\ell^t=k\right) e_\ell^t w_k^t\left(x_\ell^t\right)\right) d\ell \leq 0. \tag{7}$$

E_0 denotes expectations with respect to the idiosyncratic shocks ω. The model has no aggregate uncertainty, so that households and firms have perfect foresight of aggregate variables. δ is the discount factor, common across all countries, and ϕ^t is an intertemporal preference shifter that the household head experiences in period t. These shifters can differ across countries. The budget constraint states the family can buy consumption or bonds for next period using profits and wage income, net of interest payments (or collections) on past bonds. Let $\widetilde{\lambda}^t$ be the Lagrange multiplier on

the family's budget constraint (7).[10] The optimality condition with respect to c_ℓ^t is $u'\left(c_\ell^t\right) = \widetilde{\lambda}^t P^{F,t}$, so that individual consumption is equalized across individuals within the household: $c_\ell^t = c^t \ \forall \ell$. Henceforth, we will refer to c^t as per capita consumption. Armed with this observation, Appendix A shows that the labor supply decisions solving (6) subject to (7) and (5) can be decentralized and written recursively for unemployed and employed workers. We now turn to this recursive formulation.

Let us return to indexing countries by i. Since workers are symmetric up to x and η in each country, we stop indexing individual workers. We denote by $\widetilde{U}_{k,i}^t(\omega^t)$ the value of unemployment in sector k, country i at time t conditional on shocks ω^t, and by $W_{k,i}^t(x)$ the value of employment conditional on match-specific productivity x. If we define $\widehat{\phi}_i^{t+1} \equiv \frac{\phi_i^{t+1}}{\phi_i^t}$, the sector decision policy and the continuation rule $\widetilde{e}_k^t(.)$ must solve, conditional on shocks ω^t:

$$\widetilde{U}_{k,i}^t(\omega^t) = \max_{k'} \left(\begin{array}{c} -C_{kk',i} + \omega_{k'}^t + b_{k',i} \\ +\theta_{k',i}^t q\left(\theta_{k',i}^t\right) \delta \widehat{\phi}_i^{t+1} \int_0^\infty \max\left\{W_{k',i}^{t+1}(x), U_{k',i}^{t+1}\right\} dG_{k',i}(x) \\ +\left(1-\theta_{k',i}^t q\left(\theta_{k',i}^t\right)\right) \delta \widehat{\phi}_i^{t+1} U_{k',i}^{t+1}, \end{array} \right), \tag{8}$$

and

$$W_{k,i}^t(x) = \widetilde{\lambda}_i^t w_{k,i}^t(x) + \eta_{k,i} + \delta \widehat{\phi}_i^{t+1}(1-\chi_{k,i}) \max\left\{W_{k,i}^{t+1}(x), U_{k,i}^{t+1}\right\} + \delta \widehat{\phi}_i^{t+1} \chi_{k,i} U_{k,i}^{t+1}. \tag{9}$$

In equation (8), $U_{k,i}^t \equiv E_\omega\left(\widetilde{U}_{k,i}^t(\omega^t)\right)$ is the expected value of $\widetilde{U}_{k,i}^t(\omega^t)$ integrated over ω^t. The first line in this equation corresponds to the costs of switching sectors, $-C_{kk',i} + \omega_{k',i}^t$, as well as the sector-specific value of being unemployed in that sector $b_{k'i}$. The second line is the probability of finding a match $\theta_{k',i}^t q\left(\theta_{k',i}^t\right)$ multiplied by the discounted value of the match. Note that for low values of $W_{k',i}^{t+1}(x)$, the household head dissolves the match so that the worker obtains $U_{k',i}^{t+1}$. Finally, the third line is the discounted value of being unemployed next period if the worker fails to successfully match.

In equation (9), the first term on the left hand side is the wage function, multiplied by the household head's Lagrange multiplier on the budget constraint $\widetilde{\lambda}_i^t$. The second term is the non-pecuniary benefit of working in sector k in country i. The next terms are the continuation values: with probability $(1-\chi_{k,i})$ the match does not exogenously dissolve and the worker continues the match; with probability $\chi_{k,i}$ the match exogenously breaks and the worker receives the value of unemployment in k.

Since ω is Gumbel distributed, the policy rule for unemployed workers can be solved analytically,

[10] In an abuse of terminology we will continue to refer to $\widetilde{\lambda}^t$ as the Lagrange multiplier. However, the correct shadow price associated with period's t budget constraint is given by $\delta^t \phi^t \widetilde{\lambda}^t$.

so that transition rates between t and $t+1$ can be written as:

$$s_{kk',i}^{t,t+1} = \frac{\exp\left\{ \begin{array}{c} -C_{kk',i} + b_{k',i} + \\ \theta_{k',i}^{t} q(\theta_{k',i}^{t}) \delta \widehat{\phi}_i^{t+1} \int_0^\infty \max\left\{ W_{k',i}^{t+1}(x) - U_{k',i}^{t+1}, 0 \right\} dG_{k',i}(x) + \delta \widehat{\phi}_i^{t+1} U_{k',i}^{t+1} \end{array} \right\}}{\sum_{k''=1}^{K} \exp\left\{ \begin{array}{c} -C_{kk'',i} + b_{k'',i} + \\ \theta_{k'',i}^{t} q(\theta_{k'',i}^{t}) \delta \widehat{\phi}_i^{t+1} \int_0^\infty \max\left\{ W_{k'',i}^{t+1}(x) - U_{k'',i}^{t+1}, 0 \right\} dG_{k'',i}(x) + \delta \widehat{\phi}_i^{t+1} U_{k'',i}^{t+1} \end{array} \right\}}. \tag{10}$$

2.4 Firms

2.4.1 Incumbents

Firms produce by combining the labor of one single worker with composite intermediate inputs purchased from all sectors. All firms producing variety j in sector k and country i have access to common productivity $z_{k,i}^t(j)$ and are paid $p_{k,i}^t(j)$ for each unit of production. A firm producing variety j in sector k with match-specific productivity x and employing composite intermediate inputs $\left\{M_{\ell,i}^t\right\}_{\ell=1}^K$ obtains revenue:

$$Y_{k,i}^t(j,x) = p_{k,i}^t(j) z_{k,i}^t(j) x^{\gamma_{k,i}} \left(\prod_{\ell=1}^{K} \left(M_{\ell,i}^t\right)^{\nu_{k\ell,i}} \right)^{(1-\gamma_{k,i})}, \tag{11}$$

where $\gamma_{k,i} \in (0,1)$, $\nu_{k\ell,i} > 0$, and $\sum_{\ell=1}^{K} \nu_{k\ell,i} = 1$

Firms are price takers in product and input markets and decide on the usage of $\left\{M_{\ell,i}^t\right\}_{\ell=1}^K$ solving:

$$S_{k,i}^t(j,x) = \max_{\left\{M_{\ell,i}^t\right\}} p_{k,i}^t(j) z_{k,i}^t(j) x^{\gamma_{k,i}} \left(\prod_{\ell=1}^{K} \left(M_{\ell,i}^t\right)^{\nu_{k\ell,i}} \right)^{(1-\gamma_{k,i})} - \sum_{\ell=1}^{K} P_{\ell,i}^{I,t} M_{\ell,i}^t, \tag{12}$$

where $S_{k,i}^t(j,x)$ denotes the revenue (net of intermediate input payments) generated by the match between a firm and a worker with productivity x producing variety j. One can show that:

$$S_{k,i}^t(j,x) = \widetilde{w}_{k,i}^t(j) x, \tag{13}$$

where

$$\widetilde{w}_{k,i}^t(j) \equiv \gamma_{k,i} (1-\gamma_{k,i})^{\frac{1-\gamma_{k,i}}{\gamma_{k,i}}} \left(P_{k,i}^{M,t}\right)^{\frac{\gamma_{k,i}-1}{\gamma_{k,i}}} \left(p_{k,i}^t(j) z_{k,i}^t(j)\right)^{\frac{1}{\gamma_{k,i}}}, \tag{14}$$

$$\text{and } P_{k,i}^{M,t} \equiv \prod_{\ell=1}^{K} \left(\frac{P_{\ell,i}^t}{\nu_{k\ell,i}} \right)^{\nu_{k\ell,i}} \tag{15}$$

is the price of one unit of the intermediate input composite $im_{k,i}^t \equiv \prod_{\ell=1}^{K} \left(M_{\ell,i}^t\right)^{\nu_{k\ell,i}}$ in sector k and country i. We assume that following entry, switching across varieties within sector is costless for firms. Hence, no arbitrage across varieties will ensure that $\widetilde{w}_{k,i}^t(j) = \widetilde{w}_{k,i}^t(j')$ and $p_{k,i}^t(j) z_{k,i}^t(j) = p_{k,i}^t(j') z_{k,i}^t(j') \; \forall j, j'$. Therefore, $\widetilde{w}_{k,i}^t$ and $p_{k,i}^t z_{k,i}^t$ do not depend on the specific variety. This implies symmetry across varieties, allowing us to drop the index j identifying individual varieties. Given the expression of the surplus (13), we will henceforth refer to $\left\{\widetilde{w}_{k,i}^t\right\}$ as "sectoral surpluses". As will be become clearer in Section these sectoral surpluses will play the same role as wages do in Caliendo and Parro (2015).

Denote the wage paid by that a firm in sector k and country i with match-specific productivity x by $w_{k,i}^t(x)$. We can write the value function for incumbent firms, $J_{k,i}^t(x)$, as:

$$J_{k,i}^t(x) = \widetilde{\lambda}_i^t \left(\widetilde{w}_{k,i}^t x - w_{k,i}^t(x)\right) + (1 - \chi_{k,i}) \, \delta \widehat{\phi}_i^{t+1} \max\left\{J_{k,i}^{t+1}(x), 0\right\}. \tag{16}$$

The first term is the firm's current profit, and the second is the firm's continuation value of the match.[11] If $J_{k,i}^t(x) < 0$ the firm does not produce and exits.

2.4.2 New Entrants

Potential entrants can match with a worker by posting vacancies in k (and variety j). We assume that posting a vacancy costs $\kappa_{k,i}$ units of the final good, and so amounts to total cost $\kappa_{k,i} P_i^{F,t}$. Vacancies are posted at the interim period t_c as illustrated in Figure 1. As argued above, J is not variety specific, which means the variety distinction can also be ignored for new entrants. If a firm successfully matches with a worker at t, production starts at $t+1$. If we denote the expected value of an open vacancy by $V_{k,i}^t$, then:

$$V_{k,i}^t = -\widetilde{\lambda}_i^t \kappa_{k,i} P_i^{F,t} + \delta \widehat{\phi}_i^{t+1} \left[\begin{array}{c} q_i\left(\theta_{k,i}^t\right) \int_0^\infty \max\left\{J_{k,i}^{t+1}(s), 0\right\} dG_{k,i}(s) \\ + \left(1 - q_i\left(\theta_{k,i}^t\right)\right) \max\left\{V_{k,i}^{t+1}, 0\right\} \end{array}\right]. \tag{17}$$

The first term on the right hand side is the cost of posting vacancies, which is converted to utility units by the Lagrange multiplier $\widetilde{\lambda}_i^t$. The second term says that in the next period entrants match with probability $q_i\left(\theta_{k,i}^t\right)$ and obtain the expected value of $J_{k,i}^{t+1}$ starting in the next period, if the match is not exogenously destroyed. If they do not match (or the match is destroyed), they can post another vacancy.

There is an infinite mass of potential entrants, and there is free entry so that $V_{k,i}^t \leq 0 \; \forall k, i, t$.

[11] Firm profits are multiplied by the multiplier on the family's budget constraint in order to keep the units, utils, consistent between the firm's and worker's problem. However, if one divides $J_{k,i}^t(x)$ by $\widetilde{\lambda}_i^t$, then from the Euler Equation we derive below, it is clear that this formulation is equivalent to a risk neutral firm discounting profits using the nominal interest rate R^{t+1}.

Throughout the paper, we impose that this holds with equality in equilibrium – both in steady state and in any transitions.

2.5 Wages

The surplus of a match between a worker and a firm is defined as the utility generated by the match in excess of the parties' outside options. Imposing the free entry condition, the outside option to the firm is 0, while to the worker it is $U_{k,i}^t$. Hence, the surplus of the match is given by $\mathcal{S}_{k,i}^t(x) \equiv J_{k,i}^t(x) + W_{k,i}^t(x) - U_{k,i}^t$. We assume that firms and workers engage in Nash bargaining over the surplus, with the workers' bargaining weight given by $\beta_{k,i}$. This leads to $J_{k,i}^t(x) = \beta_{k,i}\mathcal{S}_{k,i}^t(x)$, which combined with equations (9) and (16) imply that $\mathcal{S}_{k,i}^t$, $W_{k,i}^t$, and $J_{k,i}^t$ are increasing functions of x, implying that matches only remain active at t if $x > \underline{x}_{k,i}^t$ where $\underline{x}_{k,i}^t$ solve:

$$\mathcal{S}_{k,i}^t\left(\underline{x}_{k,i}^t\right) = J_{k,i}^t\left(\underline{x}_{k,i}^t\right) = W_{k,i}^t\left(\underline{x}_{k,i}^t\right) - U_{k,i}^t = 0 \tag{18}$$

The wage equation resulting from Nash bargaing is:

$$w_{k,i}^t(x) = \beta_{k,i}\widetilde{w}_{k,i}^t x + (1-\beta_{k,i})\frac{\left(U_{k,i}^t - \delta\widehat{\phi}_i^{t+1}U_{k,i}^{t+1} - \eta_{k,i}\right)}{\widetilde{\lambda}_i^t}. \tag{19}$$

This is similar to the standard wage equation in search models: the worker's wage is a weighted average of output, and a function of their outside option. The only new term is the non-pecuniary benefit, which is subtracted from the outside option. By integrating wages across all individuals in the economy, one can solve for the family's total wage income.

2.6 Labor Market Dynamics

Since workers can switch sectors between periods t_a and t_c, the sector-specific unemployment rates actually differ at these two points in time within the same period. To this end, we first define the beginning of period sector-specific unemployment rate as $\widetilde{u}_{k,i}^{t-1}$, and labor force as $L_{k,i}^{t-1}$. After workers switch sectors, (measured before matching at t_d), we define $u_{k,i}^t$ to be the share of sector-k workers searching for a job. It is given by:

$$u_{k,i}^t = \frac{\sum_{\ell=1}^{K} L_{\ell,i}^{t-1}\widetilde{u}_{\ell,i}^{t-1}s_{\ell k,i}^{t,t+1}}{L_{k,i}^t}, \tag{20}$$

where $L_{k,i}^t$ is the number of workers in sector k at t (more precisely at t_c) and is equal to:

$$L_{k,i}^t = L_{k,i}^{t-1} + \underbrace{\sum_{\ell \neq k} L_{\ell,i}^{t-1} \widetilde{u}_{\ell,i}^{t-1} s_{\ell k,i}^{t,t+1}}_{\text{Inflow}} - \underbrace{L_{k,i}^{t-1} \widetilde{u}_{k,i}^{t-1} \left(1 - s_{kk,i}^{t,t+1}\right)}_{\text{Outflow}}, \tag{21}$$

where the second term in the right hand side is the flow of workers into sector k and the third terms is the flow of workers out of sector k.

Only firms with $x \geq \underline{x}_{k,i}^{t+1}$ produce at $t+1$. Therefore, the *number* of jobs created in sector k is given by:

$$JC_{k,i}^t = L_{k,i}^t u_{k,i}^t \theta_{k,i}^t q_i \left(\theta_{k,i}^t\right) \left(1 - G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right), \tag{22}$$

and the number of jobs destroyed is given by:

$$\begin{aligned} JD_{k,i}^t &\equiv \left(\chi_{k,i} + (1-\chi_{k,i}) \Pr\left(\underline{x}_{k,i}^t \leq x < \underline{x}_{k,i}^{t+1} | \underline{x}_{k,i}^t \leq x\right)\right) L_{k,i}^{t-1} \left(1 - \widetilde{u}_{k,i}^{t-1}\right) \\ &= \left(\chi_{k,i} + (1-\chi_{k,i}) \max\left\{\frac{G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right) - G_{k,i}\left(\underline{x}_{k,i}^t\right)}{1 - G_{k,i}\left(\underline{x}_{k,i}^t\right)}, 0\right\}\right) L_{k,i}^{t-1} \left(1 - \widetilde{u}_{k,i}^{t-1}\right), \end{aligned} \tag{23}$$

where $\Pr\left(\underline{x}_{k,i}^t \leq x < \underline{x}_{k,i}^{t+1} | \underline{x}_{k,i}^t \leq x\right)$ is the share of active firms above the productivity threshold at t but below at $t+1$ (endogenous exit). Consequently, the rate of unemployment at the end of period t, after job creation and job destruction, is given by:

$$\widetilde{u}_{k,i}^t = \frac{L_{k,i}^t u_{k,i}^t - JC_{k,i}^t + JD_{k,i}^t}{L_{k,i}^t}. \tag{24}$$

Equations (20)-(24) describe the evolution of labor market stocks over time. In any given period, these stocks are bound by the labor market clearing condition:

$$\sum_{k=1}^{K} L_{k,i}^t = \overline{L}_i. \tag{25}$$

2.7 International Trade

Our model of international trade closely follows Caliendo and Parro (2015). Varieties are traded across countries. Under the assumptions of perfect competition and iceberg trade costs, the cost of variety j from sector k produced in o can be purchased in country i at a price $p_{k,o}^t(j)\, d_{k,oi}^t$, where the first term is the domestic price of the variety and the second is the iceberg term. From (14) we can write:

$$p_{k,i}^t(j) = \frac{c_{k,i}^t}{z_{k,i}^t(j)}, \tag{26}$$

for each variety j and where

$$c_{k,i}^{t} \equiv \left(\frac{\widetilde{w}_{k,i}^{t}}{\gamma_{k,i}}\right)^{\gamma_{k,i}} \left(\frac{P_{k,i}^{M,t}}{1-\gamma_{k,i}}\right)^{1-\gamma_{k,i}}. \tag{27}$$

Recall that due to the no-arbitrage condition we impose across varieties, $c_{k,i}^{t}$ is equalized within sectors, so that price variation across varieties is dictated by $z_{k,i}^{t}(j)$.

We assume that in any country i, sector k and period t, the productivity component $z_{k,i}^{t}(j)$ is independently drawn from a Frechet distribution with scale parameter $A_{k,i}^{t}$ – which is country, sector, and time specific – and shape parameter, λ, which is time invariant.[12] Consumers buy the lowest cost variety across countries, treating the same variety from different origins as perfect substitutes. Under these assumptions, it can be shown that the resulting price index for the composite sector-specific intermediate good 1) is given by:

$$P_{k,i}^{I,t} = \Gamma_{k,i}\left[\sum_{o=1}^{N} \frac{A_{k,o}^{t}}{\left(c_{k,o}^{t} d_{k,oi}^{t}\right)^{\lambda}}\right]^{-1/\lambda}, \tag{29}$$

where $\Gamma_{k,i}$ is a constant (see Caliendo and Parro, 2015). In turn, the price of the final good (2) is given by:

$$P_{i}^{F,t} = \prod_{k=1}^{K}\left(\frac{P_{k,i}^{I,t}}{\mu_{k,i}}\right)^{\mu_{k,i}}. \tag{30}$$

If we define the object $\Phi_{k,i}^{t} = \sum_{o=1}^{N} \frac{A_{k,o}^{t}}{\left(c_{k,o}^{t} d_{k,oi}^{t}\right)^{\lambda}}$ (which is proportional to $(P_{k,i}^{I,t})^{-\lambda}$) then consumers in country i spend a share $\pi_{k,oi}^{t}$ of their sector-k expenditures on varieties from country o given by:

$$\pi_{k,oi}^{t} \equiv \frac{E_{k,oi}^{t}}{E_{k,i}^{t}} = \frac{A_{k,o}^{t}\left(c_{k,o}^{t} d_{k,oi}^{t}\right)^{-\lambda}}{\Phi_{k,i}^{t}} \tag{31}$$

where $E_{k,i}^{t}$ is total expenditure of country i on sector k varieties and $E_{k,oi}^{t}$ is total expenditure of country i on sector k varieties produced by country o: $E_{k,i}^{t} = \sum_{o=1}^{N} E_{k,oi}^{t}$. Market clearing requires that total revenue coming from the production of varieties in sector k and country o must be equal to sales to all countries $i = 1, ..., N$, and so:

[12] The CDF for the Frechet is given by,

$$F_{k,i}^{t}(z) = \exp\left(-A_{k,i}^{t} \times z^{-\lambda}\right). \tag{28}$$

$$Y_{k,o}^t = \sum_{i=1}^{N} E_{k,oi}^t = \sum_{i=1}^{N} \pi_{k,oi}^t E_{k,i}^t. \tag{32}$$

Let $C_i^t \equiv \overline{L}_i c_i^t$ denote total consumption of the final good. Write $E_{it}^C \equiv P_i^{F,t} C_i^t$ to denote total consumption expenditure with final goods, and let $E_{k,i}^{V,t}$ be the total expenditure of sector k in country i on vacancy posting costs. We can write $E_{k,i}^t$ as:

$$E_{k,i}^t = \mu_{k,i} E_i^{C,t} + \mu_{k,i} \sum_{\ell=1}^{K} E_{\ell,i}^{V,t} + \sum_{\ell=1}^{K} \left(1 - \gamma_{\ell,i}\right) \nu_{\ell k,i} Y_{\ell,i}^t. \tag{33}$$

In turn, let I_i^t denote total income in country i. Total income is given by the portion of revenue that is not devoted to intermediate good payments minus vacancy posting costs:

$$I_i^t = \sum_{\ell=1}^{K} \left(\gamma_{\ell,i} Y_{\ell,i}^t - E_{\ell,i}^{V,t} \right). \tag{34}$$

The gap between country i's total income I_i^t and total final good expenditure $E_i^{C,t}$ is a trade imbalance and is defined as:

$$NX_i^t \equiv I_i^t - E_i^{C,t}. \tag{35}$$

We can therefore rewrite (33) as:

$$E_{k,i}^t = \mu_{k,i} \left(\sum_{\ell=1}^{K} \gamma_{\ell,i} Y_{\ell,i}^t - NX_i^t \right) + \sum_{\ell=1}^{K} \left(1 - \gamma_{\ell,i}\right) \nu_{\ell k,i} Y_{\ell,i}^t. \tag{36}$$

Finally, labor market clearing dictates that total revenues coming from the production of varieties in sector k and country i is given by:

$$Y_{k,i}^t = \frac{\widetilde{w}_{k,i}^t}{\gamma_{k,i}} L_{k,i}^{t-1} \left(1 - \widetilde{u}_{k,i}^{t-1}\right) \int_{\underline{x}_{k,i}^t}^{\infty} \frac{s}{1 - G_{k,i}\left(\underline{x}_{k,i}^t\right)} dG_{k,i}(s). \tag{37}$$

We now model how NX_i^t is determined in equilibrium.

2.8 Trade Imbalances

Bonds are in zero net supply, $\sum_i B_i^t = 0$, and initial conditions are given by $W_i^0 \equiv R^0 B_i^0$ with $\sum_{i=1}^{N} W_i^0 = 0$. The solution to the household head's problem (6) must satisfy the following Euler

Equation:

$$\frac{u'(c_i^t)/P_i^{F,t}}{u'(c_i^{t+1})/P_i^{F,t+1}} = \delta \widehat{\phi}_i^{t+1} R^{t+1}. \tag{38}$$

If we assume log utility, $u(c) = \ln(c)$, the Euler equations become:

$$P_i^{F,t+1} C_i^{t+1} = \delta \widehat{\phi}_i^{t+1} R^{t+1} P_i^{F,t} C_i^t. \tag{39}$$

Therefore, bond returns are given by:

$$R^{t+1} = \frac{1}{\delta} \frac{\sum_{i=1}^N \frac{P_i^{F,t+1} C_i^{t+1}}{\widehat{\phi}_i^{t+1}}}{\sum_{i=1}^N P_i^{F,t} C_i^t} = \frac{1}{\delta} \frac{\sum_{i=1}^N \frac{E_i^{C,t+1}}{\widehat{\phi}_i^{t+1}}}{\sum_{i=1}^N E_i^{C,t}}. \tag{40}$$

Finally, we need a condition linking the financial and goods markets in each country:

$$NX_i^t = I_i^t - P_i^{F,t} C_i^t = B_i^{t+1} - R^t B_i^t. \tag{41}$$

It is worthwhile to note that the model implies that steady-state trade deficits depend on countries' initial levels of wealth. Denote T_{SS} as the time required for the steady state to be achieved, and use equation (7) to derive the intertemporal budget constraint. We obtain:

$$NX_i^{T_{SS}} = -\frac{1-\delta}{\delta} \left(\prod_{\tau=1}^{T_{SS}-1} (R^\tau)^{-1} \right)^{-1} \left(B_i^0 + \sum_{t=1}^{T_{SS}-1} \left(\prod_{\tau=1}^{t} (R^\tau)^{-1} \right) NX_i^t \right), \tag{42}$$

which means that countries with high levels of initial wealth ($R^0 B_i^0 = \frac{1}{\delta} B_i^0$) are able to sustain trade deficits in steady state ($NX_i^{T_{SS}} < 0$).

2.9 Equilibrium

An equilibrium in this model is a set of initial allocations, $\{L_{k,i}^0, \underline{x}_{k,i}^0, B_i^0, \}$, a final steady state, $\{L_{k,i}^\infty, \underline{x}_{k,i}^\infty, B_i^\infty, \}$ and sequences of policy functions for workers/firms, $\{s_{kk',i}^t, \underline{x}_{k,i}^t, w_{k,i}^t(x)\}$, value functions for workers/firms, $\{U_{k,i}^t, W_{k,i}^t, J_{k,i}^t\}$, labor market tightnesses, $\{\theta_{k,i}^t\}$, bond decisions by the households, B_i^t, allocations, $\{L_{k,i}^t, u_{k,i}^t, c_{k,i}^t\}$, profits and household consumption, $\{\Pi_i^t, C_i^t\}$, trade shares, $\left\{\pi_{k,io}^t\right\}$, and sectoral surpluses, $\{\widetilde{w}_{k,i}^t\}$ such that:

1. Worker and firms' value functions solve (8), (9), and (16).

2. Prices and labor market tightness ensure the free entry condition holds in each country and sector:

$$V_{k,i}^t = 0 \ \forall k, i, t$$

3. The wage equation solves the Nash bargaining problem and is given by (19).

4. Allocations and unemployment rates evolve according to (20), (21), (24).

5. Consumption and bonds decisions solve the household's dynamic consumption-savings problem (6)-(7) + (5).

6. Goods markets clear: equations (32)-(34) are met.

7. Labor markets clear: $\sum_{k=1}^{K} L_{k,i}^{t} = \overline{L}_i$.

8. Bonds markets clear: $\sum_{i=1}^{N} B_i^t = 0$.

9. The initial and final steady state equilibria satisfy equations (A.15)-(A.36) in Appendix B.

We set world total gross production of varieties as the numeraire, that is, $\sum_{i=1}^{N}\sum_{k=1}^{K} Y_{k,i}^{t} = 1$ for all t. Therefore, all nominal aggregate variables are expressed in terms of shares of world gross production.

3 Data and Estimation

Our main source of cross-country data is the World Input Output Database (WIOD), which compiles data from individual National Accounts combined with bilateral international trade data. These data cover 56 sectors and 44 countries, including a Rest of the World aggregate, between 2000 and 2014. We divide the economy into six sectors and six countries. We consider a world comprised by the United States, China, Europe, Asia/Oceania, the Americas and a Rest of the World aggregate constructed by the WIOD. Each country's economic activity consists of six sectors: Agriculture; Low-, Mid- and High-Tech Manufacturing; Low- and High-Tech Services. Tables 1 and 2 detail these divisions. We solve our model at the quarterly frequency.

The WIOD dataset allow us to generate various moments and statistics that are used in the method of simulated moments procedure we use to estimate a subset of the parameters (more details below): (a) employment shares across sectors and countries; (b) average wages across sectors and countries; (c) trade shares; (d) net exports.

Other moments used in the estimation procedure are obtained from ILOSTAT and the United States Current Population Survey (CPS). From ILOSTAT, we extract country-specific unemployment rates. From the CPS, we compute yearly transition rates between sectors in the United States, as well as the dispersion of wages. Table 3 summarizes the statistics used in the estimation.

Table 4 summarizes the parameters we need to numerically solve the model. We split them into three categories: (i) parameters that are fixed at values previously reported in the literature

Table 1: Country Definitions

1	USA
2	China
3	Europe
4	Asia/Oceania
5	Americas
6	Rest of the World (ROW)

Notes: Asia/Oceania = {Australia, Japan, South Korea, Taiwan}, Americas = {Brazil, Canada, Mexico}, Rest of the World ={Indonesia, India, Russia, Turkey, Rest of the World}

Table 2: Sector Definitions

1	Agriculture/Mining	Agriculture, Forestry and Fishing; Mining and quarrying
2	Low-Tech Manufacturing	Wood products; Paper, printing and publishing; Coke and refined petroleum; Basic and fabricated metals; Other manufacturing
3	Mid-Tech Manufacturing	Food, beverage and tobacco; Textiles; Leather and footwear; Rubber and plastics; Non-metallic mineral products
4	High-Tech Manufacturing	Chemical products; Machinery; Electrical and optical equipment; Transport equipment
5	Low Tech Services	Utilities; Construction; Wholesale and retail trade; Transportation; Accommodation and food service activities; Activities of households as employers
6	Hi Tech Services	Publishing; Media; Telecommunications; Financial, real estate and business services; Government, education, health

Table 3: Summary of Statistics Used in the MSM Procedure

Statistic	Source
Employment allocations across sectors and countries	WIOD
Average wages across sectors and countries	WIOD
Trade shares across country pairs and sectors	WIOD
Net exports across countries	WIOD
Unemployment rates across countries	ILOSTAT
Coefficient of variation of log-wages in the United States	CPS
Yearly transition rates between sectors and from and to unemployment for the United States	CPS

because they are difficult or impossible to be identified given the available data (Panel A); (ii) parameters that can be determined outside of the model (Panel B); and (iii) parameters that are estimated by the method of simulated moments, where we target the moments shown in Table 3. [13]

We follow Artuç et al. (2010) and impose the dispersion of ω shocks, ζ_i, to be equal to 1.63 and common across countries. Given that we solve our model at the quarterly frequency, we impose the discount factor to be $\delta = 0.9924$, leading to an annual discount factor of 0.97. The matching function is parameterized according to Coşar et al. (2016), so we impose $\xi_i = 1.63$ and common across countries. The Frechet scale parameter $\lambda = 4$ comes from Simonovska and Waugh (2014).[14] Finally, we follow standard practice in the search literature and set $\beta_{k,i} = 0.5$ ((Mortensen and Pissarides, 1999)). As described in Panel B, the WIOD dataset allows us to directly extract various parameters of the model such as country-specific final expenditure shares $\mu_{k,i}$, labor expenditure shares $\gamma_{k,i}$ and input-output matrices $\nu_{k\ell,i}$; this is due to the Cobb-Douglas assumptions we imposed in both the production functions and how final consumption is aggregated.

We impose a few restrictions on the parameters estimated by the simulated method of moments. First, the identification of mobility costs $C_{k\ell,i}$ relies on good quality data on both wages and yearly inter-sectoral transition rates. Although these data can be found in household surveys across a few of the countries we consider, we opted to identify these parameters using transition rates for the United States only. Therefore, we impose mobility costs to be common across countries. Second, the dispersion of match productivities $\sigma^2_{k,i}$ is imposed to be the same across sectors and countries, and is identified by the cross-sectional coefficient of variation of log-wages in the CPS. Third, we impose that the exogenous exit rates $\chi_{k,i}$ are decomposed into a country component χ_i and a sector component χ_k (that is, $\chi_{k,i} = \chi_i + \chi_k$).[15] Fourth, the utility of unemployment $b_{k,i}$ is imposed to be country specific (that is $b_{k,i} = b_i$). Finally, to achieve identification, we need to impose $C_{kk} = 0$ for all k and $\eta_{k_0,i} = 0$ for some sector k_0 (we picked $k_0 = Agriculture$).

We estimate the parameters described in Panel C using the method of simulated moments. Let $\Theta = (\Theta_1, ..., \Theta_N)$ be the vector of these country-specific parameters. Our estimation procedure assumes that the economy is in steady state in 2000 and conditions on the observed 2000 trade shares $\pi^{Data}_{k,oi}$ and net exports NX^{Data}_i – so these moments are perfectly matched. Appendix B summarizes the equations that must be satisfied in a steady state equilibrium. For a given guess of Θ, we use our equilibrium equations, conditional on $\pi^{Data}_{k,oi}$ and NX^{Data}_i, to generate: (a) unemployment rates across countries; (b) employment allocations and average wages across sectors and countries; (c)

[13] To be more precise, we do not target trade shares and net exports, we impose them in the estimation, so that they are matched exactly.

[14] Caliendo and Parro (2015) find a similar value for λ when it is imposed to be equal across sectors: 4.55.

[15] The sector-specific component χ_k is identified from US transitions from sector-specific employment to unemployment. The country specific components are then adjusted to better match the country-specific unemployment rates.

Table 4: Summary of Parameters

Panel A. Fixed According to the Literature			
Parameter	Value	Description	Source
δ	0.9924	Discount factor	Artuç et al. (2010)
ζ_i	1.63	Dispersion of ω shocks	Artuç et al. (2010)
ξ_i	1.84	Matching Function	Coşar et al. (2016)
λ	4	Frechet Scale Parameter	Simonovska and Waugh (2014)
$\beta_{k,i}$	0.5	Worker Bargaining Power	Mortensen and Pissarides (1999)

Panel B. Estimated Outside of the Model		
Parameter	Description	Source
$\mu_{k,i}$	Final Expenditure Shares	WIOD
$\gamma_{k,i}$	Labor Expenditure Shares	WIOD
$\nu_{k\ell,i}$	Input-Output Matrix	WIOD

Panel C. Estimated by Method of Simulated Moments	
Parameter	Description
$\widetilde{\kappa}_{k,i}$	Vacancy Costs
$\chi_{k,i}$	Exogenous Exit
$\sigma^2_{k,i}$	$G_{k,i}$
$\eta_{k,i}$	Sector-specific Utility
$C_{kk'}$	Mobility Costs
$b_{k,i}$	Unemployment Utility

Note: Artuç et al. (2010) use an annual discount factor of $\delta = 0.97$. Since we work at the quarterly frequency, we use $\delta = 0.97^{\frac{1}{4}}$.

yearly transition rates between sectors and from and to unemployment in the United States; and (d) cross-sectional wage dispersion in the United States. The estimation procedure searches for a vector of parameters Θ that minimizes the distance between the model generated moments and those we measure in the data.

A convenient aspect of our approach is that, by conditioning on the observed 2000 trade shares and normalizing world output $\sum_k \sum_i Y_{k,i} = 1$, we can solve for sector-country output levels $\{Y_{k,i}\}$ that are independent of Θ. This property allows us to estimate the model country by country, greatly simplifying the estimation procedure. Indeed, if all parameters were country specific and we had the same set of data for each country, estimation could be conducted in parallel for each country. Another convenient aspect of conducting the estimation conditional on the observed trade shares is that we do not have to estimate the technology parameters $A_{k,i}$ and trade costs $d_{k,oi}$.[16] This allows us to tractably integrate richer labor market dynamics, such as search, in a way that is consistent with the general equilibrium trade forces in the model. As we describe in the next section, we develop algorithms to perform counterfactual responses to shocks in these parameters

[16]The key assumption is that vacancies are priced in terms of the final good, as described in section 2.4.2.

relying on the exact hat algebra approach in Dekle et al. (2007) and Caliendo and Parro (2015).

Although the model could be flexibly estimated country by country if we had data on yearly transitions across sectors for all countries, we impose, in this paper, $C_{k\ell,i}$ and $\sigma^2_{k,i}$ to be constant across countries and identified off the United States' CPS transition rates and wage dispersion data. Therefore, we proceed in two steps. First, we estimate the model for the United States, back out $C_{k\ell}$, σ^2_k and χ_k. In a second step, we estimate the rest of the parameters in parallel, country by country, conditional on $C_{k\ell}$, σ_k, and χ_k.

At this point, it is worthwhile mentioning that our estimation approach, which does not estimate $A_{k,i}$ and trade costs $d_{k,oi}$, cannot recover $\kappa_{k,i}$. Instead, we are able to recover $\widetilde{\kappa}_{k,i} \equiv \frac{\kappa_{k,i} P_i^F}{\widetilde{w}_{k,i}}$. Our counterfactual algorithms then allows $\widetilde{\kappa}_{k,i}$ to respond to shocks, using exact hat algebra. It is also important to clarify that identifying $\widetilde{\kappa}_{k,i}$ is difficult in practice. Even though the objective function is not flat with respect to $\widetilde{\kappa}_{k,i}$, it tends to be relatively flat for a wide range of values. For that reason, we follow Hagedorn and Manovskii (2008) and impose that the cost of posting one vacancy $\kappa_{k,i} P_i^F$ equals $0.58 \times \overline{w}_{k,i}$, where $\overline{w}_{k,i}$ is the average wage in sector k and in country i. Hagedorn and Manovskii (2008) base their calibration on a combination of the labor and capital costs of posting vacancies in a single-sector search model.[17] While their model and ours differ on several dimensions – e.g., multiple sectors and match heterogeneity – there are two reasons we turn to their calculation. First, Hagedorn and Manovskii combine micro and macro data from multiple sources, mapping their numbers to observable costs of recruitment, capital purchasing, and other costs; second, their calibration of vacancy posting costs does not rely on other parameters of the model, just observable data (e.g., the capital share of income and the unemployment rate), so that the estimate is not contaminated by the fact that our other parameters are not always estimated to be the same as Hagedorn and Manovskii's. We impose this constraint by adding the deviations between $\left(\widetilde{\kappa}_{k,i} \times \widetilde{w}_{k,i}\right) / \overline{w}_{k,i} = \left(\kappa_{k,i} \times P_i^F\right) / \overline{w}_{k,i}$ and 0.58 in our method of simulated moments objective function. The full estimation algorithm is described in Appendix C.1.

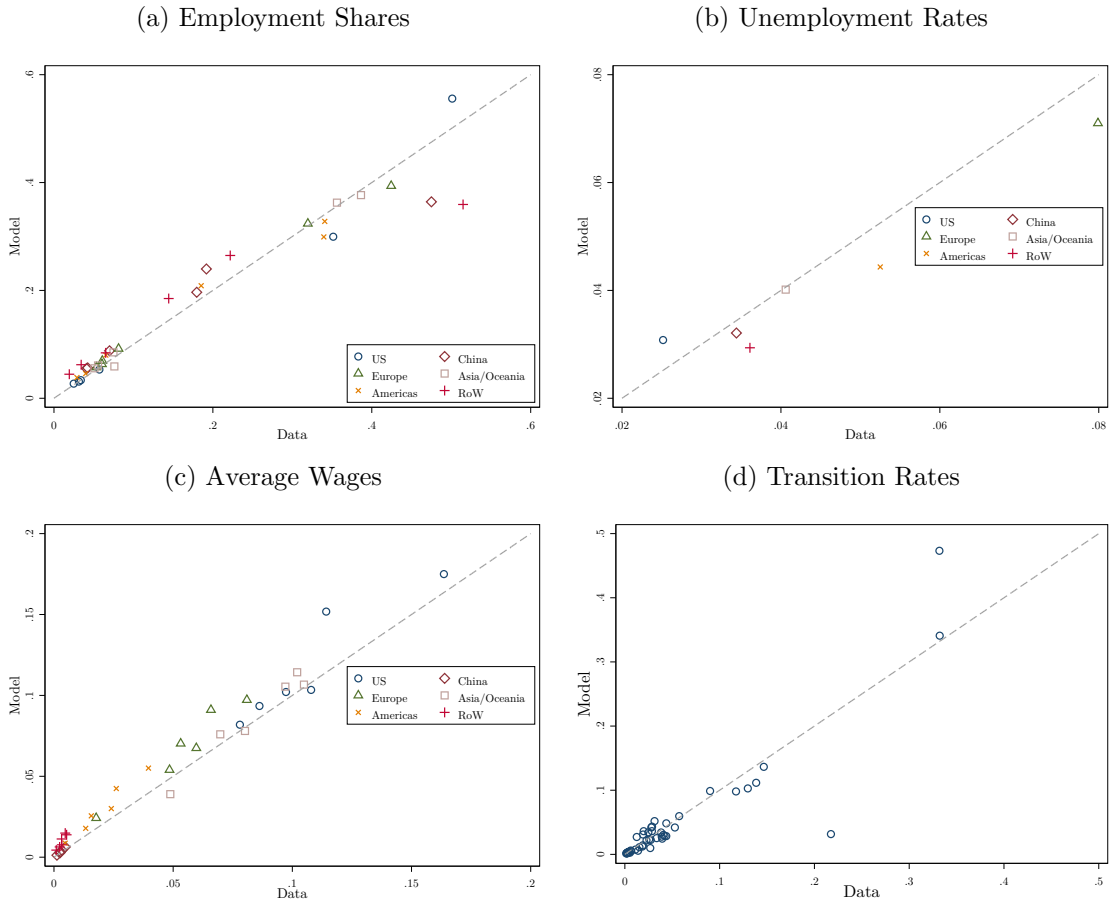
4 Estimation Results

4.1 Estimates and Model Fit

The estimated parameters can be found in the Appendix, in Tables A.1 through A.8. We first discuss the parameters that are obtained outside of the model. The online data Appendix details how these are computed with data from the WIOD. Table A.1 displays the final expenditure shares $\mu_{k,i}$. We can separate the countries in this table in two groups with similar expenditure shares: (1)

[17] As the authors point out, in the steady state of a single sector search model, capital is equivalent to interpreting vacancies as capital and reinterpreting the productivity process. In this sense, a vacancy is a metaphor for the whole cost of hiring a worker – the flow cost of capital, HR efforts, etc.. We adopt a similar view in our paper, without modeling each piece.

Figure 2: Model Fit



United States, Europe, and Americas; and (2) China and Rest of the World. The biggest difference between these two countries is that China and the Rest of the World spend a much larger share of their disposable income on Agricultural goods and significantly lower share on High-Tech Services.

Table A.2 displays the share of output devoted to labor payments. This share varies mostly within countries and across sectors and ranges between 0.24 (in High-Tech Manufacturing in China) and 0.68 (in High-Tech Services in RoW). Finally, Table A.3 displays the average across countries of the input-output matrices. As is well known, the diagonal elements tend to be larger than the off-diagonal elements.

Our mobility costs are displayed in Table A.4. Artuç et al. (2010) express their mobility costs estimates as multiples of average wages. In contrast, in our model, workers compute their value functions based on $\widetilde{\lambda}_i \times \widetilde{w}_{k,i}$, and not just based on sector-specific average wages. Therefore, to make our estimates comparable to those in Artuç et al. (2010), we express mobility costs as a multiple of $\widetilde{\lambda}_i \times \overline{w}_{US}$, where $\overline{w}_{US}$ is the average wage in the US (in the data). We obtain that our

estimated mobility costs across sectors (as a fraction of $\widetilde{\lambda}_i \times \overline{w}_{k,i}$) range from virtually zero to a maximum of 3.1. These numbers are of the same order of magnitude, but lower than what Artuç et al. (2010) estimate (in their preferred specification, mobility costs are 6 times average wages). We believe that two key differences explain this discrepancy: our model incorporates search frictions and sector-specific utility terms $\eta_{k,i}$, which are absent in their model, and should be contributing to our lower estimates of mobility costs. Finally, Table A.7 shows that the values of unemployment b_i tend to be negative across all countries. This is consistent with estimates in other models featuring search frictions and unemployment, such as Meghir et al. (2015).

Figure 2 shows the model fit for the various moments we target: (a) employment shares across sectors and countries; (b) unemployment rates across sectors and countries; (c) average wages across sectors and countries; and (d) inter-sectoral transition rates in the United States. As a general rule, the model matches the targeted moments well. Perhaps the main exception is the persistence in unemployment. 21% of unemployed workers remain unemployed the next year, whereas our model predicts that this share is much lower: 3%.

4.2 Impulse Response Functions

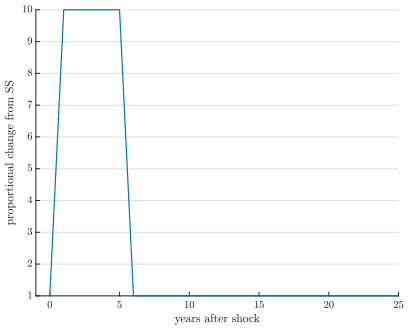
To understand the mechanisms in our model, we study the model's behavior in response to three sets of shocks. First, we subject the model to a tenfold increase in $A_{k,China}$, the productivity location parameter in China, uniform across sectors k and lasting for five years before turning back to its original level – the magnitude of this shock is in line with the size of actual changes in Chinese productivity that we recover in section 5.1. Next, we feed the model with a tenfold permanent increase in $A_{k,China}$ that happens once and for all at $t = 1$. Finally, we simulate a slow-moving and linear increase in $A_{k,China}$ that reaches a tenfold increase in 15 years. It remains at that level from there on. These shocks are illustrated in Figure 3. To highlight the quantitative and qualitative importance of modelling trade imbalances, we study the behavior of the complete model with bonds as well as the behavior of a model without bonds, where trade is balanced in every period – that is $NX_i = 0 \ \forall i$, see Appendix C.3 for details on this procedure.

We start with the temporary shock depicted in Figure 3a. The evolution of net exports in Figure 4 aligns with what we would have expected from standard macroeconomic models with inter-temporal decisions. The temporary boost in productivity, and therefore income, induces China to save in the short run by running a large trade surplus. As productivity reverts back to its initial level, China starts to sustain a small, but permanent trade deficit of 1.2% of GDP, as it consumes the return on its savings. On the other hand, the remaining countries borrow from China, running trade deficits in the short run. As the productivity increase in China vanishes, all countries start to sustain permanent trade surpluses, repaying their debts to China.

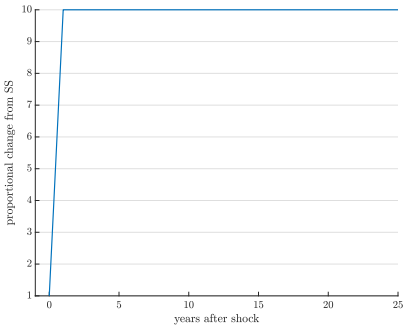
While these aggregate patterns appear standard, this productivity shock has varied impacts on

Figure 3: Shocks to Chinese Productivity $\widehat{A}_{k,China}$ – Uniform Across Sectors

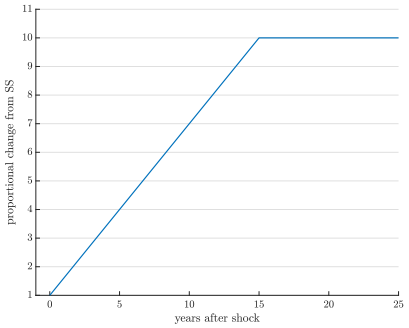
(a) Temporary



(b) Once-And-For-All



(c) Slow Moving



the more disaggregated economy. Specifically, the rise in productivity induces substantial labor reallocation across sectors. We measure reallocation according to:

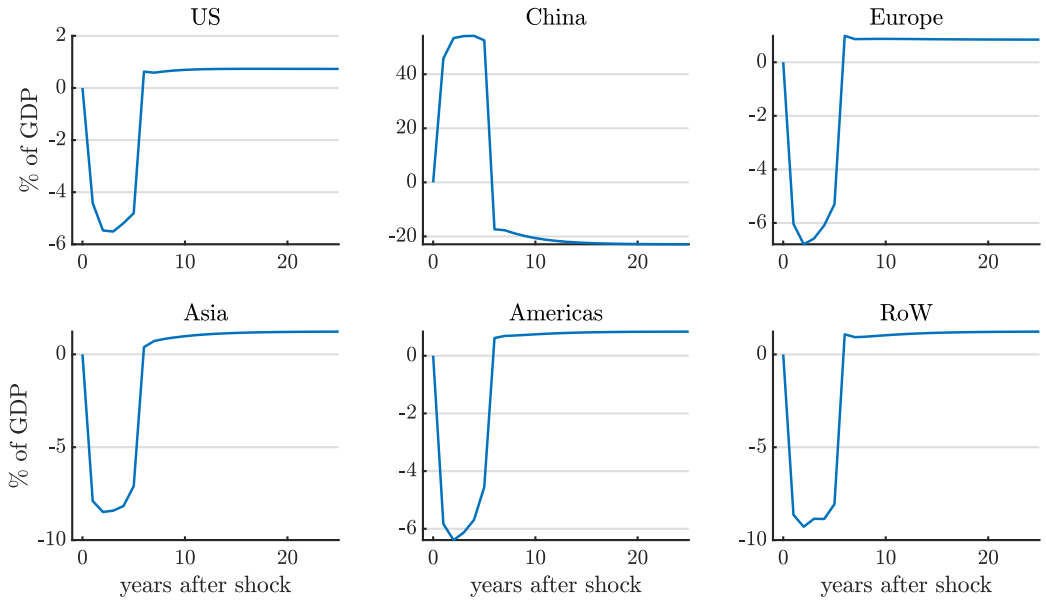
$$Reallocation_i^t = \frac{1}{2}\sum_{s=1}^{t}\sum_{k=1}^{J}\left|\frac{L_{i,k}^s}{L_i^s} - \frac{L_{i,k}^{s-1}}{L_i^{s-1}}\right|, \tag{43}$$

which accumulates yearly changes in employment shares over time.

Figure 5a plots the evolution of the reallocation index (43) across countries. When we consider our complete model with bonds, there is an immediate and steep rise in reallocation in all countries following China's productivity jump. After productivity returns to its initial level, reallocation continues for several years before reaching its final steady-state level. Even though we consider a uniform productivity shock across sectors, we observe substantial labor reallocation within China and within other countries.[18] The reason that uniform productivity changes can have large impacts

[18]Note that in a closed economy without mobility costs and search frictions, we would have seen no reallocation effects given homothetic preferences and our production structure as relative prices would not change. However, the presence of heterogeneous mobility costs across sectors could lead to some reallocation effects as the option value of search would change differentially across sectors. However, based on Shimer (2005) we expect this effect to be quantitatively small.

Figure 4: Temporary Productivity Growth in China (Figure 3a) – Net Exports Over GDP



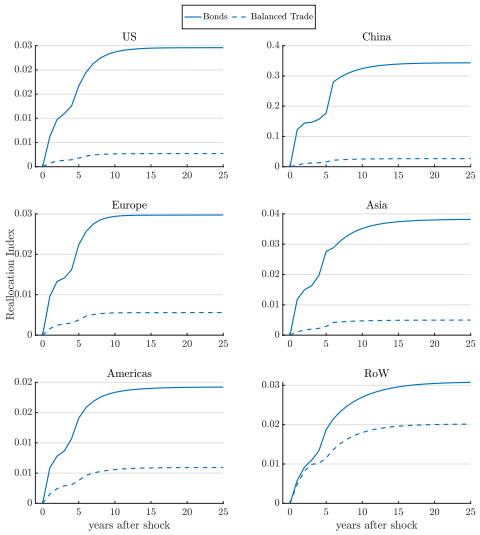
on labor reallocation is trade across countries, which adds asymmetries to how sectors respond to the same productivity shock. Our model features two sources of asymmetries. First, heterogeneity in trade costs implies that the tradability of goods differs across countries and sectors. As explained by Dornbusch et al. (1977), transfers (here, net exports) across countries alter the terms of trade and therefore the allocation of economic activity within each country. Second, countries differ in their final consumption bundles which implies that the global distribution of final consumption expenditure matters for the allocation of workers across sectors. Relatedly, heterogeneity in input-output linkages across countries also generate changes in countries' terms of trade and global expenditure patterns.

The increase in reallocation is directly tied to an increase in unemployment in all countries, as shown in Figure 5b. This highlights a key mechanism in our model: even positive labor demand shocks can increase unemployment through asymmetric shifts in labor demand. Once again, there are two reasons this can occur. First, contracting sectors will destroy low productivity jobs, dislocating workers who must switch sectors. Second, it takes time for workers who switch into expanding sectors to find new jobs.

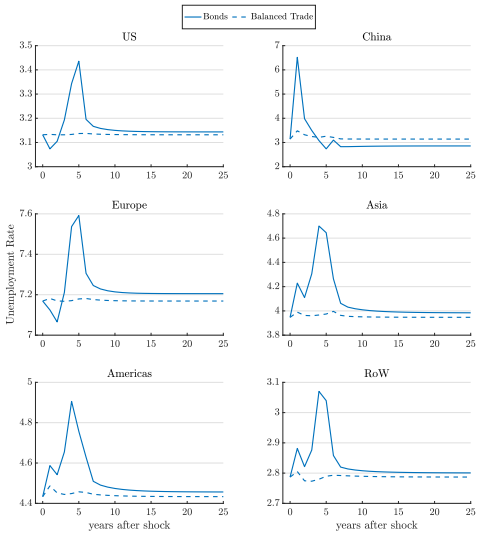
We turn to the role that trade imbalances play in our model and their interaction with trade shocks. First, imposing balanced trade leads not only to a dampening of the predicted short-run general equilibrium responses of labor markets to trade shocks, but it sometimes leads to opposite predictions. If bonds are not available and trade is balanced, China cannot smooth consumption in response to its temporary productivity boost. Therefore, short-run consumption spikes dramatically

Figure 5: Temporary Productivity Growth in China (3a) – Reallocation and Unemployment Dynamics

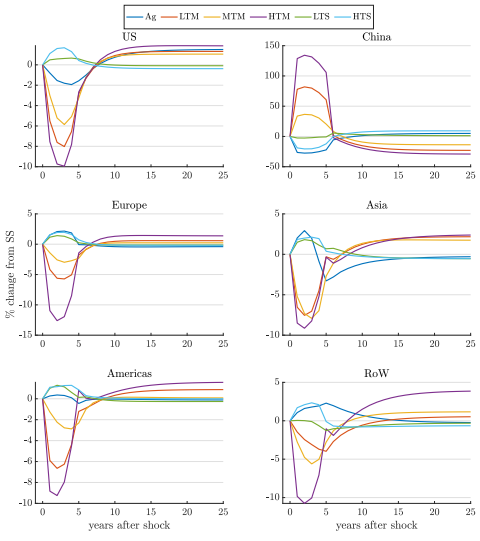
(a) Reallocation Index



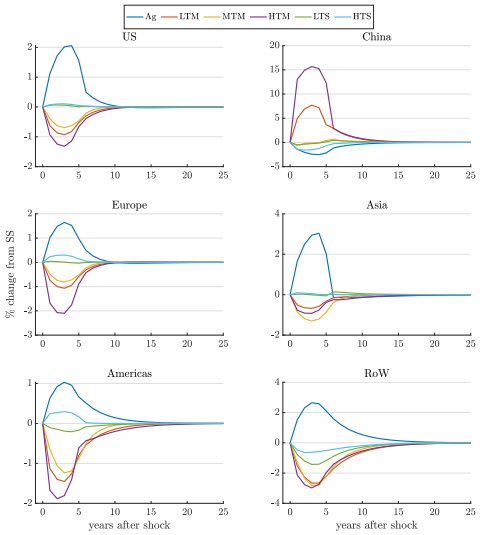
(b) Unemployment



(c) Labor Allocations – Bonds

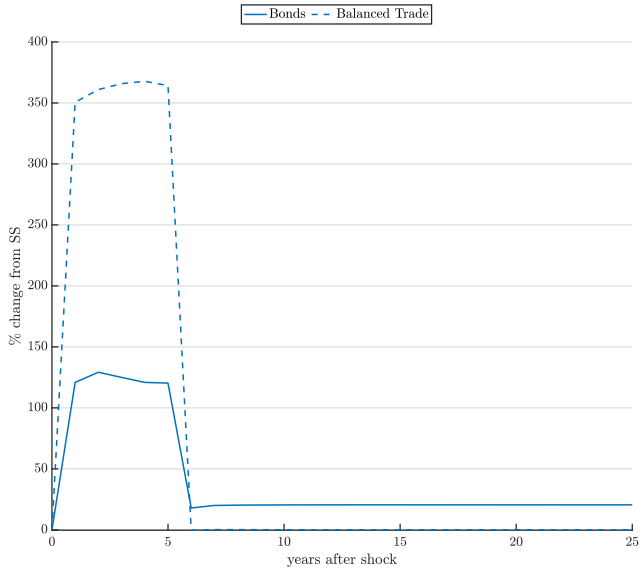


(d) Labor Allocations – No Bonds (Balanced Trade)



as illustrated in Figure 6. This leads to a low marginal value of reallocation – concave utility implies that the marginal utility gain from moving is low relative to mobility costs when consumption is high. In addition, the shock we consider quickly reverts to its initial level, therefore any short-run reallocation must be undone in the long-run, contributing to large reallocation costs relative to benefits in a balanced-trade world.

Figure 6: Temporary Productivity Growth in China (Figure 3a) – Real Expenditure in China



Our model with bonds has implications for not only the magnitude of labor reallocation, but also for the pattern of reallocation as is shown in Figures 5c and 5d. When we consider the complete model with bonds, rising imports in the US following Chinese productivity growth do not need to be immediately met with rising exports. The US imports goods in the short run and reallocates labor to service sectors, which are less tradable and initially much larger in size. However, if we impose balanced trade in every period, exports from the US must immediately rise to offset the increase in imports. In doing so, labor reallocates to the sector in which it has highest comparative advantage (high relative productivity coupled with lower trade costs), which is also the sector in which China has higher relative demand: Agriculture. Reallocation when trade is balanced is driven by sectors with lower trade costs. Hence, reallocation across goods sectors is quite important, but reallocation across service sectors is essentially unresponsive, therefore dampening aggregate reallocation.

Turning to the long run, savings and borrowing behavior leads to permanent changes in final steady-state bond positions across countries. These permanent shifts in the long-run allocation of wealth, in turn, carry implications for steady-state labor allocations and unemployment. The sources of long-run reallocation are similar to those operating in the short run, except that the

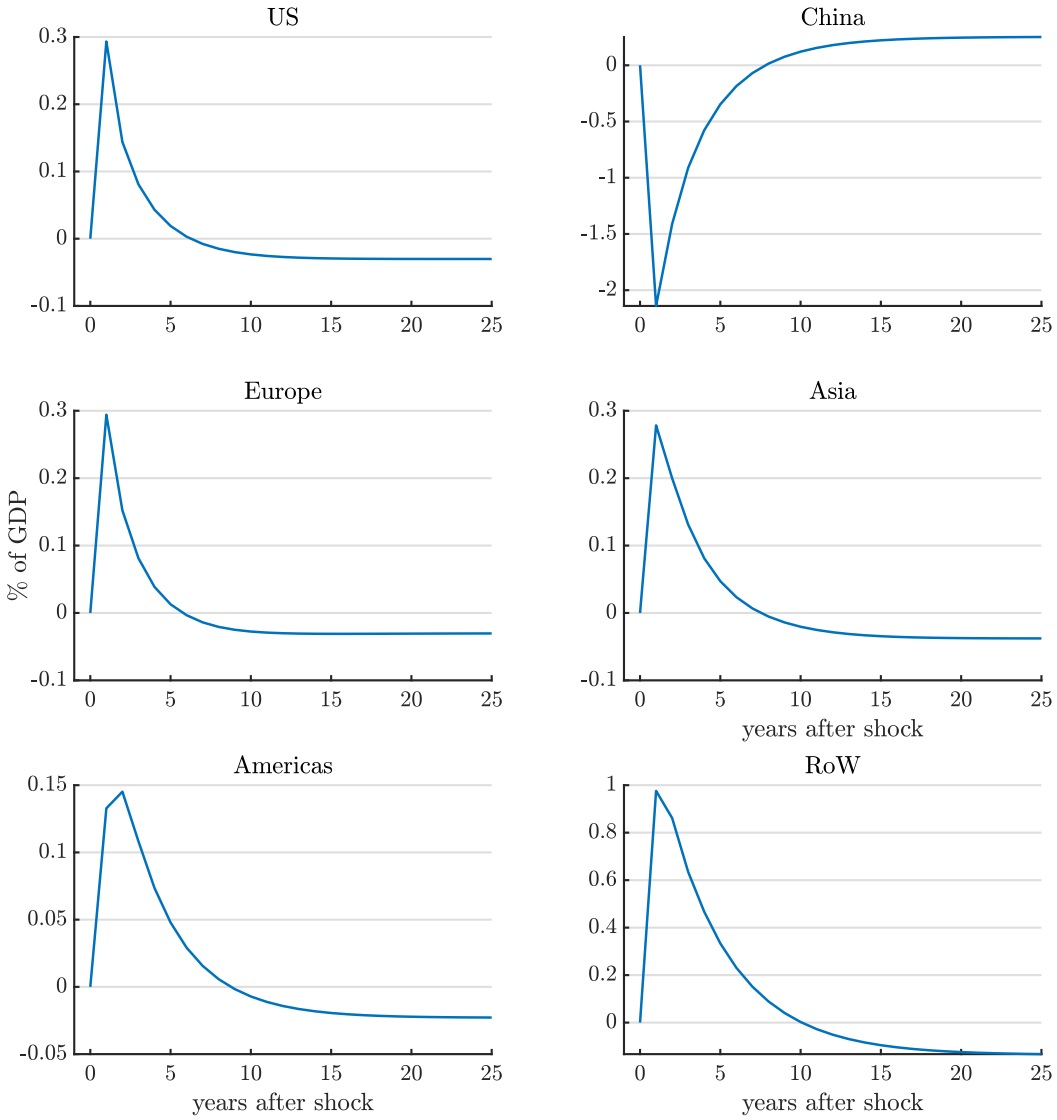
effects of the temporary shock on the distribution of bonds positions (and therefore wealth) across countries is permanent. Specifically, the US runs a large deficit in the short run, which must be paid off in the form of long-run trade surpluses. To repay its debt with China, it shifts labor back to its goods sectors which can easily ship their production internationally. In the long run, the goods sectors in the US expand relative to the initial steady state, whereas its service sectors slightly contract. These long run effects in response to a temporary shock are in sharp contrast with those in the model with balanced trade, which predicts that long-run allocations are unchanged relative to initial ones.

We next turn to the analysis of the once-and-for-all shock in Chinese productivity depicted in Figure 3b. Figure 7 highlights that the effect of this shock on net exports across countries is of much smaller magnitude. Indeed, when an economy is subject to a permanent shock the importance of savings to smooth consumption is of second order. If labor were perfectly mobile across sectors, economies would instantaneously reallocate labor and achieve their unconstrained optimal levels of expenditure. Because labor is not perfectly mobile, we do observe imbalances arising, but these are modest. It is therefore not surprising that, in the event of once-and-for-all permanent shocks, the behavior of the model with imbalances is similar to one without imbalances.

Finally, we study the behavior of our model when we feed it with a slow increase in Chinese productivity, as is illustrated in Figure 9. In this case, China heavily borrows in the short run in anticipation of its large increase in long-run productivity. All other countries lend to China by running large surpluses for over ten years. As with the temporary shock, trade imbalances greatly magnify the extent to whoch reallocation and unemployment respond relative to the trade balance benchmark. Figure 9 shows that the short-run labor market dynamics are very different across the two models. With trade imbalances, Figure 9c shows that the US reallocates resources away from its service sectors and towards its goods sectors so that their output can be shipped to China. However, in the long run, China needs to repay its debt and so ships goods back to the US. This leads to a long-run contraction of manufacturing in the US and an expansion of the goods sector. Compare this behavior with the monotonic decline in manufacturing in the US if trade is imposed to be balanced (see Figure 9d). The magnitude of the manufacturing contraction is also much more limited under trade balance.

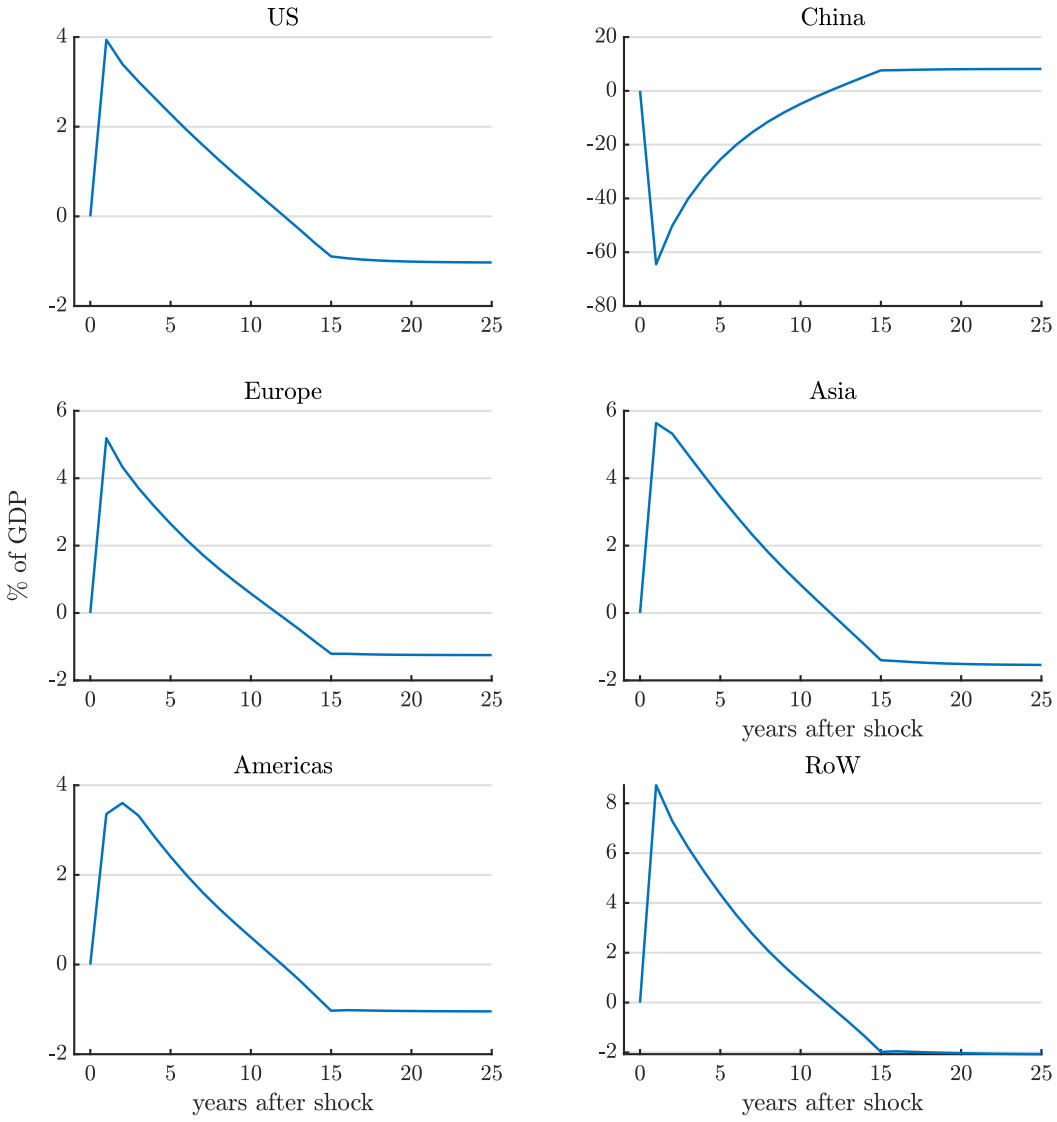
The main conclusion of this section is that, except for once-and-for-all permanent shocks, trade imbalances can significantly amplify the reallocation and unemployment responses to temporary shocks. They can also lead to very different patterns of inter-sectoral reallocation in the short and long run. Finally, our simulations highlight that, with trade imbalances, the path of shocks matter for the behavior of imbalances over time, but also for the determinantion of their long-run consequences. Given the importance of the path of shocks for trade imbalances and the adjustment process, we now turn to the analysis of the labor market consequences of the shocks the global

Figure 7: Once-and-for-all Productivity Growth in China (Figure 3b) – Net Exports Over GDP



economy endured between 2000 and 2014.

Figure 8: Slow Productivity Growth in China (9) – Net Exports Over GDP

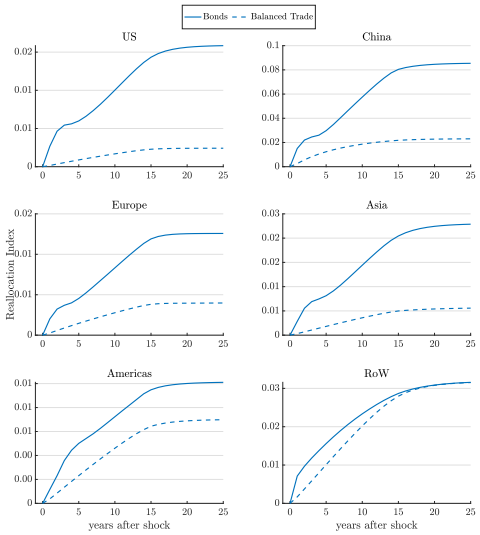


5 Counterfactuals

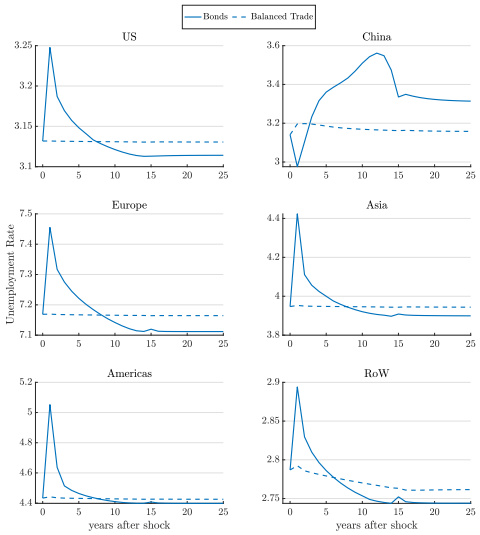
Section 4.2 showed that the exact path of shocks shape the magnitude and evolution of trade imbalances over time, directly influencing long-run outcomes through the accumulation of bonds. For this reason, we conduct an empirical exercise in which we extract the various shocks the global

Figure 9: Slow Productivity Growth in China (Figure) – Reallocation and Unemployment Dynamics

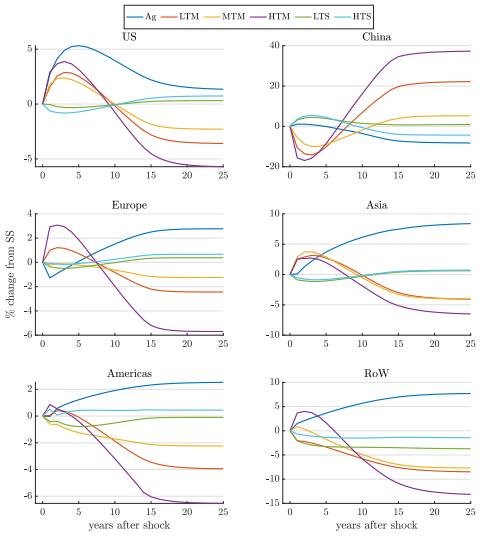
(a) Reallocation Index



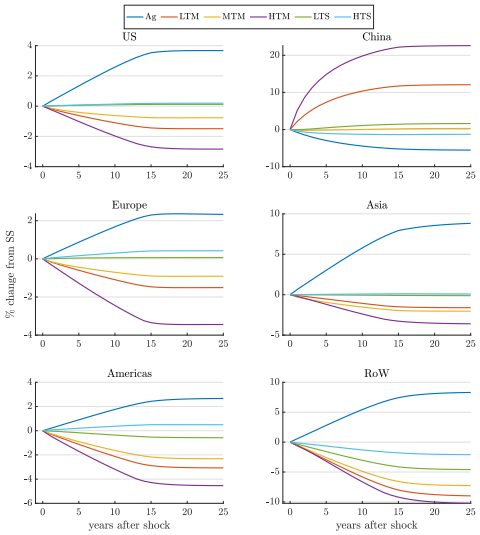
(b) Unemployment



(c) Labor Allocations - Bonds



(d) Labor Allocations - No Bonds (Balanced Trade)



economy has actually endured between 2000 and 2014. Armed with these shocks, we conduct a common exercise in the international trade literature: how do labor markets behave over time in response to the costellation of globalization shocks (shocks in productivities and trade costs)? In this exercise, we compare the labor markets responses that we obtain with our model of imbalances to the responses that we obtain imposing trade balance. Next, given the recent interest on the "China shock" on the US labor market, we use our extracted shocks to study the effects of the shocks to the Chinese economy on the behavior of unemployment and manufacturing employment in the US. We also investigate the contribution of the "China shock" for the US trade deficit and for the trade surplus in China.

5.1 Extracting Shocks from the Data

This section obtains the time series for three sets of shocks in the global economy between December of 2000 and December of 2014: changes in trade costs $\left\{\widehat{d}_{k,oi}^{t}\right\}$, inter-temporal preference shocks $\left\{\widehat{\phi}_{i}^{t}\right\}$ and productivity shocks $\left\{\widehat{A}_{k,i}^{t}\right\}$. We measure changes in trade costs and productivity relative to December of 2000 ($t=0$): $\widehat{d}_{k,oi}^{t}=\frac{d_{k,oi}^{t}}{d_{k,oi}^{0}}$, $\widehat{A}_{k,i}^{t}=\frac{A_{k,i}^{t}}{A_{k,i}^{0}}$. On the other hand, shocks to inter-temporal preferences are relative to the previous period: $\widehat{\phi}_{i}^{t+1}\equiv\frac{\phi_{i}^{t+1}}{\phi_{i}^{t}}$. [19]

We use WIOD data to construct time series of trade shares $\left\{\pi_{k,oi}^{t}\right\}$, sectoral price indices $\left\{P_{k,i}^{I,t}\right\}$ and final good expenditures $\left\{E_{i}^{C,t}\right\}$ between December of 2000 and December of 2014. Armed with these data, we can recover the changes in bilateral trade costs combining equations equations (29) and (31):

$$\widehat{d}_{k,oi}^{t}=\frac{\widehat{P}_{k,i}^{I,t}}{\widehat{P}_{k,o}^{I,t}}\left(\frac{\widehat{\pi}_{k,oo}^{t}}{\widehat{\pi}_{k,oi}^{t}}\right)^{1/\lambda}. \tag{44}$$

In turn, the Euler equation (39) allows us to recover the inter-temporal preference shocks, if we normalize $\widehat{\phi}_{US}^{t}=1$:

$$R^{t+1}=\frac{E_{US}^{C,t+1}}{\delta E_{US}^{C,t}}\text{ for }t=1,...,T-1, \tag{45}$$

$$\widehat{\phi}_{i}^{t+1}=\frac{E_{i}^{C,t+1}}{\delta E_{i}^{C,t}R^{t+1}}\text{ for }t=1,...,T-1, \tag{46}$$

where T is the last period for which we have data, that is, December of 2014. Note that we still need to determine $\widehat{\phi}_{i}^{T+1}$, but we will need to use the structure of the model to do so, as this value will depend on the model-implied steady-state value for final good expenditures.

[19] We impose $\widehat{A}_{k,i}^{t}=\widehat{A}_{k,i}^{T}$ and $\widehat{d}_{k,oi}^{t}=\widehat{d}_{k,oi}^{T}$ for all $t>T$, where T is the last period for which we have data. We also impose and $\widehat{\phi}_{i}^{t}=1$ for all $t>T+1$ – As explained below, the value of $\widehat{\phi}_{i}^{T+1}=1$ is set to gauge the model-implied steady-state value for final consumption expenditures.

Finally, we recover the productivity shocks $\left\{\widehat{A}_{k,i}^t\right\}$ using:

$$\widehat{A}_{k,i}^t = \frac{\widehat{\pi}_{k,ii}^t}{\left(\widehat{P}_{k,i}^{I,t}\right)^{\lambda}} \left(\widehat{c}_{k,i}^t\right)^{\lambda}. \tag{47}$$

Given that $\widehat{c}_{k,i}^t$ depends on $\widehat{w}_{k,i}^t$, which has no data counterpart, we need to use the full structure of the model to recover the sequence of productivity shocks. Appendix C.6 details the algorithm to recover the shocks $\left\{\widehat{A}_{k,i}^t\right\}$ as well as $\left\{\widehat{\phi}_i^{T+1}\right\}$ using the full structure of the model.

We emphasize that the procedure we use to obtain the various shocks is based on standard practice in the international trade and macro literatures. Specifically, our approach to back out shocks to inter-temporal preferences is similar to those used by Reyes-Heroles (2016) and Eaton et al. (2016); and our calculation of changes in trade costs and productivities is common in trade models that generate a gravity equation, as in Eaton et al. (2016) and Head and Ries (2001).

Figure 10a shows an increase in productivity all over the world. In particular, China has experienced a large increase in productivity, especially in manufacturing sectors. While we plot changes in the productivity location parameter $\widehat{A}$, this is not directly comparable to productivity in the classic sense of a Solow Residual. In order to make sense of the magnitudes, note that TFP growth, defined as $\widehat{c}_{k,i}^t/\widehat{P}_{k,i}^{I,t}$, can be expressed as $(\widehat{A}_{k,i}^t/\widehat{\pi}_{k,ii}^t)^{1/\lambda}$. Therefore, using our recovered values for $\widehat{A}_{k,i}^t$, data on changes in trade shares, and imposing $\lambda = 4$, the magnitude for actual annualized TFP growth in China ranges between 3 and 5% per year, depending on the sector – which is in line with growth accounting estimates discussed in Zhu (2012). Other emerging economies – which comprise the bulk of the Americas and the RoW aggregate – also experienced impressive productivity growth, while growth was more muted for advanced economies.

Turning to trade costs, we first construct a summary statistic to capture this large object. We focus on the average import cost for each country-sector pair, weighted by their initial steady state import shares:

$$\overline{d}_{k,i}^t = \sum_{o \neq i} \frac{\pi_{k,oi}^0}{1 - \pi_{k,ii}^0} \widehat{d}_{k,oi}^t. \tag{48}$$

Figure 10b plots this index for each country and sector. In general trade costs are declining for the United States and Asia, and approximately flat in Europe (with some heterogeneity across sectors). Perhaps surprisingly, starting after the 2008 financial crisis and concurrent collapse in trade, initially falling trade costs in China begin to revert and are actually larger by the end of the sample. This estimate of changes in trade costs reflects the fall in the share of trade in output, as documented in Bems et al. (2013). The sources for these increasing frictions are myriad, and include policy changes in countries like China, as well as changes in supply chain management, and other reasons. That said, our measures of frictions are a standard, straightforward, measure of the

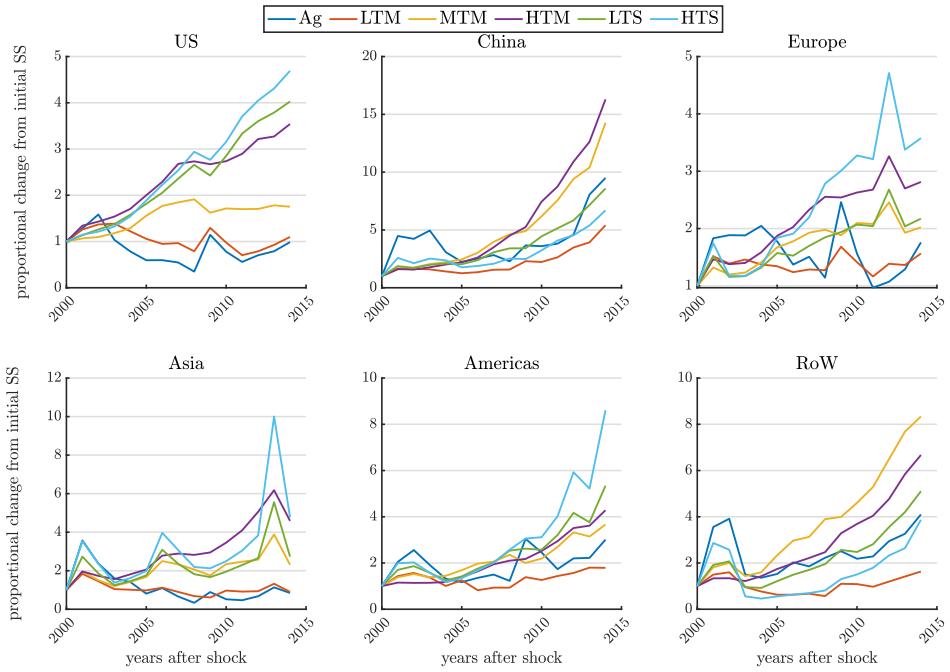
implied barriers to trade.

Finally, we turn to our measure of shocks to inter-temporal preferences, which are presented in Figure 11. The shocks in the US are normalized to 1 in every period. In Europe and Asia (except China), the discount factor shocks fluctuate around 1, suggesting little persistent deviations in consumption behavior from what would be expected with a simple consumption smoothing model. On the other hand, China, the Americas, and the aggregated remaining countries (RoW) exhibit persistent shocks to their discount factors suggesting increased patience. These persistent deviations are often referred to as the "global savings glut." The large trade surplus that China has been running since the early 2000s is a puzzle for models based on consumption smoothing behavior, such as ours. As Song et al. (2011) argue, financial frictions within China are key drivers of the Chinese savings glut. Our inter-temporal preference shocks constitute a reduced-form way to allow the model to match the time series behavior of Chinese aggregate expenditures and the rest of the world. Thus, we do not seek to explain the observed data on imbalances, instead, we take the view that consumption smoothing at least partially motivates the behavior of imbalances, and our aim is to understand how this force interacts with labor market frictions and the adjustment process to globalization shocks. In addition, we recognize that there are rich dynamics to consumption in the real world – reflecting preferences, frictions, and a myriad of other factors. We are agnostic on the correct theory, instead summarizing the effect of these channels with $\widehat{\phi}$. This is useful because it allows us to ask counterfactual questions about the dynamics of globalization shocks without the global savings glut and, in general, if the bonds market were frictionless – a useful benchmark to compare against the usual assumption in trade of no consumption smoothing whatsoever.

To extract the shocks the global economy experienced between 2000 and 2014, we have solely relied on data on trade shares, sectoral prices, and aggregate final expenditures across countries. We have not used information on labor allocations or trade imbalances. It is therefore natural to ask how the model implied behavior of labor allocations and trade imbalances compare to those in the data. A note of caution before we proceed with this comparison: given our perfect foresight assumption, once we feed the shocks into the model, responses at impact are large as agents (suddenly) fully anticipate the path of future shocks. That said, Figure 12 compares labor allocations in the United States in our model (Panel a) to labor allocations in the data (Panel b). Our model replicates the decline of manufacturing as a whole, the expansion of services, and the decline and rebound of agriculture. The magnitudes in our model are somewhat muted, but we believe this is explained by pre-existing trends driving the decline of manufacturing as well as the fact that our $\widehat{\phi}$ revert to 1 after 2014. The latter observation is important as individuals in the model start anticipate the end of the savings glut years before the end of our sample period, leading to some rebalancing happening before 2014. This observation is also important to explain the strong rebound of Low-Tech Manufacturing, for which we do not find support in the data. To

Figure 10: Extracted Globalization Shocks

(a) Productivity Shocks $\widehat{A}^t_{k,i}$



(b) Trade-Weighted Import Costs $\overline{d}^t_{k,i}$ (See Equation (48))

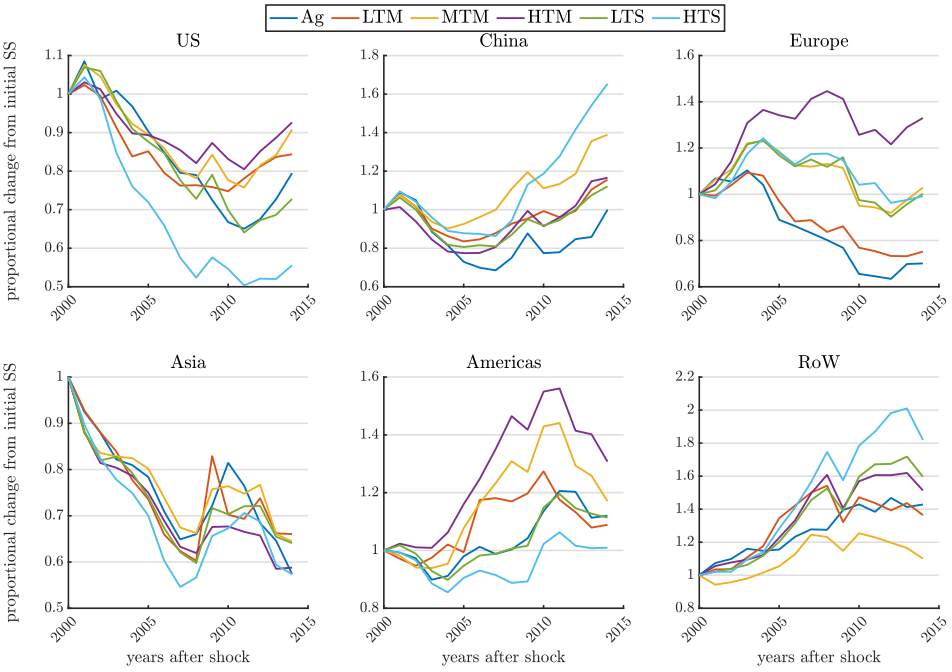
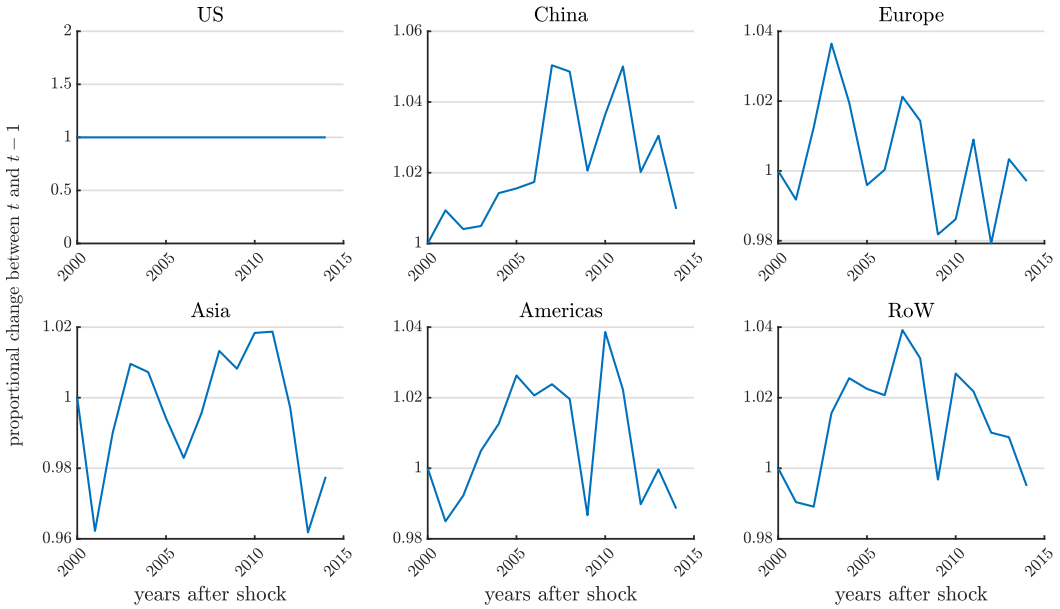


Figure 11: Extracted Inter-Temporal Preference Shocks $\widehat{\phi}_i^t$



verify the plausibility of this explanation, we simulated the same shocks we extracted with one difference: we keep the $\widehat{\phi}$ shocks fixed at their 2012 to 2014 averages for 15 more years (that is, until 2029). Indeed, not only the magnitude of the downsize of manufacturing that we obtain is larger than what is depicted in Figure 12a, but also Low-Tech steadily contracts until 2014.

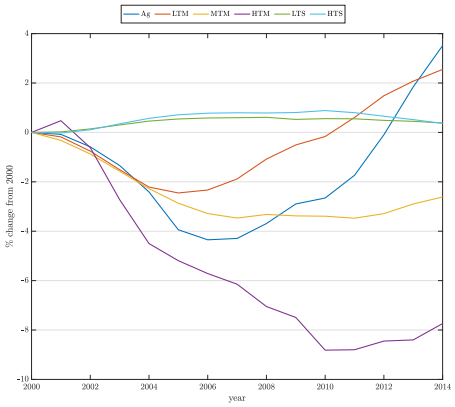
Figure 13 compares the model implied trade imbalances in the US and China to those in the data. The model is able to capture the large surplus in China as well as the persistent deficit in the US. However, the model misses the behavior of Chinese imbalances in the beginning of the period. This happens as China anticipates its massive shocks that are coming (both large increases in productivity and shocks to inter-temporal preferences), which leads to large unemployment in the short run, leading to a production shortage and deficits in the short run.

5.2 Global Technology and Trade Shocks

Our first counterfactual exercise focuses on the general equilibrium effects of changes in productivity and trade costs since 2000, with no shocks to inter-temporal preferences. In particular, we focus on the consequences of our extracted series of $\widehat{A}$ and $\widehat{d}$ above. Our goal is to isolate the quantitative significance of globalization on labor market outcomes in a setting where workers smooth their consumption over time. As we will show, the magnitude of *observed* trade shocks has dramatic impacts on global imbalances, which has implications for adjustment dynamics, unemployment, wages, and ultimately consumption.

Figure 12: Comparing Labor Allocations in the Model and Data

(a) Model



(b) Data

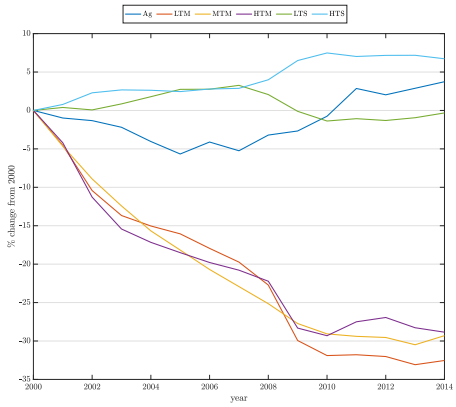
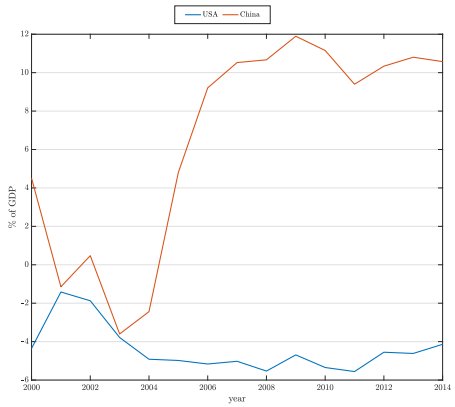
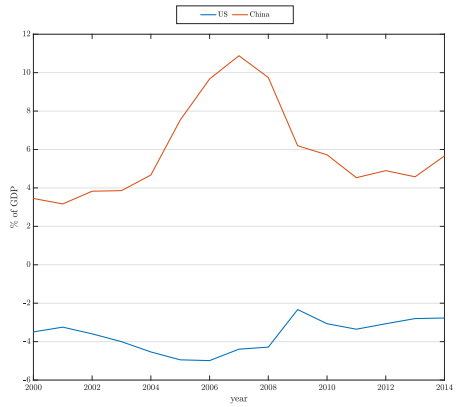


Figure 13: Comparing Net Exports in the Model and Data

(a) Model



(b) Data



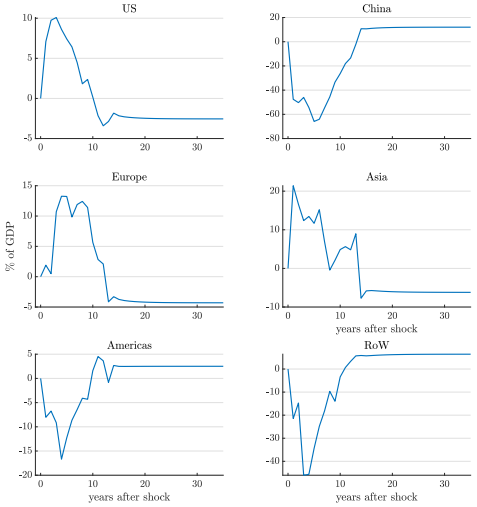
Before discussing results, it will be helpful to contrast our approach with standard practice. There are two typical approaches to dealing with imbalances in the quantitative trade literature. The first approach involves fixing imbalances in the data, or assuming trade balance. This method has been employed in both static models (e.g., Eaton and Kortum (2002) and Dekle et al. (2007)) and dynamic models (e.g., Dix-Carneiro (2014) and Traiberman (2019)). The use of this method in dynamic models is a particularly strong assumption, since it implicitly imposes that workers and governments have *no* access to borrowing or savings mechanisms. The second approach, involves assigning ownership shares of capital or fixed factors to agents at an initial point, and fixing these shares over time and in counterfactuals. Although this procedure does not follow from optimizing behavior, this method has been used in Caliendo et al. (2019), allowing for imbalances to change as returns to fixed factors change. If returns to capital increase while those to labor decline, this implies that workers have access to some social insurance. Nevertheless, agents are prevented from buying or selling shares in response to the shocks they face. These approximations may be reasonable in a situation where imbalances are small along the transition path from one equilibrium to the next. This would be the case, for example, if the primary force for changes in imbalances were shocks to aggregate consumption, and orthogonal to trade shocks (i.e., if imbalances were driven by $\widehat{\phi}$). Nevertheless, these are imperfect if the path and magnitude of trade shocks lead to large changes in consumption and savings, which we show is the case below.

To be able to compare the implications of the globalization shocks in our model of trade imbalances relative to a model without imbalances, we start both models from a steady steady with trade balance – that is $NX_i = 0 \ \forall i$, see Appendix C.3 for details on our procedure, which is based on exact hat algebra. Figure 14 shows that the globalization shocks we feed into the model can lead to substantial imbalances in the short run. These range from a trade *deficit* in China of up to 10% of GDP to a trade surplus in Asia close to 5%. The US runs a short-run surplus amounting to close to 2.5% of GDP. As expected from previous literature, if we purge the model of the intertemporal preference shocks, we see China running a large trade deficit in the short run. This is not surprising in light of our discussion in section 4.2 as Chinese productivity strongly and steadily increases between 2000 and 2014, more than in other countries.

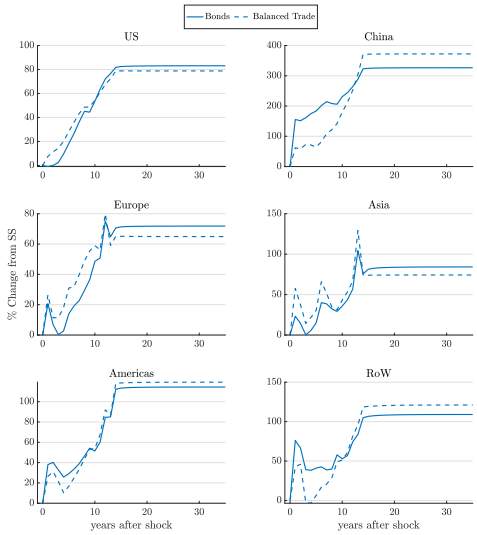
As we learned in our analysis of section 4.2, large trade imbalances amplify the amount of reallocation in the economy relative to a model where trade is balanced. This is illustrated in Figure 15a. The behavior of unemployment is also quite different, and driven by a larger amount of reallocation. Importantly, Figures 15c and 15d highlight the importance of modelling trade imbalances in understanding the adjustment process in response to globalization shocks. A model that imposes trade balance often predicts opposing patterns of short-run reallocation. For example, our model with imbalances predicts that the globalization shocks would lead to an expansion of all manufacturing sectors in the US in the short run, but a contraction in the long run. On the other

Figure 14: Aggregate Trade and Consumption Dynamics Following Globalization Shocks

(a) NX Over GDP



(b) Consumption



hand, trade balance would lead to a monotonic decline of High-Tech Manufacturig and a long-run expansion of Low-Tech Manufacturing. Other countries also display drastically different patterns of adjustment in both the short and long runs.

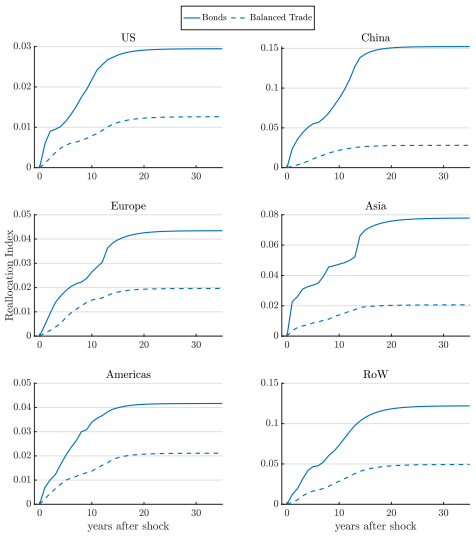
5.3 The China Shock

As a second application of our model, we investigate the role of China on the adjustment of the labor market in the US. This topic has attracted much academic interest since the work of Autor et al. (2013) and Pierce and Schott (2016). We compare the behavior of our model when we feed it with all of the shocks we recovered in Section 5.1 to the behavior of the model when (a) all global shocks to the world are neutralized, except those from China (including the savings glut); (b) all global shocks are maintained, but we neutralize those from China.

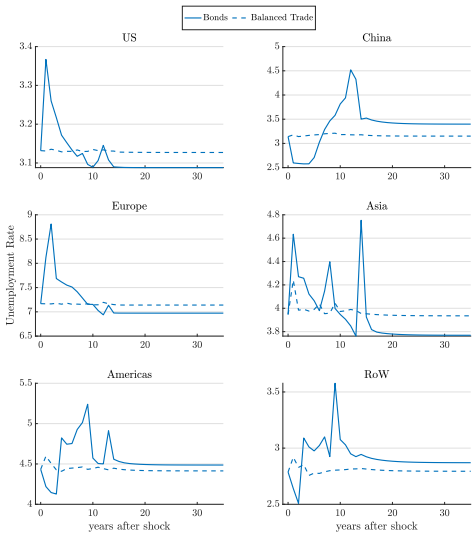
We first analyze the situation where we only feed the shocks China experienced between 2000 and 2014, including its inter-temporal preference shocks. The red dashed line in Figure 16a shows that China runs a deficit in the short run, a trade surplus in the medium run (that reaches up to 10% of GDP) and close to balanced trade in the long run. On the other hand, the deficit in the US improves for several years before increasing to up to 5% of GDP and finally settling at a long-run value close to its initial value. The reason why China initially runs a large trade deficit is because it anticipates strong productivity growth in the coming years, yet the savings glut is not yet present – $\widehat{\phi}_{China}$ only starts to become relatively large in 2005/2006, that is, at $t = 5, 6$ in Figure 16 (See

Figure 15: Reallocation and Unemployment Dynamics Following Globalization Shocks

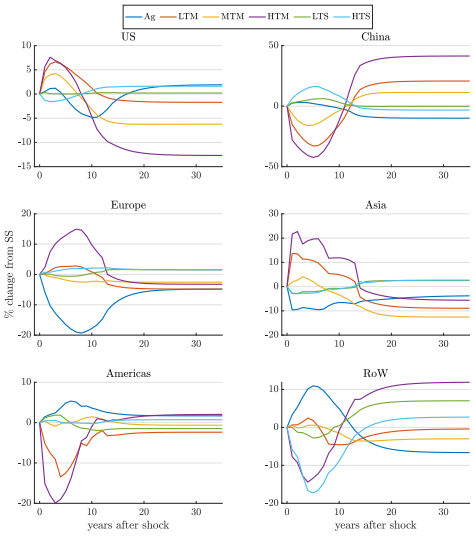
(a) Reallocation Index



(b) Unemployment



(c) Labor Allocations - Bonds



(d) Labor Allocations - No Bonds (Balanced Trade)

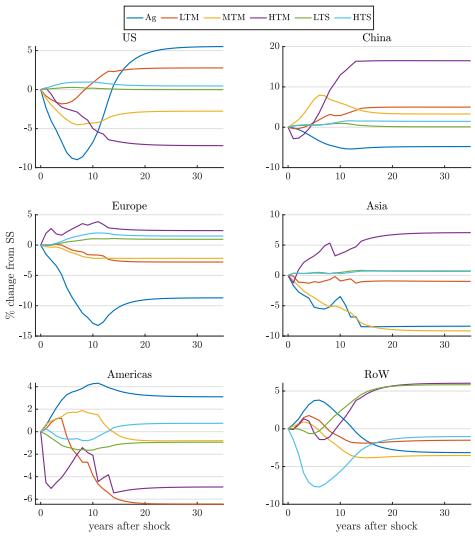
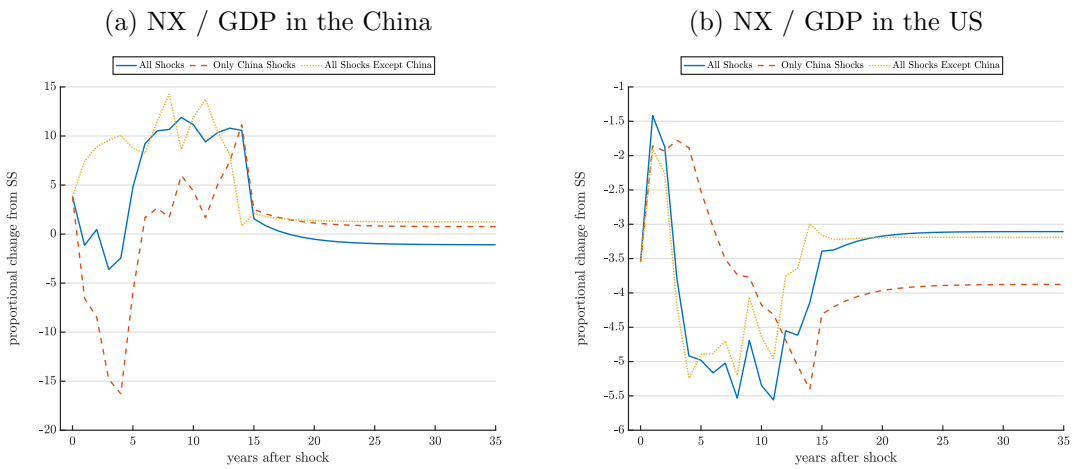


Figure 11). Interestingly, the model predicts that China would go from a large trade deficit of -15% of GDP to a large surplus of 10% of GDP within a decade. The behavior of the US trade deficit approximately mirrors the one from China. Figure 16b shows that the US experiences a significant reduction in its trade deficit in the short run, which quickly reverts to a trade deficit deterioration following the start of China's saving glut. It then stabilizes at a value close to -4% of GDP.

Figure 16: The China Shock: Net Exports

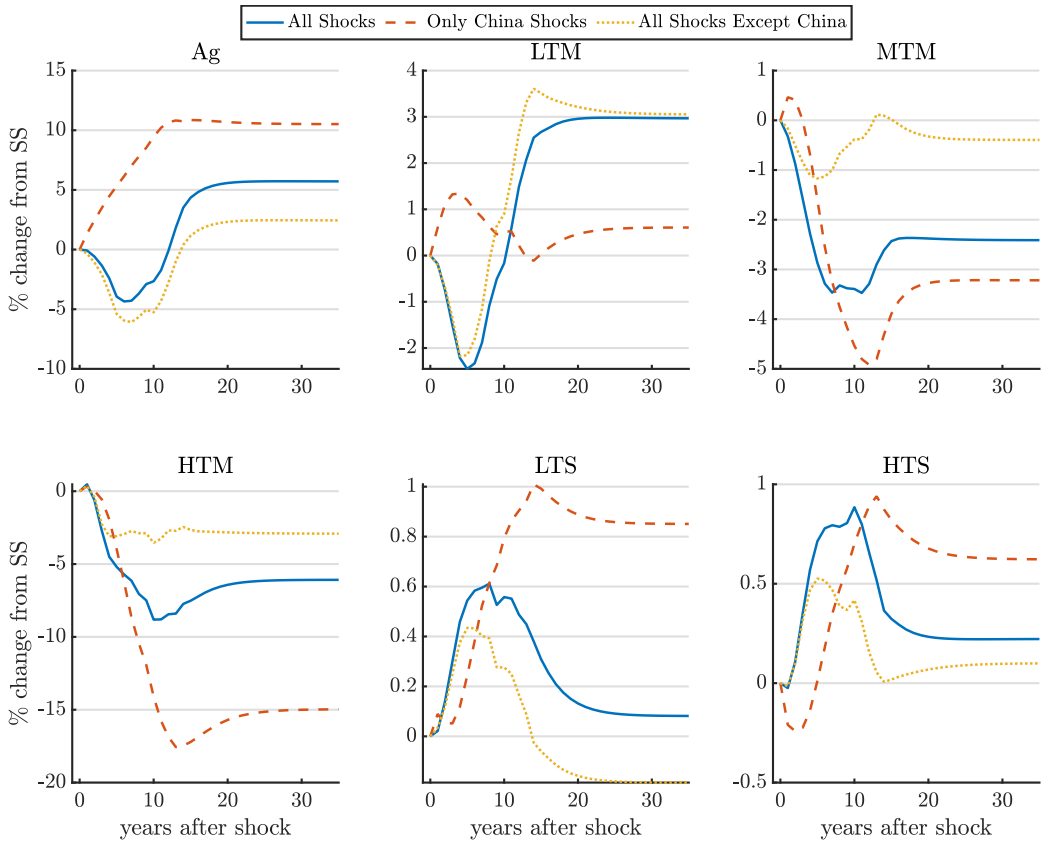


Notes: "All Shocks": model is fed with all shocks recovered in section 5.1. "Only China Shocks": model is fed with China shocks only, shocks to all other countries are neutralized. "All Shocks Except China": model is fed with all shocks recovered in section 5.1 but China's shocks are neutralized.

Figure 17 describes the impact of the China shock on sectoral reallocation in the US. The red dashed line shows that the shocks to China lead to a sharp contraction of High-Tech and Mid-Tech Manufacturing. Low-Tech Manufacturing is not as affected and actually grows slightly over the period we analyze. In contrast, Agriculture and Services expand, absorbing workers displaced from Manufacturing. Indeed, there are only two small spikes in the unemployment rate in the US in response to the China shock (as we show in Figure 18). The largest spike features an unemployment rate of 3.14%. Compared to the baseline of 3.08%, this accounts for a total of $155,181,918 \times (3.14\% - 3.08\%) \approx 93,100$ lost jobs, but for a short period of time. The average unemployment rate over this period is of 3.085%, amounting to an excess of less than 10,000 jobs lost over the period we consider.

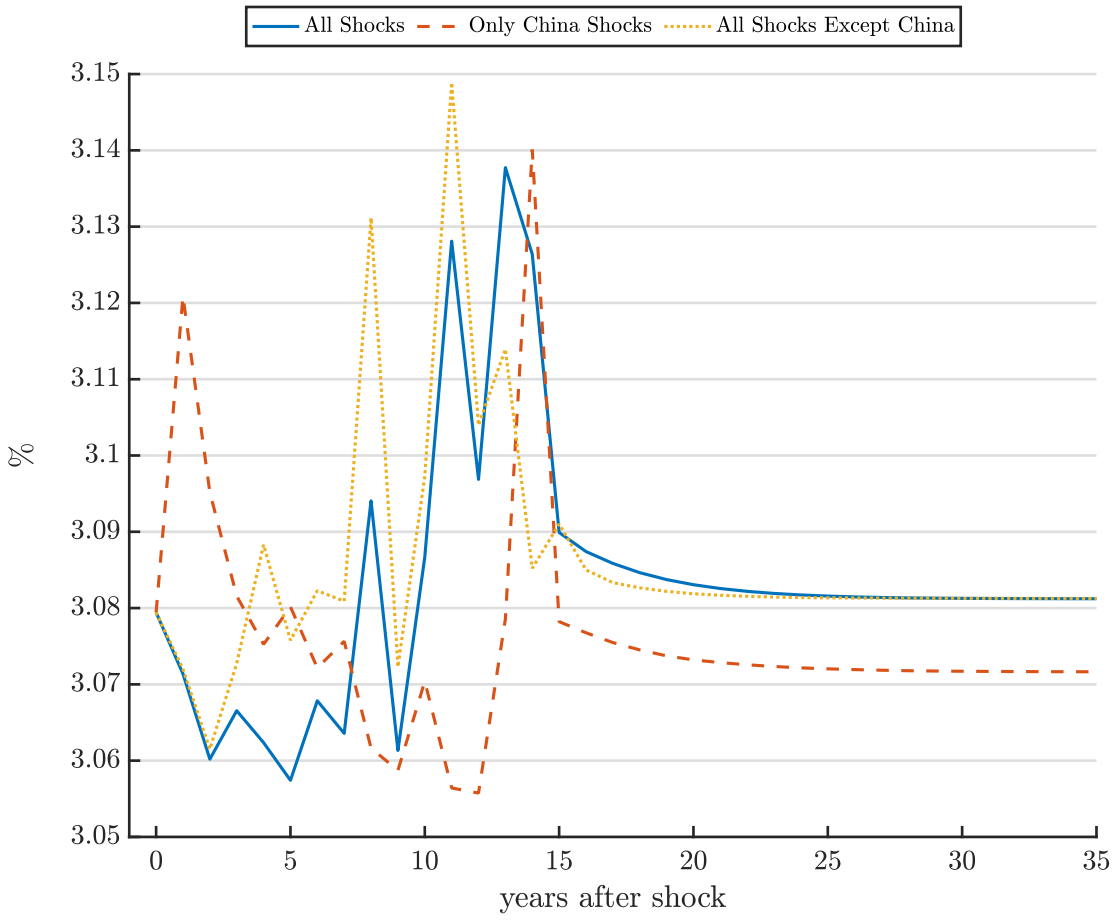
It is also instructive to compare the behavior of the model when it is subject to all the global shocks we extracted in Section 5.1 to its behavior when we subject it with the same shocks but neutralize China's shocks (that is, we keep China's productivity, inter-temporal preferences and trade costs with the rest of the world unchanged at their 2000 levels). Interestingly, the yellow dotted line in Figure 16a shows that, even in the absence of the "China shock", China would still

Figure 17: The China Shock: Labor Allocations in the US



Notes: "All Shocks": model is fed with all shocks recovered in section 5.1. "Only China Shocks": model is fed with China shocks only, shocks to all other countries are neutralized. "All Shocks Except China": model is fed with all shocks recovered in section 5.1 but China's shocks are neutralized.

Figure 18: The China Shock: Unemployment in the US



Notes: "All Shocks": model is fed with all shocks recovered in section 5.1. "Only China Shocks": model is fed with China shocks only, shocks to all other countries are neutralized. "All Shocks Except China": model is fed with all shocks recovered in section 5.1 but China's shocks are neutralized.

run a significant surplus over the period we consider. Further, if we compare the yellow dotted line to the solid blue line, we find that the Chinese surplus under "No China shock" tends to be at least as large as the one with "All shocks", except for the final 2014 year. This result indicates that the Chinese surplus over the 2000s would have occurred with or without China-specific shocks. This finding highlights the value of our multi-country approach.

The dotted yellow and solid blue lines in Figure 17 reinforce the idea that China did significantly contribute to the decline of US manufacturing, especially Mid- and High-Tech Manufacturing: the yellow dotted line is considerably above the solid blue line for these two sectors. However, as we previously noted, workers displaced from manufacturing found re-employment in Agriculture and Services. The unemployment solid blue and dotted yellow lines in Figure 18 are similarly unresponsive to global shocks, with or without China.

We now turn to the welfare consequences of the China shock. Table 5 shows the present discounted value of consumption gains of the China shock in isolation. Specifically, we calculate consumption gains relative to the initial steady state according to:

$$\Delta W_i = \exp\left\{(1-\delta)\sum_{t=0}^{\infty}\delta^t \log(C_i^t) - \log(C_i^{SS_0})\right\}, \tag{49}$$

where the first term inside the exponential is the present discounted value of utility from consumption along the transition path and the second term is initial utility from consumption. We exponentiate to move from utils to consumption. The first column shows the consumption gains accrued by the China shock across all countries. We can see that shocks to China alone were responsible to mild effects for most countries. These range from a loss of -0.4% in the Americas to a gain of 3% in the Rest of the World. The second colum shows how these gains would differ if trade was was imposed to be balanced period by period. The answer is: not too much. Finally, the third and fourth columns reproduce the first two columns, but we shut down the inter-temporal preference shocks in China ($\widehat{\phi}^t_{China} = 1\ \forall t$). In this case, the welfare gains from the China shock turn out to be a bit larger in the US, but not economically different. Importantly, this result indicates that the welfare consequence of the Chinese savings glut on US welfare was not significant.

Table 5: Global Consumption Gains of the China Shock (2000-2014)

Country	Complete Model	Balanced Trade	Complete Model $\widehat{\phi}_{China} = 1\ \forall t$	Balanced Trade $\widehat{\phi}_{China} = 1\ \forall t$
United States	1.004	1.003	1.007	1.003
Europe	1.001	1.002	1.003	1.002
Asia	1.013	1.017	1.012	1.017
Americas	0.996	0.998	0.994	0.998
RoW	1.030	1.022	1.017	1.022

We end this section asking by how much countries benefitted from globalization during the period we consider. To do so, for each country, we neutralize the shocks it experienced, but we keep the path of shocks endured by all other countries. Table 6 shows the results. Column 1 shows that all countries benefitted from globalization. China benefitted the most, with consumption gains reaching 6.6%. The Americas and Asia/Oceania gained the least: 0.03%. The magnitude of gains can be significantly affected if we impose trade balance: column 2 shows that gains in the US are reduced from 2.2% to 1.3%. Finally, we investigate the effect of inter-temporal shocks in columns 3 and 4 (with trade imbalances and without trade imbalances respectively). If we shut down the global savings glut, welfare in the US is significantly higher, reaching 3.5%, consistent with the belief that the global savings glut was detrimental to the US. However, as we highlighted in the previous section, the Chinese savings glut, by itself, did not substantially contribute to changes in US welfare.

Table 6: Globalization Consumption Gains Over 2000-2014

Country	Complete Model	Balanced Trade	Complete Model $\widehat{\phi}_i^t = 1\ \forall i,t$	Balanced Trade $\widehat{\phi}_i^t = 1\ \forall i,t$
United States	1.022	1.013	1.038	1.013
China	1.066	1.067	1.056	1.067
Europe	1.015	1.015	1.025	1.016
Asia/Oceania	1.003	1.021	0.997	1.021
Americas	1.003	1.006	1.002	1.006
RoW	1.052	1.045	1.037	1.045

6 Conclusion

In this paper, we study the implications of trade imbalances for the adjustment process in response to globalization shocks. We extend the workhorse model of trade (Eaton and Kortum, 2002, and Caliendo and Parro, 2015) to allow for inter-sectoral mobility costs (as in Artuç et al., 2010), search frictions (as in Mortensen and Pissarides, 1999), and trade imbalances (as in Reyes-Heroles, 2016). In short, we extend the state-of-the-art model of trade with the workhorse models for mobility frictions, unemployment and trade imbalances. We estimate the model using data from the WIOD, ILOSTAT, and the US CPS. Our estimation method conditions on trade shares and net exports and can be performed country by country. Even though the parameter space is large, the country by country estimation allows us to break a large optimization problem into smaller ones.

We find that (a) incorporating trade imbalances has the potential to significantly magnify the effect of globalization shocks on inter-sectoral reallocation and unemployment; (b) reallocation paths can be substantially different if we allow for trade imbalances compared to balanced trade –

specifically, sectors that expand in the short run with trade imbalances can actually contract with balanced trade; (c) the behavior of trade imbalances and of the labor market depends on the whole path of shocks. In particular, temporary shocks can have important persistent long-run effects in our model of trade imbalances, but none if we impose trade balance. This motivates to conduct an empirical application and extract the path of shocks the global economy actually endured between 2000 and 2014 using standard methods from the international trade and macro literatures.

We conduct a standard exercise in the international trade literature and subject the model to extracted global technology and trade cost shocks. Aligned with our impulse response function analyses, we find that allowing for trade imbalances have a first order effect on the behavior of labor markets around the world. Next, we investigate the "China Shock" in the context of our model. Consonant with previous literature, we find that the China shock played an important role in the contraction of US manufacturing. However, we do not find a significant effect of shocks to the Chinese economy on US unemployment. Interestingly, our model implies that China's substantial trade surplus would have occurred even in the absence of its own shocks. Therefore, shocks around the world are at least partly responsible for China's large surplus. China by itself had modest welfare effects around the world, but mostly positive. Finally, global shocks between 2000 and 2014 significantly increase welfare across all countries.

Our work shows that carefully modeling imbalances can have important implications for the adjustment process in response to globalization shocks and opens important questions for future work. Given the importance of imbalances for the reallocation process, a natural question to address next is to quantitatively characterize how trade imbalances shape the inequality effects of trade. Our model can be easily extended to allow heterogeneous workers, so this is a natural next step. In addition, we also hope to be able to add endogenous capital accumulation decisions and capital-skill complementarity, generating rich mechanisms linking globalization to the skill premium. Finally, an interesting extension of our framework would allow workers to make borrowing and savings decisions, which will aggregate into global imbalances. Even though this is a hard problem, especially regarding estimation, we believe that our method of simulated moments that can be performed country by country (conditional on trade shares and imbalances) can be applied to this situation.

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Appendix

A Household Problem

The Lagrangian of problem (6), (7) and (5) is

$$\mathcal{L} = E_0 \left\{ \begin{array}{c} \sum_{t=0}^{\infty} \left[\delta^t \phi^t \left(\overline{L} u\left(c_\ell^t\right) - \widetilde{\lambda}^t \left(\overline{L} P^{F,t} c_\ell^t + B^{t+1} - \Pi^t - R^t B^t \right) \right) \right] + \\ \int_0^{\overline{L}} \left[\sum_{t=0}^{\infty} \left[\delta^t \phi^t \left(\begin{array}{c} \left(1 - e_\ell^t\right) \left(-C_{k_\ell^t, k_\ell^{t+1}} + \omega_{\ell, k_\ell^{t+1}}^t + b_{k_\ell^{t+1}} \right) + \\ e_\ell^t \eta_{k_\ell^t} + \widetilde{\lambda}^t \left(\sum_{k=1}^{K} \mathcal{I}\left(k_\ell^{t+1} = k\right) e_\ell^t w_k^t \left(x_\ell^t\right) \right) \end{array} \right) \right] \right] d\ell \end{array} \right\} \tag{A.1}$$

Because each worker is infinitesimal, and the allocation of one worker does not interfere with the allocation/utility of other individual workers (conditional on aggregates), maximizing

$$\int_0^{\overline{L}} \left[\sum_{t=0}^{\infty} \delta^t \phi^t \left(\begin{array}{c} \left(1 - e_\ell^t\right) \left(-C_{k_\ell^t, k_\ell^{t+1}} + \omega_{\ell, k_\ell^{t+1}}^t + b_{k_\ell^{t+1}} \right) + \\ e_\ell^t \eta_{k_\ell^t} + \widetilde{\lambda}^t \left(\sum_{k=1}^{K} \mathcal{I}\left(k_\ell^{t+1} = k\right) e_\ell^t w_k^t \left(x_\ell^t\right) \right) \end{array} \right) \right] d\ell \tag{A.2}$$

means maximizing each individual term. Therefore, the planner solves, for each individual, the recursive problem:

$$\mathcal{L}_W^t \left(k_\ell^t, e_\ell^t, x_\ell^t, \omega_\ell^t\right) = \max_{k_\ell^{t+1}, \widetilde{e}_k^{t+1}(.)} \left\{ \begin{array}{c} \left(1 - e_\ell^t\right) \left(-C_{k_\ell^t, k_\ell^{t+1}} + \omega_{\ell, k_\ell^{t+1}}^t + b_{k_\ell^{t+1}} \right) + e_\ell^t \eta_{k_\ell^t} \\ + \widetilde{\lambda}_t \sum_{k=1}^{K} \mathcal{I}\left(k_\ell^t = k\right) e_\ell^t w_k^t \left(x_\ell^t\right) + \\ \delta \widehat{\phi}^{t+1} E_t \mathcal{L}_W^{t+1} \left(k_\ell^{t+1}, e_\ell^{t+1}, x_\ell^{t+1}, \omega_\ell^{t+1}\right) \end{array} \right\}, \tag{A.3}$$

where $\widehat{\phi}^{t+1} \equiv \frac{\phi^{t+1}}{\phi^t}$.

Denote by $\mathcal{F}^t$ the set of information at t. So, $E_t\left(.\right) = E\left(.|\mathcal{F}^t\right)$. For an unemployed worker in sector k at time t, $k_\ell^t = k$, $e_\ell^t = 0$:

$$\mathcal{L}_W^t \left(k_\ell^t = k, e_\ell^t = 0, x_\ell^t, \omega_\ell^t\right) = \max_{k', \{\widetilde{e}_k^{t+1}(.)\}} - C_{kk'} + \omega_{\ell,k'}^t + b_{k'} + \delta \widehat{\phi}^{t+1} E_t \mathcal{L}_W^{t+1} \left(k', e_\ell^{t+1}, x_\ell^{t+1}, \omega_\ell^{t+1}\right). \tag{A.4}$$

Using the law of iterated expectations we obtain:

$$\begin{aligned} &\mathcal{L}_W^t \left(k_\ell^t = k, e_\ell^t = 0, x_\ell^t, \omega_\ell^t\right) = \max_{k', \{\widetilde{e}_k^{t+1}(.)\}} - C_{kk'} + \omega_{\ell,k'}^t + b_{k'} \\ &+ \delta \widehat{\phi}^{t+1} E \left\{ E \left[\mathcal{L}_W^{t+1} \left(k', 1, x_\ell^{t+1}, \omega_\ell^{t+1}\right) | x_\ell^{t+1}, \mathcal{F}^t \right] \times \Pr\left(k_\ell^{t+1} = k', e_\ell^{t+1} = 1 | x_\ell^{t+1}, \mathcal{F}^t\right) | \mathcal{F}^t \right\} \\ &+ \delta \widehat{\phi}^{t+1} E \left\{ E \left[\mathcal{L}_W^{t+1} \left(k', 0, x_\ell^{t+1}, \omega_\ell^{t+1}\right) | x_\ell^{t+1}, \mathcal{F}^t \right] \times \Pr\left(k_\ell^{t+1} = k', e_\ell^{t+1} = 0 | x_\ell^{t+1}, \mathcal{F}^t\right) | \mathcal{F}^t \right\} \\ &= \max_{k', \{\widetilde{e}_k^{t+1}(.)\}} - C_{kk'} + \omega_{\ell,k'}^t + b_{k'} \\ &+ \delta \widehat{\phi}^{t+1} \theta_{k'}^t q\left(\theta_{k'}^t\right) E \left\{ \mathcal{L}_W^{t+1} \left(k', 1, x_{\ell'}^{t+1}, \omega_\ell^{t+1}\right) \widetilde{e}_{k'}^{t+1} \left(x_\ell^{t+1}\right) | \mathcal{F}^t \right\} \\ &+ \delta \widehat{\phi}^{t+1} E \left\{ \left(1 - \theta_k^t q\left(\theta_{k'}^t\right) \widetilde{e}_{k'}^{t+1} \left(x_\ell^{t+1}\right)\right) \mathcal{L}_W^{t+1} \left(s', 0, x_\ell^{t+1}, \omega_\ell^{t+1}\right) | \mathcal{F}^t \right\} \end{aligned} \tag{A.5}$$

For an employed worker in sector k, $k_\ell^t = k$, $e_\ell^t = 1$:

$$\begin{aligned}
\mathcal{L}_W^t\left(k_\ell^t = k, e_\ell^t = 1, x_\ell^t, \omega_\ell^t\right) &= \max_{\left\{\widetilde{e}_k^{t+1}(.)\right\}} \widetilde{\lambda}^t w_k^t\left(x_\ell^t\right) + \eta_k + \delta\widehat{\phi}^{t+1} E_t \mathcal{L}_W^{t+1}\left(k, e_\ell^{t+1}, x_\ell^t, \omega_\ell^{t+1}\right) \\
&= \max_{\left\{\widetilde{e}_k^{t+1}(.)\right\}} \widetilde{\lambda}^t w_k^t\left(x_\ell^t\right) + \eta_k \\
&+ \delta\widehat{\phi}^{t+1} E\left\{ \begin{array}{c} E\left[\mathcal{L}_W^{t+1}\left(k, 1, x_\ell^t, \omega_\ell^{t+1}\right) | k_\ell^{t+1} = k, , e_\ell^{t+1} = 1, x_\ell^{t+1}, \mathcal{F}^t\right] \times \\ \Pr\left(k_\ell^{t+1} = k, e_\ell^{t+1} = 1 | x_\ell^{t+1}, \mathcal{F}^t\right) | \mathcal{F}^t \end{array} \right\} \\
&+ \delta\widehat{\phi}^{t+1} E\left\{ \begin{array}{c} E\left[\mathcal{L}_W^{t+1}\left(k, 0, x_\ell^t, \omega_\ell^{t+1}\right) | k_\ell^{t+1} = k, e_\ell^{t+1} = 0, x_\ell^{t+1}, \mathcal{F}^t\right] \times \\ \Pr\left(k_\ell^{t+1} = k, e_\ell^{t+1} = 0 | x_\ell^{t+1}, \mathcal{F}^t\right) | \mathcal{F}^t \end{array} \right\} \\
&= \max_{\left\{\widetilde{e}_k^{t+1}(.)\right\}} \widetilde{\lambda}^t w_k^t\left(x_\ell^t\right) + \eta_k \\
&+ \delta\widehat{\phi}^{t+1}\left(1 - \chi_k\right) E\left[\begin{array}{c} \widetilde{e}_k^{t+1}\left(x_\ell^t\right) \mathcal{L}_W^{t+1}\left(k, 1, x_\ell^t, \omega_\ell^{t+1}\right) \\ + \left(1 - \widetilde{e}_k^{t+1}\left(x_\ell^t\right)\right) \mathcal{L}_W^{t+1}\left(k, 0, x_\ell^t, \omega_\ell^{t+1}\right) | \mathcal{F}^t \end{array} \right] \\
&+ \delta\widehat{\phi}^{t+1}\chi_k E\left[\mathcal{L}_W^{t+1}\left(k, 0, x_\ell^t, \omega_\ell^{t+1}\right) | \mathcal{F}^t\right] \qquad \text{(A.6)}
\end{aligned}$$

Make the following definitions

$$\begin{aligned}
\widetilde{U}_k^t\left(\omega_\ell^t\right) &\equiv \mathcal{L}_W^t\left(k_\ell^t = k, e_\ell^t = 0, x_\ell^t, \omega_\ell^t\right), \text{ and} \\
W_k^t\left(x\right) &\equiv \mathcal{L}_W^t\left(k_\ell^t = k, e_\ell^t = 1, x, \omega_\ell^t\right). \qquad \text{(A.7)}
\end{aligned}$$

$\widetilde{U}_k^t\left(\omega_\ell^t\right)$ is the value of unemployment in sector k, conditional on the preference shocks ω_ℓ^t, and $W_k^t\left(x\right)$ is the value of a job with match productivity x. Note that $\mathcal{L}_W^t\left(k_\ell^t = k, e_\ell^t = 0, x_\ell^t, \omega_\ell^t\right)$ does not depend on x_ℓ^t and $\mathcal{L}_W^t\left(k_\ell^t = k, e_\ell^t = 1, x, \omega_\ell^t\right)$ does not depend on ω_ℓ^t. Rewrite $\widetilde{U}_k^t\left(\omega_\ell^t\right)$ as

$$\begin{aligned}
\widetilde{U}_k^t\left(\omega_\ell^t\right) &= \max_{k', \left\{\widetilde{e}_k^{t+1}(.)\right\}} - C_{kk'} + \omega_{\ell,k'}^t + b_{k'} \\
&+ \delta\widehat{\phi}^{t+1}\theta_{k'}^t q\left(\theta_{k'}^t\right) \int W_{k'}^{t+1}\left(x\right) \widetilde{e}_{k'}^{t+1}\left(x\right) dG_{k'}\left(x\right) \\
&+ \delta\widehat{\phi}^{t+1}\left(1 - \theta_{k'}^t q\left(\theta_{k'}^t\right) \Pr\left(\widetilde{e}_{k'}^{t+1}\left(x_\ell^{t+1}\right) = 1\right)\right) E_\omega\left(\widetilde{U}_{k'}^{t+1}\left(\omega_\ell^{t+1}\right)\right), \qquad \text{(A.8)}
\end{aligned}$$

and so:

$$\begin{aligned}
\widetilde{U}_k^t\left(\omega_\ell^t\right) &= \max_{k', \left\{\widetilde{e}_k^{t+1}(.)\right\}} - C_{kk'} + \omega_{\ell,k'}^t + b_{k'} \\
&+ \delta\widehat{\phi}^{t+1}\theta_{k'}^t q\left(\theta_{k'}^t\right) \int \left(\begin{array}{c} W_{k'}^{t+1}\left(x\right) \widetilde{e}_{k'}^{t+1}\left(x\right) + \\ E_\omega\left(\widetilde{U}_{k'}^{t+1}\left(\omega_\ell^{t+1}\right)\right)\left(1 - \widetilde{e}_{k'}^{t+1}\left(x\right)\right) \end{array} \right) dG_{k'}\left(x\right) \\
&+ \delta\widehat{\phi}^{t+1}\left(1 - \theta_{k'}^t q\left(\theta_{k'}^t\right)\right) E_\omega\left(\widetilde{U}_{k'}^{t+1}\left(\omega_\ell^{t+1}\right)\right). \qquad \text{(A.9)}
\end{aligned}$$

Now, wewrite $W_k^t(x)$ as:

$$\begin{aligned} W_k^t(x) = & \max_{\{\widetilde{e}_k^{t+1}(.)\}} \widetilde{\lambda}^t w_k^t(x) + \eta_k \\ & + \delta\widehat{\phi}^{t+1}(1-\chi_k)\,\widetilde{e}_k^{t+1}(x)\,W_k^{t+1}(x) \\ & + \delta\widehat{\phi}^{t+1}\left(1-(1-\chi_k)\,\widetilde{e}_k^{t+1}(x)\right)E\left(\widetilde{U}_k^{t+1}\left(\omega_\ell^{t+1}\right)\right), \end{aligned} \tag{A.10}$$

and so

$$\begin{aligned} W_k^t(x) = & \max_{\{\widetilde{e}_k^{t+1}(.)\}} \widetilde{\lambda}^t w_k^t\left(x_\ell^t\right) + \eta_k \\ & + \delta\widehat{\phi}^{t+1}(1-\chi_k)\left(\widetilde{e}_k^{t+1}(x)\,W_k^{t+1}(x) + \left(1-\widetilde{e}_k^{t+1}(x)\right)E_\omega\left(\widetilde{U}_k^{t+1}\left(\omega_\ell^{t+1}\right)\right)\right) \\ & + \delta\widehat{\phi}^{t+1}\chi_k E_\omega\left(\widetilde{U}_k^{t+1}\left(\omega_\ell^{t+1}\right)\right). \end{aligned} \tag{A.11}$$

It is now clear that the optimal policy $\widetilde{e}_k^{t+1}(.)$ is:

$$\widetilde{e}_k^{t+1}(x) = \left\{ \begin{array}{c} 1 \text{ if } W_k^{t+1}(x) > E_\omega\left(\widetilde{U}_k^{t+1}\left(\omega_\ell^{t+1}\right)\right) \\ 0 \text{ otherwise} \end{array} \right\}. \tag{A.12}$$

Define $U_k^t \equiv E_\omega\left(\widetilde{U}_k^t\left(\omega_\ell^t\right)\right)$. We therefore have the following Bellman equations:

$$U_k^t = E_\omega\left(\begin{array}{c} \max_{k'} -C_{kk'} + b_{k'} + \omega_{\ell,k'}^t \\ +\delta\widehat{\phi}^{t+1}\theta_{k'}^t q\left(\theta_{k'}^t\right)\int \max\left\{W_{k'}^{t+1}(x), U_{k'}^{t+1}\right\} dG_{k'}(x) \\ +\delta\widehat{\phi}^{t+1}\left(1-\theta_{k'}^t q\left(\theta_{k'}^t\right)\right)U_{k'}^{t+1} \end{array}\right) \tag{A.13}$$

$$W_k^t(x) = \widetilde{\lambda}^t w_k^t(x) + \eta_k + \delta\widehat{\phi}^{t+1}(1-\chi_k)\left(\max\left\{W_k^{t+1}(x), U_k^{t+1}\right\}\right) + \delta\widehat{\phi}^{t+1}\chi_k U_k^{t+1} \tag{A.14}$$

B Steady State Equilibrium

In this section we derive the equations characterizing the steady state equilibrium. The key conditions that we impose is that variables are constant over time, inflows of workers into each sector equal outflows, and job destruction rates equal job creation rates. We also impose that the preference shifters $\left\{\phi_i^t\right\}$ are constant and equal to 1 in the long run.

Wage Equation

$$w_{k,i}(x) = \beta_{k,i}\widetilde{w}_{k,i}x + \frac{(1-\beta_{k,i})(1-\delta)U_{k,i} - (1-\beta_{k,i})\eta_{k,i}}{\widetilde{\lambda}_i} \tag{A.15}$$

Firms' value function

$$J_{k,i}(x) = \frac{1-\beta_{k,i}}{1-(1-\chi_{k,i})\delta}\widetilde{\lambda}_i\widetilde{w}_{k,i}\left(x-\underline{x}_{k,i}\right) \tag{A.16}$$

Probability of filling a vacancy

$$q_i\left(\theta_{k,i}\right) = \frac{\kappa_{k,i} P_i^F}{\widetilde{w}_{k,i}} \times \frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right) I_{k,i}\left(\underline{x}_{k,i}\right)} \tag{A.17}$$

where

$$I_{k,i}\left(\underline{x}_{k,i}\right) \equiv \int_{\underline{x}_{k,i}}^{x_{\max}} \left(s-\underline{x}_{k,i}\right) dG_{k,i}\left(s\right) \tag{A.18}$$

Unemployed workers' Bellman equation

$$U_{k,i} = \zeta_i \log\left(\sum_{k'} \exp\left\{\frac{-C_{kk',i} + b_{k',i} + \theta_{k',i}\frac{\kappa_{k',i}P_i^F}{\widetilde{w}_{k',i}} \times \widetilde{\lambda}_i \widetilde{w}_{k',i}\frac{\beta_{k',i}}{\left(1-\beta_{k',i}\right)} + \delta U_{k',i}}{\zeta_i}\right\}\right) \tag{A.19}$$

Transition rates

$$s_{k\ell,i} = \frac{\exp\left\{\frac{-C_{k\ell,i}+b_{\ell,i}+\theta_{\ell,i}\frac{\kappa_{\ell,i}P_i^F}{\widetilde{w}_{\ell,i}}\times\widetilde{\lambda}_i\widetilde{w}_{\ell,i}\frac{\beta_{\ell,i}}{\left(1-\beta_{\ell,i}\right)}+\delta U_{\ell,i}}{\zeta_i}\right\}}{\sum_{\overline{k}}\exp\left\{\frac{-C_{k\overline{k},i}+b_{\overline{k},i}+\theta_{\overline{k},i}\frac{\kappa_{\overline{k},i}P_i^F}{\widetilde{w}_{\overline{k},i}}\times\widetilde{\lambda}_i\widetilde{w}_{\overline{k},i}\frac{\beta_{\overline{k},i}}{\left(1-\beta_{\overline{k},i}\right)}+\delta U_{\overline{k},i}}{\zeta_i}\right\}} \tag{A.20}$$

Steady-state unemployment rates

$$u_{k,i} = \frac{\chi_{k,i}}{\theta_{k,i} q_i\left(\theta_{k,i}\right)\left(1-G_{k,i}\left(\underline{x}_{k,i}\right)\right)+\chi_{k,i}} \tag{A.21}$$

Trade + Price System

Input Bundle Price

$$P_{k,i}^M = \prod_{\ell=1}^{K}\left(\frac{P_{\ell,i}^I}{\nu_{k\ell,i}}\right)^{\nu_{k\ell,i}} \tag{A.22}$$

Domestic Sectoral Output Price

$$c_{k,i} = \left(\frac{\widetilde{w}_{k,i}}{\gamma_{k,i}}\right)^{\gamma_{k,i}}\left(\frac{P_{k,i}^M}{1-\gamma_{k,i}}\right)^{1-\gamma_{k,i}} \tag{A.23}$$

Price of Composite Sector-Specific Intermediate Good

$$P_{k,i}^I = \Gamma_{k,i}\left[\sum_{j=1}^{N}\frac{A_{k,j}}{\left(c_{k,j}d_{k,ji}\right)^{\lambda}}\right]^{-1/\lambda} \tag{A.24}$$

where $\Gamma_{k,i}$ is a sector and country specific constant.

Price of Final Consumption Good

$$P_i^F = \prod_{k=1}^{K} \left(\frac{P_{k,i}^I}{\mu_{k,i}} \right)^{\mu_{ki}} \tag{A.25}$$

Trade Shares

$$\pi_{k,oi} = \frac{A_{k,o} \left(c_{k,o} d_{k,oi} \right)^{-\lambda}}{\Phi_{k,i}}, \tag{A.26}$$

where

$$\Phi_{k,i} = \sum_{o=1}^{N} A_{k,o} \left(c_{k,o} d_{k,oi} \right)^{-\lambda}. \tag{A.27}$$

Zero net flows condition

$$\left(L_i . u_i \right) = \left(\sum_{\ell=1}^{K} s_{\ell k,i} L_{\ell,i} u_{\ell,i} \right)_{k=1}^{K} = s_i' \left(L_i . u_i \right) \tag{A.28}$$

Product market clearing

Gross Output

$$\gamma_{k,o} Y_{k,o} = \widetilde{w}_{k,o} L_{k,o} \left(1 - u_{k,o} \right) \int_{\underline{x}_{k,o}}^{x_{\max}} \frac{s}{1 - G_{k,i} \left(\underline{x}_{k,o} \right)} dG_{k,o} \left(s \right) \tag{A.29}$$

$$= \widetilde{w}_{k,o} \widetilde{L}_{k,o} \tag{A.30}$$

Expenditure with Vacancies

$$E_{k,o}^V = \kappa_{k,o} P_o^F \theta_{k,o} u_{k,o} L_{k,o} \tag{A.31}$$

Market Clearing System

$$Y_{k,o} = \sum_{i=1}^{N} \pi_{k,oi} E_{k,i} \tag{A.32}$$

$$E_{k,i} = \mu_{k,i} \left(\sum_{\ell=1}^{K} \gamma_{\ell,i} Y_{\ell,i} \right) + \sum_{\ell=1}^{K} \left(1 - \gamma_{\ell,i} \right) \nu_{\ell k,i} Y_{\ell,i} - \mu_{k,i} N X_i \tag{A.33}$$

Normalization: World total output is the numeraire

$$\sum_{i=1}^{N} \sum_{k=1}^{K} Y_{k,i} = 1 \tag{A.34}$$

Final Good Consumption Expenditure

$$E_i^C = \sum_{k=1}^{K} \gamma_{k,i} Y_{k,i} - \sum_{k=1}^{K} E_{k,i}^V - N X_i \tag{A.35}$$

Lagrange multipliers

$$\widetilde{\lambda}_i = \frac{\overline{L}_i}{E_i^C} \tag{A.36}$$

C Solution Methods

C.1 Estimation Algorithm

Define $I_{k,i}(x) \equiv \int_x^{x_{\max}} (s-x)\, dG_{k,i}(s)$. Imposing $G_{k,i} \sim \log \mathcal{N}\left(0, \sigma_{k,i}^2\right)$ and a bit of algebra leads to:

- $G_{k,i}(x) = \Phi\left(\frac{\ln x}{\sigma_{k,i}}\right)$
- $I_{k,i}(x) = \exp\left(\frac{\sigma_{k,i}^2}{2}\right)\Phi\left(\sigma_{k,i} - \frac{\ln x}{\sigma_{k,i}}\right) - x\Phi\left(-\frac{\ln x}{\sigma_{k,i}}\right)$
- $I_{k,i}(0) = \exp\left(\frac{\sigma_{k,i}^2}{2}\right)$
- $\int_{\underline{x}_{k,i}}^{x_{\max}} \frac{s}{1-G_{k,i}\left(\underline{x}_{k,i}\right)} dG_{k,i}(s) = \exp\left(\frac{\sigma_{k,i}^2}{2}\right) \frac{\Phi\left(\sigma_{k,i} - \frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}$

Note: The estimation procedure we describe takes trade shares $\pi_{k,oi}^{Data}$ as given.
Step 1: Solve for $\{Y_{k,i}\}$ using:

$$Y_{k,o} = \sum_{i=1}^{N}\sum_{\ell=1}^{K} \pi_{k,oi}^{Data}\left(\mu_{k,i}\gamma_{\ell,i} + (1-\gamma_{\ell,i})\,\nu_{\ell k,i}\right) Y_{\ell,i} - \sum_{i=1}^{N} \pi_{k,oi}^{Data}\mu_{k,i} N X_i$$

$$\sum_{o=1}^{N}\sum_{k=1}^{K} Y_{k,o} = 1$$

The rest of the procedure conditions on these values of $\{Y_{k,i}\}$.

Step 2: Guess model parameters Ω. We treat $\widetilde{\kappa}_{k,i} \equiv \frac{\kappa_{k,i} P_i^F}{\widetilde{w}_{k,i}}$ as parameters to be estimated.

Step 3: Define

$$\varpi_{k,i} \equiv \frac{\left(1-(1-\chi_{k,i})\,\delta\right)\widetilde{\kappa}_{k,i}}{\delta\left(1-\beta_{k,i}\right)}$$

If $\frac{\left(1-\left(1-\chi_{k,i}\right)\delta\right)\widetilde{\kappa}_{k,i}}{\delta\left(1-\beta_{k,i}\right)I_{k,i}(0)} = \frac{\varpi_{k,i}}{I_{k,i}(0)} \geq 1$, the free entry condition cannot be satisfied—$I_{k,i}$ is decreasing. Abort the procedure and highly penalize the objective function.

Step 4: Find $\underline{x}_{k,i}^{ub}$ such that $\frac{\left(1-\left(1-\chi_{k,i}\right)\delta\right)\widetilde{\kappa}_{k,i}}{\delta\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}_{k,i}^{ub}\right)} = 1 \Longleftrightarrow I_{k,i}\left(\underline{x}_{k,i}^{ub}\right) = \varpi_{k,i}$. If along the algorithm $\underline{x}_{k,i}$ goes above $\underline{x}_{k,i}^{ub}$, we update it to be equal to $\underline{x}_{k,i}^{ub}$ (minus a small number).

Step 5: Guess $\{L_{k,i}\}$, and $\left\{\underline{x}_{k,i}\right\}$

Step 6: Compute $I_{k,i}\left(\underline{x}_{k,i}\right)$, $G_{k,i}\left(\underline{x}_{k,i}\right)$, $\theta_{k,i}$ and $u_{k,i}$.

- $\theta_{k,i} = q_i^{-1}\left(\frac{\varpi_{k,i}}{I_{k,i}\left(\underline{x}_{k,i}\right)}\right)$ where $q_i^{-1}(y) = \left(\frac{1-y^{\xi_i}}{y^{\xi_i}}\right)^{1/\xi_i}$
- $u_{k,i} = \frac{\chi_{k,i}}{\theta_{k,i}q_i\left(\theta_{k,i}\right)\left(1-G_{k,i}\left(\underline{x}_{k,i}\right)\right)+\chi_{k,i}}$

Step 7: Compute $\left\{\widetilde{L}_{k,i}\right\}$

$$\begin{aligned}\widetilde{L}_{k,i} &\equiv L_{k,i}\left(1-u_{k,i}\right)\int_{\underline{x}_{k,i}}^{x_{\max}}\frac{s}{1-G_{k,i}\left(\underline{x}_{k,i}\right)}dG_{k,i}(s)\\ &= L_{k,i}\left(1-u_{k,i}\right)\exp\left(\frac{\sigma_{k,i}^2}{2}\right)\frac{\Phi\left(\sigma_{k,i}-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}\end{aligned}$$

Step 8: Compute $\{\widetilde{w}_{k,i}\}$

$$\widetilde{w}_{k,i} = \frac{\gamma_{k,i}Y_{k,i}}{\widetilde{L}_{k,i}}$$

Step 9: Compute $\left\{E_{k,i}^V\right\}$

$$E_{k,i}^V = \widetilde{\kappa}_{k,i}\widetilde{w}_{k,i}\theta_{k,i}u_{k,i}L_{k,i}$$

Step 10: Compute $\left\{E_i^C\right\}$

$$E_i^C = \sum_{k=1}^{K}\gamma_{k,i}Y_{k,i} - \sum_{k=1}^{K}E_{k,i}^V - NX_i$$

Step 11: Compute $\left\{\widetilde{\lambda}_i\right\}$

$$\widetilde{\lambda}_i = \frac{\overline{L}_i}{E_i^C}$$

Step 12: Obtain $\{U_{k,i}\}$.

- Step 12a: Guess $\left\{U_{ki}^0\right\}$
- Step 12b: Compute until convergence

$$U_{k,i}^{g+1} = \zeta_i\log\left(\sum_{\ell=1}^{K}\exp\left\{\frac{-C_{k\ell,i}+b_{\ell,i}+\theta_{\ell,i}\widetilde{\kappa}_{\ell,i}\widetilde{\lambda}_i\widetilde{w}_{\ell,i}\frac{\beta_{\ell,i}}{\left(1-\beta_{\ell,i}\right)}+\delta U_{\ell,i}-\delta U_{k,i}^g}{\zeta_i}\right\}\right)+\delta U_{k,i}^g$$

Step 13: Update $\{L_{k,i}\}$.

- Step 13a: Given knowledge of $\{U_{k,i}\}$, compute transition rates $s_{k\ell,i}$.

$$s_{k\ell,i} = \frac{\exp\left\{\frac{-C_{k\ell,i}+b_{\ell,i}+\theta_{\ell,i}\widetilde{\kappa}_{\ell,i}\widetilde{\lambda}_i\widetilde{w}_{\ell,i}\frac{\beta_{\ell,i}}{1-\beta_{\ell,i}}+\delta U_{\ell,i}}{\zeta_i}\right\}}{\sum_{\overline{k}}\exp\left\{\frac{-C_{k\overline{k},i}+b_{\overline{k},i}+\theta_{\overline{k},i}\widetilde{\kappa}_{\overline{k},i}\widetilde{\lambda}_i\widetilde{w}_{\overline{k},i}\frac{\beta_{\overline{k},i}}{1-\beta_{\overline{k},i}}+\delta U_{\overline{k},i}}{\zeta_i}\right\}}$$

- Step 13b: Find y_i such that

$$\left(I - s_i'\right) y_i = 0$$

- Step 13c: Find allocations $L_{k,i}$

$$L_{k,i}u_{k,i} = \varphi y_{k,i}$$

$$\Rightarrow L_{k,i} = \varphi \underbrace{y_{k,i}/u_{k,i}}_{\widetilde{y}_{k,i}}$$

$$\Rightarrow L_i' 1_{K\times 1} = \varphi \widetilde{y}_{k,i}' 1_{K\times 1} = \overline{L}_i$$

$$\Rightarrow \varphi = \frac{\overline{L}_i}{\widetilde{y}_{k,i}' 1_{K\times 1}}$$

$$\left(L_{k,i}\right)' = \varphi \widetilde{y}_{k,i}$$

$$L_{k,i}^{new} = \left(1-\lambda_L\right) L_{k,i} + \lambda_L \left(L_{k,i}\right)'$$

Step 14: Update $\left\{\underline{x}_{k,i}\right\}$.

Note that in equilibrium:

$$\widetilde{\lambda}_i\widetilde{w}_{k,i}\underline{x}_{k,i} = \left(1-\delta\right) U_{k,i} - \eta_{k,i} \tag{A.37}$$

So, we update $\underline{x}_{k,i}$ according to:

$$\left(\underline{x}_{k,i}\right)' = \frac{\left(1-\delta\right) U_{k,i} - \eta_{k,i}}{\widetilde{\lambda}_i\widetilde{w}_{k,i}}$$

$$\underline{x}_{k,i}^{new} = \min\left\{\left(1-\lambda_x\right)\underline{x}_{k,i} + \lambda_x\left(\underline{x}_{k,i}\right)', \underline{x}_{k,i}^{ub}\right\}$$

Step 15: Armed with $L_{k,i}^{new}$ and $\underline{x}_{k,i}^{new}$ go to Step 6 until $\left\|\left\{L_{k,i}^{new} - L_{k,i}\right\}\right\| \to 0$ and $\left\|\left\{\underline{x}_{k,i}^{new} - \underline{x}_{k,i}\right\}\right\| \to 0$.

Note that $\left\|\left\{\underline{x}_{k,i}^{new} - \underline{x}_{k,i}\right\}\right\| \to 0$ does not imply that (A.37) is satisfied. Therefore, we penalize deviations from (A.37) in the objective function.

Step 16: Generate moments, compute Loss Function, guess new parameter set Ω and go to Step 3, until objective function is minimized.

Note: Given that we condition on the trade shares $\pi_{k,oi}^{Data}$, we can estimate the model country by country, separately. However, in practice, we will first estimate the model for the US and obtain all of the US specific parameters. Next, armed with US-specific mobility costs $C_{kk'}$ and sector-specific exogenous exit components χ_k (we will impose $\chi_{k,i} = \chi_i + \chi_k$), we estimate the remaining countries' parameters separately, in parallel.

C.2 Expressions for Simulated Moments

C.2.1 Employment Shares

$$emp_{k,i} = \frac{L_{k,i}\left(1 - u_{k,i}\right)}{\sum_{k=1}^{K} L_{k,i}\left(1 - u_{k,i}\right)}$$

C.2.2 National Unemployment Rate

$$unemp_i = \frac{\sum_{k=1}^{K} L_{k,i} u_{k,i}}{\sum_{k=1}^{K} L_{k,i}}$$

C.2.3 Sector-Specific Average Wages

$$w_{k,i}\left(x\right) = \left(1 - \beta_{k,i}\right)\widetilde{w}_{k,i}\underline{x}_{k,i} + \beta_{k,i}\widetilde{w}_{k,i}x$$

$$\begin{aligned}
\overline{w}_{k,i} &= \frac{\int_{\underline{x}_{k,i}}^{x_{\max}} w_{k,i}\left(s\right) dG_{k,i}\left(s\right)}{1 - G_{k,i}\left(\underline{x}_{k,i}\right)} \\
&= \left(1 - \beta_{k,i}\right)\widetilde{w}_{k,i}\underline{x}_{k,i} + \beta_{k,i}\widetilde{w}_{k,i}\int_{\underline{x}_{k,i}}^{x_{\max}} \frac{s}{1 - G_{k,i}\left(\underline{x}_{k,i}\right)} dG_{k,i}\left(s\right) \\
&= \left(1 - \beta_{k,i}\right)\widetilde{w}_{k,i}\underline{x}_{k,i} + \beta_{k,i}\widetilde{w}_{k,i}\exp\left(\frac{\sigma_{k,i}^2}{2}\right)\frac{\Phi\left(\sigma_{k,i} - \frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}
\end{aligned}$$

C.2.4 Sector-Specific Variance of Wages

$$\begin{aligned}\sigma_{w,k,i}^2 &= \frac{\int_{\underline{x}_{k,i}}^{\infty}\left(w_{k,i}\left(s\right)-\overline{w}_{k,i}\right)^2 dG_{k,i}\left(s\right)}{1-G_{k,i}\left(\underline{x}_{k,i}\right)} \\ &= \left(\beta_{k,i}\widetilde{w}_{k,i}\right)^2 \times \frac{\int_{\underline{x}_{k,i}}^{\infty}\left(s-\exp\left(\frac{\sigma_{k,i}^2}{2}\right)\frac{\Phi\left(\sigma_{k,i}-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}\right)^2 dG_{k,i}\left(s\right)}{1-G_{k,i}\left(\underline{x}_{k,i}\right)} \\ &= \left(\beta_{k,i}\widetilde{w}_{k,i}\right)^2 \times \left(\exp\left(2\sigma_{k,i}^2\right)\frac{\Phi\left(2\sigma_{k,i}-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}-\exp\left(\sigma_{k,i}^2\right)\left(\frac{\Phi\left(\sigma_{k,i}-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}\right)^2\right)\end{aligned}$$

C.2.5 Transition Rates

Note that the transition rates in equation (10) are transitions from unemployment in sector k to search in sector k' within period t. There are no data counterfactuals for this variable. However, we can construct a matrix with transition rates between all possible (model) states between time t and time $t+N$ (where N is even)—where variables are measured at the t_a stage (which is the production stage). From this matrix, we can obtain N-period transition rates between all states observed in the data (employment in each of the sectors and unconditional unemployment). First, we obtain the one-year transition matrix $\widetilde{s}^{t,t+1}$ between states $\{\widetilde{u}_1,...,\widetilde{u}_K,1,...,K\}$. Here, we abuse notation to mean $\widetilde{u}_k$ as sector-k unemployment at the very beginning of a period.

The one-year transition rate between sector-ℓ unemployment and sector-k unemployment is given by:

$$\widetilde{s}_{\widetilde{u}_\ell\widetilde{u}_k,i}^{t,t+1} = s_{\ell k,i}^{t,t+1}\left(1-\theta_{k,i}^t q_i\left(\theta_{k,i}^t\right)\left(1-G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right)\right), \tag{A.38}$$

that is, a share $s_{\ell k,i}^t$ of individuals starting period t unemployed in sector ℓ choose to search in sector k. A fraction $\left(1-\theta_{k,i}^t q_i\left(\theta_{k,i}^t\right)\left(1-G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right)\right)$ of those do not find a match that survives until $t+1$. Similarly, the one-year transition rate between sector-ℓ unemployment and sector-k employment is given by:

$$\begin{aligned}\widetilde{s}_{\widetilde{u}_\ell k,i}^{t,t+1} &= s_{\ell k,i}^{t,t+1}\theta_{k,i}^t q_i\left(\theta_{k,i}^t\right)\left(1-G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right) \\ &= s_{\ell k,i}^{t,t+1} - \widetilde{s}_{\widetilde{u}_\ell\widetilde{u}_k,i}^{t,t+1}.\end{aligned} \tag{A.39}$$

According to the timing assumptions of the model, the one-year transition rate between employment in sector k and employment in sector k' is zero if $k \neq k'$. However, the persistence rate of employment in sector k is given by the probability that a match does not receive a death shock times the probability that the match is not dissolved because the threshold for production increases in the following period:

$$\widetilde{s}_{kk',i}^{t,t+1} = \begin{cases} 0 \text{ if } k \neq k' \\ \left(1-\chi_{k,i}\right)\Pr\left(x \geq \underline{x}_{k,i}^{t+1} | x \geq \underline{x}_{k,i}^t\right) \text{ if } k = k' \end{cases}. \tag{A.40}$$

Finally, the one-year transition rate between sector-k employment and unemployment in sector ℓ

is given by:

$$\widetilde{s}^{t,t+1}_{k\widetilde{u}_\ell,i} = \begin{cases} 0 \text{ if } k \neq \ell \\ \chi_{k,i} + (1-\chi_{k,i}) \Pr\left(x < \underline{x}^{t+1}_{k,i} | x \geq \underline{x}^{t}_{k,i}\right) \text{ if } k = \ell \end{cases}. \tag{A.41}$$

That is, if a worker is employed in sector k at t, she cannot start next period unemployed in sector ℓ if $k \neq \ell$. Otherwise, workers transition between sector k employment to sector k unemployment if their match is hit with a death shock or if their employer's productivity goes below the threshold for production at $t+1$.

We can now write the N-period transition matrix as:

$$\widetilde{s}^{t,t+N} = \widetilde{s}^{t+k-1,t+k} \times ... \times \widetilde{s}^{t+1,t+2} \times \widetilde{s}^{t,t+1}, \tag{A.42}$$

and we can write transition rates between unemployment $\widetilde{u}$ and sector-k employment between t and $t+N$ as:

$$\widetilde{s}^{t,t+N}_{\widetilde{u},k,i} = \frac{\sum_{\ell=1}^{K} L^{t-1}_{\ell,i} \widetilde{u}^{t-1}_{\ell,i} \widetilde{s}^{t,t+N}_{\widetilde{u}_\ell,k}}{\sum_{\ell=1}^{K} L^{t-1}_{\ell,i} \widetilde{u}^{t-1}_{\ell,i}}. \tag{A.43}$$

Finally, we can write transition rates between sector-k employment and unemployment $\widetilde{u}$ as:

$$\widetilde{s}^{t,t+N}_{k,\widetilde{u},i} = 1 - \sum_{k'=1}^{K} \widetilde{s}^{t,t+N}_{k,k',i}. \tag{A.44}$$

1-period transition rates

$$\widetilde{s}_{\widetilde{u}_\ell \widetilde{u}_k,i} = s_{\ell k,i} \left(1 - \theta_{k,i} q_i\left(\theta_{k,i}\right)\left(1 - G_{k,i}\left(\underline{x}_{k,i}\right)\right)\right)$$

$$\widetilde{s}_{\widetilde{u}_\ell k,i} = s_{\ell k,i} \theta_{k,i} q_i\left(\theta_{k,i}\right)\left(1 - G_{k,i}\left(\underline{x}_{k,i}\right)\right)$$

$$\widetilde{s}_{\ell k,i} = \begin{cases} 0 \text{ if } \ell \neq k \\ (1-\chi_{\ell,i}) \text{ if } \ell = k \end{cases}$$

$$\widetilde{s}_{\ell \widetilde{u}_k,i} = \begin{cases} 0 \text{ if } \ell \neq k \\ \chi_{k,i} \text{ if } \ell = k \end{cases}$$

N-period transition rates from and to unconditional unemployment: $\widetilde{s}^N$

$$\widetilde{s}^{N}_{\widetilde{u},k,i} = \frac{\sum_{\ell=1}^{K} L_{\ell,i} u_{\ell,i} \widetilde{s}^{N}_{\widetilde{u}_\ell,k,i}}{\sum_{\ell=1}^{K} L_{\ell,i} u_{\ell,i}}$$

$$\widetilde{s}^{N}_{k,\widetilde{u},i} = 1 - \sum_{\ell=1}^{K} \widetilde{s}^{N}_{k,\ell,i}.$$

C.3 Algorithm: Steady-State Equilibrium Following Shock

Consider shocks $\left\{A_{k,i}^0\right\} \to \left\{A_{k,i}^1\right\}$, $\left\{d_{k,oi}^0\right\} \to \left\{d_{k,oi}^1\right\}$, $\{NX^0\} \to \{NX_i^1\}$

We will be using 0 superscripts to denote the initial steady state, and 1 superscripts to denote the final steady state.

Start from estimated Steady State: $\left\{L_{k,i}^0\right\}$, $\left\{\underline{x}_{k,i}^0\right\}$, $\left\{\widetilde{w}_{k,i}^0\right\}$, $\left\{\pi_{k,oi}^0\right\}$

Note that $\pi_{k,oi}^0 = \pi_{k,oi}^{Data}$

We also have $\widetilde{\kappa}_{k,i}^0 = \frac{\kappa_{k,i} P_i^{F,0}}{\widetilde{w}_{k,i}^0}$, but we do not know $\left\{P_i^{F,0}\right\}$

Denote relative changes in variable a by $\widehat{a} = \frac{a^1}{a^0}$

Step 1: Guess $\left\{L_{k,i}^1\right\}$ and $\left\{\underline{x}_{k,i}^1\right\}$

Step 2: Guess $\left\{\widetilde{w}_{k,i}^1\right\}$

- Step 2a: Compute $\widehat{\widetilde{w}}_{k,i} = \frac{\widetilde{w}_{k,i}^1}{\widetilde{w}_{k,i}^0}$ and iteratively solve for $\widehat{P}_{k,i}^I$ and $\widehat{c}_{k,i}$ using the system

$$\widehat{c}_{k,i} = \left(\widehat{\widetilde{w}}_{k,i}\right)^{\gamma_{k,i}} \prod_{\ell=1}^{K} \left(\widehat{P}_{\ell,i}^I\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}}$$

$$\widehat{P}_{k,i}^I = \left(\sum_{o=1}^{N} \pi_{k,oi}^0 \widehat{A}_{k,o} \left(\widehat{c}_{k,o}\widehat{d}_{k,oi}\right)^{-\lambda}\right)^{-1/\lambda}$$

- Step 2b: Compute $\widehat{P}_{k,i}^F$:

$$\widehat{P}_i^F = \prod_{k=1}^{K} \left(\widehat{P}_{k,i}^I\right)^{\mu_{ki}}$$

- Step 2c: Compute $\widehat{\pi}_{k,oi}$:

$$\widehat{\pi}_{k,oi} = \widehat{A}_{k,o} \left(\frac{\widehat{c}_{k,o}\widehat{d}_{k,oi}}{\widehat{P}_{k,i}^I}\right)^{-\lambda}$$

- Step 2d: Compute

- $\pi^1_{k,oi} = \pi^0_{k,oi}\widehat{\pi}_{k,oi}$
- $\widetilde{\kappa}^1_{k,i} \equiv \frac{\kappa_{k,i}P^{F,1}_i}{\widetilde{w}^1_{k,i}} = \frac{\kappa_{k,i}P^{F,0}_i}{\widetilde{w}^0_{k,i}}\frac{P^{F,1}_i}{P^{F,0}_i}\frac{\widetilde{w}^0_{k,i}}{\widetilde{w}^1_{k,i}} = \widetilde{\kappa}^0_{k,i}\frac{\widehat{P}^F_i}{\widehat{\widetilde{w}}_{k,i}}$

Step 3: If $\widetilde{\kappa}^1_{k,i} \times \frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}^1_{k,i}\right)} \geq 1$ abort, set $\underline{x}^1_{k,i}$ such that $\widetilde{\kappa}^1_{k,i} \times \frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}^1_{k,i}\right)} = 1-\varepsilon$ and go back to Step 1 with this new guess. If $\widetilde{\kappa}^1_{k,i} \times \frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}^1_{k,i}\right)} < 1$, proceed to Step 4.

Step 4: Compute

$$q_i\left(\theta^1_{k,i}\right) = \widetilde{\kappa}^1_{k,i} \times \frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}^1_{k,i}\right)}$$

$$\theta^1_{k,i} = q_i^{-1}\left(\widetilde{\kappa}^1_{k,i} \times \frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}^1_{k,i}\right)}\right)$$

$$u^1_{k,i} = \frac{\chi_{k,i}}{\theta^1_{k,i}q_i\left(\theta^1_{k,i}\right)\left(1-G_{k,i}\left(\underline{x}^1_{k,i}\right)\right)+\chi_{k,i}}$$

Step 5: Solve system in $\left\{Y^1_{k,o}\right\}$

$$Y^1_{k,o} = \sum_{i=1}^N \pi^1_{k,oi}\left(\mu_{k,i}\left(\sum_{\ell=1}^K \gamma_{\ell,i}Y^1_{\ell,i}\right) + \sum_{\ell=1}^K\left(1-\gamma_{\ell,i}\right)\nu_{\ell k,i}Y^1_{\ell,i}\right) - \sum_{i=1}^N \pi^1_{k,oi}\mu_{k,i}NX^1_i$$

$$\sum_{i=1}^N\sum_{k=1}^K Y^1_{k,i} = 1$$

Step 6: Compute $\left\{\widetilde{L}^1_{k,i}\right\}$

$$\widetilde{L}^1_{k,i} \equiv L^1_{k,i}\left(1-u^1_{k,i}\right)\int_{\underline{x}^1_{k,i}}^{x_{\max}} \frac{s}{1-G_{k,i}\left(\underline{x}^1_{k,i}\right)}dG_{k,i}\left(s\right)$$

$$= L^1_{k,i}\left(1-u^1_{k,i}\right)\exp\left(\frac{\sigma^2_{k,i}}{2}\right)\frac{\Phi\left(\sigma_{k,i}-\frac{\ln \underline{x}^1_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}^1_{k,i}}{\sigma_{k,i}}\right)}$$

Step 7: Update $\left\{\widetilde{w}^1_{k,i}\right\}$

$$\left(\widetilde{w}^1_{k,i}\right)^{new} = \frac{\gamma_{k,i}Y^1_{k,i}}{\widetilde{L}^1_{k,i}}$$

Go back to Step 2a and repeat until converegence of $\left\{\widetilde{w}^1_{k,i}\right\}$.

Step 8: Compute

$$E_{k,i}^{V,1} = \widetilde{\kappa}_{k,i}^{1} \widetilde{w}_{k,i}^{1} \theta_{k,i}^{1} u_{k,i}^{1} L_{k,i}^{1}$$

$$E_{i}^{C,1} = \sum_{k=1}^{K} \gamma_{k,i} Y_{k,i}^{1} - \sum_{k=1}^{K} E_{k,i}^{V,1} - NX_{i}^{1}$$

Step 9: Obtain Lagrange Multipliers

$$\widetilde{\lambda}_{i}^{1} = \frac{\overline{L}_{i}}{E_{i}^{C,1}}$$

Step 10: Compute Bellman Equations

$$U_{k,i}^{1} = \zeta_{i} \log \left(\sum_{k'} \exp \left\{ \frac{-C_{kk',i} + b_{k',i} + \theta_{k',i}^{1} \widetilde{\kappa}_{k',i}^{1} \widetilde{\lambda}_{i}^{1} \widetilde{w}_{k',i}^{1} \frac{\beta_{k',i}}{\left(1-\beta_{k',i}\right)} + \delta U_{k',i}^{1}}{\zeta_{i}} \right\} \right)$$

Step 11: Update $\left\{ L_{k,i}^{1} \right\}$.

- Step 11a: Given knowledge of $\left\{ U_{k,i}^{1} \right\}$, compute transition rates $s_{k\ell,i}^{1}$.

$$s_{k\ell,i}^{1} = \frac{\exp \left\{ \frac{-C_{k\ell,i} + b_{\ell,i} + \theta_{\ell,i}^{1} \widetilde{\kappa}_{\ell,i}^{1} \widetilde{\lambda}_{i}^{1} \widetilde{w}_{\ell,i}^{1} \frac{\beta_{\ell,i}}{\left(1-\beta_{\ell,i}\right)} + \delta U_{\ell,i}^{1}}{\zeta_{i}} \right\}}{\sum_{\overline{k}} \exp \left\{ \frac{-C_{k\overline{k},i} + b_{\overline{k},i} + \theta_{\overline{k},i}^{1} \widetilde{\kappa}_{\overline{k},i}^{1} \widetilde{\lambda}_{i}^{1} \widetilde{w}_{\overline{k},i}^{1} \frac{\beta_{\overline{k},i}}{\left(1-\beta_{\overline{k},i}\right)} + \delta U_{\overline{k},i}^{1}}{\zeta_{i}} \right\}}$$

- Step 11b: Find y_i such that

$$\left(I - \left(s_{i}^{1} \right)^{T} \right) y_{i} = 0$$

- Step 11c: Find allocations $L_{k,i}$

$$L_{k,i}^{1} u_{k,i}^{1} = \varphi y_{k,i}$$

$$\Rightarrow L_{k,i}^{1} = \varphi \underbrace{y_{k,i} / u_{k,i}^{1}}_{\widetilde{y}_{k,i}}$$

$$\Rightarrow \left(L_{i}^{1} \right)^{T} 1_{K \times 1} = \varphi \widetilde{y}_{k,i}^{T} 1_{K \times 1} = \overline{L}_{i}$$

$$\Rightarrow \varphi = \frac{\overline{L}_{i}}{\widetilde{y}_{k,i}^{T} 1_{K \times 1}}$$

$$\left(L_{k,i}^{1}\right)' = \varphi \widetilde{y}_{k,i}$$

$$\left(L_{k,i}^{1}\right)^{new} = \left(1-\lambda_L\right) L_{k,i}^{1} + \lambda_L \left(L_{k,i}^{1}\right)'$$

Step 12: Update $\left\{\underline{x}_{k,i}^{1}\right\}$.

Note that in equilibrium:

$$\widetilde{\lambda}_i^1 \widetilde{w}_{k,i}^1 \underline{x}_{k,i}^1 = \left(1-\delta\right) U_{k,i}^1 - \eta_{k,i}$$

So, we update $\underline{x}_{k,i}^{1}$ according to:

$$\left(\underline{x}_{k,i}^{1}\right)' = \frac{\left(1-\delta\right) U_{k,i}^1 - \eta_{k,i}}{\widetilde{\lambda}_i^1 \widetilde{w}_{k,i}^1}$$

$$\left(\underline{x}_{k,i}^{1}\right)^{new} = \min\left\{\left(1-\lambda_x\right)\underline{x}_{k,i}^{1} + \lambda_x \left(\underline{x}_{k,i}^{1}\right)', \underline{x}_{k,i}^{ub}\right\}$$

Step 13: Armed with $\left(L_{k,i}^{1}\right)^{new}$ and $\left(\underline{x}_{k,i}^{1}\right)^{new}$ go to Step 2 until $\left\|\left\{\left(L_{k,i}^{1}\right)^{new} - L_{k,i}^{1}\right\}\right\| \to 0$ and $\left\|\left\{\left(\underline{x}_{k,i}^{1}\right)^{new} - \underline{x}_{k,i}^{1}\right\}\right\| \to 0$.

C.4 Algorithm: Out-of-Steady-State Transition

Inner Loop: conditional on paths for expenditures $\left\{E_i^{C,t}\right\}$ – determined in the Outer Loop below.

Consider paths $\left\{A_{k,i}^t\right\}_{t=0}^{T_{SS}}$ and $\left\{d_{o,i,k}^t\right\}_{t=0}^{T_{SS}}$ with $A_{k,i}^0 = 1$ and $d_{o,i,k}^0 = 1$. Also, consider paths $\left\{\phi_i^t\right\}_{t=0}^{T_{SS}}$ with $\phi_i^0 = 1$ and $\widehat{\phi}_i^t = 1$ for $T \leq t \leq T_{SS}$, for some $T << T_{SS}$.

Step 0: Given paths $\left\{E_i^{C,t}\right\}$, compute paths $\left\{\widetilde{\lambda}_i^t\right\}$: $\widetilde{\lambda}_i^t = \frac{\overline{L}_i}{E_i^{C,t}}$

Step 1: Guess paths $\left\{\widetilde{w}_{k,i}^t\right\}_{t=1}^{T_{SS}}$ for each sector k and country i.

Step 2: Compute $\underline{x}_{k,i}^{T_{SS}}$ consistent with $\widetilde{w}_{k,i}^{T_{SS}}$ and $\widetilde{\lambda}_i^{T_{SS}}$. Obtain $\theta_{k,i}^{T_{SS}}$, $U_{k,i}^{T_{SS}}$, $s_{k\ell,i}^{T_{SS},T_{SS}+1}$ and $\pi_{k,oi}^{T_{SS}}$.

- Step 2a: Compute $\widehat{\widetilde{w}}_{k,i} = \frac{\widetilde{w}_{k,i}^{T_{SS}}}{\widetilde{w}_{k,i}^0}$, $\widehat{A}_{k,i} = \frac{A_{k,i}^{T_{SS}}}{A_{k,i}^0}$ and $\widehat{d}_{k,i} = \frac{d_{o,i,k}^{T_{SS}}}{d_{o,i,k}^0}$. Iteratively solve for $\widehat{P}_{k,i}^I$ and $\widehat{c}_{k,i}$ using the system

$$\widehat{c}_{k,i} = \left(\widehat{\widetilde{w}}_{k,i}\right)^{\gamma_{k,i}} \prod_{\ell=1}^{K} \left(\widehat{P}_{\ell,i}^I\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}}$$

$$\widehat{P}_{k,i}^I = \left(\sum_{o=1}^{N} \pi_{k,oi}^0 \widehat{A}_{k,o} \left(\widehat{c}_{k,o}\widehat{d}_{k,oi}\right)^{-\lambda}\right)^{-1/\lambda}$$

- Step 2b: Compute $\widehat{P}_{k,i}^F$:

$$\widehat{P}_i^F = \prod_{k=1}^{K} \left(\widehat{P}_{k,i}^I\right)^{\mu_{ki}}$$

- Step 2c: Compute

$$\widehat{\pi}_{k,oi} = \widehat{A}_{k,o} \left(\frac{\widehat{c}_{k,o}\widehat{d}_{k,oi}}{\widehat{P}_{k,i}^I}\right)^{-\lambda}$$

  And obtain $\pi_{k,oi}^{T_{SS}} = \pi_{k,oi}^0 \widehat{\pi}_{k,oi}$

- Step 2d: Compute

  - $\widetilde{\kappa}_{k,i}^{T_{SS}} = \widetilde{\kappa}_{k,i}^0 \frac{\widehat{P}_i^F}{\widehat{\widetilde{w}}_{k,i}}$

- Step 2e: Guess $\left\{\underline{x}_{k,i}^{T_{SS}}\right\}$

- Step 2f: Compute

$$\theta_{k,i}^{T_{SS}} = q_i^{-1} \left(\widetilde{\kappa}_{k,i}^{T_{SS}} \times \frac{1 - \delta\left(1 - \chi_{k,i}\right)}{\delta\left(1 - \beta_{k,i}\right) I_{k,i}\left(\underline{x}_{k,i}^{T_{SS}}\right)}\right)$$

- Step 2g: Compute Bellman Equations

$$U_{k,i}^{T_{SS}} = \zeta_i \log\left(\sum_{k'} \exp\left\{\frac{-C_{kk',i} + b_{k',i} + \theta_{k',i}^{T_{SS}} \widetilde{\kappa}_{k',i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{k',i}^{T_{SS}} \frac{\beta_{k',i}}{(1-\beta_{k',i})} + \delta U_{k',i}^{T_{SS}}}{\zeta_i}\right\}\right)$$

- Step 2h: Compute

$$\left(\underline{x}_{k,i}^{T_{SS}}\right)' = \frac{(1-\delta)\, U_{k,i}^{T_{SS}} - \eta_{k,i}}{\widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{k,i}^{T_{SS}}}$$

- Step 2i: Update $\underline{x}_{k,i}^{T_{SS}} = (1-\lambda_x)\, \underline{x}_{k,i}^{T_{SS}} + \lambda_x \left(\underline{x}_{k,i}^{T_{SS}}\right)'$ and go back to Step 2d until convergence.

- Step 2j: Compute $s_{kk'}^{T_{SS}, T_{SS}+1}$

$$s_{k\ell,i}^{T_{SS},T_{SS}+1} = \frac{\exp\left\{\frac{-C_{k\ell,i} + b_{\ell,i} + \theta_{\ell,i}^{T_{SS}} \widetilde{\kappa}_{\ell,i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{\ell,i}^{T_{SS}} \frac{\beta_{\ell,i}}{1-\beta_{\ell,i}} + \delta U_{\ell,i}^{T_{SS}}}{\zeta_i}\right\}}{\sum_{\overline{k}} \exp\left\{\frac{-C_{k\overline{k},i} + b_{\overline{k},i} + \theta_{\overline{k},i}^{T_{SS}} \widetilde{\kappa}_{\overline{k},i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{\overline{k},i}^{T_{SS}} \frac{\beta_{\overline{k},i}}{1-\beta_{\overline{k},i}} + \delta U_{\overline{k},i}^{T_{SS}}}{\zeta_i}\right\}}$$

Step 3: Obtain series $\left\{\pi_{k,oi}^t\right\}_{t=0}^{T_{SS}}$, $\left\{\widetilde{\kappa}_{k,i}^t\right\}_{t=0}^{T_{SS}}$. Define $\widehat{x}^t \equiv \frac{x^t}{x^0}$.

- Step 3a: For $t = 1, ..., T_{SS} - 1$ compute $\widehat{\widetilde{w}}_{k,i}^t = \frac{\widetilde{w}_{k,i}^t}{\widetilde{w}_{k,i}^0}$ and iteratively solve for $\widehat{P}_{k,i}^{I,t}$ and $\widehat{c}_{k,i}^t$ using the system

$$\widehat{c}_{k,i}^t = \left(\widehat{\widetilde{w}}_{k,i}^t\right)^{\gamma_{k,i}} \prod_{\ell=1}^{K} \left(\widehat{P}_{\ell,i}^{I,t}\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}}$$

$$\widehat{P}_{k,i}^{I,t} = \left(\sum_{o=1}^{N} \pi_{k,oi}^0 \widehat{A}_{k,o}^t \left(\widehat{c}_{k,o}^t \widehat{d}_{k,oi}^t\right)^{-\lambda}\right)^{-1/\lambda}$$

- Step 3b: Compute $\widehat{P}_{k,i}^{F,t}$ for $t = 1, ..., T_{SS} - 1$:

$$\widehat{P}_i^{F,t} = \prod_{k=1}^{K} \left(\widehat{P}_{k,i}^{I,t}\right)^{\mu_{ki}}$$

- Step 3c: Compute $\widehat{\pi}_{k,oi}^t$ for $t = 1, ..., T_{SS} - 1$:

$$\widehat{\pi}_{k,oi}^t = \widehat{A}_{k,o}^t \left(\frac{\widehat{c}_{k,o}^t \widehat{d}_{k,oi}^t}{\widehat{P}_{k,i}^{I,t}}\right)^{-\lambda}$$

- Step 3d: Compute or $t = 1, ..., T_{SS} - 1$:

- $\pi^t_{k,oi} = \pi^0_{k,oi}\widehat{\pi}^t_{k,oi}$
- $\widetilde{\kappa}^t_{k,i} \equiv \frac{\kappa_{k,i}P^{F,t}_i}{\widetilde{w}^t_{k,i}} = \frac{\kappa_{k,i}P^{F,0}_i}{\widetilde{w}^0_{k,i}}\frac{P^{F,t}_i}{P^{F,0}_i}\frac{\widetilde{w}^0_{k,i}}{\widetilde{w}^t_{k,i}} = \widetilde{\kappa}^0_{k,i}\frac{\widehat{P}^{F,t}_i}{\widehat{\widetilde{w}}^t_{k,i}}$

Step 4: Given knowledge of $\widetilde{w}^{T_{SS}}_{k,i}$, $\widetilde{\lambda}^{T_{SS}}_i$ and $\underline{x}^{T_{SS}}_{k,i}$ (and therefore $J^{T_{SS}}_{k,i}(s)$), start at $t = T_{SS}-1$ and sequentially compute (backwards) for each $t = T_{SS}-1, ..., 1$

- Step 4a: Given $\widetilde{w}^t_{k,i}$, $\underline{x}^{t+1}_{k,i}$, $\widetilde{\kappa}^t_{k,i}$, $\widetilde{\lambda}^t_i$ and $J^{t+1}_{k,i}(s)$ compute $\theta^t_{k,i}$.

If $\frac{\widetilde{\lambda}^t_i\widetilde{\kappa}^t_{k,i}\widetilde{w}^t_{k,i}}{\delta\widehat{\phi}^{t+1}_i\int^{x^{\max}}_{\underline{x}^{t+1}_{k,i}}J^{t+1}_{k,i}(s)dG_{k,i}(s)} \leq 1$ then

$$\theta^t_{k,i} = q_i^{-1}\left(\frac{\widetilde{\lambda}^t_i\widetilde{\kappa}^t_{k,i}\widetilde{w}^t_{k,i}}{\delta\widehat{\phi}^{t+1}_i\int^{x^{\max}}_{\underline{x}^{t+1}_{k,i}}J^{t+1}_{k,i}(s)\,dG_{k,i}(s)}\right)$$

If $\frac{\widetilde{\lambda}^t_i\widetilde{\kappa}^t_{k,i}\widetilde{w}^t_{k,i}}{\delta\widehat{\phi}^{t+1}_i\int^{x^{\max}}_{\underline{x}^{t+1}_{k,i}}J^{t+1}_{k,i}(s)dG_{k,i}(s)} > 1$, it is not possible to satisfy $V^t_{k,i} = 0$, so that $V^t_{k,i} < 0$ and

$$\theta^t_{k,i} = 0.$$

- Step 4b: Given $\underline{x}^{t+1}_{k,i}$, $W^{t+1}_{k,i}(x) = \frac{\beta_{k,i}}{1-\beta_{k,i}}J^{t+1}_{k,i}(x) + U^{t+1}_{k,i}$ (for $x \geq \underline{x}^{t+1}_{k,i}$), $\theta^t_{k,i}$, $U^{t+1}_{k,i}$ compute $U^t_{k,i}$. Notice that $\int^{x^{\max}}_{\underline{x}^{t+1}_{k,i}}W^{t+1}_{k,i}(s)\,dG_{k,i}(s) = \frac{\beta_{k,i}}{1-\beta_{k,i}}\int^{x^{\max}}_{\underline{x}^{t+1}_{k,i}}J^{t+1}_{k,i}(s)\,dG_{k,i}(s) + \left(1-G_{k,i}\left(\underline{x}^{t+1}_{k,i}\right)\right)U^{t+1}_{k,i}$ so that:

$$U^t_{k,i} = \zeta_i\log\left(\sum_{k'}\exp\left\{\frac{\begin{array}{c}-C_{kk',i}+b_{k',i}\\ +\delta\widehat{\phi}^{t+1}_i\theta^t_{k',i}q_i\left(\theta^t_{k',i}\right)\frac{\beta_{k,i}}{1-\beta_{k,i}}\int^{x^{\max}}_{\underline{x}^{t+1}_{k,i}}J^{t+1}_{k,i}(s)\,dG_{k,i}(s) + \delta\widehat{\phi}^{t+1}_i U^{t+1}_{k',i}\end{array}}{\zeta_i}\right\}\right)$$

- Step 4c: Given $J^{t+1}_{k,i}(x)$, $\widetilde{w}^t_{k,i}$, $U^t_{k,i}$, $U^{t+1}_{k,i}$ and $\underline{x}^{t+1}_{k,i}$ compute $J^t_{k,i}(x)$

$$\begin{aligned}J^t_{k,i}(x) &= (1-\beta_{k,i})\widetilde{\lambda}^t_i\widetilde{w}^t_{k,i}x + (1-\beta_{k,i})\eta_{k,i}\\ &\quad -(1-\beta_{k,i})\left(U^t_{k,i} - \delta\widehat{\phi}^{t+1}_i U^{t+1}_{k,i}\right) + (1-\chi_{k,i})\delta\widehat{\phi}^{t+1}_i\max\left\{J^{t+1}_{k,i}(x), 0\right\}\end{aligned}$$

- Step 4d: Solve for $\underline{x}^t_{k,i}$: $J^t_{k,i}\left(\underline{x}^t_{k,i}\right) = 0$

Step 5: Compute transition rates $\left\{s^{t,t+1}_{kk',i}\right\}_{t=1}^{T_{SS}-1}$ for all countries i according to:

$$s^{t,t+1}_{kk',i} = \frac{\exp\left\{\begin{array}{c}-C_{kk',i}+b_{k',i}+\\ \delta\widehat{\phi}^{t+1}_i\theta^t_{k',i}q(\theta^t_{k',i})\frac{\beta_{k',i}}{1-\beta_{k',i}}\int^{x^{\max}}_{\underline{x}^{t+1}_{k',i}}J^{t+1}_{k',i}(x)\,dG_{k',i}(x) + \delta\widehat{\phi}^{t+1}_i U^{t+1}_{k',i}\end{array}\right\}}{\sum_{k''}\exp\left\{\begin{array}{c}-C_{kk'',i}+b_{k'',i}+\\ \delta\widehat{\phi}^{t+1}_i\theta^t_{k'',i}q(\theta^t_{k'',i})\frac{\beta_{k'',i}}{1-\beta_{k'',i}}\int^{x^{\max}}_{\underline{x}^{t+1}_{k'',i}}J^{t+1}_{k'',i}(x)\,dG_{k'',i}(x) + \delta\widehat{\phi}^{t+1}_i U^{t+1}_{k'',i}\end{array}\right\}}.$$

Step 6: Start loop over t going forward ($t = 0$ to $t = T_{SS} - 1$)

Initial conditions: we know $\widetilde{u}_{k,i}^{t=-1} = u_{k,i}^{t=0}$, $L_{k,i}^{t=-1} = L_{k,i}^{t=0}$, and $\theta_{k,i}^{t=0}$ from the initial steady state computation. Obtain $\widetilde{u}_{k,i}^{t}$ and $L_{k,i}^{t}$ using flow conditions and sequences $\left\{\theta_{k,i}^{t}\right\}$, $\left\{\underline{x}_{k,i}^{t}\right\}$.

- Step 6a: Compute

$$JC_{k,i}^{t} = L_{k,i}^{t} u_{k,i}^{t} \theta_{k,i}^{t} q_i\left(\theta_{k,i}^{t}\right)\left(1 - G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right)$$

$$JD_{k,i}^{t} = \left(\chi_{k,i} + (1 - \chi_{k,i}) \max\left\{\frac{G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right) - G_{k,i}\left(\underline{x}_{k,i}^{t}\right)}{1 - G_{k,i}\left(\underline{x}_{k,i}^{t}\right)}, 0\right\}\right) L_{k,i}^{t-1}\left(1 - \widetilde{u}_{k,i}^{t-1}\right)$$

$$\widetilde{u}_{k,i}^{t} = \frac{L_{k,i}^{t} u_{k,i}^{t} - JC_{k,i}^{t} + JD_{k,i}^{t}}{L_{k,i}^{t}}$$

- Step 6b: Compute

$$L_{k,i}^{t+1} = L_{k,i}^{t} + IF_{k,i}^{t+1} - OF_{k,i}^{t+1},$$

where

$$IF_{k,i}^{t+1} = \sum_{\ell \neq k} L_{\ell,i}^{t} \widetilde{u}_{\ell,i}^{t} s_{\ell k,i}^{t+1,t+2},$$

and

$$OF_{k,i}^{t+1} = L_{k,i}^{t} \widetilde{u}_{k,i}^{t}\left(1 - s_{kk,i}^{t+1,t+2}\right).$$

- Step 6c: Compute

$$u_{k,i}^{t+1} = \frac{\sum_{\ell=1}^{K} L_{\ell,i}^{t} \widetilde{u}_{\ell,i}^{t} s_{\ell k,i}^{t+1,t+2}}{L_{k,i}^{t+1}}$$

- Step 6d: Compute

$$\begin{aligned}\widetilde{L}_{k,i}^{t+1} &= L_{k,i}^{t}\left(1 - \widetilde{u}_{k,i}^{t}\right) \int_{\underline{x}_{k,i}^{t+1}}^{\infty} \frac{s}{1 - G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)} dG_{k,i}(s) \\ &= L_{k,i}^{t}\left(1 - \widetilde{u}_{k,i}^{t}\right) \exp\left(\frac{\sigma_{k,i}^{2}}{2}\right) \frac{\Phi\left(\sigma_{k,i} - \frac{\ln \underline{x}_{k,i}^{t+1}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}^{t+1}}{\sigma_{k,i}}\right)}\end{aligned}$$

- Step 6e: Compute expenditure with vacancies

$$E_{k,i}^{V,t+1} = \widetilde{\kappa}_{k,i}^{t+1} \widetilde{w}_{k,i}^{t+1} \theta_{k,i}^{t+1} u_{k,i}^{t+1} L_{k,i}^{t+1}$$

- Step 6f: Solve for $\left\{Y_{k,i}^{t+1}\right\}$ in the system

$$E_{k,i}^{t+1} = \mu_{k,i} E_i^{C,t+1} + \sum_{\ell=1}^{K} \left(\mu_{k,i} E_{\ell,i}^{V,t+1} + (1 - \gamma_{\ell,i}) \nu_{\ell k,i} Y_{\ell,i}^{t+1} \right).$$

$$Y_{k,o}^{t+1} = \sum_{i=1}^{N} \pi_{k,oi}^{t+1} E_{k,i}^{t+1}.$$

- Step 6g: Compute $\left(\widetilde{w}_{k,i}^{t+1}\right)' = \frac{\gamma_{k,i} Y_{k,i}^{t+1}}{\widetilde{L}_{k,i}^{t+1}}$

Step 7: Compute distance $dist\left(\left\{\widetilde{w}_{k,i}^{t}\right\}, \left\{\left(\widetilde{w}_{k,i}^{t}\right)'\right\}\right)$

- Step 7b: Update $\widetilde{w}_{k,i}^{t} = (1 - \lambda_w)\, \widetilde{w}_{k,i}^{t} + \lambda_w \left(\widetilde{w}_{k,i}^{t}\right)'$ $t = 1, ..., T_{SS}$
- Step 7c: At this point, we have a new series for $\left\{\widetilde{w}_{k,i}^{t}\right\}$ – go back to Step 2 until convergence of $\left\{\widetilde{w}_{k,i}^{t}\right\}$.

Step 8: Compute disposable income $\left\{I_i^t\right\}_{t=1}^{T_{SS}}$

$$I_i^t = \sum_{\ell=1}^{K} \left(\gamma_{\ell,i} Y_{\ell,i}^{t} - E_{\ell,i}^{V,t} \right)$$

Outer Loop: iteration on $\left\{NX_i^t\right\}$

Step 0: Impose a change in a subset of parameters that happens at $t = 0$, but between t_c and t_d. That is, the shock occurs **after** production, workers' decisions of where to search and after firms post vacancies at $t = 0$. Impose a large value for T_{SS}. Assume that for $t \geq T_{SS}$ the system will have converged to a new steady state. World expenditure with final goods $\sum_{i=1}^{I} E_i^{C,t}$ is normalized to 1 for every t.

Step 1: Start with estimated state equilibrium at $t = 0$. Remember that we used the normalization $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$ during the estimation procedure. **Change the normalization** from $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$ to $\sum_{i=1}^{I} E_i^C = 1$. Nominal variables to be renormalized: $\left\{Y_{k,i}^0\right\}$, $\left\{\widetilde{w}_{k,i}^0\right\}$, $\left\{E_i^{C,0}\right\}$, $\left\{NX_i^0\right\}$.

Step 2: Obtain B_i^0 with respect to the normalization $\sum_{i=1}^{I} E_i^C = 1$. Equation (XX) gives us:

$$B_i^0 = \frac{NX_i^0}{\left(1 - \frac{1}{\delta}\right)}$$

Step 3: Make initial guess for $NX_i^{T_{SS}}$ (with respect to the normalization $\sum_{i=1}^{I} E_i^C = 1$).

Step 4: Compute steady state equilibrium at T_{SS}, conditional on $NX_i^{T_{SS}}$, and the change in parameter values.

- Step 4a: Notice that steady algorithm uses the normalization $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$. Normalize $NX_i^{T_{SS}}$ with respect to normalization $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$. To perform such normalization, use output $\left\{Y_{k,i}^{T_{SS}}\right\}$ obtained in the initial steady state if this is the first outer loop iteration, otherwise use output $\left\{Y_{k,i}^{T_{SS}}\right\}$ obtained in Step 6 below.

- Step 4b: After computing the final steady state, **change the normalization** from $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$ to $\sum_{i=1}^{I} E_i^C = 1$ using $\left\{E_i^C\right\}$ obtained in Step 3a. Nominal variables to be renormalized: $\left\{Y_{k,i}^{T_{SS}}\right\}$, $\left\{\widetilde{w}_{k,i}^{T_{SS}}\right\}$, $\left\{E_i^{C,T_{SS}}\right\}$, $\left\{NX_i^{T_{SS}}\right\}$.

Step 5: Start at $t = T_{SS} - 1$ and go backward until $t = 1$ and sequentially compute:

$$R^{t+1} = \frac{1}{\delta}\frac{\sum_{i=1}^{N}\frac{E_i^{C,t+1}}{\widehat{\phi}_i^{t+1}}}{\sum_{i=1}^{N} E_i^{C,t}} = \frac{1}{\delta}\sum_{i=1}^{N}\frac{E_i^{C,t+1}}{\widehat{\phi}_i^{t+1}},$$

$$E_i^{C,t} = \frac{E_i^{C,t+1}}{\delta\widehat{\phi}_i^{t+1}R^{t+1}}$$

to obtain paths for $\left\{R^t\right\}$ and $\left\{E_i^{C,t}\right\}$. Note that, because B_i^1 is decided at $t = 0$, before the shock, $R^1 = R^0 = \frac{1}{\delta}$.

Step 6: Solve for the out-of-steady-state dynamics conditional on aggregate expenditures $\left\{E_i^{C,t}\right\}$.

Step 7: Using the path for disposable income $\left\{I_i^t\right\}_{t=1}^{T_{SS}}$ obtained in Step 6 and equation (7) compute:

$$\left(NX_i^t\right)' = I_i^t - E^{C,t} \text{ for } 1 \leq t < T_{SS}$$

$$\left(NX_i^{T_{SS}}\right)' = -\frac{1-\delta}{\delta}\frac{1}{\left(\prod_{\tau=1}^{T_{SS}-1}(R^\tau)^{-1}\right)}\left(B_i^0 + \sum_{t=1}^{T_{SS}-1}\left(\prod_{\tau=1}^{t}(R^\tau)^{-1}\right)\left(NX_i^t\right)'\right)$$

Step 8: Compute

$$dist\left(\left\{NX_i^{T_{SS}}\right\}, \left\{\left(NX_i^{T_{SS}}\right)'\right\}\right)$$

Step 9: Update $NX_i^{T_{SS}}$

$$NX_i^{T_{SS}} = (1 - \lambda_o)\, NX_i^{T_{SS}} + \lambda_o \left(NX_i^{T_{SS}} \right)'$$

Go back to Step 4 until convergence of $\left\{ NX_i^{T_{SS}} \right\}$.

C.5 Algorithm: Out-of-Steady-State Transition, Exogenous Deficits (No Bonds)

Consider paths $\left\{A_{k,i}^t\right\}_{t=0}^{T_{SS}}$ and $\left\{d_{o,i,k}^t\right\}_{t=0}^{T_{SS}}$ with $A_{k,i}^0 = 1$ and $d_{o,i,k}^0 = 1$. Also, consider paths $\left\{\phi_i^t\right\}_{t=0}^{T_{SS}}$ with $\phi_i^0 = 1$ and $\widehat{\phi}_i^t = 1$ for $T \leq t \leq T_{SS}$, for some $T << T_{SS}$.

We condition on an exogenous path for $\left\{NX_i^t\right\}_{t=1}^{T_{SS}}$.

Step 1: Guess paths $\left\{\widetilde{\lambda}_i^t\right\}_{t=1}^{T_{SS}}$ for each country i.

Step 2: Guess paths $\left\{\widetilde{w}_{k,i}^t\right\}_{t=1}^{T_{SS}}$ for each sector k and country i.

Step 3: Compute $\underline{x}_{k,i}^{T_{SS}}$ consistent with $\widetilde{w}_{k,i}^{T_{SS}}$ and $\widetilde{\lambda}_i^{T_{SS}}$. Obtain $\theta_{k,i}^{T_{SS}}$, $U_{k,i}^{T_{SS}}$, $s_{k\ell,i}^{T_{SS},T_{SS}+1}$ and $\pi_{k,oi}^{T_{SS}}$.

- Step 3a: Compute $\widehat{\widetilde{w}}_{k,i} = \frac{\widetilde{w}_{k,i}^{T_{SS}}}{\widetilde{w}_{k,i}^0}$, $\widehat{A}_{k,i} = \frac{A_{k,i}^{T_{SS}}}{A_{k,i}^0}$ and $\widehat{d}_{k,i} = \frac{d_{o,i,k}^{T_{SS}}}{d_{o,i,k}^0}$. Iteratively solve for $\widehat{P}_{k,i}^I$ and $\widehat{c}_{k,i}$ using the system

$$\widehat{c}_{k,i} = \left(\widehat{\widetilde{w}}_{k,i}\right)^{\gamma_{k,i}} \prod_{\ell=1}^{K} \left(\widehat{P}_{\ell,i}^I\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}}$$

$$\widehat{P}_{k,i}^I = \left(\sum_{o=1}^{N} \pi_{k,oi}^0 \widehat{A}_{k,o} \left(\widehat{c}_{k,o}\widehat{d}_{k,oi}\right)^{-\lambda}\right)^{-1/\lambda}$$

- Step 3b: Compute $\widehat{P}_{k,i}^F$:

$$\widehat{P}_i^F = \prod_{k=1}^{K} \left(\widehat{P}_{k,i}^I\right)^{\mu_{ki}}$$

- Step 3c: Compute

$$\widehat{\pi}_{k,oi} = \widehat{A}_{k,o} \left(\frac{\widehat{c}_{k,o}\widehat{d}_{k,oi}}{\widehat{P}_{k,i}^I}\right)^{-\lambda},$$

  and obtain $\pi_{k,oi}^{T_{SS}} = \pi_{k,oi}^0 \widehat{\pi}_{k,oi}$

- Step 3d: Compute

  - $\widetilde{\kappa}_{k,i}^{T_{SS}} = \widetilde{\kappa}_{k,i}^0 \frac{\widehat{P}_i^F}{\widehat{\widetilde{w}}_{k,i}}$

- Step 3e: Guess $\left\{\underline{x}_{k,i}^{T_{SS}}\right\}$

- Step 3f: Compute

$$\theta_{k,i}^{T_{SS}} = q_i^{-1} \left(\widetilde{\kappa}_{k,i}^{T_{SS}} \times \frac{1 - \delta\left(1 - \chi_{k,i}\right)}{\delta\left(1 - \beta_{k,i}\right) I_{k,i}\left(\underline{x}_{k,i}^{T_{SS}}\right)}\right)$$

- Step 3g: Compute Bellman Equations

$$U_{k,i}^{T_{SS}} = \zeta_i \log\left(\sum_{k'} \exp\left\{\frac{-C_{kk',i} + b_{k',i} + \theta_{k',i}^{T_{SS}} \widetilde{\kappa}_{k',i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{k',i}^{T_{SS}} \frac{\beta_{k',i}}{(1-\beta_{k',i})} + \delta U_{k',i}^{T_{SS}}}{\zeta_i}\right\}\right)$$

- Step 3h: Compute

$$\left(\underline{x}_{k,i}^{T_{SS}}\right)' = \frac{(1-\delta)\, U_{k,i}^{T_{SS}} - \eta_{k,i}}{\widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{k,i}^{T_{SS}}}$$

- Step 3i: Update $\underline{x}_{k,i}^{T_{SS}} = (1-\lambda_x)\, \underline{x}_{k,i}^{T_{SS}} + \lambda_x \left(\underline{x}_{k,i}^{T_{SS}}\right)'$ and go back to Step 2d until convergence.

- Step 3j: Compute $s_{kk'}^{T_{SS}, T_{SS}+1}$

$$s_{k\ell,i}^{T_{SS},T_{SS}+1} = \frac{\exp\left\{\frac{-C_{k\ell,i} + b_{\ell,i} + \theta_{\ell,i}^{T_{SS}} \widetilde{\kappa}_{\ell,i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{\ell,i}^{T_{SS}} \frac{\beta_{\ell,i}}{1-\beta_{\ell,i}} + \delta U_{\ell,i}^{T_{SS}}}{\zeta_i}\right\}}{\sum_{\overline{k}} \exp\left\{\frac{-C_{k\overline{k},i} + b_{\overline{k},i} + \theta_{\overline{k},i}^{T_{SS}} \widetilde{\kappa}_{\overline{k},i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{\overline{k},i}^{T_{SS}} \frac{\beta_{\overline{k},i}}{1-\beta_{\overline{k},i}} + \delta U_{\overline{k},i}^{T_{SS}}}{\zeta_i}\right\}}$$

Step 4: Obtain series $\left\{\pi_{k,oi}^t\right\}_{t=0}^{T_{SS}}$, $\left\{\widetilde{\kappa}_{k,i}^t\right\}_{t=0}^{T_{SS}}$. Define $\widehat{x}^t \equiv \frac{x^t}{x^0}$.

- Step 4a: For $t = 1, ..., T_{SS}-1$ compute $\widehat{\widetilde{w}}_{k,i}^t = \frac{\widetilde{w}_{k,i}^t}{\widetilde{w}_{k,i}^0}$ and iteratively solve for $\widehat{P}_{k,i}^{I,t}$ and $\widehat{c}_{k,i}^t$ using the system

$$\widehat{c}_{k,i}^t = \left(\widehat{\widetilde{w}}_{k,i}^t\right)^{\gamma_{k,i}} \prod_{\ell=1}^{K} \left(\widehat{P}_{\ell,i}^{I,t}\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}}$$

$$\widehat{P}_{k,i}^{I,t} = \left(\sum_{o=1}^{N} \pi_{k,oi}^0 \widehat{A}_{k,o}^t \left(\widehat{c}_{k,o}^t \widehat{d}_{k,oi}^t\right)^{-\lambda}\right)^{-1/\lambda}$$

- Step 4b: Compute $\widehat{P}_{k,i}^{F,t}$ for $t = 1, ..., T_{SS}-1$:

$$\widehat{P}_i^{F,t} = \prod_{k=1}^{K} \left(\widehat{P}_{k,i}^{I,t}\right)^{\mu_{ki}}$$

- Step 4c: Compute $\widehat{\pi}_{k,oi}^t$ for $t = 1, ..., T_{SS}-1$:

$$\widehat{\pi}_{k,oi}^t = \widehat{A}_{k,o}^t \left(\frac{\widehat{c}_{k,o}^t \widehat{d}_{k,oi}^t}{\widehat{P}_{k,i}^{I,t}}\right)^{-\lambda}$$

- Step 4d: Compute or $t = 1, ..., T_{SS}-1$:

  - $\pi_{k,oi}^t = \pi_{k,oi}^0 \widehat{\pi}_{k,oi}^t$

– $\widetilde{\kappa}_{k,i}^{t} \equiv \frac{\kappa_{k,i} P_i^{F,t}}{\widetilde{w}_{k,i}^{t}} = \frac{\kappa_{k,i} P_i^{F,0}}{\widetilde{w}_{k,i}^{0}} \frac{P_i^{F,t}}{P_i^{F,0}} \frac{\widetilde{w}_{k,i}^{0}}{\widetilde{w}_{k,i}^{t}} = \widetilde{\kappa}_{k,i}^{0} \frac{\widehat{P}_i^{F,t}}{\widehat{\widetilde{w}}_{k,i}^{t}}$

Step 5: Given knowledge of $\widetilde{w}_{k,i}^{T_{SS}}$, $\widetilde{\lambda}_i^{T_{SS}}$ and $\underline{x}_{k,i}^{T_{SS}}$ (and therefore $J_{k,i}^{T_{SS}}(s)$), start at $t = T_{SS} - 1$ and sequentially compute (backwards) for each $t = T_{SS} - 1, ..., 1$

- Step 5a: Given $\widetilde{w}_{k,i}^{t}$, $\widetilde{\lambda}_i^{t}$, $\underline{x}_{k,i}^{t+1}$, $\widetilde{\kappa}_{k,i}^{t}$ and $J_{k,i}^{t+1}(s)$ compute $\theta_{k,i}^{t}$.

If $\frac{\widetilde{\lambda}_i^t \widetilde{\kappa}_{k,i}^t \widetilde{w}_{k,i}^t}{\delta \widehat{\phi}_i^{t+1} \int_{\underline{x}_{k,i}^{t+1}}^{x^{\max}} J_{k,i}^{t+1}(s) dG_{k,i}(s)} \leq 1$ then

$$\theta_{k,i}^{t} = q_i^{-1} \left(\frac{\widetilde{\lambda}_i^t \widetilde{\kappa}_{k,i}^t \widetilde{w}_{k,i}^t}{\delta \widehat{\phi}_i^{t+1} \int_{\underline{x}_{k,i}^{t+1}}^{x^{\max}} J_{k,i}^{t+1}(s)\, dG_{k,i}(s)} \right)$$

If $\frac{\widetilde{\lambda}_i^t \widetilde{\kappa}_{k,i}^t \widetilde{w}_{k,i}^t}{\delta \widehat{\phi}_i^{t+1} \int_{\underline{x}_{k,i}^{t+1}}^{x^{\max}} J_{k,i}^{t+1}(s) dG_{k,i}(s)} > 1$, it is not possible to satisfy $V_{k,i}^{t} = 0$, so that $V_{k,i}^{t} < 0$ and

$$\theta_{k,i}^{t} = 0.$$

- Step 5b: Given $\underline{x}_{k,i}^{t+1}$, $W_{k,i}^{t+1}(x) = \frac{\beta_{k,i}}{1-\beta_{k,i}} J_{k,i}^{t+1}(x) + U_{k,i}^{t+1}$ (for $x \geq \underline{x}_{k,i}^{t+1}$), $\theta_{k,i}^{t}$, $U_{k,i}^{t+1}$ compute $U_{k,i}^{t}$. Notice that $\int_{\underline{x}_{k,i}^{t+1}}^{x^{\max}} W_{k,i}^{t+1}(s)\, dG_{k,i}(s) = \frac{\beta_{k,i}}{1-\beta_{k,i}} \int_{\underline{x}_{k,i}^{t+1}}^{x^{\max}} J_{k,i}^{t+1}(s)\, dG_{k,i}(s) + \left(1 - G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right) U_{k,i}^{t+1}$ so that:

$$U_{k,i}^{t} = \zeta_i \log \left(\sum_{k'} \exp \left\{ \frac{\begin{array}{c} -C_{kk',i} + b_{k',i} \\ +\delta \widehat{\phi}_i^{t+1} \theta_{k',i}^{t} q_i \left(\theta_{k',i}^{t}\right) \frac{\beta_{k,i}}{1-\beta_{k,i}} \int_{\underline{x}_{k,i}^{t+1}}^{x^{\max}} J_{k,i}^{t+1}(s)\, dG_{k,i}(s) + \delta \widehat{\phi}_i^{t+1} U_{k',i}^{t+1} \end{array}}{\zeta_i} \right\} \right)$$

- Step 5c: Given $\widetilde{\lambda}_i^{t}$, $J_{k,i}^{t+1}(x)$, $\widetilde{w}_{k,i}^{t}$, $\theta_{k,i}^{t}$, δ, $U_{k,i}^{t}$, $U_{k,i}^{t+1}$ and $\underline{x}_{k,i}^{t+1}$ compute $J_{k,i}^{t}(x)$

$$\begin{aligned} J_{k,i}^{t}(x) = {} & (1-\beta_{k,i}) \widetilde{\lambda}_i^t \widetilde{w}_{k,i}^t x + (1-\beta_{k,i}) \eta_{k,i} \\ & - (1-\beta_{k,i}) \left(U_{k,i}^{t} - \delta \widehat{\phi}_i^{t+1} U_{k,i}^{t+1} \right) + (1 - \chi_{k,i}) \delta \widehat{\phi}_i^{t+1} \max \left\{ J_{k,i}^{t+1}(x), 0 \right\} \end{aligned}$$

- Step 5d: Solve for $\underline{x}_{k,i}^{t}$: $J_{k,i}^{t}\left(\underline{x}_{k,i}^{t}\right) = 0$

Step 6: Compute transition rates $\left\{ s_{kk',i}^{t,t+1} \right\}_{t=1}^{T_{SS}-1}$ for all countries i according to:

$$s_{kk',i}^{t,t+1} = \frac{\exp \left\{ \begin{array}{c} -C_{kk',i} + b_{k',i} + \\ \delta \widehat{\phi}_i^{t+1} \theta_{k',i}^{t} q(\theta_{k',i}^{t}) \frac{\beta_{k',i}}{1-\beta_{k',i}} \int_{\underline{x}_{k',i}^{t+1}}^{x^{\max}} J_{k',i}^{t+1}(x)\, dG_{k',i}(x) + \delta \widehat{\phi}_i^{t+1} U_{k',i}^{t+1} \end{array} \right\}}{\sum_{k''} \exp \left\{ \begin{array}{c} -C_{kk'',i} + b_{k'',i} + \\ \delta \widehat{\phi}_i^{t+1} \theta_{k'',i}^{t} q(\theta_{k'',i}^{t}) \frac{\beta_{k'',i}}{1-\beta_{k'',i}} \int_{\underline{x}_{k'',i}^{t+1}}^{x^{\max}} J_{k'',i}^{t+1}(x)\, dG_{k'',i}(x) + \delta \widehat{\phi}_i^{t+1} U_{k'',i}^{t+1} \end{array} \right\}}.$$

Step 7: Start loop over t going forward ($t = 0$ to $t = T_{SS} - 1$)

Initial conditions: we know $\widetilde{u}_{k,i}^{t=-1} = u_{k,i}^{t=0}$, $L_{k,i}^{t=-1} = L_{k,i}^{t=0}$, and $\theta_{k,i}^{t=0}$ from the initial steady state computation. Obtain $\widetilde{u}_{k,i}^{t}$ and $L_{k,i}^{t}$ using flow conditions and sequences $\left\{\theta_{k,i}^{t}\right\}$, $\left\{\underline{x}_{k,i}^{t}\right\}$.

- Step 7a: Compute

$$JC_{k,i}^{t} = L_{k,i}^{t} u_{k,i}^{t} \theta_{k,i}^{t} q_i \left(\theta_{k,i}^{t}\right) \left(1 - G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right)$$

$$JD_{k,i}^{t} = \left(\chi_{k,i} + (1 - \chi_{k,i}) \max\left\{\frac{G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right) - G_{k,i}\left(\underline{x}_{k,i}^{t}\right)}{1 - G_{k,i}\left(\underline{x}_{k,i}^{t}\right)}, 0\right\}\right) L_{k,i}^{t-1}\left(1 - \widetilde{u}_{k,i}^{t-1}\right)$$

$$\widetilde{u}_{k,i}^{t} = \frac{L_{k,i}^{t} u_{k,i}^{t} - JC_{k,i}^{t} + JD_{k,i}^{t}}{L_{k,i}^{t}}$$

- Step 7b: Compute

$$L_{k,i}^{t+1} = L_{k,i}^{t} + IF_{k,i}^{t+1} - OF_{k,i}^{t+1},$$

where

$$IF_{k,i}^{t+1} = \sum_{\ell \neq k} L_{\ell,i}^{t} \widetilde{u}_{\ell,i}^{t} s_{\ell k,i}^{t+1,t+2},$$

and

$$OF_{k,i}^{t+1} = L_{k,i}^{t} \widetilde{u}_{k,i}^{t} \left(1 - s_{kk,i}^{t+1,t+2}\right).$$

- Step 7c: Compute

$$u_{k,i}^{t+1} = \frac{\sum_{\ell=1}^{K} L_{\ell,i}^{t} \widetilde{u}_{\ell,i}^{t} s_{\ell k,i}^{t+1,t+2}}{L_{k,i}^{t+1}}$$

- Step 7d: Compute

$$\widetilde{L}_{k,i}^{t+1} = L_{k,i}^{t}\left(1 - \widetilde{u}_{k,i}^{t}\right) \int_{\underline{x}_{k,i}^{t+1}}^{\infty} \frac{s}{1 - G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)} dG_{k,i}(s)$$

$$= L_{k,i}^{t}\left(1 - \widetilde{u}_{k,i}^{t}\right) \exp\left(\frac{\sigma_{k,i}^{2}}{2}\right) \frac{\Phi\left(\sigma_{k,i} - \frac{\ln \underline{x}_{k,i}^{t+1}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}^{t+1}}{\sigma_{k,i}}\right)}$$

$$\text{and } Y_{k,i}^{t+1} = \widetilde{w}_{k,i}^{t+1} \widetilde{L}_{k,i}^{t+1}$$

- Step 7e: Compute expenditure with vacancies

$$E_{k,i}^{V,t+1} = \widetilde{\kappa}_{k,i}^{t+1} \widetilde{w}_{k,i}^{t+1} \theta_{k,i}^{t+1} u_{k,i}^{t+1} L_{k,i}^{t+1}$$

- Step 7f: Compute $E_i^{C,t+1} = \frac{\overline{L}_i}{\widetilde{\lambda}_i^{t+1}}$

- Step 7g: Solve for $\left\{ Y_{k,i}^{t+1} \right\}$ in the system

$$E_{k,i}^{t+1} = \mu_{k,i} E_i^{C,t+1} + \sum_{\ell=1}^{K} \left(\mu_{k,i} E_{\ell,i}^{V,t+1} + (1 - \gamma_{\ell,i}) \nu_{\ell k,i} Y_{\ell,i}^{t+1} \right).$$

$$Y_{k,o}^{t+1} = \sum_{i=1}^{N} \pi_{k,oi}^{t+1} E_{k,i}^{t+1}.$$

- Step 7h: Normalize $\left\{ Y_{k,i}^{t+1} \right\}$ to make sure it sums to 1 across sectors and countries.

- Step 7i: Compute $\left(\widetilde{w}_{k,i}^{t+1} \right)' = \frac{\gamma_{k,i} Y_{k,i}^{t+1}}{\widetilde{L}_{k,i}^{t+1}}$

Step 8: Compute $dist \left(\left\{ \widetilde{w}_{k,i}^{t} \right\}_{t=1}^{T_{SS}}, \left\{ \left(\widetilde{w}_{k,i}^{t} \right)' \right\}_{t=1}^{T_{SS}} \right)$

Step 9: Update $\widetilde{w}_{k,i}^{t} = (1 - \alpha_w) \widetilde{w}_{k,i}^{t} + \alpha_w \left(\widetilde{w}_{k,i}^{t} \right)'$ for $t = 1, ..., T_{SS}$ and go back to Step 3 until convergence of $\left\{ \widetilde{w}_{k,i}^{t} \right\}$

Step 10: Compute disposable income $\left\{ I_i^t \right\}_{t=1}^{T_{SS}}$

$$I_i^t = \sum_{\ell=1}^{K} \left(\gamma_{\ell,i} Y_{\ell,i}^{t} - E_{\ell,i}^{V,t} \right)$$

Step 11: Update $\left\{ E_i^{C,t} \right\}_{t=1}^{T_{SS}}$ using

$$E_i^{C,t} = I_i^t - N X_i^t$$

Step 12a: Compute $\left(\widetilde{\lambda}_i^t \right)' = \frac{\overline{L}_i}{E_i^{C,t}}$ for all $t = 1, ..., T_{SS}$

- Step 12b: Compute $dist \left(\left\{ \widetilde{\lambda}_i^t \right\}_{t=1}^{T_{SS}}, \left\{ \left(\widetilde{\lambda}_i^t \right)' \right\}_{t=1}^{T_{SS}} \right)$

- Step 12c: Update $\widetilde{\lambda}_i^t = (1 - \alpha_\lambda) \widetilde{\lambda}_i^t + \alpha_\lambda \left(\widetilde{\lambda}_i^t \right)'$ for $t = 1, ..., T_{SS}$ and go back to Step 2 until convergence of $\left\{ \widetilde{\lambda}_i^t \right\}$.

C.6 Algorithm: Recovering Shocks

Inner Loop: conditional on paths for expenditures $\left\{E_i^{C,t}\right\}_{t=1}^{T_{SS}}$ and shocks $\left\{\widehat{\phi}_i^t\right\}_{t=2}^{T_{SS}}$, $\left\{\widehat{d}_{k,i}^t\right\}_{t=1}^{T_{SS}}$ and $\widehat{A}_{k,i} \equiv \frac{A_{k,i}^{T_{SS}}}{A_{k,i}^0}$ – determined in the Outer Loop below.

As before, we denote changes relative to $t = 0$ by $\widehat{x}^t = \frac{x^t}{x^0}$. This loop **conditions** on data on $\left\{\widehat{\pi}_{k,oi}^t\right\}_{t=1}^{T}$ and $\left\{\widehat{P}_{k,i}^{I,t}\right\}_{t=1}^{T}$, where T is the last period we have data on these variables. We assume $t = 0$ is the estimated steady state. Define $\widehat{d}_{k,i} \equiv \frac{d_{o,i,k}^{T_{SS}}}{d_{o,i,k}^0}$.

Step 1: Given paths $\left\{E_i^{C,t}\right\}$, compute paths $\left\{\widetilde{\lambda}_i^t\right\}$: $\widetilde{\lambda}_i^t = \frac{\overline{L}_i}{E_i^{C,t}}$.

Step 2: Guess paths $\left\{\widetilde{w}_{k,i}^t\right\}_{t=1}^{T_{SS}}$ for each sector k and country i.

Step 3: Compute $\underline{x}_{k,i}^{T_{SS}}$ consistent with $\widetilde{w}_{k,i}^{T_{SS}}$ and $\widetilde{\lambda}_i^{T_{SS}}$. Obtain $\theta_{k,i}^{T_{SS}}$, $U_{k,i}^{T_{SS}}$, $s_{k\ell,i}^{T_{SS},T_{SS}+1}$ and $\pi_{k,oi}^{T_{SS}}$.

- Step 3a: Compute $\widehat{\widetilde{w}}_{k,i} = \frac{\widetilde{w}_{k,i}^{T_{SS}}}{\widetilde{w}_{k,i}^0}$. Iteratively solve for $\widehat{P}_{k,i}^I$ and $\widehat{c}_{k,i}$ using the system

$$\widehat{c}_{k,i} = \left(\widehat{\widetilde{w}}_{k,i}\right)^{\gamma_{k,i}} \prod_{\ell=1}^{K} \left(\widehat{P}_{\ell,i}^I\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}}$$

$$\widehat{P}_{k,i}^I = \left(\sum_{o=1}^{N} \pi_{k,oi}^0 \widehat{A}_{k,o} \left(\widehat{c}_{k,o}\widehat{d}_{k,oi}\right)^{-\lambda}\right)^{-1/\lambda}$$

- Step 3b: Compute $\widehat{P}_{k,i}^F$:

$$\widehat{P}_i^F = \prod_{k=1}^{K} \left(\widehat{P}_{k,i}^I\right)^{\mu_{ki}}$$

- Step 3c: Compute

$$\widehat{\pi}_{k,oi} = \widehat{A}_{k,o} \left(\frac{\widehat{c}_{k,o}\widehat{d}_{k,oi}}{\widehat{P}_{k,i}^I}\right)^{-\lambda}$$

  And obtain $\pi_{k,oi}^{T_{SS}} = \pi_{k,oi}^0 \widehat{\pi}_{k,oi}$

- Step 3d: Compute

  - $\widetilde{\kappa}_{k,i}^{T_{SS}} = \widetilde{\kappa}_{k,i}^0 \frac{\widehat{P}_i^F}{\widehat{\widetilde{w}}_{k,i}}$

- Step 3e: Guess $\left\{\underline{x}_{k,i}^{T_{SS}}\right\}$

- Step 3f: Compute

$$\theta_{k,i}^{T_{SS}} = q_i^{-1}\left(\widetilde{\kappa}_{k,i}^{T_{SS}} \times \frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right) I_{k,i}\left(\underline{x}_{k,i}^{T_{SS}}\right)}\right)$$

- Step 3g: Compute Bellman Equations

$$U_{k,i}^{T_{SS}} = \zeta_i \log\left(\sum_{k'} \exp\left\{\frac{-C_{kk',i} + b_{k',i} + \theta_{k',i}^{T_{SS}} \widetilde{\kappa}_{k',i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{k',i}^{T_{SS}} \frac{\beta_{k',i}}{\left(1-\beta_{k',i}\right)} + \delta U_{k',i}^{T_{SS}}}{\zeta_i}\right\}\right)$$

- Step 3h: Compute

$$\left(\underline{x}_{k,i}^{T_{SS}}\right)' = \frac{\left(1-\delta\right) U_{k,i}^{T_{SS}} - \eta_{k,i}}{\widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{k,i}^{T_{SS}}}$$

- Step 3i: Update $\underline{x}_{k,i}^{T_{SS}} = \left(1-\lambda_x\right) \underline{x}_{k,i}^{T_{SS}} + \lambda_x \left(\underline{x}_{k,i}^{T_{SS}}\right)'$ and go back to Step 2d until convergence.

- Step 3j: Compute $s_{kk'}^{T_{SS},T_{SS}+1}$

$$s_{k\ell,i}^{T_{SS},T_{SS}+1} = \frac{\exp\left\{\frac{-C_{k\ell,i} + b_{\ell,i} + \theta_{\ell,i}^{T_{SS}} \widetilde{\kappa}_{\ell,i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{\ell,i}^{T_{SS}} \frac{\beta_{\ell,i}}{1-\beta_{\ell,i}} + \delta U_{\ell,i}^{T_{SS}}}{\zeta_i}\right\}}{\sum_{\overline{k}} \exp\left\{\frac{-C_{k\overline{k},i} + b_{\overline{k},i} + \theta_{\overline{k},i}^{T_{SS}} \widetilde{\kappa}_{\overline{k},i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{\overline{k},i}^{T_{SS}} \frac{\beta_{\overline{k},i}}{1-\beta_{\overline{k},i}} + \delta U_{\overline{k},i}^{T_{SS}}}{\zeta_i}\right\}}$$

Step 4: Obtain series $\left\{\pi_{k,oi}^t\right\}_{t=T+1}^{T_{SS}}$ and $\left\{\widetilde{\kappa}_{k,i}^t\right\}_{t=1}^{T_{SS}}$.

- Step 4a: For $t = T+1, ..., T_{SS}$ do:

Compute $\widehat{\widetilde{w}}_{k,i}^t = \frac{\widetilde{w}_{k,i}^t}{\widetilde{w}_{k,i}^0}$ and iteratively solve for $\widehat{P}_{k,i}^{I,t}$ and $\widehat{c}_{k,i}^t$ using the system

$$\widehat{c}_{k,i}^t = \left(\widehat{\widetilde{w}}_{k,i}^t\right)^{\gamma_{k,i}} \prod_{\ell=1}^{K} \left(\widehat{P}_{\ell,i}^{I,t}\right)^{\left(1-\gamma_{k,i}\right)\nu_{k\ell,i}},$$

$$\widehat{P}_{k,i}^{I,t} = \left(\sum_{o=1}^{N} \pi_{k,oi}^0 \widehat{A}_{k,o} \left(\widehat{c}_{k,o}^t \widehat{d}_{k,oi}\right)^{-\lambda}\right)^{-1/\lambda}.$$

- Step 4b: Compute $\widehat{P}_{k,i}^{F,t}$ for $t = 1, ..., T_{SS}-1$ (remember $\widehat{P}_{k,i}^{I,t}$ is data for $t = 1, ..., T$):

$$\widehat{P}_i^{F,t} = \prod_{k=1}^{K} \left(\widehat{P}_{k,i}^{I,t}\right)^{\mu_{ki}}$$

- Step 4c: Compute $\widehat{\pi}_{k,oi}^{t}$ and $\pi_{k,oi}^{t}$ for $t = T+1, ..., T_{SS}-1$.

For $t = 1, ..., T_{SS}-1$ do:
First Case: If $t \leq T$ then $\widehat{\pi}_{k,oi}^{t}$ is data, so do:

$$\pi_{k,oi}^{t} = \pi_{k,oi}^{0} \widehat{\pi}_{k,oi}^{t}$$

End of First Case
Second Case if $t \geq T+1$ do:

$$\widehat{\pi}_{k,oi}^{t} = \left(\widehat{A}_{k,o}^{t}\right)' \left(\frac{\widehat{c}_{k,o}^{t} \widehat{d}_{k,oi}^{t}}{\widehat{P}_{k,i}^{I,t}}\right)^{-\lambda}$$

$$\pi_{k,oi}^{t} = \pi_{k,oi}^{0} \widehat{\pi}_{k,oi}^{t}$$

End of Second Case

- Step 4d: Compute for $t = 1, ..., T_{SS}-1$

  - $\widetilde{\kappa}_{k,i}^{t} \equiv \frac{\kappa_{k,i} P_i^{F,t}}{\widetilde{w}_{k,i}^{t}} = \frac{\kappa_{k,i} P_i^{F,0}}{\widetilde{w}_{k,i}^{0}} \frac{P_i^{F,t}}{P_i^{F,0}} \frac{\widetilde{w}_{k,i}^{0}}{\widetilde{w}_{k,i}^{t}} = \widetilde{\kappa}_{k,i}^{0} \frac{\widehat{P}_i^{F,t}}{\widehat{\widetilde{w}}_{k,i}^{t}}$

Step 5: Given knowledge of $\widetilde{w}_{k,i}^{T_{SS}}$, $\widetilde{\lambda}_i^{T_{SS}}$ and $\underline{x}_{k,i}^{T_{SS}}$ (and therefore $J_{k,i}^{T_{SS}}(s)$), start at $t = T_{SS}-1$ and sequentially compute (backwards) for each $t = T_{SS}-1, ..., 1$

- Step 5a: Given $\widetilde{w}_{k,i}^{t}$, $\underline{x}_{k,i}^{t+1}$, $\widetilde{\kappa}_{k,i}^{t}$, $\widetilde{\lambda}_i^{t}$ and $J_{k,i}^{t+1}(s)$ compute $\theta_{k,i}^{t}$.

If $\frac{\widetilde{\lambda}_i^{t} \widetilde{\kappa}_{k,i}^{t} \widetilde{w}_{k,i}^{t}}{\delta \widehat{\phi}_i^{t+1} \int_{\underline{x}_{k,i}^{t+1}}^{x^{\max}} J_{k,i}^{t+1}(s) dG_{k,i}(s)} \leq 1$ then

$$\theta_{k,i}^{t} = q_i^{-1} \left(\frac{\widetilde{\lambda}_i^{t} \widetilde{\kappa}_{k,i}^{t} \widetilde{w}_{k,i}^{t}}{\delta \widehat{\phi}_i^{t+1} \int_{\underline{x}_{k,i}^{t+1}}^{x^{\max}} J_{k,i}^{t+1}(s)\, dG_{k,i}(s)}\right)$$

If $\frac{\widetilde{\lambda}_i^{t} \widetilde{\kappa}_{k,i}^{t} \widetilde{w}_{k,i}^{t}}{\delta \widehat{\phi}_i^{t+1} \int_{\underline{x}_{k,i}^{t+1}}^{x^{\max}} J_{k,i}^{t+1}(s) dG_{k,i}(s)} > 1$, it is not possible to satisfy $V_{k,i}^{t} = 0$, so that $V_{k,i}^{t} < 0$ and

$$\theta_{k,i}^{t} = 0.$$

- Step 5b: Given $\underline{x}_{k,i}^{t+1}$, $W_{k,i}^{t+1}(x) = \frac{\beta_{k,i}}{1-\beta_{k,i}} J_{k,i}^{t+1}(x) + U_{k,i}^{t+1}$ (for $x \geq \underline{x}_{k,i}^{t+1}$), $\theta_{k,i}^{t}$, $U_{k,i}^{t+1}$ compute $U_{k,i}^{t}$. Notice that $\int_{\underline{x}_{k,i}^{t+1}}^{x^{\max}} W_{k,i}^{t+1}(s)\, dG_{k,i}(s) = \frac{\beta_{k,i}}{1-\beta_{k,i}} \int_{\underline{x}_{k,i}^{t+1}}^{x^{\max}} J_{k,i}^{t+1}(s)\, dG_{k,i}(s) + \left(1 - G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right) U_{k,i}^{t+1}$ so that:

$$U_{k,i}^{t} = \zeta_i \log\left(\sum_{k'} \exp\left\{\frac{\begin{array}{c} -C_{kk',i} + b_{k',i} \\ +\delta \widehat{\phi}_i^{t+1} \theta_{k',i}^{t} q_i \left(\theta_{k',i}^{t}\right) \frac{\beta_{k,i}}{1-\beta_{k,i}} \int_{\underline{x}_{k,i}^{t+1}}^{x^{\max}} J_{k,i}^{t+1}(s)\, dG_{k,i}(s) + \delta \widehat{\phi}_i^{t+1} U_{k',i}^{t+1} \end{array}}{\zeta_i}\right\}\right)$$

- Step 5c: Given $J_{k,i}^{t+1}(x)$, $\widetilde{w}_{k,i}^{t}$, $U_{k,i}^{t}$, $U_{k,i}^{t+1}$ and $\underline{x}_{k,i}^{t+1}$ compute $J_{k,i}^{t}(x)$

$$\begin{aligned} J_{k,i}^{t}(x) = & (1-\beta_{k,i})\,\widetilde{\lambda}_{i}^{t}\widetilde{w}_{k,i}^{t}x + (1-\beta_{k,i})\,\eta_{k,i} \\ & - (1-\beta_{k,i})\left(U_{k,i}^{t} - \delta\widehat{\phi}_{i}^{t+1}U_{k,i}^{t+1}\right) + (1-\chi_{k,i})\,\delta\widehat{\phi}_{i}^{t+1}\max\left\{J_{k,i}^{t+1}(x),0\right\} \end{aligned}$$

- Step 5d: Solve for $\underline{x}_{k,i}^{t}$: $J_{k,i}^{t}\left(\underline{x}_{k,i}^{t}\right)=0$

Step 6: Compute transition rates $\left\{s_{kk',i}^{t,t+1}\right\}_{t=1}^{T_{SS}-1}$ for all countries i according to:

$$s_{kk',i}^{t,t+1} = \frac{\exp\left\{\begin{array}{c} -C_{kk',i} + b_{k',i} + \\ \delta\widehat{\phi}_{i}^{t+1}\theta_{k',i}^{t}q(\theta_{k',i}^{t})\frac{\beta_{k',i}}{1-\beta_{k',i}}\int_{\underline{x}_{k',i}^{t+1}}^{x_{\max}}J_{k',i}^{t+1}(x)\,dG_{k',i}(x) + \delta\widehat{\phi}_{i}^{t+1}U_{k',i}^{t+1} \end{array}\right\}}{\sum_{k''}\exp\left\{\begin{array}{c} -C_{kk'',i} + b_{k'',i} + \\ \delta\widehat{\phi}_{i}^{t+1}\theta_{k'',i}^{t}q(\theta_{k'',i}^{t})\frac{\beta_{k'',i}}{1-\beta_{k'',i}}\int_{\underline{x}_{k'',i}^{t+1}}^{x_{\max}}J_{k'',i}^{t+1}(x)\,dG_{k'',i}(x) + \delta\widehat{\phi}_{i}^{t+1}U_{k'',i}^{t+1} \end{array}\right\}}.$$

Step 7: Start loop over t going forward ($t=0$ to $t=T_{SS}-1$)

Initial conditions: we know $\widetilde{u}_{k,i}^{t=-1} = u_{k,i}^{t=0}$, $L_{k,i}^{t=-1} = L_{k,i}^{t=0}$, and $\theta_{k,i}^{t=0}$ from the initial steady state computation. Obtain $\widetilde{u}_{k,i}^{t}$ and $L_{k,i}^{t}$ using flow conditions and sequences $\left\{\theta_{k,i}^{t}\right\}$, $\left\{\underline{x}_{k,i}^{t}\right\}$.

- Step 7a: Compute

$$JC_{k,i}^{t} = L_{k,i}^{t}u_{k,i}^{t}\theta_{k,i}^{t}q_{i}\left(\theta_{k,i}^{t}\right)\left(1-G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right)$$

$$JD_{k,i}^{t} = \left(\chi_{k,i} + (1-\chi_{k,i})\max\left\{\frac{G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right) - G_{k,i}\left(\underline{x}_{k,i}^{t}\right)}{1-G_{k,i}\left(\underline{x}_{k,i}^{t}\right)},0\right\}\right)L_{k,i}^{t-1}\left(1-\widetilde{u}_{k,i}^{t-1}\right)$$

$$\widetilde{u}_{k,i}^{t} = \frac{L_{k,i}^{t}u_{k,i}^{t} - JC_{k,i}^{t} + JD_{k,i}^{t}}{L_{k,i}^{t}}$$

- Step 7b: Compute

$$L_{k,i}^{t+1} = L_{k,i}^{t} + IF_{k,i}^{t+1} - OF_{k,i}^{t+1},$$

where

$$IF_{k,i}^{t+1} = \sum_{\ell\neq k}L_{\ell,i}^{t}\widetilde{u}_{\ell,i}^{t}s_{\ell k,i}^{t+1,t+2},$$

and

$$OF_{k,i}^{t+1} = L_{k,i}^{t}\widetilde{u}_{k,i}^{t}\left(1-s_{kk,i}^{t+1,t+2}\right).$$

- Step 7c: Compute

$$u_{k,i}^{t+1} = \frac{\sum_{\ell=1}^{K} L_{\ell,i}^{t} \widetilde{u}_{\ell,i}^{t} s_{\ell k,i}^{t+1,t+2}}{L_{k,i}^{t+1}}$$

- Step 7d: Compute

$$\begin{aligned}\widetilde{L}_{k,i}^{t+1} &= L_{k,i}^{t}\left(1-\widetilde{u}_{k,i}^{t}\right)\int_{\underline{x}_{k,i}^{t+1}}^{\infty}\frac{s}{1-G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)}dG_{k,i}\left(s\right)\\ &= L_{k,i}^{t}\left(1-\widetilde{u}_{k,i}^{t}\right)\exp\left(\frac{\sigma_{k,i}^{2}}{2}\right)\frac{\Phi\left(\sigma_{k,i}-\frac{\ln \underline{x}_{k,i}^{t+1}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}^{t+1}}{\sigma_{k,i}}\right)}\end{aligned}$$

- Step 7e: Compute expenditure with vacancies

$$E_{k,i}^{V,t+1} = \widetilde{\kappa}_{k,i}^{t+1}\widetilde{w}_{k,i}^{t+1}\theta_{k,i}^{t+1}u_{k,i}^{t+1}L_{k,i}^{t+1}$$

- Step 7f: Solve for $\left\{Y_{k,i}^{t+1}\right\}$ in the system

$$E_{k,i}^{t+1} = \mu_{k,i}E_{i}^{C,t+1} + \sum_{\ell=1}^{K}\left(\mu_{k,i}E_{\ell,i}^{V,t+1} + \left(1-\gamma_{\ell,i}\right)\nu_{\ell k,i}Y_{\ell,i}^{t+1}\right).$$

$$Y_{k,o}^{t+1} = \sum_{i=1}^{N}\pi_{k,oi}^{t+1}E_{k,i}^{t+1}.$$

- Step 7g: Compute $\left(\widetilde{w}_{k,i}^{t+1}\right)' = \frac{\gamma_{k,i}Y_{k,i}^{t+1}}{\widetilde{L}_{k,i}^{t+1}}$

Step 8: Compute distance $dist\left(\left\{\widetilde{w}_{k,i}^{t}\right\}, \left\{\left(\widetilde{w}_{k,i}^{t}\right)'\right\}\right)$

- Step 8b: Update $\widetilde{w}_{k,i}^{t} = \left(1-\lambda_{w}\right)\widetilde{w}_{k,i}^{t} + \lambda_{w}\left(\widetilde{w}_{k,i}^{t}\right)'$ $t = 1, ..., T_{SS}$
- Step 8c: At this point, we have a new series for $\left\{\widetilde{w}_{k,i}^{t}\right\}$ – go back to Step 3 until convergence of $\left\{\widetilde{w}_{k,i}^{t}\right\}$.

Step 9: Compute disposable income $\left\{I_{i}^{t}\right\}_{t=1}^{T_{SS}}$

$$I_{i}^{t} = \sum_{\ell=1}^{K}\left(\gamma_{\ell,i}Y_{\ell,i}^{t} - E_{\ell,i}^{V,t}\right)$$

Step 10: Compute $\left\{\left(\widehat{A}_{k,i}^{t}\right)'\right\}$.

For $t=1,...,T$ compute $\widehat{\widetilde{w}}_{k,i}^{t}=\frac{\widetilde{w}_{k,i}^{t}}{\widetilde{w}_{k,i}^{0}}$, obtain $\widehat{c}_{k,i}^{t}$:

$$\widehat{c}_{k,i}^{t}=\left(\widehat{\widetilde{w}}_{k,i}^{t}\right)^{\gamma_{k,i}}\prod_{\ell=1}^{K}\left(\widehat{P}_{\ell,i}^{I,t}\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}},$$

and compute $\left(\widehat{A}_{k,i}^{t}\right)'$:

$$\left(\widehat{A}_{k,i}^{t}\right)'=\frac{\widehat{\pi}_{k,ii}^{t}}{\left(\widehat{P}_{k,i}^{I,t}\right)^{\lambda}}\left(\widehat{c}_{k,i}^{t}\right)^{\lambda}.$$

For $t\geq T+1$ set:

$$\left(\widehat{A}_{k,i}^{t}\right)'=\left(\widehat{A}_{k,i}^{T}\right)'$$

Feed outer loop with $\left\{\left(\widehat{A}_{k,i}^{T_{SS}}\right)'\right\}$.

Outer Loop: iteration on $\left\{NX_{i}^{t}\right\}$

Step 0: Compute changes in trade costs $\left\{\widehat{d}_{k,oi}^{t}\right\}_{t=1}^{T}$:

$$\widehat{d}_{k,oi}^{t}=\left(\frac{\widehat{\pi}_{k,oo}^{t}}{\widehat{\pi}_{k,oi}^{t}}\right)^{1/\lambda}\frac{\widehat{P}_{k,i}^{I,t}}{\widehat{P}_{k,o}^{I,t}}$$

Set $\widehat{d}_{k,oi}^{t}=\widehat{d}_{k,oi}^{T}$ for $t>T$.

Step 1: Start with estimated state equilibrium at $t=0$. Remember that we used the normalization $\sum_{i=1}^{I}\sum_{k=1}^{K}Y_{k,i}=1$ during the estimation procedure. **Change the normalization** from $\sum_{i=1}^{I}\sum_{k=1}^{K}Y_{k,i}=1$ to $\sum_{i=1}^{I}E_{i}^{C}=1$. Nominal variables to be renormalized: $\left\{Y_{k,i}^{0}\right\}$, $\left\{\widetilde{w}_{k,i}^{0}\right\}$, $\left\{E_{i}^{C,0}\right\}$, $\left\{NX_{i}^{0}\right\}$.

Step 2: Compute $E_{i}^{C,t}=\frac{E_{i}^{C,0}\left(\widehat{E}_{i}^{C,t}\right)_{Data}}{\sum_{i=1}^{N}E_{i}^{C,0}\left(\widehat{E}_{i}^{C,t}\right)_{Data}}$ for $t=1,...,T$ where $E_{i}^{C,0}$ is aggregate consumption expenditure in the estimated steady state, and $\left(\widehat{E}_{i}^{C,t}\right)_{Data}$ comes from the data.

Step 3: Normalize $\widehat{\phi}_{US}^{t}=1$ for $t=2,...,T-1$. This yields

$$R^{t+1}=\frac{E_{US}^{C,t+1}}{\delta E_{US}^{C,t}}\text{ for }t=1,...,T-1$$

Obtain remaining shocks $\left\{\widehat{\phi}_{i}^{t}\right\}_{t=2}^{T}$ using

$$\widehat{\phi}_{i}^{t+1}=\frac{E_{i}^{C,t+1}}{\delta E_{i}^{C,t}R^{t+1}}\text{ for }t=1,...,T-1$$

Set $\widehat{\phi}_i^t = 1$ for $t \geq T+2$. The value of $\widehat{\phi}_i^{T+1}$ will depend on $E_i^{C,T_{SS}}$ and will be recovered in Step 7. Note: the value of $\widehat{\phi}_i^{t=1}$ does not matter. Individuals made decisions at $t=0$ assuming $\widehat{\phi}_i^{t=1} = 1$, as the economy was assumed to be in steady state at $t=0$.

Step 4: Obtain B_i^0 with respect to the normalization $\sum_{i=1}^{I} E_i^C = 1$. Equation (XX) gives us:

$$B_i^0 = \frac{NX_i^0}{\left(1 - \frac{1}{\delta}\right)}$$

Step 5: Make initial guess for $NX_i^{T_{SS}}$ (with respect to the normalization $\sum_{i=1}^{I} E_i^C = 1$) and for $\widehat{A}_{k,i}^{T_{SS}} = \widehat{A}_{k,i}^{T} \equiv \frac{A_{k,i}^T}{A_{k,i}^0}$.

Step 6: Compute steady state equilibrium at T_{SS}, conditional on $NX_i^{T_{SS}}$, $\widehat{A}_{k,i}^{T_{SS}}$ and $\widehat{d}_{k,oi}^{T_{SS}}$.

- Step 6a: Notice that steady algorithm uses the normalization $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$. Normalize $NX_i^{T_{SS}}$ with respect to normalization $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$. To perform such normalization, use output $\left\{Y_{k,i}^{T_{SS}}\right\}$ obtained in the initial steady state if this is the first outer loop iteration, otherwise use output $\left\{Y_{k,i}^{T_{SS}}\right\}$ obtained in Step 9 below.

- Step 6b: After computing the final steady state, **change the normalization** from $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$ to $\sum_{i=1}^{I} E_i^C = 1$ using $\left\{E_i^C\right\}$ obtained in Step 3a. Nominal variables to be renormalized: $\left\{Y_{k,i}^{T_{SS}}\right\}$, $\left\{\widetilde{w}_{k,i}^{T_{SS}}\right\}$, $\left\{E_i^{C,T_{SS}}\right\}$, $\left\{NX_i^{T_{SS}}\right\}$.

Step 7: Set $E_i^{C,t} = \begin{cases} E_i^{C,0}\left(\frac{E_i^{C,2000+t}}{E_i^{C,2000}}\right)_{Data} & \text{for } t = 0,...,T, \text{ as in Step 0} \\ E_i^{C,T_{SS}} \text{ for } t = T+1,...,T_{SS} \end{cases}$

Note: $\widehat{\phi}_i^t = 1$ for $t \geq T+2 \Rightarrow E_i^{C,t} = E_i^{C,T_{SS}}$ for at $t \geq T+1$ (Euler Equation)

Step 8: Compute $\widehat{\phi}_i^{T+1}$, imposing $\widehat{\phi}_{US}^{T+1} = 1$:

$$\widehat{\phi}_i^{T+1} = \frac{E_i^{C,T_{SS}}}{\delta R^{T+1} E_i^{C,T}},$$

$$R^{T+1} = \frac{E_{US}^{C,T_{SS}}}{\delta E_{US}^{C,T}}.$$

We now have a full path for $\left\{\widehat{\phi}_i^t\right\}_{t=2}^{T_{SS}}$.

Step 9: Solve for the out-of-steady-state dynamics conditional on aggregate expenditures $\left\{E_i^{C,t}\right\}_{t=0}^{T_{SS}}$, on preference shifters $\left\{\widehat{\phi}_i^t\right\}_{t=2}^{T_{SS}}$ and steady-state productivity shocks $\left\{\widehat{A}_{k,i}^{T_{SS}}\right\}$.

Step 10: Using the path for disposable income $\left\{I_i^t\right\}_{t=1}^{T_{SS}}$ obtained in Step 9 and equation (7) compute:

$$\left(NX_i^t\right)' = I_i^t - E^{C,t} \text{ for } 1 \leq t < T_{SS}$$

$$\left(NX_i^{T_{SS}}\right)' = -\frac{1-\delta}{\delta}\frac{1}{\left(\prod_{\tau=1}^{T_{SS}-1}\left(R^\tau\right)^{-1}\right)}\left(B_i^0 + \sum_{t=1}^{T_{SS}-1}\left(\prod_{\tau=1}^{t}\left(R^\tau\right)^{-1}\right)\left(NX_i^t\right)'\right)$$

Step 11: Compute

$$dist\left(\left\{NX_i^{T_{SS}}\right\}, \left\{\left(NX_i^{T_{SS}}\right)'\right\}\right) \text{ and}$$

$$dist\left(\left\{\widehat{A}_{k,i}^{T_{SS}}\right\}, \left\{\left(\widehat{A}_{k,i}^{T_{SS}}\right)'\right\}\right),$$

using the values of $\left\{\left(\widehat{A}_{k,i}^{T_{SS}}\right)'\right\}$ obtained in Step 9.

Step 12: Update $NX_i^{T_{SS}}$

$$NX_i^{T_{SS}} = (1-\lambda_o)\, NX_i^{T_{SS}} + \lambda_o \left(NX_i^{T_{SS}}\right)'$$

and $\widehat{A}_{k,i}^{T_{SS}}$

$$\widehat{A}_{k,i}^{T_{SS}} = (1-\lambda_A)\, \widehat{A}_{k,i}^{T_{SS}} + \lambda_A \left(\widehat{A}_{k,i}^{T_{SS}}\right)'$$

Go back to Step 6 until convergence of $\left\{NX_i^{T_{SS}}\right\}$ and $\left\{\widehat{A}_{k,i}^{T_{SS}}\right\}$.

C.7 Parameter Estimates

Table A.1: Final Expenditure Shares $\mu_{k,i}$

Sector ↓ Country →	USA	China	Europe	Asia/Oceania	Americas	ROW
Agr.	0.01	0.12	0.02	0.01	0.02	0.09
LT Manuf.	0.03	0.02	0.04	0.03	0.04	0.03
MT Manuf.	0.05	0.11	0.08	0.07	0.1	0.11
HT Manuf.	0.1	0.15	0.11	0.1	0.11	0.1
LT Serv.	0.3	0.35	0.34	0.38	0.33	0.39
HT Serv.	0.51	0.25	0.41	0.41	0.4	0.29

Table A.2: Labor Shares in Production $\gamma_{k,i}$

Sector ↓ Country →	USA	China	Europe	Asia/Oceania	Americas	ROW
Agr.	0.45	0.58	0.56	0.54	0.62	0.67
LT Manuf.	0.37	0.25	0.32	0.35	0.28	0.27
MT Manuf.	0.33	0.28	0.31	0.37	0.32	0.28
HT Manuf.	0.39	0.24	0.33	0.32	0.31	0.25
LT Serv.	0.61	0.37	0.49	0.54	0.56	0.48
HT Serv.	0.62	0.55	0.63	0.67	0.67	0.68

Table A.3: Input-Output Table – Average Across Countries $\frac{1}{N}\sum_i \nu_{k\ell,i}$

User ↓ Supplier →	Agr.	LT Manuf.	MT Manuf.	HT Manuf.	LT Serv.	HT Serv.
Agr.	0.267	0.079	0.118	0.138	0.26	0.139
	(0.056)	(0.019)	(0.036)	(0.029)	(0.06)	(0.07)
LT Manuf.	0.195	0.376	0.043	0.081	0.223	0.082
	(0.047)	(0.064)	(0.009)	(0.015)	(0.041)	(0.04)
MT Manuf.	0.222	0.066	0.287	0.106	0.225	0.095
	(0.034)	(0.017)	(0.04)	(0.022)	(0.052)	(0.044)
HT Manuf.	0.022	0.157	0.067	0.463	0.184	0.106
	(0.018)	(0.022)	(0.015)	(0.052)	(0.044)	(0.05)
LT Serv.	0.057	0.136	0.105	0.101	0.343	0.259
	(0.042)	(0.03)	(0.027)	(0.034)	(0.073)	(0.11)
HT Serv.	0.007	0.076	0.033	0.11	0.267	0.507
	(0.005)	(0.023)	(0.019)	(0.071)	(0.069)	(0.171)

Note: Standard Deviation of $\nu_{k\ell}$ across countries.

Table A.4: Mobility Costs Estimates $C_{k\ell}$

From ↓ / To →	Agr.	LT Manuf.	MT Manuf.	HT Manuf.	LT Serv.	HT Serv.
Agriculture	0	0.825	1.560	0.454	0.189	1.676
LT Manufacturing	0.414	0	0.005	0.000	0.799	2.034
MT Manufacturing	2.033	0.000	0	0.002	0.866	2.646
HT Manufacturing	0.015	0.001	0.003	0	0.276	0.917
LT Services	0.268	0.972	1.221	0.466	0	0.002
HT Services	0.790	1.826	2.150	1.201	0.004	0

Table A.5: Sector-Specific Utility Estimates $\eta_{k,i}$

Sector ↓ / Country →	USA	China	Europe	Asia/Oceania	Americas	ROW
Agriculture	0	0	0	0	0	0
LT Manuf.	-0.383	-0.943	-0.428	-0.521	-0.588	-0.316
MT Manuf.	0.026	-0.566	-0.245	-0.229	-0.195	-0.072
HT Manuf.	-0.551	-1.557	-0.743	-0.607	-1.292	-0.934
LT Services	0.085	-0.454	-0.382	-0.236	-0.405	-0.588
HT Services	-0.052	-0.467	-0.603	-0.518	-0.894	-1.011

Table A.6: Exogenous Death Rates Estimates $\chi_{k,} = \chi_i + \chi_k$

Country Component χ_i	USA	0.026
	China	0.022
	Europe	0.054
	Asia/Oceania	0.021
	Americas	0.027
	ROW	0.020
Sector Component χ_k	Agriculture	0
	Low Tech Man	0.005
	Med Tech Man	0.014
	High Tech Man	0.002
	Other Services	0.008
	Hi Tech Services	-0.001

Table A.7: Unemployment Utility $b_{k,i} = b_i$

USA	-14.4
China	-12.7
Europe	-7.1
Asia/Oceania	-8.5
Americas	-8.6
ROW	-14.9

Table A.8: Vacancy Posting Costs Estimates $\widetilde{\kappa}_{k,i}$

Sector ↓ / Country →	USA	China	Europe	Asia/Oceania	Americas	ROW
Agriculture	0.539	0.450	0.451	0.664	0.451	0.452
LT Manuf.	0.632	0.810	0.681	0.865	0.809	0.583
MT Manuf.	0.472	0.615	0.549	0.660	0.540	0.447
HT Manuf.	0.698	0.984	0.821	0.922	1.076	0.830
LT Services	0.449	0.617	0.592	0.660	0.649	0.639
HT Services	0.511	0.674	0.672	0.840	0.872	0.813